Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.3-Miscellaneous/51-1.3.1-Rational-functions

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [494]. This is test number [51].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (494)	0.00 (0)
Rubi	99.39 (491)	0.61 (3)
Maple	98.99 (489)	1.01 (5)
Mupad	98.18 (485)	1.82 (9)
Fricas	92.31 (456)	7.69 (38)
Sympy	87.65 (433)	12.35 (61)
Giac	85.63 (423)	14.37 (71)
Maxima	82.79 (409)	17.21 (85)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

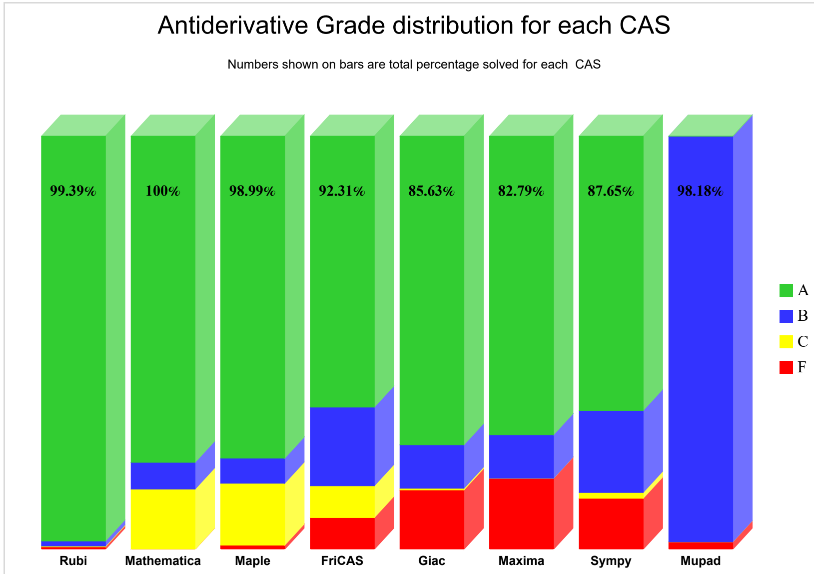
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

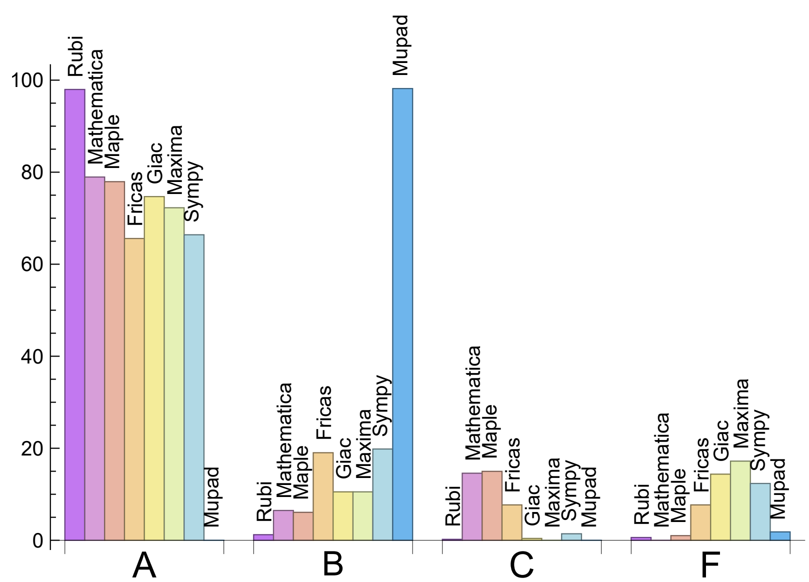
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.98	1.21	0.20	0.61
Mathematica	78.95	6.48	14.57	0.00
Maple	77.94	6.07	14.98	1.01
Giac	74.70	10.53	0.40	14.37
Maxima	72.27	10.53	0.00	17.21
Sympy	66.40	19.84	1.42	12.35
Fricas	65.59	19.03	7.69	7.69
Mupad	N/A	98.18	0.00	1.82

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	5	100.00 %	0.00 %	0.00 %
Fricas	38	10.53 %	78.95 %	10.53 %
Giac	71	94.37 %	0.00 %	5.63 %
Maxima	85	97.65 %	0.00 %	2.35 %
Sympy	61	14.75 %	81.97 %	3.28 %
Mupad	9	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

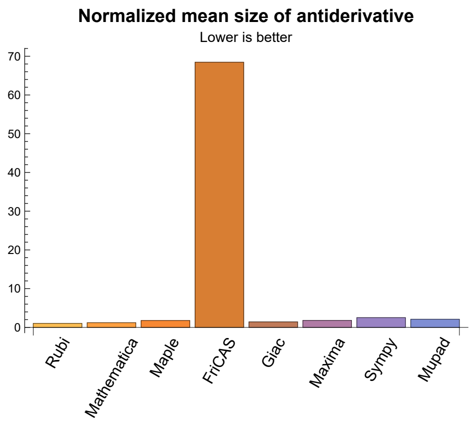
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

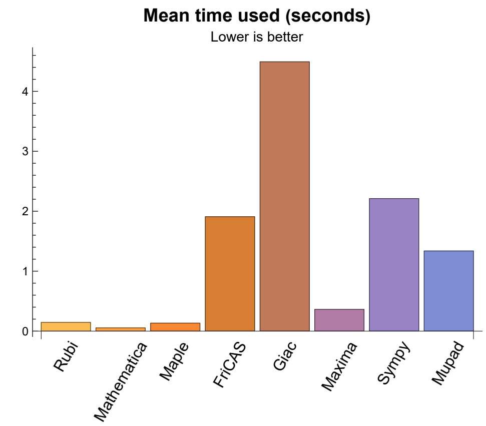
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	119.38	1.04	33.00	1.00
Mathematica	0.05	80.26	1.21	36.00	1.00
Maple	0.13	85.39	1.77	31.00	0.88
Maxima	0.36	117.82	1.80	28.00	0.88
Fricas	1.91	14150.10	68.46	39.00	1.15
Sympy	2.21	200.29	2.52	39.00	0.91
Giac	4.49	116.85	1.43	28.00	0.92
Mupad	1.34	228.31	2.10	41.00	0.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
70
60
50
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{154, 155, 156, 157\}$

Mathematica $\{31, 32\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

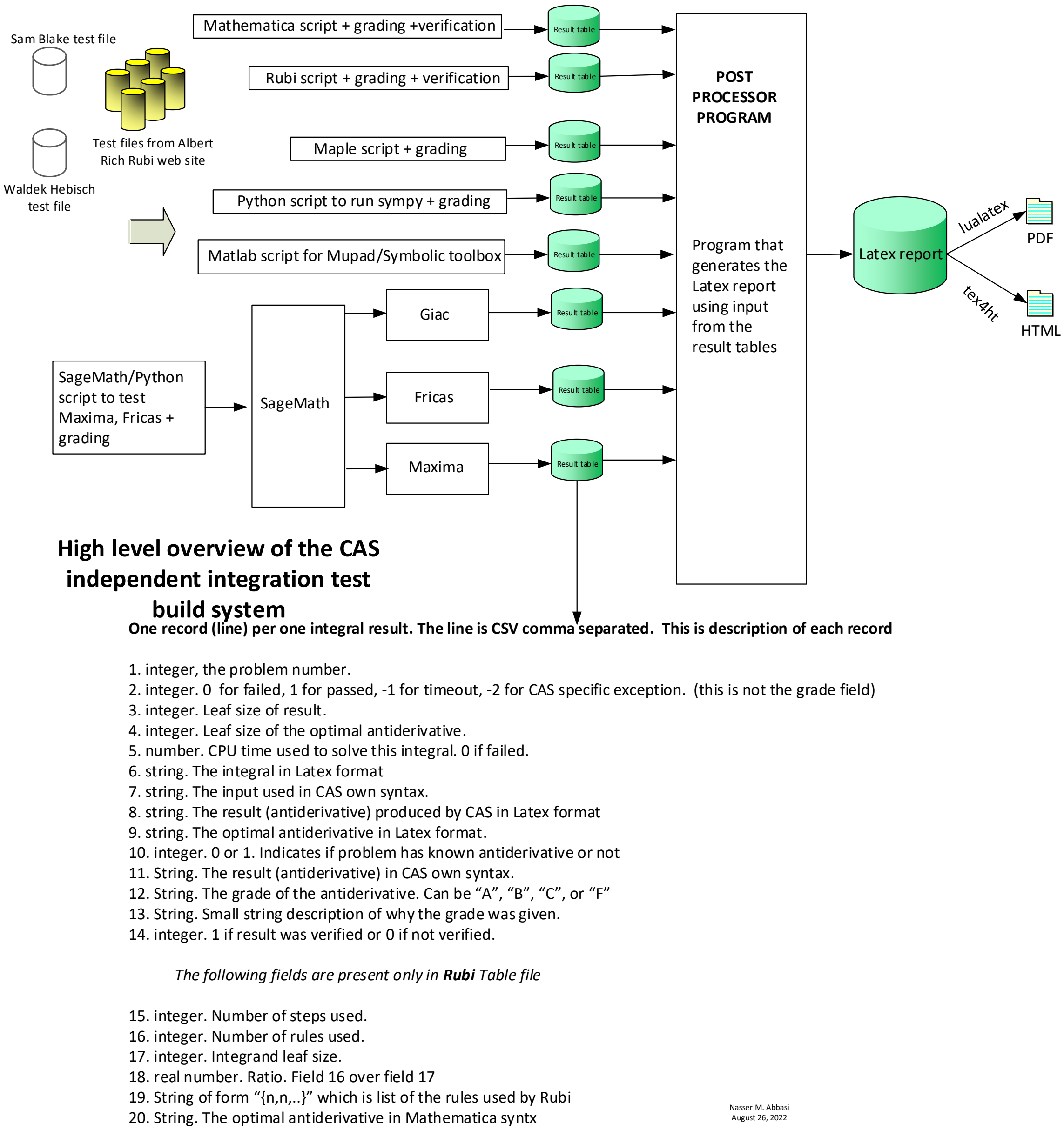
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 65, 77, 221, 222, 233, 424 }

C grade: { 174 }

F grade: { 393, 493, 494 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 106, 113, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 208, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 493, 494 }

B grade: { 65, 92, 95, 101, 102, 160, 161, 162, 192, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 221, 222, 421 }

C grade: { 12, 13, 14, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 227, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 491 }

F grade: { }

2.1.3 Maple

A grade: { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 164, 165, 166, 167, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 3, 4, 9, 15, 63, 64, 92, 95, 101, 102, 116, 161, 162, 163, 168, 169, 170, 171, 194, 197, 200, 201, 204, 205, 222, 232, 421, 424, 443, 494 }

C grade: { 1, 12, 13, 14, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 174, 250, 251, 252, 253, 254, 255, 256, 257, 387, 388, 389, 390, 391, 392, 393, 493 }

F grade: { 29, 30, 31, 32, 176 }

2.1.4 Maxima

A grade: { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 93, 96, 97, 98, 99, 100, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 160, 164, 165, 166, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 198, 199, 208, 209, 216, 217, 218, 219, 220, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492 }

B grade: { 3, 4, 7, 8, 9, 19, 20, 63, 64, 65, 66, 67, 68, 74, 87, 88, 91, 92, 94, 95, 101, 102, 161, 162, 163, 167, 169, 170, 171, 194, 197, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 421, 424, 477, 493, 494 }

C grade: { }

F grade: { 1, 12, 13, 14, 26, 29, 30, 31, 32, 37, 38, 43, 44, 49, 50, 55, 56, 61, 62, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 227, 228, 250, 251, 252, 253, 254, 255, 256, 257, 335, 336, 337, 338, 387, 388, 389, 390, 391, 392, 393, 490, 491 }

2.1.5 FriCAS

A grade: { 1, 2, 5, 6, 9, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 96, 98, 99, 100, 101, 102, 106, 113, 116, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 159, 164, 165, 166, 167, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 208, 209, 216, 217, 218, 219, 220, 224, 225, 226, 228, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 397, 404, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 3, 4, 7, 8, 12, 13, 14, 15, 37, 38, 43, 44, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 87, 91, 92, 93, 94, 95, 97, 120, 121, 122, 146, 154, 160, 161, 162, 163, 168, 169, 170, 171, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 221, 222, 223, 229, 233, 250, 251, 252, 253, 254, 255, 256, 257, 268, 277, 343, 344, 387, 388, 389, 390, 421, 423, 424, 453, 455, 477, 484, 493, 494 }

C grade: { 49, 50, 55, 56, 61, 62, 103, 104, 105, 107, 108, 109, 111, 112, 114, 115, 127, 134, 138, 334, 337, 338, 341, 342, 368, 391, 392, 393, 394, 395, 396, 398, 401, 402, 403, 408, 409, 410 }

F grade: { 19, 20, 29, 30, 31, 32, 110, 128, 129, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 227, 399, 400, 405, 406, 407, 412, 413, 414 }

2.1.6 Sympy

A grade: { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 57, 58, 59, 60, 61, 69, 70, 71, 72, 73, 75, 76, 93, 96, 97, 98, 99, 103, 104, 105, 106, 110, 111, 112, 113, 116, 117, 118, 119, 120, 123, 124, 125, 126, 127, 130, 131, 132, 133, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 177, 178, 219, 220, 224, 225, 226, 228, 229, 230, 231, 232, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 403, 404, 408, 409, 410, 411, 415, 416, 417, 418, 419, 420, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492 }

B grade: { 3, 4, 6, 7, 8, 9, 15, 26, 50, 55, 56, 62, 63, 64, 65, 66, 67, 68, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 94, 95, 121, 122, 128, 129, 134, 135, 160, 161, 162, 167, 168, 169, 170, 172, 179, 184, 185, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 233, 251, 255, 268, 339, 421, 423, 424, 459, 477, 489, 490, 493, 494 }

C grade: { 32, 89, 90, 91, 257, 279, 458 }

F grade: { 2, 18, 19, 20, 28, 29, 30, 31, 38, 44, 100, 101, 102, 107, 108, 109, 114, 115, 136, 137, 138, 139, 140, 141, 142, 163, 171, 173, 174, 175, 176, 180, 181, 182, 183, 186, 190, 218, 227, 234, 235, 236, 237, 238, 239, 240, 241, 337, 338, 340, 341, 342, 398, 399, 400, 405, 406, 407, 412, 413, 414 }

2.1.7 Giac

A grade: { 1, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 106, 113, 117, 118, 119, 123, 124, 125, 126, 130, 131, 132, 133, 158, 160, 164, 165, 166, 168, 169, 170, 172, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 189, 190, 191, 193, 194, 196, 197, 199, 200, 201, 206, 208, 209, 210, 212, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492 }

B grade: { 2, 3, 4, 9, 15, 19, 20, 43, 44, 63, 64, 65, 74, 92, 95, 116, 159, 161, 162, 173, 174, 175, 182, 188, 192, 195, 198, 202, 203, 204, 205, 207, 211, 213, 215, 234, 235, 236, 237, 238, 257, 268, 282, 324, 337, 338, 359, 421, 424, 485, 493, 494 }

C grade: { 55, 56 }

F grade: { 29, 30, 31, 32, 49, 50, 61, 62, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 115, 120, 121, 122, 127, 128, 129, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 163, 167, 171, 176, 227, 239, 240, 241, 250, 251, 252, 253, 254, 255, 256, 387, 388, 389, 390, 391, 392, 393 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494 }

C grade: { }

F grade: { 31, 100, 101, 102, 167, 171, 174, 176, 227 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	143	43	0	76	60	55	51
N.S.	1	1.00	1.86	0.56	0.00	0.99	0.78	0.71	0.66
time (sec)	N/A	0.047	0.036	0.077	0.000	0.397	0.135	3.281	0.196

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	23	26	25	43	0	73	52
N.S.	1	1.00	0.77	0.87	0.83	1.43	0.00	2.43	1.73
time (sec)	N/A	0.013	0.023	0.027	0.264	0.383	0.000	3.900	2.132

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	98	216	97	107	97	97
N.S.	1	1.00	1.00	7.00	15.43	6.93	7.64	6.93	6.93
time (sec)	N/A	0.006	0.001	0.014	0.261	0.344	0.018	3.271	2.073

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	65	99	64	66	64	64
N.S.	1	1.00	1.00	4.64	7.07	4.57	4.71	4.57	4.57
time (sec)	N/A	0.006	0.001	0.029	0.275	0.356	0.014	2.960	0.028

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	13	31	31	32	31	31
N.S.	1	1.00	1.00	0.37	0.89	0.89	0.91	0.89	0.89
time (sec)	N/A	0.005	0.000	0.023	0.273	0.374	0.006	3.642	0.037

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	24	24	26	12	26
N.S.	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86
time (sec)	N/A	0.006	0.003	0.013	0.263	0.369	0.066	3.596	0.034

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	57	57	61	12	59
N.S.	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.21
time (sec)	N/A	0.006	0.003	0.046	0.269	0.347	0.151	3.574	2.048

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	90	90	97	12	92
N.S.	1	1.00	1.00	0.93	6.43	6.43	6.93	0.86	6.57
time (sec)	N/A	0.006	0.003	0.046	0.272	0.383	0.232	3.394	2.069

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	159	295	204	156	175	166	149
N.S.	1	1.00	1.89	3.51	2.43	1.86	2.08	1.98	1.77
time (sec)	N/A	0.096	0.016	0.033	0.268	0.360	0.023	5.912	2.077

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	82	84	93	80	87	83	79
N.S.	1	1.00	1.46	1.50	1.66	1.43	1.55	1.48	1.41
time (sec)	N/A	0.054	0.008	0.013	0.256	0.348	0.015	5.714	0.039

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.004	0.000	0.011	0.264	0.349	0.008	3.950	0.038

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	63	57	0	387	53	212	174
N.S.	1	1.00	0.34	0.30	0.00	2.06	0.28	1.13	0.93
time (sec)	N/A	0.231	0.013	0.021	0.000	0.388	0.189	3.903	0.494

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	112	134	0	704	192	289	247
N.S.	1	1.00	0.46	0.55	0.00	2.87	0.78	1.18	1.01
time (sec)	N/A	0.190	0.044	0.047	0.000	0.386	0.661	4.075	2.645

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	149	279	0	1268	474	366	483
N.S.	1	1.00	0.49	0.91	0.00	4.16	1.55	1.20	1.58
time (sec)	N/A	0.240	0.065	0.053	0.000	0.396	1.442	4.039	2.983

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	653	861	483	727	1018	971	787
N.S.	1	1.00	1.81	2.39	1.34	2.01	2.82	2.69	2.18
time (sec)	N/A	0.481	0.165	0.040	0.270	0.359	0.084	3.655	2.226

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	241	188	190	269	345	346	270
N.S.	1	1.00	1.25	0.97	0.98	1.39	1.79	1.79	1.40
time (sec)	N/A	0.178	0.062	0.017	0.268	0.355	0.035	3.067	0.084

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	76	53	54	52	63	54	54
N.S.	1	1.00	1.36	0.95	0.96	0.93	1.12	0.96	0.96
time (sec)	N/A	0.012	0.000	0.012	0.262	0.373	0.008	3.272	0.045

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	87	117	112	0	137	106
N.S.	1	1.00	0.93	1.01	1.36	1.30	0.00	1.59	1.23
time (sec)	N/A	0.057	0.038	0.059	0.284	3.079	0.000	2.980	2.335

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	232	235	2261	0	0	1414	1940
N.S.	1	1.00	0.99	1.00	9.66	0.00	0.00	6.04	8.29
time (sec)	N/A	0.308	0.420	0.299	0.406	0.000	0.000	3.349	8.176

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	490	514	12087	0	0	6908	2500
N.S.	1	1.00	0.99	1.04	24.42	0.00	0.00	13.96	5.05
time (sec)	N/A	1.060	0.951	1.133	1.124	0.000	0.000	4.046	20.456

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.011	0.005	0.018	0.467	0.396	0.042	4.467	2.201

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	24	26	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.77	0.84	0.81
time (sec)	N/A	0.015	0.006	0.021	0.491	0.381	0.049	4.112	0.053

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.001	0.000	0.007	0.270	0.383	0.007	3.735	0.034

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.011	0.004	0.203	0.266	0.380	0.064	3.766	0.060

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	18	15	24	18
N.S.	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.009	0.004	0.189	0.260	0.380	0.080	4.483	2.134

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	61	0	211	564	62	213
N.S.	1	1.00	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.042	0.054	0.028	0.000	0.406	4.480	3.806	0.465

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	91	98	299	20	112	99
N.S.	1	1.00	0.77	0.79	0.85	2.60	0.17	0.97	0.86
time (sec)	N/A	0.046	0.021	0.199	0.486	0.386	0.053	4.450	0.233

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	16	0	16	16
N.S.	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00
time (sec)	N/A	0.003	0.002	0.010	0.277	0.390	0.000	4.422	2.500

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	0	0	15	0	0	56
N.S.	1	1.00	0.96	0.00	0.00	0.27	0.00	0.00	1.02
time (sec)	N/A	0.016	0.025	0.029	0.000	0.382	0.000	0.000	2.206

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	61	0	0	13	0	0	56
N.S.	1	1.11	1.15	0.00	0.00	0.25	0.00	0.00	1.06
time (sec)	N/A	0.018	0.026	0.027	0.000	0.385	0.000	0.000	2.217

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	157	0	0	18	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.14	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.255	0.010	0.000	0.379	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	196	0	0	11	34	0	41
N.S.	1	1.26	5.60	0.00	0.00	0.31	0.97	0.00	1.17
time (sec)	N/A	0.007	0.125	0.027	0.000	0.411	4.913	0.000	2.176

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	285	392	372	271	299	277	261
N.S.	1	1.00	1.06	1.45	1.38	1.00	1.11	1.03	0.97
time (sec)	N/A	0.408	0.031	0.026	0.290	0.388	0.033	3.802	2.298

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	231	205	163	180	166	160
N.S.	1	1.00	1.00	1.35	1.20	0.95	1.05	0.97	0.94
time (sec)	N/A	0.067	0.016	0.022	0.289	0.373	0.020	3.395	2.160

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	84	94	82	95	83	82
N.S.	1	1.00	1.00	0.91	1.02	0.89	1.03	0.90	0.89
time (sec)	N/A	0.031	0.008	0.018	0.272	0.353	0.013	4.559	0.036

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	28
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88
time (sec)	N/A	0.004	0.000	0.014	0.266	0.372	0.007	3.198	0.037

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	529	71	64	0	905	88	603	1551
N.S.	1	1.00	0.13	0.12	0.00	1.71	0.17	1.14	2.93
time (sec)	N/A	0.687	0.022	0.057	0.000	0.366	0.601	4.852	4.541

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	182	232	0	3222	0	1057	2500
N.S.	1	1.00	0.24	0.31	0.00	4.32	0.00	1.42	3.35
time (sec)	N/A	1.183	0.080	0.066	0.000	0.446	0.000	5.587	4.304

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	345	500	349	332	366	323	331
N.S.	1	1.00	1.17	1.69	1.18	1.13	1.24	1.09	1.12
time (sec)	N/A	0.399	0.040	0.026	0.277	0.364	0.037	6.275	0.268

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	207	288	195	198	218	187	201
N.S.	1	1.00	1.02	1.42	0.96	0.98	1.07	0.92	0.99
time (sec)	N/A	0.087	0.021	0.022	0.269	0.389	0.024	3.465	2.243

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	109	100	92	98	112	90	98
N.S.	1	1.00	1.02	0.93	0.86	0.92	1.05	0.84	0.92
time (sec)	N/A	0.038	0.010	0.018	0.265	0.354	0.014	4.464	0.044

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	30	33	36	30	33
N.S.	1	1.00	1.00	0.92	0.81	0.89	0.97	0.81	0.89
time (sec)	N/A	0.006	0.000	0.016	0.261	0.365	0.007	3.854	0.043

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	71	67	0	1115	122	505	1264
N.S.	1	1.00	0.46	0.44	0.00	7.29	0.80	3.30	8.26
time (sec)	N/A	0.196	0.018	0.081	0.000	0.405	0.989	4.602	3.731

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	234	288	0	4285	0	979	2500
N.S.	1	1.00	0.68	0.84	0.00	12.53	0.00	2.86	7.31
time (sec)	N/A	0.444	0.124	0.073	0.000	0.542	0.000	3.736	7.109

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	85	84	84	94	84	84
N.S.	1	1.00	1.00	0.89	0.88	0.88	0.98	0.88	0.88
time (sec)	N/A	0.021	0.002	0.015	0.277	0.377	0.013	4.382	0.185

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	65	64	64	71	64	64
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.96	0.86	0.86
time (sec)	N/A	0.015	0.001	0.013	0.265	0.354	0.011	3.886	0.076

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	44	44
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.011	0.002	0.012	0.274	0.376	0.009	4.077	0.032

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.002	0.000	0.009	0.272	0.362	0.006	3.743	0.031

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	45	41	0	1015	41	0	123
N.S.	1	1.00	0.17	0.15	0.00	3.79	0.15	0.00	0.46
time (sec)	N/A	0.309	0.007	0.018	0.000	1.169	0.527	0.000	2.445

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	113	83	0	1201	3834	0	176
N.S.	1	1.00	0.32	0.23	0.00	3.36	10.74	0.00	0.49
time (sec)	N/A	0.298	0.013	0.022	0.000	1.138	1.780	0.000	0.207

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	77	77	94	77	77
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.79
time (sec)	N/A	0.019	0.002	0.015	0.266	0.340	0.012	3.345	0.150

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.014	0.002	0.017	0.272	0.389	0.010	3.703	0.063

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	37	37	42	37	37
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.82
time (sec)	N/A	0.010	0.001	0.012	0.276	0.348	0.008	3.449	0.026

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	19	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.81
time (sec)	N/A	0.002	0.000	0.010	0.275	0.369	0.006	3.472	0.028

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	47	41	0	499	3432	265	87
N.S.	1	1.00	0.20	0.18	0.00	2.13	14.67	1.13	0.37
time (sec)	N/A	0.237	0.012	0.018	0.000	1.130	1.362	4.061	2.359

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	108	79	0	704	3834	315	174
N.S.	1	1.00	0.34	0.25	0.00	2.22	12.09	0.99	0.55
time (sec)	N/A	0.257	0.020	0.023	0.000	1.155	1.943	5.624	2.207

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	85	84	84	100	84	84
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.96	0.81	0.81
time (sec)	N/A	0.021	0.002	0.030	0.284	0.385	0.017	4.488	2.230

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	65	64	64	73	64	64
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84
time (sec)	N/A	0.016	0.001	0.031	0.300	0.367	0.013	5.179	0.078

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	45	44	44	49	44	44
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.94	0.85	0.85
time (sec)	N/A	0.012	0.001	0.021	0.271	0.367	0.010	5.991	0.033

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	27	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.90	0.80	0.80
time (sec)	N/A	0.003	0.000	0.009	0.271	0.387	0.006	5.349	0.019

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	55	49	0	1297	41	0	123
N.S.	1	1.00	0.21	0.19	0.00	4.93	0.16	0.00	0.47
time (sec)	N/A	0.388	0.008	0.016	0.000	1.177	1.297	0.000	0.411

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	128	96	0	1540	3839	0	181
N.S.	1	1.00	0.35	0.26	0.00	4.21	10.49	0.00	0.49
time (sec)	N/A	0.414	0.015	0.020	0.000	1.231	2.373	0.000	0.205

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	164	592	163	185	163	163
N.S.	1	1.00	1.00	11.71	42.29	11.64	13.21	11.64	11.64
time (sec)	N/A	0.012	0.002	0.021	0.303	0.376	0.028	5.261	0.167

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	109	228	108	114	108	108
N.S.	1	1.00	1.00	7.79	16.29	7.71	8.14	7.71	7.71
time (sec)	N/A	0.011	0.001	0.017	0.263	0.372	0.019	3.482	0.061

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	61	61	13	53	53	60	53	53
N.S.	1	4.36	4.36	0.93	3.79	3.79	4.29	3.79	3.79
time (sec)	N/A	0.009	0.000	0.013	0.260	0.389	0.009	5.229	0.024

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	46	46	49	12	48
N.S.	1	1.00	1.00	0.93	3.29	3.29	3.50	0.86	3.43
time (sec)	N/A	0.013	0.003	0.021	0.264	0.357	0.118	5.237	0.047

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	101	101	109	12	103
N.S.	1	1.00	1.00	0.93	7.21	7.21	7.79	0.86	7.36
time (sec)	N/A	0.012	0.003	0.054	0.308	0.360	0.290	6.008	2.105

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	156	156	168	12	158
N.S.	1	1.00	1.00	0.93	11.14	11.14	12.00	0.86	11.29
time (sec)	N/A	0.012	0.003	0.099	0.289	0.375	0.415	3.768	3.033

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	30	30	29	31	36
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.76	0.82	0.95
time (sec)	N/A	0.019	0.006	0.031	0.501	0.413	0.056	5.391	2.165

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	65	64	64	80	64	64
N.S.	1	1.00	1.00	0.77	0.76	0.76	0.95	0.76	0.76
time (sec)	N/A	0.056	0.002	0.019	0.301	0.383	0.015	6.601	2.166

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	50	49	49	60	49	49
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.051	0.002	0.016	0.277	0.380	0.012	3.889	0.053

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.045	0.001	0.017	0.293	0.374	0.010	3.373	0.024

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.002	0.000	0.031	0.273	0.360	0.006	4.046	0.032

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	62	42	54	56	63	62	27
N.S.	1	1.00	2.00	1.35	1.74	1.81	2.03	2.00	0.87
time (sec)	N/A	0.018	0.011	0.030	0.480	0.385	0.054	4.911	0.066

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	103	84	89	177	104	97	64
N.S.	1	1.00	1.16	0.94	1.00	1.99	1.17	1.09	0.72
time (sec)	N/A	0.046	0.040	0.046	0.489	0.386	0.804	3.739	0.084

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	137	126	119	282	134	112	93
N.S.	1	1.00	0.85	0.78	0.74	1.75	0.83	0.70	0.58
time (sec)	N/A	0.091	0.067	0.061	0.484	0.388	0.864	3.369	0.089

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	296	134	126
N.S.	1	2.25	1.45	1.27	1.25	2.45	3.25	1.47	1.38
time (sec)	N/A	0.100	0.067	0.045	0.477	0.393	0.803	3.606	2.184

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	111	81	198	209	77	87
N.S.	1	1.00	0.94	1.42	1.04	2.54	2.68	0.99	1.12
time (sec)	N/A	0.047	0.039	0.282	0.485	0.386	0.376	3.294	2.269

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	54	70	61	157	153	54	206
N.S.	1	1.00	1.08	1.40	1.22	3.14	3.06	1.08	4.12
time (sec)	N/A	0.029	0.022	0.314	0.497	0.413	0.228	4.949	0.090

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	54	50	136	124	43	46
N.S.	1	1.00	0.93	1.32	1.22	3.32	3.02	1.05	1.12
time (sec)	N/A	0.016	0.011	0.249	0.493	0.389	0.104	5.425	2.076

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	28	24	83	54	17	17
N.S.	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.81
time (sec)	N/A	0.007	0.004	0.336	0.478	0.366	0.070	4.023	0.043

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	74	68	154	738	62	173
N.S.	1	1.00	0.81	1.25	1.15	2.61	12.51	1.05	2.93
time (sec)	N/A	0.028	0.027	0.257	0.494	0.407	1.897	3.662	2.588

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	81	96	123	229	1620	117	425
N.S.	1	1.00	1.03	1.22	1.56	2.90	20.51	1.48	5.38
time (sec)	N/A	0.064	0.035	0.288	0.489	0.419	7.259	3.085	2.583

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	151	197	371	3284	195	573
N.S.	1	1.00	0.88	1.25	1.63	3.07	27.14	1.61	4.74
time (sec)	N/A	0.097	0.099	0.350	0.484	0.396	43.886	3.001	2.771

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	34	30	109	61	24	27
N.S.	1	1.00	1.00	1.10	0.97	3.52	1.97	0.77	0.87
time (sec)	N/A	0.012	0.008	0.409	0.478	0.383	0.079	4.430	0.057

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	86	75	253	117	65	76
N.S.	1	1.00	0.95	1.37	1.19	4.02	1.86	1.03	1.21
time (sec)	N/A	0.021	0.020	0.338	0.489	0.389	0.309	7.480	0.098

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	139	184	595	257	103	181
N.S.	1	1.00	0.82	1.53	2.02	6.54	2.82	1.13	1.99
time (sec)	N/A	0.039	0.047	0.273	0.502	0.414	0.699	6.397	2.221

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	42	66	279	92	30	31
N.S.	1	1.00	1.00	1.20	1.89	7.97	2.63	0.86	0.89
time (sec)	N/A	0.023	0.011	0.301	0.482	0.444	0.085	4.835	0.103

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	10	24	10	10
N.S.	1	1.00	1.00	1.10	1.80	1.00	2.40	1.00	1.00
time (sec)	N/A	0.002	0.004	0.284	0.474	0.383	0.061	5.837	0.040

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	59	51	55	56	41	42
N.S.	1	1.00	0.84	1.59	1.38	1.49	1.51	1.11	1.14
time (sec)	N/A	0.007	0.010	0.273	0.494	0.382	0.205	4.143	2.066

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	94	115	153	146	73	111
N.S.	1	1.00	0.87	1.57	1.92	2.55	2.43	1.22	1.85
time (sec)	N/A	0.011	0.013	0.264	0.476	0.397	0.466	3.099	0.121

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	32	26	25	22	22	27	10
N.S.	1	1.00	3.20	2.60	2.50	2.20	2.20	2.70	1.00
time (sec)	N/A	0.002	0.005	0.303	0.264	0.345	0.067	4.345	2.050

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	45	52	56	85	54	56	43
N.S.	1	1.00	1.15	1.33	1.44	2.18	1.38	1.44	1.10
time (sec)	N/A	0.009	0.017	0.247	0.271	0.393	0.218	3.627	2.064

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	78	122	220	141	88	114
N.S.	1	1.00	1.02	1.22	1.91	3.44	2.20	1.38	1.78
time (sec)	N/A	0.016	0.024	0.313	0.297	0.388	0.501	4.195	2.118

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	15	12	11	11	10	13	4
N.S.	1	1.00	3.75	3.00	2.75	2.75	2.50	3.25	1.00
time (sec)	N/A	0.001	0.002	0.227	0.276	0.364	0.029	2.640	0.149

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.005	0.013	0.194	0.262	0.369	0.036	3.432	0.066

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	37	36	44	71	44	39	36
N.S.	1	1.00	0.82	0.80	0.98	1.58	0.98	0.87	0.80
time (sec)	N/A	0.009	0.013	0.197	0.287	0.389	0.056	4.631	2.090

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	62	54	54	58	62	55
N.S.	1	1.00	1.08	1.05	0.92	0.92	0.98	1.05	0.93
time (sec)	N/A	0.042	0.018	0.253	0.267	0.370	0.059	5.950	0.050

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	12	11	11	10	11	11
N.S.	1	1.00	1.10	1.20	1.10	1.10	1.00	1.10	1.10
time (sec)	N/A	0.010	0.006	0.258	0.267	0.356	0.021	6.407	0.032

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	35	36	35	0	23	-1
N.S.	1	1.00	1.16	0.80	0.82	0.80	0.00	0.52	-0.02
time (sec)	N/A	0.020	0.055	0.179	0.478	0.371	0.000	5.579	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	163	164	139	92	0	64	-1
N.S.	1	1.00	2.43	2.45	2.07	1.37	0.00	0.96	-0.01
time (sec)	N/A	0.036	0.310	0.232	0.507	0.398	0.000	5.416	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	145	155	135	70	0	86	-1
N.S.	1	1.00	2.30	2.46	2.14	1.11	0.00	1.37	-0.02
time (sec)	N/A	0.032	0.367	0.322	0.269	0.360	0.000	3.666	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	132	108	0	6315	238	0	374
N.S.	1	1.00	0.56	0.46	0.00	26.99	1.02	0.00	1.60
time (sec)	N/A	0.265	0.038	0.028	0.000	18.072	1.672	0.000	2.460

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	81	74	0	4759	158	0	437
N.S.	1	1.00	0.39	0.35	0.00	22.66	0.75	0.00	2.08
time (sec)	N/A	0.173	0.022	0.027	0.000	1.135	0.523	0.000	2.300

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	79	72	0	1950	83	0	145
N.S.	1	1.00	0.44	0.40	0.00	10.83	0.46	0.00	0.81
time (sec)	N/A	0.114	0.019	0.023	0.000	1.093	0.344	0.000	0.257

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	116	71	0	442	26	160	144
N.S.	1	1.00	0.83	0.51	0.00	3.16	0.19	1.14	1.03
time (sec)	N/A	0.076	0.026	0.024	0.000	0.423	0.104	3.694	2.311

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	238	119	105	0	4370	0	0	553
N.S.	1	1.06	0.53	0.47	0.00	19.51	0.00	0.00	2.47
time (sec)	N/A	0.235	0.036	0.044	0.000	1.181	0.000	0.000	0.123

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	312	173	143	0	8919	0	0	1588
N.S.	1	0.99	0.55	0.46	0.00	28.40	0.00	0.00	5.06
time (sec)	N/A	0.423	0.066	0.056	0.000	1.511	0.000	0.000	2.329

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	244	200	0	14765	0	0	1328
N.S.	1	1.00	0.62	0.51	0.00	37.57	0.00	0.00	3.38
time (sec)	N/A	0.461	0.106	0.075	0.000	5.068	0.000	0.000	2.489

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	106	97	0	0	374	0	1003
N.S.	1	1.00	0.30	0.27	0.00	0.00	1.05	0.00	2.82
time (sec)	N/A	0.305	0.030	0.038	0.000	0.000	2.617	0.000	2.692

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	106	97	0	61993	274	0	625
N.S.	1	1.00	0.33	0.31	0.00	194.95	0.86	0.00	1.97
time (sec)	N/A	0.235	0.024	0.028	0.000	101.652	1.872	0.000	2.588

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	104	95	0	40785	131	0	205
N.S.	1	1.00	0.40	0.36	0.00	156.26	0.50	0.00	0.79
time (sec)	N/A	0.189	0.020	0.028	0.000	17.849	0.531	0.000	2.357

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	161	94	0	185	26	103	60
N.S.	1	1.00	0.73	0.43	0.00	0.84	0.12	0.47	0.27
time (sec)	N/A	0.138	0.056	0.023	0.000	0.432	0.143	6.675	0.116

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	163	139	0	307773	0	0	882
N.S.	1	1.00	0.41	0.35	0.00	783.14	0.00	0.00	2.24
time (sec)	N/A	0.347	0.048	0.042	0.000	5.969	0.000	0.000	2.183

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	238	184	0	1128605	0	0	2440
N.S.	1	1.00	0.48	0.37	0.00	2275.41	0.00	0.00	4.92
time (sec)	N/A	0.668	0.095	0.063	0.000	119.924	0.000	0.000	2.479

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	195	264	192	179	199	219	175
N.S.	1	1.00	1.59	2.15	1.56	1.46	1.62	1.78	1.42
time (sec)	N/A	0.170	0.022	0.020	0.280	0.365	0.037	4.046	0.215

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	114	138	119	107	114	128	108
N.S.	1	1.00	0.95	1.15	0.99	0.89	0.95	1.07	0.90
time (sec)	N/A	0.043	0.010	0.017	0.303	0.364	0.027	3.195	0.095

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	63	65	56	65	65	61
N.S.	1	1.00	0.92	0.88	0.90	0.78	0.90	0.90	0.85
time (sec)	N/A	0.022	0.006	0.031	0.266	0.376	0.021	3.525	0.039

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	22	22	22
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.85
time (sec)	N/A	0.003	0.000	0.010	0.289	0.375	0.007	4.933	0.018

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	51	0	457	66	0	571
N.S.	1	1.00	0.64	0.57	0.00	5.13	0.74	0.00	6.42
time (sec)	N/A	0.062	0.011	0.030	0.000	0.405	0.609	0.000	2.576

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	150	158	0	1948	294	0	2500
N.S.	1	1.00	0.89	0.93	0.00	11.53	1.74	0.00	14.79
time (sec)	N/A	0.214	0.043	0.031	0.000	0.386	3.770	0.000	5.352

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	254	400	0	3971	697	0	2500
N.S.	1	1.00	1.01	1.59	0.00	15.76	2.77	0.00	9.92
time (sec)	N/A	0.449	0.099	0.054	0.000	0.418	9.456	0.000	6.405

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	182	182	212	222	178
N.S.	1	1.00	0.97	1.27	0.87	0.87	1.01	1.06	0.85
time (sec)	N/A	0.162	0.021	0.063	0.271	0.374	0.034	4.406	0.217

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	130	143	113	113	128	133	113
N.S.	1	1.00	0.97	1.07	0.84	0.84	0.96	0.99	0.84
time (sec)	N/A	0.102	0.012	0.067	0.268	0.363	0.021	4.375	2.120

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	66	59	59	70	68	64
N.S.	1	1.00	0.95	0.84	0.75	0.75	0.89	0.86	0.81
time (sec)	N/A	0.054	0.007	0.058	0.274	0.369	0.013	4.109	0.038

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.008	0.002	0.023	0.269	0.383	0.007	3.932	0.022

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	59	52	0	140500	155	0	275
N.S.	1	1.00	0.51	0.45	0.00	1211.21	1.34	0.00	2.37
time (sec)	N/A	0.076	0.013	0.028	0.000	1.694	2.821	0.000	2.582

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	166	158	0	0	539	0	1167
N.S.	1	1.00	0.72	0.68	0.00	0.00	2.33	0.00	5.05
time (sec)	N/A	0.228	0.054	0.030	0.000	0.000	18.779	0.000	2.817

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	284	409	0	0	1102	0	2500
N.S.	1	1.00	0.81	1.17	0.00	0.00	3.16	0.00	7.16
time (sec)	N/A	0.436	0.105	0.049	0.000	0.000	49.636	0.000	3.394

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	267	182	182	219	222	178
N.S.	1	1.00	0.97	1.27	0.87	0.87	1.04	1.06	0.85
time (sec)	N/A	0.157	0.022	0.068	0.274	0.350	0.033	2.914	2.294

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	132	143	113	113	134	133	113
N.S.	1	1.00	0.96	1.04	0.82	0.82	0.97	0.96	0.82
time (sec)	N/A	0.107	0.012	0.064	0.285	0.349	0.021	3.664	0.092

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	66	59	59	73	68	64
N.S.	1	1.00	0.92	0.84	0.75	0.75	0.92	0.86	0.81
time (sec)	N/A	0.063	0.007	0.059	0.276	0.370	0.012	3.810	0.042

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77
time (sec)	N/A	0.007	0.002	0.010	0.268	0.351	0.007	5.195	0.021

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	61	54	0	1515766	172	0	878
N.S.	1	1.00	0.62	0.55	0.00	15310.77	1.74	0.00	8.87
time (sec)	N/A	0.068	0.012	0.020	0.000	19.572	4.676	0.000	2.783

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	182	160	0	0	561	0	1218
N.S.	1	1.00	0.81	0.71	0.00	0.00	2.49	0.00	5.41
time (sec)	N/A	0.175	0.054	0.033	0.000	0.000	21.248	0.000	2.846

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	99	93	0	0	0	0	1563
N.S.	1	1.00	0.18	0.17	0.00	0.00	0.00	0.00	2.87
time (sec)	N/A	1.461	0.044	0.043	0.000	0.000	0.000	0.000	3.175

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	99	93	0	0	0	0	1354
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.78
time (sec)	N/A	0.955	0.035	0.032	0.000	0.000	0.000	0.000	3.104

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	97	93	0	27094	0	0	825
N.S.	1	1.00	0.29	0.28	0.00	81.12	0.00	0.00	2.47
time (sec)	N/A	0.705	0.030	0.032	0.000	3.135	0.000	0.000	3.342

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	95	91	0	0	0	0	1057
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.25
time (sec)	N/A	0.894	0.030	0.027	0.000	0.000	0.000	0.000	2.903

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	99	90	0	0	0	0	1394
N.S.	1	1.00	0.19	0.17	0.00	0.00	0.00	0.00	2.67
time (sec)	N/A	1.075	0.041	0.030	0.000	0.000	0.000	0.000	0.711

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	157	134	0	0	0	0	2500
N.S.	1	1.00	0.28	0.24	0.00	0.00	0.00	0.00	4.44
time (sec)	N/A	1.288	0.070	0.061	0.000	0.000	0.000	0.000	2.546

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	640	163	133	0	0	0	0	2500
N.S.	1	0.99	0.25	0.21	0.00	0.00	0.00	0.00	3.88
time (sec)	N/A	1.543	0.086	0.051	0.000	0.000	0.000	0.000	2.721

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	61	56	0	0	70	0	427
N.S.	1	1.00	0.15	0.14	0.00	0.00	0.18	0.00	1.08
time (sec)	N/A	1.081	0.012	0.021	0.000	0.000	0.117	0.000	0.651

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	61	56	0	0	65	0	390
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	1.03
time (sec)	N/A	0.794	0.010	0.025	0.000	0.000	0.141	0.000	2.704

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	61	56	0	0	61	0	276
N.S.	1	1.00	0.17	0.16	0.00	0.00	0.17	0.00	0.76
time (sec)	N/A	0.530	0.010	0.037	0.000	0.000	0.104	0.000	0.522

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	59	56	0	1277	48	0	247
N.S.	1	1.00	0.24	0.23	0.00	5.15	0.19	0.00	1.00
time (sec)	N/A	0.350	0.009	0.017	0.000	1.108	0.075	0.000	2.679

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	57	54	0	0	61	0	176
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	0.49
time (sec)	N/A	0.547	0.009	0.020	0.000	0.000	0.122	0.000	2.423

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	62	53	0	0	65	0	306
N.S.	1	1.00	0.16	0.14	0.00	0.00	0.17	0.00	0.81
time (sec)	N/A	0.730	0.009	0.025	0.000	0.000	0.128	0.000	2.675

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	103	75	0	0	82	0	432
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.20	0.00	1.04
time (sec)	N/A	0.900	0.014	0.037	0.000	0.000	0.551	0.000	2.328

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	109	74	0	0	70	0	340
N.S.	1	1.00	0.24	0.17	0.00	0.00	0.16	0.00	0.76
time (sec)	N/A	1.071	0.014	0.030	0.000	0.000	0.177	0.000	0.292

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	167	122	0	0	112	0	388
N.S.	1	1.00	0.16	0.11	0.00	0.00	0.11	0.00	0.36
time (sec)	N/A	2.268	0.028	0.027	0.000	0.000	0.237	0.000	2.341

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1005	1005	167	122	0	0	112	0	387
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	2.149	0.020	0.026	0.000	0.000	0.287	0.000	2.304

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	677	677	167	122	0	0	112	0	388
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.17	0.00	0.57
time (sec)	N/A	1.452	0.030	0.026	0.000	0.000	0.246	0.000	2.328

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	682	167	122	0	1445	104	0	299
N.S.	1	1.00	0.24	0.18	0.00	2.12	0.15	0.00	0.44
time (sec)	N/A	1.172	0.019	0.027	0.000	1.161	0.129	0.000	0.314

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	850	850	167	122	0	0	112	0	388
N.S.	1	1.00	0.20	0.14	0.00	0.00	0.13	0.00	0.46
time (sec)	N/A	1.357	0.026	0.026	0.000	0.000	0.257	0.000	2.420

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	873	873	167	122	0	0	112	0	387
N.S.	1	1.00	0.19	0.14	0.00	0.00	0.13	0.00	0.44
time (sec)	N/A	1.770	0.019	0.024	0.000	0.000	0.293	0.000	2.416

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	A	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	986	986	167	122	0	0	112	0	388
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39
time (sec)	N/A	1.946	0.026	0.026	0.000	0.000	0.250	0.000	2.478

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.035	0.002	0.187	0.265	0.372	0.014	4.242	0.037

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	96	105	105	88	365	106
N.S.	1	1.00	0.84	1.02	1.12	1.12	0.94	3.88	1.13
time (sec)	N/A	0.085	0.024	0.206	0.277	0.398	0.126	4.622	0.059

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	14	13	154	175	13	154
N.S.	1	1.00	11.47	0.93	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.012	0.004	0.187	0.285	0.366	0.037	4.693	2.224

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	156	156
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.037	0.005	0.219	0.261	0.391	0.038	4.345	0.143

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	156	156
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.035	0.005	0.210	0.285	0.357	0.037	5.304	2.171

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	262	0	0	229
N.S.	1	1.00	1.00	10.95	10.90	12.48	0.00	0.00	10.90
time (sec)	N/A	0.021	0.012	0.210	0.277	0.370	0.000	0.000	4.019

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.003	0.003	0.191	0.284	0.374	0.038	4.372	0.050

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	18	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.87
time (sec)	N/A	0.017	0.004	0.189	0.275	0.354	0.066	4.674	2.081

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.017	0.005	0.198	0.278	0.359	0.078	4.157	0.060

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	19	18	47	23	29	0	-1
N.S.	1	1.00	1.27	1.20	3.13	1.53	1.93	0.00	-0.07
time (sec)	N/A	0.017	0.006	0.212	0.293	0.401	0.606	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	13	81	87	13	12
N.S.	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.003	0.014	0.204	0.276	0.393	0.484	4.607	4.377

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.015	0.021	0.208	0.300	0.365	0.703	4.765	2.225

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.015	0.025	0.201	0.305	0.387	0.924	3.847	7.220

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	143	0	0	-1
N.S.	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	-0.05
time (sec)	N/A	0.021	0.010	0.237	0.344	0.527	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	19	26	46	19	23
N.S.	1	1.00	0.89	1.05	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.004	0.009	0.183	0.281	0.375	0.256	3.807	2.095

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	31	35	32	0	54	45
N.S.	1	1.00	1.00	1.15	1.30	1.19	0.00	2.00	1.67
time (sec)	N/A	0.015	0.047	0.240	0.309	0.376	0.000	2.959	2.213

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	116	27	142	35	33	0	54	-1
N.S.	1	4.30	1.00	5.26	1.30	1.22	0.00	2.00	-0.04
time (sec)	N/A	0.069	0.024	0.238	0.310	0.380	0.000	3.804	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	33	35	34	0	58	49
N.S.	1	1.00	1.00	1.14	1.21	1.17	0.00	2.00	1.69
time (sec)	N/A	0.015	0.049	0.180	0.313	0.409	0.000	3.417	2.205

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	0	39	42	0	0	-1
N.S.	1	1.00	0.86	0.00	1.08	1.17	0.00	0.00	-0.03
time (sec)	N/A	0.055	0.164	0.052	0.330	0.413	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.028	0.002	0.211	0.281	0.371	0.016	3.364	2.142

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.032	0.001	0.189	0.280	0.388	0.017	3.351	0.019

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	32
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.76
time (sec)	N/A	0.033	0.011	0.167	0.477	0.371	0.124	3.462	2.130

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	25	38	0	25	54
N.S.	1	1.00	0.92	1.04	1.00	1.52	0.00	1.00	2.16
time (sec)	N/A	0.015	0.083	0.030	0.283	0.408	0.000	3.709	2.191

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	21	25	24	36	0	24	46
N.S.	1	1.00	0.88	1.04	1.00	1.50	0.00	1.00	1.92
time (sec)	N/A	0.011	0.150	0.026	0.275	0.400	0.000	4.022	2.124

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	26	39	35	0	65	51
N.S.	1	1.00	0.96	1.04	1.56	1.40	0.00	2.60	2.04
time (sec)	N/A	0.014	0.033	0.282	0.322	0.406	0.000	2.579	2.155

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	28	0	20	39
N.S.	1	1.00	1.00	1.05	1.00	1.40	0.00	1.00	1.95
time (sec)	N/A	0.007	0.026	0.023	0.277	0.383	0.000	3.965	2.143

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	26	63	19	25
N.S.	1	1.00	1.00	1.05	1.00	1.37	3.32	1.00	1.32
time (sec)	N/A	0.006	0.062	0.165	0.274	0.401	5.348	3.952	2.127

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	31	27	66	39	26
N.S.	1	1.00	1.00	1.05	1.41	1.23	3.00	1.77	1.18
time (sec)	N/A	0.006	0.024	0.217	0.308	0.394	23.870	3.547	2.161

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	0	22	43
N.S.	1	1.00	0.95	1.05	1.00	1.45	0.00	1.00	1.95
time (sec)	N/A	0.012	0.060	0.023	0.279	0.384	0.000	4.161	2.138

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	21	30	53	21	27
N.S.	1	1.00	0.90	1.05	1.00	1.43	2.52	1.00	1.29
time (sec)	N/A	0.008	0.043	0.209	0.274	0.403	0.408	3.349	2.162

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	28	32	31	56	51	28
N.S.	1	1.00	0.92	1.17	1.33	1.29	2.33	2.12	1.17
time (sec)	N/A	0.015	0.013	0.234	0.327	0.395	1.602	3.658	2.199

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	32	29	53	41	26
N.S.	1	1.00	1.00	1.05	1.45	1.32	2.41	1.86	1.18
time (sec)	N/A	0.007	0.011	0.169	0.319	0.405	1.583	4.275	2.188

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	32	32	0	22	43
N.S.	1	1.00	0.95	1.05	1.45	1.45	0.00	1.00	1.95
time (sec)	N/A	0.009	0.007	0.019	0.292	0.409	0.000	3.935	2.142

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	26	32	30	53	21	27
N.S.	1	1.00	0.90	1.24	1.52	1.43	2.52	1.00	1.29
time (sec)	N/A	0.008	0.006	0.169	0.313	0.388	0.401	4.811	2.148

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	143	20	19	1528	1771	160	1576
N.S.	1	1.00	6.81	0.95	0.90	72.76	84.33	7.62	75.05
time (sec)	N/A	0.088	0.113	0.078	0.263	0.381	0.166	3.411	2.979

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	18	441	469	18	418
N.S.	1	1.00	0.90	0.95	0.90	22.05	23.45	0.90	20.90
time (sec)	N/A	0.036	0.020	0.146	0.270	0.371	0.064	3.351	2.320

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	5596	441	441	469	18	418
N.S.	1	1.00	0.95	294.53	23.21	23.21	24.68	0.95	22.00
time (sec)	N/A	0.049	0.010	0.269	0.266	0.390	0.056	3.302	2.267

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	127	15	14	456	483	120	438
N.S.	1	1.00	7.94	0.94	0.88	28.50	30.19	7.50	27.38
time (sec)	N/A	0.017	0.045	0.092	0.266	0.381	0.059	3.408	2.628

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	98	14	13	88	97	13	88
N.S.	1	1.00	6.53	0.93	0.87	5.87	6.47	0.87	5.87
time (sec)	N/A	0.008	0.003	0.211	0.282	0.360	0.024	3.254	0.050

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	98	89	88	88	97	13	88
N.S.	1	1.00	6.12	5.56	5.50	5.50	6.06	0.81	5.50
time (sec)	N/A	0.019	0.002	0.203	0.269	0.382	0.019	3.096	0.040

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	17	16	458	484	136	440
N.S.	1	1.00	6.39	0.94	0.89	25.44	26.89	7.56	24.44
time (sec)	N/A	0.030	0.044	0.048	0.297	0.383	0.060	3.573	2.632

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	98	16	15	88	97	15	88
N.S.	1	1.00	5.76	0.94	0.88	5.18	5.71	0.88	5.18
time (sec)	N/A	0.016	0.003	0.182	0.266	0.368	0.024	4.287	2.072

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	15	88
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.041	0.003	0.188	0.270	0.385	0.022	3.421	0.047

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	15	88
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29
time (sec)	N/A	0.007	0.002	0.191	0.273	0.376	0.019	4.772	0.041

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	115	17	458	458	484	488	440
N.S.	1	1.00	6.39	0.94	25.44	25.44	26.89	27.11	24.44
time (sec)	N/A	0.039	0.008	0.030	0.266	0.373	0.057	3.329	0.568

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	16	88	88	97	88	88
N.S.	1	1.00	7.00	1.14	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.015	0.002	0.180	0.266	0.370	0.023	3.406	0.040

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	98	89	88	88	97	88	88
N.S.	1	1.00	5.44	4.94	4.89	4.89	5.39	4.89	4.89
time (sec)	N/A	0.013	0.003	0.187	0.268	0.403	0.022	3.708	0.040

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	98	89	88	88	97	88	88
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29
time (sec)	N/A	0.002	0.002	0.193	0.269	0.392	0.019	4.188	0.039

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	80	25	66	66	70	24	66
N.S.	1	1.00	2.86	0.89	2.36	2.36	2.50	0.86	2.36
time (sec)	N/A	0.019	0.004	0.181	0.270	0.363	0.019	3.458	0.051

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	108	27	187	187	194	88	180
N.S.	1	1.00	3.48	0.87	6.03	6.03	6.26	2.84	5.81
time (sec)	N/A	0.022	0.027	0.264	0.285	0.374	0.035	3.687	0.105

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	52	48	228	30	31
N.S.	1	1.00	1.00	0.91	1.53	1.41	6.71	0.88	0.91
time (sec)	N/A	0.007	0.141	0.176	0.526	0.369	23.118	3.744	2.121

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	73	33	54	52	328	32	58
N.S.	1	1.00	2.09	0.94	1.54	1.49	9.37	0.91	1.66
time (sec)	N/A	0.006	0.156	0.251	0.501	0.389	118.664	4.592	2.110

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	93	25	77	77	87	24	77
N.S.	1	1.00	3.10	0.83	2.57	2.57	2.90	0.80	2.57
time (sec)	N/A	0.015	0.006	0.177	0.269	0.356	0.024	3.876	0.055

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	140	27	280	280	314	105	266
N.S.	1	1.00	4.52	0.87	9.03	9.03	10.13	3.39	8.58
time (sec)	N/A	0.027	0.036	0.098	0.270	0.399	0.047	3.786	2.266

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	98	31	80	80	90	30	80
N.S.	1	1.00	2.88	0.91	2.35	2.35	2.65	0.88	2.35
time (sec)	N/A	0.023	0.006	0.220	0.262	0.394	0.025	3.422	0.065

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	146	33	289	289	321	126	273
N.S.	1	1.00	3.56	0.80	7.05	7.05	7.83	3.07	6.66
time (sec)	N/A	0.032	0.039	0.049	0.276	0.406	0.051	3.848	2.283

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	244	37	289	289	323	37	270
N.S.	1	1.00	5.30	0.80	6.28	6.28	7.02	0.80	5.87
time (sec)	N/A	0.034	0.049	0.197	0.271	0.372	0.054	3.614	2.237

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	248	39	773	773	930	153	753
N.S.	1	1.00	5.28	0.83	16.45	16.45	19.79	3.26	16.02
time (sec)	N/A	0.063	0.094	0.294	0.287	0.382	0.105	3.596	2.450

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	54	48	190	30	33
N.S.	1	1.00	1.06	0.91	1.59	1.41	5.59	0.88	0.97
time (sec)	N/A	0.007	0.133	0.223	0.518	0.410	41.257	3.985	2.111

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	71	57	189	36	37
N.S.	1	1.00	0.95	0.84	1.61	1.30	4.30	0.82	0.84
time (sec)	N/A	0.007	0.158	0.199	0.526	0.394	96.617	3.696	2.210

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	83	72	0	42	73
N.S.	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	1.46
time (sec)	N/A	0.007	0.195	0.036	0.528	0.401	0.000	3.018	2.170

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	18	17	29	29	30	29
N.S.	1	1.00	1.74	0.95	0.89	1.53	1.53	1.58	1.53
time (sec)	N/A	0.006	0.002	0.030	0.266	0.397	0.006	3.741	0.025

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	15	14	17	17	22	17
N.S.	1	1.00	1.31	0.94	0.88	1.06	1.06	1.38	1.06
time (sec)	N/A	0.005	0.002	0.057	0.269	0.368	0.006	4.218	0.034

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	29	86	86	94	28	86
N.S.	1	2.91	2.91	0.88	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.134	0.005	0.227	0.273	0.368	0.016	3.305	0.218

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	87	86	86	94	28	86
N.S.	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61
time (sec)	N/A	0.100	0.005	0.222	0.269	0.345	0.021	3.952	0.191

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	57	56	12	12
N.S.	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	0.86
time (sec)	N/A	0.005	0.004	0.017	0.266	0.366	0.091	3.408	2.100

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87
time (sec)	N/A	0.006	0.004	0.011	0.262	0.418	0.021	4.127	0.049

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	13
N.S.	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76
time (sec)	N/A	0.006	0.005	0.013	0.263	0.380	0.025	2.971	0.066

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	23	28	24	23	22	24	23
N.S.	1	1.00	0.58	0.70	0.60	0.58	0.55	0.60	0.58
time (sec)	N/A	0.060	0.044	0.025	0.275	0.386	15.048	4.102	0.108

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	605	98	535	0	0	0	0	-1
N.S.	1	1.00	0.16	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.285	0.056	0.057	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	53	0	83	58	58	75
N.S.	1	1.00	0.87	0.84	0.00	1.32	0.92	0.92	1.19
time (sec)	N/A	0.045	0.021	0.040	0.000	0.396	0.041	3.688	0.185

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	38	20	13	12
N.S.	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	0.86
time (sec)	N/A	0.031	0.006	0.015	0.269	0.401	0.026	3.744	0.040

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	27	32	46	29	23	21
N.S.	1	1.00	0.86	0.96	1.14	1.64	1.04	0.82	0.75
time (sec)	N/A	0.019	0.010	0.015	0.265	0.412	0.032	3.979	0.037

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	28	30	38	38	36	28	28
N.S.	1	1.00	0.47	0.51	0.64	0.64	0.61	0.47	0.47
time (sec)	N/A	0.060	0.009	0.026	0.272	0.390	0.060	4.663	0.054

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	11	11	8	11	11
N.S.	1	1.00	1.00	3.73	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.005	0.005	0.020	0.270	0.398	0.033	5.037	2.301

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	205	132	116	114	223	272	134	124
N.S.	1	2.25	1.45	1.27	1.25	2.45	2.99	1.47	1.36
time (sec)	N/A	0.106	0.067	0.044	0.486	0.386	0.798	8.596	2.369

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	44	40	0	99	49
N.S.	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.96
time (sec)	N/A	0.017	1.218	0.034	0.329	0.556	0.000	6.677	2.656

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.056	2.052	0.039	0.318	0.416	0.000	4.651	2.549

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	39	39	0	89	39
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70
time (sec)	N/A	0.040	0.963	0.035	0.302	0.421	0.000	5.188	2.319

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	37	37	0	87	37
N.S.	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.76
time (sec)	N/A	0.032	0.737	0.030	0.313	0.400	0.000	4.636	2.273

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	33	33	0	52	19
N.S.	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.00
time (sec)	N/A	0.028	0.076	0.028	0.316	0.392	0.000	5.560	2.193

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.023	0.713	0.031	0.300	0.439	0.000	0.000	3.202

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.024	0.793	0.033	0.307	0.442	0.000	0.000	3.319

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	24	36	36	0	0	23
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00
time (sec)	N/A	0.024	0.963	0.035	0.315	0.438	0.000	0.000	3.355

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	73	79	97	73	97
N.S.	1	1.00	0.86	0.76	0.75	0.81	1.00	0.75	1.00
time (sec)	N/A	0.094	0.027	0.038	0.483	0.388	0.104	4.659	0.194

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	68	74	92	68	92
N.S.	1	1.00	0.87	0.77	0.76	0.82	1.02	0.76	1.02
time (sec)	N/A	0.082	0.018	0.044	0.490	0.402	0.103	5.430	0.178

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	61	67	78	61	85
N.S.	1	1.00	0.94	0.81	0.79	0.87	1.01	0.79	1.10
time (sec)	N/A	0.082	0.023	0.033	0.498	0.380	0.095	5.426	2.290

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	56	62	75	56	80
N.S.	1	1.00	0.96	0.79	0.78	0.86	1.04	0.78	1.11
time (sec)	N/A	0.064	0.018	0.029	0.495	0.389	0.096	7.682	2.292

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	55	61	75	55	79
N.S.	1	1.00	0.92	0.79	0.77	0.86	1.06	0.77	1.11
time (sec)	N/A	0.054	0.014	0.026	0.500	0.382	0.092	8.320	0.147

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	60	59	65	78	60	83
N.S.	1	1.00	0.92	0.80	0.79	0.87	1.04	0.80	1.11
time (sec)	N/A	0.091	0.016	0.035	0.481	0.370	0.126	5.765	0.149

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	64	76	87	65	88
N.S.	1	1.00	0.93	0.77	0.76	0.90	1.04	0.77	1.05
time (sec)	N/A	0.101	0.025	0.031	0.507	0.390	0.140	6.699	2.280

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	69	89	94	70	92
N.S.	1	1.00	0.90	0.77	0.76	0.98	1.03	0.77	1.01
time (sec)	N/A	0.106	0.043	0.036	0.502	0.393	0.157	5.867	0.152

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	109	74	0	1202	61	0	128
N.S.	1	1.00	0.36	0.24	0.00	3.92	0.20	0.00	0.42
time (sec)	N/A	0.396	0.015	0.020	0.000	1.166	0.573	0.000	2.169

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	101	67	0	1145	3662	0	188
N.S.	1	1.00	0.38	0.25	0.00	4.26	13.61	0.00	0.70
time (sec)	N/A	0.270	0.012	0.019	0.000	1.182	1.625	0.000	0.134

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	94	62	0	1190	48	0	183
N.S.	1	1.00	0.41	0.27	0.00	5.17	0.21	0.00	0.80
time (sec)	N/A	0.238	0.012	0.020	0.000	1.169	0.505	0.000	0.193

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	90	58	0	1189	46	0	181
N.S.	1	1.00	0.45	0.29	0.00	6.01	0.23	0.00	0.91
time (sec)	N/A	0.142	0.010	0.015	0.000	1.184	0.496	0.000	2.342

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	101	67	0	1143	60	0	237
N.S.	1	1.00	0.41	0.27	0.00	4.67	0.24	0.00	0.97
time (sec)	N/A	0.336	0.013	0.023	0.000	1.152	9.313	0.000	2.342

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	109	72	0	1245	25507	0	242
N.S.	1	1.00	0.39	0.26	0.00	4.43	90.77	0.00	0.86
time (sec)	N/A	0.337	0.014	0.029	0.000	1.190	19.415	0.000	2.302

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	116	77	0	1274	70	0	246
N.S.	1	1.00	0.37	0.24	0.00	4.02	0.22	0.00	0.78
time (sec)	N/A	0.390	0.014	0.028	0.000	1.169	1.755	0.000	2.248

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	87	75	0	83	44	87	252
N.S.	1	1.00	4.58	3.95	0.00	4.37	2.32	4.58	13.26
time (sec)	N/A	0.069	0.033	0.102	0.000	0.374	0.600	5.452	2.270

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	34	33	47	37	34	38
N.S.	1	1.00	1.33	0.79	0.77	1.09	0.86	0.79	0.88
time (sec)	N/A	0.043	0.017	0.208	0.529	0.386	0.068	4.521	0.052

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	18	21
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.06	1.24
time (sec)	N/A	0.025	0.004	0.209	0.279	0.386	0.044	4.326	2.200

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	20	24	30
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.80	0.96	1.20
time (sec)	N/A	0.025	0.005	0.227	0.283	0.388	0.044	4.257	0.054

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	19	18	24	14	28	18
N.S.	1	1.00	0.91	0.86	0.82	1.09	0.64	1.27	0.82
time (sec)	N/A	0.009	0.007	0.226	0.274	0.369	0.034	4.032	0.036

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	38	22	21	21	26	22	25
N.S.	1	1.00	1.41	0.81	0.78	0.78	0.96	0.81	0.93
time (sec)	N/A	0.026	0.009	0.211	0.486	0.392	0.051	3.829	2.260

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.009	0.004	0.194	0.492	0.396	0.019	3.511	0.031

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	22	23	23
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.81	0.85	0.85
time (sec)	N/A	0.017	0.006	0.276	0.501	0.422	0.031	3.151	2.140

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	26	25	25	26	29	25
N.S.	1	1.00	1.00	0.67	0.64	0.64	0.67	0.74	0.64
time (sec)	N/A	0.014	0.005	0.020	0.275	0.385	0.103	3.534	2.141

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	19	18	18	17	19	18
N.S.	1	1.00	1.05	0.86	0.82	0.82	0.77	0.86	0.82
time (sec)	N/A	0.010	0.003	0.171	0.265	0.374	0.015	3.000	0.027

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87
time (sec)	N/A	0.020	0.004	0.161	0.472	0.366	0.023	2.928	2.131

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	24	21	20	26	17	22	10
N.S.	1	1.00	2.00	1.75	1.67	2.17	1.42	1.83	0.83
time (sec)	N/A	0.014	0.010	0.017	0.266	0.383	0.029	3.136	0.072

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	22	19	23	19
N.S.	1	1.00	1.00	0.88	0.84	0.88	0.76	0.92	0.76
time (sec)	N/A	0.024	0.005	0.234	0.268	0.395	0.028	2.344	2.139

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.018	0.006	0.018	0.506	0.372	0.035	4.116	0.038

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	31	35	55	36	30	35
N.S.	1	1.00	1.00	0.89	1.00	1.57	1.03	0.86	1.00
time (sec)	N/A	0.024	0.013	0.186	0.485	0.379	0.049	4.357	2.125

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.74
time (sec)	N/A	0.025	0.005	0.017	0.267	0.397	0.047	3.712	2.188

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	20	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.87	0.83
time (sec)	N/A	0.036	0.006	0.020	0.265	0.376	0.030	3.539	0.055

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	23	32	20	23	25
N.S.	1	1.00	0.86	0.83	0.79	1.10	0.69	0.79	0.86
time (sec)	N/A	0.010	0.008	0.205	0.474	0.381	0.038	2.963	2.125

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	41	58	49	43	55
N.S.	1	1.00	1.00	0.95	0.93	1.32	1.11	0.98	1.25
time (sec)	N/A	0.171	0.019	0.185	0.480	0.416	0.083	3.756	0.129

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	38	38	51	38	88
N.S.	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.092	0.014	0.348	0.482	0.398	0.083	4.950	0.160

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	34	44	72	41	44	56
N.S.	1	1.00	1.00	1.03	1.33	2.18	1.24	1.33	1.70
time (sec)	N/A	0.115	0.016	0.411	0.269	0.402	0.064	5.125	2.162

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.59	0.65	0.65
time (sec)	N/A	0.005	0.005	0.203	0.486	0.385	0.048	4.106	0.032

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	20	34	20	20
N.S.	1	1.00	0.92	0.88	0.83	0.83	1.42	0.83	0.83
time (sec)	N/A	0.013	0.007	0.202	0.491	0.405	0.066	4.055	2.134

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	23	18	17	17	17	20	19
N.S.	1	1.00	1.53	1.20	1.13	1.13	1.13	1.33	1.27
time (sec)	N/A	0.039	0.005	0.224	0.483	0.436	0.048	6.245	0.056

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	17	17	19	17	17
N.S.	1	1.00	1.00	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.008	0.008	0.207	0.486	0.429	0.052	3.653	0.050

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	27	44	29	60	35
N.S.	1	1.00	0.89	0.76	0.73	1.19	0.78	1.62	0.95
time (sec)	N/A	0.028	0.018	0.182	0.481	0.389	0.059	3.966	2.144

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	21	21	26	21	21
N.S.	1	1.00	1.00	0.85	0.81	0.81	1.00	0.81	0.81
time (sec)	N/A	0.008	0.007	0.168	0.486	0.417	0.025	3.208	0.030

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	16	10	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.33	0.83	0.92	0.83
time (sec)	N/A	0.023	0.003	0.158	0.270	0.406	0.024	2.443	2.125

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	20	21
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.95	1.00
time (sec)	N/A	0.019	0.005	0.018	0.283	0.429	0.044	3.551	2.144

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	34	17	20	20
N.S.	1	1.00	1.00	0.95	0.91	1.55	0.77	0.91	0.91
time (sec)	N/A	0.010	0.007	0.015	0.489	0.382	0.034	3.782	2.124

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	20	32	19	20	22
N.S.	1	1.00	1.00	0.88	0.83	1.33	0.79	0.83	0.92
time (sec)	N/A	0.012	0.009	0.015	0.505	0.402	0.036	4.443	0.029

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	33	33	39	33	56
N.S.	1	1.00	1.00	0.94	0.92	0.92	1.08	0.92	1.56
time (sec)	N/A	0.018	0.012	0.208	0.478	0.404	0.077	4.627	0.103

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	28	27	27	29	27	37
N.S.	1	1.00	1.00	0.76	0.73	0.73	0.78	0.73	1.00
time (sec)	N/A	0.018	0.007	0.189	0.488	0.393	0.067	6.116	2.136

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.013	0.004	0.223	0.268	0.395	0.037	5.066	2.214

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.84	0.79
time (sec)	N/A	0.010	0.003	0.188	0.276	0.401	0.018	3.863	0.034

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	34	34	46	34	36
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.12	0.83	0.88
time (sec)	N/A	0.023	0.011	0.285	0.496	0.426	0.037	3.362	0.043

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	32	31	31	34	31	31
N.S.	1	1.00	0.95	0.78	0.76	0.76	0.83	0.76	0.76
time (sec)	N/A	0.019	0.006	0.272	0.479	0.390	0.035	4.160	2.116

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	24	21	20	20	24	23	20
N.S.	1	1.00	0.80	0.70	0.67	0.67	0.80	0.77	0.67
time (sec)	N/A	0.037	0.009	0.221	0.279	0.425	0.053	3.070	2.125

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	31	30	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.89	0.86	0.77
time (sec)	N/A	0.025	0.006	0.021	0.273	0.434	0.053	2.431	0.044

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	25	24	34	27	34	26
N.S.	1	1.00	0.94	0.74	0.71	1.00	0.79	1.00	0.76
time (sec)	N/A	0.033	0.011	0.196	0.270	0.395	0.050	3.164	2.109

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	33	44	36	33	39
N.S.	1	1.00	1.00	0.83	0.79	1.05	0.86	0.79	0.93
time (sec)	N/A	0.014	0.015	0.190	0.486	0.431	0.044	3.881	2.187

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	38	37	37	46	37	41
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.94	0.76	0.84
time (sec)	N/A	0.106	0.010	0.395	0.482	0.436	0.091	3.795	0.072

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	19	19	24	22	19
N.S.	1	1.00	1.00	0.69	0.66	0.66	0.83	0.76	0.66
time (sec)	N/A	0.034	0.006	0.209	0.268	0.382	0.050	3.242	2.182

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	34	33	33	41	34	50
N.S.	1	1.00	0.93	0.74	0.72	0.72	0.89	0.74	1.09
time (sec)	N/A	0.031	0.013	0.201	0.490	0.391	0.052	3.243	0.086

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	17	10	15	14
N.S.	1	1.00	0.88	0.94	0.88	1.06	0.62	0.94	0.88
time (sec)	N/A	0.017	0.007	0.015	0.282	0.375	0.024	5.416	0.043

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	16	17	17	14	12	12
N.S.	1	1.00	0.67	0.76	0.81	0.81	0.67	0.57	0.57
time (sec)	N/A	0.013	0.003	0.011	0.279	0.389	0.022	5.927	2.085

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	23	23	17	30	15
N.S.	1	1.00	1.00	1.07	1.53	1.53	1.13	2.00	1.00
time (sec)	N/A	0.019	0.009	0.229	0.268	0.372	0.032	4.348	2.089

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	28	28	3	29	57
N.S.	1	1.00	1.00	0.94	0.90	0.90	0.10	0.94	1.84
time (sec)	N/A	0.029	0.009	0.020	0.480	0.408	0.045	3.244	0.112

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	22	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.029	0.005	0.017	0.275	0.397	0.049	2.668	0.060

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	24	36	20	26	22
N.S.	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73
time (sec)	N/A	0.020	0.012	0.021	0.270	0.415	0.026	4.116	2.108

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.026	0.004	0.186	0.482	0.404	0.047	2.852	2.121

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	77	136	88	74	96
N.S.	1	1.00	0.90	0.71	0.75	1.32	0.85	0.72	0.93
time (sec)	N/A	0.333	0.034	0.310	0.474	0.418	0.268	4.762	2.200

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	25	36	24	25	32
N.S.	1	1.00	0.91	0.85	0.76	1.09	0.73	0.76	0.97
time (sec)	N/A	0.012	0.009	0.181	0.502	0.381	0.041	5.790	0.035

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	29	44	27	30	33
N.S.	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00
time (sec)	N/A	0.028	0.015	0.185	0.485	0.398	0.052	5.829	2.113

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	19	17	26	19
N.S.	1	1.00	1.00	0.96	0.92	0.76	0.68	1.04	0.76
time (sec)	N/A	0.028	0.005	0.207	0.268	0.393	0.025	4.654	0.043

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	31	36	31	56
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56
time (sec)	N/A	0.080	0.011	0.195	0.479	0.392	0.080	3.053	0.106

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	33	22	21	23
N.S.	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79
time (sec)	N/A	0.072	0.013	0.194	0.503	0.376	0.064	3.988	0.042

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	35	39	46	36	49
N.S.	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07
time (sec)	N/A	0.041	0.019	0.023	0.491	0.417	0.062	4.522	2.169

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	26	19	18	18	17	20	14
N.S.	1	1.18	1.18	0.86	0.82	0.82	0.77	0.91	0.64
time (sec)	N/A	0.011	0.004	0.246	0.267	0.393	0.026	4.654	0.039

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.027	0.005	0.018	0.265	0.404	0.044	5.370	0.065

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	14	16	14
N.S.	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00
time (sec)	N/A	0.016	0.003	0.190	0.290	0.389	0.034	3.623	2.120

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17
N.S.	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89
time (sec)	N/A	0.018	0.006	0.019	0.271	0.428	0.044	4.015	2.116

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	19	17	33
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43
time (sec)	N/A	0.021	0.008	0.020	0.523	0.380	0.070	2.877	0.054

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	50	50	68	53	58
N.S.	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92
time (sec)	N/A	0.059	0.020	0.030	0.494	0.392	0.187	3.456	2.213

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	86	67	54	59	103	65	59	71
N.S.	1	1.25	0.97	0.78	0.86	1.49	0.94	0.86	1.03
time (sec)	N/A	0.066	0.035	0.030	0.487	0.407	0.089	4.906	2.177

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	19	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.12	0.71	0.76	0.76
time (sec)	N/A	0.009	0.005	0.200	0.499	0.379	0.031	4.506	0.033

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	15	17	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.79	0.89	0.89
time (sec)	N/A	0.018	0.004	0.201	0.484	0.364	0.029	4.575	2.098

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	28	19
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	3.11	2.11
time (sec)	N/A	0.044	0.005	0.198	0.489	0.398	0.047	4.325	2.104

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.004	0.268	0.276	0.381	0.019	3.680	0.028

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	41	51	66	99	55	48
N.S.	1	1.00	0.89	0.63	0.78	1.02	1.52	0.85	0.74
time (sec)	N/A	0.040	0.026	0.236	0.491	0.390	0.237	3.630	0.099

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	46	24	28	28
N.S.	1	1.00	1.00	1.04	1.00	1.64	0.86	1.00	1.00
time (sec)	N/A	0.016	0.009	0.276	0.483	0.370	0.040	3.682	0.038

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	26	29	26	26
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.026	0.019	0.221	0.501	0.373	0.028	4.222	0.025

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	41	39	24	23	23	26	27	23
N.S.	1	1.32	1.26	0.77	0.74	0.74	0.84	0.87	0.74
time (sec)	N/A	0.032	0.006	0.021	0.290	0.400	0.077	4.796	0.050

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	26	19	23	30
N.S.	1	1.00	1.00	0.96	0.92	1.08	0.79	0.96	1.25
time (sec)	N/A	0.119	0.007	0.210	0.478	0.434	0.054	4.485	2.129

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.017	0.009	0.195	0.930	0.374	0.041	4.889	0.033

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	35	20	19	23
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00
time (sec)	N/A	0.026	0.004	0.016	0.507	0.374	0.041	4.448	0.030

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	10	13	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	0.77	1.00	1.00
time (sec)	N/A	0.031	0.005	0.018	0.537	0.368	0.027	3.882	2.144

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	193	227	199	4545	138	208	357
N.S.	1	1.00	0.94	1.10	0.97	22.06	0.67	1.01	1.73
time (sec)	N/A	0.193	0.079	0.227	0.502	1.153	0.703	6.014	0.231

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	0	47	44	38	42
N.S.	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.93
time (sec)	N/A	0.047	0.019	0.022	0.000	0.384	0.053	4.086	2.191

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	41	0	75	46	42	47
N.S.	1	1.00	0.75	0.69	0.00	1.27	0.78	0.71	0.80
time (sec)	N/A	0.044	0.018	0.024	0.000	0.386	0.067	5.753	0.049

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	234	240	0	578003	0	1589	2500
N.S.	1	1.00	1.12	1.15	0.00	2765.56	0.00	7.60	11.96
time (sec)	N/A	0.276	0.169	0.058	0.000	28.709	0.000	6.285	3.437

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	245	253	0	540080	0	1625	3046
N.S.	1	1.00	1.09	1.13	0.00	2411.07	0.00	7.25	13.60
time (sec)	N/A	0.202	0.163	0.187	0.000	113.356	0.000	6.128	3.220

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	57	60	65	190	62	61
N.S.	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.09
time (sec)	N/A	0.029	0.020	0.223	0.276	0.384	0.606	4.561	0.213

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	75	84	162	0	85	347
N.S.	1	1.00	0.76	0.78	0.88	1.69	0.00	0.89	3.61
time (sec)	N/A	0.070	0.026	0.219	0.497	0.455	0.000	4.496	1.131

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	228	245	279	5975	0	320	570
N.S.	1	1.00	0.86	0.93	1.06	22.63	0.00	1.21	2.16
time (sec)	N/A	0.339	0.067	0.227	0.486	1.284	0.000	3.474	2.496

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	370	281	349	259898	0	401	823
N.S.	1	1.00	0.89	0.67	0.84	623.26	0.00	0.96	1.97
time (sec)	N/A	0.379	0.170	0.233	0.511	27.979	0.000	3.794	2.449

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	20	26	19	21	12
N.S.	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	0.75
time (sec)	N/A	0.006	0.008	0.200	0.294	0.370	0.030	3.521	0.039

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	23	34	20	30	17
N.S.	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	0.89
time (sec)	N/A	0.009	0.009	0.200	0.262	0.396	0.038	4.079	2.138

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	90	75	106	92	77	103
N.S.	1	1.00	0.88	0.93	0.77	1.09	0.95	0.79	1.06
time (sec)	N/A	0.050	0.035	0.209	0.478	0.395	0.173	4.234	0.177

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87
time (sec)	N/A	0.073	0.006	0.197	0.481	0.372	0.037	4.006	2.120

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85
time (sec)	N/A	0.063	0.006	0.197	0.476	0.377	0.036	3.727	0.035

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	24	24	29	24	51
N.S.	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76
time (sec)	N/A	0.081	0.012	0.200	0.479	0.394	0.074	3.929	2.153

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.008	0.007	0.290	0.499	0.380	0.042	3.780	0.042

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	7	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	0.83
time (sec)	N/A	0.007	0.003	0.209	0.264	0.369	0.023	4.423	0.037

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	17
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	1.55
time (sec)	N/A	0.024	0.004	0.196	0.489	0.389	0.044	4.407	2.166

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	13	8	11	10
N.S.	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	0.83
time (sec)	N/A	0.015	0.003	0.208	0.278	0.391	0.022	4.867	0.039

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	14	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00
time (sec)	N/A	0.012	0.003	0.184	0.272	0.381	0.030	4.743	0.047

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	16	23
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	1.35
time (sec)	N/A	0.025	0.004	0.199	0.484	0.389	0.046	4.168	2.284

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	10	14	19
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.46
time (sec)	N/A	0.022	0.007	0.200	0.491	0.380	0.044	3.639	0.060

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	22	21	20	26	20	22	20
N.S.	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	0.71
time (sec)	N/A	0.007	0.013	0.225	0.266	0.392	0.034	3.852	0.069

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	26	37	26	43	22
N.S.	1	1.00	1.00	0.84	0.81	1.16	0.81	1.34	0.69
time (sec)	N/A	0.016	0.013	0.206	0.272	0.382	0.047	3.429	2.226

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	19	19	19	20	25
N.S.	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09
time (sec)	N/A	0.023	0.007	0.207	0.479	0.388	0.047	3.911	0.049

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	36	20	47	28
N.S.	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17
time (sec)	N/A	0.023	0.012	0.215	0.501	0.399	0.048	3.178	2.111

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	39	44	39	41
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.90	0.80	0.84
time (sec)	N/A	0.099	0.011	0.351	0.489	0.390	0.088	3.700	2.156

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	19	19	19	22	19
N.S.	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76
time (sec)	N/A	0.036	0.007	0.220	0.269	0.396	0.048	3.382	2.135

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	61
N.S.	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02
time (sec)	N/A	0.173	0.040	0.329	0.489	0.393	0.106	4.410	0.130

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	5	9	6
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.55
time (sec)	N/A	0.005	0.001	0.316	0.262	0.375	0.007	5.099	0.016

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	21	21	22	23	21
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72
time (sec)	N/A	0.011	0.004	0.228	0.283	0.400	0.035	5.623	2.124

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	37	37	52	37	37	49
N.S.	1	1.00	1.00	0.82	0.82	1.16	0.82	0.82	1.09
time (sec)	N/A	0.017	0.010	0.310	0.493	0.382	0.046	7.238	0.041

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	52	29	29	44
N.S.	1	1.00	1.00	0.91	0.88	1.62	0.91	0.91	1.38
time (sec)	N/A	0.176	0.014	0.207	0.496	0.379	0.109	6.752	0.069

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	159	99	132	179	146	122	124
N.S.	1	1.00	1.07	0.67	0.89	1.21	0.99	0.82	0.84
time (sec)	N/A	0.095	0.046	0.033	0.495	0.406	0.197	7.938	2.189

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	101	74	99	515	61	92	170
N.S.	1	1.00	0.90	0.66	0.88	4.60	0.54	0.82	1.52
time (sec)	N/A	0.078	0.035	0.201	0.490	1.156	0.524	8.670	2.228

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	14	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86
time (sec)	N/A	0.011	0.003	0.201	0.261	0.401	0.030	9.070	0.044

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83
time (sec)	N/A	0.018	0.004	0.199	0.277	0.397	0.027	8.831	2.138

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	15
N.S.	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88
time (sec)	N/A	0.020	0.003	0.201	0.283	0.399	0.034	7.407	0.027

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	17	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88
time (sec)	N/A	0.020	0.004	0.200	0.287	0.409	0.027	5.462	0.033

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	25	15	23	16
N.S.	1	1.00	1.00	0.94	0.89	1.39	0.83	1.28	0.89
time (sec)	N/A	0.021	0.004	0.215	0.496	0.401	0.030	5.484	0.050

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	14	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.00
time (sec)	N/A	0.013	0.004	0.016	0.281	0.397	0.021	5.466	0.034

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71
time (sec)	N/A	0.027	0.006	0.023	0.270	0.414	0.055	5.423	0.053

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	38	38	51	38	88
N.S.	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91
time (sec)	N/A	0.090	0.014	0.312	0.481	0.424	0.083	5.090	0.002

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	17	8
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.89	0.42
time (sec)	N/A	0.042	0.003	0.185	0.277	0.438	0.033	5.704	0.084

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	30	30	34	32	26
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.015	0.004	0.020	0.269	0.416	0.039	3.804	2.093

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	25	27	27	21
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.014	0.004	0.017	0.280	0.412	0.038	4.817	0.033

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	20	22	16
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.010	0.004	0.017	0.273	0.404	0.038	3.598	2.117

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	19	13
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.005	0.003	0.014	0.282	0.386	0.033	3.302	2.110

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	19	8
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.38
time (sec)	N/A	0.007	0.003	0.014	0.266	0.389	0.032	3.382	0.078

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	24	24	17
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.011	0.004	0.016	0.270	0.374	0.050	4.668	0.095

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	26	30	31	29	22
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.020	0.004	0.020	0.284	0.415	0.059	3.920	0.035

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	39	36	34	26
N.S.	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.022	0.004	0.019	0.274	0.374	0.063	4.919	0.036

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	36	44	41	39	32
N.S.	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.028	0.004	0.020	0.277	0.388	0.069	4.689	0.037

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	54	0	1506	41	0	144
N.S.	1	1.00	0.39	0.34	0.00	9.59	0.26	0.00	0.92
time (sec)	N/A	0.117	0.012	0.030	0.000	1.148	0.098	0.000	2.745

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	61	56	0	1546	41	0	142
N.S.	1	1.00	0.39	0.36	0.00	9.85	0.26	0.00	0.90
time (sec)	N/A	0.082	0.012	0.027	0.000	1.204	0.100	0.000	2.805

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	54	0	2271	39	0	142
N.S.	1	1.00	0.32	0.29	0.00	12.08	0.21	0.00	0.76
time (sec)	N/A	0.198	0.010	0.023	0.000	1.208	0.083	0.000	2.780

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	61	56	0	2259	39	0	142
N.S.	1	1.00	0.32	0.30	0.00	12.02	0.21	0.00	0.76
time (sec)	N/A	0.124	0.010	0.021	0.000	1.247	0.084	0.000	2.740

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	322185	133	0	328
N.S.	1	1.00	0.10	0.10	0.00	485.95	0.20	0.00	0.49
time (sec)	N/A	0.934	0.027	0.101	0.000	1.898	2.158	0.000	2.699

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	63	69	0	322185	133	0	328
N.S.	1	1.00	0.10	0.10	0.00	485.95	0.20	0.00	0.49
time (sec)	N/A	0.707	0.023	0.029	0.000	1.914	2.151	0.000	0.002

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	C	A	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	5653	42	0	504
N.S.	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	3.00
time (sec)	N/A	0.278	0.044	0.300	0.000	1.118	1.081	0.000	3.082

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	322	250	306	141845	384	311	894
N.S.	1	1.00	1.01	0.78	0.96	443.27	1.20	0.97	2.79
time (sec)	N/A	0.174	0.178	0.207	0.499	20.594	27.765	3.100	2.843

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	243	230	273	86139	277	285	556
N.S.	1	1.00	0.84	0.79	0.94	296.01	0.95	0.98	1.91
time (sec)	N/A	0.145	0.070	0.203	0.506	3.035	1.705	3.574	2.657

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	184	124	209	41851	124	215	160
N.S.	1	1.00	0.84	0.57	0.95	191.10	0.57	0.98	0.73
time (sec)	N/A	0.120	0.040	0.192	0.515	1.397	0.435	3.689	2.323

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	102	169	121	20	179	33
N.S.	1	1.00	0.72	0.55	0.91	0.65	0.11	0.97	0.18
time (sec)	N/A	0.071	0.013	0.184	0.522	0.383	0.059	3.634	0.083

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	404	289	341	352864	0	371	874
N.S.	1	1.00	0.97	0.69	0.82	848.23	0.00	0.89	2.10
time (sec)	N/A	0.309	0.107	0.231	0.505	138.075	0.000	3.908	0.418

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	524	354	540	0	0	646	2436
N.S.	1	1.00	0.95	0.64	0.98	0.00	0.00	1.17	4.41
time (sec)	N/A	0.572	0.453	0.242	0.492	0.000	0.000	5.097	2.779

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	738	446	779	0	0	901	1955
N.S.	1	1.00	1.09	0.66	1.15	0.00	0.00	1.32	2.88
time (sec)	N/A	0.675	0.638	0.272	0.504	0.000	0.000	4.030	3.674

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	347	317	329	91191	350	342	670
N.S.	1	1.00	0.99	0.91	0.94	261.29	1.00	0.98	1.92
time (sec)	N/A	0.219	0.265	0.211	0.498	15.920	184.578	3.462	0.433

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	321	293	316	90963	318	323	391
N.S.	1	1.00	1.00	0.91	0.98	282.49	0.99	1.00	1.21
time (sec)	N/A	0.185	0.221	0.211	0.484	3.168	3.813	3.506	2.483

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	224	163	241	43065	155	241	282
N.S.	1	1.00	0.93	0.68	1.00	178.69	0.64	1.00	1.17
time (sec)	N/A	0.138	0.138	0.204	0.493	1.572	0.580	4.934	0.274

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	118	189	173	39	194	58
N.S.	1	1.00	0.91	0.58	0.94	0.86	0.19	0.96	0.29
time (sec)	N/A	0.081	0.074	0.188	0.477	0.410	0.128	4.352	0.089

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	855	558	430	582	0	0	771	1591
N.S.	1	1.00	0.65	0.50	0.68	0.00	0.00	0.90	1.86
time (sec)	N/A	0.608	0.273	0.251	0.501	0.000	0.000	4.312	2.995

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1141	1141	807	534	919	0	0	1104	2246
N.S.	1	1.00	0.71	0.47	0.81	0.00	0.00	0.97	1.97
time (sec)	N/A	1.168	0.570	0.273	0.517	0.000	0.000	15.447	4.104

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1384	1384	996	680	1321	0	0	1488	2500
N.S.	1	1.00	0.72	0.49	0.95	0.00	0.00	1.08	1.81
time (sec)	N/A	1.379	0.902	0.327	0.564	0.000	0.000	3.977	5.038

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	388	403	389	95566	413	389	721
N.S.	1	1.00	0.98	1.02	0.99	242.55	1.05	0.99	1.83
time (sec)	N/A	0.243	0.248	0.213	0.481	16.510	130.252	4.180	0.478

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	358	360	362	91420	374	356	676
N.S.	1	1.00	0.99	1.00	1.01	253.94	1.04	0.99	1.88
time (sec)	N/A	0.227	0.209	0.214	0.495	8.567	14.201	5.859	0.471

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	249	207	273	43180	192	260	315
N.S.	1	1.00	0.94	0.78	1.03	162.33	0.72	0.98	1.18
time (sec)	N/A	0.164	0.135	0.204	0.483	2.666	1.133	3.017	0.303

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	200	139	212	232	63	204	80
N.S.	1	1.00	0.91	0.63	0.97	1.06	0.29	0.93	0.37
time (sec)	N/A	0.097	0.060	0.196	0.494	0.433	0.215	3.769	0.096

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1352	1352	835	677	971	0	0	1259	2500
N.S.	1	1.00	0.62	0.50	0.72	0.00	0.00	0.93	1.85
time (sec)	N/A	0.990	0.500	0.287	0.532	0.000	0.000	4.063	4.406

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1830	1830	1115	829	1487	0	0	1729	2500
N.S.	1	1.00	0.61	0.45	0.81	0.00	0.00	0.94	1.37
time (sec)	N/A	1.949	1.007	0.329	0.595	0.000	0.000	81.891	5.754

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2204	2204	1338	1008	2077	0	0	2119	2500
N.S.	1	1.00	0.61	0.46	0.94	0.00	0.00	0.96	1.13
time (sec)	N/A	2.243	1.875	0.348	0.560	0.000	0.000	4.043	7.785

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.014	0.008	0.287	0.483	0.400	0.039	4.162	0.041

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	29	28	28	34	28	30
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94
time (sec)	N/A	0.020	0.004	0.194	0.480	0.402	0.035	4.268	0.035

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	26	26	36	26	30
N.S.	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.011	0.007	0.348	0.513	0.405	0.035	4.640	0.043

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	29	26	26	36	26	30
N.S.	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94
time (sec)	N/A	0.023	0.004	0.196	0.499	0.418	0.038	4.449	0.032

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.010	0.018	0.254	0.486	0.421	0.033	3.911	2.319

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76
time (sec)	N/A	0.015	0.004	0.012	0.504	0.408	0.036	5.681	0.047

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	18	17	17	15	15	6
N.S.	1	1.00	3.83	3.00	2.83	2.83	2.50	2.50	1.00
time (sec)	N/A	0.002	0.002	0.195	0.277	0.407	0.026	3.796	2.270

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	18	17	17	15	19	6
N.S.	1	1.00	1.10	0.86	0.81	0.81	0.71	0.90	0.29
time (sec)	N/A	0.002	0.002	0.204	0.259	0.387	0.028	3.385	0.146

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	24	22	9	9
N.S.	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	0.69
time (sec)	N/A	0.002	0.003	0.190	0.284	0.377	0.038	3.751	2.319

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	81	11	58	57	24	22	57	9
N.S.	1	6.23	0.85	4.46	4.38	1.85	1.69	4.38	0.69
time (sec)	N/A	0.008	0.001	0.363	0.274	0.375	0.164	3.131	0.025

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	66	66	85	68	94
N.S.	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.079	0.011	0.211	0.475	0.367	0.120	3.894	0.102

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	67	66	66	85	68	94
N.S.	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36
time (sec)	N/A	0.088	0.005	0.028	0.491	0.391	0.156	4.419	0.039

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	20	20	20	21	20
N.S.	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83
time (sec)	N/A	0.015	0.005	0.192	0.279	0.383	0.016	4.250	0.037

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20
N.S.	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77
time (sec)	N/A	0.012	0.004	0.197	0.291	0.375	0.015	3.754	0.033

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	22	19	15	18	15	15	17
N.S.	1	1.00	1.29	1.12	0.88	1.06	0.88	0.88	1.00
time (sec)	N/A	0.004	0.000	0.008	0.266	0.354	0.005	4.370	0.024

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	19	20	18	15	20	17
N.S.	1	1.00	0.92	0.79	0.83	0.75	0.62	0.83	0.71
time (sec)	N/A	0.004	0.001	0.011	0.271	0.342	0.006	4.605	0.021

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	23	22	25	20	23	28
N.S.	1	1.00	1.09	1.05	1.00	1.14	0.91	1.05	1.27
time (sec)	N/A	0.013	0.005	0.202	0.499	0.405	0.045	3.420	0.045

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	16	15	15	15	18	15
N.S.	1	1.00	1.59	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.009	0.005	0.209	0.266	0.371	0.047	3.720	0.057

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	16	13
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.84	0.68
time (sec)	N/A	0.016	0.003	0.208	0.280	0.384	0.029	3.629	2.336

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	10	10	8	13	10
N.S.	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	0.83
time (sec)	N/A	0.005	0.003	0.200	0.264	0.381	0.021	3.212	2.269

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	10	10	8	11	10
N.S.	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.00
time (sec)	N/A	0.006	0.005	0.196	0.286	0.392	0.046	3.522	0.056

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88
time (sec)	N/A	0.014	0.005	0.206	0.270	0.405	0.041	3.727	0.056

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91
time (sec)	N/A	0.014	0.004	0.206	0.492	0.407	0.043	3.525	0.049

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.006	0.005	0.013	0.274	0.391	0.022	3.627	0.042

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.015	0.004	0.018	0.273	0.385	0.045	3.819	0.067

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	14	9	9	7	9	10
N.S.	1	1.00	0.90	1.40	0.90	0.90	0.70	0.90	1.00
time (sec)	N/A	0.007	0.004	0.012	0.265	0.388	0.018	3.566	2.204

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.80	0.80	0.68
time (sec)	N/A	0.016	0.004	0.019	0.265	0.416	0.047	3.604	0.107

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00
time (sec)	N/A	0.017	0.003	0.209	0.488	0.394	0.031	3.655	2.219

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	14	30	11	16	17	11	20
N.S.	1	1.00	1.08	2.31	0.85	1.23	1.31	0.85	1.54
time (sec)	N/A	0.005	0.005	0.224	0.278	0.385	0.040	3.897	2.245

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	26	26	25	20	27	25
N.S.	1	1.00	0.96	1.00	1.00	0.96	0.77	1.04	0.96
time (sec)	N/A	0.030	0.008	0.198	0.284	0.375	0.061	3.624	2.234

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	4	4	3	5	4
N.S.	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67
time (sec)	N/A	0.006	0.001	0.009	0.262	0.380	0.011	4.436	0.018

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	20	25	19	27	18
N.S.	1	1.00	1.00	0.95	1.00	1.25	0.95	1.35	0.90
time (sec)	N/A	0.010	0.004	0.213	0.272	0.383	0.036	4.067	0.039

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	28	27	27	31	28	44
N.S.	1	1.00	1.05	0.74	0.71	0.71	0.82	0.74	1.16
time (sec)	N/A	0.019	0.008	0.216	0.481	0.400	0.050	4.004	0.158

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	14	16	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.006	0.003	0.215	0.273	0.382	0.014	3.764	0.031

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	36	22	21	21	22	22	25
N.S.	1	1.00	1.16	0.71	0.68	0.68	0.71	0.71	0.81
time (sec)	N/A	0.008	0.005	0.223	0.500	0.408	0.049	3.915	0.053

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	17	15	17	17	13
N.S.	1	1.00	1.00	0.84	0.89	0.79	0.89	0.89	0.68
time (sec)	N/A	0.011	0.005	0.203	0.270	0.394	0.036	3.854	0.090

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.004	0.004	0.214	0.267	0.392	0.022	4.827	2.217

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.009	0.006	0.023	0.502	0.396	0.056	6.415	2.216

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.016	0.015	0.221	0.266	0.409	0.075	7.595	0.042

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	8	8	7	9	8
N.S.	1	1.00	0.83	1.17	0.67	0.67	0.58	0.75	0.67
time (sec)	N/A	0.001	0.002	0.199	0.272	0.382	0.022	5.328	0.042

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	23	34	20	25	17
N.S.	1	1.00	1.29	1.33	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.002	0.006	0.207	0.271	0.394	0.032	4.598	2.216

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.003	0.006	0.195	0.486	0.387	0.029	4.945	0.027

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	9	6
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.001	0.001	0.199	0.263	0.385	0.006	5.183	0.068

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.001	0.002	0.210	0.482	0.378	0.034	3.443	2.245

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.003	0.004	0.221	0.483	0.415	0.059	4.676	2.243

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.008	0.005	0.391	0.484	0.388	0.032	4.608	0.032

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77
time (sec)	N/A	0.004	0.001	0.208	0.273	0.385	0.005	4.611	0.024

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77
time (sec)	N/A	0.004	0.001	0.216	0.272	0.358	0.005	4.291	0.029

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	10	14	15
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94
time (sec)	N/A	0.003	0.001	0.014	0.267	0.367	0.011	4.538	0.028

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	39	30	30
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.05	0.81	0.81
time (sec)	N/A	0.015	0.008	0.464	0.489	0.405	0.037	3.806	2.219

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	13
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93
time (sec)	N/A	0.003	0.000	0.189	0.260	0.380	0.004	3.921	0.024

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	7	9	6
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.64	0.82	0.55
time (sec)	N/A	0.002	0.001	0.198	0.276	0.372	0.006	4.295	0.016

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	20	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.80	0.80	1.00
time (sec)	N/A	0.007	0.005	0.212	0.501	0.391	0.046	3.560	0.049

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.007	0.005	0.214	0.485	0.389	0.047	4.689	2.228

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00
time (sec)	N/A	0.012	0.005	0.219	0.481	0.392	0.041	4.298	2.209

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	7	7	8	7	12	5	8	7
N.S.	1	0.78	0.78	0.89	0.78	1.33	0.56	0.89	0.78
time (sec)	N/A	0.004	0.003	0.203	0.260	0.390	0.015	5.359	0.023

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.008	0.010	0.023	0.489	0.404	0.053	9.612	0.055

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	44	30	30
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.19	0.81	0.81
time (sec)	N/A	0.016	0.011	0.406	0.477	0.418	0.044	5.378	0.045

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	14	16	13
N.S.	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	0.87
time (sec)	N/A	0.003	0.002	0.014	0.265	0.384	0.022	4.867	0.026

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	15	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	0.87
time (sec)	N/A	0.003	0.001	0.015	0.266	0.374	0.015	4.178	0.023

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	13	11	8	13	11
N.S.	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00
time (sec)	N/A	0.008	0.003	0.202	0.266	0.368	0.026	2.852	0.056

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	18	17	17	17	17	17
N.S.	1	1.00	0.86	0.82	0.77	0.77	0.77	0.77	0.77
time (sec)	N/A	0.003	0.001	0.036	0.276	0.373	0.005	3.419	0.030

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	19	19	19	9	9
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	0.82
time (sec)	N/A	0.001	0.001	0.204	0.270	0.369	0.006	3.949	0.142

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	16	14	13	13	12	14	11
N.S.	1	1.00	1.23	1.08	1.00	1.00	0.92	1.08	0.85
time (sec)	N/A	0.006	0.003	0.205	0.269	0.375	0.015	4.210	2.211

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	18	14	16	12
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	0.75
time (sec)	N/A	0.005	0.002	0.224	0.264	0.383	0.029	3.937	0.034

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	25	39	24	27	23
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85
time (sec)	N/A	0.007	0.012	0.017	0.276	0.379	0.035	4.270	2.228

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88
time (sec)	N/A	0.006	0.003	0.204	0.297	0.394	0.014	4.159	0.026

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	15	17	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	0.89
time (sec)	N/A	0.004	0.001	0.016	0.270	0.370	0.016	3.376	0.029

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	14	14	14	14	13
N.S.	1	1.00	1.00	1.56	0.78	0.78	0.78	0.78	0.72
time (sec)	N/A	0.007	0.001	0.025	0.485	0.370	0.005	4.157	0.024

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	34	46	31	25	23
N.S.	1	1.00	1.00	0.96	1.48	2.00	1.35	1.09	1.00
time (sec)	N/A	0.012	0.009	0.224	0.266	0.394	0.031	4.533	0.032

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	15	10	32	12
N.S.	1	1.00	0.75	0.81	0.75	0.94	0.62	2.00	0.75
time (sec)	N/A	0.008	0.006	0.217	0.509	0.387	0.033	4.155	2.219

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	26	25	25	24	26	25
N.S.	1	1.00	1.03	0.90	0.86	0.86	0.83	0.90	0.86
time (sec)	N/A	0.014	0.006	0.208	0.265	0.398	0.016	3.868	0.026

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81
time (sec)	N/A	0.005	0.001	0.200	0.276	0.371	0.007	3.424	0.021

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	17	25	14	12	17
N.S.	1	1.00	1.00	0.94	1.00	1.47	0.82	0.71	1.00
time (sec)	N/A	0.011	0.010	0.231	0.506	0.392	0.043	3.312	0.031

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	43	71	190	151	48	42
N.S.	1	1.00	1.00	0.91	1.51	4.04	3.21	1.02	0.89
time (sec)	N/A	0.046	0.019	0.348	0.272	0.402	0.140	3.118	0.097

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	61	61	0	240	294	60	82
N.S.	1	1.00	1.07	1.07	0.00	4.21	5.16	1.05	1.44
time (sec)	N/A	0.056	0.021	0.345	0.000	0.382	0.224	4.179	2.227

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	39	168	0	240	24	147	101
N.S.	1	1.00	0.21	0.89	0.00	1.28	0.13	0.78	0.54
time (sec)	N/A	0.132	0.024	0.047	0.000	0.409	0.283	4.568	2.300

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	27	28	65	65	60	30	27
N.S.	1	1.00	0.45	0.47	1.08	1.08	1.00	0.50	0.45
time (sec)	N/A	0.088	0.012	0.024	0.293	0.381	0.099	4.143	2.337

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	B	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	250	65	65	60	197	27
N.S.	1	0.00	1.00	9.26	2.41	2.41	2.22	7.30	1.00
time (sec)	N/A	0.217	0.008	0.055	0.288	0.378	0.146	5.462	0.045

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	B	B	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	112	65	65	60	111	27
N.S.	1	0.00	1.00	4.15	2.41	2.41	2.22	4.11	1.00
time (sec)	N/A	0.296	0.007	0.045	0.285	0.380	0.115	4.042	0.043

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [424] had the largest ratio of [73]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	24	0.083
2	A	3	3	1.00	29	0.103
3	A	2	2	1.00	29	0.069
4	A	2	2	1.00	29	0.069
5	A	1	0	1.00	27	0.000
6	A	2	2	1.00	29	0.069
7	A	2	2	1.00	29	0.069
8	A	2	2	1.00	29	0.069
9	A	3	2	1.00	27	0.074
10	A	3	2	1.00	27	0.074
11	A	1	0	1.00	25	0.000
12	A	7	7	1.00	27	0.259
13	A	8	8	1.00	27	0.296
14	A	9	8	1.00	27	0.296
15	A	3	2	1.00	46	0.043
16	A	3	2	1.00	46	0.043
17	A	1	0	1.00	44	0.000
18	A	2	1	1.00	46	0.022
19	A	2	1	1.00	46	0.022
20	A	2	1	1.00	46	0.022
21	A	5	4	1.00	11	0.364
22	A	5	4	1.00	17	0.235
23	A	2	2	1.00	7	0.286
24	A	3	2	1.00	13	0.154
25	A	5	5	1.00	11	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	7	1.00	16	0.438
27	A	6	6	1.00	9	0.667
28	A	2	2	1.00	7	0.286
29	A	3	3	1.00	13	0.231
30	A	3	3	1.11	11	0.273
31	A	3	3	1.00	16	0.188
32	A	2	2	1.26	9	0.222
33	A	3	2	1.00	29	0.069
34	A	2	1	1.00	29	0.034
35	A	2	1	1.00	29	0.034
36	A	1	0	1.00	27	0.000
37	A	10	6	1.00	29	0.207
38	A	11	7	1.00	29	0.241
39	A	3	2	1.00	32	0.062
40	A	2	1	1.00	32	0.031
41	A	2	1	1.00	32	0.031
42	A	1	0	1.00	30	0.000
43	A	4	3	1.00	32	0.094
44	A	5	4	1.00	32	0.125
45	A	2	1	1.00	17	0.059
46	A	2	1	1.00	17	0.059
47	A	2	1	1.00	17	0.059
48	A	1	0	1.00	15	0.000
49	A	16	9	1.00	17	0.529
50	A	18	11	1.00	17	0.647
51	A	2	1	1.00	17	0.059
52	A	2	1	1.00	17	0.059
53	A	2	1	1.00	17	0.059
54	A	1	0	1.00	15	0.000
55	A	15	9	1.00	17	0.529
56	A	17	11	1.00	17	0.647
57	A	2	1	1.00	22	0.045
58	A	2	1	1.00	22	0.045
59	A	2	1	1.00	22	0.045
60	A	1	0	1.00	20	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	16	9	1.00	22	0.409
62	A	18	11	1.00	22	0.500
63	A	2	2	1.00	51	0.039
64	A	2	2	1.00	51	0.039
65	B	1	0	4.36	49	0.000
66	A	2	2	1.00	51	0.039
67	A	2	2	1.00	51	0.039
68	A	2	2	1.00	51	0.039
69	A	6	5	1.00	13	0.385
70	A	5	3	1.00	19	0.158
71	A	5	3	1.00	19	0.158
72	A	5	3	1.00	19	0.158
73	A	1	0	1.00	17	0.000
74	A	5	2	1.00	19	0.105
75	A	7	3	1.00	19	0.158
76	A	10	3	1.00	19	0.158
77	B	15	7	2.25	17	0.412
78	A	6	5	1.00	15	0.333
79	A	6	5	1.00	15	0.333
80	A	4	4	1.00	13	0.308
81	A	2	2	1.00	11	0.182
82	A	6	6	1.00	15	0.400
83	A	7	6	1.00	15	0.400
84	A	7	6	1.00	15	0.400
85	A	2	2	1.00	13	0.154
86	A	3	3	1.00	13	0.231
87	A	4	3	1.00	13	0.231
88	A	2	2	1.00	19	0.105
89	A	2	2	1.00	11	0.182
90	A	3	3	1.00	11	0.273
91	A	4	3	1.00	11	0.273
92	A	2	2	1.00	13	0.154
93	A	3	3	1.00	13	0.231
94	A	4	3	1.00	13	0.231
95	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	11	0.273
97	A	4	3	1.00	11	0.273
98	A	3	2	1.00	15	0.133
99	A	4	3	1.00	13	0.231
100	A	4	4	1.00	17	0.235
101	A	4	4	1.00	19	0.210
102	A	4	4	1.00	17	0.235
103	A	11	10	1.00	17	0.588
104	A	9	9	1.00	17	0.529
105	A	7	7	1.00	15	0.467
106	A	7	7	1.00	13	0.538
107	A	11	10	1.06	17	0.588
108	A	11	10	0.99	17	0.588
109	A	11	10	1.00	17	0.588
110	A	16	12	1.00	17	0.706
111	A	14	10	1.00	17	0.588
112	A	14	10	1.00	15	0.667
113	A	10	7	1.00	13	0.538
114	A	18	13	1.00	17	0.765
115	A	18	13	1.00	17	0.765
116	A	3	2	1.00	22	0.091
117	A	2	1	1.00	22	0.045
118	A	2	1	1.00	22	0.045
119	A	1	0	1.00	20	0.000
120	A	4	3	1.00	22	0.136
121	A	5	4	1.00	22	0.182
122	A	6	5	1.00	22	0.227
123	A	2	1	1.00	24	0.042
124	A	2	1	1.00	24	0.042
125	A	2	1	1.00	24	0.042
126	A	2	1	1.00	22	0.045
127	A	8	7	1.00	24	0.292
128	A	10	9	1.00	24	0.375
129	A	12	10	1.00	24	0.417
130	A	2	1	1.00	26	0.038

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	1	1.00	26	0.038
132	A	2	1	1.00	26	0.038
133	A	2	1	1.00	24	0.042
134	A	9	8	1.00	26	0.308
135	A	11	10	1.00	26	0.385
136	A	14	5	1.00	46	0.109
137	A	14	5	1.00	46	0.109
138	A	8	3	1.00	46	0.065
139	A	14	5	1.00	44	0.114
140	A	14	5	1.00	42	0.119
141	A	14	5	1.00	46	0.109
142	A	14	5	0.99	46	0.109
143	A	14	6	1.00	26	0.231
144	A	14	6	1.00	26	0.231
145	A	14	6	1.00	26	0.231
146	A	8	4	1.00	26	0.154
147	A	14	6	1.00	24	0.250
148	A	14	6	1.00	22	0.273
149	A	14	6	1.00	26	0.231
150	A	14	6	1.00	26	0.231
151	A	23	7	1.00	26	0.269
152	A	23	7	1.00	26	0.269
153	A	14	6	1.00	26	0.231
154	A	17	5	1.00	26	0.192
155	A	23	7	1.00	26	0.269
156	A	23	7	1.00	26	0.269
157	A	23	7	1.00	26	0.269
158	A	2	1	1.00	52	0.019
159	A	4	3	1.00	52	0.058
160	A	1	1	1.00	18	0.056
161	A	3	3	1.00	23	0.130
162	A	3	3	1.00	23	0.130
163	A	3	3	1.00	29	0.103
164	A	1	1	1.00	18	0.056
165	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	3	1.00	20	0.150
167	A	4	3	1.00	22	0.136
168	A	1	1	1.00	18	0.056
169	A	3	3	1.00	23	0.130
170	A	3	3	1.00	23	0.130
171	A	3	3	1.00	29	0.103
172	A	1	1	1.00	18	0.056
173	A	1	1	1.00	25	0.040
174	C	7	3	4.30	38	0.079
175	A	1	1	1.00	27	0.037
176	A	1	1	1.00	31	0.032
177	A	2	1	1.00	54	0.019
178	A	3	1	1.00	54	0.019
179	A	5	4	1.00	54	0.074
180	A	1	1	1.00	30	0.033
181	A	1	1	1.00	29	0.034
182	A	1	1	1.00	28	0.036
183	A	1	1	1.00	21	0.048
184	A	1	1	1.00	20	0.050
185	A	1	1	1.00	21	0.048
186	A	1	1	1.00	26	0.038
187	A	1	1	1.00	25	0.040
188	A	2	2	1.00	26	0.077
189	A	1	1	1.00	24	0.042
190	A	1	1	1.00	24	0.042
191	A	1	1	1.00	23	0.043
192	A	1	1	1.00	30	0.033
193	A	1	1	1.00	29	0.034
194	A	1	1	1.00	28	0.036
195	A	1	1	1.00	21	0.048
196	A	1	1	1.00	20	0.050
197	A	2	2	1.00	21	0.095
198	A	1	1	1.00	26	0.038
199	A	1	1	1.00	25	0.040
200	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	1	1	1.00	22	0.045
202	A	1	1	1.00	24	0.042
203	A	2	2	1.00	23	0.087
204	A	1	1	1.00	23	0.043
205	A	1	1	1.00	19	0.053
206	A	2	1	1.00	22	0.045
207	A	2	1	1.00	23	0.043
208	A	2	1	1.00	22	0.045
209	A	2	1	1.00	23	0.043
210	A	2	1	1.00	24	0.042
211	A	2	1	1.00	25	0.040
212	A	2	1	1.00	31	0.032
213	A	2	1	1.00	32	0.031
214	A	2	1	1.00	35	0.029
215	A	2	1	1.00	36	0.028
216	A	2	1	1.00	24	0.042
217	A	2	1	1.00	31	0.032
218	A	2	1	1.00	35	0.029
219	A	1	1	1.00	22	0.045
220	A	1	1	1.00	18	0.056
221	B	3	2	2.91	26	0.077
222	B	2	1	2.91	28	0.036
223	A	1	1	1.00	18	0.056
224	A	1	1	1.00	20	0.050
225	A	1	1	1.00	21	0.048
226	A	3	3	1.00	52	0.058
227	A	9	5	1.00	38	0.132
228	A	3	2	1.00	32	0.062
229	A	4	3	1.00	33	0.091
230	A	3	2	1.00	34	0.059
231	A	6	6	1.00	43	0.140
232	A	1	1	1.00	16	0.062
233	B	15	7	2.25	25	0.280
234	A	1	1	1.00	56	0.018
235	A	1	1	1.00	51	0.020

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	1	1	1.00	49	0.020
237	A	1	1	1.00	46	0.022
238	A	2	2	1.00	48	0.042
239	A	1	1	1.00	49	0.020
240	A	1	1	1.00	48	0.021
241	A	1	1	1.00	48	0.021
242	A	10	5	1.00	35	0.143
243	A	10	5	1.00	35	0.143
244	A	10	5	1.00	35	0.143
245	A	10	5	1.00	33	0.152
246	A	10	5	1.00	32	0.156
247	A	13	6	1.00	35	0.171
248	A	13	6	1.00	35	0.171
249	A	13	6	1.00	35	0.171
250	A	13	6	1.00	35	0.171
251	A	13	6	1.00	35	0.171
252	A	11	6	1.00	33	0.182
253	A	9	5	1.00	32	0.156
254	A	13	6	1.00	35	0.171
255	A	13	6	1.00	35	0.171
256	A	13	6	1.00	35	0.171
257	A	2	2	1.00	40	0.050
258	A	6	5	1.00	20	0.250
259	A	3	2	1.00	20	0.100
260	A	3	2	1.00	20	0.100
261	A	2	1	1.00	16	0.062
262	A	5	4	1.00	22	0.182
263	A	3	2	1.00	21	0.095
264	A	6	5	1.00	26	0.192
265	A	2	1	1.00	20	0.050
266	A	2	1	1.00	11	0.091
267	A	4	3	1.00	22	0.136
268	A	3	2	1.00	21	0.095
269	A	3	2	1.00	25	0.080
270	A	6	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	5	5	1.00	31	0.161
272	A	3	2	1.00	21	0.095
273	A	4	3	1.00	33	0.091
274	A	4	4	1.00	14	0.286
275	A	7	6	1.00	33	0.182
276	A	7	6	1.00	29	0.207
277	A	6	5	1.00	44	0.114
278	A	3	2	1.00	15	0.133
279	A	5	4	1.00	15	0.267
280	A	4	3	1.00	18	0.167
281	A	3	2	1.00	20	0.100
282	A	5	4	1.00	26	0.154
283	A	3	2	1.00	13	0.154
284	A	3	2	1.00	18	0.111
285	A	2	1	1.00	26	0.038
286	A	5	5	1.00	19	0.263
287	A	5	5	1.00	24	0.208
288	A	8	6	1.00	20	0.300
289	A	8	6	1.00	18	0.333
290	A	5	3	1.00	19	0.158
291	A	4	3	1.00	13	0.231
292	A	6	5	1.00	22	0.227
293	A	6	5	1.00	24	0.208
294	A	2	1	1.00	29	0.034
295	A	2	1	1.00	30	0.033
296	A	2	1	1.00	19	0.053
297	A	4	4	1.00	16	0.250
298	A	10	5	1.00	36	0.139
299	A	2	1	1.00	21	0.048
300	A	5	4	1.00	16	0.250
301	A	2	1	1.00	24	0.042
302	A	2	1	1.00	21	0.048
303	A	2	1	1.00	24	0.042
304	A	6	5	1.00	26	0.192
305	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	2	1	1.00	29	0.034
307	A	6	5	1.00	20	0.250
308	A	14	10	1.00	32	0.312
309	A	4	4	1.00	23	0.174
310	A	6	5	1.00	26	0.192
311	A	4	3	1.00	26	0.115
312	A	8	4	1.00	25	0.160
313	A	6	3	1.00	23	0.130
314	A	7	6	1.00	23	0.261
315	A	5	3	1.18	20	0.150
316	A	3	2	1.00	25	0.080
317	A	3	2	1.00	22	0.091
318	A	2	1	1.00	24	0.042
319	A	7	6	1.00	24	0.250
320	A	6	5	1.00	43	0.116
321	A	7	5	1.25	50	0.100
322	A	3	2	1.00	16	0.125
323	A	6	5	1.00	15	0.333
324	A	5	4	1.00	20	0.200
325	A	3	2	1.00	24	0.083
326	A	5	3	1.00	27	0.111
327	A	5	5	1.00	26	0.192
328	A	3	2	1.00	16	0.125
329	A	11	8	1.32	22	0.364
330	A	5	4	1.00	24	0.167
331	A	4	3	1.00	26	0.115
332	A	5	3	1.00	36	0.083
333	A	4	3	1.00	26	0.115
334	A	10	9	1.00	20	0.450
335	A	6	6	1.00	27	0.222
336	A	7	7	1.00	20	0.350
337	A	8	7	1.00	25	0.280
338	A	8	7	1.00	22	0.318
339	A	2	1	1.00	18	0.056
340	A	5	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	10	9	1.00	20	0.450
342	A	16	12	1.00	20	0.600
343	A	3	2	1.00	14	0.143
344	A	2	2	1.00	20	0.100
345	A	14	8	1.00	20	0.400
346	A	4	3	1.00	26	0.115
347	A	4	3	1.00	24	0.125
348	A	6	4	1.00	30	0.133
349	A	4	4	1.00	21	0.190
350	A	2	1	1.00	15	0.067
351	A	4	3	1.00	18	0.167
352	A	3	2	1.00	22	0.091
353	A	3	2	1.00	16	0.125
354	A	6	5	1.00	16	0.312
355	A	3	2	1.00	25	0.080
356	A	3	2	1.00	19	0.105
357	A	2	1	1.00	23	0.043
358	A	5	4	1.00	23	0.174
359	A	5	4	1.00	21	0.190
360	A	10	6	1.00	28	0.214
361	A	2	1	1.00	24	0.042
362	A	6	5	1.00	26	0.192
363	A	2	1	1.00	14	0.071
364	A	5	3	1.00	16	0.188
365	A	5	5	1.00	16	0.312
366	A	7	5	1.00	43	0.116
367	A	17	13	1.00	26	0.500
368	A	18	13	1.00	16	0.812
369	A	3	2	1.00	15	0.133
370	A	3	2	1.00	15	0.133
371	A	3	2	1.00	17	0.118
372	A	3	2	1.00	15	0.133
373	A	4	3	1.00	15	0.200
374	A	4	3	1.00	18	0.167
375	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	7	6	1.00	29	0.207
377	A	4	3	1.00	22	0.136
378	A	6	4	1.00	16	0.250
379	A	6	4	1.00	16	0.250
380	A	5	4	1.00	14	0.286
381	A	4	3	1.00	12	0.250
382	A	4	3	1.00	16	0.188
383	A	6	5	1.00	16	0.312
384	A	4	3	1.00	16	0.188
385	A	4	3	1.00	16	0.188
386	A	4	3	1.00	16	0.188
387	A	8	5	1.00	17	0.294
388	A	8	5	1.00	19	0.263
389	A	8	5	1.00	15	0.333
390	A	8	5	1.00	17	0.294
391	A	16	10	1.00	23	0.435
392	A	17	10	1.00	21	0.476
393	F	0	0	N/A	0.	N/A
394	A	15	11	1.00	17	0.647
395	A	13	9	1.00	17	0.529
396	A	13	9	1.00	15	0.600
397	A	9	6	1.00	9	0.667
398	A	17	12	1.00	17	0.706
399	A	17	12	1.00	17	0.706
400	A	17	12	1.00	17	0.706
401	A	16	12	1.00	17	0.706
402	A	14	10	1.00	17	0.588
403	A	14	10	1.00	15	0.667
404	A	10	7	1.00	9	0.778
405	A	31	14	1.00	17	0.824
406	A	31	14	1.00	17	0.824
407	A	31	14	1.00	17	0.824
408	A	15	11	1.00	17	0.647
409	A	15	10	1.00	17	0.588
410	A	15	10	1.00	15	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	11	7	1.00	9	0.778
412	A	46	15	1.00	17	0.882
413	A	46	15	1.00	17	0.882
414	A	46	15	1.00	17	0.882
415	A	4	4	1.00	14	0.286
416	A	5	5	1.00	13	0.385
417	A	4	4	1.00	16	0.250
418	A	5	5	1.00	18	0.278
419	A	3	2	1.00	14	0.143
420	A	4	3	1.00	16	0.188
421	A	2	2	1.00	11	0.182
422	A	1	0	1.00	17	0.000
423	A	1	1	1.00	11	0.091
424	B	1	0	6.23	73	0.000
425	A	11	7	1.00	13	0.538
426	A	13	9	1.00	19	0.474
427	A	3	2	1.00	15	0.133
428	A	3	2	1.00	11	0.182
429	A	1	0	1.00	11	0.000
430	A	1	0	1.00	11	0.000
431	A	5	4	1.00	16	0.250
432	A	2	1	1.00	16	0.062
433	A	4	3	1.00	15	0.200
434	A	1	1	1.00	15	0.067
435	A	1	1	1.00	20	0.050
436	A	3	2	1.00	15	0.133
437	A	6	5	1.00	13	0.385
438	A	1	1	1.00	22	0.045
439	A	3	2	1.00	18	0.111
440	A	3	3	1.00	20	0.150
441	A	3	2	1.00	16	0.125
442	A	6	6	1.00	17	0.353
443	A	1	1	1.00	17	0.059
444	A	4	3	1.00	25	0.120
445	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	3	2	1.00	18	0.111
447	A	6	5	1.00	15	0.333
448	A	2	1	1.00	11	0.091
449	A	5	5	1.00	13	0.385
450	A	3	2	1.00	20	0.100
451	A	2	1	1.00	16	0.062
452	A	4	3	1.00	16	0.188
453	A	2	1	1.00	16	0.062
454	A	1	1	1.00	9	0.111
455	A	2	2	1.00	7	0.286
456	A	2	2	1.00	11	0.182
457	A	1	1	1.00	7	0.143
458	A	1	1	1.00	9	0.111
459	A	1	1	1.00	9	0.111
460	A	2	2	1.00	10	0.200
461	A	2	1	1.00	13	0.077
462	A	2	1	1.00	11	0.091
463	A	2	1	1.00	14	0.071
464	A	4	4	1.00	16	0.250
465	A	2	1	1.00	7	0.143
466	A	2	1	1.00	11	0.091
467	A	5	5	1.00	13	0.385
468	A	5	5	1.00	13	0.385
469	A	5	4	1.00	14	0.286
470	A	2	1	0.78	13	0.077
471	A	3	2	1.00	20	0.100
472	A	4	4	1.00	18	0.222
473	A	2	1	1.00	12	0.083
474	A	2	1	1.00	10	0.100
475	A	3	2	1.00	16	0.125
476	A	2	1	1.00	11	0.091
477	A	1	1	1.00	7	0.143
478	A	2	1	1.00	15	0.067
479	A	2	1	1.00	12	0.083
480	A	3	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	2	1	1.00	11	0.091
482	A	2	1	1.00	17	0.059
483	A	2	1	1.00	29	0.034
484	A	2	1	1.00	18	0.056
485	A	3	2	1.00	14	0.143
486	A	2	1	1.00	24	0.042
487	A	2	1	1.00	11	0.091
488	A	3	2	1.00	18	0.111
489	A	3	3	1.00	15	0.200
490	A	3	3	1.00	16	0.188
491	A	10	7	1.00	15	0.467
492	A	5	2	1.00	50	0.040
493	F	0	0	N/A	0.	N/A
494	F	0	0	N/A	0.	N/A

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{1}{2\sqrt{3}\,b^{3/2}-9bx+9x^3}\,dx$

Optimal. Leaf size=77

$$\frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}$$

```
[Out] -1/27*ln(-x*3^(1/2)+b^(1/2))/b+1/27*ln(x*3^(1/2)+2*b^(1/2))/b+1/9*3^(1/2)/b
^(1/2)/(-3*x+3^(1/2)*b^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2088, 46}

$$\frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/(2
7*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2088

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[1/(3^(3*p)*
a^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d},
 x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{2\sqrt{3}\,b^{3/2}-9bx+9x^3}\,dx = (324b^3)\int \frac{1}{\left(6\sqrt{3}\,b^{3/2}-18bx\right)^2\left(6\sqrt{3}\,b^{3/2}+9bx\right)}\,dx$$

$$= (324b^3)\int \left(\frac{1}{324\sqrt{3}\,b^{7/2}\left(\sqrt{3}\,\sqrt{b}-3x\right)^2}+\frac{1}{2916b^4\left(\sqrt{3}\,\sqrt{b}-3x\right)}+\frac{}{2916}\right.$$

$$= \frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}$$

Mathematica [A]
time = 0.04, size = 143, normalized size = 1.86

$$-\frac{\left(-\sqrt{3}\,\sqrt{b}+3x\right)\left(2\sqrt{3}\,\sqrt{b}+3x\right)\left(3\sqrt{3}\,\sqrt{b}+\left(-\sqrt{3}\,\sqrt{b}+3x\right)\log\left(-\sqrt{3}\,\sqrt{b}+3x\right)+\left(\sqrt{3}\,\sqrt{b}-3x\right)\log\left(2\sqrt{3}\,\sqrt{b}+3x\right)\right)}{81b\left(2\sqrt{3}\,b^{3/2}-9bx+9x^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
[Out] -1/81*((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqrt
[b] + (-(Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]*S
qrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(b*(2*Sqrt[3]*b^(3/2) - 9*b*x
+ 9*x^3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 43, normalized size = 0.56

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(-9b\_Z+9\_Z^3+2b^{\frac{3}{2}}\sqrt{3}\right)}\frac{\ln\left(x-\_R\right)}{3\_R^2-b}\right)}{9}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

Fricas [A]
time = 0.40, size = 76, normalized size = 0.99

$$-\frac{3\sqrt{3}\sqrt{b}\,x-(3\,x^2-b)\log\left(2\sqrt{3}\sqrt{b}+3\,x\right)+(3\,x^2-b)\log\left(-\sqrt{3}\sqrt{b}+3\,x\right)+3\,b}{27\,(3\,bx^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/27*(3*sqrt(3)*sqrt(b)*x - (3*x^2 - b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (3*
x^2 - b)*log(-sqrt(3)*sqrt(b) + 3*x) + 3*b)/(3*b*x^2 - b^2)
```

Sympy [A]
time = 0.13, size = 60, normalized size = 0.78

$$-\frac{3\sqrt{3}}{81\sqrt{b}\,x-27\sqrt{3}\,b}+\frac{-\frac{\log\left(-\frac{\sqrt{3}\sqrt{b}}{3}+x\right)}{27}+\frac{\log\left(\frac{2\sqrt{3}\sqrt{b}}{3}+x\right)}{27}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
```

```
[Out] -3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)/3 + x)/27
 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b
```

Giac [A]
time = 3.28, size = 55, normalized size = 0.71

$$\frac{\log\left(\left|9\sqrt{3}\,x+18\sqrt{b}\right|\right)}{27\,b}-\frac{\log\left(\left|-\sqrt{3}\,x+\sqrt{b}\right|\right)}{27\,b}-\frac{1}{9\left(\sqrt{3}\,x-\sqrt{b}\right)\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="giac")
```

```
[Out] 1/27*log(abs(9*sqrt(3)*x + 18*sqrt(b)))/b - 1/27*log(abs(-sqrt(3)*x + sqrt(
b)))/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))
```

Mupad [B]
time = 0.20, size = 51, normalized size = 0.66

$$\frac{2\sqrt{3}\,\sqrt{27}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{27}}{27}+\frac{2\sqrt{27}\,x}{9\sqrt{b}}\right)}{243\,b}-\frac{\sqrt{3}}{27\sqrt{b}\,\left(x-\frac{\sqrt{3}\,\sqrt{b}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3),x)
[Out] (2*3^(1/2)*27^(1/2)*atanh((3^(1/2)*27^(1/2))/27 + (2*27^(1/2)*x)/(9*b^(1/2)
)))/(243*b) - 3^(1/2)/(27*b^(1/2)*(x - (3^(1/2)*b^(1/2))/3))
```

3.2 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$

Optimal. Leaf size=30

$$\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{1+3p}$$

```
[Out] (a/b+x)*(b^3*(a/b+x)^3)^p/(1+3*p)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2092, 15, 30}

$$\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{3p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

```
[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx = \text{Subst}\left(\int \left(b^3x^3\right)^p\,dx, x, \frac{a}{b}+x\right)$$
$$= \left(\left(\frac{a}{b}+x\right)^{-3p}\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p\right)\text{Subst}\left(\int x^{3p}\,dx, x, \frac{a}{b}+x\right)$$
$$= \frac{(a+bx)\left((a+bx)^3\right)^p}{b(1+3p)}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.77

$$\frac{(a+bx)\left((a+bx)^3\right)^p}{b+3bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
[Out] ((a + b*x)*((a + b*x)^3)^p)/(b + 3*b*p)
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.87

method	result	size
risch	$\frac{(bx+a)\left((bx+a)^3\right)^p}{b(1+3p)}$	26
gosper	$\frac{(bx+a)\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p}{b(1+3p)}$	46
norman	$\frac{x\,{\rm e}^{p\ln\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}}{1+3p}+\frac{a\,{\rm e}^{p\ln\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}}{b(1+3p)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x,method=_RETURNVERBOSE)
[Out] (b*x+a)/b/(1+3*p)*((b*x+a)^3)^p
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.83

$$\frac{(bx+a)(bx+a)^{3\,p}}{b(3\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))
```

Fricas [A]
time = 0.38, size = 43, normalized size = 1.43

$$\frac{(bx+a)(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^p}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{\sqrt[3]{a^3}} & \text{for } b=0 \wedge p=-\frac{1}{3} \\ x(a^3)^p & \text{for } b=0 \\ \int \frac{1}{\sqrt[3]{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx & \text{for } p=-\frac{1}{3} \\ \frac{a(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b}+\frac{bx(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)
```

```
[Out] Piecewise((x/(a**3)**(1/3), Eq(b, 0) & Eq(p, -1/3)), (x*(a**3)**p, Eq(b, 0)
), (Integral((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**(-1/3), x), E
q(p, -1/3)), (a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p +
 b) + b*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b), T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(30) = 60.
time = 3.90, size = 73, normalized size = 2.43

$$\frac{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^pbx+(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)^pa}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")
```

```
[Out] ((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)
```

Mupad [B]
time = 2.13, size = 52, normalized size = 1.73

$$\left(\frac{x}{3\,p+1}+\frac{a}{b\,\left(3\,p+1\right)}\right)\,{\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)
[Out] (x/(3*p + 1) + a/(b*(3*p + 1)))*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p
```

3.3 $\int (a^3+3a^2bx+3ab^2x^2+b^3x^3)^3 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{10}}{10b}$$

```
[Out] 1/10*(b*x+a)^10/b
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2084, 32}

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3 \, dx &= \int (a+bx)^9 \, dx\\ &= \frac{(a+bx)^{10}}{10b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

```
[Out] (a + b*x)^10/(10*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.
time = 0.01, size = 98, normalized size = 7.00

method	result
default	$\frac{1}{10}b^9x^{10} + a\,b^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8b\,x^2 +$
norman	$\frac{1}{10}b^9x^{10} + a\,b^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8b\,x^2 +$
risch	$\frac{1}{10}b^9x^{10} + a\,b^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8b\,x^2 +$
gosper	$\frac{x(b^9x^9+10a\,b^8x^8+45a^2b^7x^7+120a^3b^6x^6+210a^4b^5x^5+252a^5b^4x^4+210a^6b^3x^3+120a^7b^2x^2+45a^8bx+10a^9)}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^9*x^10+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5*x^6+126/5
*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^2+a^9*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(12) = 24$.
time = 0.26, size = 216, normalized size = 15.43

$$\frac{1}{10}\,b^9x^{10} + ab^8x^9 + \frac{27}{8}\,a^2b^7x^8 + \frac{27}{7}\,a^3b^6x^7 + \frac{27}{4}\,a^6b^3x^4 + a^9x + \frac{3}{4}\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2\right)a^6 + \frac{9}{10}\left(5\,b^3x^6 + 18\,ab^2x^5\right)a^4b^2 + \frac{3}{70}\left(10\,b^6x^7 + 70\,ab^5x^6 + 126\,a^2b^4x^5 + 210\,a^4b^2x^3 + 21\left(4\,b^3x^5 + 15\,ab^2x^4\right)a^2b\right)a^3 + \frac{9}{56}\left(7\,b^6x^8 + 48\,ab^5x^7 + 84\,a^2b^4x^6\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 + 27/4*a^6*
b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^6 + 9/10*(5*b
^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6*x^7 + 70*a*b^5*x^6 + 126*a^2*
b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 + 9/56
*(7*b^6*x^8 + 48*a*b^5*x^7 + 84*a^2*b^4*x^6)*a^2*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.
time = 0.34, size = 97, normalized size = 6.93

$$\frac{1}{10}\,b^9x^{10} + ab^8x^9 + \frac{9}{2}\,a^2b^7x^8 + 12\,a^3b^6x^7 + 21\,a^4b^5x^6 + \frac{126}{5}\,a^5b^4x^5 + 21\,a^6b^3x^4 + 12\,a^7b^2x^3 + \frac{9}{2}\,a^8bx^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x
^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 +
a^9*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(8) = 16$.
time = 0.02, size = 107, normalized size = 7.64

$$a^9x + \frac{9a^8bx^2}{2} + 12a^7b^2x^3 + 21a^6b^3x^4 + \frac{126a^5b^4x^5}{5} + 21a^4b^5x^6 + 12a^3b^6x^7 + \frac{9a^2b^7x^8}{2} + ab^8x^9 + \frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**5
*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2 +
 a*b**8*x**9 + b**9*x**10/10
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.
time = 3.27, size = 97, normalized size = 6.93

$$\frac{1}{10}\,b^9x^{10} + ab^8x^9 + \frac{9}{2}\,a^2b^7x^8 + 12\,a^3b^6x^7 + 21\,a^4b^5x^6 + \frac{126}{5}\,a^5b^4x^5 + 21\,a^6b^3x^4 + 12\,a^7b^2x^3 + \frac{9}{2}\,a^8bx^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x
^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 +
a^9*x
```

Mupad [B]
time = 2.07, size = 97, normalized size = 6.93

$$a^9\,x + \frac{9\,a^8\,b\,x^2}{2} + 12\,a^7\,b^2\,x^3 + 21\,a^6\,b^3\,x^4 + \frac{126\,a^5\,b^4\,x^5}{5} + 21\,a^4\,b^5\,x^6 + 12\,a^3\,b^6\,x^7 + \frac{9\,a^2\,b^7\,x^8}{2} + a\,b^8\,x^9 + \frac{b^9\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] a^9*x + (b^9*x^10)/10 + (9*a^8*b*x^2)/2 + a*b^8*x^9 + 12*a^7*b^2*x^3 + 21*a
^6*b^3*x^4 + (126*a^5*b^4*x^5)/5 + 21*a^4*b^5*x^6 + 12*a^3*b^6*x^7 + (9*a^2
*b^7*x^8)/2
```

3.4 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^7}{7b}$$

[Out] 1/7*(b*x+a)^7/b

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2084, 32}

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] (a + b*x)^7/(7*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \, dx &= \int (a+bx)^6 \, dx \\ &= \frac{(a+bx)^7}{7b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.
time = 0.03, size = 65, normalized size = 4.64

method	result	size
default	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$	65
norman	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$	65
risch	$\frac{1}{7}b^6x^7 + a\,b^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5b\,x^2 + a^6x$	65
gosper	$\frac{x(b^6x^6+7a\,b^5x^5+21a^2b^4x^4+35a^3b^3x^3+35a^4b^2x^2+21a^5bx+7a^6)}{7}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2
+a^6*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(12) = 24$.
time = 0.27, size = 99, normalized size = 7.07

$$\frac{1}{7}\,b^6x^7 + ab^5x^6 + \frac{9}{5}\,a^2b^4x^5 + 3\,a^4b^2x^3 + a^6x + \frac{1}{2}\,\left(b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2\right)a^3 + \frac{3}{10}\,\left(4\,b^3x^5 + 15\,ab^2x^4\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b^
3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*a^
2*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.
time = 0.36, size = 64, normalized size = 4.57

$$\frac{1}{7}\,b^6x^7 + ab^5x^6 + 3\,a^2b^4x^5 + 5\,a^3b^3x^4 + 5\,a^4b^2x^3 + 3\,a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(8) = 16$.
time = 0.01, size = 66, normalized size = 4.71

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*
x**5 + a*b**5*x**6 + b**6*x**7/7
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(12) = 24$.
time = 2.96, size = 64, normalized size = 4.57

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

Mupad [B]
time = 0.03, size = 64, normalized size = 4.57

$$a^6\,x + 3\,a^5\,b\,x^2 + 5\,a^4\,b^2\,x^3 + 5\,a^3\,b^3\,x^4 + 3\,a^2\,b^4\,x^5 + a\,b^5\,x^6 + \frac{b^6\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

```
[Out] a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*x
^4 + 3*a^2*b^4*x^5
```

3.5 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$

Optimal. Leaf size=35

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

```
[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi steps

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.37

method	result	size
default	$\frac{(bx+a)^4}{4b}$	13
norman	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32

risch	$a^3x+\frac{3}{2}a^2b\,x^2+a\,b^2x^3+\frac{1}{4}b^3x^4$	32
gosper	$\frac{x\left(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3\right)}{4}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^4/b
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.89

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Fricas [A]
time = 0.37, size = 31, normalized size = 0.89

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [A]
time = 0.01, size = 32, normalized size = 0.91

$$a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

Giac [A]
time = 3.64, size = 31, normalized size = 0.89

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x^3+\frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x,x)
```

```
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

3.6 $\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

```
[Out] -1/2/b/(b*x+a)^2
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2083, 32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx &= \int \frac{1}{(a+bx)^3} \, dx \\ &= -\frac{1}{2b(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

Maple [A]
time = 0.01, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{1}{2b(bx+a)^{2}}$	13
norman	$-\frac{1}{2b(bx+a)^{2}}$	13
gosper	$-\frac{1}{2b(b^{2}x^{2}+2abx+a^{2})}$	24
risch	$-\frac{1}{2b(b^{2}x^{2}+2abx+a^{2})}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(b*x+a)^2
```

Maxima [A]
time = 0.26, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Fricas [A]
time = 0.37, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.07, size = 26, normalized size = 1.86

$$-\frac{1}{2a^{2}b+4ab^{2}x+2b^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Giac [A]
time = 3.60, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(bx+a\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

Mupad [B]
time = 0.03, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
```

```
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

3.7 $\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

```
[Out] -1/5/b/(b*x+a)^5
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2083, 32}

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

```
[Out] -1/5*1/(b*(a + b*x)^5)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx &= \int \frac{1}{(a+bx)^6}\,dx\\ &= -\frac{1}{5b(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
```

```
[Out] -1/5*1/(b*(a + b*x)^5)
```

Maple [A]
time = 0.05, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{1}{5b(bx+a)^5}$	13
norman	$-\frac{1}{5b(bx+a)^5}$	13
risch	$-\frac{1}{5b(b^2x^2+2abx+a^2)^2(bx+a)}$	31
gosper	$-\frac{1}{5(b^2x^2+2abx+a^2)(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)b}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b/(b*x+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.
time = 0.27, size = 57, normalized size = 4.07

$$-\frac{1}{5\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.
time = 0.35, size = 57, normalized size = 4.07

$$-\frac{1}{5\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.
time = 0.15, size = 61, normalized size = 4.36

$$-\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] -1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*
a*b**5*x**4 + 5*b**6*x**5)
```

Giac [A]
time = 3.57, size = 12, normalized size = 0.86

$$-\frac{1}{5\,(bx+a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] -1/5/((b*x + a)^5*b)
```

Mupad [B]
time = 2.05, size = 59, normalized size = 4.21

$$-\frac{1}{5\,a^5\,b+25\,a^4\,b^2\,x+50\,a^3\,b^3\,x^2+50\,a^2\,b^4\,x^3+25\,a\,b^5\,x^4+5\,b^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

```
[Out] -1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 50
*a^2*b^4*x^3)
```

3.8 $\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{8b(a+bx)^8}$$

```
[Out] -1/8/b/(b*x+a)^8
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2083, 32}

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/8*1/(b*(a + b*x)^8)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx &= \int \frac{1}{(a+bx)^9}\,dx\\ &= -\frac{1}{8b(a+bx)^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/8*1/(b*(a + b*x)^8)
```

Maple [A]
time = 0.05, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{1}{8b(bx+a)^8}$	13
norman	$-\frac{1}{8b(bx+a)^8}$	13
risch	$-\frac{1}{8b(b^2x^2+2abx+a^2)^3(bx+a)^2}$	31
gosper	$-\frac{1}{8(b^2x^2+2abx+a^2)(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)^2b}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b/(b*x+a)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(12) = 24.
time = 0.27, size = 90, normalized size = 6.43

$$-\frac{1}{8\left(b^9x^8+8\,ab^8x^7+28\,a^2b^7x^6+56\,a^3b^6x^5+70\,a^4b^5x^4+56\,a^5b^4x^3+28\,a^6b^3x^2+8\,a^7b^2x+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(12) = 24.
time = 0.38, size = 90, normalized size = 6.43

$$-\frac{1}{8\left(b^9x^8+8\,ab^8x^7+28\,a^2b^7x^6+56\,a^3b^6x^5+70\,a^4b^5x^4+56\,a^5b^4x^3+28\,a^6b^3x^2+8\,a^7b^2x+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.
time = 0.23, size = 97, normalized size = 6.93

$$-\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] -1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**4*x**3 + 5
60*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*x**6 + 64*a*b**8*x**
7 + 8*b**9*x**8)
```

Giac [**A**]
time = 3.39, size = 12, normalized size = 0.86

$$-\frac{1}{8\,(bx+a)^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] -1/8/((b*x + a)^8*b)
```

Mupad [**B**]
time = 2.07, size = 92, normalized size = 6.57

$$-\frac{1}{8\,a^8\,b+64\,a^7\,b^2\,x+224\,a^6\,b^3\,x^2+448\,a^5\,b^4\,x^3+560\,a^4\,b^5\,x^4+448\,a^3\,b^6\,x^5+224\,a^2\,b^7\,x^6+64\,a\,b^8\,x^7+8\,b^9\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] -1/(8*a^8*b + 8*b^9*x^8 + 64*a^7*b^2*x + 64*a*b^8*x^7 + 224*a^6*b^3*x^2 + 4
48*a^5*b^4*x^3 + 560*a^4*b^5*x^4 + 448*a^3*b^6*x^5 + 224*a^2*b^7*x^6)
```

3.9 $\int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^3 \, dx$

Optimal. Leaf size=84

$$-\frac{b^3(b^2-3ac)^3 x}{c^3}+\frac{3b^2(b^2-3ac)^2 (b+cx)^4}{4c^4}-\frac{3b(b^2-3ac)(b+cx)^7}{7c^4}+\frac{(b+cx)^{10}}{10c^4}$$

```
[Out] -b^3*(-3*a*c+b^2)^3*x/c^3+3/4*b^2*(-3*a*c+b^2)^2*(c*x+b)^4/c^4-3/7*b*(-3*a*
c+b^2)*(c*x+b)^7/c^4+1/10*(c*x+b)^10/c^4
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2085, 200}

$$-\frac{3b(b^2-3ac)(b+cx)^7}{7c^4}+\frac{3b^2(b^2-3ac)^2 (b+cx)^4}{4c^4}-\frac{b^3x(b^2-3ac)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)^4)/(4*c^4
) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)^10/(10*c^4)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2085

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + c^2*(x^3/b), x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\int (3ab+3b^2x+3bcx^2+c^2x^3)^3 \, dx = \frac{1}{27}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^3 dx, x, \frac{b}{c}+x\right)$$

$$= \frac{1}{27}\text{Subst}\left(\int \left(\frac{27(-b^3+3abc)^3}{c^3}+81(b^3-3abc)^2 x^3-81bc^3(b^2-3a\right.\right.$$

$$= -\frac{b^3(b^2-3ac)^3 x}{c^3}+\frac{3b^2(b^2-3ac)^2 (b+cx)^4}{4c^4}-\frac{3b(b^2-3ac)(b+cx)^7}{7c^4}$$

Mathematica [A]
time = 0.02, size = 159, normalized size = 1.89

$$27a^3b^3x+\frac{81}{2}a^2b^4x^2+27ab^3\left(b^2+ac\right)x^3+\frac{27}{4}b^2\left(b^4+6ab^2c+a^2c^2\right)x^4+\frac{27}{5}b^3c\left(3b^2+5ac\right)x^5+9b^2c^2\left(2b^2+ac\right)x^6+\frac{9}{7}bc^3\left(9b^2+ac\right)x^7+\frac{9}{2}b^2c^4x^8+bc^5x^9+\frac{c^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] 27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^4
 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2*c
^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)*x^7)/7 + (9*b^2*c^4*x^8)/2 +
b*c^5*x^9 + (c^6*x^10)/10
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(78) = 156.
time = 0.03, size = 295, normalized size = 3.51

method	result
norman	$\frac{c^6x^{10}}{10}+b\,c^5x^9+\frac{9b^2c^4x^8}{2}+\left(\frac{9}{7}ab\,c^4+\frac{81}{7}b^3c^3\right)x^7+\left(9a\,b^2c^3+18b^4c^2\right)x^6+\left(27a\,b^3c^2+\frac{81}{5}b^5c\right)x^5+($
gosper	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x^5b^5c+$
risch	$\frac{1}{10}c^6x^{10}+b\,c^5x^9+\frac{9}{2}b^2c^4x^8+\frac{9}{7}x^7ab\,c^4+\frac{81}{7}x^7b^3c^3+9a\,b^2c^3x^6+18b^4c^2x^6+27x^5a\,b^3c^2+\frac{81}{5}x^5b^5c+$
default	$\frac{c^6x^{10}}{10}+b\,c^5x^9+\frac{9b^2c^4x^8}{2}+\frac{\left(3ab\,c^4+63b^3c^3+c^2\left(6ab\,c^2+18b^3c\right)\right)x^7}{7}+\frac{\left(18a\,b^2c^3+45b^4c^2+3bc\left(6ab\,c^2+18b^3c\right)+c^2\left(18a\,b^2c+9b^4\right.\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+1/7*(3*a*b*c^4+63*b^3*c^3+c^2*(6*a*
b*c^2+18*b^3*c))*x^7+1/6*(18*a*b^2*c^3+45*b^4*c^2+3*b*c*(6*a*b*c^2+18*b^3*c
)+c^2*(18*a*b^2*c+9*b^4))*x^6+1/5*(63*a*b^3*c^2+3*b^2*(6*a*b*c^2+18*b^3*c)+
3*b*c*(18*a*b^2*c+9*b^4))*x^5+1/4*(3*a*b*(6*a*b*c^2+18*b^3*c)+3*b^2*(18*a*b
^2*c+9*b^4)+54*a*b^4*c+9*a^2*b^2*c^2)*x^4+1/3*(3*a*b*(18*a*b^2*c+9*b^4)+54*
a*b^5+27*a^2*b^3*c)*x^3+81/2*a^2*b^4*x^2+27*a^3*b^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(78) = 156.
time = 0.27, size = 204, normalized size = 2.43

$$\frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{27}{8}\,b^2c^4x^8+\frac{27}{7}\,b^3c^3x^7+\frac{27}{4}\,b^6x^4+27\,a^3b^3x+\frac{27}{4}\,\left(c^2x^4+4\,bcx^3+6\,b^2x^2\right)a^2b^2+\frac{9}{10}\,\left(5\,c^2x^6+18\,bcx^5\right)b^4+\frac{9}{70}\,\left(10\,c^4x^7+70\,bc^3x^6+126\,b^2c^2x^5+210\,b^4x^3+21\,\left(4\,c^2x^5+15\,bcx^4\right)b^2\right)ab+\frac{9}{56}\,\left(7\,c^4x^8+48\,bc^3x^7+84\,b^2c^2x^6\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6*
x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/10*
```

```
(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2*c^
2*x^5 + 210*b^4*x^3 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^
8 + 48*b*c^3*x^7 + 84*b^2*c^2*x^6)*b^2
```

Fricas [A]
time = 0.36, size = 156, normalized size = 1.86

$$\frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{9}{2}\,b^2c^4x^8+\frac{81}{2}\,a^2b^4x^2+\frac{9}{7}\,(9\,b^3c^3+abc^4)x^7+27\,a^3b^3x+9\,(2\,b^4c^2+ab^2c^3)x^6+\frac{27}{5}\,(3\,b^5c+5\,ab^3c^2)x^5+\frac{27}{4}\,(b^6+6\,ab^4c+a^2b^2c^2)x^4+27\,(ab^5+a^2b^3c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/2*a^2*b^4*x^2 + 9/7*(9*b^3
*c^3 + a*b*c^4)*x^7 + 27*a^3*b^3*x + 9*(2*b^4*c^2 + a*b^2*c^3)*x^6 + 27/5*(
3*b^5*c + 5*a*b^3*c^2)*x^5 + 27/4*(b^6 + 6*a*b^4*c + a^2*b^2*c^2)*x^4 + 27*
(a*b^5 + a^2*b^3*c)*x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(80) = 160.
time = 0.02, size = 175, normalized size = 2.08

$$27a^3b^3x+\frac{81a^2b^4x^2}{2}+\frac{9b^2c^4x^8}{2}+bc^5x^9+\frac{c^6x^{10}}{10}+x^7\cdot\left(\frac{9abc^4}{7}+\frac{81b^3c^3}{7}\right)+x^6\cdot\left(9ab^2c^3+18b^4c^2\right)+x^5\cdot\left(27ab^3c^2+\frac{81b^5c}{5}\right)+x^4\cdot\left(\frac{27a^2b^2c^2}{4}+\frac{81ab^4c}{2}+\frac{27b^6}{4}\right)+x^3\cdot\left(27a^2b^3c+27ab^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] 27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 + c
**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3 +
 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**2*c
**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.
time = 5.91, size = 166, normalized size = 1.98

$$\frac{1}{10}\,c^6x^{10}+bc^5x^9+\frac{9}{2}\,b^2c^4x^8+\frac{81}{7}\,b^3c^3x^7+\frac{9}{7}\,abc^4x^7+18\,b^4c^2x^6+9\,ab^2c^3x^6+\frac{81}{5}\,b^5cx^5+27\,ab^3c^2x^5+\frac{27}{4}\,b^6x^4+\frac{81}{2}\,ab^4cx^4+\frac{27}{4}\,a^2b^2c^2x^4+27\,ab^5x^3+27\,a^2b^3cx^3+\frac{81}{2}\,a^2b^4x^2+27\,a^3b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c^
4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*x^
5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^3 +
 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x
```

Mupad [B]
time = 2.08, size = 149, normalized size = 1.77

$$x^4\left(\frac{27\,a^2\,b^2\,c^2}{4}+\frac{81\,a\,b^4\,c}{2}+\frac{27\,b^6}{4}\right)+\frac{c^6\,x^{10}}{10}+27\,a^3\,b^3\,x+b\,c^5\,x^9+\frac{81\,a^2\,b^4\,x^2}{2}+\frac{9\,b^2\,c^4\,x^8}{2}+9\,b^2\,c^2\,x^6\,\left(2\,b^2+a\,c\right)+27\,a\,b^3\,x^3\,\left(b^2+a\,c\right)+\frac{27\,b^3\,c\,x^5\,\left(3\,b^2+5\,a\,c\right)}{5}+\frac{9\,b\,c^3\,x^7\,\left(9\,b^2+a\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] x^4*((27*b^6)/4 + (27*a^2*b^2*c^2)/4 + (81*a*b^4*c)/2) + (c^6*x^10)/10 + 27
*a^3*b^3*x + b*c^5*x^9 + (81*a^2*b^4*x^2)/2 + (9*b^2*c^4*x^8)/2 + 9*b^2*c^2
*x^6*(a*c + 2*b^2) + 27*a*b^3*x^3*(a*c + b^2) + (27*b^3*c*x^5*(5*a*c + 3*b^
2))/5 + (9*b*c^3*x^7*(a*c + 9*b^2))/7
```

3.10 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{b^2\left(b^2-3ac\right)^2x}{c^2}-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{(b+cx)^7}{7c^3}$$

```
[Out] b^2*(-3*a*c+b^2)^2*x/c^2-1/2*b*(-3*a*c+b^2)*(c*x+b)^4/c^3+1/7*(c*x+b)^7/c^3
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2085, 200}

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b + 
c*x)^7/(7*c^3)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2085

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + c^2*(x^3/b), x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 \, dx &= \frac{1}{9}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^2 \, dx, x, \frac{b}{c}+x\right)\\
&= \frac{1}{9}\text{Subst}\left(\int \left(\frac{9\left(-b^3+3abc\right)^2}{c^2}-18bc\left(b^2-3ac\right)x^3+9c^4x^6\right) dx, x, \frac{b}{c}\right.\\
&= \frac{b^2\left(b^2-3ac\right)^2x}{c^2}-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{(b+cx)^7}{7c^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.46

$$9a^2b^2x + 9ab^3x^2 + 3b^2(b^2 + 2ac)x^3 + \frac{3}{2}bc(3b^2 + ac)x^4 + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] 9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)*
x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7
```

Maple [A]
time = 0.01, size = 84, normalized size = 1.50

method	result	size
norman	$\frac{c^4x^7}{7} + b\,c^3x^6 + 3b^2c^2x^5 + \left(\frac{3}{2}ab\,c^2 + \frac{9}{2}b^3c\right)x^4 + \left(6a\,b^2c + 3b^4\right)x^3 + 9a\,b^3x^2 + 9a^2b^2x$	82
gosper	$\frac{1}{7}c^4x^7 + b\,c^3x^6 + 3b^2c^2x^5 + \frac{3}{2}x^4ab\,c^2 + \frac{9}{2}x^4b^3c + 6a\,b^2c\,x^3 + 3b^4x^3 + 9a\,b^3x^2 + 9a^2b^2x$	84
default	$\frac{c^4x^7}{7} + b\,c^3x^6 + 3b^2c^2x^5 + \frac{(6ab\,c^2+18b^3c)x^4}{4} + \frac{(18a\,b^2c+9b^4)x^3}{3} + 9a\,b^3x^2 + 9a^2b^2x$	84
risch	$\frac{1}{7}c^4x^7 + b\,c^3x^6 + 3b^2c^2x^5 + \frac{3}{2}x^4ab\,c^2 + \frac{9}{2}x^4b^3c + 6a\,b^2c\,x^3 + 3b^4x^3 + 9a\,b^3x^2 + 9a^2b^2x$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c^4*x^7+b*c^3*x^6+3*b^2*c^2*x^5+1/4*(6*a*b*c^2+18*b^3*c)*x^4+1/3*(18*a*
b^2*c+9*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x
```

Maxima [A]
time = 0.26, size = 93, normalized size = 1.66

$$\frac{1}{7}c^4x^7 + bc^3x^6 + \frac{9}{5}b^2c^2x^5 + 3b^4x^3 + 9a^2b^2x + \frac{3}{2}(c^2x^4 + 4bcx^3 + 6b^2x^2)ab + \frac{3}{10}(4c^2x^5 + 15bcx^4)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*(
c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2
```

Fricas [A]
time = 0.35, size = 80, normalized size = 1.43

$$\frac{1}{7}c^4x^7 + bc^3x^6 + 3b^2c^2x^5 + 9ab^3x^2 + 9a^2b^2x + \frac{3}{2}(3b^3c + abc^2)x^4 + 3(b^4 + 2ab^2c)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9*a*b^3*x^2 + 9*a^2*b^2*x + 3/2*(
3*b^3*c + a*b*c^2)*x^4 + 3*(b^4 + 2*a*b^2*c)*x^3
```

Sympy [A]
time = 0.02, size = 87, normalized size = 1.55

$$9a^2b^2x + 9ab^3x^2 + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7} + x^4 \cdot \left(\frac{3abc^2}{2} + \frac{9b^3c}{2}\right) + x^3 \cdot \left(6ab^2c + 3b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7/
7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)
```

Giac [A]
time = 5.71, size = 83, normalized size = 1.48

$$\frac{1}{7}\,c^4x^7 + bc^3x^6 + 3\,b^2c^2x^5 + \frac{9}{2}\,b^3cx^4 + \frac{3}{2}\,abc^2x^4 + 3\,b^4x^3 + 6\,ab^2cx^3 + 9\,ab^3x^2 + 9\,a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4 +
 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x
```

Mupad [B]
time = 0.04, size = 79, normalized size = 1.41

$$x^3\,\left(3\,b^4 + 6\,a\,c\,b^2\right) + \frac{c^4\,x^7}{7} + 9\,a^2\,b^2\,x + 9\,a\,b^3\,x^2 + b\,c^3\,x^6 + 3\,b^2\,c^2\,x^5 + \frac{3\,b\,c\,x^4\,\left(3\,b^2 + a\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] x^3*(3*b^4 + 6*a*b^2*c) + (c^4*x^7)/7 + 9*a^2*b^2*x + 9*a*b^3*x^2 + b*c^3*x
^6 + 3*b^2*c^2*x^5 + (3*b*c*x^4*(a*c + 3*b^2))/2
```

3.11 $\int (3ab+3b^2x+3bcx^2+c^2x^3) \, dx$

Optimal. Leaf size=32

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4

Rubi steps

$$\int (3ab+3b^2x+3bcx^2+c^2x^3) \, dx = 3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$3abx+\frac{3b^2x^2}{2}+bcx^3+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4

Maple [A]
time = 0.01, size = 29, normalized size = 0.91

method	result	size
gosper	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29
default	$3abx+\frac{3}{2}b^2x^2+bc\,x^3+\frac{1}{4}c^2x^4$	29

norman	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29
risch	$3abx + \frac{3}{2}b^2x^2 + bc\,x^3 + \frac{1}{4}c^2x^4$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

Maxima [A]
time = 0.26, size = 28, normalized size = 0.88

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Fricas [A]
time = 0.35, size = 28, normalized size = 0.88

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Sympy [A]
time = 0.01, size = 31, normalized size = 0.97

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)
```

```
[Out] 3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4
```

Giac [A]
time = 3.95, size = 28, normalized size = 0.88

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.88

$$\frac{3\,b^2\,x^2}{2}+b\,c\,x^3+3\,a\,b\,x+\frac{c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2,x)
```

```
[Out] (3*b^2*x^2)/2 + (c^2*x^4)/4 + 3*a*b*x + b*c*x^3
```

3.12 $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\,(\ldots\right.}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

```
[Out] 1/3*ln(b-b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/6*ln(
b^(2/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)
^2)/b^(2/3)/(-3*a*c+b^2)^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^
2)^(1/3))/b^(1/3)*3^(1/2))/b^(2/3)/(-3*a*c+b^2)^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2092, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))]/(
Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3)
 + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) +
 b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2
 - 3*a*c)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,dx, x, \frac{b}{c}+x\right)$$

$$= \frac{c^{2/3}\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}\,dx, x, \frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{c^{2/3}\text{Subst}\left(\int \frac{}{b^{2/3}(}\right.}{}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[3]{c}\sqrt[3]{b^2-}}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\sqrt[3]{c}}\right.}{6b^{2/3}\left(b^2\right.}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b}\right.}{6b^{2/3}\left(b^2-\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 63, normalized size = 0.34

$$\frac{1}{3}\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&, \frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b
*c*#1 + c^2*#1^2) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 57, normalized size = 0.30

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^2c^2+2\_Rbc+b^2}\right)}{3}$	57

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^2c^2+2\_Rbc+b^2}\right)}{3}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*
_Z*b^2+3*a*b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")
```

```
[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(151) = 302$.
time = 0.39, size = 387, normalized size = 2.06

$$-\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{6}}\left(b^3-3abc\right)\arctan\left(\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)+\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)}{3\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{5}{6}}}\right)+\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\log\left(-b^5+3ab^3c-\left(b^3c^2-3abc^3\right)x^2-2\left(b^4c-3ab^2c^2\right)x-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)\right)-2\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\log\left(-b^4+3ab^2c-\left(b^3c-3abc^2\right)x+\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\right)}{6\left(b^6-6ab^4c+9a^2b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(b^3 - 3*a*b*c)*arc
tan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt
(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4
*c + 9*a^2*b^2*c^2)^(5/6)) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b
^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*
b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)
*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^
2*c^2)^(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)
```

Sympy [A]
time = 0.19, size = 53, normalized size = 0.28

$$\mathrm{RootSum}\left(t^3\cdot\left(243a^2b^2c^2-162ab^4c+27b^6\right)-1,\left(t\mapsto t\log\left(x+\frac{9tabc-3tb^3+b}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

```
[Out] RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t,
 _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))
```

Giac [A]
time = 3.90, size = 212, normalized size = 1.13

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,cx+\sqrt{3}\,b-\sqrt{3}\,(-b^3+3\,abc)^{\frac{1}{3}}}{cx+b+(-b^3+3\,abc)^{\frac{1}{3}}}\right)}{3\,(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}\,cx+\sqrt{3}\,b-\sqrt{3}\,(-b^3+3\,abc)^{\frac{1}{3}}\right)^2+4\left(cx+b+(-b^3+3\,abc)^{\frac{1}{3}}\right)^2\right)}{6\,(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{3}}}+\frac{\log\left(\left|cx+b+(-b^3+3\,abc)^{\frac{1}{3}}\right|\right)}{3\,(b^6-6\,ab^4c+9\,a^2b^2c^2)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3
))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1
/3) - 1/6*log(4*(sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^
2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^
2)^(1/3) + 1/3*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c
+ 9*a^2*b^2*c^2)^(1/3)
```

Mupad [B]
time = 0.49, size = 174, normalized size = 0.93

$$\frac{\ln\left(b+b^{1/3}\,(3\,a\,c-b^2)^{1/3}+c\,x\right)}{3\,b^{2/3}\,(3\,a\,c-b^2)^{2/3}}+\frac{\ln\left(3\,b\,c^3+3\,c^4\,x+\frac{3\,b^{1/3}\,c^3\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(3\,a\,c-b^2)^{1/3}}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{2/3}\,(3\,a\,c-b^2)^{2/3}}-\frac{\ln\left(3\,b\,c^3+3\,c^4\,x-\frac{3\,b^{1/3}\,c^3\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(3\,a\,c-b^2)^{1/3}}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,b^{2/3}\,(3\,a\,c-b^2)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)
```

```
[Out] log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x)/(3*b^(2/3)*(3*a*c - b^2)^(2/3))
+ (log(3*b*c^3 + 3*c^4*x + (3*b^(1/3)*c^3*(3^(1/2)*1i - 1)*(3*a*c - b^2)^(1
/3))/2)*(3^(1/2)*1i - 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3)) - (log(3*b*c^3 +
3*c^4*x - (3*b^(1/3)*c^3*(3^(1/2)*1i + 1)*(3*a*c - b^2)^(1/3))/2)*(3^(1/2)*
1i + 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3))
```

3.13 $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{2c\tan^{-1}\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}\,b^{5/3}\left(b^2-3ac\right)^{5/3}}-\frac{2c\log\left(b-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+c\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

```
[Out] -1/3*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)-2/9*c*ln(b-
b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(5/3)/(-3*a*c+b^2)^(5/3)+1/9*c*ln(b^(2/3)
*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)^2)/b^(
5/3)/(-3*a*c+b^2)^(5/3)+2/9*c*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^2)^(1
/3))/b^(1/3)*3^(1/2))/b^(5/3)/(-3*a*c+b^2)^(5/3)*3^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2092, 205, 206, 31, 648, 631, 210, 642}

$$\frac{2c\text{ArcTan}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}\,b^{5/3}\left(b^2-3ac\right)^{5/3}}-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}-\frac{2c\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -1/3*(c*(b/c + x))/(b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)
) + (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1
/3))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2*c*Log[b - b^(1/3)*(b^2
- 3*a*c)^(1/3) + c*x])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^
2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^
2])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^2}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{(2c)\text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c \ldots}\right.}{3b\left(b^2-3 \ldots\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{\left(2c^{5/3}\right)\text{Subst}\left(\int \frac{\ldots}{-\sqrt[3]{b} \ldots}\right.}{9b^{5/3} \ldots}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2 \ldots}\right.\right.}{9b^{5/3}\left(b^2-3a \ldots\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2 \ldots}\right.\right.}{9b^{5/3}\left(b^2-3a \ldots\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{2c\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2- \ldots}}}{\sqrt{3}}\right.}{3\sqrt{3}\,b^{5/3}\left(b^2-3ac \ldots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 112, normalized size = 0.46

$$-\frac{\frac{3(b+cx)}{3ab+x(3b^2+3bcx+c^2x^2)}+2c\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&,\frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]}{9\left(b^3-3abc\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -1/9*((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[3
*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c
^2*#1^2) & ])/(b^3 - 3*a*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 134, normalized size = 0.55

method	result	size
default	$\frac{\frac{cx}{9b\left(3ac-b^2\right)}+\frac{1}{27ac-9b^2}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{2c\left(\sum_{\_R=\mathrm{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\_R^2c^2+2\_Rbc+b^2}\right)}{9b\left(3ac-b^2\right)}$	134
risch	$\frac{\frac{cx}{9b\left(3ac-b^2\right)}+\frac{1}{27ac-9b^2}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{2c\left(\sum_{\_R=\mathrm{RootOf}\left(c^2\_Z^3+3b\_Z^2c+3b^2\_Z+3ab\right)}\frac{\ln\left(x-\_R\right)}{\left(3ac-b^2\right)\left(\_R^2c^2+2\_Rbc+b^2\right)}\right)}{9b}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/9*c/b/(3*a*c-b^2)*x+1/9/(3*a*c-b^2))/(1/3*c^2*x^3+b*c*x^2+b^2*x+a*b)+2/9
*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+
3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] -2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*b
*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*
(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(204) = 408.
time = 0.39, size = 704, normalized size = 2.87

$$-\frac{3b^7-18ab^5c+27a^2b^3c^2-2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{6}}\left(3ab^4c-9a^2b^2c^2+\left(b^3c^3-3abc^4\right)x^3+3\left(b^4c^2-3ab^2c^3\right)x^2+3\left(b^5c-3ab^3c^2\right)x\right)\arctan\left(\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)+\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)}{3\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{5}{6}}}\right)-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\left(c^3x^3+3bc^2x^2+3b^2cx+3abc\right)\log\left(-b^5+3ab^3c-\left(b^3c^2-3abc^3\right)x^2-2\left(b^4c-3ab^2c^2\right)x-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)\right)+2\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\left(c^3x^3+3bc^2x^2+3b^2cx+3abc\right)\log\left(-b^4+3ab^2c-\left(b^3c-3abc^2\right)x+\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\right)+3\left(b^6c-6ab^4c^2+9a^2b^2c^3\right)x}{9\left(3ab^{10}-27a^2b^8c+81a^3b^6c^2-81a^4b^4c^3+\left(b^9c^2-9ab^7c^3+27a^2b^5c^4-27a^3b^3c^5\right)x^3+3\left(b^{10}c-9ab^8c^2+27a^2b^6c^3-27a^3b^4c^4\right)x^2+3\left(b^{11}-9ab^9c+27a^2b^7c^2-27a^3b^5c^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(1/6)*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 +
 3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*arctan(1/3*(2*s
qrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(3)*(b^6 - 6
*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*a^2*b
^2*c^2)^(5/6)) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2
*x^2 + 3*b^2*c*x + 3*a*b*c)*log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^
```

```
2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*
x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) + 2*(b^6
- 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a
*b*c)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a
^2*b^2*c^2)^(2/3)) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)/(3*a*b^10 -
 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9*a*b^7*c^3 +
27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^10*c - 9*a*b^8*c^2 + 27*a^2*b^6
*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^11 - 9*a*b^9*c + 27*a^2*b^7*c^2 - 27*a^3*
b^5*c^3)*x)
```

Sympy [A]
time = 0.66, size = 192, normalized size = 0.78

$$\frac{b+cx}{27a^2b^2c-9ab^4+x^3\cdot(9abc^3-3b^3c^2)+x^2\cdot(27ab^2c^2-9b^4c)+x\,(27ab^3c-9b^5)}+\operatorname{RootSum}\left(t^3\cdot(177147a^5b^5c^5-295245a^4b^7c^4+196830a^3b^9c^3-65610a^2b^{11}c^2+10935ab^{13}c-729b^{15})-8c^3,\left(t\mapsto t\log\left(x+\frac{81ta^2b^2c^2-54tab^4c+9tb^6+2bc}{2c^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] (b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x*
*2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + RootSum(_t**3*
(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 - 65
610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, Lambda(_t, _t*
log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2
))))
```

Giac [A]
time = 4.07, size = 289, normalized size = 1.18

$$-\frac{2\sqrt{3}\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}}{cx+b+(-b^3+3abc)^{\frac{1}{3}}}\right)-\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}\right)^2+4\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)^2\right)+2\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(\left|cx+b+(-b^3+3abc)^{\frac{1}{3}}\right|\right)}{9(b^3-3abc)}-\frac{cx+b}{3(c^2x^3+3bcx^2+3b^2x+3ab)(b^3-3abc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^3 - 3*a*b*c) - 1/3*(c*x + b)/((c
^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)*(b^3 - 3*a*b*c))
```

Mupad [B]
time = 2.65, size = 247, normalized size = 1.01

$$\frac{\frac{1}{3(3ac-b^2)}+\frac{cx}{3b(3ac-b^2)}}{3b^2x+3bcx^2+3ab+c^2x^3}+\frac{2c\ln\left(b+b^{1/3}(3ac-b^2)^{1/3}+cx\right)}{9b^{5/3}(3ac-b^2)^{5/3}}-\frac{\ln\left(2b-b^{1/3}(3ac-b^2)^{1/3}+2cx-\sqrt{3}\,b^{1/3}(3ac-b^2)^{1/3}\,1\mathrm{i}\right)\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{9b^{5/3}(3ac-b^2)^{5/3}}-\frac{\ln\left(2b-b^{1/3}(3ac-b^2)^{1/3}+2cx+\sqrt{3}\,b^{1/3}(3ac-b^2)^{1/3}\,1\mathrm{i}\right)\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{9b^{5/3}(3ac-b^2)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] (1/(3*(3*a*c - b^2)) + (c*x)/(3*b*(3*a*c - b^2)))/(3*a*b + 3*b^2*x + c^2*x^
3 + 3*b*c*x^2) + (2*c*log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x))/(9*b^(5/3
)*(3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x - 3
^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c + 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*
c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x + 3^(1/2)*
b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c - 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*c - b^2
)^(5/3))
```

3.14 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

Optimal. Leaf size=305

$$-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{5c^2\tan^{-1}\left(\frac{\sqrt[3]{b}}{\cdots}\right)}{9\sqrt{3}\,b^{8/3}\,(b\cdots}$$

```
[Out] -1/6*c*(b/c+x)/b/(-3*a*c+b^2)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/18*c^2*
(b/c+x)/b^2/(-3*a*c+b^2)^2/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+5/27*c^2*ln(b-
b^(1/3)*(-3*a*c+b^2)^(1/3)+c*x)/b^(8/3)/(-3*a*c+b^2)^(8/3)-5/54*c^2*ln(b^(2
/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*c*(-3*a*c+b^2)^(1/3)*(b/c+x)+c^2*(b/c+x)^2)/
b^(8/3)/(-3*a*c+b^2)^(8/3)-5/27*c^2*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b
^2)^(1/3))/b^(1/3)*3^(1/2))/b^(8/3)/(-3*a*c+b^2)^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2092, 205, 206, 31, 648, 631, 210, 642}

$$-\frac{5c^2\,\text{ArcTan}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}\,b^{8/3}\left(b^2-3ac\right)^{8/3}}+\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}}-\frac{5c^2\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{54b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] -1/6*(c*(b/c + x))/(b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)
^2) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^
2 + c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))
/(Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b
 - b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (
5*c^2*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c
+ x) + c^2*(b/c + x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^3}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} - \frac{(5c)\text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)\right.}\right.}{6b\left(b^2-\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b-}{18b^2\left(b^2-3ac\right)^2(3ab+}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b-}{18b^2\left(b^2-3ac\right)^2(3ab+}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b-}{18b^2\left(b^2-3ac\right)^2(3ab+}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b-}{18b^2\left(b^2-3ac\right)^2(3ab+}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2} + \frac{5c(b-}{18b^2\left(b^2-3ac\right)^2(3ab+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.06, size = 149, normalized size = 0.49

$$\frac{-\frac{3(b+cx)(3b^3-15b^2cx-5c^3x^3-3bc(8a+5cx^2))}{(3ab+x(3b^2+3bcx+c^2x^2))^2}+10c^2\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&,\frac{\log(x-\#1)}{b^2+2bc\#1+c^2\#1^2}\&\right]}{54\left(b^3-3abc\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(3
*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1 +
```

```
 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) & ])/(54
*(b^3 - 3*a*b*c)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 279, normalized size = 0.91

method	result
risch	$\frac{\frac{5c^4x^4}{18\left(9a^2c^2-6a\,b^2c+b^4\right)b^2}+\frac{10c^3x^3}{9b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{5c^2x^2}{3\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{2\left(2ac+b^2\right)cx}{3b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{9\left(8ac-b^2\right)}{486a^2c^2-324a\,b^2c+54b^4}}{\left(c^2x^3+3bc\,x^2+3b^2x+3ab\right)^2}+\frac{5c^2\left(\sum_{\_R=\mathrm{R}} \right.}{}$
default	$\frac{\frac{5c^4x^4}{18\left(9a^2c^2-6a\,b^2c+b^4\right)b^2}+\frac{10c^3x^3}{9b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{5c^2x^2}{3\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{2\left(2ac+b^2\right)cx}{3b\left(9a^2c^2-6a\,b^2c+b^4\right)}+\frac{9\left(8ac-b^2\right)}{486a^2c^2-324a\,b^2c+54b^4}}{\left(c^2x^3+3bc\,x^2+3b^2x+3ab\right)^2}+\frac{5c^2\left(\sum_{\_R=\mathrm{R}} \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 9*(5/162*c^4/(9*a^2*c^2-6*a*b^2*c+b^4)/b^2*x^4+10/81/b*c^3/(9*a^2*c^2-6*a*b
^2*c+b^4)*x^3+5/27*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/27*(2*a*c+b^2)*c/b/(
9*a^2*c^2-6*a*b^2*c+b^4)*x+1/54*(8*a*c-b^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(c^2
*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2+5/3*c^2/(81*a^2*c^2-54*a*b^2*c+9*b^4)/b^2*s
um(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^
2+3*a*b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 5/9*c^2*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a*
b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 -
3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x)/(9*a^2*b^8 - 54*a^3*b^6*c +
81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 -
 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c
^4)*x^4 + 6*(3*b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 +
 9*(b^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9 - 6*
a^2*b^7*c + 9*a^3*b^5*c^2)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(262) = 524$.
time = 0.40, size = 1268, normalized size = 4.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(9*b^10 - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*c
^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b
^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*sqrt(3)*(
9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5 - 3
*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*b^4*c
^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2 + 18*
(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*arct
an(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(
3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(5/6)) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*
a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*
a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^5 + 3*a*b^3*
c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3
)*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*
c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*
c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^4 + 3*a*b^2*c - (b
^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)) - 36*(b^9*c
- 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^2*b^14 - 108*a^3*b^
12*c + 486*a^4*b^10*c^2 - 972*a^5*b^8*c^3 + 729*a^6*b^6*c^4 + (b^12*c^4 - 1
2*a*b^10*c^5 + 54*a^2*b^8*c^6 - 108*a^3*b^6*c^7 + 81*a^4*b^4*c^8)*x^6 + 6*(
b^13*c^3 - 12*a*b^11*c^4 + 54*a^2*b^9*c^5 - 108*a^3*b^7*c^6 + 81*a^4*b^5*c^
7)*x^5 + 15*(b^14*c^2 - 12*a*b^12*c^3 + 54*a^2*b^10*c^4 - 108*a^3*b^8*c^5 +
 81*a^4*b^6*c^6)*x^4 + 6*(3*b^15*c - 35*a*b^13*c^2 + 150*a^2*b^11*c^3 - 270
*a^3*b^9*c^4 + 135*a^4*b^7*c^5 + 81*a^5*b^5*c^6)*x^3 + 9*(b^16 - 10*a*b^14*
c + 30*a^2*b^12*c^2 - 135*a^4*b^8*c^4 + 162*a^5*b^6*c^5)*x^2 + 18*(a*b^15 -
 12*a^2*b^13*c + 54*a^3*b^11*c^2 - 108*a^4*b^9*c^3 + 81*a^5*b^7*c^4)*x)
```

Sympy [A]
time = 1.44, size = 474, normalized size = 1.55

$$\frac{24ab^2c - 3b^4 + 30b^2c^2x^2 + 20bc^3x^3 + 5c^4x^4 + x(24abc^2 + 12b^3c)}{1458a^4b^4c^2 - 972a^3b^6c + 162a^2b^8 + x^6 \cdot (162a^2b^2c^6 - 108ab^4c^5 + 18b^6c^4) + x^5 \cdot (972a^2b^3c^5 - 648ab^5c^4 + 108b^7c^3) + x^4 \cdot (2430a^2b^4c^4 - 1620ab^6c^3 + 270b^8c^2) + x^3 \cdot (972a^3b^3c^4 + 2268a^2b^5c^3 - 1836ab^7c^2 + 324b^9c) + x^2 \cdot (2916a^3b^4c^3 - 486a^2b^6c^2 - 648ab^8c + 162b^{10}) + x(2916a^3b^5c^2 - 1944a^2b^7c + 324ab^9)} + \operatorname{RootSum}\left(t^3 \cdot (129140163a^8b^8c^8 - 344373768a^7b^{10}c^7 + 401769396a^6b^{12}c^6 - 267846264a^5b^{14}c^5 + 111602610a^4b^{16}c^4 - 29760696a^3b^{18}c^3 + 4960116a^2b^{20}c^2 - 472392ab^{22}c + 19683b^{24}) - 125c^6, \left(t \mapsto t\log\left(x + \frac{729ta^3b^3c^3 - 729ta^2b^5c^2 + 243tab^7c - 27tb^9 + 5bc^2}{5c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] (24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c**4*x**4 +
x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972*a**3*b**6*c + 162*a
**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b**4*c**5 + 18*b**6*c**4) + x**
5*(972*a**2*b**3*c**5 - 648*a*b**5*c**4 + 108*b**7*c**3) + x**4*(2430*a**2*
b**4*c**4 - 1620*a*b**6*c**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c**4 +
2268*a**2*b**5*c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**3*b**4
```

```
*c**3 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**3*b**5*
c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(129140163*a**8*b**8
*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6*b**12*c**6 - 267846264*a
**5*b**14*c**5 + 111602610*a**4*b**16*c**4 - 29760696*a**3*b**18*c**3 + 496
0116*a**2*b**20*c**2 - 472392*a*b**22*c + 19683*b**24) - 125*c**6, Lambda(_
t, _t*log(x + (729*_t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 + 243*_t*a*b**
7*c - 27*_t*b**9 + 5*b*c**2)/(5*c**3))))
```

Giac [A]
time = 4.04, size = 366, normalized size = 1.20

$$\frac{5\left(2\sqrt{3}\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}}{cx+b+(-b^3+3abc)^{\frac{1}{3}}}\right)-\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}\right)^2+4\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)^2\right)+2\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(\left|cx+b+(-b^3+3abc)^{\frac{1}{3}}\right|\right)\right)}{54\,(b^6-6ab^4c+9a^2b^2c^2)}+\frac{5c^4x^4+20bc^3x^3+30b^2c^2x^2+12b^3cx+24abc^2x-3b^4+24ab^2c}{18\,(b^6-6ab^4c+9a^2b^2c^2)(c^2x^3+3bcx^2+3b^2x+3ab)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 5/54*(2*sqrt(3)*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2) +
 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 + 12*b^3*c*x + 24*a*b*c^2*
x - 3*b^4 + 24*a*b^2*c)/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(c^2*x^3 + 3*b*c
*x^2 + 3*b^2*x + 3*a*b)^2)
```

Mupad [B]
time = 2.98, size = 483, normalized size = 1.58

$$\frac{\frac{8ac-b^2}{6(9a^2c^2-6ab^2c+b^4)}+\frac{5c^2x^2}{3(9a^2c^2-6ab^2c+b^4)}+\frac{10c^3x^3}{9b(9a^2c^2-6ab^2c+b^4)}+\frac{5c^4x^4}{18b^2(9a^2c^2-6ab^2c+b^4)}+\frac{2cx(b^2+2ac)}{3b(9a^2c^2-6ab^2c+b^4)}}{x^2(9b^4+18acb^2)+9a^2b^2+c^4x^6+x^3(18b^3c+6abc^2)+6bc^3x^5+15b^2c^2x^4+18ab^3x}+\frac{5c^2\ln\left(b(3ac-b^2)^{8/3}-b^{19/3}+cx(3ac-b^2)^{8/3}+27a^3b^{1/3}c^3-27a^2b^{7/3}c^2+9ab^{13/3}c\right)}{27b^{8/3}(3ac-b^2)^{8/3}}-\frac{5c^2\ln\left(2b-b^{1/3}(3ac-b^2)^{1/3}+2cx-\sqrt{3}\,b^{1/3}(3ac-b^2)^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27b^{8/3}(3ac-b^2)^{8/3}}+\frac{5c^2\ln\left(2b-b^{1/3}(3ac-b^2)^{1/3}+2cx+\sqrt{3}\,b^{1/3}(3ac-b^2)^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{27b^{8/3}(3ac-b^2)^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] ((8*a*c - b^2)/(6*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (5*c^2*x^2)/(3*(b^4 + 9*
a^2*c^2 - 6*a*b^2*c)) + (10*c^3*x^3)/(9*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) +
(5*c^4*x^4)/(18*b^2*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/
(3*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)))/(x^2*(9*b^4 + 18*a*b^2*c) + 9*a^2*b^2
+ c^4*x^6 + x^3*(18*b^3*c + 6*a*b*c^2) + 6*b*c^3*x^5 + 15*b^2*c^2*x^4 + 18*
a*b^3*x) + (5*c^2*log(b*(3*a*c - b^2)^(8/3) - b^(19/3) + c*x*(3*a*c - b^2)^
(8/3) + 27*a^3*b^(1/3)*c^3 - 27*a^2*b^(7/3)*c^2 + 9*a*b^(13/3)*c))/(27*b^(8
/3)*(3*a*c - b^2)^(8/3)) - (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2
*c*x - 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2))/(27*
b^(8/3)*(3*a*c - b^2)^(8/3)) + (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3)
 + 2*c*x + 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2))/
(27*b^(8/3)*(3*a*c - b^2)^(8/3))
```

3.15 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 +$

Optimal. Leaf size=361

$$\frac{(bc-ad)^3(be-af)^3(a+bx)^4}{4b^7}+\frac{3(bc-ad)^2(be-af)^2(bde+bcf-2adf)(a+bx)^5}{5b^7}+\frac{(bc-ad)(be-af)\left(5a^2\right.}{}$$

```
[Out] 1/4*(-a*d+b*c)^3*(-a*f+b*e)^3*(b*x+a)^4/b^7+3/5*(-a*d+b*c)^2*(-a*f+b*e)^2*(
-2*a*d*f+b*c*f+b*d*e)*(b*x+a)^5/b^7+1/2*(-a*d+b*c)*(-a*f+b*e)*(5*a^2*d^2*f^
2-5*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^6/b^7+1/7*(-
2*a*d*f+b*c*f+b*d*e)*(10*a^2*d^2*f^2-10*a*b*d*f*(c*f+d*e)+b^2*(c^2*f^2+8*c*
d*e*f+d^2*e^2))*(b*x+a)^7/b^7+3/8*d*f*(5*a^2*d^2*f^2-5*a*b*d*f*(c*f+d*e)+b^
2*(c^2*f^2+3*c*d*e*f+d^2*e^2))*(b*x+a)^8/b^7+1/3*d^2*f^2*(-2*a*d*f+b*c*f+b*
d*e)*(b*x+a)^9/b^7+1/10*d^3*f^3*(b*x+a)^10/b^7
```

Rubi [A]
time = 0.48, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2084, 90}

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2(c^2f^2+3cdef+d^2e^2)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10abdf(cf+de)+b^2(c^2f^2+8cdef+d^2e^2)\right)}{7b^7}+\frac{(a+bx)^6(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(cf+de)+b^2(c^2f^2+3cdef+d^2e^2)\right)}{2b^7}+\frac{d^2f^2(a+bx)^9(-2adf+bcf+bde)}{3b^7}+\frac{3(a+bx)^5(bc-ad)^2(be-af)^2(-2adf+bcf+bde)}{5b^7}+\frac{(a+bx)^4(bc-ad)^3(be-af)^3}{4b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^3,x]
```

```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e -
 a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e
- a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2
 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^
7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a
*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx=\int (a+bx)^3(c+dx)^3(e+fx)^3\,dx$$
$$=\int \left(\frac{(bc-ad)^3(be-af)^3(a+bx)^3}{b^6}+\right.$$
$$=\frac{(bc-ad)^3(be-af)^3(a+bx)^4}{4b^7}+\frac{3(bc}{}$$

Mathematica [**A**]
time = 0.17, size = 653, normalized size = 1.81

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^3,x]
```

```
[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2*
c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^3
+ ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))
*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f) + b
^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c*d*e*f +
 c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^6)
/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3
 + (b^3*d^3*f^3*x^10)/10
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(347) = 694$.
time = 0.04, size = 861, normalized size = 2.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/10*d^3*f^3*b^3*x^10+1/3*(a*d*f+b*c*f+b*d*e)*b^2*d^2*f^2*x^9+1/8*((a*c*f+a
*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b*d*f*(2*(a*c*f+a*d*e
```

```
+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))*x^8+1/7*(a*b^2*c*d^2*e*f^2+2*(a*c*f+a
*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f+b*d*e)*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*
e)*(a*d*f+b*c*f+b*d*e)))*x^7+1/6*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*c*f+
a*d*e+b*c*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*d*f+b*c
*f+b*d*e)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+b*d*f*(
2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2))*x^6+1/5*(a*c*e*(2*(a*c*
f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*b*
d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+(a*d*f+b*c*f+b*d*e)*(2*a*c*e
*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*b*d*f*a*c*e*(a*c*f+a*d*e+b*c*
e))*x^5+1/4*(a*c*e*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)
)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2
*(a*d*f+b*c*f+b*d*e)*a*c*e*(a*c*f+a*d*e+b*c*e)+a^2*b*c^2*d*e^2*f)*x^4+1/3*(
a*c*e*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*(a*c*f+a*d*e+b*
c*e)^2*a*c*e+(a*d*f+b*c*f+b*d*e)*e^2*c^2*a^2)*x^3+3/2*a^2*c^2*e^2*(a*c*f+a*
d*e+b*c*e)*x^2+c^3*e^3*a^3*x
```

Maxima [A]
time = 0.27, size = 483, normalized size = 1.34

$$\frac{1}{10}b^3d^3f^3x^{10}+\frac{1}{3}(bcf+adf+bde)b^2d^2f^2x^9+\frac{3}{8}(bcf+adf+bde)^2bdfx^8+\frac{1}{7}(bcf+adf+bde)^3x^7+a^3c^3xe^3+\frac{1}{4}(acf+bce+ade)^3x^4+\frac{1}{4}\left(3bdfx^4+4(bcf+adf+bde)x^3+6(acf+bce+ade)x^2\right)a^2c^2e^2+\frac{1}{70}\left(30b^2d^2f^2x^7+70(bcf+adf+bde)bdfx^6+42(bcf+adf+bde)^2x^5+70(acf+bce+ade)^2x^3+21\left(4bdfx^5+5(bde+(bc+ad)f)x^4\right)(acf+bce+ade)\right)ace+\frac{1}{10}\left(5bdfx^6+6(bde+(bc+ad)f)x^5\right)(acf+bce+ade)^2+\frac{1}{56}\left(21b^2d^2f^2x^8+48\left(b^2d^2fe+(b^2cd+abd^2)f^2\right)x^7+28\left(b^2d^2e^2+(b^2c^2+2abcd+a^2d^2)f^2+2(b^2cde+abd^2e)f\right)x^6\right)(acf+bce+ade)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*c*f + a*d*f + b*d*e)*b^2*d^2*f^2*x^9 + 3/8*(
b*c*f + a*d*f + b*d*e)^2*b*d*f*x^8 + 1/7*(b*c*f + a*d*f + b*d*e)^3*x^7 + a^
3*c^3*x*e^3 + 1/4*(a*c*f + b*c*e + a*d*e)^3*x^4 + 1/4*(3*b*d*f*x^4 + 4*(b*c
*f + a*d*f + b*d*e)*x^3 + 6*(a*c*f + b*c*e + a*d*e)*x^2)*a^2*c^2*e^2 + 1/70
*(30*b^2*d^2*f^2*x^7 + 70*(b*c*f + a*d*f + b*d*e)*b*d*f*x^6 + 42*(b*c*f + a
*d*f + b*d*e)^2*x^5 + 70*(a*c*f + b*c*e + a*d*e)^2*x^3 + 21*(4*b*d*f*x^5 +
5*(b*d*e + (b*c + a*d)*f)*x^4)*(a*c*f + b*c*e + a*d*e))*a*c*e + 1/10*(5*b*d
*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(a*c*f + b*c*e + a*d*e)^2 + 1/56*(2
1*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*f*e + (b^2*c*d + a*b*d^2)*f^2)*x^7 + 28*(b^
2*d^2*e^2 + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2 + 2*(b^2*c*d*e + a*b*d^2*e)
*f)*x^6)*(a*c*f + b*c*e + a*d*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(347) = 694$.
time = 0.36, size = 727, normalized size = 2.01

$$\frac{1}{10}b^3d^3f^3x^{10}+a^3c^3e^3x+\frac{1}{3}\left(b^3d^3ef^2+(b^3cd^2+ab^2d^3)f^3\right)x^9+\frac{3}{8}\left(b^3d^3e^2f+3(b^3cd^2+ab^2d^3)ef^2+(b^3c^2d+3ab^2cd^2+a^2bd^3)f^3\right)x^8+\frac{1}{7}\left(b^3d^3e^3+9(b^3cd^2+ab^2d^3)e^2f+9(b^3c^2d+3ab^2cd^2+a^2bd^3)ef^2+(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3)f^3\right)x^7+\frac{1}{2}\left((b^3cd^2+ab^2d^3)e^3+3(b^3c^2d+3ab^2cd^2+a^2bd^3)e^2f+(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3)ef^2+(ab^2c^3+3a^2bc^2d+a^3cd^2)f^3\right)x^6+\frac{3}{5}\left((b^3c^2d+3ab^2cd^2+a^2bd^3)e^3+(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3)e^2f+3(ab^2c^3+3a^2bc^2d+a^3cd^2)ef^2+(a^2bc^3+a^3c^2d)f^3\right)x^5+\frac{1}{4}\left(a^3c^3f^3+(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3)e^3+9(ab^2c^3+3a^2bc^2d+a^3cd^2)e^2f+9(a^2bc^3+a^3c^2d)ef^2\right)x^4+\left(a^3c^3ef^2+(ab^2c^3+3a^2bc^2d+a^3cd^2)e^3+3(a^2bc^3+a^3c^2d)e^2f\right)x^3+\frac{3}{2}\left(a^3c^3e^2f+(a^2bc^3+a^3c^2d)e^3\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="fricas")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + a^3*c^3*e^3*x + 1/3*(b^3*d^3*e*f^2 + (b^3*c*d^2 + a
*b^2*d^3)*f^3)*x^9 + 3/8*(b^3*d^3*e^2*f + 3*(b^3*c*d^2 + a*b^2*d^3)*e*f^2 +
 (b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^8 + 1/7*(b^3*d^3*e^3 + 9*(b
^3*c*d^2 + a*b^2*d^3)*e^2*f + 9*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e*f
^2 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3)*x^7 + 1/2*((b
^3*c*d^2 + a*b^2*d^3)*e^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^2*f
 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*f^2 + (a*b^2*c^3 +
 3*a^2*b*c^2*d + a^3*c*d^2)*f^3)*x^6 + 3/5*((b^3*c^2*d + 3*a*b^2*c*d^2 + a^
2*b*d^3)*e^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^2*f +
3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e*f^2 + (a^2*b*c^3 + a^3*c^2*d)*f
^3)*x^5 + 1/4*(a^3*c^3*f^3 + (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3
*d^3)*e^3 + 9*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f + 9*(a^2*b*c^3
+ a^3*c^2*d)*e*f^2)*x^4 + (a^3*c^3*e*f^2 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3
*c*d^2)*e^3 + 3*(a^2*b*c^3 + a^3*c^2*d)*e^2*f)*x^3 + 3/2*(a^3*c^3*e^2*f + (
a^2*b*c^3 + a^3*c^2*d)*e^3)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(364) = 728$.
time = 0.08, size = 1018, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**3,x)
```

```
[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b**
3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*b*
*2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**3*c
*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*
c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2
*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*
d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c*d**2
*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*d**2*e*f
**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**2*d*e*f**
2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e*f**2/2 +
3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 +
 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27*
a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 +
 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*d**2*e**3/
5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**3*c**3*f**3/4 +
 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**3*e**3/4 + 9*a**
2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9*
```

```
a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a
**3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e
**2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e**2*f
/2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 971 vs. 2(356) = 712.
time = 3.65, size = 971, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="giac")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*b^3*c*d^2*f^3*x^9 + 1/3*a*b^2*d^3*f^3*x^9 + 1/3
*b^3*d^3*f^2*x^9*e + 3/8*b^3*c^2*d*f^3*x^8 + 9/8*a*b^2*c*d^2*f^3*x^8 + 3/8*
a^2*b*d^3*f^3*x^8 + 9/8*b^3*c*d^2*f^2*x^8*e + 9/8*a*b^2*d^3*f^2*x^8*e + 1/7
*b^3*c^3*f^3*x^7 + 9/7*a*b^2*c^2*d*f^3*x^7 + 9/7*a^2*b*c*d^2*f^3*x^7 + 1/7*
a^3*d^3*f^3*x^7 + 3/8*b^3*d^3*f*x^8*e^2 + 9/7*b^3*c^2*d*f^2*x^7*e + 27/7*a*
b^2*c*d^2*f^2*x^7*e + 9/7*a^2*b*d^3*f^2*x^7*e + 1/2*a*b^2*c^3*f^3*x^6 + 3/2
*a^2*b*c^2*d*f^3*x^6 + 1/2*a^3*c*d^2*f^3*x^6 + 9/7*b^3*c*d^2*f*x^7*e^2 + 9/
7*a*b^2*d^3*f*x^7*e^2 + 1/2*b^3*c^3*f^2*x^6*e + 9/2*a*b^2*c^2*d*f^2*x^6*e +
 9/2*a^2*b*c*d^2*f^2*x^6*e + 1/2*a^3*d^3*f^2*x^6*e + 3/5*a^2*b*c^3*f^3*x^5
+ 3/5*a^3*c^2*d*f^3*x^5 + 1/7*b^3*d^3*x^7*e^3 + 3/2*b^3*c^2*d*f*x^6*e^2 + 9
/2*a*b^2*c*d^2*f*x^6*e^2 + 3/2*a^2*b*d^3*f*x^6*e^2 + 9/5*a*b^2*c^3*f^2*x^5*
e + 27/5*a^2*b*c^2*d*f^2*x^5*e + 9/5*a^3*c*d^2*f^2*x^5*e + 1/4*a^3*c^3*f^3*
x^4 + 1/2*b^3*c*d^2*x^6*e^3 + 1/2*a*b^2*d^3*x^6*e^3 + 3/5*b^3*c^3*f*x^5*e^2
 + 27/5*a*b^2*c^2*d*f*x^5*e^2 + 27/5*a^2*b*c*d^2*f*x^5*e^2 + 3/5*a^3*d^3*f*
x^5*e^2 + 9/4*a^2*b*c^3*f^2*x^4*e + 9/4*a^3*c^2*d*f^2*x^4*e + 3/5*b^3*c^2*d
*x^5*e^3 + 9/5*a*b^2*c*d^2*x^5*e^3 + 3/5*a^2*b*d^3*x^5*e^3 + 9/4*a*b^2*c^3*
f*x^4*e^2 + 27/4*a^2*b*c^2*d*f*x^4*e^2 + 9/4*a^3*c*d^2*f*x^4*e^2 + a^3*c^3*
f^2*x^3*e + 1/4*b^3*c^3*x^4*e^3 + 9/4*a*b^2*c^2*d*x^4*e^3 + 9/4*a^2*b*c*d^2
*x^4*e^3 + 1/4*a^3*d^3*x^4*e^3 + 3*a^2*b*c^3*f*x^3*e^2 + 3*a^3*c^2*d*f*x^3*
e^2 + a*b^2*c^3*x^3*e^3 + 3*a^2*b*c^2*d*x^3*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a
^3*c^3*f*x^2*e^2 + 3/2*a^2*b*c^3*x^2*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + a^3*c^3*
x*e^3
```

Mupad [B]
time = 2.23, size = 787, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^3,x)
```

```
[Out] x^7*((a^3*d^3*f^3)/7 + (b^3*c^3*f^3)/7 + (b^3*d^3*e^3)/7 + (9*a*b^2*c^2*d*f
^3)/7 + (9*a^2*b*c*d^2*f^3)/7 + (9*a*b^2*d^3*e^2*f)/7 + (9*a^2*b*d^3*e*f^2)
/7 + (9*b^3*c*d^2*e^2*f)/7 + (9*b^3*c^2*d*e*f^2)/7 + (27*a*b^2*c*d^2*e*f^2)
/7) + x^5*((3*a^2*b*c^3*f^3)/5 + (3*a^2*b*d^3*e^3)/5 + (3*a^3*c^2*d*f^3)/5
+ (3*b^3*c^2*d*e^3)/5 + (3*a^3*d^3*e^2*f)/5 + (3*b^3*c^3*e^2*f)/5 + (9*a*b^
2*c*d^2*e^3)/5 + (9*a*b^2*c^3*e*f^2)/5 + (9*a^3*c*d^2*e*f^2)/5 + (27*a*b^2*
c^2*d*e^2*f)/5 + (27*a^2*b*c*d^2*e^2*f)/5 + (27*a^2*b*c^2*d*e*f^2)/5) + x^6
*((a*b^2*c^3*f^3)/2 + (a*b^2*d^3*e^3)/2 + (a^3*c*d^2*f^3)/2 + (b^3*c*d^2*e^
3)/2 + (a^3*d^3*e*f^2)/2 + (b^3*c^3*e*f^2)/2 + (3*a^2*b*c^2*d*f^3)/2 + (3*a
^2*b*d^3*e^2*f)/2 + (3*b^3*c^2*d*e^2*f)/2 + (9*a*b^2*c*d^2*e^2*f)/2 + (9*a*
b^2*c^2*d*e*f^2)/2 + (9*a^2*b*c*d^2*e*f^2)/2) + x^4*((a^3*c^3*f^3)/4 + (a^3
*d^3*e^3)/4 + (b^3*c^3*e^3)/4 + (9*a*b^2*c^2*d*e^3)/4 + (9*a^2*b*c*d^2*e^3)
/4 + (9*a*b^2*c^3*e^2*f)/4 + (9*a^2*b*c^3*e*f^2)/4 + (9*a^3*c*d^2*e^2*f)/4
+ (9*a^3*c^2*d*e*f^2)/4 + (27*a^2*b*c^2*d*e^2*f)/4) + a^3*c^3*e^3*x + (b^3*
d^3*f^3*x^10)/10 + (3*a^2*c^2*e^2*x^2*(a*c*f + a*d*e + b*c*e))/2 + (b^2*d^2
*f^2*x^9*(a*d*f + b*c*f + b*d*e))/3 + a*c*e*x^3*(a^2*c^2*f^2 + a^2*d^2*e^2
+ b^2*c^2*e^2 + 3*a*b*c*d*e^2 + 3*a*b*c^2*e*f + 3*a^2*c*d*e*f) + (3*b*d*f*x
^8*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 3*a*b*c*d*f^2 + 3*a*b*d^2*e*f
 + 3*b^2*c*d*e*f))/8
```

3.16 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 +$

Optimal. Leaf size=193

$$\frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5}+\frac{(bc-ad)(be-af)(bde+bcf-2adf)(a+bx)^4}{2b^5}+\frac{(6a^2d^2f^2-6abdf(de+cf)}{}$$

```
[Out] 1/3*(-a*d+b*c)^2*(-a*f+b*e)^2*(b*x+a)^3/b^5+1/2*(-a*d+b*c)*(-a*f+b*e)*(-2*a
*d*f+b*c*f+b*d*e)*(b*x+a)^4/b^5+1/5*(6*a^2*d^2*f^2-6*a*b*d*f*(c*f+d*e)+b^2*
(c^2*f^2+4*c*d*e*f+d^2*e^2))*(b*x+a)^5/b^5+1/3*d*f*(-2*a*d*f+b*c*f+b*d*e)*(
b*x+a)^6/b^5+1/7*d^2*f^2*(b*x+a)^7/b^5
```

Rubi [A]

time = 0.18, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2084, 90}

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2(c^2f^2+4cdef+d^2e^2)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(bc-ad)(be-af)(-2adf+bcf+bde)}{2b^5}+\frac{(a+bx)^3(bc-ad)^2(be-af)^2}{3b^5}+\frac{d^2f^2(a+bx)^7}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^2,x]
```

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f
)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*
d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5)
 + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)
^7)/(7*b^5)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx=\int (a+bx)^2(c+dx)^2(e+fx)^2\,dx$$
$$=\int \left(\frac{(bc-ad)^2(be-af)^2(a+bx)^2}{b^4}+\right.$$
$$=\frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5}+\frac{(bc-}{}$$

Mathematica [A]
time = 0.06, size = 241, normalized size = 1.25

$$a^2c^2e^2x+ace(bce+ade+acf)x^2+\frac{1}{3}\left(b^2c^2e^2+4abce(de+cf)+a^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^3+\frac{1}{2}\left(b^2ce(de+cf)+a^2df(de+cf)+ab\left(d^2e^2+4cdef+c^2f^2\right)\right)x^4+\frac{1}{5}\left(a^2d^2f^2+4abdf(de+cf)+b^2\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5+\frac{1}{3}bdf(bde+bcf+adf)x^6+\frac{1}{7}b^2d^2f^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^2,x]
```

```
[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*c
*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*(d
*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^4)
/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7
```

Maple [A]
time = 0.02, size = 188, normalized size = 0.97

method	result
default	$\frac{d^2f^2b^2x^7}{7}+\frac{(daf+fbc+deb)bdf\,x^6}{3}+\frac{\left(2(acf+ade+bce)bdf+(daf+fbc+deb)^2\right)x^5}{5}+\frac{(2acebdf+2(acf+ade+bce)(daf+fbc+de}{4}$
norman	$\frac{d^2f^2b^2x^7}{7}+\left(\frac{1}{3}d^2f^2ab+\frac{1}{3}b^2cd\,f^2+\frac{1}{3}b^2d^2ef\right)x^6+\left(\frac{1}{5}a^2d^2f^2+\frac{4}{5}abcd\,f^2+\frac{4}{5}ab\,d^2ef+\frac{1}{5}b^2c^2f^2+\frac{4}{5}b\right.$
gosper	$\frac{x\left(30d^2f^2b^2x^6+70x^5d^2f^2ab+70x^5b^2cd\,f^2+70x^5b^2d^2ef+42x^4a^2d^2f^2+168x^4abcd\,f^2+168x^4ab\,d^2ef+42x^4b^2c^2f^2+168x^4b^2cdef+42\right.}{}$
risch	$e^2c^2a^2x+\frac{1}{7}d^2f^2b^2x^7+\frac{1}{5}x^5a^2d^2f^2+\frac{1}{5}x^5b^2c^2f^2+\frac{1}{3}ab\,d^2f^2x^6+\frac{1}{2}x^4a^2cd\,f^2+\frac{1}{2}x^4a^2d^2ef+\frac{1}{2}x^4ab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 1/7*d^2*f^2*b^2*x^7+1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6+1/5*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5+1/4*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*
c*e)*(a*d*f+b*c*f+b*d*e))*x^4+1/3*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e
+b*c*e)^2)*x^3+a*c*e*(a*c*f+a*d*e+b*c*e)*x^2+e^2*c^2*a^2*x
```

Maxima [A]
time = 0.27, size = 190, normalized size = 0.98

$$\frac{1}{7}b^2d^2f^2x^7+\frac{1}{3}(bcf+adf+bde)bdfx^6+\frac{1}{5}(bcf+adf+bde)^2x^5+a^2c^2xe^2+\frac{1}{3}(acf+bce+ade)^2x^3+\frac{1}{6}\left(3\,bdfx^4+4\,(bcf+adf+bde)x^3+6\,(acf+bce+ade)x^2\right)ace+\frac{1}{10}\left(4\,bdfx^5+5\,(bde+(bc+ad)f)x^4\right)(acf+bce+ade)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(b*c*f + a*d*f + b*d*e)*b*d*f*x^6 + 1/5*(b*c*f +
a*d*f + b*d*e)^2*x^5 + a^2*c^2*x*e^2 + 1/3*(a*c*f + b*c*e + a*d*e)^2*x^3 +
1/6*(3*b*d*f*x^4 + 4*(b*c*f + a*d*f + b*d*e)*x^3 + 6*(a*c*f + b*c*e + a*d*e
)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(a*c*f +
b*c*e + a*d*e)
```

Fricas [A]
time = 0.35, size = 269, normalized size = 1.39

$$\frac{1}{7}b^2d^2f^2x^7+a^2c^2e^2x+\frac{1}{3}\left(b^2d^2ef+(b^2cd+abd^2)f^2\right)x^6+\frac{1}{5}\left(b^2d^2e^2+4\,(b^2cd+abd^2)ef+(b^2c^2+4\,abcd+a^2d^2)f^2\right)x^5+\frac{1}{2}\left((b^2cd+abd^2)e^2+(b^2c^2+4\,abcd+a^2d^2)ef+(abc^2+a^2cd)f^2\right)x^4+\frac{1}{3}\left(a^2c^2f^2+(b^2c^2+4\,abcd+a^2d^2)e^2+4\,(abc^2+a^2cd)ef\right)x^3+\left(a^2c^2ef+(abc^2+a^2cd)e^2\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="fricas")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + a^2*c^2*e^2*x + 1/3*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2
)*f^2)*x^6 + 1/5*(b^2*d^2*e^2 + 4*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*f^2)*x^5 + 1/2*((b^2*c*d + a*b*d^2)*e^2 + (b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*e*f + (a*b*c^2 + a^2*c*d)*f^2)*x^4 + 1/3*(a^2*c^2*f^2 + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*e^2 + 4*(a*b*c^2 + a^2*c*d)*e*f)*x^3 + (a^2*c^
2*e*f + (a*b*c^2 + a^2*c*d)*e^2)*x^2
```

Sympy [A]
time = 0.03, size = 345, normalized size = 1.79

$$a^2c^2e^2x+\frac{b^2d^2f^2x^7}{7}+x^6\left(\frac{abd^2f^2}{3}+\frac{b^2cdf^2}{3}+\frac{b^2d^2ef}{3}\right)+x^5\left(\frac{a^2d^2f^2}{5}+\frac{4abcdf^2}{5}+\frac{4abd^2ef}{5}+\frac{b^2c^2f^2}{5}+\frac{4b^2cdef}{5}+\frac{b^2d^2e^2}{5}\right)+x^4\left(\frac{a^2cdf^2}{2}+\frac{a^2d^2ef}{2}+\frac{abc^2f^2}{2}+2abcdef+\frac{abd^2e^2}{2}+\frac{b^2c^2ef}{2}+\frac{b^2cde^2}{2}\right)+x^3\left(\frac{a^2c^2f^2}{3}+\frac{4a^2cdef}{3}+\frac{a^2d^2e^2}{3}+\frac{4abc^2ef}{3}+\frac{4abcde^2}{3}+\frac{b^2c^2e^2}{3}\right)+x^2\left(a^2c^2ef+a^2cde^2+abc^2e^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**2,x)
```

```
[Out] a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*d
*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*
a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5) +
x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f +
 a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**
```

```
2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e*
*2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c**2*e
**2)
```

Giac [A]
time = 3.07, size = 346, normalized size = 1.79

$$\frac{1}{7}b^2d^2f^2x^7+\frac{1}{3}b^2cdf^2x^6+\frac{1}{3}abd^2f^2x^6+\frac{1}{3}b^2d^2fx^6e+\frac{1}{5}b^2c^2f^2x^5+\frac{4}{5}abcdf^2x^5+\frac{1}{5}a^2d^2f^2x^5+\frac{4}{5}b^2cdfx^5e+\frac{4}{5}abd^2fx^5e+\frac{1}{2}abc^2f^2x^4+\frac{1}{2}a^2cdf^2x^4+\frac{1}{5}b^2d^2x^5e^2+\frac{1}{2}b^2c^2fx^4e+2abcdfx^4e+\frac{1}{2}a^2d^2fx^4e+\frac{1}{3}a^2c^2f^2x^3+\frac{1}{2}b^2cdx^4e^2+\frac{1}{2}abd^2x^4e^2+\frac{4}{3}abc^2fx^3e+\frac{4}{3}a^2cdfx^3e+\frac{1}{3}b^2c^2x^3e^2+\frac{4}{3}abcdx^3e^2+\frac{1}{3}a^2d^2x^3e^2+a^2c^2fx^2e+abc^2x^2e^2+a^2cdx^2e^2+a^2c^2xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*d^2*f^2*x^6 + 1/3*b^2*d
^2*f*x^6*e + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5 + 1/5*a^2*d^2*f^2*x^
5 + 4/5*b^2*c*d*f*x^5*e + 4/5*a*b*d^2*f*x^5*e + 1/2*a*b*c^2*f^2*x^4 + 1/2*a
^2*c*d*f^2*x^4 + 1/5*b^2*d^2*x^5*e^2 + 1/2*b^2*c^2*f*x^4*e + 2*a*b*c*d*f*x^
4*e + 1/2*a^2*d^2*f*x^4*e + 1/3*a^2*c^2*f^2*x^3 + 1/2*b^2*c*d*x^4*e^2 + 1/2
*a*b*d^2*x^4*e^2 + 4/3*a*b*c^2*f*x^3*e + 4/3*a^2*c*d*f*x^3*e + 1/3*b^2*c^2*
x^3*e^2 + 4/3*a*b*c*d*x^3*e^2 + 1/3*a^2*d^2*x^3*e^2 + a^2*c^2*f*x^2*e + a*b
*c^2*x^2*e^2 + a^2*c*d*x^2*e^2 + a^2*c^2*x*e^2
```

Mupad [B]
time = 0.08, size = 270, normalized size = 1.40

$$x^4\left(\frac{a^2\,c\,d\,f^2}{2}+\frac{a^2\,d^2\,e\,f}{2}+\frac{a\,b\,c^2\,f^2}{2}+2\,a\,b\,c\,d\,e\,f+\frac{a\,b\,d^2\,e^2}{2}+\frac{b^2\,c^2\,e\,f}{2}+\frac{b^2\,c\,d\,e^2}{2}\right)+x^3\left(\frac{a^2\,c^2\,f^2}{3}+\frac{4\,a^2\,c\,d\,e\,f}{3}+\frac{a^2\,d^2\,e^2}{3}+\frac{4\,a\,b\,c^2\,e\,f}{3}+\frac{4\,a\,b\,c\,d\,e^2}{3}+\frac{b^2\,c^2\,e^2}{3}\right)+x^5\left(\frac{a^2\,d^2\,f^2}{5}+\frac{4\,a\,b\,c\,d\,f^2}{5}+\frac{4\,a\,b\,d^2\,e\,f}{5}+\frac{b^2\,c^2\,f^2}{5}+\frac{4\,b^2\,c\,d\,e\,f}{5}+\frac{b^2\,d^2\,e^2}{5}\right)+a^2\,c^2\,e^2\,x+\frac{b^2\,d^2\,f^2\,x^7}{7}+a\,c\,e\,x^2\,(a\,c\,f+a\,d\,e+b\,c\,e)+\frac{b\,d\,f\,x^6\,(a\,d\,f+b\,c\,f+b\,d\,e)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^2,x)
```

```
[Out] x^4*((a*b*c^2*f^2)/2 + (a*b*d^2*e^2)/2 + (a^2*c*d*f^2)/2 + (b^2*c*d*e^2)/2
 + (a^2*d^2*e*f)/2 + (b^2*c^2*e*f)/2 + 2*a*b*c*d*e*f) + x^3*((a^2*c^2*f^2)/3
 + (a^2*d^2*e^2)/3 + (b^2*c^2*e^2)/3 + (4*a*b*c*d*e^2)/3 + (4*a*b*c^2*e*f)/
3 + (4*a^2*c*d*e*f)/3) + x^5*((a^2*d^2*f^2)/5 + (b^2*c^2*f^2)/5 + (b^2*d^2*
e^2)/5 + (4*a*b*c*d*f^2)/5 + (4*a*b*d^2*e*f)/5 + (4*b^2*c*d*e*f)/5) + a^2*c
^2*e^2*x + (b^2*d^2*f^2*x^7)/7 + a*c*e*x^2*(a*c*f + a*d*e + b*c*e) + (b*d*f
*x^6*(a*d*f + b*c*f + b*d*e))/3
```

3.17 $\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 +$

Optimal. Leaf size=56

$$ace x + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{4}bdf x^4$$

```
[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x
^4
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdf x^4$$

Antiderivative was successfully verified.

```
[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f
*x^3,x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3
 + (b*d*f*x^4)/4
```

Rubi steps

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdf x^3\right)\, dx = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde +$$

Mathematica [A]
time = 0.00, size = 76, normalized size = 1.36

$$acex + \frac{1}{2}bcex^2 + \frac{1}{2}adex^2 + \frac{1}{2}acf x^2 + \frac{1}{3}bdex^3 + \frac{1}{3}bcf x^3 + \frac{1}{3}adf x^3 + \frac{1}{4}bdf x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 +
 b*d*f*x^3,x]
```

```
[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 + (
b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4
```

Maple [A]
time = 0.01, size = 53, normalized size = 0.95

method	result	size
default	$\frac{bdf\,x^4}{4}+\frac{((cf+de)b+daf)x^3}{3}+\frac{(bce+(cf+de)a)x^2}{2}+acex$	53
norman	$\frac{bdf\,x^4}{4}+\left(\frac{1}{3}daf+\frac{1}{3}fbc+\frac{1}{3}deb\right)x^3+\left(\frac{1}{2}acf+\frac{1}{2}ade+\frac{1}{2}bce\right)x^2+acex$	55
gosper	$\frac{x(3bdf\,x^3+4adf\,x^2+4bcf\,x^2+4bde\,x^2+6acfx+6adex+6bcex+12ace)}{12}$	60
risch	$acex+\frac{1}{2}acf\,x^2+\frac{1}{2}ade\,x^2+\frac{1}{2}bce\,x^2+\frac{1}{3}adf\,x^3+\frac{1}{3}bcf\,x^3+\frac{1}{3}bde\,x^3+\frac{1}{4}bdf\,x^4$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,method=
_RETURNVERBOSE)
```

```
[Out] 1/4*b*d*f*x^4+1/3*((c*f+d*e)*b+d*a*f)*x^3+1/2*(b*c*e+(c*f+d*e)*a)*x^2+a*c*e
*x
```

Maxima [A]
time = 0.26, size = 54, normalized size = 0.96

$$\frac{1}{4}\,bdfx^4+\frac{1}{3}\,(bcf+adf+bde)x^3+acxe+\frac{1}{2}\,(acf+bce+ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="maxima")
```

```
[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(a*c*f + b*
c*e + a*d*e)*x^2
```

Fricas [A]
time = 0.37, size = 52, normalized size = 0.93

$$\frac{1}{4}\,bdfx^4+acex+\frac{1}{3}\,(bde+(bc+ad)f)x^3+\frac{1}{2}\,(acf+(bc+ad)e)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="fricas")
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + (b*c + a*d)*f)*x^3 + 1/2*(a*c*f + (b
*c + a*d)*e)*x^2
```

Sympy [A]
time = 0.01, size = 63, normalized size = 1.12

$$acex+\frac{bdfx^4}{4}+x^3\left(\frac{adf}{3}+\frac{bcf}{3}+\frac{bde}{3}\right)+x^2\left(\frac{acf}{2}+\frac{ade}{2}+\frac{bce}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x
)
```

```
[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/2
 + a*d*e/2 + b*c*e/2)
```

Giac [A]
time = 3.27, size = 54, normalized size = 0.96

$$\frac{1}{4}\,bdfx^4 + \frac{1}{3}\,(bcf + adf + bde)x^3 + acxe + \frac{1}{2}\,(acf + bce + ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="giac")
```

```
[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(a*c*f + b*
c*e + a*d*e)*x^2
```

Mupad [B]
time = 0.04, size = 54, normalized size = 0.96

$$\frac{b\,d\,f\,x^4}{4} + \left(\frac{a\,d\,f}{3} + \frac{b\,c\,f}{3} + \frac{b\,d\,e}{3}\right)\,x^3 + \left(\frac{a\,c\,f}{2} + \frac{a\,d\,e}{2} + \frac{b\,c\,e}{2}\right)\,x^2 + a\,c\,e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f
*x^3,x)
```

```
[Out] x^2*((a*c*f)/2 + (a*d*e)/2 + (b*c*e)/2) + x^3*((a*d*f)/3 + (b*c*f)/3 + (b*d
*e)/3) + a*c*e*x + (b*d*f*x^4)/4
```

3.18 $\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$

Optimal. Leaf size=86

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

```
[Out] b*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)-d*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)+f*ln(f*
x+e)/(-a*f+b*e)/(-c*f+d*e)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2083}

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-1),x]
```

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*
(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \int \left(\frac{b^2}{(bc-ad)(be-af)(a+bx)}+\frac{}{(bc-}\right.$$
$$=\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.93

$$\frac{b(-de+cf)\log(a+bx)+d(be-af)\log(c+dx)+(-bc+ad)f\log(e+fx)}{(bc-ad)(be-af)(-de+cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-1),x]
```

```
[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*d
)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))
```

Maple [A]
time = 0.06, size = 87, normalized size = 1.01

method	result	size
default	$\frac{f\ln(fx+e)}{(cf-de)(fa-eb)}-\frac{d\ln(dx+c)}{(cf-de)(ad-bc)}+\frac{b\ln(bx+a)}{(fa-eb)(ad-bc)}$	87
norman	$\frac{f\ln(fx+e)}{c\,f^{2}a-adef-bcef+d\,e^{2}b}+\frac{b\ln(bx+a)}{(fa-eb)(ad-bc)}-\frac{d\ln(dx+c)}{(cf-de)(ad-bc)}$	94
risch	$\frac{b\ln(bx+a)}{a^{2}df-abcf-abde+b^{2}ce}-\frac{d\ln(dx+c)}{acdf-a\,d^{2}e-b\,c^{2}f+bcde}+\frac{f\ln(-fx-e)}{c\,f^{2}a-adef-bcef+d\,e^{2}b}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x,met
hod=_RETURNVERBOSE)
```

```
[Out] f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b/(a*f-b*e)
/(a*d-b*c)*ln(b*x+a)
```

Maxima [A]
time = 0.28, size = 117, normalized size = 1.36

$$\frac{b\log\left(bx+a\right)}{b^{2}ce-abde-\left(abc-a^{2}d\right)f}-\frac{d\log\left(dx+c\right)}{bcde-ad^{2}e-\left(bc^{2}-acd\right)f}+\frac{f\log\left(fx+e\right)}{acf^{2}+bde^{2}-\left(bce+ade\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(b^2*c*e - a*b*d*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/(b*
c*d*e - a*d^2*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(a*c*f^2 + b*d*e^2 -
(b*c*e + a*d*e)*f)
```

Fricas [A]
time = 3.08, size = 112, normalized size = 1.30

$$\frac{(bc-ad)f\log\left(fx+e\right)+(bde-bcf)\log\left(bx+a\right)-(bde-adf)\log\left(dx+c\right)}{(b^{2}cd-abd^{2})e^{2}-(b^{2}c^{2}-a^{2}d^{2})ef+(abc^{2}-a^{2}cd)f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*f
)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b*c
^2 - a^2*c*d)*f^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.98, size = 137, normalized size = 1.59

$$-\frac{b^2\log\left(|bx+a|\right)}{ab^2cf-a^2bdf-b^3ce+ab^2de}+\frac{d^2\log\left(|dx+c|\right)}{bc^2df-acd^2f-bcd^2e+ad^3e}+\frac{f^2\log\left(|fx+e|\right)}{acf^3-bcf^2e-adf^2e+bdfe^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/(a*b^2*c*f - a^2*b*d*f - b^3*c*e + a*b^2*d*e) + d^2*
log(abs(d*x + c))/(b*c^2*d*f - a*c*d^2*f - b*c*d^2*e + a*d^3*e) + f^2*log(a
bs(f*x + e))/(a*c*f^3 - b*c*f^2*e - a*d*f^2*e + b*d*f*e^2)
```

Mupad [**B**]
time = 2.33, size = 106, normalized size = 1.23

$$\frac{b\,\ln\left(a+b\,x\right)}{b^2\,c\,e+a^2\,d\,f-a\,b\,c\,f-a\,b\,d\,e}+\frac{d\,\ln\left(c+d\,x\right)}{a\,d^2\,e+b\,c^2\,f-a\,c\,d\,f-b\,c\,d\,e}+\frac{f\,\ln\left(e+f\,x\right)}{a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3),x)
```

```
[Out] (b*log(a + b*x))/(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e) + (d*log(c + d*x))
/(a*d^2*e + b*c^2*f - a*c*d*f - b*c*d*e) + (f*log(e + f*x))/(a*c*f^2 + b*d*
e^2 - a*d*e*f - b*c*e*f)
```

3.19 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2} dx$

Optimal. Leaf size=234

$$-\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)}-\frac{d^3}{(bc-ad)^2(de-cf)^2(c+dx)}-\frac{f^3}{(be-af)^2(de-cf)^2(e+fx)}-\frac{2b^3(bde+bc}{(bc-}$$

```
[Out] -b^3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)-d^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x+c
)-f^3/(-a*f+b*e)^2/(-c*f+d*e)^2/(f*x+e)-2*b^3*(-2*a*d*f+b*c*f+b*d*e)*ln(b*x
+a)/(-a*d+b*c)^3/(-a*f+b*e)^3+2*d^3*(a*d*f-2*b*c*f+b*d*e)*ln(d*x+c)/(-a*d+b
*c)^3/(-c*f+d*e)^3+2*f^3*(-a*d*f-b*c*f+2*b*d*e)*ln(f*x+e)/(-a*f+b*e)^3/(-c*
f+d*e)^3
```

Rubi [A]

time = 0.31, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2083}

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(de-cf)^3}-\frac{f^3}{(e+fx)(be-af)^2(de-cf)^2}+\frac{2f^3\log(e+fx)(-adf-bcf+2bde)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (
2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2} dx = \int \left(\frac{b^4}{(bc-ad)^2(be-af)^2(a+bx)^2}-\right.$$

$$= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)}-\frac{}{(bc-}$$

Mathematica [A]
time = 0.42, size = 232, normalized size = 0.99

$$-\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)}-\frac{d^3}{(bc-ad)^2(de-cf)^2(c+dx)}-\frac{f^3}{(be-af)^2(de-cf)^2(e+fx)}-\frac{2b^3(bde+bcf-2adf)\log(a+bx)}{(bc-ad)^3(be-af)^3}-\frac{2d^3(bde-2bcf+adf)\log(c+dx)}{(bc-ad)^3(-de+cf)^3}-\frac{2f^3(-2bde+bcf+adf)\log(e+fx)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) - (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(-(d*e) + c*f)^3)
- (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f
)^3)
```

Maple [A]
time = 0.30, size = 235, normalized size = 1.00

method	result
default	$-\frac{f^3}{(cf-de)^2(fa-eb)^2(fx+e)}-\frac{2f^3(daf+fbc-2deb)\ln(fx+e)}{(cf-de)^3(fa-eb)^3}-\frac{d^3}{(cf-de)^2(ad-bc)^2(dx+c)}+\frac{2d^3(daf-2fbc+deb)\ln(dx+c)}{(cf-de)^3(ad-bc)^3}$
norman	$\frac{\frac{-a^3bcd^3f^4-a^3bd^4ef^3+2a^2b^2c^2d^2f^4+2a^2b^2d^4e^2f^2-ab^3c^3df^4-ab^3d^4e^3f-b^4c^3def^3+2b^4c^2d^2e^2f^2-b^4cd^3e^3f}{fdb\left(a^2c^2f^4-2a^2cdef^3+a^2d^2e^2f^2-2abc^2ef^3+4abcde^2f^2-2abd^2e^3f+b^2c^2e^2f^2-2b^2cde^3f+b^2d^2e^4\right)\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{}{fdb\left(a^2c^2f^4-2a^2\right.}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^3*(a*d*f+b*c*f-2*b*d*e)/(c*f-d*e)^
3/(a*f-b*e)^3*ln(f*x+e)-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^3*(a*d*f-2*
b*c*f+b*d*e)/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)-b^3/(a*f-b*e)^2/(a*d-b*c)^2/
(b*x+a)+2*b^3*(2*a*d*f-b*c*f-b*d*e)/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2261 vs. $2(247) = 494$.
time = 0.41, size = 2261, normalized size = 9.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="maxima")
```

```
[Out] -2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/(b^6*c^3*e^3 - 3*a*b^5*c^
2*d*e^3 + 3*a^2*b^4*c*d^2*e^3 - a^3*b^3*d^3*e^3 - (a^3*b^3*c^3 - 3*a^4*b^2*
c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*f^3 + 3*(a^2*b^4*c^3*e - 3*a^3*b^3*c^2*d*e
 + 3*a^4*b^2*c*d^2*e - a^5*b*d^3*e)*f^2 - 3*(a*b^5*c^3*e^2 - 3*a^2*b^4*c^2*
d*e^2 + 3*a^3*b^3*c*d^2*e^2 - a^4*b^2*d^3*e^2)*f) + 2*(b*d^4*e - (2*b*c*d^3
 - a*d^4)*f)*log(d*x + c)/(b^3*c^3*d^3*e^3 - 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*
c*d^5*e^3 - a^3*d^6*e^3 - (b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*
c^3*d^3)*f^3 + 3*(b^3*c^5*d*e - 3*a*b^2*c^4*d^2*e + 3*a^2*b*c^3*d^3*e - a^3
*c^2*d^4*e)*f^2 - 3*(b^3*c^4*d^2*e^2 - 3*a*b^2*c^3*d^3*e^2 + 3*a^2*b*c^2*d^
4*e^2 - a^3*c*d^5*e^2)*f) + 2*(2*b*d*f^3*e - (b*c + a*d)*f^4)*log(f*x + e)/
(a^3*c^3*f^6 + b^3*d^3*e^6 - 3*(a^2*b*c^3*e + a^3*c^2*d*e)*f^5 + 3*(a*b^2*c
^3*e^2 + 3*a^2*b*c^2*d*e^2 + a^3*c*d^2*e^2)*f^4 - (b^3*c^3*e^3 + 9*a*b^2*c^
2*d*e^3 + 9*a^2*b*c*d^2*e^3 + a^3*d^3*e^3)*f^3 + 3*(b^3*c^2*d*e^4 + 3*a*b^2
*c*d^2*e^4 + a^2*b*d^3*e^4)*f^2 - 3*(b^3*c*d^2*e^5 + a*b^2*d^3*e^5)*f) - (b
^3*c*d^2*e^3 + a*b^2*d^3*e^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3
+ (b^3*c^3*e + a^3*d^3*e)*f^2 + 2*(b^3*d^3*f*e^2 + (b^3*c^2*d - a*b^2*c*d^2
 + a^2*b*d^3)*f^3 - (b^3*c*d^2*e + a*b^2*d^3*e)*f^2)*x^2 - 2*(b^3*c^2*d*e^2
 + a^2*b*d^3*e^2)*f + (2*b^3*d^3*e^3 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d
^2 + 2*a^3*d^3)*f^3 - (b^3*c^2*d*e + a^2*b*d^3*e)*f^2 - (b^3*c*d^2*e^2 + a*
b^2*d^3*e^2)*f)*x)/(a*b^4*c^3*d^2*e^5 - 2*a^2*b^3*c^2*d^3*e^5 + a^3*b^2*c*d
^4*e^5 + (a^3*b^2*c^5*e - 2*a^4*b*c^4*d*e + a^5*c^3*d^2*e)*f^4 - 2*(a^2*b^3
*c^5*e^2 - a^3*b^2*c^4*d*e^2 - a^4*b*c^3*d^2*e^2 + a^5*c^2*d^3*e^2)*f^3 + (
(a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*f^5 - 2*(a*b^4*c^4*d*e
- a^2*b^3*c^3*d^2*e - a^3*b^2*c^2*d^3*e + a^4*b*c*d^4*e)*f^4 + (b^5*c^4*d*e
^2 + 2*a*b^4*c^3*d^2*e^2 - 6*a^2*b^3*c^2*d^3*e^2 + 2*a^3*b^2*c*d^4*e^2 + a^
4*b*d^5*e^2)*f^3 - 2*(b^5*c^3*d^2*e^3 - a*b^4*c^2*d^3*e^3 - a^2*b^3*c*d^4*e
^3 + a^3*b^2*d^5*e^3)*f^2 + (b^5*c^2*d^3*e^4 - 2*a*b^4*c*d^4*e^4 + a^2*b^3*
d^5*e^4)*f)*x^3 + (a*b^4*c^5*e^3 + 2*a^2*b^3*c^4*d*e^3 - 6*a^3*b^2*c^3*d^2*
e^3 + 2*a^4*b*c^2*d^3*e^3 + a^5*c*d^4*e^3)*f^2 + (b^5*c^2*d^3*e^5 - 2*a*b^4
*c*d^4*e^5 + a^2*b^3*d^5*e^5 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2
 + a^5*c^2*d^3)*f^5 - (2*a*b^4*c^5*e - a^2*b^3*c^4*d*e - 2*a^3*b^2*c^3*d^2*
e - a^4*b*c^2*d^3*e + 2*a^5*c*d^4*e)*f^4 + (b^5*c^5*e^2 + a*b^4*c^4*d*e^2 -
 2*a^2*b^3*c^3*d^2*e^2 - 2*a^3*b^2*c^2*d^3*e^2 + a^4*b*c*d^4*e^2 + a^5*d^5*
e^2)*f^3 - (b^5*c^4*d*e^3 - 2*a*b^4*c^3*d^2*e^3 + 2*a^2*b^3*c^2*d^3*e^3 - 2
*a^3*b^2*c*d^4*e^3 + a^4*b*d^5*e^3)*f^2 - (b^5*c^3*d^2*e^4 - a*b^4*c^2*d^3*
e^4 - a^2*b^3*c*d^4*e^4 + a^3*b^2*d^5*e^4)*f)*x^2 - 2*(a*b^4*c^4*d*e^4 - a^
2*b^3*c^3*d^2*e^4 - a^3*b^2*c^2*d^3*e^4 + a^4*b*c*d^4*e^4)*f + (b^5*c^3*d^2
*e^5 - a*b^4*c^2*d^3*e^5 - a^2*b^3*c*d^4*e^5 + a^3*b^2*d^5*e^5 + (a^3*b^2*c
^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5 - (a^2*b^3*c^5*e - a^3*b^2*c^4*d*e -
a^4*b*c^3*d^2*e + a^5*c^2*d^3*e)*f^4 - (a*b^4*c^5*e^2 - 2*a^2*b^3*c^4*d*e^2
 + 2*a^3*b^2*c^3*d^2*e^2 - 2*a^4*b*c^2*d^3*e^2 + a^5*c*d^4*e^2)*f^3 + (b^5*
c^5*e^3 + a*b^4*c^4*d*e^3 - 2*a^2*b^3*c^3*d^2*e^3 - 2*a^3*b^2*c^2*d^3*e^3 +
 a^4*b*c*d^4*e^3 + a^5*d^5*e^3)*f^2 - (2*b^5*c^4*d*e^4 - a*b^4*c^3*d^2*e^4
- 2*a^2*b^3*c^2*d^3*e^4 - a^3*b^2*c*d^4*e^4 + 2*a^4*b*d^5*e^4)*f)*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(247) = 494$.
time = 3.35, size = 1414, normalized size = 6.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="giac")
```

```
[Out] 2*(b^5*c*f - 2*a*b^4*d*f + b^5*d*e)*log(abs(b*x + a))/(a^3*b^4*c^3*f^3 - 3*
a^4*b^3*c^2*d*f^3 + 3*a^5*b^2*c*d^2*f^3 - a^6*b*d^3*f^3 - 3*a^2*b^5*c^3*f^2
*e + 9*a^3*b^4*c^2*d*f^2*e - 9*a^4*b^3*c*d^2*f^2*e + 3*a^5*b^2*d^3*f^2*e +
3*a*b^6*c^3*f*e^2 - 9*a^2*b^5*c^2*d*f*e^2 + 9*a^3*b^4*c*d^2*f*e^2 - 3*a^4*b
^3*d^3*f*e^2 - b^7*c^3*e^3 + 3*a*b^6*c^2*d*e^3 - 3*a^2*b^5*c*d^2*e^3 + a^3*
b^4*d^3*e^3) + 2*(2*b*c*d^4*f - a*d^5*f - b*d^5*e)*log(abs(d*x + c))/(b^3*c
^6*d*f^3 - 3*a*b^2*c^5*d^2*f^3 + 3*a^2*b*c^4*d^3*f^3 - a^3*c^3*d^4*f^3 - 3*
b^3*c^5*d^2*f^2*e + 9*a*b^2*c^4*d^3*f^2*e - 9*a^2*b*c^3*d^4*f^2*e + 3*a^3*c
^2*d^5*f^2*e + 3*b^3*c^4*d^3*f*e^2 - 9*a*b^2*c^3*d^4*f*e^2 + 9*a^2*b*c^2*d^
5*f*e^2 - 3*a^3*c*d^6*f*e^2 - b^3*c^3*d^4*e^3 + 3*a*b^2*c^2*d^5*e^3 - 3*a^2
*b*c*d^6*e^3 + a^3*d^7*e^3) - 2*(b*c*f^5 + a*d*f^5 - 2*b*d*f^4*e)*log(abs(f
*x + e))/(a^3*c^3*f^7 - 3*a^2*b*c^3*f^6*e - 3*a^3*c^2*d*f^6*e + 3*a*b^2*c^3
*f^5*e^2 + 9*a^2*b*c^2*d*f^5*e^2 + 3*a^3*c*d^2*f^5*e^2 - b^3*c^3*f^4*e^3 -
```

```
9*a*b^2*c^2*d*f^4*e^3 - 9*a^2*b*c*d^2*f^4*e^3 - a^3*d^3*f^4*e^3 + 3*b^3*c^2
*d*f^3*e^4 + 9*a*b^2*c*d^2*f^3*e^4 + 3*a^2*b*d^3*f^3*e^4 - 3*b^3*c*d^2*f^2*
e^5 - 3*a*b^2*d^3*f^2*e^5 + b^3*d^3*f*e^6) - (2*b^3*c^2*d*f^3*x^2 - 2*a*b^2
*c*d^2*f^3*x^2 + 2*a^2*b*d^3*f^3*x^2 - 2*b^3*c*d^2*f^2*x^2*e - 2*a*b^2*d^3*
f^2*x^2*e + 2*b^3*c^3*f^3*x - a*b^2*c^2*d*f^3*x - a^2*b*c*d^2*f^3*x + 2*a^3
*d^3*f^3*x + 2*b^3*d^3*f*x^2*e^2 - b^3*c^2*d*f^2*x*e - a^2*b*d^3*f^2*x*e +
a*b^2*c^3*f^3 - 2*a^2*b*c^2*d*f^3 + a^3*c*d^2*f^3 - b^3*c*d^2*f*x*e^2 - a*b
^2*d^3*f*x*e^2 + b^3*c^3*f^2*e + a^3*d^3*f^2*e + 2*b^3*d^3*x*e^3 - 2*b^3*c^
2*d*f*e^2 - 2*a^2*b*d^3*f*e^2 + b^3*c*d^2*e^3 + a*b^2*d^3*e^3)/((a^2*b^2*c^
4*f^4 - 2*a^3*b*c^3*d*f^4 + a^4*c^2*d^2*f^4 - 2*a*b^3*c^4*f^3*e + 2*a^2*b^2
*c^3*d*f^3*e + 2*a^3*b*c^2*d^2*f^3*e - 2*a^4*c*d^3*f^3*e + b^4*c^4*f^2*e^2
+ 2*a*b^3*c^3*d*f^2*e^2 - 6*a^2*b^2*c^2*d^2*f^2*e^2 + 2*a^3*b*c*d^3*f^2*e^2
 + a^4*d^4*f^2*e^2 - 2*b^4*c^3*d*f*e^3 + 2*a*b^3*c^2*d^2*f*e^3 + 2*a^2*b^2*
c*d^3*f*e^3 - 2*a^3*b*d^4*f*e^3 + b^4*c^2*d^2*e^4 - 2*a*b^3*c*d^3*e^4 + a^2
*b^2*d^4*e^4)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f*x^2 + b*d*x^2*e + a*c*f*x + b*
c*x*e + a*d*x*e + a*c*e))
```

Mupad [B]
time = 8.18, size = 1940, normalized size = 8.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^2,x)
```

```
[Out] - ((a*b^2*c^3*f^3 + a*b^2*d^3*e^3 + a^3*c*d^2*f^3 + b^3*c*d^2*e^3 + a^3*d^3
*e*f^2 + b^3*c^3*e*f^2 - 2*a^2*b*c^2*d*f^3 - 2*a^2*b*d^3*e^2*f - 2*b^3*c^2*
d*e^2*f)/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2
*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*
d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c
^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^
3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e
*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) + (2*x^2*(a^2*b*d^3*f^3 + b^3*c^2*d*f^3 +
 b^3*d^3*e^2*f - a*b^2*c*d^2*f^3 - a*b^2*d^3*e*f^2 - b^3*c*d^2*e*f^2))/(a^2
*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^
4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b
^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f +
2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a
^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*
b^2*c^2*d^2*e^2*f^2) - (x*(a*b^2*c^2*d*f^3 - 2*b^3*c^3*f^3 - 2*b^3*d^3*e^3
- 2*a^3*d^3*f^3 + a^2*b*c*d^2*f^3 + a*b^2*d^3*e^2*f + a^2*b*d^3*e*f^2 + b^3
*c*d^2*e^2*f + b^3*c^2*d*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c
^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*
c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a
```

```
^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*
e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2
*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2))/(x^2*(a*d*f + b*
c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3) - (log(a + b*
x)*(b^4*(2*c*f + 2*d*e) - 4*a*b^3*d*f))/(b^6*c^3*e^3 + a^6*d^3*f^3 - a^3*b^
3*c^3*f^3 - a^3*b^3*d^3*e^3 - 3*a*b^5*c^2*d*e^3 - 3*a^5*b*c*d^2*f^3 - 3*a*b
^5*c^3*e^2*f - 3*a^5*b*d^3*e*f^2 + 3*a^2*b^4*c*d^2*e^3 + 3*a^4*b^2*c^2*d*f^
3 + 3*a^2*b^4*c^3*e*f^2 + 3*a^4*b^2*d^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*a
^3*b^3*c*d^2*e^2*f - 9*a^3*b^3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^2*e*f^2) - (log(
c + d*x)*(d^4*(2*a*f + 2*b*e) - 4*b*c*d^3*f))/(a^3*d^6*e^3 + b^3*c^6*f^3 -
a^3*c^3*d^3*f^3 - b^3*c^3*d^3*e^3 - 3*a^2*b*c*d^5*e^3 - 3*a*b^2*c^5*d*f^3 -
 3*a^3*c*d^5*e^2*f - 3*b^3*c^5*d*e*f^2 + 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*c^4*
d^2*f^3 + 3*a^3*c^2*d^4*e*f^2 + 3*b^3*c^4*d^2*e^2*f - 9*a*b^2*c^3*d^3*e^2*f
 + 9*a*b^2*c^4*d^2*e*f^2 + 9*a^2*b*c^2*d^4*e^2*f - 9*a^2*b*c^3*d^3*e*f^2) -
 (log(e + f*x)*(f^4*(2*a*d + 2*b*c) - 4*b*d*e*f^3))/(a^3*c^3*f^6 + b^3*d^3*
e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*e
^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^2*
b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 + 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^2*e
^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*
f^4)
```

3.20 $\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$

Optimal. Leaf size=495

$$-\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{3b^5(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)}+\frac{d^5}{2(bc-ad)^3(de-cf)^3(c+dx)^2}+\frac{3d^5(b}{(bc-ad}$$

```
[Out] -1/2*b^5/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2+3*b^5*(-2*a*d*f+b*c*f+b*d*e)/(
-a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)+1/2*d^5/(-a*d+b*c)^3/(-c*f+d*e)^3/(d*x+c)^
2+3*d^5*(a*d*f-2*b*c*f+b*d*e)/(-a*d+b*c)^4/(-c*f+d*e)^4/(d*x+c)-1/2*f^5/(-a
*f+b*e)^3/(-c*f+d*e)^3/(f*x+e)^2-3*f^5*(-a*d*f-b*c*f+2*b*d*e)/(-a*f+b*e)^4/
(-c*f+d*e)^4/(f*x+e)+3*b^5*(7*a^2*d^2*f^2-7*a*b*d*f*(c*f+d*e)+b^2*(2*c^2*f^
2+3*c*d*e*f+2*d^2*e^2))*ln(b*x+a)/(-a*d+b*c)^5/(-a*f+b*e)^5-3*d^5*(2*a^2*d^
2*f^2+a*b*d*f*(-7*c*f+3*d*e)+b^2*(7*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*ln(d*x+c)
/(-a*d+b*c)^5/(-c*f+d*e)^5+3*f^5*(2*a^2*d^2*f^2-a*b*d*f*(-3*c*f+7*d*e)+b^2*
(2*c^2*f^2-7*c*d*e*f+7*d^2*e^2))*ln(f*x+e)/(-a*f+b*e)^5/(-c*f+d*e)^5
```

Rubi [A]
time = 1.06, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = $\{2083\}$

$$\frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2(2c^2f^2-7cdef+7d^2e^2)\right)}{(be-af)^5(de-cf)^5}-\frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2(7c^2f^2-7cdef+2d^2e^2)\right)}{(bc-ad)^5(de-cf)^5}+\frac{3b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2(2c^2f^2+3cdef+2d^2e^2)\right)}{(bc-ad)^5(be-af)^5}+\frac{3b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4}-\frac{b^5}{2(a+bx)^2(bc-ad)^3(be-af)^3}+\frac{3d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4}+\frac{d^5}{2(c+dx)^2(bc-ad)^3(de-cf)^3}-\frac{3f^5(-adf-bcf+2bde)}{(e+fx)(be-af)^4(de-cf)^4}-\frac{f^5}{2(e+fx)^2(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-3),x]
```

```
[Out] -1/2*b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f
- 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*
(d*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)
^4*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^
2) - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*
x)) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*
d*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2
*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2
*f^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2
- a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e
+ f*x])/((b*e - a*f)^5*(d*e - c*f)^5)
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \int \left(\frac{b^6}{(bc-ad)^3(be-af)^3(a+bx)^3} - \right.$$
$$= -\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2} + \frac{}{(b}$$

Mathematica [A]
time = 0.95, size = 490, normalized size = 0.99

$$\frac{1}{2}\left(-\frac{b^5}{(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{6b^5(bde+bcf-2adf)}{(bc-ad)^4(be-af)^4(a+bx)}-\frac{d^5}{(bc-ad)^3(-de+cf)^3(c+dx)^2}+\frac{6d^5(bde-2bcf+adf)}{(bc-ad)^4(de-cf)^4(c+dx)}-\frac{f^5}{(be-af)^3(de-cf)^3(e+fx)^2}+\frac{6f^5(-2bde+bcf+adf)}{(be-af)^4(de-cf)^4(e+fx)}+\frac{6b^5(7a^2d^2f^2-7abdf(de+cf)+b^2(2d^2e^2+3cdef+2c^2f^2))\log(a+bx)}{(bc-ad)^5(be-af)^5}+\frac{6d^5(2a^2d^2f^2+abdf(3de-7cf)+b^2(2d^2e^2-7cdef+7c^2f^2))\log(c+dx)}{(bc-ad)^5(-de+cf)^5}+\frac{6f^5(2a^2d^2f^2+abdf(-7de+3cf)+b^2(7d^2e^2-7cdef+2c^2f^2))\log(e+fx)}{(be-af)^5(de-cf)^5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-3),x]
```

```
[Out] (-(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f -
 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(-(
d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)
^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x
)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d
*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^5*(2*
a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*
f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a^2*d^2*f^
2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log
[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2
```

Maple [A]
time = 1.13, size = 514, normalized size = 1.04

method	result
default	$-\frac{f^5}{2(cf-de)^3(fa-eb)^3(fx+e)^2}+\frac{3f^5(daf+fbc-2deb)}{(cf-de)^4(fa-eb)^4(fx+e)}+\frac{3f^5(2a^2d^2f^2+3abcd\,f^2-7ab\,d^2ef+2b^2c^2f^2-7b^2cdef+7b^2d^2e^2)}{(cf-de)^5(fa-eb)^5}$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/2*f^5/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^2+3*f^5*(a*d*f+b*c*f-2*b*d*e)/(c*f
-d*e)^4/(a*f-b*e)^4/(f*x+e)+3*f^5*(2*a^2*d^2*f^2+3*a*b*c*d*f^2-7*a*b*d^2*e*
```

```
f+2*b^2*c^2*f^2-7*b^2*c*d*e*f+7*b^2*d^2*e^2)/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x
+e)+1/2*d^5/(c*f-d*e)^3/(a*d-b*c)^3/(d*x+c)^2+3*d^5*(a*d*f-2*b*c*f+b*d*e)/(
c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)-3*d^5*(2*a^2*d^2*f^2-7*a*b*c*d*f^2+3*a*b*d^2
*e*f+7*b^2*c^2*f^2-7*b^2*c*d*e*f+2*b^2*d^2*e^2)/(c*f-d*e)^5/(a*d-b*c)^5*ln(
d*x+c)-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2-3*b^5*(2*a*d*f-b*c*f-b*d*e
)/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)+3*b^5*(7*a^2*d^2*f^2-7*a*b*c*d*f^2-7*a*b*
d^2*e*f+2*b^2*c^2*f^2+3*b^2*c*d*e*f+2*b^2*d^2*e^2)/(a*f-b*e)^5/(a*d-b*c)^5*
ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 12087 vs. $2(510) = 1020$.
time = 1.12, size = 12087, normalized size = 24.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="maxima")
```

```
[Out] 3*(2*b^7*d^2*e^2 + (2*b^7*c^2 - 7*a*b^6*c*d + 7*a^2*b^5*d^2)*f^2 + (3*b^7*c
*d*e - 7*a*b^6*d^2*e)*f)*log(b*x + a)/(b^10*c^5*e^5 - 5*a*b^9*c^4*d*e^5 + 1
0*a^2*b^8*c^3*d^2*e^5 - 10*a^3*b^7*c^2*d^3*e^5 + 5*a^4*b^6*c*d^4*e^5 - a^5*
b^5*d^5*e^5 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*
b^2*c^2*d^3 + 5*a^9*b*c*d^4 - a^10*d^5)*f^5 + 5*(a^4*b^6*c^5*e - 5*a^5*b^5*
c^4*d*e + 10*a^6*b^4*c^3*d^2*e - 10*a^7*b^3*c^2*d^3*e + 5*a^8*b^2*c*d^4*e -
 a^9*b*d^5*e)*f^4 - 10*(a^3*b^7*c^5*e^2 - 5*a^4*b^6*c^4*d*e^2 + 10*a^5*b^5*
c^3*d^2*e^2 - 10*a^6*b^4*c^2*d^3*e^2 + 5*a^7*b^3*c*d^4*e^2 - a^8*b^2*d^5*e^
2)*f^3 + 10*(a^2*b^8*c^5*e^3 - 5*a^3*b^7*c^4*d*e^3 + 10*a^4*b^6*c^3*d^2*e^3
 - 10*a^5*b^5*c^2*d^3*e^3 + 5*a^6*b^4*c*d^4*e^3 - a^7*b^3*d^5*e^3)*f^2 - 5*
(a*b^9*c^5*e^4 - 5*a^2*b^8*c^4*d*e^4 + 10*a^3*b^7*c^3*d^2*e^4 - 10*a^4*b^6*
c^2*d^3*e^4 + 5*a^5*b^5*c*d^4*e^4 - a^6*b^4*d^5*e^4)*f) - 3*(2*b^2*d^7*e^2
+ (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2 - (7*b^2*c*d^6*e - 3*a*b*d^
7*e)*f)*log(d*x + c)/(b^5*c^5*d^5*e^5 - 5*a*b^4*c^4*d^6*e^5 + 10*a^2*b^3*c^
3*d^7*e^5 - 10*a^3*b^2*c^2*d^8*e^5 + 5*a^4*b*c*d^9*e^5 - a^5*d^10*e^5 - (b^
5*c^10 - 5*a*b^4*c^9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*
c^6*d^4 - a^5*c^5*d^5)*f^5 + 5*(b^5*c^9*d*e - 5*a*b^4*c^8*d^2*e + 10*a^2*b^
3*c^7*d^3*e - 10*a^3*b^2*c^6*d^4*e + 5*a^4*b*c^5*d^5*e - a^5*c^4*d^6*e)*f^4
 - 10*(b^5*c^8*d^2*e^2 - 5*a*b^4*c^7*d^3*e^2 + 10*a^2*b^3*c^6*d^4*e^2 - 10*
a^3*b^2*c^5*d^5*e^2 + 5*a^4*b*c^4*d^6*e^2 - a^5*c^3*d^7*e^2)*f^3 + 10*(b^5*
c^7*d^3*e^3 - 5*a*b^4*c^6*d^4*e^3 + 10*a^2*b^3*c^5*d^5*e^3 - 10*a^3*b^2*c^4
*d^6*e^3 + 5*a^4*b*c^3*d^7*e^3 - a^5*c^2*d^8*e^3)*f^2 - 5*(b^5*c^6*d^4*e^4
- 5*a*b^4*c^5*d^5*e^4 + 10*a^2*b^3*c^4*d^6*e^4 - 10*a^3*b^2*c^3*d^7*e^4 + 5
*a^4*b*c^2*d^8*e^4 - a^5*c*d^9*e^4)*f) + 3*(7*b^2*d^2*f^5*e^2 + (2*b^2*c^2
+ 3*a*b*c*d + 2*a^2*d^2)*f^7 - 7*(b^2*c*d*e + a*b*d^2*e)*f^6)*log(f*x + e)/
(a^5*c^5*f^10 + b^5*d^5*e^10 - 5*(a^4*b*c^5*e + a^5*c^4*d*e)*f^9 + 5*(2*a^3
```

```
*b^2*c^5*e^2 + 5*a^4*b*c^4*d*e^2 + 2*a^5*c^3*d^2*e^2)*f^8 - 10*(a^2*b^3*c^5
*e^3 + 5*a^3*b^2*c^4*d*e^3 + 5*a^4*b*c^3*d^2*e^3 + a^5*c^2*d^3*e^3)*f^7 + 5
*(a*b^4*c^5*e^4 + 10*a^2*b^3*c^4*d*e^4 + 20*a^3*b^2*c^3*d^2*e^4 + 10*a^4*b*
c^2*d^3*e^4 + a^5*c*d^4*e^4)*f^6 - (b^5*c^5*e^5 + 25*a*b^4*c^4*d*e^5 + 100*
a^2*b^3*c^3*d^2*e^5 + 100*a^3*b^2*c^2*d^3*e^5 + 25*a^4*b*c*d^4*e^5 + a^5*d^
5*e^5)*f^5 + 5*(b^5*c^4*d*e^6 + 10*a*b^4*c^3*d^2*e^6 + 20*a^2*b^3*c^2*d^3*e
^6 + 10*a^3*b^2*c*d^4*e^6 + a^4*b*d^5*e^6)*f^4 - 10*(b^5*c^3*d^2*e^7 + 5*a*
b^4*c^2*d^3*e^7 + 5*a^2*b^3*c*d^4*e^7 + a^3*b^2*d^5*e^7)*f^3 + 5*(2*b^5*c^2
*d^3*e^8 + 5*a*b^4*c*d^4*e^8 + 2*a^2*b^3*d^5*e^8)*f^2 - 5*(b^5*c*d^4*e^9 +
a*b^4*d^5*e^9)*f) - 1/2*(b^7*c^3*d^4*e^7 - 7*a*b^6*c^2*d^5*e^7 - 7*a^2*b^5*
c*d^6*e^7 + a^3*b^4*d^7*e^7 + (a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^
5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*f^7 - 7*(a^2*b^5*c^7*e - 3*a^3*b^4*c
^6*d*e + 2*a^4*b^3*c^5*d^2*e + 2*a^5*b^2*c^4*d^3*e - 3*a^6*b*c^3*d^4*e + a^
7*c^2*d^5*e)*f^6 - (7*a*b^6*c^7*e^2 - 26*a^2*b^5*c^6*d*e^2 + 52*a^3*b^4*c^5
*d^2*e^2 - 78*a^4*b^3*c^4*d^3*e^2 + 52*a^5*b^2*c^3*d^4*e^2 - 26*a^6*b*c^2*d
^5*e^2 + 7*a^7*c*d^6*e^2)*f^5 - 6*(2*b^7*d^7*f^2*e^5 + (2*b^7*c^5*d^2 - 5*a
*b^6*c^4*d^3 + 2*a^2*b^5*c^3*d^4 + 2*a^3*b^4*c^2*d^5 - 5*a^4*b^3*c*d^6 + 2*
a^5*b^2*d^7)*f^7 - (5*b^7*c^4*d^3*e - 16*a*b^6*c^3*d^4*e + 12*a^2*b^5*c^2*d
^5*e - 16*a^3*b^4*c*d^6*e + 5*a^4*b^3*d^7*e)*f^6 + 2*(b^7*c^3*d^4*e^2 - 6*a
*b^6*c^2*d^5*e^2 - 6*a^2*b^5*c*d^6*e^2 + a^3*b^4*d^7*e^2)*f^5 + 2*(b^7*c^2*
d^5*e^3 + 8*a*b^6*c*d^6*e^3 + a^2*b^5*d^7*e^3)*f^4 - 5*(b^7*c*d^6*e^4 + a*b
^6*d^7*e^4)*f^3)*x^5 + (b^7*c^7*e^3 + 21*a*b^6*c^6*d*e^3 - 52*a^2*b^5*c^5*d
^2*e^3 - 52*a^5*b^2*c^2*d^5*e^3 + 21*a^6*b*c*d^6*e^3 + a^7*d^7*e^3)*f^4 - 3
*(8*b^7*d^7*f*e^6 + (8*b^7*c^6*d - 14*a*b^6*c^5*d^2 - 7*a^2*b^5*c^4*d^3 + 1
4*a^3*b^4*c^3*d^4 - 7*a^4*b^3*c^2*d^5 - 14*a^5*b^2*c*d^6 + 8*a^6*b*d^7)*f^7
 - 2*(7*b^7*c^5*d^2*e - 17*a*b^6*c^4*d^3*e - 3*a^2*b^5*c^3*d^4*e - 3*a^3*b^
4*c^2*d^5*e - 17*a^4*b^3*c*d^6*e + 7*a^5*b^2*d^7*e)*f^6 - (7*b^7*c^4*d^3*e^
2 - 6*a*b^6*c^3*d^4*e^2 + 78*a^2*b^5*c^2*d^5*e^2 - 6*a^3*b^4*c*d^6*e^2 + 7*
a^4*b^3*d^7*e^2)*f^5 + 2*(7*b^7*c^3*d^4*e^3 + 3*a*b^6*c^2*d^5*e^3 + 3*a^2*b
^5*c*d^6*e^3 + 7*a^3*b^4*d^7*e^3)*f^4 - (7*b^7*c^2*d^5*e^4 - 34*a*b^6*c*d^6
*e^4 + 7*a^2*b^5*d^7*e^4)*f^3 - 14*(b^7*c*d^6*e^5 + a*b^6*d^7*e^5)*f^2)*x^4
 - 2*(2*b^7*c^6*d*e^4 + 7*a*b^6*c^5*d^2*e^4 - 39*a^2*b^5*c^4*d^3*e^4 - 39*a
^4*b^3*c^2*d^5*e^4 + 7*a^5*b^2*c*d^6*e^4 + 2*a^6*b*d^7*e^4)*f^3 - 2*(6*b^7*
d^7*e^7 + (6*b^7*c^7 + 3*a*b^6*c^6*d - 37*a^2*b^5*c^5*d^2 + 19*a^3*b^4*c^4*
d^3 + 19*a^4*b^3*c^3*d^4 - 37*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d^6 + 6*a^7*d^7)*
f^7 + (3*b^7*c^6*d*e - 28*a*b^6*c^5*d^2*e + 86*a^2*b^5*c^4*d^3*e - 68*a^3*b
^4*c^3*d^4*e + 86*a^4*b^3*c^2*d^5*e - 28*a^5*b^2*c*d^6*e + 3*a^6*b*d^7*e)*f
^6 - (37*b^7*c^5*d^2*e^2 - 86*a*b^6*c^4*d^3*e^2 + 52*a^2*b^5*c^3*d^4*e^2 +
52*a^3*b^4*c^2*d^5*e^2 - 86*a^4*b^3*c*d^6*e^2 + 37*a^5*b^2*d^7*e^2)*f^5 + (
19*b^7*c^4*d^3*e^3 - 68*a*b^6*c^3*d^4*e^3 - 52*a^2*b^5*c^2*d^5*e^3 - 68*a^3
*b^4*c*d^6*e^3 + 19*a^4*b^3*d^7*e^3)*f^4 + (19*...
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6908 vs. $2(510) = 1020$.
time = 4.05, size = 6908, normalized size = 13.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="giac")
```

```
[Out] -3*(2*b^8*c^2*f^2 - 7*a*b^7*c*d*f^2 + 7*a^2*b^6*d^2*f^2 + 3*b^8*c*d*f*e - 7
*a*b^7*d^2*f*e + 2*b^8*d^2*e^2)*log(abs(b*x + a))/(a^5*b^6*c^5*f^5 - 5*a^6*
b^5*c^4*d*f^5 + 10*a^7*b^4*c^3*d^2*f^5 - 10*a^8*b^3*c^2*d^3*f^5 + 5*a^9*b^2
*c*d^4*f^5 - a^10*b*d^5*f^5 - 5*a^4*b^7*c^5*f^4*e + 25*a^5*b^6*c^4*d*f^4*e
- 50*a^6*b^5*c^3*d^2*f^4*e + 50*a^7*b^4*c^2*d^3*f^4*e - 25*a^8*b^3*c*d^4*f^
4*e + 5*a^9*b^2*d^5*f^4*e + 10*a^3*b^8*c^5*f^3*e^2 - 50*a^4*b^7*c^4*d*f^3*e
^2 + 100*a^5*b^6*c^3*d^2*f^3*e^2 - 100*a^6*b^5*c^2*d^3*f^3*e^2 + 50*a^7*b^4
*c*d^4*f^3*e^2 - 10*a^8*b^3*d^5*f^3*e^2 - 10*a^2*b^9*c^5*f^2*e^3 + 50*a^3*b
^8*c^4*d*f^2*e^3 - 100*a^4*b^7*c^3*d^2*f^2*e^3 + 100*a^5*b^6*c^2*d^3*f^2*e^
3 - 50*a^6*b^5*c*d^4*f^2*e^3 + 10*a^7*b^4*d^5*f^2*e^3 + 5*a*b^10*c^5*f*e^4
- 25*a^2*b^9*c^4*d*f*e^4 + 50*a^3*b^8*c^3*d^2*f*e^4 - 50*a^4*b^7*c^2*d^3*f*
e^4 + 25*a^5*b^6*c*d^4*f*e^4 - 5*a^6*b^5*d^5*f*e^4 - b^11*c^5*e^5 + 5*a*b^1
0*c^4*d*e^5 - 10*a^2*b^9*c^3*d^2*e^5 + 10*a^3*b^8*c^2*d^3*e^5 - 5*a^4*b^7*c
*d^4*e^5 + a^5*b^6*d^5*e^5) + 3*(7*b^2*c^2*d^6*f^2 - 7*a*b*c*d^7*f^2 + 2*a^
2*d^8*f^2 - 7*b^2*c*d^7*f*e + 3*a*b*d^8*f*e + 2*b^2*d^8*e^2)*log(abs(d*x +
c))/(b^5*c^10*d*f^5 - 5*a*b^4*c^9*d^2*f^5 + 10*a^2*b^3*c^8*d^3*f^5 - 10*a^3
*b^2*c^7*d^4*f^5 + 5*a^4*b*c^6*d^5*f^5 - a^5*c^5*d^6*f^5 - 5*b^5*c^9*d^2*f^
4*e + 25*a*b^4*c^8*d^3*f^4*e - 50*a^2*b^3*c^7*d^4*f^4*e + 50*a^3*b^2*c^6*d^
```

5*f^4*e - 25*a^4*b*c^5*d^6*f^4*e + 5*a^5*c^4*d^7*f^4*e + 10*b^5*c^8*d^3*f^3
*e^2 - 50*a*b^4*c^7*d^4*f^3*e^2 + 100*a^2*b^3*c^6*d^5*f^3*e^2 - 100*a^3*b^2
*c^5*d^6*f^3*e^2 + 50*a^4*b*c^4*d^7*f^3*e^2 - 10*a^5*c^3*d^8*f^3*e^2 - 10*b
^5*c^7*d^4*f^2*e^3 + 50*a*b^4*c^6*d^5*f^2*e^3 - 100*a^2*b^3*c^5*d^6*f^2*e^3
 + 100*a^3*b^2*c^4*d^7*f^2*e^3 - 50*a^4*b*c^3*d^8*f^2*e^3 + 10*a^5*c^2*d^9*
f^2*e^3 + 5*b^5*c^6*d^5*f*e^4 - 25*a*b^4*c^5*d^6*f*e^4 + 50*a^2*b^3*c^4*d^7
*f*e^4 - 50*a^3*b^2*c^3*d^8*f*e^4 + 25*a^4*b*c^2*d^9*f*e^4 - 5*a^5*c*d^10*f
*e^4 - b^5*c^5*d^6*e^5 + 5*a*b^4*c^4*d^7*e^5 - 10*a^2*b^3*c^3*d^8*e^5 + 10*
a^3*b^2*c^2*d^9*e^5 - 5*a^4*b*c*d^10*e^5 + a^5*d^11*e^5) + 3*(2*b^2*c^2*f^8
 + 3*a*b*c*d*f^8 + 2*a^2*d^2*f^8 - 7*b^2*c*d*f^7*e - 7*a*b*d^2*f^7*e + 7*b^
2*d^2*f^6*e^2)*log(abs(f*x + e))/(a^5*c^5*f^11 - 5*a^4*b*c^5*f^10*e - 5*a^5
*c^4*d*f^10*e + 10*a^3*b^2*c^5*f^9*e^2 + 25*a^4*b*c^4*d*f^9*e^2 + 10*a^5*c^
3*d^2*f^9*e^2 - 10*a^2*b^3*c^5*f^8*e^3 - 50*a^3*b^2*c^4*d*f^8*e^3 - 50*a^4*
b*c^3*d^2*f^8*e^3 - 10*a^5*c^2*d^3*f^8*e^3 + 5*a*b^4*c^5*f^7*e^4 + 50*a^2*b
^3*c^4*d*f^7*e^4 + 100*a^3*b^2*c^3*d^2*f^7*e^4 + 50*a^4*b*c^2*d^3*f^7*e^4 +
 5*a^5*c*d^4*f^7*e^4 - b^5*c^5*f^6*e^5 - 25*a*b^4*c^4*d*f^6*e^5 - 100*a^2*b
^3*c^3*d^2*f^6*e^5 - 100*a^3*b^2*c^2*d^3*f^6*e^5 - 25*a^4*b*c*d^4*f^6*e^5 -
 a^5*d^5*f^6*e^5 + 5*b^5*c^4*d*f^5*e^6 + 50*a*b^4*c^3*d^2*f^5*e^6 + 100*a^2
*b^3*c^2*d^3*f^5*e^6 + 50*a^3*b^2*c*d^4*f^5*e^6 + 5*a^4*b*d^5*f^5*e^6 - 10*
b^5*c^3*d^2*f^4*e^7 - 50*a*b^4*c^2*d^3*f^4*e^7 - 50*a^2*b^3*c*d^4*f^4*e^7 -
 10*a^3*b^2*d^5*f^4*e^7 + 10*b^5*c^2*d^3*f^3*e^8 + 25*a*b^4*c*d^4*f^3*e^8 +
 10*a^2*b^3*d^5*f^3*e^8 - 5*b^5*c*d^4*f^2*e^9 - 5*a*b^4*d^5*f^2*e^9 + b^5*d
^5*f*e^10) + 1/2*(12*b^7*c^5*d^2*f^7*x^5 - 30*a*b^6*c^4*d^3*f^7*x^5 + 12*a^
2*b^5*c^3*d^4*f^7*x^5 + 12*a^3*b^4*c^2*d^5*f^7*x^5 - 30*a^4*b^3*c*d^6*f^7*x
^5 + 12*a^5*b^2*d^7*f^7*x^5 - 30*b^7*c^4*d^3*f^6*x^5*e + 96*a*b^6*c^3*d^4*f
^6*x^5*e - 72*a^2*b^5*c^2*d^5*f^6*x^5*e + 96*a^3*b^4*c*d^6*f^6*x^5*e - 30*a
^4*b^3*d^7*f^6*x^5*e + 24*b^7*c^6*d*f^7*x^4 - 42*a*b^6*c^5*d^2*f^7*x^4 - 21
*a^2*b^5*c^4*d^3*f^7*x^4 + 42*a^3*b^4*c^3*d^4*f^7*x^4 - 21*a^4*b^3*c^2*d^5*
f^7*x^4 - 42*a^5*b^2*c*d^6*f^7*x^4 + 24*a^6*b*d^7*f^7*x^4 + 12*b^7*c^3*d^4*
f^5*x^5*e^2 - 72*a*b^6*c^2*d^5*f^5*x^5*e^2 - 72*a^2*b^5*c*d^6*f^5*x^5*e^2 +
 12*a^3*b^4*d^7*f^5*x^5*e^2 - 42*b^7*c^5*d^2*f^6*x^4*e + 102*a*b^6*c^4*d^3*
f^6*x^4*e + 18*a^2*b^5*c^3*d^4*f^6*x^4*e + 18*a^3*b^4*c^2*d^5*f^6*x^4*e + 1
02*a^4*b^3*c*d^6*f^6*x^4*e - 42*a^5*b^2*d^7*f^6*x^4*e + 12*b^7*c^7*f^7*x^3
+ 6*a*b^6*c^6*d*f^7*x^3 - 74*a^2*b^5*c^5*d^2*f^7*x^3 + 38*a^3*b^4*c^4*d^3*f
^7*x^3 + 38*a^4*b^3*c^3*d^4*f^7*x^3 - 74*a^5*b^2*c^2*d^5*f^7*x^3 + 6*a^6*b*
c*d^6*f^7*x^3 + 12*a^7*d^7*f^7*x^3 + 12*b^7*c^2*d^5*f^4*x^5*e^3 + 96*a*b^6*
c*d^6*f^4*x^5*e^3 + 12*a^2*b^5*d^7*f^4*x^5*e^3 - 21*b^7*c^4*d^3*f^5*x^4*e^2
 + 18*a*b^6*c^3*d^4*f^5*x^4*e^2 - 234*a^2*b^5*c^2*d^5*f^5*x^4*e^2 + 18*a^3*
b^4*c*d^6*f^5*x^4*e^2 - 21*a^4*b^3*d^7*f^5*x^4*e^2 + 6*b^7*c^6*d*f^6*x^3*e
- 56*a*b^6*c^5*d^2*f^6*x^3*e + 172*a^2*b^5*c^4*d^3*f^6*x^3*e - 136*a^3*b^4*
c^3*d^4*f^6*x^3*e + 172*a^4*b^3*c^2*d^5*f^6*x^3*e - 56*a^5*b^2*c*d^6*f^6*x^
3*e + 6*a^6*b*d^7*f^6*x^3*e + 18*a*b^6*c^7*f^7*x^2 - 37*a^2*b^5*c^6*d*f^7*x
^2 - 3*a^3*b^4*c^5*d^2*f^7*x^2 + 32*a^4*b^3*c^4*d^3*f^7*x^2 - 3*a^5*b^2*c^3
*d^4*f^7*x^2 - 37*a^6*b*c^2*d^5*f^7*x^2 + 18*a^7*c*d^6*f^7*x^2 - 30*b^7*c*d
^6*f^3*x^5*e^4 - 30*a*b^6*d^7*f^3*x^5*e^4 + 42*b^7*c^3*d^4*f^4*x^4*e^3 + 18

```
*a*b^6*c^2*d^5*f^4*x^4*e^3 + 18*a^2*b^5*c*d^6*f^4*x^4*e^3 + 42*a^3*b^4*d^7*
f^4*x^4*e^3 - 74*b^7*c^5*d^2*f^5*x^3*e^2 + 172*a*b^6*c^4*d^3*f^5*x^3*e^2 -
104*a^2*b^5*c^3*d^4*f^5*x^3*e^2 - 104*a^3*b^4*c^2*d^5*f^5*x^3*e^2 + 172*a^4
*b^3*c*d^6*f^5*x^3*e^2 - 74*a^5*b^2*d^7*f^5*x^3...
```

Mupad [B]
time = 20.46, size = 2500, normalized size = 5.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^3,x)
```

```
[Out] symsum(log(root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8*
c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^11
*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573300
*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3 - 3
43980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*f^10*
z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d^11*e^
11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^11*c^10*
d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340*a^13*b^
7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3 + 326340*
a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f^10*z^3 +
326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^10*e^8*f^12
*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^10*c^12*d^8*e
^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a^8*b^12*c^12
*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 245700*a^14*b
^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^3 + 245700*a
^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^12*z^3 + 245
700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e^12*f^8*z^3
- 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d^12*e^9*f^11
*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*c^7*d^13*e^1
1*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11*b^9*c^7*d^1
3*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100*a^9*b^11*c^
8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 191100*a^8*b^
12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^3 - 191100*a
^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^13*z^3 - 123
900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e^6*f^14*z^3
- 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*d^7*e^6*f^14
*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*c^14*d^6*e^7
*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*b^13*c^14*d^
6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900*a^6*b^14*c^
13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 101700*a^15*b
^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^3 + 101700*a
```

```
^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f^9*z^3 + 101
700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e^14*f^6*z^3
+ 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d^14*e^15*f^5
*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*c^9*d^11*e^1
5*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*b^15*c^11*d^
9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*a^14*b^6*c^6
*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 65820*a^10*b^
10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^3 - 65820*a^
6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^13*z^3 - 5670
0*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*f^12*z^3 + 5
6700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e^5*f^15*z^3
- 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^16*e^13*f^7*z
^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*d^15*e^13*f^
7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c^7*d^13*e^15
*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^13*c^15*d^5*e
^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7*b^13*c^7*d^1
3*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a^5*b^15*c^12
*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 48252*a^15*b^5*c
^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 - 48252*a^10*
b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5*z^3 - 48252*
a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15*f^5*z^3 - 3
2400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e^8*f^12*z^3
- 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^16*e^12*f^8*z
^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4*d^16*e^14*f^
6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c^6*d^14*e^16
*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^14*c^8*d^12*e
^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4*b^16*c^12*d^
8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565*a^16*b^4*c^4
*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 20565*a^10*b^
10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^3 + 20565*a^
4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^15*z^3 + 1566
0*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3*f^17*z^3 + 1
5660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e^3*f^17*z^3
+ 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^3*e^5*f^15*z
^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*a^5*b^15*c^17*d^3*e^8*f^1
2*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + ...
```

3.21 $\int \frac{1}{1+x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {2083, 649, 209, 266}

$$\frac{\text{ArcTan}(x)}{2}-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x+x^2+x^3}\,dx &= \int \left(\frac{1}{2(1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right)\,dx\\ &= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1-x}{1+x^2}\,dx\\ &= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.80

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln(x^2+1)}{4}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A]
time = 0.47, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A]
time = 0.40, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.76

$$\frac{\log\left(x+1\right)}{2}-\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2+x+1),x)
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Giac [A]
time = 4.47, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

Mupad [B]
time = 2.20, size = 25, normalized size = 1.00

$$\frac{\ln\left(x+1\right)}{2}+\ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2 + x^3 + 1),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.22 $\int \frac{1}{-1+4x-4x^2+16x^3}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{10}\tan^{-1}(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)$$

```
[Out] -1/10*arctan(2*x)+1/5*ln(1-4*x)-1/10*ln(4*x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {2083, 649, 209, 266}

$$-\frac{1}{10}\text{ArcTan}(2x)-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

```
[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{-1+4x-4x^2+16x^3}\,dx = \int \left(\frac{4}{5(-1+4x)}+\frac{-1-4x}{5\,(1+4x^2)}\right)dx$$
$$= \frac{1}{5}\log(1-4x)+\frac{1}{5}\int \frac{-1-4x}{1+4x^2}\,dx$$
$$= \frac{1}{5}\log(1-4x)-\frac{1}{5}\int \frac{1}{1+4x^2}\,dx-\frac{4}{5}\int \frac{x}{1+4x^2}\,dx$$
$$= -\frac{1}{10}\tan^{-1}(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{10}\tan^{-1}(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

```
[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.84

method	result	size
default	$-\frac{\ln(4x^2+1)}{10}-\frac{\arctan(2x)}{10}+\frac{\ln(4x-1)}{5}$	26
risch	$-\frac{\ln(4x^2+1)}{10}-\frac{\arctan(2x)}{10}+\frac{\ln(4x-1)}{5}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(16*x^3-4*x^2+4*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*ln(4*x^2+1)-1/10*arctan(2*x)+1/5*ln(4*x-1)
```

Maxima [A]
time = 0.49, size = 25, normalized size = 0.81

$$-\frac{1}{10}\arctan\left(2\,x\right)-\frac{1}{10}\log\left(4\,x^2+1\right)+\frac{1}{5}\log\left(4\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")
```

```
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Fricas [A]
time = 0.38, size = 25, normalized size = 0.81

$$-\frac{1}{10}\arctan\left(2\,x\right)-\frac{1}{10}\log\left(4\,x^{2}+1\right)+\frac{1}{5}\log\left(4\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="fricas")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.77

$$\frac{\log\left(x-\frac{1}{4}\right)}{5}-\frac{\log\left(x^{2}+\frac{1}{4}\right)}{10}-\frac{\operatorname{atan}\left(2x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x**3-4*x**2+4*x-1),x)
[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

Giac [A]
time = 4.11, size = 26, normalized size = 0.84

$$-\frac{1}{10}\arctan\left(2\,x\right)-\frac{1}{10}\log\left(4\,x^{2}+1\right)+\frac{1}{5}\log\left(|4\,x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(x-\frac{1}{4}\right)}{5}+\ln\left(x-\frac{1}{2}\mathrm{i}\right)\,\left(-\frac{1}{10}+\frac{1}{20}\mathrm{i}\right)+\ln\left(x+\frac{1}{2}\mathrm{i}\right)\,\left(-\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 4*x^2 + 16*x^3 - 1),x)
[Out] log(x - 1/4)/5 - log(x - 1i/2)*(1/10 - 1i/20) - log(x + 1i/2)*(1/10 + 1i/20
)
```

3.23 $\int \frac{1}{dx^3} dx$

Optimal. Leaf size=10

$$-\frac{1}{2dx^2}$$

[Out] -1/2/d/x^2

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {12, 30}

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(d*x^3),x]

[Out] -1/2*1/(d*x^2)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{dx^3} dx &= \frac{\int \frac{1}{x^3} dx}{d}\\ &= -\frac{1}{2dx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(d*x^3),x]
```

```
[Out] -1/2*1/(d*x^2)
```

Maple [A]
time = 0.01, size = 9, normalized size = 0.90

method	result	size
gosper	$-\frac{1}{2 d \,x^{2}}$	9
default	$-\frac{1}{2 d \,x^{2}}$	9
norman	$-\frac{1}{2 d \,x^{2}}$	9
risch	$-\frac{1}{2 d \,x^{2}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/d/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/x^2
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/(d*x^2)
```

Fricas [A]
time = 0.38, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="fricas")
```

```
[Out] -1/2/(d*x^2)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x**3,x)
```

```
[Out] -1/(2*d*x**2)
```

Giac [A]
time = 3.74, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="giac")
```

```
[Out] -1/2/(d*x^2)
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{2\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3),x)
```

```
[Out] -1/(2*d*x^2)
```

3.24 $\int \frac{1}{cx^2+dx^3}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}$$

```
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 46}

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{cx^2+dx^3}\,dx &= \int \frac{1}{x^2(c+dx)}\,dx\\ &= \int \left(\frac{1}{cx^2}-\frac{d}{c^2x}+\frac{d^2}{c^2(c+dx)}\right)dx\\ &= -\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^(-1),x]
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Maple [A]
time = 0.20, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(dx+c)}{c^2}$	29
norman	$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(dx+c)}{c^2}$	29
risch	$-\frac{1}{cx}-\frac{d\ln(x)}{c^2}+\frac{d\ln(-dx-c)}{c^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2),x,method=_RETURNVERBOSE)
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

Maxima [A]
time = 0.27, size = 28, normalized size = 1.00

$$\frac{d\log(dx+c)}{c^2}-\frac{d\log(x)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")
[Out] d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)
```

Fricas [A]
time = 0.38, size = 26, normalized size = 0.93

$$\frac{dx\log(dx+c)-dx\log(x)-c}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")
[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)
```

Sympy [A]
time = 0.06, size = 19, normalized size = 0.68

$$-\frac{1}{cx}+\frac{d\left(-\log\left(x\right)+\log\left(\frac{c}{d}+x\right)\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2),x)
```

```
[Out] -1/(c*x) + d*(-log(x) + log(c/d + x))/c**2
```

Giac [A]
time = 3.77, size = 30, normalized size = 1.07

$$\frac{d\log\left(|dx+c|\right)}{c^2}-\frac{d\log\left(|x|\right)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="giac")
```

```
[Out] d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)
```

Mupad [B]
time = 0.06, size = 25, normalized size = 0.89

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{2\,d\,x}{c}+1\right)}{c^2}-\frac{1}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2 + d*x^3),x)
```

```
[Out] (2*d*atanh((2*d*x)/c + 1))/c^2 - 1/(c*x)
```

3.25 $\int \frac{1}{bx+dx^3}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

[Out] ln(x)/b-1/2*ln(d*x^2+b)/b

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1607, 272, 36, 29, 31}

$$\frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(b*x + d*x^3)^(-1),x]

[Out] Log[x]/b - Log[b + d*x^2]/(2*b)

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{bx+dx^3}\,dx &= \int \frac{1}{x\,(b+dx^2)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(b+dx)}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2b}-\frac{d\text{Subst}\left(\int \frac{1}{b+dx}\,dx,x,x^2\right)}{2b}\\ &= \frac{\log(x)}{b}-\frac{\log(b+dx^2)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b}-\frac{\log(b+dx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)^(-1),x]
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Maple [A]
time = 0.19, size = 21, normalized size = 0.95

method	result	size
default	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^2+b)}{2b}$	21
norman	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^2+b)}{2b}$	21
risch	$\frac{\ln(x)}{b}-\frac{\ln(d\,x^2+b)}{2b}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

Maxima [A]
time = 0.26, size = 20, normalized size = 0.91

$$-\frac{\log(dx^2+b)}{2\,b}+\frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(d*x^2 + b)/b + log(x)/b
```

Fricas [A]
time = 0.38, size = 18, normalized size = 0.82

$$-\frac{\log\left(dx^2+b\right)-2\,\log\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.68

$$\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{b}{d}+x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+b*x),x)
```

```
[Out] log(x)/b - log(b/d + x**2)/(2*b)
```

Giac [A]
time = 4.48, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,b}-\frac{\log\left(\left|dx^2+b\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b
```

Mupad [B]
time = 2.13, size = 18, normalized size = 0.82

$$-\frac{\ln\left(d\,x^2+b\right)-2\,\ln\left(x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + d*x^3),x)
```

```
[Out] -(log(b + d*x^2) - 2*log(x))/(2*b)
```

3.26 $\int \frac{1}{bx+cx^2+dx^3} dx$

Optimal. Leaf size=62

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}+\frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}$$

```
[Out] ln(x)/b-1/2*ln(d*x^2+c*x+b)/b+c*arctanh((2*d*x+c)/(-4*b*d+c^2)^(1/2))/b/(-4
*b*d+c^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1608, 719, 29, 648, 632, 212, 642}

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d]) + Log[x]/b
 - Log[b + c*x + d*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{bx+cx^2+dx^3}\,dx &= \int \frac{1}{x(b+cx+dx^2)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{b}+\frac{\int \frac{-c-dx}{b+cx+dx^2}\,dx}{b}\\
&= \frac{\log(x)}{b}-\frac{\int \frac{c+2dx}{b+cx+dx^2}\,dx}{2b}-\frac{c\int \frac{1}{b+cx+dx^2}\,dx}{2b}\\
&= \frac{\log(x)}{b}-\frac{\log(b+cx+dx^2)}{2b}+\frac{c\text{Subst}\left(\int \frac{1}{c^2-4bd-x^2}\,dx,x,c+2dx\right)}{b}\\
&= \frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}+\frac{\log(x)}{b}-\frac{\log(b+cx+dx^2)}{2b}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.98

$$-\frac{\frac{2c\tan^{-1}\left(\frac{c+2dx}{\sqrt{-c^2+4bd}}\right)}{\sqrt{-c^2+4bd}}-2\log(x)+\log(b+x(c+dx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -1/2*((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d] - 2*L
og[x] + Log[b + x*(c + d*x)])/b
```

Maple [A]
time = 0.03, size = 61, normalized size = 0.98

method	result	siz
default	$\frac{\ln(x)}{b}+\frac{-\frac{\ln\left(d\,x^{2}+cx+b\right)}{2}-\frac{c\arctan\left(\frac{2dx+c}{\sqrt{4bd-c^{2}}}\right)}{\sqrt{4bd-c^{2}}}}{b}$	61
risch	$\frac{\ln(x)}{b}+\left(\sum_{\_R=\mathrm{RootOf}\left((4b^{2}d-b\,c^{2})\_Z^{2}+(4bd-c^{2})\_Z+d\right)}\_R\ln\left(\left(\left(6bd-2c^{2}\right)\_R+3d\right)x-bc\_R+c\right)\right)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2+b*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/b+1/b*(-1/2*ln(d*x^2+c*x+b)-c/(4*b*d-c^2)^(1/2)*arctan((2*d*x+c)/(4*b
*d-c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b*d-c^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{c^{2}-4\,bd}\,c\log\left(\frac{2\,d^{2}x^{2}+2\,cdx+c^{2}-2\,bd+\sqrt{c^{2}-4\,bd}\,(2\,dx+c)}{dx^{2}+cx+b}\right)-(c^{2}-4\,bd)\log\left(dx^{2}+cx+b\right)+2\,(c^{2}-4\,bd)\log\left(x\right)}{2\,(bc^{2}-4\,b^{2}d)},\ \frac{2\,\sqrt{-c^{2}+4\,bd}\,c\arctan\left(-\frac{\sqrt{-c^{2}+4\,bd}\,(2\,dx+c)}{c^{2}-4\,bd}\right)-(c^{2}-4\,bd)\log\left(dx^{2}+cx+b\right)+2\,(c^{2}-4\,bd)\log\left(x\right)}{2\,(bc^{2}-4\,b^{2}d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c^2 - 4*b*d)*c*log((2*d^2*x^2 + 2*c*d*x + c^2 - 2*b*d + sqrt(c^2
 - 4*b*d)*(2*d*x + c))/(d*x^2 + c*x + b)) - (c^2 - 4*b*d)*log(d*x^2 + c*x +
 b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d), 1/2*(2*sqrt(-c^2 + 4*b*d)*
c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 - 4*b*d)) - (c^2 - 4*b*d)*log
(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.
time = 4.48, size = 564, normalized size = 9.10

$$\left(-\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)}-\frac{1}{2b}\right)\log\left(x+\frac{24b^4d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-14b^3c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-12b^3d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)+2b^2c^4\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2+3b^2c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)-12b^2d^2+11bc^2d-2c^4}{9bcd^2-2c^3d}\right)+\left(\frac{c\sqrt{-4bd+c^2}}{2b\,(4bd-c^2)}-\frac{1}{2b}\right)\log\left(x+\frac{24b^4d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-14b^3c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2-12b^3d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)+2b^2c^4\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)^2+3b^2c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b(4bd-c^2)}-\frac{1}{2b}\right)-12b^2d^2+11bc^2d-2c^4}{9bcd^2-2c^3d}\right)+\frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2+b*x),x)
```

```
[Out] (-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**
2*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b
*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d)) + (c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
 c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c*
*3*d)) + log(x)/b
```

Giac [**A**]
time = 3.81, size = 62, normalized size = 1.00

$$-\frac{c\arctan\left(\frac{2\,dx+c}{\sqrt{-c^2+4\,bd}}\right)}{\sqrt{-c^2+4\,bd}\,b}-\frac{\log\left(dx^2+cx+b\right)}{2\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -c*arctan((2*d*x + c)/sqrt(-c^2 + 4*b*d))/(sqrt(-c^2 + 4*b*d)*b) - 1/2*log(
d*x^2 + c*x + b)/b + log(abs(x))/b
```

Mupad [**B**]
time = 0.47, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{b}-\ln\left(\left(x\,\left(6\,b\,d^2-2\,c^2\,d\right)-b\,c\,d\right)\,\left(\frac{1}{2\,b}-\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)-c\,d-3\,d^2\,x\right)\,\left(\frac{1}{2\,b}-\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)-\ln\left(\left(x\,\left(6\,b\,d^2-2\,c^2\,d\right)-b\,c\,d\right)\,\left(\frac{1}{2\,b}+\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)-c\,d-3\,d^2\,x\right)\,\left(\frac{1}{2\,b}+\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2 + d*x^3),x)
```

```
[Out] log(x)/b - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d)))
```

3.27 $\int \frac{1}{a+dx^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}$$

```
[Out] 1/3*ln(a^(1/3)+d^(1/3)*x)/a^(2/3)/d^(1/3)-1/6*ln(a^(2/3)-a^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/a^(2/3)/d^(1/3)-1/3*arctan(1/3*(a^(1/3)-2*d^(1/3)*x)/a^(1/3)*3
^(1/2))/a^(2/3)/d^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3
))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+dx^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{d}\,x}\,dx}{3a^{2/3}} + \frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{d}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3a^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{2\sqrt[3]{a}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{d}+2d^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{d}}\\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{d}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 89, normalized size = 0.77

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right) + \log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + d^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(a^(2/3)*d^
(1/3))
```

Maple [A]
time = 0.20, size = 91, normalized size = 0.79

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}}{3d}$	27
default	$\frac{\ln\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{d}\right)^{\frac{1}{3}}x+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{a}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d/(a/d)^(2/3)*ln(x+(a/d)^(1/3))-1/6/d/(a/d)^(2/3)*ln(x^2-(a/d)^(1/3)*x+
(a/d)^(2/3))+1/3/d/(a/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/d)^(1/3)*x-
1))
```

Maxima [A]
time = 0.49, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{2}-x\left(\frac{a}{d}\right)^{\frac{1}{3}}+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/d)^(1/3))/(a/d)^(1/3))/(d*(a/d)^(2
/3)) - 1/6*log(x^2 - x*(a/d)^(1/3) + (a/d)^(2/3))/(d*(a/d)^(2/3)) + 1/3*log
(x + (a/d)^(1/3))/(d*(a/d)^(2/3))
```

Fricas [A]
time = 0.39, size = 299, normalized size = 2.60

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ad\sqrt{-\frac{\left(a^{2}d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,adx^{3}-3\left(a^{2}d\right)^{\frac{1}{3}}ax-a^{2}+3\sqrt{\frac{1}{3}}\left(2\,adx^{2}+\left(a^{2}d\right)^{\frac{2}{3}}x-\left(a^{2}d\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}d\right)^{\frac{1}{3}}}{d}}}{dx^{3}+a}\right)-\left(a^{2}d\right)^{\frac{2}{3}}\log\left(adx^{2}-\left(a^{2}d\right)^{\frac{2}{3}}x+\left(a^{2}d\right)^{\frac{1}{3}}a\right)+2\left(a^{2}d\right)^{\frac{2}{3}}\log\left(adx+\left(a^{2}d\right)^{\frac{2}{3}}\right)}{6\,a^{2}d},\ \frac{6\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{\left(a^{2}d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^{2}d\right)^{\frac{2}{3}}x-\left(a^{2}d\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^{2}d\right)^{\frac{1}{3}}}{d}}}{a^{2}}\right)-\left(a^{2}d\right)^{\frac{2}{3}}\log\left(adx^{2}-\left(a^{2}d\right)^{\frac{2}{3}}x+\left(a^{2}d\right)^{\frac{1}{3}}a\right)+2\left(a^{2}d\right)^{\frac{2}{3}}\log\left(adx+\left(a^{2}d\right)^{\frac{2}{3}}\right)}{6\,a^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*d*sqrt(-(a^2*d)^(1/3)/d)*log((2*a*d*x^3 - 3*(a^2*d)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*d*x^2 + (a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a)*
sqrt(-(a^2*d)^(1/3)/d))/(d*x^3 + a)) - (a^2*d)^(2/3)*log(a*d*x^2 - (a^2*d)^
(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x + (a^2*d)^(2/3)))/(a
^2*d), 1/6*(6*sqrt(1/3)*a*d*sqrt((a^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(a^2*
d)^(2/3)*x - (a^2*d)^(1/3)*a)*sqrt((a^2*d)^(1/3)/d)/a^2) - (a^2*d)^(2/3)*lo
g(a*d*x^2 - (a^2*d)^(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x
+ (a^2*d)^(2/3)))/(a^2*d)]
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3a^2d - 1, \left(t \mapsto t\log\left(3ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Giac [A]
time = 4.45, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\,\left(-ad^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,ad}+\frac{\left(-ad^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{d}\right)^{\frac{1}{3}}+\left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/d)^(1/3)*log(abs(x - (-a/d)^(1/3)))/a + 1/3*sqrt(3)*(-a*d^2)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/d)^(1/3))/(-a/d)^(1/3))/(a*d) + 1/6*(-a*d^2)
^(1/3)*log(x^2 + x*(-a/d)^(1/3) + (-a/d)^(2/3))/(a*d)
```

Mupad [B]
time = 0.23, size = 99, normalized size = 0.86

$$\frac{\ln\left(d^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,d^{1/3}}+\frac{\ln\left(3\,d^2\,x+\frac{3\,a^{1/3}\,d^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}-\frac{\ln\left(3\,d^2\,x-\frac{3\,a^{1/3}\,d^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + d*x^3),x)
```

```
[Out] log(d^(1/3)*x + a^(1/3))/(3*a^(2/3)*d^(1/3)) + (log(3*d^2*x + (3*a^(1/3)*d^
(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*d^(1/3)) - (log(3*d
^2*x - (3*a^(1/3)*d^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)
*d^(1/3))
```

3.28 $\int (dx^3)^n \, dx$

Optimal. Leaf size=16

$$\frac{x(dx^3)^n}{1+3n}$$

[Out] x*(d*x^3)^n/(1+3*n)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {15, 30}

$$\frac{x(dx^3)^n}{3n+1}$$

Antiderivative was successfully verified.

[In] Int[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx^3)^n \, dx &= \left(x^{-3n}(dx^3)^n\right)\int x^{3n}\,dx\\ &= \frac{x(dx^3)^n}{1+3n}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{x(dx^3)^n}{1+3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^3)^n,x]
```

```
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Maple [A]
time = 0.01, size = 17, normalized size = 1.06

method	result	size
gosper	$\frac{x\left(d\,x^{3}\right)^{n}}{1+3n}$	17
risch	$\frac{x\left(d\,x^{3}\right)^{n}}{1+3n}$	17
norman	$\frac{x\,{\mathrm e}^{n\ln\left(d\,x^{3}\right)}}{1+3n}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)^n,x,method=_RETURNVERBOSE)
```

```
[Out] x*(d*x^3)^n/(1+3*n)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 1.06

$$\frac{d^{n}xx^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="maxima")
```

```
[Out] d^n*x*x^(3*n)/(3*n + 1)
```

Fricas [A]
time = 0.39, size = 16, normalized size = 1.00

$$\frac{\left(dx^{3}\right)^{n}x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x(dx^{3})^{n}}{3n+1} & \text{for } n \neq -\frac{1}{3} \\ \int \frac{1}{\sqrt[3]{dx^{3}}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3)**n,x)
```

```
[Out] Piecewise((x*(d*x**3)**n/(3*n + 1), Ne(n, -1/3)), (Integral((d*x**3)**(-1/3
), x), True))
```

Giac [A]
time = 4.42, size = 16, normalized size = 1.00

$$\frac{(dx^3)^n x}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="giac")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

Mupad [B]
time = 2.50, size = 16, normalized size = 1.00

$$\frac{x\,(d\,x^3)^n}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)^n,x)
```

```
[Out] (x*(d*x^3)^n)/(3*n + 1)
```

3.29 $\int (cx^2 + dx^3)^n \, dx$

Optimal. Leaf size=55

$$\frac{x\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n {}_2F_1\left(-n,1+2n;2(1+n);-\frac{dx}{c}\right)}{1+2n}$$

```
[Out] x*(d*x^3+c*x^2)^n*hypergeom([-n, 1+2*n],[2+2*n],-d*x/c)/(1+2*n)/((1+d*x/c)^
n)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2036, 68, 66}

$$\frac{x\left(\frac{dx}{c}+1\right)^{-n}\left(cx^2+dx^3\right)^n {}_2F_1\left(-n,2n+1;2(n+1);-\frac{dx}{c}\right)}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(c*x^2 + d*x^3)^n*Hypergeometric2F1[-n, 1 + 2*n, 2*(1 + n), -((d*x)/c)])
/((1 + 2*n)*(1 + (d*x)/c)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \left(cx^2+dx^3\right)^n\,dx &= \left(x^{-2n}(c+dx)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}(c+dx)^n\,dx\\ &= \left(x^{-2n}\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\right)\int x^{2n}\left(1+\frac{dx}{c}\right)^n\,dx\\ &= \frac{x\left(1+\frac{dx}{c}\right)^{-n}\left(cx^2+dx^3\right)^n\,{}_2F_1\left(-n,1+2n;2(1+n);-\frac{dx}{c}\right)}{1+2n}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 53, normalized size = 0.96

$$\frac{x\left(x^2(c+dx)\right)^n\left(1+\frac{dx}{c}\right)^{-n}\,{}_2F_1\left(-n,1+2n;2+2n;-\frac{dx}{c}\right)}{1+2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^n,x]
[Out] (x*(x^2*(c + d*x))^n*Hypergeometric2F1[-n, 1 + 2*n, 2 + 2*n, -((d*x)/c)])/(
(1 + 2*n)*(1 + (d*x)/c)^n)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(d\,x^3+c\,x^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2)^n,x)
[Out] int((d*x^3+c*x^2)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="maxima")
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

Fricas [F]
time = 0.38, size = 15, normalized size = 0.27

$$\text{integral}\left(\left(dx^3+cx^2\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="fricas")
[Out] integral((d*x^3 + c*x^2)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2+dx^3\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2)**n,x)
[Out] Integral((c*x**2 + d*x**3)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2)^n,x, algorithm="giac")
[Out] integrate((d*x^3 + c*x^2)^n, x)
```

Mupad [B]
time = 2.21, size = 56, normalized size = 1.02

$$\frac{x\left(d\,x^3+c\,x^2\right)^n\,{}_2\mathrm{F}_1\left(2\,n+1,-n;\ 2\,n+2;\ -\frac{d\,x}{c}\right)}{\left(2\,n+1\right)\,\left(\frac{d\,x}{c}+1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2 + d*x^3)^n,x)
[Out] (x*(c*x^2 + d*x^3)^n*hypergeom([2*n + 1, -n], 2*n + 2, -(d*x)/c))/((2*n + 1
)*((d*x)/c + 1)^n)
```

3.30 $\int (bx + dx^3)^n \, dx$

Optimal. Leaf size=53

$$\frac{x(b + dx^2)\left(bx + dx^3\right)^n {}_2F_1\left(1, \frac{3(1+n)}{2}; \frac{3+n}{2}; -\frac{dx^2}{b}\right)}{b(1 + n)}$$

```
[Out] x*(d*x^2+b)*(d*x^3+b*x)^n*hypergeom([1, 3/2+3/2*n],[3/2+1/2*n],-d*x^2/b)/b/
(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2036, 372, 371}

$$\frac{x\left(\frac{dx^2}{b} + 1\right)^{-n} \left(bx + dx^3\right)^n {}_2F_1\left(-n, \frac{n+1}{2}; \frac{n+3}{2}; -\frac{dx^2}{b}\right)}{n + 1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)^n,x]
```

```
[Out] (x*(b*x + d*x^3)^n*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)
])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \left(bx+dx^3\right)^n\,dx = \left(x^{-n}\left(b+dx^2\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(b+dx^2\right)^n\,dx$$
$$= \left(x^{-n}\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\right)\int x^n\left(1+\frac{dx^2}{b}\right)^n\,dx$$
$$= \frac{x\left(1+\frac{dx^2}{b}\right)^{-n}\left(bx+dx^3\right)^n\,{}_2F_1\left(-n,\frac{1+n}{2};\frac{3+n}{2};-\frac{dx^2}{b}\right)}{1+n}$$

Mathematica [**A**]
time = 0.03, size = 61, normalized size = 1.15

$$\frac{x\left(x\left(b+dx^2\right)\right)^n\left(1+\frac{dx^2}{b}\right)^{-n}\,{}_2F_1\left(-n,\frac{1+n}{2};1+\frac{1+n}{2};-\frac{dx^2}{b}\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)^n,x]
[Out] (x*(x*(b + d*x^2))^n*Hypergeometric2F1[-n, (1 + n)/2, 1 + (1 + n)/2, -((d*x
^2)/b)])/((1 + n)*(1 + (d*x^2)/b)^n)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(d\,x^3+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+b*x)^n,x)
[Out] int((d*x^3+b*x)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="maxima")
[Out] integrate((d*x^3 + b*x)^n, x)
```

Fricas [F]
time = 0.38, size = 13, normalized size = 0.25

$$\text{integral}\left(\left(dx^3+bx\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + b*x)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx+dx^3\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+b*x)**n,x)
```

```
[Out] Integral((b*x + d*x**3)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+b*x)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + b*x)^n, x)
```

Mupad [B]
time = 2.22, size = 56, normalized size = 1.06

$$\frac{x\,{\left(d\,x^3+b\,x\right)}^n\,{}_2\mathrm{F}_1\left(\frac{n}{2}+\frac{1}{2},-n;\ \frac{n}{2}+\frac{3}{2};\ -\frac{d\,x^2}{b}\right)}{{\left(\frac{d\,x^2}{b}+1\right)}^n\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^n,x)
```

```
[Out] (x*(b*x + d*x^3)^n*hypergeom([n/2 + 1/2, -n], n/2 + 3/2, -(d*x^2)/b))/(((d*
x^2)/b + 1)^n*(n + 1))
```

3.31 $\int (bx + cx^2 + dx^3)^n \, dx$

Optimal. Leaf size=132

$$\frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(bx+cx^2+dx^3)^n F_1\left(1+n;-n,-n;2+n;-\frac{2dx}{c-\sqrt{c^2-4bd}}\right.}{1+n}$$

```
[Out] x*(d*x^3+c*x^2+b*x)^n*AppellF1(1+n,-n,-n,2+n,-2*d*x/(c-(-4*b*d+c^2)^(1/2)),
-2*d*x/(c+(-4*b*d+c^2)^(1/2)))/(1+n)/((1+2*d*x/(c-(-4*b*d+c^2)^(1/2)))^n)/(
(1+2*d*x/(c+(-4*b*d+c^2)^(1/2)))^n)
```

Rubi [A]
time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1922, 773, 138}

$$\frac{x\left(\frac{2dx}{c-\sqrt{c^2-4bd}}+1\right)^{-n}\left(\frac{2dx}{\sqrt{c^2-4bd}+c}+1\right)^{-n}(bx+cx^2+dx^3)^n F_1\left(n+1;-n,-n;n+2;-\frac{2dx}{c-\sqrt{c^2-4bd}},-\frac{2dx}{c+\sqrt{c^2-4bd}}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b*x + c*x^2 + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c - Sqr
t[c^2 - 4*b*d]), (-2*d*x)/(c + Sqrt[c^2 - 4*b*d])])/((1 + n)*(1 + (2*d*x)/(
c - Sqrt[c^2 - 4*b*d]))^n*(1 + (2*d*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 773

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p), Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d -
 e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 1922

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n - q) + c*x
^(2*(n - q)))^p), Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p]
```

Rubi steps

$$\int (bx+cx^2+dx^3)^n\,dx = \left(x^{-n}(b+cx+dx^2)^{-n}(bx+cx^2+dx^3)^n\right)\int x^n(b+cx+dx^2)^n\,dx$$
$$= \left(x^{-n}\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(bx+cx^2+dx^3)^n\right)\text{Su}$$
$$= \frac{x\left(1+\frac{2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(1+\frac{2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(bx+cx^2+dx^3)^n F_1\left(1+n;-\right.}{1+n}$$

Mathematica [A]
time = 0.26, size = 157, normalized size = 1.19

$$\frac{x\left(\frac{c-\sqrt{c^2-4bd}+2dx}{c-\sqrt{c^2-4bd}}\right)^{-n}\left(\frac{c+\sqrt{c^2-4bd}+2dx}{c+\sqrt{c^2-4bd}}\right)^{-n}(x(b+x(c+dx)))^n F_1\left(1+n;-n,-n;2+n;-\frac{2dx}{c+\sqrt{c^2-4bd}},\frac{2dx}{-c+\sqrt{c^2-4bd}}\right)}{1+n}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + x*(c + d*x)))^n*AppellF1[1 + n, -n, -n, 2 + n, (-2*d*x)/(c + Sqr
t[c^2 - 4*b*d]), (2*d*x)/(-c + Sqrt[c^2 - 4*b*d])])/((1 + n)*((c - Sqrt[c^2
 - 4*b*d] + 2*d*x)/(c - Sqrt[c^2 - 4*b*d]))^n*((c + Sqrt[c^2 - 4*b*d] + 2*d
*x)/(c + Sqrt[c^2 - 4*b*d]))^n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(d\,x^3+c\,x^2+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x)^n,x)
```

```
[Out] int((d*x^3+c*x^2+b*x)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

Fricas [F]
time = 0.38, size = 18, normalized size = 0.14

$$\mathrm{integral}\left(\left(dx^3+cx^2+bx\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
[Out] integral((d*x^3 + c*x^2 + b*x)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx+cx^2+dx^3\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x)**n,x)
[Out] Integral((b*x + c*x**2 + d*x**3)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
[Out] integrate((d*x^3 + c*x^2 + b*x)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(d\,x^3+c\,x^2+b\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2 + d*x^3)^n,x)
[Out] int((b*x + c*x^2 + d*x^3)^n, x)
```

3.32 $\int (a+dx^3)^n \, dx$

Optimal. Leaf size=35

$$\frac{x(a+dx^3)^{1+n} \, {}_2F_1\left(1,\frac{4}{3}+n;\frac{4}{3};-\frac{dx^3}{a}\right)}{a}$$

```
[Out] x*(d*x^3+a)^(1+n)*hypergeom([1, 4/3+n],[4/3],-d*x^3/a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {252, 251}

$$x(a+dx^3)^n \left(\frac{dx^3}{a}+1\right)^{-n} {}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)^n,x]
```

```
[Out] (x*(a + d*x^3)^n*Hypergeometric2F1[1/3, -n, 4/3, -((d*x^3)/a)])/(1 + (d*x^3
)/a)^n
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+dx^3)^n \, dx &= \left((a+dx^3)^n \left(1+\frac{dx^3}{a}\right)^{-n}\right) \int \left(1+\frac{dx^3}{a}\right)^n dx \\ &= x(a+dx^3)^n \left(1+\frac{dx^3}{a}\right)^{-n} {}_2F_1\left(\frac{1}{3},-n;\frac{4}{3};-\frac{dx^3}{a}\right)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.13, size = 196, normalized size = 5.60

$$\frac{2^{-n}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{d}\,x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{d}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-n}\left(\frac{i\left(1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-n}(a+dx^3)^n F_1\left(1+n;-n,-n;2+n;-\frac{i\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{\sqrt{3}\sqrt[3]{a}},\frac{i+\sqrt{3}-\frac{2i\sqrt[3]{d}\,x}{\sqrt[3]{a}}}{3i+\sqrt{3}}\right)}{\sqrt[3]{d}\,(1+n)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + d*x^3)^n,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + d^(1/3)*x)*(a + d*x^3)^n*AppellF1[1 + n, -n, -n, 2 +
 n, ((-I)*((-1)^(2/3)*a^(1/3) + d^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + Sqrt[3]
 - ((2*I)*d^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^n*d^(1/3)*(1 + n)*((a^(1
/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^n*((I*(1 + (d^(1/3)
*x)/a^(1/3)))/(3*I + Sqrt[3]))^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(d\,x^3+a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+a)^n,x)
```

```
[Out] int((d*x^3+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + a)^n, x)
```

Fricas [F]
time = 0.41, size = 11, normalized size = 0.31

$$\text{integral}\left(\left(dx^3+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+a)^n,x, algorithm="fricas")
```

[Out] integral((d*x^3 + a)^n, x)

Sympy [C] Result contains complex when optimal does not.
time = 4.91, size = 34, normalized size = 0.97

$$\frac{a^n x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -n \\ \frac{4}{3} \end{matrix}\middle| \frac{dx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x**3+a)**n,x)

[Out] a**n*x*gamma(1/3)*hyper((1/3, -n), (4/3,), d*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+a)^n,x, algorithm="giac")

[Out] integrate((d*x^3 + a)^n, x)

Mupad [B]
time = 2.18, size = 41, normalized size = 1.17

$$\frac{x\left(d\,x^3+a\right)^n {}_2\mathrm{F}_1\left(\frac{1}{3},-n;\ \frac{4}{3};\ -\frac{d\,x^3}{a}\right)}{\left(\frac{d\,x^3}{a}+1\right)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + d*x^3)^n,x)

[Out] (x*(a + d*x^3)^n*hypergeom([1/3, -n], 4/3, -(d*x^3)/a))/((d*x^3)/a + 1)^n

3.33 $\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^4 \, dx$

Optimal. Leaf size=270

$$\frac{c^4(c^3+4ad^2)^4 x}{d^8} - \frac{8c^5(c^3+4ad^2)^3 \left(\frac{c}{d}+x\right)^3}{3d^6} + \frac{4c^3(c^3+4ad^2)^2 (7c^3+4ad^2) \left(\frac{c}{d}+x\right)^5}{5d^4} - \frac{8c^4(c^3+4ad^2)(7c^3+ \dots}{7d^2}$$

```
[Out] c^4*(4*a*d^2+c^3)^4*x/d^8-8/3*c^5*(4*a*d^2+c^3)^3*(c/d+x)^3/d^6+4/5*c^3*(4*
a*d^2+c^3)^2*(4*a*d^2+7*c^3)*(c/d+x)^5/d^4-8/7*c^4*(4*a*d^2+c^3)*(12*a*d^2+
7*c^3)*(c/d+x)^7/d^2+2/9*c^2*(48*a^2*d^4+120*a*c^3*d^2+35*c^6)*(c/d+x)^9-8/
11*c^3*d^2*(12*a*d^2+7*c^3)*(c/d+x)^11+4/13*c*d^4*(4*a*d^2+7*c^3)*(c/d+x)^1
3-8/15*c^2*d^6*(c/d+x)^15+1/17*d^8*(c/d+x)^17
```

Rubi [A]

time = 0.41, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1120, 1104}

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9 - \frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11} + \frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13} + \frac{4c^3(4ad^2+c^3)^2(4ad^2+7c^3)\left(\frac{c}{d}+x\right)^5}{5d^4} - \frac{8c^5(4ad^2+c^3)^3\left(\frac{c}{d}+x\right)^3}{3d^6} - \frac{8c^4(4ad^2+c^3)(12ad^2+7c^3)\left(\frac{c}{d}+x\right)^7}{7d^2} + \frac{c^4x(4ad^2+c^3)^4}{d^8} - \frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15} + \frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)^3)/(3*d^
6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^4) - (8*c
^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*c^2*(35*c^6
 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^
2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*
d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^4 dx, x, \frac{c}{d}+x\right)$$
$$= \text{Subst}\left(\int \left(\frac{\left(c^4+4acd^2\right)^4}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3x^2}{d^6}+\frac{24c^6\left(c^3+4ad^2\right)^2}{d^4}\right.\right.$$
$$= \frac{c^4\left(c^3+4ad^2\right)^4x}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}+\frac{4c^3\left(c^3+4ad^2\right)^2\left(7c^3\right.}{5d^4}$$

Mathematica [A]
time = 0.03, size = 285, normalized size = 1.06

$$256a^4c^4x+\frac{1024}{3}a^3c^5x^3+256a^3c^4dx^4+\frac{256}{5}a^2c^3\left(6c^3+ad^2\right)x^5+512a^2c^5dx^6+\frac{256}{7}ac^4\left(4c^3+9ad^2\right)x^7+96ac^3d\left(4c^3+ad^2\right)x^8+\frac{32}{9}c^2\left(8c^6+120ac^3d^2+3a^2d^4\right)x^9+\frac{256}{5}c^4d\left(2c^3+5ad^2\right)x^{10}+\frac{64}{11}c^3d^2\left(28c^3+15ad^2\right)x^{11}+\frac{16}{3}c^2d^3\left(28c^3+3ad^2\right)x^{12}+\frac{16}{13}cd^4\left(70c^3+ad^2\right)x^{13}+32c^3d^5x^{14}+\frac{112}{15}c^2d^6x^{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6*
c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^7)
/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3*a^
2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3
+ 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(
70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*
x^16 + (d^8*x^17)/17
```

Maple [A]
time = 0.03, size = 392, normalized size = 1.45

method	result
norman	$256a^4c^4x+\frac{1024a^3c^5x^3}{3}+256a^3c^4d\,x^4+\left(\frac{256}{5}a^3c^3d^2+\frac{1536}{5}a^2c^6\right)x^5+512a^2c^5d\,x^6+\left(\frac{2304}{7}a^2c^4d^2+\right.$
gosper	$\frac{16}{13}x^{13}ac\,d^6+96a^2c^3d^3x^8+384a\,c^6d\,x^8+\frac{960}{11}x^{11}a\,c^3d^4+16x^{12}a\,c^2d^5+\frac{32}{3}x^9a^2c^2d^4+\frac{1280}{3}x^9a\,c^5d^2+$
risch	$\frac{16}{13}x^{13}ac\,d^6+96a^2c^3d^3x^8+384a\,c^6d\,x^8+\frac{960}{11}x^{11}a\,c^3d^4+16x^{12}a\,c^2d^5+\frac{32}{3}x^9a^2c^2d^4+\frac{1280}{3}x^9a\,c^5d^2+$
default	$\frac{d^8x^{17}}{17}+c\,d^7x^{16}+\frac{112c^2d^6x^{15}}{15}+32c^3d^5x^{14}+\frac{\left(2\left(8ac\,d^2+16c^4\right)d^4+1088c^4d^4\right)x^{13}}{13}+\frac{\left(64a\,c^2d^5+16\left(8ac\,d^2+16c^4\right)c\,d^3+\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*d^8*x^17+c*d^7*x^16+112/15*c^2*d^6*x^15+32*c^3*d^5*x^14+1/13*(2*(8*a*c
*d^2+16*c^4)*d^4+1088*c^4*d^4)*x^13+1/12*(64*a*c^2*d^5+16*(8*a*c*d^2+16*c^4
```

```
)*c*d^3+1536*c^5*d^3)*x^12+1/11*(576*a*c^3*d^4+48*(8*a*c*d^2+16*c^4)*c^2*d^
2+1024*c^6*d^2)*x^11+1/10*(2048*a*c^4*d^3+64*(8*a*c*d^2+16*c^4)*c^3*d)*x^10
+1/9*(32*a^2*c^2*d^4+3584*a*c^5*d^2+(8*a*c*d^2+16*c^4)^2)*x^9+1/8*(256*a^2*
c^3*d^3+2048*a*c^6*d+64*a*c^2*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(1792*a^2*c^4*d
^2+64*a*c^3*(8*a*c*d^2+16*c^4))*x^7+512*a^2*c^5*d*x^6+1/5*(32*a^2*c^2*(8*a*
c*d^2+16*c^4)+1024*a^2*c^6)*x^5+256*a^3*c^4*d*x^4+1024/3*a^3*c^5*x^3+256*a^
4*c^4*x
```

Maxima [A]
time = 0.29, size = 372, normalized size = 1.38

$$\frac{1}{17}d^8x^{17}+cd^7x^{16}+\frac{32}{5}c^2d^6x^{15}+\frac{128}{7}c^3d^5x^{14}+\frac{256}{13}c^4d^4x^{13}+\frac{256}{9}c^8x^9+256\,a^4c^4x+\frac{256}{15}\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)a^3c^3+\frac{256}{55}\left(5\,d^2x^{11}+22\,cdx^{10}\right)c^6+\frac{32}{105}\left(35\,d^4x^9+315\,cd^3x^8+720\,c^2d^2x^7+1008\,c^4x^5+120\left(3\,d^2x^7+14\,cdx^6\right)c^2\right)a^2c^2+\frac{32}{143}\left(33\,d^4x^{13}+286\,cd^3x^{12}+624\,c^2d^2x^{11}\right)c^4+\frac{16}{15015}\left(1155\,d^6x^{13}+15015\,cd^5x^{12}+65520\,c^2d^4x^{11}+96096\,c^3d^3x^{10}+137280\,c^6x^7+40040\left(2\,d^2x^9+9\,cdx^8\right)c^4+364\left(45\,d^4x^{11}+396\,cd^3x^{10}+880\,c^2d^2x^9\right)c^2\right)ac+\frac{16}{1365}\left(91\,d^6x^{15}+1170\,cd^5x^{14}+5040\,c^2d^4x^{13}+7280\,c^3d^3x^{12}\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/1
3*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c*d
*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/105
*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*
x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624
*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2
*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9*c*d*
x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2)*a*c +
16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*c^3*d^3*x
^12)*c^2
```

Fricas [A]
time = 0.39, size = 271, normalized size = 1.00

$$\frac{1}{17}d^8x^{17}+cd^7x^{16}+\frac{112}{15}c^2d^6x^{15}+32\,c^3d^5x^{14}+512\,a^2c^5dx^6+\frac{16}{13}\left(70\,c^4d^4+acd^6\right)x^{13}+\frac{16}{3}\left(28\,c^5d^3+3\,ac^2d^5\right)x^{12}+256\,a^3c^4dx^4+\frac{64}{11}\left(28\,c^6d^2+15\,ac^3d^4\right)x^{11}+\frac{1024}{3}a^3c^5x^3+\frac{256}{5}\left(2\,c^7d+5\,ac^4d^3\right)x^{10}+\frac{32}{9}\left(8\,c^8+120\,ac^5d^2+3\,a^2c^2d^4\right)x^9+256\,a^4c^4x+96\left(4\,ac^6d+a^2c^3d^3\right)x^8+\frac{256}{7}\left(4\,ac^7+9\,a^2c^4d^2\right)x^7+\frac{256}{5}\left(6\,a^2c^6+a^3c^3d^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 512*a^
2*c^5*d*x^6 + 16/13*(70*c^4*d^4 + a*c*d^6)*x^13 + 16/3*(28*c^5*d^3 + 3*a*c^
2*d^5)*x^12 + 256*a^3*c^4*d*x^4 + 64/11*(28*c^6*d^2 + 15*a*c^3*d^4)*x^11 +
1024/3*a^3*c^5*x^3 + 256/5*(2*c^7*d + 5*a*c^4*d^3)*x^10 + 32/9*(8*c^8 + 120
*a*c^5*d^2 + 3*a^2*c^2*d^4)*x^9 + 256*a^4*c^4*x + 96*(4*a*c^6*d + a^2*c^3*d
^3)*x^8 + 256/7*(4*a*c^7 + 9*a^2*c^4*d^2)*x^7 + 256/5*(6*a^2*c^6 + a^3*c^3*
d^2)*x^5
```

Sympy [A]
time = 0.03, size = 299, normalized size = 1.11

$$256a^4c^4x+\frac{1024a^3c^5x^3}{3}+256a^3c^4dx^4+512a^2c^5dx^6+32c^3d^5x^{14}+\frac{112c^2d^6x^{15}}{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}+x^{13}\cdot\left(\frac{16acd^6}{13}+\frac{1120c^4d^4}{13}\right)+x^{12}\cdot\left(16ac^2d^5+\frac{448c^5d^3}{3}\right)+x^{11}\cdot\left(\frac{960ac^3d^4}{11}+\frac{1792c^6d^2}{11}\right)+x^{10}\cdot\left(256ac^4d^3+\frac{512c^7d}{5}\right)+x^9\cdot\left(\frac{32a^2c^2d^4}{3}+\frac{1280ac^5d^2}{3}+\frac{256c^8}{9}\right)+x^8\cdot\left(96a^2c^3d^3+384ac^6d\right)+x^7\cdot\left(\frac{2304a^2c^4d^2}{7}+\frac{1024ac^7}{7}\right)+x^5\cdot\left(\frac{256a^3c^3d^2}{5}+\frac{1536a^2c^6}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

```
[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*c
**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 + d
**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*c**
2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/11)
+ x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280
*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x*
*7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/5 + 1
536*a**2*c**6/5)
```

Giac [A]
time = 3.80, size = 277, normalized size = 1.03

$$\frac{1}{17}d^8x^{17}+cd^7x^{16}+\frac{112}{15}c^2d^6x^{15}+32\,c^3d^5x^{14}+\frac{1120}{13}c^4d^4x^{13}+\frac{16}{13}acd^6x^{13}+\frac{448}{3}c^5d^3x^{12}+16\,ac^2d^5x^{12}+\frac{1792}{11}c^6d^2x^{11}+\frac{960}{11}ac^3d^4x^{11}+\frac{512}{5}c^7dx^{10}+256\,ac^4d^3x^{10}+\frac{256}{9}c^8x^9+\frac{1280}{3}ac^5d^2x^9+\frac{32}{3}a^2c^2d^4x^9+384\,ac^6dx^8+96\,a^2c^3d^3x^8+\frac{1024}{7}ac^7x^7+\frac{2304}{7}a^2c^4d^2x^7+512\,a^2c^5dx^6+\frac{1536}{5}a^2c^6x^5+\frac{256}{5}a^3c^3d^2x^5+256\,a^3c^4dx^4+\frac{1024}{3}a^3c^5x^3+256\,a^4c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/1
3*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x^1
2 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 256*a
*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9
 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4
*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 +
 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

Mupad [B]
time = 2.30, size = 261, normalized size = 0.97

$$x^{10}\left(\frac{512\,c^7\,d}{5}+256\,a\,c^4\,d^3\right)+x^{13}\left(\frac{1120\,c^4\,d^4}{13}+\frac{16\,a\,c\,d^6}{13}\right)+x^9\left(\frac{32\,a^2\,c^2\,d^4}{3}+\frac{1280\,a\,c^5\,d^2}{3}+\frac{256\,c^8}{9}\right)+x^{12}\left(\frac{448\,c^5\,d^3}{3}+16\,a\,c^2\,d^5\right)+x^{11}\left(\frac{1792\,c^6\,d^2}{11}+\frac{960\,a\,c^3\,d^4}{11}\right)+\frac{d^8\,x^{17}}{17}+256\,a^4\,c^4\,x+c\,d^7\,x^{16}+\frac{1024\,a^3\,c^5\,x^3}{3}+32\,c^3\,d^5\,x^{14}+\frac{112\,c^2\,d^6\,x^{15}}{15}+256\,a^3\,c^4\,d\,x^4+512\,a^2\,c^5\,d\,x^6+\frac{256\,a\,c^4\,x^7\,\left(4\,c^3+9\,a\,d^2\right)}{7}+\frac{256\,a^2\,c^3\,x^5\,\left(6\,c^3+a\,d^2\right)}{5}+96\,a\,c^3\,d\,x^8\,\left(4\,c^3+a\,d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)
```

```
[Out] x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d^
6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^12
*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3*d^
4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3)/3
+ 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d*x^4 + 512*a^2*c^5
*d*x^6 + (256*a*c^4*x^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^2 + 6*
c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)
```

3.34 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx$

Optimal. Leaf size=171

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5+64ac^4dx^6+\frac{32}{7}c^3\left(2c^3+9ad^2\right)x^7+12c^2d\left(2c^3+ad^2\right)x^8$$

```
[Out] 64*a^3*c^3*x+64*a^2*c^4*x^3+48*a^2*c^3*d*x^4+48/5*a*c^2*(a*d^2+4*c^3)*x^5+6
4*a*c^4*d*x^6+32/7*c^3*(9*a*d^2+2*c^3)*x^7+12*c^2*d*(a*d^2+2*c^3)*x^8+4/3*c
*d^2*(a*d^2+20*c^3)*x^9+16*c^3*d^3*x^10+60/11*c^2*d^4*x^11+c*d^5*x^12+1/13*
d^6*x^13
```

Rubi [A]
time = 0.07, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.034$, Rules used = {2086}

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+4c^3\right)+16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx &= \int \left(64a^3c^3+192a^2c^4x^2+192a^2c^3dx^3+48ac^2\left(4c^3+ad^2\right)x^4+384a\right.\\ &= 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5+64ac^4dx\end{aligned}$$

Mathematica [A]
time = 0.02, size = 171, normalized size = 1.00

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5+64ac^4dx^6+\frac{32}{7}c^3\left(2c^3+9ad^2\right)x^7+12c^2d\left(2c^3+ad^2\right)x^8+\frac{4}{3}cd^2\left(20c^3+ad^2\right)x^9+16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Maple [A]
time = 0.02, size = 231, normalized size = 1.35

method	result
norman	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\left(\frac{4}{3}ac\,d^4+\frac{80}{3}c^4d^2\right)x^9+\left(12a\,c^2d^3+24c^5d\right)x^8+\left(\frac{288}{7}a\,c^3d$
gosper	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9c^4d^2+12a\,c^2d^3x^8+24c^5d\,x^8+\frac{288}{7}x$
risch	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9ac\,d^4+\frac{80}{3}x^9c^4d^2+12a\,c^2d^3x^8+24c^5d\,x^8+\frac{288}{7}x$
default	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\frac{\left(4ac\,d^4+224c^4d^2+d^2\left(8ac\,d^2+16c^4\right)\right)x^9}{9}+\frac{(64a\,c^2d^3+128c^5d+4cd(8ac\,d^2+}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+1/9*(4*a*c*d^4+
224*c^4*d^2+d^2*(8*a*c*d^2+16*c^4))*x^9+1/8*(64*a*c^2*d^3+128*c^5*d+4*c*d*(
8*a*c*d^2+16*c^4))*x^8+1/7*(256*a*c^3*d^2+4*c^2*(8*a*c*d^2+16*c^4))*x^7+64*
a*c^4*d*x^6+1/5*(4*a*c*(8*a*c*d^2+16*c^4)+128*a*c^5+16*a^2*c^2*d^2)*x^5+48*
a^2*c^3*d*x^4+64*a^2*c^4*x^3+64*a^3*c^3*x
```

Maxima [A]
time = 0.29, size = 205, normalized size = 1.20

$$\frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{48}{11}\,c^2d^4x^{11}+\frac{32}{5}\,c^3d^3x^{10}+\frac{64}{7}\,c^6x^7+64\,a^3c^3x+\frac{16}{5}\,\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)a^2c^2+\frac{8}{3}\,\left(2\,d^2x^9+9\,cdx^8\right)c^4+\frac{4}{105}\,\left(35\,d^4x^9+315\,cd^3x^8+720\,c^2d^2x^7+1008\,c^4x^5+120\,\left(3\,d^2x^7+14\,cdx^6\right)c^2\right)ac+\frac{4}{165}\,\left(45\,d^4x^{11}+396\,cd^3x^{10}+880\,c^2d^2x^9\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7*
c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c^2
 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 72
0*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/16
5*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
```

Fricas [A]
time = 0.37, size = 163, normalized size = 0.95

$$\frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{60}{11}\,c^2d^4x^{11}+16\,c^3d^3x^{10}+64\,ac^4dx^6+48\,a^2c^3dx^4+\frac{4}{3}\,\left(20\,c^4d^2+acd^4\right)x^9+64\,a^2c^4x^3+12\,\left(2\,c^5d+ac^2d^3\right)x^8+\frac{32}{7}\,\left(2\,c^6+9\,ac^3d^2\right)x^7+64\,a^3c^3x+\frac{48}{5}\,\left(4\,ac^5+a^2c^2d^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 64*a*c^
4*d*x^6 + 48*a^2*c^3*d*x^4 + 4/3*(20*c^4*d^2 + a*c*d^4)*x^9 + 64*a^2*c^4*x^
3 + 12*(2*c^5*d + a*c^2*d^3)*x^8 + 32/7*(2*c^6 + 9*a*c^3*d^2)*x^7 + 64*a^3*
c^3*x + 48/5*(4*a*c^5 + a^2*c^2*d^2)*x^5
```

Sympy [A]
time = 0.02, size = 180, normalized size = 1.05

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + cd^5x^{12} + \frac{d^6x^{13}}{13} + x^9 \cdot \left(\frac{4acd^4}{3} + \frac{80c^4d^2}{3}\right) + x^8 \cdot (12ac^2d^3 + 24c^5d) + x^7 \cdot \left(\frac{288ac^3d^2}{7} + \frac{64c^6}{7}\right) + x^5 \cdot \left(\frac{48a^2c^2d^2}{5} + \frac{192ac^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

```
[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**6
 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13/1
3 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**5*d
) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*
a*c**5/5)
```

Giac [A]
time = 3.39, size = 166, normalized size = 0.97

$$\frac{1}{13}\,d^6x^{13} + cd^5x^{12} + \frac{60}{11}\,c^2d^4x^{11} + 16\,c^3d^3x^{10} + \frac{80}{3}\,c^4d^2x^9 + \frac{4}{3}\,acd^4x^9 + 24\,c^5dx^8 + 12\,ac^2d^3x^8 + \frac{64}{7}\,c^6x^7 + \frac{288}{7}\,ac^3d^2x^7 + 64\,ac^4dx^6 + \frac{192}{5}\,ac^5x^5 + \frac{48}{5}\,a^2c^2d^2x^5 + 48\,a^2c^3dx^4 + 64\,a^2c^4x^3 + 64\,a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c^
4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*x^
7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^2*d
^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x
```

Mupad [B]
time = 2.16, size = 160, normalized size = 0.94

$$x^8\,\left(24\,c^5\,d+12\,a\,c^2\,d^3\right)+x^9\,\left(\frac{80\,c^4\,d^2}{3}+\frac{4\,a\,c\,d^4}{3}\right)+\frac{d^6\,x^{13}}{13}+x^7\,\left(\frac{64\,c^6}{7}+\frac{288\,a\,c^3\,d^2}{7}\right)+64\,a^3\,c^3\,x+c\,d^5\,x^{12}+64\,a^2\,c^4\,x^3+16\,c^3\,d^3\,x^{10}+\frac{60\,c^2\,d^4\,x^{11}}{11}+48\,a^2\,c^3\,d\,x^4+\frac{48\,a\,c^2\,x^5\,\left(4\,c^3+a\,d^2\right)}{5}+64\,a\,c^4\,d\,x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)
```

```
[Out] x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^6
*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x^1
2 + 64*a^2*c^4*x^3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + 48*a^2*c^3*d*
x^4 + (48*a*c^2*x^5*(a*d^2 + 4*c^3))/5 + 64*a*c^4*d*x^6
```

3.35 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx$

Optimal. Leaf size=92

$$16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c\left(2c^3+ad^2\right)x^5+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

```
[Out] 16*a^2*c^2*x+32/3*a*c^3*x^3+8*a*c^2*d*x^4+8/5*c*(a*d^2+2*c^3)*x^5+16/3*c^3*
d*x^6+24/7*c^2*d^2*x^7+c*d^3*x^8+1/9*d^4*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2086}

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx = \int \left(16a^2c^2+32ac^3x^2+32ac^2dx^3+8c\left(2c^3+ad^2\right)x^4+32c^3dx^5+24\right.$$

$$=16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c\left(2c^3+ad^2\right)x^5+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2$$

Mathematica [A]
time = 0.01, size = 92, normalized size = 1.00

$$16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c\left(2c^3+ad^2\right)x^5+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9

Maple [A]
time = 0.02, size = 84, normalized size = 0.91

method	result	siz
norman	$\frac{d^4x^9}{9}+c\,d^3x^8+\frac{24c^2d^2x^7}{7}+\frac{16c^3d\,x^6}{3}+\left(\frac{8}{5}ac\,d^2+\frac{16}{5}c^4\right)x^5+8a\,c^2d\,x^4+\frac{32a\,c^3x^3}{3}+16a^2c^2x$	83
gosper	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5ac\,d^2+\frac{16}{5}x^5c^4+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2x$	84
default	$\frac{d^4x^9}{9}+c\,d^3x^8+\frac{24c^2d^2x^7}{7}+\frac{16c^3d\,x^6}{3}+\frac{(8ac\,d^2+16c^4)x^5}{5}+8a\,c^2d\,x^4+\frac{32a\,c^3x^3}{3}+16a^2c^2x$	84
risch	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5ac\,d^2+\frac{16}{5}x^5c^4+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2x$	84

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)

[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+1/5*(8*a*c*d^2+16*c^4
)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x

Maxima [A]
time = 0.27, size = 94, normalized size = 1.02

$$\frac{1}{9}\,d^4x^9+cd^3x^8+\frac{16}{7}\,c^2d^2x^7+\frac{16}{5}\,c^4x^5+16\,a^2c^2x+\frac{8}{15}\,\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)ac+\frac{8}{21}\,\left(3\,d^2x^7+14\,cdx^6\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")

[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x +
8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d*x
^6)*c^2

Fricas [A]
time = 0.35, size = 82, normalized size = 0.89

$$\frac{1}{9}\,d^4x^9+cd^3x^8+\frac{24}{7}\,c^2d^2x^7+\frac{16}{3}\,c^3dx^6+8\,ac^2dx^4+\frac{32}{3}\,ac^3x^3+\frac{8}{5}\,\left(2\,c^4+acd^2\right)x^5+16\,a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")

[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 8*a*c^2*d*x^4
 + 32/3*a*c^3*x^3 + 8/5*(2*c^4 + a*c*d^2)*x^5 + 16*a^2*c^2*x

Sympy [A]
time = 0.01, size = 95, normalized size = 1.03

$$16a^2c^2x+\frac{32ac^3x^3}{3}+8ac^2dx^4+\frac{16c^3dx^6}{3}+\frac{24c^2d^2x^7}{7}+cd^3x^8+\frac{d^4x^9}{9}+x^5\cdot\left(\frac{8acd^2}{5}+\frac{16c^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 24
*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c**
4/5)
```

Giac [A]
time = 4.56, size = 83, normalized size = 0.90

$$\frac{1}{9}\,d^4x^9 + cd^3x^8 + \frac{24}{7}\,c^2d^2x^7 + \frac{16}{3}\,c^3dx^6 + \frac{16}{5}\,c^4x^5 + \frac{8}{5}\,acd^2x^5 + 8\,ac^2dx^4 + \frac{32}{3}\,ac^3x^3 + 16\,a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5
+ 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x
```

Mupad [B]
time = 0.04, size = 82, normalized size = 0.89

$$x^5\left(\frac{16\,c^4}{5}+\frac{8\,a\,c\,d^2}{5}\right)+\frac{d^4\,x^9}{9}+16\,a^2\,c^2\,x+\frac{32\,a\,c^3\,x^3}{3}+\frac{16\,c^3\,d\,x^6}{3}+c\,d^3\,x^8+\frac{24\,c^2\,d^2\,x^7}{7}+8\,a\,c^2\,d\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*x
^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4
```

3.36 $\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)\ dx$

Optimal. Leaf size=32

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

```
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi steps

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)\ dx = 4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Maple [A]
time = 0.01, size = 29, normalized size = 0.91

method	result	size
gosper	$4acx + \frac{4}{3}c^2x^3 + cd\,x^4 + \frac{1}{5}x^5d^2$	29
default	$4acx + \frac{4}{3}c^2x^3 + cd\,x^4 + \frac{1}{5}x^5d^2$	29

norman	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}x^5d^2$	29
risch	$4acx+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}x^5d^2$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x,method=_RETURNVERBOSE)
```

```
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*x^5*d^2
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")
```

```
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Fricas [A]
time = 0.37, size = 28, normalized size = 0.88

$$\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")
```

```
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Sympy [A]
time = 0.01, size = 31, normalized size = 0.97

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
```

```
[Out] 4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

Giac [A]
time = 3.20, size = 28, normalized size = 0.88

$$\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")
```

```
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.88

$$\frac{4\,c^2\,x^3}{3}+c\,d\,x^4+4\,a\,c\,x+\frac{d^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)
```

```
[Out] (4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4
```

3.37 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=529

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{2}\,c+\sqrt[4]{c}\,\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}\,dx}{\sqrt[4]{c}\,\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\,\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}\,(c+dx)}{\sqrt[4]{c}\,\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\,\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}$$

```
[Out] -1/4*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))
^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d
^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/4*d*arctanh((-(d*x+c)*2
^(1/2)+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d
^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)-(4*a*d^2
+c^3)^(1/2))^(1/2)-1/8*d*ln(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/
4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(
4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/8*d*ln(d^2*(c/d+x)
^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a*d^2+
c^3)^(1/2))^(1/2))/c^(3/4)*2^(1/2)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^
3)^(1/2))^(1/2)
```

Rubi [A]

time = 0.69, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1120, 1108, 648, 632, 212, 642}

$$-\frac{d\log\left(\sqrt{c}\,\sqrt{4ad^2+c^3}-\sqrt{2}\,\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\,\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{c}\,\sqrt{4ad^2+c^3}+\sqrt{2}\,\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\,\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}-\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}\,c+\sqrt{2}\,dx}{\sqrt[4]{c}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}\,(c+dx)}{\sqrt[4]{c}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] -1/2*(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] +
Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(Sqrt[2]*c^(3/
4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c
^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sq
rt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]
*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2]
- Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/
d + x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 +
 4*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c
^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(4*Sqrt[2]*c^(3
/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Subst}\left(\int \frac{1}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{d\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}-x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,x}{d}+x^2}\,dx, x, \frac{c}{d}\right.}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,x}{d}+x^2}\,dx, x, \frac{c}{d}\right.}{4\sqrt{c}\sqrt{c^3+4ad^2}}$$

$$= -\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,(c+dx)+(c\right.}{4\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$= -\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}\,c^{3/4}-\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}\,dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}} - \frac{d\tanh^{-}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 71, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&, \frac{\log(x-\#1)}{2c^2\#1+3cd\#1^2+d^2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*#
1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 64, normalized size = 0.12

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^{2} \_Z^{4}+4cd\_Z^{3}+4c^{2}\_Z^{2}+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}d^{2}+3\_R^{2}cd+2\_Rc^{2}}}{4}$	64
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^{2} \_Z^{4}+4cd\_Z^{3}+4c^{2}\_Z^{2}+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}d^{2}+3\_R^{2}cd+2\_Rc^{2}}}{4}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3
*c*d+4*_Z^2*c^2+4*a*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(408) = 816$.
time = 0.37, size = 905, normalized size = 1.71

$$\frac{1}{8}\sqrt{-\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}+1}{ac^{3}+4a^{2}d^{2}}}\log\left(d^{2}x+cd+\left(2acd^{2}+\left(ac^{7}+4a^{2}c^{4}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}\right)\sqrt{-\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}+1}{ac^{3}+4a^{2}d^{2}}}\right)-\frac{1}{8}\sqrt{-\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}+1}{ac^{3}+4a^{2}d^{2}}}\log\left(d^{2}x+cd-\left(2acd^{2}+\left(ac^{7}+4a^{2}c^{4}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}\right)\sqrt{-\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}+1}{ac^{3}+4a^{2}d^{2}}}\right)+\frac{1}{8}\sqrt{\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}-1}{ac^{3}+4a^{2}d^{2}}}\log\left(d^{2}x+cd+\left(2acd^{2}-\left(ac^{7}+4a^{2}c^{4}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}\right)\sqrt{\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}-1}{ac^{3}+4a^{2}d^{2}}}\right)-\frac{1}{8}\sqrt{\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}-1}{ac^{3}+4a^{2}d^{2}}}\log\left(d^{2}x+cd-\left(2acd^{2}-\left(ac^{7}+4a^{2}c^{4}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}\right)\sqrt{\frac{2\left(ac^{3}+4a^{2}d^{2}\right)\sqrt{-\frac{d^{2}}{ac^{9}+8a^{2}c^{6}d^{2}+16a^{3}c^{3}d^{4}}}-1}{ac^{3}+4a^{2}d^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*
c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7 +
 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-
(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4))
+ 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*
c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x
+ c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d
^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^
2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt((2*(a*c^
3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*
```

c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sq
rt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d
^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2
*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2
+ 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 -
 (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)
))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^
3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))

Sympy [A]
time = 0.60, size = 88, normalized size = 0.17

$$\operatorname{RootSum}\left(t^{4}\cdot\left(16384a^{3}c^{3}d^{2}+4096a^{2}c^{6}\right)+128t^{2}ac^{3}+1,\ \left(t\mapsto t\log\left(x+\frac{-1024t^{3}a^{2}c^{4}d^{2}-256t^{3}ac^{7}+16tacd^{2}-4tc^{4}+cd}{d^{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

```
[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 +
1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 + 1
6*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

Giac [A]
time = 4.85, size = 603, normalized size = 1.14

$$-\frac{\log\left(x+\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{4\left(d^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{3}-3cd\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)\right)}+\frac{\log\left(x-\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{4\left(d^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{3}+3cd\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)\right)}-\frac{\log\left(x+\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{4\left(d^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{3}-3cd\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)\right)}+\frac{\log\left(x-\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{4\left(d^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{3}+3cd\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")
```

```
[Out] -1/4*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*
d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)
*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d))
 + 1/4*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^
2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*
c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d
)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((
c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c
/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt
((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt
(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) -
 c/d))
```

Mupad [B]
time = 4.54, size = 1551, normalized size = 2.93

$$\operatorname{atan}\left(\frac{\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}-64acd^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)1i+\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}+64acd^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)1i}{\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}-64acd^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)-\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}+64acd^6\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)}\right)\sqrt{-\frac{2d\sqrt{-a^3c^3}+ac^3}{64(4a^3c^3d^2+a^2c^6)}}\,2i+\operatorname{atan}\left(\frac{\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}-64acd^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)1i+\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}+64acd^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)1i}{\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}-64acd^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)-\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}\left(\left(\left(256ac^4d^5+256axc^3d^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}+64acd^6\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}+4cd^5+4d^6x\right)}\right)\sqrt{\frac{2d\sqrt{-a^3c^3}-ac^3}{64(4a^3c^3d^2+a^2c^6)}}\,2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3),x)
```

```
[Out] atan(((-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2
)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*
(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a
*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i + (-(2*
d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*
c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 +
4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-(2*d*(-a^3*c^3
)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 2
56*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d
^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4
*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - (-(2*d*(-a^3*c^3)^(1/2) + a*c^
3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x
)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))
)^(1/2) + 4*c*d^5 + 4*d^6*x)))*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^
6 + 4*a^3*c^3*d^2)))^(1/2)*2i + atan((((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*
(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^
6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
4*c*d^5 + 4*d^6*x)*1i + ((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/
2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3
*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^
6*x)*1i)/(((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(
1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(6
4*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) -
a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - ((2*d*(
-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4
*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c
^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)))*((2*d*(-a^3*c^3)^(1/2) -
 a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*2i
```

3.38 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$

Optimal. Leaf size=746

$$-\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}-\frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,c+\sqrt[4]{c}\,\sqrt{c^3}}{\sqrt[4]{c}\,\sqrt{c}}\right)}{32\sqrt{2}\,ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}-\sqrt{c^3}}}$$

```
[Out] -1/16*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*
d*x^3+4*c^2*x^2+4*a*c)-1/64*d*arctanh((c*2^(1/2)+d*x*2^(1/2)+c^(1/4)*(c^(3/
2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2))
*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2
^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)+1/64*d*arctanh((-(d*x+c)*2^(1/2)
+c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/(c^(3/2)-(4*a*d^2+c^3
)^(1/2))^(1/2))*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4)/(4*a*d
^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)-(4*a*d^2+c^3)^(1/2))^(1/2)-1/128*d*ln(d^2*(c
/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)-c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3/2)+(4*a
*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))/a/c^(7/4
)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/128*d*l
n(d^2*(c/d+x)^2+c^(1/2)*(4*a*d^2+c^3)^(1/2)+c^(1/4)*d*(c/d+x)*2^(1/2)*(c^(3
/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*(c^3+12*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))/
a/c^(7/4)/(4*a*d^2+c^3)^(3/2)*2^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)
```

Rubi [A]
time = 1.18, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1120, 1106, 1183, 648, 632, 212, 642}

$$-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}-\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}\,c+\sqrt{2}\,dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}+\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}\,(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] -1/16*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)*
(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a*d^2 + c^(3/2)*S
qrt[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 +
4*a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(3
2*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2
]]) + (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sq
rt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2
) - Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqr
t[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 +
 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2)
```

```
 + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)
*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*
a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt
[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)
^2])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 +
4*a*d^2]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^2}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{\text{Subst}\left(\int \frac{4c^4-2c(4}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{2}\,\sqrt[4]{\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{\left(d\left(c^3+12ad^2-\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2-c\cdots\right.}{\cdots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2+c\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.08, size = 182, normalized size = 0.24

$$\frac{\frac{4(c+dx)(4ad+cx(2c+dx))}{4ac+x^2(2c+dx)^2}+\text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&,\frac{2c^3\log(x-\#1)+12ad^2\log(x-\#1)+2c^2d\log(x-\#1)\#1+cd^2\log(x-\#1)\#1^2}{2c^2\#1+3cd\#1^2+d^2\#1^3}\&\right]}{64ac\left(c^3+4ad^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] ((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + Root
Sum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c^3*Log[x - #1] + 12*
a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*c^2
*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(64*a*c*(c^3 + 4*a*d^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 232, normalized size = 0.31

method	result
default	$\frac{\frac{d^2x^3}{64a\left(4a\,d^2+c^3\right)}+\frac{3cd\,x^2}{64a\left(4a\,d^2+c^3\right)}+\frac{\left(2a\,d^2+c^3\right)x}{32c\left(4a\,d^2+c^3\right)a}+\frac{d}{64a\,d^2+16c^3}}{\frac{1}{4}x^4d^2+cd\,x^3+c^2x^2+ac}+\frac{\sum_{\_R=\text{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\left(\_R^2c\,d^2+2\_R\,c^2\ldots\right.}{\_R^3d^2+3\ldots}}{16(16a\,d^2+4c^3)ac}$
risch	$\frac{\frac{d^2x^3}{64a\left(4a\,d^2+c^3\right)}+\frac{3cd\,x^2}{64a\left(4a\,d^2+c^3\right)}+\frac{\left(2a\,d^2+c^3\right)x}{32c\left(4a\,d^2+c^3\right)a}+\frac{d}{64a\,d^2+16c^3}}{\frac{1}{4}x^4d^2+cd\,x^3+c^2x^2+ac}+\frac{\sum_{\_R=\text{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\left(\frac{d^2\_R^2}{4a\,d^2+c^3}+\frac{2cd\_R\ldots}{4a\,d^2+c^3}\right.}{\_R^3d^2+3\ldots}}{64a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/64*d^2/a/(4*a*d^2+c^3)*x^3+3/64/a*c*d/(4*a*d^2+c^3)*x^2+1/32/c*(2*a*d^2+
c^3)/(4*a*d^2+c^3)/a*x+1/16*d/(4*a*d^2+c^3))/(1/4*x^4*d^2+c*d*x^3+c^2*x^2+a
*c)+1/16/(16*a*d^2+4*c^3)/a/c*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_
R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*
c^2+4*a*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 +
 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^
```

```
3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*c^2
*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c
^4 + 4*a^2*c*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3222 vs. $2(608) = 1216$.
time = 0.45, size = 3222, normalized size = 4.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2 +
 (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6
 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11
 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 36
0*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4
 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^
7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*lo
g(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 32
4*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^1
9 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8
)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^2
2*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8
*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 +
2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c
^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a
^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 +
4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c
^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4
*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6
+ 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4
 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*
c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^
13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d
^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a
^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96
*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13
*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^
8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^
6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*
sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*
a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*
d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 +
```

```
3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 1
2*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + (4*a^2*c^5 + 16*a^3*c^
2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4
*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a
^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d
^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c
^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096
*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d
^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d
^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (
a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*
c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*
a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6
144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2
*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt
(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2
+ 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*
d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 6
4*a^6*c^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*
x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt
(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*
c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10
)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840
*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^
4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5
+ 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d
^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a
^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a
*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 +
1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d
^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^
2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 12
96*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16
*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c
^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.59, size = 1057, normalized size = 1.42

$$-\frac{\frac{\left(cd^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{2}-2c^{2}d\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)+2c^{3}+12ad^{2}\right)\log\left(x+\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{d^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{3}-3cd\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}-\frac{\left(cd^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{2}+2c^{2}d\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)+2c^{3}+12ad^{2}\right)\log\left(x-\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{d^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{3}+3cd\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}+2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)}+\frac{\left(cd^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{2}-2c^{2}d\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)+2c^{3}+12ad^{2}\right)\log\left(x+\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{d^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{3}-3cd\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}-\frac{\left(cd^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{2}+2c^{2}d\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)+2c^{3}+12ad^{2}\right)\log\left(x-\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}+\frac{c}{d}\right)}{d^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{3}+3cd\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)^{2}+2c^{2}\left(\sqrt{\frac{c^{2}d^{2}-2\sqrt{-ac}\,d^{3}}{d^{4}}}-\frac{c}{d}\right)}}{64\left(ac^{4}+4a^{2}cd^{2}\right)}+\frac{cd^{2}x^{3}+3c^{2}dx^{2}+2c^{3}x+4ad^{2}x+4acd}{16\left(d^{2}x^{4}+4cdx^{3}+4c^{2}x^{2}+4ac\right)\left(ac^{4}+4a^{2}cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] -1/64*((c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(s
qrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sq
rt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a
*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/
d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt
((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 + 2*sq
rt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 + 2*sqr
t(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d
)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt(
(c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) + (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(
-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4)
 + c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) +
 c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((
c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c
/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12
*a*d^2)*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c
^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a
*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/
d)))/(a*c^4 + 4*a^2*c*d^2) + 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 2*c^3*x + 4*a*
d^2*x + 4*a*c*d)/((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)*(a*c^4 + 4*a^2*
c*d^2))
```

Mupad [B]
time = 4.30, size = 2500, normalized size = 3.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] (d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 + c
^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c +
 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan(((-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^
7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(40
96*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*
((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*
```

(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768
*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c
^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a
^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 +
 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*
c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*
d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*
c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48
*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 +
704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36
*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*
c^2*d^4)))*1i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2
+ 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13
*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 +
2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 +
 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c
^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10
*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^
9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*
c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d
^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^
3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c
^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7
*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^
3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a
*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*d^8 +
 c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^11 +
10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-
a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^
9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*
a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a
^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3
*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15
*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16
+ 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c
^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^
5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^
4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 1
2*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5
+ 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5
*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^
3*c^5*d^2 + 16*a^4*c^2*d^4))) + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) +
 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^
16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144
*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 +

```
 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d
^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*
(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4
 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^
10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 +
196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(
a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 +
72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*
d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^
4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10
+ c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))
))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*
d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8
*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*2i - atan(((-(a^3*c^11 - 10*c^3*d^3*(-a
^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2)
)/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^...
```

3.39 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx$

Optimal. Leaf size=295

$$\frac{\left(5d^4+256ae^3\right)^4x}{1048576e^4}-\frac{d^2\left(5d^4+256ae^3\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2}+\frac{\left(5d^4+256ae^3\right)^2\left(59d^4+256ae^3\right)\left(\frac{d}{4e}+x\right)^5}{5120}-\frac{9}{224}d^2e^2\left(5d^4\right.$$

```
[Out] 1/1048576*(256*a*e^3+5*d^4)^4*x/e^4-1/8192*d^2*(256*a*e^3+5*d^4)^3*(1/4*d/e
+x)^3/e^2+1/5120*(256*a*e^3+5*d^4)^2*(256*a*e^3+59*d^4)*(1/4*d/e+x)^5-9/224
*d^2*e^2*(256*a*e^3+5*d^4)*(256*a*e^3+17*d^4)*(1/4*d/e+x)^7+1/24*e^4*(65536
*a^2*e^6+20992*a*d^4*e^3+601*d^8)*(1/4*d/e+x)^9-72/11*d^2*e^6*(256*a*e^3+17
*d^4)*(1/4*d/e+x)^11+64/13*e^8*(256*a*e^3+59*d^4)*(1/4*d/e+x)^13-2048/5*d^2
*e^10*(1/4*d/e+x)^15+4096/17*e^12*(1/4*d/e+x)^17
```

Rubi [A]
time = 0.40, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1120, 1104}

$$\frac{1}{24}e^4\left(65536a^2e^6+20992ad^4e^3+601d^8\right)\left(\frac{d}{4e}+x\right)^9+\frac{\left(256ae^3+5d^4\right)^2\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^5}{5120}+\frac{64}{13}e^8\left(256ae^3+59d^4\right)\left(\frac{d}{4e}+x\right)^{13}+\frac{x\left(256ae^3+5d^4\right)^4}{1048576e^4}-\frac{72}{11}d^2e^6\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^{11}-\frac{9}{224}d^2e^2\left(256ae^3+5d^4\right)\left(256ae^3+17d^4\right)\left(\frac{d}{4e}+x\right)^7-\frac{d^2\left(256ae^3+5d^4\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2}-\frac{2048}{5}d^2e^{10}\left(\frac{d}{4e}+x\right)^{15}+\frac{4096}{17}e^{12}\left(\frac{d}{4e}+x\right)^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*
e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e
) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*
e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e)
+ x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^
8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^
15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = \text{Subst}\left(\int \left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^4\,dx, x, \frac{d}{4e}+x\right.$$

$$= \text{Subst}\left(\int \left(\frac{\left(5d^4+256ae^3\right)^4}{1048576e^4}-\frac{3d^2\left(5d^4+256ae^3\right)^3x^2}{8192e^2}+\frac{27}{512}d^4\left(5d^4\right.\right.\right.$$

$$= \frac{\left(5d^4+256ae^3\right)^4x}{1048576e^4}-\frac{d^2\left(5d^4+256ae^3\right)^3\left(\frac{d}{4e}+x\right)^3}{8192e^2}+\frac{\left(5d^4+256ae^3\right)}{}$$

Mathematica [A]
time = 0.04, size = 345, normalized size = 1.17

$$4096a^4e^8x-1024a^3d^3e^6x^2+128a^2d^6e^4x^3+8ade^2\left(-d^8+512a^2e^6\right)x^4+\frac{1}{5}\left(d^{12}-6144a^2d^4e^6+16384a^3e^9\right)x^5-128ad^3e^4\left(-d^4+8ae^3\right)x^6-\frac{32}{7}d^2e^2\left(d^8-24ad^4e^3-768a^2e^6\right)x^7-4de^3\left(d^8+192ad^4e^3-1536a^2e^6\right)x^8+\frac{128}{3}e^4\left(d^8-32ad^4e^3+64a^2e^6\right)x^9+\frac{128}{5}d^3e^5\left(3d^4+40ae^3\right)x^{10}+\frac{128}{11}d^2e^6\left(-13d^4+384ae^3\right)x^{11}-512de^7\left(d^4-8ae^3\right)x^{12}+\frac{2048}{13}e^8\left(-d^4+8ae^3\right)x^{13}+1024d^3e^9x^{14}+\frac{8192}{5}d^2e^{10}x^{15}+1024de^{11}x^{16}+\frac{4096e^{12}x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d
^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 -
 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768
*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*
e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e
^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4
- 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 +
 (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17
```

Maple [A]
time = 0.03, size = 500, normalized size = 1.69

method	result
norman	$4096a^4e^8x-1024a^3e^6d^3x^2+128a^2e^4d^6x^3+\left(4096a^3e^8d-8a\,d^9e^2\right)x^4+\left(\frac{16384}{5}a^3e^9-\frac{6144}{5}a^2d^4e^6\right.$
gosper	$128a\,d^7e^4x^6+6144a^2d\,e^9x^8-768a\,d^5e^6x^8+4096ad\,e^{10}x^{12}-1024a^2d^3e^7x^6+1024x^{10}a\,d^3e^8+\frac{491}{1}$
risch	$128a\,d^7e^4x^6+6144a^2d\,e^9x^8-768a\,d^5e^6x^8+4096ad\,e^{10}x^{12}-1024a^2d^3e^7x^6+1024x^{10}a\,d^3e^8+\frac{491}{1}$
default	$\frac{4096e^{12}x^{17}}{17}+1024d\,e^{11}x^{16}+\frac{8192d^2e^{10}x^{15}}{5}+1024d^3e^9x^{14}+\frac{128\left(128e^5a-16d^4e^2\right)e^6x^{13}}{13}+\frac{\left(16384d\,e^{10}a+256\left(128e^5\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4096/17*e^12*x^17+1024*d*e^11*x^16+8192/5*d^2*e^10*x^15+1024*d^3*e^9*x^14+1
28/13*(128*a*e^5-16*d^4*e^2)*e^6*x^13+1/12*(16384*d*e^10*a+256*(128*a*e^5-1
6*d^4*e^2)*d*e^5-2048*d^5*e^7)*x^12+1/11*(384*d^6*e^6+32768*d^2*e^9*a+128*(
128*a*e^5-16*d^4*e^2)*d^2*e^4)*x^11+1/10*(14336*a*d^3*e^8+256*d^7*e^5-32*(1
28*a*e^5-16*d^4*e^2)*d^3*e^3)*x^10+1/9*(8192*a^2*e^10-8192*a*d^4*e^7+128*d^
8*e^4+(128*a*e^5-16*d^4*e^2)^2)*x^9+1/8*(16384*a^2*d*e^9-2048*a*d^5*e^6-32*
d^9*e^3+256*d*e^4*a*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(24576*a^2*d^2*e^8+512*
a*d^6*e^5+2*d^6*(128*a*e^5-16*d^4*e^2))*x^7+1/6*(-2048*a^2*e^7*d^3-32*a*d^3
*e^2*(128*a*e^5-16*d^4*e^2)+256*d^7*e^4*a)*x^6+1/5*(128*a^2*e^4*(128*a*e^5-
16*d^4*e^2)-4096*a^2*d^4*e^6+d^12)*x^5+1/4*(16384*a^3*d*e^8-32*a*d^9*e^2)*x
^4+128*a^2*e^4*d^6*x^3-1024*a^3*e^6*d^3*x^2+4096*a^4*e^8*x
```

Maxima [A]
time = 0.28, size = 349, normalized size = 1.18

$$\frac{4096}{17}x^{17}e^{12}+1024\,dx^{16}e^{11}+\frac{8192}{5}d^2x^{15}e^{10}+\frac{8192}{7}d^3x^{14}e^9+\frac{4096}{13}d^4x^{13}e^8+\frac{1}{5}d^{12}x^5-\frac{4}{7}\left(7x^8e^3+8\,dx^7e^2\right)d^9+\frac{128}{165}\left(45x^{11}e^6+99\,dx^{10}e^5+55\,d^2x^9e^4\right)d^6+4096\,a^4xe^8+\frac{1024}{5}\left(16x^5e^3+20\,dx^4e^2-5\,d^3x^2\right)a^3e^6-\frac{512}{1001}\left(286x^{14}e^9+924\,dx^{13}e^8+1001\,d^2x^{12}e^7+364\,d^3x^{11}e^6\right)d^3+\frac{128}{105}\left(2240x^9e^6+5040\,dx^8e^5+2880\,d^2x^7e^4+105\,d^6x^3-168\left(5x^6e^3+6\,dx^5e^2\right)d^3\right)a^2e^4+\frac{8}{15015}\left(2365440x^{13}e^9+7687680\,dx^{12}e^8+8386560\,d^2x^{11}e^7+3075072\,d^3x^{10}e^6-15015\,d^9x^4+34320\left(6x^7e^3+7\,dx^6e^2\right)d^6-32032\left(36x^{10}e^6+80\,dx^9e^5+45\,d^2x^8e^4\right)d^3\right)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 8192/7*d^3*x^
14*e^9 + 4096/13*d^4*x^13*e^8 + 1/5*d^12*x^5 - 4/7*(7*x^8*e^3 + 8*d*x^7*e^2
)*d^9 + 128/165*(45*x^11*e^6 + 99*d*x^10*e^5 + 55*d^2*x^9*e^4)*d^6 + 4096*a
^4*x*e^8 + 1024/5*(16*x^5*e^3 + 20*d*x^4*e^2 - 5*d^3*x^2)*a^3*e^6 - 512/100
1*(286*x^14*e^9 + 924*d*x^13*e^8 + 1001*d^2*x^12*e^7 + 364*d^3*x^11*e^6)*d^
3 + 128/105*(2240*x^9*e^6 + 5040*d*x^8*e^5 + 2880*d^2*x^7*e^4 + 105*d^6*x^3
 - 168*(5*x^6*e^3 + 6*d*x^5*e^2)*d^3)*a^2*e^4 + 8/15015*(2365440*x^13*e^9 +
 7687680*d*x^12*e^8 + 8386560*d^2*x^11*e^7 + 3075072*d^3*x^10*e^6 - 15015*d
^9*x^4 + 34320*(6*x^7*e^3 + 7*d*x^6*e^2)*d^6 - 32032*(36*x^10*e^6 + 80*d*x^
9*e^5 + 45*d^2*x^8*e^4)*d^3)*a*e^2
```

Fricas [A]
time = 0.36, size = 332, normalized size = 1.13

$$\frac{4096}{17}e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}d^2e^{10}x^{15}+1024\,d^3e^9x^{14}+128\,a^2d^6e^4x^3-1024\,a^3d^3e^6x^2-\frac{2048}{13}\left(d^4e^8-8\,ae^{11}\right)x^{13}+4096\,a^4e^8x-512\left(d^5e^7-8\,ade^{10}\right)x^{12}-\frac{128}{11}\left(13\,d^6e^6-384\,ad^2e^9\right)x^{11}+\frac{128}{5}\left(3\,d^7e^5+40\,ad^3e^8\right)x^{10}+\frac{128}{3}\left(d^8e^4-32\,ad^4e^7+64\,a^2e^{10}\right)x^9-4\left(d^9e^3+192\,ad^5e^6-1536\,a^2de^9\right)x^8-\frac{32}{7}\left(d^{10}e^2-24\,ad^6e^5-768\,a^2d^2e^8\right)x^7+128\left(ad^7e^4-8\,a^2d^3e^7\right)x^6+\frac{1}{5}\left(d^{12}-6144\,a^2d^4e^6+16384\,a^3e^9\right)x^5-8\left(ad^9e^2-512\,a^3de^8\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 1024*d^3*e^9*
x^14 + 128*a^2*d^6*e^4*x^3 - 1024*a^3*d^3*e^6*x^2 - 2048/13*(d^4*e^8 - 8*a*
e^11)*x^13 + 4096*a^4*e^8*x - 512*(d^5*e^7 - 8*a*d*e^10)*x^12 - 128/11*(13*
d^6*e^6 - 384*a*d^2*e^9)*x^11 + 128/5*(3*d^7*e^5 + 40*a*d^3*e^8)*x^10 + 128
/3*(d^8*e^4 - 32*a*d^4*e^7 + 64*a^2*e^10)*x^9 - 4*(d^9*e^3 + 192*a*d^5*e^6
- 1536*a^2*d*e^9)*x^8 - 32/7*(d^10*e^2 - 24*a*d^6*e^5 - 768*a^2*d^2*e^8)*x^
```

```
7 + 128*(a*d^7*e^4 - 8*a^2*d^3*e^7)*x^6 + 1/5*(d^12 - 6144*a^2*d^4*e^6 + 16
384*a^3*e^9)*x^5 - 8*(a*d^9*e^2 - 512*a^3*d*e^8)*x^4
```

Sympy [A]
time = 0.04, size = 366, normalized size = 1.24

$$4096a^4e^8x - 1024a^3d^3e^6x^2 + 128a^2d^6e^4x^3 + 1024d^3e^9x^{14} + \frac{8192d^2e^{10}x^{15}}{5} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17} + x^{13}\cdot\left(\frac{16384ae^{11}}{13} - \frac{2048d^4e^8}{13}\right) + x^{12}\cdot\left(4096ade^{10} - 512d^5e^7\right) + x^{11}\cdot\left(\frac{49152ad^2e^9}{11} - \frac{1664d^6e^6}{11}\right) + x^{10}\cdot\left(1024ad^3e^8 + \frac{384d^7e^5}{5}\right) + x^9\cdot\left(\frac{8192a^2e^{10}}{3} - \frac{4096ad^4e^7}{3} + \frac{128d^8e^4}{3}\right) + x^8\cdot\left(6144a^2de^9 - 768ad^5e^6 - 4d^9e^3\right) + x^7\cdot\left(\frac{24576a^2d^2e^8}{7} + \frac{768ad^6e^5}{7} - \frac{32d^{10}e^2}{7}\right) + x^6\left(-1024a^2d^3e^7 + 128ad^7e^4\right) + x^5\cdot\left(\frac{16384a^3e^9}{5} - \frac{6144a^2d^4e^6}{5} + \frac{d^{12}}{5}\right) + x^4\cdot\left(4096a^3de^8 - 8ad^9e^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

```
[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 102
4*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e**
12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096*a*
d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6/11)
 + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4
096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*
e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7 - 32
*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x**5*(1638
4*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3*d*e**8 -
 8*a*d**9*e**2)
```

Giac [A]
time = 6.28, size = 323, normalized size = 1.09

$$\frac{4096}{17}x^{17}e^{12} + 1024\,dx^{16}e^{11} + \frac{8192}{5}d^2x^{15}e^{10} + 1024\,d^3x^{14}e^9 - \frac{2048}{13}d^4x^{13}e^8 - 512\,d^5x^{12}e^7 - \frac{1664}{11}d^6x^{11}e^6 + \frac{384}{5}d^7x^{10}e^5 + \frac{128}{3}d^8x^9e^4 - 4\,d^9x^8e^3 - \frac{32}{7}d^{10}x^7e^2 + \frac{1}{5}d^{12}x^5 + \frac{16384}{13}ax^{13}e^{11} + 4096\,adx^{12}e^{10} + \frac{49152}{11}ad^2x^{11}e^9 + 1024\,ad^3x^{10}e^8 - \frac{4096}{3}ad^4x^9e^7 - 768\,ad^5x^8e^6 + \frac{768}{7}ad^6x^7e^5 + 128\,ad^7x^6e^4 - 8\,ad^9x^4e^2 + \frac{8192}{3}a^2x^9e^{10} + 6144\,a^2dx^8e^9 + \frac{24576}{7}a^2d^2x^7e^8 - 1024\,a^2d^3x^6e^7 - \frac{6144}{5}a^2d^4x^5e^6 + 128\,a^2d^6x^3e^4 + \frac{16384}{5}a^3x^5e^9 + 4096\,a^3dx^4e^8 - 1024\,a^3d^3x^2e^6 + 4096\,a^4xe^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 1024*d^3*x^14
*e^9 - 2048/13*d^4*x^13*e^8 - 512*d^5*x^12*e^7 - 1664/11*d^6*x^11*e^6 + 384
/5*d^7*x^10*e^5 + 128/3*d^8*x^9*e^4 - 4*d^9*x^8*e^3 - 32/7*d^10*x^7*e^2 + 1
/5*d^12*x^5 + 16384/13*a*x^13*e^11 + 4096*a*d*x^12*e^10 + 49152/11*a*d^2*x^
11*e^9 + 1024*a*d^3*x^10*e^8 - 4096/3*a*d^4*x^9*e^7 - 768*a*d^5*x^8*e^6 + 7
68/7*a*d^6*x^7*e^5 + 128*a*d^7*x^6*e^4 - 8*a*d^9*x^4*e^2 + 8192/3*a^2*x^9*e
^10 + 6144*a^2*d*x^8*e^9 + 24576/7*a^2*d^2*x^7*e^8 - 1024*a^2*d^3*x^6*e^7 -
 6144/5*a^2*d^4*x^5*e^6 + 128*a^2*d^6*x^3*e^4 + 16384/5*a^3*x^5*e^9 + 4096*
a^3*d*x^4*e^8 - 1024*a^3*d^3*x^2*e^6 + 4096*a^4*x*e^8
```

Mupad [B]
time = 0.27, size = 331, normalized size = 1.12

$$x^5\left(\frac{16384\,a^3\,e^9}{5} - \frac{6144\,a^2\,d^4\,e^6}{5} + \frac{d^{12}}{5}\right) + x^{10}\left(\frac{384\,d^7\,e^5}{5} + 1024\,a\,d^3\,e^8\right) - x^{11}\left(\frac{1664\,d^6\,e^6}{11} - \frac{49152\,a\,d^2\,e^9}{11}\right) + \frac{4096\,e^{12}\,x^{17}}{17} + \frac{2048\,e^8\,x^{13}\,\left(8\,a\,e^3 - d^4\right)}{13} + \frac{128\,e^4\,x^9\,\left(64\,a^2\,e^6 - 32\,a\,d^4\,e^3 + d^8\right)}{3} + 4096\,a^4\,e^8\,x + 1024\,d\,e^{11}\,x^{16} + 1024\,d^3\,e^9\,x^{14} + \frac{8192\,d^2\,e^{10}\,x^{15}}{5} + 512\,d\,e^7\,x^{12}\,\left(8\,a\,e^3 - d^4\right) + \frac{32\,d^2\,e^2\,x^7\,\left(768\,a^2\,e^6 + 24\,a\,d^4\,e^3 - d^8\right)}{7} - 1024\,a^3\,d^3\,e^6\,x^2 + 128\,a^2\,d^6\,e^4\,x^3 - 4\,d\,e^3\,x^8\,\left(-1536\,a^2\,e^6 + 192\,a\,d^4\,e^3 + d^8\right) - 128\,a\,d^3\,e^4\,x^6\,\left(8\,a\,e^3 - d^4\right) - 8\,a\,d\,e^2\,x^4\,\left(d^8 - 512\,a^2\,e^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)
```

```
[Out] x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e^
5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11) +
(4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*(d^
8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 + 102
4*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^4) +
(32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*e^6*x^
2 + 128*a^2*d^6*e^4*x^3 - 4*d*e^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^4*e^3)
- 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d*e^2*x^4*(d^8 - 512*a^2*e^6)
```

3.40 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx$

Optimal. Leaf size=203

$$512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3-\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4-\frac{384}{5}ae^4\left(d^4-4ae^3\right)x^5+4d^3e^2\left(d^4-16ae^3\right)x^6+\frac{24}{7}d$$

```
[Out] 512*a^3*e^6*x-96*a^2*d^3*e^4*x^2+8*a*d^6*e^2*x^3-1/4*d*(-1536*a^2*e^6+d^8)*
x^4-384/5*a*e^4*(-4*a*e^3+d^4)*x^5+4*d^3*e^2*(-16*a*e^3+d^4)*x^6+24/7*d^2*e
^3*(64*a*e^3+d^4)*x^7-24*d*e^4*(-16*a*e^3+d^4)*x^8-128/3*e^5*(-4*a*e^3+d^4)
*x^9+32*d^3*e^6*x^10+1536/11*d^2*e^7*x^11+128*d*e^8*x^12+512/13*e^9*x^13
```

Rubi [A]
time = 0.09, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{2086\}$

$$512a^3e^6x-\frac{1}{4}dx^4\left(d^8-1536a^2e^6\right)-96a^2d^3e^4x^2+8ad^6e^2x^3-\frac{128}{3}e^5x^9\left(d^4-4ae^3\right)-24de^4x^8\left(d^4-16ae^3\right)-\frac{384}{5}ae^4x^5\left(d^4-4ae^3\right)+4d^3e^2x^6\left(d^4-16ae^3\right)+\frac{24}{7}d^2e^3x^7\left(64ae^3+d^4\right)+32d^3e^6x^{10}+\frac{1536}{11}d^2e^7x^{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)
*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
- (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/1
1 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=\int \left(512a^3e^6-192a^2d^3e^4x+24ad^6e^2x^2-d\left(d^8-1536a^2e^6\right)x^3-38\right.$$

$$=512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3-\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4-\frac{384}{5}a$$

Mathematica [A]
time = 0.02, size = 207, normalized size = 1.02

$$512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3-\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4+\frac{384}{5}ae^4\left(-d^4+4ae^3\right)x^5+4d^3e^2\left(d^4-16ae^3\right)x^6+\frac{24}{7}d^2e^3\left(d^4+64ae^3\right)x^7-24de^4\left(d^4-16ae^3\right)x^8+\frac{128}{3}e^5\left(-d^4+4ae^3\right)x^9+32d^3e^6x^{10}+\frac{1536}{11}d^2e^7x^{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3
)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)
/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Maple [A]
time = 0.02, size = 288, normalized size = 1.42

method	result
norman	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\left(\frac{512}{3}a\,e^8-\frac{128}{3}d^4e^5\right)x^9+\left(384a\,e^7d-24d^5e^4\right)x^8$
gosper	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24d^5e^4$
risch	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-24d^5e^4$
default	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\frac{\left(512a\,e^8-256d^4e^5+8e^3\left(128e^5a-16d^4e^2\right)\right)x^9}{9}+\frac{(2048a\,e^7d-64}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+1/9*(51
2*a*e^8-256*d^4*e^5+8*e^3*(128*a*e^5-16*d^4*e^2))*x^9+1/8*(2048*a*e^7*d-64*
d^5*e^4+8*d*e^2*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(1536*a*d^2*e^6+24*d^6*e^3)
*x^7+1/6*(-256*a*e^5*d^3-d^3*(128*a*e^5-16*d^4*e^2)+8*d^7*e^2)*x^6+1/5*(8*a
*e^2*(128*a*e^5-16*d^4*e^2)-256*d^4*e^4*a+512*a^2*e^7)*x^5+1/4*(1536*a^2*d*
e^6-d^9)*x^4+8*a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+512*a^3*e^6*x
```

Maxima [A]
time = 0.27, size = 195, normalized size = 0.96

$$\frac{512}{13}\,x^{13}e^9+128\,dx^{12}e^8+\frac{1536}{11}\,d^2x^{11}e^7+\frac{256}{5}\,d^3x^{10}e^6-\frac{1}{4}\,d^9x^4+\frac{4}{7}\,(6\,x^7e^3+7\,dx^6e^2)d^6+512\,a^3xe^6-\frac{8}{15}\,(36\,x^{10}e^6+80\,dx^9e^5+45\,d^2x^8e^4)d^3+\frac{96}{5}\,(16\,x^5e^3+20\,dx^4e^2-5\,d^3x^2)a^2e^4+\frac{8}{105}\,(2240\,x^9e^6+5040\,dx^8e^5+2880\,d^2x^7e^4+105\,d^6x^3-168\,(5\,x^6e^3+6\,dx^5e^2)d^3)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13*e^9 + 128*d*x^12*e^8 + 1536/11*d^2*x^11*e^7 + 256/5*d^3*x^10*e^
6 - 1/4*d^9*x^4 + 4/7*(6*x^7*e^3 + 7*d*x^6*e^2)*d^6 + 512*a^3*x*e^6 - 8/15*
(36*x^10*e^6 + 80*d*x^9*e^5 + 45*d^2*x^8*e^4)*d^3 + 96/5*(16*x^5*e^3 + 20*d
*x^4*e^2 - 5*d^3*x^2)*a^2*e^4 + 8/105*(2240*x^9*e^6 + 5040*d*x^8*e^5 + 2880
*d^2*x^7*e^4 + 105*d^6*x^3 - 168*(5*x^6*e^3 + 6*d*x^5*e^2)*d^3)*a*e^2
```

Fricas [A]
time = 0.39, size = 198, normalized size = 0.98

$$\frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+32\,d^3e^6x^{10}+8\,ad^6e^2x^3-96\,a^2d^3e^4x^2+512\,a^3e^6x-\frac{128}{3}\,(d^4e^5-4\,ae^8)x^9-24\,(d^5e^4-16\,ade^7)x^8+\frac{24}{7}\,(d^6e^3+64\,ad^2e^6)x^7+4\,(d^7e^2-16\,ad^3e^5)x^6-\frac{384}{5}\,(ad^4e^4-4\,a^2e^7)x^5-\frac{1}{4}\,(d^9-1536\,a^2de^6)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 32*d^3*e^6*x^10 +
 8*a*d^6*e^2*x^3 - 96*a^2*d^3*e^4*x^2 + 512*a^3*e^6*x - 128/3*(d^4*e^5 - 4*
a*e^8)*x^9 - 24*(d^5*e^4 - 16*a*d*e^7)*x^8 + 24/7*(d^6*e^3 + 64*a*d^2*e^6)*
x^7 + 4*(d^7*e^2 - 16*a*d^3*e^5)*x^6 - 384/5*(a*d^4*e^4 - 4*a^2*e^7)*x^5 -
1/4*(d^9 - 1536*a^2*d*e^6)*x^4
```

Sympy [A]
time = 0.02, size = 218, normalized size = 1.07

$$512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + 32d^3e^6x^{10} + \frac{1536d^2e^7x^{11}}{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13} + x^9 \cdot \left(\frac{512ae^8}{3} - \frac{128d^4e^5}{3}\right) + x^8 \cdot (384ade^7 - 24d^5e^4) + x^7 \cdot \left(\frac{1536ad^2e^6}{7} + \frac{24d^6e^3}{7}\right) + x^6(-64ad^3e^5 + 4d^7e^2) + x^5 \cdot \left(\frac{1536a^2e^7}{5} - \frac{384ad^4e^4}{5}\right) + x^4 \cdot \left(384a^2de^6 - \frac{d^9}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

```
[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e**
6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13 +
x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**4)
+ x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d*
*7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e
**6 - d**9/4)
```

Giac [A]
time = 3.47, size = 187, normalized size = 0.92

$$\frac{512}{13}\,x^{13}e^9 + 128\,dx^{12}e^8 + \frac{1536}{11}\,d^2x^{11}e^7 + 32\,d^3x^{10}e^6 - \frac{128}{3}\,d^4x^9e^5 - 24\,d^5x^8e^4 + \frac{24}{7}\,d^6x^7e^3 + 4\,d^7x^6e^2 - \frac{1}{4}\,d^9x^4 + \frac{512}{3}\,ax^9e^8 + 384\,adx^8e^7 + \frac{1536}{7}\,ad^2x^7e^6 - 64\,ad^3x^6e^5 - \frac{384}{5}\,ad^4x^5e^4 + 8\,ad^6x^3e^2 + \frac{1536}{5}\,a^2x^5e^7 + 384\,a^2dx^4e^6 - 96\,a^2d^3x^2e^4 + 512\,a^3xe^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13*e^9 + 128*d*x^12*e^8 + 1536/11*d^2*x^11*e^7 + 32*d^3*x^10*e^6 -
 128/3*d^4*x^9*e^5 - 24*d^5*x^8*e^4 + 24/7*d^6*x^7*e^3 + 4*d^7*x^6*e^2 - 1/
4*d^9*x^4 + 512/3*a*x^9*e^8 + 384*a*d*x^8*e^7 + 1536/7*a*d^2*x^7*e^6 - 64*a
*d^3*x^6*e^5 - 384/5*a*d^4*x^5*e^4 + 8*a*d^6*x^3*e^2 + 1536/5*a^2*x^5*e^7 +
 384*a^2*d*x^4*e^6 - 96*a^2*d^3*x^2*e^4 + 512*a^3*x*e^6
```

Mupad [B]
time = 2.24, size = 201, normalized size = 0.99

$$\frac{512\,e^9\,x^{13}}{13} - x^4\,\left(\frac{d^9}{4} - 384\,a^2\,d\,e^6\right) + \frac{128\,e^5\,x^9\,\left(4\,a\,e^3 - d^4\right)}{3} + 512\,a^3\,e^6\,x + 128\,d\,e^8\,x^{12} + 32\,d^3\,e^6\,x^{10} + \frac{1536\,d^2\,e^7\,x^{11}}{11} + 8\,a\,d^6\,e^2\,x^3 + \frac{384\,a\,e^4\,x^5\,\left(4\,a\,e^3 - d^4\right)}{5} + 24\,d\,e^4\,x^8\,\left(16\,a\,e^3 - d^4\right) + \frac{24\,d^2\,e^3\,x^7\,\left(d^4 + 64\,a\,e^3\right)}{7} - 96\,a^2\,d^3\,e^4\,x^2 - 4\,d^3\,e^2\,x^6\,\left(16\,a\,e^3 - d^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^3,x)
```

```
[Out] (512*e^9*x^13)/13 - x^4*(d^9/4 - 384*a^2*d*e^6) + (128*e^5*x^9*(4*a*e^3 - d
^4))/3 + 512*a^3*e^6*x + 128*d*e^8*x^12 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x
^11)/11 + 8*a*d^6*e^2*x^3 + (384*a*e^4*x^5*(4*a*e^3 - d^4))/5 + 24*d*e^4*x^
8*(16*a*e^3 - d^4) + (24*d^2*e^3*x^7*(64*a*e^3 + d^4))/7 - 96*a^2*d^3*e^4*x
^2 - 4*d^3*e^2*x^6*(16*a*e^3 - d^4)
```

3.41 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 \, dx$

Optimal. Leaf size=107

$$64a^2e^4x-8ad^3e^2x^2+\frac{d^6x^3}{3}+32ade^4x^4-\frac{16}{5}e^2\left(d^4-8ae^3\right)x^5-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d^2e^4x^7+16de^5x^8+\frac{64e^6x^9}{9}$$

```
[Out] 64*a^2*e^4*x-8*a*d^3*e^2*x^2+1/3*d^6*x^3+32*a*d*e^4*x^4-16/5*e^2*(-8*a*e^3+
d^4)*x^5-8/3*d^3*e^3*x^6+64/7*d^2*e^4*x^7+16*d*e^5*x^8+64/9*e^6*x^9
```

Rubi [A]
time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {2086}

$$64a^2e^4x-\frac{16}{5}e^2x^5\left(d^4-8ae^3\right)-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3}-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d^2e^4x^7+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^
4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8
 + (64*e^6*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 \, dx = \int \left(64a^2e^4-16ad^3e^2x+d^6x^2+128ade^4x^3-16e^2\left(d^4-8ae^3\right)x^4-\right.$$

$$= 64a^2e^4x-8ad^3e^2x^2+\frac{d^6x^3}{3}+32ade^4x^4-\frac{16}{5}e^2\left(d^4-8ae^3\right)x^5-\frac{8}{3}d$$

Mathematica [A]
time = 0.01, size = 109, normalized size = 1.02

$$64a^2e^4x-8ad^3e^2x^2+\frac{d^6x^3}{3}+32ade^4x^4+\frac{16}{5}e^2\left(-d^4+8ae^3\right)x^5-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d^2e^4x^7+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-d
^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^
8 + (64*e^6*x^9)/9
```

Maple [A]
time = 0.02, size = 100, normalized size = 0.93

method	result
norman	$\frac{64e^6x^9}{9}+16d\,e^5x^8+\frac{64d^2e^4x^7}{7}-\frac{8d^3e^3x^6}{3}+\left(\frac{128}{5}e^5a-\frac{16}{5}d^4e^2\right)x^5+32ad\,e^4x^4+\frac{d^6x^3}{3}-8a\,d^3e^2x^2+6$
gosper	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5e^5a-\frac{16}{5}x^5d^4e^2+32ad\,e^4x^4+\frac{1}{3}d^6x^3-8a\,d^3e^2x$
default	$\frac{64e^6x^9}{9}+16d\,e^5x^8+\frac{64d^2e^4x^7}{7}-\frac{8d^3e^3x^6}{3}+\frac{(128e^5a-16d^4e^2)x^5}{5}+32ad\,e^4x^4+\frac{d^6x^3}{3}-8a\,d^3e^2x^2+64a^2e^4$
risch	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5e^5a-\frac{16}{5}x^5d^4e^2+32ad\,e^4x^4+\frac{1}{3}d^6x^3-8a\,d^3e^2x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+1/5*(128*a*e^5-1
6*d^4*e^2)*x^5+32*a*d*e^4*x^4+1/3*d^6*x^3-8*a*d^3*e^2*x^2+64*a^2*e^4*x
```

Maxima [A]
time = 0.26, size = 92, normalized size = 0.86

$$\frac{64}{9}\,x^9e^6+16\,dx^8e^5+\frac{64}{7}\,d^2x^7e^4+\frac{1}{3}\,d^6x^3-\frac{8}{15}\left(5\,x^6e^3+6\,dx^5e^2\right)d^3+64\,a^2xe^4+\frac{8}{5}\left(16\,x^5e^3+20\,dx^4e^2-5\,d^3x^2\right)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 + 1/3*d^6*x^3 - 8/15*(5*x^6*
e^3 + 6*d*x^5*e^2)*d^3 + 64*a^2*x*e^4 + 8/5*(16*x^5*e^3 + 20*d*x^4*e^2 - 5*
d^3*x^2)*a*e^2
```

Fricas [A]
time = 0.35, size = 98, normalized size = 0.92

$$\frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7-\frac{8}{3}\,d^3e^3x^6+32\,ade^4x^4+\frac{1}{3}\,d^6x^3-8\,ad^3e^2x^2+64\,a^2e^4x-\frac{16}{5}\left(d^4e^2-8\,ae^5\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 + 32*a*d*e
^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x^2 + 64*a^2*e^4*x - 16/5*(d^4*e^2 - 8*a
*e^5)*x^5
```

Sympy [A]
time = 0.01, size = 112, normalized size = 1.05

$$64a^2e^4x - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} + 16de^5x^8 + \frac{64e^6x^9}{9} + x^5 \cdot \left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] 64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d*
*3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 + x*
*5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

Giac [A]
time = 4.46, size = 90, normalized size = 0.84

$$\frac{64}{9}\,x^9e^6 + 16\,dx^8e^5 + \frac{64}{7}\,d^2x^7e^4 - \frac{8}{3}\,d^3x^6e^3 - \frac{16}{5}\,d^4x^5e^2 + \frac{1}{3}\,d^6x^3 + \frac{128}{5}\,ax^5e^5 + 32\,adx^4e^4 - 8\,ad^3x^2e^2 + 64\,a^2xe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 - 8/3*d^3*x^6*e^3 - 16/5*d^4
*x^5*e^2 + 1/3*d^6*x^3 + 128/5*a*x^5*e^5 + 32*a*d*x^4*e^4 - 8*a*d^3*x^2*e^2
 + 64*a^2*x*e^4
```

Mupad [B]
time = 0.04, size = 98, normalized size = 0.92

$$x^5\left(\frac{128\,a\,e^5}{5} - \frac{16\,d^4\,e^2}{5}\right) + \frac{d^6\,x^3}{3} + \frac{64\,e^6\,x^9}{9} + 64\,a^2\,e^4\,x + 16\,d\,e^5\,x^8 - \frac{8\,d^3\,e^3\,x^6}{3} + \frac{64\,d^2\,e^4\,x^7}{7} - 8\,a\,d^3\,e^2\,x^2 + 32\,a\,d\,e^4\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a^
2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3*e
^2*x^2 + 32*a*d*e^4*x^4
```

3.42 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx$

Optimal. Leaf size=37

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

```
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi steps

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) \, dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.00

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.92

method	result	size
gosper	$8a\,e^2x - \frac{1}{2}x^2d^3 + 2x^4d\,e^2 + \frac{8}{5}x^5e^3$	34
default	$8a\,e^2x - \frac{1}{2}x^2d^3 + 2x^4d\,e^2 + \frac{8}{5}x^5e^3$	34

norman	$8a\,e^2x-\frac{1}{2}x^2d^3+2x^4d\,e^2+\frac{8}{5}x^5e^3$	34
risch	$8a\,e^2x-\frac{1}{2}x^2d^3+2x^4d\,e^2+\frac{8}{5}x^5e^3$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8*a*e^2*x-1/2*x^2*d^3+2*x^4*d*e^2+8/5*x^5*e^3
```

Maxima [A]
time = 0.26, size = 30, normalized size = 0.81

$$\frac{8}{5}\,x^5e^3+2\,dx^4e^2-\frac{1}{2}\,d^3x^2+8\,axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")
```

```
[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2
```

Fricas [A]
time = 0.37, size = 33, normalized size = 0.89

$$\frac{8}{5}\,e^3x^5+2\,de^2x^4-\frac{1}{2}\,d^3x^2+8\,ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")
```

```
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Sympy [A]
time = 0.01, size = 36, normalized size = 0.97

$$8ae^2x-\frac{d^3x^2}{2}+2de^2x^4+\frac{8e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
```

```
[Out] 8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

Giac [A]
time = 3.85, size = 30, normalized size = 0.81

$$\frac{8}{5}\,x^5e^3+2\,dx^4e^2-\frac{1}{2}\,d^3x^2+8\,axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")
[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.89

$$-\frac{d^3\,x^2}{2}+2\,d\,e^2\,x^4+\frac{8\,e^3\,x^5}{5}+8\,a\,e^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)
[Out] (8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x
```

3.43 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal. Leaf size=153

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

```
[Out] 2*arctanh((4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^
(1/2)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2)-2*arctanh((4*e*x+d)/(3*d^2+2*(-
64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(1/2)/(3*d^2+2*(-64*a*e^3+d^4)^
(1/2))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1120, 1107, 214}

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 6
4*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqr
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = \text{Subst}\left(\int \frac{1}{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx,x,\frac{d}{4e}+x\right)$$

$$=\frac{(4e^2)\,\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}-e\sqrt{d^4-64ae^3}+8e^3x^2}\,dx,x,\frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}}-\frac{(4e^2)\,\text{Sub}}{}$$

$$=\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\,\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-}}}\right.}{\sqrt{d^4-64ae^3}\,\sqrt{3d^2+2\sqrt{d}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 71, normalized size = 0.46

$$-\text{RootSum}\left[8ae^2-d^3\#1+8de^2\#1^3+8e^3\#1^4\&,\frac{\log(x-\#1)}{d^3-24de^2\#1^2-32e^3\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3
- 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 67, normalized size = 0.44

method	result	size
default	$\sum_{\_R=\text{RootOf}(8e^3\_Z^4+8de^2\_Z^3-d^3\_Z+8ae^2)}\frac{\ln(x-\_R)}{32\_R^3e^3+24\_R^2de^2-d^3}$	67

risch	$\sum_{\_R=\mathrm{RootOf}\left(8e^3\_Z^4+8d\,e^2\_Z^3-d^3\_Z+8a\,e^2\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3e^3+24\_R^2d\,e^2-d^3}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x,method=_RETURNVERBOSE)
```

```
[Out] sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*
d*e^2-_Z*d^3+8*a*e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. 2(133) = 266.
time = 0.41, size = 1115, normalized size = 7.29

$$-\sqrt{\frac{3d^2+\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}\log\left(8ex+2\left(2d^4-128ae^3-\frac{3\left(5d^{10}-64ad^6e^3-16384a^2d^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}\right)\sqrt{\frac{3d^2+\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}+2d\right)+\sqrt{\frac{3d^2+\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}\log\left(8ex-2\left(2d^4-128ae^3-\frac{3\left(5d^{10}-64ad^6e^3-16384a^2d^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}\right)\sqrt{\frac{3d^2+\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}+2d\right)-\sqrt{\frac{3d^2-\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}\log\left(8ex+2\left(2d^4-128ae^3+\frac{3\left(5d^{10}-64ad^6e^3-16384a^2d^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}\right)\sqrt{\frac{3d^2-\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}+2d\right)+\sqrt{\frac{3d^2-\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}\log\left(8ex-2\left(2d^4-128ae^3+\frac{3\left(5d^{10}-64ad^6e^3-16384a^2d^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}\right)\sqrt{\frac{3d^2-\frac{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)}{\sqrt{25d^{12}+960ad^8e^3-98304a^2d^4e^6-4194304a^3e^9}}}{5d^8-64ad^4e^3-16384a^2e^6}}+2d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")
```

```
[Out] -sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*
a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 1
6384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3
- 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419
4304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(
25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64
*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3
 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419430
4*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 -
128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 96
0*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8
- 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^
4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) -
sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a
*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16
384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 -
```

```
16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194
304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(2
5*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*
a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3
- 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304
*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 - 1
28*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960
*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 -
64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4
*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d)
```

Sympy [A]
time = 0.99, size = 122, normalized size = 0.80

$$\operatorname{RootSum}\left(t^{4}\cdot\left(1048576a^{3}e^{9}-12288a^{2}d^{4}e^{6}-384ad^{8}e^{3}+5d^{12}\right)+t^{2}\cdot\left(384ad^{2}e^{3}-6d^{6}\right)+1,\left(t\mapsto t\log\left(x+\frac{-49152t^{3}a^{2}d^{2}e^{6}-192t^{3}ad^{6}e^{3}+15t^{3}d^{10}+256tae^{3}-13td^{4}+2d}{8e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

```
[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3 +
5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (-4
9152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256*_t
*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(129) = 258.
time = 4.60, size = 505, normalized size = 3.30

$$-\frac{2\log\left(\frac{1}{4}de^{(-1)}+\frac{1}{4}\sqrt{3d^{2}e^{2}+2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}+x\right)}{\left(de^{(-1)}+\sqrt{3d^{2}e^{2}+2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{3}e^{3}-3\left(de^{(-1)}+\sqrt{3d^{2}e^{2}+2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{2}de^{2}+2d^{3}}-\frac{2\log\left(\frac{1}{4}de^{(-1)}-\frac{1}{4}\sqrt{3d^{2}e^{2}+2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}+x\right)}{\left(de^{(-1)}-\sqrt{3d^{2}e^{2}+2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{3}e^{3}-3\left(de^{(-1)}-\sqrt{3d^{2}e^{2}+2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{2}de^{2}+2d^{3}}-\frac{2\log\left(\frac{1}{4}de^{(-1)}+\frac{1}{4}\sqrt{3d^{2}e^{2}-2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}+x\right)}{\left(de^{(-1)}+\sqrt{3d^{2}e^{2}-2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{3}e^{3}-3\left(de^{(-1)}+\sqrt{3d^{2}e^{2}-2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{2}de^{2}+2d^{3}}-\frac{2\log\left(\frac{1}{4}de^{(-1)}-\frac{1}{4}\sqrt{3d^{2}e^{2}-2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}+x\right)}{\left(de^{(-1)}-\sqrt{3d^{2}e^{2}-2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{3}e^{3}-3\left(de^{(-1)}-\sqrt{3d^{2}e^{2}-2\sqrt{d^{4}-64ae^{3}}e^{2}}e^{(-2)}\right)^{2}de^{2}+2d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")
```

```
[Out] -2*log(1/4*d*e^(-1) + 1/4*sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-
2) + x)/((d*e^(-1) + sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^3
*e^3 - 3*(d*e^(-1) + sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2
*d*e^2 + 2*d^3) - 2*log(1/4*d*e^(-1) - 1/4*sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64
*a*e^3)*e^2)*e^(-2) + x)/((d*e^(-1) - sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^
3)*e^2)*e^(-2))^3*e^3 - 3*(d*e^(-1) - sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^
3)*e^2)*e^(-2))^2*d*e^2 + 2*d^3) - 2*log(1/4*d*e^(-1) + 1/4*sqrt(3*d^2*e^2
- 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2) + x)/((d*e^(-1) + sqrt(3*d^2*e^2 - 2*s
qrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^3*e^3 - 3*(d*e^(-1) + sqrt(3*d^2*e^2 - 2*s
qrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d*e^2 + 2*d^3) - 2*log(1/4*d*e^(-1) - 1/
4*sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2) + x)/((d*e^(-1) - sqr
t(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^3*e^3 - 3*(d*e^(-1) - sqr
t(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d*e^2 + 2*d^3)
```

Mupad [B]
time = 3.73, size = 1264, normalized size = 8.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)
```

```
[Out] atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)
*3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*e*
x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^
(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*
d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 10485
76*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*e^9*(-
(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^
6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^
4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 +
 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^
9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^6*(-(2*(d^1
2 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192
*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))
^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(
1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 1
2288*a^2*d^4*e^6))^(1/2)*2i - atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*
e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i + d^9*2i - a*d^5*e^3*256i + a^2*d*e^6*819
2i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a
*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i - a*d^4*e^4*x*1024i)/(5*d^12*((2*(d
^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 1
92*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6
))^(1/2) + 1048576*a^3*e^9*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 122
88*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 -
384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*((2*(d^12 - 26214
4*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^
3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) -
12288*a^2*d^4*e^6*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^
4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8
*e^3 - 12288*a^2*d^4*e^6))^(1/2)))*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e
^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^
3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)*2i
```

3.44 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

Optimal. Leaf size=342

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8-64ad^4e^3-16384a^2e^6)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)}-\frac{24e\left(d^4+128ae^3-d^2\sqrt{d^4-64ae^3}\right)\tanh^{-1}}{(d^4-64ae^3)^{3/2}\,(5d^4+256ae^3)\,\sqrt{3c}}$$

```
[Out] 2*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^6
-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)-24*e*arctanh((4*
e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))*(d^4+128*a*e^3-d^2*(-64*a*e^3
+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e^3+5*d^4)/(3*d^2-2*(-64*a*e^3+d^
4)^(1/2))^(1/2)+24*e*arctanh((4*e*x+d)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2
))*(d^4+128*a*e^3+d^2*(-64*a*e^3+d^4)^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e
^3+5*d^4)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1120, 1106, 1180, 214}

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\,(8ae^2-d^3x+8de^2x^3+8e^3x^4)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\,\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}+\frac{24e\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{(d^4-64ae^3)^{3/2}\,(256ae^3+5d^4)\,\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*
x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 +
256*a*e^3)*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 +
 d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*Sqrt[
d^4 - 64*a*e^3]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
```

```
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^2}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{4\text{Su}\ldots}{\ldots}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} + \frac{(48\ldots}{\ldots}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{24e\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 234, normalized size = 0.68

$$\frac{(d+4ex)\left(5d^4-128ae^3-12d^3ex-24d^2e^2x^2\right)}{\left(d^4-64ae^3\right)\left(5d^4+256ae^3\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}+\frac{48e^2\text{RootSum}\left[8ae^2-d^3\#1+8de^2\#1^3+8e^3\#1^4\&,\frac{32ae^2\log(x-\#1)+d^3\log(x-\#1)\#1+2d^2e\log(x-\#1)\#1^2}{-d^3+24de^2\#1^2+32e^3\#1^3}\&\right]}{-5d^8+64ad^4e^3+16384a^2e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64*
a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) + (
48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^2*L
og[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^
2*#1^2 + 32*e^3*#1^3) & ])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 288, normalized size = 0.84

method	result
default	$\frac{\frac{12d^2e^3x^3}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{9d^3e^2x^2}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{ex}{256a\,e^3+5d^4}+\frac{d(128a\,e^3-5d^4)}{131072a^2e^6+512a\,d^4e^3-40d^8}}{e^3x^4+d\,e^2x^3-\frac{1}{8}d^3x+a\,e^2}+\frac{384e^2\left(\sum_{\_R=\mathrm{RootOf}(8e^3\_Z^4+\cdots}\right.}{\cdots}$
risch	$\frac{\frac{12d^2e^3x^3}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{9d^3e^2x^2}{(256a\,e^3+5d^4)(64a\,e^3-d^4)}+\frac{ex}{256a\,e^3+5d^4}+\frac{d(128a\,e^3-5d^4)}{131072a^2e^6+512a\,d^4e^3-40d^8}}{e^3x^4+d\,e^2x^3-\frac{1}{8}d^3x+a\,e^2}+48e^2\left(\sum_{\_R=\mathrm{RootOf}(8e^3\_Z\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^4
)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384*a
^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/(20
48*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*_R^3
*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+8
*a*e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] -48*e^2*integrate((2*d^2*x^2*e + d^3*x + 32*a*e^2)/(8*x^4*e^3 + 8*d*x^3*e^2
 - d^3*x + 8*a*e^2), x)/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6) - (96*d^2*x^
3*e^3 + 72*d^3*x^2*e^2 - 5*d^5 + 128*a*d*e^3 - 8*(d^4*e - 64*a*e^4)*x)/(40*
a*d^8*e^2 - 512*a^2*d^4*e^5 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*
x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - 131072*a^3*e^8 -
 (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. $2(316) = 632$.
time = 0.54, size = 4285, normalized size = 12.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2)*(40*a*
d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 -
16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (
5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^
5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6
+ 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^1
5 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)
/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^
3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27
44381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 51881467707
30811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a
*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d
^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*
d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 1382
4*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*
e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22
*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a
^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
86976294838206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^
2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d
^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804651110
4*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1
800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150
994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a
^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4
*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167
360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 515396
07552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 -
 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2
*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 -
64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960
*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 3119513
```

```
6*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980
46511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d
^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^
9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443810229
28896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*
a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3
 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 -
 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 +
 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)
*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 26
8435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 56
6493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e
^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^
8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 -
115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e
^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931
694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737869762948
38206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (
125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 +
 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^1
8)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*
d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000
*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e
^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7
3786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d
^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5
*d^4*e^15 - 4398046511104*a^6*e^18))) + 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*
d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4
 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*
e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*
e^8 - (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(300) = 600.
time = 3.74, size = 979, normalized size = 2.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")

[Out] 12*(((d*e^(-1) + sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d^2*e^3 - 2*(d*e^(-1) + sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))*d^3*e^2 + 256*a*e^4)*log(1/4*d*e^(-1) + 1/4*sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2) + x)/((d*e^(-1) + sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^3*e^3 - 3*(d*e^(-1) + sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d*e^2 + 2*d^3) + ((d*e^(-1) - sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d^2*e^3 - 2*(d*e^(-1) - sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))*d^3*e^2 + 256*a*e^4)*log(1/4*d*e^(-1) - 1/4*sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2) + x)/((d*e^(-1) - sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^3*e^3 - 3*(d*e^(-1) - sqrt(3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d*e^2 + 2*d^3) + ((d*e^(-1) + sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d^2*e^3 - 2*(d*e^(-1) + sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))*d^3*e^2 + 256*a*e^4)*log(1/4*d*e^(-1) + 1/4*sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2) + x)/((d*e^(-1) + sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^3*e^3 - 3*(d*e^(-1) + sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d*e^2 + 2*d^3) + ((d*e^(-1) - sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d^2*e^3 - 2*(d*e^(-1) - sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))*d^3*e^2 + 256*a*e^4)*log(1/4*d*e^(-1) - 1/4*sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2) + x)/((d*e^(-1) - sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^3*e^3 - 3*(d*e^(-1) - sqrt(3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)*e^(-2))^2*d*e^2 + 2*d^3))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6) - (96*d^2*x^3*e^3 + 72*d^3*x^2*e^2 - 8*d^4*x*e - 5*d^5 + 512*a*x*e^4 + 128*a*d*e^3)/((5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2))

Mupad [B]
time = 7.11, size = 2500, normalized size = 7.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)

[Out] ((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)) + (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3) + atan((((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20

*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837
1998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(6871
9476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 -
5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5
*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 1342
17728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^
19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^
5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 -
2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a
^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^
15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*
e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6
+ 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/
2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440
*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^
(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 12902
40*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9663
6764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*
d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 616038
4*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a
e^5(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27
 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250
489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*
e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2
) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^
2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2
*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(78643
2*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12
- 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + ((288*(d
^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d
^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*
d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460
6846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*
d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062
494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8
*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e
^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e
^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) -
((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2
*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17
179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d
^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 +
24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 2
5769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 171798691

```
84*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9
+ 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 11878
4*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 1717986
9184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744
*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3
- d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^1
1 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e
^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^3
2*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^2
0*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7916483
71998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22
*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14
*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2
*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))...
```

3.45 $\int (8+8x-x^3+8x^4)^4 \, dx$

Optimal. Leaf size=96

$$4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1408x^9+\frac{21488x^{10}}{5}+\frac{25312x^{11}}{11}-448x$$

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2086}

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int (8+8x-x^3+8x^4)^4 \, dx = \int (4096+16384x+24576x^2+14336x^3+14336x^4+43008x^5+47488x^6+$$
$$=4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8$$

Mathematica [A]
time = 0.00, size = 96, normalized size = 1.00

$$4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1408x^9+\frac{21488x^{10}}{5}+\frac{25312x^{11}}{11}-448x^{12}+\frac{10241x^{13}}{13}+1168x^{14}+\frac{128x^{15}}{5}-128x^{16}+\frac{4096x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Maple [A]
time = 0.02, size = 85, normalized size = 0.89

method	result
gosper	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{21488}{5}x^{10}$
default	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{21488}{5}x^{10}$
norman	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{21488}{5}x^{10}$
risch	$4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336}{5}x^5 + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{21488}{5}x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

Maxima [A]
time = 0.28, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336}{5}x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Fricas [A]
time = 0.38, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336}{5}x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Sympy [A]
time = 0.01, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 448
*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x**7
 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x
```

Giac [A]
time = 4.38, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336}{5}x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Mupad [B]
time = 0.19, size = 84, normalized size = 0.88

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

3.46 $\int (8+8x-x^3+8x^4)^3\,dx$

Optimal. Leaf size=74

$$512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2086}

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int (8+8x-x^3+8x^4)^3\,dx = \int (512+1536x+1536x^2+320x^3+1152x^4+2880x^5+1560x^6-360x^7+1$$

$$= 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+$$

Mathematica [A]
time = 0.00, size = 74, normalized size = 1.00

$$512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Maple [A]
time = 0.01, size = 65, normalized size = 0.88

method	result
gosper	$512x + 768x^2 + 512x^3 + 80x^4 + \frac{1152}{5}x^5 + 480x^6 + \frac{1560}{7}x^7 - 45x^8 + 128x^9 + \frac{307}{2}x^{10} + \frac{24}{11}x^{11} - 16$
default	$512x + 768x^2 + 512x^3 + 80x^4 + \frac{1152}{5}x^5 + 480x^6 + \frac{1560}{7}x^7 - 45x^8 + 128x^9 + \frac{307}{2}x^{10} + \frac{24}{11}x^{11} - 16$
norman	$512x + 768x^2 + 512x^3 + 80x^4 + \frac{1152}{5}x^5 + 480x^6 + \frac{1560}{7}x^7 - 45x^8 + 128x^9 + \frac{307}{2}x^{10} + \frac{24}{11}x^{11} - 16$
risch	$512x + 768x^2 + 512x^3 + 80x^4 + \frac{1152}{5}x^5 + 480x^6 + \frac{1560}{7}x^7 - 45x^8 + 128x^9 + \frac{307}{2}x^{10} + \frac{24}{11}x^{11} - 16$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^3,x,method=_RETURNVERBOSE)
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.86

$$\frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^{9} - 45\,x^{8} + \frac{1560}{7}\,x^{7} + 480\,x^{6} + \frac{1152}{5}\,x^{5} + 80\,x^{4} + 512\,x^{3} + 768\,x^{2} + 512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Fricas [A]
time = 0.35, size = 64, normalized size = 0.86

$$\frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^{9} - 45\,x^{8} + \frac{1560}{7}\,x^{7} + 480\,x^{6} + \frac{1152}{5}\,x^{5} + 80\,x^{4} + 512\,x^{3} + 768\,x^{2} + 512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Sympy [A]
time = 0.01, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^{9} - 45x^{8} + \frac{1560x^{7}}{7} + 480x^{6} + \frac{1152x^{5}}{5} + 80x^{4} + 512x^{3} + 768x^{2} + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 +
1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 512*
x
```

Giac [A]
time = 3.89, size = 64, normalized size = 0.86

$$\frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^{9}-45\,x^{8}+\frac{1560}{7}\,x^{7}+480\,x^{6}+\frac{1152}{5}\,x^{5}+80\,x^{4}+512\,x^{3}+768\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Mupad [B]
time = 0.08, size = 64, normalized size = 0.86

$$\frac{512\,x^{13}}{13}-16\,x^{12}+\frac{24\,x^{11}}{11}+\frac{307\,x^{10}}{2}+128\,x^{9}-45\,x^{8}+\frac{1560\,x^{7}}{7}+480\,x^{6}+\frac{1152\,x^{5}}{5}+80\,x^{4}+512\,x^{3}+768\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^3,x)
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

3.47 $\int \left(8+8x-x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=54

$$64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2086}

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8+8x-x^3+8x^4\right)^2 \, dx &= \int \left(64+128x+64x^2-16x^3+112x^4+128x^5+x^6-16x^7+64x^8\right) dx\\ &= 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 54, normalized size = 1.00

$$64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Maple [A]
time = 0.01, size = 45, normalized size = 0.83

method	result	size
gosper	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
default	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
norman	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
risch	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.81

$$\frac{64}{9}\,x^9-2\,x^8+\frac{1}{7}\,x^7+\frac{64}{3}\,x^6+\frac{112}{5}\,x^5-4\,x^4+\frac{64}{3}\,x^3+64\,x^2+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Fricas [A]
time = 0.38, size = 44, normalized size = 0.81

$$\frac{64}{9}\,x^9-2\,x^8+\frac{1}{7}\,x^7+\frac{64}{3}\,x^6+\frac{112}{5}\,x^5-4\,x^4+\frac{64}{3}\,x^3+64\,x^2+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.91

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3 +
 64*x**2 + 64*x
```

Giac [A]
time = 4.08, size = 44, normalized size = 0.81

$$\frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.81

$$\frac{64\,x^9}{9} - 2\,x^8 + \frac{x^7}{7} + \frac{64\,x^6}{3} + \frac{112\,x^5}{5} - 4\,x^4 + \frac{64\,x^3}{3} + 64\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

3.48 $\int (8+8x-x^3+8x^4)\,dx$

Optimal. Leaf size=23

$$8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

[In] Int[8 + 8*x - x^3 + 8*x^4,x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5

Rubi steps

$$\int (8+8x-x^3+8x^4)\,dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[8 + 8*x - x^3 + 8*x^4,x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5

Maple [A]
time = 0.01, size = 20, normalized size = 0.87

method	result	size
gosper	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
default	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20

norman	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20
risch	$8x + 4x^2 - \frac{1}{4}x^4 + \frac{8}{5}x^5$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-x^3+8*x+8,x,method=_RETURNVERBOSE)
```

```
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.83

$$\frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Fricas [A]
time = 0.36, size = 19, normalized size = 0.83

$$\frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.83

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-x**3+8*x+8,x)
```

```
[Out] 8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

Giac [A]
time = 3.74, size = 19, normalized size = 0.83

$$\frac{8}{5}\,x^5 - \frac{1}{4}\,x^4 + 4\,x^2 + 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.83

$$\frac{8\,x^5}{5}-\frac{x^4}{4}+4\,x^2+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x - x^3 + 8*x^4 + 8,x)
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

3.49 $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

Optimal. Leaf size=268

$$-\frac{\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}}-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{2-\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}$$

```
[Out] -1/84*arctan(1/42*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/29232*ln((1+4/x)^2+3*29^
(1/2)-(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/2))^(1/2)+1/29232*
ln((1+4/x)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-132762+81606*29^(1/
2))^(1/2)-1/14616*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2)
)*(132762+81606*29^(1/2))^(1/2)-1/14616*arctan((2+8/x+(6+6*29^(1/2))^(1/2))
/(-6+6*29^(1/2))^(1/2))*(132762+81606*29^(1/2))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2094, 12, 1687, 1183, 648, 632, 210, 642, 1121}

$$-\frac{\text{ArcTan}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}}-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\text{ArcTan}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\text{ArcTan}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] -1/12*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/Sqrt[7] - (Sqrt[(109 + 67*Sqrt[
29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])
]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]
 + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log
[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(
-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x)
 + (1 + 4/x)^2])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = -\left(1024\text{Subst}\left(\int \frac{(8-32x)^2}{8\left(1069056-393216x^2+1048576x^4\right)}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\text{Subst}\left(\int \frac{(8-32x)^2}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\text{Subst}\left(\int -\frac{512x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right) - 128\text{Su}$$

$$\text{Subst}\left(\int -\right.$$

$$= 65536\text{Subst}\left(\int \frac{x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right) - $$

$$\left(87+\right.$$

$$= 32768\text{Subst}\left(\int \frac{1}{1069056-393216x+1048576x^2}\,dx, x, \left(\frac{1}{4}+\frac{1}{x}\right)^2\right) - $$

$$= -\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\,\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right.$$

$$= -\frac{\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\,\tan^{-1}\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 45, normalized size = 0.17

$$\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&, \frac{\log(x-\#1)}{8-3\#1^2+32\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) &]

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 41, normalized size = 0.15

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-\_Z^3+8\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-3\_R^2+8}$	41
risch	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-\_Z^3+8\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-3\_R^2+8}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-x^3+8*x+8),x,method=_RETURNVERBOSE)

[Out] sum(1/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")

[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)

Fricas [C] Result contains complex when optimal does not.
time = 1.17, size = 1015, normalized size = 3.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")

[Out] -1/168*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696))*log(287314195392*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 - 12038906880*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 + 16878104*x + 4897683*I*sqrt(7) - 411405372*sqrt(65/43848*I*sqrt(7) - 109/87696) + 6055613) - 1/168*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696))*log(-35914274424*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 + 16443*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696) - 91520) + 609*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) + 2109763

```
*x - 1911147/8*I*sqrt(7) + 40134087/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
- 1461344) + 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/43
848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848
*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7
) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) - 7)
 + 261*sqrt(65/43848*I*sqrt(7) - 109/87696) + 261*sqrt(-65/43848*I*sqrt(7)
- 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7)
 - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/
87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt
(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/8
7696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/876
96))^2 + 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7)
- 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) -
 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) -
109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/
43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/438
48*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt
(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) -
7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt(7) - 10
9/87696) + 220336) - 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sq
rt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-
65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*
I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/8769
6)) - 7) - 261*sqrt(65/43848*I*sqrt(7) - 109/87696) - 261*sqrt(-65/43848*I*
sqrt(7) - 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I
*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7
) - 109/87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/4384
8*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7)
 - 109/87696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) -
 109/87696))^2 - 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*
sqrt(7) - 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*s
qrt(7) - 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sq
rt(7) - 109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*
sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt
(-65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/4384
8*I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87
696)) - 7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt
(7) - 109/87696) + 220336)
```

Sympy [A]
time = 0.53, size = 41, normalized size = 0.15

$$\operatorname{RootSum}\left(66298176t^4 + 74088t^2 + 4095t + 64, \left(t \mapsto t \log\left(\frac{35914274424t^3}{2109763} - \frac{1504863360t^2}{2109763} + \frac{102851343t}{2109763} + x + \frac{6055613}{16878104}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8),x)
```

```
[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(3591
4274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 + x
 + 6055613/16878104)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

Mupad [B]
time = 2.45, size = 123, normalized size = 0.46

$$\sum_{k=1}^{4}\ln\left(-\frac{\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)\,\left(8064\,\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)+256\,x+\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)\,x\,12285+\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)^2\,x\,148176+198072\,\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)^2-8\right)}{4096}\right)\,\mathrm{root}\left(z^4+\frac{7\,z^2}{6264}+\frac{65\,z}{1052352}+\frac{1}{1035909},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8),x)
```

```
[Out] symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(8
064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x + 1
2285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148176
*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 198072*r
oot(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/4096)*ro
ot(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)
```

3.50 $\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$

Optimal. Leaf size=357

$$-\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}+\frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}-\frac{17\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}}-\cdots$$

```
[Out] 1/336*(-207-29*(1+4/x)^2)/(261-6*(1+4/x)^2+(1+4/x)^4)+5/87696*(5157+199*(1+
4/x)^2)*(1+4/x)/(261-6*(1+4/x)^2+(1+4/x)^4)-17/7056*arctan(1/42*(3-(1+4/x)^
2)*7^(1/2))*7^(1/2)-1/213627456*ln((1+4/x)^2+3*29^(1/2)-(1+4/x)*(6+6*29^(1/
2))^(1/2))*(-220437694722+55934612286*29^(1/2))^(1/2)+1/213627456*ln((1+4/x
)^2+3*29^(1/2)+(1+4/x)*(6+6*29^(1/2))^(1/2))*(-220437694722+55934612286*29^
(1/2))^(1/2)-1/106813728*arctan((2+8/x-(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2)
)^(1/2))*(220437694722+55934612286*29^(1/2))^(1/2)-1/106813728*arctan((2+8/
x+(6+6*29^(1/2))^(1/2))/(-6+6*29^(1/2))^(1/2))*(220437694722+55934612286*29
^(1/2))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2094, 12, 1687, 1692, 1183, 648, 632, 210, 642, 1677, 1674}

$$-\frac{17\text{ArcTan}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\,\text{ArcTan}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\,\text{ArcTan}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}+\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] -1/336*(207 + 29*(1 + 4/x)^2)/(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4) + (5*(515
7 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])/(1008*Sqrt[7]) - (Sqrt[(18098
3329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/S
qrt[6*(-1 + Sqrt[29])]])/87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218
]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696
 - (Sqrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1
+ Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-180983329 + 4592332
7*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 +
4/x)^2])/175392
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = -\left(1024\text{Subst}\left(\int \frac{(8-32x)^6}{64\,(1069056-393216x^2+1048576x^4)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int \frac{(8-32x)^6}{(1069056-393216x^2+1048576x^4)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int \frac{x(-6291456-335544320x^2-1610612736x^4)}{(1069056-393216x^2+1048576x^4)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right.$$

$$= \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{278927740761415247462 4+775800880}{1069056-393216x^2+1048}\right.}{57853663025666}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 113, normalized size = 0.32

$$\frac{544+1539x-1146x^2+784x^3}{43848\,(8+8x-x^3+8x^4)} + \frac{\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&, \frac{2243\log(x-\#1)-1097\log(x-\#1)\#1+392\log(x-\#1)\#1^2}{8-3\#1^2+32\#1^3}\&\right]}{21924}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + RootS
um[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 + 3
92*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) & ]/21924
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 83, normalized size = 0.23

method	result	size
default	$\frac{\frac{7}{3132}x^3-\frac{191}{58464}x^2+\frac{57}{12992}x+\frac{17}{10962}}{x^4-\frac{1}{8}x^3+x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\left(392\_R^2-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}\right)}{21924}$	83
risch	$\frac{\frac{7}{3132}x^3-\frac{191}{58464}x^2+\frac{57}{12992}x+\frac{17}{10962}}{x^4-\frac{1}{8}x^3+x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-\_Z^3+8\_Z+8\right)}\frac{\left(392\_R^2-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^3-3\_R^2+8}\right)}{21924}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*su
m((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3
+8*_Z+8))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/219
24*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.14, size = 1201, normalized size = 3.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 1/213627456*(3819648*x^3 - 15138*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(621
7850567873065654359973859328*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/3345405
96096*I*sqrt(7) - 180983329/4683568345344))^3 - 100287672431797174786327756
80*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 + 67481665655469287031416*x + 320944207138750561964778*
I*sqrt(7) - 133210725033589645013145504*sqrt(4550065/334540596096*I*sqrt(7)
 - 180983329/4683568345344) + 333979081113202533090737) - 15138*(8*x^4 - x^
3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344))*log(-777231320984133206794996732416*(17/14112*I*sqr
t(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^
3 + 878169064752*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sq
rt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqrt(7) + 44252943
5492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) -
 1427510892508480) + 7569*(7276511507810430573072*(17/14112*I*sqrt(7) - 1/2
*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 233594
23554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344)) + 8435208206933660878927*x - 1484491951413286827726
33/4*I*sqrt(7) + 15403787072311988024172036*sqrt(4550065/334540596096*I*sqr
t(7) - 180983329/4683568345344) - 47393606606696595067616) - 5583312*x^2 +
(56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-125452723536*(17/14112*I*sqrt(7)
 - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 -
125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7
) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-45
50065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 66301
91) + 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/33454
0596096*I*sqrt(7) - 180983329/4683568345344)) + 7569*(8*x^4 - x^3 + 8*x + 8
)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468
3568345344)))*log(-439084532376*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/33
4540596096*I*sqrt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqr
t(7) + 442529435492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4
683568345344) - 1427510892508480) - 7569/2*(7276511507810430573072*(17/1411
2*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356834
5344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/3345405960
96*I*sqrt(7) - 180983329/4683568345344)) + 626797952698732342414548480*(17/
14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835
68345344))^2 + 1/16*(261*(62716756730859*sqrt(87)*(-17*I*sqrt(7) + 7056*sqr
t(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 142751089250
8480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) -
180983329/4683568345344)) - 372580342944713280*sqrt(87)*(-17*I*sqrt(7) + 70
56*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 104650
21752358451264*sqrt(87))*sqrt(-125452723536*(17/14112*I*sqrt(7) - 1/2*sqrt(
4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 125452723536
*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 18098332
```

```
9/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540
596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7056*sqrt(455
0065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 6630191) + 843520
8206933660878927*x - 3005727107011649552439/2*I*sqrt(7) + 62377677844335880
1235576*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) + 22
95910220839785410704) - (56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-12545272
3536*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983
329/4683568345344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-45500
65/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I
*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345
344))*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329
/4683568345344)) - 6630191) - 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) +
7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 756
9*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I
*sqrt(7) - 180983329/4683568345344)))*log(-439084532376*(-17/14112*I*sqrt(7
) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2*
(-1066184864424603*I*sqrt(7) + 442529435492941104*sqrt(4550065/334540596096
*I*sqrt(7) - 180983329/4683568345344) - 1427510892508480) - 7569/2*(7276511
507810430573072*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(
7) - 180983329/4683568345344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*
sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 62679795
2698732342414548480*(17/14112*I*sqrt(7) - 1/2*s...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. $2(274) = 548$.
time = 1.78, size = 3834, normalized size = 10.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x
+ 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/1292018853888
)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329
 + 45923327*sqrt(29)) + 40699873480352667)/227008323264998681573683424 - 26
7658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-
180983329 + 45923327*sqrt(29)) + 40699873480352667)/8435208206933660878927
- 2157374520970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/11
3504161632499340786841712 + 3881045239007430*sqrt(29)/5326727264361229 + 43
5853770857118353330297/33740832827734643515708 + 20905585576953*sqrt(42)*sq
rt(-180983329 + 45923327*sqrt(29))/85227636229779664) - 2942814074101429415
084030510182204250067556953*sqrt(214095423017213*sqrt(29) + 47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/888496186
```

751485201253966401139075287452416534006272 - 14257625632856314835831142972765102609010539559351093/27765505835983912539186450035596102732888016687696
- 75184631502818837388875900060881355871*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/30637799543154662112205737970312940946635052896768 - 9633141817961412597488587661065704878094062299*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/30637799543154662112205737970312940946635052896768 - 1398888334001652366855237255*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/359456428291497016547944746810895370264 + 91245981690030498967778233214015591679*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/23005211410655809059068463795897303696896 + 10304175351841941260676745569701505519*sqrt(29)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/19347382796361535418676578052349632409089536 + 639111088489748962499984017403917984374085485*sqrt(29)/4836845699090383854669144513087408102272384) + sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/1292018853888)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/227008323264998681573683424 - 20905585576953*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/85227636229779664 + 3881045239007430*sqrt(29)/5326727264361229 + 267658292345340*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/8435208206933660878927 + 2157374520970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/113504161632499340786841712 + 435853770857118353330297/33740832827734643515708) - 14257625632856314835831142972765102609010539559351093/27765505835983912539186450035596102732888016687696 - 10304175351841941260676745569701505519*sqrt(29)*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/19347382796361535418676578052349632409089536 - 91245981690030498967778233214015591679*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/23005211410655809059068463795897303696896 - 75184631502818837388875900060881355871*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/30637799543154662112205737970312940946635052896768 - 1398888334001652366855237255*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/359456428291497016547944746810895370264 + 9633141817961412597488587661065704878094062299*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/30637799543154662112205737970312940946635052896768 + 2942814074101429415084030510182204250067556953*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/888496186751485201253966401139075287452416534006272 + 639111088489748962499984017403917984374085485*sqrt(29)/4836845699090383854669144513087408102272384) - 2*sqrt(199631405/37468546762752 + sqr

```
t(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230
17213*sqrt(29) + 40699873480352667)/9367136690688 + 1583563*sqrt(29)/430672
951296)*atan(454016646529997363147366848*x/(-450967351627246742986O*sqrt(12
18)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*s
qrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(19
9631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29
)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) +
20905585576953*sqrt(1218)*sqrt(199631405 + 4*sq...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

Mupad [B]
time = 0.21, size = 176, normalized size = 0.49

$$\left(\sum_{k=1}^{4}\ln\left(\frac{2615257\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)}{72918171648}+\frac{4225\,x}{40375589184}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)x\,34885379}{72918171648}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2x\,191555}{475136}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3x\,9261}{256}-\frac{11205\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2}{59392}-\frac{24759\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3}{512}+\frac{10901}{107668237824}\right)\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\right)+\frac{\frac{7\,x^3}{3132}-\frac{191\,x^2}{58464}+\frac{57\,x}{12992}+\frac{17}{10962}}{x^4-\frac{x^3}{8}+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)
[Out] symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/67
4433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/40
375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (191555
*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 111
4096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191*z^2)/
167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z,
 k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (24759*root(z
^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13
723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (6630191*z^2
)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709,
z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/3132 + 17/10962
)/(x - x^3/8 + x^4 + 1)
```

3.51 $\int (1+4x+4x^2+4x^4)^4 \, dx$

Optimal. Leaf size=97

$$x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512}{}$$

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2086}

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)^4 \, dx = \int \left(1+16x+112x^2+448x^3+1136x^4+1984x^5+2752x^6+3584x^7+4192\right.$$
$$= x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+$$

Mathematica [A]
time = 0.00, size = 97, normalized size = 1.00

$$x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Maple [A]
time = 0.02, size = 78, normalized size = 0.80

method	result
gosper	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+256x$
default	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+256x$
norman	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+256x$
risch	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}+256x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^9+448x^8+\frac{2752}{7}x^7+\frac{992}{3}x^6+\frac{1136}{5}x^5+112x^4+\frac{112}{3}x^3+8x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Fricas [A]
time = 0.34, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^9+448x^8+\frac{2752}{7}x^7+\frac{992}{3}x^6+\frac{1136}{5}x^5+112x^4+\frac{112}{3}x^3+8x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Sympy [A]
time = 0.01, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^{9}}{9}+448x^{8}+\frac{2752x^{7}}{7}+\frac{992x^{6}}{3}+\frac{1136x^{5}}{5}+112x^{4}+\frac{112x^{3}}{3}+8x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

```
[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 33
28*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6/3
 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x
```

Giac [A]
time = 3.35, size = 77, normalized size = 0.79

$$\frac{256}{17}\,x^{17}+\frac{1024}{15}\,x^{15}+\frac{512}{7}\,x^{14}+\frac{1792}{13}\,x^{13}+256\,x^{12}+\frac{3328}{11}\,x^{11}+384\,x^{10}+\frac{4192}{9}\,x^{9}+448\,x^{8}+\frac{2752}{7}\,x^{7}+\frac{992}{3}\,x^{6}+\frac{1136}{5}\,x^{5}+112\,x^{4}+\frac{112}{3}\,x^{3}+8\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Mupad [B]
time = 0.15, size = 77, normalized size = 0.79

$$\frac{256\,x^{17}}{17}+\frac{1024\,x^{15}}{15}+\frac{512\,x^{14}}{7}+\frac{1792\,x^{13}}{13}+256\,x^{12}+\frac{3328\,x^{11}}{11}+384\,x^{10}+\frac{4192\,x^{9}}{9}+448\,x^{8}+\frac{2752\,x^{7}}{7}+\frac{992\,x^{6}}{3}+\frac{1136\,x^{5}}{5}+112\,x^{4}+\frac{112\,x^{3}}{3}+8\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

3.52 $\int \left(1+4x+4x^2+4x^4\right)^3 \, dx$

Optimal. Leaf size=69

$$x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x^{13}}{13}$$

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {2086}

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \int \left(1+12x+60x^2+160x^3+252x^4+288x^5+352x^6+384x^7+240x^8+1\right.$$

$$= x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.84

method	result	size
gosper	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58
default	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58
norman	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58
risch	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.83

$$\frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^{9}+48\,x^{8}+\frac{352}{7}\,x^{7}+48\,x^{6}+\frac{252}{5}\,x^{5}+40\,x^{4}+20\,x^{3}+6\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Fricas [A]
time = 0.39, size = 57, normalized size = 0.83

$$\frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^{9}+48\,x^{8}+\frac{352}{7}\,x^{7}+48\,x^{6}+\frac{252}{5}\,x^{5}+40\,x^{4}+20\,x^{3}+6\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Sympy [A]
time = 0.01, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^{9}}{3}+48x^{8}+\frac{352x^{7}}{7}+48x^{6}+\frac{252x^{5}}{5}+40x^{4}+20x^{3}+6x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

```
[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7
+ 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

Giac [A]
time = 3.70, size = 57, normalized size = 0.83

$$\frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^{9}+48\,x^{8}+\frac{352}{7}\,x^{7}+48\,x^{6}+\frac{252}{5}\,x^{5}+40\,x^{4}+20\,x^{3}+6\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Mupad [B]
time = 0.06, size = 57, normalized size = 0.83

$$\frac{64\,x^{13}}{13}+\frac{192\,x^{11}}{11}+\frac{96\,x^{10}}{5}+\frac{80\,x^{9}}{3}+48\,x^{8}+\frac{352\,x^{7}}{7}+48\,x^{6}+\frac{252\,x^{5}}{5}+40\,x^{4}+20\,x^{3}+6\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

3.53 $\int \left(1+4x+4x^2+4x^4\right)^2 \, dx$

Optimal. Leaf size=45

$$x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

```
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2086}

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+4x+4x^2+4x^4\right)^2 \, dx &= \int \left(1+8x+24x^2+32x^3+24x^4+32x^5+32x^6+16x^8\right) \, dx\\ &= x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.00

$$x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Maple [A]
time = 0.01, size = 38, normalized size = 0.84

method	result	size
gosper	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38
default	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38
norman	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38
risch	$x+4x^{2}+8x^{3}+8x^{4}+\frac{24}{5}x^{5}+\frac{16}{3}x^{6}+\frac{32}{7}x^{7}+\frac{16}{9}x^{9}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.82

$$\frac{16}{9}\,x^{9}+\frac{32}{7}\,x^{7}+\frac{16}{3}\,x^{6}+\frac{24}{5}\,x^{5}+8\,x^{4}+8\,x^{3}+4\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.82

$$\frac{16}{9}\,x^{9}+\frac{32}{7}\,x^{7}+\frac{16}{3}\,x^{6}+\frac{24}{5}\,x^{5}+8\,x^{4}+8\,x^{3}+4\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Sympy [A]
time = 0.01, size = 42, normalized size = 0.93

$$\frac{16x^{9}}{9}+\frac{32x^{7}}{7}+\frac{16x^{6}}{3}+\frac{24x^{5}}{5}+8x^{4}+8x^{3}+4x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 +
x
```

Giac [A]
time = 3.45, size = 37, normalized size = 0.82

$$\frac{16}{9}\,x^{9}+\frac{32}{7}\,x^{7}+\frac{16}{3}\,x^{6}+\frac{24}{5}\,x^{5}+8\,x^{4}+8\,x^{3}+4\,x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.82

$$\frac{16\,x^9}{9}+\frac{32\,x^7}{7}+\frac{16\,x^6}{3}+\frac{24\,x^5}{5}+8\,x^4+8\,x^3+4\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

3.54 $\int (1 + 4x + 4x^2 + 4x^4) \, dx$

Optimal. Leaf size=21

$$x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

```
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Antiderivative was successfully verified.

```
[In] Int[1 + 4*x + 4*x^2 + 4*x^4,x]
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi steps

$$\int (1 + 4x + 4x^2 + 4x^4) \, dx = x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.86

method	result	size
gosper	$x + 2x^2 + \frac{4}{3}x^3 + \frac{4}{5}x^5$	18
default	$x + 2x^2 + \frac{4}{3}x^3 + \frac{4}{5}x^5$	18

norman	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
risch	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^4+4*x^2+4*x+1,x,method=_RETURNVERBOSE)
```

```
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$\frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Fricas [A]
time = 0.37, size = 17, normalized size = 0.81

$$\frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.90

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**4+4*x**2+4*x+1,x)
```

```
[Out] 4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

Giac [A]
time = 3.47, size = 17, normalized size = 0.81

$$\frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.81

$$\frac{4\,x^5}{5}+\frac{4\,x^3}{3}+2\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 4*x^2 + 4*x^4 + 1,x)
```

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

3.55 $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

Optimal. Leaf size=234

$$\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}$$

```
[Out] 1/2*arctan(-1/2+1/2*(1+1/x)^2)-1/20*ln((1+1/x)^2+5^(1/2)-(1+1/x)*(2+2*5^(1/
2))^(1/2))*(-10+5*5^(1/2))^(1/2)+1/20*ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(
1/2))^(1/2))*(-10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/
(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)-1/10*arctan((2+2/x+(2+2*5^(1/2))
^(1/2))/(-2+2*5^(1/2))^(1/2))*(10+5*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$,
Rules used = {2094, 1687, 1183, 648, 632, 210, 642, 12, 1121}

$$\frac{1}{2}\text{ArcTan}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt
[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5
]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sq
rt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1
+ x^(-1))^2])/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5]
)]*(1 + x^(-1)) + (1 + x^(-1))^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = -\left(16\text{Subst}\left(\int \frac{(4-4x)^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int -\frac{32x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)\right) - 16\text{Subst}\left(\int \frac{1 \cdots}{1280 - \cdots}\right.$$

$$= 512\text{Subst}\left(\int \frac{x}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{2\left(1+\sqrt{5}\right)} \cdots}{\sqrt{5}-\sqrt{2\left(1 \cdots\right.}}\right.}{32\sqrt{10 \cdots}}$$

$$= 256\text{Subst}\left(\int \frac{1}{1280-512x+256x^2}\,dx, x, \left(1+\frac{1}{x}\right)^2\right) + \frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int - \cdots\right.}{4 \cdots}$$

$$= -\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)+\frac{1}{4}\sqrt{- \cdots}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) - \frac{\left(1+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x} \cdots}{\sqrt{2\left(-1+\sqrt{5}\right)} \cdots}\right.}{4\sqrt{10} \cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 47, normalized size = 0.20

$$\frac{1}{4}\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\&, \frac{\log(x-\#1)}{1+2\#1+4\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) & ]/
4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 41, normalized size = 0.18

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{4}$	41
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{4}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.13, size = 499, normalized size = 2.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] -1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*
I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I
)^2 - 9) - 5*sqrt(1/10*I - 1/5) - 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/
10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I -
 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^
2 + ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1
/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I
- 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)
- 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(sqrt(10)*sqrt(
-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt
(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 5*sqrt(1/1
0*I - 1/5) + 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/5) - I)^2*(1
```

```
2*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1
/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 - ((6*sqrt(10)*(2*s
qrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) - I) - sqrt(10)*(
2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(
2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*
I - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*sqrt(1/10*I - 1/5) - I)*log(-5*(2*
sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) - 30*(2*sqrt(1
/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 30*(2*sqrt(-1/10*I - 1/5)
 + I)^3 + 8*x - 216*sqrt(-1/10*I - 1/5) - 108*I + 21) - 1/4*(2*sqrt(-1/10*I
 - 1/5) + I)*log(30*(2*sqrt(-1/10*I - 1/5) + I)^3 + 5*(2*sqrt(-1/10*I - 1/5
) + I)^2 + 8*x + 216*sqrt(-1/10*I - 1/5) + 108*I - 27)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3432 vs. $2(190) = 380$.
time = 1.36, size = 3432, normalized size = 14.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

```
[Out] sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(-8 - 21*sqrt(5)*sqrt(-2 + sqrt(5))/1
0 - sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 - sqrt(5)/2 + 12*s
qrt(-2 + sqrt(5)) + 9*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 +
sqrt(5)) + sqrt(5) + 19)/5) - 841*sqrt(5)*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)
) + sqrt(5) + 19)/20 - 14351/40 - 441*sqrt(-2 + sqrt(5))/4 - 75*sqrt(5)*sqr
t(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 3*sq
rt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 301*s
qrt(5)*sqrt(-2 + sqrt(5))/10 + 7407*sqrt(5)/40 + 3913*sqrt(-2*sqrt(5)*sqrt(
-2 + sqrt(5)) + sqrt(5) + 19)/40) - sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(
-8 - 12*sqrt(-2 + sqrt(5)) - sqrt(5)/2 + 21*sqrt(5)*sqrt(-2 + sqrt(5))/10 +
 sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 + 9*sqrt(5)*sqrt(-2 +
sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/5) - 3913*sqrt(2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40 - 14351/40 - 75*sqrt(5)*sqrt
(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 301*sq
rt(5)*sqrt(-2 + sqrt(5))/10 - 3*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 +
 sqrt(5)) + sqrt(5) + 19) + 441*sqrt(-2 + sqrt(5))/4 + 7407*sqrt(5)/40 + 84
1*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/20) - 2*sqrt(3/
80 + 3*sqrt(5)/80 + sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40)*
atan(-20*x/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(
-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(
5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*
sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 18*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
```

```
 + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt
(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 120*
sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt
(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) +
 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 +
 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*
sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(5)/(-27*sqrt(5)*sqrt(3
 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sq
rt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2
 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
 19)) + 5*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*s
qrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) +
 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt
(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 21*sqrt(5)*sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
+ 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(
5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) +
 sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 80/(-
27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-
2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqr
t(5)) + sqrt(5) + 19))) - 2*sqrt(-sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(
5) + 19)/40 + 3/80 + 3*sqrt(5)/80)*atan(20*x/(5*sqrt(-2 + sqrt(5))*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sqr
t(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt
(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 80/(5
*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(
5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5))) - 120*sqrt(-2 + sqrt(5))/(5*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sq
rt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqr
t(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 5*sq
rt(5)/(5*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt...
```

Giac [C] Result contains complex when optimal does not.

time = 4.06, size = 265, normalized size = 1.13

$$-\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(\frac{i}{\sqrt{5}-2}+1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)x+29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}+(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right)-\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(-\frac{i}{\sqrt{5}-2}-1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)x-29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}-(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right)-\frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(-\frac{i}{\sqrt{5}+2}-1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)x+13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}+(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right)-\frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(\frac{i}{\sqrt{5}+2}+1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)x-13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}-(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] -1/20*((I + 2)*sqrt(sqrt(5) - 2)*(I/(sqrt(5) - 2) + 1) + 5*I)*log((406*I +
174)*sqrt(5)*x + (868*I + 372)*x + 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I
 - 203)*sqrt(5) + (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*(
(I + 2)*sqrt(sqrt(5) - 2)*(-I/(sqrt(5) - 2) - 1) + 5*I)*log((406*I + 174)*s
qrt(5)*x + (868*I + 372)*x - 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I - 203
)*sqrt(5) - (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*((2*I +
 1)*sqrt(sqrt(5) + 2)*(-I/(sqrt(5) + 2) - 1) - 5*I)*log((26*I + 130)*sqrt(5
)*x - (44*I + 220)*x + 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(
5) + (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22) - 1/20*((2*I + 1)*sqrt
(sqrt(5) + 2)*(I/(sqrt(5) + 2) + 1) - 5*I)*log((26*I + 130)*sqrt(5)*x - (44
*I + 220)*x - 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(5) - (19*
I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22)
```

Mupad [B]
time = 2.36, size = 87, normalized size = 0.37

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4+\frac{9z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(\frac{x}{4}+\mathrm{root}\left(z^4+\frac{9z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(6x+\mathrm{root}\left(z^4+\frac{9z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)(36x+16)\right)\right)\right)\mathrm{root}\left(z^4+\frac{9z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)
```

```
[Out] symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 +
(9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1/1
280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k), k,
1, 4)
```

3.56 $\int \frac{1}{(1+4x+4x^2+4x^4)^2} \, dx$

Optimal. Leaf size=317

$$-\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right)-\frac{1}{20}$$

```
[Out] 1/2*(-17+(1+1/x)^2)/(5-2*(1+1/x)^2+(1+1/x)^4)+1/10*(59-17*(1+1/x)^2)*(1+1/x
)/(5-2*(1+1/x)^2+(1+1/x)^4)+7/4*arctan(-1/2+1/2*(1+1/x)^2)+1/400*ln((1+1/x)
^2+5^(1/2)-(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1/2)-1/400*
ln((1+1/x)^2+5^(1/2)+(1+1/x)*(2+2*5^(1/2))^(1/2))*(-59590+26650*5^(1/2))^(1
/2)-1/200*arctan((2+2/x-(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))^(1/2))*(59590+2
6650*5^(1/2))^(1/2)-1/200*arctan((2+2/x+(2+2*5^(1/2))^(1/2))/(-2+2*5^(1/2))
^(1/2))*(59590+26650*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2094, 1687, 1692, 1183, 648, 632, 210, 642, 1677, 1674, 12}

$$\frac{7}{4}\text{ArcTan}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\text{ArcTan}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\text{ArcTan}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)-\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] -1/2*(17 - (1 + x^(-1))^2)/(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4) + ((59 -
 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/1
0]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (
Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqr
t[2*(-1 + Sqrt[5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - S
qrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959 + 26
65*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(
-1))^2])/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = -\left(16\text{Subst}\left(\int \frac{(4-4x)^6}{(1280-512x^2+256x^4)^2}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\left(16\text{Subst}\left(\int \frac{x(-24576-81920x^2-24576x^4)}{(1280-512x^2+256x^4)^2}\,dx, x, 1+\frac{1}{x}\right)\right) - 16\text{Subst}$$

$$= \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4}\,dx, x, 1\right.}{167772160}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\text{Subst}}{}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + 896\text{Sul}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{1}{40}\sqrt{-}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{7}{4}\tan^{-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 108, normalized size = 0.34

$$\frac{1}{40}\left(\frac{38+84x-32x^2+72x^3}{1+4x+4x^2+4x^4}+\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\&,\frac{27\log(x-\#1)-16\log(x-\#1)\#1+18\log(x-\#1)\#1^2}{1+2\#1+4\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #1]
*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 79, normalized size = 0.25

method	result	size
default	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79
risch	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+27
)/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integra
te((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.16, size = 704, normalized size = 2.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000
) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/4*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt(19
/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 + 4*x
+ 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) + 74744
88*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730*(4*sqr
t(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000)
```

```
 - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) - 451608564
96*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-3
75/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I -
5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-
19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-1
9/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000
) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/
8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 1
/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19
/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32
*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 59
59/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 8
78404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 3843
0175/2*I + 213096267) + (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2
000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/100
0*I - 5959/2000) - 7*I)^2 - 3021) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000
*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 74
74488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*s
qrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 595
9/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 1/2*sqr
t(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*
I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sq
rt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/200
0) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*
sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2
*I + 213096267) + 840*x + 380)/(4*x^4 + 4*x^2 + 4*x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. $2(257) = 514$.

time = 1.94, size = 3834, normalized size = 12.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] (36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - sqrt(-595
9/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-1601676*sqrt(10)*sqrt(-5959 + 26
65*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
```

+ 36004639)/13543383425 - 1067784*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/101638
9 + 3131659367*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/
1083470674 + 470215*sqrt(5)/2032778 + 94043*sqrt(-665*sqrt(10)*sqrt(-5959 +
 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337) - 40634464149111451*
sqrt(5)*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36
004639)/27530691871904650 - 2885835544225227917282997/146738587677251784500
 - 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 - 5020
8805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093 - 53848575489193
3*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*
sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450 - 92532195509690
1411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/29347717535450356900 + 48430461193
8766076267*sqrt(5)/55061383743809300 + 22013036087014785403*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/666993580351144
4750) + sqrt(-5959/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-94043*sqrt(665*
sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337
- 1601676*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sqrt(-5959
+ 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/13543383425 - 3131659367*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/1083470674 + 1067784
*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/1016389 + 470215*sqrt(5)/2032778) - 220
13036087014785403*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)/6669935803511444750 - 2885835544225227917282997/1467385876
77251784500 - 50208805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093
 - 538485754891933*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450
+ 925321955096901411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/293477175354503569
00 + 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 + 48
4304611938766076267*sqrt(5)/55061383743809300 + 40634464149111451*sqrt(5)*s
qrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/27
530691871904650) + 2*sqrt(6291/16000 + 1599*sqrt(5)/3200 + sqrt(-665*sqrt(1
0)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/4000)*atan(54173
533700*x/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt
(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt
(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqr
t(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 2
21195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
221195*sqrt(5) + 36004639)) - 3203352*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*s
qrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/(
-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2
665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2
665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7
995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt

```
(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)) - 28456443600*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/(-644057
0878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqr
t(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqr
t(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) +
36004639)) + 6263318734*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/(-6440570878*sq
rt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sq
rt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqrt(5))*s
qrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqrt(5) +
4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3600463
9)) + 7292234875/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-6
65*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351
075*sqrt(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)...
```

Giac [C] Result contains complex when optimal does not.
time = 5.62, size = 315, normalized size = 0.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] -1/400*(-(I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1)
- 350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*
I + 26022625108780)*x + 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118)
- (8426908086005*I - 1267318112395)*sqrt(5) + (8166407345*I - 7795873310)*s
qrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*
((I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1) - 350*I)
*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*I + 2602
2625108780)*x - 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118) - (84269
08086005*I - 1267318112395)*sqrt(5) - (8166407345*I - 7795873310)*sqrt(1424
281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*((3*I +
1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((
16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 381438558
5140)*x + 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (12404110944
55*I - 8361475596225)*sqrt(5) + (8153361745*I + 7721428310)*sqrt(1416617*sq
rt(5) + 2178118) + 1907192792570*I - 12856178136150) - 1/400*(-(3*I + 1)*sq
rt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((16722
951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 3814385585140)
```

```
*x - 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (1240411094455*I
- 8361475596225)*sqrt(5) - (8153361745*I + 7721428310)*sqrt(1416617*sqrt(5)
 + 2178118) + 1907192792570*I - 12856178136150) + 1/20*(36*x^3 - 16*x^2 + 4
2*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1)
```

Mupad [B]
time = 2.21, size = 174, normalized size = 0.55

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{169\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)}{100}+\frac{11\,x}{1600}+\frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\,x\,131}{100}-\frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2\,x\,72}{5}-\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3\,x\,36+\frac{59\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2}{20}-16\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3+\frac{27}{1600}\right)\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\right)+\frac{\frac{9\,x^3}{20}-\frac{x^2}{5}+\frac{21\,x}{40}+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

```
[Out] symsum(log((11*x)/1600 - (169*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 2
9/64000, z, k))/100 + (131*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/6
4000, z, k)*x)/100 - (72*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/640
00, z, k)^2*x)/5 - 36*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000,
 z, k)^3*x + (59*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k
)^2)/20 - 16*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3
+ 27/1600)*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k), k,
1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1/4)
```

3.57 $\int (8+24x+8x^2-15x^3+8x^4)^4\,dx$

Optimal. Leaf size=104

$$4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9}-\frac{169584x^{10}}{5}-$$

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

Rubi [A]
time = 0.02, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2086}

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4\,dx = \int \left(4096+49152x+237568x^2+559104x^3+538624x^4-184320x^5\right.$$

$$= 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-$$

Mathematica [A]
time = 0.00, size = 104, normalized size = 1.00

$$4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9}-\frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13}-\frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Maple [A]
time = 0.03, size = 85, normalized size = 0.82

method	result
gosper	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641152}{9}x^9$
default	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641152}{9}x^9$
norman	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641152}{9}x^9$
risch	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641152}{9}x^9$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

Maxima [A]
time = 0.28, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^9+36384x^8-\frac{566912}{7}x^7-30720x^6+\frac{538624}{5}x^5+139776x^4+\frac{237568}{3}x^3+24576x^2+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

Fricas [A]
time = 0.38, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^9+36384x^8-\frac{566912}{7}x^7-30720x^6+\frac{538624}{5}x^5+139776x^4+\frac{237568}{3}x^3+24576x^2+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

Sympy [A]
time = 0.02, size = 100, normalized size = 0.96

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^{9}}{9} + 36384x^{8} - \frac{566912x^{7}}{7} - 30720x^{6} + \frac{538624x^{5}}{5} + 139776x^{4} + \frac{237568x^{3}}{3} + 24576x^{2} + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13/
13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 36384
*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 237568*x
**3/3 + 24576*x**2 + 4096*x
```

Giac [A]
time = 4.49, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^{9} + 36384x^{8} - \frac{566912}{7}x^{7} - 30720x^{6} + \frac{538624}{5}x^{5} + 139776x^{4} + \frac{237568}{3}x^{3} + 24576x^{2} + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

Mupad [B]
time = 2.23, size = 84, normalized size = 0.81

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^{9}}{9} + 36384x^{8} - \frac{566912x^{7}}{7} - 30720x^{6} + \frac{538624x^{5}}{5} + 139776x^{4} + \frac{237568x^{3}}{3} + 24576x^{2} + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

3.58 $\int (8+24x+8x^2-15x^3+8x^4)^3 \, dx$

Optimal. Leaf size=76

$$512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+$$

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2086}

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int (8+24x+8x^2-15x^3+8x^4)^3 \, dx = \int (512+4608x+15360x^2+20160x^3-384x^4-17856x^5+5528$$
$$= 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+$$

Mathematica [A]
time = 0.00, size = 76, normalized size = 1.00

$$512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Maple [A]
time = 0.03, size = 65, normalized size = 0.86

method	result
gosper	$512x + 2304x^2 + 5120x^3 + 5040x^4 - \frac{384}{5}x^5 - 2976x^6 + \frac{5528}{7}x^7 + 2097x^8 - \frac{2936}{3}x^9 - \frac{4527}{10}x^{10} + \frac{6936}{11}$
default	$512x + 2304x^2 + 5120x^3 + 5040x^4 - \frac{384}{5}x^5 - 2976x^6 + \frac{5528}{7}x^7 + 2097x^8 - \frac{2936}{3}x^9 - \frac{4527}{10}x^{10} + \frac{6936}{11}$
norman	$512x + 2304x^2 + 5120x^3 + 5040x^4 - \frac{384}{5}x^5 - 2976x^6 + \frac{5528}{7}x^7 + 2097x^8 - \frac{2936}{3}x^9 - \frac{4527}{10}x^{10} + \frac{6936}{11}$
risch	$512x + 2304x^2 + 5120x^3 + 5040x^4 - \frac{384}{5}x^5 - 2976x^6 + \frac{5528}{7}x^7 + 2097x^8 - \frac{2936}{3}x^9 - \frac{4527}{10}x^{10} + \frac{6936}{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Maxima [A]
time = 0.30, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Fricas [A]
time = 0.37, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Sympy [A]
time = 0.01, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 20
97*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 + 23
04*x**2 + 512*x
```

Giac [A]
time = 5.18, size = 64, normalized size = 0.84

$$\frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^{9}+2097\,x^{8}+\frac{5528}{7}\,x^{7}-2976\,x^{6}-\frac{384}{5}\,x^{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

Mupad [B]
time = 0.08, size = 64, normalized size = 0.84

$$\frac{512\,x^{13}}{13}-240\,x^{12}+\frac{6936\,x^{11}}{11}-\frac{4527\,x^{10}}{10}-\frac{2936\,x^{9}}{3}+2097\,x^{8}+\frac{5528\,x^{7}}{7}-2976\,x^{6}-\frac{384\,x^{5}}{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

3.59 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx$

Optimal. Leaf size=52

$$64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2086}

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \int \left(64+384x+704x^2+144x^3-528x^4+144x^5+353x^6-240x^7+\right.$$
$$= 64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+$$

Mathematica [A]
time = 0.00, size = 52, normalized size = 1.00

$$64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.87

method	result	size
gosper	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
default	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
norman	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
risch	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.85

$$\frac{64}{9}\,x^9-30\,x^8+\frac{353}{7}\,x^7+24\,x^6-\frac{528}{5}\,x^5+36\,x^4+\frac{704}{3}\,x^3+192\,x^2+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Fricas [A]
time = 0.37, size = 44, normalized size = 0.85

$$\frac{64}{9}\,x^9-30\,x^8+\frac{353}{7}\,x^7+24\,x^6-\frac{528}{5}\,x^5+36\,x^4+\frac{704}{3}\,x^3+192\,x^2+64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.94

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x**
3/3 + 192*x**2 + 64*x
```

Giac [A]
time = 5.99, size = 44, normalized size = 0.85

$$\frac{64}{9}\,x^{9} - 30\,x^{8} + \frac{353}{7}\,x^{7} + 24\,x^{6} - \frac{528}{5}\,x^{5} + 36\,x^{4} + \frac{704}{3}\,x^{3} + 192\,x^{2} + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

Mupad [B]
time = 0.03, size = 44, normalized size = 0.85

$$\frac{64\,x^9}{9} - 30\,x^8 + \frac{353\,x^7}{7} + 24\,x^6 - \frac{528\,x^5}{5} + 36\,x^4 + \frac{704\,x^3}{3} + 192\,x^2 + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

3.60 $\int (8+24x+8x^2-15x^3+8x^4)\,dx$

Optimal. Leaf size=30

$$8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

```
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Antiderivative was successfully verified.

```
[In] Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi steps

$$\int (8+24x+8x^2-15x^3+8x^4)\,dx=8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.83

method	result	size
gosper	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
default	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25

norman	$8x + 12x^2 + \frac{8}{3}x^3 - \frac{15}{4}x^4 + \frac{8}{5}x^5$	25
risch	$8x + 12x^2 + \frac{8}{3}x^3 - \frac{15}{4}x^4 + \frac{8}{5}x^5$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-15*x^3+8*x^2+24*x+8,x,method=_RETURNVERBOSE)
```

```
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Fricas [A]
time = 0.39, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Sympy [A]
time = 0.01, size = 27, normalized size = 0.90

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
```

```
[Out] 8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

Giac [A]
time = 5.35, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Mupad [B]
time = 0.02, size = 24, normalized size = 0.80

$$\frac{8\,x^5}{5}-\frac{15\,x^4}{4}+\frac{8\,x^3}{3}+12\,x^2+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)
```

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

3.61 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

Optimal. Leaf size=263

$$-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{6-\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{6+\sqrt{2\left(1\right.}}{\sqrt{2\left(-\right.}}\right.$$

```
[Out] 1/52*arctan(1/39*(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/322608*ln((3+4/x)
^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))*(-208364442+9476610*517^(1/2))
^(1/2)+1/322608*ln((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-20
8364442+9476610*517^(1/2))^(1/2)-1/161304*arctan((6+8/x-(38+2*517^(1/2))^(1
/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9476610*517^(1/2))^(1/2)-1/161304*
arctan((6+8/x+(38+2*517^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(208364442+9
476610*517^(1/2))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2094, 12, 1687, 1183, 648, 632, 210, 642, 1121}

$$\frac{1}{4}\sqrt{\frac{3}{13}}\,\text{ArcTan}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\,\text{ArcTan}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\,\text{ArcTan}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] -1/4*(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517]
)] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]]) - (Sqrt[(5167 + 235*Sqrt[517])/40326]
*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4
+ (Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 +
 235*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqr
t[2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
```

```
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = -\left(1024\text{Subst}\left(\int \frac{(24-32x)^2}{8\,(2117632-2490368x^2+1048576x^4)}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\text{Subst}\left(\int \frac{(24-32x)^2}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\text{Subst}\left(\int -\frac{1536x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= 196608\text{Subst}\left(\int \frac{x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right) -$$

$$= 98304\text{Subst}\left(\int \frac{1}{2117632-2490368x+1048576x^2}\,dx, x, \left(\frac{3}{4}+\frac{1}{x}\right)^2\right)$$

$$= -\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)\right.$$

$$= -\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{19-\left(3+\frac{4}{x}\right)^2}{2\sqrt{39}}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{6}{-}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 0.21

$$\text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&, \frac{\log(x-\#1)}{24+16\#1-45\#1^2+32\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1 -
 45*#1^2 + 32*#1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 49, normalized size = 0.19

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$	49
risch	$\sum_{\_R=\mathrm{RootOf}(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8)} \frac{\ln(x-\_R)}{32\_R^3-45\_R^2+16\_R+24}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x,method=_RETURNVERBOSE)
```

```
[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.18, size = 1297, normalized size = 4.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

```
[Out] -1/104*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/
322608))*log(37895495846208*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304
*I*sqrt(13)*sqrt(3) - 5167/322608))^3 - 537872704512*(1/104*I*sqrt(13)*sqrt
(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 + 5614027117
*I*sqrt(13)*sqrt(3) + 1789133960*x - 291929410084*sqrt(109/161304*I*sqrt(13
)*sqrt(3) - 5167/322608) + 2270349121) - 1/104*(I*sqrt(13)*sqrt(3) + 52*sqr
t(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*log(-4736936980776*(1/104*
```

I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))
^3 + 20163*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqr
t(3) - 5167/322608))^2*(-2258963*I*sqrt(13)*sqrt(3) + 117466076*sqrt(109/16
1304*I*sqrt(13)*sqrt(3) - 5167/322608) - 3334528) + 517*(88099557*(1/104*I*
sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2
 - 16507)*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 51
67/322608)) - 5545754165/8*I*sqrt(13)*sqrt(3) + 223641745*x + 72094804145/2
*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) - 916562824) + 1/80652*(
sqrt(40326)*sqrt(-120978*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*
sqrt(13)*sqrt(3) - 5167/322608))^2 - 120978*(-1/104*I*sqrt(13)*sqrt(3) - 1/
2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 1551/208*(I*sqrt(
13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*(-I*sq
rt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 24
55) + 20163*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) + 20163*sqrt(
-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*log(-60489/2*(-1/104*I*sqrt(
13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2*(-2
258963*I*sqrt(13)*sqrt(3) + 117466076*sqrt(109/161304*I*sqrt(13)*sqrt(3) -
5167/322608) - 3334528) - 1551/2*(88099557*(1/104*I*sqrt(13)*sqrt(3) - 1/2*
sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 16507)*(I*sqrt(13)*s
qrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) + 100851132
096*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 51
67/322608))^2 + 1/416*(3*(2258963*sqrt(40326)*(-I*sqrt(13)*sqrt(3) + 52*sqr
t(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 3334528*sqrt(40326))*(I*s
qrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) -
10003584*sqrt(40326)*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*s
qrt(3) - 5167/322608)) + 13733824*sqrt(40326))*sqrt(-120978*(1/104*I*sqrt(1
3)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 120
978*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) -
5167/322608))^2 - 1551/208*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt
(13)*sqrt(3) - 5167/322608))*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sq
rt(13)*sqrt(3) - 5167/322608)) - 2455) - 25602357/2*I*sqrt(13)*sqrt(3) + 67
0925235*x + 665661282*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608) + 3
20161368) - 1/80652*(sqrt(40326)*sqrt(-120978*(1/104*I*sqrt(13)*sqrt(3) - 1
/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 120978*(-1/104*I*
sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^
2 - 1551/208*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) -
 5167/322608))*(-I*sqrt(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3)
 - 5167/322608)) - 2455) - 20163*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/
322608) - 20163*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*log(-60
489/2*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*I*sqrt(13)*sqrt(3)
 - 5167/322608))^2*(-2258963*I*sqrt(13)*sqrt(3) + 117466076*sqrt(109/161304*
I*sqrt(13)*sqrt(3) - 5167/322608) - 3334528) - 1551/2*(88099557*(1/104*I*sq
rt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))^2 -
 16507)*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(3) - 5167
/322608)) + 100851132096*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*

```
sqrt(13)*sqrt(3) - 5167/322608))^2 - 1/416*(3*(2258963*sqrt(40326)*(-I*sqrt
(13)*sqrt(3) + 52*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 3334
528*sqrt(40326))*(I*sqrt(13)*sqrt(3) + 52*sqrt(-109/161304*I*sqrt(13)*sqrt(
3) - 5167/322608)) - 10003584*sqrt(40326)*(-I*sqrt(13)*sqrt(3) + 52*sqrt(10
9/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) + 13733824*sqrt(40326))*sqrt(-1
20978*(1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(109/161304*I*sqrt(13)*sqrt(3) -
5167/322608))^2 - 120978*(-1/104*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-109/161304*
I*sqrt(13)*sqrt(3) - 5167/322608))^2 - 1551/208*(I*sqrt(13)*sqrt(3) + 52*sq
rt(-109/161304*I*sqrt(13)*sqrt(3) - 5167/322608))*(-I*sqrt(13)*sqrt(3) + 52
*sqrt(109/161304*I*sqrt(13)*sqrt(3) - 5167/322608)) - 2455) - 25602357/2*I*
sqrt(13)*sqrt(3) + 670925235*x + 665661282*sqrt(109/161304*I*sqrt(13)*sqrt(
3) - 5167/322608) + 320161368)
```

Sympy [A]
time = 1.30, size = 41, normalized size = 0.16

$$\mathrm{RootSum}\left(50326848t^4+765960t^2+12753t+64,\left(t\mapsto t\log\left(\frac{100785893208t^3}{4758335}-\frac{1430512512t^2}{4758335}+\frac{72982352521t}{223641745}+x+\frac{2270349121}{1789133960}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

```
[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(10
0785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/223641
745 + x + 2270349121/1789133960)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Mupad [B]
time = 0.41, size = 123, normalized size = 0.47

$$\sum_{k=1}^{4}\ln\left(-\frac{\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)\,\left(2184\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)+256\,x+\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)\,x\,38259+\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)^2\,x\,1531920+805896\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)^2-120\right)}{4096}\right)\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8),x)
```

```
[Out] symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
)*(2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) + 2
56*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
```

```
)*x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z,
k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z
, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/7863
57, z, k), k, 1, 4)
```

3.62 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2} \, dx$

Optimal. Leaf size=366

$$-\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}+\frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)}-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\,(167818}{}$$

```
[Out] -3/208*(3359-107*(3+4/x)^2)/(517-38*(3+4/x)^2+(3+4/x)^4)+1/322608*(3327931-
129631*(3+4/x)^2)*(3+4/x)/(517-38*(3+4/x)^2+(3+4/x)^4)+73/2704*arctan(1/39*
(-5*x^2+12*x+8)/x^2*39^(1/2))*39^(1/2)-1/26018980416*arctan((6+8/x-(38+2*51
7^(1/2))^(1/2))/(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+
40326*517^(1/2))^(1/2)-1/26018980416*arctan((6+8/x+(38+2*517^(1/2))^(1/2))/
(-38+2*517^(1/2))^(1/2))*(1678181+74897*517^(1/2))*(766194+40326*517^(1/2))
^(1/2)-1/26018980416*ln((3+4/x)^2+517^(1/2)-(3+4/x)*(38+2*517^(1/2))^(1/2))
*(-2405208568240933026+105781971094684170*517^(1/2))^(1/2)+1/26018980416*ln
((3+4/x)^2+517^(1/2)+(3+4/x)*(38+2*517^(1/2))^(1/2))*(-2405208568240933026+
105781971094684170*517^(1/2))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2094, 12, 1687, 1692, 1183, 648, 632, 210, 642, 1677, 1674}

$$\frac{73}{208}\sqrt{\frac{3}{13}}\,\text{ArcTan}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\text{ArcTan}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\text{ArcTan}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}-\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}+\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) +
((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 +
(3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*A
rcTan[(6 - Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/6452
16 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 +
Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sq
rt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-596441146
71451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[5
17])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-59644114671451 + 262317043
8295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/645216
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = -\left(1024\text{Subst}\left(\int \frac{(24-32x)^6}{64\,(2117632-2490368x^2+1048576x^4)^2}\,dx, x, \frac{3}{4}\right.\right.$$

$$= -\left(16\text{Subst}\left(\int \frac{(24-32x)^6}{(2117632-2490368x^2+1048576x^4)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right.$$

$$= -\left(16\text{Subst}\left(\int \frac{x(-1528823808-9059696640x^2-4831838208x^4)}{(2117632-2490368x^2+1048576x^4)^2}\right.\right.$$

$$= \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{120925685220163941}{2117632}\right.}{70}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\right.\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\right.\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\right.\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\right.\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 128, normalized size = 0.35

$$\frac{72888+89033x-94314x^2+39280x^3}{161304\,(8+24x+8x^2-15x^3+8x^4)} + \frac{\text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&, \frac{74897\log(x-\#1)-57489\log(x-\#1)\#1+19640\log(x-\#1)\#1^2}{24+16\#1-45\#1^2+32\#1^3}\&\right]}{80652}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Log[
x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 45*#
1^2 + 32*#1^3) & ]/80652
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 96, normalized size = 0.26

method	result
default	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}\right)}{80652}$
risch	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}\right)}{80652}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x^
2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+24)
*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2
+ 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15*x
^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.23, size = 1540, normalized size = 4.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] 1/26018980416*(6336021120*x^3 - 4811202*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)
*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sq
rt(3) - 59644114671451/2098482808511232))*log(-131155175531952*(21770028828
7626155772963*I*sqrt(13)*sqrt(3) + 806385725383207020835742 4*sqrt(-53750875
7/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232) - 29
04532176689925771712)*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(537508757/2690
3625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 + 21152
332271818991653597636374906968232 96*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt
(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209848280851
1232))^3 - 801867*(48777466142704809483333222033 6*(-73/5408*I*sqrt(13)*sqrt
(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 5964411467145
1/2098482808511232))^2 + 32817075305772686518 99)*(-73*I*sqrt(13)*sqrt(3) +
2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098
482808511232)) - 3246265196161156614776051552784488493/4*I*sqrt(13)*sqrt(3)
 + 1509305314029940798815339032152 65*x - 3006130510417728591217275136551115
3716*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/209
8482808511232) + 1090507114917617311013907313810175 2) - 4811202*(8*x^4 - 15
*x^3 + 8*x^2 + 24*x + 8)*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/2690
3625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))*log(-1692
186581745519332287810909992557458636 8*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sq
rt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808
511232))^3 + 304755541977575868624365442970993459 2*(-73/5408*I*sqrt(13)*sqr
t(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146714
51/2098482808511232))^2 + 649252885553041733545293676366844411 4*I*sqrt(13)*
sqrt(3) + 120744425122395263905227122572212 0*x + 240490383908962307877599 19
1903554423072*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146
71451/2098482808511232) - 907404798791025007870824774437492 95) - 1521322545
6*x^2 - (8*sqrt(20163)*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*sqrt(-3934655265
95856*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(537508757/26903625750144*I*sqr
t(13)*sqrt(3) - 59644114671451/2098482808511232))^2 - 393465526595856*(-73/
5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757/26903625750144*I*sqrt(13)*sqr
t(3) - 59644114671451/2098482808511232))^2 - 3731087151/416*(73*I*sqrt(13)*
sqrt(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114
671451/2098482808511232))*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/269
03625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)) - 149116
25619311) - 2405601*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*(73*I*sqrt(13)*sqrt
(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146714
51/2098482808511232)) - 2405601*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*(-73*I*
sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) -
59644114671451/2098482808511232)))*log(196732763297928*(2177002882876261557
72963*I*sqrt(13)*sqrt(3) + 806385725383207020835742 4*sqrt(-537508757/269036
25750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232) - 2904532176
689925771712)*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(537508757/269036257501
```

```
44*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 - 5714166412079
54753670685205570612736*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757/
26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 + 2
405601/2*(487774661427048094833332220336*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2
*sqrt(-537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482
808511232))^2 + 3281707530577268651899)*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt
(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511
232)) + 1/208*sqrt(-393465526595856*(73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(
537508757/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/20984828085112
32))^2 - 393465526595856*(-73/5408*I*sqrt(13)*sqrt(3) - 1/2*sqrt(-537508757
/26903625750144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))^2 -
3731087151/416*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/26903625750144
*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232))*(-73*I*sqrt(13)*sqr
t(3) + 2704*sqrt(537508757/26903625750144*I*sqrt(13)*sqrt(3) - 596441146714
51/2098482808511232)) - 14911625619311)*(4653*(2982195729967481585931*sqrt(
20163)*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/26903625750144*I*sqrt(
13)*sqrt(3) - 59644114671451/2098482808511232)) - 2904532176689925771712*sq
rt(20163))*(-73*I*sqrt(13)*sqrt(3) + 2704*sqrt(537508757/26903625750144*I*s
qrt(13)*sqrt(3) - 59644114671451/2098482808511232)) - 135147882181382246157
75936*sqrt(20163)*(73*I*sqrt(13)*sqrt(3) + 2704*sqrt(-537508757/26903625750
144*I*sqrt(13)*sqrt(3) - 59644114671451/2098482808511232)) - 27303806654402
87518379968*sqrt(20163)) + 576296960960287187378212699827/2*I*sqrt(13)*sqrt
(3) + 452791594208982239644601709645795*x + 1067333549614120927856635027624
8*sqrt(-537508757/26903625750144*I*sqrt(13)*sqr...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3839 vs. $2(292) = 584$.
time = 2.37, size = 3839, normalized size = 10.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/1678786246808985
6 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-1123969950204685033
06932567484755463/603722125611976319526135612861060 - 296438698298128332309
07750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/
1936419398792394461637855141912238396080 - 181533261043120360732*sqrt(-7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/1509305314
02994079881533903215265 - 46926347979646613249222*sqrt(517)/297468603626329
12338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 + 2623170438
295*sqrt(517))/1427849297406379792240272 + 994065243322493861977*sqrt(40326
```

)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 75)/12909462658615963077585700946081589 30720) - 459714970677306696892185479122356023880918931359173517602 9*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/1843276718699869862645060404837476389014874805380627572841955840 - 10221327637202671758827804250636131310886019359583038780811587109497154599674114864 47/302201812380681690634631534385892067350866441656553353409624708723614680800 - 1063809471733428012617611152668277664372838283556533836993*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/689619370306997243674472351962660786270618994938298086656 0 - 8903603892985006467318455955930346703260445958708241693 13*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/935247388412167707960174961348988989867284925822989101461120955430915 8400 - 4511397632748880932509450163382601467 1791*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/10775302661046831932413630499416574785478421795910907604 0 + 42698009636515468718900942734274005212255282299552802371283821308121207*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/93524738841216770796017496134898898986728492582298910146112095543091 58400 + 6854877670966967408189285140741320937321800706093435313715257320940507 3*sqrt(517)/460819179674967465661265101209369097253718701345156893210488960 + 2741964319335541530074345707646806021327350986246447585831575286311670 33*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/4835228998090907050154104550174273077613863066504853654553995339577834892800) - sqrt(-59644114671451/16787862468089856 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-11239699502046850330693256748475546 3/603722125611976319526135612861060 - 994065243322493861977*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/1427849297406379792240272 - 4692634797964661324 9222*sqrt(517)/29746860362632912338339 + 18153326104312036073 2*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/1509305314029940798815339032152 65 + 2964386982981283323090775077773395 7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/1936419398792394461637855141912238396080 + 994065243322493861977*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/12909462658615963077585700946081589 30720) - 27419643193355415300743457076468060213273509862464475858315752863116703 3*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/4835228998090907

```
050154104550174273077613863066504853654553995339577834892800 - 102213276372
0267175882780425063613131088601935958303878081158710949715459967411486447/3
02201812380681690634631534385892067350866441656553353409624708723614680800
- 890360389298500646731845595593034670326044595870824169313*sqrt(40326)*sqr
t(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004293
5*sqrt(517) + 3557579971691991294769382675)/9352473884121677079601749613489
889898672849258229891014611209554309158400 - 426980096365154687189009427342
74005212255282299552802371283821308121207*sqrt(40326)*sqrt(-59644114671451
+ 2623170438295*sqrt(517))/935247388412167707960174961348988989867284925822
9891014611209554309158400 - 4511397632748880932...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)
```

Mupad [B]
time = 0.21, size = 181, normalized size = 0.49

$$\frac{\frac{2455\,x^3}{80652}-\frac{1429\,x^2}{19552}+\frac{89033\,x}{1290432}+\frac{3037}{53768}}{x^4-\frac{15\,x^3}{8}+x^2+3\,x+1}+\left(\sum_{k=1}^{4}\ln\left(\frac{2146659825\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)}{2960381771776}+\frac{2222183\,x}{338246745408}+\frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)\,x\,924124364159}{26643435945984}-\frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2\,x\,72451101}{8470528}-\frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3\,x\,95745}{256}+\frac{389551\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2}{264704}-\frac{100737\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3}{512}+\frac{271033}{624455529984}\right)\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] ((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*x
 + x^2 - (15*x^3)/8 + x^4 + 1) + symsum(log((2146659825*root(z^4 + (1491162
5619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/
44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (92412
4364159*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3
730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984 - (72
451101*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37
30636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (95745*ro
ot(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37306361040
19968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*root(z^4 + (14
911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 4302
3440/44204510553294663, z, k)^2)/264704 - (100737*root(z^4 + (1491162561931
1*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/442045
10553294663, z, k)^3)/512 + 271033/624455529984)*root(z^4 + (14911625619311
*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/4420451
0553294663, z, k), k, 1, 4)
```

3.63 $\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)$

Optimal. Leaf size=14

$$\frac{(a+bx)^{16}}{16b}$$

[Out] 1/16*(b*x+a)^16/b

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2084, 32}

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3,x]

[Out] (a + b*x)^16/(16*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx = \int (a+bx)^{15}\,dx$$
$$= \frac{(a+bx)^{16}}{16b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(12) = 24$.
time = 0.02, size = 164, normalized size = 11.71

method	result
default	$a^{15}x + \frac{15}{2}a^{14}b\,x^2 + 35a^{13}b^2x^3 + \frac{455}{4}a^{12}b^3x^4 + 273a^{11}b^4x^5 + \frac{1001}{2}a^{10}b^5x^6 + 715a^9b^6x^7 + \frac{6435}{8}a^8b^7x^8$
norman	$a^{15}x + \frac{15}{2}a^{14}b\,x^2 + 35a^{13}b^2x^3 + \frac{455}{4}a^{12}b^3x^4 + 273a^{11}b^4x^5 + \frac{1001}{2}a^{10}b^5x^6 + 715a^9b^6x^7 + \frac{6435}{8}a^8b^7x^8$
risch	$a^{15}x + \frac{15}{2}a^{14}b\,x^2 + 35a^{13}b^2x^3 + \frac{455}{4}a^{12}b^3x^4 + 273a^{11}b^4x^5 + \frac{1001}{2}a^{10}b^5x^6 + 715a^9b^6x^7 + \frac{6435}{8}a^8b^7x^8$
gosper	$\frac{x(b^{15}x^{15}+16a\,b^{14}x^{14}+120a^2b^{13}x^{13}+560a^3b^{12}x^{12}+1820a^4b^{11}x^{11}+4368a^5b^{10}x^{10}+8008a^6b^9x^9+11440a^7b^8x^8+12870a^8b^7x^7+1144}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] a^15*x+15/2*a^14*b*x^2+35*a^13*b^2*x^3+455/4*a^12*b^3*x^4+273*a^11*b^4*x^5+
1001/2*a^10*b^5*x^6+715*a^9*b^6*x^7+6435/8*a^8*b^7*x^8+715*a^7*b^8*x^9+1001
/2*a^6*b^9*x^10+273*a^5*b^10*x^11+455/4*a^4*b^11*x^12+35*a^3*b^12*x^13+15/2
*a^2*b^13*x^14+a*b^14*x^15+1/16*b^15*x^16
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(12) = 24$.
time = 0.30, size = 592, normalized size = 42.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b^12*x^13 +
 100*a^6*b^9*x^10 + 1000/7*a^9*b^6*x^7 + 125/4*a^12*b^3*x^4 + a^15*x + 1/2*
(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^
10 + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)
*a^8*b^2 + 5/3*(18*b^5*x^10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25
/11*(11*b^5*x^12 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 1386*a*b
^9*x^10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 11550*
a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*
```

```
x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 3
85*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^10*x^12 + 840*a*b^
9*x^11 + 4158*a^2*b^8*x^10 + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*
a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 + 2145*a*b^9*
x^12 + 10530*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6*x^9)*a^3*b^2
 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*a^2*b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(12) = 24$.
time = 0.38, size = 163, normalized size = 11.64

$$\frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{15}{2}\,a^{2}b^{13}x^{14}+35\,a^{3}b^{12}x^{13}+\frac{455}{4}\,a^{4}b^{11}x^{12}+273\,a^{5}b^{10}x^{11}+\frac{1001}{2}\,a^{6}b^{9}x^{10}+715\,a^{7}b^{8}x^{9}+\frac{6435}{8}\,a^{8}b^{7}x^{8}+715\,a^{9}b^{6}x^{7}+\frac{1001}{2}\,a^{10}b^{5}x^{6}+273\,a^{11}b^{4}x^{5}+\frac{455}{4}\,a^{12}b^{3}x^{4}+35\,a^{13}b^{2}x^{3}+\frac{15}{2}\,a^{14}bx^{2}+a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="fricas")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/
4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9
 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^
4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(8) = 16$.
time = 0.03, size = 185, normalized size = 13.21

$$a^{15}x+\frac{15a^{14}bx^{2}}{2}+35a^{13}b^{2}x^{3}+\frac{455a^{12}b^{3}x^{4}}{4}+273a^{11}b^{4}x^{5}+\frac{1001a^{10}b^{5}x^{6}}{2}+715a^{9}b^{6}x^{7}+\frac{6435a^{8}b^{7}x^{8}}{8}+715a^{7}b^{8}x^{9}+\frac{1001a^{6}b^{9}x^{10}}{2}+273a^{5}b^{10}x^{11}+\frac{455a^{4}b^{11}x^{12}}{4}+35a^{3}b^{12}x^{13}+\frac{15a^{2}b^{13}x^{14}}{2}+ab^{14}x^{15}+\frac{b^{15}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**3,x)
```

```
[Out] a**15*x + 15*a**14*b*x**2/2 + 35*a**13*b**2*x**3 + 455*a**12*b**3*x**4/4 +
273*a**11*b**4*x**5 + 1001*a**10*b**5*x**6/2 + 715*a**9*b**6*x**7 + 6435*a*
*8*b**7*x**8/8 + 715*a**7*b**8*x**9 + 1001*a**6*b**9*x**10/2 + 273*a**5*b**
10*x**11 + 455*a**4*b**11*x**12/4 + 35*a**3*b**12*x**13 + 15*a**2*b**13*x**
14/2 + a*b**14*x**15 + b**15*x**16/16
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(12) = 24$.
time = 5.26, size = 163, normalized size = 11.64

$$\frac{1}{16}\,b^{15}x^{16}+ab^{14}x^{15}+\frac{15}{2}\,a^{2}b^{13}x^{14}+35\,a^{3}b^{12}x^{13}+\frac{455}{4}\,a^{4}b^{11}x^{12}+273\,a^{5}b^{10}x^{11}+\frac{1001}{2}\,a^{6}b^{9}x^{10}+715\,a^{7}b^{8}x^{9}+\frac{6435}{8}\,a^{8}b^{7}x^{8}+715\,a^{9}b^{6}x^{7}+\frac{1001}{2}\,a^{10}b^{5}x^{6}+273\,a^{11}b^{4}x^{5}+\frac{455}{4}\,a^{12}b^{3}x^{4}+35\,a^{13}b^{2}x^{3}+\frac{15}{2}\,a^{14}bx^{2}+a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="giac")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/
4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9
 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^
4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

Mupad [B]
time = 0.17, size = 163, normalized size = 11.64

$$a^{15}\,x+\frac{15\,a^{14}\,b\,x^2}{2}+35\,a^{13}\,b^2\,x^3+\frac{455\,a^{12}\,b^3\,x^4}{4}+273\,a^{11}\,b^4\,x^5+\frac{1001\,a^{10}\,b^5\,x^6}{2}+715\,a^9\,b^6\,x^7+\frac{6435\,a^8\,b^7\,x^8}{8}+715\,a^7\,b^8\,x^9+\frac{1001\,a^6\,b^9\,x^{10}}{2}+273\,a^5\,b^{10}\,x^{11}+\frac{455\,a^4\,b^{11}\,x^{12}}{4}+35\,a^3\,b^{12}\,x^{13}+\frac{15\,a^2\,b^{13}\,x^{14}}{2}+a\,b^{14}\,x^{15}+\frac{b^{15}\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^3,x)
```

```
[Out] a^15*x + (b^15*x^16)/16 + (15*a^14*b*x^2)/2 + a*b^14*x^15 + 35*a^13*b^2*x^3
 + (455*a^12*b^3*x^4)/4 + 273*a^11*b^4*x^5 + (1001*a^10*b^5*x^6)/2 + 715*a^
9*b^6*x^7 + (6435*a^8*b^7*x^8)/8 + 715*a^7*b^8*x^9 + (1001*a^6*b^9*x^10)/2
+ 273*a^5*b^10*x^11 + (455*a^4*b^11*x^12)/4 + 35*a^3*b^12*x^13 + (15*a^2*b^
13*x^14)/2
```

3.64 $\int (a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

```
[Out] 1/11*(b*x+a)^11/b
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2084, 32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx &= \int (a+bx)^{10}\,dx\\ &= \frac{(a+bx)^{11}}{11b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(12) = 24$.
time = 0.02, size = 109, normalized size = 7.79

method	result
default	$\frac{1}{11}b^{10}x^{11} + a\,b^{9}x^{10} + 5a^{2}b^{8}x^{9} + 15a^{3}b^{7}x^{8} + 30a^{4}b^{6}x^{7} + 42a^{5}b^{5}x^{6} + 42a^{6}b^{4}x^{5} + 30a^{7}b^{3}x^{4} + 15a^{8}b^{2}$
norman	$\frac{1}{11}b^{10}x^{11} + a\,b^{9}x^{10} + 5a^{2}b^{8}x^{9} + 15a^{3}b^{7}x^{8} + 30a^{4}b^{6}x^{7} + 42a^{5}b^{5}x^{6} + 42a^{6}b^{4}x^{5} + 30a^{7}b^{3}x^{4} + 15a^{8}b^{2}$
risch	$\frac{1}{11}b^{10}x^{11} + a\,b^{9}x^{10} + 5a^{2}b^{8}x^{9} + 15a^{3}b^{7}x^{8} + 30a^{4}b^{6}x^{7} + 42a^{5}b^{5}x^{6} + 42a^{6}b^{4}x^{5} + 30a^{7}b^{3}x^{4} + 15a^{8}b^{2}$
gosper	$\frac{x\left(b^{10}x^{10}+11a\,b^{9}x^{9}+55a^{2}b^{8}x^{8}+165a^{3}b^{7}x^{7}+330a^{4}b^{6}x^{6}+462a^{5}b^{5}x^{5}+462a^{6}b^{4}x^{4}+330a^{7}b^{3}x^{3}+165a^{8}b^{2}x^{2}+55a^{9}bx+11a^{10}\right)}{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 1/11*b^10*x^11+a*b^9*x^10+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a^
5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(12) = 24$.
time = 0.26, size = 228, normalized size = 16.29

$$\frac{1}{11}\,b^{10}x^{11} + ab^{9}x^{10} + \frac{25}{9}\,a^{2}b^{8}x^{9} + \frac{100}{7}\,a^{4}b^{6}x^{7} + 20\,a^{6}b^{4}x^{5} + \frac{25}{3}\,a^{8}b^{2}x^{3} + a^{10}x + \frac{1}{3}\,\left(b^{5}x^{6} + 6\,ab^{4}x^{5} + 15\,a^{2}b^{3}x^{4} + 20\,a^{3}b^{2}x^{3} + 15\,a^{4}bx^{2}\right)a^{5} + \frac{5}{21}\,\left(6\,b^{5}x^{7} + 35\,ab^{4}x^{6} + 84\,a^{2}b^{3}x^{5} + 105\,a^{3}b^{2}x^{4}\right)a^{4}b + \frac{5}{42}\,\left(21\,b^{5}x^{8} + 120\,ab^{4}x^{7} + 280\,a^{2}b^{3}x^{6}\right)a^{3}b^{2} + \frac{5}{18}\,\left(8\,b^{5}x^{9} + 45\,ab^{4}x^{8}\right)a^{2}b^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^6
*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^10*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*
b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^4*x
^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4
*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(12) = 24$.
time = 0.37, size = 108, normalized size = 7.71

$$\frac{1}{11}\,b^{10}x^{11} + ab^{9}x^{10} + 5\,a^{2}b^{8}x^{9} + 15\,a^{3}b^{7}x^{8} + 30\,a^{4}b^{6}x^{7} + 42\,a^{5}b^{5}x^{6} + 42\,a^{6}b^{4}x^{5} + 30\,a^{7}b^{3}x^{4} + 15\,a^{8}b^{2}x^{3} + 5\,a^{9}bx^{2} + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(8) = 16$.
time = 0.02, size = 114, normalized size = 8.14

$$a^{10}x + 5a^9bx^2 + 15a^8b^2x^3 + 30a^7b^3x^4 + 42a^6b^4x^5 + 42a^5b^5x^6 + 30a^4b^6x^7 + 15a^3b^7x^8 + 5a^2b^8x^9 + ab^9x^{10} + \frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**2,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(12) = 24$.
time = 3.48, size = 108, normalized size = 7.71

$$\frac{1}{11}b^{10}x^{11} + ab^9x^{10} + 5\,a^2b^8x^9 + 15\,a^3b^7x^8 + 30\,a^4b^6x^7 + 42\,a^5b^5x^6 + 42\,a^6b^4x^5 + 30\,a^7b^3x^4 + 15\,a^8b^2x^3 + 5\,a^9bx^2 + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="giac")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

Mupad [B]
time = 0.06, size = 108, normalized size = 7.71

$$a^{10}\,x + 5\,a^9\,b\,x^2 + 15\,a^8\,b^2\,x^3 + 30\,a^7\,b^3\,x^4 + 42\,a^6\,b^4\,x^5 + 42\,a^5\,b^5\,x^6 + 30\,a^4\,b^6\,x^7 + 15\,a^3\,b^7\,x^8 + 5\,a^2\,b^8\,x^9 + a\,b^9\,x^{10} + \frac{b^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^2,x)
```

```
[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^
7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x
^8 + 5*a^2*b^8*x^9
```

3.65 $\int (a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

```
[Out] 1/6*(b*x+a)^6/b
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 61 vs. 2(14) = 28.
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x
^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Rubi steps

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx=a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 61 vs. 2(14) = 28.
time = 0.00, size = 61, normalized size = 4.36

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 +
 b^5*x^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Maple [A]
time = 0.01, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^6}{6b}$	13
norman	$a^5x+\frac{5}{2}b\,a^4x^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+a\,b^4x^5+\frac{1}{6}b^5x^6$	54
risch	$a^5x+\frac{5}{2}b\,a^4x^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+a\,b^4x^5+\frac{1}{6}b^5x^6$	54
gosper	$\frac{x(b^5x^5+6a\,b^4x^4+15a^2b^3x^3+20a^3b^2x^2+15a^4bx+6a^5)}{6}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x,metho
d=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x+a)^6/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.
time = 0.26, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.
time = 0.39, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="fricas")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(8) = 16$.
time = 0.01, size = 60, normalized size = 4.29

$$a^5x+\frac{5a^4bx^2}{2}+\frac{10a^3b^2x^3}{3}+\frac{5a^2b^3x^4}{2}+ab^4x^5+\frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
4*b*x+a**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.
time = 5.23, size = 53, normalized size = 3.79

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Mupad [B]
time = 0.02, size = 53, normalized size = 3.79

$$a^5\,x + \frac{5\,a^4\,b\,x^2}{2} + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^4}{2} + a\,b^4\,x^5 + \frac{b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b
*x,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5
*a^2*b^3*x^4)/2
```

3.66 $\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4b(a+bx)^4}$$

```
[Out] -1/4/b/(b*x+a)^4
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2083, 32}

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-1),x]
```

```
[Out] -1/4*1/(b*(a + b*x)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx &= \int \frac{1}{(a+bx)^5}\,dx\\ &= -\frac{1}{4b(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-1),x]
```

```
[Out] -1/4*1/(b*(a + b*x)^4)
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{1}{4b(bx+a)^{4}}$	13
norman	$-\frac{1}{4b(bx+a)^{4}}$	13
gosper	$-\frac{1}{4b(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4})}$	46
risch	$-\frac{1}{4b(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4})}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/4/b/(b*x+a)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(12) = 24.
time = 0.26, size = 46, normalized size = 3.29

$$-\frac{1}{4\left(b^{5}x^{4}+4\,ab^{4}x^{3}+6\,a^{2}b^{3}x^{2}+4\,a^{3}b^{2}x+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="maxima")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(12) = 24.
time = 0.36, size = 46, normalized size = 3.29

$$-\frac{1}{4\left(b^{5}x^{4}+4\,ab^{4}x^{3}+6\,a^{2}b^{3}x^{2}+4\,a^{3}b^{2}x+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="fricas")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.
time = 0.12, size = 49, normalized size = 3.50

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5),x)
```

```
[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5
*x**4)
```

Giac [A]
time = 5.24, size = 12, normalized size = 0.86

$$-\frac{1}{4(bx + a)^4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="giac")
```

```
[Out] -1/4/((b*x + a)^4*b)
```

Mupad [B]
time = 0.05, size = 48, normalized size = 3.43

$$-\frac{1}{4\,a^4\,b + 16\,a^3\,b^2\,x + 24\,a^2\,b^3\,x^2 + 16\,a\,b^4\,x^3 + 4\,b^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x),x)
```

```
[Out] -1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)
```

3.67 $\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/9/b/(b*x+a)^9
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2083, 32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-2),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx &= \int \frac{1}{(a+bx)^{10}}\,dx\\ &= -\frac{1}{9b(a+bx)^9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-2),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Maple [A]
time = 0.05, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{1}{9b(bx+a)^{9}}$	13
norman	$-\frac{1}{9b(bx+a)^{9}}$	13
risch	$-\frac{1}{9b(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4})^{2}(bx+a)}$	53
gosper	$-\frac{1}{9(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4})(b^{5}x^{5}+5a\,b^{4}x^{4}+10a^{2}b^{3}x^{3}+10a^{3}b^{2}x^{2}+5a^{4}bx+a^{5})b}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x
,method=_RETURNVERBOSE)
```

```
[Out] -1/9/b/(b*x+a)^9
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(12) = 24.
time = 0.31, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^{9}+9\,ab^{9}x^{8}+36\,a^{2}b^{8}x^{7}+84\,a^{3}b^{7}x^{6}+126\,a^{4}b^{6}x^{5}+126\,a^{5}b^{5}x^{4}+84\,a^{6}b^{4}x^{3}+36\,a^{7}b^{3}x^{2}+9\,a^{8}b^{2}x+a^{9}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="maxima")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(12) = 24.
time = 0.36, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^{9}+9\,ab^{9}x^{8}+36\,a^{2}b^{8}x^{7}+84\,a^{3}b^{7}x^{6}+126\,a^{4}b^{6}x^{5}+126\,a^{5}b^{5}x^{4}+84\,a^{6}b^{4}x^{3}+36\,a^{7}b^{3}x^{2}+9\,a^{8}b^{2}x+a^{9}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(12) = 24$.
time = 0.29, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**2,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

Giac [A]
time = 6.01, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

Mupad [B]
time = 2.10, size = 103, normalized size = 7.36

$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8\,x^7+81\,a\,b^9\,x^8+9\,b^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^2,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

3.68 $\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{14b(a+bx)^{14}}$$

```
[Out] -1/14/b/(b*x+a)^14
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2083, 32}

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-3),x]
```

```
[Out] -1/14*1/(b*(a + b*x)^14)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5)^3}\,dx &= \int \frac{1}{(a+bx)^{15}}\,dx\\ &= -\frac{1}{14b(a+bx)^{14}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-3),x]
```

```
[Out] -1/14*1/(b*(a + b*x)^14)
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{1}{14b(bx+a)^{14}}$	13
norman	$-\frac{1}{14b(bx+a)^{14}}$	13
risch	$-\frac{1}{14b(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)^3(bx+a)^2}$	53
gosper	$-\frac{1}{14(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4)(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)^2b}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x
,method=_RETURNVERBOSE)
```

```
[Out] -1/14/b/(b*x+a)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(12) = 24.
time = 0.29, size = 156, normalized size = 11.14

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^2b^{13}x^{12}+364\,a^3b^{12}x^{11}+1001\,a^4b^{11}x^{10}+2002\,a^5b^{10}x^9+3003\,a^6b^9x^8+3432\,a^7b^8x^7+3003\,a^8b^7x^6+2002\,a^9b^6x^5+1001\,a^{10}b^5x^4+364\,a^{11}b^4x^3+91\,a^{12}b^3x^2+14\,a^{13}b^2x+a^{14}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="maxima")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(12) = 24.
time = 0.37, size = 156, normalized size = 11.14

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^2b^{13}x^{12}+364\,a^3b^{12}x^{11}+1001\,a^4b^{11}x^{10}+2002\,a^5b^{10}x^9+3003\,a^6b^9x^8+3432\,a^7b^8x^7+3003\,a^8b^7x^6+2002\,a^9b^6x^5+1001\,a^{10}b^5x^4+364\,a^{11}b^4x^3+91\,a^{12}b^3x^2+14\,a^{13}b^2x+a^{14}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="fricas")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(12) = 24$.
time = 0.42, size = 168, normalized size = 12.00

$$-\frac{1}{14a^{14}b + 196a^{13}b^2x + 1274a^{12}b^3x^2 + 5096a^{11}b^4x^3 + 14014a^{10}b^5x^4 + 28028a^9b^6x^5 + 42042a^8b^7x^6 + 48048a^7b^8x^7 + 42042a^6b^9x^8 + 28028a^5b^{10}x^9 + 14014a^4b^{11}x^{10} + 5096a^3b^{12}x^{11} + 1274a^2b^{13}x^{12} + 196ab^{14}x^{13} + 14b^{15}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**3,x)
```

```
[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4*
x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**6
+ 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 + 140
14*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 196*a
*b**14*x**13 + 14*b**15*x**14)
```

Giac [A]
time = 3.77, size = 12, normalized size = 0.86

$$-\frac{1}{14\,(bx+a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="giac")
```

```
[Out] -1/14/((b*x + a)^14*b)
```

Mupad [B]
time = 3.03, size = 158, normalized size = 11.29

$$-\frac{1}{14\,a^{14}\,b + 196\,a^{13}\,b^2\,x + 1274\,a^{12}\,b^3\,x^2 + 5096\,a^{11}\,b^4\,x^3 + 14014\,a^{10}\,b^5\,x^4 + 28028\,a^9\,b^6\,x^5 + 42042\,a^8\,b^7\,x^6 + 48048\,a^7\,b^8\,x^7 + 42042\,a^6\,b^9\,x^8 + 28028\,a^5\,b^{10}\,x^9 + 14014\,a^4\,b^{11}\,x^{10} + 5096\,a^3\,b^{12}\,x^{11} + 1274\,a^2\,b^{13}\,x^{12} + 196\,a\,b^{14}\,x^{13} + 14\,b^{15}\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^3,x)
```

```
[Out] -1/(14*a^14*b + 14*b^15*x^14 + 196*a^13*b^2*x + 196*a*b^14*x^13 + 1274*a^12
*b^3*x^2 + 5096*a^11*b^4*x^3 + 14014*a^10*b^5*x^4 + 28028*a^9*b^6*x^5 + 420
42*a^8*b^7*x^6 + 48048*a^7*b^8*x^7 + 42042*a^6*b^9*x^8 + 28028*a^5*b^10*x^9
 + 14014*a^4*b^11*x^10 + 5096*a^3*b^12*x^11 + 1274*a^2*b^13*x^12)
```

3.69 $\int \frac{1}{1+x^2+x^3+x^5} \, dx$

Optimal. Leaf size=38

$$\frac{1}{2}\tan^{-1}(x)+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.385$, Rules used = {2083, 649, 209, 266, 642}

$$\frac{\text{ArcTan}(x)}{2}+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+x^3+x^5}\,dx &= \int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx\\ &= \frac{1}{6}\log(1+x)+\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx\\ &= \frac{1}{6}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x)+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3 + x^5)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Maple [A]
time = 0.03, size = 31, normalized size = 0.82

method	result	size
default	$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{6}+\frac{\ln(x^2+1)}{4}-\frac{\ln(x^2-x+1)}{3}$	31
risch	$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{6}+\frac{\ln(x^2+1)}{4}-\frac{\ln(x^2-x+1)}{3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5+x^3+x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/6*ln(1+x)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

Maxima [A]
time = 0.50, size = 30, normalized size = 0.79

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

Fricas [A]
time = 0.41, size = 30, normalized size = 0.79

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{3}\log\left(x^{2}-x+1\right)+\frac{1}{4}\log\left(x^{2}+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

Sympy [A]
time = 0.06, size = 29, normalized size = 0.76

$$\frac{\log\left(x+1\right)}{6}+\frac{\log\left(x^{2}+1\right)}{4}-\frac{\log\left(x^{2}-x+1\right)}{3}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**5+x**3+x**2+1),x)
[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2
```

Giac [A]
time = 5.39, size = 31, normalized size = 0.82

$$\frac{1}{2}\arctan\left(x\right)-\frac{1}{3}\log\left(x^{2}-x+1\right)+\frac{1}{4}\log\left(x^{2}+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(abs(x + 1
))
```

Mupad [B]
time = 2.16, size = 36, normalized size = 0.95

$$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x^{2}-x+1\right)}{3}+\ln\left(x-\mathrm{i}\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^3 + x^5 + 1),x)
[Out] log(x + 1)/6 + log(x - 1i)*(1/4 - 1i/4) + log(x + 1i)*(1/4 + 1i/4) - log(x^
2 - x + 1)/3
```

3.70 $\int (3-19x^2+32x^4-16x^6)^4\,dx$

Optimal. Leaf size=84

$$81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}-\frac{1094656x^{15}}{3}+\frac{5633536x^{17}}{17}-\frac{4014080}{19}$$

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2084, 531, 535}

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx = \int (-1+x)^4(1+x)^4(-1+2x)^4(1+2x)^4\left(-3+4x^2\right)^4 \, dx$$
$$= \int (-1+2x)^4(1+2x)^4\left(-1+x^2\right)^4\left(-3+4x^2\right)^4 \, dx$$
$$= \int \left(-1+x^2\right)^4\left(-3+4x^2\right)^4\left(-1+4x^2\right)^4 \, dx$$
$$= \int (81-2052x^2+22950x^4-149700x^6+634321x^8-1841600x^{10}+37$$
$$= 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764}{}$$

Mathematica [A]
time = 0.00, size = 84, normalized size = 1.00

$$81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}-\frac{1094656x^{15}}{3}+\frac{5633536x^{17}}{17}-\frac{4014080x^{19}}{19}+\frac{1884160x^{21}}{21}-\frac{524288x^{23}}{23}+\frac{65536x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Maple [A]
time = 0.02, size = 65, normalized size = 0.77

method	result
default	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}x^{17}-$
norman	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}x^{17}-$
risch	$81x-684x^3+4590x^5-\frac{149700}{7}x^7+\frac{634321}{9}x^9-\frac{1841600}{11}x^{11}+\frac{3764416}{13}x^{13}-\frac{1094656}{3}x^{15}+\frac{5633536}{17}x^{17}-$
gosper	$x(4386184298496x^{24}-38140733030400x^{22}+150122379264000x^{20}-353491826688000x^{18}+554471344627200x^{16}-610524871756800$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

Maxima [A]
time = 0.30, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Fricas [A]
time = 0.38, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Sympy [A]
time = 0.01, size = 80, normalized size = 0.95

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^{9}}{9}-\frac{149700x^{7}}{7}+4590x^{5}-684x^{3}+81x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

```
[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 56
33536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 + 63
4321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x
```

Giac [A]
time = 6.60, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

Mupad [B]
time = 2.17, size = 64, normalized size = 0.76

$$\frac{65536\,x^{25}}{25}-\frac{524288\,x^{23}}{23}+\frac{1884160\,x^{21}}{21}-\frac{4014080\,x^{19}}{19}+\frac{5633536\,x^{17}}{17}-\frac{1094656\,x^{15}}{3}+\frac{3764416\,x^{13}}{13}-\frac{1841600\,x^{11}}{11}+\frac{634321\,x^9}{9}-\frac{149700\,x^7}{7}+4590\,x^5-684\,x^3+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^4,x)
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

3.71 $\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx$

Optimal. Leaf size=63

$$27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-\frac{21248x^{15}}{5}+\frac{24576x^{17}}{17}-\frac{4096x^{19}}{19}$$

```
[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21
248/5*x^15+24576/17*x^17-4096/19*x^19
```

Rubi [A]

time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2084, 531, 535}

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\,dx=-\int (-1+x)^3(1+x)^3(-1+2x)^3(1+2x)^3\left(-3+4x^2\right)^3\,dx$$
$$=-\int (-1+2x)^3(1+2x)^3\left(-1+x^2\right)^3\left(-3+4x^2\right)^3\,dx$$
$$=-\int \left(-1+x^2\right)^3\left(-3+4x^2\right)^3\left(-1+4x^2\right)^3\,dx$$
$$=-\int \left(-27+513x^2-4113x^4+18235x^6-49344x^8+84912x^{10}-9344\right.$$
$$=27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-$$

Mathematica [A]
time = 0.00, size = 63, normalized size = 1.00

$$27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-\frac{21248x^{15}}{5}+\frac{24576x^{17}}{17}-\frac{4096x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Maple [A]
time = 0.02, size = 50, normalized size = 0.79

method	result
default	$27x-171x^3+\frac{4113}{5}x^5-2605x^7+\frac{16448}{3}x^9-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}x^{19}$
norman	$27x-171x^3+\frac{4113}{5}x^5-2605x^7+\frac{16448}{3}x^9-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}x^{19}$
risch	$27x-171x^3+\frac{4113}{5}x^5-2605x^7+\frac{16448}{3}x^9-\frac{84912}{11}x^{11}+\frac{93440}{13}x^{13}-\frac{21248}{5}x^{15}+\frac{24576}{17}x^{17}-\frac{4096}{19}x^{19}$
gosper	$-\frac{x\left(149360640x^{18}-1001594880x^{16}+2944271616x^{14}-4979884800x^{12}+5348182320x^{10}-3798583360x^8+1804835175x^6-569926071\right.}{692835}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21
248/5*x^15+24576/17*x^17-4096/19*x^19
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.78

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^9-2605\,x^7+\frac{4113}{5}\,x^5-171\,x^3+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

Fricas [A]
time = 0.38, size = 49, normalized size = 0.78

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

Sympy [A]
time = 0.01, size = 60, normalized size = 0.95

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^{9}}{3}-2605x^{7}+\frac{4113x^{5}}{5}-171x^{3}+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13 - 84912*x*
*11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x**3 + 27*x
```

Giac [A]
time = 3.89, size = 49, normalized size = 0.78

$$-\frac{4096}{19}\,x^{19}+\frac{24576}{17}\,x^{17}-\frac{21248}{5}\,x^{15}+\frac{93440}{13}\,x^{13}-\frac{84912}{11}\,x^{11}+\frac{16448}{3}\,x^{9}-2605\,x^{7}+\frac{4113}{5}\,x^{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

Mupad [B]
time = 0.05, size = 49, normalized size = 0.78

$$-\frac{4096\,x^{19}}{19}+\frac{24576\,x^{17}}{17}-\frac{21248\,x^{15}}{5}+\frac{93440\,x^{13}}{13}-\frac{84912\,x^{11}}{11}+\frac{16448\,x^{9}}{3}-2605\,x^{7}+\frac{4113\,x^{5}}{5}-171\,x^{3}+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

3.72 $\int \left(3-19 x^2+32 x^4-16 x^6\right)^2 \, dx$

Optimal. Leaf size=44

$$9 x-38 x^3+\frac{553 x^5}{5}-\frac{1312 x^7}{7}+\frac{544 x^9}{3}-\frac{1024 x^{11}}{11}+\frac{256 x^{13}}{13}$$

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2084, 531, 535}

$$\frac{256 x^{13}}{13}-\frac{1024 x^{11}}{11}+\frac{544 x^9}{3}-\frac{1312 x^7}{7}+\frac{553 x^5}{5}-38 x^3+9 x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2084

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(3-19x^2+32x^4-16x^6\right)^2 \, dx = \int (-1+x)^2(1+x)^2(-1+2x)^2(1+2x)^2 \left(-3+4x^2\right)^2 \, dx$$
$$= \int (-1+2x)^2(1+2x)^2 \left(-1+x^2\right)^2 \left(-3+4x^2\right)^2 \, dx$$
$$= \int \left(-1+x^2\right)^2 \left(-3+4x^2\right)^2 \left(-1+4x^2\right)^2 \, dx$$
$$= \int \left(9-114x^2+553x^4-1312x^6+1632x^8-1024x^{10}+256x^{12}\right) \, dx$$
$$= 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.80

method	result	size
default	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
norman	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
risch	$9x-38x^3+\frac{553}{5}x^5-\frac{1312}{7}x^7+\frac{544}{3}x^9-\frac{1024}{11}x^{11}+\frac{256}{13}x^{13}$	35
gosper	$\frac{x\left(295680x^{12}-1397760x^{10}+2722720x^8-2814240x^6+1660659x^4-570570x^2+135135\right)}{15015}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.77

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Fricas [A]
time = 0.37, size = 34, normalized size = 0.77

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^{9}-\frac{1312}{7}\,x^{7}+\frac{553}{5}\,x^{5}-38\,x^{3}+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Sympy [A]
time = 0.01, size = 41, normalized size = 0.93

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^{9}}{3}-\frac{1312x^{7}}{7}+\frac{553x^{5}}{5}-38x^{3}+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*x
**3 + 9*x
```

Giac [A]
time = 3.37, size = 34, normalized size = 0.77

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^{9}-\frac{1312}{7}\,x^{7}+\frac{553}{5}\,x^{5}-38\,x^{3}+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

Mupad [B]
time = 0.02, size = 34, normalized size = 0.77

$$\frac{256\,x^{13}}{13}-\frac{1024\,x^{11}}{11}+\frac{544\,x^{9}}{3}-\frac{1312\,x^{7}}{7}+\frac{553\,x^{5}}{5}-38\,x^{3}+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

3.73 $\int (3-19x^2+32x^4-16x^6)\,dx$

Optimal. Leaf size=25

$$3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

```
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi steps

$$\int (3-19x^2+32x^4-16x^6)\,dx = 3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.80

method	result	size
default	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20
norman	$3x-\frac{19}{3}x^3+\frac{32}{5}x^5-\frac{16}{7}x^7$	20

risch	$3x - \frac{19}{3}x^3 + \frac{32}{5}x^5 - \frac{16}{7}x^7$	20
gosper	$-\frac{x(240x^6-672x^4+665x^2-315)}{105}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-16*x^6+32*x^4-19*x^2+3,x,method=_RETURNVERBOSE)
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

Maxima [**A**]
time = 0.27, size = 19, normalized size = 0.76

$$-\frac{16}{7}\,x^{7}+\frac{32}{5}\,x^{5}-\frac{19}{3}\,x^{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Fricas [**A**]
time = 0.36, size = 19, normalized size = 0.76

$$-\frac{16}{7}\,x^{7}+\frac{32}{5}\,x^{5}-\frac{19}{3}\,x^{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Sympy [**A**]
time = 0.01, size = 22, normalized size = 0.88

$$-\frac{16x^{7}}{7}+\frac{32x^{5}}{5}-\frac{19x^{3}}{3}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x**6+32*x**4-19*x**2+3,x)
[Out] -16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

Giac [**A**]
time = 4.05, size = 19, normalized size = 0.76

$$-\frac{16}{7}\,x^{7}+\frac{32}{5}\,x^{5}-\frac{19}{3}\,x^{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")
```

```
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$-\frac{16\,x^7}{7}+\frac{32\,x^5}{5}-\frac{19\,x^3}{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(32*x^4 - 19*x^2 - 16*x^6 + 3,x)
```

```
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

3.74 $\int \frac{1}{3-19x^2+32x^4-16x^6} \, dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(x)+1/3*arctanh(2*x)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2082, 213}

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3-19x^2+32x^4-16x^6} \, dx &= \int \left(-\frac{1}{3(-1+x^2)}+\frac{2}{-3+4x^2}-\frac{2}{3(-1+4x^2)}\right) dx \\
&= -\left(\frac{1}{3}\int \frac{1}{-1+x^2} \, dx\right)-\frac{2}{3}\int \frac{1}{-1+4x^2} \, dx+2\int \frac{1}{-3+4x^2} \, dx \\
&= \frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 2.00

$$\frac{1}{6}\left(\sqrt{3}\log\left(\sqrt{3}-2x\right)-\sqrt{3}\log\left(\sqrt{3}+2x\right)-\log\left(1-3x+2x^2\right)+\log\left(1+3x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2*
x^2] + Log[1 + 3*x + 2*x^2])/6
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.35

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{2x\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(2x+1)}{6}-\frac{\ln(-1+x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(1+x)}{6}$	42
risch	$\frac{\sqrt{3}\ln\left(2x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\ln\left(2x+\sqrt{3}\right)}{6}+\frac{\ln(2x^2+3x+1)}{6}-\frac{\ln(2x^2-3x+1)}{6}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3),x,method=_RETURNVERBOSE)
[Out] -1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)+1/6*ln(2*x+1)-1/6*ln(-1+x)-1/6*ln(2*x-1
)+1/6*ln(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(23) = 46.
time = 0.48, size = 54, normalized size = 1.74

$$\frac{1}{6}\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}}{2\,x+\sqrt{3}}\right)+\frac{1}{6}\log\left(2\,x+1\right)-\frac{1}{6}\log\left(2\,x-1\right)+\frac{1}{6}\log\left(x+1\right)-\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*l
og(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.
time = 0.39, size = 56, normalized size = 1.81

$$\frac{1}{6}\sqrt{3}\log\left(\frac{4\,x^2-4\,\sqrt{3}\,x+3}{4\,x^2-3}\right)+\frac{1}{6}\log\left(2\,x^2+3\,x+1\right)-\frac{1}{6}\log\left(2\,x^2-3\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3*
x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.
time = 0.05, size = 63, normalized size = 2.03

$$\frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{6}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
```

```
[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3*
x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(23) = 46$.
time = 4.91, size = 62, normalized size = 2.00

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)+\frac{1}{6}\log\left(|2x+1|\right)-\frac{1}{6}\log\left(|2x-1|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{6}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2*
x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

Mupad [B]
time = 0.07, size = 27, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{x}{4608\left(\frac{x^2}{6912}+\frac{1}{13824}\right)}\right)}{3}-\frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3),x)
```

```
[Out] atanh(x/(4608*(x^2/6912 + 1/13824)))/3 - (3^(1/2)*atanh((2*3^(1/2)*x)/3))/3
```

3.75 $\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\,(3-4x^2)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 1/18/(1-2*x)+1/36/(1-x)-1/36/(1+x)-1/18/(1+2*x)+2/3*x/(-4*x^2+3)+67/54*arct
anh(x)-7/27*arctanh(2*x)-5/9*arctanh(2/3*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2082, 213, 205}

$$\frac{2x}{3\,(3-4x^2)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*
x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[
(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2}\,dx = \int \left(\frac{1}{36(-1+x)^2}+\frac{1}{36(1+x)^2}+\frac{1}{9(-1+2x)^2}+\frac{1}{9(1+2x)^2}-\frac{6}{54\,(-1}\right.$$

$$= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{14}{27}\int\frac{1}{-1+4x^2}$$

$$= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\,(3-4x^2)}+\frac{67}{54}$$

$$= \frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\,(3-4x^2)}+\frac{67}{54}$$

Mathematica [A]
time = 0.04, size = 103, normalized size = 1.16

$$\frac{1}{108}\left(-\frac{6x(27-104x^2+80x^4)}{-3+19x^2-32x^4+16x^6}+14\log(1-2x)+30\sqrt{3}\log\left(\sqrt{3}-2x\right)-67\log(1-x)+67\log(1+x)-14\log(1+2x)-30\sqrt{3}\log\left(\sqrt{3}+2x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1
- 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] - 14
*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108
```

Maple [A]
time = 0.05, size = 84, normalized size = 0.94

method	result
default	$-\frac{x}{6(x^2-\frac{3}{4})}-\frac{5\,\mathrm{arctanh}\left(\frac{2x\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{1}{18(2x+1)}-\frac{7\ln(2x+1)}{54}-\frac{1}{36(-1+x)}-\frac{67\ln(-1+x)}{108}-\frac{1}{18(2x-1)}+\frac{7\ln(2x}{54}$
risch	$\frac{-\frac{5}{18}x^5+\frac{13}{36}x^3-\frac{3}{32}x}{x^6-2x^4+\frac{19}{16}x^2-\frac{3}{16}}+\frac{5\sqrt{3}\,\ln\left(2x-\sqrt{3}\right)}{18}-\frac{5\sqrt{3}\,\ln\left(2x+\sqrt{3}\right)}{18}-\frac{67\ln(-1+x)}{108}-\frac{7\ln(2x+1)}{54}+\frac{7\ln(2x-1)}{54}+67\,l$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x/(x^2-3/4)-5/9*arctanh(2/3*x*3^(1/2))*3^(1/2)-1/18/(2*x+1)-7/54*ln(2*
x+1)-1/36/(-1+x)-67/108*ln(-1+x)-1/18/(2*x-1)+7/54*ln(2*x-1)-1/36/(1+x)+67/
108*ln(1+x)
```

Maxima [A]
time = 0.49, size = 89, normalized size = 1.00

$$\frac{5}{18}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)-\frac{80x^5-104x^3+27x}{18(16x^6-32x^4+19x^2-3)}-\frac{7}{54}\log(2x+1)+\frac{7}{54}\log(2x-1)+\frac{67}{108}\log(x+1)-\frac{67}{108}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5 - 104*x^3
+ 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x + 1) + 7/54*log(2*x -
 1) + 67/108*log(x + 1) - 67/108*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(67) = 134.
time = 0.39, size = 177, normalized size = 1.99

$$-\frac{480x^5-624x^3-30\sqrt{3}(16x^6-32x^4+19x^2-3)\log\left(\frac{4x^2-4\sqrt{3}x+3}{4x^2-3}\right)+14(16x^6-32x^4+19x^2-3)\log(2x+1)-14(16x^6-32x^4+19x^2-3)\log(2x-1)-67(16x^6-32x^4+19x^2-3)\log(x+1)+67(16x^6-32x^4+19x^2-3)\log(x-1)+162x}{108(16x^6-32x^4+19x^2-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(480*x^5 - 624*x^3 - 30*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log((
4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*l
og(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) - 67*(16*x^6 -
 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x
- 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)
```

Sympy [A]
time = 0.80, size = 104, normalized size = 1.17

$$\frac{-80x^5+104x^3-27x}{288x^6-576x^4+342x^2-54}-\frac{67\log(x-1)}{108}+\frac{7\log\left(x-\frac{1}{2}\right)}{54}-\frac{7\log\left(x+\frac{1}{2}\right)}{54}+\frac{67\log(x+1)}{108}+\frac{5\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{18}-\frac{5\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] (-80*x**5 + 104*x**3 - 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*log
(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 + 5
*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18
```

Giac [A]
time = 3.74, size = 97, normalized size = 1.09

$$\frac{5}{18}\sqrt{3}\log\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)-\frac{80x^5-104x^3+27x}{18(16x^6-32x^4+19x^2-3)}-\frac{7}{54}\log(|2x+1|)+\frac{7}{54}\log(|2x-1|)+\frac{67}{108}\log(|x+1|)-\frac{67}{108}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18*(80*x^5
- 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(abs(2*x + 1)) +
 7/54*log(abs(2*x - 1)) + 67/108*log(abs(x + 1)) - 67/108*log(abs(x - 1))
```

Mupad [B]
time = 0.08, size = 64, normalized size = 0.72

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\,67\mathrm{i}}{54}+\frac{\operatorname{atan}(x\,2\mathrm{i})\,7\mathrm{i}}{27}-\frac{\frac{5\,x^5}{18}-\frac{13\,x^3}{36}+\frac{3\,x}{32}}{x^6-2\,x^4+\frac{19\,x^2}{16}-\frac{3}{16}}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\,5\mathrm{i}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] (atan(x*2i)*7i)/27 - (atan(x*1i)*67i)/54 - ((3*x)/32 - (13*x^3)/36 + (5*x^5
)/18)/((19*x^2)/16 - 2*x^4 + x^6 - 3/16) + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*
5i)/9
```

3.76 $\int \frac{1}{(3-19x^2+32x^4-16x^6)^3} \, dx$

Optimal. Leaf size=161

$$\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{67}{432(1+x)}-\frac{1}{108(1+2x)^2}+\frac{7}{108(1+2}$$

```
[Out] 1/108/(1-2*x)^2-7/108/(1-2*x)+1/432/(1-x)^2+67/432/(1-x)-1/432/(1+x)^2-67/4
32/(1+x)-1/108/(1+2*x)^2+7/108/(1+2*x)-2/3*x/(-4*x^2+3)^2+5/3*x/(-4*x^2+3)+
3913/648*arctanh(x)+67/162*arctanh(2*x)-67/18*arctanh(2/3*x*3^(1/2))*3^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2082, 213, 205}

$$\frac{5x}{3(3-4x^2)}-\frac{2x}{3(3-4x^2)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)}-\frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2}+\frac{1}{432(1-x)^2}-\frac{1}{432(x+1)^2}-\frac{1}{108(2x+1)^2}+\frac{3913}{648}\tanh^{-1}(x)+\frac{67}{162}\tanh^{-1}(2x)-4\sqrt{3}\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 -
x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(
1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh
[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) -
 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(3-19x^2+32x^4-16x^6)^3}\,dx = \int \left(-\frac{1}{216(-1+x)^3}+\frac{67}{432(-1+x)^2}+\frac{1}{216(1+x)^3}+\frac{67}{432(1+x)^2}-\right.$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x}$$

$$=\frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x}$$

Mathematica [A]
time = 0.07, size = 137, normalized size = 0.85

$$\frac{\frac{36x(27-104x^2+80x^4)}{(3-19x^2+32x^4-16x^6)^2}-\frac{6x(345-2384x^2+2288x^4)}{-3+19x^2-32x^4+16x^6}-268\log(1-2x)+2412\sqrt{3}\log\left(\sqrt{3}-2x\right)-3913\log(1-x)+3913\log(1+x)+268\log(1+2x)-2412\sqrt{3}\log\left(\sqrt{3}+2x\right)}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(34
5 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2*x
] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x] + 2
68*Log[1 + 2*x] - 2412*Sqrt[3]*Log[Sqrt[3] + 2*x])/1296
```

Maple [A]
time = 0.06, size = 126, normalized size = 0.78

method	result
risch	$\frac{-\frac{4576}{27}x^{11}+\frac{4640}{9}x^9-580x^7+\frac{7960}{27}x^5-\frac{4777}{72}x^3+\frac{133}{24}x}{(16x^6-32x^4+19x^2-3)^2}-\frac{67\ln(2x-1)}{324}+\frac{67\ln(2x+1)}{324}-\frac{3913\ln(-1+x)}{1296}+\frac{3913\ln(1+x)}{1296}+\frac{67\sqrt{}}{}$

default	$\frac{-\frac{20}{3}x^3+\frac{13}{3}x}{(4x^2-3)^2}-\frac{67\,\mathrm{arctanh}\left(\frac{2x\sqrt{3}}{3}\right)\sqrt{3}}{18}-\frac{1}{108(2x+1)^2}+\frac{7}{108(2x+1)}+\frac{67\ln(2x+1)}{324}+\frac{1}{432(-1+x)^2}-\frac{67}{432(-1+x)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64*(-5/48*x^3+13/192*x)/(4*x^2-3)^2-67/18*arctanh(2/3*x*3^(1/2))*3^(1/2)-1/
108/(2*x+1)^2+7/108/(2*x+1)+67/324*ln(2*x+1)+1/432/(-1+x)^2-67/432/(-1+x)-3
913/1296*ln(-1+x)+1/108/(2*x-1)^2+7/108/(2*x-1)-67/324*ln(2*x-1)-1/432/(1+x
)^2-67/432/(1+x)+3913/1296*ln(1+x)
```

Maxima [A]
time = 0.48, size = 119, normalized size = 0.74

$$\frac{67}{36}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)-\frac{36608x^{11}-111360x^9+125280x^7-63680x^5+14331x^3-1197x}{216\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)}+\frac{67}{324}\log(2x+1)-\frac{67}{324}\log(2x-1)+\frac{3913}{1296}\log(x+1)-\frac{3913}{1296}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/216*(36608*x^11 - 11
1360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(256*x^12 - 1024*x^
10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9) + 67/324*log(2*x + 1) - 6
7/324*log(2*x - 1) + 3913/1296*log(x + 1) - 3913/1296*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(111) = 222$.
time = 0.39, size = 282, normalized size = 1.75

$$-\frac{219648x^{11}-668160x^9+751680x^7-382080x^5+85986x^3-2412\sqrt{3}\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)\log\left(\frac{4x^2-4\sqrt{3}x+3}{4x^2-3}\right)-268\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)\log(2x+1)+268\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)\log(2x-1)-3913\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)\log(x+1)+3913\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)\log(x-1)-7182x}{1296\,(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 - 2
412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2
 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*x^1
0 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(256*x^
12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x - 1)
- 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)
*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 -
114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x
^6 + 553*x^4 - 114*x^2 + 9)
```

Sympy [A]
time = 0.86, size = 134, normalized size = 0.83

$$-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{55296x^{12}-221184x^{10}+352512x^{8}-283392x^{6}+119448x^{4}-24624x^{2}+1944}-\frac{3913\log(x-1)}{1296}-\frac{67\log\left(x-\frac{1}{2}\right)}{324}+\frac{67\log\left(x+\frac{1}{2}\right)}{324}+\frac{3913\log(x+1)}{1296}+\frac{67\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{36}-\frac{67\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -(36608*x**11 - 111360*x**9 + 125280*x**7 - 63680*x**5 + 14331*x**3 - 1197*
x)/(55296*x**12 - 221184*x**10 + 352512*x**8 - 283392*x**6 + 119448*x**4 -
24624*x**2 + 1944) - 3913*log(x - 1)/1296 - 67*log(x - 1/2)/324 + 67*log(x
+ 1/2)/324 + 3913*log(x + 1)/1296 + 67*sqrt(3)*log(x - sqrt(3)/2)/36 - 67*s
qrt(3)*log(x + sqrt(3)/2)/36
```

Giac [A]
time = 3.37, size = 112, normalized size = 0.70

$$\frac{67}{36}\sqrt{3}\log\left(\frac{\left|8x-4\sqrt{3}\right|}{\left|8x+4\sqrt{3}\right|}\right)-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{216\left(16x^{6}-32x^{4}+19x^{2}-3\right)^{2}}+\frac{67}{324}\log\left(\left|2x+1\right|\right)-\frac{67}{324}\log\left(\left|2x-1\right|\right)+\frac{3913}{1296}\log\left(\left|x+1\right|\right)-\frac{3913}{1296}\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 67/36*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/216*(36608
*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(16*x^6 -
 32*x^4 + 19*x^2 - 3)^2 + 67/324*log(abs(2*x + 1)) - 67/324*log(abs(2*x - 1
)) + 3913/1296*log(abs(x + 1)) - 3913/1296*log(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 93, normalized size = 0.58

$$\frac{-\frac{143x^{11}}{216}+\frac{145x^{9}}{72}-\frac{145x^{7}}{64}+\frac{995x^{5}}{864}-\frac{4777x^{3}}{18432}+\frac{133x}{6144}}{x^{12}-4x^{10}+\frac{51x^{8}}{8}-\frac{41x^{6}}{8}+\frac{553x^{4}}{256}-\frac{57x^{2}}{128}+\frac{9}{256}}-\frac{\operatorname{atan}(x\,2\mathrm{i})\,67\mathrm{i}}{162}-\frac{\operatorname{atan}(x\,1\mathrm{i})\,3913\mathrm{i}}{648}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)67\mathrm{i}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

```
[Out] ((133*x)/6144 - (4777*x^3)/18432 + (995*x^5)/864 - (145*x^7)/64 + (145*x^9)
/72 - (143*x^11)/216)/((553*x^4)/256 - (57*x^2)/128 - (41*x^6)/8 + (51*x^8)
/8 - 4*x^10 + x^12 + 9/256) - (atan(x*2i)*67i)/162 - (atan(x*1i)*3913i)/648
 + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*67i)/18
```

3.77 $\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{32(1-x^2)}+\frac{x(99-17x^2)}{128(1-6x^2+x^4)}+\frac{5}{32}\tanh^{-1}(x)+\frac{1}{512}\left(-4+3\sqrt{2}\right)\tanh^{-1}\left(\left(-1+\sqrt{2}\right)x\right)+\frac{1}{512}\left(4+3\sqrt{2}\right.$$

```
[Out] 1/32*x/(-x^2+1)+1/128*x*(-17*x^2+99)/(x^4-6*x^2+1)+5/32*arctanh(x)+1/512*ar
ctanh(x*(2^(1/2)-1))*(-4+3*2^(1/2))+1/512*arctanh(x*(1+2^(1/2)))*(4+3*2^(1/
2))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 205 vs. $2(91) = 182$.
time = 0.10, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$,
Rules used = {2082, 213, 652, 632, 212, 646, 31}

$$-\frac{41-17x}{256(-x^2+2x+1)}+\frac{17x+41}{256(-x^2-2x+1)}+\frac{1}{64(1-x)}-\frac{1}{64(x+1)}+\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)-\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)-\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)-\frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{256\sqrt{2}}+\frac{5}{32}\tanh^{-1}(x)+\frac{17\tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] 1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41 -
 17*x)/(256*(1 + 2*x - x^2)) - (17*ArcTanh[(1 - x)/Sqrt[2]])/(256*Sqrt[2])
+ (5*ArcTanh[x])/32 + (17*ArcTanh[(1 + x)/Sqrt[2]])/(256*Sqrt[2]) + ((2 - 7
*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x]
)/512 - ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] + x])/512 - ((2 + 7*Sqrt[2])*Log[1
 + Sqrt[2] + x])/512
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx &= \int \left(\frac{1}{64(-1+x)^2}+\frac{1}{64(1+x)^2}-\frac{5}{32\left(-1+x^2\right)}+\frac{29-12x}{64\left(-1-2x+x^2\right)^2}+\ldots\right. \\
&= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{1}{128}\int \frac{6+x}{-1-2x+x^2}\,dx+\frac{1}{128}\int \frac{6-x}{-1+2x+x^2}\ldots \\
&= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tan\ldots \\
&= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tan\ldots \\
&= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}-\frac{17\tan\ldots}{\ldots}
\end{aligned}$$

Mathematica [**A**]

time = 0.07, size = 132, normalized size = 1.45

$$\frac{-\frac{8x(103-140x^2+21x^4)}{-1+7x^2-7x^4+x^6}-80\log(1-x)-\left(4+3\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)+\left(4-3\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right)+80\log(1+x)+\left(4+3\sqrt{2}\right)\log\left(-1+\sqrt{2}+x\right)+\left(-4+3\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x]
 - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2]
- x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*Sqrt
[2])*Log[1 + Sqrt[2] + x])/1024
```

Maple [A]
time = 0.04, size = 116, normalized size = 1.27

method	result
default	$-\frac{\frac{17x}{2}+\frac{41}{2}}{128(x^2+2x-1)}-\frac{\ln(x^2+2x-1)}{256}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512}-\frac{1}{64(-1+x)}-\frac{5\ln(-1+x)}{64}+\frac{-\frac{17x}{2}+\frac{41}{2}}{128x^2-256x-128}+$ ln
risch	$\frac{-\frac{21}{128}x^5+\frac{35}{32}x^3-\frac{103}{128}x}{x^6-7x^4+7x^2-1}-\frac{5\ln(-1+x)}{64}+\frac{3\ln\left(3x+3\sqrt{2}+3\right)\sqrt{2}}{1024}-\frac{\ln\left(3x+3\sqrt{2}+3\right)}{256}-\frac{\ln\left(3x+3-3\sqrt{2}\right)}{256}-$ 3 ln(3x+3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-7*x^4+7*x^2-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(17/2*x+41/2)/(x^2+2*x-1)-1/256*ln(x^2+2*x-1)+3/512*2^(1/2)*arctanh(
1/4*(2*x+2)*2^(1/2))-1/64/(-1+x)-5/64*ln(-1+x)+1/128*(-17/2*x+41/2)/(x^2-2*
x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2))-1/64/(1
+x)+5/64*ln(1+x)
```

Maxima [A]
time = 0.48, size = 114, normalized size = 1.25

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{21x^5-140x^3+103x}{128(x^6-7x^4+7x^2-1)}-\frac{1}{256}\log(x^2+2x-1)+\frac{1}{256}\log(x^2-2x-1)+\frac{5}{64}\log(x+1)-\frac{5}{64}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -3/1024*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) - 3/1024*sqrt(2)*l
og((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/
(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2*x
- 1) + 5/64*log(x + 1) - 5/64*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(71) = 142.
time = 0.39, size = 223, normalized size = 2.45

$$-\frac{168x^5-1120x^3-3\sqrt{2}(x^6-7x^4+7x^2-1)\log\left(\frac{x^2+2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)-3\sqrt{2}(x^6-7x^4+7x^2-1)\log\left(\frac{x^2+2\sqrt{2}(x-1)-2x+3}{x^2-2x-1}\right)+4(x^6-7x^4+7x^2-1)\log(x^2+2x-1)-4(x^6-7x^4+7x^2-1)\log(x^2-2x-1)-80(x^6-7x^4+7x^2-1)\log(x+1)+80(x^6-7x^4+7x^2-1)\log(x-1)+824x}{1024(x^6-7x^4+7x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2
+ 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4 +
7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*(x^
6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log
(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 - 7*x^4
 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(75) = 150$.
time = 0.80, size = 296, normalized size = 3.25

$$\frac{-21x^5+140x^3-103x}{128x^6-896x^4+896x^2-128}-\frac{5\log(x-1)}{64}+\frac{5\log(x+1)}{64}+\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001}{202624020}-\frac{471550901878784\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}+\frac{8071264001\sqrt{2}}{270165360}\right)+\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}-\frac{8071264001}{202624020}+\frac{1299552375287054336\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^5}{50656005}-\frac{471550901878784\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^3}{2979765}\right)+\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}+\frac{1299552375287054336\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)^5}{50656005}-\frac{471550901878784\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{8071264001}{202624020}\right)+\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{471550901878784\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}+\frac{8071264001}{202624020}+\frac{8071264001\sqrt{2}}{270165360}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] (-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*lo
g(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 807126400
1/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 129955
2375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2)/27
0165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165360 -
 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)**5/50
656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/256 - 3
*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287054336*(
1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*sqrt(2)/10
24)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)*log(x - 4
71550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 1299552375287054336*(1
/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 8071264001*sqrt
(2)/270165360)
```

Giac [A]
time = 3.61, size = 134, normalized size = 1.47

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{21x^5-140x^3+103x}{128(x^6-7x^4+7x^2-1)}-\frac{1}{256}\log\left(\left|x^2+2x-1\right|\right)+\frac{1}{256}\log\left(\left|x^2-2x-1\right|\right)+\frac{5}{64}\log\left(\left|x+1\right|\right)-\frac{5}{64}\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")
```

```
[Out] -3/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) - 3/
1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/128
*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(abs(x^2 +
```

```
 2*x - 1)) + 1/256*log(abs(x^2 - 2*x - 1)) + 5/64*log(abs(x + 1)) - 5/64*lo
g(abs(x - 1))
```

Mupad [B]
time = 2.18, size = 126, normalized size = 1.38

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{32}-\frac{\frac{21\,x^5}{128}-\frac{35\,x^3}{32}+\frac{103\,x}{128}}{x^6-7\,x^4+7\,x^2-1}-\operatorname{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}-\frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}-\frac{1}{128}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)}+\frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}+\frac{1}{128}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] - (atan(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 -
7*x^4 + x^6 - 1) - atan((x*940311i)/(134217728*((275445*2^(1/2))/134217728
- 389421/134217728)) - (2^(1/2)*x*332433i)/(67108864*((275445*2^(1/2))/1342
17728 - 389421/134217728)))*((2^(1/2)*3i)/512 - 1i/128) - atan((x*940311i)/
(134217728*((275445*2^(1/2))/134217728 + 389421/134217728)) + (2^(1/2)*x*33
2433i)/(67108864*((275445*2^(1/2))/134217728 + 389421/134217728)))*((2^(1/2
)*3i)/512 + 1i/128)
```

3.78 $\int \frac{x^3}{c+(a+bx)^2} \, dx$

Optimal. Leaf size=78

$$-\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{a(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(3a^2-c)\log\left(c+(a+bx)^2\right)}{2b^4}$$

```
[Out] -3*a*x/b^3+1/2*(b*x+a)^2/b^4+1/2*(3*a^2-c)*ln(c+(b*x+a)^2)/b^4-a*(a^2-3*c)*
arctan((b*x+a)/c^(1/2))/b^4/c^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {378, 716, 649, 209, 266}

$$-\frac{a(a^2-3c)\,\text{ArcTan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(3a^2-c)\log\left((a+bx)^2+c\right)}{2b^4}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c
]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*b^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^3}{c+x^2}\,dx, x, a+bx\right)}{b^4}\\ &= \frac{\text{Subst}\left(\int \left(-3a+x-\frac{a^3-3ac-(3a^2-c)x}{c+x^2}\right)dx, x, a+bx\right)}{b^4}\\ &= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\text{Subst}\left(\int \frac{a^3-3ac-(3a^2-c)x}{c+x^2}\,dx, x, a+bx\right)}{b^4}\\ &= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{(a(a^2-3c))\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^4}+\frac{(3a^2-c)\,\text{Subst}\left(\int \frac{x}{c+x^2}\right.}{b^4}\\ &= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{a(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(3a^2-c)\log\left(c+(a+bx)^2\right)}{2b^4}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 0.94

$$\frac{bx(-4a+bx)-\frac{2(a^3-3ac)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+(3a^2-c)\log\left(a^2+c+2abx+b^2x^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + (
3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)
```

Maple [A]
time = 0.28, size = 111, normalized size = 1.42

method	result
default	$-\frac{-\frac{1}{2}b\,x^{2}+2ax}{b^{3}}+\frac{\frac{\left(3a^{2}b-bc\right)\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{2b^{2}}+\frac{\left(2a^{3}+2ac-\frac{\left(3a^{2}b-bc\right)a}{b}\right)\arctan\left(\frac{2b^{2}x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}}{b^{3}}$
risch	$\frac{x^{2}}{2b^{2}}-\frac{2ax}{b^{3}}+\frac{3\ln\left(-a^{3}c-\sqrt{-a^{2}c\left(a^{2}-3c\right)^{2}}\,bx+3c^{2}a-\sqrt{-a^{2}c\left(a^{2}-3c\right)^{2}}\,a\right)a^{2}}{2b^{4}}-\frac{c\ln\left(-a^{3}c-\sqrt{-a^{2}c\left(a^{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/2*b*x^2+2*a*x)+1/b^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+a
^2+c)+(2*a^3+2*a*c-(3*a^2*b-b*c)*a/b)/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/
b/c^(1/2)))
```

Maxima [A]

time = 0.49, size = 81, normalized size = 1.04

$$\frac{bx^{2}-4\,ax}{2\,b^{3}}+\frac{\left(3\,a^{2}-c\right)\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\,b^{4}}-\frac{\left(a^{3}-3\,ac\right)\arctan\left(\frac{b^{2}x+ab}{b\sqrt{c}}\right)}{b^{4}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/
b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))
```

Fricas [A]

time = 0.39, size = 198, normalized size = 2.54

$$\left[\frac{b^{2}cx^{2}-4\,abcx+\left(a^{3}-3\,ac\right)\sqrt{-c}\,\log\left(\frac{b^{2}x^{2}+2\,abx+a^{2}-2\,(bx+a)\sqrt{-c}-c}{b^{2}x^{2}+2\,abx+a^{2}+c}\right)+\left(3\,a^{2}c-c^{2}\right)\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\,b^{4}c},\ \frac{b^{2}cx^{2}-4\,abcx-2\left(a^{3}-3\,ac\right)\sqrt{c}\,\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+\left(3\,a^{2}c-c^{2}\right)\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\,b^{4}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x
 + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^2*
c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a*b*
c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^2)*l
og(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(71) = 142$.
time = 0.38, size = 209, normalized size = 2.68

$$-\frac{2ax}{b^3}+\left(-\frac{a\sqrt{-c}\,(a^2-3c)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)\log\left(x+\frac{a^4-2b^4c\left(-\frac{a\sqrt{-c}\,(a^2-3c)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)-c^2}{a^3b-3abc}\right)+\left(\frac{a\sqrt{-c}\,(a^2-3c)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)\log\left(x+\frac{a^4-2b^4c\left(\frac{a\sqrt{-c}\,(a^2-3c)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)-c^2}{a^3b-3abc}\right)+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4))
*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 -
c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b**4
*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3
*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + x**2/
(2*b**2)
```

Giac [A]
time = 3.29, size = 77, normalized size = 0.99

$$\frac{(3\,a^2-c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4}-\frac{(a^3-3\,ac)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan
((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

Mupad [B]
time = 2.27, size = 87, normalized size = 1.12

$$\frac{x^2}{2\,b^2}-\frac{2\,a\,x}{b^3}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)\,\left(4\,b^4\,c^2-12\,a^2\,b^4\,c\right)}{8\,b^8\,c}+\frac{a\,\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)\,\left(3\,c-a^2\right)}{b^4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c + (a + b*x)^2),x)
```

```
[Out] x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 -
12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c^(
1/2))
```

3.79 $\int \frac{x^2}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b^2}+\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}$$

```
[Out] x/b^2-a*ln(c+(b*x+a)^2)/b^3+(a^2-c)*arctan((b*x+a)/c^(1/2))/b^3/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {378, 716, 649, 209, 266}

$$\frac{(a^2-c)\,\text{ArcTan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + (a + b*x)^2),x]
```

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a
 + b*x)^2])/b^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a^2-c-2ax}{c+x^2}\right)\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{\text{Subst}\left(\int \frac{a^2-c-2ax}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}-\frac{(2a)\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^3}+\frac{(a^2-c)\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^3}\\
&= \frac{x}{b^2}+\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.08

$$\frac{bx+\frac{(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-a\log\left(a^2+c+2abx+b^2x^2\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + (a + b*x)^2),x]
[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a*
b*x + b^2*x^2])/b^3
```

Maple [A]
time = 0.31, size = 70, normalized size = 1.40

method	result

default	$\frac{x}{b^2}+\frac{-\frac{a\ln\left(b^2x^2+2abx+a^2+c\right)}{b}+\frac{\left(a^2-c\right)\arctan\left(\frac{2b^2x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}}{b^2}$
risch	$\frac{x}{b^2}-\frac{\ln\left(-\sqrt{-c\left(a^2-c\right)^2}\,bx+a^2c-\sqrt{-c\left(a^2-c\right)^2}\,a-c^2\right)a}{b^3}+\frac{\ln\left(-\sqrt{-c\left(a^2-c\right)^2}\,bx+a^2c-\sqrt{-c\left(a^2-\right.}\right.}{2b^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/b^2+1/b^2*(-a/b*ln(b^2*x^2+2*a*b*x+a^2+c)+(a^2-c)/b/c^(1/2)*arctan(1/2*(2
*b^2*x+2*a*b)/b/c^(1/2)))
```

Maxima [A]
time = 0.50, size = 61, normalized size = 1.22

$$\frac{x}{b^2}-\frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3}+\frac{\left(a^2-c\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x +
a*b)/(b*sqrt(c)))/(b^3*sqrt(c))
```

Fricas [A]
time = 0.41, size = 157, normalized size = 3.14

$$\left[\frac{2\,bcx-2\,ac\log\left(b^2x^2+2\,abx+a^2+c\right)+\left(a^2-c\right)\sqrt{-c}\,\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,b^3c},\ \frac{bcx-ac\log\left(b^2x^2+2\,abx+a^2+c\right)+\left(a^2-c\right)\sqrt{c}\,\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c)
*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*
x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^
2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(44) = 88$.
time = 0.23, size = 153, normalized size = 3.06

$$\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+(b*x+a)**2),x)
```

```
[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(
-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + sqr
t(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + sqrt
(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2
```

Giac [A]
time = 4.95, size = 54, normalized size = 1.08

$$\frac{x}{b^2} - \frac{a\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{b^3} + \frac{\left(a^2 - c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a)
/sqrt(c))/(b^3*sqrt(c))
```

Mupad [B]
time = 0.09, size = 206, normalized size = 4.12

$$\frac{x}{b^2} - \frac{a\,\ln\left(a^2 + 2\,a\,b\,x + b^2\,x^2 + c\right)}{b^3} + \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,\left(c-a^2\right)} - \frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}} - \frac{a\,\sqrt{c}}{c-a^2} + \frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3} - \frac{a^2\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,\left(c-a^2\right)} - \frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}} - \frac{a\,\sqrt{c}}{c-a^2} + \frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c + (a + b*x)^2),x)
```

```
[Out] x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1/
2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)
/(c^(1/2)*(c/b - a^2/b))))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2)) - (c^(1/
2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b
))))/(b^3*c^(1/2))
```

3.80 $\int \frac{x}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=41

$$-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(c+(a+bx)^2\right)}{2b^2}$$

[Out] 1/2*ln(c+(b*x+a)^2)/b^2-a*arctan((b*x+a)/c^(1/2))/b^2/c^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {378, 649, 209, 266}

$$\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\text{ArcTan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[x/(c + (a + b*x)^2),x]

[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^2)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{-a+x}{c+x^2}\,dx, x, a+bx\right)}{b^2}\\ &= \frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^2}\\ &= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log\left(c+(a+bx)^2\right)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.93

$$\frac{-\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + \log\left(c+(a+bx)^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + (a + b*x)^2),x]
[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)
```

Maple [A]
time = 0.25, size = 54, normalized size = 1.32

method	result
default	$\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2b^2} - \frac{a\arctan\left(\frac{2b^2x+2ab}{2b\sqrt{c}}\right)}{b^2\sqrt{c}}$
risch	$\frac{\ln\left(-\sqrt{-c}\,bx-a\sqrt{-c}-c\right)a\sqrt{-c}}{2c\,b^2} + \frac{\ln\left(-\sqrt{-c}\,bx-a\sqrt{-c}-c\right)}{2b^2} - \frac{\ln\left(\sqrt{-c}\,bx+a\sqrt{-c}-c\right)a\sqrt{-c}}{2c\,b^2} + \frac{\ln\left(\sqrt{-\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] 1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/
b/c^(1/2))
```

Maxima [A]
time = 0.49, size = 50, normalized size = 1.22

$$-\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*
b*x + a^2 + c)/b^2
```

Fricas [A]
time = 0.39, size = 136, normalized size = 3.32

$$\left[-\frac{a\sqrt{-c}\,\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c},\,-\frac{2\,a\sqrt{c}\,\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c
), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x +
a^2 + c))/(b^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(36) = 72$.
time = 0.10, size = 124, normalized size = 3.02

$$\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)+\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-a*sqrt(-c)
/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)
)*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b))
```

Giac [A]
time = 5.43, size = 43, normalized size = 1.05

$$-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a^
2 + c)/b^2
```

Mupad [B]
time = 2.08, size = 46, normalized size = 1.12

$$\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{2\,b^2}-\frac{a\,\mathrm{atan}\left(\frac{a}{\sqrt{c}}+\frac{b\,x}{\sqrt{c}}\right)}{b^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + (a + b*x)^2),x)
```

```
[Out] log(c + a^2 + b^2*x^2 + 2*a*b*x)/(2*b^2) - (a*atan(a/c^(1/2) + (b*x)/c^(1/2
)))/(b^2*c^(1/2))
```

3.81 $\int \frac{1}{c+(a+bx)^2}\,dx$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

```
[Out] arctan((b*x+a)/c^(1/2))/b/c^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {253, 209}

$$\frac{\text{ArcTan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b}\\ &= \frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a + b*x)^2)^(-1),x]
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Maple [A]
time = 0.34, size = 28, normalized size = 1.33

method	result	size
default	$\frac{\arctan\left(\frac{2b^2x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}$	28
risch	$-\frac{\ln\left(bx+\sqrt{-c}+a\right)}{2\sqrt{-c}\,b}+\frac{\ln\left(-bx+\sqrt{-c}-a\right)}{2\sqrt{-c}\,b}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
[Out] 1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

Maxima [A]
time = 0.48, size = 24, normalized size = 1.14

$$\frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")
[Out] arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))
```

Fricas [A]
time = 0.37, size = 83, normalized size = 3.95

$$\left[-\frac{\sqrt{-c}\,\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,bc},\ \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^
2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.
time = 0.07, size = 54, normalized size = 2.57

$$\frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)**2),x)
```

```
[Out] (-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*sq
rt(-1/c))/b)/2)/b
```

Giac [A]
time = 4.02, size = 17, normalized size = 0.81

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

Mupad [B]
time = 0.04, size = 17, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)}{b\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + (a + b*x)^2),x)
```

```
[Out] atan((a + b*x)/c^(1/2))/(b*c^(1/2))
```

3.82 $\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=59

$$-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}$$

```
[Out] ln(x)/(a^2+c)-1/2*ln(c+(b*x+a)^2)/(a^2+c)-a*arctan((b*x+a)/c^(1/2))/(a^2+c)
/c^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {378, 720, 31, 649, 209, 266}

$$-\frac{a\text{ArcTan}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2+c\right)}-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^2 + c) - L
og[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx &= \text{Subst}\left(\int \frac{1}{(-a+x)\left(c+x^2\right)}\,dx,x,a+bx\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-a+x}\,dx,x,a+bx\right)}{a^2+c}+\frac{\text{Subst}\left(\int \frac{-a-x}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}\\ &= \frac{\log(x)}{a^2+c}-\frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}-\frac{a\text{Subst}\left(\int \frac{1}{c+x^2}\,dx,x,a+bx\right)}{a^2+c}\\ &= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}-\frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.81

$$-\frac{\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)+\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a + b
*x)^2])/(a^2 + c)
```

Maple [A]
time = 0.26, size = 74, normalized size = 1.25

method	result
default	$-\frac{b\left(\frac{\ln\left(b^{2} x^{2}+2 a b x+a^{2}+c\right)}{2 b}+\frac{a \arctan\left(\frac{2 b^{2} x+2 a b}{2 b \sqrt{c}}\right)}{b \sqrt{c}}\right)}{a^{2}+c}+\frac{\ln(x)}{a^{2}+c}$
risch	$\frac{\ln\left(\left(-\sqrt{-c}\, a^{2} b+3 \sqrt{-c}\, b c+4 a b c\right) x-\sqrt{-c}\, a^{3}-\sqrt{-c}\, a c+3 a^{2} c+3 c^{2}\right) a \sqrt{-c}}{2 c\left(a^{2}+c\right)}-\frac{\ln\left(\left(-\sqrt{-c}\, a^{2} b+3 \sqrt{-c}\, b c+4 a b c\right.\right.}{2(a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a^2+c)*(1/2/b*ln(b^2*x^2+2*a*b*x+a^2+c)+a/b/c^(1/2)*arctan(1/2*(2*b^2*x
+2*a*b)/b/c^(1/2)))+ln(x)/(a^2+c)
```

Maxima [A]
time = 0.49, size = 68, normalized size = 1.15

$$-\frac{a \arctan\left(\frac{b^{2} x+a b}{b \sqrt{c}}\right)}{\left(a^{2}+c\right) \sqrt{c}}-\frac{\log\left(b^{2} x^{2}+2 a b x+a^{2}+c\right)}{2\left(a^{2}+c\right)}+\frac{\log(x)}{a^{2}+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2
+ 2*a*b*x + a^2 + c)/(a^2 + c) + log(x)/(a^2 + c)
```

Fricas [A]
time = 0.41, size = 154, normalized size = 2.61

$$\left[-\frac{a \sqrt{-c} \log\left(\frac{b^{2} x^{2}+2 a b x+a^{2}+2 (b x+a) \sqrt{-c}-c}{b^{2} x^{2}+2 a b x+a^{2}+c}\right)+c \log\left(b^{2} x^{2}+2 a b x+a^{2}+c\right)-2 c \log(x)}{2\left(a^{2} c+c^{2}\right)}, -\frac{2 a \sqrt{c} \arctan\left(\frac{b x+a}{\sqrt{c}}\right)+c \log\left(b^{2} x^{2}+2 a b x+a^{2}+c\right)-2 c \log(x)}{2\left(a^{2} c+c^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*l
og(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*log(b
^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(49) = 98$.
time = 1.90, size = 738, normalized size = 12.51

$$\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)\log\left(x+\frac{-4a^6c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2+4a^4c^2\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6a^4c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+20a^2c^3\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-12a^2c^2\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+10a^2c+12c^4\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6c^3\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)-6c^2}{a^3b+9abc}\right)+\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)\log\left(x+\frac{-4a^6c\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2+4a^4c^2\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6a^4c\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+20a^2c^3\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-12a^2c^2\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+10a^2c+12c^4\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6c^3\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)-6c^2}{a^3b+9abc}\right)+\frac{\log\left(x+\frac{-\frac{4a^6c}{(a^2+c)^2}+\frac{4a^4c^2}{(a^2+c)^2}-\frac{6a^4c}{a^2+c}+\frac{20a^2c^3}{(a^2+c)^2}-\frac{12a^2c^2}{a^2+c}+10a^2c+\frac{12c^4}{(a^2+c)^2}-\frac{6c^3}{a^2+c}-6c^2}{a^3b+9abc}\right)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**6*c*(-a*sq
rt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)/(
2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**2 +
 c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(
2*(a**2 + c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 +
c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) - 6*c**2
)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*lo
g(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a*
*4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*sqrt(-c)/(2*
c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)/(2*c*(a**2
+ c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**6*c/(a**2 + c)**2 +
4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c**3/(a**2 + c)**
2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 + c)**2 - 6*c**3/(a
**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
```

Giac [A]
time = 3.66, size = 62, normalized size = 1.05

$$-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^2+c)\sqrt{c}}-\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^2+c)}+\frac{\log\left(|x|\right)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b*
x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)
```

Mupad [B]
time = 2.59, size = 173, normalized size = 2.93

$$\frac{\ln(x)}{a^2+c}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c+a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,(a^2+c)}\right)\,\left(c+a\,\sqrt{-c}\right)}{2\,(a^2\,c+c^2)}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\,\left(c-a\,\sqrt{-c}\right)\,\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\,(a^2+c)}\right)\,\left(c-a\,\sqrt{-c}\right)}{2\,(a^2\,c+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)/(c + a^2) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c + a*(-c)^(1/2))*(a*c +
a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c + a*(-c)^(1/2)))/(2*(a^2*c + c^
2)) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c - a*(-c)^(1/2))*(a*c + a^3 - 3*b*c*x
 + a^2*b*x))/(c*(c + a^2)))*(c - a*(-c)^(1/2)))/(2*(a^2*c + c^2))
```

3.83 $\int \frac{1}{x^2(c+(a+bx)^2)} \, dx$

Optimal. Leaf size=79

$$-\frac{1}{(a^2+c)\,x}+\frac{b(a^2-c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2+c\right)^2}-\frac{2ab\log(x)}{(a^2+c)^2}+\frac{ab\log\left(c+(a+bx)^2\right)}{(a^2+c)^2}$$

```
[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+a*b*ln(c+(b*x+a)^2)/(a^2+c)^2+b*(a^2-c)*
arctan((b*x+a)/c^(1/2))/(a^2+c)^2/c^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {378, 724, 815, 649, 209, 266}

$$\frac{b(a^2-c)\,\text{ArcTan}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2+c\right)^2}-\frac{2ab\log(x)}{(a^2+c)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{(a^2+c)^2}-\frac{1}{x\,(a^2+c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2
+ c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a + b*x)^2])/(a^2 + c)
^2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx &= b\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(c+x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\text{Subst}\left(\int \frac{-a-x}{(-a+x)\left(c+x^2\right)}\,dx,x,a+bx\right)}{a^2+c}\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\text{Subst}\left(\int \left(\frac{2a}{\left(a^2+c\right)(a-x)}+\frac{a^2-c+2ax}{\left(a^2+c\right)\left(c+x^2\right)}\right)dx,x,a+bx\right)}{a^2+c}\\
&= -\frac{1}{\left(a^2+c\right)x}-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{b\text{Subst}\left(\int \frac{a^2-c+2ax}{c+x^2}\,dx,x,a+bx\right)}{\left(a^2+c\right)^2}\\
&= -\frac{1}{\left(a^2+c\right)x}-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{(2ab)\text{Subst}\left(\int \frac{x}{c+x^2}\,dx,x,a+bx\right)}{\left(a^2+c\right)^2}+\frac{\left(b\left(a^2-c\right)\right)\text{Subst}}{(a}\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left(c+(a+bx)^2\right)}{\left(a^2+c\right)^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 1.03

$$\frac{b\left(a^2-c\right)x\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)-\sqrt{c}\left(a^2+c+2abx\log(x)-abx\log\left(a^2+c+2abx+b^2x^2\right)\right)}{\sqrt{c}\left(a^2+c\right)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[x
] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)
```

Maple [A]
time = 0.29, size = 96, normalized size = 1.22

method	result
default	$\frac{b^{2}\left(\frac{a\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{b}+\frac{\left(a^{2}-c\right)\arctan\left(\frac{2b^{2}x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}\right)}{\left(a^{2}+c\right)^{2}}-\frac{1}{\left(a^{2}+c\right)x}-\frac{2ab\ln\left(x\right)}{\left(a^{2}+c\right)^{2}}$
risch	$-\frac{1}{\left(a^{2}+c\right)x}-\frac{2ab\ln\left(x\right)}{a^{4}+2a^{2}c+c^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(c\,a^{4}+2a^{2}c^{2}+c^{3}\right)\_Z^{2}-4abc\_Z+b^{2}\right)}\_R\ln\left(\left(\left(-a^{6}b+a^{4}bc+5a^{2}b\,c^{2}+3b\,c^{3}\right)\_R^{2}+(-\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a^2+c)^2*(a/b*ln(b^2*x^2+2*a*b*x+a^2+c)+(a^2-c)/b/c^(1/2)*arctan(1/2*(
2*b^2*x+2*a*b)/b/c^(1/2)))-1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2
```

Maxima [A]
time = 0.49, size = 123, normalized size = 1.56

$$\frac{ab\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{a^{4}+2\,a^{2}c+c^{2}}-\frac{2\,ab\log\left(x\right)}{a^{4}+2\,a^{2}c+c^{2}}+\frac{\left(a^{2}b^{2}-b^{2}c\right)\arctan\left(\frac{b^{2}x+ab}{b\sqrt{c}}\right)}{\left(a^{4}+2\,a^{2}c+c^{2}\right)b\sqrt{c}}-\frac{1}{\left(a^{2}+c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(x)/(
a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/
((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

Fricas [A]
time = 0.42, size = 229, normalized size = 2.90

$$\left[\frac{2\,abcx\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)-4\,abcx\log\left(x\right)+\left(a^{2}b-bc\right)\sqrt{-c}\,x\log\left(\frac{b^{2}x^{2}+2\,abx+a^{2}+2\,(bx+a)\sqrt{-c}-c}{b^{2}x^{2}+2\,abx+a^{2}+c}\right)-2\,a^{2}c-2\,c^{2}}{2\,\left(a^{4}c+2\,a^{2}c^{2}+c^{3}\right)x},\ \frac{abcx\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)-2\,abcx\log\left(x\right)+\left(a^{2}b-bc\right)\sqrt{c}\,x\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-a^{2}c-c^{2}}{\left(a^{4}c+2\,a^{2}c^{2}+c^{3}\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2*
b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c
)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2 + c
^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) + (a^2
*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c + 2*a^
2*c^2 + c^3)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(73) = 146$.
time = 7.26, size = 1620, normalized size = 20.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c**2/(a**2 +
 c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**2*c/(a**2 + c)**2 +
608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b**2*c**2/(a**2 + c)**2 + 432*a*
*5*b**2*c**5/(a**2 + c)**4 - 72*a**5*b**2*c**3/(a**2 + c)**2 + 36*a**5*b**2
*c + 112*a**3*b**2*c**6/(a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**2 - 8
8*a**3*b**2*c**2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6*b**3
+ 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (a*b/(a*
*2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*log(x +
(-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*
c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2
*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 - b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a**2 +
c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32*a**6*
b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c
**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a*
*2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3*b**2*
c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a
**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*(a*b/(
a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 2
*b*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c +
c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)) + (a
*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*lo
g(x + (-4*a**11*c*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2
*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 -
 c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 + b
*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a
```

```
**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32
*a**6*b*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2
*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 + b*sqrt(-
c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**
2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3
*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*
(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 + b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*
(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 2*b*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**
2*c + c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)
) - 1/(x*(a**2 + c))
```

Giac [A]
time = 3.09, size = 117, normalized size = 1.48

$$\frac{ab\log\left(b^2x^2+2\,abx+a^2+c\right)}{a^4+2\,a^2c+c^2}-\frac{2\,ab\log\left(|x|\right)}{a^4+2\,a^2c+c^2}+\frac{\left(a^2b^2-b^2c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^4+2\,a^2c+c^2\right)b\sqrt{c}}-\frac{1}{\left(a^2+c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs(
x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/((a
^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

Mupad [B]
time = 2.58, size = 425, normalized size = 5.38

$$\frac{\ln\left({\left(-c\right)}^{13/2}-35\,a^2\,{\left(-c\right)}^{11/2}+34\,a^4\,{\left(-c\right)}^{9/2}+34\,a^6\,{\left(-c\right)}^{7/2}-35\,a^8\,{\left(-c\right)}^{5/2}+a^{10}\,{\left(-c\right)}^{3/2}+a\,c^6-a^{11}\,c+35\,a^3\,c^5+34\,a^5\,c^4-34\,a^7\,c^3-35\,a^9\,c^2+b\,c^6\,x-a^{10}\,b\,c\,x+35\,a^2\,b\,c^5\,x+34\,a^4\,b\,c^4\,x-34\,a^6\,b\,c^3\,x-35\,a^8\,b\,c^2\,x\right)\,\left(b\,{\left(-c\right)}^{3/2}+2\,a\,b\,c+a^2\,b\,\sqrt{-c}\right)}{2\,\left(a^4\,c+2\,a^2\,c^2+c^3\right)}-\frac{1}{x\,\left(a^2+c\right)}-\frac{\ln\left({\left(-c\right)}^{13/2}-35\,a^2\,{\left(-c\right)}^{11/2}+34\,a^4\,{\left(-c\right)}^{9/2}+34\,a^6\,{\left(-c\right)}^{7/2}-35\,a^8\,{\left(-c\right)}^{5/2}+a^{10}\,{\left(-c\right)}^{3/2}-a\,c^6+a^{11}\,c-35\,a^3\,c^5-34\,a^5\,c^4+34\,a^7\,c^3+35\,a^9\,c^2-b\,c^6\,x+a^{10}\,b\,c\,x-35\,a^2\,b\,c^5\,x-34\,a^4\,b\,c^4\,x+34\,a^6\,b\,c^3\,x+35\,a^8\,b\,c^2\,x\right)\,\left(b\,{\left(-c\right)}^{3/2}-2\,a\,b\,c+a^2\,b\,\sqrt{-c}\right)}{2\,\left(a^4\,c+2\,a^2\,c^2+c^3\right)}-\frac{2\,a\,b\,\ln\left(x\right)}{{\left(a^2+c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + (a + b*x)^2)),x)
```

```
[Out] (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/
2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) + a*c^6 - a^11*c + 35*a^3*c^5 + 34
*a^5*c^4 - 34*a^7*c^3 - 35*a^9*c^2 + b*c^6*x - a^10*b*c*x + 35*a^2*b*c^5*x
+ 34*a^4*b*c^4*x - 34*a^6*b*c^3*x - 35*a^8*b*c^2*x)*(b*(-c)^(3/2) + 2*a*b*c
 + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2)) - (lo
g((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/2)
- 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) - a*c^6 + a^11*c - 35*a^3*c^5 - 34*a^
5*c^4 + 34*a^7*c^3 + 35*a^9*c^2 - b*c^6*x + a^10*b*c*x - 35*a^2*b*c^5*x - 3
4*a^4*b*c^4*x + 34*a^6*b*c^3*x + 35*a^8*b*c^2*x)*(b*(-c)^(3/2) - 2*a*b*c +
a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - (2*a*b*log(x))/(c + a^2)
^2
```

3.84 $\int \frac{1}{x^3(c+(a+bx)^2)}\,dx$

Optimal. Leaf size=121

$$-\frac{1}{2(a^2+c)x^2}+\frac{2ab}{(a^2+c)^2 x}-\frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^3}+\frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3}-\frac{b^2(3a^2-c)\log\left(c+(a+bx)^2\right)}{2(a^2+c)^3}$$

```
[Out] -1/2/(a^2+c)/x^2+2*a*b/(a^2+c)^2/x+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3-1/2*b^2*(3
*a^2-c)*ln(c+(b*x+a)^2)/(a^2+c)^3-a*b^2*(a^2-3*c)*arctan((b*x+a)/c^(1/2))/(
a^2+c)^3/c^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {378, 724, 815, 649, 209, 266}

$$-\frac{ab^2(a^2-3c)\,\text{ArcTan}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2+c)^3}+\frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3}-\frac{b^2(3a^2-c)\log\left((a+bx)^2+c\right)}{2(a^2+c)^3}+\frac{2ab}{x(a^2+c)^2}-\frac{1}{2x^2(a^2+c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*1/((a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*c)*ArcTa
n[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2 - c)*Log[x])/(a^2
 + c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx &= b^2\text{Subst}\left(\int \frac{1}{(-a+x)^3\left(c+x^2\right)}\,dx,x,a+bx\right)\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{b^2\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2(c+x^2)}\,dx,x,a+bx\right)}{a^2+c}\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{b^2\text{Subst}\left(\int \left(-\frac{2a}{(a^2+c)(a-x)^2}+\frac{-3a^2+c}{(a^2+c)^2(a-x)}+\frac{-a(a^2-3c)-(3a^2-c)x}{(a^2+c)^2(c+x^2)}\right) d\right.}{a^2+c}\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}+\frac{b^2(3a^2-c)\log(x)}{\left(a^2+c\right)^3}+\frac{b^2\text{Subst}\left(\int \frac{-a(a^2-3c)-(3a^2-c)}{c+x^2}\right.}{\left(a^2+c\right)^3}\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}+\frac{b^2(3a^2-c)\log(x)}{\left(a^2+c\right)^3}-\frac{(ab^2(a^2-3c))\,\text{Subst}\left(\int \frac{1}{c+x^2}\right.}{\left(a^2+c\right)^3}\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}-\frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2+c\right)^3}+\frac{b^2(3a^2-c)\log(x)}{\left(a^2+c\right)^3}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 106, normalized size = 0.88

$$-\frac{\frac{(a^2+c)(a^2+c-4abx)}{x^2}+\frac{2ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+2b^2(-3a^2+c)\log(x)+b^2(3a^2-c)\log\left(a^2+c+2abx+b^2x^2\right)}{2\left(a^2+c\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a
+ b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Log[
a^2 + c + 2*a*b*x + b^2*x^2])/(a^2 + c)^3
```

Maple [A]
time = 0.35, size = 151, normalized size = 1.25

method	result
default	$-\frac{b^{3}\left(\frac{\left(3a^{2}b-bc\right)\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{2b^{2}}+\frac{\left(4a^{3}-4ac-\frac{\left(3a^{2}b-bc\right)a}{b}\right)\arctan\left(\frac{2b^{2}x+2ab}{2b\sqrt{c}}\right)}{b\sqrt{c}}\right)}{\left(a^{2}+c\right)^{3}}-\frac{1}{2\left(a^{2}+c\right)x^{2}}+\frac{b^{2}\left(3a^{2}-c\right)\ln\left(x\right)}{\left(a^{2}+c\right)^{3}}+\frac{2ab}{(a^2+c)}$
risch	$\frac{\frac{2abx}{a^{4}+2a^{2}c+c^{2}}-\frac{1}{2\left(a^{2}+c\right)}}{x^{2}}+\frac{3b^{2}\ln\left(x\right)a^{2}}{a^{6}+3c\,a^{4}+3a^{2}c^{2}+c^{3}}-\frac{b^{2}\ln\left(x\right)c}{a^{6}+3c\,a^{4}+3a^{2}c^{2}+c^{3}}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{6}c+3a^{4}c^{2}+3a^{2}c^{3}+c^{4}\right)\_Z^{2}+\left(6a^{2}b^{2}c-2b^{2}\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c+(b*x+a)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a^2+c)^3*(1/2*(3*a^2*b-b*c)/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)+(4*a^3-4*a*
c-(3*a^2*b-b*c)*a/b)/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2)))-1/2/(
a^2+c)/x^2+b^2*(3*a^2-c)*ln(x)/(a^2+c)^3+2*a*b/(a^2+c)^2/x
```

Maxima [A]
time = 0.48, size = 197, normalized size = 1.63

$$-\frac{(3\,a^2b^2-b^2c)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,(a^6+3\,a^4c+3\,a^2c^2+c^3)}+\frac{(3\,a^2b^2-b^2c)\log\left(x\right)}{a^6+3\,a^4c+3\,a^2c^2+c^3}-\frac{(a^3b^3-3\,ab^3c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^6+3\,a^4c+3\,a^2c^2+c^3)b\sqrt{c}}+\frac{4\,abx-a^2-c}{2\,(a^4+2\,a^2c+c^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2 +
c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^6 + 3*a^
4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a^2*c
 + c^2)*x^2)
```

Fricas [A]
time = 0.40, size = 371, normalized size = 3.07

$$\left[-\frac{a^4c-(a^3b^2-3\,ab^2c)\sqrt{-c}\,x^2\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,(3\,a^2b^2c-b^2c^2)x^2\log\left(x\right)+c^3-4\,(a^3bc+abc^2)x}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2},\ -\frac{a^4c+2\,(a^3b^2-3\,ab^2c)\sqrt{c}\,x^2\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+2\,a^2c^2+(3\,a^2b^2c-b^2c^2)x^2\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,(3\,a^2b^2c-b^2c^2)x^2\log\left(x\right)+c^3-4\,(a^3bc+abc^2)x}{2\,(a^6c+3\,a^4c^2+3\,a^2c^3+c^4)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x +
a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^2
+ (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b
^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4
*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sqrt(c)
*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*lo
g(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3
 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3284 vs. $2(109) = 218$.
time = 43.89, size = 3284, normalized size = 27.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c+(b*x+a)**2),x)
```

```
[Out] b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3*a
**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 + 56
8*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3*a**
2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**6 - 4
2*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a**6*b**
4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)/(a**2
+ c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(a**2 + c
)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4*c**3 - 2
4*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**6*(3*a**2
- c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 - c)**2/(a**2
 + c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5)/(a**9*b**5
 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**
4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**
16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c
**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14*c**2*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) + 216*a**12*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 +
 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2
- 44*a**10*b**2*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 568*a**10*c*
```

```
*4*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**
3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 78*a**8*b**4*c - 42*a**8*b*
*2*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 720*a**8*c**5*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*b**4*c**2 - 8*a**6*b**2*c**4*(-a
*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -
b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a**6*c**6*(-a*b**2*sqrt(-c)*(a**
2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(
2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2*a**4*b**2*c**5*(-a*b**2*sqrt(
-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**
2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2
*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c
)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c**6*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) - 24*a**2*c**8*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3
*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 +
6*b**4*c**5 - 6*b**2*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**
4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 12*c**9
*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)
) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*b**5 + 72*a**7*b**5*c -
270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**4)) + (a*b**2*sqrt(-
c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2
 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**16*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/
(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 +
 c)**3))**2 + 24*a**14*c**2*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a*
*4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*
a**12*b**2*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c
**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 216*a**12*c**3*(a*b**
2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2
*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 44*a**10*b**2*c**2*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3)) + 568*a**10*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a
**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)
)**2 + 78*a**8*b**4*c - 42*a**8*b**2*c**3*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*
c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)
**3)) + 720*a**8*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*
b**4*c**2 - 8*a**6*b**2*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a
**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a
**6*c**6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2
*a**4*b**2*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(...
```

Giac [A]

time = 3.00, size = 195, normalized size = 1.61

$$-\frac{(3\,a^{2}b^{2}-b^{2}c)\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\,(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3})}+\frac{(3\,a^{2}b^{2}-b^{2}c)\log\left(|x|\right)}{a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}}-\frac{(a^{3}b^{3}-3\,ab^{3}c)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3})b\sqrt{c}}-\frac{a^{4}+2\,a^{2}c+c^{2}-4\,(a^{3}b+abc)x}{2\,(a^{2}+c)^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2*c
^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^4*c
 + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*
c)*x)/((a^2 + c)^3*x^2)
```

Mupad [B]
time = 2.77, size = 573, normalized size = 4.74

$$\ln(x)\left(\frac{3\,b^{2}}{(a^{2}+c)^{2}}-\frac{4\,b^{2}\,c}{(a^{2}+c)^{3}}\right)-\frac{\frac{1}{2\,(a^{2}+c)}-\frac{2\,a\,b\,x}{(a^{2}+c)^{2}}}{x^{2}}-\frac{\ln\left(27\,(-c)^{15/2}+90\,a^{2}\,(-c)^{13/2}+9\,a^{4}\,(-c)^{11/2}-324\,a^{6}\,(-c)^{9/2}+125\,a^{8}\,(-c)^{7/2}+74\,a^{10}\,(-c)^{5/2}-a^{12}\,(-c)^{3/2}-27\,a\,c^{7}+a^{13}\,c+90\,a^{3}\,c^{6}-9\,a^{5}\,c^{5}-324\,a^{7}\,c^{4}-125\,a^{9}\,c^{3}+74\,a^{11}\,c^{2}-27\,b\,c^{7}\,x+a^{12}\,b\,c\,x+90\,a^{2}\,b\,c^{6}\,x-9\,a^{4}\,b\,c^{5}\,x-324\,a^{6}\,b\,c^{4}\,x-125\,a^{8}\,b\,c^{3}\,x+74\,a^{10}\,b\,c^{2}\,x\right)\left(a^{3}\,b^{2}\,\sqrt{-c}-b^{2}\,c^{2}+3\,a^{2}\,b^{2}\,c+3\,a\,b^{2}\,(-c)^{3/2}\right)}{2\,(a^{6}\,c+3\,a^{4}\,c^{2}+3\,a^{2}\,c^{3}+c^{4})}+\frac{\ln\left(27\,(-c)^{15/2}+90\,a^{2}\,(-c)^{13/2}+9\,a^{4}\,(-c)^{11/2}-324\,a^{6}\,(-c)^{9/2}+125\,a^{8}\,(-c)^{7/2}+74\,a^{10}\,(-c)^{5/2}-a^{12}\,(-c)^{3/2}+27\,a\,c^{7}-a^{13}\,c-90\,a^{3}\,c^{6}+9\,a^{5}\,c^{5}+324\,a^{7}\,c^{4}+125\,a^{9}\,c^{3}-74\,a^{11}\,c^{2}+27\,b\,c^{7}\,x-a^{12}\,b\,c\,x-90\,a^{2}\,b\,c^{6}\,x+9\,a^{4}\,b\,c^{5}\,x+324\,a^{6}\,b\,c^{4}\,x+125\,a^{8}\,b\,c^{3}\,x-74\,a^{10}\,b\,c^{2}\,x\right)\left(b^{2}\,c^{2}+a^{3}\,b^{2}\,\sqrt{-c}-3\,a^{2}\,b^{2}\,c+3\,a\,b^{2}\,(-c)^{3/2}\right)}{2\,(a^{6}\,c+3\,a^{4}\,c^{2}+3\,a^{2}\,c^{3}+c^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)*((3*b^2)/(c + a^2)^2 - (4*b^2*c)/(c + a^2)^3) - (1/(2*(c + a^2)) - (
2*a*b*x)/(c + a^2)^2)/x^2 - (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) + 9*a^
4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(5/2
) - a^12*(-c)^(3/2) - 27*a*c^7 + a^13*c + 90*a^3*c^6 - 9*a^5*c^5 - 324*a^7*
c^4 - 125*a^9*c^3 + 74*a^11*c^2 - 27*b*c^7*x + a^12*b*c*x + 90*a^2*b*c^6*x
- 9*a^4*b*c^5*x - 324*a^6*b*c^4*x - 125*a^8*b*c^3*x + 74*a^10*b*c^2*x)*(a^3
*b^2*(-c)^(1/2) - b^2*c^2 + 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*c +
c^4 + 3*a^2*c^3 + 3*a^4*c^2)) + (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) +
9*a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^
(5/2) - a^12*(-c)^(3/2) + 27*a*c^7 - a^13*c - 90*a^3*c^6 + 9*a^5*c^5 + 324*
a^7*c^4 + 125*a^9*c^3 - 74*a^11*c^2 + 27*b*c^7*x - a^12*b*c*x - 90*a^2*b*c^
6*x + 9*a^4*b*c^5*x + 324*a^6*b*c^4*x + 125*a^8*b*c^3*x - 74*a^10*b*c^2*x)*
(b^2*c^2 + a^3*b^2*(-c)^(1/2) - 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*
c + c^4 + 3*a^2*c^3 + 3*a^4*c^2))
```

3.85 $\int \frac{1}{a+b(c+dx)^2} \, dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

```
[Out] arctan((d*x+c)*b^(1/2)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {253, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b(c+dx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} \, dx, x, c+dx\right)}{d} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A]
time = 0.41, size = 34, normalized size = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{2d^2bx+2bcd}{2d\sqrt{ab}}\right)}{d\sqrt{ab}}$	34
risch	$-\frac{\ln\left(bdx+bc+\sqrt{-ab}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(-bdx-bc+\sqrt{-ab}\right)}{2\sqrt{-ab}\,d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 30, normalized size = 0.97

$$\frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="maxima")
[Out] arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*d)
```

Fricas [A]
time = 0.38, size = 109, normalized size = 3.52

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}\,(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{2\,abd},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,(dx+c)}{a}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c
) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqrt(
a*b)*(d*x + c)/a)/(a*b*d)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.
time = 0.08, size = 61, normalized size = 1.97

$$\frac{-\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{-a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2),x)
```

```
[Out] (-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(a*b))*log(
x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

Giac [A]
time = 4.43, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)
```

Mupad [B]
time = 0.06, size = 27, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2),x)
```

```
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

3.86 $\int \frac{1}{\left(a+b(c+dx)^2\right)^2} \, dx$

Optimal. Leaf size=63

$$\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}$$

```
[Out] 1/2*(d*x+c)/a/d/(a+b*(d*x+c)^2)+1/2*arctan((d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)
/d/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{(a+b(c+dx)^2)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{c+dx}{2ad\,(a+b(c+dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{2ad}$$
$$= \frac{c+dx}{2ad\,(a+b(c+dx)^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.95

$$\frac{\frac{\sqrt{a}\,(c+dx)}{a+b(c+dx)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-2),x]
[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[
a]]/Sqrt[b])/(2*a^(3/2)*d)
```

Maple [A]
time = 0.34, size = 86, normalized size = 1.37

method	result	size
default	$\frac{2d^2bx+2bcd}{4d^2ab(b\,d^2x^2+2bcdx+b\,c^2+a)} + \frac{\arctan\left(\frac{2d^2bx+2bcd}{2d\sqrt{ab}}\right)}{2da\sqrt{ab}}$	86
risch	$\frac{\frac{x}{2a}+\frac{c}{2ad}}{b\,d^2x^2+2bcdx+b\,c^2+a} - \frac{\ln\left(bdx+bc+\sqrt{-ab}\right)}{4\sqrt{-ab}\,da} + \frac{\ln\left(-bdx-bc+\sqrt{-ab}\right)}{4\sqrt{-ab}\,da}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(2*b*d^2*x+2*b*c*d)/d^2/a/b/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)
^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 75, normalized size = 1.19

$$\frac{dx+c}{2\,(abd^3x^2+2\,abcd^2x+(abc^2+a^2)d)} + \frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{2\,\sqrt{ab}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d) + 1/2*arcta
n((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d)
```

Fricas [**A**]
time = 0.39, size = 253, normalized size = 4.02

$$\left[\frac{2\,abdx + 2\,abc - (bd^2x^2 + 2\,bcdx + bc^2 + a)\sqrt{-ab}\,\log\left(\frac{bd^2x^2 + 2\,bcdx + bc^2 - 2\,\sqrt{-ab}\,(dx+c) - a}{bd^2x^2 + 2\,bcdx + bc^2 + a}\right)}{4\,(a^2b^2d^3x^2 + 2\,a^2b^2cd^2x + (a^2b^2c^2 + a^3b)d)}, \frac{abdx + abc + (bd^2x^2 + 2\,bcdx + bc^2 + a)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,(dx+c)}{a}\right)}{2\,(a^2b^2d^3x^2 + 2\,a^2b^2cd^2x + (a^2b^2c^2 + a^3b)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)*
log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2
 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2
*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d
^2*x + (a^2*b^2*c^2 + a^3*b)*d)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(51) = 102$.
time = 0.31, size = 117, normalized size = 1.86

$$\frac{c + dx}{2a^2d + 2abc^2d + 4abcd^2x + 2abd^3x^2} + \frac{-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x + \frac{-a^2\sqrt{-\frac{1}{a^3b}} + c}{d}\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(x + \frac{a^2\sqrt{-\frac{1}{a^3b}} + c}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (-
sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)/d)/4 + sqrt(-1/(a**
3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)/4)/d
```

Giac [**A**]
time = 7.48, size = 65, normalized size = 1.03

$$\frac{\arctan\left(\frac{bdx + bc}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ad} + \frac{dx + c}{2\,(bd^2x^2 + 2\,bcdx + bc^2 + a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^2
*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)
```

Mupad [B]
time = 0.10, size = 76, normalized size = 1.21

$$\frac{\frac{x}{2\,a}+\frac{c}{2\,a\,d}}{b\,c^2+2\,b\,c\,d\,x+b\,d^2\,x^2+a}+\frac{\mathrm{atan}\left(2\,a\,\left(\frac{\sqrt{b}\,c}{2\,a^{3/2}}+\frac{\sqrt{b}\,d\,x}{2\,a^{3/2}}\right)\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2)^2,x)
```

```
[Out] (x/(2*a) + c/(2*a*d))/(a + b*c^2 + b*d^2*x^2 + 2*b*c*d*x) + atan(2*a*((b^(1
/2)*c)/(2*a^(3/2)) + (b^(1/2)*d*x)/(2*a^(3/2))))/(2*a^(3/2)*b^(1/2)*d)
```

3.87 $\int \frac{1}{\left(a+b(c+dx)^2\right)^3}\,dx$

Optimal. Leaf size=91

$$\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}+\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}$$

```
[Out] 1/4*(d*x+c)/a/d/(a+b*(d*x+c)^2)^2+3/8*(d*x+c)/a^2/d/(a+b*(d*x+c)^2)+3/8*arc
tan((d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/d/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 205, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*(a + b*(c
+ d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, c+dx\right)}{4ad}$$
$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{8a^2d}$$
$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}$$

Mathematica [A]
time = 0.05, size = 75, normalized size = 0.82

$$\frac{\frac{\sqrt{a}\,(c+dx)\left(5a+3b(c+dx)^2\right)}{\left(a+b(c+dx)^2\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*Arc
Tan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b])/(8*a^(5/2)*d)
```

Maple [A]
time = 0.27, size = 139, normalized size = 1.53

method	result	size
default	$\frac{2d^2bx+2bcd}{8d^2ab(b\,d^2x^2+2bcdx+b\,c^2+a)^2}+\frac{\frac{3\left(2d^2bx+2bcd\right)}{16d^2ab\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)}+\frac{3\arctan\left(\frac{2d^2bx+2bcd}{2d\sqrt{ab}}\right)}{8da\sqrt{ab}}}{a}$	139
risch	$\frac{\frac{3b\,d^2x^3}{8a^2}+\frac{9c\,x^2bd}{8a^2}+\frac{\left(9b\,c^2+5a\right)x}{8a^2}+\frac{c\left(3b\,c^2+5a\right)}{8d\,a^2}}{\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^2}-\frac{3\ln\left(bdx+bc+\sqrt{-ab}\right)}{16\sqrt{-ab}\,d\,a^2}+\frac{3\ln\left(-bdx-bc+\sqrt{-ab}\right)}{16\sqrt{-ab}\,d\,a^2}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*b*d^2*x+2*b*c*d)/d^2/a/b/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3/4/a*(1/4*
(2*b*d^2*x+2*b*c*d)/d^2/a/b/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)^(1/
2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(a*b)^(1/2)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(77) = 154$.
time = 0.50, size = 184, normalized size = 2.02

$$\frac{3\,bd^3x^3 + 9\,bcd^2x^2 + 3\,bc^3 + (9\,bc^2 + 5\,a)dx + 5\,ac}{8\,(a^2b^2d^5x^4 + 4\,a^2b^2cd^4x^3 + 2\,(3\,a^2b^2c^2 + a^3b)d^3x^2 + 4\,(a^2b^2c^3 + a^3bc)d^2x + (a^2b^2c^4 + 2\,a^3bc^2 + a^4)d)} + \frac{3\,\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{8\,\sqrt{ab}\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c)/(
a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2 + a^3*b)*d^3*x^2 +
 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4 + 2*a^3*b*c^2 + a^4)*d) + 3
/8*arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a^2*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(77) = 154$.
time = 0.41, size = 595, normalized size = 6.54

$$\left[\frac{6\,ab^2d^3x^3 + 18\,ab^2cd^2x^2 + 6\,ab^2c^3 + 10\,a^2bc + 2\,(9\,ab^2c^2 + 5\,a^2b)dx - 3\,(b^2d^4x^4 + 4\,b^2cd^3x^3 + b^2c^4 + 2\,(3\,b^2c^2 + ab)d^2x^2 + 2\,abc^2 + 4\,(b^2c^3 + abc)dx + a^2)\sqrt{-ab}\,\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}\,(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{16\,(a^3b^3d^5x^4 + 4\,a^3b^3cd^4x^3 + 2\,(3\,a^3b^3c^2 + a^4b^2)d^3x^2 + 4\,(a^3b^3c^3 + a^4b^2c)d^2x + (a^3b^3c^4 + 2\,a^4b^2c^2 + a^5b)d)}, \frac{3\,ab^2d^3x^3 + 9\,ab^2cd^2x^2 + 3\,ab^2c^3 + 5\,a^2bc + (9\,ab^2c^2 + 5\,a^2b)dx + 3\,(b^2d^4x^4 + 4\,b^2cd^3x^3 + b^2c^4 + 2\,(3\,b^2c^2 + ab)d^2x^2 + 2\,abc^2 + 4\,(b^2c^3 + abc)dx + a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,(dx+c)}{a}\right)}{8\,(a^3b^3d^5x^4 + 4\,a^3b^3cd^4x^3 + 2\,(3\,a^3b^3c^2 + a^4b^2)d^3x^2 + 4\,(a^3b^3c^3 + a^4b^2c)d^2x + (a^3b^3c^4 + 2\,a^4b^2c^2 + a^5b)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2*
(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 +
2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sq
rt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(
b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3
 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x
+ (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9*a*b^2*
c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x + 3*(b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c
^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)
)/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*
x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^
5*b)*d)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(82) = 164$.
time = 0.70, size = 257, normalized size = 2.82

$$\frac{5ac + 3bc^3 + 9bcd^2x^2 + 3bd^3x^3 + x(5ad + 9bc^2d)}{8a^4d + 16a^3bc^2d + 8a^2b^2c^4d + 32a^2b^2cd^4x^3 + 8a^2b^2d^5x^4 + x^2 \cdot (16a^3bd^3 + 48a^2b^2c^2d^3) + x\,(32a^3bcd^2 + 32a^2b^2c^3d^2)} + \frac{-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x + \frac{-3a^3\sqrt{-\frac{1}{a^5b}} + 3c}{3d}\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x + \frac{3a^3\sqrt{-\frac{1}{a^5b}} + 3c}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**3,x)
```

```
[Out] (5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d + 9*b*c**2*d
))/(8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d + 32*a**2*b**2*c*d**4*
x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*b*d**3 + 48*a**2*b**2*c**2*d**
3) + x*(32*a**3*b*c*d**2 + 32*a**2*b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5*b))
*log(x + (-3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16 + 3*sqrt(-1/(a**5*b))*
log(x + (3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16)/d
```

Giac [A]
time = 6.40, size = 103, normalized size = 1.13

$$\frac{3\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^2d}+\frac{3\,bd^3x^3+9\,bcd^2x^2+9\,bc^2dx+3\,bc^3+5\,adx+5\,ac}{8\left(bd^2x^2+2\,bcdx+bc^2+a\right)^2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*d^3*x^3 +
9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c)/((b*d^2*x^2 + 2*b*
c*d*x + b*c^2 + a)^2*a^2*d)
```

Mupad [B]
time = 2.22, size = 181, normalized size = 1.99

$$\frac{\frac{x\,\left(9\,b\,c^2+5\,a\right)}{8\,a^2}+\frac{3\,b\,c^3+5\,a\,c}{8\,a^2\,d}+\frac{3\,b\,d^2\,x^3}{8\,a^2}+\frac{9\,b\,c\,d\,x^2}{8\,a^2}}{x^2\,\left(6\,b^2\,c^2\,d^2+2\,a\,b\,d^2\right)+x\,\left(4\,d\,b^2\,c^3+4\,a\,d\,b\,c\right)+a^2+b^2\,c^4+b^2\,d^4\,x^4+2\,a\,b\,c^2+4\,b^2\,c\,d^3\,x^3}+\frac{3\,\mathrm{atan}\left(\frac{8\,a^2\,\left(\frac{3\,\sqrt{b}\,c}{8\,a^{5/2}}+\frac{3\,\sqrt{b}\,d\,x}{8\,a^{5/2}}\right)}{3}\right)}{8\,a^{5/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2)^3,x)
```

```
[Out] ((x*(5*a + 9*b*c^2))/(8*a^2) + (5*a*c + 3*b*c^3)/(8*a^2*d) + (3*b*d^2*x^3)/
(8*a^2) + (9*b*c*d*x^2)/(8*a^2))/(x^2*(6*b^2*c^2*d^2 + 2*a*b*d^2) + x*(4*b^
2*c^3*d + 4*a*b*c*d) + a^2 + b^2*c^4 + b^2*d^4*x^4 + 2*a*b*c^2 + 4*b^2*c*d^
3*x^3) + (3*atan((8*a^2*((3*b^(1/2)*c)/(8*a^(5/2)) + (3*b^(1/2)*d*x)/(8*a^(
5/2))))/3))/(8*a^(5/2)*b^(1/2)*d)
```

3.88 $\int \frac{1}{\sqrt{-a}+b(c+dx)^2} \, dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\,\sqrt{b}\,d}$$

```
[Out] arctan((d*x+c)*b^(1/2)/(-a)^(1/4))/(-a)^(1/4)/d/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {253, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-a}+b(c+dx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+bx^2} \, dx, x, c+dx\right)}{d} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\,\sqrt{b}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Maple [A]
time = 0.30, size = 42, normalized size = 1.20

method	result	size
default	$\frac{\arctan\left(\frac{2d^2bx+2bcd}{2d\sqrt{\sqrt{-a}\,b}}\right)}{d\sqrt{\sqrt{-a}\,b}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(d*x+c)^2+(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/((-a)^(1/2)*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/((-a)^(1/2)*b)^(1
/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.
time = 0.48, size = 66, normalized size = 1.89

$$\frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}\,b}\,d}{bd^2x+bcd+\sqrt{-\sqrt{-a}\,b}\,d}\right)}{2\sqrt{-\sqrt{-a}\,b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-sq
rt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)
```

Fricas [A]
time = 0.44, size = 279, normalized size = 7.97

$$\left[\frac{\sqrt{\frac{\sqrt{-a}}{ab}}\log\left(\frac{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4-2\,(bd^2x^2+2\,bcdx+bc^2)\sqrt{-a}+2\left(abdx+abc+(b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b^2c^3)\sqrt{-a}\right)\sqrt{\frac{\sqrt{-a}}{ab}}-a}{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4+a}\right)}{2\,d},\ \frac{\sqrt{-\frac{\sqrt{-a}}{ab}}\arctan\left((bdx+bc)\sqrt{-\frac{\sqrt{-a}}{ab}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^
2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-a
) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/(a*b)
)*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(31) = 62$.
time = 0.08, size = 92, normalized size = 2.63

$$\frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c-\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c+\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

```
[Out] (-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2 +
 sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2)/d
```

Giac [A]
time = 4.83, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)
```

Mupad [B]
time = 0.10, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c + d*x)^2 + (-a)^(1/2)),x)
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/(-a)^(1/4))/((-a)^(1/4)*b^(1/2)*d)
```

3.89 $\int \frac{1}{1+(c+dx)^2} dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}(c+dx)}{d}$$

```
[Out] arctan(d*x+c)/d
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {253, 209}

$$\frac{\text{ArcTan}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+(c+dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

Maple [A]
time = 0.28, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\arctan(dx+c)}{d}$	11
risch	$\frac{\arctan(dx+c)}{d}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(d*x+c)/d
```

Maxima [A]
time = 0.47, size = 18, normalized size = 1.80

$$\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan((d^2*x + c*d)/d)/d
```

Fricas [A]
time = 0.38, size = 10, normalized size = 1.00

$$\frac{\arctan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] arctan(d*x + c)/d
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 24, normalized size = 2.40

$$\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{2}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2),x)
```

```
[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d
```

Giac [A]
time = 5.84, size = 10, normalized size = 1.00

$$\frac{\arctan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(d*x + c)/d
```

Mupad [B]
time = 0.04, size = 10, normalized size = 1.00

$$\frac{\mathrm{atan}(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1),x)
```

```
[Out] atan(c + d*x)/d
```

3.90 $\int \frac{1}{(1+(c+dx)^2)^2}\,dx$

Optimal. Leaf size=37

$$\frac{c+dx}{2d\,(1+(c+dx)^2)}+\frac{\tan^{-1}(c+dx)}{2d}$$

```
[Out] 1/2*(d*x+c)/d/(1+(d*x+c)^2)+1/2*arctan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 205, 209}

$$\frac{\text{ArcTan}(c+dx)}{2d}+\frac{c+dx}{2d\,((c+dx)^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{c+dx}{2d\,(1+(c+dx)^2)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{2d}\\
&= \frac{c+dx}{2d\,(1+(c+dx)^2)}+\frac{\tan^{-1}(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.84

$$\frac{\frac{c+dx}{1+(c+dx)^2}+\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)
```

Maple [A]
time = 0.27, size = 59, normalized size = 1.59

method	result	size
risch	$\frac{\frac{x}{2}+\frac{c}{2d}}{d^2x^2+2cdx+c^2+1}+\frac{\arctan(dx+c)}{2d}$	43
default	$\frac{2d^2x+2cd}{4d^2(d^2x^2+2cdx+c^2+1)}+\frac{\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{2d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+1/2/d*arctan(1/2*(2*d^2*x+2
*c*d)/d)
```

Maxima [A]
time = 0.49, size = 51, normalized size = 1.38

$$\frac{dx+c}{2\left(d^3x^2+2\,cd^2x+(c^2+1)d\right)}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d^2*x + c*d
)/d)/d
```

Fricas [A]
time = 0.38, size = 55, normalized size = 1.49

$$\frac{dx + (d^2x^2 + 2\,cdx + c^2 + 1)\arctan\left(dx + c\right) + c}{2\left(d^3x^2 + 2\,cd^2x + (c^2 + 1)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^3*x^2 + 2*
c*d^2*x + (c^2 + 1)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.21, size = 56, normalized size = 1.51

$$\frac{c + dx}{2c^2d + 4cd^2x + 2d^3x^2 + 2d} + \frac{-\frac{i\log\left(x + \frac{c-i}{d}\right)}{4} + \frac{i\log\left(x + \frac{c+i}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*log(x + (c - I)
/d)/4 + I*log(x + (c + I)/d)/4)/d
```

Giac [A]
time = 4.14, size = 41, normalized size = 1.11

$$\frac{\arctan\left(dx + c\right)}{2\,d} + \frac{dx + c}{2\left(d^2x^2 + 2\,cdx + c^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)
```

Mupad [B]
time = 2.07, size = 42, normalized size = 1.14

$$\frac{\frac{x}{2} + \frac{c}{2\,d}}{c^2 + 2\,c\,d\,x + d^2\,x^2 + 1} + \frac{\mathrm{atan}(c + d\,x)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1)^2,x)
```

```
[Out] (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x + 1) + atan(c + d*x)/(2*d)
```

3.91 $\int \frac{1}{\left(1+(c+dx)^2\right)^3} \, dx$

Optimal. Leaf size=60

$$\frac{c+dx}{4d\left(1+(c+dx)^2\right)^2}+\frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)}+\frac{3\tan^{-1}(c+dx)}{8d}$$

```
[Out] 1/4*(d*x+c)/d/(1+(d*x+c)^2)^2+3/8*(d*x+c)/d/(1+(d*x+c)^2)+3/8*arctan(d*x+c)
/d
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {253, 205, 209}

$$\frac{3\text{ArcTan}(c+dx)}{8d}+\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2))
 + (3*ArcTan[c + d*x])/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{(1+(c+dx)^2)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{c+dx}{4d\,(1+(c+dx)^2)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, c+dx\right)}{4d}$$
$$= \frac{c+dx}{4d\,(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d\,(1+(c+dx)^2)} + \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{8d}$$
$$= \frac{c+dx}{4d\,(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d\,(1+(c+dx)^2)} + \frac{3\tan^{-1}(c+dx)}{8d}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.87

$$\frac{\frac{2(c+dx)}{(1+(c+dx)^2)^2} + \frac{3(c+dx)}{1+(c+dx)^2} + 3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*Ar
cTan[c + d*x])/(8*d)
```

Maple [A]
time = 0.26, size = 94, normalized size = 1.57

method	result	size
risch	$\frac{\frac{3x^3d^2}{8}+\frac{9cd\,x^2}{8}+\left(\frac{9c^2}{8}+\frac{5}{8}\right)x+\frac{c(3c^2+5)}{8d}}{(d^2x^2+2cdx+c^2+1)^2}+\frac{3\arctan(dx+c)}{8d}$	71
default	$\frac{2d^2x+2cd}{8d^2(d^2x^2+2cdx+c^2+1)^2}+\frac{\frac{3}{8}d^2x+\frac{3}{8}cd}{d^2(d^2x^2+2cdx+c^2+1)}+\frac{3\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{8d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)^2+3/16*(2*d^2*x+2*c*d)/d^2/
(d^2*x^2+2*c*d*x+c^2+1)+3/8/d*arctan(1/2*(2*d^2*x+2*c*d)/d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(54) = 108$.

time = 0.48, size = 115, normalized size = 1.92

$$\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+(9\,c^2+5)dx+5\,c}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2+1)d^3x^2+4\,(c^3+c)d^2x+(c^4+2\,c^2+1)d)}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^5*x^4 + 4*
c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d
) + 3/8*arctan((d^2*x + c*d)/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.
time = 0.40, size = 153, normalized size = 2.55

$$\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+(9\,c^2+5)dx+3\,(d^4x^4+4\,cd^3x^3+2\,(3\,c^2+1)d^2x^2+c^4+4\,(c^3+c)dx+2\,c^2+1)\arctan\left(dx+c\right)+5\,c}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2+1)d^3x^2+4\,(c^3+c)d^2x+(c^4+2\,c^2+1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*d
^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*arctan(
d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c
)*d^2*x + (c^4 + 2*c^2 + 1)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 146, normalized size = 2.43

$$\frac{3c^3+9cd^2x^2+5c+3d^3x^3+x(9c^2d+5d)}{8c^4d+16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\cdot(48c^2d^3+16d^3)+x\,(32c^3d^2+32cd^2)}+\frac{-\frac{3i\log\left(x+\frac{3c-3i}{3d}\right)}{16}+\frac{3i\log\left(x+\frac{3c+3i}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**3,x)
```

```
[Out] (3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))/(8*c**4*d
 + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 + 16
*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*I*log(x + (3*c - 3*I)/(3*d))/1
6 + 3*I*log(x + (3*c + 3*I)/(3*d))/16)/d
```

Giac [A]
time = 3.10, size = 73, normalized size = 1.22

$$\frac{3\,\arctan\left(dx+c\right)}{8\,d}+\frac{3\,d^3x^3+9\,cd^2x^2+9\,c^2dx+3\,c^3+5\,dx+5\,c}{8\,(d^2x^2+2\,cdx+c^2+1)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 +
5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)
```

Mupad [B]
time = 0.12, size = 111, normalized size = 1.85

$$\frac{3\,\mathrm{atan}(c+d\,x)}{8\,d}+\frac{x\,\left(\frac{9\,c^2}{8}+\frac{5}{8}\right)+\frac{3\,c^3+5\,c}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{x^2\,\left(6\,c^2\,d^2+2\,d^2\right)+2\,c^2+c^4+x\,\left(4\,d\,c^3+4\,d\,c\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1)^3,x)
```

```
[Out] (3*atan(c + d*x))/(8*d) + (x*((9*c^2)/8 + 5/8) + (5*c + 3*c^3)/(8*d) + (3*d
^2*x^3)/8 + (9*c*d*x^2)/8)/(x^2*(2*d^2 + 6*c^2*d^2) + 2*c^2 + c^4 + x*(4*c*
d + 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

3.92 $\int \frac{1}{1-(c+dx)^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}(c+dx)}{d}$$

```
[Out] arctanh(d*x+c)/d
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {253, 212}

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTanh[c + d*x]/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(c+dx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, c+dx\right)}{d} \\ &= \frac{\tanh^{-1}(c+dx)}{d} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.
time = 0.00, size = 32, normalized size = 3.20

$$-\frac{\log(1-c-dx)}{2d} + \frac{\log(1+c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] -1/2*Log[1 - c - d*x]/d + Log[1 + c + d*x]/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 0.30, size = 26, normalized size = 2.60

method	result	size
default	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(dx+c+1)}{2d}$	26
norman	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(dx+c+1)}{2d}$	26
risch	$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(-dx-c-1)}{2d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*ln(d*x+c-1)+1/2/d*ln(d*x+c+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 0.26, size = 25, normalized size = 2.50

$$\frac{\log\left(dx+c+1\right)}{2\,d}-\frac{\log\left(dx+c-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.34, size = 22, normalized size = 2.20

$$\frac{\log\left(dx+c+1\right)-\log\left(dx+c-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.
time = 0.07, size = 22, normalized size = 2.20

$$-\frac{\frac{\log\left(x+\frac{c-1}{d}\right)}{2}-\frac{\log\left(x+\frac{c+1}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2),x)
```

```
[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 4.35, size = 27, normalized size = 2.70

$$\frac{\log\left(|dx+c+1|\right)}{2\,d}-\frac{\log\left(|dx+c-1|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d
```

Mupad [B]
time = 2.05, size = 10, normalized size = 1.00

$$\frac{\operatorname{atanh}(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((c + d*x)^2 - 1),x)
```

```
[Out] atanh(c + d*x)/d
```

3.93 $\int \frac{1}{\left(1-(c+dx)^2\right)^2} dx$

Optimal. Leaf size=39

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

```
[Out] 1/2*(d*x+c)/d/(1-(d*x+c)^2)+1/2*arctanh(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 205, 212}

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{(1-(c+dx)^2)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, c+dx\right)}{d}$$
$$= \frac{c+dx}{2d\,(1-(c+dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{2d}$$
$$= \frac{c+dx}{2d\,(1-(c+dx)^2)} + \frac{\tanh^{-1}(c+dx)}{2d}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.15

$$\frac{-\frac{2(c+dx)}{-1+(c+dx)^2} - \log(1-c-dx) + \log(1+c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/(
4*d)
```

Maple [A]
time = 0.25, size = 52, normalized size = 1.33

method	result	size
default	$-\frac{1}{4d(dx+c-1)} - \frac{\ln(dx+c-1)}{4d} - \frac{1}{4d(dx+c+1)} + \frac{\ln(dx+c+1)}{4d}$	52
norman	$\frac{-\frac{c}{2d}-\frac{x}{2}}{d^2x^2+2cdx+c^2-1} - \frac{\ln(dx+c-1)}{4d} + \frac{\ln(dx+c+1)}{4d}$	56
risch	$\frac{-\frac{c}{2d}-\frac{x}{2}}{d^2x^2+2cdx+c^2-1} - \frac{\ln(dx+c-1)}{4d} + \frac{\ln(-dx-c-1)}{4d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c+1)
```

Maxima [A]
time = 0.27, size = 56, normalized size = 1.44

$$-\frac{dx+c}{2\left(d^3x^2+2\,cd^2x+(c^2-1)d\right)} + \frac{\log\left(dx+c+1\right)}{4\,d} - \frac{\log\left(dx+c-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x + c + 1)/d
 - 1/4*log(d*x + c - 1)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(33) = 66$.
time = 0.39, size = 85, normalized size = 2.18

$$-\frac{2\,dx - (d^2x^2 + 2\,cdx + c^2 - 1)\log\left(dx + c + 1\right) + (d^2x^2 + 2\,cdx + c^2 - 1)\log\left(dx + c - 1\right) + 2\,c}{4\,(d^3x^2 + 2\,cd^2x + (c^2 - 1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*
d)
```

Sympy [A]
time = 0.22, size = 54, normalized size = 1.38

$$\frac{-c - dx}{2c^2d + 4cd^2x + 2d^3x^2 - 2d} + \frac{-\frac{\log\left(x + \frac{c-1}{d}\right)}{4} + \frac{\log\left(x + \frac{c+1}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**2,x)
```

```
[Out] (-c - d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x + (c - 1)/
d)/4 + log(x + (c + 1)/d)/4)/d
```

Giac [A]
time = 3.63, size = 56, normalized size = 1.44

$$\frac{\log\left(|dx + c + 1|\right)}{4\,d} - \frac{\log\left(|dx + c - 1|\right)}{4\,d} - \frac{dx + c}{2\,(d^2x^2 + 2\,cdx + c^2 - 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(d*x + c + 1))/d - 1/4*log(abs(d*x + c - 1))/d - 1/2*(d*x + c)/(
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
```

Mupad [B]
time = 2.06, size = 43, normalized size = 1.10

$$\frac{\mathrm{atanh}(c + d\,x)}{2\,d} - \frac{\frac{x}{2} + \frac{c}{2\,d}}{c^2 + 2\,c\,d\,x + d^2\,x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 - 1)^2,x)
```

```
[Out] atanh(c + d*x)/(2*d) - (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x - 1)
```

3.94 $\int \frac{1}{\left(1-(c+dx)^2\right)^3} \, dx$

Optimal. Leaf size=64

$$\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

```
[Out] 1/4*(d*x+c)/d/(1-(d*x+c)^2)^2+3/8*(d*x+c)/d/(1-(d*x+c)^2)+3/8*arctanh(d*x+c
)/d
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 205, 212}

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2))
 + (3*ArcTanh[c + d*x])/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{(1-(c+dx)^2)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3}\,dx, x, c+dx\right)}{d}$$
$$= \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, c+dx\right)}{4d}$$
$$= \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{8d}$$
$$= \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{3\tanh^{-1}(c+dx)}{8d}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.02

$$\frac{\frac{4(c+dx)}{(-1+(c+dx)^2)^2} - \frac{6(c+dx)}{-1+(c+dx)^2} - 3\log(1-c-dx) + 3\log(1+c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-3),x]
[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3*
Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)
```

Maple [A]
time = 0.31, size = 78, normalized size = 1.22

method	result	size
default	$\frac{1}{16d(dx+c-1)^2} - \frac{3}{16d(dx+c-1)} - \frac{3\ln(dx+c-1)}{16d} - \frac{1}{16d(dx+c+1)^2} - \frac{3}{16d(dx+c+1)} + \frac{3\ln(dx+c+1)}{16d}$	78
risch	$\frac{-\frac{3x^3d^2}{8} - \frac{9cd\,x^2}{8} + \left(-\frac{9c^2}{8}+\frac{5}{8}\right)x - \frac{c\left(3c^2-5\right)}{8d}}{\left(d^2x^2+2cdx+c^2-1\right)^2} + \frac{3\ln(-dx-c-1)}{16d} - \frac{3\ln(dx+c-1)}{16d}$	87
norman	$\frac{\frac{-3c^3d^3+5c\,d^3}{8d^4} + \frac{\left(-9c^2d^3+5d^3\right)x}{8d^3} - \frac{3x^3d^2}{8} - \frac{9cd\,x^2}{8}}{\left(d^2x^2+2cdx+c^2-1\right)^2} - \frac{3\ln(dx+c-1)}{16d} + \frac{3\ln(dx+c+1)}{16d}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
[Out] 1/16/d/(d*x+c-1)^2-3/16/d/(d*x+c-1)-3/16/d*ln(d*x+c-1)-1/16/d/(d*x+c+1)^2-3
/16/d/(d*x+c+1)+3/16/d*ln(d*x+c+1)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(54) = 108$.
time = 0.30, size = 122, normalized size = 1.91

$$-\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+(9\,c^2-5)dx-5\,c}{8\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2-1)d^3x^2+4\,(c^3-c)d^2x+(c^4-2\,c^2+1)d)}+\frac{3\,\log\left(dx+c+1\right)}{16\,d}-\frac{3\,\log\left(dx+c-1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 + 4
*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*
d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x + c - 1)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(54) = 108$.
time = 0.39, size = 220, normalized size = 3.44

$$-\frac{6\,d^3x^3+18\,cd^2x^2+6\,c^3+2\,(9\,c^2-5)dx-3\,(d^4x^4+4\,cd^3x^3+2\,(3\,c^2-1)d^2x^2+c^4+4\,(c^3-c)dx-2\,c^2+1)\log\left(dx+c+1\right)+3\,(d^4x^4+4\,cd^3x^3+2\,(3\,c^2-1)d^2x^2+c^4+4\,(c^3-c)dx-2\,c^2+1)\log\left(dx+c-1\right)-10\,c}{16\,(d^5x^4+4\,cd^4x^3+2\,(3\,c^2-1)d^3x^2+4\,(c^3-c)d^2x+(c^4-2\,c^2+1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 +
4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*lo
g(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4
*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3
 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(53) = 106$.
time = 0.50, size = 141, normalized size = 2.20

$$-\frac{3c^3+9cd^2x^2-5c+3d^3x^3+x(9c^2d-5d)}{8c^4d-16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\cdot(48c^2d^3-16d^3)+x\,(32c^3d^2-32cd^2)}-\frac{\frac{3\log\left(x+\frac{3c-3}{3d}\right)}{16}-\frac{3\log\left(x+\frac{3c+3}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**3,x)
```

```
[Out] -(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d))/(8*c**4*
d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 - 1
6*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*log(x + (3*c - 3)/(3*d))/16 -
3*log(x + (3*c + 3)/(3*d))/16)/d
```

Giac [**A**]
time = 4.20, size = 88, normalized size = 1.38

$$\frac{3\,\log\left(|dx+c+1|\right)}{16\,d}-\frac{3\,\log\left(|dx+c-1|\right)}{16\,d}-\frac{3\,d^3x^3+9\,cd^2x^2+9\,c^2dx+3\,c^3-5\,dx-5\,c}{8\,(d^2x^2+2\,cdx+c^2-1)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(d*x + c + 1))/d - 3/16*log(abs(d*x + c - 1))/d - 1/8*(3*d^3*x^
3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x + c^
2 - 1)^2*d)
```

Mupad [B]
time = 2.12, size = 114, normalized size = 1.78

$$\frac{3\,\mathrm{atanh}(c+d\,x)}{8\,d}-\frac{x\,\left(\frac{9\,c^2}{8}-\frac{5}{8}\right)-\frac{5\,c-3\,c^3}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{c^4-2\,c^2-x^2\,\left(2\,d^2-6\,c^2\,d^2\right)-x\,\left(4\,c\,d-4\,c^3\,d\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((c + d*x)^2 - 1)^3,x)
```

```
[Out] (3*atanh(c + d*x))/(8*d) - (x*((9*c^2)/8 - 5/8) - (5*c - 3*c^3)/(8*d) + (3*
d^2*x^3)/8 + (9*c*d*x^2)/8)/(c^4 - 2*c^2 - x^2*(2*d^2 - 6*c^2*d^2) - x*(4*c
*d - 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

3.95 $\int \frac{1}{1-(1+x)^2} dx$

Optimal. Leaf size=4

$$\tanh^{-1}(1+x)$$

[Out] arctanh(1+x)

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {253, 212}

$$\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] ArcTanh[1 + x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(1+x)^2} dx &= \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, 1+x\right) \\ &= \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 15 vs. 2(4) = 8.
time = 0.00, size = 15, normalized size = 3.75

$$-\frac{\log(x)}{2} + \frac{1}{2}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] -1/2*Log[x] + Log[2 + x]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.
time = 0.23, size = 12, normalized size = 3.00

method	result	size
default	$\frac{\ln(x+2)}{2} - \frac{\ln(x)}{2}$	12
norman	$\frac{\ln(x+2)}{2} - \frac{\ln(x)}{2}$	12
risch	$\frac{\ln(x+2)}{2} - \frac{\ln(x)}{2}$	12
meijerg	$\frac{\ln(1+\frac{x}{2})}{2} - \frac{\ln(x)}{2} + \frac{\ln(2)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x+2)-1/2*ln(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.
time = 0.28, size = 11, normalized size = 2.75

$$\frac{1}{2}\log(x+2) - \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(4) = 8$.
time = 0.36, size = 11, normalized size = 2.75

$$\frac{1}{2}\log(x+2) - \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(3) = 6$.
time = 0.03, size = 10, normalized size = 2.50

$$-\frac{\log(x)}{2} + \frac{\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2),x)
```

```
[Out] -log(x)/2 + log(x + 2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(4) = 8$.
time = 2.64, size = 13, normalized size = 3.25

$$\frac{1}{2}\log\left(|x+2|\right) - \frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 2)) - 1/2*log(abs(x))
```

Mupad [B]
time = 0.15, size = 4, normalized size = 1.00

$$\operatorname{atanh}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^2 - 1),x)
```

```
[Out] atanh(x + 1)
```

3.96 $\int \frac{1}{\left(1-(1+x)^2\right)^2}\,dx$

Optimal. Leaf size=27

$$\frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\tanh^{-1}(1+x)$$

[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 205, 212}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{(1-(1+x)^2)^2} \, dx = \text{Subst}\left(\int \frac{1}{(1-x^2)^2} \, dx, x, 1+x\right)$$
$$= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, 1+x\right)$$
$$= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2}\tanh^{-1}(1+x)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)} - \log(x) + \log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A]
time = 0.19, size = 24, normalized size = 0.89

method	result	size
default	$-\frac{1}{4(x+2)}+\frac{\ln(x+2)}{4}-\frac{1}{4x}-\frac{\ln(x)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
meijerg	$\frac{3x}{16(3+\frac{3x}{2})}+\frac{\ln(1+\frac{x}{2})}{4}-\frac{1}{8}-\frac{\ln(x)}{4}+\frac{\ln(2)}{4}-\frac{1}{4x}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(x+2)+1/4*ln(x+2)-1/4/x-1/4*ln(x)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.93

$$-\frac{x+1}{2(x^2+2x)} + \frac{1}{4}\log(x+2) - \frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Fricas [A]
time = 0.37, size = 39, normalized size = 1.44

$$\frac{(x^2+2\,x)\log\left(x+2\right)-(x^2+2\,x)\log\left(x\right)-2\,x-2}{4\,(x^2+2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x}-\frac{\log\left(x\right)}{4}+\frac{\log\left(x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**2,x)
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Giac [A]
time = 3.43, size = 27, normalized size = 1.00

$$-\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\log\left(|x+2|\right)-\frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

Mupad [B]
time = 0.07, size = 23, normalized size = 0.85

$$\frac{\mathrm{atanh}(x+1)}{2}-\frac{x+1}{2\,\left({(x+1)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^2 - 1)^2,x)
[Out] atanh(x + 1)/2 - (x + 1)/(2*((x + 1)^2 - 1))
```

3.97 $\int \frac{1}{\left(1-(1+x)^2\right)^3} \, dx$

Optimal. Leaf size=45

$$\frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3(1+x)}{8\left(1-(1+x)^2\right)}+\frac{3}{8}\tanh^{-1}(1+x)$$

```
[Out] 1/4*(1+x)/(1-(1+x)^2)^2+3/8*(1+x)/(1-(1+x)^2)+3/8*arctanh(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 205, 212}

$$\frac{3(x+1)}{8\left(1-(x+1)^2\right)}+\frac{x+1}{4\left(1-(x+1)^2\right)^2}+\frac{3}{8}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTan
h[1 + x])/8
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{(1-(1+x)^2)^3}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)^3}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{4(1-(1+x)^2)^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{4(1-(1+x)^2)^2} + \frac{3(1+x)}{8(1-(1+x)^2)} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{4(1-(1+x)^2)^2} + \frac{3(1+x)}{8(1-(1+x)^2)} + \frac{3}{8}\tanh^{-1}(1+x)$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.82

$$\frac{1}{16}\left(\frac{1}{x^2} - \frac{3}{x} - \frac{1}{(2+x)^2} - \frac{3}{2+x} - 3\log(x) + 3\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-3),x]
[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16
```

Maple [A]
time = 0.20, size = 36, normalized size = 0.80

method	result	size
default	$-\frac{1}{16(x+2)^2}-\frac{3}{16(x+2)}+\frac{3\ln(x+2)}{16}+\frac{1}{16x^2}-\frac{3}{16x}-\frac{3\ln(x)}{16}$	36
norman	$\frac{\frac{1}{4}-\frac{9}{8}x^2-\frac{3}{8}x^3-\frac{1}{2}x}{x^2(x+2)^2}-\frac{3\ln(x)}{16}+\frac{3\ln(x+2)}{16}$	36
risch	$\frac{\frac{1}{4}-\frac{9}{8}x^2-\frac{3}{8}x^3-\frac{1}{2}x}{x^2(x+2)^2}-\frac{3\ln(x)}{16}+\frac{3\ln(x+2)}{16}$	36
meijerg	$\frac{x\left(\frac{7x}{2}+8\right)}{128\left(1+\frac{x}{2}\right)^2}+\frac{3\ln\left(1+\frac{x}{2}\right)}{16}-\frac{7}{64}-\frac{3\ln(x)}{16}+\frac{3\ln(2)}{16}+\frac{1}{16x^2}-\frac{3}{16x}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/16/(x+2)^2-3/16/(x+2)+3/16*ln(x+2)+1/16/x^2-3/16/x-3/16*ln(x)
```

Maxima [A]
time = 0.29, size = 44, normalized size = 0.98

$$-\frac{3\,x^3+9\,x^2+4\,x-2}{8\,(x^4+4\,x^3+4\,x^2)}+\frac{3}{16}\,\log\left(x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x + 2) - 3/
16*log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(35) = 70$.
time = 0.39, size = 71, normalized size = 1.58

$$-\frac{6\,x^3+18\,x^2-3\,(x^4+4\,x^3+4\,x^2)\log{(x+2)}+3\,(x^4+4\,x^3+4\,x^2)\log{(x)}+8\,x-4}{16\,(x^4+4\,x^3+4\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x^4 + 4*x^3
 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)
```

Sympy [A]
time = 0.06, size = 44, normalized size = 0.98

$$-\frac{3\log{(x)}}{16}+\frac{3\log{(x+2)}}{16}-\frac{3x^3+9x^2+4x-2}{8x^4+32x^3+32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**3,x)
```

```
[Out] -3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*x
**3 + 32*x**2)
```

Giac [A]
time = 4.63, size = 39, normalized size = 0.87

$$-\frac{3\,x^3+9\,x^2+4\,x-2}{8\,(x^2+2\,x)^2}+\frac{3}{16}\,\log{(|x+2|)}-\frac{3}{16}\,\log{(|x|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*log(abs(x + 2)) - 3/16*
log(abs(x))
```

Mupad [B]
time = 2.09, size = 36, normalized size = 0.80

$$\frac{3\,\mathrm{atanh}(x+1)}{8}+\frac{\frac{5\,x}{8}-\frac{3\,(x+1)^3}{8}+\frac{5}{8}}{(x+1)^4-2\,(x+1)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^2 - 1)^3,x)
```

```
[Out] (3*atanh(x + 1))/8 + ((5*x)/8 - (3*(x + 1)^3)/8 + 5/8)/((x + 1)^4 - 2*(x +
1)^2 + 1)
```

3.98 $\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx$

Optimal. Leaf size=59

$$a\left(2+a^2\right)bx+\frac{1}{2}\left(2+a^2\right)(a+bx)^2+\frac{1}{3}a(a+bx)^3+\frac{1}{4}(a+bx)^4+\left(1+a^2\right)^2\log(x)$$

```
[Out] a*(a^2+2)*b*x+1/2*(a^2+2)*(b*x+a)^2+1/3*a*(b*x+a)^3+1/4*(b*x+a)^4+(a^2+1)^2
*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {378, 711}

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)
^4/4 + (1 + a^2)^2*Log[x]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx&=\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{-a+x}\,dx,x,a+bx\right)\\&=\text{Subst}\left(\int \left(a\left(2+a^2\right)-\frac{\left(1+a^2\right)^2}{a-x}+\left(2+a^2\right)x+ax^2+x^3\right)dx,x,a+bx\right)\\&=a\left(2+a^2\right)bx+\frac{1}{2}\left(2+a^2\right)(a+bx)^2+\frac{1}{3}a(a+bx)^3+\frac{1}{4}(a+bx)^4+\left(1+a^2\right)^2\log(x\end{aligned}$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 1.08

$$a(2+a^2)(a+bx)+\frac{1}{2}(2+a^2)(a+bx)^2+\frac{1}{3}a(a+bx)^3+\frac{1}{4}(a+bx)^4+(1+a^2)^2\log(bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
+ b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

Maple [A]
time = 0.25, size = 62, normalized size = 1.05

method	result	size
norman	$(3a^2b^2+b^2)x^2+(4a^3b+4ab)x+\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+(a^4+2a^2+1)\ln(x)$	61
default	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+4a^3bx+b^2x^2+4abx+(a^4+2a^2+1)\ln(x)$	62
risch	$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+b^2x^2+4a^3bx+4abx+a^4\ln(x)+2a^2\ln(x)+\ln(x)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b*x+a)^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^4*x^4+4/3*a*b^3*x^3+3*a^2*b^2*x^2+4*a^3*b*x+b^2*x^2+4*a*b*x+(a^4+2*a^
2+1)*ln(x)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.92

$$\frac{1}{4}b^4x^4+\frac{4}{3}ab^3x^3+(3a^2+1)b^2x^2+4(a^3+a)bx+(a^4+2a^2+1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.92

$$\frac{1}{4}b^4x^4+\frac{4}{3}ab^3x^3+(3a^2+1)b^2x^2+4(a^3+a)bx+(a^4+2a^2+1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

Sympy [A]
time = 0.06, size = 58, normalized size = 0.98

$$\frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2 \cdot \left(3a^2b^2 + b^2\right) + x\left(4a^3b + 4ab\right) + \left(a^2 + 1\right)^2 \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b + 4
*a*b) + (a**2 + 1)**2*log(x)
```

Giac [A]
time = 5.95, size = 62, normalized size = 1.05

$$\frac{1}{4}\,b^4x^4 + \frac{4}{3}\,ab^3x^3 + 3\,a^2b^2x^2 + 4\,a^3bx + b^2x^2 + 4\,abx + \left(a^4 + 2\,a^2 + 1\right)\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x
 + (a^4 + 2*a^2 + 1)*log(abs(x))
```

Mupad [B]
time = 0.05, size = 55, normalized size = 0.93

$$\ln\left(x\right)\,\left(a^4 + 2\,a^2 + 1\right) + \frac{b^4\,x^4}{4} + \frac{4\,a\,b^3\,x^3}{3} + b^2\,x^2\,\left(3\,a^2 + 1\right) + 4\,a\,b\,x\,\left(a^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^2/x,x)
```

```
[Out] log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2 +
 1) + 4*a*b*x*(a^2 + 1)
```

3.99 $\int \frac{x^2}{1+(-1+x)^2} \, dx$

Optimal. Leaf size=10

$$x + \log\left(1 + (-1 + x)^2\right)$$

[Out] x+ln(1+(-1+x)^2)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {378, 716, 266}

$$x + \log\left((x-1)^2 + 1\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + (-1 + x)^2),x]

[Out] x + Log[1 + (-1 + x)^2]

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 716

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1+(-1+x)^2}\,dx &= \text{Subst}\left(\int \frac{(1+x)^2}{1+x^2}\,dx, x, -1+x\right)\\ &= \text{Subst}\left(\int \left(1+\frac{2x}{1+x^2}\right)\,dx, x, -1+x\right)\\ &= x + 2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, -1+x\right)\\ &= x + \log\left(1+(-1+x)^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.10

$$x + \log\left(2 - 2x + x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x)^2),x]
[Out] x + Log[2 - 2*x + x^2]
```

Maple [A]
time = 0.26, size = 12, normalized size = 1.20

method	result	size
default	$x+\ln\left(x^2-2x+2\right)$	12
norman	$x+\ln\left(x^2-2x+2\right)$	12
risch	$x+\ln\left(x^2-2x+2\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(-1+x)^2),x,method=_RETURNVERBOSE)
[Out] x+ln(x^2-2*x+2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.10

$$x + \log\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")
[Out] x + log(x^2 - 2*x + 2)
```

Fricas [A]
time = 0.36, size = 11, normalized size = 1.10

$$x + \log\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="fricas")
[Out] x + log(x^2 - 2*x + 2)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 1.00

$$x + \log\left(x^2 - 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(-1+x)**2),x)
[Out] x + log(x**2 - 2*x + 2)
```

Giac [A]
time = 6.41, size = 11, normalized size = 1.10

$$x + \log\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")
[Out] x + log(x^2 - 2*x + 2)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 1.10

$$x + \ln\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)^2 + 1),x)
[Out] x + log(x^2 - 2*x + 2)
```

3.100 $\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx$

Optimal. Leaf size=44

$$\frac{3}{2}\sqrt{1-(1+x)^2}-\frac{1}{2}x\sqrt{1-(1+x)^2}+\frac{3}{2}\sin^{-1}(1+x)$$

```
[Out] 3/2*arcsin(1+x)+3/2*(1-(1+x)^2)^(1/2)-1/2*x*(1-(1+x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {378, 685, 655, 222}

$$\frac{3}{2}\text{ArcSin}(x+1)-\frac{1}{2}\sqrt{1-(x+1)^2}\,x+\frac{3}{2}\sqrt{1-(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (1 + x)^2],x]
```

```
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 685

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[2*c*
d*((m + p)/(c*(m + 2*p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
```

```
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx &= \text{Subst}\left(\int \frac{(-1+x)^2}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= -\frac{1}{2}x\sqrt{1-(1+x)^2} - \frac{3}{2}\text{Subst}\left(\int \frac{-1+x}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2}\sin^{-1}(1+x)
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 51, normalized size = 1.16

$$\frac{x(-6-x+x^2)+6\sqrt{x}\,\sqrt{2+x}\,\tanh^{-1}\left(\sqrt{\frac{x}{2+x}}\right)}{2\sqrt{-x(2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (1 + x)^2],x]
[Out] (x*(-6 - x + x^2) + 6*Sqrt[x]*Sqrt[2 + x]*ArcTanh[Sqrt[x/(2 + x)]])/(2*Sqrt
[-(x*(2 + x))])
```

Maple [**A**]
time = 0.18, size = 35, normalized size = 0.80

method	result	size
risch	$\frac{(x-3)x(x+2)}{2\sqrt{-x(x+2)}}+\frac{3\arcsin(1+x)}{2}$	25
default	$-\frac{x\sqrt{-x^2-2x}}{2}+\frac{3\sqrt{-x^2-2x}}{2}+\frac{3\arcsin(1+x)}{2}$	35
meijerg	$-\frac{4i\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-5x+15)\sqrt{1+\frac{x}{2}}}{40}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{4}\right)}{\sqrt{\pi}}$	45

trager	$\left(-\frac{x}{2}+\frac{3}{2}\right)\sqrt{-x^2-2x}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(x\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-2x}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(1+x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*x*(-x^2-2*x)^(1/2)+3/2*(-x^2-2*x)^(1/2)+3/2*arcsin(1+x)
```

Maxima [A]
time = 0.48, size = 36, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2-2\,x}\,x+\frac{3}{2}\sqrt{-x^2-2\,x}-\frac{3}{2}\arcsin\left(-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)
```

Fricas [A]
time = 0.37, size = 35, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-x^2-2\,x}\,(x-3)-3\arctan\left(\frac{\sqrt{-x^2-2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{-x\,(x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(1+x)**2)**(1/2),x)
[Out] Integral(x**2/sqrt(-x*(x + 2)), x)
```

Giac [A]
time = 5.58, size = 23, normalized size = 0.52

$$-\frac{1}{2}\sqrt{-x^2-2\,x}\,(x-3)+\frac{3}{2}\arcsin\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) + 3/2*arcsin(x + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{1-(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - (x + 1)^2)^(1/2),x)
[Out] int(x^2/(1 - (x + 1)^2)^(1/2), x)
```

3.101 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx$

Optimal. Leaf size=67

$$\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}+\frac{(1+2a^2)\sin^{-1}(a+bx)}{2b^3}$$

```
[Out] 1/2*(2*a^2+1)*arcsin(b*x+a)/b^3+3/2*a*(1-(b*x+a)^2)^(1/2)/b^3-1/2*x*(1-(b*x
+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {378, 757, 655, 222}

$$\frac{(2a^2+1)\,\mathrm{ArcSin}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (
(1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
```

```
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b^3}\\
&= -\frac{x\sqrt{1-(a+bx)^2}}{2b^2} - \frac{\text{Subst}\left(\int \frac{-1-2a^2+3ax}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3}\\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3}\\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\sin^{-1}(a+bx)}{2b^3}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 163 vs. 2(67) = 134.
time = 0.31, size = 163, normalized size = 2.43

$$
\frac{-2b(-3a+bx)\sqrt{1-a^2-2abx-b^2x^2}-2(1+2a^2)\,b\tan^{-1}\left(\frac{-\sqrt{-b^2}\,x+\sqrt{1-a^2-2abx-b^2x^2}}{a}\right)+(1+2a^2)\sqrt{-b^2}\,\log\left(-1+2abx+2b^2x^2+2\sqrt{-b^2}\,x\sqrt{1-a^2-2abx-b^2x^2}\right)}{4b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (-2*b*(-3*a + b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] - 2*(1 + 2*a^2)*b*ArcT
an[(-(Sqrt[-b^2]*x) + Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a] + (1 + 2*a^2)*S
qrt[-b^2]*Log[-1 + 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 - a^2 - 2*a*
b*x - b^2*x^2]])/(4*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(57) = 114.
time = 0.23, size = 164, normalized size = 2.45

method	result

risch	$-\frac{(-bx+3a)(b^2x^2+2abx+a^2-1)}{2b^3\sqrt{-b^2x^2-2abx-a^2+1}}+\frac{\arctan\left(\frac{\sqrt{b^2}\,\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)a^2}{b^2\sqrt{b^2}}+\frac{\arctan\left(\frac{\sqrt{b^2}\,\left(\frac{a}{b}+\right.}{\sqrt{-b^2x^2-2abx}}\right.}{2b^2\sqrt{b^2}}$
default	$-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}-\frac{3a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a\arctan\left(\frac{\sqrt{b^2}\,\left(\frac{a}{b}+x\right)}{\sqrt{-b^2x^2-2abx-a^2+}}\right.}{b\sqrt{b^2}}\right.}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-3/2*a/b*(-1/b^2*(-b^2*x^2-2*a*b*x
-a^2+1)^(1/2)-a/b/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-
a^2+1)^(1/2)))+1/2*(-a^2+1)/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^
2*x^2-2*a*b*x-a^2+1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(57) = 114$.
time = 0.51, size = 139, normalized size = 2.07

$$-\frac{3\,a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3}-\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,x}{2\,b^2}+\frac{(a^2-1)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3}+\frac{3\sqrt{-b^2x^2-2\,abx-a^2+1}\,a}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 1/2*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + 1/2*(a^2 - 1)*arcsin(-(b^2*x + a*b)
/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*a/b^3
```

Fricas [A]
time = 0.40, size = 92, normalized size = 1.37

$$-\frac{(2\,a^2+1)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*
x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))
/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

Giac [A]
time = 5.42, size = 64, normalized size = 0.96

$$-\frac{1}{2}\sqrt{-b^2x^2-2\,abx-a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{(2\,a^2+1)\arcsin\left(-bx-a\right)\operatorname{sgn}(b)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)
*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{1-(a+b\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - (a + b*x)^2)^(1/2),x)
```

```
[Out] int(x^2/(1 - (a + b*x)^2)^(1/2), x)
```

3.102 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx$

Optimal. Leaf size=63

$$-\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}$$

```
[Out] -1/2*(-2*a^2+1)*arcsinh(b*x+a)/b^3-3/2*a*(1+(b*x+a)^2)^(1/2)/b^3+1/2*x*(1+(
b*x+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {378, 757, 655, 221}

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/(2*b^2) -
((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
```

```
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^3}$$

$$= \frac{x\sqrt{1+(a+bx)^2}}{2b^2} + \frac{\text{Subst}\left(\int \frac{-1+2a^2-3ax}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^3}$$

$$= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{(1-2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a\right.}{2b^3}$$

$$= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3} + \frac{x\sqrt{1+(a+bx)^2}}{2b^2} - \frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 145 vs. 2(63) = 126.
time = 0.37, size = 145, normalized size = 2.30

$$-\frac{-2b(-3a+bx)\sqrt{1+a^2+2abx+b^2x^2}+(-1+2a^2)\left(b+\sqrt{b^2}\right)\log\left(-a-\sqrt{b^2}\,x+\sqrt{1+a^2+2abx+b^2x^2}\right)+(-1+2a^2)\left(-b+\sqrt{b^2}\right)\log\left(a-\sqrt{b^2}\,x+\sqrt{1+a^2+2abx+b^2x^2}\right)}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] -1/4*(-2*b*(-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 + 2*a^2)*(b
 + Sqrt[b^2])*Log[-a - Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]] + (
-1 + 2*a^2)*(-b + Sqrt[b^2])*Log[a - Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x +
 b^2*x^2]])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(53) = 106.
time = 0.32, size = 155, normalized size = 2.46

method	result
risch	$-\frac{(-bx+3a)\sqrt{b^2x^2+2abx+a^2+1}}{2b^3}+\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)a^2}{b^2\sqrt{b^2}}-\frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2a\right.}{2b^2\sqrt{b^2}}$

default	$\frac{x\sqrt{b^2x^2+2abx+a^2+1}}{2b^2}-\frac{3a\left(\frac{\sqrt{b^2x^2+2abx+a^2+1}}{b^2}-\frac{a\ln\left(\frac{b^2x+ab}{\sqrt{b^2}}+\sqrt{b^2x^2+2abx+a^2+1}\right)}{b\sqrt{b^2}}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b*(1/b^2*(b^2*x^2+2*a*b*x+a^2
+1)^(1/2)-a/b*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^
2)^(1/2))-1/2*(a^2+1)/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1
)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.
time = 0.27, size = 135, normalized size = 2.14

$$\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3}+\frac{\sqrt{b^2x^2+2\,abx+a^2+1}\,x}{2\,b^2}-\frac{(a^2+1)\,\mathrm{arsinh}\left(\frac{2\,(b^2x+ab)}{\sqrt{-4\,a^2b^2+4\,(a^2+1)b^2}}\right)}{2\,b^3}-\frac{3\,\sqrt{b^2x^2+2\,abx+a^2+1}\,a}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 1
/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 1/2*(a^2 + 1)*arcsinh(2*(b^2*x
 + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a/b^3
```

Fricas [A]
time = 0.36, size = 70, normalized size = 1.11

$$-\frac{(2\,a^2-1)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

Giac [A]
time = 3.67, size = 86, normalized size = 1.37

$$\frac{1}{2}\sqrt{b^2x^2+2\,abx+a^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{(2\,a^2-1)\log\left(-ab-\left(x|b|-\sqrt{b^2x^2+2\,abx+a^2+1}\right)|b|\right)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*l
og(-a*b - (x*abs(b) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1))*abs(b))/(b^2*abs(b
))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{{(a+b\,x)}^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^2 + 1)^(1/2),x)
```

```
[Out] int(x^2/((a + b*x)^2 + 1)^(1/2), x)
```

3.103 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=234

$$\frac{x}{bd^3}+\frac{\left(a-3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}d^4}-\frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}$$

```
[Out] x/b/d^3-1/3*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(
2/3)/b^(4/3)/d^4+1/6*(a+3*a^(1/3)*b^(2/3)*c^2+b*c^3)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(4/3)/d^4-c*ln(a+b*(d*x+c)^3)/b/d
^4+1/3*(a-3*a^(1/3)*b^(2/3)*c^2+b*c^3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c
))/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)/d^4*3^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {378, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\left(-3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}d^4}-\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4}-\frac{c\log\left(a+b(c+dx)^3\right)}{bd^4}+\frac{x}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^(1/3
)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)*d^4) - ((a + 3*a^
(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(
4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a +
b*(c + d*x)^3])/(b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a+bc^3-3bc^2x+3bcx^2}{b(a+bx^3)}\right)\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x+3bcx^2}{a+bx^3}\,dx, x, c+dx\right)}{bd^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x}{a+bx^3}\,dx, x, c+dx\right)}{bd^4} - \frac{(3c)\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-3\sqrt[3]{a}\,bc^2+2\sqrt[3]{b}\,(a+bc^3)\right)+\sqrt[3]{b}\left(-3\sqrt[3]{a}\,bc^2\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\right.}{3a^{2/3}b^{4/3}d^4}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} -$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)}{}$$

$$= \frac{x}{bd^3} + \frac{\left(a-3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}d^4} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log}{3a^{2/3}b^{4/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 132, normalized size = 0.56

$$-\frac{-3bdx+\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&, \frac{a\log(x-\#1)+bc^3\log(x-\#1)+3bc^2d\log(x-\#1)\#1+3bcd^2\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] -1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*
#1^3 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 + 3*
b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(b^2*d^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 108, normalized size = 0.46

method	result	size
default	$\frac{x}{d^3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\left(-3\_R^2bc\,d^2-3\_Rb\,c^2d-b\,c^3-a\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3b^2d^4}$	108
risch	$\frac{x}{d^3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\left(-3\_R^2bc\,d^2-3\_Rb\,c^2d-b\,c^3-a\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3b^2d^4}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] x/d^3/b+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2*
_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3
+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)
```

Fricas [C] Result contains complex when optimal does not.
time = 18.07, size = 6315, normalized size = 26.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)
)/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 +
 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2
*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(
b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*b*d^4*log(-3/4*((-I*s
```

```
qrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^
12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 +
3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2
*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3
)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + b^3*c^10 - 9*a*b
^2*c^7 - 12*a^2*b*c^4 + 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(
3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12)
- 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^
2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*
c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a
^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*d^4 - 2*a^3*c + (b^3*c^9 - 24*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)*d*x) - 12*d*x - (((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) +
(b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(
a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^1
2) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3
) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^1
2) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54
*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(
b*d^4))*b*d^4 - 3*sqrt(1/3)*b*d^4*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8)
 + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*
c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*
d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(
1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*
d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1
/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*
c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2
*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12
) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*
(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*s
qrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9
- 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*
b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*d^8)) - 18*c)*log(3/4*((-I*sqrt(3) +
1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2
*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3
)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^
4*d^12))^(1/3) + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + 2*b^3*c^10 - 63*a*b^2*c^7
 + 21*a^2*b*c^4 - 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(3) + 1
```

```
)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*
(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^
3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/
(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4
*d^12))^(1/3) + 6*c/(b*d^4))*d^4 + 5*a^3*c + 2*(b^3*c^9 - 24*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)*d*x + 3/4*sqrt(1/3)*(3*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8)
+ (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c
/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d
^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1
/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d
^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/
54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c
/(b*d^4))*a^2*b^3*c^2*d^8 + 2*(a*b^3*c^6 - 7*a^...
```

Sympy [**A**]
time = 1.67, size = 238, normalized size = 1.02

$$\operatorname{RootSum}\left(27t^3a^2b^4d^{12} + 81t^2a^2b^3cd^8 + t\left(54a^2b^2c^2d^4 - 27ab^3c^5d^4\right) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto t\log\left(x + \frac{-27t^2a^2b^3c^2d^8 - 3ta^3bd^4 - 60ta^2b^2c^3d^4 - 3tab^3c^6d^4 - 2a^3c - 12a^2bc^4 - 9ab^2c^7 + b^3c^{10}}{a^3d + 3a^2bc^3d - 24ab^2c^6d + b^3c^9d}\right)\right)\right) + \frac{x}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2*
b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c**
6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*_t*
a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c
- 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d -
24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^3*b + a), x)
```

Mupad [**B**]
time = 2.46, size = 374, normalized size = 1.60

$$\left(\sum_{k=1}^{3}\ln\left(\frac{3\,\left(b\,c^5+a\,c^2\right)}{d^2}-\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\,\left(\frac{3\,\left(b^2\,c^4\,d^4-5\,a\,b\,c\,d^4\right)}{d^2}+\frac{3\,x\,\left(b^2\,c^3\,d^4+a\,b\,d^4\right)}{d}-\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\,a\,b^2\,d^6\,9\right)-\frac{3\,x\,\left(a\,c-2\,b\,c^4\right)}{d}\right)\,\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\right)+\frac{x}{b\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^3),x)

[Out] symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*
c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + b^3*c^9 + a^3, z, k)*((3*(b^2*c^4*d^4 - 5*a*b*c*d^4))/d^2 + (3*x*(
b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d^8*z
^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ b^3*c^9 + a^3, z, k)*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*a^2*b^
4*d^12*z^3 + 81*a^2*b^3*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4
*z + 3*a*b^2*c^6 + 3*a^2*b*c^3 + b^3*c^9 + a^3, z, k), k, 1, 3) + x/(b*d^3)
```

3.104 $\int \frac{x^2}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=210

$$\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)}{}$$

```
[Out] 1/3*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^3
-1/6*c*(2*a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*
x+c)^2)/a^(2/3)/b^(2/3)/d^3+1/3*ln(a+b*(d*x+c)^3)/b/d^3+1/3*c*(2*a^(1/3)-b^
(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^
(2/3)/d^3*3^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {378, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3) + b^(1/3)*c)*Log[a
^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^3) - (c*(2*a^(1/3) + b^(1
/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a
^(2/3)*b^(2/3)*d^3) + Log[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\int \frac{x^2}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3} + \frac{\text{Subst}\left(\int \frac{c^2-2cx}{a+bx^3}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{a}\,c+2\sqrt[3]{b}\,c^2\right)+\sqrt[3]{b}\left(-2\sqrt[3]{a}\,c-\sqrt[3]{b}\,c^2\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c \ldots\right.}{3a^{2/3}\sqrt[3]{b}\,d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} - \frac{\left(c\left(\frac{2}{\sqrt[3]{b}} - \frac{c}{\sqrt[3]{a}} \ldots\right.\right.}{\ldots}$$

$$= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(\ldots\right.}{6a^{2/3}b^{2/3}d^3}$$

$$= \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx\ldots\right.}{3a^{2/3}b^{2/3}d^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 81, normalized size = 0.39

$$\frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&, \frac{\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 74, normalized size = 0.35

method	result	size

default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	74
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z
^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.14, size = 4759, normalized size = 22.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^
2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/
(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d
^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b
*d^3))*b*d^3*log(-1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2
*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*
b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) +
 (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) - 2/(b*d^3))^2*a^2*b^2*d^6 + b^2*c^6 - a*b*c^3 - 1/2*(a*b^2*c^3 + 4*
a^2*b)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*
d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b
```

^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I
sqrt(3) + 1)((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9
) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d
^3))*d^3 + (b^2*c^5 - 8*a*b*c^2)*d*x - 2*a^2) - ((2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2
d^9) + 3(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/
(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*
b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 - 3*sqrt(1/3)*b*d^3*sq
rt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^
6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3
*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9)
 + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3
))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^
6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3
*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a
)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1
/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) + 6)*log(1/2*(2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3
 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^
2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))^2*a^2*b^
2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*d^3 + 2*(b^2*c^5 -
 8*a*b*c^2)*d*x + 2*a^2 + 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((
2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) +
3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^
2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2
d^9) + 3(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b^2*c^3 - 2*a^2*b)*
d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a
*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)
/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/

```
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*
c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d
^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6))) - ((2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 -
 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9)
 + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 + 3*
sqrt(1/3)*b*d^3*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(...
```

Sympy [A]
time = 0.52, size = 158, normalized size = 0.75

$$\operatorname{RootSum}\left(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t\left(9a^2bd^3 - 18ab^2c^3d^3\right) - a^2 - 2abc^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{18t^2a^2b^2d^6 - 12ta^2bd^3 - 3tab^2c^3d^3 + 2a^2 + abc^3 - b^2c^6}{8abc^2d - b^2c^5d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d*
*3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*lo
g(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d**3
+ 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

Mupad [B]
time = 2.30, size = 437, normalized size = 2.08

$$\sum_{k=1}^{3}\ln\left(a + b\,c^3 - \mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\right)\,a\,b\,d^3\,6 + 3\,b\,c^2\,d\,x + \mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\right)^2\,a\,b^2\,d^6\,9 + \mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\right)\,b^2\,c^3\,d^3\,3 + \mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\right)\,b^2\,c^2\,d^4\,x\,3\right)\,\mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3 - 27\,a^2\,b^2\,d^6\,z^2 - 18\,a\,b^2\,c^3\,d^3\,z + 9\,a^2\,b\,d^3\,z - 2\,a\,b\,c^3 - b^2\,c^6 - a^2, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*
a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*a*b*d^3
+ 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c
^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6 + 3
*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*
```

```
b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^2*b^3*
d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c
^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b
^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2
, z, k), k, 1, 3)
```

3.105 $\int \frac{x}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log}{}$$

```
[Out] -1/3*(a^(1/3)+b^(1/3)*c)*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(2/3)/d^2+1/
6*(a^(1/3)+b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)
/a^(2/3)/b^(2/3)/d^2-1/3*(a^(1/3)-b^(1/3)*c)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(2/3)/d^2*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {378, 1874, 31, 648, 631, 210, 642}

$$-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^3),x]
```

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b^(1/3)*c)*Log[a^(1/3
) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^2) + ((a^(1/3) + b^(1/3)*c)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^
(2/3)*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^3}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}c\right)+\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}\,d^2} - \frac{\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\text{Subst}$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\frac{1}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}\right.}{2d^2}$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c\right.}{6a^{2/3}b^{2/3}d^2}$$

$$= -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 79, normalized size = 0.44

$$\frac{\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&, \frac{\log(x-\#1)\#1}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 72, normalized size = 0.40

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)} \frac{\_R\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	72

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)

[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] integrate(x/((d*x + c)^3*b + a), x)

Fricas [C] Result contains complex when optimal does not.
time = 1.09, size = 1950, normalized size = 10.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)
/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3
 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3)
 + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2
*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*
(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*
log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/
(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*
b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d
^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*
c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a
)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1
)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6
))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(

```
3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b
^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4))) + 1/36*(9*(I*sqrt(3) + 1)*(
-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + 3
*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/5
4*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*
c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4
- 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b
^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*log(1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)
 + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*
(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*s
qrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c - 1/
12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b
*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*
a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1
/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(
b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^
4 - 144*c)/(a*b*d^4))) - 1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^
4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))
)*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 +
a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 1/
6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + b*c^4 + (b*c
^3 - a)*d*x + a*c)
```

Sympy [A]
time = 0.34, size = 83, normalized size = 0.46

$$\text{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**3),x)
[Out] RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t,
_t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d -
 b*c**3*d))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

Mupad [B]
time = 0.26, size = 145, normalized size = 0.81

$$\sum_{k=1}^{3} \ln\left(-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\,\left(3\,b^2\,c^2\,d^4-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\,a\,b^2\,d^6\,9+3\,b^2\,c\,d^5\,x\right)+b\,d^3\,x\right)\,\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z
, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a
, z, k)*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z
 + b*c^3 + a, z, k), k, 1, 3)
```

3.106 $\int \frac{1}{a+b(c+dx)^3} \, dx$

Optimal. Leaf size=140

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)*b^
(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*(a^(1/3)-
2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {253, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(
1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}\,d} - \frac{\text{Subst}(}{}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+d\right.}{6a^{2/3}\sqrt[3]{b}}$$

Mathematica [A]
time = 0.03, size = 116, normalized size = 0.83

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*L
og[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 71, normalized size = 0.51

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}}{3bd}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*
b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

Fricas [A]
time = 0.42, size = 442, normalized size = 3.16

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abd^3x^3+6abcd^2x^2+6abc^2dx+2abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2abd^2x^2+4abcdx+2abc^2+(a^2b)^{\frac{2}{3}}(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3(a^2b)^{\frac{1}{3}}(adx+ac)}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abd^2x^2+2abcdx+abc^2-(a^2b)^{\frac{2}{3}}(dx+c)+(a^2b)^{\frac{1}{3}}a\right)+2(a^2b)^{\frac{2}{3}}\log\left(abdx+abc+(a^2b)^{\frac{2}{3}}\right)}{6a^2bd},\ \frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(a^2b)^{\frac{2}{3}}(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-(a^2b)^{\frac{2}{3}}\log\left(abd^2x^2+2abcdx+abc^2-(a^2b)^{\frac{2}{3}}(dx+c)+(a^2b)^{\frac{1}{3}}a\right)+2(a^2b)^{\frac{2}{3}}\log\left(abdx+abc+(a^2b)^{\frac{2}{3}}\right)}{6a^2bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d
^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a
*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(
a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
 a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

Sympy [A]
time = 0.10, size = 26, normalized size = 0.19

$$\frac{\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

Giac [A]
time = 3.69, size = 160, normalized size = 1.14

$$\frac{1}{3}\,\sqrt{3}\left(\frac{1}{a^{2}bd^{3}}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^{2}\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\,\left(ab^{2}\right)^{\frac{1}{3}}}\right)-\frac{1}{6}\left(\frac{1}{a^{2}bd^{3}}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\,\left(ab^{2}\right)^{\frac{1}{3}}\right)^{2}+4\left(bdx+bc+\left(ab^{2}\right)^{\frac{1}{3}}\right)^{2}\right)+\frac{1}{3}\left(\frac{1}{a^{2}bd^{3}}\right)^{\frac{1}{3}}\log\left(\left|bdx+bc+\left(ab^{2}\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sq
rt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*
c + (a*b^2)^(1/3)))
```

Mupad [B]
time = 2.31, size = 144, normalized size = 1.03

$$\frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d}+\frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x+\frac{3\,a^{1/3}\,b^{5/3}\,d^5\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}-\frac{\ln\left(3\,b^2\,c\,d^5+3\,b^2\,d^6\,x-\frac{3\,a^{1/3}\,b^{5/3}\,d^5\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*c
*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1
i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1/3)
*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

3.107 $\int \frac{1}{x(a+b(c+dx)^3)}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt[3]{b}\,c\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,c+b^{2/3}c^2\right)}+\frac{\log(x)}{a+bc^3}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)}-\frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\right.}$$

```
[Out] ln(x)/(b*c^3+a)-1/3*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(a^(1/3)+b^(1/3)*c)
-1/6*(2*a^(1/3)-b^(1/3)*c)*ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+
c)^2)/a^(2/3)/(a^(2/3)-a^(1/3)*b^(1/3)*c+b^(2/3)*c^2)+1/3*b^(1/3)*c*arctan(
1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^(2/3)-a^(1/3)*b
^(1/3)*c+b^(2/3)*c^2)*3^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {378, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)}-\frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)}-\frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)}+\frac{\log(x)}{a+bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]/(a + b*c^3) + (b^(1
/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a
 + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c +
 d*x)^3]/(3*(a + b*c^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\,(a+b(c+dx)^3)}\,dx = \text{Subst}\left(\int \frac{1}{(-c+x)\,(a+bx^3)}\,dx, x, c+dx\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{(a+bc^3)\,(c-x)} - \frac{b(c^2+cx+x^2)}{(a+bc^3)\,(a+bx^3)}\right)\,dx, x, c+dx\right)$$
$$= \frac{\log(x)}{a+bc^3} - \frac{b\text{Subst}\left(\int \frac{c^2+cx+x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$
$$= \frac{\log(x)}{a+bc^3} - \frac{b\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3} - \frac{b\text{Subst}\left(\int \frac{c^2+cx}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}$$
$$= \frac{\log(x)}{a+bc^3} - \frac{\log\left(a+b(c+dx)^3\right)}{3\,(a+bc^3)} - \frac{b^{2/3}\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}\,c+2\sqrt[3]{b}\,c^2\right)+\sqrt[3]{b}\left(\sqrt[3]{a}\,c-\sqrt[3]{\right.}}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}}{3a^{2/3}\,(a+bc^3)}$$
$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\,(a+bc^3)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\,(a+bc^3)}$$
$$= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\,(a+bc^3)} - \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\text{l}}{}$$
$$= \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\,(a+bc^3)} + \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\right.}{3a^{2/3}\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 119, normalized size = 0.53

$$-\frac{-3\log(x)+\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&, \frac{3c^2\log(x-\#1)+3cd\log(x-\#1)\#1+d^2\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3\,(a+bc^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/3*(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3
*#1^3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^2)
/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(a + b*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 105, normalized size = 0.47

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(d^{3} b \_Z^{3}+3 b c\, d^{2} \_Z^{2}+3 b\, c^{2} d \_Z+b\, c^{3}+a\right)} \frac{\left(d^{2} \_R^{2}+3 c d \_R+3 c^{2}\right) \ln \left(x-\_R\right)}{d^{2} \_R^{2}+2 c d \_R+c^{2}}}{3\left(b\, c^{3}+a\right)}+\frac{\ln \left(x\right)}{b\, c^{3}+a}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^{2} b\, c^{3}+a^{3}\right) \_Z^{3}+3 a^{2} \_Z^{2}+3 a \_Z\right)} \_R \ln \left(\left(\left(2 a b\, c^{3} d-4 d\, a^{2}\right) \_R^{2}+\left(b\, c^{3} d-8 a d\right) \_R-4 d\right) x+\left(a b\, c^{4}+a^{2} c\right) \_R^{2}+\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=Root
Of(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)+ln(x)/(b*c^3+
a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 + a)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.18, size = 4370, normalized size = 19.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
```

```
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))*log(b*c^2*d*x + b*c^3
+ 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2)
- 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a
*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3
) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2
)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 - 1/2*(a*b*c^3 -
 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^
2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + a) - ((b*c^3 + a)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)
^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3
+ a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - 2/(b*c^3 + a)) + 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 +
 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^
3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a
)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 +
a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^
3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) +
6)*log(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2)
- 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^
(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c
^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b
*c^3 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*
b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1
/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((
a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 3/4
*sqrt(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*
c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/
2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) +
 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))
+ 2*a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2
```

```
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a
^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3)
- 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(
a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^
(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/
((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4
*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) - ((b*c^3 + a)*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a
^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)
^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2
*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) -
2/(b*c^3 + a)) - 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a
^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c
^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 +
 a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(
b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3
 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x), x)
```

Mupad [**B**]
time = 0.12, size = 553, normalized size = 2.47

$$\frac{\ln(x)}{b\,c^3+a}+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,b^4\,c^4\,d^6\,3-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,c\,d^6\,3-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,d^6\,x\,4-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,a\,b^3\,c\,d^6\,6-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,a\,b^3\,d^6\,x\,24+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a^2\,b^3\,c\,d^6\,9+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a\,b^4\,c^4\,d^6\,9-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a^2\,b^3\,d^6\,x\,36+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,b^4\,c^3\,d^6\,x\,3+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a\,b^4\,c^3\,d^6\,x\,18\right)\,\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^3)),x)

[Out] log(x)/(a + b*c^3) + symsum(log(3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a
^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*root(27*a^2*b*c^3*z^3 + 27*a^3*
z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*root(27*a^2*b*c^3*z^3 + 2
7*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*root(27*a^2*b*c^3*z
^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*root(27*
a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^9*x +
9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*
b^3*c*d^8 + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1,
z, k)^3*a*b^4*c^4*d^8 - 36*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2
+ 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 +
 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*root(27*a^2*b*c^3*z^3 +
 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*root(27*a^2*
b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1, 3)
```

3.108 $\int \frac{1}{x^2(a+b(c+dx)^3)}\,dx$

Optimal. Leaf size=314

$$-\frac{1}{(a+bc^3)\,x}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3 d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\,(a+bc^3)^2}-\frac{3bc^2d\log(x)}{(a+bc^3)^2}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}\,(a\right.}{}$$

```
[Out] -1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)*(-2*b*c^3+a
)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^2
-1/6*b^(1/3)*(a^(1/3)*(-2*b*c^3+a)-b^(1/3)*c*(-b*c^3+2*a))*d*ln(a^(2/3)-a^(
1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^2+b*c^2*d*ln(a+b*
(d*x+c)^3)/(b*c^3+a)^2+1/3*b^(1/3)*(a^(1/3)-b^(1/3)*c)*(a^(1/3)+b^(1/3)*c)^
3*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+
a)^2*3^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {378, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\,(a+bc^3)^2}+\frac{b^{2/3}d\left(-\frac{\sqrt[3]{a}\,(a-2bc^3)}{\sqrt[3]{b}}+2ac-bc^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\,(a+bc^3)^2}+\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}\,(a-2bc^3)-\sqrt[3]{b}\,c(2a-bc^3)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\,(a+bc^3)^2}-\frac{1}{x\,(a+bc^3)}-\frac{3bc^2d\log(x)}{(a+bc^3)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{(a+bc^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1/3)*c)
^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^(1/3)*(a^(1/3)
*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*(a - 2*b*
c^3))/b^(1/3))*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x
)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c
^3)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \left(a+b(c+dx)^3\right)} dx &= d\text{Subst}\left(\int \frac{1}{(-c+x)^2 \left(a+bx^3\right)} dx, x, c+dx\right) \\
&= d\text{Subst}\Bigg(\int \Bigg(\frac{1}{\left(a+bc^3\right)(c-x)^2} + \frac{3bc^2}{\left(a+bc^3\right)^2 (c-x)} + \frac{b(-c(2a-bc^3)-(a-2 \cdots}{\left(a+bc^3\right)^2 (a \cdots} \\
&= -\frac{1}{\left(a+bc^3\right)x} - \frac{3bc^2 d\log(x)}{\left(a+bc^3\right)^2} + \frac{(bd)\text{Subst}\Big(\int \frac{-c(2a-bc^3)-(a-2bc^3)x+3bc^2x^2}{a+bx^3} dx, x, c+ \cdots}{\left(a+bc^3\right)^2} \\
&= -\frac{1}{\left(a+bc^3\right)x} - \frac{3bc^2 d\log(x)}{\left(a+bc^3\right)^2} + \frac{(bd)\text{Subst}\Big(\int \frac{-c(2a-bc^3)+(-a+2bc^3)x}{a+bx^3} dx, x, c+dx\Big)}{\left(a+bc^3\right)^2} \cdots \\
&= -\frac{1}{\left(a+bc^3\right)x} - \frac{3bc^2 d\log(x)}{\left(a+bc^3\right)^2} + \frac{bc^2 d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2} + \frac{\left(b^{2/3}d\right)\text{Subst}\Bigg(\int \frac{\sqrt[3]{\cdots}}{\cdots}}{\cdots} \\
&= -\frac{1}{\left(a+bc^3\right)x} - \frac{3bc^2 d\log(x)}{\left(a+bc^3\right)^2} - \frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\,(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3a^{2/3}\left(a+bc^3\right)^2} \\
&= -\frac{1}{\left(a+bc^3\right)x} - \frac{3bc^2 d\log(x)}{\left(a+bc^3\right)^2} - \frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\,(a-2bc^3)}{\sqrt[3]{b}}\right) d\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3a^{2/3}\left(a+bc^3\right)^2} \\
&= -\frac{1}{\left(a+bc^3\right)x} + \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3 d\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^2} - \cdots
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.07, size = 173, normalized size = 0.55

$$\frac{-3\left(a+bc^3+3bc^2dx\log(x)\right)+dx\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{-3ac\log(x-\#1)+6bc^4\log(x-\#1)-ad\log(x-\#1)\#1+8bc^3d\log(x-\#1)\#1+3bc^2d^2\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{3\left(a+bc^3\right)^2 x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#1
 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x - #1
] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x - #1]
*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 143, normalized size = 0.46

method	result
default	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(d^3 b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2 d\_Z+b\,c^3+a\right)}\frac{\left(-3\_R^2 b\,c^2 d^2-8\_R b\,c^3 d-6b\,c^4+\_R a d+3ac\right)\ln\left(x-\_R\right)}{d^2\_R^2+2cd\_R+c^2}\right)}{3(b\,c^3+a)^2}-\frac{1}{(b\,c^3+a)}$
risch	$-\frac{1}{(b\,c^3+a)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2b^2c^6+2a^3b\,c^3+a^4\right)\_Z^3-9a^2b\,c^2 d\_Z^2+6abc\,d^2\_Z-d^3 b\right)}\_R\ln\left(\left(\left(2a\,b^3c^9 d-6a^3b\,c^3 d-4a^4 d\right)\_R^3+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d*sum((-3*_R^2*b*c^2*d^2-8*_R*b*c^3*d-6*b*c^4+_R*a*d+3*a*c)/(_R^2*d^2+
2*_R*c*d+c^2)*ln(x-_R),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c
^3+a))/(b*c^3+a)^2-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((3*b*c^2*d^
2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*c^3 + a^2) - 1/((b*c^3 + a)*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.51, size = 8919, normalized size = 28.40

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(36*b*c^2*d*x*log(x) + 12*b*c^3 - 2*(b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(
I*sqrt(3) + 1))*x*log((b^3*c^6 - a^2*b)*d^3*x + 1/4*(2*a^2*b^3*c^9 + 3*a^3*
b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^
2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^
3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)
) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a
)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a
^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
+ a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 1
0*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*
c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3
+ a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18
*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2))
+ b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^
3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2
)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 +
a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (b^3*c^7 + 5*a*b^2*c^4 - 5*a^2*b*c)*d
^2) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a
*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2
- 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^
6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*
c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
+ a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*x -
3*sqrt(1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2)*x*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^
9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^
2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2
/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6
 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^
2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^
3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*
c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*
```

```
(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b
*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 12*(a*b^3*c
^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*
(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2
*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*
b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 +
 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*
b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2
*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^
6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 -
a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + 4*(8*b^3*c^7 - 11*
a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4
*a^4*b*c^3 + a^5)))*log(2*(b^3*c^6 - a^2*b)*d^3*x - 1/4*(2*a^2*b^3*c^9 + 3*
a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(
9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*
b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^
3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^
2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3
 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*
c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 1/2...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
[Out] integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

Mupad [B]
time = 2.33, size = 1588, normalized size = 5.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((b^4*d^12*x - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*
a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^3*b^3*d^9
- 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*
d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^9*d^9 - 9*root(27*a^2*b^2*c^6
*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z
- b*d^3, z, k)*b^5*c^5*d^10 + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3
 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^
4*c^3*d^9 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81
*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^4*d^8 + 27*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2*b^2*c^6*z^
3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b
*d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 +
 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*b^5*c^4*d^
11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b
*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8 + 18*root(27*a^2
*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*
c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3
*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k
)^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*d^9*x -
 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^8*d^10*x + 48*root(27*a^2*b^2*
c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2
*z - b*d^3, z, k)^2*a*b^5*c^5*d^10*x + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*
b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)
^3*a*b^6*c^9*d^9*x - 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*a*b^4*c*d^11*x +
51*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^4*c^2*d^10*x - 54*root(27*a^2*
b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c
*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^3*d^9*x)/(a^2 + b^2*c^6 + 2*a*b*c^3))*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k), k, 1, 3) - 1/(a*x + b*c^3*x) - (3*b*c^2*d*l
og(x))/(a^2 + b^2*c^6 + 2*a*b*c^3)
```

3.109 $\int \frac{1}{x^3(a+b(c+dx)^3)}\,dx$

Optimal. Leaf size=393

$$-\frac{1}{2\left(a+bc^3\right)x^2}+\frac{3bc^2d}{\left(a+bc^3\right)^2x}+\frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}\,c+bc^3\right)d^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}-\frac{3bc}{}$$

```
[Out] -1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b*
c^3+a)^3-1/3*b^(2/3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)
*c^5+b^2*c^6)*d^2*ln(a^(1/3)+b^(1/3)*(d*x+c))/a^(2/3)/(b*c^3+a)^3+1/6*b^(2/
3)*(a^2+6*a^(4/3)*b^(2/3)*c^2-7*a*b*c^3-3*a^(1/3)*b^(5/3)*c^5+b^2*c^6)*d^2*
ln(a^(2/3)-a^(1/3)*b^(1/3)*(d*x+c)+b^(2/3)*(d*x+c)^2)/a^(2/3)/(b*c^3+a)^3+b
*c*(-2*b*c^3+a)*d^2*ln(a+b*(d*x+c)^3)/(b*c^3+a)^3+1/3*b^(2/3)*(a^(1/3)+b^(1
/3)*c)^3*(a-3*a^(2/3)*b^(1/3)*c+b*c^3)*d^2*arctan(1/3*(a^(1/3)-2*b^(1/3)*(d
*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(b*c^3+a)^3*3^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {378, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{b^{2/3}d^2\left(-3a^{2/3}\sqrt[3]{b}\,c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^3}-\frac{3bcd^2\log(x)\left(a-2bc^3\right)}{\left(a+bc^3\right)^3}+\frac{bcd^2\left(a-2bc^3\right)\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^3}-\frac{1}{2x^2\left(a+bc^3\right)}+\frac{3bc^2d}{x\left(a+bc^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/2*1/((a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)*(a^(1/3
) + b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*ArcTan[(a^(1/3) - 2*
b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^3) - (3
*b*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^
(2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) +
b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b
^(2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3
) + (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
```

```
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = d^2\text{Subst}\left(\int \frac{1}{(-c+x)^3\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= d^2\text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^3\right)(c-x)^3} - \frac{3bc^2}{\left(a+bc^3\right)^2(c-x)^2} - \frac{3bc\left(-a+2bc^3\right)}{\left(a+bc^3\right)^3(c-x)} + \cdots\right.\right.$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{-a^2+7\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{-a^2+7\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{bc\left(a-2bc^3\right)d^2\log\left(a\cdots\right.}{\left(a+bc^3\right.\cdots}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2\cdots\right.}{\cdots}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2\cdots\right.}{\cdots}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} + \frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}\,c+bc^3\right)d^2\,\text{t}\cdots}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.11, size = 244, normalized size = 0.62

$$-\frac{3(a+bc^3)\left(a+bc^2(c-6dx)\right)+18bc\left(a-2bc^3\right)d^2x^2\log(x)+2d^2x^2\text{RootSum}\left[a+bc^3+3bc^2d\#1+3bcd^2\#1^2+bd^3\#1^3\&,\frac{a^2\log(x-\#1)-16abc^3\log(x-\#1)+10b^2c^6\log(x-\#1)-12abc^2d\log(x-\#1)\#1+15b^2c^5d\log(x-\#1)\#1-3abcd^2\log(x-\#1)\#1^2+6b^2c^4d^2\log(x-\#1)\#1^2}{c^2+2cd\#1+d^2\#1^2}\&\right]}{6\left(a+bc^3\right)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/6*(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2*
Log[x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^
3*#1^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x - #1] + 10*b^2*c^6*Log[x - #
1] - 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b^2*c^5*d*Log[x - #1]*#1 - 3*a*b*c*d^
2*Log[x - #1]*#1^2 + 6*b^2*c^4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*
#1^2) & ])/((a + b*c^3)^3*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 200, normalized size = 0.51

method	result
default	$-\frac{d^2\left(\sum_{\_R=\text{RootOf}\left(d^3b\_Z^3+3bc\,d^2\_Z^2+3b\,c^2d\_Z+b\,c^3+a\right)}\frac{\left(6\_R^2b^2c^4d^2+15\_Rb^2c^5d+10b^2c^6-3\_R^2abc\,d^2-12\_Rab\,c^2d-16ab\,c^3\right.}{d^2\_R^2+2cd\_R+c^2}\right.}{3(b\,c^3+a)^3}$
risch	$\frac{\frac{3b\,c^2dx}{b^2c^6+2ab\,c^3+a^2}-\frac{1}{2\left(b\,c^3+a\right)}}{x^2}+\frac{6b^2c^4d^2\ln(-x)}{b^3c^9+3a\,b^2c^6+3a^2b\,c^3+a^3}-\frac{3bc\,d^2\ln(-x)a}{b^3c^9+3a\,b^2c^6+3a^2b\,c^3+a^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^2b^3c^9+3a^3b^2c^6+3a^4b\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*d^2*sum((6*_R^2*b^2*c^4*d^2+15*_R*b^2*c^5*d+10*b^2*c^6-3*_R^2*a*b*c*d^
2-12*_R*a*b*c^2*d-16*a*b*c^3+a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(x-_R),_R=RootO
f(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))/(b*c^3+a)^3-1/2/(b*c^3+a
)/x^2+3*b*c^2*d/(b*c^3+a)^2/x-3*b*c*(-2*b*c^3+a)*d^2*ln(x)/(b*c^3+a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2 +
 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*b^2
```

```
*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 1/2*
(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 5.07, size = 14765, normalized size = 37.57

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*c^6 - 36*(2*b^2*c^4 - a*b*c)*d^2*x^2*log(x) + 12*a*b*c^3 - 2*(
b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b
^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)
^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 +  ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

Mupad [B]
time = 2.49, size = 1328, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((6*b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((a^3*b^4*d^12 + 19*b^7*c^9*d^
12 + 12*a*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*(root(81*a^3*b^2*c^6*z^3 + 27*
a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*
a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((9*a^6*b^3*c*d^8
 + 9*a*b^8*c^16*d^8 + 45*a^5*b^4*c^4*d^8 + 90*a^4*b^5*c^7*d^8 + 90*a^3*b^6*
c^10*d^8 + 45*a^2*b^7*c^13*d^8)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9
 + 6*a^2*b^2*c^6) - (x*(36*a^6*b^3*d^9 - 18*a*b^8*c^15*d^9 + 126*a^5*b^4*c^
3*d^9 + 144*a^4*b^5*c^6*d^9 + 36*a^3*b^6*c^9*d^9 - 36*a^2*b^7*c^12*d^9))/(a
^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)) + (3*b^8*c^14*d
^10 - 42*a*b^7*c^11*d^10 + 30*a^4*b^4*c^2*d^10 + 12*a^3*b^5*c^5*d^10 - 63*a
^2*b^6*c^8*d^10)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^
6) + (x*(3*b^8*c^13*d^11 + 66*a^4*b^4*c*d^11 - 87*a*b^7*c^10*d^11 + 39*a^3*
b^5*c^4*d^11 - 117*a^2*b^6*c^7*d^11))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b
^3*c^9 + 6*a^2*b^2*c^6)) + (x*(18*b^7*c^8*d^13 + 90*a*b^6*c^5*d^13 - 9*a^2*
b^5*c^2*d^13))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)
) - (x*(a*b^5*d^15 + b^6*c^3*d^15))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3
*c^9 + 6*a^2*b^2*c^6))*root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^
4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 2
7*a*b^2*c^2*d^4*z + b^2*d^6, z, k), k, 1, 3) - 1/(2*(a*x^2 + b*c^3*x^2)) +
(3*b*c^2*d)/(a^2*x + b^2*c^6*x + 2*a*b*c^3*x) + (6*b^2*c^4*d^2*log(x))/(a^3
 + b^3*c^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6) - (3*a*b*c*d^2*log(x))/(a^3 + b^3*c
^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6)
```

3.110 $\int \frac{x^3}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=356

$$\frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^4}+\frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4}-\frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4}$$

```
[Out] 1/4*ln(a+b*(d*x+c)^4)/b/d^4+3/2*c^2*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^4/a
^(1/2)/b^(1/2)-1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*
b^(1/2))*(3*a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)+1/8*c*ln(a^(1/
4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(3*a^(1/2)-b^(1/2)*c^
2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1
/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2^(1/2)-1/4*c*arctan(1+b^(
1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(3*a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^4*2
^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {378, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$\frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4}-\frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4}-\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{3c^2\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^4}+\frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*d^4) + (c*
(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(
3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) +
Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)^4]/(4*b*d^4)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{x(3c^2+x^2)}{a+bx^4} + \frac{-c^3-3cx^2}{a+bx^4}\right)dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \frac{x(3c^2+x^2)}{a+bx^4}\,dx, x, c+dx\right)}{d^4} + \frac{\text{Subst}\left(\int \frac{-c^3-3cx^2}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \frac{3c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{\left(c\left(3-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c\right.}{2bd^4}$$

$$\left(c\left(3\sqrt{a}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} - $$

$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^4} - \frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}$$

$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 106, normalized size = 0.30

$$\frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&, \frac{\log(x-\#1)\#1^3}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 97, normalized size = 0.27

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^{4}b\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	97
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^{4}b\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(
_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 2.62, size = 374, normalized size = 1.05

$$\mathrm{RootSum}\left(256t^{4}a^{3}b^{4}d^{16}-256t^{3}a^{3}b^{3}d^{12}+t^{2}\cdot\left(96a^{3}b^{2}d^{8}+480a^{2}b^{3}c^{4}d^{8}\right)+t\left(-16a^{3}bd^{4}+192a^{2}b^{2}c^{4}d^{4}-48ab^{3}c^{8}d^{4}\right)+a^{3}+3a^{2}bc^{4}+3ab^{2}c^{8}+b^{3}c^{12},\left(t\mapsto t\log\left(x+\frac{-1728t^{3}a^{4}b^{3}d^{12}-960t^{3}a^{3}b^{4}c^{4}d^{12}+1296t^{2}a^{4}b^{2}d^{8}+2016t^{2}a^{3}b^{3}c^{4}d^{8}-48t^{2}a^{2}b^{4}c^{8}d^{8}-324ta^{4}bd^{4}-4716ta^{3}b^{2}c^{4}d^{4}-1452ta^{2}b^{3}c^{8}d^{4}-4tab^{4}c^{12}d^{4}+27a^{4}-390a^{3}bc^{4}-444a^{2}b^{2}c^{8}-26ab^{3}c^{12}+b^{4}c^{16}}{729a^{3}bc^{3}d-1053a^{2}b^{2}c^{7}d-117ab^{3}c^{11}d+b^{4}c^{15}d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*a
**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2*b
```

```
**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c**8
 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960*_t*
*3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*
c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**
3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**4 + 2
7*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b**4*c**1
6)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d + b**4*c
**15*d))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

Mupad [B]
time = 2.69, size = 1003, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*root(256*
a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*
b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z +
 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^5*d^4 + 32*root(25
6*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^
3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z
 + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*c*d^8 + 24*roo
t(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 9
6*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d
^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*d^9*x - 2*
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^4*d^5*x +
 38*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*
z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*
a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*c*d^4 +
 6*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z
^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a
^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*d^5*x))*
```

```
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k), k, 1, 4)
```

3.111 $\int \frac{x^2}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=318

$$-\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3}-\frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^3}+\frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}}{\sqrt[4]{}}\right.}{2\sqrt{2}\,a^{3/4}b^{3/4}d^3}$$

```
[Out] -c*arctan((d*x+c)^2*b^(1/2)/a^(1/2))/d^3/a^(1/2)/b^(1/2)+1/8*ln(-a^(1/4)*b^
(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3
/4)/b^(3/4)/d^3*2^(1/2)-1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x
+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)+1/4*arctan
(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/b^(3/4)/
d^3*2^(1/2)+1/4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*
c^2)/a^(3/4)/b^(3/4)/d^3*2^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {378, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^3}+\frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^3}+\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}-\frac{c\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)) - ((Sqrt
[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^4}\,dx,x,c+dx\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{2cx}{a+bx^4}+\frac{c^2+x^2}{a+bx^4}\right)\,dx,x,c+dx\right)}{d^3}$$

$$= \frac{\text{Subst}\left(\int \frac{c^2+x^2}{a+bx^4}\,dx,x,c+dx\right)}{d^3} - \frac{(2c)\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx,x,c+dx\right)}{d^3}$$

$$= -\frac{c\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,(c+dx)^2\right)}{d^3} - \frac{\left(1-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx,x,c+\right.}{2bd^3}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} + \frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx,x,c+\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} + \frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\sqrt{b}\,(\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3}$$

$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} - \frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 106, normalized size = 0.33

$$\frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1^2}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 97, normalized size = 0.31

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^4b\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^4b\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(
_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 101.65, size = 61993, normalized size = 194.95

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/24*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((a*b*d^3*sqrt(-1/(a*b))*sqrt((b*c^4 -
 2*a*b*c^2*sqrt(-1/(a*b)) - a)/(a^2*b^2*d^6*sqrt(-1/(a*b)))) - 2*c)^2/(a*b*
d^6) + 3*(b*c^4 - (2*(2*c*d^3*sqrt((b*c^4 - 2*a*b*c^2*sqrt(-1/(a*b)) - a)/(
```

```
a^2*b^2*d^6*sqrt(-1/(a*b)))) - 3*c^2*sqrt(-1/(a*b)))*b + 1)*a)/(a^2*b^2*d^6
*sqrt(-1/(a*b))))/(9*(a*b*d^3*sqrt(-1/(a*b))*sqrt((b*c^4 - 2*a*b*c^2*sqrt(-
1/(a*b)) - a)/(a^2*b^2*d^6*sqrt(-1/(a*b)))) - 2*c)*(b*c^4 - (2*(2*c*d^3*sqr
t((b*c^4 - 2*a*b*c^2*sqrt(-1/(a*b)) - a)/(a^2*b^2*d^6*sqrt(-1/(a*b)))) - 3*
c^2*sqrt(-1/(a*b)))*b + 1)*a)/(a^2*b^2*d^9) + 27*(a^2*b^2*d^9*sqrt(-1/(a*b)
)*((b*c^4 - 2*a*b*c^2*sqrt(-1/(a*b)) - a)/(a^2*b^2*d^6*sqrt(-1/(a*b))))^(3/
2) - 2*b*c^5*sqrt(-1/(a*b)) - 4*c^3 + 2*a*c*sqrt(-1/(a*b)))/(a^2*b^2*d^9*sq
rt(-1/(a*b))) + 2*(a*b*d^3*sqrt(-1/(a*b))*sqrt((b*c^4 - 2*a*b*c^2*sqrt(-1/(
a*b)) - a)/(a^2*b^2*d^6*sqrt(-1/(a*b)))) - 2*c)^3/(a^3*b^3*d^9*(-1/(a*b))^(
3/2)) + 2*sqrt(-12*(a^8*b^8*(-1/(a*b))^(11/2) + 3*a^7*b^7*(-1/(a*b))^(9/2)
- 29*a^5*b^5*(-1/(a*b))^( ...
```

Sympy [A]
time = 1.87, size = 274, normalized size = 0.86

$$\operatorname{RootSum}\left(256t^4a^3b^3d^{12} + 192t^2a^2b^2c^2d^6 + t\left(-32a^2bcd^3 + 32ab^2c^5d^3\right) + a^2 + 2abc^4 + b^2c^8, \left(t \mapsto t\log\left(x + \frac{64t^3a^4b^2d^9 + 448t^3a^3b^3c^4d^9 + 160t^2a^3b^2c^3d^6 - 32t^2a^2b^3c^7d^6 + 60ta^3bc^2d^3 + 256ta^2b^2c^6d^3 + 4tab^3c^{10}d^3 - 5a^3c - 9a^2bc^5 - 3ab^2c^9 + b^3c^{13}}{a^3d - 33a^2bc^4d - 33ab^2c^8d + b^3c^{12}d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-32
*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d**9
+ 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3
*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3
*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d
 - 33*a*b**2*c**8*d + b**3*c**12*d))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

Mupad [B]
time = 2.59, size = 625, normalized size = 1.97

$$\sum_{k=1}^{4}\ln\Big(-b\,d^4\,\big(a+b\,c^4+4\,b\,c^3\,d\,x+\mathrm{root}(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k)\,b^2\,c^5\,d^3\,4+\mathrm{root}(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k)\,b^2\,c^4\,d^4\,x\,4-\mathrm{root}(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k)\,a\,b\,c\,d^3\,20-\mathrm{root}(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k)\,a\,b\,d^4\,x\,4+\mathrm{root}(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k)^2\,a\,b^2\,c^2\,d^6\,48+\mathrm{root}(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k)^2\,a\,b^2\,c\,d^7\,x\,32\big)\Big)\,\mathrm{root}(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(-b*d^4*(a + b*c^4 + 4*b*c^3*d*x + 4*root(256*a^3*b^3*d^12*z^4 +
192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4
 + b^2*c^8 + a^2, z, k)*b^2*c^5*d^3 + 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2
*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*
c^8 + a^2, z, k)*b^2*c^4*d^4*x - 20*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2
*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8
+ a^2, z, k)*a*b*c*d^3 - 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*
z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z,
 k)*a*b*d^4*x + 48*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32
*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*
b^2*c^2*d^6 + 32*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a
*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*b^
2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*
c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k), k, 1, 4)
```

3.112 $\int \frac{x}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=261

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}+\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2}-\frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2}+\frac{c\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\ \cdots\right.}{4\sqrt{2}\ \cdots}$$

```
[Out] -1/4*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^(1/
2)-1/4*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d^2*2^(1
/2)+1/8*c*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))/a^
(3/4)/b^(1/4)/d^2*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+
(d*x+c)^2*b^(1/2))/a^(3/4)/b^(1/4)/d^2*2^(1/2)+1/2*arctan((d*x+c)^2*b^(1/2)
/a^(1/2))/d^2/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {378, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$\frac{c\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2}-\frac{c\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2}+\frac{c\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2}-\frac{c\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) -
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1
/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+b(c+dx)^4}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{-c+x}{a+bx^4}\,dx, x, c+dx\right)}{d^2}\\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{c}{a+bx^4}+\frac{x}{a+bx^4}\right)\,dx, x, c+dx\right)}{d^2}\\
&= \frac{\operatorname{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^2} - \frac{c\operatorname{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d^2}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^2} - \frac{c\operatorname{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d^2} - \frac{c\operatorname{Subst}\left(\right.}{}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\,d^2} - \frac{c\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}\,d^2} - \frac{c\operatorname{Subst}\left(\int\right.}{}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\,d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,(c+dx)+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2} - \frac{c\log}{}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\,d^2} + \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2} - \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d^2}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 104, normalized size = 0.40

$$\frac{\operatorname{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&, \frac{\log(x-\#1)\#1}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &
]/(4*b*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 95, normalized size = 0.36

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(d^{4} b \_Z^{4}+4 b c\, d^{3} \_Z^{3}+6 b\, c^{2} d^{2} \_Z^{2}+4 b\, c^{3} d \_Z+b\, c^{4}+a\right)} \frac{\_R \ln \left(x-\_R\right)}{d^{3} \_R^{3}+3 c\, d^{2} \_R^{2}+3 c^{2} d \_R+c^{3}}}{4 b d}$	95
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(d^{4} b \_Z^{4}+4 b c\, d^{3} \_Z^{3}+6 b\, c^{2} d^{2} \_Z^{2}+4 b\, c^{3} d \_Z+b\, c^{4}+a\right)} \frac{\_R \ln \left(x-\_R\right)}{d^{3} \_R^{3}+3 c\, d^{2} \_R^{2}+3 c^{2} d \_R+c^{3}}}{4 b d}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z
^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 17.85, size = 40785, normalized size = 156.26

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c^2/(a^2*b*d^4*sqrt(-1/(a*b)))) + sqrt(-1/(a*b))/d^2)*log(2*a^3
*b*d^6*(sqrt(-c^2/(a^2*b*d^4*sqrt(-1/(a*b)))) + sqrt(-1/(a*b))/d^2)^3 - a^2
*b*c^2*d^4*(sqrt(-c^2/(a^2*b*d^4*sqrt(-1/(a*b)))) + sqrt(-1/(a*b))/d^2)^2 +
 b*c^6 + (a*b*c^4 + 2*a^2)*d^2*(sqrt(-c^2/(a^2*b*d^4*sqrt(-1/(a*b)))) + sqr
t(-1/(a*b))/d^2) + a*c^2 + (b*c^5 - 4*a*c)*d*x) + 1/288*((-I*sqrt(3) + 1)*(
(a*b*d^2*sqrt(-1/(a*b))*sqrt(-c^2/(a^2*b*d^4*sqrt(-1/(a*b)))) - 1)^2/(a*b*d
```

```
^4) - 3*((2*d^2*sqrt(-c^2/(a^2*b*d^4*sqrt(-1/(a*b)))) - sqrt(-1/(a*b)))*a +
 c^2)/(a^2*b*d^4*sqrt(-1/(a*b))))/(-1/384*(a*b*d^2*sqrt(-1/(a*b))*sqrt(-c^2
/(a^2*b*d^4*sqrt(-1/(a*b)))) - 1)*((2*d^2*sqrt(-c^2/(a^2*b*d^4*sqrt(-1/(a*b
)))) - sqrt(-1/(a*b)))*a + c^2)/(a^2*b*d^6) + 1/128*(a^2*b^2*d^6*sqrt(-1/(a
*b))*(-c^2/(a^2*b*d^4*sqrt(-1/(a*b))))^(3/2) - a*b*d^2*sqrt(-1/(a*b))*sqrt(
-c^2/(a^2*b*d^4*sqrt(-1/(a*b)))) + b*c^2*sqrt(-1/(a*b)) - 1)/(a^2*b^2*d^6*s
qrt(-1/(a*b))) + 1/1728*(a*b*d^2*sqrt(-1/(a*b))*sqrt(-c^2/(a^2*b*d^4*sqrt(-
1/(a*b)))) - 1)^3/(a^3*b^ ...
```

Sympy [A]
time = 0.53, size = 131, normalized size = 0.50

$$\operatorname{RootSum}\left(256t^4a^3b^2d^8+32t^2a^2bd^4-16tabc^2d^2+a+bc^4,\left(t\mapsto t\log\left(x+\frac{128t^3a^3bd^6+16t^2a^2bc^2d^4+8ta^2d^2+4tabc^4d^2-ac^2-bc^6}{4acd-bc^5d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d*
*2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*a*
*2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/(4*
a*c*d - b*c**5*d))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

Mupad [B]
time = 2.36, size = 205, normalized size = 0.79

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\,\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\,\left(16\,a\,x\,b^3\,d^{12}+32\,a\,c\,b^3\,d^{11}\right)+4\,b^3\,c^3\,d^9+4\,b^3\,c^2\,d^{10}\,x\right)+b^2\,d^8\,x\right)\,\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b
*c^2*d^2*z + b*c^4 + a, z, k)*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 + 3
2*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k)*(32*a*b^3*c*d^11 + 16
*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4
*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)
```

3.113 $\int \frac{1}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=221

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)+1/
4*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/d*2^(1/2)-1/8*l
n(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))/a^(3/4)/b^(1/
4)/d*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/
2))/a^(3/4)/b^(1/4)/d*2^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {253, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1/
4)*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
b^(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c +
 d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c-\right.}{4\sqrt{a}\sqrt{b}\,d}$$
$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{}}$$

Mathematica [A]
time = 0.06, size = 161, normalized size = 0.73

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d
*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x
) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 94, normalized size = 0.43

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^4 b\_Z^4+4bc\,d^3\_Z^3+6b\,c^2 d^2\_Z^2+4b\,c^3 d\_Z+b\,c^4+a\right)} \frac{\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2 d\_R+c^3}}{4bd}$	94

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(d^4b\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^
4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^4*b + a), x)
```

Fricas [A]
time = 0.43, size = 185, normalized size = 0.84

$$\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2d^2\sqrt{-\frac{1}{a^3bd^4}}+d^2x^2+2\,cdx+c^2}\,a^2bd^3\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}-\left(a^2bd^4x+a^2bcd^3\right)\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*b*d^4))^(1/4)*arctan(sqrt(a^2*d^2*sqrt(-1/(a^3*b*d^4)) + d^2*x^2 +
 2*c*d*x + c^2)*a^2*b*d^3*(-1/(a^3*b*d^4))^(3/4) - (a^2*b*d^4*x + a^2*b*c*d
^3)*(-1/(a^3*b*d^4))^(3/4)) + 1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/(a^3*b
*d^4))^(1/4) + d*x + c) - 1/4*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^3*b*d^
4))^(1/4) + d*x + c)
```

Sympy [A]
time = 0.14, size = 26, normalized size = 0.12

$$\frac{\mathrm{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(x+\frac{4ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d
```

Giac [A]
time = 6.67, size = 103, normalized size = 0.47

$$-\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{bdx+bc}{(-ab^3)^{\frac{1}{4}}}\right)+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|bdx+bc+(-ab^3)^{\frac{1}{4}}\right|\right)-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|-bdx-bc+(-ab^3)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(-1/(a^3*b*d^4))^(1/4)*arctan(-(b*d*x + b*c)/(-a*b^3)^(1/4)) + 1/4*(-1
/(a^3*b*d^4))^(1/4)*log(abs(b*d*x + b*c + (-a*b^3)^(1/4))) - 1/4*(-1/(a^3*b
*d^4))^(1/4)*log(abs(-b*d*x - b*c + (-a*b^3)^(1/4)))
```

Mupad [B]
time = 0.12, size = 60, normalized size = 0.27

$$-\frac{\operatorname{atan}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,b^{1/4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^4),x)
```

```
[Out] -(atan((b^(1/4)*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)) + atanh((b^(1/4)*
c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)))/(2*(-a)^(3/4)*b^(1/4)*d)
```

3.114 $\int \frac{1}{x(a+b(c+dx)^4)}\,dx$

Optimal. Leaf size=393

$$-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+bc^4)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,(a+bc^4)}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}}{2\sqrt{2}\,a^{3/4}\,(}$$

```
[Out] ln(x)/(b*c^4+a)-1/4*ln(a+b*(d*x+c)^4)/(b*c^4+a)-1/2*c^2*arctan((d*x+c)^2*b^
(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)/a^(1/2)-1/8*b^(1/4)*c*ln(-a^(1/4)*b^(1/4)*
(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b
*c^4+a)*2^(1/2)+1/8*b^(1/4)*c*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d
*x+c)^2*b^(1/2))*(a^(1/2)-b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4
)*c*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*(a^(1/2)+b^(1/2)*c^2)/a^(3/4
)/(b*c^4+a)*2^(1/2)-1/4*b^(1/4)*c*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))
*(a^(1/2)+b^(1/2)*c^2)/a^(3/4)/(b*c^4+a)*2^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {378, 6857, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,(a+bc^4)}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\,(a+bc^4)}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\,(a+bc^4)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\,(a+bc^4)}-\frac{\sqrt{b}\,c^2\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+bc^4)}-\frac{\log\left(a+b(c+dx)^4\right)}{4\,(a+bc^4)}+\frac{\log(x)}{a+bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -1/2*(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c^
4)) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d
*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + Sqrt
[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(a + b*c^4)) + Log[x]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sq
rt[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(
3/4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx &= \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^4\right)}\,dx, x, c+dx\right) \\
&= \text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^4\right)(c-x)}-\frac{b(c^3+c^2x+cx^2+x^3)}{\left(a+bc^4\right)\left(a+bx^4\right)}\right)dx, x, c+dx\right) \\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{c^3+c^2x+cx^2+x^3}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4} \\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \left(\frac{x(c^2+x^2)}{a+bx^4}+\frac{c^3+cx^2}{a+bx^4}\right)dx, x, c+dx\right)}{a+bc^4} \\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{x(c^2+x^2)}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{c^3+cx^2}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4} \\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)}+\frac{\left(c\left(1-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{}}{a+b}\right.}{2\left(a+bc^4\right)} \\
&= \frac{\log(x)}{a+bc^4}-\frac{b\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)}-\frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right.}{2\left(a+bc^4\right)} \\
&= -\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(a+bc^4\right)}+\frac{\log(x)}{a+bc^4}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{}\right.}{4\sqrt{2}\,a^{3/4}\left(a\right.} \\
&= -\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(a+bc^4\right)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 163, normalized size = 0.41

$$
-\frac{-4\log(x)+\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&, \frac{4c^3\log(x-\#1)+6c^2d\log(x-\#1)\#1+4cd^2\log(x-\#1)\#1^2+d^3\log(x-\#1)\#1^3}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4\left(a+bc^4\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -1/4*(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b
*c*d^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1 +
```

4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &])/(a + b*c^4)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 139, normalized size = 0.35

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(d^4 b\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\left(d^3\_R^3+4c\,d^2\_R^2+6c^2d\_R+4c^3\right)\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4(b\,c^4+a)}+\frac{\ln(x)}{b\,c^4+a}$
risch	$\frac{\ln(x)}{b\,c^4+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^3b\,c^4+a^4\right)\_Z^4+4\_Z^3a^3+6a^2\_Z^2+4a\_Z\right)}\_R\ln\left(\left(\left(-3a^2b\,c^4d+5a^3d\right)\_R^3+\left(-3ab\,c^4d+15d\,a^2\right)\_R\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)

[Out] -1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))+ln(x)/(b*c^4+a)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] -b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b*c^4 + a) + log(x)/(b*c^4 + a)

Fricas [C] Result contains complex when optimal does not.
time = 5.97, size = 307773, normalized size = 783.14

too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")

[Out] 1/48*(2*(b*c^4 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((b*c^2 - (a^2*sqrt((b^2*c^6 - 2*a*b*c^4*sqrt(-b/a) - a*b*c^2)/(a^2*b^2*c^8*sqrt(-b/a) + 2*a^3*b*c^4*sqrt(-b/a) + a^4*sqrt(-b/a))) + (b*c^4*sqrt((b^2*c^6 - 2*a*b*c^4*sqrt(-b/a) - a*b*c^2)/(a^2*b^2*c^8*sqrt(-b/a) + 2*a^3*b*c^4*sqrt(-b/a) + a^4*sqrt(

```
-b/a))) - 3)*a)*sqrt(-b/a))^2*a/((a*b*c^4 + a^2)^2*b) - 3*(2*a*b*c^2*sqrt((
b^2*c^6 - 2*a*b*c^4*sqrt(-b/a) - a*b*c^2)/(a^2*b^2*c^8*sqrt(-b/a) + 2*a^3*b
*c^4*sqrt(-b/a) + a^4*sqrt(-b/a))) - b*c^2 + (2*a^2*sqrt((b^2*c^6 - 2*a*b*c
^4*sqrt(-b/a) - a*b*c^2)/(a^2*b^2*c^8*sqrt(-b/a) + 2*a^3*b*c^4*sqrt(-b/a) +
 a^4*sqrt(-b/a))) - 3*a)*sqrt(-b/a))/((a^2*b*c^4 + a^3)*sqrt(-b/a)))/(9*(2*
a*b*c^2*sqrt((b^2*c^6 - 2*a*b*c^4*sqrt(-b/a) - a*b*c^2)/(a^2*b^2*c^8*sqrt(-
b/a) + 2*a^3*b*c^4*sqrt(-b/a) + a^4*sqrt(-b/a))) - b*c^2 + (2*a^2*sqrt((b^2
*c^6 - 2*a*b*c^4*sqrt(-b/a) - a*b*c^2)/(a^2*b^2*c^8*sqrt(-b/a) + 2*a^3*b*c^
4*sqrt(-b/a) + a^4*sqrt(-b/a))) - 3*a)*sqrt(-b/a))*(b*c^2 - (a^2*sqrt((b^2*
c^6 - 2*a*b*c^4*sqrt(-b/a ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x), x)
```

Mupad [B]
time = 2.18, size = 882, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^4)),x)
```

```
[Out] log(x)/(a + b*c^4) + symsum(log(4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 25
6*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*c*d^15 - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
5*d^15 + 5*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2
+ 16*a*z + 1, z, k)*b^4*d^16*x - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 +
256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^5*d^15 + 28*root(2
56*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z,
```

```
k)^2*a*b^4*c*d^15 + 60*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 +
 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*d^16*x + 32*root(256*a^3*b*c^4*z^4
+ 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^2*b^4*c*d^
15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 1
6*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4
 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^15 + 240*root
(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z
, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*
z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z
^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256*a^3*b*c^4*z^4 + 25
6*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^4*d^16
*x)*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*
z + 1, z, k), k, 1, 4)
```

3.115 $\int \frac{1}{x^2(a+b(c+dx)^4)}\,dx$

Optimal. Leaf size=496

$$-\frac{1}{(a+bc^4)\,x}-\frac{\sqrt{b}\,c(a-bc^4)\,d\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2}+\frac{\sqrt[4]{b}\left(\sqrt{a}\,(a-3bc^4)+\sqrt{b}\,c^2(3a-bc^4)\right)d\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}\,(a+bc^4)^2}$$

```
[Out] -1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2+b*c^3*d*ln(a+b*(d*x+c)^4)/(b*c^4
+a)^2-c*(-b*c^4+a)*d*arctan((d*x+c)^2*b^(1/2)/a^(1/2))*b^(1/2)/(b*c^4+a)^2/
a^(1/2)-1/8*b^(1/4)*d*ln(-a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)^2
*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)
^2*2^(1/2)+1/8*b^(1/4)*d*ln(a^(1/4)*b^(1/4)*(d*x+c)*2^(1/2)+a^(1/2)+(d*x+c)
^2*b^(1/2))*((-3*b*c^4+a)*a^(1/2)-c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+
a)^2*2^(1/2)-1/4*b^(1/4)*d*arctan(-1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*
b*c^4+a)*a^(1/2)+c^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)-1/4*
b^(1/4)*d*arctan(1+b^(1/4)*(d*x+c)*2^(1/2)/a^(1/4))*((-3*b*c^4+a)*a^(1/2)+c
^2*(-b*c^4+3*a)*b^(1/2))/a^(3/4)/(b*c^4+a)^2*2^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {378, 6857, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,(a-3bc^4)+\sqrt{b}\,c^2(3a-bc^4)\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,(a+bc^4)^2}-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,(a-3bc^4)+\sqrt{b}\,c^2(3a-bc^4)\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\,(a+bc^4)^2}-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,(a-3bc^4)-\sqrt{b}\,c^2(3a-bc^4)\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\,(a+bc^4)^2}+\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\,(a-3bc^4)-\sqrt{b}\,c^2(3a-bc^4)\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\,(a+bc^4)^2}-\frac{\sqrt{b}\,cd(a-bc^4)\,\text{ArcTan}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2}-\frac{1}{x\,(a+bc^4)}-\frac{4bc^3d\log(x)}{(a+bc^4)^2}+\frac{bc^3d\log\left(a+b(c+dx)^4\right)}{(a+bc^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(c + d*x)^2
)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqr
t[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[
b]*c^2*(3*a - b*c^4))*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/
4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a
 + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))
*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/
(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b
*c^4)^2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (a+b(c+dx)^4)}\,dx = d\text{Subst}\left(\int \frac{1}{(-c+x)^2 (a+bx^4)}\,dx, x, c+dx\right)$$

$$= d\text{Subst}\left(\int \left(\frac{1}{(a+bc^4)(c-x)^2} + \frac{4bc^3}{(a+bc^4)^2 (c-x)} + \frac{b(-c^2(3a-bc^4) - 2c(a}{(a+}\right.\right.$$

$$= -\frac{1}{(a+bc^4)x} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \frac{-c^2(3a-bc^4)-2c(a-bc^4)x-(a-3bc^4)x^2+4bc^3}{a+bx^4}\right.}{(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)x} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \left(\frac{x(-2c(a-bc^4)+4bc^3x^2)}{a+bx^4} + \frac{-c^2(3a-bc^4)+}{a+b}\right.\right.}{(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)x} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \frac{x(-2c(a-bc^4)+4bc^3x^2)}{a+bx^4}\,dx, x, c+dx\right)}{(a+bc^4)^2}$$

$$= -\frac{1}{(a+bc^4)x} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2} + \frac{(bd)\text{Subst}\left(\int \frac{-2c(a-bc^4)+4bc^3x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2(a+bc^4)^2} +$$

$$= -\frac{1}{(a+bc^4)x} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2} + \frac{(2b^2c^3d)\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{(a+bc^4)^2} - \frac{(bc(a}{}$$

$$= -\frac{1}{(a+bc^4)x} - \frac{\sqrt{b}\,c(a-bc^4)\,d\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2} - \frac{4bc^3 d\log(x)}{(a+bc^4)^2} - \frac{\sqrt[4]{b}\,\left(a\right.}{}$$

$$= -\frac{1}{(a+bc^4)x} - \frac{\sqrt{b}\,c(a-bc^4)\,d\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2} + \frac{\sqrt[4]{b}\,\left(a-3bc^4+\frac{\sqrt{b}\,c^2(}{\sqrt{}}\right.}{2\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 238, normalized size = 0.48

$$\frac{-4(a+bc^4+4bc^3dx\log(x)) + dx\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&, \frac{-6ac^2\log(x-\#1)+10bc^6\log(x-\#1)-4acd\log(x-\#1)\#1+20bc^5d\log(x-\#1)\#1-ad^2\log(x-\#1)\#1^2+15bc^4d^2\log(x-\#1)\#1^2+4bc^3d^3\log(x-\#1)\#1^3}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4(a+bc^4)^2 x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] (-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*b*c^3*d*#1
 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (-6*a*c^2*Log[x - #1]
```

```
 + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]*#1 + 20*b*c^5*d*Log[x - #1]*#
1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^3*Lo
g[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(a + b
*c^4)^2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 184, normalized size = 0.37

method	result
default	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(d^4 b\_Z^4+4bc\,d^3\_Z^3+6b\,c^2 d^2\_Z^2+4b\,c^3 d\_Z+b\,c^4+a\right)}\frac{\left(-4d^3 b\,c^3\_R^3+d^2\left(-15b\,c^4+a\right)\_R^2+4cd\left(-5b\,c^4+a\right)\_R-10b\,c^{6}\right.}{d^3\_R^3+3c\,d^2\_R^2+3c^2 d\_R+c^3}\right.}{4\left(b\,c^4+a\right)^2}$
risch	$-\frac{1}{\left(b\,c^4+a\right)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(b^2 a^3 c^8+2a^4 b\,c^4+a^5\right)\_Z^4-16a^3 b\,c^3 d\_Z^3+20a^2 b\,c^2 d^2\_Z^2-8abc\,d^3\_Z+d^4 b\right)}\_R\ln\left(\left(\left(\left(-3a^2 b^3 c^{12} d-a^3 b\right.\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/(b*c^4+a)^2*sum((-4*d^3*b*c^3*_R^3+d^2*(-15*b*c^4+a)*_R^2+4*c*d*(-5*
b*c^4+a)*_R-10*b*c^6+6*c^2*a)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_
R),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+
a))-1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -4*b*c^3*d*log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*integrate((4*b*c^3*d^
3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d*x
)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a),
x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)
```

Fricas [C] Result contains complex when optimal does not.
time = 119.92, size = 1128605, normalized size = 2275.41

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/48*(192*b*c^3*d*x*log(x) + 48*b*c^4 - 12*(b^2*c^8 + 2*a*b*c^4 + a^2)*(4*
b*c^3*d/(b^2*c^8 + 2*a*b*c^4 + a^2) - 2*(b*c^4 - a)*c*d*sqrt(-b/a)/(b*c^4 +
 a)^2 - sqrt(32*b^2*c^6*d^2/(b^2*c^8 + 2*a*b*c^4 + a^2)^2 - 6*b*c^2*d^2/(a*
b^2*c^8 + 2*a^2*b*c^4 + a^3) - (64*b^3*c^9*d^3/(b^2*c^8 + 2*a*b*c^4 + a^2)^
3 - 20*b^2*c^5*d^3/((a*b^2*c^8 + 2*a^2*b*c^4 + a^3)*(b^2*c^8 + 2*a*b*c^4 +
a^2)) + b*c*d^3/(a^2*b^2*c^8 + 2*a^3*b*c^4 + a^4))*(b*c^4 + a)^2/((b*c^4 -
a)*c*d*sqrt(-b/a))))*x*log((b^3*c^8 - a^2*b)*d^4*x + (5*a^3*b^3*c^12 + 11*a
^4*b^2*c^8 + 7*a^5*b*c^4 + a^6)*(4*b*c^3*d/(b^2*c^8 + 2*a*b*c^4 + a^2) - 2*
(b*c^4 - a)*c*d*sqrt(-b/a)/(b*c^4 + a)^2 - sqrt(32*b^2*c^6*d^2/(b^2*c^8 + 2
*a*b*c^4 + a^2)^2 - 6*b*c^2*d^2/(a*b^2*c^8 + 2*a^2*b*c^4 + a^3) - (64*b^3*c
^9*d^3/(b^2*c^8 + 2*a*b*c^4 + a^2)^3 - 20*b^2*c^5*d^3/((a*b^2*c^8 + 2*a^2*b
*c^4 + a^3)*(b^2*c^8 + 2*a*b*c^4 + a^2)) + b*c*d^3/(a^2*b^2*c^8 + 2*a^3*b*c
^4 + a^4))*(b*c^4 + a)^2/((b*c^4 - a)*c*d*sqrt(-b/a))))^3 + 2*(a^2*b^3*c^11
 - 38*a^3*b^2*c^7 - 7*a^4 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)
```

Mupad [**B**]
time = 2.48, size = 2440, normalized size = 4.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^4)),x)
```

```
[Out] symsum(log(-(4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
k)^2*b^7*c^11*d^17 - 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*
```

a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^4*b^4*d^16 - b^5*d^20*x + 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^18 - 60*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^17 + 176*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^4*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*c^5*d^15 + 144*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^8*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^9*d^15 + 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^13*d^15 + 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^5*d^19*x + 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*c*d^15 - 184*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^7*d^17 - 48*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^12*d^16 - 320*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*d^16*x + 4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*b^7*c^10*d^18*x - 248*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^6*d^18*x - 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^11*d^17*x + 32*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*a*b^5*c*d^19*x - 316*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^2*d^18*x + 704*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^3*d^17*x - 448*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*c^4*d^16*x + 640*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^7*d^17*x + 64*root(256*a^3*b^2*

```
c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*
b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^8*d^16*x + 192*ro
ot(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d
*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^12
*d^16*x)/(a^2 + b^2*c^8 + 2*a*b*c^4))*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*
c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a
*b*c*d^3*z + b*d^4, z, k), k, 1, 4) - 1/(a*x + b*c^4*x) - (4*b*c^3*d*log(x)
)/(a^2 + b^2*c^8 + 2*a*b*c^4)
```

3.116 $\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=123

$$-\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5+\frac{8}{7}(3+a)(5+3a)(-1+x)^7-\frac{2}{9}\left(37+6a-3a^2\right)(-1+x)^9-\frac{8}{11}(5+$$

```
[Out] -8/3*(3+a)^3*(-1+x)^3+4/5*(3-a)*(3+a)^2*(-1+x)^5+8/7*(3+a)*(5+3*a)*(-1+x)^7
-2/9*(-3*a^2+6*a+37)*(-1+x)^9-8/11*(5+3*a)*(-1+x)^11+4/13*(3-a)*(-1+x)^13+8
/15*(-1+x)^15+1/17*(-1+x)^17+(3+a)^4*x
```

Rubi [A]
time = 0.17, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1120, 1104}

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}(a+3)^3(x-1)^3+(a+3)^4x+\frac{1}{17}(x-1)^{17}+\frac{8}{15}(x-1)^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 +
a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 +
3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 + (8*(-1 + x)^15)/15 + (-
1 + x)^17/17 + (3 + a)^4*x
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^4\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \left(81\left(1+\frac{1}{81}a\left(108+54a+12a^2+a^3\right)\right)-216\left(1+a\left(1+\right.\right.\right.\right.$$

$$= -\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5+\frac{8}{7}(3+a)(5+3a$$

Mathematica [A]
time = 0.02, size = 195, normalized size = 1.59

$$a^4x+16a^3x^2-\frac{32}{3}(-12+a)a^2x^3+4a\left(128-48a+a^2\right)x^4-\frac{4}{5}\left(-1024+1536a-192a^2+a^3\right)x^5-\frac{16}{3}\left(512-288a+15a^2\right)x^6+\frac{64}{7}\left(512-140a+3a^2\right)x^7-6\left(896-128a+a^2\right)x^8+\frac{2}{9}\left(20480-1536a+3a^2\right)x^9+\frac{16}{5}(-928+35a)x^{10}-\frac{32}{11}(-524+9a)x^{11}+\frac{4}{3}(-464+3a)x^{12}-\frac{4}{13}(-640+a)x^{13}-48x^{14}+\frac{128x^{15}}{15}-x^{16}+\frac{x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4
- (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)*x
^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (2*(
20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524 + 9*
a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^14 +
(128*x^15)/15 - x^16 + x^17/17
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(107) = 214$.
time = 0.02, size = 264, normalized size = 2.15

method	result
norman	$a^4x+16a^3x^2+\left(-\frac{32}{3}a^3+128a^2\right)x^3+\left(4a^3-192a^2+512a\right)x^4+\left(-\frac{4}{5}a^3+\frac{768}{5}a^2-\frac{6144}{5}a+\frac{4096}{5}\right.$
gosper	$-5376x^8+\frac{40960}{9}x^9+\frac{32768}{7}x^7-\frac{8192}{3}x^6-\frac{4}{13}x^{13}a-6a^2x^8+768a\,x^8-\frac{288}{11}x^{11}a+4x^{12}a-\frac{1024}{3}x^9a$
risch	$-5376x^8+\frac{40960}{9}x^9+\frac{32768}{7}x^7-\frac{8192}{3}x^6-\frac{4}{13}x^{13}a-6a^2x^8+768a\,x^8-\frac{288}{11}x^{11}a+4x^{12}a-\frac{1024}{3}x^9a$
default	$\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+\frac{(-4a+2560)x^{13}}{13}+\frac{(48a-7424)x^{12}}{12}+\frac{(-288a+16768)x^{11}}{11}+\frac{(1120a-29696)x^{10}}{10}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*x^17-x^16+128/15*x^15-48*x^14+1/13*(-4*a+2560)*x^13+1/12*(48*a-7424)*x
^12+1/11*(-288*a+16768)*x^11+1/10*(1120*a-29696)*x^10+1/9*(2*a^2-2560*a+245
76+(-2*a+128)^2)*x^9+1/8*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128))*x^8+
1/7*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^7+1/6*(-160*a^2+3
```

```
2*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^6+1/5*(2*a^2*(-2*a+128)+32*a*(8*a-
128)+(-16*a+64)^2)*x^5+1/4*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^4+1/3*(2*a^2
*(-16*a+64)+256*a^2)*x^3+16*a^3*x^2+a^4*x
```

Maxima [A]
time = 0.28, size = 192, normalized size = 1.56

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{2560}{13}x^{13} - \frac{1856}{3}x^{12} + \frac{16768}{11}x^{11} - \frac{14848}{5}x^{10} + \frac{40960}{9}x^9 - 5376x^8 + \frac{32768}{7}x^7 - \frac{8192}{3}x^6 + a^4x + \frac{4096}{5}x^5 - \frac{4}{15}(3x^5 - 15x^4 + 40x^3 - 60x^2)a^3 + \frac{2}{105}(35x^9 - 315x^8 + 1440x^7 - 4200x^6 + 8064x^5 - 10080x^4 + 6720x^3)a^2 - \frac{4}{2145}(165x^{13} - 2145x^{12} + 14040x^{11} - 60060x^{10} + 183040x^9 - 411840x^8 + 686400x^7 - 823680x^6 + 658944x^5 - 274560x^4)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 167
68/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3*x
^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 + 2/1
05*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^
3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 183040*x^
9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

Fricas [A]
time = 0.36, size = 179, normalized size = 1.46

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - \frac{4}{13}(a - 640)x^{13} - 48x^{14} + \frac{4}{3}(3a - 464)x^{12} - \frac{32}{11}(9a - 524)x^{11} + \frac{16}{5}(35a - 928)x^{10} + \frac{2}{9}(3a^2 - 1536a + 20480)x^9 - 6(a^2 - 128a + 896)x^8 + \frac{64}{7}(3a^2 - 140a + 512)x^7 - \frac{16}{3}(15a^2 - 288a + 512)x^6 - \frac{4}{5}(a^3 - 192a^2 + 1536a - 1024)x^5 + a^4x + 16a^3x^2 + 4(a^3 - 48a^2 + 128a)x^4 - \frac{32}{3}(a^3 - 12a^2)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*(a - 640)*x^13 - 48*x^14 + 4/3*(3*a -
 464)*x^12 - 32/11*(9*a - 524)*x^11 + 16/5*(35*a - 928)*x^10 + 2/9*(3*a^2 -
 1536*a + 20480)*x^9 - 6*(a^2 - 128*a + 896)*x^8 + 64/7*(3*a^2 - 140*a + 51
2)*x^7 - 16/3*(15*a^2 - 288*a + 512)*x^6 - 4/5*(a^3 - 192*a^2 + 1536*a - 10
24)*x^5 + a^4*x + 16*a^3*x^2 + 4*(a^3 - 48*a^2 + 128*a)*x^4 - 32/3*(a^3 - 1
2*a^2)*x^3
```

Sympy [A]
time = 0.04, size = 199, normalized size = 1.62

$$a^4x + 16a^3x^2 + \frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 48x^{14} + x^{13}\cdot\left(\frac{2560}{13} - \frac{4a}{13}\right) + x^{12}\cdot\left(4a - \frac{1856}{3}\right) + x^{11}\cdot\left(\frac{16768}{11} - \frac{288a}{11}\right) + x^{10}\cdot\left(112a - \frac{14848}{5}\right) + x^9\cdot\left(\frac{2a^2}{3} - \frac{1024a}{3} + \frac{40960}{9}\right) + x^8(-6a^2 + 768a - 5376) + x^7\cdot\left(\frac{192a^2}{7} - 1280a + \frac{32768}{7}\right) + x^6\left(-80a^2 + 1536a - \frac{8192}{3}\right) + x^5\left(-\frac{4a^3}{5} + \frac{768a^2}{5} - \frac{6144a}{5} + \frac{4096}{5}\right) + x^4\cdot(4a^3 - 192a^2 + 512a) + x^3\left(-\frac{32a^3}{3} + 128a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13*
(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) + x
**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6*a*
*2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a**2
```

```
+ 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5) + x*
*4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(103) = 206.
time = 4.05, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - \frac{4}{13}ax^{13} - 48x^{14} + 4ax^{12} + \frac{2560}{13}x^{13} - \frac{288}{11}ax^{11} - \frac{1856}{3}x^{12} + \frac{2}{3}a^2x^9 + 112ax^{10} + \frac{16768}{11}x^{11} - 6a^2x^8 - \frac{1024}{3}ax^9 - \frac{14848}{5}x^{10} + \frac{192}{7}a^2x^7 + 768ax^8 + \frac{40960}{9}x^9 - \frac{4}{5}a^3x^5 - 80a^2x^6 - 1280ax^7 - 5376x^8 + 4a^3x^4 + \frac{768}{5}a^2x^5 + 1536ax^6 + \frac{32768}{7}x^7 - \frac{32}{3}a^3x^3 - 192a^2x^4 - \frac{6144}{5}ax^5 - \frac{8192}{3}x^6 + a^4x + 16a^3x^2 + 128a^2x^3 + 512ax^4 + \frac{4096}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12 + 2560/13
*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112*a*x^10 + 16768/11*x
^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10 + 192/7*a^2*x^7 + 768*a*x^8 +
 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3*x^4
 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x^4 -
6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*x^4 +
4096/5*x^5
```

Mupad [B]
time = 0.21, size = 175, normalized size = 1.42

$$x^{12}\left(4a - \frac{1856}{3}\right) - x^{13}\left(\frac{4a}{13} - \frac{2560}{13}\right) + x^{10}\left(112a - \frac{14848}{5}\right) - x^{11}\left(\frac{288a}{11} - \frac{16768}{11}\right) - x^8\left(6a^2 - 768a + 5376\right) - x^6\left(80a^2 - 1536a + \frac{8192}{3}\right) + x^7\left(\frac{192a^2}{7} - 1280a + \frac{32768}{7}\right) + x^9\left(\frac{2a^2}{3} - \frac{1024a}{3} + \frac{40960}{9}\right) - x^5\left(\frac{4a^3}{5} - \frac{768a^2}{5} + \frac{6144a}{5} - \frac{4096}{5}\right) + a^4x - 48x^{14} + \frac{128x^{15}}{15} - x^{16} + \frac{x^{17}}{17} + 16a^3x^2 + 4ax^4\left(a^2 - 48a + 128\right) - \frac{32a^2x^3(a-12)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^12*(4*a - 1856/3) - x^13*((4*a)/13 - 2560/13) + x^10*(112*a - 14848/5) -
x^11*((288*a)/11 - 16768/11) - x^8*(6*a^2 - 768*a + 5376) - x^6*(80*a^2 - 1
536*a + 8192/3) + x^7*((192*a^2)/7 - 1280*a + 32768/7) + x^9*((2*a^2)/3 - (
1024*a)/3 + 40960/9) - x^5*((6144*a)/5 - (768*a^2)/5 + (4*a^3)/5 - 4096/5)
+ a^4*x - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17 + 16*a^3*x^2 + 4*a*x^4*(
a^2 - 48*a + 128) - (32*a^2*x^3*(a - 12))/3
```

3.117 $\int (a+8x-8x^2+4x^3-x^4)^3 \, dx$

Optimal. Leaf size=120

$$a^3x+12a^2x^2+8(8-a)ax^3+\left(128-96a+3a^2\right)x^4-\frac{3}{5}\left(512-128a+a^2\right)x^5+8(48-5a)x^6-\frac{32}{7}(70-3a)x^7+3(64-$$

```
[Out] a^3*x+12*a^2*x^2+8*(8-a)*a*x^3+(3*a^2-96*a+128)*x^4-3/5*(a^2-128*a+512)*x^5
+8*(48-5*a)*x^6-32/7*(70-3*a)*x^7+3*(64-a)*x^8-1/3*(256-a)*x^9+28*x^10-72/1
1*x^11+x^12-1/13*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2086}

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48-5a)x^6+8(8-a)ax^3-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 -
 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a)*x^7)/7 + 3*(64 - a
)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = \int \left(a^3+24a^2x+24(8-a)ax^2+4\left(128-96a+3a^2\right)x^3-3\left(512-128\right.\right.$$
$$= a^3x+12a^2x^2+8(8-a)ax^3+\left(128-96a+3a^2\right)x^4-\frac{3}{5}\left(512-128a+\right.$$

Mathematica [A]
time = 0.01, size = 114, normalized size = 0.95

$$a^3x+12a^2x^2-8(-8+a)ax^3+\left(128-96a+3a^2\right)x^4-\frac{3}{5}\left(512-128a+a^2\right)x^5-8(-48+5a)x^6+\frac{32}{7}(-70+3a)x^7-3(-64+a)x^8+\frac{1}{3}(-256+a)x^9+28x^{10}-\frac{72x^{11}}{11}+x^{12}-\frac{x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512
- 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-64
 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Maple [A]
time = 0.02, size = 138, normalized size = 1.15

method	result
norman	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\left(\frac{a}{3}-\frac{256}{3}\right)x^{9}+\left(-3a+192\right)x^{8}+\left(\frac{96a}{7}-320\right)x^{7}+\left(-40a+384\right)x$
gosper	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}x^{7}a-320x^{7}-40a\,x^{6}+384$
risch	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^{9}a-\frac{256}{3}x^{9}-3a\,x^{8}+192x^{8}+\frac{96}{7}x^{7}a-320x^{7}-40a\,x^{6}+384$
default	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\frac{(3a-768)x^{9}}{9}+\frac{(-24a+1536)x^{8}}{8}+\frac{(96a-2240)x^{7}}{7}+\frac{(-240a+2304)x^{6}}{6}+\frac{(a(-2a+12$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*x^13+x^12-72/11*x^11+28*x^10+1/9*(3*a-768)*x^9+1/8*(-24*a+1536)*x^8+1
/7*(96*a-2240)*x^7+1/6*(-240*a+2304)*x^6+1/5*(a*(-2*a+128)+256*a-1536-a^2)*
x^5+1/4*(a*(8*a-128)-256*a+512+4*a^2)*x^4+1/3*(a*(-16*a+64)+128*a-8*a^2)*x^
3+12*a^2*x^2+a^3*x
```

Maxima [A]
time = 0.30, size = 119, normalized size = 0.99

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}-\frac{256}{3}x^{9}+192x^{8}-320x^{7}+384x^{6}-\frac{1536}{5}x^{5}+a^{3}x+128x^{4}-\frac{1}{5}\left(3x^{5}-15x^{4}+40x^{3}-60x^{2}\right)a^{2}+\frac{1}{105}\left(35x^{9}-315x^{8}+1440x^{7}-4200x^{6}+8064x^{5}-10080x^{4}+6720x^{3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 - 320*x^7 +
384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 60*
x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080
*x^4 + 6720*x^3)*a
```

Fricas [A]
time = 0.36, size = 107, normalized size = 0.89

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+\frac{1}{3}(a-256)x^{9}+28x^{10}-3(a-64)x^{8}+\frac{32}{7}(3a-70)x^{7}-8(5a-48)x^{6}-\frac{3}{5}\left(a^{2}-128a+512\right)x^{5}+\left(3a^{2}-96a+128\right)x^{4}+a^{3}x+12a^{2}x^{2}-8\left(a^{2}-8a\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*(a - 256)*x^9 + 28*x^10 - 3*(a - 64)*x
^8 + 32/7*(3*a - 70)*x^7 - 8*(5*a - 48)*x^6 - 3/5*(a^2 - 128*a + 512)*x^5 +
 (3*a^2 - 96*a + 128)*x^4 + a^3*x + 12*a^2*x^2 - 8*(a^2 - 8*a)*x^3
```

Sympy [A]
time = 0.03, size = 114, normalized size = 0.95

$$a^3x + 12a^2x^2 - \frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10} + x^9\left(\frac{a}{3} - \frac{256}{3}\right) + x^8 \cdot (192 - 3a) + x^7 \cdot \left(\frac{96a}{7} - 320\right) + x^6 \cdot (384 - 40a) + x^5\left(-\frac{3a^2}{5} + \frac{384a}{5} - \frac{1536}{5}\right) + x^4 \cdot (3a^2 - 96a + 128) + x^3(-8a^2 + 64a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(a
/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a) +
x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*(-8
*a**2 + 64*a)
```

Giac [A]
time = 3.19, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + \frac{1}{3}ax^9 + 28x^{10} - 3ax^8 - \frac{256}{3}x^9 + \frac{96}{7}ax^7 + 192x^8 - \frac{3}{5}a^2x^5 - 40ax^6 - 320x^7 + 3a^2x^4 + \frac{384}{5}ax^5 + 384x^6 - 8a^2x^3 - 96ax^4 - \frac{1536}{5}x^5 + a^3x + 12a^2x^2 + 64ax^3 + 128x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9
+ 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 384
/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*x^2
 + 64*a*x^3 + 128*x^4
```

Mupad [B]
time = 0.10, size = 108, normalized size = 0.90

$$x^9\left(\frac{a}{3} - \frac{256}{3}\right) - x^8(3a - 192) - x^6(40a - 384) + x^7\left(\frac{96a}{7} - 320\right) + x^4(3a^2 - 96a + 128) - x^5\left(\frac{3a^2}{5} - \frac{384a}{5} + \frac{1536}{5}\right) + a^3x + 28x^{10} - \frac{72x^{11}}{11} + x^{12} - \frac{x^{13}}{13} + 12a^2x^2 - 8ax^3(a - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 32
0) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^3*
x + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13 + 12*a^2*x^2 - 8*a*x^3*(a - 8)
```

3.118 $\int \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx$

Optimal. Leaf size=72

$$a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4+\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

```
[Out] a^2*x+8*a*x^2+16/3*(4-a)*x^3-2*(16-a)*x^4+2/5*(64-a)*x^5-40/3*x^6+32/7*x^7-
x^8+1/9*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2086}

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5
- (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx = \int \left(a^2+16ax+16(4-a)x^2-8(16-a)x^3+2(64-a)x^4-80x^5+3\right.$$

$$= a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4+\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32}{7}$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 0.92

$$a^2x+8ax^2-\frac{16}{3}(-4+a)x^3+2(-16+a)x^4-\frac{2}{5}(-64+a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5)
/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Maple [A]
time = 0.03, size = 63, normalized size = 0.88

method	result	size
norman	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\left(-\frac{2a}{5}+\frac{128}{5}\right)x^5+\left(2a-32\right)x^4+\left(-\frac{16a}{3}+\frac{64}{3}\right)x^3+8a\,x^2+a^2x$	60
default	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\frac{(-2a+128)x^5}{5}+\frac{(8a-128)x^4}{4}+\frac{(-16a+64)x^3}{3}+8a\,x^2+a^2x$	63
gosper	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66
risch	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+a^2x$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+1/5*(-2*a+128)*x^5+1/4*(8*a-128)*x^4+1/3*(-16
*a+64)*x^3+8*a*x^2+a^2*x
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.90

$$\frac{1}{9}\,x^9-x^8+\frac{32}{7}\,x^7-\frac{40}{3}\,x^6+\frac{128}{5}\,x^5-32\,x^4+a^2x+\frac{64}{3}\,x^3-\frac{2}{15}\left(3\,x^5-15\,x^4+40\,x^3-60\,x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^3
 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

Fricas [A]
time = 0.38, size = 56, normalized size = 0.78

$$\frac{1}{9}\,x^9-x^8+\frac{32}{7}\,x^7-\frac{2}{5}\,(a-64)x^5-\frac{40}{3}\,x^6+2\,(a-16)x^4-\frac{16}{3}\,(a-4)x^3+a^2x+8\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*(a - 64)*x^5 - 40/3*x^6 + 2*(a - 16)*x^4 - 1
6/3*(a - 4)*x^3 + a^2*x + 8*a*x^2
```

Sympy [A]
time = 0.02, size = 65, normalized size = 0.90

$$a^2x+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+x^5\cdot\left(\frac{128}{5}-\frac{2a}{5}\right)+x^4\cdot(2a-32)+x^3\cdot\left(\frac{64}{3}-\frac{16a}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 - 2
*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)
```

Giac [A]
time = 3.52, size = 65, normalized size = 0.90

$$\frac{1}{9}\,x^{9}-x^{8}+\frac{32}{7}\,x^{7}-\frac{2}{5}\,ax^{5}-\frac{40}{3}\,x^{6}+2\,ax^{4}+\frac{128}{5}\,x^{5}-\frac{16}{3}\,ax^{3}-32\,x^{4}+a^{2}x+8\,ax^{2}+\frac{64}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16/
3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

Mupad [B]
time = 0.04, size = 61, normalized size = 0.85

$$x^4\,\left(2\,a-32\right)-x^3\,\left(\frac{16\,a}{3}-\frac{64}{3}\right)-x^5\,\left(\frac{2\,a}{5}-\frac{128}{5}\right)+8\,a\,x^2+a^2\,x-\frac{40\,x^6}{3}+\frac{32\,x^7}{7}-x^8+\frac{x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 +
a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

3.119 $\int \left(a+8x-8x^2+4x^3-x^4\right) dx$

Optimal. Leaf size=26

$$ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

`[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5`

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Antiderivative was successfully verified.

`[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]`

`[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5`

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right) dx = ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

Antiderivative was successfully verified.

`[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]`

`[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5`

Maple [A]
time = 0.01, size = 23, normalized size = 0.88

method	result	size
gosper	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
default	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23

norman	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23
risch	$ax+4x^2-\frac{8}{3}x^3+x^4-\frac{1}{5}x^5$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4+4*x^3-8*x^2+a+8*x,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.85

$$-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Fricas [A]
time = 0.37, size = 22, normalized size = 0.85

$$-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.85

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
```

```
[Out] a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

Giac [A]
time = 4.93, size = 22, normalized size = 0.85

$$-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Mupad [B]
time = 0.02, size = 22, normalized size = 0.85

$$-\frac{x^5}{5}+x^4-\frac{8\,x^3}{3}+4\,x^2+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + 8*x - 8*x^2 + 4*x^3 - x^4,x)
```

```
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

3.120 $\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=89

$$-\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\,\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\,\sqrt{1+\sqrt{4+a}}}$$

```
[Out] -1/2*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)
+1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1120, 1107, 210}

$$\frac{\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\,\sqrt{\sqrt{a+4}+1}}-\frac{\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\,\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[
4 + a]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
```

```
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx,\right.}{2\sqrt{4+a}}\\ &= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 57, normalized size = 0.64

$$-\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{\log(x-\#1)}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1 -
 3*#1^2 + #1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 51, normalized size = 0.57

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	51
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-
a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(65) = 130$.
time = 0.41, size = 457, normalized size = 5.13

$$\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)-\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(-\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)+\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)-\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(-\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a +
 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((
a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x -
 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 +
 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*
sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)
) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1
)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7
*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 +
33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
/(a^2 + 7*a + 12)) + x - 1)
```

Sympy [A]
time = 0.61, size = 66, normalized size = 0.74

$$-\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2(-32a-128)-1,\left(t\mapsto t\log\left(64t^3a^2+448t^3a+768t^3-4ta-20t+x-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 12
8) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*a
- 20*_t + x - 1)))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa]=[86]Warning, nee
d to choose a br
```

Mupad [B]
time = 2.58, size = 571, normalized size = 6.42

$$-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}+x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}-\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2\,x\,1\mathrm{i}+a^2\,1\mathrm{i}+16\mathrm{i}}{44\,a^2\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+160\,a\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}}\right)\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}\,2\mathrm{i}-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}-x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}+\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2\,x\,1\mathrm{i}+a^2\,1\mathrm{i}+16\mathrm{i}}{160\,a\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+44\,a^2\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}}\right)\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] - atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (4
8*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a -
(48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/
2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 1
6*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(64
0*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3 + 64)
^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a + 12*a^2
+ a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i - atan(-(
a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (48*a + 12*
a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a + (48*a + 12
*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*(
(a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768)
)^(1/2) + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a
^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) +
4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a + (48*a + 12*a^2 + a^3 + 6
4)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i
```

3.121 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$

Optimal. Leaf size=169

$$\frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)}-\frac{\left(10+3a+\sqrt{4+a}\right)\tan^{-1}\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}}+\frac{(1\ldots$$

```
[Out] 1/4*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)-1/8*arctan
((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1-(4
+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))*(10+3*a-(4+a)^(1/
2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1120, 1106, 1180, 210}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[
4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sq
rt[1 + Sqrt[4 + a]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)\cdots}{3+\cdots}\right.}{8\cdots}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} - \frac{\left(10+3a-\sqrt{4+\cdots}\right.}{\cdots}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{\left(10+3a+\sqrt{4+\cdots}\right.}{8(3+a)(\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 150, normalized size = 0.89

$$\frac{(-1+x)(6+a-2x+x^2)}{4(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{12\log(x-\#1)+3a\log(x-\#1)-2\log(x-\#1)\#1+\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{16(12+7a+a^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
+ x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (12*Log[x - #1] +
3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2
+ #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 158, normalized size = 0.93

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(3+a)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\_R^2-2\_R+3a+12\right)\ln\left(x-\right.}{-\_R^3+3\_R^2-4\_R+}}{16(4+a)(3+a)}$
risch	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(3+a)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{a^2+7a+12}-\frac{2\_R}{a^2+7a+12}+\frac{3}{3+}\right.}{-\_R^3+3\_R^2-4}\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/(a^2+7*a+12)*x^3-3/4/(4+a)/(3+a)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*(6+a
)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(4+a)/(3+a)*sum((_R^2-2*_R+3*
a+12)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a
 + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x -
12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x
), x)/(a^2 + 7*a + 12)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. 2(139) = 278.
time = 0.39, size = 1948, normalized size = 11.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

[Out] -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 +

```
128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*
a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 40
8*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 214
20*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 5800*a + 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3
 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39
9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304
*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 +
3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x - 12*x^2 - 4*a - 24)/((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(144) = 288$.
time = 3.77, size = 294, normalized size = 1.74

$$\frac{a-x^3+3x^2+x(-a-8)+6}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x(-32a^2-224a-384)}+\operatorname{RootSum}\left(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4+12952010752a^3+20082327552a^2+18119393280a+7247757312)+t^2(-7680a^5-145920a^4-1107968a^3-4202496a^2-7962624a-6029312)-81a^2-576a-1024,\left(t\mapsto t\log\left(x+\frac{-16384t^3a^7-401408t^3a^6-4202496t^3a^5-24371200t^3a^4-84549632t^3a^3-175472640t^3a^2-201719808t^3a-99090432t^3+432ta^4+7488ta^3+47024ta^2+128096ta+128512t-81a^2-567a-992}{81a^2+567a+992}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a*
*2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 3
84) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**
8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12
952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-7
680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 6029312)
 - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7 - 40140
8*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*_t**3*a*
*3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 432*_t*a**
4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a**2 - 567*
a - 992)/(81*a**2 + 567*a + 992))))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa]=[86]Warning, nee
d to choose a br
```

Mupad [B]
time = 5.35, size = 2500, normalized size = 14.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] atan(-(((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2
+ 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632
*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592))
)^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^
5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576
)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/
(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2
) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/
(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6
 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 1067
52*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2)
 + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*
a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^
9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a
+ 73*a^2 + 14*a^3 + a^4 + 144)))*1i + ((15552*a - 9*a*((a + 4)^9)^(1/2) - 3
1*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*
(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 48
3*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997
696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2
 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 +
 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*(
(15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a
^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8
1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)
+ (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(
816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4
115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 1
44*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(
61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i)/((9*a +
 32)/(32*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a - 9*
a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4
+ 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22
488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*
```

a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 943718
4)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 1
17760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 +
14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1
/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 3064
32*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 +
 a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 +
768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((
15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^
3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81
744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) +
 (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a
^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a
^4 + 144))) - ((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 82
08*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 +
 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 1
10592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9
0112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^
5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 14
7456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4
115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2
 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^
3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9
)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a +
306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a
^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*
a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*
(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))))*((155...

3.122 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal. Leaf size=252

$$\frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2}+\frac{((6+a)(25+7a)+6(7+2a)(-1+x)^2)(-1+x)}{32(3+a)^2(4+a)^2(3+a-2(-1+x)^2-(-1+x)^4)}-$$

```
[Out] 1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+1/32*((6
+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+
x)^4)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2)+a*
(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arctan((
-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+a)^2/
(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1120, 1106, 1192, 1180, 210}

$$-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a
^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt
[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*
a - (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]
)/(64*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
```

```
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)$$

$$= \frac{(5+a+(-1+x)^2)\,(-1+x)}{8\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^2} - \frac{\text{Subst}\Big(\int \frac{4+2(3+a}{(3}}{1}$$

$$= -\frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)\,(1-x)}{32\,(12+7a+a^2)^2\,(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a}{8\,(12+7a+a^2)\,(}$$

$$= -\frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)\,(1-x)}{32\,(12+7a+a^2)^2\,(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a}{8\,(12+7a+a^2)\,(}$$

$$= -\frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)\,(1-x)}{32\,(12+7a+a^2)^2\,(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a}{8\,(12+7a+a^2)\,(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.10, size = 254, normalized size = 1.01

$$\frac{1}{128}\left(\frac{16(-1+x)\,(6+a-2x+x^2)}{(3+a)(4+a)\,(a-x\,(-8+8x-4x^2+x^3))^2}+\frac{4(-1+x)\,(7a^2+6(32-14x+7x^2)+a(79-24x+12x^2))}{(3+a)^2(4+a)^2\,(a-x\,(-8+8x-4x^2+x^3))}-\frac{3\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{108\log(x-\#1)+55a\log(x-\#1)+7a^2\log(x-\#1)-28\log(x-\#1)\#1-8a\log(x-\#1)\#1+14\log(x-\#1)\#1^2+4a\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{(12+7a+a^2)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^
2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x +
12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootS
um[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (108*Log[x - #1] + 55*a*Log[x - #1
] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x -
 #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*
a + a^2)^2)/128
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 400, normalized size = 1.59

method	result

default	$-\frac{\frac{3(7+2a)x^7}{16(a^4+14a^3+73a^2+168a+144)}-\frac{21(7+2a)x^6}{16(a^2+8a+16)(a^2+6a+9)}+\frac{(7a^2+343a+1116)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{5(7a^2+175a+528)x^4}{32(a^4+14a^3+73a^2+168a+144)}+\frac{}{16a^4+}}{(-x^4+4x^3-8x^2+a}$
risch	$\frac{-\frac{3(7+2a)x^7}{16(a^4+14a^3+73a^2+168a+144)}+\frac{21(7+2a)x^6}{16(a^2+8a+16)(a^2+6a+9)}-\frac{(7a^2+343a+1116)x^5}{32(a^4+14a^3+73a^2+168a+144)}+\frac{5(7a^2+175a+528)x^4}{32(a^4+14a^3+73a^2+168a+144)}-\frac{(34a^2}{16(a^4+14}}{(-x^4+4x^3-8x^2+a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16)
/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^5-
5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+679*a
+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a
^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2
+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(3+a)/(a^3+11*a^2+40*a+48))/(
-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(4+a)*sum((-108+2*(-2*
a-7)*_R^2+4*(7+2*a)*_R-7*a^2-55*a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf
(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5*
(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(32*
a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x + 40
8*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 7
3*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a
^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^
4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 15
1*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a
^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a + 7)*x +
55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 16
8*a + 144)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3971 vs. $2(220) = 440$.

time = 0.42, size = 3971, normalized size = 15.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^5
 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3 -
8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2
 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 +
14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^
4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16
*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 +
14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 +
 (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4
+ 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 +
 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 2
41870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^
2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 +
5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 +
725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^
4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6
+ 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11
 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410
692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 338
41152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 +
 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^
9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940
*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a
 + 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8
+ 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2
+ 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 3
46921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^1
0 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a
^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176
)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129
367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 1122
8868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 +
 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 1
44)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(
a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 -
 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a
```

```
^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*
a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*
a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3
 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2
+ 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11
 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6
 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 27713664
0*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 3
1085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 2
48832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^
3 + 177061*a^2 + 415884*a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5
 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10
 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 11
7844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt(
(2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1
165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320
*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 78207
1200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*
sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 +
 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a +
 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15
 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*
a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 7143179
40*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a
 + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 37
3020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 992
3472) - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^
6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(230) = 460$.
time = 9.46, size = 697, normalized size = 2.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -(11*a**3 + 131*a**2 + 408*a + x**7*(12*a + 42) + x**6*(-84*a - 294) + x**5
*(7*a**2 + 343*a + 1116) + x**4*(-35*a**2 - 875*a - 2640) + x**3*(68*a**2 +
 1358*a + 3936) + x**2*(-64*a**2 - 1246*a - 3600) + x*(-11*a**3 - 107*a**2
+ 84*a + 1152) + 288)/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 + 4608*a*
*2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-256*a**
4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14336*a**
```

```
3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3 - 18688
0*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**3 + 288
256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a**3 - 25
6000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*a**3 +
63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a**3 + 86
016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*a**14 +
 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 52787590
8304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 5453715112
7224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 55115219
3655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2 + 28499
3413919539200*a + 68398419340689408) + _t**2*(-30965760*a**9 - 1052835840*a
**8 - 15910207488*a**7 - 140262506496*a**6 - 795007254528*a**5 - 3004516270
080*a**4 - 7571263979520*a**3 - 12268037210112*a**2 - 11598827618304*a - 48
75324751872) - 194481*a**4 - 2762424*a**3 - 14762736*a**2 - 35178624*a - 31
539456, Lambda(_t, _t*log(x + (23068672*_t**3*a**12 + 968884224*_t**3*a**11
 + 18624806912*_t**3*a**10 + 216677744640*_t**3*a**9 + 1699123036160*_t**3*
a**8 + 9461389328384*_t**3*a**7 + 38361186172928*_t**3*a**6 + 1141074915491
84*_t**3*a**5 + 247138458009600*_t**3*a**4 + 380084473036800*_t**3*a**3 + 3
94002582994944*_t**3*a**2 + 247177515368448*_t**3*a + 70970039599104*_t**3
- 395136*_t*a**7 - 11676672*_t*a**6 - 144076032*_t*a**5 - 969518592*_t*a**4
 - 3861475200*_t*a**3 - 9133300224*_t*a**2 - 11906574336*_t*a - 6611337216*
_t - 64827*a**4 - 907578*a**3 - 4780647*a**2 - 11228868*a - 9923472)/(64827
*a**4 + 907578*a**3 + 4780647*a**2 + 11228868*a + 9923472)))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [sageVARa]=[86]Warning, nee
d to choose a br
```

Mupad [B]
time = 6.41, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] atan(((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^
4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^
9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 +
 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (
(4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^
4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8
 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(94
0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5
564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038
4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74
38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2
 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9
*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*(
(a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^
5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168
3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5
+ 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^1
0 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1
/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) -
49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
95744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384
*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703
040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 196
6491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548039
68)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
4016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) - (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*
(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7
+ a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^
15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187
840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531
456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a
^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442
00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*1i - (((52357496832*a + 57139003392*a^2 + 363221483
52*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 +
5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 89
9328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^
9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336*a^2 + 5025917042688
*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 598621552
64*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 13
91569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149
208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3
510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*
```

```
a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(2
56*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a
^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a +
4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10
187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16
531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 205320192
0*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119
44200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 +
 a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 58
9*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*
a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*
a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2
053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a
^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 5
5*a^14 + a^15 + 254803968)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088
*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
 582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 +
441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15
)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + ...
```

3.123 $\int x(a+8x-8x^2+4x^3-x^4)^4\,dx$

Optimal. Leaf size=210

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5+\frac{2}{3}\left(1024-1536a+192a^2-a^3\right)x^6-\frac{32}{7}\left(512-288a\right.$$

```
[Out] 1/2*a^4*x^2+32/3*a^3*x^3+8*(12-a)*a^2*x^4+16/5*a*(a^2-48*a+128)*x^5+2/3*(-a
^3+192*a^2-1536*a+1024)*x^6-32/7*(15*a^2-288*a+512)*x^7+8*(128-3*a)*(4-a)*x
^8-16/3*(a^2-128*a+896)*x^9+1/5*(3*a^2-1536*a+20480)*x^10-32/11*(928-35*a)*
x^11+8/3*(524-9*a)*x^12-16/13*(464-3*a)*x^13+2/7*(640-a)*x^14-224/5*x^15+8*
x^16-16/17*x^17+1/18*x^18
```

Rubi [A]
time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6874}

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5+8(12-a)a^2x^4+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)x^6+\frac{2}{7}(640-a)x^{14}-\frac{16}{13}(464-3a)x^{13}+\frac{8}{3}(524-9a)x^{12}-\frac{32}{11}(928-35a)x^{11}+8(128-3a)(4-a)x^8+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^2
)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a + 1
5*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)/3
+ ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(524 -
 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15
)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \int \left(a^4x+32a^3x^2-32(-12+a)a^2x^3+16a\left(128-48a+a^2\right)x^4-4(\right.\\ &= \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5+\frac{2}{3}\left(1024\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 204, normalized size = 0.97

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}-8(-12+a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5-\frac{2}{3}\left(-1024+1536a-192a^2+a^3\right)x^6-\frac{32}{7}\left(512-288a+15a^2\right)x^7+8\left(512-140a+3a^2\right)x^8-\frac{16}{3}\left(896-128a+a^2\right)x^9+\frac{1}{5}\left(20480-1536a+3a^2\right)x^{10}+\frac{32}{11}(-928+35a)x^{11}-\frac{8}{3}(-524+9a)x^{12}+\frac{16}{13}(-464+3a)x^{13}-\frac{2}{7}(-640+a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 - 48*a + a^
2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3 - (32*(512 - 288*a +
 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8 - (16*(896 - 128*a + a^2)*x^9
)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 + (32*(-928 + 35*a)*x^11)/11 - (8*(
-524 + 9*a)*x^12)/3 + (16*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/7 - (
224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Maple [A]
time = 0.06, size = 267, normalized size = 1.27

method	result
norman	$\frac{a^{4} x^{2}}{2}+\frac{32 a^{3} x^{3}}{3}+\left(-8 a^{3}+96 a^{2}\right) x^{4}+\left(\frac{16}{5} a^{3}-\frac{768}{5} a^{2}+\frac{2048}{5} a\right) x^{5}+\left(-\frac{2}{3} a^{3}+128 a^{2}-1024 a+\frac{2048}{3}\right) x$
gosper	$4096 x^{8}-\frac{14336}{3} x^{9}-\frac{16384}{7} x^{7}+\frac{2048}{3} x^{6}+\frac{1}{2} a^{4} x^{2}+24 a^{2} x^{8}-1120 a\, x^{8}+\frac{1120}{11} x^{11} a-24 x^{12} a+\frac{2048}{3} x^{9} a$
risch	$4096 x^{8}-\frac{14336}{3} x^{9}-\frac{16384}{7} x^{7}+\frac{2048}{3} x^{6}+\frac{1}{2} a^{4} x^{2}+24 a^{2} x^{8}-1120 a\, x^{8}+\frac{1120}{11} x^{11} a-24 x^{12} a+\frac{2048}{3} x^{9} a$
default	$\frac{x^{18}}{18}-\frac{16 x^{17}}{17}+8 x^{16}-\frac{224 x^{15}}{5}+\frac{\left(-4 a+2560\right) x^{14}}{14}+\frac{\left(48 a-7424\right) x^{13}}{13}+\frac{\left(-288 a+16768\right) x^{12}}{12}+\frac{\left(1120 a-29696\right) x^{11}}{11}+\frac{\left(2 a^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*x^18-16/17*x^17+8*x^16-224/5*x^15+1/14*(-4*a+2560)*x^14+1/13*(48*a-742
4)*x^13+1/12*(-288*a+16768)*x^12+1/11*(1120*a-29696)*x^11+1/10*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^10+1/9*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128)
)*x^9+1/8*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^8+1/7*(-160
*a^2+32*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^7+1/6*(2*a^2*(-2*a+128)+32*a
*(8*a-128)+(-16*a+64)^2)*x^6+1/5*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^5+1/4*
(2*a^2*(-16*a+64)+256*a^2)*x^4+32/3*a^3*x^3+1/2*a^4*x^2
```

Maxima [A]
time = 0.27, size = 182, normalized size = 0.87

$$\frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}(a-640)x^{14}-\frac{224}{5}x^{15}+\frac{16}{13}(3a-464)x^{13}-\frac{8}{3}(9a-524)x^{12}+\frac{32}{11}(35a-928)x^{11}+\frac{1}{5}(3a^2-1536a+20480)x^{10}-\frac{16}{3}(a^2-128a+896)x^9+8(3a^2-140a+512)x^8-\frac{32}{7}(15a^2-288a+512)x^7-\frac{2}{3}(a^3-192a^2+1536a-1024)x^6+\frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+\frac{16}{5}(a^3-48a^2+128a)x^5-8(a^3-12a^2)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(
3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a
^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a
 + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a
```

```
 - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5
 - 8*(a^3 - 12*a^2)*x^4
```

Fricas [A]
time = 0.37, size = 182, normalized size = 0.87

$$\frac{1}{18}\,x^{18} - \frac{16}{17}\,x^{17} + 8\,x^{16} - \frac{2}{7}\,(a - 640)x^{14} - \frac{224}{5}\,x^{15} + \frac{16}{13}\,(3\,a - 464)x^{13} - \frac{8}{3}\,(9\,a - 524)x^{12} + \frac{32}{11}\,(35\,a - 928)x^{11} + \frac{1}{5}\,(3\,a^2 - 1536\,a + 20480)x^{10} - \frac{16}{3}\,(a^2 - 128\,a + 896)x^9 + 8\,(3\,a^2 - 140\,a + 512)x^8 - \frac{32}{7}\,(15\,a^2 - 288\,a + 512)x^7 - \frac{2}{3}\,(a^3 - 192\,a^2 + 1536\,a - 1024)x^6 + \frac{1}{2}\,a^4x^2 + \frac{32}{3}\,a^3x^3 + \frac{16}{5}\,(a^3 - 48\,a^2 + 128\,a)x^5 - 8\,(a^3 - 12\,a^2)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(
3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a
^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a
 + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a
 - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5
 - 8*(a^3 - 12*a^2)*x^4
```

Sympy [A]
time = 0.03, size = 212, normalized size = 1.01

$$\frac{a^4x^2}{2} + \frac{32a^3x^3}{3} + \frac{x^{18}}{18} - \frac{16x^{17}}{17} + 8x^{16} - \frac{224x^{15}}{5} + x^{14}\cdot\left(\frac{1280}{7} - \frac{2a}{7}\right) + x^{13}\cdot\left(\frac{48a}{13} - \frac{7424}{13}\right) + x^{12}\cdot\left(\frac{4192}{3} - 24a\right) + x^{11}\cdot\left(\frac{1120a}{11} - \frac{29696}{11}\right) + x^{10}\cdot\left(\frac{3a^2}{5} - \frac{1536a}{5} + 4096\right) + x^9\left(-\frac{16a^2}{3} + \frac{2048a}{3} - \frac{14336}{3}\right) + x^8\cdot(24a^2 - 1120a + 4096) + x^7\left(-\frac{480a^2}{7} + \frac{9216a}{7} - \frac{16384}{7}\right) + x^6\left(-\frac{2a^3}{3} + 128a^2 - 1024a + \frac{2048}{3}\right) + x^5\cdot\left(\frac{16a^3}{5} - \frac{768a^2}{5} + \frac{2048a}{5}\right) + x^4(-8a^3 + 96a^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**15
/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 - 2
4*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096) +
x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096) + x
**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*
a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96
*a**2)
```

Giac [A]
time = 4.41, size = 222, normalized size = 1.06

$$\frac{1}{18}\,x^{18} - \frac{16}{17}\,x^{17} + 8\,x^{16} - \frac{2}{7}\,ax^{14} - \frac{224}{5}\,x^{15} + \frac{48}{13}\,ax^{13} + \frac{1280}{7}\,x^{14} - 24\,ax^{12} - \frac{7424}{13}\,x^{13} + \frac{3}{5}\,a^2x^{10} + \frac{1120}{11}\,ax^{11} + \frac{4192}{3}\,x^{12} - \frac{16}{3}\,a^2x^9 - \frac{1536}{5}\,ax^{10} - \frac{29696}{11}\,x^{11} + 24\,a^2x^8 + \frac{2048}{3}\,ax^9 + 4096\,x^{10} - \frac{2}{3}\,a^3x^6 - \frac{480}{7}\,a^2x^7 - 1120\,ax^8 - \frac{14336}{3}\,x^9 + \frac{16}{5}\,a^3x^5 + 128\,a^2x^6 + \frac{9216}{7}\,ax^7 + 4096\,x^8 - 8\,a^3x^4 - \frac{768}{5}\,a^2x^5 - 1024\,ax^6 - \frac{16384}{7}\,x^7 + \frac{1}{2}\,a^4x^2 + \frac{32}{3}\,a^3x^3 + 96\,a^2x^4 + \frac{2048}{5}\,ax^5 + \frac{2048}{3}\,x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13*a*x^13 +
1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 + 1120/11*a*x^11 + 41
92/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29696/11*x^11 + 24*a^2*x^8 + 204
8/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 14336/3*
```

```
x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4 - 76
8/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^
2*x^4 + 2048/5*a*x^5 + 2048/3*x^6
```

Mupad [B]
time = 0.22, size = 178, normalized size = 0.85

$$x^{13}\left(\frac{48\,a}{13}-\frac{7424}{13}\right)-x^{12}\left(24\,a-\frac{4192}{3}\right)-x^{14}\left(\frac{2\,a}{7}-\frac{1280}{7}\right)+x^{11}\left(\frac{1120\,a}{11}-\frac{29696}{11}\right)+x^8\left(24\,a^2-1120\,a+4096\right)+x^{10}\left(\frac{3\,a^2}{5}-\frac{1536\,a}{5}+4096\right)-x^9\left(\frac{16\,a^2}{3}-\frac{2048\,a}{3}+\frac{14336}{3}\right)-x^7\left(\frac{480\,a^2}{7}-\frac{9216\,a}{7}+\frac{16384}{7}\right)-x^6\left(\frac{2\,a^3}{3}-128\,a^2+1024\,a-\frac{2048}{3}\right)-\frac{224\,x^{15}}{5}+8\,x^{16}-\frac{16\,x^{17}}{17}+\frac{x^{18}}{18}+\frac{32\,a^3\,x^3}{3}+\frac{a^4\,x^2}{2}+\frac{16\,a\,x^5\,\left(a^2-48\,a+128\right)}{5}-8\,a^2\,x^4\,\left(a-12\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7)
 + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((3*
a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) - x^7
*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)/3 -
 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^3)/3
+ (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)
```

3.124 $\int x(a+8x-8x^2+4x^3-x^4)^3\,dx$

Optimal. Leaf size=134

$$\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}\left(512-128a+a^2\right)x^6+\frac{48}{7}(48-5a)x^7-4(70-3a)x^8+\frac{8}{3}$$

```
[Out] 1/2*a^3*x^2+8*a^2*x^3+6*(8-a)*a*x^4+4/5*(3*a^2-96*a+128)*x^5-1/2*(a^2-128*a
+512)*x^6+48/7*(48-5*a)*x^7-4*(70-3*a)*x^8+8/3*(64-a)*x^9-3/10*(256-a)*x^10
+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

Rubi [A]
time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6874}

$$\frac{a^3x^2}{2}-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-5a)x^7+6(8-a)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
- ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8 +
(8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (12*x
^13)/13 - x^14/14
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\int \left(a^3x+24a^2x^2-24(-8+a)ax^3+4\left(128-96a+3a^2\right)x^4-3(512\right.$$
$$=\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}(512-12$$

Mathematica [A]
time = 0.01, size = 130, normalized size = 0.97

$$\frac{a^3x^2}{2}+8a^2x^3-6(-8+a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5+\frac{1}{2}\left(-512+128a-a^2\right)x^6-\frac{48}{7}(-48+5a)x^7+4(-70+3a)x^8-\frac{8}{3}(-64+a)x^9+\frac{3}{10}(-256+a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/7 + 4*(-70 + 3*a)*x^
8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)/10 + (280*x^11)/11 - 6*x^12 +
 (12*x^13)/13 - x^14/14
```

Maple [A]
time = 0.07, size = 143, normalized size = 1.07

method	result
norman	$\frac{a^3 x^2}{2}+8a^2x^3+\left(-6a^2+48a\right)x^4+\left(\frac{12}{5}a^2-\frac{384}{5}a+\frac{512}{5}\right)x^5+\left(-\frac{1}{2}a^2+64a-256\right)x^6+\left(-\frac{240a}{7}+2\right.$
gosper	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{240}{7}x^7a$
risch	$\frac{1}{2}a^3x^2+8a^2x^3-6a^2x^4+48a\,x^4+\frac{12}{5}a^2x^5-\frac{384}{5}a\,x^5+\frac{512}{5}x^5-\frac{1}{2}a^2x^6+64a\,x^6-256x^6-\frac{240}{7}x^7a$
default	$-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+\frac{(3a-768)x^{10}}{10}+\frac{(-24a+1536)x^9}{9}+\frac{(96a-2240)x^8}{8}+\frac{(-240a+2304)x^7}{7}+\frac{(a(-2a+1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*x^14+12/13*x^13-6*x^12+280/11*x^11+1/10*(3*a-768)*x^10+1/9*(-24*a+153
6)*x^9+1/8*(96*a-2240)*x^8+1/7*(-240*a+2304)*x^7+1/6*(a*(-2*a+128)+256*a-15
36-a^2)*x^6+1/5*(a*(8*a-128)-256*a+512+4*a^2)*x^5+1/4*(a*(-16*a+64)+128*a-8
*a^2)*x^4+8*a^2*x^3+1/2*a^3*x^2
```

Maxima [A]
time = 0.27, size = 113, normalized size = 0.84

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{3}{10}(a-256)x^{10}+\frac{280}{11}x^{11}-\frac{8}{3}(a-64)x^9+4(3a-70)x^8-\frac{48}{7}(5a-48)x^7-\frac{1}{2}(a^2-128a+512)x^6+\frac{4}{5}(3a^2-96a+128)x^5+\frac{1}{2}a^3x^2+8a^2x^3-6(a^2-8a)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3*
(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a +
512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2
- 8*a)*x^4
```

Fricas [A]
time = 0.36, size = 113, normalized size = 0.84

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{3}{10}(a-256)x^{10}+\frac{280}{11}x^{11}-\frac{8}{3}(a-64)x^9+4(3a-70)x^8-\frac{48}{7}(5a-48)x^7-\frac{1}{2}(a^2-128a+512)x^6+\frac{4}{5}(3a^2-96a+128)x^5+\frac{1}{2}a^3x^2+8a^2x^3-6(a^2-8a)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3*
(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a +
512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2
- 8*a)*x^4
```

Sympy [A]
time = 0.02, size = 128, normalized size = 0.96

$$\frac{a^3x^2}{2} + 8a^2x^3 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + x^{10} \cdot \left(\frac{3a}{10} - \frac{384}{5}\right) + x^9 \cdot \left(\frac{512}{3} - \frac{8a}{3}\right) + x^8 \cdot (12a - 280) + x^7 \cdot \left(\frac{2304}{7} - \frac{240a}{7}\right) + x^6\left(-\frac{a^2}{2} + 64a - 256\right) + x^5 \cdot \left(\frac{12a^2}{5} - \frac{384a}{5} + \frac{512}{5}\right) + x^4(-6a^2 + 48a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/11
 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x**7
*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 384*a
/5 + 512/5) + x**4*(-6*a**2 + 48*a)
```

Giac [A]
time = 4.38, size = 133, normalized size = 0.99

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}ax^{10} + \frac{280}{11}x^{11} - \frac{8}{3}ax^9 - \frac{384}{5}x^{10} + 12ax^8 + \frac{512}{3}x^9 - \frac{1}{2}a^2x^6 - \frac{240}{7}ax^7 - 280x^8 + \frac{12}{5}a^2x^5 + 64ax^6 + \frac{2304}{7}x^7 - 6a^2x^4 - \frac{384}{5}ax^5 - 256x^6 + \frac{1}{2}a^3x^2 + 8a^2x^3 + 48ax^4 + \frac{512}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/3*a*x^9 -
384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 + 1
2/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6 + 1
/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5
```

Mupad [B]
time = 2.12, size = 113, normalized size = 0.84

$$x^8\,(12\,a-280)+x^{10}\,\left(\frac{3\,a}{10}-\frac{384}{5}\right)-x^9\,\left(\frac{8\,a}{3}-\frac{512}{3}\right)-x^7\,\left(\frac{240\,a}{7}-\frac{2304}{7}\right)-x^6\,\left(\frac{a^2}{2}-64\,a+256\right)+x^5\,\left(\frac{12\,a^2}{5}-\frac{384\,a}{5}+\frac{512}{5}\right)+\frac{280\,x^{11}}{11}-6\,x^{12}+\frac{12\,x^{13}}{13}-\frac{x^{14}}{14}+8\,a^2\,x^3+\frac{a^3\,x^2}{2}-6\,a\,x^4\,(a-8)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^8*(12*a - 280) + x^10*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*((
240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)/5
 + 512/5) + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14 + 8*a^2*x^3 + (
a^3*x^2)/2 - 6*a*x^4*(a - 8)
```

3.125 $\int x(a+8x-8x^2+4x^3-x^4)^2\,dx$

Optimal. Leaf size=79

$$\frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+16/3*a*x^3+4*(4-a)*x^4-8/5*(16-a)*x^5+1/3*(64-a)*x^6-80/7*x^7+4
*x^8-8/9*x^9+1/10*x^10
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.042$, Rules used = {6874}

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)
*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x(a+8x-8x^2+4x^3-x^4)^2\,dx=\int (a^2x+16ax^2-16(-4+a)x^3+8(-16+a)x^4-2(-64+a)x^5-8$$
$$=\frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 0.95

$$\frac{a^2x^2}{2}+\frac{16ax^3}{3}-4(-4+a)x^4+\frac{8}{5}(-16+a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 -
a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Maple [A]
time = 0.06, size = 66, normalized size = 0.84

method	result	siz
norman	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\left(\frac{64}{3}-\frac{a}{3}\right)x^6+\left(\frac{8a}{5}-\frac{128}{5}\right)x^5+\left(-4a+16\right)x^4+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$	63
default	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\frac{(-2a+128)x^6}{6}+\frac{(8a-128)x^5}{5}+\frac{(-16a+64)x^4}{4}+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$	66
gosper	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$	69
risch	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}a\,x^6+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x^2$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+1/6*(-2*a+128)*x^6+1/5*(8*a-128)*x^5+1/4*(
-16*a+64)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.75

$$\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^
5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Fricas [A]
time = 0.37, size = 59, normalized size = 0.75

$$\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^
5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Sympy [A]
time = 0.01, size = 70, normalized size = 0.89

$$\frac{a^2x^2}{2}+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+x^6\cdot\left(\frac{64}{3}-\frac{a}{3}\right)+x^5\cdot\left(\frac{8a}{5}-\frac{128}{5}\right)+x^4\cdot(16-4a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**6
*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)
```

Giac [A]
time = 4.11, size = 68, normalized size = 0.86

$$\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^{9}+4\,x^{8}-\frac{1}{3}\,ax^{6}-\frac{80}{7}\,x^{7}+\frac{8}{5}\,ax^{5}+\frac{64}{3}\,x^{6}-4\,ax^{4}-\frac{128}{5}\,x^{5}+\frac{1}{2}\,a^{2}x^{2}+\frac{16}{3}\,ax^{3}+16\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6 -
 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4
```

Mupad [B]
time = 0.04, size = 64, normalized size = 0.81

$$x^5\left(\frac{8\,a}{5}-\frac{128}{5}\right)-x^6\left(\frac{a}{3}-\frac{64}{3}\right)-x^4\,\left(4\,a-16\right)+\frac{16\,a\,x^3}{3}-\frac{80\,x^7}{7}+4\,x^8-\frac{8\,x^9}{9}+\frac{x^{10}}{10}+\frac{a^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 -
(80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2
```

3.126 $\int x(a+8x-8x^2+4x^3-x^4)\,dx$

Optimal. Leaf size=35

$$\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

```
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx &= \int \left(ax+8x^2-8x^3+4x^4-x^5\right)\,dx\\ &= \frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.80

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
default	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
norman	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28
risch	$\frac{1}{2}a\,x^2+\frac{8}{3}x^3-2x^4+\frac{4}{5}x^5-\frac{1}{6}x^6$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.77

$$-\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Fricas [A]
time = 0.38, size = 27, normalized size = 0.77

$$-\frac{1}{6}\,x^6+\frac{4}{5}\,x^5-2\,x^4+\frac{1}{2}\,ax^2+\frac{8}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.83

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

Giac [A]
time = 3.93, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Mupad [B]
time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3} + \frac{ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

3.127 $\int \frac{x}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}+\frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

```
[Out] 1/2*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a
)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)+1/2*arctan((-1+x)/(1+(4+a
)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1694, 1687, 1107, 210, 1121, 632, 212}

$$-\frac{\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[
4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{1+x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{3+a-2x^2-x^4}$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right) - \frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\right.}{2\sqrt{4+a}}$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} - \text{Subst}\left(\int \frac{}{4(4}\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1}{\sqrt{4+}}\right.}{2\sqrt{4+a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 59, normalized size = 0.51

$$-\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{\log(x-\#1)\#1}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 + 4
*#1 - 3*#1^2 + #1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 52, normalized size = 0.45

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)} \frac{\_R\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	52

risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.69, size = 140500, normalized size = 1211.21

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] -1/24*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(((a + 4)^(3/2)*sqrt((sqrt(a + 4) + 1
)/(a^2 + 7*a + 12)) + a + 4)^2/(a + 4)^3 + 3*((2*a + 7)*(a + 4)^(3/2) - a^2
 - (a^2 + 7*a + 12)*sqrt(a + 4) - 2*(a^3 + 11*a^2 + 40*a + 48)*sqrt((sqrt(a
 + 4) + 1)/(a^2 + 7*a + 12)) - 8*a - 16)/((a^2 + 7*a + 12)*(a + 4)^(3/2)))/
(2*((a + 4)^(3/2)*sqrt((sqrt(a + 4) + 1)/(a^2 + 7*a + 12)) + a + 4)^3/(a +
4)^(9/2) + 27*(((a^2 + 7*a + 12)*((sqrt(a + 4) + 1)/(a^2 + 7*a + 12))^(3/2)
 - 2*(a + 4)*sqrt((sqrt(a + 4) + 1)/(a^2 + 7*a + 12)) - 4)*(a + 4)^(3/2) -
a^2 + 3*((a^2 + 7*a + 12)*sqrt((sqrt(a + 4) + 1)/(a^2 + 7*a + 12)) + a + 4)
*sqrt(a + 4) - 6*a - 8)/((a^2 + 7*a + 12)*(a + 4)^(3/2)) + 9*((2*a + 7)*(a
+ 4)^(3/2) - a^2 - (a^2 + 7*a + 12)*sqrt(a + 4) - 2*(a^3 + 11*a^2 + 40*a +
48)*sqrt((sqrt(a + 4) + 1)/(a^2 + 7*a + 12)) - 8*a - 16)*((a + 4)^(3/2)*sqr
t((sqrt(a + 4) + 1)/(a^2 + 7*a + 12)) + a + 4)/((a^2 + 7*a + 12)*(a + 4)^3)
 + 9*sqrt(1/3)*sqrt((4*(7*(a + 4)^(3/2)*((sqrt(a + 4) + 1)/(a^2 + 7*a + 12)
)^(3/2) + (2*sqrt(a + 4)  ...
```

Sympy [A]
time = 2.82, size = 155, normalized size = 1.34

$$-\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t\left(-16a-64\right)+a,\left(t\mapsto t\log\left(x+\frac{-128t^3a^4-1728t^3a^3-8640t^3a^2-18944t^3a-15360t^3+48t^2a^3+464t^2a^2+1472t^2a+1536t^2+8ta^3+88ta^2+312ta+352t-a^2-2a}{4a^2+21a+28}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2 -
 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a**4
 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*_t*
*2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a*
*2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Mupad [B]
time = 2.58, size = 275, normalized size = 2.37

$$\sum_{k=1}^{4}\ln\left(-x-\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)\,\left(\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)\,\left(32\,a-\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)\,\left(64\,a-x\,\left(64\,a+256\right)+256\right)-x\,\left(16\,a+64\right)+128\right)-8\right)\right)\,\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] symsum(log(- x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k)*(root(2816*a^
2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 51
2*z^2 + 16*a*z + 64*z + a, z, k)*(32*a - root(2816*a^2*z^4 + 256*a^3*z^4 +
10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z
+ a, z, k)*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8))*root(
2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z
^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)
```

3.128 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal. Leaf size=231

$$\frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}+\frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}-\frac{\left(10+3a \cdots\right.}{8 \cdots}$$

```
[Out] 1/4*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(5+a+(-1+x)^2)*(-1+x)/
(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*arctanh((1+(-1+x)^2)/(4+a)^(1/2)
)/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(10+3*a+(4+a)^(1/2))
/(3+a)/(4+a)^(3/2)/(1-(4+a)^(1/2))^(1/2)+1/8*arctan((-1+x)/(1+(4+a)^(1/2))^
(1/2))*(10+3*a-(4+a)^(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1694, 1687, 1106, 1180, 210, 1121, 628, 632, 212}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{4(a+4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
+ (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x
)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(
8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*
ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + S
qrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{(3+a-2x$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{1}{2}\text{Subst}\left(\int \frac{}{(3+$$

$$= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x}{4(12+7a+a^2)(3+a-2($$

$$= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x}{4(12+7a+a^2)(3+a-2($$

$$= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x}{4(12+7a+a^2)(3+a-2($$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 166, normalized size = 0.72

$$\frac{a+2x-ax+ax^2+x^3}{4(3+a)(4+a)(a-x(-8+8x-4x^2+x^3))}-\frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{6\log(x-\#1)+a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{16(12+7a+a^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (6*Log[x - #1] + a*
Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2
 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 158, normalized size = 0.68

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(4+a)(3+a)}-\frac{(a-2)x}{4\left(a^2+7a+12\right)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(6+\_R^2+2(a+2)\_R+a\right)\ln\left(x\cdots\right.}{-\_R^3+3\_R^2-4\_R+\cdots}}{16a^2+112a+192}$
risch	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(4+a)(3+a)}-\frac{(a-2)x}{4\left(a^2+7a+12\right)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{a^2+7a+12}+\frac{2(a+2)\_R}{a^2+7a+12}+\frac{\cdots}{a^2\cdots}\right.}{-\_R^3+3\_R^2-\cdots}\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/(a^2+7*a+12)*x^3+1/4*a/(4+a)/(3+a)*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4*a/
(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(a^2+7*a+12)*sum((6+_R^2+2*(a+2
)*_R+a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 1
2)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a
) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a - 8*
x), x)/(a^2 + 7*a + 12)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(197) = 394.
time = 18.78, size = 539, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] (-a*x**2 - a - x**3 + x*(a - 2))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-2048*a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155776*a - 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 253952) + 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t**3*a**12 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a**9 + 5839372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*a**6 + 303864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t**3*a**3 + 683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t**3 + 30208*_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156928*_t**2*a**7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312*_t**2*a**4 + 12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600*_t**2*a + 2944401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**7 - 6200656*_t*a**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*a**3 - 228144640*_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a**6 + 71132*a**5 + 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 956416)/(576*a**7 + 10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3 + 1826304*a**2 + 1867776*a + 917504))))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)

Mupad [B]
time = 2.82, size = 1167, normalized size = 5.05

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 +

```
 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z
^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*
z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 141
55776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 26419
2*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a + 3600*a
^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))
 + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^
9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357
174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 284
2624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z
^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 2
53952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(12952010752*a^3*z^4 +
 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2
0082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^
6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2
 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2
*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16
*a^3 - 2064, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(3932160*a + 2719744*a^2 + 999424*a^3 + 205824*a^4 + 22528
*a^5 + 1024*a^6 + 2359296))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 +
 576))) - (1150976*a + 631808*a^2 + 172800*a^3 + 23552*a^4 + 1280*a^5 + 835
584)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a +
 58880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(16*(816*a + 460*a^2
+ 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(864*a + 228*a^2 + 20*a^3 + 1088))/(
16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + (x*(9*a + 2*a^2 + 8
))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*root(12952010752*
a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*
a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269
680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 52070
4*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 1
02912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57
*a^2 + 16*a^3 - 2064, z, k), k, 1, 4) + (x^3/(4*(7*a + a^2 + 12)) + a/(4*(a
 + 3)*(a + 4)) - (x*(a - 2))/(4*(a + 3)*(a + 4)) + (a*x^2)/(4*(a + 3)*(a +
4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)
```

3.129 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal. Leaf size=349

$$\frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2}+\frac{3(1+(-1+x)^2)}{16(4+a)^2\left(3+a-2(-1+x)^2-(-1+x)^4\right)}+\frac{(5}{8\left(12+7a+\right.}$$

```
[Out] 1/8*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^2+3/16*(1+(-1+x)^2)/(4+a)^
2/(3+a-2*(-1+x)^2-(-1+x)^4)+1/8*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(
-1+x)^2-(-1+x)^4)^2+1/32*((6+a)*(25+7*a)+6*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*
a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)+3/16*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4
+a)^(5/2)-3/64*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(80+7*a^2+14*(4+a)^(1/2
)+a*(47+4*(4+a)^(1/2)))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)-3/64*arct
an((-1+x)/(1+(4+a)^(1/2))^(1/2))*(14+4*a+(-7*a^2-47*a-80)/(4+a)^(1/2))/(3+a
)^2/(4+a)^2/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1694, 1687, 1106, 1192, 1180, 210, 1121, 628, 632, 212}

$$-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{3((x-1)^2+1)}{16(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{8(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{3\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{16(a+4)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (3*(1
 + (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
 + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 +
x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(32*(3
+ a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14
*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]
]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a - (80
 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(64*(
3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1 + x)^2)/Sqr
t[4 + a]])/(16*(4 + a)^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
```

```
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^3}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{(3+a-2x^2$$

$$= \frac{(5+a+(-1+x)^2)\,(-1+x)}{8\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{1}{2}\text{Subst}\left(\int \frac{}{(3+}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^2} - \frac{((6+a)(25+7a)+6(7+}{32\,(12+7a+a^2)^2\,(3+a-}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2}{16(4+a)^2\,(3+a-2(1-x}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2}{16(4+a)^2\,(3+a-2(1-x}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2}{16(4+a)^2\,(3+a-2(1-x}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.10, size = 284, normalized size = 0.81

$$\frac{1}{128}\left(\frac{16(a+2x-ax+ax^2+x^3)}{(3+a)(4+a)\,(a-x\,(-8+8x-4x^2+x^3))^2} + \frac{4(a^2(5-5x+6x^2)+6(-14+28x-12x^2+7x^3)+a(-7+31x+12x^3))}{(3+a)^2(4+a)^2\,(a-x\,(-8+8x-4x^2+x^3))} - \frac{3\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{72\log(x-\#1)+31a\log(x-\#1)+3a^2\log(x-\#1)+8\log(x-\#1)\#1+16a\log(x-\#1)\#1+4a^2\log(x-\#1)\#1+14\log(x-\#1)\#1^2+4a\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{(12+7a+a^2)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x
^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^3)
 + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (72*Log[x - #1]
+ 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1
]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(
-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 409, normalized size = 1.17

method	result
default	$-\frac{\frac{3(7+2a)x^7}{16(a^4+14a^3+73a^2+168a+144)}+\frac{3(a^2-8a-40)x^6}{16(a^4+14a^3+73a^2+168a+144)}-\frac{(29a^2-127a-792)x^5}{32(a^4+14a^3+73a^2+168a+144)}+\frac{(73a^2-227a-1668)x^4}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{}{16\cdots}}{(-x^4+4x^3-8x^2\cdots}$
risch	$\frac{-\frac{3(7+2a)x^7}{16(a^4+14a^3+73a^2+168a+144)}-\frac{3(a^2-8a-40)x^6}{16(a^4+14a^3+73a^2+168a+144)}+\frac{(29a^2-127a-792)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{(73a^2-227a-1668)x^4}{32(a^4+14a^3+73a^2+168a+144)}+\frac{}{16\cdots}}{(-x^4+4x^3-8x^2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14*
a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*a+
144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16*(6
2*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*
a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)/(a^4
+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+168*a+
144))/(-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)*sum((-
72+2*(-2*a-7)*_R^2+4*(-a^2-4*a-2)*_R-3*a^2-31*a)/(-_R^3+3*_R^2-4*_R+2)*ln(x
-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 + 6*(a^2 - 8*a - 40)*x^6 - (29*a^2 - 127*a - 792)*x^
5 + (73*a^2 - 227*a - 1668)*x^4 - 2*(62*a^2 - 103*a - 1104)*x^3 - 9*a^3 - 2
*(5*a^3 - 26*a^2 + 140*a + 1008)*x^2 - 21*a^2 + 3*(3*a^3 - 17*a^2 - 40*a +
192)*x + 36*a)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3
 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6
 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 5
0*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4
- 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17
*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 1
68*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 3*a^2 + 4*(a^2 + 4*a
```

```
 + 2)*x + 31*a + 72)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. $2(318) = 636$.
time = 49.64, size = 1102, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
[Out] -(-9*a**3 - 21*a**2 + 36*a + x**7*(12*a + 42) + x**6*(6*a**2 - 48*a - 240)
+ x**5*(-29*a**2 + 127*a + 792) + x**4*(73*a**2 - 227*a - 1668) + x**3*(-12
4*a**2 + 206*a + 2208) + x**2*(-10*a**3 + 52*a**2 - 280*a - 2016) + x*(9*a*
*3 - 51*a**2 - 120*a + 576))/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 +
4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-
256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14
336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3
- 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**
3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a*
*3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*
a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a*
*3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*
a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 5
27875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 545
37151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 5
51152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2
+ 284993413919539200*a + 68398419340689408) + _t**2*(-4718592*a**10 - 19611
6480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 286939938816*a**6 - 140543
7345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3 - 16752587046912*a*
*2 - 15023392948224*a - 6049461436416) + _t*(-2709504*a**7 - 72880128*a**6
- 839890944*a**5 - 5375877120*a**4 - 20640890880*a**3 - 47542173696*a**2 -
60827369472*a - 33351008256) + 20736*a**5 - 155601*a**4 - 4706424*a**3 - 29
249424*a**2 - 74027520*a - 68345856, Lambda(_t, _t*log(x + (-469762048*_t**
3*a**20 - 31417434112*_t**3*a**19 - 992305217536*_t**3*a**18 - 196635766292
```

```
48*_t**3*a**17 - 273880031690752*_t**3*a**16 - 2846116194287616*_t**3*a**15
 - 22853982892326912*_t**3*a**14 - 144840417605582848*_t**3*a**13 - 7331931
54773123072*_t**3*a**12 - 2977941469704224768*_t**3*a**11 - 967719737311730
0736*_t**3*a**10 - 24850421452415959040*_t**3*a**9 - 48984708931769073664*_
t**3*a**8 - 69124682329943441408*_t**3*a**7 - 54921507243737219072*_t**3*a*
*6 + 18833423088924753920*_t**3*a**5 + 128767022044444360704*_t**3*a**4 + 1
97893824476545548288*_t**3*a**3 + 170576989286005997568*_t**3*a**2 + 837098
68624400351232*_t**3*a + 18392762450832261120*_t**3 + 136642560*_t**2*a**17
 + 7616593920*_t**2*a**16 + 198980665344*_t**2*a**15 + 3234300690432*_t**2*
a**14 + 36614363283456*_t**2*a**13 + 306155605721088*_t**2*a**12 + 19563396
56687616*_t**2*a**11 + 9747894775578624*_t**2*a**10 + 38291841445330944*_t*
*2*a**9 + 119050488573591552*_t**2*a**8 + 292236772188880896*_t**2*a**7 + 5
61261720373297152*_t**2*a**6 + 828898581078343680*_t**2*a**5 + 914439454498
750464*_t**2*a**4 + 718255692208668672*_t**2*a**3 + 369227414724673536*_t**
2*a**2 + 104815442748506112*_t**2*a + 10263520138493952*_t**2 + 4128768*_t*
a**15 + 235608192*_t*a**14 + 6050117376*_t*a**13 + 92875570560*_t*a**12 + 9
50838962688*_t*a**11 + 6825858397056*_t*a**10 + 34932826734336*_t*a**9 + 12
5262778564224*_t*a**8 + 287989861404672*_t*a**7 + 257684685023232*_t*a**6 -
 836263788945408*_t*a**5 - 4002432415137792*_t*a**4 - 8409454278082560*_t*a
**3 - 10371340262965248*_t*a**2 - 7285247072796672*_t*a - 2270140431335424*
_t + 1000512*a**12 + 42546357*a**11 + 777344580*a**10 + 7998006582*a**9 + 5
0045408388*a**8 + 182866499613*a**7 + 247394170512*a**6 - 1063305068832*a**
5 - 6960658344192*a**4 - 19132655580288*a**3 - 30001872614400*a**2 - 261928
92672000*a - 9953981595648)/(1354752*a**12 + 44550027*a**11 + 663517980*a**
10 + 5951170602*a**9 + 36270700668*a**8 + 162289912419*a**7 + 567868212432*
a**6 + 1626099007104*a**5 + 3825839091456*a**4 + 7035734732544*a**3 + 92167
60449024*a**2 + 7467334520832*a + 2773884911616))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)
```

Mupad [B]
time = 3.39, size = 2500, normalized size = 7.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] symsum(log(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153
980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a
^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 59995324
41600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206
246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378
493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a
^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2
- 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11
043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 60494614
36416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 728801
28*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3335100825
6*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 68
345856, z, k)*((242823168*a + 170044416*a^2 + 63509760*a^3 + 13340736*a^4 +
 1494144*a^5 + 69696*a^6 + 144506880)/(16384*(940032*a + 1195776*a^2 + 8993
28*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9
+ a^10 + 331776)) + root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*
z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 55115219365
5275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 +
 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z
^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*
z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 -
4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 150233929482
24*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7
*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 -
 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z
 + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3
3351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736
*a^5 - 68345856, z, k)*(root(15003759578972160*a^8*z^4 + 54537151127224320*
a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 5511521
93655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z
^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200
*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a
^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^
4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392
948224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608
*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z
^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a
^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z
 + 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 2
0736*a^5 - 68345856, z, k)*((4290672328704*a + 6001143054336*a^2 + 50259170
42688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 5986
2155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11
 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4
+ 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) -
(x*(536334041088*a + 750142881792*a^2 + 628239630336*a^3 + 350065000448*a^4
```

```
 + 136275034112*a^5 + 37819514880*a^6 + 7482769408*a^7 + 1034420224*a^8 + 9
5158272*a^9 + 5242880*a^10 + 131072*a^11 + 173946175488))/(2048*(940032*a +
 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7
+ 582*a^8 + 36*a^9 + a^10 + 331776))) - (73421291520*a + 81260445696*a^2 +
52393672704*a^3 + 21688418304*a^4 + 5977620480*a^5 + 1096949760*a^6 + 12924
5184*a^7 + 8871936*a^8 + 270336*a^9 + 29444014080)/(16384*(940032*a + 11957
76*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*
a^8 + 36*a^9 + a^10 + 331776)) + (x*(2632974336*a + 3015180288*a^2 + 200894
0544*a^3 + 858243072*a^4 + 243806208*a^5 + 46055424*a^6 + 5578752*a^7 + 393
216*a^8 + 12288*a^9 + 1019215872))/(2048*(940032*a + 1195776*a^2 + 899328*a
^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^
10 + 331776))) - (x*(10805760*a + 7173504*a^2 + 2539872*a^3 + 505800*a^4 +
53712*a^5 + 2376*a^6 + 6782976))/(2048*(940032*a + 1195776*a^2 + 899328*a^3
 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10
 + 331776))) - (133812*a + 56187*a^2 + 10098*a^3 + 648*a^4 + 115776)/(16384
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(1971*a^2 - 1539*a +
918*a^3 + 108*a^4 - 6372))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442
864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331
776)))*root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 1539804
18717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z
^4 + 664192984106926080*a^3*z^4 + 5533622120271...
```

3.130 $\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx$

Optimal. Leaf size=210

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6+\frac{4}{7}\left(1024-1536a+192a^2-a^3\right)x^7-4\left(512-288a+1\right.$$

```
[Out] 1/3*a^4*x^3+8*a^3*x^4+32/5*(12-a)*a^2*x^5+8/3*a*(a^2-48*a+128)*x^6+4/7*(-a^
3+192*a^2-1536*a+1024)*x^7-4*(15*a^2-288*a+512)*x^8+64/9*(128-3*a)*(4-a)*x^
9-24/5*(a^2-128*a+896)*x^10+2/11*(3*a^2-1536*a+20480)*x^11-8/3*(928-35*a)*x
^12+32/13*(524-9*a)*x^13-8/7*(464-3*a)*x^14+4/15*(640-a)*x^15-42*x^16+128/1
7*x^17-8/9*x^18+1/19*x^19
```

Rubi [A]
time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6874}

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+128\right)x^6+\frac{32}{5}(12-a)a^2x^5+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right)x^7+\frac{4}{15}(640-a)x^{15}-\frac{8}{7}(464-3a)x^{14}+\frac{32}{13}(524-9a)x^{13}-\frac{8}{3}(928-35a)x^{12}+\frac{64}{9}(128-3a)(4-a)x^9+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2)
*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15*a
^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)/5
+ (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524
 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16
 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \int \left(a^4x^2+32a^3x^3-32(-12+a)a^2x^4+16a\left(128-48a+a^2\right)x^5-4\right.\\ &= \frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6+\frac{4}{7}\left(1024\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 204, normalized size = 0.97

$$\frac{a^4x^3}{3}+8a^3x^4-\frac{32}{5}(-12+a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6-\frac{4}{7}\left(-1024+1536a-192a^2+a^3\right)x^7-4\left(512-288a+15a^2\right)x^8+\frac{64}{9}\left(512-140a+3a^2\right)x^9-\frac{24}{5}\left(896-128a+a^2\right)x^{10}+\frac{2}{11}\left(20480-1536a+3a^2\right)x^{11}+\frac{8}{3}(-928+35a)x^{12}-\frac{32}{13}(-524+9a)x^{13}+\frac{8}{7}(-464+3a)x^{14}-\frac{4}{15}(-640+a)x^{15}-42x^{16}+\frac{128x^{17}}{17}-\frac{8x^{18}}{9}+\frac{x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2
)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 - 4*(512 - 288*a + 15
*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10
)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 + (8*(-928 + 35*a)*x^12)/3 - (32
*(-524 + 9*a)*x^13)/13 + (8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)/15 -
 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Maple [A]
time = 0.07, size = 267, normalized size = 1.27

method	result
norman	$\frac{a^4 x^3}{3}+8a^3x^4+\left(-\frac{32}{5}a^3+\frac{384}{5}a^2\right)x^5+\left(\frac{8}{3}a^3-128a^2+\frac{1024}{3}a\right)x^6+\left(-\frac{4}{7}a^3+\frac{768}{7}a^2-\frac{6144}{7}a+\frac{4096}{7}\right.$
gosper	$-\frac{4}{7}x^7a^3-\frac{32}{5}x^5a^3-2048x^8+\frac{32768}{9}x^9+\frac{4096}{7}x^7-60a^2x^8+1152a\,x^8-\frac{3072}{11}x^{11}a+\frac{280}{3}x^{12}a-\frac{8960}{9}$
risch	$-\frac{4}{7}x^7a^3-\frac{32}{5}x^5a^3-2048x^8+\frac{32768}{9}x^9+\frac{4096}{7}x^7-60a^2x^8+1152a\,x^8-\frac{3072}{11}x^{11}a+\frac{280}{3}x^{12}a-\frac{8960}{9}$
default	$\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+\frac{(-4a+2560)x^{15}}{15}+\frac{(48a-7424)x^{14}}{14}+\frac{(-288a+16768)x^{13}}{13}+\frac{(1120a-29696)x^{12}}{12}+\frac{(2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*x^19-8/9*x^18+128/17*x^17-42*x^16+1/15*(-4*a+2560)*x^15+1/14*(48*a-742
4)*x^14+1/13*(-288*a+16768)*x^13+1/12*(1120*a-29696)*x^12+1/11*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^11+1/10*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128
))*x^10+1/9*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^9+1/8*(-1
60*a^2+32*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^8+1/7*(2*a^2*(-2*a+128)+32
*a*(8*a-128)+(-16*a+64)^2)*x^7+1/6*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^6+1/
5*(2*a^2*(-16*a+64)+256*a^2)*x^5+8*a^3*x^4+1/3*a^4*x^3
```

Maxima [A]
time = 0.27, size = 182, normalized size = 0.87

$$\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}(a-640)x^{15}-42x^{16}+\frac{8}{7}(3a-464)x^{14}-\frac{32}{13}(9a-524)x^{13}+\frac{8}{3}(35a-928)x^{12}+\frac{2}{11}(3a^2-1536a+20480)x^{11}-\frac{24}{5}(a^2-128a+896)x^{10}+\frac{64}{9}(3a^2-140a+512)x^9-4(15a^2-288a+512)x^8-\frac{4}{7}(a^3-192a^2+1536a-1024)x^7+\frac{1}{3}a^4x^3+8a^3x^4+\frac{8}{3}(a^3-48a^2+128a)x^6-\frac{32}{5}(a^3-12a^2)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3
*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a
^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 1
40*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*
```

```
a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 -
32/5*(a^3 - 12*a^2)*x^5
```

Fricas [A]
time = 0.35, size = 182, normalized size = 0.87

$$\frac{1}{19}\,x^{19}-\frac{8}{9}\,x^{18}+\frac{128}{17}\,x^{17}-\frac{4}{15}\,(a-640)x^{15}-42\,x^{16}+\frac{8}{7}\,(3\,a-464)x^{14}-\frac{32}{13}\,(9\,a-524)x^{13}+\frac{8}{3}\,(35\,a-928)x^{12}+\frac{2}{11}\,(3\,a^2-1536\,a+20480)x^{11}-\frac{24}{5}\,(a^2-128\,a+896)x^{10}+\frac{64}{9}\,(3\,a^2-140\,a+512)x^{9}-4\,(15\,a^2-288\,a+512)x^{8}-\frac{4}{7}\,(a^3-192\,a^2+1536\,a-1024)x^{7}+\frac{1}{3}\,a^4x^3+8\,a^3x^4+\frac{8}{3}\,(a^3-48\,a^2+128\,a)x^{6}-\frac{32}{5}\,(a^3-12\,a^2)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3
*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a
^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 1
40*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*
a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 -
32/5*(a^3 - 12*a^2)*x^5
```

Sympy [A]
time = 0.03, size = 219, normalized size = 1.04

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+x^{15}\cdot\left(\frac{512}{3}-\frac{4a}{15}\right)+x^{14}\cdot\left(\frac{24a}{7}-\frac{3712}{7}\right)+x^{13}\cdot\left(\frac{16768}{13}-\frac{288a}{13}\right)+x^{12}\cdot\left(\frac{280a}{3}-\frac{7424}{3}\right)+x^{11}\cdot\left(\frac{6a^2}{11}-\frac{3072a}{11}+\frac{40960}{11}\right)+x^{10}\left(-\frac{24a^2}{5}+\frac{3072a}{5}-\frac{21504}{5}\right)+x^{9}\cdot\left(\frac{64a^2}{3}-\frac{8960a}{9}+\frac{32768}{9}\right)+x^{8}(-60a^2+1152a-2048)+x^{7}\left(-\frac{4a^3}{7}+\frac{768a^2}{7}-\frac{6144a}{7}+\frac{4096}{7}\right)+x^{6}\cdot\left(\frac{8a^3}{3}-128a^2+\frac{1024a}{3}\right)+x^{5}\left(-\frac{32a^3}{5}+\frac{384a^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16
+ x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 288*
a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/11)
 + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 3
2768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 -
6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3
/5 + 384*a**2/5)
```

Giac [A]
time = 2.91, size = 222, normalized size = 1.06

$$\frac{1}{19}\,x^{19}-\frac{8}{9}\,x^{18}+\frac{128}{17}\,x^{17}-\frac{4}{15}\,ax^{15}-42\,x^{16}+\frac{24}{7}\,ax^{14}+\frac{512}{3}\,x^{15}-\frac{288}{13}\,ax^{13}-\frac{3712}{7}\,x^{14}+\frac{6}{11}\,a^2x^{11}+\frac{280}{3}\,ax^{12}+\frac{16768}{13}\,x^{13}-\frac{24}{5}\,a^2x^{10}-\frac{3072}{11}\,ax^{11}-\frac{7424}{3}\,x^{12}+\frac{64}{3}\,a^2x^9+\frac{3072}{5}\,ax^{10}+\frac{40960}{11}\,x^{11}-\frac{4}{7}\,a^3x^7-60\,a^2x^8-\frac{8960}{9}\,ax^9-\frac{21504}{5}\,x^{10}+\frac{8}{3}\,a^3x^6+\frac{768}{7}\,a^2x^7+1152\,ax^8+\frac{32768}{9}\,x^9-\frac{32}{5}\,a^3x^5-128\,a^2x^6-\frac{6144}{7}\,ax^7-2048\,x^8+\frac{1}{3}\,a^4x^3+8\,a^3x^4+\frac{384}{5}\,a^2x^5+\frac{1024}{3}\,ax^6+\frac{4096}{7}\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7*a*x^14 +
512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^11 + 280/3*a*x^12 + 1
6768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11 - 7424/3*x^12 + 64/3*a^2*x^9
+ 3072/5*a*x^10 + 40960/11*x^11 - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x^9 -
```

```
 21504/5*x^10 + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9 - 32
/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^
4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

Mupad [B]
time = 2.29, size = 178, normalized size = 0.85

$$x^{14}\left(\frac{24\,a}{7}-\frac{3712}{7}\right)-x^{15}\left(\frac{4\,a}{15}-\frac{512}{3}\right)+x^{12}\left(\frac{280\,a}{3}-\frac{7424}{3}\right)-x^{13}\left(\frac{288\,a}{13}-\frac{16768}{13}\right)-x^8\left(60\,a^2-1152\,a+2048\right)-x^{10}\left(\frac{24\,a^2}{5}-\frac{3072\,a}{5}+\frac{21504}{5}\right)+x^9\left(\frac{64\,a^2}{3}-\frac{8960\,a}{9}+\frac{32768}{9}\right)+x^{11}\left(\frac{6\,a^2}{11}-\frac{3072\,a}{11}+\frac{40960}{11}\right)-x^7\left(\frac{4\,a^3}{7}-\frac{768\,a^2}{7}+\frac{6144\,a}{7}-\frac{4096}{7}\right)-42\,x^{16}+\frac{128\,x^{17}}{17}-\frac{8\,x^{18}}{9}+\frac{x^{19}}{19}+8\,a^3\,x^4+\frac{a^4\,x^3}{3}+\frac{8\,a\,x^6\,\left(a^2-48\,a+128\right)}{3}-\frac{32\,a^2\,x^5\,\left(a-12\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^14*((24*a)/7 - 3712/7) - x^15*((4*a)/15 - 512/3) + x^12*((280*a)/3 - 7424
/3) - x^13*((288*a)/13 - 16768/13) - x^8*(60*a^2 - 1152*a + 2048) - x^10*((
24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9*((64*a^2)/3 - (8960*a)/9 + 32768/9)
 + x^11*((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7*((6144*a)/7 - (768*a^2)
/7 + (4*a^3)/7 - 4096/7) - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19 +
 8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x^5*(a
- 12))/5
```

3.131 $\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx$

Optimal. Leaf size=138

$$\frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7+6(48-5a)x^8-\frac{32}{9}(70-3a)x^9+\frac{1}{5}$$

```
[Out] 1/3*a^3*x^3+6*a^2*x^4+24/5*(8-a)*a*x^5+2/3*(3*a^2-96*a+128)*x^6-3/7*(a^2-12
8*a+512)*x^7+6*(48-5*a)*x^8-32/9*(70-3*a)*x^9+12/5*(64-a)*x^10-3/11*(256-a)
*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6874}

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6(48-5a)x^8+\frac{24}{5}(8-a)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^
6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^
9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^1
3)/13 + (6*x^14)/7 - x^15/15
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\int \left(a^3x^2+24a^2x^3-24(-8+a)ax^4+4\left(128-96a+3a^2\right)x^5-3(512$$

$$=\frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}(512-1$$

Mathematica [A]
time = 0.01, size = 132, normalized size = 0.96

$$\frac{a^3x^3}{3}+6a^2x^4-\frac{24}{5}(-8+a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7-6(-48+5a)x^8+\frac{32}{9}(-70+3a)x^9-\frac{12}{5}(-64+a)x^{10}+\frac{3}{11}(-256+a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x
^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*a)
*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 - (7
2*x^13)/13 + (6*x^14)/7 - x^15/15
```

Maple [A]
time = 0.06, size = 143, normalized size = 1.04

method	result
norman	$\frac{a^3x^3}{3}+6a^2x^4+\left(-\frac{24}{5}a^2+\frac{192}{5}a\right)x^5+\left(2a^2-64a+\frac{256}{3}\right)x^6+\left(-\frac{3}{7}a^2+\frac{384}{7}a-\frac{1536}{7}\right)x^7+(-30a+$
gosper	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}x^7a-\frac{1536}{7}x^7-30a\,x$
risch	$\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}a^2x^5+\frac{192}{5}a\,x^5+2a^2x^6-64a\,x^6+\frac{256}{3}x^6-\frac{3}{7}a^2x^7+\frac{384}{7}x^7a-\frac{1536}{7}x^7-30a\,x$
default	$-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+\frac{(3a-768)x^{11}}{11}+\frac{(-24a+1536)x^{10}}{10}+\frac{(96a-2240)x^9}{9}+\frac{(-240a+2304)x^8}{8}+\frac{(a(-2a+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*x^15+6/7*x^14-72/13*x^13+70/3*x^12+1/11*(3*a-768)*x^11+1/10*(-24*a+15
36)*x^10+1/9*(96*a-2240)*x^9+1/8*(-240*a+2304)*x^8+1/7*(a*(-2*a+128)+256*a-
1536-a^2)*x^7+1/6*(a*(8*a-128)-256*a+512+4*a^2)*x^6+1/5*(a*(-16*a+64)+128*a
-8*a^2)*x^5+6*a^2*x^4+1/3*a^3*x^3
```

Maxima [A]
time = 0.29, size = 113, normalized size = 0.82

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8-\frac{3}{7}(a^2-128a+512)x^7+\frac{2}{3}(3a^2-96a+128)x^6+\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}(a^2-8a)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/5
*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a
+ 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*
(a^2 - 8*a)*x^5
```

Fricas [A]
time = 0.35, size = 113, normalized size = 0.82

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8-\frac{3}{7}(a^2-128a+512)x^7+\frac{2}{3}(3a^2-96a+128)x^6+\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}(a^2-8a)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/5
*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a
+ 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*
(a^2 - 8*a)*x^5
```

Sympy [A]
time = 0.02, size = 134, normalized size = 0.97

$$\frac{a^3x^3}{3}+6a^2x^4-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+x^{11}\cdot\left(\frac{3a}{11}-\frac{768}{11}\right)+x^{10}\cdot\left(\frac{768}{5}-\frac{12a}{5}\right)+x^9\cdot\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^8\cdot(288-30a)+x^7\left(-\frac{3a^2}{7}+\frac{384a}{7}-\frac{1536}{7}\right)+x^6\cdot\left(2a^2-64a+\frac{256}{3}\right)+x^5\left(-\frac{24a^2}{5}+\frac{192a}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/3
 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240/9
) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a**2
- 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)
```

Giac [A]
time = 3.66, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}ax^{11}+\frac{70}{3}x^{12}-\frac{12}{5}ax^{10}-\frac{768}{11}x^{11}+\frac{32}{3}ax^9+\frac{768}{5}x^{10}-\frac{3}{7}a^2x^7-30\,ax^8-\frac{2240}{9}x^9+2\,a^2x^6+\frac{384}{7}ax^7+288\,x^8-\frac{24}{5}a^2x^5-64\,ax^6-\frac{1536}{7}x^7+\frac{1}{3}a^3x^3+6\,a^2x^4+\frac{192}{5}ax^5+\frac{256}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12/5*a*x^10
- 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9*x
^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/7*x
^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6
```

Mupad [B]
time = 0.09, size = 113, normalized size = 0.82

$$x^{11}\left(\frac{3\,a}{11}-\frac{768}{11}\right)-x^{10}\left(\frac{12\,a}{5}-\frac{768}{5}\right)-x^8\,(30\,a-288)+x^9\left(\frac{32\,a}{3}-\frac{2240}{9}\right)+x^6\left(2\,a^2-64\,a+\frac{256}{3}\right)-x^7\left(\frac{3\,a^2}{7}-\frac{384\,a}{7}+\frac{1536}{7}\right)+\frac{70\,x^{12}}{3}-\frac{72\,x^{13}}{13}+\frac{6\,x^{14}}{7}-\frac{x^{15}}{15}+6\,a^2\,x^4+\frac{a^3\,x^3}{3}-\frac{24\,a\,x^5\,(a-8)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^11*((3*a)/11 - 768/11) - x^10*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^9
*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384*a
)/7 + 1536/7) + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15 + 6*a^2*x
^4 + (a^3*x^3)/3 - (24*a*x^5*(a - 8))/5
```

3.132 $\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx$

Optimal. Leaf size=79

$$\frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+\frac{x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+4*a*x^4+16/5*(4-a)*x^5-4/3*(16-a)*x^6+2/7*(64-a)*x^7-10*x^8+32/
9*x^9-4/5*x^10+1/11*x^11
```

Rubi [A]
time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6874}

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx &= \int \left(a^2x^2+16ax^3-16(-4+a)x^4+8(-16+a)x^5-2(-64+a)x^6\right.\\ &= \frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3}+4ax^4-\frac{16}{5}(-4+a)x^5+\frac{4}{3}(-16+a)x^6-\frac{2}{7}(-64+a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+\frac{x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-64
 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Maple [A]
time = 0.06, size = 66, normalized size = 0.84

method	result	siz
norman	$\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^{9}}{9}-10x^{8}+\left(-\frac{2a}{7}+\frac{128}{7}\right)x^{7}+\left(\frac{4a}{3}-\frac{64}{3}\right)x^{6}+\left(-\frac{16a}{5}+\frac{64}{5}\right)x^{5}+4a\,x^{4}+\frac{a^{2}x^{3}}{3}$	63
default	$\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^{9}}{9}-10x^{8}+\frac{(-2a+128)x^{7}}{7}+\frac{(8a-128)x^{6}}{6}+\frac{(-16a+64)x^{5}}{5}+4a\,x^{4}+\frac{a^{2}x^{3}}{3}$	66
gosper	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}x^{7}a+\frac{128}{7}x^{7}+\frac{4}{3}a\,x^{6}-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}x^{3}$	69
risch	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}x^{7}a+\frac{128}{7}x^{7}+\frac{4}{3}a\,x^{6}-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}x^{3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+1/7*(-2*a+128)*x^7+1/6*(8*a-128)*x^6+1/5
*(-16*a+64)*x^5+4*a*x^4+1/3*a^2*x^3
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.75

$$\frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^{9}-\frac{2}{7}\,(a-64)x^{7}-10\,x^{8}+\frac{4}{3}\,(a-16)x^{6}-\frac{16}{5}\,(a-4)x^{5}+\frac{1}{3}\,a^{2}x^{3}+4\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*
x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Fricas [A]
time = 0.37, size = 59, normalized size = 0.75

$$\frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^{9}-\frac{2}{7}\,(a-64)x^{7}-10\,x^{8}+\frac{4}{3}\,(a-16)x^{6}-\frac{16}{5}\,(a-4)x^{5}+\frac{1}{3}\,a^{2}x^{3}+4\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*
x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Sympy [A]
time = 0.01, size = 73, normalized size = 0.92

$$\frac{a^{2}x^{3}}{3}+4ax^{4}+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^{9}}{9}-10x^{8}+x^{7}\cdot\left(\frac{128}{7}-\frac{2a}{7}\right)+x^{6}\cdot\left(\frac{4a}{3}-\frac{64}{3}\right)+x^{5}\cdot\left(\frac{64}{5}-\frac{16a}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7*
(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)
```

Giac [A]
time = 3.81, size = 68, normalized size = 0.86

$$\frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^{9}-\frac{2}{7}\,ax^{7}-10\,x^{8}+\frac{4}{3}\,ax^{6}+\frac{128}{7}\,x^{7}-\frac{16}{5}\,ax^{5}-\frac{64}{3}\,x^{6}+\frac{1}{3}\,a^{2}x^{3}+4\,ax^{4}+\frac{64}{5}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x^
7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5
```

Mupad [B]
time = 0.04, size = 64, normalized size = 0.81

$$x^6\left(\frac{4\,a}{3}-\frac{64}{3}\right)-x^5\left(\frac{16\,a}{5}-\frac{64}{5}\right)-x^7\left(\frac{2\,a}{7}-\frac{128}{7}\right)+4\,a\,x^4-10\,x^8+\frac{32\,x^9}{9}-\frac{4\,x^{10}}{5}+\frac{x^{11}}{11}+\frac{a^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a*
x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3
```

3.133 $\int x^2(a+8x-8x^2+4x^3-x^4)\,dx$

Optimal. Leaf size=35

$$\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

```
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx&=\int \left(ax^2+8x^3-8x^4+4x^5-x^6\right)\,dx\\&=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Maple [A]
time = 0.01, size = 28, normalized size = 0.80

method	result	size
gosper	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
default	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
norman	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
risch	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.77

$$-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Fricas [A]
time = 0.35, size = 27, normalized size = 0.77

$$-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.83

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

Giac [A]
time = 5.19, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7+\frac{2}{3}x^6-\frac{8}{5}x^5+\frac{1}{3}ax^3+2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Mupad [B]
time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4+\frac{ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

3.134 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=99

$$-\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}-\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}+\frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}$$

```
[Out] arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)-1/2*arctan((-1+x)/(1-(4+a)^(1
/2))^(1/2))/(1-(4+a)^(1/2))^(1/2)-1/2*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2))/
(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1694, 1687, 1180, 210, 12, 1121, 632, 212}

$$-\frac{\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/Sqrt[1 - Sqrt[4 + a]] - ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-
1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{2x}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{3+a-2x^2-x^4}\right.$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+\sqrt{4}}\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} + \text{Subst}\left(\int \frac{}{3+}\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} - 2\text{Subst}\left(\int \frac{}{4(}\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-}{\sqrt{4}}\right.}{\sqrt{4+a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.62

$$-\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)/(-2 +
 4*#1 - 3*#1^2 + #1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 54, normalized size = 0.55

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^2/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*
_Z-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 19.57, size = 1515766, normalized size = 15310.77

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] -1/192*(128*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(3*((sqrt(2)*(9*a^2 + 55*a) + 84*s
qrt(2))*(a + 4)^(3/2) - 84*sqrt(2)*a^2 - 4*sqrt(2)*(a^4 + 14*a^3 + 73*a^2 +
 168*a + 144)*sqrt((sqrt(a + 4) + 1)/(a + 3)) - sqrt(2)*(a^4 + 15*a^3) - 4*
(10*sqrt(2)*a^2 + sqrt(2)*(a^3 + 33*a) + 36*sqrt(2))*sqrt(a + 4) - 208*sqrt
(2)*a - 192*sqrt(2))/((10*sqrt(2)*a^2 + sqrt(2)*(a^3 + 33*a) + 36*sqrt(2))*
(a + 4)^(3/2)) + (28*sqrt(2)*a^3 + sqrt(2)*(a^3 + 10*a^2 + 33*a + 36)*(a +
4)^(3/2)*sqrt((sqrt(a + 4) + 1)/(a + 3)) + 2*sqrt(2)*(a^4 + 73*a^2) + 336*s
qrt(2)*a + 288*sqrt(2))^2/((10*sqrt(2)*a^2 + sqrt(2)*(a^3 + 33*a) + 36*sqrt
(2))^2*(a + 4)^3))/(13824*((sqrt(2)*(a^3 + 10*a^2 + 33*a + 36)*((sqrt(a + 4
) + 1)/(a + 3))^(3/2) - 2*sqrt(2)*(5*a^2 + 31*a + 48)*sqrt((sqrt(a + 4) + 1
)/(a + 3)) - 8*sqrt(2)*(a^2 + 7*a) - 96*sqrt(2))*(a + 4)^(3/2) - 2*sqrt(2)*
(3*a^3 + 29*a^2) + 6*(2*sqrt(2)*(a^3 + 10*a^2 + 33*a + 36)*sqrt((sqrt(a + 4
```

```
) + 1)/(a + 3)) + sqrt(2)*(a^3 + 11*a^2) + 40*sqrt(2)*a + 48*sqrt(2))*sqrt(
a + 4) - 184*sqrt(2)*a -  ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(82) = 164$.
time = 4.68, size = 172, normalized size = 1.74

$$-\operatorname{RootSum}\left(t^4\cdot(256a^3+2816a^2+10240a+12288)+t^2(-160a^2-1152a-2048)+t(-32a^2-256a-512)-a^2,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4-448t^3a^3-256t^3a^2+3584t^3a+6144t^3-224t^2a^3-2208t^2a^2-7168t^2a-7680t^2+56ta^3+400ta^2+864ta+512t+5a^3+34a^2+56a}{a^3+60a^2+320a+448}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
- 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(x
+ (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 6144*_
t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t
*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 +
60*a**2 + 320*a + 448))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Mupad [B]
time = 2.78, size = 878, normalized size = 8.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] symsum(log(64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 1
60*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k
) - a - 8*x + 20*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)*a - 48*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160
*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^
2*a + 64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^
2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a
 + 128*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*
```

```
z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*x -
 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^
2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*x - 1
92*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2
- 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2 + 256*r
oot(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 11
52*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3 - 4*root(28
16*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z
^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a*x + 32*root(2816*
a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2
- 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*a*x - 64*root(2816*a
^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 -
 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a*x)*root(2816*a^2*z^
4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048
*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k), k, 1, 4)
```

3.135 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal. Leaf size=225

$$\frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}+\frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)}-\frac{\left(4+a+\right.}{8}$$

```
[Out] 1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/4*(4+a)*(2+(-1+x)^2)*(-1
+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)+1/2*arctanh((1+(-1+x)^2)/(4+a)^(
1/2))/(4+a)^(3/2)-1/8*arctan((-1+x)/(1-(4+a)^(1/2))^(1/2))*(4+a+(4+a)^(1/2)
)/(3+a)/(4+a)/(1-(4+a)^(1/2))^(1/2)-1/8*arctan((-1+x)/(1+(4+a)^(1/2))^(1/2)
)*(4+a-(4+a)^(1/2))/(3+a)/(4+a)/(1+(4+a)^(1/2))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1694, 1687, 1192, 1180, 210, 12, 1121, 628, 632, 212}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}}-\frac{\left(a-\sqrt{a+4}+4\right)\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)\sqrt{\sqrt{a+4}+1}}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2(a+4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)
*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/
(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(
-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) +
 ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
```

```
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \mathrm{Subst}\left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right)$$

$$= \mathrm{Subst}\left(\int \frac{2x}{(3+a-2x^2-x^4)^2}\,dx, x, -1+x\right) + \mathrm{Subst}\left(\int \frac{1+}{(3+a-2}\right.$$

$$= \frac{(4+a)\,(2+(-1+x)^2)\,(-1+x)}{4\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)} + 2\mathrm{Subst}\left(\int \frac{}{(3+}\right.$$

$$= \frac{(4+a)\,(2+(-1+x)^2)\,(-1+x)}{4\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{\left(4+a-\sqrt{4+}\right.}{}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\,(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)\,(2+(-}{4\,(12+7a+a^2)\,(3+a}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\,(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)\,(2+(-}{4\,(12+7a+a^2)\,(3+a}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\,(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)\,(2+(-}{4\,(12+7a+a^2)\,(3+a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.05, size = 182, normalized size = 0.81

$$\frac{2x(4-3x+2x^2)+a(1+x-x^2+x^3)}{4(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)}-\frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{-a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+4\log(x-\#1)\#1^2+a\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{16\left(12+7a+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*(
-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (
-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]*#1
^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^
2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 160, normalized size = 0.71

method	result
default	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(3+a)(4+a)}+\frac{(a+8)x}{4(3+a)(4+a)}+\frac{a}{4(4+a)(3+a)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\_R^2(4+a)+2(a+2)\_R-a\right)\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}}{16(4+a)(3+a)}$
risch	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(3+a)(4+a)}+\frac{(a+8)x}{4(3+a)(4+a)}+\frac{a}{4(4+a)(3+a)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{3+a}+\frac{2(a+2)\_R}{(3+a)(4+a)}-\frac{a}{(4+a)(3+a)}\right)\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4/(3+a)*x^3-1/4*(6+a)/(3+a)/(4+a)*x^2+1/4*(a+8)/(3+a)/(4+a)*x+1/4*a/(4+a
)/(3+a))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(4+a)/(3+a)*sum((_R^2*(4+a)+2*(a+2)*
_R-a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a + 4)*x^3 - (a + 6)*x^2 + (a + 8)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*
(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a
+ 12)*x - 12*a) - 1/4*integrate(((a + 4)*x^2 + 2*(a + 2)*x - a)/(x^4 - 4*x^
3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(185) = 370$.
time = 21.25, size = 561, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (-a + x**3*(-a - 4) + x**2*(a + 6) + x*(-a - 8))/(-4*a**3 - 28*a**2 - 48*a
+ x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2
 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 +
 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 53571747
84*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312)
 + _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24215552
*a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a**3 + 23
7568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792*a, Lamb
da(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*_t**3*a**
10 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t**3*a**7 -
 42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832*_t**3*a**4
 - 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 439848271872*_t**3*a
 - 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9 - 15400960*_t
**2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414427648*_t**2*a
**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938255360*_t**2*a*
*2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 - 6096*_t*a**8 +
174608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 119009280*_t*a**4 + 3
32017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 225837056*_t + 11*a
**8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4 + 1567552*a**3 + 20
49792*a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**6 + 165176*a**5 + 82456
0*a**4 + 2452288*a**3 + 4340224*a**2 + 4229120*a + 1748992)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
[Out] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

Mupad [B]
time = 2.85, size = 1218, normalized size = 5.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
[Out] symsum(log((x*(40*a + 7*a^2 + 56))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
 a^5 + 576)) - (48*a + 12*a^2 - a^3)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^
4 + a^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8
*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568
*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 242155
52*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*
z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*z + 5888*a^4
*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 144*a^3 - a^4,
 z, k)*((28160*a + 11328*a^2 + 2064*a^3 + 144*a^4 + 26624)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + root(12952010752*a^3*z^4 + 31653888
*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2008232755
2*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7
247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 2094
08*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z +
53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 102
4*a^2 + 144*a^3 - a^4, z, k)*(root(12952010752*a^3*z^4 + 31653888*a^7*z^4 +
 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4
+ 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*
z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2
 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*
z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 14
4*a^3 - a^4, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(1966080*a + 1359872*a^2 + 499712*a^3 + 102912*a^4 + 11264
*a^5 + 512*a^6 + 1179648))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 5
76))) - (1359872*a + 749568*a^2 + 205824*a^3 + 28160*a^4 + 1536*a^5 + 98304
0)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a + 5
8880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(8*(816*a + 460*a^2 + 1
29*a^3 + 18*a^4 + a^5 + 576))) + (x*(448*a + 104*a^2 - 2*a^3 - 2*a^4 + 512)
)/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))))*root(12952010752*a
^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a
*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 2696
```

```
80640*a^6*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 18780
16*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2
+ 237568*a^2*z + 53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752
*z + 1792*a + 1024*a^2 + 144*a^3 - a^4, z, k), k, 1, 4) + (x^3/(4*(a + 3))
+ a/(4*(a + 3)*(a + 4)) - (x^2*(a + 6))/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(
4*(a + 3)*(a + 4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)
```

3.136 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=545

$$-\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}\,b+3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{\quad}{\sqrt{3}\sqrt{\ }}\right.}{9\sqrt{3}\,a^{5/6}b^2\sqrt{4b-3\sqrt[3]{\ }}}$$

[Out] -1/18*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3)+1/6*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(2/3)/b^2/c^(1/3)+1/18*(-1)^(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3)-1/27*(2*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(2/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9*(-1)^(1/3)*(2*(-1)^(1/3)*b+3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(5/6)/b^2/c^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)

Rubi [A]
time = 1.46, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\text{ArcTan}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\text{ArcTan}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{(-1)^{2/3}\left(3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+2b\right)\text{ArcTan}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}-\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]

[Out] -1/3*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]])/(9*Sqrt[3]*a^(5/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^

```
(1/3))^2*a^(2/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*
c^(1/3)*x + b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}\;-\sqrt[3]{c}\;}{59049\left(1+\sqrt[3]{-1}\right)^2 a^{20/3}bc^{2/3}\left(-3a+3\sqrt[3]{\ }\right.}\right.$$

$$= \frac{\int \frac{-\sqrt[3]{a}-\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{9a^{2/3}bc^{2/3}} - \frac{(-1)^{2/3}\int \frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{c}\,x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}}{9a^{2/3}bc^{2/3}}$$

$$= \frac{\left(3-\frac{2b}{\sqrt[3]{a}\,c^{2/3}}\right)\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{18b^2} + \frac{\left(3-\frac{2(-1)^{2/3}b}{\sqrt[3]{a}\,c^{2/3}}\right)}{}$$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}} + \frac{\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{\ }\right.}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}}$$

$$= -\frac{\left(3ib+\sqrt{3}\left(b+3\sqrt[3]{a}\,c^{2/3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}}{\sqrt{3}\sqrt{a}\sqrt{4b-3(}}\right.}{27a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 99, normalized size = 0.18

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)\#1^3}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 93, normalized size = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9a\,b^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Mupad [B]
time = 3.18, size = 1563, normalized size = 2.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 -
387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z
^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*
a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 15
94323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 3280
5*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048*a^5*b
^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*
a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z
+ 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^
6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*
c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^3*c^2 - 19
683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^
4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^
2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*root(918330048*a^
5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 10235
16*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c
*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b^9*c^4*z^6 - 3874
20489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 -
 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*
b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^
6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3
+ 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^5*b^6*c^5 + 108*root(9
18330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*
z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2
+ 324*a*b*c*z + 1, z, k)^2*a*b^5*c*x + 108*root(918330048*a^5*b^9*c^4*z^6 -
```

387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*a*b*c^2*x + 1458*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^2*c^3*x - 2916*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^2*b^6*c^2*x + 78732*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^4*a^3*b^7*c^3*x + 1062882*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*b^8*c^4*x))*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k), k, 1, 6)

3.137 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=487

$$-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{7/6}b\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}}+\frac{\sqrt[3]{-}\cdots}{3\sqrt{3}\,(1-\cdots}$$

```
[Out] 1/54*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c^(2/3)-1/18*(-1)^(2/3)*ln
(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(4/3)/b/c^(2/
3)+1/54*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c
^(2/3)-1/27*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(
1/3)*c^(2/3))^(1/2))/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2
)+1/9*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^
(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3
))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/9
*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1
)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(7/6)/b/c^(1/3)*3^(1/2)/
(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A]
time = 0.96, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\text{ArcTan}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{7/6}b\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{\sqrt[3]{-1}\,\text{ArcTan}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}+\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -1/3*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*
b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*S
qrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7
/6)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(
2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1
/3)*c^(2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sq
rt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/
3)*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1
)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^
(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = (19683a^6)\int \left(\frac{(-1)^{2/3}x}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{}\right.}\right.$$

$$= \frac{\int \frac{x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}}{27a^{4/3}c^{2/3}}$$

$$= \frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}}{54a^{4/3}bc^{2/3}}$$

$$= \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}\right.}{18\left(1+\sqrt[3]{-1}\right)^2}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 99, normalized size = 0.20

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)\#1^2}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 93, normalized size = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9ab^{2}\_Z^{4}+27a^{2}c\_Z^{3}+27a^{2}b\_Z^{2}+27a^{3}\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}ab^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Mupad [B]
time = 3.10, size = 1354, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(4782969*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c
^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4
*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^9*b^6*c^3 - 729*a^5*b^7*x + 12914
0163*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348
907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a
^3*b*c^2*z^2 - 1, z, k)^3*a^10*b^8*c^3 + 1549681956*root(10460353203*a^9*b^
3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928
*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^
11*b^10*c^3 + 167365651248*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a
^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441
*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^12*c^3 - 94143178827
*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*
a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b
*c^2*z^2 - 1, z, k)^5*a^13*b^9*c^5 + 98415*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^7*c + 4
374*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143489
07*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^
3*b*c^2*z^2 - 1, z, k)*a^6*b^9*x - 2125764*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b^9*c -
 59049*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143
48907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683
*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^6*c^2*x - 531441*root(10460353203*a^9*b^3*c
^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^
```

4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b ^8*c^2*x - 688747536*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6 *c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c ^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^10*b^12*c^2*x + 1162261467*root (10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b ^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2* z^2 - 1, z, k)^4*a^11*b^9*c^4*x - 20920706406*root(10460353203*a^9*b^3*c^6* z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b ^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^1 1*c^4*x)*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 1 4348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 196 83*a^3*b*c^2*z^2 - 1, z, k), k, 1, 6)

3.138 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=334

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{11/6}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}+\frac{2(-1)^{2}\cdots}{9\sqrt{3}\left(1-\sqrt[3]{\cdots}\right.}$$

```
[Out] 2/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^
(2/3))^(1/2))/a^(11/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+2/27*(
-1)^(2/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(
4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^
(11/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+2/27*(-1)^(
2/3)*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3
*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(11/6)/c^(2/3)*3^(1/
2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A]

time = 0.70, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2122, 632, 210}

$$\frac{2(-1)^{2/3}\text{ArcTan}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{2\text{ArcTan}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{2(-1)^{2/3}\text{ArcTan}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a
]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2
*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*ArcTan[(3*
a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/
(27*Sqrt[3]*a^(11/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)
*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3
))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{(-1)^{2/3}}{177147\left(1+\sqrt[3]{-1}\right)^2a^{22/3}c^{2/3}\left(-3a+3\right.}\right.$$

$$= \frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+b}}{27a^{4/3}c^{2/3}}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}\,c^{2/3}\right)-x^2}\,dx, x, 3a^{2/3}\sqrt[3]{c}+2bx\right.}{27a^{4/3}c^{2/3}}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 97, normalized size = 0.29

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)\#1}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &
]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 93, normalized size = 0.28

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9a\,b^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9a\,b^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),
_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.14, size = 27094, normalized size = 81.12

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] -1/1458*sqrt(2)*sqrt(1/6)*sqrt(((64*a^3*b^3 - 27*a^4*c^2)*((-I*sqrt(3) + 1)
*(1/(64*a^7*b^3*c^2 - 27*a^8*c^4) + 27/(64*a^3*b^3 - 27*a^4*c^2)^2)/(-1/209
```

```
20706406/(64*a^11*b^3*c^4 - 27*a^12*c^6) - 1/258280326/((64*a^7*b^3*c^2 - 2
7*a^8*c^4)*(64*a^3*b^3 - 27*a^4*c^2)) - 1/14348907/(64*a^3*b^3 - 27*a^4*c^2
)^3 + 32/10460353203*b^3/((64*b^3 - 27*a*c^2)^2*a^11*c^4))^(1/3) + 1594323*
(I*sqrt(3) + 1)*(-1/20920706406/(64*a^11*b^3*c^4 - 27*a^12*c^6) - 1/2582803
26/((64*a^7*b^3*c^2 - 27*a^8*c^4)*(64*a^3*b^3 - 27*a^4*c^2)) - 1/14348907/(
64*a^3*b^3 - 27*a^4*c^2)^3 + 32/10460353203*b^3/((64*b^3 - 27*a*c^2)^2*a^11
*c^4))^(1/3) + 13122/(64*a^3*b^3 - 27*a^4*c^2)) + 3*sqrt(1/3)*(64*a^3*b^3 -
 27*a^4*c^2)*sqrt(-((4096*a^7*b^6*c^2 - 3456*a^8*b^3*c^4 + 729*a^9*c^6)*((-
I*sqrt(3) + 1)*(1/(64*a^7*b^3*c^2 - 27*a^8*c^4) + 27/(64*a^3*b^3 - 27*a^4*c
^2)^2)/(-1/20920706406/(64*a^11*b^3*c^4 - 27*a^12*c^6) - 1/258280326/((64*a
^7*b^3*c^2 - 27*a^8*c^4)*(64*a^3*b^3 - 27*a^4*c^2)) - 1/14348907/(64*a^3*b^
3 - 27*a^4*c^2)^3 + 32/10 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

Mupad [B]
time = 3.34, size = 825, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 28242
9536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)
```

```
^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12
*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c^3 -
 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784401*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(669462604992*
a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 1968
3*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^4*z^6 - 28
2429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
 k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^
12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^3
*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^
6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^3*c^3
+ 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^2*c*x + 708588*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x + 38263752*root(66
9462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4
*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^2*c^3*x + 774840978*root(669462
604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4
 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^2*c^4*x + 1))*root(669462604992*a^1
1*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a
^4*c^2*z^2 + 1, z, k), k, 1, 6)
```

3.139 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=469

$$-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2 a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{13/6}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}}+\frac{\sqrt[3]{-}}{9\sqrt{3}\,(1-}$$

```
[Out] -1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2/3)+1/54*(-1)^(2/3)*ln
(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(7/3)/c^(2/3)
-1/162*(-1)^(2/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2
/3)-1/81*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3
)*c^(2/3))^(1/2))/a^(13/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/
27*(-1)^(1/3)*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/
2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^
2/a^(13/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/27*ar
ctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(
2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(13/6)/c^(1/3)*3^(1/2)/(4*b
-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2122, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2 a^{13/6}\sqrt[3]{c}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\text{ArcTan}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{13/6}\sqrt[3]{c}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{\sqrt[3]{-1}\,\text{ArcTan}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{9\sqrt{3}\,\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{13/6}\sqrt[3]{c}\,\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}-\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2 a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -1/9*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*
b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(13/6)*Sq
rt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3)
 + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(1
3/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2
/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/
3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt
[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)
*x + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2
/3)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/
3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = (19683a^6)\int \left(\frac{-3a^{2/3}\sqrt[3]{c}\,-(-1)^{2/}}{531441\left(1+\sqrt[3]{-1}\right)^2 a^{25/3}c^{2/3}\left(-3a+3\right.}\right.$$

$$= \frac{\int \frac{-3a^{2/3}\sqrt[3]{c}\,-bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,+bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+b}}{81a^{7/3}c^{2/3}}$$

$$= -\frac{\int \frac{3a^{2/3}\sqrt[3]{c}\,+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{162a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,+2}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x}}{162a^{7/3}c^{2/3}}$$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-}\right.}{54\left(1+\sqrt[3]{-1}\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2 a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 95, normalized size = 0.20

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),
x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 91, normalized size = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	91

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9a\,b^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R
=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Mupad [B]
time = 2.90, size = 1057, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(b^12*x + 1033121304*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4*a^10*b^11*c
^3 + 167365651248*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15
*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a
^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^12*c^3 - 94143178
827*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 116
2261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 1
77147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^13*b^9*c^5 + 54*root(18075490334784*
a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 +
 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 +
b^3, z, k)*a^2*b^13*x + 177147*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^2*a^5*b^11*c^
2*x + 17006112*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^
6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*
c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^7*b^12*c^2*x - 14348907*r
oot(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 11622614
67*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147
*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^8*b^9*c^4*x + 229582512*root(180754903347
84*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^
4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2
 + b^3, z, k)^4*a^9*b^13*c^2*x + 387420489*root(18075490334784*a^14*b^3*c^4
*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7
*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4
```

```
*a^10*b^10*c^4*x - 20920706406*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^11*c
^4*x)*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1
162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 +
 177147*a^5*b^2*c^2*z^2 + b^3, z, k), k, 1, 6)
```

3.140 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=522

$$-\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}\,b+3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{\cdots}{\sqrt{3}\,\cdots}\right)}{81\sqrt{3}\,a^{17/6}\sqrt{4b-3\cdots}}$$

```
[Out] 1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/54*ln(3*a-3*(-1)^
(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(8/3)/c^(1/3)-1/162*(-1)^
(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/243*(2
*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/
(4*b-3*a^(1/3)*c^(2/3))^(1/2))/a^(17/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2
/3))^(1/2)-1/81*(2*(-1)^(2/3)*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)
*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^
(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(17/6)/c^(2/3)*3^(1/2)/(4*b+3*(-1)
^(1/3)*a^(1/3)*c^(2/3))^(1/2)-1/81*(-1)^(1/3)*(2*(-1)^(1/3)*b+3*a^(1/3)*c^(
2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-
3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(17/6)/c^(2/3)*3^(1
/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A]

time = 1.08, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2095, 648, 632, 210, 642}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\text{ArcTan}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\text{ArcTan}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\,a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}\,c^{2/3}\right)\text{ArcTan}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{27\sqrt{3}\,\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}+\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}-\frac{\sqrt[3]{-1}\,\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1),x]
```

```
[Out] -1/27*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)
*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*
c^(2/3)])])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*
a^(17/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1
/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]
*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 - (-1)^(1/3))*(
1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)
) + Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)) - Log[3*a
- 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^
```

```
(1/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(16
2*a^(8/3)*c^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2095

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}\,b-3\sqrt[3]{-1}\,a^{2/3}c\ldots}{531441\left(1+\sqrt[3]{-1}\right)^2a^{26/3}c^{2/3}\left(-3a+3\ldots\right.}\right.$$

$$=\frac{\int\frac{-\sqrt[3]{a}\,b+3a^{2/3}c^{2/3}+b\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{8/3}c^{2/3}}-\frac{\int\frac{(-1)^{2/3}\sqrt[3]{a}\,b-3a^{2/3}c^{2/3}+\ldots}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\ldots}}{81a^{8/3}c^{2/3}\ldots}$$

$$=-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\int\frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{162a^{7/3}c^{2/3}}-\frac{\left(2(-1)^{2/3}b\ldots\right.}{\ldots}$$

$$=\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\ldots\right.}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/\ldots}}$$

$$=-\frac{\left(2(-1)^{2/3}b+3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\ldots}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-\ldots}}\right.}{27\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 99, normalized size = 0.19

$$\frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)}{18a^2b\#1+27a^2c\#1^2+12ab^2\#1^3+2b^3\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-
1),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &
]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 90, normalized size = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(b^3\_Z^6+9a\,b^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	90

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9a\,b^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=
RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

Mupad [B]
time = 0.71, size = 1394, normalized size = 2.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
[Out] symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^1
8*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 3874
20489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k
)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209464
9*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 +
 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6,
 z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209
4649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^
3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b
^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^3*c^4*z^6
- 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a
^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^
3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(488038239039168*a^17
*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4
- 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*
z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 167365651248*root(488
038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*
a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 265
7205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^12*c^3 - 9414
3178827*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^
10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^13
*b^9*c^5 + 14580*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^
18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387
420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z,
```

```
k)^2*a^3*b^14*c*x - 10628820*root(488038239039168*a^17*b^3*c^4*z^6 - 205891
132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c
^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*
z + b^6, z, k)^3*a^6*b^13*c^2*x + 2238429492*root(488038239039168*a^17*b^3*
c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746
143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 -
 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^12*c^3*x - 1162261467*root(488038239
039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b
^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a
^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^9*c^5*x - 209207064
06*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 1
0460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^
5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^11
*c^4*x)*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^
10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k), k, 1,
 6)
```

3.141 $\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=563

$$\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}}+\frac{\left(b-\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+\cdots}{\sqrt{3}\sqrt{a}\sqrt{4b-\cdots}}\right)}{27\sqrt{3}\,a^{19/6}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}\,\cdots}$$

```
[Out] 1/27*ln(x)/a^3-1/486*(3*a^(1/3)-b/c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2
)/a^(10/3)-1/486*(3*a^(1/3)-(-1)^(2/3)*b/c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/
3)*c^(1/3)*x+b*x^2)/a^(10/3)-1/972*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*
x^2)*(b+6*a^(1/3)*c^(2/3)+I*b*3^(1/2))/a^(10/3)/c^(2/3)+1/81*(b-a^(1/3)*c^(
2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c
^(2/3))^(1/2))/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/27*
(-1)^(2/3)*((-1)^(2/3)*b-a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*
c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1
-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^
(1/3)*c^(2/3))^(1/2)+1/27*(b-(-1)^(2/3)*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)
^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(
2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(19/6)/c^(1/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*a^
(1/3)*c^(2/3))^(1/2)
```

Rubi [A]

time = 1.29, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\text{ArcTan}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{19/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(b-\sqrt[3]{a}\,c^{2/3}\right)\text{ArcTan}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{19/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}\,c^{2/3}\right)\text{ArcTan}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{19/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}\,c^{2/3}+i\sqrt{3}\,b+b\right)\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{10/3}}+\frac{\log(x)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1
/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a
^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2
*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^
3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*
a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a
```

```
^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/
3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x}+\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)}{4782969a^{28/3}c^{2/3}\left(3\right.}\right.$$

$$=\frac{\log(x)}{27a^3}+\frac{\int\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\sqrt[3]{c}+b\left(b-3\sqrt[3]{a}\,c^{2/3}\right)x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{243a^{10/3}c^{2/3}}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{486a^{10/3}}-\frac{\left(3\right.}{}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{486a^{10/3}}$$

$$=\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1}}\right.}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.07, size = 157, normalized size = 0.28

$$-\frac{-3\log(x)+\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{27a^2b\log(x-\#1)+27a^2c\log(x-\#1)\#1+9ab^2\log(x-\#1)\#1^2+b^3\log(x-\#1)\#1^4}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6)),x]
```

```
[Out] -1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2
*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*
b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 12*a
*b^2*#1^2 + 2*b^3*#1^4) & ])/a^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 134, normalized size = 0.24

method	result

default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9ab^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\left(\_R^5b^3+9\_R^3ab^2+27\_R^2a^2c+27a^2b\_R\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3ab^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}+\frac{\ln(x)}{27a^3}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(27a^{21}c^6-64a^{20}b^3c^4\right)\_Z^6+\left(243a^{18}c^6-576a^{17}b^3c^4\right)\_Z^5+\left(729a^{15}c^6-1755a^{14}c^4b^3\right)\_Z^4+\left(729a^{12}c^6-1917a^{11}c^4b^3+16a^{10}c^2b^6\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*b)/(2*_R^5*b^3
+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(_Z^6*b^3+9*_Z^
4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))+1/27*ln(x)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] -1/27*integrate((b^3*x^5 + 9*a*b^2*x^3 + 27*a^2*c*x^2 + 27*a^2*b*x)/(b^3*x^
6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 + 1/27*log(
x)/a^3
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x), x)
```

Mupad [B]
time = 2.55, size = 2500, normalized size = 4.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),x)
```

```
[Out] log(x)/(27*a^3) + symsum(log(7*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)*b^18*x - 162*root(13177032454057536*a^20*b^3
*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5
 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 25418658
28329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*
z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^
6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^3*b^18*x + 86093442*root(13
177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 48803823
9039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^1
4*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 -
 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c
^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^8*b
^13*c^3 + 34867844010*root(13177032454057536*a^20*b^3*c^4*z^6 - 55590605665
55523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^1
8*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 2
7506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12
*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b
```

^6*c^2*z + b^9, z, k)^4*a^11*b^13*c^3 - 10460353203*root(13177032454057536*
a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3
*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 -
2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*
b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009737
9*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^12*b^10*c^5 + 15062
90861232*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^
6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6
119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^
11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14
348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^5*a^14*b^13*c^3 - 564859072962*root(13177032454057536*a^20*b^3*c^4
*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2
05891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 254186582832
9*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3
- 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^
2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^10*c^5 - 67783088755440*ro
ot(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488
038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930662375
5*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*
z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*
b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*
a^17*b^13*c^3 + 22876792454961*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^10*c^5 + 17496*root(131770324540575
36*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*
b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4
 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^
10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009
7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^4*b^16*c - 47239
2*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 +
 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 61193066
23755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*
c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*
a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k
)^3*a^7*b^16*c - 39366*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566
555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^
18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 +
27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^1
2*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*
b^6*c^2*z + b^9, z, k)^2*a^4*b^15*c^2*x + 51372630*root(13177032454057536*a
^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*

```
c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930...
```

3.142 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=645

$$-\frac{1}{27a^3x}+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\,\sqrt[3]{a}\,bc^{2/3}+9a^{2/3}c^{4/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}+\frac{(2b^2-}{}$$

```
[Out] -1/27/a^3/x-1/486*(2*b-3*a^(1/3)*c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)
/a^(11/3)/c^(1/3)+1/162*(2*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))*ln(3*a-3*(-1)^(1
/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(11/3)/c^(1/3)+1/486*(-1)^(
1/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)
*x+b*x^2)/a^(11/3)/c^(1/3)+1/729*(2*b^2-12*a^(1/3)*b*c^(2/3)+9*a^(2/3)*c^(4
/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^
(2/3))^(1/2))/a^(23/6)/c^(2/3)*3^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)+1/243*
(-1)^(2/3)*(2*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*(-1)^(2/3)*a^(2/3)*c^(4
/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3
*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(23/6
)/c^(2/3)*3^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)+1/243*(2*(-1)^(2
/3)*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*a^(2/3)*c^(4/3))*arctan(1/3*(3*(-
1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c
^(2/3))^(1/2))/(1+(-1)^(1/3))^2/a^(23/6)/c^(2/3)*3^(1/2)/(4*b-3*(-1)^(2/3)*
a^(1/3)*c^(2/3))^(1/2)
```

Rubi [A]

time = 1.54, antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2122, 648, 632, 210, 642}

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\,\sqrt[3]{a}\,bc^{2/3}+2(-1)^{2/3}b^2\right)\text{ArcTan}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\,\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}\,bc^{2/3}+2b^2\right)\text{ArcTan}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{243\sqrt{3}\,a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(-9\sqrt[3]{-1}\,a^{2/3}c^{4/3}-12\sqrt[3]{a}\,bc^{2/3}+2(-1)^{2/3}b^2\right)\text{ArcTan}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{81\sqrt{3}\,\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2a^{11/3}\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\left(3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+2b\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}-\frac{1}{27a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),
x]
```

```
[Out] -1/27*1/(a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2/3) + 9*
a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqr
t[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]])/(81*Sqrt[3]*(1 + (-1)^(1/3
))^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 -
 12*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*
x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(243*Sqrt[3]*a^(23/6)*
Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*(-1)^(2/3)*b^2 - 12*a^(1/3)*b*
c^(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3
```

```
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(81
*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b + 3*(-1)^(1/
3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a^(2/
3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/3)*a^(1/3
)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*(1 + (-1
)^(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2
/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/
3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x^2}+\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}\,bc\right.}{1594323\left(1-\sqrt[3]{-1}\right)}\right.$$

$$= -\frac{1}{27a^3x}+\frac{\int\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}\,bc^{2/3}+9a^{2/3}c^{4/3}\right)-b\left(2b-3\sqrt[3]{a}\,c\right.}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}}{243a^{11/3}c^{2/3}}$$

$$= -\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}dx}{486a^{11/3}\sqrt[3]{c}}+$$

$$= -\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+b\right.}{486a^{11/3}\sqrt[3]{c}}$$

$$= -\frac{1}{27a^3x}+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\,\sqrt[3]{a}\,bc^{2/3}+9a^{2/3}c\right.}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.09, size = 163, normalized size = 0.25

$$-\frac{3+x\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{27a^2b\log(x-\#1)+27a^2c\log(x-\#1)\#1+9ab^2\log(x-\#1)\#1^2+b^3\log(x-\#1)\#1^4}{18a^2b\#1+27a^2c\#1^2+12ab^2\#1^3+2b^3\#1^5}\&\right]}{81a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*
x^6)),x]
```

```
[Out] -1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4
+ b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Lo
g[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*
b^2*#1^3 + 2*b^3*#1^5) & ])/(a^3*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 133, normalized size = 0.21

method	result

default	$-\frac{1}{27a^3x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9a\,b^2\_Z^4+27a^2c\_Z^3+27a^2b\_Z^2+27a^3\right)}\frac{\left(-\_R^4b^3-9\_R^2a\,b^2-27\_R\,a^2c-27a^2b\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}$
risch	$-\frac{1}{27a^3x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^{24}c^6-1728a^{23}b^3c^4\right)\_Z^6+\left(13122a^{17}b\,c^6-31347a^{16}b^4c^4\right)\_Z^4+\left(-19683c^7a^{14}+52488c^5b^3a^{13}-14472c^3b^6a^{1}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/27/a^3/x+1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_
R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(_Z^6*b^
3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="maxima")
```

```
[Out] -1/27*integrate((b^3*x^4 + 9*a*b^2*x^2 + 27*a^2*c*x + 27*a^2*b)/(b^3*x^6 +
9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 - 1/27/(a^3*x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
a**3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x^2), x)
```

Mupad [B]
time = 2.72, size = 2500, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),
x)
```

```
[Out] symsum(log(-282429536481*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946
35296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*
a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^
3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a
^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^23*b^9*(2*b^10*x + 2541
865828329*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^2
4*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^
4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 2824295
36481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2
+ 17496*a^4*b^10*c*z + b^12, z, k)^4*a^17*c^5 - 45*a*b^8*c + 387420489*root
(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45
753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 7531454306
16*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7
*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^
10*c*z + b^12, z, k)^2*a^10*c^6*x - 401769396*root(355779876259553472*a^23*
b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z
^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207
657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*
c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^
9*b^4*c^3 - 2066242608*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635
```

296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^
16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*
z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8
*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^12*b^5*c^2 + 6973568802
*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6
 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 75314
5430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^1
4*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a
^4*b^10*c*z + b^12, z, k)^3*a^13*b^2*c^4 - 4518872583696*root(3557798762595
53472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a
^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^
5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 2582803
26*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12,
 z, k)^4*a^16*b^3*c^3 - 328050*root(355779876259553472*a^23*b^3*c^4*z^6 - 1
50094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 1093002306
18147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*
b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 10044
2349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^5*b^6*c^2 - 17714
7*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^
6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 7531
45430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^
14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*
a^4*b^10*c*z + b^12, z, k)*a^6*b^3*c^4 + 387420489*root(355779876259553472*
a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*
c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3
 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9
*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)
^2*a^10*b*c^5 + 23328*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946352
96999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^1
6*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z
^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*
b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^4*b^8*c*x + 196830*root(35
5779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753
584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*
a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^
3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*
c*z + b^12, z, k)*a^5*b^5*c^3*x - 20920706406*root(355779876259553472*a^23*
b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z
^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207
657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*
c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^
13*b*c^5*x + 74401740*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946352
96999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^1
6*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z
^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*

```
b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^8*b^6*c^2*x - 746143164*
root(355779876259553472*a^23*b^3*c^4*z^6 - 1500...
```

3.143 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=395

$$-\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{\sqrt[6]{\frac{3}{2}}\left(1-(-3)^{2/3}\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}\right.}{\sqrt{3\left(4-3(-3\right.}}\right.}{\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$

```
[Out] 1/108*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/108*(18-2^
(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/216*ln(6-3*(-3)^(1/3)*2^(2/3
)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))+1/2*3^(1/6)*2^(5/6)*(1-(-3)^(2/
3)*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3
))^(1/2))/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/6*(1-2^(1/3)*3^
(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))
*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/3*(-2)^(1/3)*(1+(-2)^(1/3)*
3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2)
)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]

time = 1.08, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$-\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}\,3^{2/3}\right)\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{\sqrt[6]{\frac{3}{2}}\left(1-(-3)^{2/3}\sqrt[3]{2}\right)\text{ArcTan}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)+\frac{1}{108}\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)-\frac{\left(1-\sqrt[3]{2}\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/
Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6
) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 - (-3)^(2/3)*2^(1/3))*ArcTan[(2^
(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1
+ (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/
(2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3)*(1
+ I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 - (-2)^(2/3
)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 - 2^(2/3)*3^(1
/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int \left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}\,2^{2/3}+\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{3779136\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-3}\,2^{2/3}x+\right.}\right.$$

$$= \frac{1}{54}\int \frac{6\sqrt[3]{2}\,3^{2/3}+\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx + \frac{(-1)^{2/3}\int \frac{3(-2)^{2/3}\sqrt[3]{3}}{6+3(-2}}{9\sqrt[3]{2}}$$

$$= \frac{\left((-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\right)\int \frac{-3\sqrt[3]{-3}\,2^{2/3}+2x}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{6\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\left((-1)^{2}\right.}{}$$

$$= \frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}+i2^{2/3}3^{5/6}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)+\frac{1}{10}$$

$$= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{6^{5/6}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.15

$$\frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^4}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 56, normalized size = 0.14

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\_R^5 \ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\_R^5 \ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.12, size = 70, normalized size = 0.18

$$\operatorname{RootSum}\left(72662865048t^6-72662865048t^5+24163559388t^4-2646786132t^3-6626610t^2-4374t-1,\left(t\mapsto t\log\left(-\frac{89236417131047376t^5}{833243797}+\frac{89301949532998128t^4}{833243797}-\frac{29740560281805852t^3}{833243797}+\frac{192466080408420t^2}{49014341}+\frac{5867255361684t}{833243797}+x+\frac{5365044886}{2499731391}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 2646786
132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-892364171310473
76*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 29740560281805852*
```

```
_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/8332437
97 + x + 5365044886/2499731391)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B]
time = 0.65, size = 427, normalized size = 1.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 26467
86132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 19131876
*x - 6482268*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 2416355
9388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 742851*x*root(z^6 + 4374*
z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 7266
2865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^
3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(z^6 + 4
374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z -
72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4 + 264678
6132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 1704742
2*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72
662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5 + 6626610*z^
4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4
 + 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 +
 72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 + 4374*z^5 + 662
6610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048,
z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24
163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*root(z^6 - z^5 + (421
*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 - z/16612452 - 1/7266
2865048, z, k), k, 1, 6)
```

3.144 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2\cdots}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3\cdots\right.}}\right.}{27\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)}}$$

```
[Out] 1/36*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2+1/10
8*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)-1/108*ln(6+3*
2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/27*(-1)^(2/3)*(3*(-3)^(2/3)-2^(2/3
))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(5
/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/27*(9-2^(2/3)*3^(1/3)
)*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(-24
+18*2^(1/3)*3^(2/3))^(1/2)+1/27*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-2)^(2/3)
*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^(1/6)+9*
2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]

time = 0.79, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2122, 648, 632, 210, 642, 212}

$$\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{27\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)}}+\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{18\,2^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{18\,2^{2/3}\sqrt[3]{3}}-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{27\sqrt{6\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4
- 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*
3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(
1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6
*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6*2^(2
/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3
^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712 \int \left(\frac{(-1)^{2/3}\left(-2+\sqrt[3]{-3}\,2^{2/3}x\right)}{7558272\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x - \right.} \right.$$

$$= -\frac{(-1)^{2/3}\int \frac{2+(-2)^{2/3}\sqrt[3]{3}\,x}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{18\sqrt[3]{2}\,3^{2/3}} - \frac{\int \frac{\sqrt[3]{2}+\sqrt[3]{3}\,x}{6+3\;2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{9\;6^{2/3}} + \frac{(-1)^{2}}{6}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{18\;2^{2/3}} - \frac{\int \frac{3\;2^{2/3}\sqrt[3]{3}+2x}{6+3\;2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{18\;2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{3\sqrt[3]{}}{-6+3\sqrt[3]{}}}{6\;2^{2/3}\sqrt[3]{3}}$$

$$= \frac{\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{6\;2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\sqrt[3]{-\frac{1}{3}}\,\log\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x\right.}{18\;2^{2/3}}$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{((}{}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.16

$$\frac{1}{6}\mathrm{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^3}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

<u>**Maple**</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 56, normalized size = 0.15

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^4\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^4\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.14, size = 65, normalized size = 0.17

$$\mathrm{RootSum}\left(15695178850368t^6-2066242608t^4+1845163152t^3-1180980t^2-1944t-1,\left(t\mapsto t\log\left(\frac{614714526178551746208t^5}{57121295165}-\frac{1270857362386176t^4}{57121295165}-\frac{80483053187684376t^3}{57121295165}+\frac{724313183251038 84t^2}{57121295165}-\frac{453586026890 88t}{57121295165}+x-\frac{44532180783}{57121295165}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 118098
0*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(614714526178551746208*_t**5/571212
95165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/571212
```

```
95165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/57121295165
 + x - 44532180783/57121295165)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B]
time = 2.70, size = 390, normalized size = 1.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 11809
80*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976*x
*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1569
5178850368, z, k)^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 18451631
52*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 + 1946*x*root(z^6 + 1944*
z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)
^4 + 3*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^
2 - 15695178850368, z, k)^5 - 4251528*root(z^6 + 1944*z^5 + 1180980*z^4 - 1
845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 3927852*root(z^6
 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1569517885036
8, z, k)^3 - 1188*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066
242608*z^2 - 15695178850368, z, k)^4 - root(z^6 + 1944*z^5 + 1180980*z^4 -
1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 + 7558272*root(z^
6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 156951788503
68, z, k) + 33519046752))/root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^
3 + 2066242608*z^2 - 15695178850368, z, k)^5)*root(z^6 - z^4/7596 + (217*z^
3)/1845828 - (5*z^2)/66449808 - z/8073651672 - 1/15695178850368, z, k), k,
1, 6)
```

3.145 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\,\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{9\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\,(\ldots}{\sqrt{3\left(-\ldots\right.}}\right.}{18\sqrt[6]{2}\,3^{5/6}\sqrt{-\ldots}}$$

```
[Out] -1/216*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^
(1/3))^2+1/648*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)+
1/648*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/36*arctan((3*(-3)^(1/
3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-1)^(
1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/108*arctanh(2^(1/6)*(3*3^(1/3)+2^(
1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3
))^(1/2)+1/54*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3
)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))
^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$-\frac{\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\,\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{9\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{18\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -1/6*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]
]/(2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-
1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/
3))]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3
)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]]/(18*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[
6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
+ ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1/3)*3^(2/3))
+ Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712 \int \left(\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x\right.}\right.$$

$$= \frac{\int \frac{x}{6+3\ 2^{2/3}\sqrt[3]{3}\ x+x^2}dx}{54\sqrt[3]{2}\,3^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{6+3(-2)^{2/3}\sqrt[3]{3}\ x+x^2}dx}{54\sqrt[3]{2}\,3^{2/3}} + \frac{(-1)^{2/}}{18}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}\ x+x^2}dx}{18\ 2^{2/3}} + \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}\ x+x^2}dx}{108\sqrt[3]{2}\,3^{2/3}} + \frac{(-1)^{2/3}}{}$$

$$= -\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{(-1)^{2/3}\log\left(6+3(-2)^{2}\right.}{108\sqrt[3]{2}\,3^{2/}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\right.}{18\sqrt[6]{2}\,3^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.17

$$\frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^2}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 56, normalized size = 0.16

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^3\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^3\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}}{6}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.10, size = 61, normalized size = 0.17

$$\mathrm{RootSum}\left(3390158631679488t^6-74384733888t^4-1332145440t^3-1417176t^2-1,\left(t\mapsto t\log\left(-\frac{8482372214243328t^5}{415817}+\frac{2216055910930560t^4}{415817}-\frac{2062546612992t^3}{415817}-\frac{57027208896t^2}{415817}-\frac{416583756t}{415817}+x-\frac{89938}{415817}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 141
7176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 22160559
10930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4158
17 - 416583756*_t/415817 + x - 89938/415817)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B]
time = 0.52, size = 276, normalized size = 0.76

$$\sum_{k=1}^{6}\ln\left(-\frac{23328\left(297538935552\,x-7992872640\,x\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)+52488\,x\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)^3+2904\,x\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)^4+x\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)^5-153055008\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)^2-2764368\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)^3-1620\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)^4-3673320192\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)+7240114098432\right)}{\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k)^5}\right)\mathrm{root}\left(z^6-\frac{z^4}{45576}-\frac{235\,z^3}{598048272}-\frac{z^2}{2392193088}-\frac{1}{3390158631679488},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 +
1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root(z
^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z,
k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3
390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 7438
4733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 1417176*z^4
+ 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 2764368*ro
ot(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
 z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 -
 3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 + 1332145440
*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432))/root(z^6
 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)
^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/2392193088 - 1/3390158
631679488, z, k), k, 1, 6)
```

3.146 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=248

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\cdots\right.}{\sqrt{3\left(-\cdots\right.}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{-\cdots}}$$

```
[Out] 1/162*(-1)^(2/3)*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3
))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1
/486*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2
^(1/6)*3^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/486*(-1)^(2/3)*arctan((3*(-2)
^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(5/6)/(8+9*
I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2122, 632, 210, 212}

$$\frac{(-1)^{2/3}\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^
(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1
/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^
(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^
(1/3)*3^(2/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int \left(\frac{(-1)^{2/3}}{22674816\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x \cdots\right.}\right.$$

$$= \frac{\int \frac{1}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}} + \frac{(-1)^{2/3}\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}} + \frac{(-1)^{2/\cdots}}{18\cdots}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-6\left(4-3\sqrt[3]{2}\,3^{2/3}\right)-x^2}\,dx, x, 3\,2^{2/3}\sqrt[3]{3}+2x\right)}{27\sqrt[3]{2}\,3^{2/3}} - \frac{(-1)^{2/\cdots}}{\cdots}$$

$$= \frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3}\tan^{-1}\left(\cdots\right.}{81\,2^{5/6}\sqrt[6]{3}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.01, size = 59, normalized size = 0.24

$$\frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(36
 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 56, normalized size = 0.23

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. 2(162) = 324.
time = 1.11, size = 1277, normalized size = 5.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)*
(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867
)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt(1/6
33)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(
2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 72
9/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)*sqrt(-2
/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 4
8*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(
1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36
*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)
 - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273
*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*
18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3)
+ sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) +
 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt
(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*
(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/
3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) +
48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(
1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))
*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(
2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3
888*18^(1/3) - 16578) - 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27
*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*
18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18
^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3)
+ 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 -
 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3)
+ 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 124
7*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*1
8^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18
^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3)
 - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3)
 + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 1
8)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18^(2/3) + 8*1
8^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3)
 + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 9*sqrt(-1/2
7*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sq
```

```
rt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)
*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) - sq
rt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371
) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578)
```

Sympy [A]
time = 0.08, size = 48, normalized size = 0.19

$$\operatorname{RootSum}\left(732274264442769408t^6 - 2677850419968t^4 + 2834352t^2 - 1, \left(t \mapsto t\log\left(10170475895038464t^5 - 5231726283456t^4 - 31809932496t^3 + 19131876t^2 + 19683t + x - \frac{27}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1,
 Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 31809932
496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B]
time = 2.68, size = 247, normalized size = 1.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 26
77850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 2834352*
z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2834
352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 - 2834
352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 132239526912*x*
root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 20
4073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408,
z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 73227426444
2769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274
264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 2677850419968
*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 - 2834352*z^4
 + 2677850419968*z^2 - 732274264442769408, z, k)^5)*root(z^6 - z^4/273456 +
 z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)
```

3.147 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{54\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\ldots}{\sqrt{3\,(\ldots}}\right.}{108\sqrt[6]{2}\,3^{5/6}\ldots}$$

```
[Out] 1/1296*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^
(1/3))^2-1/3888*(-1)^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)
-1/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/216*arctan((3*(-3)^
(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(1/6)/(1+(-1
)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/648*arctanh(2^(1/6)*(3*3^(1/3)+
2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(
2/3))^(1/2)+1/324*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^
(1/3)*3^(2/3))^(1/2))*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2
/3))^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2122, 648, 632, 210, 642, 212}

$$-\frac{\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\,\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{54\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{108\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -1/36*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))
]]/(2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((
-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2
/3))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2
/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^
(2/3))]]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*L
og[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))
^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/
3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int \left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}\,2^{2/3}-x\right)}{136048896\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}\right.}\right.$$

$$= -\frac{(-1)^{2/3}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+x}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{324\sqrt[3]{2}\,3^{2/3}} - \frac{\int \frac{6\sqrt[3]{3}+\sqrt[3]{2}\,x}{6+3\ 2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{324\ 6^{2/3}} + \frac{(-1}{1}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{108\ 2^{2/3}} - \frac{\int \frac{3\ 2^{2/3}\sqrt[3]{3}+2x}{6+3\ 2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{648\sqrt[3]{2}\,3^{2/3}} - \frac{(-1)^{2/}}{}$$

$$= \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\right.}{648\sqrt[3]{2}\,3^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\,\tan^{-1}\left(\right.}{108\sqrt[6]{2}\,3^{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 57, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 16
2*#1 + 12*#1^2 + #1^4) & ]/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 54, normalized size = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	54

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.12, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(158171241119638192128t^{6}-96402615118848t^{4}+287743415040t^{3}-51018336t^{2}-1,\ \left(t\mapsto t\log\left(\frac{65418399445721140961280t^{5}}{415817}+\frac{248092645742510284 8t^{4}}{415817}-\frac{39451802929737984t^{3}}{415817}+\frac{118071997444800t^{2}}{415817}-\frac{16745884920t}{415817}+x-\frac{268790}{415817}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*_
t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(65418399445721140961280*_t**5/
415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/415817
+ 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415817)
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B]
time = 2.42, size = 176, normalized size = 0.49

$$\sum_{k=1}^{6}\ln\left(x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(216\,x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(51018336\,x-\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(277947894528\,x-\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(33192121254912\,x-\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)(6940988288557056\,x+1688973816882216960)+28563737812992\right)\right)\right)\right)\right)\,\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
2242048 - 1/158171241119638192128, z, k)*(216*x + root(z^6 - z^4/1640736 +
(235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)
*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
2242048 - 1/158171241119638192128, z, k)*(277947894528*x - root(z^6 - z^4/1
640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/1581712411196381921
28, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/1291784267
52 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(6940988288557056*x
 + 1688973816882216960) + 28563737812992)))))*root(z^6 - z^4/1640736 + (235*
z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k), k,
1, 6)
```

3.148 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3\right.}}\right.}{972\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)}}$$

```
[Out] -1/1296*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2-1
/3888*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)+1/3888*ln
(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)+1/972*(-1)^(2/3)*(3*(-3)^(2/3)-
2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2)
)*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*(9-2^(2/3)*
3^(1/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))/(-24+18*2^(1/3)*3^(2/3))^(1/2)+1/972*(9-(-2)^(2/3)*3^(1/3))*arctan((3*(-
2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(24+27*I*2^(1/3)*3^
(1/6)+9*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2095, 648, 632, 210, 642, 212}

$$\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{972\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)}}-\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{648\,2^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{648\,2^{2/3}\sqrt[3]{3}}-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{972\sqrt{6\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(972*Sqrt[3*(8 + (9*I
)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*Sq
rt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(21
6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2
^(2/3)*3^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2095

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int \left(\frac{(-1)^{2/3}\left(-2+6(-3)^{2/3}\sqrt[3]{2}-\sqrt[3]{-3}\,2^{2/3}x\right)}{272097792\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x \cdots\right.}\cdots\right.$$

$$= \frac{\int \frac{18-2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}\,3^{2/3}x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{1944} - \frac{\int \frac{2(-1)^{2/3}-6\sqrt[3]{2}\,3^{2/3}+\sqrt[3]{-3}\,2^{2/3}x}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{648\sqrt[3]{2}\,3^{2/3}} + \cdots$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{648\,2^{2/3}} + \frac{\int \frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{648\,2^{2/3}\sqrt[3]{3}} - \frac{\int \cdots}{216\,2^{2\cdots}}\cdots$$

$$= -\frac{\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+\cdots\right.}{648\,2^{2/3}}\cdots$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 62, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1 +
 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 53, normalized size = 0.14

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	53
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324
*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.13, size = 65, normalized size = 0.17

$$\mathrm{RootSum}\left(341649880818418494996 48t^{6}-3470494144278528t^{4}-86087932019712t^{3}-1530550080t^{2}+69984t-1,\left(t\mapsto t\log\left(\frac{185904446699109611410573787136t^{5}}{57121295165}+\frac{637730125326791738276 6592t^{4}}{57121295165}-\frac{18904636002388564311552t^{3}}{57121295165}-\frac{469080552915181723968t^{2}}{57121295165}-\frac{24358640509989936t}{57121295165}+x+\frac{152427895956}{57121295165}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(341649880818418494996 48*_t**6 - 3470494144278528*_t**4 - 8608793201
9712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(18590444669
9109611410573787136*_t**5/57121295165 + 6377301253267917382766592*_t**4/571
```

```
21295165 - 18904636002388564311552*_t**3/57121295165 - 46908055291518172396
8*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895956/5
7121295165)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B]
time = 2.67, size = 306, normalized size = 0.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
[Out] symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/
111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2
*x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071
3728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 61222003
20*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728
 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 2582637960
59136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713
728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x - 6940988
288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5*x + 944
784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071372
8 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 - 16529940864
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^3 - 33192121254912
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^4 - 16889738168822
1696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607137
28 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5)*root(z^6 -
z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842
961846272 - 1/34164988081841849499648, z, k), k, 1, 6)
```

3.149 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=415

$$\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{216\sqrt[3]{2}\,3^{5/6}\sqrt{8+9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{\cdots}\right.}{\sqrt{3\left(4-3\cdots\right.}}\right.}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2\cdots}}}$$

```
[Out] 1/216*ln(x)-1/23328*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2
)-1/23328*(18-2^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/46656*ln(6-3
*(-3)^(1/3)*2^(2/3)*x+x^2)*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))-1/1296*(-1)^(
2/3)*((-3)^(1/3)+3*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(
-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))
^(1/2)-1/1296*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-1
2+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/
1296*(-1)^(2/3)*((-2)^(2/3)-2*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(2
4+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^
(1/3)*3^(2/3))^(1/2)
```

Rubi [A]

time = 0.90, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{216\sqrt[3]{2}\,3^{5/6}\sqrt{8+9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\text{ArcTan}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{23328}-\frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{23328}+\frac{\log(x)}{216}-\frac{\left(1-\sqrt[3]{2}\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{216\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))
]])/(216*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 -
 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + Lo
g[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
 x^2])/23328
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = 1259712\int \left(\frac{1}{272097792x} + \frac{(-1)^{2/3}\left(6\left(9+\sqrt[3]{-3}\,2^{2/3}\right) - \right.}{816293376\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)}\right.$$

$$= \frac{\log(x)}{216} + \frac{\int \frac{-6\sqrt[3]{6}\left(9\sqrt[3]{2}-2\sqrt[3]{3}\right)-\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\ 2^{2/3}\sqrt[3]{3}\,x+x^2}dx}{11664} + \frac{(-1)^{2/3}}{}$$

$$= \frac{\log(x)}{216} + \frac{\left(\left(-\frac{1}{6}\right)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\right)\int \frac{1}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}dx}{72\left(1+\sqrt[3]{-1}\right)^2}$$

$$= \frac{\log(x)}{216} - \frac{(-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+\right.}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{216\ 6^{5/6}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 103, normalized size = 0.25

$$\frac{\log(x)}{216} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{36+162\#1+12\#1^2+\#1^4}\&\right]}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 75, normalized size = 0.18

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(136728\_Z^{6}+1230552\_Z^{5}+3682908\_Z^{4}+3630708\_Z^{3}-81810\_Z^{2}+486\_Z-1\right)}\_R\ln\left(-23672342955240\_R^{5}-2130562779\right.\right.}{1944}$
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(\_R^{5}+18\_R^{3}+324\_R^{2}+108\_R\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{1296}+\frac{\ln\left(x\right)}{216}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1296*sum((_R^5+18*_R^3+324*_R^2+108*_R)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln
(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))+1/216*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3 +
 108*x^2 + 216), x) + 1/216*log(x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.55, size = 82, normalized size = 0.20

$$\frac{\log(x)}{216}+\mathrm{RootSum}\left(7379637425677839491923968t^{6}+3416498808184184949964 8t^{5}+525988092506853703 68t^{4}+26673506015311872t^{3}-309171116160t^{2}+944784t-1,\ \left(t\mapsto t\log\left(\frac{814557009966881793678336211511929736056 0128t^{6}}{14342579930905244 0063}+\frac{977068766770806381087358257564745728t^{5}}{143425799309052440063}-\frac{116529526608851264288400971539061538816t^{4}}{143425799309052440063}-\frac{239359794985242202542501440710766592t^{3}}{143425799309052440063}-\frac{136678312638137094439887341418240t^{2}}{143425799309052440063}+\frac{156311556906766379573541369 6t}{143425799309052440063}+x-\frac{316444631507523619004 4}{143425799309052440063}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 34164988081841849499
648*_t**5 + 52598809250685370368*_t**4 + 26673506015311872*_t**3 - 30917111
```

```
6160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(814557009966881793678336211511
9297360560128*_t**6/143425799309052440063 + 9770687667708063810873582575647
45728*_t**5/143425799309052440063 - 1165295266088512642884009715390615388
16*_t**4/143425799309052440063 - 239359794985242202542501440710766592*_t**3/
143425799309052440063 - 1366783126381370944398873414182 40*_t**2/14342579930
9052440063 + 1563115569067663795735413696*_t/143425799309052440063 + x - 31
644463150752361900 44/143425799309052440063)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

Mupad [B]
time = 2.33, size = 432, normalized size = 1.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] log(x)/216 + symsum(log(7*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853
*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352
- 1/7379637425677839491923968, z, k)*x - 5670000*root(z^6 + z^5/216 + (421*
z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 +
 z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 154687594
7520*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^3*x - 106961147905609728*root(z^6 + z^5/216 + (421*z^4)/590
66496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/78109
25487389540352 - 1/7379637425677839491923968, z, k)^4*x - 14051199585413401
8048*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^5*x - 4560729056738761900 0320*root(z^6 + z^5/216 + (421*z^4
)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/
7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x + 839808*root(
z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2
)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968,
z, k)^2 + 594896472576*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^
3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1
/7379637425677839491923968, z, k)^3 - 8483430130458624*root(z^6 + z^5/216 +
```

```
 (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1205389735708
2624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4 - 38314
25535283494912*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902
540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637
425677839491923968, z, k)^5 + 1217393817906599165952*root(z^6 + z^5/216 + (
421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/120538973570826
24 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6)*root(z^6
 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1
2053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z,
k), k, 1, 6)
```

3.150 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=448

$$-\frac{1}{216x}-\frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\ 3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\ 3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\ 3^{2/3}}}-\frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\cdots\right.}{1944\sqrt[6]{6}\left(1+\sqrt[3]{\cdots}\right.}$$

[Out] -1/216/x-1/7776*(-1)^(2/3)*(9+(-3)^(1/3)*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2+1/23328*(3*(-6)^(2/3)+2*(-2)^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*3^(2/3)-1/23328*(2^(2/3)-3*3^(2/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*6^(2/3)-1/11664*(-1)^(2/3)*(6*(-6)^(2/3)+27*(-3)^(1/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/34992*(2^(1/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/17496*(27*(-6)^(1/3)-(-2)^(2/3)+12*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)

Rubi [A]
time = 1.07, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 648, 632, 210, 642, 212}

$$-\frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\ 3^{2/3}\right)\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\ 3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\ 3^{2/3}}}-\frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\text{ArcTan}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\ 2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}\ 2^{2/3}x+6\right)}{1296\sqrt[3]{2}\ 3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{3888\sqrt[3]{6}}-\frac{1}{216x}-\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\ 3^{2/3}-4\right)}}\right)}{5832\sqrt[6]{6}\sqrt{3\sqrt[3]{2}\ 3^{2/3}-4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] -1/216*1/x - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(5832*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(6*(-6)^(2/3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7776*3^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(3888*6^(1/3))

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx = 1259712\int\left(\frac{1}{272097792x^2}+\frac{(-1)^{2/3}\left(-1+9(-3)^{2/3}\sqrt[3]{2}\right.}{816293376\sqrt[3]{2}\,3^{2/3}\left(1+\right.}\right.$$

$$=-\frac{1}{216x}+\frac{\int\frac{-54+2^{2/3}\sqrt[3]{3}+54\sqrt[3]{2}\,3^{2/3}-6^{2/3}(2^{2/3}-3\,3^{2/3})x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{11664}+\frac{(-}{}$$

$$=-\frac{1}{216x}-\frac{\left((-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\right)\int\frac{-3\sqrt[3]{-3}\,2^{2/3}+2x}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=-\frac{1}{216x}-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x\right.}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$

$$=-\frac{1}{216x}+\frac{(-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-}}{5832\,2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 109, normalized size = 0.24

$$-\frac{1}{216x}-\frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 74, normalized size = 0.17

method	result

risch	$-\frac{1}{216x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(633\_Z^{6}+204849\_Z^{4}-5446980\_Z^{3}-80433\_Z^{2}-72\_Z-1\right)}\_R\ln\left(-462040439801351484393\_R^{5}+13642318\right.\right.}{}$
default	$-\frac{1}{216x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(-\_R^{4}-18\_R^{2}-324\_R-108\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{1296}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
[Out] -1/216/x+1/1296*sum((-_R^4-18*_R^2-324*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_R
)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 324
*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A]
time = 0.18, size = 70, normalized size = 0.16

$$\operatorname{RootSum}\left(159400168394641333025557708 8t^{6}+379161202646033163878 4t^{4}-8643672699589509120t^{3}-10942820851968t^{2}-839808t-1,\left(t\mapsto t\log\left(-\frac{498755327619024960032935612369144680284 16t^{5}}{12350449784703991795}+\frac{1262548987243162038800597520049766 4t^{4}}{12350449784703991795}-\frac{1186376926075737712385507988526448 64t^{3}}{12350449784703991795}+\frac{270486324927832147818193778754816t^{2}}{12350449784703991795}+\frac{273914194897479402961199352t}{12350449784703991795}+x-\frac{12798926329353908292}{12350449784703991795}\right)\right)\right)-\frac{1}{216x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
[Out] RootSum(1594001683946413330255577088*_t**6 + 3791612026460331638784*_t**4 -
 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_
```

```
t, _t*log(-49875532761902496003293561236914468028416*_t**5/1235044978470399
1795 + 126254898724316203880059752004976 64*_t**4/12350449784703991795 - 118
637692607573771238550798852644864*_t**3/12350449784703991795 + 270486324927
832147818193778754816*_t**2/12350449784703991795 + 2739141948974794029611 99
352*_t/12350449784703991795 + x - 1279892632935390829 2/12350449784703991795
))) - 1/(216*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

Mupad [B]
time = 0.29, size = 340, normalized size = 0.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
(331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413
330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300
846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594
001683946413330255577088, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/118132
992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189805489
3435658305536 - 1/1594001683946413330255577088, z, k)^2*x - 598229670528*ro
ot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155
89428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z,
 k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008
46726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/15940
01683946413330255577088, z, k)^4*x - 6940988288557056*root(z^6 + (281*z^4)/
118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189
8054893435658305536 - 1/1594001683946413330255577088, z, k)^5*x + 2344464*r
oot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215
589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z
, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008467
26144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/15940016
83946413330255577088, z, k)^3 - 10535082310656*root(z^6 + (281*z^4)/1181329
92 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893
435658305536 - 1/1594001683946413330255577088, z, k)^4 - 1688973816882216 96
```

```
*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482
15589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088,
 z, k)^5)*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331
*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641333302
55577088, z, k), k, 1, 6) - 1/(216*x)
```

3.151 $\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=1064

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-2^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}\right)\right)x\right)}{162\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)+\left(2+\right.\right.}{729\,2^{2/3}\left(8+9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3\right.}$$

```
[Out] -1/972*(-1)^(1/3)*3^(2/3)*(54+9*(-3)^(1/3)*2^(2/3)+(2-2^(2/3)*(6*(-6)^(2/3)+27*(-3)^(1/3)))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/4374*(-1)^(1/3)*3^(2/3)*(54-9*(-2)^(2/3)*3^(1/3)+(2+27*(-2)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/8748*(54-9*2^(2/3)*3^(1/3)+(2+2^(2/3)*(27*3^(1/3)-6*6^(2/3)))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/972*(-1)^(1/3)*(2+27*(-2)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/5832*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/5832*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/52488*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/486*(-1)^(1/3)*(6*(-6)^(2/3)+27*(-3)^(1/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)+1/486*(2^(1/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)*2^(1/2)+1/972*(I*2^(2/3)-9*3^(1/6)-3*I*3^(2/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/972*I*((-2)^(2/3)+6*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/8748*(1+3*2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]

time = 2.27, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2122, 652, 632, 210, 648, 642, 212}

Antiderivative was successfully verified.

```
[In] Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -1/162*((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 3*2^(2/3)*(2*(-6)^(
2/3) + 9*(-3)^(1/3)))*x))/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(
1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^(1/3)*(9*(6 - (-2)^(2/3
)*3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2
^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^
(1/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6
^(2/3)))*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^
(1/3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^
(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1
)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)
*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[
6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1
+ (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/
3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/S
qrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3)
)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1
/3))]])/(162*2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/
3)]) - ((1 + 3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/S
qrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)
*3^(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3
) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(81*Sqrt[2]*3^(5/6)*(1 -
 (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - Log[6 -
 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(972*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + (
(I/972)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(
1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = 1586874322944\int\left(\frac{\sqrt[3]{-\frac{1}{3}}\left(-1+3(-3)^{2/3}\sqrt[3]{2}+(9+\sqrt[3]{\cdots}\right.}{42845606719488\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(6-3\cdots\right.}\right.$$

$$=\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{-1-3\sqrt[3]{-2}\,3^{2/3}+\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}dx}{243\ 2^{2/3}}-\frac{\int\frac{-27+x}{6+3\ 2^{2/3}\sqrt[3]{3}\,x+x\cdots}}{4374\sqrt[3]{2}\,3^{2/\cdots}}$$

$$=-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right.\right.\right.}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

$$=-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right.\right.\right.}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

$$=-\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\ 2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right.\right.\right.}{162\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 167, normalized size = 0.16

$$\frac{-7884+324x-3990x^2-11610x^3-203x^4-9x^5}{34182\,(216+108x^2+324x^3+18x^4+x^6)}-\frac{\mathrm{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{324\log(x-\#1)-96\log(x-\#1)\#1+324\log(x-\#1)\#1^2+406\log(x-\#1)\#1^3+9\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{205092}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108*
x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^
4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 +
406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/205092
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 122, normalized size = 0.11

method	result
default	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4-406\_R^3-324\_\right.}{\_R^5+12\_R^3}\right.}{205092}$
risch	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4-406\_R^3-324\_\right.}{\_R^5+12\_R^3}\right.}{205092}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+1
8*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+1
08*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18*
x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324*x
^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.24, size = 112, normalized size = 0.11

$$\mathrm{RootSum}\left(85256017052964187415123360664576t^6+50105191533385434568704t^4+48885748051277486016t^3+865447782603408t^2+3220532460t+4513,\left(t\mapsto t\log\left(\frac{35492036204084174404119193135483487466590764032t^5}{356900697070792948475845}-\frac{19474160067218837086826809631017022308224t^4}{71380139414158589695169}+\frac{20779963076545132233894582764903396544t^3}{356900697070792948475845}+\frac{20265219154367004972162198012037344t^2}{356900697070792948475845}+\frac{275192468949210532049075145372t}{356900697070792948475845}+x+\frac{1290285191292177289622012}{1070702091212378845427535}\right)\right)\right)+\frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182x^6+615276x^4+11074968x^3+3691656x^2+7383312}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(852560170529641874151233606645 76*_t**6 + 5010519153338543456870 4*_t
**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t +
4513, Lambda(_t, _t*log(35492036204084174404119193135483487466590764032*_t*
*5/35690069707079294847 5845 - 1947416006721883708682680963101702230822 4*_t*
*4/7138013941415858969516 9 + 2077996307654513223389458276490339654 4*_t**3/3
569006970707929484758 45 + 20265219154367004972162198012037344*_t**2/3569006
97070792948475845 + 27519246894921053204907514537 2*_t/3569006970707929484 75
845 + x + 129028519129217728962201 2/1070702091212378845427535)) + (-9*x**5
 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 + 615276*x
**4 + 11074968*x**3 + 3691656*x**2 + 7383312)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [**B**]
time = 2.34, size = 388, normalized size = 0.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((239491904*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141
49992416343982992 + (5171*z^2)/5093997269883833877 12 + (505*z)/133686864350
83133627113728 + 4513/852560170529641874151233606645 76, z, k)*x)/876306843
- (275536*x)/638827688547 - (3848128*root(z^6 + (326*z^4)/554702231619 + (8
113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*
z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k
))/3606201 - (152363520*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1
4149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868643
5083133627113728 + 4513/85256017052964187415123360664576, z, k)^2*x)/44521
- (698075283456*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141499924
16343982992 + (5171*z^2)/509399726988383387712 + (505*z)/133686864350831336
27113728 + 4513/85256017052964187415123360664576, z, k)^3*x)/44521 + (13078
9789876224*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343
982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868643508313362711 3
728 + 4513/85256017052964187415123360664576, z, k)^4*x)/211 - 6940988288557
```

```
056*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992
+ (5171*z^2)/5093997269883833877l2 + (505*z)/133686864350831336271137 28 + 4
513/8525601705296418741512336066457 6, z, k)^5*x - (4264220928*root(z^6 + (3
26*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093
99726988383387712 + (505*z)/133686864350831336271137 28 + 4513/8525601705296
418741512336066457 6, z, k)^2)/44521 - (5086414725120*root(z^6 + (326*z^4)/5
54702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883
83387712 + (505*z)/133686864350831336271137 28 + 4513/8525601705296418741512
3360664576, z, k)^3)/44521 + (243585208571904*root(z^6 + (326*z^4)/55470223
1619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/50939972698838338771
2 + (505*z)/133686864350831336271137 28 + 4513/852560170529641874151233606 64
576, z, k)^4)/211 - 168897381688221696*root(z^6 + (326*z^4)/554702231619 +
(8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (50
5*z)/133686864350831336271137 28 + 4513/852560170529641874151233606645 76, z,
 k)^5 - 48160/23660284761)*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3
)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868
643508313362711372 8 + 4513/852560170529641874151233606645 76, z, k), k, 1, 6
) - ((665*x^2)/5697 - (2*x)/211 + (215*x^3)/633 + (203*x^4)/34182 + x^5/379
8 + 146/633)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

3.152 $\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=1005

$$-\frac{2\left(2\sqrt[3]{-1}\,3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)x}{972\;2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)-9(1}{4374\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)(6}$$

```
[Out] 1/1944*(-4*(-1)^(1/3)*3^(2/3)-18*6^(1/3)+9*((-2)^(2/3)+2*(-1)^(1/3)*3^(2/3)
)*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2
/3)*x+x^2)+1/4374*(-(-6)^(1/3)*(9*(-2)^(1/3)+2*3^(1/3))+9*(1+(-2)^(1/3)*3^(
2/3))*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*
x+x^2)+1/17496*(4-6*2^(1/3)*3^(2/3)-3*(6-2^(2/3)*3^(1/3))*x)*2^(1/3)*3^(2/3
)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/3888*I*ln(6-3*(-3)^(1
/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-1/104976*ln(6+3*2^(2/3)
*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)-1/324*(-1)^(1/3)*((-3)^(1/3)+3*2^(1/3))*arc
tan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/
6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)*2^(1/2)-1/7776*ln(6+3*(-
2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/324*
(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*
3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^
(3/2)*6^(1/2)+1/324*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*
x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2
^(1/3)*3^(2/3))^(3/2)*6^(1/2)+1/5832*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-
18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I+3^(1/3)*(2*I*2^(2/3)-9*3^(1/6)+2*2^(2/3)
*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)+1/5832*(9*3^(1/6
)+I*(4*2^(2/3)-3*3^(2/3)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1
/3)*3^(2/3))^(1/2))*3^(1/3)/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3))^(1/2)
+1/78732*(2*2^(2/3)+3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9
*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]
time = 2.15, antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2122, 648, 632, 210, 642, 652, 212}

Antiderivative was successfully verified.

```
[In] Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -1/972*(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(1/3)
*3^(2/3))*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*
(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) - 9*(
1 + (-2)^(1/3)*3^(2/3))*x)/(4374*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(
2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*
(6 - 2^(2/3)*3^(1/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 +
 3*2^(2/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6) +
 2*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)
^(2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3
))]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6
*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/
3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*3^(1/6) + I*(4*2^(2/3) - 3*3^
(2/3)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(1944*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))])
- ((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1
/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(54*Sqrt[2]*3^(5/6)*(1 + (-1)^
(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh
[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*S
qrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)
) + ((2*2^(2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt
[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3
))]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1
+ (-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(1
296*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2
]/(17496*2^(2/3)*3^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = 1586874322944 \int \left(\frac{-27+3\ 2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}}{9254651051409408\left(1+\sqrt[3]{-1}\right)^5(-6+3} \right.$$

$$= \frac{\int \frac{-18-2\ 2^{2/3}\sqrt[3]{3}-\sqrt[3]{2}\ 3^{2/3}x}{6+3\ 2^{2/3}\sqrt[3]{3}\ x+x^2}\,dx}{52488} + \frac{\int \frac{9\ 2^{2/3}+\sqrt[3]{-1}\ 3^{2/3}\left(1+3\sqrt[3]{-2}\ 3^{2/}\right.}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\ x+x^2\right)^2}}{4374\ 2^{2/3}}$$

$$= -\frac{2\left(2\sqrt[3]{-1}\ 3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\ 3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\ 2^{2/3}x\right.}$$

$$= -\frac{2\left(2\sqrt[3]{-1}\ 3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\ 3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\ 2^{2/3}x\right.}$$

$$= -\frac{2\left(2\sqrt[3]{-1}\ 3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\ 3^{2/3}\right)x}{972\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\ 2^{2/3}x\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 167, normalized size = 0.17

$$\frac{648-96x+432x^2+908x^3-18x^4+73x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{96\log(x-\#1)-216\log(x-\#1)\#1+96\log(x-\#1)\#1^2-36\log(x-\#1)\#1^3+73\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{410184}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 +
324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1
^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*Log[
x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]
/410184
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 122, normalized size = 0.12

method	result
default	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(73\_R^4-36\_R^3+96\_R^2\right.}{\_R^5+12\_R^3+1}\right.}{410184}$
risch	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(73\_R^4-36\_R^3+96\_R^2\right.}{\_R^5+12\_R^3+1}\right.}{410184}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x^
4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(_R
^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^
2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 +
324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 - 21
6*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.29, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(58928958987008846341333266891354931 2t^6-53964029026607524840573747 2t^4+921826381685096823920 64t^3-5532414420691704 96t^2-3759837842016t-7197829,\left(t\mapsto t\log\left(\frac{4299602763972744771400374330516074611101843850102599932 3136t^5}{1542060097910520444906943803032 37521}-\frac{42584766259508194684689715474422251405157209835847680t^4}{154206009791052044490694380303237521}-\frac{3751244612884958815010836944932375407831734108 2112t^3}{154206009791052044490694380303237521}+\frac{71520375940216752676388907155316724819202221 44t^2}{154206009791052044490694380303237521}-\frac{4422754699883529772383029179497431052403 2t}{154206009791052044490694380303237521}+x-\frac{1745733490366760477341325695830248 55}{154206009791052044490694380303237521}\right)\right)\right)+\frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(589289589870088463413332668913549312*_t**6 - 539640290266075248405737472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 - 3759837842016*_t - 7197829, Lambda(_t, _t*log(4299602763972744771400374330516074611101843850102599932313 6*_t**5/1542060097910520444906943803032375 21 - 4258476625950819468468971547444222514051572098358476 80*_t**4/154206009791052044490694380303237521 - 3751244612884958815010836944932375407831734108 2112*_t**3/154206009791052044490694380303237521 + 7152037594021675267638890715531672481920222144*_t**2/154206009791052044490694380303237521 - 44227546998835297723830291794974310524032*_t/154206009791052044490694380303237521 + x - 1745733490366760477341325695830248 55/154206009791052044490694380303237521)))
 + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B]
time = 2.30, size = 387, normalized size = 0.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012426 24 - (2479189*z^2)/264072818470777948189900 8 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546662622050124 2624 - (2479189*z^2)/264072818470777948189900 8 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)*x)/7886761587 - (207565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012426 24 - (2479189*z^2)/264072818470777948189900 8 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012426 24 - (2479189*z^2)/264072818470777948189900 8 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463
```

13332668913549312, z, k)^3*x)/44521 - (147138513610752*root(z^6 - (292589*z
^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2
6407281847077794818990 08 - (1989787*z)/31186471715761934125330904 6784 - 719
7829/5892895898700884634133326689135493 12, z, k)^4*x)/211 - 694098828855705
6*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546662622050124
2624 - (2479189*z^2)/264072818470777948189 9008 - (1989787*z)/31186471715761
9341253309046784 - 7197829/5892895898700884634133326689135493 12, z, k)^5*x
- (1156135728*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546
6626220501242624 - (2479189*z^2)/264072818470777948189 9008 - (1989787*z)/31
186471715761934125330904 6784 - 7197829/5892895898700884634133326689135493 12
, z, k)^2)/44521 + (6458021903232*root(z^6 - (292589*z^4)/319508485412544 +
(11805253*z^3)/75466626220501242624 - (2479189*z^2)/264072818470777948189 9
008 - (1989787*z)/31186471715761934125330904 6784 - 7197829/5892895898700884
6341333266891354931 2, z, k)^3)/44521 - (102226052063232*root(z^6 - (292589*
z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/
26407281847077794818990 08 - (1989787*z)/31186471715761934125330904 6784 - 71
97829/5892895898700884634133326689135493 12, z, k)^4)/211 - 1688973816882216
96*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012
42624 - (2479189*z^2)/264072818470777948189 9008 - (1989787*z)/3118647171576
19341253309046784 - 7197829/5892895898700884634133326689135493 12, z, k)^5 +
2207561/7665932262564)*root(z^6 - (292589*z^4)/319508485412544 + (11805253
*z^3)/75466626220501242624 - (2479189*z^2)/264072818470777948189 9008 - (198
9787*z)/31186471715761934125330904 6784 - 7197829/589289589870088463413332 66
8913549312, z, k), k, 1, 6) + ((4*x^2)/633 - (8*x)/5697 + (227*x^3)/17091 -
x^4/3798 + (73*x^5)/68364 + 2/211)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216
)

3.153 $\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=677

$$\frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\frac{9\ 2^{2/3}+\sqrt[3]{-1}\,3^{2/3}(2+\cdots}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/\cdots}\right.}$$

```
[Out] 1/5832*(9*(-2)^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3))*x)*2^(1/3)/(1+(-1)^(1/3
))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/26244*(9*2^(
2/3)+(-1)^(1/3)*3^(2/3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*
3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(3*2^(2/3
)*3^(1/3)-(2-3*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6
+3*2^(2/3)*3^(1/3)*x+x^2)+1/2916*(-1)^(1/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((
3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*6^(1/6)/(1+(-1)
^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)+1/2916*(3*(-3)^(2/3)+(-1)^(1/3)*2^
(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*
6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/
2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/
3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*
3^(2/3))^(3/2)+1/34992*(-1)^(1/6)*3^(5/6)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*
2^(2/3)/(1+(-1)^(1/3))^5-1/34992*I*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)
*3^(5/6)/(1+(-1)^(1/3))^5+1/314928*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^
(1/3)
```

Rubi [A]

time = 1.45, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2122, 652, 632, 210, 642, 212}

$$\frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}\,2^{2/3}\right)\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)^{3/2}}+\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x+9(-2)^{2/3}}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\,3^{2/3}\left(2+3\sqrt[3]{-2}\,3^{2/3}\right)x+9\ 2^{2/3}}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}+\frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}\,3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}\,x+6\right)}+\frac{\sqrt[6]{-\frac{1}{3}}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5}-\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{5832\sqrt[3]{2}\,\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}\,x+6\right)}{52488\sqrt[3]{2}\,3^{2/3}}-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(3\sqrt[3]{2}\,3^{2/3}-4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) +
(9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13122*2^(2/3
)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)
*x + x^2)) + (3*2^(2/3)*3^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*
3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1
```

```
/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4
- 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/
3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-
1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3
) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x
^2])/(5832*2^(1/3)*(1 + (-1)^(1/3))^5) - ((I/5832)*Log[6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(
1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
```

```
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{-2\sqrt[3]{-1}\,3^{2/3}+3(-}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4(}\right.$$

$$= -\frac{\int\frac{-2\sqrt[3]{-1}\,3^{2/3}+3\ 2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{8748\ 2^{2/3}} - \frac{\int\frac{2+2^{2/3}\sqrt[3]{3}\,x}{\left(6+3\ 2^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{2916\ 2^{2/3}\sqrt[3]{3}} + \frac{\int}{6}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 167, normalized size = 0.25

$$\frac{-96+108x-64x^2-72x^3+73x^4-3x^5}{68364\left(216+108x^2+324x^3+18x^4+x^6\right)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{108\log(x-\#1)-32\log(x-\#1)\#1+108\log(x-\#1)\#1^2-146\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{410184}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 32
4*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6
```

```
 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*Log[
x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/
410184
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 122, normalized size = 0.18

method	result
default	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3-108 \ldots\right.}{\_R^5+12\_ \ldots}\right.}{410184}$
risch	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3-108 \ldots\right.}{\_R^5+12\_ \ldots}\right.}{410184}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3-108*_R^2+32*_R-108)
/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108
*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146*x^3 + 108*x^2 - 32*
x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.25, size = 112, normalized size = 0.17

$$\operatorname{RootSum}\left(397770473162309712803999551516645785 6 t^{6} - 1010314319415295961050951680 t^{4} - 2016822447709395715 1232 t^{3} - 112582856818899648 t^{2} - 50648453064 t - 880007, \left(t \mapsto t \log \left(- \frac{2736555670900189915706499414143955609861996880406446 08 t^{5}}{497978553961399002675733956 95} + \frac{1183700847019604608530864623076435429280504457011 2 t^{4}}{49797855396139900267573395695} - \frac{10570581900446717266374077482873315047787008 t^{3}}{49797855396139900267573395695} - \frac{15525474115694698723875632187927893233 92 t^{2}}{49797855396139900267573395695} - \frac{1254292379115914082690900325029592 8 t}{49797855396139900267573395695} + x - \frac{2306653387032032241083434829 6}{49797855396139900267573395695} \right) \right)\right) + \frac{-3x^{5} + 73x^{4} - 72x^{3} - 64x^{2} + 108x - 96}{68364x^{6} + 1230552x^{4} + 22149936x^{3} + 7383312x^{2} + 14766624}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(397770473162309712803999551516645785 6*_t**6 - 10103143194152959610509516 80*_t**4 - 2016822447709395715 1232*_t**3 - 112582856818899648*_t**2 - 50648453064*_t - 880007, Lambda(_t, _t*log(-27365556709001899157064994141439556098619968804064460 8*_t**5/49797855396139900267573395695 + 11837008470196046085308646230764354292805044570112*_t**4/49797855396139900267573395695 - 10570581900446717266374077482873315047787008*_t**3/49797855396139900267573395695 - 1552547411569469872387563218792789323392*_t**2/49797855396139900267573395695 - 12542923791159140826909003250295928*_t/49797855396139900267573395695 + x - 23066533870320322410834348296/49797855396139900267573395695))) + (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B]
time = 2.33, size = 388, normalized size = 0.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((7028852*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305639836193030032 6272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309712803999551516645785 6, z, k))/2628920529 - (1980083*x)/310470256633842 - (235710556*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305639836193030032 6272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309712803999551516645785 6, z, k)*x)/70980854283 - (
```

6628544*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619
30300326272 - (168169*z^2)/594163841559250383427276 8 - (3971*z)/31186471715
761934125330904 6784 - 880007/3977704731623097128039995515166457856, z, k)^2
*x)/44521 - (141776759808*root(z^6 - (60865*z^4)/239631364059408 - (1549690
9*z^3)/30563983619303003262 72 - (168169*z^2)/5941638415592503834272768 - (3
971*z)/3118647171576193412533090 46784 - 880007/39777047316230971280399955 15
166457856, z, k)^3*x)/44521 + (183701926508544*root(z^6 - (60865*z^4)/23963
1364059408 - (15496909*z^3)/30563983619303003 26272 - (168169*z^2)/594163841
5592503834272768 - (3971*z)/3118647171576193412533090 46784 - 880007/3977704
7316230971280399955151664 57856, z, k)^4*x)/211 - 6940988288557056*root(z^6
- (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619303003 26272 - (16
8169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090467
84 - 880007/3977704731623097128039995515166457856, z, k)^5*x + (100886752*r
oot(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305639836193030032 62
72 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31186471715761934125
3309046784 - 880007/3977704731623097128039995515166457856, z, k)^2)/133563
+ (1715052538368*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30
563983619303003 26272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31
186471715761934125330904 6784 - 880007/3977704731623097128039995515166457856
, z, k)^3)/44521 + (115004308571136*root(z^6 - (60865*z^4)/239631364059408
- (15496909*z^3)/30563983619303003 26272 - (168169*z^2)/59416384155925038342
72768 - (3971*z)/3118647171576193412533090 46784 - 880007/397770473162309712
8039995515166457856, z, k)^4)/211 - 1688973816882216 96*root(z^6 - (60865*z^
4)/239631364059408 - (15496909*z^3)/30563983619303003 26272 - (168169*z^2)/5
941638415592503834272768 - (3971*z)/3118647171576193412533090 46784 - 880007
/3977704731623097128039995515166457856, z, k)^5 - 265/5749449196923)*root(z
^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619303003 26272 -
(168169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090
46784 - 880007/3977704731623097128039995515166457856, z, k), k, 1, 6) - ((1
6*x^2)/17091 - x/633 + (2*x^3)/1899 - (73*x^4)/68364 + x^5/22788 + 8/5697)/
(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

3.154 $\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=682

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left(4+(-2)^{2}\right.}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right.}$$

```
[Out] 1/11664*(-1)^(1/3)*3^(2/3)*(4-(-3)^(1/3)*2^(2/3)*x)*2^(1/3)/(1+(-1)^(1/3))^
4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/52488*(-1)^(1/3
)*3^(2/3)*(4+(-2)^(2/3)*3^(1/3)*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)
*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-4-2^(2/3)*3^(1/3)*x)*2^
(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/13122*arc
tan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(
1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)-1/13122*arctan((3*(-2)^(2/3)*
3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(
1/3)*3^(2/3))^(3/2)*3^(1/2)-1/52488*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(
-12+9*2^(1/3)*3^(2/3))^(1/2))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/26244*
arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)
*3^(5/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/8748*I*arctan((3
*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(1/3)/
(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/236196*arctanh(2^(1/6)*(3
*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5/6)/(-4+3*2^
(1/3)*3^(2/3))^(1/2)
```

Rubi [A]
time = 1.17, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2122, 652, 632, 210, 212}

$$\frac{\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)^{3/2}}-\frac{\text{ArcTan}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)^{3/2}}-\frac{i\text{ArcTan}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}+\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}x+4\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{2^{2/3}\sqrt[3]{3}x+4}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{8748\sqrt{6}\left(3\sqrt[3]{2}\,3^{2/3}-4\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + ((-1/3)^(1
/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/
3)*x)/(17496*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)
*x + x^2)) - ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2
```

```
^(1/3))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^
(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1
/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
 - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3
^(2/3))]])/(2^(5/6)*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]]/(4374*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/
(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3)
 + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(39366*2^(5/6)*3^(1/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
```

```
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}\,x}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4 (\cdots}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{8748\ 2^{2/3}} + \frac{\int \frac{1}{6+3\ 2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{26244\sqrt[3]{2}\ 3^{2/3}} + \cdots$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x \cdots\right.}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x \cdots\right.}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x \cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 167, normalized size = 0.24

$$\frac{972-144x+648x^2+729x^3-27x^4+4x^5}{615276\left(216+108x^2+324x^3+18x^4+x^6\right)}+\frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{144\log(x-\#1)-324\log(x-\#1)\#1+2043\log(x-\#1)\#1^2-54\log(x-\#1)\#1^3+4\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{3691656}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2 +
 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #
```

```
1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 - 54*
Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/3691656
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 122, normalized size = 0.18

method	result
default	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+2043\_ \right.}{\_R^5+12\_R^3}\right.}{3691656}$
risch	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+2043\_ \right.}{\_R^5+12\_R^3}\right.}{3691656}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^4
+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)/(
_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_
Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4 +
 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2 -
 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(463) = 926.
time = 1.16, size = 1445, normalized size = 2.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] 1/28041818976*(182304*x^5 - 1230552*x^4 + 33224904*x^3 + 422*sqrt(1/633)*(x
^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(
```

1/3) + 44687457)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3)
+ 9367856*18^(1/3) + 44687457)^2 - 44790237484004061769673*18^(2/3) - 83
343065225076612586451 12*18^(1/3) - 268625598114228853471 20477)*sqrt(5034474
*18^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 513225545496
0803463351330/9393931*18^(2/3) + 95498020363770460407535 20/9393931*18^(1/3)
+ 2727892823303394003242 5830/9393931) - 422*sqrt(1/633)*(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
*log(-2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)^2 - 44790237484004061769673*18^(2/3) - 83343065225076612
58645112*18^(1/3) - 268625598114228853471 20477)*sqrt(5034474*18^(2/3) + 936
7856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 9367856*
18^(1/3) + 44687457)^2 + 247458158879850620*x + 51322554549608034633513 30/9
393931*18^(2/3) + 95498020363770460407535 20/9393931*18^(1/3) + 272789282330
33940032425830/9393931) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 2
16)*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/
3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397
4962699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 3064230/211*sqrt(-1/19683*(5034474*
18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787
01088/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*
18^(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 243*sqrt(-1/19683*(5034474*18^(2/3)
+ 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81
18^(1/3) + 273974962699)(14476041114*sqrt(422)*(5034474*18^(2/3) + 936785
6*18^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 177934129698570542
9*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805
69051992480475*sqrt(422))*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387
65959317812080*x - 10264510909921606926702660/211*18^(2/3) - 19099604072754
092081507040/211*18^(1/3) - 54557856466067880064851660/211) + 9*sqrt(422)*(
x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) + sqrt(-1/1968
3*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3)
+ 3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 36779
8)*log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
3064230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895
278433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*
(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
243*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228
60116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*
sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061
5865*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856
*18^(1/3) + 44687457) + 26505855880569051992480475*sqrt(422))*sqrt(-20718*1

```
8^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 936785
6/243*18^(1/3) + 367798) + 44068338765959317812080*x - 10264510909921606926
702660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466
067880064851660/211) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)
*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3)
+ 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397496
2699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 3064230/211*sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*18^
(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 243*sqrt(-1/19683*(5034474*18^(2/3) +
9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18
^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*1
8^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 1779341296985705429*s
qrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805690
51992480475*sqrt(422))*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/
3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/
81*18^(1/3) + 273974962699) - 9367856/243*18^(1...
```

Sympy [**A**]
time = 0.13, size = 104, normalized size = 0.15

$$\operatorname{RootSum}\left(27493895104978847349012449000830556700672t^6 - 1318718189226950088862983192576t^4 + 12120917704776776448t^2 - 39753025, \left(t \mapsto t\log\left(\frac{947842259001288723909832054550209950242045952t^5}{61864539719962655} - \frac{243458646817775607639654889480814592t^4}{9811980923071} - \frac{41682556475067500431787310779667456t^3}{61864539719962655} + \frac{12026877442664328616462272t^2}{9811980923071} + \frac{216142618488859793668428t}{61864539719962655} + x - \frac{308574300024117}{39247923692284}\right)\right)\right) + \frac{4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972}{615276x^6 + 11074968x^4 + 199349424x^3 + 66449808x^2 + 132899616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(27493895104978847349012449000830556700672*_t**6 - 13187181892269500
88862983192576*_t**4 + 12120917704776776448*_t**2 - 39753025, Lambda(_t, _t
*log(947842259001288723909832054550209950242045952*_t**5/61864539719962655
- 243458646817775607639654889480814592*_t**4/9811980923071 - 41682556475067
500431787310779667456*_t**3/61864539719962655 + 12026877442664328616462272*
_t**2/9811980923071 + 216142618488859793668428*_t/61864539719962655 + x - 3
08574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 648*x**2
- 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 66449808*x**
2 + 132899616)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B]
time = 0.31, size = 299, normalized size = 0.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
[Out] symsum(log((6305*x)/4967524106141472 - (4477969*root(z^6 - (183899*z^4)/383
4101824950528 + (6209*z^2)/1408388365177482390346l376 - 39753025/2749389510
4978847349012449000830556700672, z, k))/189282278088 - (16340881*root(z^6 -
 (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39
753025/27493895104978847349012449000830556700672, z, k)*x)/5110621508376 -
(43348696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651
774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)
^2*x)/10818603 - (65333687616*root(z^6 - (183899*z^4)/3834101824950528 + (6
209*z^2)/14083883651774823903461376 - 39753025/274938951049788473490124490
0830556700672, z, k)^3*x)/44521 - (40024496812032*root(z^6 - (183899*z^4)/3
834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895
104978847349012449000830556700672, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376
- 39753025/27493895104978847349012449000830556700672, z, k)^5*x + (5943884*
root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/140838836517748239034
61376 - 39753025/27493895104978847349012449000830556700672, z, k)^2)/400689
 + (224442467136*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/1408
3883651774823903461376 - 39753025/27493895104978847349012449000830556700672
, z, k)^3)/44521 - (137087493272064*root(z^6 - (183899*z^4)/383410182495052
8 + (6209*z^2)/14083883651774823903461376 - 39753025/274938951049788473490l
2449000830556700672, z, k)^4)/211 - 1688973816882216 96*root(z^6 - (183899*z
^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/274
93895104978847349012449000830556700672, z, k)^5 - 13082875/1788308678210929
92)*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823
903461376 - 39753025/27493895104978847349012449000830556700672, z, k), k, 1
, 6) + ((2*x^2)/1899 - (4*x)/17091 + x^3/844 - x^4/22788 + x^5/153819 + 1/6
33)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

3.155 $\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=850

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}-\frac{\sqrt[3]{-\frac{1}{3}}\left(3(-2)^{2/3}\sqrt[3]{\ }\right.}{26244\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right.}$$

```
[Out] 1/34992*(-1)^(1/3)*3^(2/3)*(3*(-3)^(1/3)*2^(2/3)-2*x)*2^(1/3)/(1+(-1)^(1/3)
)^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/157464*(-1)^(
1/3)*3^(2/3)*(3*(-2)^(2/3)*3^(1/3)+2*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^
(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(-3*3^(1/3)-2^(1/3)*x
)/(9*2^(1/3)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/4374*(-1)^(1/3)*arcta
n((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/3)*3^(1
/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)-1/1749
6*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(
1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(
2/3))^(3/2)+1/157464*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3
^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/209952*ln(6-3
*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/209952*I*ln(6
+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/1889568*ln(
6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/34992*I*arctan(2^(1/6)*(3*(-3)
^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^
(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/69984*arctan((3*(-2)^(2/3)*3^(1/3
)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1+(-1
)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)+1/314928*arctanh(2^(1/6)*(3*3^(1/
3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*
3^(2/3))^(1/2)
```

Rubi [A]
time = 1.36, antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2122, 628, 632, 210, 648, 642, 212}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-1)^(1/3))^
4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^
```

```
(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/
6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) +
2^(1/3)*x)/(52488*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2))
+ ((-1)^(1/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*
2^(1/3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1
/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3)
+ 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*2^(1/6)*3^(5/6)*(1 - (-1)
^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt
[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]
])/(11664*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]
) - ((I/5832)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(
52488*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) -
 Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = 1586874322944\int\left(-\frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(\right.}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{8748\ 2^{2/3}} - \frac{\int\frac{3\ 2^{2/3}\sqrt[3]{3}+x}{6+3\ 2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{157464\sqrt[3]{2}\,3^{2/3}} +$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 167, normalized size = 0.20

$$\frac{-288+324x-1458x^2-216x^3+8x^4-9x^5}{1230552\left(216+108x^2+324x^3+18x^4+x^6\right)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{324\log(x-\#1)-2628\log(x-\#1)\#1+324\log(x-\#1)\#1^2-16\log(x-\#1)\#1^3+9\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{7383312}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^2
 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
 #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 1
6*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5
) & ]/7383312
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 122, normalized size = 0.14

method	result
default	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3-32\right.}{\_R^5+12\_}\right.}{7383312}$
risch	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3-32\right.}{\_R^5+12\_}\right.}{7383312}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+1
08*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^4
 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x^2
 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.26, size = 112, normalized size = 0.13

$$\operatorname{RootSum}\left(185583791958607219605834030755606257729536t^6-1309367357962223565522033377280t^4+43563364870522947446666112t^3-405298284548038732 8t^2+303890718384t-880007,\left(t\mapsto t\log\left(\frac{39083462657955593476841044707333565976412952759280634691584t^5}{4979785539613990026757339 5695}+\frac{883697934622378553891281760141471110239680446 2575616t^4}{49797855396139900267573395695}-\frac{2649305813483085325888442495971346957068050677 76t^3}{49797855396139900267573395695}+\frac{886135333547363185201515109826158376250624t^2}{49797855396139900267573395695}-\frac{682321479574909906511394635855601936t}{49797855396139900267573395695}+x-\frac{21375560770846486224291519568}{49797855396139900267573395695}\right)\right)\right)+\frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552x^6+22149936x^4+398698848x^3+132899616x^2+265799232}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(185583791958607219605834030755606257729536*_t**6 - 1309367357962223565522033377280*_t**4 + 4356336487052294744666112*_t**3 - 4052982845480387328*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(39083462657955593476841044707333565976412952759280634691584*_t**5/49797855396139900267573395695 + 8836979346223785538912817601414711102396804462575616*_t**4/49797855396139900267573395695 - 264930581348308532588844249597134695706805067776*_t**3/49797855396139900267573395695 + 886135333547363185201515109826158376250624*_t**2/49797855396139900267573395695 - 682321479574909906511394635855601936*_t/49797855396139900267573395695 + x - 21375560770846486224291519568/49797855396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x**2 + 324*x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899616*x**2 + 265799232)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

Mupad [B]
time = 2.42, size = 388, normalized size = 0.46

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 - (1090723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^2*x)/3606201 - (64554687936*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617

647997585731147792384 - 880007/185583791958607219605834030755606257729536,
z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/8626729106138688
+ (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770036338660788496
9217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791
958607219605834030755606257729536, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752
 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997
585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^
5*x - (1697552*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660
182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z
)/2425060040617647997585731147792384 - 880007/18558379195860721960583403075
5606257729536, z, k)^2)/10818603 + (12229983936*root(z^6 - (60865*z^4)/8626
729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/77003
63386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880
007/185583791958607219605834030755606257729536, z, k)^3)/44521 + (253679492
45952*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
040617647997585731147792384 - 880007/18558379195860721960583403075560625772
9536, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^4)/862672910613
8688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607
884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/1855
83791958607219605834030755606257729536, z, k)^5 - 971/22353858477636624)*ro
ot(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/66018204617694487047
4752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/242506004061764
7997585731147792384 - 880007/185583791958607219605834030755606257729536, z,
 k), k, 1, 6) - (x^2/844 - x/3798 + x^3/5697 - x^4/153819 + x^5/136728 + 4/
17091)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

3.156 $\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=873

$$\frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}-\frac{\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)+}{157464\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6+3\right.}$$

```
[Out] 1/157464*((-6)^(1/3)*(2*(-3)^(1/3)+9*2^(1/3))-3*x)/(8-9*I*2^(1/3)*3^(1/6)+3
*2^(1/3)*3^(2/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/157464*(-(-6)^(1/3)*(9*(
-2)^(1/3)+2*3^(1/3))-3*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-
2)^(2/3)*3^(1/3)*x+x^2)+1/104976*(-2*2^(1/3)+3*6^(2/3)+3^(1/3)*x)/(9*2^(1/3
)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/139968*I*ln(6-3*(-3)^(1/3)*2^(2/
3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/3779136*ln(6+3*2^(2/3)*3^(1/3)
*x+x^2)*2^(1/3)*3^(2/3)+1/78732*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-
3)^(2/3)*2^(1/3))^(1/2))/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^
(1/2)-1/78732*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^
(1/2))/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)*3^(1/2)+1/279936*ln(
6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*(3^(1/2)+I)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5-
1/314928*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))/(-4+3*2^(1/3)*3^(2/3))^(3/2)*6^(1/2)-1/209952*arctan((3*(-3)^(1/3)*2^(2/
3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(9*I-3^(1/3)*(2*I*2^(2/3)+9*3^(1/
6)+2*2^(2/3)*3^(1/2)))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)+1/20
9952*(9*I+3^(1/3)*(4*I*2^(2/3)-9*3^(1/6)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x
)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3
))^(1/2)+1/2834352*(2*2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)
*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]
time = 1.77, antiderivative size = 873, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2122, 652, 632, 210, 648, 642, 212}

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3
)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*
```

```
2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/
3) - 3^(1/3)*x)/(104976*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x +
x^2)) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
- ((9*I - 3^(1/3)*((2*I)*2^(2/3) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(
3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(209952*(1
 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)
*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*
I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((4*I)*2^(
2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3
))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^
(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]])/(944784*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) +
((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3^(5/6
)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(629856*2^(2/3)*
3^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = 1586874322944 \int \left(-\frac{9(-2)^{2/3}-\sqrt[3]{-1}\,3}{27763953154228224\;2^{2/3}\left(1+\sqrt[3]{-1}\right)^4 (}\right.$$

$$= \frac{\int \frac{18-2\;2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}\;3^{2/3}x}{6+3\;2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{1889568} - \frac{\int \frac{9\;2^{2/3}-\sqrt[3]{-1}\;3^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{157464\;2^{2/3}} - $$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 167, normalized size = 0.19

$$\frac{972-3942x+648x^2+96x^3-27x^4+4x^5}{3691656\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{1971\log(x-\#1)-162\log(x-\#1)\#1+72\log(x-\#1)\#1^2-27\log(x-\#1)\#1^3+2\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{11074968}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2
+ 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 - 27*
Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/11074968
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 122, normalized size = 0.14

method	result
default	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{(2\_R^4-27\_R^3+}{\_R^5+1}\right.}{11074968}$
risch	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{(2\_R^4-27\_R^3+}{\_R^5+1}\right.}{11074968}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6+
18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R+1
971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3
+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4
+ 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2 -
 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.29, size = 112, normalized size = 0.13

RootSum(...) [illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 906388465775544244426251149770752*_t**4 - 4300873166389987741684137984*_t**3 - 717000908921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17257935592810449901409556597891882995604001083339368041361480613888*_t**5/154206009791052044490694380303237521 + 2389607400620985524376358853572652207181956324560587684052992*_t**4/154206009791052044490694380303237521 - 12286072160883283930711715948878260078996992193488388096*_t**3/154206009791052044490694380303237521 - 59490553573959173161125496013527909754156558410752*_t**2/154206009791052044490694380303237521 - 17520149679836691112367064197713753004827200*_t/154206009791052044490694380303237521 + x + 766422988707229615055855287040887332/154206009791052044490694380303237521))) + (4*x**5 - 27*x**4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196096544*x**3 + 398698848*x**2 + 797397696)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

Mupad [B]
time = 2.42, size = 387, normalized size = 0.44

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k))/30663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)*x)/6623365474855296 + (6603523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^2*x)/584204562 - (1762321104*root(z^6 - (29

2589*z^4)/414082997094657024 - (11805253*z^3)/35209709129437059758653444 - (
2479189*z^2)/44354093106861417422692842209228 + (1989787*z)/1885726687584283
0829226645405233577984 - 7197829/128275517001789310191552482058275045342655
2832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589*z^4)/4140829970
94657024 - (11805253*z^3)/35209709129437059758653444 - (2479189*z^2)/4435409
310686141742269284220928 + (1989787*z)/188572668758428308292266454052335779
84 - 7197829/1282755170017893101915524820582750453426552832, z, k)^4*x)/211
 - 6940988288557056*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*
z^3)/35209709129437059758653444 - (2479189*z^2)/4435409310686141742269284220
928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/12827551
70017893101915524820582750453426552832, z, k)^5*x + (166697*root(z^6 - (292
589*z^4)/414082997094657024 - (11805253*z^3)/35209709129437059758653444 - (2
479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830
829226645405233577984 - 7197829/1282755170017893101915524820582750453426552
832, z, k)^2)/43274412 + (639193032*root(z^6 - (292589*z^4)/414082997094657
024 - (11805253*z^3)/35209709129437059758653444 - (2479189*z^2)/443540931068
6141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 -
7197829/1282755170017893101915524820582750453426552832, z, k)^3)/44521 - (9
815247601920*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/35
209709129437059758653444 - (2479189*z^2)/4435409310686141742269284220928 + (
1989787*z)/18857266875842830829226645405233577984 - 7197829/128275517001789
3101915524820582750453426552832, z, k)^4)/211 - 168897381688221696*root(z^6
 - (292589*z^4)/414082997094657024 - (11805253*z^3)/35209709129437059758653
44 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/1885726687
5842830829226645405233577984 - 7197829/128275517001789310191552482058275045
3426552832, z, k)^5 + 661/28970600587017064704)*root(z^6 - (292589*z^4)/414
082997094657024 - (11805253*z^3)/35209709129437059758653444 - (2479189*z^2)/
4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405
233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k),
k, 1, 6) + (x^2/5697 - (73*x)/68364 + (4*x^3)/153819 - x^4/136728 + x^5/922
914 + 1/3798)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

3.157 $\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=986

$$-\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\,\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}-\frac{27\,2^{2/3}\left(1+\sqrt[3]{-2}\,3^{2/3}\right)-\sqrt[3]{\cdots}}{472392\,2^{2/3}\left(8+9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{\cdots}\right.}$$

```
[Out] 1/209952*(-27*(-2)^(2/3)-54*(-1)^(1/3)*3^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3
))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(
2/3)*x+x^2)+1/944784*(-27*2^(2/3)-54*(-1)^(1/3)*3^(2/3)+(-1)^(1/3)*3^(2/3)*
(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/1889568*(54-9*2^(2/3)*3^(1/3)-(2-3*2^(1
/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*
x+x^2)+1/104976*(3*(-3)^(2/3)+(-1)^(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1
/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)
^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/104976*(2^(2/3)-3*3^(2/3))*arcta
nh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-
(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/1259712*I*ln(
6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/11337408*l
n(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/52488*arctan((3*(-3)^(1/3)*2
^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*(1+3*2^(1/3)*3^(2/3)+I*3^(1/2
))*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3
))^(3/2)+1/2519424*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*(3^(1/2)+I)*2^(2/3)*3^(
5/6)/(1+(-1)^(1/3))^5+1/104976*I*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(1
2-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(
2/3)*2^(1/3))^(1/2)+1/209952*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^
(1/3)*3^(2/3))^(1/2))*(3^(1/2)+I)*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2
)^(1/3)*3^(2/3))^(1/2)-1/944784*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+
9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Rubi [A]
time = 1.95, antiderivative size = 986, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2122, 652, 632, 210, 648, 642, 212}

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -1/104976*(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3)
*2^(2/3))*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*
(-3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 + (-2)^(1/3)*3^(2/3)) - (-1)^
(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(
6 - 2^(2/3)*3^(1/3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(
4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3] +
 3*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^
(2/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/
3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3
))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])
/(17496*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(
2/3))^(3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4
 + 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[
4 + 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(
1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1
/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^
(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^
(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
 - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((I + Sqrt[3])*
Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/3)*3^(1/6)*(1 + (-1)^(1
/3))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/
6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3
)*3^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2122

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = 1586874322944 \int \left(\frac{2\sqrt[3]{-1}\,3^{2/3}+18\sqrt[3]{6}+3}{55527906308456448\;2^{2/3}\left(1+\sqrt[3]{-1}\right)^4 (\cdots}\right.$$

$$= \frac{\int \frac{2\sqrt[3]{-1}\,3^{2/3}+18(-1)^{2/3}\sqrt[3]{6}+3\;2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}dx}{314928\;2^{2/3}} + \frac{\int \frac{-2+6\sqrt[3]{2}\,3^{2/3}+2^{2/3}\sqrt[3]{3}\cdots}{\left(6+3\;2^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\cdots}{104976\;2^{2/3}\sqrt[3]{3}\cdots}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\cdots}{104976\;2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2\cdots\right.}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\cdots}{104976\;2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2\cdots\right.}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\cdots}{104976\;2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 167, normalized size = 0.17

$$\frac{-7884+324x-2724x^2-216x^3+8x^4-9x^5}{7383312(216+108x^2+324x^3+18x^4+x^6)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{324\log(x-\#1)+2436\log(x-\#1)\#1+324\log(x-\#1)\#1^2-16\log(x-\#1)\#1^3+9\log(x-\#1)\#1^4}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]}{44299872}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x^
2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
+ #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 -
16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^
5) & ]/44299872
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 122, normalized size = 0.12

method	result
default	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R\right.}{\_R^5}\right.}{44299872}$
risch	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R\right.}{\_R^5}\right.}{44299872}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)/
(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^4+16*_R^3-324*_R^2-2
436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+3
24*_Z^3+108*_Z^2+216))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x^
4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*x^
2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.25, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(8658597397620778437929792538933565560629231616t^6+109068095871770168248838645612544t^4-492655707593366915713499136t^3+40378331745144603648t^2-695635011360t+4513,\left(t\mapsto t\log\left(\frac{101442531561804181113161287039859349851881619653631712165888t^5}{356900697070792948475845}-\frac{149796550082359335112709434971975088967050210050048t^4}{356900697070792948475845}+\frac{12224097544582728185058987776867078361723699 2t^3}{356900697070792948475845}-\frac{5775055524251595723022901938558261453824t^2}{356900697070792948475845}+\frac{961652422002602657656039304704 32t}{713801394141585896951 69}+x-\frac{17059152341129698120545584}{1070702091212378845427535}\right)\right)\right)+\frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312x^6+132899616x^4+2392193088x^3+797397696x^2+1594795392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(865859739762077843792979253893356556062923161 6*_t**6 + 109068095871
770168248838645612544*_t**4 - 492655707593366915713499136*_t**3 + 403783317
45144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(1014425315618
04181113161287039859349851881619653631712165888*_t**5/3569006970707929484 75
845 - 1497965500823593351127094349719750889670502100 50048*_t**4/35690069707
0792948475845 + 122240975445827281850589877768670783617236992*_t**3/356900
697070792948475845 - 57750555242515957230229019385582614 53824*_t**2/3569006
97070792948475845 + 961652422002602657656039304 70432*_t/7138013941415858969
5169 + x - 170591523411296981205455 84/107070209121237884542753 5))) + (-9*x*
*5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6 + 13289961
6*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B]
time = 2.48, size = 388, normalized size = 0.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((4897*x)/18772949180387057928192 - (8147*root(z^6 + (163*z^4)/12
940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/11
0885232767153543556732105523 2 - (505*z)/628575562528094360974221513507785 93
28 + 4513/865859739762077843792979253893356556062923161 6, z, k))/1103894245
809216 - (1197643*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/14
2599321974220092022546432 + (5171*z^2)/110885232767153543556732105523 2 - (5
05*z)/62857556252809436097422151350778593 28 + 4513/865859739762077843792979
253893356556062923161 6, z, k)*x)/29805144636848832 + (452809*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/110885232767153543556732105523 2 - (505*z)/628575562528094360974221 51
35077859328 + 4513/865859739762077843792979253893356556062923161 6, z, k)^2*
x)/194734854 - (1241776944*root(z^6 + (163*z^4)/12940093659208032 - (811359
7*z^3)/142599321974220092022546432 + (5171*z^2)/110885232767153543556732105
5232 - (505*z)/62857556252809436097422151350778593 28 + 4513/865859739762077
```

```
8437929792538933565560629231616, z, k)^3*x)/44521 + (452407928832*root(z^6
+ (163*z^4)/12940093659208032 - (8113597*z^3)/1425993219742200920225464 32 +
 (5171*z^2)/1108852327671535435567321055232 - (505*z)/628575562528094360974
2215135077859328 + 4513/8658597397620778437929792538933565560629231616, z,
k)^4*x)/211 - 6940988288557056*root(z^6 + (163*z^4)/12940093659208032 - (81
13597*z^3)/14259932197422009202254 6432 + (5171*z^2)/110885232767153543556 73
21055232 - (505*z)/6285755625280943609742215135077859328 + 4513/86585973976
20778437929792538933565560629231616, z, k)^5*x + (114155*root(z^6 + (163*z^
4)/12940093659208032 - (8113597*z^3)/14259932197422009202254 6432 + (5171*z^
2)/1108852327671535435567321055232 - (505*z)/62857556252809436097422151350 7
7859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2)/292
102281 - (163984176*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/
14259932197422009202254 6432 + (5171*z^2)/1108852327671535435567321055232 -
(505*z)/6285755625280943609742215135077859328 + 4513/865859739762077843792 9
79253893356556062923 1616, z, k)^3)/44521 + (94281884928*root(z^6 + (163*z^4
)/12940093659208032 - (8113597*z^3)/14259932197422009202254 6432 + (5171*z^2
)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077
859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^4)/211
- 1688973816882216 96*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)
/14259932197422009202254 6432 + (5171*z^2)/1108852327671535435567321055232 -
 (505*z)/6285755625280943609742215135077859328 + 4513/865859739762077843792
9792538933565560629231616, z, k)^5 + 1/19313733724678043136)*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/14259932197422009202254 6432 + (517
1*z^2)/1108852327671535435567321055232 - (505*z)/62857556252809436097422151
35077859328 + 4513/8658597397620778437929792538933565560629231616, z, k), k
, 1, 6) - ((227*x^2)/615276 - x/22788 + x^3/34182 - x^4/922914 + x^5/820368
 + 73/68364)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

3.158 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx} dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1600}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} dx &= \int \left(a^2 + 2abx^2 + b^2x^4\right) dx\\ &= a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A]
time = 0.19, size = 22, normalized size = 0.88

method	result	size
default	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
norman	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
gosper	$\frac{x\left(3b^2x^4+10ab\,x^2+15a^2\right)}{15}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x,m
ethod=_RETURNVERBOSE)
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="fricas")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.88

$$a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c),x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Giac [A]
time = 4.24, size = 21, normalized size = 0.84

$$\frac{1}{5} b^2 x^5 + \frac{2}{3} abx^3 + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B]
time = 0.04, size = 21, normalized size = 0.84

$$a^2 x + \frac{2 a b x^3}{3} + \frac{b^2 x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x),x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.159 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{bc(bc^2+2ad^2)\,x}{d^4}+\frac{b(bc^2+2ad^2)\,x^2}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+\frac{(bc^2+ad^2)^2\log(c+dx)}{d^5}$$

```
[Out] -b*c*(2*a*d^2+b*c^2)*x/d^4+1/2*b*(2*a*d^2+b*c^2)*x^2/d^3-1/3*b^2*c*x^3/d^2+
1/4*b^2*x^4/d+(a*d^2+b*c^2)^2*ln(d*x+c)/d^5
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {1600, 28, 711}

$$\frac{(ad^2+bc^2)^2\log(c+dx)}{d^5}-\frac{bcx(2ad^2+bc^2)}{d^4}+\frac{bx^2(2ad^2+bc^2)}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x)^2,x]
```

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2
*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \int \frac{a^2+2abx^2+b^2x^4}{c+dx}\,dx$$
$$= \frac{\int \frac{(ab+b^2x^2)^2}{c+dx}\,dx}{b^2}$$
$$= \frac{\int \left(-\frac{b^3c(bc^2+2ad^2)}{d^4} + \frac{b^3(bc^2+2ad^2)x}{d^3} - \frac{b^4cx^2}{d^2} + \frac{b^4x^3}{d} + \frac{b^2(}{c}\right.}{b^2}$$
$$= -\frac{bc(bc^2+2ad^2)\,x}{d^4} + \frac{b(bc^2+2ad^2)\,x^2}{2d^3} - \frac{b^2cx^3}{3d^2} + \frac{b^2}{4}$$

Mathematica [A]
time = 0.02, size = 79, normalized size = 0.84

$$\frac{bdx(12ad^2(-2c+dx)+b(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3))+12(bc^2+ad^2)^2\log(c+dx)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x)^2,x]
```

```
[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^
3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

Maple [A]
time = 0.21, size = 96, normalized size = 1.02

method	result	size
default	$-\frac{b\left(-\frac{b\,x^4 d^3}{4}+\frac{bc\,d^2 x^3}{3}-\frac{\left(2a\,d^2+b\,c^2\right)x^2 d}{2}+c\left(2a\,d^2+b\,c^2\right)x\right)}{d^4}+\frac{\left(a^2 d^4+2ab\,c^2 d^2+b^2 c^4\right)\ln\left(dx+c\right)}{d^5}$	96
risch	$\frac{b^2x^4}{4d}-\frac{b^2c\,x^3}{3d^2}+\frac{ba\,x^2}{d}+\frac{b^2c^2x^2}{2d^3}-\frac{2bacx}{d^2}-\frac{b^2c^3x}{d^4}+\frac{\ln(dx+c)a^2}{d}+\frac{2\ln(dx+c)ab\,c^2}{d^3}+\frac{\ln(dx+c)b^2c^4}{d^5}$	114
norman	$\frac{\frac{c\left(2ab\,c^2d^2+b^2c^4\right)}{d^5}+\frac{b^2x^5}{4}+\frac{b\left(6a\,d^2+b\,c^2\right)x^3}{6d^2}-\frac{b^2c\,x^4}{12d}-\frac{bc\left(2a\,d^2+b\,c^2\right)x^2}{2d^3}}{dx+c}+\frac{\left(a^2d^4+2ab\,c^2d^2+b^2c^4\right)\ln\left(dx+c\right)}{d^5}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x
,method=_RETURNVERBOSE)
```

```
[Out] -b/d^4*(-1/4*b*x^4*d^3+1/3*b*c*d^2*x^3-1/2*(2*a*d^2+b*c^2)*x^2*d+c*(2*a*d^2
+b*c^2)*x)+(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)/d^5*ln(d*x+c)
```

Maxima [A]
time = 0.28, size = 105, normalized size = 1.12

$$\frac{3\,b^2d^3x^4-4\,b^2cd^2x^3+6\,(b^2c^2d+2\,abd^3)x^2-12\,(b^2c^3+2\,abcd^2)x}{12\,d^4}+\frac{(b^2c^4+2\,abc^2d^2+a^2d^4)\log\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12*
(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*
x + c)/d^5
```

Fricas [A]
time = 0.40, size = 105, normalized size = 1.12

$$\frac{3\,b^2d^4x^4-4\,b^2cd^3x^3+6\,(b^2c^2d^2+2\,abd^4)x^2-12\,(b^2c^3d+2\,abcd^3)x+12\,(b^2c^4+2\,abc^2d^2+a^2d^4)\log\left(dx+c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 - 1
2*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(
d*x + c))/d^5
```

Sympy [A]
time = 0.13, size = 88, normalized size = 0.94

$$-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+x^2\left(\frac{ab}{d}+\frac{b^2c^2}{2d^3}\right)+x\left(-\frac{2abc}{d^2}-\frac{b^2c^3}{d^4}\right)+\frac{\left(ad^2+bc^2\right)^2\log\left(c+dx\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3))
 + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)/d
**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(88) = 176.
time = 4.62, size = 365, normalized size = 3.88

$$-\frac{1}{12}\,b^2d\left(\frac{(dx+c)^4\left(\frac{20\,c}{dx+c}-\frac{60\,c^2}{(dx+c)^2}+\frac{120\,c^3}{(dx+c)^3}-3\right)}{d^6}+\frac{60\,c^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6}-\frac{12\,c^5}{(dx+c)d^6}\right)-\frac{1}{3}\,b^2c\left(\frac{(dx+c)^3\left(\frac{6\,c}{dx+c}-\frac{18\,c^2}{(dx+c)^2}-1\right)}{d^5}-\frac{12\,c^3\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5}+\frac{3\,c^4}{(dx+c)d^5}\right)-abd\left(\frac{(dx+c)^2\left(\frac{6\,c}{dx+c}-1\right)}{d^4}+\frac{6\,c^2\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4}-\frac{2\,c^3}{(dx+c)d^4}\right)+2\,abc\left(\frac{2\,c\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3}+\frac{dx+c}{d^3}-\frac{c^2}{(dx+c)d^3}\right)-a^2\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)-\frac{a^2c}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="giac")
```

```
[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d*
x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 12*
c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d*x
+ c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4
/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(
abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a*b*c*(
2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x
+ c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d
)) - a^2*c/((d*x + c)*d)
```

Mupad [B]
time = 0.06, size = 106, normalized size = 1.13

$$x^2\left(\frac{b^2\,c^2}{2\,d^3}+\frac{a\,b}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4\right)}{d^5}+\frac{b^2\,x^4}{4\,d}-\frac{b^2\,c\,x^3}{3\,d^2}-\frac{c\,x\,\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x)^2,x)
```

```
[Out] x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a*
b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)/d
^3 + (2*a*b)/d))/d
```

3.160 $\int (b+2cx) (bx+cx^2)^{13} \, dx$

Optimal. Leaf size=15

$$\frac{1}{14}(bx+cx^2)^{14}$$

[Out] 1/14*(c*x^2+b*x)^14

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {643}

$$\frac{1}{14}(bx+cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b*x + c*x^2)^14/14

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx) (bx+cx^2)^{13} \, dx = \frac{1}{14}(bx+cx^2)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(15) = 30.
time = 0.00, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c

```
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [A]
time = 0.19, size = 14, normalized size = 0.93

method	result
gosper	$\frac{(cx+b)^{14}x^{14}}{14}$
default	$\frac{(c\,x^2+bx)^{14}}{14}$
norman	$b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^{17}+\frac{143}{2}x^{18}b^{10}c^4+143b^9c^5x^{19}+$
risch	$b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^{17}+\frac{143}{2}x^{18}b^{10}c^4+143b^9c^5x^{19}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x,method=_RETURNVERBOSE)
[Out] 1/14*(c*x^2+b*x)^14
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
[Out] 1/14*(c*x^2 + b*x)^14
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(13) = 26$.
time = 0.37, size = 154, normalized size = 10.27

$$\frac{1}{14}c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}b^{10}c^4x^{18}+26\,b^{11}c^3x^{17}+\frac{13}{2}b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.
time = 0.04, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143b^5c^9x^{23}+\frac{143b^4c^{10}x^{24}}{2}+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Giac [A]
time = 4.69, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

Mupad [B]
time = 2.22, size = 154, normalized size = 10.27

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

3.161 $\int x^{14}(b+2cx^2)\left(bx+cx^3\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] 1/28*x^28*(c*x^2+b)^14
```

Rubi [A]
time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 457, 75}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx=\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)$$
$$=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.
time = 0.00, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.
time = 0.22, size = 157, normalized size = 9.81

method	result
gosper	$\frac{x^{28}\left(c\,x^2+b\right)^{14}}{28}$
default	$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}x^{52}b^2c^{12}+\frac{143}{2}x^{38}b^9c^5+\frac{429}{4}x^{40}b^8c^6+\frac{858}{7}x^{42}b^7c^7+\frac{429}{4}x^{44}b^6c^8+\frac{143}{2}x^{46}b^5c$
risch	$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}x^{52}b^2c^{12}+\frac{143}{2}x^{38}b^9c^5+\frac{429}{4}x^{40}b^8c^6+\frac{858}{7}x^{42}b^7c^7+\frac{429}{4}x^{44}b^6c^8+\frac{143}{2}x^{46}b^5c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*x^52*b^2*c^12+143/2*x^38*b^9*c^5+429/4*
x^40*b^8*c^6+858/7*x^42*b^7*c^7+429/4*x^44*b^6*c^8+143/2*x^46*b^5*c^9+143/4
*x^48*b^4*c^10+13*x^50*b^3*c^11+1/2*x^30*b^13*c+13/4*x^32*b^12*c^2+13*x^34*
b^11*c^3+143/4*x^36*b^10*c^4+1/28*x^28*b^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.

time = 0.26, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}+\frac{143}{2}b^9c^5x^{38}+\frac{143}{4}b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}b^{12}c^2x^{32}+\frac{1}{2}b^{13}cx^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.39, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}+\frac{143}{2}b^9c^5x^{38}+\frac{143}{4}b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}b^{12}c^2x^{32}+\frac{1}{2}b^{13}cx^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.
time = 0.04, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 4.34, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}+\frac{143}{2}b^9c^5x^{38}+\frac{143}{4}b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}b^{12}c^2x^{32}+\frac{1}{2}b^{13}cx^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Mupad [B]
time = 0.14, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c^8\,x^{44}}{4}+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}+13\,b^3\,c^{11}\,x^{50}+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

3.162 $\int x^{28}(b+2cx^3)\left(bx+cx^4\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A]
time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 457, 75}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx=\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$$
$$=\frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)$$
$$=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 186 vs. $2(16) = 32$.
time = 0.00, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.21, size = 157, normalized size = 9.81

method	result
gosper	$\frac{x^{42}\left(c\,x^3+b\right)^{14}}{42}$
default	$\frac{26}{3}x^{75}b^3c^{11}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{1}{42}c^{14}x^{84}+\frac{143}{2}x^{66}b^6c^8+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{143}{2}x^{60}b^8c$
risch	$\frac{26}{3}x^{75}b^3c^{11}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{1}{42}c^{14}x^{84}+\frac{143}{2}x^{66}b^6c^8+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{143}{2}x^{60}b^8c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 26/3*x^75*b^3*c^11+13/6*x^78*b^2*c^12+1/3*b*c^13*x^81+1/42*c^14*x^84+143/2*
x^66*b^6*c^8+143/3*x^69*b^5*c^9+143/6*x^72*b^4*c^10+143/2*x^60*b^8*c^6+572/
7*x^63*b^7*c^7+143/3*x^57*b^9*c^5+26/3*x^51*b^11*c^3+143/6*x^54*b^10*c^4+13
/6*x^48*b^12*c^2+1/42*x^42*b^14+1/3*x^45*b^13*c
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

time = 0.28, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.
time = 0.36, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(12) = 24.
time = 0.04, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^{2}x^{48}}{6}+\frac{26b^{11}c^{3}x^{51}}{3}+\frac{143b^{10}c^{4}x^{54}}{6}+\frac{143b^{9}c^{5}x^{57}}{3}+\frac{143b^{8}c^{6}x^{60}}{2}+\frac{572b^{7}c^{7}x^{63}}{7}+\frac{143b^{6}c^{8}x^{66}}{2}+\frac{143b^{5}c^{9}x^{69}}{3}+\frac{143b^{4}c^{10}x^{72}}{6}+\frac{26b^{3}c^{11}x^{75}}{3}+\frac{13b^{2}c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.
time = 5.30, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Mupad [B]
time = 2.17, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

3.163 $\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1598, 457, 75}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^{14(-1+n)}(b+2cx^n)\left(bx+cx^{1+n}\right)^{13}\,dx=\int x^{13+14(-1+n)}(b+cx^n)^{13}\,(b+2cx^n)\,dx$$
$$=\frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}$$
$$=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.
time = 0.21, size = 230, normalized size = 10.95

method	result
risch	$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13b^{2}c^{12}x^{26n}}{2n}+\frac{26b^{3}c^{11}x^{25n}}{n}+\frac{143b^{4}c^{10}x^{24n}}{2n}+\frac{143b^{5}c^{9}x^{23n}}{n}+\frac{429b^{6}c^{8}x^{22n}}{2n}+\frac{1716b^{7}c^{7}x^{21n}}{7n}+$ 4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.
time = 0.28, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^{2}c^{12}x^{26\,n}}{2\,n}+\frac{26\,b^{3}c^{11}x^{25\,n}}{n}+\frac{143\,b^{4}c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^{5}c^{9}x^{23\,n}}{n}+\frac{429\,b^{6}c^{8}x^{22\,n}}{2\,n}+\frac{1716\,b^{7}c^{7}x^{21\,n}}{7\,n}+\frac{429\,b^{8}c^{6}x^{20\,n}}{2\,n}+\frac{143\,b^{9}c^{5}x^{19\,n}}{n}+\frac{143\,b^{10}c^{4}x^{18\,n}}{2\,n}+\frac{26\,b^{11}c^{3}x^{17\,n}}{n}+\frac{13\,b^{12}c^{2}x^{16\,n}}{2\,n}+\frac{b^{13}cx^{15\,n}}{n}+\frac{b^{14}x^{14\,n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(19) = 38$.
time = 0.37, size = 262, normalized size = 12.48

$$\frac{b^{14}x^{14}x^{14n+14} + 14\,b^{13}cx^{13}x^{15n+15} + 91\,b^{12}c^2x^{12}x^{16n+16} + 364\,b^{11}c^3x^{11}x^{17n+17} + 1001\,b^{10}c^4x^{10}x^{18n+18} + 2002\,b^9c^5x^9x^{19n+19} + 3003\,b^8c^6x^8x^{20n+20} + 3432\,b^7c^7x^7x^{21n+21} + 3003\,b^6c^8x^6x^{22n+22} + 2002\,b^5c^9x^5x^{23n+23} + 1001\,b^4c^{10}x^4x^{24n+24} + 364\,b^3c^{11}x^3x^{25n+25} + 91\,b^2c^{12}x^2x^{26n+26} + 14\,bc^{13}xx^{27n+27} + c^{14}x^{28n+28}}{14\,nx^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2*
x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10*x
^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n +
20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 200
2*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^1
1*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n +
 27) + c^14*x^(28*n + 28))/(n*x^28)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

```
[Out] integrate((b*x + c*x^(n + 1))^13*(2*c*x^n + b)*x^(14*n - 14), x)
```

Mupad [B]
time = 4.02, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.164 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal. Leaf size=10

$$\log \left(bx+cx^2\right)$$

```
[Out] ln(c*x^2+b*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {642}

$$\log \left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.90

$$\log(x)+\log(b+cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

Maple [A]
time = 0.19, size = 9, normalized size = 0.90

method	result	size
default	$\ln(x(cx+b))$	9
norman	$\ln(x)+\ln(cx+b)$	10
derivativedivides	$\ln(c\,x^2+bx)$	11
risch	$\ln(c\,x^2+bx)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x*(c*x+b))
```

Maxima [A]
time = 0.28, size = 10, normalized size = 1.00

$$\log(cx^2+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
[Out] log(c*x^2 + b*x)
```

Fricas [A]
time = 0.37, size = 10, normalized size = 1.00

$$\log(cx^2+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
[Out] log(c*x^2 + b*x)
```

Sympy [A]
time = 0.04, size = 8, normalized size = 0.80

$$\log(bx+cx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
[Out] log(b*x + c*x**2)
```

Giac [A]
time = 4.37, size = 11, normalized size = 1.10

$$\log(|cx^2+bx|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] log(abs(c*x^2 + b*x))
```

Mupad [B]
time = 0.05, size = 8, normalized size = 0.80

$$\ln\left(x\left(b+c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
```

```
[Out] log(x*(b + c*x))
```

3.165 $\int \frac{b+2cx^2}{bx+cx^3} \, dx$

Optimal. Leaf size=15

$$\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

[Out] ln(x)+1/2*ln(c*x^2+b)

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1607, 457, 78}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(b*x + c*x^3),x]

[Out] Log[x] + Log[b + c*x^2]/2

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+2cx^2}{bx+cx^3}\,dx &= \int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx,x,x^2\right) \\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.00, size = 15, normalized size = 1.00

$$\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [**A**]
time = 0.19, size = 14, normalized size = 0.93

method	result	size
default	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
norman	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14
risch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{2}+b\right)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/(c*x^3+b*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Maxima [**A**]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A]
time = 0.07, size = 12, normalized size = 0.80

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)
[Out] log(x) + log(b/c + x**2)/2
```

Giac [A]
time = 4.67, size = 18, normalized size = 1.20

$$\frac{1}{2}\,\log\left(x^2\right)+\frac{1}{2}\,\log\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

Mupad [B]
time = 2.08, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(b*x + c*x^3),x)
[Out] log(b + c*x^2)/2 + log(x)
```

3.166 $\int \frac{b+2cx^3}{bx+cx^4}\,dx$

Optimal. Leaf size=15

$$\log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1607, 457, 78}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{b+2cx^3}{bx+cx^4}\,dx &= \int \frac{b+2cx^3}{x\,(b+cx^3)}\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx,x,x^3\right)\\
&= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$
\log(x)+\frac{1}{3}\log\left(b+cx^3\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A]
time = 0.20, size = 14, normalized size = 0.93

method	result	size
default	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
norman	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14
risch	$\ln\left(x\right)+\frac{\ln\left(c\,x^{3}+b\right)}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/(c*x^4+b*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$
\frac{1}{3}\,\log\left(cx^3+b\right)+\log\left(x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A]
time = 0.08, size = 12, normalized size = 0.80

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/(c*x**4+b*x),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

Giac [A]
time = 4.16, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

Mupad [B]
time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(b*x + c*x^4),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

3.167 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

Optimal. Leaf size=15

$$\log(x)+\frac{\log\left(b+cx^n\right)}{n}$$

[Out] ln(x)+ln(b+c*x^n)/n

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1607, 457, 78}

$$\frac{\log\left(b+cx^n\right)}{n}+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]

[Out] Log[x] + Log[b + c*x^n]/n

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx = \int \frac{b+2cx^n}{x(b+cx^n)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^n\right)}{n}$$
$$= \log(x) + \frac{\log(b+cx^n)}{n}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.27

$$\frac{\log(x^n)+\log(n(b+cx^n))}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

```
[Out] (Log[x^n] + Log[n*(b + c*x^n)])/n
```

Maple [A]
time = 0.21, size = 18, normalized size = 1.20

method	result	size
norman	$\ln(x)+\frac{\ln\left(c\,e^{n\ln(x)}+b\right)}{n}$	18
risch	$\ln(x)+\frac{\ln\left(x^n+\frac{b}{c}\right)}{n}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.29, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A]
time = 0.40, size = 23, normalized size = 1.53

$$\frac{(n-1)\log(x) + \log(bx + cx^{n+1})}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")
[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.61, size = 29, normalized size = 1.93

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge (c = 0 \vee n = 0) \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c} + x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
[Out] Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
+ c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{b + 2\,c\,x^n}{b\,x + c\,x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)
[Out] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)
```

3.168 $\int \frac{b+2cx}{(bx+cx^2)^8} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

```
[Out] -1/7/(c*x^2+b*x)^7
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{643\}$

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(b*x + c*x^2)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} \, dx = -\frac{1}{7\left(bx+cx^2\right)^7}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(13) = 26$.
time = 0.20, size = 177, normalized size = 11.80

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
derivativedivides	$-\frac{1}{7(c x^2+bx)^7}$
default	$\frac{132c^7}{b^{13}(cx+b)} + \frac{66c^7}{b^{12}(cx+b)^2} + \frac{30c^7}{b^{11}(cx+b)^3} + \frac{12c^7}{b^{10}(cx+b)^4} + \frac{4c^7}{b^9(cx+b)^5} + \frac{c^7}{b^8(cx+b)^6} + \frac{c^7}{7b^7(cx+b)^7} - \frac{1}{7b^7x^7} - \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7-1/7
/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b
^9*c^2/x^5+1/b^8*c/x^6
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.
time = 0.39, size = 81, normalized size = 5.40

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.
time = 0.48, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Giac [**A**]
time = 4.61, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2 + bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Mupad [**B**]
time = 4.38, size = 12, normalized size = 0.80

$$-\frac{1}{7\,x^7\,\left(b + c\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

3.169 $\int \frac{b+2cx^2}{x^7(bx+cx^3)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 457, 75}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx = \int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)$$
$$= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(14) = 28$.
time = 0.21, size = 197, normalized size = 12.31

method	result
gosper	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
norman	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
risch	$-\frac{1}{14x^{14}\left(c\,x^2+b\right)^7}$
default	$-\frac{c^8\left(-\frac{4b^4}{c\left(c\,x^2+b\right)^5}-\frac{30b^2}{c\left(c\,x^2+b\right)^3}-\frac{132}{c\left(c\,x^2+b\right)}-\frac{66b}{c\left(c\,x^2+b\right)^2}-\frac{b^6}{7c\left(c\,x^2+b\right)^7}-\frac{b^5}{c\left(c\,x^2+b\right)^6}-\frac{12b^3}{c\left(c\,x^2+b\right)^4}\right)}{2b^{13}}-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c^8/b^13*(-4*b^4/c/(c*x^2+b)^5-30/c*b^2/(c*x^2+b)^3-132/c/(c*x^2+b)-66
*b/c/(c*x^2+b)^2-1/7*b^6/c/(c*x^2+b)^7-b^5/c/(c*x^2+b)^6-12*b^3/c/(c*x^2+b)
^4)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

time = 0.30, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.37, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.
time = 0.70, size = 87, normalized size = 5.44

$$-\frac{1}{14b^{7}x^{14}+98b^{6}cx^{16}+294b^{5}c^{2}x^{18}+490b^{4}c^{3}x^{20}+490b^{3}c^{4}x^{22}+294b^{2}c^{5}x^{24}+98bc^{6}x^{26}+14c^{7}x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

Giac [A]
time = 4.76, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^{4}+bx^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

Mupad [B]
time = 2.22, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\,{\left(c\,x^2+b\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

3.170 $\int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 457, 75}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx = \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)$$
$$= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(14) = 28$.
time = 0.20, size = 197, normalized size = 12.31

method	result
gosper	$-\frac{1}{21x^{21}\left(c\,x^3+b\right)^7}$
risch	$-\frac{1}{21x^{21}\left(c\,x^3+b\right)^7}$
default	$-\frac{c^8\left(-\frac{4b^4}{c\left(c\,x^3+b\right)^5}-\frac{30b^2}{c\left(c\,x^3+b\right)^3}-\frac{132}{c\left(c\,x^3+b\right)}-\frac{66b}{c\left(c\,x^3+b\right)^2}-\frac{b^6}{7c\left(c\,x^3+b\right)^7}-\frac{b^5}{c\left(c\,x^3+b\right)^6}-\frac{12b^3}{c\left(c\,x^3+b\right)^4}\right)}{3b^{13}}-\frac{1}{21b^7x^{21}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c^8/b^13*(-4*b^4/c/(c*x^3+b)^5-30/c*b^2/(c*x^3+b)^3-132/c/(c*x^3+b)-66
*b/c/(c*x^3+b)^2-1/7*b^6/c/(c*x^3+b)^7-b^5/c/(c*x^3+b)^6-12*b^3/c/(c*x^3+b)
^4)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.30, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.39, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.
time = 0.92, size = 87, normalized size = 5.44

$$-\frac{1}{21b^{7}x^{21}+147b^{6}cx^{24}+441b^{5}c^{2}x^{27}+735b^{4}c^{3}x^{30}+735b^{3}c^{4}x^{33}+441b^{2}c^{5}x^{36}+147bc^{6}x^{39}+21c^{7}x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

Giac [A]
time = 3.85, size = 15, normalized size = 0.94

$$-\frac{1}{21\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

Mupad [B]
time = 7.22, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,\left(c\,x^{3}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

3.171 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A]

time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1598, 457, 75}

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8}\,dx = \int \frac{x^{-8-7(-1+n)}(b+2cx^n)}{(b+cx^n)^8}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(21) = 42.
time = 0.24, size = 203, normalized size = 9.67

method	result
risch	$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7(924x^{6n}c^6+6006b\,c^5x^{5n}+16380b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.
time = 0.34, size = 612, normalized size = 29.14

$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13n}+2342340\,bc^{12}x^{12n}+6426420\,b^2c^{11}x^{11n}+9579570\,b^3c^{10}x^{10n}+8270262\,b^4c^9x^{9n}+4018014\,b^5c^8x^{8n}+934362\,b^6c^7x^{7n}+45045\,b^7c^6x^{6n}-5005\,b^8c^5x^{5n}+1001\,b^9c^4x^{4n}-273\,b^{10}c^3x^{3n}+91\,b^{11}c^2x^{2n}-35\,b^{12}cx^n+15\,b^{13}}{b^{14}c^7nx^{14n}+7\,b^{15}c^6nx^{13n}+21\,b^{16}c^5nx^{12n}+35\,b^{17}c^4nx^{11n}+35\,b^{18}c^3nx^{10n}+21\,b^{19}c^2nx^{9n}+7\,b^{20}cnx^{8n}+b^{21}nx^{7n}}+\frac{360360\,c^7\log(x)}{b^{15}}-\frac{360360\,c^7\log\left(\frac{cx^n+b}{c}\right)}{b^{15}n}\right)+\frac{1}{105}c\left(\frac{360360\,c^{12}x^{12n}+2342340\,bc^{11}x^{11n}+6426420\,b^2c^{10}x^{10n}+9579570\,b^3c^9x^{9n}+8270262\,b^4c^8x^{8n}+4018014\,b^5c^7x^{7n}+934362\,b^6c^6x^{6n}+45045\,b^7c^5x^{5n}-5005\,b^8c^4x^{4n}+1001\,b^9c^3x^{3n}-273\,b^{10}c^2x^{2n}+91\,b^{11}cx^n-35\,b^{12}}{b^{13}c^7nx^{13n}+7\,b^{14}c^6nx^{12n}+21\,b^{15}c^5nx^{11n}+35\,b^{16}c^4nx^{10n}+35\,b^{17}c^3nx^{9n}+21\,b^{18}c^2nx^{8n}+7\,b^{19}cnx^{7n}+b^{20}nx^{6n}}+\frac{360360\,c^6\log(x)}{b^{14}}-\frac{360360\,c^6\log\left(\frac{cx^n+b}{c}\right)}{b^{14}n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(21) = 42$.
time = 0.53, size = 143, normalized size = 6.81

$$-\frac{x^{14}}{7\left(b^{7}nx^{7}x^{7n+7}+7b^{6}cnx^{6}x^{8n+8}+21b^{5}c^{2}nx^{5}x^{9n+9}+35b^{4}c^{3}nx^{4}x^{10n+10}+35b^{3}c^{4}nx^{3}x^{11n+11}+21b^{2}c^{5}nx^{2}x^{12n+12}+7bc^{6}nxx^{13n+13}+c^{7}nx^{14n+14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas")
```

```
[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*n
*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(11*
n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c^7*
n*x^(14*n + 14))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{7-7\,n}\,\left(b+2\,c\,x^n\right)}{{\left(b\,x+c\,x^{n+1}\right)}^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)
[Out] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)
```

3.172 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

```
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{643\}$

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx = \frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{1+p}}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A]
time = 0.18, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\left(c\,x^{2}+b x \right)^{1+p}}{1+p}$	20
default	$\frac{\left(c\,x^{2}+b x \right)^{1+p}}{1+p}$	20
risch	$\frac{x(cx+b)(x(cx+b))^{p}}{1+p}$	22
gosper	$\frac{x(cx+b)\left(c\,x^{2}+b x \right)^{p}}{1+p}$	24
norman	$\frac{b x\,{\mathrm e}^{p \ln \left(c\,x^{2}+b x \right)}}{1+p}+\frac{c\,x^{2} {\mathrm e}^{p \ln \left(c\,x^{2}+b x \right)}}{1+p}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 1.00

$$\frac{(cx^2 + bx)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Fricas [A]
time = 0.37, size = 26, normalized size = 1.37

$$\frac{(cx^2 + bx)(cx^2 + bx)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.
time = 0.26, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for}\ p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Giac [A]
time = 3.81, size = 19, normalized size = 1.00

$$\frac{(cx^2+bx)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.10, size = 23, normalized size = 1.21

$$\frac{x\,(c\,x^2+b\,x)^p\,(b+c\,x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

```
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

3.173 $\int x^{1+p}(b+2cx^2)\left(bx+cx^3\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{x^{1+p}(bx+cx^3)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*x^(1+p)*(c*x^3+b*x)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1604}

$$\frac{x^{p+1}(bx+cx^3)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{1+p}\left(b+2cx^2\right)\left(bx+cx^3\right)^p\,dx=\frac{x^{1+p}(bx+cx^3)^{1+p}}{2(1+p)}$$

Mathematica [A]
time = 0.05, size = 27, normalized size = 1.00

$$\frac{x^{1+p}(x(b+cx^2))^{1+p}}{2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(1 + p)*(x*(b + c*x^2))^(1 + p))/(2*(1 + p))
```

Maple [A]
time = 0.24, size = 31, normalized size = 1.15

method	result
gosper	$\frac{x^{2+p}\left(c\,x^{2}+b\right)\left(c\,x^{3}+bx\right)^{p}}{2+2p}$
risch	$\frac{\left(c\,x^{2}+b\right)x\,x^{1+p}\,\mathrm{e}^{\frac{p\left(-i\pi\,\mathrm{csgn}\left(ix\left(c\,x^{2}+b\right)\right)^{3}+i\pi\,\mathrm{csgn}\left(ix\left(c\,x^{2}+b\right)\right)^{2}\mathrm{csgn}(ix)+i\pi\,\mathrm{csgn}\left(ix\left(c\,x^{2}+b\right)\right)^{2}\mathrm{csgn}\left(i\left(c\,x^{2}+b\right)\right)-i\pi\,\mathrm{csgn}\left(ix\left(c\,x^{2}+b\right)\right)\mathrm{csgn}(ix)\mathrm{c}}{2}}}{2+2p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(2+p)*(c*x^2+b)/(1+p)*(c*x^3+b*x)^p
```

Maxima [A]
time = 0.31, size = 35, normalized size = 1.30

$$\frac{(cx^4 + bx^2)e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A]
time = 0.38, size = 32, normalized size = 1.19

$$\frac{(cx^3 + bx)(cx^3 + bx)^p x^{p+1}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.
time = 2.96, size = 54, normalized size = 2.00

$$\frac{cx^3e^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}+bxe^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
+ b) + 2*p*log(x) + log(x)))/(p + 1)
```

Mupad [B]
time = 2.21, size = 45, normalized size = 1.67

$$\left(c\,x^3+b\,x\right)^p\,\left(\frac{b\,x\,x^{p+1}}{2\,p+2}+\frac{c\,x^{p+1}\,x^3}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)
```

```
[Out] (b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))
```

3.174 $\int \left(bx^{1+p}(bx+cx^3)^p+2cx^{3+p}(bx+cx^3)^p\right)\,dx$

Optimal. Leaf size=27

$$\frac{x^{1+p}(bx+cx^3)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*x^(1+p)*(c*x^3+b*x)^(1+p)/(1+p)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 7, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2057, 372, 371}

$$\frac{bx^{p+2}(bx+cx^3)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)}{2(p+1)}+\frac{cx^{p+4}(bx+cx^3)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)}{p+2}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/
b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hypergeom
etric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)\,dx = b\int x^{1+p}\left(bx+cx^3\right)^p\,dx+(2c)\int x^{3+p}\left(bx+cx^3\right)^p\,dx$$

$$= \left(bx^{-p}\left(b+cx^2\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(b+cx^2\right)^p\,dx+($$

$$= \left(bx^{-p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(1+\frac{cx^2}{b}\right)^p\,d$$

$$= \frac{bx^{2+p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\ {}_2F_1\left(-p,1+p;2+p;-\frac{cx^2}{b}\right.}{2(1+p)}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 1.00

$$\frac{x^{1+p}\left(x\left(b+cx^2\right)\right)^{1+p}}{2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
[Out] (x^(1 + p)*(x*(b + c*x^2))^(1 + p))/(2*(1 + p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 142, normalized size = 5.26

method	result
risch	$\frac{(c\,x^2+b)x\,x^{1+p}\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)^3+i\pi\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)^2\mathrm{csgn}(ix)+i\pi\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)^2\mathrm{csgn}\left(i\left(c\,x^2+b\right)\right)-i\pi\,\mathrm{csgn}\left(ix\left(c\,x^2+b\right)\right)\mathrm{csgn}(i}{2}}}{2+2p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x,method=_RETURNVERBO
SE)
[Out] 1/2*(c*x^2+b)*x*x^(1+p)/(1+p)*exp(1/2*p*(-I*Pi*csgn(I*x*(c*x^2+b))^3+I*Pi*c
sgn(I*x*(c*x^2+b))^2*csgn(I*x)+I*Pi*csgn(I*x*(c*x^2+b))^2*csgn(I*(c*x^2+b))
-I*Pi*csgn(I*x*(c*x^2+b))*csgn(I*x)*csgn(I*(c*x^2+b))+2*ln(c*x^2+b)+2*ln(x)
))
```

Maxima [A]
time = 0.31, size = 35, normalized size = 1.30

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="m
axima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A]
time = 0.38, size = 33, normalized size = 1.22

$$\frac{(cx^2+b)(cx^3+bx)^p x^{p+3}}{2\,(p+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="f
ricas")
```

```
[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int xx^p\left(x\left(b+cx^2\right)\right)^p\left(b+2cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

```
[Out] Integral(x*x**p*(x*(b + c*x**2))**p*(b + 2*c*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.
time = 3.80, size = 54, normalized size = 2.00

$$\frac{cx^3e^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}+bxe^{(p\log(cx^2+b)+2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="g
iac")
```

```
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
+ b) + 2*p*log(x) + log(x)))/(p + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int b\,x^{p+1}\,\left(c\,x^3+b\,x\right)^p+2\,c\,x^{p+3}\,\left(c\,x^3+b\,x\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p,x)
```

```
[Out] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)
```

3.175 $\int x^{2(1+p)}(b+2cx^3)\left(bx+cx^4\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*x^(2+2*p)*(c*x^4+b*x)^(1+p)/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1604}

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

Mathematica [A]
time = 0.05, size = 29, normalized size = 1.00

$$\frac{x^{2+2p}\left(x\left(b+cx^3\right)\right)^{1+p}}{3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
[Out] (x^(2 + 2*p)*(x*(b + c*x^3))^(1 + p))/(3*(1 + p))
```

Maple [A]
time = 0.18, size = 33, normalized size = 1.14

method	result
gosper	$\frac{x^{3+2p}\left(c\,x^{3}+b\right)\left(c\,x^{4}+bx\right)^{p}}{3+3p}$
risch	$\frac{\left(c\,x^{3}+b\right)x\,x^{2+2p}\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}\left(ix\left(c\,x^{3}+b\right)\right)^{3}+i\pi\mathrm{csgn}\left(ix\left(c\,x^{3}+b\right)\right)^{2}\mathrm{csgn}(ix)+i\pi\mathrm{csgn}\left(ix\left(c\,x^{3}+b\right)\right)^{2}\mathrm{csgn}\left(i\left(c\,x^{3}+b\right)\right)-i\pi\,\mathrm{csgn}\left(ix\left(c\,x^{3}+b\right)\right)\mathrm{csgn}(ix}{2}}}{3+3p}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x,method=_RETURNVERBOSE)
[Out] 1/3*x^(3+2*p)*(c*x^3+b)/(1+p)*(c*x^4+b*x)^p
```

Maxima [A]
time = 0.31, size = 35, normalized size = 1.21

$$\frac{(cx^{6}+bx^{3})e^{\left(p\log\left(cx^{3}+b\right)+3\,p\log\left(x\right)\right)}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A]
time = 0.41, size = 34, normalized size = 1.17

$$\frac{(cx^{4}+bx)(cx^{4}+bx)^{p}x^{2\,p+2}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(27) = 54.
time = 3.42, size = 58, normalized size = 2.00

$$\frac{cx^4e^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)+2\,\log\left(x\right)\right)}+bxe^{\left(p\log\left(cx^3+b\right)+3\,p\log\left(x\right)+2\,\log\left(x\right)\right)}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x^
3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)
```

Mupad [B]
time = 2.21, size = 49, normalized size = 1.69

$$\left(c\,x^4+b\,x\right)^p\,\left(\frac{c\,x^{2\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{2\,p+2}}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)
```

```
[Out] (b*x + c*x^4)^p*((c*x^(2*p + 2)*x^4)/(3*p + 3) + (b*x*x^(2*p + 2))/(3*p + 3
))
```

3.176 $\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx$

Optimal. Leaf size=36

$$\frac{x^{-((1-n)(1+p))}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (b*x+c*x^(1+n))^(1+p)/n/(1+p)/(x^((1-n)*(1+p)))
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2061}

$$\frac{x^{-((1-n)(p+1))}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rule 2061

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Rubi steps

$$\int x^{(-1+n)(1+p)}(b+2cx^n)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-(1-n)(1+p)}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

Mathematica [A]
time = 0.16, size = 31, normalized size = 0.86

$$\frac{x^{(-1+n)(1+p)}(x(b+cx^n))^{1+p}}{n(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

[Out] (x^((-1 + n)*(1 + p))*(x*(b + c*x^n))^(1 + p))/(n*(1 + p))

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{(-1+n)(1+p)}(b+2c\,x^{n})\left(bx+c\,x^{1+n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
[Out] int(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

Maxima [A]
time = 0.33, size = 39, normalized size = 1.08

$$\frac{(cx^{2\,n}+bx^{n})e^{(np\log(x)+p\log(cx^{n}+b))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="maxi
ma")
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Fricas [A]
time = 0.41, size = 42, normalized size = 1.17

$$\frac{(bx+cx^{n+1})(bx+cx^{n+1})^{p}x^{(n-1)p+n-1}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fric
as")
[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="giac
")
```

```
[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^{(n-1)\,(p+1)}\,{\left(b\,x+c\,x^{n+1}\right)}^p\,\left(b+2\,c\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n),x)
```

```
[Out] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n), x)
```

3.177 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx$

Optimal. Leaf size=32

$$acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1600}

$$acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx &= \int \left(ac+adx+bcx^2+bdx^3\right)\,dx\\ &= acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Maple [A]
time = 0.21, size = 27, normalized size = 0.84

method	result	size
default	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
norman	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
risch	$acx+\frac{1}{2}ad\,x^{2}+\frac{1}{3}bc\,x^{3}+\frac{1}{4}bd\,x^{4}$	27
gosper	$\frac{x\left(3bd\,x^{3}+4bc\,x^{2}+6adx+12ac\right)}{12}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x
,method=_RETURNVERBOSE)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^{4}+\frac{1}{3}\,bcx^{3}+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Fricas [A]
time = 0.37, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^{4}+\frac{1}{3}\,bcx^{3}+\frac{1}{2}\,adx^{2}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A]
time = 0.02, size = 29, normalized size = 0.91

$$acx+\frac{adx^{2}}{2}+\frac{bcx^{3}}{3}+\frac{bdx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

Giac [A]
time = 3.36, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^4+\frac{1}{3}\,bcx^3+\frac{1}{2}\,adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B]
time = 2.14, size = 26, normalized size = 0.81

$$\frac{b\,d\,x^4}{4}+\frac{b\,c\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2),x)
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

3.178 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2
```

Rubi [A]
time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1600}

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx &= \int \frac{ac+adx+bcx^2+bdx^3}{a+bx^2}\,dx\\ &= \int (c+dx)\,dx\\ &= cx+\frac{dx^2}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Maple [A]
time = 0.19, size = 11, normalized size = 0.92

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$cx+\frac{1}{2}d\,x^{2}$	11
risch	$cx+\frac{1}{2}d\,x^{2}$	11
norman	$\frac{acx+bc\,x^{3}-\frac{a^{2}d}{2b}+\frac{bd\,x^{4}}{2}}{b\,x^{2}+a}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2
,x,method=_RETURNVERBOSE)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^{2}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A]
time = 0.39, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^{2}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + c*x
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**2,x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A]
time = 3.35, size = 10, normalized size = 0.83

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2} + c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^2,x)
```

```
[Out] c*x + (d*x^2)/2
```

3.179 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=42

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*d*ln(b*x^2+a)/b+c*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1600, 649, 211, 266}

$$\frac{c\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx &= \int \frac{ac+adx+bcx^2+bdx^3}{(a+bx^2)^2}\,dx\\ &= \int \frac{c+dx}{a+bx^2}\,dx\\ &= c\int \frac{1}{a+bx^2}\,dx+d\int \frac{x}{a+bx^2}\,dx\\ &= \frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Maple [A]
time = 0.17, size = 32, normalized size = 0.76

method	result	size
default	$\frac{d\ln(b\,x^2+a)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	32
risch	$\frac{\ln\left(-\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)d}{2b}-\frac{\ln\left(\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(\sqrt{-ab}\,x+a\right)d}{2b}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3
,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 31, normalized size = 0.74

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="maxima")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

Fricas [A]
time = 0.37, size = 98, normalized size = 2.33

$$\left[\frac{ad\log\left(bx^2+a\right)-\sqrt{-ab}\,c\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{ad\log\left(bx^2+a\right)+2\,\sqrt{ab}\,c\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*b)
*x/a))/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(37) = 74$.
time = 0.12, size = 124, normalized size = 2.95

$$\left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}-\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right)+\left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x+\frac{2ab\left(\frac{d}{2b}+\frac{c\sqrt{-ab^3}}{2ab^2}\right)-ad}{bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**3,x)
```

```
[Out] (d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a*
b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*lo
g(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))
```

Giac [A]
time = 3.46, size = 31, normalized size = 0.74

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log\left(bx^2 + a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

Mupad [B]
time = 2.13, size = 32, normalized size = 0.76

$$\frac{d \ln\left(b\,x^2 + a\right)}{2\,b} + \frac{c \operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^3,x)
```

```
[Out] (d*log(a + b*x^2))/(2*b) + (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.180 $\int (b+2cx+3dx^2)(a+bx+cx^2+dx^3)^n\,dx$

Optimal. Leaf size=25

$$\frac{(a+bx+cx^2+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1602}

$$\frac{(a+bx+cx^2+dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (b+2cx+3dx^2)\left(a+bx+cx^2+dx^3\right)^n\,dx = \frac{(a+bx+cx^2+dx^3)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.08, size = 23, normalized size = 0.92

$$\frac{(a+x(b+x(c+dx)))^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A]
time = 0.03, size = 26, normalized size = 1.04

method	result	siz
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{1+n}}{1+n}$	26
derivativedivides	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{1+n}}{1+n}$	26
default	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{1+n}}{1+n}$	26
risch	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{n}}{1+n}$	39
norman	$\frac{a\,\mathrm{e}^{n\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{1+n}+\frac{bx\,\mathrm{e}^{n\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{1+n}+\frac{c\,x^{2}\mathrm{e}^{n\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{1+n}+\frac{d\,x^{3}\mathrm{e}^{n\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{1+n}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+b*x+a)^(1+n)/(1+n)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

Fricas [A]
time = 0.41, size = 38, normalized size = 1.52

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.71, size = 25, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.19, size = 54, normalized size = 2.16

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^n\,\left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{c\,x^2}{n+1}+\frac{d\,x^3}{n+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^n*(a/(n + 1) + (b*x)/(n + 1) + (c*x^2)/(n + 1) +
(d*x^3)/(n + 1))
```

3.181 $\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^n \, dx$

Optimal. Leaf size=24

$$\frac{(bx+cx^2+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+b*x)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1602}

$$\frac{(bx+cx^2+dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^n \, dx = \frac{(bx+cx^2+dx^3)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.15, size = 21, normalized size = 0.88

$$\frac{(x(b+x(c+dx)))^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

Maple [A]
time = 0.03, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$\frac{(d x^3+c x^2+b x)^{1+n}}{1+n}$	25
default	$\frac{(d x^3+c x^2+b x)^{1+n}}{1+n}$	25
risch	$\frac{x(d x^2+c x+b)\,(x(d x^2+c x+b))^{n}}{1+n}$	32
gosper	$\frac{x(d x^2+c x+b)\,(d x^3+c x^2+b x)^{n}}{1+n}$	34
norman	$\frac{b x\,\mathrm{e}^{n \ln\left(d x^3+c x^2+b x\right)}}{1+n}+\frac{c x^2\mathrm{e}^{n \ln\left(d x^3+c x^2+b x\right)}}{1+n}+\frac{d x^3\mathrm{e}^{n \ln\left(d x^3+c x^2+b x\right)}}{1+n}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+b*x)^(1+n)/(1+n)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.00

$$\frac{(dx^3+cx^2+bx)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

Fricas [A]
time = 0.40, size = 36, normalized size = 1.50

$$\frac{(dx^3+cx^2+bx)(dx^3+cx^2+bx)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.02, size = 24, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.12, size = 46, normalized size = 1.92

$$\left(\frac{b\,x}{n+1} + \frac{c\,x^2}{n+1} + \frac{d\,x^3}{n+1}\right)\,\left(d\,x^3 + c\,x^2 + b\,x\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x)
```

```
[Out] ((b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(b*x + c*x^2 + d*x^3)^n
```

3.182 $\int x^n (b+cx+dx^2)^n (b+2cx+3dx^2)\, dx$

Optimal. Leaf size=25

$$\frac{x^{1+n}(b+cx+dx^2)^{1+n}}{1+n}$$

[Out] x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1604}

$$\frac{x^{n+1}(b+cx+dx^2)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right)\, dx = \frac{x^{1+n}(b+cx+dx^2)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 0.96

$$\frac{x^{1+n}(b+x(c+dx))^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A]
time = 0.28, size = 26, normalized size = 1.04

method	result	size
gosper	$\frac{x^{1+n}\left(d\,x^{2}+cx+b\right)^{1+n}}{1+n}$	26
risch	$\frac{x\left(d\,x^{2}+cx+b\right)x^{n}\left(d\,x^{2}+cx+b\right)^{n}}{1+n}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+n)*(d*x^2+c*x+b)^(1+n)/(1+n)
```

Maxima [A]
time = 0.32, size = 39, normalized size = 1.56

$$\frac{(dx^3 + cx^2 + bx)e^{(n\log(dx^2+cx+b)+n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)
```

Fricas [A]
time = 0.41, size = 35, normalized size = 1.40

$$\frac{(dx^3 + cx^2 + bx)(dx^2 + cx + b)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.
time = 2.58, size = 65, normalized size = 2.60

$$\frac{(dx^2 + cx + b)^n dx^3 x^n + (dx^2 + cx + b)^n cx^2 x^n + (dx^2 + cx + b)^n bx x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 + c
*x + b)^n*b*x*x^n)/(n + 1)
```

Mupad [B]
time = 2.15, size = 51, normalized size = 2.04

$$\left(\frac{c\,x^n\,x^2}{n+1}+\frac{d\,x^n\,x^3}{n+1}+\frac{b\,x\,x^n}{n+1}\right)\,{\left(d\,x^2+c\,x+b\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x)
```

```
[Out] ((c*x^n*x^2)/(n + 1) + (d*x^n*x^3)/(n + 1) + (b*x*x^n)/(n + 1))*(b + c*x +
d*x^2)^n
```

3.183 $\int (b+3dx^2)(a+bx+dx^3)^n \, dx$

Optimal. Leaf size=20

$$\frac{(a+bx+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+b*x+a)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1602}

$$\frac{(a+bx+dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (b+3dx^2)(a+bx+dx^3)^n \, dx = \frac{(a+bx+dx^3)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$\frac{(a+bx+dx^3)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)

Maple [A]
time = 0.02, size = 21, normalized size = 1.05

method	result	size
gosper	$\frac{\left(d\,x^{3}+b x+a\right)^{1+n}}{1+n}$	21
derivativedivides	$\frac{\left(d\,x^{3}+b x+a\right)^{1+n}}{1+n}$	21
default	$\frac{\left(d\,x^{3}+b x+a\right)^{1+n}}{1+n}$	21
risch	$\frac{\left(d\,x^{3}+b x+a\right)\left(d\,x^{3}+b x+a\right)^{n}}{1+n}$	29
norman	$\frac{a\,{\mathrm e}^{n \ln\left(d\,x^{3}+b x+a\right)}}{1+n}+\frac{b x\,{\mathrm e}^{n \ln\left(d\,x^{3}+b x+a\right)}}{1+n}+\frac{d\,x^{3} {\mathrm e}^{n \ln\left(d\,x^{3}+b x+a\right)}}{1+n}$	69

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x,method=_RETURNVERBOSE)

[Out] (d*x^3+b*x+a)^(1+n)/(1+n)

Maxima [A]
time = 0.28, size = 20, normalized size = 1.00

$$\frac{(dx^3+bx+a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)

Fricas [A]
time = 0.38, size = 28, normalized size = 1.40

$$\frac{(dx^3+bx+a)(dx^3+bx+a)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")

[Out] (d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.96, size = 20, normalized size = 1.00

$$\frac{(dx^3+bx+a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.14, size = 39, normalized size = 1.95

$$\left(\frac{a}{n+1}+\frac{b\,x}{n+1}+\frac{d\,x^3}{n+1}\right)\,{\left(d\,x^3+b\,x+a\right)}^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^n,x)
```

```
[Out] (a/(n + 1) + (b*x)/(n + 1) + (d*x^3)/(n + 1))*(a + b*x + d*x^3)^n
```

3.184 $\int (b+3dx^2) (bx+dx^3)^n \, dx$

Optimal. Leaf size=19

$$\frac{(bx+dx^3)^{1+n}}{1+n}$$

[Out] (d*x^3+b*x)^(1+n)/(1+n)

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1602\}$

$$\frac{(bx+dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] (b*x + d*x^3)^(1 + n)/(1 + n)

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (b+3dx^2) (bx+dx^3)^n \, dx = \frac{(bx+dx^3)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.06, size = 19, normalized size = 1.00

$$\frac{(x(b+dx^2))^{1+n}}{1+n}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

```
[Out] (x*(b + d*x^2))^(1 + n)/(1 + n)
```

Maple [A]
time = 0.16, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\left(d\,x^{3}+b x\right)^{1+n}}{1+n}$	20
default	$\frac{\left(d\,x^{3}+b x\right)^{1+n}}{1+n}$	20
gosper	$\frac{x\left(d\,x^{2}+b\right)\left(d\,x^{3}+b x\right)^{n}}{1+n}$	26
risch	$\frac{x\left(d\,x^{2}+b\right)\left(x\left(d\,x^{2}+b\right)\right)^{n}}{1+n}$	26
norman	$\frac{b x\,{\mathrm e}^{n \ln \left(d\,x^{3}+b x\right)}}{1+n}+\frac{d\,x^{3} {\mathrm e}^{n \ln \left(d\,x^{3}+b x\right)}}{1+n}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+b*x)^(1+n)/(1+n)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.00

$$\frac{(dx^3+bx)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

Fricas [A]
time = 0.40, size = 26, normalized size = 1.37

$$\frac{(dx^3+bx)(dx^3+bx)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")
[Out] (d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(14) = 28$.
time = 5.35, size = 63, normalized size = 3.32

$$\begin{cases} \frac{bx(bx+dx^3)^n}{n+1}+\frac{dx^3(bx+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ \log(x)+\log\left(x-\sqrt{-\frac{b}{d}}\right)+\log\left(x+\sqrt{-\frac{b}{d}}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)
```

```
[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1)
, Ne(n, -1)), (log(x) + log(x - sqrt(-b/d)) + log(x + sqrt(-b/d)), True))
```

Giac [A]
time = 3.95, size = 19, normalized size = 1.00

$$\frac{(dx^3 + bx)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.13, size = 25, normalized size = 1.32

$$\frac{x\,(d\,x^3 + b\,x)^n\,(d\,x^2 + b)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^n*(b + 3*d*x^2),x)
```

```
[Out] (x*(b*x + d*x^3)^n*(b + d*x^2))/(n + 1)
```

3.185 $\int x^n(b+dx^2)^n\left(b+3dx^2\right)\,dx$

Optimal. Leaf size=22

$$\frac{x^{1+n}(b+dx^2)^{1+n}}{1+n}$$

[Out] x^(1+n)*(d*x^2+b)^(1+n)/(1+n)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {460}

$$\frac{x^{n+1}(b+dx^2)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^n\left(b+dx^2\right)^n\left(b+3dx^2\right)\,dx=\frac{x^{1+n}(b+dx^2)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{x^{1+n}(b+dx^2)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)

Maple [A]
time = 0.22, size = 23, normalized size = 1.05

method	result	size
gosper	$\frac{x^{1+n}\left(d\,x^{2}+b\right)^{1+n}}{1+n}$	23
risch	$\frac{x\left(d\,x^{2}+b\right)x^{n}\left(d\,x^{2}+b\right)^{n}}{1+n}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x,method=_RETURNVERBOSE)
[Out] x^(1+n)*(d*x^2+b)^(1+n)/(1+n)
```

Maxima [A]
time = 0.31, size = 31, normalized size = 1.41

$$\frac{(dx^3+bx)e^{(n\log(dx^2+b)+n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")
[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)
```

Fricas [A]
time = 0.39, size = 27, normalized size = 1.23

$$\frac{(dx^3+bx)(dx^2+b)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="fricas")
[Out] (d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(17) = 34$.
time = 23.87, size = 66, normalized size = 3.00

$$\begin{cases} \frac{bxx^n(b+dx^2)^n}{n+1} + \frac{dx^3x^n(b+dx^2)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(x - \sqrt{-\frac{b}{d}}\right) + \log\left(x + \sqrt{-\frac{b}{d}}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)
```

```
[Out] Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/(
n + 1), Ne(n, -1)), (log(x) + log(x - sqrt(-b/d)) + log(x + sqrt(-b/d)), Tr
ue))
```

Giac [A]
time = 3.55, size = 39, normalized size = 1.77

$$\frac{\left(dx^{2}+b\right)^{n}dx^{3}x^{n}+\left(dx^{2}+b\right)^{n}bxx^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)
```

Mupad [B]
time = 2.16, size = 26, normalized size = 1.18

$$\frac{x\,x^{n}\,\left(d\,x^{2}+b\right)^{n}\,\left(d\,x^{2}+b\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(b + d*x^2)^n*(b + 3*d*x^2),x)
```

```
[Out] (x*x^n*(b + d*x^2)^n*(b + d*x^2))/(n + 1)
```

3.186 $\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1602}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.06, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A]
time = 0.02, size = 23, normalized size = 1.05

method	result	size
gosper	$\frac{(d\,x^3+c\,x^2+a)^{1+n}}{1+n}$	23
derivativedivides	$\frac{(d\,x^3+c\,x^2+a)^{1+n}}{1+n}$	23
default	$\frac{(d\,x^3+c\,x^2+a)^{1+n}}{1+n}$	23
risch	$\frac{(d\,x^3+c\,x^2+a)^{n}(d\,x^3+c\,x^2+a)}{1+n}$	33
norman	$\frac{a\,e^{n\ln(d\,x^3+c\,x^2+a)}}{1+n}+\frac{c\,x^2 e^{n\ln(d\,x^3+c\,x^2+a)}}{1+n}+\frac{d\,x^3 e^{n\ln(d\,x^3+c\,x^2+a)}}{1+n}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Maxima [A]
time = 0.28, size = 22, normalized size = 1.00

$$\frac{(dx^3+cx^2+a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

Fricas [A]
time = 0.38, size = 32, normalized size = 1.45

$$\frac{(dx^3+cx^2+a)(dx^3+cx^2+a)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.16, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{c\,x^2}{n+1} + \frac{d\,x^3}{n+1}\right)\,\left(d\,x^3 + c\,x^2 + a\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x)
```

```
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

3.187 $\int (2cx+3dx^2)(cx^2+dx^3)^n\,dx$

Optimal. Leaf size=21

$$\frac{(cx^2+dx^3)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1602}

$$\frac{(cx^2+dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (2cx+3dx^2)\left(cx^2+dx^3\right)^n\,dx = \frac{(cx^2+dx^3)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.90

$$\frac{(x^2(c+dx))^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)

Maple [A]
time = 0.21, size = 22, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\left(d\,x^{3}+c\,x^{2}\right)^{1+n}}{1+n}$	22
default	$\frac{\left(d\,x^{3}+c\,x^{2}\right)^{1+n}}{1+n}$	22
risch	$\frac{x^{2}(dx+c)\left(x^{2}(dx+c)\right)^{n}}{1+n}$	26
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}\right)^{n}x^{2}(dx+c)}{1+n}$	28
norman	$\frac{c\,x^{2}{\mathrm e}^{n\ln\left(d\,x^{3}+c\,x^{2}\right)}}{1+n}+\frac{d\,x^{3}{\mathrm e}^{n\ln\left(d\,x^{3}+c\,x^{2}\right)}}{1+n}$	52

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x,method=_RETURNVERBOSE)

[Out] (d*x^3+c*x^2)^(1+n)/(1+n)

Maxima [A]
time = 0.27, size = 21, normalized size = 1.00

$$\frac{(dx^3+cx^2)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)

Fricas [A]
time = 0.40, size = 30, normalized size = 1.43

$$\frac{(dx^3+cx^2)(dx^3+cx^2)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.41, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [A]
time = 3.35, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.16, size = 27, normalized size = 1.29

$$\frac{x^2 \left(d\,x^3 + c\,x^2\right)^n (c + d\,x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x)
```

```
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

3.188 $\int x^n(cx+dx^2)^n(2cx+3dx^2)\,dx$

Optimal. Leaf size=24

$$\frac{x^{1+n}(cx+dx^2)^{1+n}}{1+n}$$

```
[Out] x^(1+n)*(d*x^2+c*x)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 777}

$$\frac{x^{n+1}(cx+dx^2)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 777

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[g*(e*x)^m*((b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 2))), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^n(cx+dx^2)^n(2cx+3dx^2)\,dx &= \int x^{1+n}(2c+3dx)(cx+dx^2)^n\,dx\\ &= \frac{x^{1+n}(cx+dx^2)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.92

$$\frac{x^{1+n}(x(c+dx))^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
[Out] (x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)
```

Maple [A]
time = 0.23, size = 28, normalized size = 1.17

method	result
gosper	$\frac{(d\,x^2+cx)^n x^{2+n}(dx+c)}{1+n}$
risch	$\frac{(dx+c)x^2x^n\mathrm{e}^{\frac{n\left(-i\pi\mathrm{csgn}(ix(dx+c))^3+i\pi\mathrm{csgn}(ix(dx+c))^2\mathrm{csgn}(ix)+i\pi\mathrm{csgn}(ix(dx+c))^2\mathrm{csgn}(i(dx+c))-i\pi\,\mathrm{csgn}(ix(dx+c))\mathrm{csgn}(ix)\mathrm{csgn}(i(dx+c))+2\ln(}{2}}}{1+n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
[Out] (d*x^2+c*x)^n*x^(2+n)*(d*x+c)/(1+n)
```

Maxima [A]
time = 0.33, size = 32, normalized size = 1.33

$$\frac{(dx^3+cx^2)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 1.29

$$\frac{(dx^3+cx^2)(dx^2+cx)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(19) = 38.
time = 1.60, size = 56, normalized size = 2.33

$$\begin{cases}\frac{cx^2x^n(cx+dx^2)^n}{n+1}+\frac{dx^3x^n(cx+dx^2)^n}{n+1} & \text{for}\ n\neq-1\\ 2\log(x)+\log\left(\frac{c}{d}+x\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x**
2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.
time = 3.66, size = 51, normalized size = 2.12

$$\frac{dx^3x^ne^{(n\log(dx+c)+n\log(x))} + cx^2x^ne^{(n\log(dx+c)+n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] (d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n*
log(x)))/(n + 1)
```

Mupad [B]
time = 2.20, size = 28, normalized size = 1.17

$$\frac{x^n\,x^2\,{\left(d\,x^2+c\,x\right)}^n\,\left(c+d\,x\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x)
```

```
[Out] (x^n*x^2*(c*x + d*x^2)^n*(c + d*x))/(n + 1)
```

3.189 $\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) \, dx$

Optimal. Leaf size=22
$$\frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

```
[Out] x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{859\}$

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rule 859

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[c*x^(m + 2)*((f + g*x)^(n + 1)/(g*(m + n + 3))), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) \, dx = \frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^{2+2n}(c+dx)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)
```

Maple [A]
time = 0.17, size = 23, normalized size = 1.05

method	result	size
gosper	$\frac{x^{2+2n}(dx+c)^{1+n}}{1+n}$	23
risch	$\frac{(dx+c)^{n}x^{2n}x^{2}(dx+c)}{1+n}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

```
[Out] x^(2+2*n)*(d*x+c)^(1+n)/(1+n)
```

Maxima [A]
time = 0.32, size = 32, normalized size = 1.45

$$\frac{(dx^3+cx^2)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A]
time = 0.40, size = 29, normalized size = 1.32

$$\frac{(dx^3+cx^2)(dx+c)^n x^{2\,n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.
time = 1.58, size = 53, normalized size = 2.41

$$\begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for}\ n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x)
**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [A]
time = 4.27, size = 41, normalized size = 1.86

$$\frac{(dx + c)^n dx^3 x^{2n} + (dx + c)^n cx^2 x^{2n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)
```

Mupad [B]
time = 2.19, size = 26, normalized size = 1.18

$$\frac{x^{2n}\,x^2\,(c + d\,x)^n\,(c + d\,x)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(2*c*x + 3*d*x^2)*(c + d*x)^n,x)
```

```
[Out] (x^(2*n)*x^2*(c + d*x)^n*(c + d*x))/(n + 1)
```

3.190 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1602}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A]
time = 0.02, size = 23, normalized size = 1.05

method	result	size
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}+a\right)^{1+n}}{1+n}$	23
risch	$\frac{\left(d\,x^{3}+c\,x^{2}+a\right)^{n}\left(d\,x^{3}+c\,x^{2}+a\right)}{1+n}$	33
norman	$\frac{a\,\mathrm{e}^{n\ln\left(d\,x^{3}+c\,x^{2}+a\right)}}{1+n}+\frac{c\,x^{2}\mathrm{e}^{n\ln\left(d\,x^{3}+c\,x^{2}+a\right)}}{1+n}+\frac{d\,x^{3}\mathrm{e}^{n\ln\left(d\,x^{3}+c\,x^{2}+a\right)}}{1+n}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x,method=_RETURNVERBOSE)
[Out] (d*x^3+c*x^2+a)^(1+n)/(1+n)
```

Maxima [A]
time = 0.29, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Fricas [A]
time = 0.41, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.94, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

Mupad [**B**]
time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{c\,x^2}{n+1} + \frac{d\,x^3}{n+1}\right)\,\left(d\,x^3 + c\,x^2 + a\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x)
```

```
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

3.191 $\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

```
[Out] (d*x^3+c*x^2)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{1602\}$

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$\frac{\left(x^2(c+dx)\right)^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

Maple [A]
time = 0.17, size = 26, normalized size = 1.24

method	result	size
risch	$\frac{x^{2}(dx+c)\left(x^{2}(dx+c)\right)^{n}}{1+n}$	26
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}\right)^{n}x^{2}(dx+c)}{1+n}$	28
norman	$\frac{c\,x^{2}{\mathrm e}^{n\ln\left(d\,x^{3}+c\,x^{2}\right)}}{1+n}+\frac{d\,x^{3}{\mathrm e}^{n\ln\left(d\,x^{3}+c\,x^{2}\right)}}{1+n}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(d*x+c)/(1+n)*(x^2*(d*x+c))^n
```

Maxima [A]
time = 0.31, size = 32, normalized size = 1.52

$$\frac{(dx^3+cx^2)e^{(n\log(dx+c)+2\,n\log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

Fricas [A]
time = 0.39, size = 30, normalized size = 1.43

$$\frac{(dx^3+cx^2)(dx^3+cx^2)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.40, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1}+\frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n\neq -1 \\ 2\log(x)+\log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

Giac [A]
time = 4.81, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.15, size = 27, normalized size = 1.29

$$\frac{x^2\,\left(d\,x^3+c\,x^2\right)^n\,\left(c+d\,x\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x)
```

```
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

3.192 $\int (b+2cx+3dx^2) (a+bx+cx^2+dx^3)^7 \, dx$

Optimal. Leaf size=21

$$\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+b*x+a)^8
```

Rubi [A]
time = 0.09, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{1602\}$

$$\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right) \left(a+bx+cx^2+dx^3\right)^7 \, dx = \frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(21) = 42.
time = 0.11, size = 143, normalized size = 6.81

$$\frac{1}{8}x(b+x(c+dx))\left(8a^7+28a^6x(b+x(c+dx))+56a^5x^2(b+x(c+dx))^2+70a^4x^3(b+x(c+dx))^3+56a^3x^4(b+x(c+dx))^4+28a^2x^5(b+x(c+dx))^5+8ax^6(b+x(c+dx))^6+x^7(b+x(c+dx))^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b +
x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c + d*
```

```
x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x^7*
(b + x*(c + d*x))^7))/8
```

Maple [A]
time = 0.08, size = 20, normalized size = 0.95

method	result	size
default	$\frac{(d\,x^3+c\,x^2+bx+a)^8}{8}$	20
norman	Expression too large to display	1579
gosper	Expression too large to display	1957
risch	Expression too large to display	1962

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x^3+c*x^2+b*x+a)^8
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.90

$$\frac{1}{8}\left(dx^3+cx^2+bx+a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1528 vs. $2(19) = 38$.
time = 0.38, size = 1528, normalized size = 72.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + (7*c^3*d^5 + 7
*b*c*d^6 + a*d^7)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*(b^2 + 2*a*c)*d^
6)*x^20 + 7*(c^5*d^3 + 5*b*c^3*d^4 + a*b*d^6 + 3*(b^2*c + a*c^2)*d^5)*x^19
+ 7/2*(c^6*d^2 + 10*b*c^4*d^3 + a^2*d^6 + 2*(b^3 + 6*a*b*c)*d^5 + 5*(3*b^2*
c^2 + 2*a*c^3)*d^4)*x^18 + (c^7*d + 21*b*c^5*d^2 + 21*(a*b^2 + a^2*c)*d^5 +
 35*(b^3*c + 3*a*b*c^2)*d^4 + 35*(2*b^2*c^3 + a*c^4)*d^3)*x^17 + 1/8*(c^8 +
 56*b*c^6*d + 168*a^2*b*d^5 + 70*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4 + 560*(
b^3*c^2 + 2*a*b*c^3)*d^3 + 84*(5*b^2*c^4 + 2*a*c^5)*d^2)*x^16 + (b*c^7 + 7*
```

```
a^3*d^5 + 35*(a*b^3 + 3*a^2*b*c)*d^4 + 35*(b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)
*d^3 + 35*(2*b^3*c^3 + 3*a*b*c^4)*d^2 + 7*(3*b^2*c^5 + a*c^6)*d)*x^15 + 1/2
*(7*b^2*c^6 + 2*a*c^7 + 35*(3*a^2*b^2 + 2*a^3*c)*d^4 + 14*(b^5 + 20*a*b^3*c
 + 30*a^2*b*c^2)*d^3 + 105*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2 + 14*(5*b^
3*c^4 + 6*a*b*c^5)*d)*x^14 + 7*(b^3*c^5 + a*b*c^6 + 5*a^3*b*d^4 + 5*(a*b^4
+ 6*a^2*b^2*c + 2*a^3*c^2)*d^3 + 3*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^
2 + (5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*d)*x^13 + 7/4*(5*b^4*c^4 + 12*a*
b^2*c^5 + 2*a^2*c^6 + 5*a^4*d^4 + 40*(a^2*b^3 + 2*a^3*b*c)*d^3 + 2*(b^6 + 3
0*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^2 + 4*(3*b^5*c^2 + 20*a*b^3*c^3
+ 15*a^2*b*c^4)*d)*x^12 + 7*(b^5*c^3 + 5*a*b^3*c^4 + 3*a^2*b*c^5 + 5*(2*a^3
*b^2 + a^4*c)*d^3 + 3*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^2 + (b^6*c +
15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*d)*x^11 + 1/2*(7*b^6*c^2 + 70*a*
b^4*c^3 + 105*a^2*b^2*c^4 + 14*a^3*c^5 + 70*a^4*b*d^3 + 105*(a^2*b^4 + 4*a^
3*b^2*c + a^4*c^2)*d^2 + 2*(b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*
c^3)*d)*x^10 + (b^7*c + 21*a*b^5*c^2 + 70*a^2*b^3*c^3 + 35*a^3*b*c^4 + 7*a^
5*d^3 + 35*(2*a^3*b^3 + 3*a^4*b*c)*d^2 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b
^2*c^2 + 5*a^4*c^3)*d)*x^9 + a^7*b*x + 1/8*(b^8 + 56*a*b^6*c + 420*a^2*b^4*
c^2 + 560*a^3*b^2*c^3 + 70*a^4*c^4 + 84*(5*a^4*b^2 + 2*a^5*c)*d^2 + 56*(3*a
^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*d)*x^8 + (a*b^7 + 21*a^2*b^5*c + 70*a
^3*b^3*c^2 + 35*a^4*b*c^3 + 21*a^5*b*d^2 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*
a^5*c^2)*d)*x^7 + 7/2*(a^2*b^6 + 10*a^3*b^4*c + 15*a^4*b^2*c^2 + 2*a^5*c^3
+ a^6*d^2 + 2*(5*a^4*b^3 + 6*a^5*b*c)*d)*x^6 + 7*(a^3*b^5 + 5*a^4*b^3*c + 3
*a^5*b*c^2 + (3*a^5*b^2 + a^6*c)*d)*x^5 + 7/4*(5*a^4*b^4 + 12*a^5*b^2*c + 2
*a^6*c^2 + 4*a^6*b*d)*x^4 + (7*a^5*b^3 + 7*a^6*b*c + a^7*d)*x^3 + 1/2*(7*a^
6*b^2 + 2*a^7*c)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(17) = 34$.
time = 0.17, size = 1771, normalized size = 84.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

```
[Out] a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x
**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 7*b**2*d**6/
2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(7*a*b*d**6 + 21*a*c**2*d**5 +
 21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 42
*a*b*c*d**5 + 35*a*c**3*d**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c*
*4*d**3 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 105*a*b
*c**2*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**
5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**4/2 + 105*a*b**
2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35*b**4*d**4/4 + 70*b**3*c*
*2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(7*a**3*d**5
```

```
+ 105*a**2*b*c*d**4 + 70*a**2*c**3*d**3 + 35*a*b**3*d**4 + 210*a*b**2*c**2*
d**3 + 105*a*b*c**4*d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**3*c**3*d**2
+ 21*b**2*c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*b**2*d**4/2 +
 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d**3 + 210*a*b*
*2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 105*b**4*c**2*d**2/2
+ 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35*a**3*b*d**4 + 70*a**3*c**2*d*
*3 + 210*a**2*b**2*c*d**3 + 210*a**2*b*c**3*d**2 + 21*a**2*c**5*d + 35*a*b*
*4*d**3 + 210*a*b**3*c**2*d**2 + 105*a*b**2*c**4*d + 7*a*b*c**6 + 21*b**5*c
*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d**4/4 + 140*a**3*b*
c*d**3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315*a**2*b**2*c**2*d**2 +
105*a**2*b*c**4*d + 7*a**2*c**6/2 + 105*a*b**4*c*d**2 + 140*a*b**3*c**3*d +
 21*a*b**2*c**5 + 7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*
(35*a**4*c*d**3 + 70*a**3*b**2*d**3 + 210*a**3*b*c**2*d**2 + 35*a**3*c**4*d
 + 210*a**2*b**3*c*d**2 + 210*a**2*b**2*c**3*d + 21*a**2*b*c**5 + 21*a*b**5
*d**2 + 105*a*b**4*c**2*d + 35*a*b**3*c**4 + 7*b**6*c*d + 7*b**5*c**3) + x*
*10*(35*a**4*b*d**3 + 105*a**4*c**2*d**2/2 + 210*a**3*b**2*c*d**2 + 140*a**
3*b*c**3*d + 7*a**3*c**5 + 105*a**2*b**4*d**2/2 + 210*a**2*b**3*c**2*d + 10
5*a**2*b**2*c**4/2 + 42*a*b**5*c*d + 35*a*b**4*c**3 + b**7*d + 7*b**6*c**2/
2) + x**9*(7*a**5*d**3 + 105*a**4*b*c*d**2 + 35*a**4*c**3*d + 70*a**3*b**3*
d**2 + 210*a**3*b**2*c**2*d + 35*a**3*b*c**4 + 105*a**2*b**4*c*d + 70*a**2*
b**3*c**3 + 7*a*b**6*d + 21*a*b**5*c**2 + b**7*c) + x**8*(21*a**5*c*d**2 +
105*a**4*b**2*d**2/2 + 105*a**4*b*c**2*d + 35*a**4*c**4/4 + 140*a**3*b**3*c
*d + 70*a**3*b**2*c**3 + 21*a**2*b**5*d + 105*a**2*b**4*c**2/2 + 7*a*b**6*c
 + b**8/8) + x**7*(21*a**5*b*d**2 + 21*a**5*c**2*d + 105*a**4*b**2*c*d + 35
*a**4*b*c**3 + 35*a**3*b**4*d + 70*a**3*b**3*c**2 + 21*a**2*b**5*c + a*b**7
) + x**6*(7*a**6*d**2/2 + 42*a**5*b*c*d + 7*a**5*c**3 + 35*a**4*b**3*d + 10
5*a**4*b**2*c**2/2 + 35*a**3*b**4*c + 7*a**2*b**6/2) + x**5*(7*a**6*c*d + 2
1*a**5*b**2*d + 21*a**5*b*c**2 + 35*a**4*b**3*c + 7*a**3*b**5) + x**4*(7*a*
*6*b*d + 7*a**6*c**2/2 + 21*a**5*b**2*c + 35*a**4*b**4/4) + x**3*(a**7*d +
7*a**6*b*c + 7*a**5*b**3) + x**2*(a**7*c + 7*a**6*b**2/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(19) = 38.
time = 3.41, size = 160, normalized size = 7.62

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8+\left(dx^3+cx^2+bx\right)^7a+\frac{7}{2}\left(dx^3+cx^2+bx\right)^6a^2+7\left(dx^3+cx^2+bx\right)^5a^3+\frac{35}{4}\left(dx^3+cx^2+bx\right)^4a^4+7\left(dx^3+cx^2+bx\right)^3a^5+\frac{7}{2}\left(dx^3+cx^2+bx\right)^2a^6+\left(dx^3+cx^2+bx\right)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8 + (d*x^3 + c*x^2 + b*x)^7*a + 7/2*(d*x^3 + c*x^
2 + b*x)^6*a^2 + 7*(d*x^3 + c*x^2 + b*x)^5*a^3 + 35/4*(d*x^3 + c*x^2 + b*x)
^4*a^4 + 7*(d*x^3 + c*x^2 + b*x)^3*a^5 + 7/2*(d*x^3 + c*x^2 + b*x)^2*a^6 +
(d*x^3 + c*x^2 + b*x)*a^7
```

Mupad [B]

time = 2.98, size = 1576, normalized size = 75.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + (35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*
a*b^2*c^5 + 21*b^5*c^2*d + 70*a^2*b^3*d^3 + 70*a^3*c^3*d^2 + 315*a^2*b^2*c^
2*d^2 + 140*a*b^3*c^3*d + 105*a*b^4*c*d^2 + 105*a^2*b*c^4*d + 140*a^3*b*c*d
^3) + x^11*(7*b^5*c^3 + 35*a*b^3*c^4 + 21*a^2*b*c^5 + 21*a*b^5*d^2 + 35*a^3
*c^4*d + 35*a^4*c*d^3 + 70*a^3*b^2*d^3 + 7*b^6*c*d + 210*a^2*b^2*c^3*d + 21
0*a^2*b^3*c*d^2 + 210*a^3*b*c^2*d^2 + 105*a*b^4*c^2*d) + x^13*(7*b^3*c^5 +
35*a*b^4*d^3 + 35*a^3*b*d^4 + 21*a^2*c^5*d + 35*b^4*c^3*d + 21*b^5*c*d^2 +
70*a^3*c^2*d^3 + 7*a*b*c^6 + 210*a*b^3*c^2*d^2 + 210*a^2*b*c^3*d^2 + 210*a^
2*b^2*c*d^3 + 105*a*b^2*c^4*d) + x^5*(7*a^3*b^5 + 35*a^4*b^3*c + 21*a^5*b*c
^2 + 21*a^5*b^2*d + 7*a^6*c*d) + x^19*(7*c^5*d^3 + 21*a*c^2*d^5 + 35*b*c^3*
d^4 + 21*b^2*c*d^5 + 7*a*b*d^6) + x^8*(b^8/8 + (35*a^4*c^4)/4 + 21*a^2*b^5*
d + 21*a^5*c*d^2 + (105*a^2*b^4*c^2)/2 + 70*a^3*b^2*c^3 + (105*a^4*b^2*d^2)
/2 + 7*a*b^6*c + 140*a^3*b^3*c*d + 105*a^4*b*c^2*d) + x^9*(b^7*c + 7*a^5*d^
3 + 21*a*b^5*c^2 + 35*a^3*b*c^4 + 35*a^4*c^3*d + 70*a^2*b^3*c^3 + 70*a^3*b^
3*d^2 + 7*a*b^6*d + 210*a^3*b^2*c^2*d + 105*a^2*b^4*c*d + 105*a^4*b*c*d^2)
+ x^16*(c^8/8 + (35*b^4*d^4)/4 + 21*a^2*b*d^5 + 21*a*c^5*d^2 + (105*a^2*c^2
*d^4)/2 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d + 140*a*b*c^3*d^
3 + 105*a*b^2*c*d^4) + x^10*(b^7*d + 7*a^3*c^5 + (7*b^6*c^2)/2 + 35*a*b^4*c
^3 + 35*a^4*b*d^3 + (105*a^2*b^2*c^4)/2 + (105*a^2*b^4*d^2)/2 + (105*a^4*c^
2*d^2)/2 + 210*a^2*b^3*c^2*d + 210*a^3*b^2*c*d^2 + 42*a*b^5*c*d + 140*a^3*b
*c^3*d) + x^15*(b*c^7 + 7*a^3*d^5 + 35*a*b^3*d^4 + 21*b^2*c^5*d + 35*b^4*c*
d^3 + 70*a^2*c^3*d^3 + 70*b^3*c^3*d^2 + 7*a*c^6*d + 210*a*b^2*c^2*d^3 + 105
*a*b*c^4*d^2 + 105*a^2*b*c*d^4) + x^14*(a*c^7 + (7*b^2*c^6)/2 + 7*b^5*d^3 +
 35*a^3*c*d^4 + 35*b^3*c^4*d + (105*a^2*b^2*d^4)/2 + (105*a^2*c^4*d^2)/2 +
(105*b^4*c^2*d^2)/2 + 210*a*b^2*c^3*d^2 + 210*a^2*b*c^2*d^3 + 42*a*b*c^5*d
+ 140*a*b^3*c*d^3) + x^4*((35*a^4*b^4)/4 + (7*a^6*c^2)/2 + 21*a^5*b^2*c + 7
*a^6*b*d) + x^20*((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5 + 7*a*c*d^6
) + x^6*((7*a^2*b^6)/2 + 7*a^5*c^3 + (7*a^6*d^2)/2 + 35*a^3*b^4*c + 35*a^4*
b^3*d + (105*a^4*b^2*c^2)/2 + 42*a^5*b*c*d) + x^7*(a*b^7 + 21*a^2*b^5*c + 3
5*a^4*b*c^3 + 35*a^3*b^4*d + 21*a^5*b*d^2 + 21*a^5*c^2*d + 70*a^3*b^3*c^2 +
 105*a^4*b^2*c*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5 + (7*c^6*d^2)/2 + 35*a*
c^3*d^4 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2 + 42*a*b*c*d^5) + x^17*(c^7*d
+ 21*a*b^2*d^5 + 35*a*c^4*d^3 + 21*a^2*c*d^5 + 21*b*c^5*d^2 + 35*b^3*c*d^4
+ 70*b^2*c^3*d^3 + 105*a*b*c^2*d^4) + x^3*(a^7*d + 7*a^5*b^3 + 7*a^6*b*c) +
 (d^8*x^24)/8 + x^2*(a^7*c + (7*a^6*b^2)/2) + c*d^7*x^23 + d^5*x^21*(a*d^2
+ 7*c^3 + 7*b*c*d) + (d^6*x^22*(2*b*d + 7*c^2))/2 + a^7*b*x
```

3.193 $\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^7\,dx$

Optimal. Leaf size=20

$$\frac{1}{8}(bx+cx^2+dx^3)^8$$

[Out] 1/8*(d*x^3+c*x^2+b*x)^8

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1602}

$$\frac{1}{8}(bx+cx^2+dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] (b*x + c*x^2 + d*x^3)^8/8

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (b+2cx+3dx^2)(bx+cx^2+dx^3)^7\,dx = \frac{1}{8}(bx+cx^2+dx^3)^8$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.90

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] (x^8*(b + x*(c + d*x))^8)/8

Maple [A]
time = 0.15, size = 19, normalized size = 0.95

method	result
gosper	$\frac{x^{8}\left(d\,x^{2}+cx+b\right)^{8}}{8}$
default	$\frac{\left(d\,x^{3}+c\,x^{2}+bx\right)^{8}}{8}$
norman	$\left(\frac{7}{2}b^{2}d^{6}+21b\,c^{2}d^{5}+\frac{35}{4}c^{4}d^{4}\right)x^{20}+\left(7bc\,d^{6}+7c^{3}d^{5}\right)x^{21}+\left(b\,d^{7}+\frac{7}{2}c^{2}d^{6}\right)x^{22}+c\,d^{7}x^{23}+\frac{d^{8}x^{24}}{8}+($
risch	$35b\,c^{3}d^{4}x^{19}+35b^{3}c\,d^{4}x^{17}+7b^{3}c^{5}x^{13}+b\,c^{7}x^{15}+c^{7}d\,x^{17}+7c^{5}d^{3}x^{19}+7x^{18}b^{3}d^{5}+\frac{7}{2}x^{18}c^{6}d^{2}+\frac{7}{2}x^{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x,method=_RETURNVERBOSE)
[Out] 1/8*(d*x^3+c*x^2+b*x)^8
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.90

$$\frac{1}{8}\left(dx^{3}+cx^{2}+bx\right)^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(18) = 36.
time = 0.37, size = 441, normalized size = 22.05

$$\frac{1}{8}d^{8}x^{24}+cd^{7}x^{23}+\frac{1}{2}\left(7c^{2}d^{6}+2bd^{7}\right)x^{22}+7\left(c^{3}d^{5}+bcd^{6}\right)x^{21}+\frac{7}{4}\left(5c^{4}d^{4}+12bc^{2}d^{5}+2b^{2}d^{6}\right)x^{20}+7\left(c^{5}d^{3}+5bc^{3}d^{4}+3b^{2}cd^{5}\right)x^{19}+\frac{7}{2}\left(c^{6}d^{2}+10bc^{4}d^{3}+15b^{2}c^{2}d^{4}+2b^{3}d^{5}\right)x^{18}+\left(c^{7}d+21bc^{5}d^{2}+70b^{2}c^{3}d^{3}+35b^{3}cd^{4}\right)x^{17}+b^{7}cx^{9}+\frac{1}{8}\left(c^{8}+56bc^{6}d+420b^{2}c^{4}d^{2}+560b^{3}c^{2}d^{3}+70b^{4}d^{4}\right)x^{16}+\frac{1}{8}b^{8}x^{8}+\left(bc^{7}+21b^{2}c^{5}d+70b^{3}c^{3}d^{2}+35b^{4}cd^{3}\right)x^{15}+\frac{7}{2}\left(b^{2}c^{6}+10b^{3}c^{4}d+15b^{4}c^{2}d^{2}+2b^{5}d^{3}\right)x^{14}+7\left(b^{3}c^{5}+5b^{4}c^{3}d+3b^{5}cd^{2}\right)x^{13}+\frac{7}{4}\left(5b^{4}c^{4}+12b^{5}c^{2}d+2b^{6}d^{2}\right)x^{12}+7\left(b^{5}c^{3}+b^{6}cd\right)x^{11}+\frac{1}{2}\left(7b^{6}c^{2}+2b^{7}d\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(15) = 30$.
time = 0.06, size = 469, normalized size = 23.45

$$\frac{b^8x^8}{8} + b^7cx^9 + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{22}\left(bd^7 + \frac{7c^2d^6}{2}\right) + x^{21}\cdot(7bcd^6 + 7c^3d^5) + x^{20}\cdot\left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4}\right) + x^{19}\cdot(21b^2cd^5 + 35bc^3d^4 + 7c^5d^3) + x^{18}\cdot\left(7b^3d^5 + \frac{105b^2c^2d^4}{2} + 35bc^4d^3 + \frac{7c^6d^2}{2}\right) + x^{17}\cdot(35b^3cd^4 + 70b^2c^3d^3 + 21bc^5d^2 + c^7d) + x^{16}\cdot\left(\frac{35b^4d^4}{4} + 70b^3c^2d^3 + \frac{105b^2c^4d^2}{2} + 7bc^6d + \frac{c^8}{8}\right) + x^{15}\cdot(35b^4cd^3 + 70b^3c^3d^2 + 21b^2c^5d + bc^7) + x^{14}\cdot\left(7b^5d^3 + \frac{105b^4c^2d^2}{2} + 35b^3c^4d + \frac{7b^2c^6}{2}\right) + x^{13}\cdot(21b^5cd^2 + 35b^4c^3d + 7b^3c^5) + x^{12}\cdot\left(\frac{7b^6d^2}{2} + 21b^5c^2d + \frac{35b^4c^4}{4}\right) + x^{11}\cdot(7b^6cd + 7b^5c^3) + x^{10}\left(b^7d + \frac{7b^6c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

Giac [A]
time = 3.35, size = 18, normalized size = 0.90

$$\frac{1}{8}\left(dx^3 + cx^2 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Mupad [B]
time = 2.32, size = 418, normalized size = 20.90

$$x^{14}\left(7b^5d^3 + \frac{105b^4c^2d^2}{2} + 35b^3c^4d + \frac{7b^2c^6}{2}\right) + x^{18}\left(7b^3d^5 + \frac{105b^2c^2d^4}{2} + 35bc^4d^3 + \frac{7c^6d^2}{2}\right) + x^{12}\left(\frac{7b^6d^2}{2} + 21b^5c^2d + \frac{35b^4c^4}{4}\right) + x^{20}\left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4}\right) + x^{16}\left(\frac{35b^4d^4}{4} + 70b^3c^2d^3 + \frac{105b^2c^4d^2}{2} + 7bc^6d + \frac{c^8}{8}\right) + \frac{b^8x^8}{8} + \frac{d^8x^{24}}{8} + x^{10}\left(db^7 + \frac{7b^6c^2}{2}\right) + b^7cx^9 + cd^7x^{23} + \frac{d^6x^{22}(7c^2 + 2bd)}{2} + 7b^3cx^{13}(3b^2d^2 + 5bc^2d + c^4) + 7cd^3x^{19}(3b^2d^2 + 5bc^2d + c^4) + bcx^{15}(35b^3d^3 + 70b^2c^2d^2 + 21bc^4d + c^6) + cdx^{17}(35b^3d^3 + 70b^2c^2d^2 + 21bc^4d + c^6) + 7b^5cx^{11}(c^2 + bd) + 7cd^5x^{21}(c^2 + bd)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
```

```
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

3.194 $\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx$

Optimal. Leaf size=19

$$\frac{1}{8}x^8(b+cx+dx^2)^8$$

```
[Out] 1/8*x^8*(d*x^2+c*x+b)^8
```

Rubi [A]
time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1602}

$$\frac{1}{8}x^8(b+cx+dx^2)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7(b+cx+dx^2)^7(b+2cx+3dx^2)\,dx = \frac{1}{8}x^8(b+cx+dx^2)^8$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.95

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5595 vs. $2(17) = 34$.
time = 0.27, size = 5596, normalized size = 294.53

method	result
norman	$\left(\frac{7}{2}b^2d^6 + 21b\,c^2d^5 + \frac{35}{4}c^4d^4\right)x^{20} + \left(7bc\,d^6 + 7c^3d^5\right)x^{21} + \left(b\,d^7 + \frac{7}{2}c^2d^6\right)x^{22} + c\,d^7x^{23} + \frac{d^8x^{24}}{8} + ($
gosper	$35b\,c^3d^4x^{19} + 35b^3c\,d^4x^{17} + 7b^3c^5x^{13} + b\,c^7x^{15} + c^7d\,x^{17} + 7c^5d^3x^{19} + 7x^{18}b^3d^5 + \frac{7}{2}x^{18}c^6d^2 + \frac{7}{2}x^2$
risch	$35b\,c^3d^4x^{19} + 35b^3c\,d^4x^{17} + 7b^3c^5x^{13} + b\,c^7x^{15} + c^7d\,x^{17} + 7c^5d^3x^{19} + 7x^{18}b^3d^5 + \frac{7}{2}x^{18}c^6d^2 + \frac{7}{2}x^2$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(17) = 34$.
time = 0.27, size = 441, normalized size = 23.21

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}\left(7c^2d^6 + 2bd^7\right)x^{22} + 7\left(c^3d^5 + bcd^6\right)x^{21} + \frac{7}{4}\left(5c^4d^4 + 12bc^2d^5 + 2b^2d^6\right)x^{20} + 7\left(c^5d^3 + 5bc^3d^4 + 3b^2cd^5\right)x^{19} + \frac{7}{2}\left(c^6d^2 + 10bc^4d^3 + 15b^2c^2d^4 + 2b^3d^5\right)x^{18} + \left(c^7d + 21bc^5d^2 + 70b^2c^3d^3 + 35b^3cd^4\right)x^{17} + b^7cx^9 + \frac{1}{8}\left(c^8 + 56bc^6d + 420b^2c^4d^2 + 560b^3c^2d^3 + 70b^4d^4\right)x^{16} + \frac{1}{8}b^8x^8 + \left(bc^7 + 21b^2c^5d + 70b^3c^3d^2 + 35b^4cd^3\right)x^{15} + \frac{7}{2}\left(b^2c^6 + 10b^3c^4d + 15b^4c^2d^2 + 2b^5d^3\right)x^{14} + 7\left(b^3c^5 + 5b^4c^3d + 3b^5cd^2\right)x^{13} + \frac{7}{4}\left(5b^4c^4 + 12b^5c^2d + 2b^6d^2\right)x^{12} + 7\left(b^5c^3 + b^6cd\right)x^{11} + \frac{1}{2}\left(7b^6c^2 + 2b^7d\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(17) = 34$.
time = 0.39, size = 441, normalized size = 23.21

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}\left(7c^2d^6 + 2bd^7\right)x^{22} + 7\left(c^3d^5 + bcd^6\right)x^{21} + \frac{7}{4}\left(5c^4d^4 + 12bc^2d^5 + 2b^2d^6\right)x^{20} + 7\left(c^5d^3 + 5bc^3d^4 + 3b^2cd^5\right)x^{19} + \frac{7}{2}\left(c^6d^2 + 10bc^4d^3 + 15b^2c^2d^4 + 2b^3d^5\right)x^{18} + \left(c^7d + 21bc^5d^2 + 70b^2c^3d^3 + 35b^3cd^4\right)x^{17} + b^7cx^9 + \frac{1}{8}\left(c^8 + 56bc^6d + 420b^2c^4d^2 + 560b^3c^2d^3 + 70b^4d^4\right)x^{16} + \frac{1}{8}b^8x^8 + \left(bc^7 + 21b^2c^5d + 70b^3c^3d^2 + 35b^4cd^3\right)x^{15} + \frac{7}{2}\left(b^2c^6 + 10b^3c^4d + 15b^4c^2d^2 + 2b^5d^3\right)x^{14} + 7\left(b^3c^5 + 5b^4c^3d + 3b^5cd^2\right)x^{13} + \frac{7}{4}\left(5b^4c^4 + 12b^5c^2d + 2b^6d^2\right)x^{12} + 7\left(b^5c^3 + b^6cd\right)x^{11} + \frac{1}{2}\left(7b^6c^2 + 2b^7d\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(15) = 30$.
time = 0.06, size = 469, normalized size = 24.68

$$\frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\cdot\left(7bcd^6+7c^3d^5\right)+x^{20}\cdot\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\cdot\left(21b^2cd^5+35bc^3d^4+7c^5d^3\right)+x^{18}\cdot\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{17}\cdot\left(35b^3cd^4+70b^2c^3d^3+21bc^5d^2+c^7d\right)+x^{16}\cdot\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+x^{15}\cdot\left(35b^4cd^3+70b^3c^3d^2+21b^2c^5d+bc^7\right)+x^{14}\cdot\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{13}\cdot\left(21b^5cd^2+35b^4c^3d+7b^3c^5\right)+x^{12}\cdot\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{11}\cdot\left(7b^6cd+7b^5c^3\right)+x^{10}\left(b^7d+\frac{7b^6c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

Giac [A]
time = 3.30, size = 18, normalized size = 0.95

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

Mupad [B]
time = 2.27, size = 418, normalized size = 22.00

$$x^{14}\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{18}\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{12}\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{20}\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{16}\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+\frac{b^8x^8}{8}+\frac{d^8x^{24}}{8}+x^{10}\left(db^7+\frac{7b^6c^2}{2}\right)+b^7cx^9+cd^7x^{23}+\frac{d^6x^{22}\left(7c^2+2bd\right)}{2}+7b^3cx^{13}\left(3b^2d^2+5bc^2d+c^4\right)+7cd^3x^{19}\left(3b^2d^2+5bc^2d+c^4\right)+bcx^{15}\left(35b^3d^3+70b^2c^2d^2+21bc^4d+c^6\right)+cdx^{17}\left(35b^3d^3+70b^2c^2d^2+21bc^4d+c^6\right)+7b^5cx^{11}\left(c^2+bd\right)+7cd^5x^{21}\left(c^2+bd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

3.195 $\int (b+3dx^2)(a+bx+dx^3)^7\,dx$

Optimal. Leaf size=16

$$\frac{1}{8}\left(a+bx+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+b*x+a)^8
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1602}

$$\frac{1}{8}\left(a+bx+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (a + b*x + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+dx^3\right)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(16) = 32.
time = 0.05, size = 127, normalized size = 7.94

$$\frac{1}{8}x\left(b+dx^2\right)\left(8a^7+28a^6x\left(b+dx^2\right)+56a^5x^2\left(b+dx^2\right)^2+70a^4x^3\left(b+dx^2\right)^3+56a^3x^4\left(b+dx^2\right)^4+28a^2x^5\left(b+dx^2\right)^5+8ax^6\left(b+dx^2\right)^6+x^7\left(b+dx^2\right)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 + 7
0*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^2)
^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7))/8
```

Maple [A]
time = 0.09, size = 15, normalized size = 0.94

method	result
default	$\frac{\left(d\,x^{3}+b x+a\right)^{8}}{8}$
norman	$7ab\,d^{6}x^{19}+\frac{7x^{20}b^{2}d^{6}}{2}+a\,d^{7}x^{21}+x^{22}b\,d^{7}+\frac{d^{8}x^{24}}{8}+21a\,b^{2}d^{5}x^{17}+\left(\frac{7}{2}a^{2}d^{6}+7b^{3}d^{5}\right)x^{18}+\left(7a^{3}d^{5}+\right.$
gosper	$7a^{3}d^{5}x^{15}+21a^{5}b^{2}d\,x^{5}+7x^{18}b^{3}d^{5}+\frac{7}{2}x^{20}b^{2}d^{6}+7a^{3}b^{5}x^{5}+a\,b^{7}x^{7}+7a^{5}d^{3}x^{9}+x^{22}b\,d^{7}+\frac{7}{2}x^{2}b^{2}a^{6}$
risch	$7a^{3}d^{5}x^{15}+21a^{5}b^{2}d\,x^{5}+7x^{18}b^{3}d^{5}+\frac{7}{2}x^{20}b^{2}d^{6}+7a^{3}b^{5}x^{5}+a\,b^{7}x^{7}+7a^{5}d^{3}x^{9}+x^{22}b\,d^{7}+\frac{7}{2}x^{2}b^{2}a^{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x^3+b*x+a)^8
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$\frac{1}{8}\left(dx^{3}+bx+a\right)^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x + a)^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(14) = 28.
time = 0.38, size = 456, normalized size = 28.50

$$\frac{1}{8}d^{8}x^{24}+bd^{7}x^{22}+ad^{7}x^{21}+\frac{7}{2}b^{2}d^{6}x^{20}+7abd^{6}x^{19}+21ab^{2}d^{5}x^{17}+\frac{7}{2}\left(2b^{3}d^{5}+a^{2}d^{6}\right)x^{18}+\frac{7}{4}\left(5b^{4}d^{4}+12a^{2}bd^{5}\right)x^{16}+7\left(5ab^{3}d^{4}+a^{3}d^{5}\right)x^{15}+\frac{7}{2}\left(2b^{5}d^{3}+15a^{2}b^{2}d^{4}\right)x^{14}+35\left(ab^{4}d^{3}+a^{3}bd^{4}\right)x^{13}+\frac{7}{4}\left(2b^{6}d^{2}+40a^{2}b^{3}d^{3}+5a^{4}d^{4}\right)x^{12}+7\left(3ab^{5}d^{2}+10a^{3}b^{2}d^{3}\right)x^{11}+\frac{1}{2}\left(2b^{7}d+105a^{2}b^{4}d^{2}+70a^{4}bd^{3}\right)x^{10}+\frac{7}{2}a^{6}b^{2}x^{2}+7\left(ab^{6}d+10a^{3}b^{3}d^{2}+a^{5}d^{3}\right)x^{9}+a^{7}bx+\frac{1}{8}\left(b^{8}+168a^{2}b^{5}d+420a^{4}b^{2}d^{2}\right)x^{8}+\left(ab^{7}+35a^{3}b^{4}d+21a^{5}bd^{2}\right)x^{7}+\frac{7}{2}\left(a^{2}b^{6}+10a^{4}b^{3}d+a^{6}d^{2}\right)x^{6}+7\left(a^{3}b^{5}+3a^{5}b^{2}d\right)x^{5}+\frac{7}{4}\left(5a^{4}b^{4}+4a^{6}bd\right)x^{4}+\left(7a^{5}b^{3}+a^{7}d\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + a*d^7*x^21 + 7/2*b^2*d^6*x^20 + 7*a*b*d^6*x^19
+ 21*a*b^2*d^5*x^17 + 7/2*(2*b^3*d^5 + a^2*d^6)*x^18 + 7/4*(5*b^4*d^4 + 12*
a^2*b*d^5)*x^16 + 7*(5*a*b^3*d^4 + a^3*d^5)*x^15 + 7/2*(2*b^5*d^3 + 15*a^2*
b^2*d^4)*x^14 + 35*(a*b^4*d^3 + a^3*b*d^4)*x^13 + 7/4*(2*b^6*d^2 + 40*a^2*b
^3*d^3 + 5*a^4*d^4)*x^12 + 7*(3*a*b^5*d^2 + 10*a^3*b^2*d^3)*x^11 + 1/2*(2*b
^7*d + 105*a^2*b^4*d^2 + 70*a^4*b*d^3)*x^10 + 7/2*a^6*b^2*x^2 + 7*(a*b^6*d
```

```
+ 10*a^3*b^3*d^2 + a^5*d^3)*x^9 + a^7*b*x + 1/8*(b^8 + 168*a^2*b^5*d + 420*
a^4*b^2*d^2)*x^8 + (a*b^7 + 35*a^3*b^4*d + 21*a^5*b*d^2)*x^7 + 7/2*(a^2*b^6
 + 10*a^4*b^3*d + a^6*d^2)*x^6 + 7*(a^3*b^5 + 3*a^5*b^2*d)*x^5 + 7/4*(5*a^4
*b^4 + 4*a^6*b*d)*x^4 + (7*a^5*b^3 + a^7*d)*x^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(12) = 24$.
time = 0.06, size = 483, normalized size = 30.19

$$a^7bx + \frac{7a^6b^2x^2}{2} + 21ab^2d^5x^{17} + 7abd^6x^{19} + ad^7x^{21} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8} + x^{18}\cdot\left(\frac{7a^2d^6}{2} + 7b^3d^5\right) + x^{16}\cdot\left(21a^2bd^5 + \frac{35b^4d^4}{4}\right) + x^{15}\cdot\left(7a^3d^5 + 35ab^3d^4\right) + x^{14}\cdot\left(\frac{105a^2b^2d^4}{2} + 7b^5d^3\right) + x^{13}\cdot\left(35a^3bd^4 + 35ab^4d^3\right) + x^{12}\cdot\left(\frac{35a^4d^4}{4} + 70a^2b^3d^3 + \frac{7b^6d^2}{2}\right) + x^{11}\cdot\left(70a^3b^2d^3 + 21ab^5d^2\right) + x^{10}\cdot\left(35a^4bd^3 + \frac{105a^2b^4d^2}{2} + b^7d\right) + x^9\cdot\left(7a^5d^3 + 70a^3b^3d^2 + 7ab^6d\right) + x^8\cdot\left(\frac{105a^4b^2d^2}{2} + 21a^2b^5d + \frac{b^8}{8}\right) + x^7\cdot\left(21a^5bd^2 + 35a^3b^4d + ab^7\right) + x^6\cdot\left(\frac{7a^6d^2}{2} + 35a^4b^3d + \frac{7a^2b^6}{2}\right) + x^5\cdot\left(21a^5b^2d + 7a^3b^5\right) + x^4\cdot\left(7a^6bd + \frac{35a^4b^4}{4}\right) + x^3\left(a^7d + 7a^5b^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)
```

```
[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 + a
*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(7*
a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x**1
5*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**
3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a
**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2)
 + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a**5*d*
*3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 + 21*a**2
*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7) + x**6*
(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2*b**6/2) + x**5*(21*a**5*b**2*d + 7
*a**3*b**5) + x**4*(7*a**6*b*d + 35*a**4*b**4/4) + x**3*(a**7*d + 7*a**5*b*
*3)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(14) = 28$.
time = 3.41, size = 120, normalized size = 7.50

$$\frac{1}{8}\left(dx^3 + bx\right)^8 + \left(dx^3 + bx\right)^7 a + \frac{7}{2}\left(dx^3 + bx\right)^6 a^2 + 7\left(dx^3 + bx\right)^5 a^3 + \frac{35}{4}\left(dx^3 + bx\right)^4 a^4 + 7\left(dx^3 + bx\right)^3 a^5 + \frac{7}{2}\left(dx^3 + bx\right)^2 a^6 + \left(dx^3 + bx\right)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8 + (d*x^3 + b*x)^7*a + 7/2*(d*x^3 + b*x)^6*a^2 + 7*(d*x^
3 + b*x)^5*a^3 + 35/4*(d*x^3 + b*x)^4*a^4 + 7*(d*x^3 + b*x)^3*a^5 + 7/2*(d*
x^3 + b*x)^2*a^6 + (d*x^3 + b*x)*a^7
```

Mupad [**B**]
time = 2.63, size = 438, normalized size = 27.38

$$x^{12}\left(\frac{35\,a^4\,d^4}{4} + 70\,a^2\,b^3\,d^3 + \frac{7\,b^6\,d^2}{2}\right) + x^4\left(7\,d\,a^6\,b + \frac{35\,a^4\,b^4}{4}\right) + x^{18}\left(\frac{7\,a^2\,d^6}{2} + 7\,b^3\,d^5\right) + x^6\left(\frac{7\,a^6\,d^2}{2} + 35\,a^4\,b^3\,d + \frac{7\,a^2\,b^6}{2}\right) + x^8\left(\frac{105\,a^4\,b^2\,d^2}{2} + 21\,a^2\,b^5\,d + \frac{b^8}{8}\right) + \frac{d^8\,x^{24}}{8} + x^3\left(d\,a^7 + 7\,a^5\,b^3\right) + a\,d^7\,x^{21} + b\,d^7\,x^{22} + \frac{7\,a^6\,b^2\,x^2}{2} + \frac{7\,b^2\,d^6\,x^{20}}{2} + a^7\,b\,x + 21\,a\,b^2\,d^5\,x^{17} + a\,b\,x^7\left(21\,a^4\,d^2 + 35\,a^2\,b^3\,d + b^6\right) + 7\,a\,d\,x^9\left(a^4\,d^2 + 10\,a^2\,b^3\,d + b^6\right) + 7\,a^3\,b^2\,x^5\left(3\,d\,a^2 + b^3\right) + 7\,a\,d^4\,x^{15}\left(d\,a^2 + 5\,b^3\right) + \frac{7\,b\,d^4\,x^{16}\left(12\,d\,a^2 + 5\,b^3\right)}{4} + \frac{b\,d\,x^{10}\left(70\,a^4\,d^2 + 105\,a^2\,b^3\,d + 2\,b^6\right)}{2} + 7\,a\,b\,d^6\,x^{19} + \frac{7\,b^2\,d^3\,x^{14}\left(15\,d\,a^2 + 2\,b^3\right)}{2} + 7\,a\,b^2\,d^2\,x^{11}\left(10\,d\,a^2 + 3\,b^3\right) + 35\,a\,b\,d^3\,x^{13}\left(d\,a^2 + b^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x)

[Out] x^12*((35*a^4*d^4)/4 + (7*b^6*d^2)/2 + 70*a^2*b^3*d^3) + x^4*((35*a^4*b^4)/
4 + 7*a^6*b*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5) + x^6*((7*a^2*b^6)/2 + (7
*a^6*d^2)/2 + 35*a^4*b^3*d) + x^8*(b^8/8 + 21*a^2*b^5*d + (105*a^4*b^2*d^2)
/2) + (d^8*x^24)/8 + x^3*(a^7*d + 7*a^5*b^3) + a*d^7*x^21 + b*d^7*x^22 + (7
*a^6*b^2*x^2)/2 + (7*b^2*d^6*x^20)/2 + a^7*b*x + 21*a*b^2*d^5*x^17 + a*b*x^
7*(b^6 + 21*a^4*d^2 + 35*a^2*b^3*d) + 7*a*d*x^9*(b^6 + a^4*d^2 + 10*a^2*b^3
*d) + 7*a^3*b^2*x^5*(3*a^2*d + b^3) + 7*a*d^4*x^15*(a^2*d + 5*b^3) + (7*b*d
^4*x^16*(12*a^2*d + 5*b^3))/4 + (b*d*x^10*(2*b^6 + 70*a^4*d^2 + 105*a^2*b^3
*d))/2 + 7*a*b*d^6*x^19 + (7*b^2*d^3*x^14*(15*a^2*d + 2*b^3))/2 + 7*a*b^2*d
^2*x^11*(10*a^2*d + 3*b^3) + 35*a*b*d^3*x^13*(a^2*d + b^3)
```

3.196 $\int (b+3dx^2) (bx+dx^3)^7 \, dx$

Optimal. Leaf size=15

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

[Out] 1/8*(d*x^3+b*x)^8

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1602}

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

[Out] (b*x + d*x^3)^8/8

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right) \left(bx+dx^3\right)^7 \, dx = \frac{1}{8}\left(bx+dx^3\right)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(15) = 30.
time = 0.00, size = 98, normalized size = 6.53

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

Maple [A]
time = 0.21, size = 14, normalized size = 0.93

method	result	s
default	$\frac{(d\,x^3+bx)^8}{8}$	1
gosper	$\frac{x^8(d\,x^2+b)^8}{8}$	1
norman	$x^{22}b\,d^7+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6$	8
risch	$x^{22}b\,d^7+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x,method=_RETURNVERBOSE)
[Out] 1/8*(d*x^3+b*x)^8
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{1}{8}\left(dx^3+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")
[Out] 1/8*(d*x^3 + b*x)^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(13)=26$.
time = 0.36, size = 88, normalized size = 5.87

$$\frac{1}{8}d^8x^{24}+bd^7x^{22}+\frac{7}{2}b^2d^6x^{20}+7b^3d^5x^{18}+\frac{35}{4}b^4d^4x^{16}+7b^5d^3x^{14}+\frac{7}{2}b^6d^2x^{12}+b^7dx^{10}+\frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10)=20$.
time = 0.02, size = 97, normalized size = 6.47

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7b^6d^2x^{12}}{2}+7b^5d^3x^{14}+\frac{35b^4d^4x^{16}}{4}+7b^3d^5x^{18}+\frac{7b^2d^6x^{20}}{2}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

Giac [A]
time = 3.25, size = 13, normalized size = 0.87

$$\frac{1}{8}\left(dx^3 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

Mupad [B]
time = 0.05, size = 88, normalized size = 5.87

$$\frac{b^8\,x^8}{8}+b^7\,d\,x^{10}+\frac{7\,b^6\,d^2\,x^{12}}{2}+7\,b^5\,d^3\,x^{14}+\frac{35\,b^4\,d^4\,x^{16}}{4}+7\,b^3\,d^5\,x^{18}+\frac{7\,b^2\,d^6\,x^{20}}{2}+b\,d^7\,x^{22}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^7*(b + 3*d*x^2),x)
```

```
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

3.197 $\int x^7(b+dx^2)^7(b+3dx^2)\,dx$

Optimal. Leaf size=16

$$\frac{1}{8}x^8\left(b+dx^2\right)^8$$

[Out] 1/8*x^8*(d*x^2+b)^8

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {457, 75}

$$\frac{1}{8}x^8\left(b+dx^2\right)^8$$

Antiderivative was successfully verified.

[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]

[Out] (x^8*(b + d*x^2)^8)/8

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(b+dx^2\right)^7\left(b+3dx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x^3(b+dx)^7(b+3dx)\,dx,x,x^2\right)\\ &= \frac{1}{8}x^8\left(b+dx^2\right)^8\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(16) = 32.

time = 0.00, size = 98, normalized size = 6.12

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.
time = 0.20, size = 89, normalized size = 5.56

method	result	siz
gosper	$x^{22}b\,d^7+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6$	89
default	$x^{22}b\,d^7+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6$	89
norman	$x^{22}b\,d^7+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6$	89
risch	$x^{22}b\,d^7+\frac{1}{8}d^8x^{24}+\frac{1}{8}x^8b^8+x^{10}b^7d+\frac{7}{2}x^{12}b^6d^2+7x^{14}b^5d^3+\frac{35}{4}x^{16}b^4d^4+7x^{18}b^3d^5+\frac{7}{2}x^{20}b^2d^6$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] x^22*b*d^7+1/8*d^8*x^24+1/8*x^8*b^8+x^10*b^7*d+7/2*x^12*b^6*d^2+7*x^14*b^5*
d^3+35/4*x^16*b^4*d^4+7*x^18*b^3*d^5+7/2*x^20*b^2*d^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.
time = 0.27, size = 88, normalized size = 5.50

$$\frac{1}{8}\,d^8x^{24}+bd^7x^{22}+\frac{7}{2}\,b^2d^6x^{20}+7\,b^3d^5x^{18}+\frac{35}{4}\,b^4d^4x^{16}+7\,b^5d^3x^{14}+\frac{7}{2}\,b^6d^2x^{12}+b^7dx^{10}+\frac{1}{8}\,b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(14) = 28$.
time = 0.38, size = 88, normalized size = 5.50

$$\frac{1}{8}\,d^8x^{24}+bd^7x^{22}+\frac{7}{2}\,b^2d^6x^{20}+7\,b^3d^5x^{18}+\frac{35}{4}\,b^4d^4x^{16}+7\,b^5d^3x^{14}+\frac{7}{2}\,b^6d^2x^{12}+b^7dx^{10}+\frac{1}{8}\,b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.
time = 0.02, size = 97, normalized size = 6.06

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

Giac [A]
time = 3.10, size = 13, normalized size = 0.81

$$\frac{1}{8}\left(dx^3 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

Mupad [B]
time = 0.04, size = 88, normalized size = 5.50

$$\frac{b^8\,x^8}{8} + b^7\,d\,x^{10} + \frac{7\,b^6\,d^2\,x^{12}}{2} + 7\,b^5\,d^3\,x^{14} + \frac{35\,b^4\,d^4\,x^{16}}{4} + 7\,b^3\,d^5\,x^{18} + \frac{7\,b^2\,d^6\,x^{20}}{2} + b\,d^7\,x^{22} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b + d*x^2)^7*(b + 3*d*x^2),x)
```

```
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

3.198 $\int (2cx+3dx^2)(a+cx^2+dx^3)^7 \, dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1602}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7 \, dx = \frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(18) = 36.
time = 0.04, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8

Maple [A]
time = 0.05, size = 17, normalized size = 0.94

method	result
default	$\frac{\left(d\,x^{3}+c\,x^{2}+a\right)^{8}}{8}$
norman	$\frac{d^{8} x^{24}}{8}+\left(7 a c\,d^{6}+\frac{35}{4} c^{4} d^{4}\right) x^{20}+\left(d^{7} a+7 c^{3} d^{5}\right) x^{21}+\frac{7 x^{22} c^{2} d^{6}}{2}+c\,d^{7} x^{23}+\left(21 a^{5} c\,d^{2}+\frac{35}{4} a^{4} c^{4}\right) x^{8}+$
gosper	$\frac{x^{2}\left(d^{8} x^{22}+8 c\,d^{7} x^{21}+28 x^{20} c^{2} d^{6}+8 a\,d^{7} x^{19}+56 c^{3} d^{5} x^{19}+56 x^{18} a c\,d^{6}+70 x^{18} c^{4} d^{4}+168 a\,c^{2} d^{5} x^{17}+56 c^{5} d^{3} x^{17}+28 x^{16} a^{2} d^{6}+280 x^{16} a\,c^{3}\right.}{}$
risch	$7 a^{3} d^{5} x^{15}+c^{7} d\,x^{17}+7 c^{5} d^{3} x^{19}+\frac{7}{2} x^{18} c^{6} d^{2}+\frac{35}{4} x^{20} c^{4} d^{4}+7 a^{5} d^{3} x^{9}+21 a^{5} c^{2} d\,x^{7}+\frac{7}{2} x^{22} c^{2} d^{6}+\frac{7}{2} x^{1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x,method=_RETURNVERBOSE)

[Out] 1/8*(d*x^3+c*x^2+a)^8

Maxima [A]
time = 0.30, size = 16, normalized size = 0.89

$$\frac{1}{8}\left(dx^{3}+cx^{2}+a\right)^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2 + a)^8

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(16) = 32.
time = 0.38, size = 458, normalized size = 25.44

$$\frac{1}{8} d^{8} x^{24}+c d^{7} x^{23}+\frac{7}{2} c^{2} d^{6} x^{22}+\left(7 c^{3} d^{5}+a d^{7}\right) x^{21}+\frac{7}{4}\left(5 c^{4} d^{4}+4 a c d^{6}\right) x^{20}+7\left(c^{5} d^{3}+3 a c^{2} d^{5}\right) x^{19}+\frac{7}{2}\left(c^{6} d^{2}+10 a c^{3} d^{4}+a^{2} d^{6}\right) x^{18}+\left(c^{7} d+35 a c^{4} d^{3}+21 a^{2} c d^{5}\right) x^{17}+\frac{1}{8}\left(c^{8}+168 a c^{5} d^{2}+420 a^{2} c^{2} d^{4}\right) x^{16}+7\left(a c^{6} d+10 a^{2} c^{3} d^{3}+a^{3} d^{5}\right) x^{15}+21 a^{5} c^{2} d x^{7}+\frac{1}{2}\left(2 a c^{7}+105 a^{2} c^{4} d^{2}+70 a^{3} c d^{4}\right) x^{14}+7\left(3 a^{2} c^{5} d+10 a^{3} c^{2} d^{3}\right) x^{13}+7 a^{6} c d x^{5}+\frac{7}{4}\left(2 a^{2} c^{6}+40 a^{3} c^{3} d^{2}+5 a^{4} d^{4}\right) x^{12}+\frac{7}{2} a^{6} c^{2} x^{4}+35\left(a^{3} c^{4} d+a^{4} c d^{3}\right) x^{11}+a^{7} d x^{3}+\frac{7}{2}\left(2 a^{3} c^{5}+15 a^{4} c^{2} d^{2}\right) x^{10}+a^{7} c x^{2}+7\left(5 a^{4} c^{3} d+a^{5} d^{3}\right) x^{9}+\frac{7}{4}\left(5 a^{4} c^{4}+12 a^{5} c d^{2}\right) x^{8}+\frac{7}{2}\left(2 a^{5} c^{3}+a^{6} d^{2}\right) x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c

```
*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^
4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c
^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 +
 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(14) = 28$.
time = 0.06, size = 484, normalized size = 26.89

$$a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}\cdot(ad^7 + 7c^3d^5) + x^{20}\cdot\left(7acd^6 + \frac{35c^4d^4}{4}\right) + x^{19}\cdot(21ac^2d^5 + 7c^5d^3) + x^{18}\cdot\left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + x^{17}\cdot(21a^2cd^5 + 35ac^4d^3 + c^7d) + x^{16}\cdot\left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{15}\cdot(7a^3d^5 + 70a^2c^3d^3 + 7ac^6d) + x^{14}\cdot\left(35a^3cd^4 + \frac{105a^2c^4d^2}{2} + ac^7\right) + x^{13}\cdot(70a^3c^2d^3 + 21a^2c^5d) + x^{12}\cdot\left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^{11}\cdot(35a^4cd^3 + 35a^3c^4d) + x^{10}\cdot\left(\frac{105a^4c^2d^2}{2} + 7a^3c^5\right) + x^9\cdot(7a^5d^3 + 35a^4c^3d) + x^8\cdot\left(21a^5cd^2 + \frac{35a^4c^4}{4}\right) + x^6\cdot\left(\frac{7a^6d^2}{2} + 7a^5c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(16) = 32$.
time = 3.57, size = 136, normalized size = 7.56

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8 + \left(dx^3 + cx^2\right)^7 a + \frac{7}{2}\left(dx^3 + cx^2\right)^6 a^2 + 7\left(dx^3 + cx^2\right)^5 a^3 + \frac{35}{4}\left(dx^3 + cx^2\right)^4 a^4 + 7\left(dx^3 + cx^2\right)^3 a^5 + \frac{7}{2}\left(dx^3 + cx^2\right)^2 a^6 + \left(dx^3 + cx^2\right)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8 + (d*x^3 + c*x^2)^7*a + 7/2*(d*x^3 + c*x^2)^6*a^2 + 7
*(d*x^3 + c*x^2)^5*a^3 + 35/4*(d*x^3 + c*x^2)^4*a^4 + 7*(d*x^3 + c*x^2)^3*a
^5 + 7/2*(d*x^3 + c*x^2)^2*a^6 + (d*x^3 + c*x^2)*a^7
```

Mupad [**B**]
time = 2.63, size = 440, normalized size = 24.44

$$x^{12}\left(\frac{35\,a^4\,d^4}{4} + 70\,a^3\,c^3\,d^2 + \frac{7\,a^2\,c^6}{2}\right) + x^6\left(\frac{7\,a^6\,d^2}{2} + 7\,a^5\,c^3\right) + x^{20}\left(\frac{35\,c^4\,d^4}{4} + 7\,a\,c\,d^6\right) + x^{16}\left(\frac{105\,a^2\,c^2\,d^4}{2} + 21\,a\,c^5\,d^2 + \frac{c^8}{8}\right) + x^{18}\left(\frac{7\,a^2\,d^6}{2} + 35\,a\,c^3\,d^4 + \frac{7\,c^6\,d^2}{2}\right) + \frac{d^8\,x^{24}}{8} + x^{21}\left(7\,c^3\,d^5 + a\,d^7\right) + a^7\,c\,x^2 + a^7\,d\,x^3 + c\,d^7\,x^{23} + \frac{7\,a^6\,c^2\,x^4}{2} + \frac{7\,c^2\,d^6\,x^{22}}{2} + 21\,a^5\,c^2\,d\,x^7 + 7\,a\,d\,x^{15}\left(a^2\,d^4 + 10\,a\,c^3\,d^2 + c^6\right) + c\,d\,x^{17}\left(21\,a^2\,d^4 + 35\,a\,c^3\,d^2 + c^6\right) + \frac{7\,a^4\,c\,x^8\left(5\,c^3 + 12\,a\,d^2\right)}{4} + 7\,a^4\,d\,x^9\left(5\,c^3 + a\,d^2\right) + 7\,c^2\,d^3\,x^{19}\left(c^3 + 3\,a\,d^2\right) + \frac{a\,c\,x^{14}\left(70\,a^2\,d^4 + 105\,a\,c^3\,d^2 + 2\,c^6\right)}{2} + 7\,a^6\,c\,d\,x^5 + \frac{7\,a^3\,c^2\,x^{10}\left(2\,c^3 + 15\,a\,d^2\right)}{2} + 7\,a^2\,c^2\,d\,x^{13}\left(3\,c^3 + 10\,a\,d^2\right) + 35\,a^3\,c\,d\,x^{11}\left(c^3 + a\,d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (
7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*
d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3
*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c
*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a
*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*
c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3)
 + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c
^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*
c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

3.199 $\int (2cx + 3dx^2)(cx^2 + dx^3)^7 \, dx$

Optimal. Leaf size=17

$$\frac{1}{8}\left(cx^2 + dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2)^8
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1602}

$$\frac{1}{8}\left(cx^2 + dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (2cx + 3dx^2)\left(cx^2 + dx^3\right)^7 \, dx = \frac{1}{8}\left(cx^2 + dx^3\right)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(17) = 34.
time = 0.00, size = 98, normalized size = 5.76

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7}{2}c^6d^2x^{18} + 7c^5d^3x^{19} + \frac{35}{4}c^4d^4x^{20} + 7c^3d^5x^{21} + \frac{7}{2}c^2d^6x^{22} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [A]
time = 0.18, size = 16, normalized size = 0.94

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{(d\,x^3+c\,x^2)^8}{8}$
norman	$\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^{21}+\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}$
risch	$\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^{21}+\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x,method=_RETURNVERBOSE)
[Out] 1/8*(d*x^3+c*x^2)^8
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.88

$$\frac{1}{8}\left(dx^3+cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.
time = 0.37, size = 88, normalized size = 5.18

$$\frac{1}{8}d^8x^{24}+cd^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(12) = 24$.
time = 0.02, size = 97, normalized size = 5.71

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [A]
time = 4.29, size = 15, normalized size = 0.88

$$\frac{1}{8}\left(dx^3+cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Mupad [B]
time = 2.07, size = 88, normalized size = 5.18

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.200 $\int x^7(cx+dx^2)^7(2cx+3dx^2)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

[Out] 1/8*x^16*(d*x+c)^8

Rubi [A]
time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1598, 859}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 859

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[c*x^(m + 2)*((f + g*x)^(n + 1)/(g*(m + n + 3))), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^7\left(cx+dx^2\right)^7\left(2cx+3dx^2\right)\,dx &= \int x^{14}(c+dx)^7\left(2cx+3dx^2\right)\,dx\\ &= \frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.
time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.19, size = 89, normalized size = 6.36

method	result	si
gosper	$\frac{x^{16}(dx+c)^{8}}{8}$	1
default	$\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x^{21}+\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}$	8
norman	$\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x^{21}+\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}$	8
risch	$\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x^{21}+\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*c^8+c^7*d*x^17+7/2*x^18*c^6*d^2+7*c^5*d^3*x^19+35/4*x^20*c^4*d^4+7
*c^3*d^5*x^21+7/2*x^22*c^2*d^6+c*d^7*x^23+1/8*d^8*x^24
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.27, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^{8}x^{24}+cd^{7}x^{23}+\frac{7}{2}\,c^{2}d^{6}x^{22}+7\,c^{3}d^{5}x^{21}+\frac{35}{4}\,c^{4}d^{4}x^{20}+7\,c^{5}d^{3}x^{19}+\frac{7}{2}\,c^{6}d^{2}x^{18}+c^{7}dx^{17}+\frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.39, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^{8}x^{24}+cd^{7}x^{23}+\frac{7}{2}\,c^{2}d^{6}x^{22}+7\,c^{3}d^{5}x^{21}+\frac{35}{4}\,c^{4}d^{4}x^{20}+7\,c^{5}d^{3}x^{19}+\frac{7}{2}\,c^{6}d^{2}x^{18}+c^{7}dx^{17}+\frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.
time = 0.02, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [A]
time = 3.42, size = 15, normalized size = 1.07

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Mupad [B]
time = 0.05, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.201 $\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) \, dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {859}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 859

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[c*x^(m + 2)*((f + g*x)^(n + 1)/(g*(m + n + 3))), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) \, dx = \frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.
time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.19, size = 89, normalized size = 6.36

method	result
gosper	$\frac{x^{16}\left(d^{8} x^{8}+8 c \,d^{7} x^{7}+28 x^{6} c^{2} d^{6}+56 c^{3} d^{5} x^{5}+70 x^{4} c^{4} d^{4}+56 c^{5} d^{3} x^{3}+28 x^{2} c^{6} d^{2}+8 c^{7} d x +c^{8}\right)}{8}$
default	$\frac{1}{8} x^{16} c^{8}+c^{7} d \,x^{17}+\frac{7}{2} x^{18} c^{6} d^{2}+7 c^{5} d^{3} x^{19}+\frac{35}{4} x^{20} c^{4} d^{4}+7 c^{3} d^{5} x^{21}+\frac{7}{2} x^{22} c^{2} d^{6}+c \,d^{7} x^{23}+\frac{1}{8} d^{8} x^{24}$
norman	$\frac{1}{8} x^{16} c^{8}+c^{7} d \,x^{17}+\frac{7}{2} x^{18} c^{6} d^{2}+7 c^{5} d^{3} x^{19}+\frac{35}{4} x^{20} c^{4} d^{4}+7 c^{3} d^{5} x^{21}+\frac{7}{2} x^{22} c^{2} d^{6}+c \,d^{7} x^{23}+\frac{1}{8} d^{8} x^{24}$
risch	$\frac{1}{8} x^{16} c^{8}+c^{7} d \,x^{17}+\frac{7}{2} x^{18} c^{6} d^{2}+7 c^{5} d^{3} x^{19}+\frac{35}{4} x^{20} c^{4} d^{4}+7 c^{3} d^{5} x^{21}+\frac{7}{2} x^{22} c^{2} d^{6}+c \,d^{7} x^{23}+\frac{1}{8} d^{8} x^{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*c^8+c^7*d*x^17+7/2*x^18*c^6*d^2+7*c^5*d^3*x^19+35/4*x^20*c^4*d^4+7
*c^3*d^5*x^21+7/2*x^22*c^2*d^6+c*d^7*x^23+1/8*d^8*x^24
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.27, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^{8}x^{24} + cd^{7}x^{23} + \frac{7}{2}\,c^{2}d^{6}x^{22} + 7\,c^{3}d^{5}x^{21} + \frac{35}{4}\,c^{4}d^{4}x^{20} + 7\,c^{5}d^{3}x^{19} + \frac{7}{2}\,c^{6}d^{2}x^{18} + c^{7}dx^{17} + \frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.38, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^{8}x^{24} + cd^{7}x^{23} + \frac{7}{2}\,c^{2}d^{6}x^{22} + 7\,c^{3}d^{5}x^{21} + \frac{35}{4}\,c^{4}d^{4}x^{20} + 7\,c^{5}d^{3}x^{19} + \frac{7}{2}\,c^{6}d^{2}x^{18} + c^{7}dx^{17} + \frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.
time = 0.02, size = 97, normalized size = 6.93

$$\frac{c^{8}x^{16}}{8} + c^{7}dx^{17} + \frac{7c^{6}d^{2}x^{18}}{2} + 7c^{5}d^{3}x^{19} + \frac{35c^{4}d^{4}x^{20}}{4} + 7c^{3}d^{5}x^{21} + \frac{7c^{2}d^{6}x^{22}}{2} + cd^{7}x^{23} + \frac{d^{8}x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [A]
time = 4.77, size = 15, normalized size = 1.07

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

Mupad [B]
time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(2*c*x + 3*d*x^2)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.202 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

```
[Out] 1/8*(d*x^3+c*x^2+a)^8
```

Rubi [A]
time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1602}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(18) = 36.
time = 0.01, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8

Maple [A]
time = 0.03, size = 17, normalized size = 0.94

method	result
default	$\frac{(d\,x^3+c\,x^2+a)^8}{8}$
norman	$\frac{d^8x^{24}}{8} + \left(7ac\,d^6 + \frac{35}{4}c^4d^4\right)x^{20} + \left(d^7a + 7c^3d^5\right)x^{21} + \frac{7x^{22}c^2d^6}{2} + c\,d^7x^{23} + \left(21a^5c\,d^2 + \frac{35}{4}a^4c^4\right)x^8 + ($
gosper	$7a^3d^5x^{15} + c^7d\,x^{17} + 7c^5d^3x^{19} + \frac{7}{2}x^{18}c^6d^2 + \frac{35}{4}x^{20}c^4d^4 + 7a^5d^3x^9 + 21a^5c^2d\,x^7 + 35a^3c^4d\,x^{11} + \frac{7}{2}x$
risch	$7a^3d^5x^{15} + c^7d\,x^{17} + 7c^5d^3x^{19} + \frac{7}{2}x^{18}c^6d^2 + \frac{35}{4}x^{20}c^4d^4 + 7a^5d^3x^9 + 21a^5c^2d\,x^7 + 35a^3c^4d\,x^{11} + \frac{7}{2}x$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x,method=_RETURNVERBOSE)

[Out] 1/8*(d*x^3+c*x^2+a)^8

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(16) = 32$.
time = 0.27, size = 458, normalized size = 25.44

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + (7c^3d^5 + ad^7)x^{21} + \frac{7}{4}(5c^4d^4 + 4acd^6)x^{20} + 7(c^5d^3 + 3ac^2d^5)x^{19} + \frac{7}{2}(c^6d^2 + 10ac^3d^4 + a^2d^6)x^{18} + (c^7d + 35ac^4d^3 + 21a^2cd^5)x^{17} + \frac{1}{8}(c^8 + 168ac^5d^2 + 420a^2c^2d^4)x^{16} + 7(ac^6d + 10a^2c^3d^3 + a^3d^5)x^{15} + 21a^5c^2dx^7 + \frac{1}{2}(2ac^7 + 105a^2c^4d^2 + 70a^3cd^4)x^{14} + 7(3a^2c^5d + 10a^3c^2d^3)x^{13} + 7a^6cdx^5 + \frac{7}{4}(2a^2c^6 + 40a^3c^3d^2 + 5a^4d^4)x^{12} + \frac{7}{2}a^6c^2x^4 + 35(a^3c^4d + a^4cd^3)x^{11} + a^7dx^3 + \frac{7}{2}(2a^3c^5 + 15a^4c^2d^2)x^{10} + a^7cx^2 + 7(5a^4c^3d + a^5d^3)x^9 + \frac{7}{4}(5a^4c^4 + 12a^5cd^2)x^8 + \frac{7}{2}(2a^5c^3 + a^6d^2)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 + 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(16) = 32$.
time = 0.37, size = 458, normalized size = 25.44

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + (7c^3d^5 + ad^7)x^{21} + \frac{7}{4}(5c^4d^4 + 4acd^6)x^{20} + 7(c^5d^3 + 3ac^2d^5)x^{19} + \frac{7}{2}(c^6d^2 + 10ac^3d^4 + a^2d^6)x^{18} + (c^7d + 35ac^4d^3 + 21a^2cd^5)x^{17} + \frac{1}{8}(c^8 + 168ac^5d^2 + 420a^2c^2d^4)x^{16} + 7(ac^6d + 10a^2c^3d^3 + a^3d^5)x^{15} + 21a^5c^2dx^7 + \frac{1}{2}(2ac^7 + 105a^2c^4d^2 + 70a^3cd^4)x^{14} + 7(3a^2c^5d + 10a^3c^2d^3)x^{13} + 7a^6cdx^5 + \frac{7}{4}(2a^2c^6 + 40a^3c^3d^2 + 5a^4d^4)x^{12} + \frac{7}{2}a^6c^2x^4 + 35(a^3c^4d + a^4cd^3)x^{11} + a^7dx^3 + \frac{7}{2}(2a^3c^5 + 15a^4c^2d^2)x^{10} + a^7cx^2 + 7(5a^4c^3d + a^5d^3)x^9 + \frac{7}{4}(5a^4c^4 + 12a^5cd^2)x^8 + \frac{7}{2}(2a^5c^3 + a^6d^2)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7
/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6
*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)
*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*
a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4
*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c
*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^
4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c
^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 +
 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(14) = 28$.
time = 0.06, size = 484, normalized size = 26.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(16) = 32$.
time = 3.33, size = 488, normalized size = 27.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21
+ 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19 +
```

```
7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 35*
a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105
/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15
 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x
^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 35/4*a
^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^10 + 105/
2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 +
21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a
^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2
```

Mupad [B]
time = 0.57, size = 440, normalized size = 24.44

$$x^{12}\left(\frac{35\,a^4\,d^4}{4}+70\,a^3\,c^3\,d^2+\frac{7\,a^2\,c^6}{2}\right)+x^6\left(\frac{7\,a^6\,d^2}{2}+7\,a^5\,c^3\right)+x^{20}\left(\frac{35\,c^4\,d^4}{4}+7\,a\,c\,d^6\right)+x^{16}\left(\frac{105\,a^2\,c^2\,d^4}{2}+21\,a\,c^5\,d^2+\frac{c^8}{8}\right)+x^{18}\left(\frac{7\,a^2\,d^6}{2}+35\,a\,c^3\,d^4+\frac{7\,c^6\,d^2}{2}\right)+\frac{d^8\,x^{24}}{8}+x^{21}\left(7\,c^3\,d^5+a\,d^7\right)+a^7\,c\,x^2+a^7\,d\,x^3+c\,d^7\,x^{23}+\frac{7\,a^6\,c^2\,x^4}{2}+\frac{7\,c^2\,d^6\,x^{22}}{2}+21\,a^5\,c^2\,d\,x^7+7\,a\,d\,x^{15}\left(a^2\,d^4+10\,a\,c^3\,d^2+c^6\right)+c\,d\,x^{17}\left(21\,a^2\,d^4+35\,a\,c^3\,d^2+c^6\right)+\frac{7\,a^4\,c\,x^8\left(5\,c^3+12\,a\,d^2\right)}{4}+7\,a^4\,d\,x^9\left(5\,c^3+a\,d^2\right)+7\,c^2\,d^3\,x^{19}\left(c^3+3\,a\,d^2\right)+\frac{a\,c\,x^{14}\left(70\,a^2\,d^4+105\,a\,c^3\,d^2+2\,c^6\right)}{2}+7\,a^6\,c\,d\,x^5+\frac{7\,a^3\,c^2\,x^{10}\left(2\,c^3+15\,a\,d^2\right)}{2}+7\,a^2\,c^2\,d\,x^{13}\left(3\,c^3+10\,a\,d^2\right)+35\,a^3\,c\,d\,x^{11}\left(c^3+a\,d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (
7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*
d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3
*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c
*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a
*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*
c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3)
 + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c
^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*
c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

3.203 $\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

```
[Out] 1/8*x^16*(d*x+c)^8
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {1598, 75}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx&=\int x^{15}(c+dx)^7(2c+3dx)\,dx\\&=\frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.
time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

Maple [A]
time = 0.18, size = 16, normalized size = 1.14

method	result	si
gosper	$\frac{x^{16}(dx+c)^{8}}{8}$	1
default	$\frac{(d\,x^{3}+c\,x^{2})^{8}}{8}$	1
norman	$\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x^{21}+\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}$	8
risch	$\frac{1}{8}x^{16}c^{8}+c^{7}d\,x^{17}+\frac{7}{2}x^{18}c^{6}d^{2}+7c^{5}d^{3}x^{19}+\frac{35}{4}x^{20}c^{4}d^{4}+7c^{3}d^{5}x^{21}+\frac{7}{2}x^{22}c^{2}d^{6}+c\,d^{7}x^{23}+\frac{1}{8}d^{8}x^{24}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x^3+c*x^2)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.27, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^{8}x^{24}+cd^{7}x^{23}+\frac{7}{2}\,c^{2}d^{6}x^{22}+7\,c^{3}d^{5}x^{21}+\frac{35}{4}\,c^{4}d^{4}x^{20}+7\,c^{5}d^{3}x^{19}+\frac{7}{2}\,c^{6}d^{2}x^{18}+c^{7}dx^{17}+\frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.37, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^{8}x^{24}+cd^{7}x^{23}+\frac{7}{2}\,c^{2}d^{6}x^{22}+7\,c^{3}d^{5}x^{21}+\frac{35}{4}\,c^{4}d^{4}x^{20}+7\,c^{5}d^{3}x^{19}+\frac{7}{2}\,c^{6}d^{2}x^{18}+c^{7}dx^{17}+\frac{1}{8}\,c^{8}x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.
time = 0.02, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 3.41, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Mupad [B]
time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.204 $\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

[Out] 1/8*x^8*(d*x^2+c*x)^8

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {777}

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Antiderivative was successfully verified.

[In] Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]

[Out] (x^8*(c*x + d*x^2)^8)/8

Rule 777

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[g*(e*x)^m*((b*x + c*x^2)^(p + 1)/(c*(m + 2*p + 2))), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(18) = 36.
time = 0.00, size = 98, normalized size = 5.44

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.
time = 0.19, size = 89, normalized size = 4.94

method	result
gosper	$\frac{x^{16}(dx+c)^8}{8}$
default	$\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^{21}+\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}$
norman	$\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^{21}+\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}$
risch	$\frac{1}{8}x^{16}c^8+c^7d\,x^{17}+\frac{7}{2}x^{18}c^6d^2+7c^5d^3x^{19}+\frac{35}{4}x^{20}c^4d^4+7c^3d^5x^{21}+\frac{7}{2}x^{22}c^2d^6+c\,d^7x^{23}+\frac{1}{8}d^8x^{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*c^8+c^7*d*x^17+7/2*x^18*c^6*d^2+7*c^5*d^3*x^19+35/4*x^20*c^4*d^4+7
*c^3*d^5*x^21+7/2*x^22*c^2*d^6+c*d^7*x^23+1/8*d^8*x^24
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.
time = 0.27, size = 88, normalized size = 4.89

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.
time = 0.40, size = 88, normalized size = 4.89

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(14) = 28$.
time = 0.02, size = 97, normalized size = 5.39

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(16) = 32$.
time = 3.71, size = 88, normalized size = 4.89

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Mupad [B]
time = 0.04, size = 88, normalized size = 4.89

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x + d*x^2)^7*(2*c + 3*d*x),x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.205 $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

[Out] 1/8*x^16*(d*x+c)^8

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {75}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

[In] Int[x^15*(c + d*x)^7*(2*c + 3*d*x),x]

[Out] (x^16*(c + d*x)^8)/8

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx = \frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(14) = 28.
time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x),x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.19, size = 89, normalized size = 6.36

method	result	si
gosper	$\frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x^{21} + \frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24}$	8
default	$\frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x^{21} + \frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24}$	8
norman	$\frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x^{21} + \frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24}$	8
risch	$\frac{1}{8}x^{16}c^8 + c^7d\,x^{17} + \frac{7}{2}x^{18}c^6d^2 + 7c^5d^3x^{19} + \frac{35}{4}x^{20}c^4d^4 + 7c^3d^5x^{21} + \frac{7}{2}x^{22}c^2d^6 + c\,d^7x^{23} + \frac{1}{8}d^8x^{24}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(d*x+c)^7*(3*d*x+2*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^16*c^8+c^7*d*x^17+7/2*x^18*c^6*d^2+7*c^5*d^3*x^19+35/4*x^20*c^4*d^4+7
*c^3*d^5*x^21+7/2*x^22*c^2*d^6+c*d^7*x^23+1/8*d^8*x^24
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.27, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.39, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="fricas")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(10) = 20$.
time = 0.02, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.
time = 4.19, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

Mupad [B]
time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(2*c + 3*d*x)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

3.206 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal. Leaf size=28

$$ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5$$

```
[Out] a*x+1/2*b*x^2+1/160*x^5*(b*x+2*a)^5
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1605}

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx&=\text{Subst}\left(\int \left(1+x^4\right)\,dx,x,ax+\frac{bx^2}{2}\right)\\&=ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.
time = 0.00, size = 80, normalized size = 2.86

$$ax+\frac{bx^2}{2}+\frac{a^5x^5}{5}+\frac{1}{2}a^4bx^6+\frac{1}{2}a^3b^2x^7+\frac{1}{4}a^2b^3x^8+\frac{1}{16}ab^4x^9+\frac{b^5x^{10}}{160}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3*
x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

Maple [A]
time = 0.18, size = 25, normalized size = 0.89

method	result	size
default	$\frac{\left(a x+\frac{1}{2} b \,x^{2}\right)^{5}}{5}+a x+\frac{b \,x^{2}}{2}$	25
gosper	$\frac{x \left(b^{5} x^{9}+10 b^{4} a \,x^{8}+40 a^{2} b^{3} x^{7}+80 b^{2} a^{3} x^{6}+80 a^{4} b \,x^{5}+32 a^{5} x^{4}+80 b x+160 a \right)}{160}$	67
norman	$a x+\frac{1}{5} a^{5} x^{5}+\frac{1}{2} b \,x^{2}+\frac{1}{160} b^{5} x^{10}+\frac{1}{16} a \,b^{4} x^{9}+\frac{1}{4} a^{2} b^{3} x^{8}+\frac{1}{2} a^{3} b^{2} x^{7}+\frac{1}{2} a^{4} b \,x^{6}$	67
risch	$a x+\frac{1}{5} a^{5} x^{5}+\frac{1}{2} b \,x^{2}+\frac{1}{160} b^{5} x^{10}+\frac{1}{16} a \,b^{4} x^{9}+\frac{1}{4} a^{2} b^{3} x^{8}+\frac{1}{2} a^{3} b^{2} x^{7}+\frac{1}{2} a^{4} b \,x^{6}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(a*x+1/2*b*x^2)^5+a*x+1/2*b*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.
time = 0.27, size = 66, normalized size = 2.36

$$\frac{1}{160}\,b^{5}x^{10}+\frac{1}{16}\,ab^{4}x^{9}+\frac{1}{4}\,a^{2}b^{3}x^{8}+\frac{1}{2}\,a^{3}b^{2}x^{7}+\frac{1}{2}\,a^{4}bx^{6}+\frac{1}{5}\,a^{5}x^{5}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.
time = 0.36, size = 66, normalized size = 2.36

$$\frac{1}{160}\,b^{5}x^{10}+\frac{1}{16}\,ab^{4}x^{9}+\frac{1}{4}\,a^{2}b^{3}x^{8}+\frac{1}{2}\,a^{3}b^{2}x^{7}+\frac{1}{2}\,a^{4}bx^{6}+\frac{1}{5}\,a^{5}x^{5}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(22) = 44$.
time = 0.02, size = 70, normalized size = 2.50

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
```

```
[Out] a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4*
x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

Giac [A]
time = 3.46, size = 24, normalized size = 0.86

$$\frac{1}{160}\left(bx^2 + 2\,ax\right)^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.05, size = 66, normalized size = 2.36

$$\frac{a^5\,x^5}{5} + \frac{a^4\,b\,x^6}{2} + \frac{a^3\,b^2\,x^7}{2} + \frac{a^2\,b^3\,x^8}{4} + \frac{a\,b^4\,x^9}{16} + a\,x + \frac{b^5\,x^{10}}{160} + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (b^5*x^10)/160 + (a^4*b*x^6)/2 + (a*b^4*x^9
)/16 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4
```

3.207 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal. Leaf size=31

$$ax+\frac{bx^2}{2}+\frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5$$

```
[Out] a*x+1/2*b*x^2+1/5*(c+a*x+1/2*b*x^2)^5
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1605}

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx &= \text{Subst}\left(\int \left(1+x^4\right)\,dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(31) = 62.
time = 0.03, size = 108, normalized size = 3.48

$$\frac{1}{160}x(2a+bx)\left(80+80c^4+16a^4x^4+32a^3bx^5+24a^2b^2x^6+8ab^3x^7+b^4x^8+80c^3x(2a+bx)+40c^2x^2(2a+bx)^2+10cx^3(2a+bx)^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 +
8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 + 1
0*c*x^3*(2*a + b*x)^3))/160
```

Maple [A]
time = 0.26, size = 27, normalized size = 0.87

method	result
default	$\frac{\left(c+ax+\frac{1}{2}b\,x^{2}\right)^{5}}{5}+c+ax+\frac{b\,x^{2}}{2}$
norman	$\left(\frac{1}{4}a^{2}b^{3}+\frac{1}{16}b^{4}c\right)x^{8}+\left(\frac{1}{2}b^{2}a^{3}+\frac{1}{2}a\,b^{3}c\right)x^{7}+\left(\frac{1}{5}a^{5}+2c\,a^{3}b+\frac{3}{2}a\,b^{2}c^{2}\right)x^{5}+\left(2a^{2}c^{3}+\frac{1}{2}b\,c^{4}+\frac{1}{2}b\right)x^{2}$
gosper	$\frac{x\left(b^{5}x^{9}+10b^{4}a\,x^{8}+40a^{2}b^{3}x^{7}+10b^{4}c\,x^{7}+80b^{2}a^{3}x^{6}+80a\,b^{3}c\,x^{6}+80a^{4}b\,x^{5}+240a^{2}b^{2}c\,x^{5}+40b^{3}c^{2}x^{5}+32a^{5}x^{4}+320x^{4}c\,a^{3}b+240x^{4}a\,b^{2}c^{2}\right.}{160}$
risch	$\frac{1}{160}b^{5}x^{10}+\frac{1}{16}a\,b^{4}x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{16}x^{8}b^{4}c+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}x^{7}a\,b^{3}c+\frac{1}{2}a^{4}b\,x^{6}+\frac{3}{2}x^{6}a^{2}b^{2}c+\frac{1}{4}x^{6}b^{3}c^{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(c+a*x+1/2*b*x^2)^5+c+a*x+1/2*b*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.
time = 0.29, size = 187, normalized size = 6.03

$$\frac{1}{160}\,b^{5}x^{10}+\frac{1}{16}\,ab^{4}x^{9}+\frac{1}{16}\,(4\,a^{2}b^{3}+b^{4}c)x^{8}+\frac{1}{2}\,(a^{3}b^{2}+ab^{3}c)x^{7}+\frac{1}{4}\,(2\,a^{4}b+6\,a^{2}b^{2}c+b^{3}c^{2})x^{6}+\frac{1}{10}\,(2\,a^{5}+20\,a^{3}bc+15\,ab^{2}c^{2})x^{5}+\frac{1}{2}\,(2\,a^{4}c+6\,a^{2}bc^{2}+b^{2}c^{3})x^{4}+2\,(a^{3}c^{2}+abc^{3})x^{3}+\frac{1}{2}\,(4\,a^{2}c^{3}+bc^{4}+b)x^{2}+(ac^{4}+a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b
^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5
 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x
^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 +
 a)*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.
time = 0.37, size = 187, normalized size = 6.03

$$\frac{1}{160}\,b^{5}x^{10}+\frac{1}{16}\,ab^{4}x^{9}+\frac{1}{16}\,(4\,a^{2}b^{3}+b^{4}c)x^{8}+\frac{1}{2}\,(a^{3}b^{2}+ab^{3}c)x^{7}+\frac{1}{4}\,(2\,a^{4}b+6\,a^{2}b^{2}c+b^{3}c^{2})x^{6}+\frac{1}{10}\,(2\,a^{5}+20\,a^{3}bc+15\,ab^{2}c^{2})x^{5}+\frac{1}{2}\,(2\,a^{4}c+6\,a^{2}bc^{2}+b^{2}c^{3})x^{4}+2\,(a^{3}c^{2}+abc^{3})x^{3}+\frac{1}{2}\,(4\,a^{2}c^{3}+bc^{4}+b)x^{2}+(ac^{4}+a)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b
^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5
 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x
^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 +
 a)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(24) = 48$.
time = 0.04, size = 194, normalized size = 6.26

$$\frac{ab^4x^9}{16} + \frac{b^5x^{10}}{160} + x^8\left(\frac{a^2b^3}{4} + \frac{b^4c}{16}\right) + x^7\left(\frac{a^3b^2}{2} + \frac{ab^3c}{2}\right) + x^6\left(\frac{a^4b}{2} + \frac{3a^2b^2c}{2} + \frac{b^3c^2}{4}\right) + x^5\left(\frac{a^5}{5} + 2a^3bc + \frac{3ab^2c^2}{2}\right) + x^4\left(a^4c + 3a^2bc^2 + \frac{b^2c^3}{2}\right) + x^3\cdot\left(2a^3c^2 + 2abc^3\right) + x^2\cdot\left(2a^2c^3 + \frac{bc^4}{2} + \frac{b}{2}\right) + x\left(ac^4 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

```
[Out] a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a*
*3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4) +
 x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2) + x**4*(a**4*c + 3*a**2*b*c**
2 + b**2*c**3/2) + x**3*(2*a**3*c**2 + 2*a*b*c**3) + x**2*(2*a**2*c**3 + b*
c**4/2 + b/2) + x*(a*c**4 + a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.
time = 3.69, size = 88, normalized size = 2.84

$$\frac{1}{160}\left(bx^2 + 2\,ax\right)^5 + \frac{1}{16}\left(bx^2 + 2\,ax\right)^4 c + \frac{1}{4}\left(bx^2 + 2\,ax\right)^3 c^2 + \frac{1}{2}\left(bx^2 + 2\,ax\right)^2 c^3 + \frac{1}{2}\left(bx^2 + 2\,ax\right)c^4 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3*
c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.10, size = 180, normalized size = 5.81

$$x^6\left(\frac{a^4\,b}{2} + \frac{3\,a^2\,b^2\,c}{2} + \frac{b^3\,c^2}{4}\right) + x^4\left(a^4\,c + 3\,a^2\,b\,c^2 + \frac{b^2\,c^3}{2}\right) + x^2\left(2\,a^2\,c^3 + \frac{b\,c^4}{2} + \frac{b}{2}\right) + x^5\left(\frac{a^5}{5} + 2\,a^3\,b\,c + \frac{3\,a\,b^2\,c^2}{2}\right) + \frac{b^5\,x^{10}}{160} + x^8\left(\frac{a^2\,b^3}{4} + \frac{c\,b^4}{16}\right) + \frac{a\,b^4\,x^9}{16} + a\,x\,\left(c^4 + 1\right) + \frac{a\,b^2\,x^7\,\left(a^2 + b\,c\right)}{2} + 2\,a\,c^2\,x^3\,\left(a^2 + b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2
+ 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^2*
c^2)/2 + 2*a^3*b*c) + (b^5*x^10)/160 + x^8*((b^4*c)/16 + (a^2*b^3)/4) + (a*
b^4*x^9)/16 + a*x*(c^4 + 1) + (a*b^2*x^7*(b*c + a^2))/2 + 2*a*c^2*x^3*(b*c
+ a^2)
```

3.208 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal. Leaf size=34

$$ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1605}

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 34, normalized size = 1.00

$$\frac{x(2a+bx)\left(1+n+\left(ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Maple [A]
time = 0.18, size = 31, normalized size = 0.91

method	result	size
derivativedivides	$a x+\frac{b \,x^{2}}{2}+\frac{\left(a x+\frac{1}{2} b \,x^{2}\right)^{1+n}}{1+n}$	31
default	$a x+\frac{b \,x^{2}}{2}+\frac{\left(a x+\frac{1}{2} b \,x^{2}\right)^{1+n}}{1+n}$	31
risch	$a x+\frac{b \,x^{2}}{2}+\frac{x (b x+2 a) \left(\frac{1}{2}\right)^{n} \left(x (b x+2 a)\right)^{n}}{2+2 n}$	40
norman	$a x+\frac{a x \,{\mathrm e}^{n \ln \left(a x+\frac{1}{2} b \,x^{2}\right)}}{1+n}+\frac{b \,x^{2}}{2}+\frac{b \,x^{2} {\mathrm e}^{n \ln \left(a x+\frac{1}{2} b \,x^{2}\right)}}{2+2 n}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Maxima [A]
time = 0.53, size = 52, normalized size = 1.53

$$\frac{1}{2} \, b x^{2} + a x + \frac{{\left(b x^{2} + 2 \, a x\right)} e^{\left(n \log\left(b x + 2 \, a\right) + n \log\left(x\right)\right)}}{2^{n + 1} n + 2^{n + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))/(2^(n + 1
)*n + 2^(n + 1))
```

Fricas [A]
time = 0.37, size = 48, normalized size = 1.41

$$\frac{{\left(b n + b\right)} x^{2} + {\left(b x^{2} + 2 \, a x\right)} {\left(\frac{1}{2} \, b x^{2} + a x\right)}^{n} + 2 \, {\left(a n + a\right)} x}{2 \, {\left(n + 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/(
n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(26) = 52$.
time = 23.12, size = 228, normalized size = 6.71

$$\begin{cases} a\left(x+\frac{\log(x)}{a}\right) & \text{for } b=0 \wedge n=-1 \\ a\left(\frac{nx}{n+1}+\frac{x(ax)^n}{n+1}+\frac{x}{n+1}\right) & \text{for } b=0 \\ ax+\frac{bx^2}{2}+\log(x)+\log\left(\frac{2a}{b}+x\right) & \text{for } n=-1 \\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2\cdot 2^n abx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2 nx^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2 x^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2abx(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{b^2x^2(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)
```

```
[Out] Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(n*x/(n + 1) + x*(a*
x)**n/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a/b
 + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*x/(
2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**n*b*
*2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*b*n +
 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), True))
```

Giac [A]
time = 3.74, size = 30, normalized size = 0.88

$$\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.12, size = 31, normalized size = 0.91

$$\frac{x\,(2\,a+b\,x)\,\left(n+{\left(\frac{b\,x^2}{2}+a\,x\right)}^n+1\right)}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

```
[Out] (x*(2*a + b*x)*(n + (a*x + (b*x^2)/2)^n + 1))/(2*(n + 1))
```

3.209 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal. Leaf size=35

$$ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1605}

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.
time = 0.16, size = 73, normalized size = 2.09

$$\frac{2c\left(c+ax+\frac{bx^2}{2}\right)^n+2ax\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)+bx^2\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]

[Out] (2*c*(c + a*x + (b*x^2)/2)^n + 2*a*x*(1 + n + (c + a*x + (b*x^2)/2)^n) + b*x^2*(1 + n + (c + a*x + (b*x^2)/2)^n))/(2*(1 + n))

Maple [A]
time = 0.25, size = 33, normalized size = 0.94

method	result	size
derivativedivides	$c+ax+\frac{b\,x^{2}}{2}+\frac{\left(c+ax+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$	33
default	$c+ax+\frac{b\,x^{2}}{2}+\frac{\left(c+ax+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$	33
risch	$ax+\frac{b\,x^{2}}{2}+\frac{\left(b\,x^{2}+2ax+2c\right)\left(b\,x^{2}+2ax+2c\right)^{n}\left(\frac{1}{2}\right)^{n}}{2+2n}$	49
norman	$ax+\frac{c\,{\mathrm e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^{2}\right)}}{1+n}+\frac{ax\,{\mathrm e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^{2}\right)}}{1+n}+\frac{b\,x^{2}}{2}+\frac{b\,x^{2}{\mathrm e}^{n\ln\left(c+ax+\frac{1}{2}b\,x^{2}\right)}}{2+2n}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)

[Out] c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)

Maxima [A]
time = 0.50, size = 54, normalized size = 1.54

$$\frac{1}{2}\,bx^2+ax+\frac{(bx^2+2\,ax+2\,c)(bx^2+2\,ax+2\,c)^n}{2^{n+1}n+2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")

[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)*n + 2^(n + 1))

Fricas [A]
time = 0.39, size = 52, normalized size = 1.49

$$\frac{(bn+b)x^2+(bx^2+2\,ax+2\,c)\left(\frac{1}{2}\,bx^2+ax+c\right)^n+2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")

[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*n + a)*x)/(n + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(27) = 54$.
time = 118.66, size = 328, normalized size = 9.37

$$\begin{cases} a\left(x + \frac{\log\left(x + \frac{c}{a}\right)}{a}\right) & \text{for } b = 0 \wedge n = -1 \\ a\left(\frac{anx}{an+a} + \frac{ax(ax+c)^n}{an+a} + \frac{ax}{an+a} + \frac{c(ax+c)^n}{an+a}\right) & \text{for } b = 0 \\ ax + \frac{bx^2}{2} + \log\left(\frac{a}{b} + x - \frac{\sqrt{a^2-2bc}}{b}\right) + \log\left(\frac{a}{b} + x + \frac{\sqrt{a^2-2bc}}{b}\right) & \text{for } n = -1 \\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn + 2\cdot 2^n b} + \frac{2\cdot 2^n abx}{2\cdot 2^n bn + 2\cdot 2^n b} + \frac{2^n b^2 nx^2}{2\cdot 2^n bn + 2\cdot 2^n b} + \frac{2^n b^2 x^2}{2\cdot 2^n bn + 2\cdot 2^n b} + \frac{2abx(2ax+bx^2+2c)^n}{2\cdot 2^n bn + 2\cdot 2^n b} + \frac{b^2x^2(2ax+bx^2+2c)^n}{2\cdot 2^n bn + 2\cdot 2^n b} + \frac{2bc(2ax+bx^2+2c)^n}{2\cdot 2^n bn + 2\cdot 2^n b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

```
[Out] Piecewise((a*(x + log(x + c/a)/a), Eq(b, 0) & Eq(n, -1)), (a*(a*n*x/(a*n +
a) + a*x*(a*x + c)**n/(a*n + a) + a*x/(a*n + a) + c*(a*x + c)**n/(a*n + a))
, Eq(b, 0)), (a*x + b*x**2/2 + log(a/b + x - sqrt(a**2 - 2*b*c)/b) + log(a/
b + x + sqrt(a**2 - 2*b*c)/b), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*
2**n*b) + 2*2**n*a*b*x/(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b
*n + 2*2**n*b) + 2**n*b**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x +
b*x**2 + 2*c)**n/(2*2**n*b*n + 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2 + 2*c)
**n/(2*2**n*b*n + 2*2**n*b) + 2*b*c*(2*a*x + b*x**2 + 2*c)**n/(2*2**n*b*n +
 2*2**n*b), True))
```

Giac [A]
time = 4.59, size = 32, normalized size = 0.91

$$\frac{1}{2}bx^2 + ax + c + \frac{\left(\frac{1}{2}bx^2 + ax + c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.11, size = 58, normalized size = 1.66

$$a\,x + \left(\frac{b\,x^2}{2} + a\,x + c\right)^n\left(\frac{2\,c}{2\,n+2} + \frac{b\,x^2}{2\,n+2} + \frac{2\,a\,x}{2\,n+2}\right) + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

```
[Out] a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*x
)/(2*n + 2)) + (b*x^2)/2
```

3.210 $\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=30

$$ax+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1605}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int (1+x^5)\,dx, x, ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(30) = 60.
time = 0.01, size = 93, normalized size = 3.10

$$ax+\frac{cx^3}{3}+\frac{a^6x^6}{6}+\frac{1}{3}a^5cx^8+\frac{5}{18}a^4c^2x^{10}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}a^2c^4x^{14}+\frac{1}{243}ac^5x^{16}+\frac{c^6x^{18}}{4374}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*a
^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/4374
```

Maple [A]
time = 0.18, size = 25, normalized size = 0.83

method	result	si
default	$ax+\frac{c\,x^{3}}{3}+\frac{\left(ax+\frac{1}{3}c\,x^{3}\right)^{6}}{6}$	2
norman	$ax+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{10}{81}a^{3}c^{3}x^{12}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}a^{5}c\,x^{8}$	7
risch	$ax+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{10}{81}a^{3}c^{3}x^{12}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}a^{5}c\,x^{8}$	7
gosper	$\frac{x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+135a^{2}c^{4}x^{13}+540a^{3}c^{3}x^{11}+1215a^{4}c^{2}x^{9}+1458a^{5}c\,x^{7}+729a^{6}x^{5}+1458c\,x^{2}+4374a\right)}{4374}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/3*c*x^3+1/6*(a*x+1/3*c*x^3)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(24) = 48$.
time = 0.27, size = 77, normalized size = 2.57

$$\frac{1}{4374}\,c^{6}x^{18}+\frac{1}{243}\,ac^{5}x^{16}+\frac{5}{162}\,a^{2}c^{4}x^{14}+\frac{10}{81}\,a^{3}c^{3}x^{12}+\frac{5}{18}\,a^{4}c^{2}x^{10}+\frac{1}{3}\,a^{5}cx^{8}+\frac{1}{6}\,a^{6}x^{6}+\frac{1}{3}\,cx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(24) = 48$.
time = 0.36, size = 77, normalized size = 2.57

$$\frac{1}{4374}\,c^{6}x^{18}+\frac{1}{243}\,ac^{5}x^{16}+\frac{5}{162}\,a^{2}c^{4}x^{14}+\frac{10}{81}\,a^{3}c^{3}x^{12}+\frac{5}{18}\,a^{4}c^{2}x^{10}+\frac{1}{3}\,a^{5}cx^{8}+\frac{1}{6}\,a^{6}x^{6}+\frac{1}{3}\,cx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(22) = 44$.

time = 0.02, size = 87, normalized size = 2.90

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81
+ 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x**3
/3
```

Giac [A]

time = 3.88, size = 24, normalized size = 0.80

$$\frac{1}{4374}\left(cx^3 + 3\,ax\right)^6 + \frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x
```

Mupad [B]

time = 0.05, size = 77, normalized size = 2.57

$$\frac{a^6\,x^6}{6} + \frac{a^5\,c\,x^8}{3} + \frac{5\,a^4\,c^2\,x^{10}}{18} + \frac{10\,a^3\,c^3\,x^{12}}{81} + \frac{5\,a^2\,c^4\,x^{14}}{162} + \frac{a\,c^5\,x^{16}}{243} + a\,x + \frac{c^6\,x^{18}}{4374} + \frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)
```

```
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (a^5*c*x^8)/3 + (a*c^5*x^
16)/243 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162
```

3.211 $\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=31

$$ax+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/3*c*x^3+1/6*(d+a*x+1/3*c*x^3)^6
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.040$, Rules used = {1605}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int (1+x^5)\,dx, x, d+ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(31) = 62.
time = 0.04, size = 140, normalized size = 4.52

$$\frac{x(3a+cx^2)\left(1458+1458d^5+243a^5x^5+405a^4cx^7+270a^3c^2x^9+90a^2c^3x^{11}+15ac^4x^{13}+c^5x^{15}+1215d^4(3ax+cx^3)+540d^3(3ax+cx^3)^2+135d^2(3ax+cx^3)^3+18d(3ax+cx^3)^4\right)}{4374}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*c
^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c*x
^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*x +
 c*x^3)^4))/4374
```

Maple [A]
time = 0.10, size = 27, normalized size = 0.87

method	result
default	$\frac{\left(d+ax+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+d+ax+\frac{c\,x^{3}}{3}$
norman	$\left(\frac{10}{81}a^{3}c^{3}+\frac{5}{162}c^{4}d^{2}\right)x^{12}+\left(\frac{5}{18}a^{4}c^{2}+\frac{10}{27}a\,c^{3}d^{2}\right)x^{10}+\left(\frac{5}{2}a^{4}d^{2}+\frac{5}{3}ac\,d^{4}\right)x^{4}+\left(\frac{1}{3}a^{5}c+\frac{5}{3}a^{2}c^{2}d^{2}\right)x^{8}+($
risch	$\frac{1}{3}c\,x^{3}+\frac{1}{6}a^{6}x^{6}+\frac{5}{162}x^{12}c^{4}d^{2}+\frac{10}{81}x^{9}c^{3}d^{3}+ax+\frac{5}{81}a\,c^{4}d\,x^{13}+\frac{5}{2}a^{2}d^{4}x^{2}+\frac{1}{243}c^{5}d\,x^{15}+\frac{1}{3}a^{5}c\,x^{8}+\frac{5}{18}a$
gosper	$x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+18c^{5}d\,x^{14}+135a^{2}c^{4}x^{13}+270a\,c^{4}d\,x^{12}+540a^{3}c^{3}x^{11}+135x^{11}c^{4}d^{2}+1620a^{2}c^{3}d\,x^{10}+1215a^{4}c^{2}x^{9}+1620x^{9}a\,c^{3}d^{2}+4$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d+a*x+1/3*c*x^3)^6+d+a*x+1/3*c*x^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(27) = 54.
time = 0.27, size = 280, normalized size = 9.03

$$\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}ac^{5}x^{16}+\frac{1}{243}c^{5}dx^{15}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{5}{81}ac^{4}dx^{13}+\frac{10}{27}a^{2}c^{3}dx^{11}+\frac{5}{162}\left(4\,a^{3}c^{3}+c^{4}d^{2}\right)x^{12}+\frac{5}{54}\left(3\,a^{4}c^{2}+4\,ac^{3}d^{2}\right)x^{10}+\frac{10}{81}\left(9\,a^{3}c^{2}d+c^{3}d^{3}\right)x^{9}+\frac{1}{3}\left(a^{5}c+5\,a^{2}c^{2}d^{2}\right)x^{8}+\frac{5}{2}a^{2}d^{4}x^{2}+\frac{5}{9}\left(3\,a^{4}cd+2\,ac^{2}d^{3}\right)x^{7}+\frac{1}{18}\left(3\,a^{6}+60\,a^{3}cd^{2}+5\,c^{2}d^{4}\right)x^{6}+\frac{1}{3}\left(3\,a^{5}d+10\,a^{2}cd^{3}\right)x^{5}+\frac{5}{6}\left(3\,a^{4}d^{2}+2\,acd^{4}\right)x^{4}+\frac{1}{3}\left(10\,a^{3}d^{3}+cd^{5}+c\right)x^{3}+\left(ad^{5}+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^
12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^
9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*
a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d
 + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 +
c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(27) = 54.
time = 0.40, size = 280, normalized size = 9.03

$$\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}ac^{5}x^{16}+\frac{1}{243}c^{5}dx^{15}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{5}{81}ac^{4}dx^{13}+\frac{10}{27}a^{2}c^{3}dx^{11}+\frac{5}{162}\left(4\,a^{3}c^{3}+c^{4}d^{2}\right)x^{12}+\frac{5}{54}\left(3\,a^{4}c^{2}+4\,ac^{3}d^{2}\right)x^{10}+\frac{10}{81}\left(9\,a^{3}c^{2}d+c^{3}d^{3}\right)x^{9}+\frac{1}{3}\left(a^{5}c+5\,a^{2}c^{2}d^{2}\right)x^{8}+\frac{5}{2}a^{2}d^{4}x^{2}+\frac{5}{9}\left(3\,a^{4}cd+2\,ac^{2}d^{3}\right)x^{7}+\frac{1}{18}\left(3\,a^{6}+60\,a^{3}cd^{2}+5\,c^{2}d^{4}\right)x^{6}+\frac{1}{3}\left(3\,a^{5}d+10\,a^{2}cd^{3}\right)x^{5}+\frac{5}{6}\left(3\,a^{4}d^{2}+2\,acd^{4}\right)x^{4}+\frac{1}{3}\left(10\,a^{3}d^{3}+cd^{5}+c\right)x^{3}+\left(ad^{5}+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^
12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^
9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*
a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d
 + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 +
c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(24) = 48$.
time = 0.05, size = 314, normalized size = 10.13

$$\frac{5a^2c^4x^{14}}{162} + \frac{10a^2c^3dx^{11}}{27} + \frac{5a^2d^4x^2}{2} + \frac{ac^5x^{16}}{243} + \frac{5ac^4dx^{13}}{81} + \frac{c^6x^{18}}{4374} + \frac{c^5dx^{15}}{243} + x^{12} \cdot \left(\frac{10a^3c^3}{81} + \frac{5c^4d^2}{162}\right) + x^{10} \cdot \left(\frac{5a^4c^2}{18} + \frac{10ac^3d^2}{27}\right) + x^9 \cdot \left(\frac{10a^3c^2d}{9} + \frac{10c^3d^3}{81}\right) + x^8\left(\frac{a^5c}{3} + \frac{5a^2c^2d^2}{3}\right) + x^7 \cdot \left(\frac{5a^4cd}{3} + \frac{10ac^2d^3}{9}\right) + x^6\left(\frac{a^6}{6} + \frac{10a^3cd^2}{3} + \frac{5c^2d^4}{18}\right) + x^5\left(a^5d + \frac{10a^2cd^3}{3}\right) + x^4 \cdot \left(\frac{5a^4d^2}{2} + \frac{5acd^4}{3}\right) + x^3 \cdot \left(\frac{10a^3d^3}{3} + \frac{cd^5}{3} + \frac{c}{3}\right) + x(ad^5 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

```
[Out] 5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c*
*5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 + x
**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*c**
3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5
*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6
+ 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x
**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 + c/3)
 + x*(a*d**5 + a)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(27) = 54$.
time = 3.79, size = 105, normalized size = 3.39

$$\frac{1}{4374}\left(cx^3 + 3\,ax\right)^6 + \frac{1}{243}\left(cx^3 + 3\,ax\right)^5 d + \frac{5}{162}\left(cx^3 + 3\,ax\right)^4 d^2 + \frac{10}{81}\left(cx^3 + 3\,ax\right)^3 d^3 + \frac{5}{18}\left(cx^3 + 3\,ax\right)^2 d^4 + \frac{1}{3}\left(cx^3 + 3\,ax\right)d^5 + \frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*x
)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*(c
*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x
```

Mupad [**B**]
time = 2.27, size = 266, normalized size = 8.58

$$x^5\left(a^5\,d + \frac{10\,c\,a^2\,d^3}{3}\right) + x^4\left(\frac{5\,a^4\,d^2}{2} + \frac{5\,c\,a\,d^4}{3}\right) + x^3\left(\frac{10\,a^3\,d^3}{3} + \frac{c\,d^5}{3} + \frac{c}{3}\right) + x^6\left(\frac{a^6}{6} + \frac{10\,a^3\,c\,d^2}{3} + \frac{5\,c^2\,d^4}{18}\right) + \frac{c^6\,x^{18}}{4374} + \frac{a\,c^5\,x^{16}}{243} + a\,x\,\left(d^5 + 1\right) + \frac{c^5\,d\,x^{15}}{243} + \frac{5\,a^2\,c^4\,x^{14}}{162} + \frac{5\,a^2\,d^4\,x^2}{2} + \frac{5\,c^3\,x^{12}\,\left(4\,a^3 + c\,d^2\right)}{162} + \frac{a^2\,c\,x^8\,\left(a^3 + 5\,c\,d^2\right)}{3} + \frac{10\,a^2\,c^3\,d\,x^{11}}{27} + \frac{5\,a\,c^2\,x^{10}\,\left(3\,a^3 + 4\,c\,d^2\right)}{54} + \frac{10\,c^2\,d\,x^9\,\left(9\,a^3 + c\,d^2\right)}{81} + \frac{5\,a\,c^4\,d\,x^{13}}{81} + \frac{5\,a\,c\,d\,x^7\,\left(3\,a^3 + 2\,c\,d^2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2),x)
```

```
[Out] x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3*
(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^3*
c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*x^1
5)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2 + 4*
a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (5*a*c
^2*x^10*(4*c*d^2 + 3*a^3))/54 + (10*c^2*d*x^9*(c*d^2 + 9*a^3))/81 + (5*a*c^
4*d*x^13)/81 + (5*a*c*d*x^7*(2*c*d^2 + 3*a^3))/9
```

3.212 $\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=34

$$\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+1/279936*x^12*(2*c*x+3*b)^6
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1605}

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int (1+x^5)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(34) = 68.
time = 0.01, size = 98, normalized size = 2.88

$$\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{b^6x^{12}}{384}+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c^2x^{14}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{486}bc^5x^{17}+\frac{c^6x^{18}}{4374}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14)
/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (c^
6*x^18)/4374
```

Maple [A]
time = 0.22, size = 31, normalized size = 0.91

method	result
default	$\frac{\left(\frac{1}{2} b \,x^{2}+\frac{1}{3} c \,x^{3}\right)^{6}}{6}+\frac{b \,x^{2}}{2}+\frac{c \,x^{3}}{3}$
gosper	$\frac{x^{2} \left(64 c^{6} x^{16}+576 b \,c^{5} x^{15}+2160 b^{2} c^{4} x^{14}+4320 b^{3} c^{3} x^{13}+4860 b^{4} c^{2} x^{12}+2916 b^{5} c \,x^{11}+729 b^{6} x^{10}+93312 c x +139968 b \right)}{279936}$
norman	$\frac{1}{2} b \,x^{2}+\frac{1}{384} b^{6} x^{12}+\frac{1}{3} c \,x^{3}+\frac{1}{4374} c^{6} x^{18}+\frac{1}{486} b \,c^{5} x^{17}+\frac{5}{648} b^{2} c^{4} x^{16}+\frac{5}{324} b^{3} c^{3} x^{15}+\frac{5}{288} b^{4} c^{2} x^{14}+\frac{1}{96} b^{5} c \,x$
risch	$\frac{1}{2} b \,x^{2}+\frac{1}{384} b^{6} x^{12}+\frac{1}{3} c \,x^{3}+\frac{1}{4374} c^{6} x^{18}+\frac{1}{486} b \,c^{5} x^{17}+\frac{5}{648} b^{2} c^{4} x^{16}+\frac{5}{324} b^{3} c^{3} x^{15}+\frac{5}{288} b^{4} c^{2} x^{14}+\frac{1}{96} b^{5} c \,x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1/2*b*x^2+1/3*c*x^3)^6+1/2*b*x^2+1/3*c*x^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.
time = 0.26, size = 80, normalized size = 2.35

$$\frac{1}{4374} \, c^{6} x^{18} + \frac{1}{486} \, b c^{5} x^{17} + \frac{5}{648} \, b^{2} c^{4} x^{16} + \frac{5}{324} \, b^{3} c^{3} x^{15} + \frac{5}{288} \, b^{4} c^{2} x^{14} + \frac{1}{96} \, b^{5} c x^{13} + \frac{1}{384} \, b^{6} x^{12} + \frac{1}{3} \, c x^{3} + \frac{1}{2} \, b x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.
time = 0.39, size = 80, normalized size = 2.35

$$\frac{1}{4374} \, c^{6} x^{18} + \frac{1}{486} \, b c^{5} x^{17} + \frac{5}{648} \, b^{2} c^{4} x^{16} + \frac{5}{324} \, b^{3} c^{3} x^{15} + \frac{5}{288} \, b^{4} c^{2} x^{14} + \frac{1}{96} \, b^{5} c x^{13} + \frac{1}{384} \, b^{6} x^{12} + \frac{1}{3} \, c x^{3} + \frac{1}{2} \, b x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(27) = 54$.
time = 0.02, size = 90, normalized size = 2.65

$$\frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**1
5/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/43
74 + c*x**3/3
```

Giac [A]
time = 3.42, size = 30, normalized size = 0.88

$$\frac{1}{279936}\left(2\,cx^3 + 3\,bx^2\right)^6 + \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B]
time = 0.07, size = 80, normalized size = 2.35

$$\frac{b^6\,x^{12}}{384} + \frac{b^5\,c\,x^{13}}{96} + \frac{5\,b^4\,c^2\,x^{14}}{288} + \frac{5\,b^3\,c^3\,x^{15}}{324} + \frac{5\,b^2\,c^4\,x^{16}}{648} + \frac{b\,c^5\,x^{17}}{486} + \frac{b\,x^2}{2} + \frac{c^6\,x^{18}}{4374} + \frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (c^6*x^18)/4374 + (b^5*c*x^13)/96
+ (b*c^5*x^17)/486 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c
^4*x^16)/648
```

3.213 $\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=41

$$\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+1/6*(d+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = $\{1605\}$

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int (1+x^5)\,dx, x, d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(41) = 82.
time = 0.04, size = 146, normalized size = 3.56

$$\frac{x^2(3b+2cx)\left(46656+46656d^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+32c^5x^{15}+19440d^4x^2(3b+2cx)+4320d^3x^4(3b+2cx)^2+540d^2x^6(3b+2cx)^3+36dx^8(3b+2cx)^4\right)}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 108
0*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*d^
4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b + 2*c
*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936
```

Maple [A]
time = 0.05, size = 33, normalized size = 0.80

method	result
default	$\frac{\left(d+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+d+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}$
norman	$\left(\frac{5}{324}b^{3}c^{3}+\frac{1}{243}c^{5}d\right)x^{15}+\left(\frac{5}{12}b^{3}d^{3}+\frac{5}{18}c^{2}d^{4}\right)x^{6}+\left(\frac{5}{288}b^{4}c^{2}+\frac{5}{162}b\,c^{4}d\right)x^{14}+\left(\frac{5}{32}b^{4}d^{2}+\frac{5}{9}b\,c^{2}d^{3}\right)x^{8}$
gosper	$\frac{x^{2}\left(64c^{6}x^{16}+576b\,c^{5}x^{15}+2160b^{2}c^{4}x^{14}+4320b^{3}c^{3}x^{13}+1152c^{5}d\,x^{13}+4860b^{4}c^{2}x^{12}+8640b\,c^{4}d\,x^{12}+2916b^{5}c\,x^{11}+25920b^{2}c^{3}d\,x^{11}+729\right.}{}$
risch	$\frac{1}{3}c\,x^{3}+\frac{5}{162}x^{12}c^{4}d^{2}+\frac{10}{81}x^{9}c^{3}d^{3}+\frac{5}{48}x^{11}dc\,b^{4}+\frac{5}{27}x^{11}b\,c^{3}d^{2}+\frac{5}{12}x^{10}d^{2}c^{2}b^{2}+\frac{5}{12}x^{9}b^{3}c\,d^{2}+\frac{5}{9}x^{8}b\,c^{2}d^{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(d+1/2*b*x^2+1/3*c*x^3)^6+d+1/2*b*x^2+1/3*c*x^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(33) = 66.
time = 0.28, size = 289, normalized size = 7.05

$$\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}bc^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{1}{972}\left(15b^{3}c^{3}+4c^{5}d\right)x^{15}+\frac{5}{2592}\left(9b^{4}c^{2}+16bc^{4}d\right)x^{14}+\frac{1}{864}\left(9b^{5}c+80b^{2}c^{3}d\right)x^{13}+\frac{5}{6}b^{2}cd^{3}x^{7}+\frac{1}{10368}\left(27b^{6}+1440b^{3}c^{2}d+320c^{4}d^{2}\right)x^{12}+\frac{5}{432}\left(9b^{4}cd+16bc^{3}d^{2}\right)x^{11}+\frac{5}{6}bcd^{4}x^{5}+\frac{1}{96}\left(3b^{5}d+40b^{2}c^{2}d^{2}\right)x^{10}+\frac{5}{8}b^{2}d^{4}x^{4}+\frac{5}{324}\left(27b^{3}cd^{2}+8c^{3}d^{3}\right)x^{9}+\frac{5}{288}\left(9b^{4}d^{2}+32bc^{2}d^{3}\right)x^{8}+\frac{5}{36}\left(3b^{3}d^{3}+2c^{2}d^{4}\right)x^{6}+\frac{1}{3}\left(cd^{5}+c\right)x^{3}+\frac{1}{2}\left(bd^{5}+b\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^3
 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c +
80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*d +
 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^
5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*
c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3
*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(33) = 66.
time = 0.41, size = 289, normalized size = 7.05

$$\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}bc^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{1}{972}\left(15b^{3}c^{3}+4c^{5}d\right)x^{15}+\frac{5}{2592}\left(9b^{4}c^{2}+16bc^{4}d\right)x^{14}+\frac{1}{864}\left(9b^{5}c+80b^{2}c^{3}d\right)x^{13}+\frac{5}{6}b^{2}cd^{3}x^{7}+\frac{1}{10368}\left(27b^{6}+1440b^{3}c^{2}d+320c^{4}d^{2}\right)x^{12}+\frac{5}{432}\left(9b^{4}cd+16bc^{3}d^{2}\right)x^{11}+\frac{5}{6}bcd^{4}x^{5}+\frac{1}{96}\left(3b^{5}d+40b^{2}c^{2}d^{2}\right)x^{10}+\frac{5}{8}b^{2}d^{4}x^{4}+\frac{5}{324}\left(27b^{3}cd^{2}+8c^{3}d^{3}\right)x^{9}+\frac{5}{288}\left(9b^{4}d^{2}+32bc^{2}d^{3}\right)x^{8}+\frac{5}{36}\left(3b^{3}d^{3}+2c^{2}d^{4}\right)x^{6}+\frac{1}{3}\left(cd^{5}+c\right)x^{3}+\frac{1}{2}\left(bd^{5}+b\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^3
 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c +
80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*d +
 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^
5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*
c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3
*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(31) = 62$.
time = 0.05, size = 321, normalized size = 7.83

$$\frac{5b^2c^4x^{16}}{648} + \frac{5b^2cd^3x^7}{6} + \frac{5b^2d^4x^4}{8} + \frac{bc^5x^{17}}{486} + \frac{5bcd^4x^5}{6} + \frac{c^6x^{18}}{4374} + x^{15}\cdot\left(\frac{5b^3c^3}{324} + \frac{c^5d}{243}\right) + x^{14}\cdot\left(\frac{5b^4c^2}{288} + \frac{5bc^4d}{162}\right) + x^{13}\left(\frac{b^5c}{96} + \frac{5b^2c^3d}{54}\right) + x^{12}\left(\frac{b^6}{384} + \frac{5b^3c^2d}{36} + \frac{5c^4d^2}{162}\right) + x^{11}\cdot\left(\frac{5b^4cd}{48} + \frac{5bc^3d^2}{27}\right) + x^{10}\left(\frac{b^5d}{32} + \frac{5b^2c^2d^2}{12}\right) + x^9\cdot\left(\frac{5b^3cd^2}{12} + \frac{10c^3d^3}{81}\right) + x^8\cdot\left(\frac{5b^4d^2}{32} + \frac{5bc^2d^3}{9}\right) + x^6\cdot\left(\frac{5b^3d^3}{12} + \frac{5c^2d^4}{18}\right) + x^3\left(\frac{cd^5}{3} + \frac{c}{3}\right) + x^2\left(\frac{bd^5}{2} + \frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] 5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5*
x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324 +
c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/96 +
 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162)
+ x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2*c**2
*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*d**2/3
2 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3*(c*d**
5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(33) = 66$.
time = 3.85, size = 126, normalized size = 3.07

$$\frac{1}{279936}\left(2\,cx^3 + 3\,bx^2\right)^6 + \frac{1}{7776}\left(2\,cx^3 + 3\,bx^2\right)^5 d + \frac{5}{2592}\left(2\,cx^3 + 3\,bx^2\right)^4 d^2 + \frac{5}{324}\left(2\,cx^3 + 3\,bx^2\right)^3 d^3 + \frac{5}{72}\left(2\,cx^3 + 3\,bx^2\right)^2 d^4 + \frac{1}{6}\left(2\,cx^3 + 3\,bx^2\right) d^5 + \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/7776*(2*c*x^3 + 3*b*x^2)^5*d + 5/2592*(2
*c*x^3 + 3*b*x^2)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2)^3*d^3 + 5/72*(2*c*x^3 +
 3*b*x^2)^2*d^4 + 1/6*(2*c*x^3 + 3*b*x^2)*d^5 + 1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B]
time = 2.28, size = 273, normalized size = 6.66

$$x^{13}\left(\frac{b^5\,c}{96} + \frac{5\,d\,b^2\,c^3}{54}\right) + x^{14}\left(\frac{5\,b^4\,c^2}{288} + \frac{5\,d\,b\,c^4}{162}\right) + x^{12}\left(\frac{b^6}{384} + \frac{5\,b^3\,c^2\,d}{36} + \frac{5\,c^4\,d^2}{162}\right) + \frac{c^6\,x^{18}}{4374} + x^{15}\left(\frac{5\,b^3\,c^3}{324} + \frac{d\,c^5}{243}\right) + \frac{5\,d^3\,x^6\,(3\,b^3 + 2\,d\,c^2)}{36} + \frac{b\,c^5\,x^{17}}{486} + \frac{5\,b^2\,c^4\,x^{16}}{648} + \frac{b\,x^2\,(d^5 + 1)}{2} + \frac{5\,b^2\,d^4\,x^4}{8} + \frac{c\,x^3\,(d^5 + 1)}{3} + \frac{5\,b^2\,c\,d^3\,x^7}{6} + \frac{5\,b\,d^2\,x^8\,(9\,b^3 + 32\,d\,c^2)}{288} + \frac{b^2\,d\,x^{10}\,(3\,b^3 + 40\,d\,c^2)}{96} + \frac{5\,c\,d^2\,x^9\,(27\,b^3 + 8\,d\,c^2)}{324} + \frac{5\,b\,c\,d^4\,x^5}{6} + \frac{5\,b\,c\,d\,x^{11}\,(9\,b^3 + 16\,d\,c^2)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] x^13*((b^5*c)/96 + (5*b^2*c^3*d)/54) + x^14*((5*b^4*c^2)/288 + (5*b*c^4*d)/
162) + x^12*(b^6/384 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36) + (c^6*x^18)/437
4 + x^15*((c^5*d)/243 + (5*b^3*c^3)/324) + (5*d^3*x^6*(2*c^2*d + 3*b^3))/36
 + (b*c^5*x^17)/486 + (5*b^2*c^4*x^16)/648 + (b*x^2*(d^5 + 1))/2 + (5*b^2*d
^4*x^4)/8 + (c*x^3*(d^5 + 1))/3 + (5*b^2*c*d^3*x^7)/6 + (5*b*d^2*x^8*(32*c^
2*d + 9*b^3))/288 + (b^2*d*x^10*(40*c^2*d + 3*b^3))/96 + (5*c*d^2*x^9*(8*c^
2*d + 27*b^3))/324 + (5*b*c*d^4*x^5)/6 + (5*b*c*d*x^11*(16*c^2*d + 9*b^3))/
432
```

3.214 $\int (a+bx+cx^2) \left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx$

Optimal. Leaf size=46

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1605}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right) dx &= \text{Subst}\left(\int (1+x^5)\, dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(46) = 92.
time = 0.05, size = 244, normalized size = 5.30

$$\frac{a^6x^6}{6}+\frac{1}{6}a^5x^7(3b+2cx)+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5a^2x^{10}(3b+2cx)^4}{2592}+a\left(x+\frac{b^5x^{11}}{32}+\frac{5}{48}b^4cx^{12}+\frac{5}{36}b^3c^2x^{13}+\frac{5}{54}b^2c^3x^{14}+\frac{5}{162}bc^4x^{15}+\frac{c^5x^{16}}{243}\right)+\frac{x^2(729b^6x^{10}+2916b^5cx^{11}+4860b^4c^2x^{12}+4320b^3c^3x^{13}+2160b^2c^4x^{14}+576b(243+c^5x^{15})+64cx(1458+c^5x^{15}))}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 +
(5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(x
+ (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x^14
)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5
*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b
*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936
```

Maple [A]
time = 0.20, size = 37, normalized size = 0.80

method	result
default	$ax+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{\left(ax+\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3\right)^6}{6}$
norman	$ax+\left(\frac{1}{243}a\,c^5+\frac{5}{648}b^2c^4\right)x^{16}+\left(\frac{1}{3}a^5c+\frac{5}{8}a^4b^2\right)x^8+\left(\frac{5}{162}ab\,c^4+\frac{5}{324}b^3c^3\right)x^{15}+\left(\frac{5}{6}a^4bc+\frac{5}{12}a^3b^3\right)x$
risch	$\frac{1}{3}c\,x^3+\frac{1}{6}a^6x^6+\frac{1}{2}b\,x^2+ax+\frac{1}{96}b^5c\,x^{13}+\frac{1}{3}a^5c\,x^8+\frac{5}{18}a^4c^2x^{10}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}a^2c^4x^{14}+\frac{1}{243}a\,c$
gosper	$x(64c^6x^{17}+576b\,c^5x^{16}+1152a\,c^5x^{15}+2160b^2c^4x^{15}+8640ab\,c^4x^{14}+4320b^3c^3x^{14}+8640a^2c^4x^{13}+25920a\,b^2c^3x^{13}+4860b^4c^2x^{13}+518$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(37) = 74.
time = 0.27, size = 289, normalized size = 6.28

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}(15b^2c^4+8ac^5)x^{16}+\frac{5}{324}(b^3c^3+2abc^4)x^{15}+\frac{5}{2592}(9b^4c^2+48ab^2c^3+16a^2c^4)x^{14}+\frac{1}{864}(9b^5c+120ab^3c^2+160a^2bc^3)x^{13}+\frac{1}{2}a^5bx^7+\frac{1}{10368}(27b^6+1080ab^4c+4320a^2b^2c^2+1280a^3c^3)x^{12}+\frac{1}{6}a^6x^6+\frac{1}{288}(9ab^5+120a^2b^3c+160a^3bc^2)x^{11}+\frac{5}{288}(9a^2b^4+48a^3b^2c+16a^4c^2)x^{10}+\frac{5}{12}(a^3b^3+2a^4bc)x^9+\frac{1}{24}(15a^4b^2+8a^5c)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxim
a")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5
/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2
*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5
*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*
x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 +
 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4
*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(37) = 74$.
time = 0.37, size = 289, normalized size = 6.28

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}\left(15b^2c^4+8ac^5\right)x^{16}+\frac{5}{324}\left(b^3c^3+2abc^4\right)x^{15}+\frac{5}{2592}\left(9b^4c^2+48ab^2c^3+16a^2c^4\right)x^{14}+\frac{1}{864}\left(9b^5c+120ab^3c^2+160a^2bc^3\right)x^{13}+\frac{1}{2}a^5bx^7+\frac{1}{10368}\left(27b^6+1080ab^4c+4320a^2b^2c^2+1280a^3c^3\right)x^{12}+\frac{1}{6}a^6x^6+\frac{1}{288}\left(9ab^5+120a^2b^3c+160a^3bc^2\right)x^{11}+\frac{5}{288}\left(9a^2b^4+48a^3b^2c+16a^4c^2\right)x^{10}+\frac{5}{12}\left(a^3b^3+2a^4bc\right)x^9+\frac{1}{24}\left(15a^4b^2+8a^5c\right)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="frica
s")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5
/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2
*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5
*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*
x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 +
 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4
*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(36) = 72$.
time = 0.05, size = 323, normalized size = 7.02

$$\frac{a^6x^6}{6}+\frac{a^5bx^7}{2}+ax+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\cdot\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}\right)+x^{14}\cdot\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}\right)+x^{13}\cdot\left(\frac{5a^2bc^3}{27}+\frac{5ab^3c^2}{36}+\frac{b^5c}{96}\right)+x^{12}\cdot\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{b^6}{384}\right)+x^{11}\cdot\left(\frac{5a^3bc^2}{9}+\frac{5a^2b^3c}{12}+\frac{ab^5}{32}\right)+x^{10}\cdot\left(\frac{5a^4c^2}{18}+\frac{5a^3b^2c}{6}+\frac{5a^2b^4}{32}\right)+x^9\cdot\left(\frac{5a^4bc}{6}+\frac{5a^3b^3}{12}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^4b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**1
8/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**
4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b*
*4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x*
*12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x*
*11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/1
8 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12
) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

Giac [A]
time = 3.61, size = 37, normalized size = 0.80

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac"
)
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B]
time = 2.24, size = 270, normalized size = 5.87

$$x^{12}\left(\frac{10\,a^3\,c^3}{81}+\frac{5\,a^2\,b^2\,c^2}{12}+\frac{5\,a\,b^4\,c}{48}+\frac{b^6}{384}\right)+a\,x+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{a^6\,x^6}{6}+\frac{c^6\,x^{18}}{4374}+\frac{5\,a^2\,x^{10}\,\left(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4\right)}{288}+\frac{5\,c^2\,x^{14}\,\left(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4\right)}{2592}+\frac{a^5\,b\,x^7}{2}+\frac{b\,c^5\,x^{17}}{486}+\frac{a^4\,x^8\,\left(15\,b^2+8\,a\,c\right)}{24}+\frac{c^4\,x^{16}\,\left(15\,b^2+8\,a\,c\right)}{1944}+\frac{a\,b\,x^{11}\,\left(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4\right)}{288}+\frac{b\,c\,x^{13}\,\left(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4\right)}{864}+\frac{5\,a^3\,b\,x^9\,\left(b^2+2\,a\,c\right)}{12}+\frac{5\,b\,c^3\,x^{15}\,\left(b^2+2\,a\,c\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a*
x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(9*
b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 + 48*
a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 15*b^
2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2*c^2
 + 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/864 +
(5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324
```

3.215 $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=47

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6
```

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}=0.028$, Rules used = {1605}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int (1+x^5)\,dx, x, d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(47) = 94.
time = 0.09, size = 248, normalized size = 5.28

$$\frac{x(6a+x(3b+2cx))\left(46656+46656d^5+7776a^5x^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+32c^5x^{15}+6480a^4x^6(3b+2cx)+2160a^3x^7(3b+2cx)^2+360a^2x^8(3b+2cx)^3+30ax^9(3b+2cx)^4+19440d^4x(6a+x(3b+2cx))+4320d^3x^2(6a+x(3b+2cx))^2+540d^2x^3(6a+x(3b+2cx))^3+36dx^4(6a+x(3b+2cx))^4\right)}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^10
 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 +
 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2 +
360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a +
 x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6
*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

Maple [**A**]
time = 0.29, size = 39, normalized size = 0.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*(d+a*x+1/2*b*x^2+1/3*c*x^3)^6+d+a*x+1/2*b*x^2+1/3*c*x^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.
time = 0.29, size = 773, normalized size = 16.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="max
ima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1
/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^
2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +
480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a
*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8
*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b
*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^
2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b
^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b*
c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c +
32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*
b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1
/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15
*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 3
0*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2
```

```
+ 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 +
2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.
time = 0.38, size = 773, normalized size = 16.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fri
cas")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1
/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^
2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +
480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a
*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8
*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b
*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^
2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b
^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b*
c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c +
32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*
b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1
/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15
*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 3
0*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2
+ 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 +
2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(37) = 74$.
time = 0.10, size = 930, normalized size = 19.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648) +
x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c**4
/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a**2
*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/5
4) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*
```

```
c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*
b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2*c**2
*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 + 5*a**3*b
**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10*a*c**3*d
**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 + 5*a**3*b**
3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d*
*2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8
 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2
/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*c*d/3 + 5*
a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**2*d**3/9 + 5*b*
*2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2/3 + 15*a**2*b**
2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*
d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x*
*4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(1
0*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 + b*d*
*5/2 + b/2) + x*(a*d**5 + a)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(40) = 80.
time = 3.60, size = 153, normalized size = 3.26

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2+6\,ax\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2+6\,ax\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\,bx^2+6\,ax\right)^3d^3+\frac{5}{72}\left(2\,cx^3+3\,bx^2+6\,ax\right)^2d^4+\frac{1}{6}\left(2\,cx^3+3\,bx^2+6\,ax\right)d^5+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gia
c")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/7776*(2*c*x^3 + 3*b*x^2 + 6*a*x)
^5*d + 5/2592*(2*c*x^3 + 3*b*x^2 + 6*a*x)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2
+ 6*a*x)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2 + 6*a*x)^2*d^4 + 1/6*(2*c*x^3 + 3*
b*x^2 + 6*a*x)*d^5 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [**B**]
time = 2.45, size = 753, normalized size = 16.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^10*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10*
a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3) +
 x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*b*c
^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2) + x^9
*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/12 +
(5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/2) + x
```

```
^14*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4*d)/162
) + x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36 +
(5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27) + x^6*(a^6/6 + (5*
b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2)/4 + (5*a
^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3 + (5*
a*b*d^4)/2) + x^11*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^2)/9 + (10*a
^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^2*d)/6) + x^7
*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d^3)/9 + (5*b^2
*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d^5)/2 + (5*a^2*
d^4)/2) + x^13*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^3)/27 + (5*b^2*c
^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2 + 5*a^3*b*d^2 + (
10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d^2*x^4*(12*a^4 + 3
*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) + (b*c^5*x^17)/486 +
 (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4*x^16*(8*a*c + 15*b^2))
/1944
```

3.216 $\int (a+cx^2) \left(1+\left(ax+\frac{cx^3}{3}\right)^n\right) \, dx$

Optimal. Leaf size=34

$$ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1605\}$

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right) dx &= \text{Subst}\left(\int (1+x^n)\, dx, x, ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 36, normalized size = 1.06

$$\frac{x(3a+cx^2)\left(1+n+\left(ax+\frac{cx^3}{3}\right)^n\right)}{3(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

Maple [A]
time = 0.22, size = 31, normalized size = 0.91

method	result	size
derivativedivides	$a x +\frac{c \,x^{3}}{3}+\frac{\left(a x +\frac{1}{3} c \,x^{3}\right)^{1+n}}{1+n}$	31
default	$a x +\frac{c \,x^{3}}{3}+\frac{\left(a x +\frac{1}{3} c \,x^{3}\right)^{1+n}}{1+n}$	31
risch	$a x +\frac{c \,x^{3}}{3}+\frac{x \left(c \,x^{2}+3 a \right) \left(x \left(c \,x^{2}+3 a \right)\right)^{n} \left(\frac{1}{3}\right)^{n}}{3+3 n}$	44
norman	$a x +\frac{a x \,{\mathrm e}^{n \ln \left(a x +\frac{1}{3} c \,x^{3}\right)}}{1+n}+\frac{c \,x^{3}}{3}+\frac{c \,x^{3} {\mathrm e}^{n \ln \left(a x +\frac{1}{3} c \,x^{3}\right)}}{3+3 n}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A]
time = 0.52, size = 54, normalized size = 1.59

$$\frac{1}{3}\,cx^3+ax+\frac{(cx^3+3\,ax)e^{(n\log(cx^2+3\,a)+n\log(x))}}{3^{n+1}n+3^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")
[Out] 1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n +
 1)*n + 3^(n + 1))
```

Fricas [A]
time = 0.41, size = 48, normalized size = 1.41

$$\frac{(cn+c)x^3+(cx^3+3\,ax)\left(\frac{1}{3}\,cx^3+ax\right)^n+3\,(an+a)x}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")
[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/(
n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(26) = 52$.
time = 41.26, size = 190, normalized size = 5.59

$$\begin{cases} \frac{3\cdot 3^n anx}{3\cdot 3^n n+3\cdot 3^n} + \frac{3\cdot 3^n ax}{3\cdot 3^n n+3\cdot 3^n} + \frac{3^n cnx^3}{3\cdot 3^n n+3\cdot 3^n} + \frac{3^n cx^3}{3\cdot 3^n n+3\cdot 3^n} + \frac{3ax(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n} + \frac{cx^3(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n} & \text{for } n \neq -1 \\ ax + \frac{cx^3}{3} + \log(x) + \log\left(x - \sqrt{3}\sqrt{-\frac{a}{c}}\right) + \log\left(x + \sqrt{3}\sqrt{-\frac{a}{c}}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

```
[Out] Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**n
) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) + 3
*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**n/(
3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(x - sqrt(3)*
sqrt(-a/c)) + log(x + sqrt(3)*sqrt(-a/c)), True))
```

Giac [A]
time = 3.99, size = 30, normalized size = 0.88

$$\frac{1}{3}cx^3 + ax + \frac{\left(\frac{1}{3}cx^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

Mupad [B]
time = 2.11, size = 33, normalized size = 0.97

$$\frac{x\left(c\,x^2+3\,a\right)\,\left(n+{\left(\frac{c\,x^3}{3}+a\,x\right)}^n+1\right)}{3\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)
```

```
[Out] (x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))
```

3.217 $\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=44

$$\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

```
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.032$, Rules used = {1605}

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 42, normalized size = 0.95

$$\frac{x^2(3b+2cx)\left(1+n+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))
```

Maple [A]
time = 0.20, size = 37, normalized size = 0.84

method	result	size
derivativedivides	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$	37
default	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$	37
risch	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{x^{2}(2cx+3b)\left(\frac{1}{3}\right)^{n}\left(\frac{1}{2}\right)^{n}\left(x^{2}(2cx+3b)\right)^{n}}{6n+6}$	52
norman	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{b\,x^{2}\mathrm{e}^{n\ln\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{2+2n}+\frac{c\,x^{3}\mathrm{e}^{n\ln\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{3+3n}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A]
time = 0.53, size = 71, normalized size = 1.61

$$\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+\frac{(2\,cx^{3}+3\,bx^{2})e^{(n\log(2\,cx+3\,b)+2\,n\log(x))}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b) + 2*n*log
(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Fricas [A]
time = 0.39, size = 57, normalized size = 1.30

$$\frac{2\,(cn+c)x^{3}+3\,(bn+b)x^{2}+(2\,cx^{3}+3\,bx^{2})\left(\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}\right)^{n}}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 + 1
/2*b*x^2)^n)/(n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(32) = 64$.

time = 96.62, size = 189, normalized size = 4.30

$$\begin{cases} \frac{3 \cdot 6^{n} b n x^{2}}{6 \cdot 6^{n} n + 6 \cdot 6^{n}} + \frac{3 \cdot 6^{n} b x^{2}}{6 \cdot 6^{n} n + 6 \cdot 6^{n}} + \frac{2 \cdot 6^{n} c n x^{3}}{6 \cdot 6^{n} n + 6 \cdot 6^{n}} + \frac{2 \cdot 6^{n} c x^{3}}{6 \cdot 6^{n} n + 6 \cdot 6^{n}} + \frac{3 b x^{2} \left(3 b x^{2} + 2 c x^{3}\right)^{n}}{6 \cdot 6^{n} n + 6 \cdot 6^{n}} + \frac{2 c x^{3} \left(3 b x^{2} + 2 c x^{3}\right)^{n}}{6 \cdot 6^{n} n + 6 \cdot 6^{n}} & \text{for}\: n \neq -1 \\ \frac{b x^{2}}{2} + \frac{c x^{3}}{3} + 2 \log{\left(x \right)} + \log{\left(\frac{3 b}{2 c} + x \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Piecewise((3*6**n*b*n*x**2/(6*6**n*n + 6*6**n) + 3*6**n*b*x**2/(6*6**n*n +
6*6**n) + 2*6**n*c*n*x**3/(6*6**n*n + 6*6**n) + 2*6**n*c*x**3/(6*6**n*n + 6
*6**n) + 3*b*x**2*(3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n) + 2*c*x**3*(
3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n), Ne(n, -1)), (b*x**2/2 + c*x**3
/3 + 2*log(x) + log(3*b/(2*c) + x), True))
```

Giac [A]

time = 3.70, size = 36, normalized size = 0.82

$$\frac{1}{3} \, c x^{3} + \frac{1}{2} \, b x^{2} + \frac{{\left(\frac{1}{3} \, c x^{3} + \frac{1}{2} \, b x^{2}\right)}^{n + 1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)
```

Mupad [B]

time = 2.21, size = 37, normalized size = 0.84

$$\frac{x^2\,\left(3\,b+2\,c\,x\right)\,\left(n+{\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}\right)}^n+1\right)}{6\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)
```

```
[Out] (x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))
```

3.218 $\int (a+bx+cx^2) \left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) dx$

Optimal. Leaf size=50

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1605}

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) dx = \text{Subst}\left(\int (1+x^n)\,dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$
$$= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

Mathematica [A]
time = 0.19, size = 49, normalized size = 0.98

$$\frac{x(6a+x(3b+2cx))\left(1+n+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n))/(6*(1
 + n))
```

Maple [A]
time = 0.04, size = 43, normalized size = 0.86

method	result	size
derivativedivides	$a x +\frac{b \,x^{2}}{2}+\frac{c \,x^{3}}{3}+\frac{\left(a x +\frac{1}{2} b \,x^{2}+\frac{1}{3} c \,x^{3}\right)^{1+n}}{1+n}$	43
default	$a x +\frac{b \,x^{2}}{2}+\frac{c \,x^{3}}{3}+\frac{\left(a x +\frac{1}{2} b \,x^{2}+\frac{1}{3} c \,x^{3}\right)^{1+n}}{1+n}$	43
risch	$a x +\frac{b \,x^{2}}{2}+\frac{c \,x^{3}}{3}+\frac{x \left(2 c \,x^{2}+3 b x +6 a \right) \left(\frac{1}{3}\right)^{n} \left(\frac{1}{2}\right)^{n} \left(x \left(2 c \,x^{2}+3 b x +6 a \right)\right)^{n}}{6 n +6}$	63
norman	$a x +\frac{a x \,{\mathrm e}^{n \ln \left(a x +\frac{1}{2} b \,x^{2}+\frac{1}{3} c \,x^{3}\right)}}{1+n}+\frac{b \,x^{2}}{2}+\frac{c \,x^{3}}{3}+\frac{b \,x^{2} {\mathrm e}^{n \ln \left(a x +\frac{1}{2} b \,x^{2}+\frac{1}{3} c \,x^{3}\right)}}{2+2 n}+\frac{c \,x^{3} {\mathrm e}^{n \ln \left(a x +\frac{1}{2} b \,x^{2}+\frac{1}{3} c \,x^{3}\right)}}{3+3 n}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Maxima [A]
time = 0.53, size = 83, normalized size = 1.66

$$\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax+\frac{(2\,cx^{3}+3\,bx^{2}+6\,ax)e^{(n\log(2\,cx^{2}+3\,bx+6\,a)+n\log(x))}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxim
a")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2
+ 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Fricas [A]
time = 0.40, size = 72, normalized size = 1.44

$$\frac{2\,(cn+c)x^{3}+3\,(bn+b)x^{2}+(2\,cx^{3}+3\,bx^{2}+6\,ax)\left(\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax\right)^{n}+6\,(an+a)x}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*c
*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
[Out] Timed out
```

Giac [**A**]
time = 3.02, size = 42, normalized size = 0.84

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac"
)
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

Mupad [**B**]
time = 2.17, size = 73, normalized size = 1.46

$$a\,x+\left(\frac{3\,b\,x^2}{6\,n+6}+\frac{2\,c\,x^3}{6\,n+6}+\frac{6\,a\,x}{6\,n+6}\right)\,{\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x\right)}^n+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)
[Out] a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x
+ (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3
```

3.219 $\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx$

Optimal. Leaf size=19

$$\frac{1}{6}(5-12x+6x^2+x^3)^2$$

```
[Out] 1/6*(x^3+6*x^2-12*x+5)^2
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1602}

$$\frac{1}{6}(x^3+6x^2-12x+5)^2$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx = \frac{1}{6}(5-12x+6x^2+x^3)^2$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.74

$$-20x+34x^2-\frac{67x^3}{3}+2x^4+2x^5+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

Maple [A]
time = 0.03, size = 18, normalized size = 0.95

method	result	size
default	$\frac{\left(x^3+6x^2-12x+5\right)^2}{6}$	18
gosper	$\frac{x\left(x^5+12x^4+12x^3-134x^2+204x-120\right)}{6}$	27
norman	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$	30
risch	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x+\frac{25}{6}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x,method=_RETURNVERBOSE)
[Out] 1/6*(x^3+6*x^2-12*x+5)^2
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.89

$$\frac{1}{6}\left(x^3+6\,x^2-12\,x+5\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")
[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

Fricas [A]
time = 0.40, size = 29, normalized size = 1.53

$$\frac{1}{6}\,x^6+2\,x^5+2\,x^4-\frac{67}{3}\,x^3+34\,x^2-20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

Sympy [A]
time = 0.01, size = 29, normalized size = 1.53

$$\frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
```

```
[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

Giac [**A**]
time = 3.74, size = 30, normalized size = 1.58

$$\frac{5}{3}\,x^3+\frac{1}{6}\,\left(x^3+6\,x^2-12\,x\right)^2+10\,x^2-20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x
```

Mupad [**B**]
time = 0.03, size = 29, normalized size = 1.53

$$\frac{x^6}{6}+2\,x^5+2\,x^4-\frac{67\,x^3}{3}+34\,x^2-20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)
```

```
[Out] 34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

3.220 $\int (2x + x^3)(1 + 4x^2 + x^4)\, dx$

Optimal. Leaf size=16

$$\frac{1}{8}(1+4x^2+x^4)^2$$

[Out] 1/8*(x^4+4*x^2+1)^2

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1602}

$$\frac{1}{8}(x^4+4x^2+1)^2$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] (1 + 4*x^2 + x^4)^2/8

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (2x+x^3)(1+4x^2+x^4)\, dx = \frac{1}{8}(1+4x^2+x^4)^2$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.31

$$x^2+\frac{9x^4}{4}+x^6+\frac{x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8

Maple [A]
time = 0.06, size = 15, normalized size = 0.94

method	result	size
default	$\frac{\left(x^{4}+4x^{2}+1\right)^{2}}{8}$	15
norman	$\frac{1}{8}x^{8}+x^{6}+\frac{9}{4}x^{4}+x^{2}$	18
risch	$\frac{1}{8}x^{8}+x^{6}+\frac{9}{4}x^{4}+x^{2}+\frac{1}{8}$	19
gosper	$\frac{x^{2}\left(x^{6}+8x^{4}+18x^{2}+8\right)}{8}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)*(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/8*(x^4+4*x^2+1)^2
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$\frac{1}{8}\left(x^{4}+4\,x^{2}+1\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")
[Out] 1/8*(x^4 + 4*x^2 + 1)^2
```

Fricas [A]
time = 0.37, size = 17, normalized size = 1.06

$$\frac{1}{8}\,x^{8}+x^{6}+\frac{9}{4}\,x^{4}+x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

Sympy [A]
time = 0.01, size = 17, normalized size = 1.06

$$\frac{x^{8}}{8}+x^{6}+\frac{9x^{4}}{4}+x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)*(x**4+4*x**2+1),x)
```

```
[Out] x**8/8 + x**6 + 9*x**4/4 + x**2
```

Giac [**A**]
time = 4.22, size = 22, normalized size = 1.38

$$\frac{1}{4}x^4 + \frac{1}{8}\left(x^4 + 4x^2\right)^2 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2
```

Mupad [**B**]
time = 0.03, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^3)*(4*x^2 + x^4 + 1),x)
```

```
[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

3.221 $\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2 dx$

Optimal. Leaf size=33

$$81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

```
[Out] 81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33)=66$.
time = 0.13, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1607, 1626}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2 dx &= \int x^3(1+x)^3(1+2x)\left(-18+7\left(x+x^2\right)^3\right)^2 dx\\ &= \int \left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8\right.\\ &= 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x}{10}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.
time = 0.00, size = 96, normalized size = 2.91

$$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Maple [A]
time = 0.23, size = 29, normalized size = 0.88

method	result
default	$\frac{49(x^2+x)^{10}}{10}-36(x^2+x)^7+81(x^2+x)^4$
gosper	$\frac{(1+x)^3(49x^{13}+343x^{12}+1029x^{11}+1715x^{10}+1715x^9+1029x^8-17x^7-1391x^6-2160x^5-1440x^4-360x^3+810x+810)x^4}{10}$
norman	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+993$
risch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+993$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 49/10*(x^2+x)^10-36*(x^2+x)^7+81*(x^2+x)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.
time = 0.27, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.
time = 0.37, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(31) = 62$.
time = 0.02, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Giac [A]
time = 3.30, size = 28, normalized size = 0.85

$$\frac{49}{10}\left(x^2+x\right)^{10} - 36\left(x^2+x\right)^7 + 81\left(x^2+x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

Mupad [B]
time = 0.22, size = 86, normalized size = 2.61

$$\frac{49\,x^{20}}{10} + 49\,x^{19} + \frac{441\,x^{18}}{2} + 588\,x^{17} + 1029\,x^{16} + \frac{6174\,x^{15}}{5} + 993\,x^{14} + 336\,x^{13} - \frac{1071\,x^{12}}{2} - 1211\,x^{11} - \frac{12551\,x^{10}}{10} - 756\,x^9 - 171\,x^8 + 288\,x^7 + 486\,x^6 + 324\,x^5 + 81\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.222 $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

Optimal. Leaf size=33

$$81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

```
[Out] 81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.
time = 0.10, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.036$, Rules used = {1626}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx=\int\left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804\right.$$

$$=81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{1255}{10}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.
time = 0.00, size = 96, normalized size = 2.91

$$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.
time = 0.22, size = 87, normalized size = 2.64

method	result
gosper	$\frac{x^4(49x^{16}+490x^{15}+2205x^{14}+5880x^{13}+10290x^{12}+12348x^{11}+9930x^{10}+3360x^9-5355x^8-12110x^7-12551x^6-7560x^5-1710x^4+2880}{10}$
default	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+993x$
norman	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+993x$
risch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}+993x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^3*(2*x+1)*(-18+7*x^3*(1+x)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 81*x^4+324*x^5+486*x^6+288*x^7-171*x^8-756*x^9-12551/10*x^10-1211*x^11-1071
/2*x^12+336*x^13+993*x^14+6174/5*x^15+1029*x^16+588*x^17+441/2*x^18+49*x^19
+49/10*x^20
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.
time = 0.27, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.
time = 0.34, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(31) = 62$.
time = 0.02, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Giac [A]
time = 3.95, size = 28, normalized size = 0.85

$$\frac{49}{10}\left(x^2+x\right)^{10} - 36\left(x^2+x\right)^7 + 81\left(x^2+x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

Mupad [B]
time = 0.19, size = 86, normalized size = 2.61

$$\frac{49\,x^{20}}{10} + 49\,x^{19} + \frac{441\,x^{18}}{2} + 588\,x^{17} + 1029\,x^{16} + \frac{6174\,x^{15}}{5} + 993\,x^{14} + 336\,x^{13} - \frac{1071\,x^{12}}{2} - 1211\,x^{11} - \frac{12551\,x^{10}}{10} - 756\,x^9 - 171\,x^8 + 288\,x^7 + 486\,x^6 + 324\,x^5 + 81\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

3.223 $\int \frac{2-x^2}{(1-6x+x^3)^5} \, dx$

Optimal. Leaf size=14

$$\frac{1}{12(1-6x+x^3)^4}$$

```
[Out] 1/12/(x^3-6*x+1)^4
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1602}

$$\frac{1}{12(x^3-6x+1)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{2-x^2}{(1-6x+x^3)^5} \, dx = \frac{1}{12(1-6x+x^3)^4}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{12(1-6x+x^3)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{1}{12(x^3-6x+1)^4}$	13
default	$\frac{1}{12(x^3-6x+1)^4}$	13
norman	$\frac{1}{12(x^3-6x+1)^4}$	13
risch	$\frac{1}{12(x^3-6x+1)^4}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)/(x^3-6*x+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/12/(x^3-6*x+1)^4
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{1}{12\,(x^3-6\,x+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")
[Out] 1/12/(x^3 - 6*x + 1)^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.
time = 0.37, size = 57, normalized size = 4.07

$$\frac{1}{12\,(x^{12}-24\,x^{10}+4\,x^9+216\,x^8-72\,x^7-858\,x^6+432\,x^5+1224\,x^4-860\,x^3+216\,x^2-24\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")
[Out] 1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224*
x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(12) = 24.
time = 0.09, size = 56, normalized size = 4.00

$$\frac{1}{12x^{12}-288x^{10}+48x^9+2592x^8-864x^7-10296x^6+5184x^5+14688x^4-10320x^3+2592x^2-288x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)/(x**3-6*x+1)**5,x)
```

```
[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 518
4*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)
```

Giac [A]
time = 3.41, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

Mupad [B]
time = 2.10, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)
```

```
[Out] 1/(12*(x^3 - 6*x + 1)^4)
```

3.224 $\int \frac{2x+x^2}{4+3x^2+x^3} dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1601}

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{1}{3}\log\left(4+3x^2+x^3\right)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
[Out] Log[4 + 3*x^2 + x^3]/3
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\ln(x^3+3x^2+4)}{3}$	14
norman	$\frac{\ln(x^3+3x^2+4)}{3}$	14
risch	$\frac{\ln(x^3+3x^2+4)}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x^3+3*x^2+4)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Fricas [A]
time = 0.42, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.80

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
[Out] log(x**3 + 3*x**2 + 4)/3
```

Giac [A]
time = 4.13, size = 14, normalized size = 0.93

$$\frac{1}{3}\,\log\left(\left|x^3+3\,x^2+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

Mupad [B]
time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^3+3\,x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
[Out] log(3*x^2 + x^3 + 4)/3
```

3.225 $\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

```
[Out] 1/4*ln(x^4+2*x^2+4*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1601}

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.18

$$\frac{\log(x)}{4}+\frac{1}{4}\log\left(4+2x+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\ln(x(x^3+2x+4))}{4}$	14
risch	$\frac{\ln(x^4+2x^2+4x)}{4}$	16
norman	$\frac{\ln(x)}{4}+\frac{\ln(x^3+2x+4)}{4}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x*(x^3+2*x+4))
```

Maxima [A]
time = 0.26, size = 15, normalized size = 0.88

$$\frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Fricas [A]
time = 0.38, size = 15, normalized size = 0.88

$$\frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4+2x^2+4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

Giac [A]
time = 2.97, size = 18, normalized size = 1.06

$$\frac{1}{4}\,\log\left(4\left|\frac{1}{4}\,x^{4}+\frac{1}{2}\,x^{2}+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
```

```
[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

Mupad [B]
time = 0.07, size = 13, normalized size = 0.76

$$\frac{\ln\left(x\,\left(x^{3}+2\,x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
```

```
[Out] log(x*(2*x + x^3 + 4))/4
```

3.226 $\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{\left(c+dx+ex^2+fx^3\right)^2}\,dx$

Optimal. Leaf size=40

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

```
[Out] a/(f*x^3+e*x^2+d*x+c)+b*x/(f*x^3+e*x^2+d*x+c)
```

Rubi [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {6, 2127, 1602}

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x]
```

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq, x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2127

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn, x, n])), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m + n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn + (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx = \int \frac{bc-ad-2aex+(-be-3af)x^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx$$
$$= \frac{bx}{c+dx+ex^2+fx^3} - \frac{\int \frac{2adf+4aefx+6af^2x^2}{(c+dx+ex^2+fx^3)^2}\,dx}{2f}$$
$$= \frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.58

$$\frac{a+bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x
+ e*x^2 + f*x^3)^2,x]
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.70

method	result	size
gosper	$\frac{bx+a}{f\,x^3+e\,x^2+dx+c}$	24
norman	$\frac{bx+a}{f\,x^3+e\,x^2+dx+c}$	24
risch	$\frac{bx+a}{f\,x^3+e\,x^2+dx+c}$	24
default	$-\frac{-bx-a}{f\,x^3+e\,x^2+dx+c}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x,
method=_RETURNVERBOSE)
```

```
[Out] -(-b*x-a)/(f*x^3+e*x^2+d*x+c)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.60

$$\frac{bx+a}{fx^3+x^2e+dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)/(f*x^3 + x^2*e + d*x + c)
```

Fricas [A]
time = 0.39, size = 23, normalized size = 0.58

$$\frac{bx+a}{fx^3+ex^2+dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="fricas")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

Sympy [A]
time = 15.05, size = 22, normalized size = 0.55

$$-\frac{-a-bx}{c+dx+ex^2+fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+
d*x+c)**2,x)
```

```
[Out] -(-a - b*x)/(c + d*x + e*x**2 + f*x**3)
```

Giac [A]
time = 4.10, size = 24, normalized size = 0.60

$$\frac{bx+a}{fx^3+x^2e+dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/(f*x^3 + x^2*e + d*x + c)
```

Mupad [B]
time = 0.11, size = 23, normalized size = 0.58

$$\frac{a+b\,x}{f\,x^3+e\,x^2+d\,x+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*d - b*c + 2*a*e*x + 3*a*f*x^2 + b*e*x^2 + 2*b*f*x^3)/(c + d*x + e*x
^2 + f*x^3)^2,x)
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

3.227 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$

Optimal. Leaf size=605

$$\frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-4ac}\,C+2cD\right)\right)}{\sqrt{2}\,a\sqrt{8a^2+b^2-4ac}\,\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2+b^2-\right.}}}$$

```
[Out] -1/4*ln(2*a+2*a*x^2+x*(b-(8*a^2-4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b-(8*a^2-4
*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c+b^2)^(1/2)+1/4*ln(2*a+2*a*x^2+x*(b+(8*a^2-
4*a*c+b^2)^(1/2)))*(2*a*(A-C)+D*(b+(8*a^2-4*a*c+b^2)^(1/2)))/a/(8*a^2-4*a*c
+b^2)^(1/2)+1/2*arctan(1/2*(b+4*a*x-(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4*a^2
+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b-(8*a^2-4*a*c+b
^2)^(1/2))-a*(b*C+2*c*D+A*(b-(8*a^2-4*a*c+b^2)^(1/2))-C*(8*a^2-4*a*c+b^2)^(
1/2)))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2
)^(1/2)))^(1/2)-1/2*arctan(1/2*(b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))*2^(1/2)/(4
*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*(4*a^2*B+b*D*(b+(8*a^2-4*a
*c+b^2)^(1/2))-a*(b*C+2*c*D+C*(8*a^2-4*a*c+b^2)^(1/2)+A*(b+(8*a^2-4*a*c+b^2
)^(1/2))))/a*2^(1/2)/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c
+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 3.29, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2111, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2B\right)}{\sqrt{2}\,a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}-\frac{\text{ArcTan}\left(\frac{\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}\right)\left(-a\left(A\left(\sqrt{8a^2-4ac+b^2}+b\right)+C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2B\right)}{\sqrt{2}\,a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}-\frac{\log\left(x\left(b-\sqrt{8a^2-4ac+b^2}\right)+2ax^2+2a\right)\left(D\left(b-\sqrt{8a^2-4ac+b^2}\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}+\frac{\log\left(x\left(\sqrt{8a^2-4ac+b^2}+b\right)+2ax^2+2a\right)\left(D\left(\sqrt{8a^2-4ac+b^2}+b\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 + b^
2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sqrt[
8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a
^2 + b^2 - 4*a*c])])])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*
a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2
+ b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(
Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]
*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 -
 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (
```

```
b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]
) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[
8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2111

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = -\frac{\int \frac{Ab-2aB-A\sqrt{8a^2+b^2-4ac}+2aD+\left(2aA-2aC+bD-\sqrt{8a^2+b^2-4ac}\,D\right)x}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2} \cdots}{\sqrt{8a^2+b^2-4ac}}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\int \frac{b-\sqrt{8a^2+b^2-4ac}+\cdots}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)\cdots}}{4a\sqrt{8a^2+b^2-4ac}}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2}\cdots\right.\right.}{4a\sqrt{8a^2+b^2-4ac}}$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+\cdots\right.\right.}{\sqrt{2}\,a\sqrt{8a^2+b^2-4ac}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.06, size = 98, normalized size = 0.16

$$\text{RootSum}\left[a+b\#1+c\#1^2+b\#1^3+a\#1^4\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2+D\log(x-\#1)\#1^3}{b+2c\#1+3b\#1^2+4a\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x -
#1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2 +
4*a*#1^3) & ]
```

Maple [A]
time = 0.06, size = 535, normalized size = 0.88

method	result

default	$4a\left(\frac{-\frac{\left(2aA-2aC-\sqrt{8a^2-4ac+b^2}\,D+Db\right)\ln\left(-2a\,x^2+\sqrt{8a^2-4ac+b^2}\,x-bx-2a\right)}{4a}+\frac{2\left(\left(2aA-2aC-\sqrt{8a^2-4ac\ldots}\right.\right.}{\ldots}}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*a*(1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(-1/4*(2*a*A-2*a*C-(8*a^2-4*a*c+b^2)^(1/
2)*D+D*b)/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)+2*(1/4*(2*a*A-2*
a*C-(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*((8*a^2-4*a*c+b^2)^(1/2)-b)-(8*a^2-4*a
*c+b^2)^(1/2)*A+A*b-2*a*B+2*D*a)/(8*a^2+4*a*c-2*b^2+2*(8*a^2-4*a*c+b^2)^(1/
2)*b)^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*
(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2)))+1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(1/4*(2*a*
A-2*a*C+(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)
*x+b*x+2*a)+2*(-1/4*(2*a*A-2*a*C+(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*(b+(8*a^2
-4*a*c+b^2)^(1/2))+(8*a^2-4*a*c+b^2)^(1/2)*A+A*b-2*a*B+2*D*a)/(8*a^2+4*a*c-
2*b^2-2*(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^
(1/2))/(8*a^2+4*a*c-2*b^2-2*(8*a^2-4*a*c+b^2)^(1/2)*b)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxim
a")
```

```
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.77Not invertible E
rror: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x+C\,x^2+x^3\,D}{a\,x^4+b\,x^3+c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

3.228 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}}-\frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}$$

```
[Out] -2*ln(2+2*x^2-x*(-5^(1/2)+1))/(-5^(1/2)+1)-2*ln(2+2*x^2-x*(5^(1/2)+1))/(5^(
1/2)+1)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2111, 642}

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt[
5])*x + 2*x^2])/(1 + Sqrt[5])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2111

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right)x}{2+\left(-1-\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}} + \frac{\int \frac{2\sqrt{5}+\left(10+2\sqrt{5}\right)x}{2+\left(-1+\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}}$$
$$= -\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}} - \frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.87

$$\frac{1}{2}\left(-\left(\left(-1+\sqrt{5}\right)\log\left(-2+x+\sqrt{5}\,x-2x^2\right)\right)+\left(1+\sqrt{5}\right)\log\left(2+\left(-1+\sqrt{5}\right)x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 +
(-1 + Sqrt[5])*x + 2*x^2])/2
```

Maple [A]
time = 0.04, size = 53, normalized size = 0.84

method	result
default	$2\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(x\sqrt{5}+2x^2-x+2\right)-2\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(-x\sqrt{5}+2x^2-x+2\right)$
risch	$\frac{\ln\left(2+2x^2+\left(\sqrt{5}-1\right)x\right)}{2}+\frac{\ln\left(2+2x^2+\left(\sqrt{5}-1\right)x\right)\sqrt{5}}{2}+\frac{\ln\left(2+2x^2+\left(-\sqrt{5}-1\right)x\right)}{2}-\frac{\ln\left(2+2x^2+\left(-\sqrt{5}-1\right)x\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/4*5^(1/2)+1/4)*ln(x*5^(1/2)+2*x^2-x+2)-2*(1/4*5^(1/2)-1/4)*ln(-x*5^(1/
2)+2*x^2-x+2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

Fricas [A]
time = 0.40, size = 83, normalized size = 1.32

$$\frac{1}{2}\sqrt{5}\log\left(\frac{2x^4-2x^3+7x^2+\sqrt{5}(2x^3-x^2+2x)-2x+2}{x^4-x^3+x^2-x+1}\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
[Out] 1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x
+ 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

Sympy [A]
time = 0.04, size = 58, normalized size = 0.92

$$\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+1\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)*
log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

Giac [A]
time = 3.69, size = 58, normalized size = 0.92

$$-\frac{1}{2}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{2}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")
[Out] -1/2*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*log(x^2 + 1/2
*x*(sqrt(5) - 1) + 1) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

Mupad [B]
time = 0.18, size = 75, normalized size = 1.19

$$\frac{\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}-\frac{\sqrt{5}\,\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\sqrt{5}\,\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)
[Out] log(x^2 - (5^(1/2)*x)/2 - x/2 + 1)/2 + log((5^(1/2)*x)/2 - x/2 + x^2 + 1)/2
 - (5^(1/2)*log(x^2 - (5^(1/2)*x)/2 - x/2 + 1))/2 + (5^(1/2)*log((5^(1/2)*x
)/2 - x/2 + x^2 + 1))/2
```

3.229 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3(1+x)^3}+\log(1+x)$$

```
[Out] 1/3/(1+x)^3+ln(1+x)
```

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1608, 1694, 14}

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \int \frac{x(3+3x+x^2)}{1+4x+6x^2+4x^3+x^4}\,dx$$
$$= \text{Subst}\left(\int \frac{-1+x^3}{x^4}\,dx, x, 1+x\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x}\right)\,dx, x, 1+x\right)$$
$$= \frac{1}{3(1+x)^3} + \log(1+x)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3(1+x)^3} + \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.93

method	result	size
default	$\frac{1}{3(1+x)^3}+\ln(1+x)$	13
norman	$\frac{1}{3(1+x)^3}+\ln(1+x)$	13
risch	$\frac{1}{3x^3+9x^2+9x+3}+\ln(1+x)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
[Out] 1/3/(1+x)^3+ln(1+x)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 1.57

$$\frac{1}{3\,(x^3+3\,x^2+3\,x+1)} + \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.
time = 0.40, size = 38, normalized size = 2.71

$$\frac{3\,(x^3+3\,x^2+3\,x+1)\log{(x+1)}+1}{3\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 1.43

$$\log{(x+1)}+\frac{1}{3x^3+9x^2+9x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

Giac [A]
time = 3.74, size = 13, normalized size = 0.93

$$\frac{1}{3\,(x+1)^3}+\log{(|x+1|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] 1/3/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B]
time = 0.04, size = 12, normalized size = 0.86

$$\ln{(x+1)}+\frac{1}{3\,(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

```
[Out] log(x + 1) + 1/(3*(x + 1)^3)
```

3.230 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=28

$$\frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)$$

```
[Out] 8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1694, 45}

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx &= \text{Subst}\left(\int \frac{(-2+x)^3}{x^4}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \left(-\frac{8}{x^4}+\frac{12}{x^3}-\frac{6}{x^2}+\frac{1}{x}\right)dx, x, 1+x\right)\\
&= \frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.86

$$\frac{2(4+9x+9x^2)}{3(1+x)^3}+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A]
time = 0.02, size = 27, normalized size = 0.96

method	result	size
norman	$\frac{6x+6x^2+\frac{8}{3}}{(1+x)^3}+\ln(1+x)$	22
default	$\frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\ln(1+x)$	27
risch	$\frac{6x+6x^2+\frac{8}{3}}{x^3+3x^2+3x+1}+\ln(1+x)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
[Out] 8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Maxima [A]
time = 0.26, size = 32, normalized size = 1.14

$$\frac{2\,(9\,x^2+9\,x+4)}{3\,(x^3+3\,x^2+3\,x+1)}+\log{(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Fricas [A]
time = 0.41, size = 46, normalized size = 1.64

$$\frac{18\,x^2+3\,(x^3+3\,x^2+3\,x+1)\log{(x+1)}+18\,x+8}{3\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2
 + 3*x + 1)
```

Sympy [A]
time = 0.03, size = 29, normalized size = 1.04

$$\frac{18x^2 + 18x + 8}{3x^3 + 9x^2 + 9x + 3} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

Giac [A]
time = 3.98, size = 23, normalized size = 0.82

$$\frac{2(9x^2 + 9x + 4)}{3(x + 1)^3} + \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B]
time = 0.04, size = 21, normalized size = 0.75

$$\ln(x + 1) + \frac{6x^2 + 6x + \frac{8}{3}}{(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

```
[Out] log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

3.231 $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=59

$$\frac{2(1-2x^2)}{(3+2x^2+x^4)^2}-\frac{2x(18+13x^2)}{(3+2x^2+x^4)^2}+\frac{13x}{3+2x^2+x^4}$$

```
[Out] 2*(-2*x^2+1)/(x^4+2*x^2+3)^2-2*x*(13*x^2+18)/(x^4+2*x^2+3)^2+13*x/(x^4+2*x^
2+3)
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {1687, 1692, 1602, 1677, 1674, 8}

$$\frac{13x}{x^4+2x^2+3}-\frac{2(13x^2+18)\,x}{(x^4+2x^2+3)^2}+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x
^4)^3,x]
```

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4)
^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx &= \int \frac{x(-40+24x^4)}{(3+2x^2+x^4)^3}\,dx+\int \frac{9-18x^2+174x^4+26x^6}{(3+2x^2+x^4)^3}\\ &= -\frac{2x(18+13x^2)}{(3+2x^2+x^4)^2}+\frac{1}{96}\int \frac{3744-2496x^2-3744x^4}{(3+2x^2+x^4)^2}\\ &= \frac{2(1-2x^2)}{(3+2x^2+x^4)^2}-\frac{2x(18+13x^2)}{(3+2x^2+x^4)^2}+\frac{13x}{3+2x^2+x^4}\\ &= \frac{2(1-2x^2)}{(3+2x^2+x^4)^2}-\frac{2x(18+13x^2)}{(3+2x^2+x^4)^2}+\frac{13x}{3+2x^2+x^4}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 28, normalized size = 0.47

$$\frac{2+3x-4x^2+13x^5}{(3+2x^2+x^4)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x
^2 + x^4)^3,x]
```

```
[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

Maple [A]
time = 0.03, size = 30, normalized size = 0.51

method	result	size
gosper	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
norman	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
risch	$\frac{13x^5-4x^2+3x+2}{(x^4+2x^2+3)^2}$	29
default	$-\frac{-13x^5+4x^2-3x-2}{(x^4+2x^2+3)^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,method=
_RETURNVERBOSE)
```

```
[Out] -(-13*x^5+4*x^2-3*x-2)/(x^4+2*x^2+3)^2
```

Maxima [A]
time = 0.27, size = 38, normalized size = 0.64

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="maxima")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Fricas [A]
time = 0.39, size = 38, normalized size = 0.64

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="fricas")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A]
time = 0.06, size = 36, normalized size = 0.61

$$-\frac{-13x^5+4x^2-3x-2}{x^8+4x^6+10x^4+12x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3
)**3,x)
```

```
[Out] -(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

Giac [A]
time = 4.66, size = 28, normalized size = 0.47

$$\frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="giac")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.47

$$\frac{13\,x^5-4\,x^2+3\,x+2}{\left(x^4+2\,x^2+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4 +
 3)^3,x)
```

```
[Out] (3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2
```

3.232 $\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx$

Optimal. Leaf size=11

$$-\frac{x}{1+x+x^5}$$

[Out] -x/(x^5+x+1)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1602}

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{1+x+x^5}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{1+x+x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.
time = 0.02, size = 41, normalized size = 3.73

method	result	size
gosper	$-\frac{x}{x^5+x+1}$	12
norman	$-\frac{x}{x^5+x+1}$	12
risch	$-\frac{x}{x^5+x+1}$	12
default	$\frac{-1-3x}{7x^2+7x+7} - \frac{-3x^2+5x-1}{7(x^3-x^2+1)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/7*(-1-3*x)/(x^2+x+1)-1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
[Out] -x/(x^5 + x + 1)
```

Fricas [A]
time = 0.40, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
[Out] -x/(x^5 + x + 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

Giac [A]
time = 5.04, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

Mupad [B]
time = 2.30, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

```
[Out] -x/(x + x^5 + 1)
```

3.233 $\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{16\left(1-x^2\right)}+\frac{x\left(29-5x^2\right)}{32\left(1-6x^2+x^4\right)}+\frac{1}{4}\tanh^{-1}(x)+\frac{1}{64}\left(\left(3-2\sqrt{2}\right)\tanh^{-1}\left(\left(-1+\sqrt{2}\right)x\right)-\left(3+2\sqrt{2}\right)\tanh\right.$$

```
[Out] 1/16*x/(-x^2+1)+1/32*x*(-5*x^2+29)/(x^4-6*x^2+1)+1/4*arctanh(x)+1/64*arctan
h(x*(2^(1/2)-1))*(3-2*2^(1/2))-1/64*arctanh(x*(1+2^(1/2)))*(3+2*2^(1/2))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 205 vs. $2(91) = 182$.
time = 0.11, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2098, 213, 652, 632, 212, 646, 31}

$$-\frac{12-5x}{64\left(-x^2+2x+1\right)}+\frac{5x+12}{64\left(-x^2-2x+1\right)}+\frac{1}{32(1-x)}-\frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)-\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)+\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)-\frac{5\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)+\frac{5\tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 - 5
*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])/(64*Sqrt[2]) + ArcT
anh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*Sqrt[2]) - (3*(2 + 3*Sqrt[2])*L
og[1 - Sqrt[2] - x])/256 - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (
3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sq
rt[2] + x])/256
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \int \left(\frac{1}{32(-1+x)^2}+\frac{1}{32(1+x)^2}-\frac{1}{4\,(-1+x^2)}+\frac{17-7x}{32\,(-1-2x+x^2)^2}-\frac{}{64\,(}\right.$$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{1}{32}\int\frac{17-7x}{(-1-2x+x^2)^2}\,dx+\frac{1}{32}\int\frac{17+7x}{(-1+2x+x^2}$$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\,(1-2x-x^2)}-\frac{12-5x}{64\,(1+2x-x^2)}+\frac{1}{4}\tanh^{-1}($$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\,(1-2x-x^2)}-\frac{12-5x}{64\,(1+2x-x^2)}+\frac{1}{4}\tanh^{-1}($$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\,(1-2x-x^2)}-\frac{12-5x}{64\,(1+2x-x^2)}-\frac{5\tanh^{-1}\Big(}{64\sqrt{}}$$

Mathematica [A]

time = 0.07, size = 132, normalized size = 1.45

$$\frac{1}{128}\left(-\frac{4x(31-46x^2+7x^4)}{-1+7x^2-7x^4+x^6}-16\log(1-x)+\left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)+\left(-3+2\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right)+16\log(1+x)-\left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}+x\right)+\left(3-2\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] +
(3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2] -
x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt[2]
)*Log[1 + Sqrt[2] + x])/128
```

Maple [A]
time = 0.04, size = 116, normalized size = 1.27

method	result
default	$\frac{-5x-12}{64x^2+128x-64}+\frac{3\ln(x^2+2x-1)}{128}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32}-\frac{1}{32(-1+x)}-\frac{\ln(-1+x)}{8}-\frac{-12+5x}{64(x^2-2x-1)}-\frac{3\ln(x^2-2}{128}$
risch	$\frac{-\frac{7}{32}x^5+\frac{23}{16}x^3-\frac{31}{32}x}{x^6-7x^4+7x^2-1}+\frac{\ln(1+x)}{8}-\frac{3\ln\left(2x-2-2\sqrt{2}\right)}{128}+\frac{\ln\left(2x-2-2\sqrt{2}\right)\sqrt{2}}{64}-\frac{3\ln\left(2x-2+2\sqrt{2}\right)}{128}-\frac{\ln\left(2x-2+2\sqrt{\right.}}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*
x+2)*2^(1/2))-1/32/(-1+x)-1/8*ln(-1+x)-1/64*(-12+5*x)/(x^2-2*x-1)-3/128*ln(
x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2))-1/32/(1+x)+1/8*ln(1+x)
```

Maxima [A]
time = 0.49, size = 114, normalized size = 1.25

$$\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{7x^5-46x^3+31x}{32(x^6-7x^4+7x^2-1)}+\frac{3}{128}\log\left(x^2+2x-1\right)-\frac{3}{128}\log\left(x^2-2x-1\right)+\frac{1}{8}\log\left(x+1\right)-\frac{1}{8}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((x
 - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*
x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) + 1/
8*log(x + 1) - 1/8*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(71) = 142$.
time = 0.39, size = 223, normalized size = 2.45

$$-\frac{28x^5-184x^3-2\sqrt{2}(x^6-7x^4+7x^2-1)\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)-2\sqrt{2}(x^6-7x^4+7x^2-1)\log\left(\frac{x^2-2\sqrt{2}(x-1)-2x+3}{x^2-2x-1}\right)-3(x^6-7x^4+7x^2-1)\log\left(x^2+2x-1\right)+3(x^6-7x^4+7x^2-1)\log\left(x^2-2x-1\right)-16(x^6-7x^4+7x^2-1)\log\left(x+1\right)+16(x^6-7x^4+7x^2-1)\log\left(x-1\right)+124x}{128(x^6-7x^4+7x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2
*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x
^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^6 -
 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^
2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x^4 +
7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(73) = 146$.
time = 0.80, size = 272, normalized size = 2.99

$$\frac{-7x^5+46x^3-31x}{32x^6-224x^4+224x^2-32}-\frac{\log\left(x-1\right)}{8}+\frac{\log\left(x+1\right)}{8}+\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}-\frac{38423555\sqrt{2}}{1363992}+\frac{9549859782656\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499}-\frac{56267374592\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833}\right)+\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}+\frac{9549859782656\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^5}{170499}-\frac{56267374592\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^3}{56833}+\frac{38423555\sqrt{2}}{1363992}\right)+\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555\sqrt{2}}{1363992}-\frac{56267374592\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833}+\frac{9549859782656\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499}+\frac{38423555}{909328}\right)+\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)\log\left(x-\frac{56267374592\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^3}{56833}+\frac{9549859782656\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^5}{170499}+\frac{38423555\sqrt{2}}{1363992}+\frac{38423555}{909328}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] (-7*x**5 + 46*x**3 - 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x - 1
)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 384235
55*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562673
74592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 38423
555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 56267374592*(-
3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - sqrt(2)
/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)/64)**3
/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/909328) +
(sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/56833 + 95
49859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/1363992 + 384
23555/909328)
```

Giac [A]
time = 8.60, size = 134, normalized size = 1.47

$$\frac{1}{64}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)}+\frac{3}{128}\log\left(\left|x^2+2x-1\right|\right)-\frac{3}{128}\log\left(\left|x^2-2x-1\right|\right)+\frac{1}{8}\log\left(\left|x+1\right|\right)-\frac{1}{8}\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64*
sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*x^
5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x - 1)
) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1
))
```

Mupad [B]
time = 2.37, size = 124, normalized size = 1.36

$$-\frac{\mathrm{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}-\frac{\frac{7x^5}{32}-\frac{23x^3}{16}+\frac{31x}{32}}{x^6-7x^4+7x^2-1}+\mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\left(\frac{27309\sqrt{2}}{32768}-\frac{19317}{16384}\right)}-\frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\left(\frac{27309\sqrt{2}}{32768}-\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}-\frac{3}{64}\mathrm{i}\right)+\mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\left(\frac{27309\sqrt{2}}{32768}+\frac{19317}{16384}\right)}+\frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\left(\frac{27309\sqrt{2}}{32768}+\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}+\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] atan((x*23313i)/(8192*((27309*2^(1/2))/32768 - 19317/16384)) - (2^(1/2)*x*6
5943i)/(32768*((27309*2^(1/2))/32768 - 19317/16384)))*((2^(1/2)*1i)/32 - 3i
/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1) - (
atan(x*1i)*1i)/4 + atan((x*23313i)/(8192*((27309*2^(1/2))/32768 + 19317/163
84)) + (2^(1/2)*x*65943i)/(32768*((27309*2^(1/2))/32768 + 19317/16384)))*((
2^(1/2)*1i)/32 + 3i/64)
```

3.234 $\int x^m(a+bx+cx^2+dx^3)^p\,(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

Optimal. Leaf size=25

$$x^{1+m}\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {1604}

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3
 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^m\left(a+bx+cx^2+dx^3\right)^p\left(a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x))\right)dx=x^{1+}$$

Mathematica [A]
time = 1.22, size = 23, normalized size = 0.92

$$x^{1+m}(a+x(b+x(c+dx)))^{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x
*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A]
time = 0.03, size = 26, normalized size = 1.04

method	result	size
gosper	$x^{1+m}(d\,x^{3}+c\,x^{2}+bx+a)^{1+p}$	26
risch	$(d\,x^{3}+c\,x^{2}+bx+a)^{p}\,x^{m}x(d\,x^{3}+c\,x^{2}+bx+a)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3
*p)*x))),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A]
time = 0.33, size = 44, normalized size = 1.76

$$(dx^4+cx^3+bx^2+ax)e^{(p\log(dx^3+cx^2+bx+a)+m\log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x))
```

Fricas [A]
time = 0.56, size = 40, normalized size = 1.60

$$(dx^4+cx^3+bx^2+ax)(dx^3+cx^2+bx+a)^p x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p
)+d*(4+m+3*p)*x))),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(25) = 50$.
time = 6.68, size = 99, normalized size = 3.96

$$(dx^3 + cx^2 + bx + a)^p dx^4x^m + (dx^3 + cx^2 + bx + a)^p cx^3x^m + (dx^3 + cx^2 + bx + a)^p bx^2x^m + (dx^3 + cx^2 + bx + a)^p axx^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*x
^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*a*
x*x^m
```

Mupad [**B**]
time = 2.66, size = 49, normalized size = 1.96

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(a\,x\,x^m+b\,x^m\,x^2+c\,x^m\,x^3+d\,x^m\,x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p
+ 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)
```

3.235 $\int x^2(a+bx+cx^2+dx^3)^p\,(3a+b(4+p)x+c(5+2p$

Optimal. Leaf size=23

$$x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A]
time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {1602}

$$x^3\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + 
d*(6 + 3*p)*x^3),x]
```

```
[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+bx+cx^2+dx^3\right)^p\left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)\,dx=x^3\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A]
time = 2.05, size = 21, normalized size = 0.91

$$x^3(a+x(b+x(c+dx)))^{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*
x^2 + d*(6 + 3*p)*x^3),x]
```

```
[Out] x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A]
time = 0.04, size = 24, normalized size = 1.04

method	result
gosper	$x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{1+p}$
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)$
norman	$a\,x^{3}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+b\,x^{4}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{5}\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+x^{6}d\,\mathrm{e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x
,method=_RETURNVERBOSE)
```

```
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A]
time = 0.32, size = 39, normalized size = 1.70

$$\left(dx^{6}+cx^{5}+bx^{4}+ax^{3}\right)\left(dx^{3}+cx^{2}+bx+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="maxima")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A]
time = 0.42, size = 39, normalized size = 1.70

$$\left(dx^{6}+cx^{5}+bx^{4}+ax^{3}\right)\left(dx^{3}+cx^{2}+bx+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="fricas")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+
3*p)*x**3),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(23) = 46$.
time = 4.65, size = 89, normalized size = 3.87

$$(dx^3 + cx^2 + bx + a)^p dx^6 + (dx^3 + cx^2 + bx + a)^p cx^5 + (dx^3 + cx^2 + bx + a)^p bx^4 + (dx^3 + cx^2 + bx + a)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

Mupad [B]
time = 2.55, size = 39, normalized size = 1.70

$$(d\,x^3 + c\,x^2 + b\,x + a)^p\,(d\,x^6 + c\,x^5 + b\,x^4 + a\,x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) +
d*x^3*(3*p + 6)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)
```

3.236 $\int x(a+bx+cx^2+dx^3)^p\,(2a+b(3+p)x+c(4+2p$

Optimal. Leaf size=23

$$x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{1602\}$

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*
(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx=x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A]
time = 0.96, size = 21, normalized size = 0.91

$$x^2(a+x(b+x(c+dx)))^{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^
2 + d*(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A]
time = 0.04, size = 24, normalized size = 1.04

method	result
gosper	$x^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{1+p}$
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} x^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)$
norman	$a\,x^{2}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+b\,x^{3}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{4}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+d\,x^{5}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A]
time = 0.30, size = 39, normalized size = 1.70

$$\left(dx^{5}+cx^{4}+bx^{3}+ax^{2}\right)\left(dx^{3}+cx^{2}+bx+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="maxima")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A]
time = 0.42, size = 39, normalized size = 1.70

$$\left(dx^{5}+cx^{4}+bx^{3}+ax^{2}\right)\left(dx^{3}+cx^{2}+bx+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="fricas")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p
)*x**3),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(23) = 46.
time = 5.19, size = 89, normalized size = 3.87

$$(dx^3 + cx^2 + bx + a)^p dx^5 + (dx^3 + cx^2 + bx + a)^p cx^4 + (dx^3 + cx^2 + bx + a)^p bx^3 + (dx^3 + cx^2 + bx + a)^p ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

Mupad [B]
time = 2.32, size = 39, normalized size = 1.70

$$(d x^3 + c x^2 + b x + a)^p (d x^5 + c x^4 + b x^3 + a x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d*
x^3*(3*p + 5)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

3.237 $\int (a+bx+cx^2+dx^3)^p (a+b(2+p)x+c(3+2p)x^2$

Optimal. Leaf size=21

$$x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A]
time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = $\{1602\}$

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 +
 3*p)*x^3),x]
```

```
[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right)\,dx = x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A]
time = 0.74, size = 19, normalized size = 0.90

$$x(a+x(b+x(c+dx)))^{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
d*(4 + 3*p)*x^3),x]
```

```
[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A]
time = 0.03, size = 22, normalized size = 1.05

method	result
gosper	$x\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{1+p}$
risch	$\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}x\left(d\,x^{3}+c\,x^{2}+bx+a\right)$
norman	$ax\,{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+b\,x^{2}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{3}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+d\,x^{4}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x,metho
d=_RETURNVERBOSE)
```

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A]
time = 0.31, size = 37, normalized size = 1.76

$$(dx^4+cx^3+bx^2+ax)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A]
time = 0.40, size = 37, normalized size = 1.76

$$(dx^4+cx^3+bx^2+ax)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x*
*3),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(21) = 42$.
time = 4.64, size = 87, normalized size = 4.14

$$(dx^3 + cx^2 + bx + a)^p dx^4 + (dx^3 + cx^2 + bx + a)^p cx^3 + (dx^3 + cx^2 + bx + a)^p bx^2 + (dx^3 + cx^2 + bx + a)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

Mupad [B]
time = 2.27, size = 37, normalized size = 1.76

$$(d\,x^3 + c\,x^2 + b\,x + a)^p\,(d\,x^4 + c\,x^3 + b\,x^2 + a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3*
(3*p + 4)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

3.238 $\int \frac{(a+bx+cx^2+dx^3)^p(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx$

Optimal. Leaf size=19

$$\left(a+bx+cx^2+dx^3\right)^{1+p}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1599, 1602}

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*
p)*x^3))/x,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx+cx^2+dx^3)^p\,(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x}\,dx &= \int (b(1+p)+c(2+2p)x+d(3+3\\ &= \left(a+bx+cx^2+dx^3\right)^{1+p}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 17, normalized size = 0.89

$$(a+x(b+x(c+dx)))^{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(
3 + 3*p)*x^3))/x,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)
```

Maple [A]
time = 0.03, size = 20, normalized size = 1.05

method	result	size
gosper	$(d\,x^{3}+c\,x^{2}+bx+a)^{1+p}$	20
risch	$(d\,x^{3}+c\,x^{2}+bx+a)^{p}\,(d\,x^{3}+c\,x^{2}+bx+a)$	34
norman	$a\,e^{p\ln(d\,x^{3}+c\,x^{2}+bx+a)}+bx\,e^{p\ln(d\,x^{3}+c\,x^{2}+bx+a)}+c\,x^{2}e^{p\ln(d\,x^{3}+c\,x^{2}+bx+a)}+d\,x^{3}e^{p\ln(d\,x^{3}+c\,x^{2}+bx+a)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x,metho
d=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)
```

Maxima [A]
time = 0.32, size = 33, normalized size = 1.74

$$(dx^{3}+cx^{2}+bx+a)\,(dx^{3}+cx^{2}+bx+a)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Fricas [A]
time = 0.39, size = 33, normalized size = 1.74

$$(dx^{3}+cx^{2}+bx+a)\,(dx^{3}+cx^{2}+bx+a)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3
)/x,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.
time = 5.56, size = 52, normalized size = 2.74

$$\frac{(dx^3+cx^2+bx+a)^{p+1}p}{p+1}+\frac{(dx^3+cx^2+bx+a)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
+ 1)/(p + 1)
```

Mupad [B]
time = 2.19, size = 19, normalized size = 1.00

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 + d
*x^3)^p)/x,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)
```

3.239 $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = $\{1604\}$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p
)*x^3))/x^2,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p\left(-a+bpx+c(1+2p)x^2+d(2+3p)x^3\right)}{x^2}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

Mathematica [A]
time = 0.71, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{1+p}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2
 + 3*p)*x^3))/x^2,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x
```

Maple [A]
time = 0.03, size = 24, normalized size = 1.04

method	result	size
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{1+p}}{x}$	24
risch	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}}{x}$	37
norman	$\frac{a\,{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+bx\,{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+c\,x^{2}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}+d\,x^{3}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{x}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x,meth
od=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x
```

Maxima [A]
time = 0.30, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Fricas [A]
time = 0.44, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)
/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
b*x + a)^p/x^2, x)
```

Mupad [B]
time = 3.20, size = 23, normalized size = 1.00

$$\frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p
+ 2)))/x^2,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x
```

3.240 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = $\{1604\}$

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 +
3*p)*x^3))/x^3,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

Mathematica [A]
time = 0.79, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{1+p}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d
*(1 + 3*p)*x^3))/x^3,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

Maple [A]
time = 0.03, size = 24, normalized size = 1.04

method	result	size
gosper	$\frac{\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{1+p}}{x^{2}}$	24
risch	$\frac{\left(d\,x^{3}+c\,x^{2}+b x+a\right)\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{p}}{x^{2}}$	37
norman	$\frac{a\,{\mathrm e}^{p \ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}+b x\,{\mathrm e}^{p \ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}+c\,x^{2} {\mathrm e}^{p \ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}+d\,x^{3} {\mathrm e}^{p \ln\left(d\,x^{3}+c\,x^{2}+b x+a\right)}}{x^{2}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^2
```

Maxima [A]
time = 0.31, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Fricas [A]
time = 0.44, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x*
*3)/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^3, x)
```

Mupad [B]
time = 3.32, size = 23, normalized size = 1.00

$$\frac{(d\,x^3+c\,x^2+b\,x+a)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3*
p + 1)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2
```

3.241 $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^3
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1604}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 +
3*d*p*x^3))/x^4,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p\,(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

Mathematica [A]
time = 0.96, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{1+p}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*
x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

Maple [A]
time = 0.04, size = 24, normalized size = 1.04

method	result	size
gosper	$\frac{(d x^3+c x^2+bx+a)^{1+p}}{x^3}$	24
risch	$\frac{(d x^3+c x^2+bx+a)(d x^3+c x^2+bx+a)^p}{x^3}$	37
norman	$\frac{a\,e^{p\ln(d x^3+c x^2+bx+a)}+bx\,e^{p\ln(d x^3+c x^2+bx+a)}+c\,x^2 e^{p\ln(d x^3+c x^2+bx+a)}+d\,x^3 e^{p\ln(d x^3+c x^2+bx+a)}}{x^3}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x,
method=_RETURNVERBOSE)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(1+p)/x^3
```

Maxima [A]
time = 0.31, size = 36, normalized size = 1.57

$$\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Fricas [A]
time = 0.44, size = 36, normalized size = 1.57

$$\frac{(dx^3+cx^2+bx+a)(dx^3+cx^2+bx+a)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x
**3)/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="giac")
```

```
[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^4, x)
```

Mupad [B]
time = 3.35, size = 23, normalized size = 1.00

$$\frac{\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2*
p - 1)))/x^4,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3
```

3.242 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right)$$

```
[Out] 5/4*x-3/4*x^2+1/3*x^3+1/4*x^4+1/3*ln(x^2+x+1)-13/48*ln(2*x^2-x+2)+1/72*arct
an(1/15*(1-4*x)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2100, 648, 632, 210, 642}

$$\frac{1}{24}\sqrt{\frac{5}{3}}\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]
)/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (1
3*Log[2 - x + 2*x^2])/48
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(\frac{5}{4}-\frac{3x}{2}+x^2+x^3+\frac{2(-2+x)}{3(1+x+x^2)}+\frac{2-13x}{12(2-x+2x^2)}\right)dx \\
&= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{12}\int \frac{2-13x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx \\
&= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx-\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \\
&= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right)+\frac{5}{24}\text{Sub} \\
&= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\text{lo}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 0.86

$$\frac{1}{144}\left(180x-108x^2+48x^3+36x^4-160\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+48\log\left(1+x+x^2\right)-39\log\left(2-x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
- 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2 -
 x + 2*x^2])/144
```

Maple [A]
time = 0.04, size = 74, normalized size = 0.76

method	result

default	$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{13\ln(2x^2-x+2)}{48}-\frac{\sqrt{15}\ \arctan\left(\frac{(4x-1)\sqrt{1}}{15}\right.}{72}$
risch	$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\ln(16x^2-8x+16)}{48}-\frac{\sqrt{15}\ \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}+\frac{\ln(4x^2+4x+4)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{}}{3}\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+1/3*x^3-3/4*x^2+5/4*x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/
2))*3^(1/2)-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Maxima [A]
time = 0.48, size = 73, normalized size = 0.75

$$\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{3}{4}x^2-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{5}{4}x-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

Fricas [A]
time = 0.39, size = 79, normalized size = 0.81

$$\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{3}{4}x^2-\frac{1}{72}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}\,(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{5}{4}x-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt
(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48
*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Sympy [A]
time = 0.10, size = 97, normalized size = 1.00

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\ \operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{72}-\frac{10\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 + log(x**2 +
 x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 - 10*sqrt(3)*at
an(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A]
time = 4.66, size = 73, normalized size = 0.75

$$\frac{1}{4}\,x^{4}+\frac{1}{3}\,x^{3}-\frac{3}{4}\,x^{2}-\frac{1}{72}\,\sqrt{15}\,\arctan\left(\frac{1}{15}\,\sqrt{15}\,(4\,x-1)\right)-\frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{5}{4}\,x-\frac{13}{48}\,\log\left(2\,x^{2}-x+2\right)+\frac{1}{3}\,\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

Mupad [B]
time = 0.19, size = 97, normalized size = 1.00

$$\frac{5\,x}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(-\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)-\frac{3\,x^{2}}{4}+\frac{x^{3}}{3}+\frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] (5*x)/4 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3
^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*1i)/144 - 13/48) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i
)/144 + 13/48) - (3*x^2)/4 + x^3/3 + x^4/4
```

3.243 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=90

$$-\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)$$

```
[Out] -3/2*x+1/2*x^2+1/3*x^3+2/3*ln(x^2+x+1)-1/24*ln(2*x^2-x+2)+5/36*arctan(1/15*
(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2100, 648, 632, 210, 642}

$$\frac{5}{12}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (8
*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 -
x + 2*x^2]/24
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(-\frac{3}{2}+x+x^2+\frac{2(3+2x)}{3\,(1+x+x^2)}+\frac{-6-x}{6\,(2-x+2x^2)}\right)dx\\ &= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{1}{6}\int \frac{-6-x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx\\ &= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int \frac{}{2-}\\ &= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)+\frac{25}{12}\text{Subst}\Big(\int\\ &= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log(1+\end{aligned}$$

Mathematica [A]
time = 0.02, size = 78, normalized size = 0.87

$$\frac{1}{72}\left(-108x+36x^2+24x^3+64\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-10\sqrt{15}\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+48\log\left(1+x+x^2\right)-3\log\left(2-x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt[
15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2
])/72
```

Maple [A]
time = 0.04, size = 69, normalized size = 0.77

method	result
default	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}+\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}$
risch	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}+\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(16x^2-8x+16)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right.}{36}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+1/2*x^2-3/2*x+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/
2)-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Maxima [A]
time = 0.49, size = 68, normalized size = 0.76

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{15}\,\arctan\left(\frac{1}{15}\,\sqrt{15}\,(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

Fricas [A]
time = 0.40, size = 74, normalized size = 0.82

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{5}\,\sqrt{3}\,\arctan\left(\frac{1}{15}\,\sqrt{5}\,\sqrt{3}\,(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x -
 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 -
 x + 2) + 2/3*log(x^2 + x + 1)
```

Sympy [A]
time = 0.10, size = 92, normalized size = 1.02

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x + 1)/3 -
5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*sqrt(3)*atan(2*sqrt(3
)*x/3 + sqrt(3)/3)/9
```

Giac [A]
time = 5.43, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

Mupad [B]
time = 0.18, size = 92, normalized size = 1.02

$$\frac{x^2}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5i}{72}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5i}{72}\right)-\frac{3x}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - (3*x)/2 + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*5i)/72 - 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5i)/
72 + 1/24) + x^2/2 + x^3/3
```

3.244 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=77

$$x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)$$

```
[Out] x+1/2*x^2-ln(x^2+x+1)+1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*
15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2100, 648, 632, 210, 642}

$$\frac{1}{6}\sqrt{\frac{5}{3}}\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(1+x-\frac{2(1+3x)}{3\,(1+x+x^2)}+\frac{-2+3x}{3\,(2-x+2x^2)}\right)dx\\ &= x+\frac{x^2}{2}+\frac{1}{3}\int \frac{-2+3x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx\\ &= x+\frac{x^2}{2}+\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx\\ &= x+\frac{x^2}{2}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.\\ &= x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(1+x+x^2\right)+\end{aligned}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.94

$$\frac{1}{36}\left(8\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+9\left(2x(2+x)-4\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36
```

Maple [A]
time = 0.03, size = 62, normalized size = 0.81

method	result

default	$x+\frac{x^2}{2}-\ln(x^2+x+1)+\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}$
risch	$x+\frac{x^2}{2}+\frac{\ln(16x^2-8x+16)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}-\ln(4x^2+4x+4)+\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2-ln(x^2+x+1)+2/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/4*ln(2*x^2-
x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Maxima [A]
time = 0.50, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{15}\,\arctan\left(\frac{1}{15}\sqrt{15}\,(4x-1)\right)+\frac{2}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Fricas [A]
time = 0.38, size = 67, normalized size = 0.87

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{5}\,\sqrt{3}\,\arctan\left(\frac{1}{15}\sqrt{5}\,\sqrt{3}\,(4x-1)\right)+\frac{2}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9
*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x
^2 + x + 1)
```

Sympy [A]
time = 0.10, size = 78, normalized size = 1.01

$$\frac{x^2}{2}+x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right)-\frac{\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A]
time = 5.43, size = 61, normalized size = 0.79

$$\frac{1}{2}\,x^2 - \frac{1}{18}\,\sqrt{15}\,\arctan\left(\frac{1}{15}\,\sqrt{15}\,(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right) + x + \frac{1}{4}\,\log\left(2\,x^2 - x + 2\right) - \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Mupad [B]
time = 2.29, size = 85, normalized size = 1.10

$$x - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}\ 1i}{2}\right)\left(1 + \frac{\sqrt{3}\ 1i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}\ 1i}{2}\right)\left(-1 + \frac{\sqrt{3}\ 1i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}\ 1i}{4}\right)\left(\frac{1}{4} + \frac{\sqrt{15}\ 1i}{36}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}\ 1i}{4}\right)\left(-\frac{1}{4} + \frac{\sqrt{15}\ 1i}{36}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x + (3^(1/2)*1
i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/36 + 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 - 1/4) +
 x^2/2
```

3.245 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=72

$$x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)$$

```
[Out] x+1/3*ln(x^2+x+1)+1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1
/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2100, 648, 632, 210, 642}

$$-\frac{1}{3}\sqrt{\frac{5}{3}}\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]
])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(1+\frac{2(-2+x)}{3(1+x+x^2)}+\frac{2(1+x)}{3(2-x+2x^2)}\right) dx \\
&= x+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx+\frac{2}{3}\int \frac{1+x}{2-x+2x^2}\,dx \\
&= x+\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx-\frac{5}{3}\int \\
&= x+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,\right. \\
&= x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\text{lo}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 69, normalized size = 0.96

$$\frac{1}{18}\left(-20\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+3\left(6x+2\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18
```

Maple [A]
time = 0.03, size = 57, normalized size = 0.79

method	result	size

default	$x+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}$	57
risch	$x+\frac{\ln(4x^2+4x+4)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(16x^2-8x+16)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/6*ln(2*x^2-x+2
)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Maxima [A]
time = 0.49, size = 56, normalized size = 0.78

$$\frac{1}{9}\sqrt{15}\,\arctan\left(\frac{1}{15}\sqrt{15}\,(4x-1)\right)-\frac{10}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Fricas [A]
time = 0.39, size = 62, normalized size = 0.86

$$\frac{1}{9}\sqrt{5}\sqrt{3}\,\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}\,(4x-1)\right)-\frac{10}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x
 + 1)
```

Sympy [A]
time = 0.10, size = 75, normalized size = 1.04

$$x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*
x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A]
time = 7.68, size = 56, normalized size = 0.78

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Mupad [B]
time = 2.29, size = 80, normalized size = 1.11

$$x+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3^(1/2)
*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^
(1/2)*1i)/18 - 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 + 1/
6)
```

3.246 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)$$

```
[Out] 2/3*ln(x^2+x+1)-1/6*ln(2*x^2-x+2)-1/9*arctan(1/15*(1-4*x)*15^(1/2))*15^(1/2
)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2099, 648, 632, 210, 642}

$$-\frac{1}{3}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -1/3*(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]) + (8*ArcTan[(1 + 2*x)/Sqrt[3]])
/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx &= \int \left(\frac{2(3+2x)}{3\,(1+x+x^2)}+\frac{3-2x}{3\,(2-x+2x^2)}\right)dx\\ &= \frac{1}{3}\int \frac{3-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx\\ &= -\left(\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx\right)+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx+\frac{4}{3}\\ &= \frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1\right.\\ &= -\frac{1}{3}\sqrt{\frac{5}{3}}\,\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 0.92

$$\frac{1}{18}\left(16\sqrt{3}\,\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+2\sqrt{15}\,\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+12\log\left(1+x+x^2\right)-3\log\left(2-x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

Maple [A]
time = 0.03, size = 56, normalized size = 0.79

method	result	size

default	$\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(2x^2-x+2)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}$	56
risch	$-\frac{\ln(16x^2-8x+16)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}+\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*ln(x^2+x+1)+8/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/6*ln(2*x^2-x+2)+1
/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Maxima [A]
time = 0.50, size = 55, normalized size = 0.77

$$\frac{1}{9}\,\sqrt{15}\,\arctan\left(\frac{1}{15}\,\sqrt{15}\,(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{1}{6}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Fricas [A]
time = 0.38, size = 61, normalized size = 0.86

$$\frac{1}{9}\,\sqrt{5}\,\sqrt{3}\,\arctan\left(\frac{1}{15}\,\sqrt{5}\,\sqrt{3}\,(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-\frac{1}{6}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Sympy [A]
time = 0.09, size = 75, normalized size = 1.06

$$-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*x
/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A]
time = 8.32, size = 55, normalized size = 0.77

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Mupad [B]
time = 0.15, size = 79, normalized size = 1.11

$$-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/18 + 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6)
```

3.247 $\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=75

$$\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)$$

```
[Out] 5/2*ln(x)-ln(x^2+x+1)-1/4*ln(2*x^2-x+2)+1/18*arctan(1/15*(1-4*x)*15^(1/2))*
15^(1/2)+2/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 210, 642}

$$\frac{1}{6}\sqrt{\frac{5}{3}}\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx &= -\left(\frac{1}{3}\int \frac{-6+4x}{x\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x\left(4+4x+4x^2\right)}\,dx \\
&= \frac{1}{3}\int \left(\frac{6}{x}-\frac{2(1+3x)}{1+x+x^2}\right)\,dx-\frac{1}{3}\int \left(-\frac{3}{2x}+\frac{1+6x}{2\left(2-x+2x^2\right)}\right)\,dx \\
&= \frac{5\log(x)}{2}-\frac{1}{6}\int \frac{1+6x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx \\
&= \frac{5\log(x)}{2}-\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2
\end{aligned}$$

$$\begin{aligned}
&= \frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5\log(x)}{2}-\log\left(1+x+x^2
\end{aligned}$$

Mathematica [**A**]

time = 0.02, size = 69, normalized size = 0.92

$$\frac{1}{36}\left(8\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+90\log(x)-36\log\left(1+x+x^2\right)-9\log\left(2-x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

Maple [A]
time = 0.04, size = 60, normalized size = 0.80

method	result
risch	$-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}-\frac{\ln\left(16x^2-8x+16\right)}{4}+\frac{5\ln(x)}{2}-\ln\left(x^2+x+1\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$
default	$-\ln\left(x^2+x+1\right)+\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln\left(2x^2-x+2\right)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}+\frac{5\ln(x)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x^2+x+1)+2/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*ln(2*x^2-x+2)-1/18
*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+5/2*ln(x)
```

Maxima [A]
time = 0.48, size = 59, normalized size = 0.79

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

Fricas [A]
time = 0.37, size = 65, normalized size = 0.87

$$-\frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) +
 5/2*log(x)
```

Sympy [A]
time = 0.13, size = 78, normalized size = 1.04

$$\frac{5\log{(x)}}{2} - \frac{\log{\left(x^2 - \frac{x}{2} + 1\right)}}{4} - \log{\left(x^2 + x + 1\right)} - \frac{\sqrt{15}\,\operatorname{atan}{\left(\frac{4\sqrt{15}\,x}{15} - \frac{\sqrt{15}}{15}\right)}}{18} + \frac{2\sqrt{3}\,\operatorname{atan}{\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A]
time = 5.77, size = 60, normalized size = 0.80

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4\,x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right) - \frac{1}{4}\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(abs(x
))
```

Mupad [B]
time = 0.15, size = 83, normalized size = 1.11

$$\frac{5\ln(x)}{2} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x +
 (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4
)*((15^(1/2)*1i)/36 - 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/
36 + 1/4)
```

3.248 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=84

$$-\frac{5}{2x}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{3\log(x)}{4}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{24}\log\left(2-x+2x^2\right)$$

```
[Out] -5/2/x-3/4*ln(x)+1/3*ln(x^2+x+1)+1/24*ln(2*x^2-x+2)+5/36*arctan(1/15*(1-4*x
)*15^(1/2))*15^(1/2)-10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 210, 642}

$$\frac{5}{12}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x +
2*x^2]/24
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx &= -\left(\frac{1}{3}\int \frac{-6+4x}{x^2\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^2\left(4+4x+4x^2\right)}\,dx \\
&= \frac{1}{3}\int \left(\frac{6}{x^2}-\frac{2}{x}+\frac{2(-2+x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x^2}+\frac{1}{4x}+\frac{13-2x}{4\left(2-x+2x^2\right.}\right. \\
&= -\frac{5}{2x}-\frac{3\log(x)}{4}-\frac{1}{12}\int \frac{13-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx \\
&= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int \\
&= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{24}\log\left(2-x+2x^2\right)+\frac{25}{12}\text{Subs} \\
&= -\frac{5}{2x}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{3\log(x)}{4}+\frac{1}{3}\log
\end{aligned}$$

Mathematica [A]

time = 0.02, size = 78, normalized size = 0.93

$$-\frac{180+80\sqrt{3}\,x\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+10\sqrt{15}\,x\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+54x\log(x)-24x\log\left(1+x+x^2\right)-3x\log\left(2-x+2x^2\right)}{72x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan[
(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 - x
+ 2*x^2])/x
```

Maple [A]
time = 0.03, size = 65, normalized size = 0.77

method	result
default	$\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}-\frac{5}{2x}-\frac{3\ln(x)}{4}$
risch	$-\frac{5}{2x}-\frac{3\ln(x)}{4}-\frac{5\sqrt{15}\,\arctan\left(\frac{2\left(10x-\frac{5}{2}\right)\sqrt{15}}{75}\right)}{36}+\frac{\ln(100x^2-50x+100)}{24}+\frac{\ln(25x^2+25x+25)}{3}-\frac{10\sqrt{3}\,\arctan\left(2(\ldots\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/24*ln(2*x^2-x+2)
-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/2/x-3/4*ln(x)
```

Maxima [A]
time = 0.51, size = 64, normalized size = 0.76

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{3}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(x)
```

Fricas [A]
time = 0.39, size = 76, normalized size = 0.90

$$-\frac{10\sqrt{5}\sqrt{3}\,x\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}\,(4x-1)\right)+80\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-3x\log\left(2x^2-x+2\right)-24x\log\left(x^2+x+1\right)+54x\log(x)+180}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sqr
t(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^
2 + x + 1) + 54*x*log(x) + 180)/x
```

Sympy [A]
time = 0.14, size = 87, normalized size = 1.04

$$-\frac{3\log(x)}{4}+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\ \mathrm{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*ata
n(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(
3)/3)/9 - 5/(2*x)
```

Giac [A]
time = 6.70, size = 65, normalized size = 0.77

$$-\frac{5}{36}\sqrt{15}\ \arctan\left(\frac{1}{15}\sqrt{15}\,(4\,x-1)\right)-\frac{10}{9}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{5}{2\,x}+\frac{1}{24}\log\left(2\,x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{3}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(abs(x))
```

Mupad [B]
time = 2.28, size = 88, normalized size = 1.05

$$-\frac{3\ln(x)}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\frac{5}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - (3*log(x))/4 - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 -
 1/4)*((15^(1/2)*5i)/72 + 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)
*5i)/72 - 1/24) - 5/(2*x)
```

3.249 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} \, dx$

Optimal. Leaf size=91

$$-\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x\right.$$

```
[Out] -5/4/x^2+3/4/x-15/8*ln(x)+2/3*ln(x^2+x+1)+13/48*ln(2*x^2-x+2)+1/72*arctan(1
/15*(1-4*x)*15^(1/2))*15^(1/2)+8/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 210, 642}

$$\frac{1}{24}\sqrt{\frac{5}{3}}\text{ArcTan}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/3
+ (13*Log[2 - x + 2*x^2])/48
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx &= -\left(\frac{1}{3}\int \frac{-6+4x}{x^3\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^3\left(4+4x+4x^2\right)}\,dx \\
&= \frac{1}{3}\int \left(\frac{6}{x^3}-\frac{2}{x^2}-\frac{4}{x}+\frac{2(3+2x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x^3}+\frac{1}{4x^2}+\frac{13}{8x}+\cdots\right. \\
&= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{1}{24}\int \frac{9-26x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx \\
&= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx+\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,d\cdots \\
&= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)+\cdots \\
&= -\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{15\log(x)}{8}\cdots
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 82, normalized size = 0.90

$$\frac{1}{144}\left(128\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{-1+4x}{\sqrt{15}}\right)+3\left(-\frac{60}{x^2}+\frac{36}{x}-90\log(x)+32\log\left(1+x+x^2\right)+13\log\left(2-x+2x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x +
 2*x^2]))/144
```

Maple [A]
time = 0.04, size = 70, normalized size = 0.77

method	result
default	$\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{13\ln(2x^2-x+2)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}-\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\ln\cdots}{8}$
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}+\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{15\ln(x)}{8}-\frac{\sqrt{15}\,\arctan\left(\frac{2\left(2x-\frac{1}{2}\right)\sqrt{15}}{15}\right)}{72}+\frac{13\ln(4x^2-2\cdots}{48}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*ln(x^2+x+1)+8/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+13/48*ln(2*x^2-x+2)
-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/4/x^2+3/4/x-15/8*ln(x)
```

Maxima [A]
time = 0.50, size = 69, normalized size = 0.76

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{15}{8}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(x)
```

Fricas [A]
time = 0.39, size = 89, normalized size = 0.98

$$-\frac{2\sqrt{5}\sqrt{3}\,x^2\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}\,(4x-1)\right)-128\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-39x^2\log\left(2x^2-x+2\right)-96x^2\log\left(x^2+x+1\right)+270x^2\log\left(x\right)-108x+180}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128*
sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*
x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2
```

Sympy [A]
time = 0.16, size = 94, normalized size = 1.03

$$-\frac{15\log(x)}{8}+\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}+\frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sq
rt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

Giac [A]
time = 5.87, size = 70, normalized size = 0.77

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4\,x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{3\,x-5}{4\,x^2}+\frac{13}{48}\log\left(2\,x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{15}{8}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(abs(x))
```

Mupad [B]
time = 0.15, size = 92, normalized size = 1.01

$$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}-\frac{15\ln(x)}{8}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] ((3*x)/4 - 5/4)/x^2 - (15*log(x))/8 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/
2)*4i)/9 - 2/3) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) + lo
g(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 + 13/48) - log(x + (15^(1/2
)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 - 13/48)
```

3.250 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=307

$$-\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{42}$$

```
[Out] 1/28*x^2*(7-5*I*7^(1/2))+1/42*x^3*(7-5*I*7^(1/2))+1/28*x^2*(7+5*I*7^(1/2))+
1/42*x^3*(7+5*I*7^(1/2))-1/28*x*(35-9*I*7^(1/2))-1/28*x*(35+9*I*7^(1/2))+3/
112*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7-11*I*7^(1/2))+3/112*ln(4+4*x^2+x*(1+I*7^
(1/2)))*(7+11*I*7^(1/2))-11/4*arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/
2))*(9*I-5*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+11/4*arctan((1+8*x-I*7^(1/2))/
(70+2*I*7^(1/2))^(1/2))*(9*I+5*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 210, 642}

$$\frac{11\left(5\sqrt{7}+9i\right)\text{ArcTan}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{11\left(-5\sqrt{7}+9i\right)\text{ArcTan}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)-\frac{1}{28}\left(35+9i\sqrt{7}\right)x-\frac{1}{28}\left(35-9i\sqrt{7}\right)x$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -1/28*((35 - (9*I)*Sqrt[7])*x) - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7 - (5*I)*
Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (5*I)*Sqrt[7])*x^3)
/42 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I + 5*Sqrt[7])*ArcTan[(1 - I*Sq
rt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11
*(9*I - 5*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(4*Sqrt[14*(35 - I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt
[7])*x + 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x +
4*x^2])/112
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^3\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^3\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{4}\left(-9+5i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{1}{2}\left(5-i\sqrt{7}\right)x^2+\frac{2\left(9-5i\sqrt{7}\right)-3}{2\left(4+\left(1-i\sqrt{}\right.\right.}\right.}{\sqrt{7}}$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+\right.$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+\right.$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+\right.$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 109, normalized size = 0.36

$$\frac{1}{6}\left(x(-15+3x+2x^2)+3\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{10\log(x-\#1)+\log(x-\#1)\#1+19\log(x-\#1)\#1^2+3\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (10*
Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/(1
 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 74, normalized size = 0.24

method	result	size
default	$\frac{x^{3}}{3}+\frac{x^{2}}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(2\_Z^{4}+\_Z^{3}+5\_Z^{2}+\_Z+2\right)}\frac{\left(3\_R^{3}+19\_R^{2}+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^{3}+3\_R^{2}+10\_R+1}\right)}{2}$	74
risch	$\frac{x^{3}}{3}+\frac{x^{2}}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(2\_Z^{4}+\_Z^{3}+5\_Z^{2}+\_Z+2\right)}\frac{\left(3\_R^{3}+19\_R^{2}+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^{3}+3\_R^{2}+10\_R+1}\right)}{2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+1
)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4
+ x^3 + 5*x^2 + x + 2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(199) = 398$.
time = 1.17, size = 1202, normalized size = 3.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 1/112*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55
/32) - 21)*log(23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/
32) + 3/16)^3 - 23765*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^2 + 7744*x + 19470*I*sqrt(7) - 33040*sqrt(2101/1568*I*sqrt(7)
- 55/32) + 38950) - 1/112*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*log(-23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^3 + 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) -
```

55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) -
869) + 1/256*(53312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) -
36681)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 17493*
(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 7744*
x - 15708*I*sqrt(7) + 26656*sqrt(2101/1568*I*sqrt(7) - 55/32) - 29132) + 1/
112*(2*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) -
 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7)
- 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99
/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 28*sqrt(210
1/1568*I*sqrt(7) - 55/32) + 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 21)*log
(-49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(5
3312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 -
11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sq
rt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7
) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((17*sqrt(7)*(-
33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(3
3*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-3
3*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*s
qrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)
^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 8
4*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) +
6384*sqrt(2101/1568*I*sqrt(7) - 55/32) - 5946) - 1/112*(2*sqrt(7)*sqrt(-336
*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*
(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/5
6*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7
) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 1859/2) - 28*sqrt(2101/1568*I*sqrt(7) - 55/32
) - 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log(-49/4*(-33/112*I*sqrt(7
) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(53312*(33/112*I*sqrt(7) -
 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 11781*I*sqrt(7) + 19992*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I
sqrt(7) - 55/32) + 3/16)^2 - 1/256((17*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(21
01/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*sqrt(-336*(33/112*I*sqrt(
7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(
7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7)
+ 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101

```
/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(
7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) + 6384*sqrt(2101/1568*I*sq
rt(7) - 55/32) - 5946) - 5/2*x
```

Sympy [A]
time = 0.57, size = 61, normalized size = 0.20

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \mathrm{RootSum}\left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t\log\left(\frac{5831t^3}{1936} - \frac{23765t^2}{7744} + \frac{2065t}{242} + x + \frac{415}{121}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 26
88*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*_t
/242 + x + 415/121)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B]
time = 2.17, size = 128, normalized size = 0.42

$$\left(\sum_{k=1}^{4}\ln\left(-29\,x + \mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\left(-\frac{289\,x}{4} + \mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\left(\frac{581\,x}{16} - \mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\left(\frac{147\,x}{4} - \frac{49}{16}\right) + \frac{1141}{64}\right) + \frac{47}{4}\right) + 7\right)\mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\right) - \frac{5\,x}{2} + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] symsum(log(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*(
root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((581*x)/16
- root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((147*x)/4
 - 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*root(z^4 - (3*z^3)/4 +
 (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 + x^3/
3
```

3.251 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=269

$$\frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2-\frac{\left(53i+\sqrt{7}\right)\tan^{-1}\left(\frac{1 \cdots}{\sqrt{2(\cdots}}\right.}{2\sqrt{14\left(35+i\sqrt{\cdots}\right.}}$$

```
[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*x^2*(7-5*I*7^(1/2))+1/14*x*(7+5*I*7^(1/2))+1/28
*x^2*(7+5*I*7^(1/2))-1/56*ln(4+4*x^2+x*(1+I*7^(1/2)))*(35-9*I*7^(1/2))-1/56
*ln(4+4*x^2+x*(1-I*7^(1/2)))*(35+9*I*7^(1/2))+1/2*arctan((1+8*x+I*7^(1/2))/
(70-2*I*7^(1/2))^(1/2))*(53*I-7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/2*arctan(
(1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(53*I+7^(1/2))/(490+14*I*7^(1/2))
^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 210, 642}

$$-\frac{\left(\sqrt{7}+53i\right)\text{ArcTan}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-\sqrt{7}+53i\right)\text{ArcTan}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[
7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 -
I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) +
 ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2
])/56
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^2\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^2\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{2}\left(5-i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{i\left(2\left(5i+\sqrt{7}\right)+\left(9i+5\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\right)dx}{\sqrt{7}} - \cdots$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\cdots\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\cdots\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\cdots\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\cdots\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 101, normalized size = 0.38

$$x+\frac{x^2}{2}-\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{2\log(x-\#1)+3\log(x-\#1)\#1+\log(x-\#1)\#1^2+5\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3*
Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2
 + 8*#1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 67, normalized size = 0.25

method	result	size
default	$x+\frac{x^2}{2}+\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(-5\_R^3-\_R^2-3\_R-2)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67
risch	$x+\frac{x^2}{2}+\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(-5\_R^3-\_R^2-3\_R-2)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=Roo
tOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x +
2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(171) = 342$.
time = 1.18, size = 1145, normalized size = 4.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(
49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*s
qrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 10290*(-9/56*I*sqrt(
7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 25725*(-9/56*I*sqrt(7)
- 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 3/64*(3920*(-9/56*I*sqrt(7
) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sq
rt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt
(7) + 79/56) + 35) + 8384*x + 6615/2*I*sqrt(7) + 10290*sqrt(-37/392*I*sqrt(
```

```
7) + 79/56) + 13373/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*
I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt
(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79
/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14
*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*s
qrt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(135*I*s
qrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*
sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392
*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/5
6) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5
/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*
I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*
sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*
I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56
) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776)
+ 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391)
- 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/
8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 -
 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(
7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-3
7/392*I*sqrt(7) + 79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqr
t(-37/392*I*sqrt(7) + 79/56) + 5)*log(-49/4*(135*I*sqrt(7) + 420*sqrt(-37/3
92*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) +
 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5
587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-1
2*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56
*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt
(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/
392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7)
+ 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 14
59)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(
7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt
(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/56*(9*I*sqrt(7) + 2
8*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(10290*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 1421*(-9/56*I*sqrt(7) - 1/2*sqrt(-
37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x + 3267/2*I*sqrt(7) + 5082*sqrt(
-37/392*I*sqrt(7) + 79/56) + 13793/2) + x
```

<u>**Sympy**</u> **[B]** Leaf count of result is larger than twice the leaf count of optimal. 3662 vs. $2(219) = 438$.

time = 1.62, size = 3662, normalized size = 13.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**2/2 + x + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-1459*sqrt(1
4)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 15*s
qrt(77)*sqrt(553 + 64*sqrt(77))/2096 - 10391*sqrt(553 + 64*sqrt(77))/268288
 + 1459*sqrt(77)/8384 + 522933/268288 + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))
*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576) - 51089
5297*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71
978450944 - 6009493*sqrt(22)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 76
48*sqrt(77))/1124663296 - 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/2249326
592 - 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 153195*sqrt(22)*sqrt
(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7))/2249326592 + 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 + 290832444193/359892
25472 + 2303470247*sqrt(77)/2249326592) + (-5/8 - sqrt(79/448 + sqrt(77)/49
))*log(x**2 + x*(-45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 1459*sqrt(14)*sqrt(333*sqrt(5
53 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 + 10391*sqrt(553 + 64*sqr
t(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 15*sqrt(77)*sqrt(553 +
 64*sqrt(77))/2096) - 510895297*sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) +
 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*sqrt(333*sqrt(553 +
64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 8313499*sqrt(14)*sqrt(55
3 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/
71978450944 - 153195*sqrt(22)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77))/2249326592 + 4417610843*sqrt(553 + 64*
sqrt(77))/35989225472 + 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/224932659
2 + 290832444193/35989225472 + 2303470247*sqrt(77)/2249326592) + 2*sqrt(-sq
rt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1568 + 5/1
4 + 3*sqrt(77)/49)*atan(1073152*x/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqr
t(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqr
t(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77
)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqr
t(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 45*sqrt
(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) +
 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sq
rt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)
)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*
sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 2
```

```
1975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 1459*sqrt(14)*sqrt(333*sqrt(5
53 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqr
t(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
 + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(
333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)))
+ 20782*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(
7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))
 + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(
77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 93376*sqr
t(77)/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333
*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt
(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 1045866/(4313*sqrt(2)*sqrt(-sqrt(14
)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt
(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21
975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*
sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(7
7))) + 3840*sqrt(77)*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*s
qrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77
)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7
648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B]
time = 0.13, size = 188, normalized size = 0.70

$$x+\frac{x^2}{2}+\left(\sum_{k=1}^{4}\ln\left(-\frac{179\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8}-7\,x-\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,459}{8}-\frac{{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^2\,x\,665}{8}-\frac{{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^3\,x\,147}{4}-\frac{35\,{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^2}{32}+\frac{49\,{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^3}{16}-15\right)\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + x^2/2 + symsum(log((49*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^3)/16 - 7*x - (459*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)*x)/8 - (665*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^2*x)/8 - (147*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
 k)^3*x)/4 - (35*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^
2)/32 - (179*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 -
 15)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)
```

3.252 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=230

$$\frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x-\frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(19i-7\sqrt{7}\right)\tan^{-1}\left(\right.}{\sqrt{14\left(35\right.}}$$

```
[Out] 1/14*x*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/1
4*x*(7+5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(1/2)))*(7+5*I*7^(1/2))+arctan
((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19*I-7*7^(1/2))/(490-14*I*7^(1/
2))^(1/2)-arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(19*I+7*7^(1/2))
/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2112, 787, 648, 632, 210, 642}

$$-\frac{\left(7\sqrt{7}+19i\right)\text{ArcTan}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-7\sqrt{7}+19i\right)\text{ArcTan}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7
])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*
Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 -
I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
+ 4*x^2])/28
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{i\int \frac{-4\left(10-2i\sqrt{7}\right)+\left(-\left(1-i\sqrt{7}\right)\left(10\right.\right.}{4+\left(1-i\sqrt{7}\right)}}{4\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\surd}{4+\left(1+i\surd\right.}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\surd\right.\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}}{\sqrt{2\left(35+\right.}}\right.}{\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 94, normalized size = 0.41

$$x+2\mathrm{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{-\log(x-\#1)+2\log(x-\#1)\#1-2\log(x-\#1)\#1^2+\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x -
 #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1
^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 62, normalized size = 0.27

method	result	size
default	$x+2\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{\left(\_R^3-2\_R^2+2\_R-1\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62

risch	$x+2\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(\_R^3-2\_R^2+2\_R-1)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_
Z^4+_Z^3+5*_Z^2+_Z+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(151) = 302.
time = 1.17, size = 1190, normalized size = 5.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I*
sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqr
t(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/9
8*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqr
t(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt
(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)
*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 1155/2*I*s
qrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt(7)*sqrt
(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/
28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/
2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*
log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*s
qrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7)
```

```
- 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*
I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) -
7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/
4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/
56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7)
 - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14
) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)
 + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 3456*s
qrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt(7) - 1/14) + 636) -
 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1
/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(
196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqr
t(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98
*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I
*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7)
- 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21
)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7
*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(5
3/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I
*sqrt(7) - 1/14) - 7) + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7)
- 1/14) - 7) + 3456*sqrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqr
t(7) - 1/14) + 636) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) -
 7)*log(3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 -
 1029*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x
- 715/2*I*sqrt(7) - 1001*sqrt(53/98*I*sqrt(7) - 1/14) - 2871/2) + x
```

Sympy [A]
time = 0.51, size = 48, normalized size = 0.21

$$x + \operatorname{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t\log\left(\frac{3773t^3}{304} - \frac{1029t^2}{304} + \frac{1001t}{152} + x - \frac{121}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t
*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B]
time = 0.19, size = 183, normalized size = 0.80

$$x+\left(\sum_{k=1}^{4}\ln\left(\frac{115\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8}+15\,x-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8}+\frac{{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}^2\,x\,133}{8}-\frac{{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}^3\,x\,147}{4}-\frac{189\,{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}^2}{16}+\frac{49\,{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}^3}{16}-4\right)\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)
)/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x
)/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/8 -
 (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 - (189
*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z
^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4 - z^3
 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

3.253 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=198

$$\frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{\left(19i-7\sqrt{7}\right)\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{28}\left(7+5i\sqrt{7}\right)$$

```
[Out] 1/28*ln(4+4*x^2+x*(1+I*7^(1/2)))*(7-5*I*7^(1/2))+1/28*ln(4+4*x^2+x*(1-I*7^(
1/2)))*(7+5*I*7^(1/2))-arctan((1+8*x+I*7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(19
*I-7*7^(1/2))/(490-14*I*7^(1/2))^(1/2)+arctan((1+8*x-I*7^(1/2))/(70+2*I*7^(
1/2))^(1/2))*(19*I+7*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2111, 648, 632, 210, 642}

$$\frac{\left(7\sqrt{7}+19i\right)\text{ArcTan}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{\left(-7\sqrt{7}+19i\right)\text{ArcTan}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])
/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*
x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[
7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (
1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2111

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= -\left(\frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx\right) + \frac{1}{28}\left(7+5i\sqrt{7}\right)\int$$

$$= \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(\right.\right.$$

$$= \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{\left(19i-7\sqrt{7}\right)\tan^{-1}\left(\frac{1+}{\sqrt{2\left(\right.}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 90, normalized size = 0.45

$$\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{5\log(x-\#1)+\log(x-\#1)\#1+3\log(x-\#1)\#1^2+2\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#1
 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &
 ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 58, normalized size = 0.29

method	result	size
default	$\sum_{\_R=\text{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(2\_R^3+3\_R^2+\_R+5)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$	58
risch	$\sum_{\_R=\text{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(2\_R^3+3\_R^2+\_R+5)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4+
_Z^3+5*_Z^2+_Z+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1189 vs. 2(129) = 258.
time = 1.18, size = 1189, normalized size = 6.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")

[Out] -1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1
/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4
116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sq
rt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53
/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/9
8*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(
7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) +
 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7)
 - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqr
t(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/9
8*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(
7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*
sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sq
rt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98
*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) -
 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*
sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) -
 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(49/4*(105*I*sqrt(7) + 294
*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sq
rt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-
5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I
*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(
53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/
14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt
(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*
I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sq
rt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) - 1/28*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(105*I*sqrt(7) + 294*sqrt(53/9
8*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/
14) + 1/4)^2 + 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^3 - 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
```

```
 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x - 2205/2*I*sqrt(7) - 3087*sqrt(5
3/98*I*sqrt(7) - 1/14) - 3025/2) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt
(7) - 1/14) - 7)*log(-7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/
14) + 1/4)^3 + 2303*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^2 + 304*x + 1555/2*I*sqrt(7) + 2177*sqrt(53/98*I*sqrt(7) - 1/14) + 5823
/2)
```

Sympy [**A**]
time = 0.50, size = 46, normalized size = 0.23

$$\operatorname{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t\log\left(-\frac{7203t^3}{304} + \frac{2303t^2}{304} - \frac{2177t}{152} + x + \frac{250}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log
(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [**B**]
time = 2.34, size = 181, normalized size = 0.91

$$\sum_{k=1}^{4}\ln\left(-\frac{193\,\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)}{8}+4x-\frac{\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)x\,137}{8}+\frac{\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^2x\,651}{16}-\frac{\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^3x\,147}{4}+\frac{273\,\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^2}{16}+\frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^3}{16}+7\right)\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z,
k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8 +
 (651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 - (14
7*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273*roo
t(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z^4 -
 z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z^3 + (
6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

3.254 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)} dx$

Optimal. Leaf size=245

$$-\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{1}{28}\left(35-9i\sqrt{7}\right.$$

```
[Out] 1/28*ln(x)*(35-9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(35-9*I*7^(1
/2))+1/28*ln(x)*(35+9*I*7^(1/2))-1/56*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I
*7^(1/2))-1/2*arctanh((I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(53+I*7^(1/
2))/(490-14*I*7^(1/2))^(1/2)+1/2*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))
^(1/2))*(53-I*7^(1/2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 212, 642}

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)-\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/2*((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[
7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] +
 (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 -
 (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I
)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7
])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x}+\frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{2\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x}+\frac{3\left(11\right.}{2\left(\right.}\right.}{}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)-\frac{i\int \frac{3\left(11i+\sqrt{7}\right)-2\left(9\right.}{4i+\left(i-\sqrt{7}\right)\right.}}{2\sqrt{7}}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\int$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\mathrm{lo}$$

$$= -\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\right.}{2\sqrt{14\left(35-\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 101, normalized size = 0.41

$$\frac{5\log(x)}{2}-\frac{1}{2}\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{3\log(x-\#1)+19\log(x-\#1)\#1+\log(x-\#1)\#1^2+10\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1] +
 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 + 3
*#1^2 + 8*#1^3) & ]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 67, normalized size = 0.27

method	result
risch	$\left(\sum_{\_R=\mathrm{RootOf}(686\_Z^4+1715\_Z^3+1372\_Z^2+448\_Z+256)} \_R\ln\left(2058\_R^3+20825\_R^2+25844\_R+8384x\right.\right.$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)} \frac{(-10\_R^3-\_R^2-19\_R-3)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}+\frac{5\ln(x)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((-10*_R^3-_R^2-19*_R-3)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(
2*_Z^4+_Z^3+5*_Z^2+_Z+2))+5/2*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
+ 5/2*log(x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1143 vs. 2(148) = 296.
time = 1.15, size = 1143, normalized size = 4.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(49/4*(27*
I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/
2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 2058*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 5145*(-9/56*I*sqrt(7) - 1/2*sqrt(-3
7/392*I*sqrt(7) + 79/56) - 5/8)^2 + 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(
-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*s
qrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56)
+ 35) + 8384*x + 1323/2*I*sqrt(7) + 2058*sqrt(-37/392*I*sqrt(7) + 79/56) +
16089/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) -
```

```
 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-
9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 1
0*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sqrt(7) + 79/56
) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(27*I*sqrt(7) + 84*sqr
t(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sq
rt(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(
7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqr
t(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/5
6) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 7/64*
sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12
*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9
*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*s
qrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*s
qrt(7) + 79/56) + 11/2)*((27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56)
 + 1385)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*
sqrt(7) + 35840*sqrt(-37/392*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I
*sqrt(7) + 10864*sqrt(-37/392*I*sqrt(7) + 79/56) + 5484) - 1/8*(2*sqrt(-12*
(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I
*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7
) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/39
2*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) +
79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqrt(-37/392*I*sqrt(7
) + 79/56) + 5)*log(-49/4*(27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56
) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1
5680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64
*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 94
5*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) +
 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) -
1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((2
7*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(-9*I*sqrt(7) + 28
*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*sqrt(7) + 35840*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I*sqrt(7) + 10864*sqrt(-37/3
92*I*sqrt(7) + 79/56) + 5484) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(
7) + 79/56) + 35)*log(2058*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^3 + 20825*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/
56) - 5/8)^2 + 8384*x - 8307/2*I*sqrt(7) - 12922*sqrt(-37/392*I*sqrt(7) + 7
9/56) - 18673/2) + 5/2*log(x)
```

Sympy [A]
time = 9.31, size = 60, normalized size = 0.24

$$\frac{5\log(x)}{2} + \operatorname{RootSum}\left(686t^4 + 1715t^3 + 1372t^2 + 448t + 256, \left(t \mapsto t\log\left(-\frac{160344611t^4}{532759184} - \frac{16880402t^3}{33297449} + \frac{4010520787t^2}{2131036736} + \frac{1537535671t}{532759184} + x + \frac{46660495}{66594898}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, La
mbda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 4010
520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/66594898))
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)
```

Mupad [B]
time = 2.34, size = 237, normalized size = 0.97

$$\frac{5\,\ln\left(x\right)}{2}+\left(\sum_{k=1}^{4}\ln\left(\frac{223\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8}-\frac{31\,x}{2}+\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,71}{16}-\frac{{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^2\,x\,4463}{64}+\frac{{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^3\,x\,1449}{16}+\frac{{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^4\,x\,3675}{32}+\frac{257\,{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^2}{32}+\frac{1673\,{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^3}{64}-\frac{441\,{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}^4}{32}+10\right)\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 12
8/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k
, 1, 4)
```

3.255 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=281

$$-\frac{35-9i\sqrt{7}}{28x}-\frac{35+9i\sqrt{7}}{28x}+\frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{11\left(9-5i\sqrt{7}\right)\tanh^{-1}\left(\frac{i\cdots}{\sqrt{2\left(\cdots\right.}}\right.}{4\sqrt{14\left(35+i\sqrt{7}\right.}}$$

```
[Out] 1/28*(-35+9*I*7^(1/2))/x+1/28*(-35-9*I*7^(1/2))/x-3/56*ln(x)*(7-11*I*7^(1/2
))+3/112*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(7-11*I*7^(1/2))-3/56*ln(x)*(7+11*I*
7^(1/2))+3/112*ln(4*I+4*I*x^2+x*(I-7^(1/2)))*(7+11*I*7^(1/2))+11/4*arctanh(
(I+8*I*x-7^(1/2))/(70-2*I*7^(1/2))^(1/2))*(9+5*I*7^(1/2))/(490-14*I*7^(1/2)
)^(1/2)-11/4*arctanh((I+8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(9-5*I*7^(1/
2))/(490+14*I*7^(1/2))^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 212, 642}

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35-9i\sqrt{7}}{28x}-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)+\frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{11\left(9-5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/28*(35 - (9*I)*Sqrt[7])/x - (35 + (9*I)*Sqrt[7])/(28*x) + (11*(9 + (5*I)
*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqr
t[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8
*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (
11*I)*Sqrt[7])*Log[x])/56 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (1
1*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*
Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^2\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^2}+\frac{3\left(11-i\sqrt{7}\right)}{8x}+\frac{-7\left(9i-5\sqrt{7}\right)-6\left(11i+\sqrt{7}\right)x}{4\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx \quad i\int \left(\right.}{\sqrt{7}} + \text{—}$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{}\right.$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{}\right.$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{}\right.$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} + \frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{}\right.}}\right.}{4\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 109, normalized size = 0.39

$$-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{4}\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{-35\log(x-\#1)+13\log(x-\#1)\#1-17\log(x-\#1)\#1^2+6\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35*
Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^3)
/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 72, normalized size = 0.26

method	result
risch	$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(686\_Z^4-1029\_Z^3+6272\_Z^2+10752\_Z+4096\right)}\_R\ln\left(-45962\_R^3+98735\_R^2-497168\_R+\right.\right.}{2}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(6\_R^3-17\_R^2+13\_R-35\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{4}-\frac{5}{2x}-\frac{3\ln(x)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((6*_R^3-17*_R^2+13*_R-35)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootO
f(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/2/x-3/4*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3 + 5*x^2 +
x + 2), x) - 3/4*log(x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1245 vs. $2(172) = 344$.
time = 1.19, size = 1245, normalized size = 4.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/224*(2*x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log
(91924*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3
- 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839) - 1/256
*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33
*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 68943*(33/112*I*
```

sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x + 61908
*I*sqrt(7) - 105056*sqrt(2101/1568*I*sqrt(7) - 55/32) + 123428) + 2*x*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21)*log(-91924*(33/112*I
*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3 + 98735*(33/112*
I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x - 146
487/2*I*sqrt(7) + 124292*sqrt(2101/1568*I*sqrt(7) - 55/32) - 285347/2) + (4
*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32
) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) -
 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*s
qrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2)*x - x*(33*I*sqrt(7)
 + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - x*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) - 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1
/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqr
t(2101/1568*I*sqrt(7) - 55/32) - 3839) + 1/256*(210112*(33/112*I*sqrt(7) -
1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 46431*I*sqrt(7) + 78792*s
qrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) - 29792*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*
I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(
-2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) + 147456*sqrt(7))*sqrt(-336*(33/112*I*s
qrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*s
qrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt
(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*s
qrt(7) - 55/32) - 1859/2) + 30976*x + 22671/2*I*sqrt(7) - 19236*sqrt(2101/1
568*I*sqrt(7) - 55/32) + 53979/2) - (4*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7)
- 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7)
- 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 5
6*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/15
68*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7)
- 55/32) - 1859/2)*x + x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/
32) - 21) + x*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839
) + 1/256*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 11
7483)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 29792*(
33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/256*
((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) -
2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) -
 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 147456*sqrt(7))*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7)
 - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7

```
) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) -
55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) +
99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 30976*x +
 22671/2*I*sqrt(7) - 19236*sqrt(2101/1568*I*sqrt(7) - 55/32) + 53979/2) + 1
68*x*log(x) + 560)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 25507 vs. $2(241) = 482$.
time = 19.42, size = 25507, normalized size = 90.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + (3/16 - sqrt(-55/256 + 11*sqrt(77)/196))*log(x**2 + x*(108964
79943156192*sqrt(77)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 81
5992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628
044800*sqrt(77)) + 1720992726634016*sqrt(7)*sqrt(-245 + 64*sqrt(77))/(-3936
5093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 69742908928
00*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) + 39603456
8160*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*
sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983
777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 +
6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77))
 + 1300300581888*sqrt(154)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) -
21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-39365
093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 697429089280
0*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) - 278094051
039*sqrt(22)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 59837
77)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 6
974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77))
- 29480043023893*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21
120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-3936509
3785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 6974290892800*
sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) - 49949438061
3858*sqrt(11)*sqrt(-245 + 64*sqrt(77))/(-39365093785600*sqrt(7)*sqrt(-245 +
 64*sqrt(77)) - 815992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) + 225454628044800*sqrt(77)) - 133336449027059894/(-39365093785600*sq
rt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 6974290892800*sqrt(11)*s
qrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77))) + 62476107871936200684
235707503295184*sqrt(77)/(-12820275149960147338206904320000*sqrt(77) - 9780
98111454293303592222720000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 3531716786284
21216922828800000*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 137638843164853174995
```

```
608862720000) + 3325655347490676642136637231706384*sqrt(7)*sqrt(-245 + 64*s
qrt(77))/(-12820275149960147338206904320000*sqrt(77) - 97809811145429330359
2222720000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 35317167862842121692282880000
0*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 137638843164853174995608862720000) +
12591448063677487443028673736328*sqrt(154)*sqrt(-62589*sqrt(11)*sqrt(-245 +
 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) +
5983777)/(-12820275149960147338206904320000*sqrt(77) - 97809811145429330359
2222720000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 35317167862842121692282880000
0*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 137638843164853174995608862720000) +
1275262686986013252063099749736*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-625
89*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77
)) - 103712*sqrt(77) + 5983777)/(-12820275149960147338206904320000*sqrt(77)
 - 978098111454293303592222720000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 353171
678628421216922828800000*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 13763884316485
3174995608862720000) - 1213346648248587045336001776855*sqrt(22)*sqrt(-245 +
 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-128202751499601473
38206904320000*sqrt(77) - 978098111454293303592222720000*sqrt(7)*sqrt(-245
+ 64*sqrt(77)) + 353171678628421216922828800000*sqrt(11)*sqrt(-245 + 64*sqr
t(77)) + 137638843164853174995608862720000) - 98833524199287508514073622742
205*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*s
qrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983777)/(-12820275149960147338
206904320000*sqrt(77) - 978098111454293303592222720000*sqrt(7)*sqrt(-245 +
64*sqrt(77)) + 353171678628421216922828800000*sqrt(11)*sqrt(-245 + 64*sqrt(
77)) + 137638843164853174995608862720000) - 3523282605099669306216811941636
850*sqrt(11)*sqrt(-245 + 64*sqrt(77))/(-12820275149960147338206904320000*sq
rt(77) - 978098111454293303592222720000*sqrt(7)*sqrt(-245 + 64*sqrt(77)) +
353171678628421216922828800000*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 13763884
3164853174995608862720000) - 496941299152482355176771113608017254/(-1282027
5149960147338206904320000*sqrt(77) - 978098111454293303592222720000*sqrt(7)
*sqrt(-245 + 64*sqrt(77)) + 353171678628421216922828800000*sqrt(11)*sqrt(-2
45 + 64*sqrt(77)) + 137638843164853174995608862720000)) + (3/16 + sqrt(-55/
256 + 11*sqrt(77)/196))*log(x**2 + x*(133336449027059894/(-225454628044800*
sqrt(77) - 39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) + 6974290892800*
sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 815992034457600) + 1720992726634016*sqr
t(7)*sqrt(-245 + 64*sqrt(77))/(-225454628044800*sqrt(77) - 39365093785600*s
qrt(7)*sqrt(-245 + 64*sqrt(77)) + 6974290892800...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)
```

Mupad [B]
time = 2.30, size = 242, normalized size = 0.86

$$\left(\sum_{k=1}^{4}\ln\left(\frac{1199\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)}{32}+25x+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)x\,4169}{32}+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^2x\,43993}{256}+\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^3x\,28+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^4x\,3675}{32}+\frac{11647\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^2}{128}+\frac{7273\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^3}{128}-\frac{441\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^4}{32}+\frac{21}{4}\right)\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)\right)-\frac{3\ln(x)}{4}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] symsum(log((1199*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z
, k))/32 + 25*x + (4169*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128
/343, z, k)*x)/32 + (43993*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 +
128/343, z, k)^2*x)/256 + 28*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
+ 128/343, z, k)^3*x + (3675*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
+ 128/343, z, k)^4*x)/32 + (11647*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z
)/49 + 128/343, z, k)^2)/128 + (7273*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (9
6*z)/49 + 128/343, z, k)^3)/128 - (441*root(z^4 - (3*z^3)/4 + (16*z^2)/7 +
(96*z)/49 + 128/343, z, k)^4)/32 + 21/4)*root(z^4 - (3*z^3)/4 + (16*z^2)/7
+ (96*z)/49 + 128/343, z, k), k, 1, 4) - (3*log(x))/4 - 5/(2*x)
```

3.256 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=317

$$-\frac{35-9i\sqrt{7}}{56x^2}-\frac{35+9i\sqrt{7}}{56x^2}+\frac{3\left(7-11i\sqrt{7}\right)}{56x}+\frac{3\left(7+11i\sqrt{7}\right)}{56x}+\frac{\left(355-73i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right.}{8\sqrt{14\left(35-i\sqrt{7}\right)}}$$

```
[Out] 1/56*(-35+9*I*7^(1/2))/x^2-1/16*ln(x)*(35-9*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+
x*(I-7^(1/2)))*(35-9*I*7^(1/2))+1/56*(-35-9*I*7^(1/2))/x^2-1/16*ln(x)*(35+9
*I*7^(1/2))+1/32*ln(4*I+4*I*x^2+x*(I+7^(1/2)))*(35+9*I*7^(1/2))+3/56*(7-11*
I*7^(1/2))/x+3/56*(7+11*I*7^(1/2))/x+1/8*arctanh((I+8*I*x-7^(1/2))/(70-2*I*
7^(1/2))^(1/2))*(355-73*I*7^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/8*arctanh((I+
8*I*x+7^(1/2))/(70+2*I*7^(1/2))^(1/2))*(355+73*I*7^(1/2))/(490+14*I*7^(1/2)
)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 814, 648, 632, 212, 642}

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{3\left(7+11i\sqrt{7}\right)}{56x}+\frac{3\left(7-11i\sqrt{7}\right)}{56x}-\frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x)-\frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x)+\frac{\left(355-73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{\left(355+73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/56*(35 - (9*I)*Sqrt[7])/x^2 - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (1
1*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sq
rt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[1
4*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)
*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)
*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt
[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log
[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2112

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[x^m*((b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x)/(2*a + (b + q)*x + 2*a*x^2)), x], x] - Dist[1/q, Int[x^m*((b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2)),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^3\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^3\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^3} + \frac{3\left(11-i\sqrt{7}\right)}{8x^2} - \frac{7i\left(-9i+5\sqrt{7}\right)}{16x} + \frac{-223i-61\sqrt{7}+14\left(9i-5\sqrt{7}\right.}{8\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right.}\right.}{\sqrt{7}}$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} + \frac{(3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 116, normalized size = 0.37

$$-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\log(x)}{8}+\frac{1}{8}\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{61\log(x-\#1)+141\log(x-\#1)\#1+47\log(x-\#1)\#1^2+70\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#
1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*Log
[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 77, normalized size = 0.24

method	result
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(686\_Z^4-12005\_Z^3+73696\_Z^2-50176\_Z+65536\right)}\_R\ln\left(-2261742\_R^3+41411909\_R^2-249593568\_R\right.\right.}{4}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(70\_R^3+47\_R^2+141\_R+61\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{8}-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\ln\left(x\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum((70*_R^3+47*_R^2+141*_R+61)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=Roo
tOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/4/x^2+3/4/x-35/8*ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 + x
^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1274 vs. 2(196) = 392.
time = 1.17, size = 1274, normalized size = 4.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/448*(14*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
)*log(-49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896
) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35
/32)^2 + 9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 35/32)^3 - 39580485*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) +
 2815/896) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803
/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt
```

```
(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/
6272*I*sqrt(7) + 2815/896) - 35) + 9662336*x - 68336919/4*I*sqrt(7) - 30371
964*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 257023549/4) + 14*x^2*(9*I*sqrt
(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-9046968*(-9/32*I*
sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3 + 41411909*(
-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 96
62336*x + 70198191/4*I*sqrt(7) + 31199196*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 240366533/4) + 1960*x^2*log(x) + (4*sqrt(7)*sqrt(-1344*(9/32*I*sqrt(
7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqr
t(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*s
qrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9
803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 - 7*x^2*(9*I*sqrt(7) + 16*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 7*x^2*(-9*I*sqrt(7) + 16*sqrt(980
3/6272*I*sqrt(7) + 2815/896) - 35) - 980*x^2)*log(49/4*(207711*I*sqrt(7) +
369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/
2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sqrt(7
) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(1378585
6*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 +
 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 6848
8563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 1/102
4*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 3
5/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) +
 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/
2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(2307
9*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
49504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sqrt(-
9803/6272*I*sqrt(7) + 2815/896) + 666914) - (4*sqrt(7)*sqrt(-1344*(9/32*I*s
qrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*
I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I
*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) +
16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqr
t(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 + 7*x^2*(9*I*sqrt(7) + 16*
sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) + 7*x^2*(-9*I*sqrt(7) + 16*sqrt
(9803/6272*I*sqrt(7) + 2815/896) - 35) + 980*x^2)*log(49/4*(207711*I*sqrt(7
) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7)
- 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(137
85856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)
^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) -
68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
/1024*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896)
```

```
 + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2
205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(
23079*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)
 - 149504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896)
- 35) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sq
rt(-9803/6272*I*sqrt(7) + 2815/896) + 666914) - 336*x + 560)/x^2
```

Sympy [A]
time = 1.76, size = 70, normalized size = 0.22

$$-\frac{35\log(x)}{8} + \operatorname{RootSum}\left(2744t^4 - 12005t^3 + 18424t^2 - 3136t + 1024, \left(t \mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586} + \frac{944515214496t^3}{45953952180793} + \frac{16572327093911939t^2}{5882105879141504} - \frac{4564471749800865t}{735263234892688} + x + \frac{70084064010625}{91907904361586}\right)\right)\right) + \frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t + 1
024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 944515214496*
_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447174
9800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*x -
5)/(4*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)
```

Mupad [B]
time = 2.25, size = 246, normalized size = 0.78

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{8939\,\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)}{128}-\frac{69x}{8}+\frac{\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)x\,14945}{128}-\frac{\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)^2x\,269991}{1024}-\frac{\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)^3x\,1393}{8}+\frac{\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)^4x\,3675}{32}-\frac{35697\,\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)^2}{512}-\frac{18487\,\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)^3}{256}-\frac{441\,\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)^4}{32}+\frac{245}{8}\right)\mathrm{root}\left(z^4-\frac{35z^3}{8}+\frac{47z^2}{7}-\frac{8z}{7}+\frac{128}{343},z,k\right)\right)-\frac{35\ln(x)}{8}+\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z
, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7
 + 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)
/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/7 -
```

```
(8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^2)/7
- (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (47*z^2
)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8 + (47*
z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)/8 + (47
*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z^3)/8 + (
47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 + ((3*x)/4 -
 5/4)/x^2
```

3.257 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

[Out] arctan(c*x^3/(b*x^2+a))/c

Rubi [A]
time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2119, 211}

$$\frac{\text{ArcTan}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]

[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx &= (3a^2)\,\text{Subst}\left(\int \frac{1}{a^2+9a^2c^2x^2}\,dx, x, \frac{x^3}{3a+3bx^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 87, normalized size = 4.58

$$\frac{1}{2}\text{RootSum}\left[a^2+2ab\#1^2+b^2\#1^4+c^2\#1^6\&,\frac{3a\log(x-\#1)\#1+b\log(x-\#1)\#1^3}{2ab+2b^2\#1^2+3c^2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #1]*#1 + b*
Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 75, normalized size = 3.95

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^2\_Z^6+b^2\_Z^4+2a\_Z^2b+a^2\right)}\frac{\left(\_R^4b+3\_R^2a\right)\ln\left(x-\_R\right)}{3\_R^5c^2+2\_R^3b^2+2ab\_R}\right)}{2}$	75
risch	$-\frac{\arctan\left(\frac{c\,x^5b}{a^2}-\frac{c\,x^3}{a}+\frac{b^3x^3}{a^2c}+\frac{b^2x}{ac}\right)}{c}-\frac{\arctan\left(-\frac{c\,x^3}{a}+\frac{cx}{b}-\frac{b^2x}{ac}\right)}{c}+\frac{\arctan\left(\frac{cx}{b}\right)}{c}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/2*sum((_R^4*b+3*_R^2*a)/(3*_R^5*c^2+2*_R^3*b^2+2*_R*a*b)*ln(x-_R),_R=Root
Of(_Z^6*c^2+_Z^4*b^2+2*_Z^2*a*b+a^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(19) = 38.
time = 0.37, size = 83, normalized size = 4.37

$$\frac{\arctan\left(\frac{cx}{b}\right)-\arctan\left(\frac{bc^2x^5+ab^2x+(b^3-ac^2)x^3}{a^2c}\right)+\arctan\left(\frac{bc^2x^3+(b^3-ac^2)x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fri
cas")
```

```
[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c))
+ arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 44, normalized size = 2.32

$$\frac{-\frac{i\log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i\log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2)
/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(19) = 38.
time = 5.45, size = 87, normalized size = 4.58

$$\frac{\arctan\left(\frac{cx}{b}\right)+\arctan\left(-\frac{bc^2x^5+b^3x^3-ac^2x^3+ab^2x}{a^2c}\right)-\arctan\left(-\frac{bc^2x^3+b^3x-ac^2x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="gia
c")
```

```
[Out] (arctan(c*x/b) + arctan(-(b*c^2*x^5 + b^3*x^3 - a*c^2*x^3 + a*b^2*x)/(a^2*c
)) - arctan(-(b*c^2*x^3 + b^3*x - a*c^2*x)/(a*b*c)))/c
```

Mupad [B]
time = 2.27, size = 252, normalized size = 13.26

$$\frac{\mathrm{atan}\left(\frac{27\,a\,c^5\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{27\,b\,c^5\,x^5}{27\,a^2\,c^4-4\,a\,b^3\,c^2}-\frac{31\,b^3\,c^3\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^6\,c\,x^3}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}+\frac{4\,b^5\,c\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2}+\frac{4\,b^4\,c^3\,x^5}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2}-\frac{27\,a\,b^2\,c^3\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2}\right)+\mathrm{atan}\left(\frac{c\,x^3}{a}-\frac{c\,x}{b}+\frac{b^2\,x}{a\,c}\right)+\mathrm{atan}\left(\frac{c\,x}{b}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)
```

```
[Out] (atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c^
4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c*x
^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2) +
 (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^2*c^
4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((c*x)/
b))/c
```

3.258 $\int \frac{1-3x^4}{(-2+x)(1+x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{1-2x}{5(1+x^2)}-\frac{46}{25}\tan^{-1}(x)-\frac{47}{25}\log(2-x)-\frac{14}{25}\log\left(1+x^2\right)$$

```
[Out] 1/5*(-1+2*x)/(x^2+1)-46/25*arctan(x)-47/25*ln(2-x)-14/25*ln(x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1661, 1643, 649, 209, 266}

$$-\frac{46\text{ArcTan}(x)}{25}-\frac{1-2x}{5(x^2+1)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] -1/5*(1 - 2*x)/(1 + x^2) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log
[1 + x^2])/25
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-3x^4}{(-2+x)\,(1+x^2)^2}\,dx &= -\frac{1-2x}{5\,(1+x^2)}-\frac{1}{2}\int \frac{-\frac{18}{5}-\frac{4x}{5}+6x^2}{(-2+x)\,(1+x^2)}\,dx\\
&= -\frac{1-2x}{5\,(1+x^2)}-\frac{1}{2}\int \left(\frac{94}{25(-2+x)}+\frac{4(23+14x)}{25\,(1+x^2)}\right)dx\\
&= -\frac{1-2x}{5\,(1+x^2)}-\frac{47}{25}\log(2-x)-\frac{2}{25}\int \frac{23+14x}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\,(1+x^2)}-\frac{47}{25}\log(2-x)-\frac{28}{25}\int \frac{x}{1+x^2}\,dx-\frac{46}{25}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1-2x}{5\,(1+x^2)}-\frac{46}{25}\tan^{-1}(x)-\frac{47}{25}\log(2-x)-\frac{14}{25}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 1.33

$$\frac{3+2(-2+x)}{5\,(5+4(-2+x)+(-2+x)^2)}-\frac{46}{25}\tan^{-1}(x)-\frac{14}{25}\log\left(5+4(-2+x)+(-2+x)^2\right)-\frac{47}{25}\log(-2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (1
4*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25
```

Maple [A]
time = 0.21, size = 34, normalized size = 0.79

method	result	size
risch	$\frac{\frac{2x}{5}-\frac{1}{5}}{x^2+1}-\frac{14\ln(x^2+1)}{25}-\frac{46\arctan(x)}{25}-\frac{47\ln(x-2)}{25}$	33

default	$-\frac{47\ln(x-2)}{25}-\frac{2\left(-5x+\frac{5}{2}\right)}{25(x^2+1)}-\frac{14\ln(x^2+1)}{25}-\frac{46\arctan(x)}{25}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^4+1)/(x-2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -47/25*ln(x-2)-2/25*(-5*x+5/2)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)
```

Maxima [A]
time = 0.53, size = 33, normalized size = 0.77

$$\frac{2x-1}{5(x^2+1)}-\frac{46}{25}\arctan(x)-\frac{14}{25}\log(x^2+1)-\frac{47}{25}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
x - 2)
```

Fricas [A]
time = 0.39, size = 47, normalized size = 1.09

$$-\frac{46(x^2+1)\arctan(x)+14(x^2+1)\log(x^2+1)+47(x^2+1)\log(x-2)-10x+5}{25(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*lo
g(x - 2) - 10*x + 5)/(x^2 + 1)
```

Sympy [A]
time = 0.07, size = 37, normalized size = 0.86

$$-\frac{1-2x}{5x^2+5}-\frac{47\log(x-2)}{25}-\frac{14\log(x^2+1)}{25}-\frac{46\operatorname{atan}(x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
```

```
[Out] -(1 - 2*x)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 - 46*atan(
x)/25
```

Giac [A]
time = 4.52, size = 34, normalized size = 0.79

$$\frac{2x-1}{5(x^2+1)}-\frac{46}{25}\arctan(x)-\frac{14}{25}\log(x^2+1)-\frac{47}{25}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
abs(x - 2))
```

Mupad [B]
time = 0.05, size = 38, normalized size = 0.88

$$\frac{\frac{2x}{5}-\frac{1}{5}}{x^2+1}-\frac{47\ln(x-2)}{25}+\ln(x-\mathrm{i})\left(-\frac{14}{25}+\frac{23}{25}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{14}{25}-\frac{23}{25}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x)
```

```
[Out] ((2*x)/5 - 1/5)/(x^2 + 1) - log(x - 1i)*(14/25 - 23i/25) - log(x + 1i)*(14/
25 + 23i/25) - (47*log(x - 2))/25
```

3.259 $\int \frac{-9-9x+2x^2}{-9x+x^3} \, dx$

Optimal. Leaf size=17

$$-\log(3-x)+\log(x)+2\log(3+x)$$

```
[Out] -ln(3-x)+ln(x)+2*ln(3+x)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1607, 1816}

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-9-9x+2x^2}{-9x+x^3} \, dx &= \int \frac{-9-9x+2x^2}{x\left(-9+x^2\right)} \, dx \\
&= \int \left(\frac{1}{3-x}+\frac{1}{x}+\frac{2}{3+x}\right) dx \\
&= -\log(3-x)+\log(x)+2\log(3+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\log(3-x)+\log(x)+2\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Maple [A]
time = 0.21, size = 16, normalized size = 0.94

method	result	size
default	$-\ln(x-3)+\ln(x)+2\ln(3+x)$	16
norman	$-\ln(x-3)+\ln(x)+2\ln(3+x)$	16
risch	$-\ln(x-3)+\ln(x)+2\ln(3+x)$	16
meijerg	$\frac{\ln\left(1-\frac{x^2}{9}\right)}{2}+\ln(x)-\ln(3)+\frac{i\pi}{2}+3\,\mathrm{arctanh}\left(\frac{x}{3}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-9*x-9)/(x^3-9*x),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x-3)+ln(x)+2*ln(3+x)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.88

$$2\log(x+3)-\log(x-3)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Fricas [A]
time = 0.39, size = 15, normalized size = 0.88

$$2\log(x+3)-\log(x-3)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="fricas")
```

```
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.82

$$\log(x)-\log(x-3)+2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-9*x-9)/(x**3-9*x),x)
[Out] log(x) - log(x - 3) + 2*log(x + 3)
```

Giac [A]
time = 4.33, size = 18, normalized size = 1.06

$$2 \log(|x + 3|) - \log(|x - 3|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="giac")
[Out] 2*log(abs(x + 3)) - log(abs(x - 3)) + log(abs(x))
```

Mupad [B]
time = 2.20, size = 21, normalized size = 1.24

$$2 \ln(x + 3) - 2 \operatorname{atanh}\left(\frac{1296}{18x + 162} - 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x - 2*x^2 + 9)/(9*x - x^3),x)
[Out] 2*log(x + 3) - 2*atanh(1296/(18*x + 162) - 7)
```

3.260 $\int \frac{1+2x^2+x^5}{-x+x^3} \, dx$

Optimal. Leaf size=25

$$x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)$$

[Out] x+1/3*x^3+2*ln(1-x)-ln(x)+ln(1+x)

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1607, 1816}

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2 + x^5)/(-x + x^3),x]

[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2+x^5}{-x+x^3} \, dx &= \int \frac{1+2x^2+x^5}{x\left(-1+x^2\right)} \, dx\\ &= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}+x^2+\frac{1}{1+x}\right) dx\\ &= x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$x + \frac{x^3}{3} + 2\log(1-x) - \log(x) + \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^5)/(-x + x^3),x]
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Maple [A]
time = 0.23, size = 22, normalized size = 0.88

method	result	size
default	$\frac{x^3}{3} + x + 2\ln(-1+x) - \ln(x) + \ln(1+x)$	22
norman	$\frac{x^3}{3} + x + 2\ln(-1+x) - \ln(x) + \ln(1+x)$	22
risch	$\frac{x^3}{3} + x + 2\ln(-1+x) - \ln(x) + \ln(1+x)$	22
meijerg	$\frac{3\ln(-x^2+1)}{2} - \ln(x) - \frac{i\pi}{2} + \frac{i\left(-\frac{2ix(5x^2+15)}{15} + 2i\,\mathrm{arctanh}(x)\right)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+2*x^2+1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+x+2*ln(-1+x)-ln(x)+ln(1+x)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.84

$$\frac{1}{3}\,x^3 + x + \log(x+1) + 2\,\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Fricas [A]
time = 0.39, size = 21, normalized size = 0.84

$$\frac{1}{3}\,x^3 + x + \log(x+1) + 2\,\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.80

$$\frac{x^3}{3} + x - \log(x) + 2\log(x-1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+2*x**2+1)/(x**3-x),x)
[Out] x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)
```

Giac [A]
time = 4.26, size = 24, normalized size = 0.96

$$\frac{1}{3}x^3 + x + \log(|x+1|) + 2\,\log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="giac")
[Out] 1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B]
time = 0.05, size = 30, normalized size = 1.20

$$x + 2\,\ln(x-1) + \frac{x^3}{3} + \mathrm{atan}\left(\frac{48\mathrm{i}}{11\,(22\,x-2)} + \frac{13}{11}\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + x^5 + 1)/(x - x^3),x)
[Out] x + 2*log(x - 1) + atan(48i/(11*(22*x - 2)) + 13i/11)*2i + x^3/3
```

3.261 $\int \frac{3+2x^2}{(-1+x)^2x}\,dx$

Optimal. Leaf size=22

$$\frac{5}{1-x}-\log(1-x)+3\log(x)$$

```
[Out] 5/(1-x)-ln(1-x)+3*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {908}

$$\frac{5}{1-x}-\log(1-x)+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

```
[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x^2}{(-1+x)^2x}\,dx &= \int \left(\frac{1}{1-x}+\frac{5}{(-1+x)^2}+\frac{3}{x}\right)dx\\ &= \frac{5}{1-x}-\log(1-x)+3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.91

$$-\frac{5}{-1+x}-\log(1-x)+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x),x]
```

```
[Out] -5/(-1 + x) - Log[1 - x] + 3*Log[x]
```

Maple [A]
time = 0.23, size = 19, normalized size = 0.86

method	result	size
default	$-\frac{5}{-1+x}-\ln\left(-1+x\right)+3\ln\left(x\right)$	19
norman	$-\frac{5}{-1+x}-\ln\left(-1+x\right)+3\ln\left(x\right)$	19
risch	$-\frac{5}{-1+x}-\ln\left(-1+x\right)+3\ln\left(x\right)$	19
meijerg	$\frac{2x}{1-x}-\ln\left(1-x\right)+\frac{6x}{2-2x}+3+3\ln\left(x\right)+3i\pi$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(-1+x)^2/x,x,method=_RETURNVERBOSE)
[Out] -5/(-1+x)-ln(-1+x)+3*ln(x)
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.82

$$-\frac{5}{x-1}-\log\left(x-1\right)+3\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")
[Out] -5/(x - 1) - log(x - 1) + 3*log(x)
```

Fricas [A]
time = 0.37, size = 24, normalized size = 1.09

$$-\frac{(x-1)\log\left(x-1\right)-3\left(x-1\right)\log\left(x\right)+5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")
[Out] -((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.64

$$3\log\left(x\right)-\log\left(x-1\right)-\frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(-1+x)**2/x,x)
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

Giac [A]
time = 4.03, size = 28, normalized size = 1.27

$$-\frac{5}{x-1}+2\,\log\left(|x-1|\right)+3\,\log\left(\left|-\frac{1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")
[Out] -5/(x - 1) + 2*log(abs(x - 1)) + 3*log(abs(-1/(x - 1) - 1))
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.82

$$3\,\ln\left(x\right)-\ln\left(x-1\right)-\frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3)/(x*(x - 1)^2),x)
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

3.262 $\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx$

Optimal. Leaf size=27

$$\frac{3}{17}\tan^{-1}(x)-\frac{7}{34}\log(1-4x)+\frac{6}{17}\log\left(1+x^2\right)$$

```
[Out] 3/17*arctan(x)-7/34*ln(1-4*x)+6/17*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1643, 649, 209, 266}

$$\frac{3\text{ArcTan}(x)}{17}+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

```
[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+2x^2}{(-1+4x)(1+x^2)}\,dx = \int \left(-\frac{14}{17(-1+4x)}+\frac{3(1+4x)}{17(1+x^2)}\right)dx$$
$$= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1+4x}{1+x^2}\,dx$$
$$= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1}{1+x^2}\,dx+\frac{12}{17}\int \frac{x}{1+x^2}\,dx$$
$$= \frac{3}{17}\tan^{-1}(x)-\frac{7}{34}\log(1-4x)+\frac{6}{17}\log\left(1+x^2\right)$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.41

$$\frac{3}{17}\tan^{-1}(x)-\frac{7}{34}\log(-1+4x)+\frac{6}{17}\log\left(17+2(-1+4x)+(-1+4x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

```
[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 +
4*x)^2])/17
```

Maple [A]
time = 0.21, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{7\ln(4x-1)}{34}+\frac{6\ln(x^2+1)}{17}+\frac{3\arctan(x)}{17}$	22
risch	$-\frac{7\ln(4x-1)}{34}+\frac{6\ln(x^2+1)}{17}+\frac{3\arctan(x)}{17}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(4*x-1)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -7/34*ln(4*x-1)+6/17*ln(x^2+1)+3/17*arctan(x)
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.78

$$\frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="maxima")
```

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)

Fricas [A]
time = 0.39, size = 21, normalized size = 0.78

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log\left(x^2 + 1\right) - \frac{7}{34} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="fricas")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)

Sympy [A]
time = 0.05, size = 26, normalized size = 0.96

$$-\frac{7\log\left(x - \frac{1}{4}\right)}{34} + \frac{6\log\left(x^2 + 1\right)}{17} + \frac{3\operatorname{atan}(x)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)

[Out] -7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17

Giac [A]
time = 3.83, size = 22, normalized size = 0.81

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log\left(x^2 + 1\right) - \frac{7}{34} \log(|4x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="giac")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))

Mupad [B]
time = 2.26, size = 25, normalized size = 0.93

$$-\frac{7\ln\left(x - \frac{1}{4}\right)}{34} + \ln(x - \mathrm{i})\left(\frac{6}{17} - \frac{3}{34}\mathrm{i}\right) + \ln(x + 1\mathrm{i})\left(\frac{6}{17} + \frac{3}{34}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 - 1)/((4*x - 1)*(x^2 + 1)),x)

[Out] log(x - 1i)*(6/17 - 3i/34) - (7*log(x - 1/4))/34 + log(x + 1i)*(6/17 + 3i/3
4)

3.263 $\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx$

Optimal. Leaf size=21

$$-3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1824, 266}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx &= \int \left(-3+x+\frac{x}{1+x^2}\right)dx\\ &= -3x+\frac{x^2}{2}+\int \frac{x}{1+x^2}\,dx\\ &= -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Maple [A]
time = 0.19, size = 18, normalized size = 0.86

method	result	size
default	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
norman	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
meijerg	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18
risch	$-3x+\frac{x^2}{2}+\frac{\ln(x^2+1)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+2*x-3)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2-3x+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.40, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2-3x+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.02, size = 15, normalized size = 0.71

$$\frac{x^2}{2}-3x+\frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)
```

```
[Out] x**2/2 - 3*x + log(x**2 + 1)/2
```

Giac [A]
time = 3.51, size = 17, normalized size = 0.81

$$\frac{1}{2}\,x^2 - 3\,x + \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.81

$$\frac{\ln\left(x^2+1\right)}{2} - 3\,x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3*x^2 + x^3 - 3)/(x^2 + 1),x)
```

```
[Out] log(x^2 + 1)/2 - 3*x + x^2/2
```

3.264 $\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} \, dx$

Optimal. Leaf size=27

$$\frac{x^3}{3}-3\tan^{-1}(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)$$

```
[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1671, 648, 632, 210, 642}

$$-3\text{ArcTan}(x+3)+\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx &= \int \left(x^2+\frac{x}{10+6x+x^2}\right)dx \\
&= \frac{x^3}{3}+\int \frac{x}{10+6x+x^2}\,dx \\
&= \frac{x^3}{3}+\frac{1}{2}\int \frac{6+2x}{10+6x+x^2}\,dx-3\int \frac{1}{10+6x+x^2}\,dx \\
&= \frac{x^3}{3}+\frac{1}{2}\log\left(10+6x+x^2\right)+6\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,6+2x\right) \\
&= \frac{x^3}{3}-3\tan^{-1}(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^3}{3}-3\tan^{-1}(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Maple [A]
time = 0.28, size = 24, normalized size = 0.89

method	result	size
default	$\frac{x^3}{3}-3\arctan\left(3+x\right)+\frac{\ln\left(x^2+6x+10\right)}{2}$	24
risch	$\frac{x^3}{3}-3\arctan\left(3+x\right)+\frac{\ln\left(x^2+6x+10\right)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3-3*arctan(3+x)+1/2*ln(x^2+6*x+10)
```

Maxima [A]
time = 0.50, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Fricas [A]
time = 0.42, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="fricas")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.81

$$\frac{x^3}{3} + \frac{\log\left(x^2 + 6x + 10\right)}{2} - 3\operatorname{atan}(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)
[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

Giac [A]
time = 3.15, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3 - 3\arctan(x+3) + \frac{1}{2}\log\left(x^2 + 6x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="giac")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

Mupad [B]
time = 2.14, size = 23, normalized size = 0.85

$$\frac{\ln\left(x^2 + 6x + 10\right)}{2} - 3\operatorname{atan}(x+3) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 10*x^2 + 6*x^3 + x^4)/(6*x + x^2 + 10),x)
[Out] log(6*x + x^2 + 10)/2 - 3*atan(x + 3) + x^3/3
```

3.265 $\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)$$

```
[Out] 1/8*ln(1-x)-1/5*ln(2-x)+1/12*ln(3-x)-1/120*ln(3+x)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2083}

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx &= \int \left(\frac{1}{12(-3+x)}-\frac{1}{5(-2+x)}+\frac{1}{8(-1+x)}-\frac{1}{120(3+x)}\right)dx\\ &= \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.67

method	result	size
default	$\frac{\ln(x-3)}{12}-\frac{\ln(x-2)}{5}+\frac{\ln(-1+x)}{8}-\frac{\ln(3+x)}{120}$	26
norman	$\frac{\ln(x-3)}{12}-\frac{\ln(x-2)}{5}+\frac{\ln(-1+x)}{8}-\frac{\ln(3+x)}{120}$	26
risch	$\frac{\ln(x-3)}{12}-\frac{\ln(x-2)}{5}+\frac{\ln(-1+x)}{8}-\frac{\ln(3+x)}{120}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^3-7*x^2+27*x-18),x,method=_RETURNVERBOSE)
[Out] 1/12*ln(x-3)-1/5*ln(x-2)+1/8*ln(-1+x)-1/120*ln(3+x)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.64

$$-\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1)-\frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

Fricas [A]
time = 0.38, size = 25, normalized size = 0.64

$$-\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1)-\frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="fricas")
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

Sympy [A]
time = 0.10, size = 26, normalized size = 0.67

$$\frac{\log(x-3)}{12}-\frac{\log(x-2)}{5}+\frac{\log(x-1)}{8}-\frac{\log(x+3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120
```

Giac [A]
time = 3.53, size = 29, normalized size = 0.74

$$-\frac{1}{120}\log\left(|x+3|\right)+\frac{1}{8}\log\left(|x-1|\right)-\frac{1}{5}\log\left(|x-2|\right)+\frac{1}{12}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="giac")
```

```
[Out] -1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*l
og(abs(x - 3))
```

Mupad [B]
time = 2.14, size = 25, normalized size = 0.64

$$\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x-2\right)}{5}+\frac{\ln\left(x-3\right)}{12}-\frac{\ln\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(7*x^2 - 27*x + 3*x^3 - x^4 + 18),x)
```

```
[Out] log(x - 1)/8 - log(x - 2)/5 + log(x - 3)/12 - log(x + 3)/120
```

3.266 $\int \frac{1+x^3}{-2+x} dx$

Optimal. Leaf size=22

$$4x + x^2 + \frac{x^3}{3} + 9\log(2-x)$$

```
[Out] 4*x+x^2+1/3*x^3+9*ln(2-x)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1864}

$$\frac{x^3}{3} + x^2 + 4x + 9\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-2 + x),x]
[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{-2+x} dx &= \int \left(4 + \frac{9}{-2+x} + 2x + x^2\right) dx \\ &= 4x + x^2 + \frac{x^3}{3} + 9\log(2-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.05

$$-\frac{44}{3} + 4x + x^2 + \frac{x^3}{3} + 9\log(-2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-2 + x),x]
[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]
```

Maple [A]
time = 0.17, size = 19, normalized size = 0.86

method	result	size
default	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
norman	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
risch	$\frac{x^3}{3}+x^2+4x+9\ln\left(x-2\right)$	19
meijerg	$9\ln\left(1-\frac{x}{2}\right)+\frac{x\left(x^2+3x+12\right)}{3}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x-2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+x^2+4*x+9*ln(x-2)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.82

$$\frac{1}{3}\,x^3+x^2+4\,x+9\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Fricas [A]
time = 0.37, size = 18, normalized size = 0.82

$$\frac{1}{3}\,x^3+x^2+4\,x+9\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

Sympy [A]
time = 0.02, size = 17, normalized size = 0.77

$$\frac{x^3}{3}+x^2+4x+9\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(-2+x),x)
```

```
[Out] x**3/3 + x**2 + 4*x + 9*log(x - 2)
```

Giac [A]
time = 3.00, size = 19, normalized size = 0.86

$$\frac{1}{3}\,x^3 + x^2 + 4\,x + 9\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))
```

Mupad [B]
time = 0.03, size = 18, normalized size = 0.82

$$4\,x + 9\,\ln\left(x-2\right) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x - 2),x)
```

```
[Out] 4*x + 9*log(x - 2) + x^2 + x^3/3
```

3.267 $\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx$

Optimal. Leaf size=15

$$-4x+\frac{3x^2}{2}+4\tan^{-1}(x)$$

```
[Out] -4*x+3/2*x^2+4*arctan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1608, 1816, 209}

$$4\text{ArcTan}(x)+\frac{3x^2}{2}-4x$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx &= \int \frac{x(3-4x+3x^2)}{1+x^2}\,dx\\ &= \int \left(-4+3x+\frac{4}{1+x^2}\right)dx\\ &= -4x+\frac{3x^2}{2}+4\int \frac{1}{1+x^2}\,dx\\ &= -4x+\frac{3x^2}{2}+4\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-4x+\frac{3x^2}{2}+4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Maple [A]
time = 0.16, size = 14, normalized size = 0.93

method	result	size
default	$-4x+\frac{3x^2}{2}+4\arctan(x)$	14
meijerg	$-4x+\frac{3x^2}{2}+4\arctan(x)$	14
risch	$-4x+\frac{3x^2}{2}+4\arctan(x)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x^2+3*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -4*x+3/2*x^2+4*arctan(x)
```

Maxima [A]
time = 0.47, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2-4\,x+4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Sympy [A]
time = 0.02, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4\,\text{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)
```

```
[Out] 3*x**2/2 - 4*x + 4*atan(x)
```

Giac [A]
time = 2.93, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2 - 4\,x + 4\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

Mupad [B]
time = 2.13, size = 13, normalized size = 0.87

$$4\,\text{atan}(x) - 4\,x + \frac{3\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2 + 3*x^3)/(x^2 + 1),x)
```

```
[Out] 4*atan(x) - 4*x + (3*x^2)/2
```

3.268 $\int \frac{5+3x}{1-x-x^2+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

```
[Out] 4/(1-x)+arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2099, 212}

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] 4/(1 - x) + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{5+3x}{1-x-x^2+x^3} \, dx &= \int \left(\frac{4}{(-1+x)^2}+\frac{1}{1-x^2}\right) dx\\ &= \frac{4}{1-x}+\int \frac{1}{1-x^2} \, dx\\ &= \frac{4}{1-x}+\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 2.00

$$-\frac{4}{-1+x}-\frac{1}{2}\log(-1+x)+\frac{1}{2}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2
```

Maple [A]
time = 0.02, size = 21, normalized size = 1.75

method	result	size
default	$-\frac{4}{-1+x}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	21
norman	$-\frac{4}{-1+x}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	21
risch	$-\frac{4}{-1+x}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+3*x)/(x^3-x^2-x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(-1+x)-1/2*ln(-1+x)+1/2*ln(1+x)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 1.67

$$-\frac{4}{x-1}+\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")
```

```
[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.38, size = 26, normalized size = 2.17

$$\frac{(x-1)\log\left(x+1\right)-(x-1)\log\left(x-1\right)-8}{2\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.
time = 0.03, size = 17, normalized size = 1.42

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}-\frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x**3-x**2-x+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.
time = 3.14, size = 22, normalized size = 1.83

$$-\frac{4}{x-1}+\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="giac")
```

```
[Out] -4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.07, size = 10, normalized size = 0.83

$$\operatorname{atanh}(x)-\frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + 5)/(x + x^2 - x^3 - 1),x)
```

```
[Out] atanh(x) - 4/(x - 1)
```

3.269 $\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)$$

```
[Out] -1/x+1/2*x^2-2*ln(1-x)+2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1607, 1634}

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-x-x^3+x^4}{-x^2+x^3} \, dx &= \int \frac{-1-x-x^3+x^4}{(-1+x)x^2} \, dx \\
&= \int \left(-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}+x\right) \, dx \\
&= -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$-\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Maple [A]
time = 0.23, size = 22, normalized size = 0.88

method	result	size
default	$\frac{x^2}{2}-2\ln\left(-1+x\right)-\frac{1}{x}+2\ln\left(x\right)$	22
risch	$\frac{x^2}{2}-2\ln\left(-1+x\right)-\frac{1}{x}+2\ln\left(x\right)$	22
norman	$\frac{-1+\frac{x^3}{2}}{x}+2\ln\left(x\right)-2\ln\left(-1+x\right)$	23
meijerg	$2\ln\left(x\right)+2i\pi-\frac{1}{x}+\frac{x(3x+6)}{6}-x-2\ln\left(1-x\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3-x-1)/(x^3-x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-2*ln(-1+x)-1/x+2*ln(x)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.84

$$\frac{1}{2}\,x^2-\frac{1}{x}-2\,\log\left(x-1\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)
```

Fricas [A]
time = 0.40, size = 22, normalized size = 0.88

$$\frac{x^3-4\,x\log\left(x-1\right)+4\,x\log\left(x\right)-2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^2}{2} + 2\log(x) - 2\log(x-1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3-x-1)/(x**3-x**2),x)
[Out] x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x
```

Giac [A]
time = 2.34, size = 23, normalized size = 0.92

$$\frac{1}{2}x^2 - \frac{1}{x} - 2\log(|x-1|) + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="giac")
[Out] 1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))
```

Mupad [B]
time = 2.14, size = 19, normalized size = 0.76

$$4\,\mathrm{atanh}(2\,x-1) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 - x^4 + 1)/(x^2 - x^3),x)
[Out] 4*atanh(2*x - 1) - 1/x + x^2/2
```

3.270 $\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}(x)+\frac{1}{2}\log\left(2+x^2\right)$$

[Out] arctan(x)+1/2*ln(x^2+2)

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1687, 1163, 209, 1261, 640, 31}

$$\text{ArcTan}(x)+\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]

[Out] ArcTan[x] + Log[2 + x^2]/2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 640

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx &= \int \frac{x(1+x^2)}{2+3x^2+x^4}\,dx + \int \frac{2+x^2}{2+3x^2+x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{2+3x+x^2}\,dx, x, x^2\right) + \int \frac{1}{1+x^2}\,dx\\ &= \tan^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)\\ &= \tan^{-1}(x) + \frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\tan^{-1}(x) + \frac{1}{2}\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.92

method	result	size
default	$\arctan(x) + \frac{\ln(x^2+2)}{2}$	12
risch	$\arctan(x) + \frac{\ln(x^2+2)}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+2)/(x^4+3*x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)+1/2*ln(x^2+2)
```

Maxima [A]
time = 0.51, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Fricas [A]
time = 0.37, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="fricas")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$\frac{\log(x^2 + 2)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
```

```
[Out] log(x**2 + 2)/2 + atan(x)
```

Giac [A]
time = 4.12, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="giac")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

Mupad [B]
time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln(x^2+2)}{2}+\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 2)/(3*x^2 + x^4 + 2),x)
```

```
[Out] log(x^2 + 2)/2 + atan(x)
```

3.271 $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx$

Optimal. Leaf size=35

$$-\frac{1}{(2+x^2)^2}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1828, 1600, 649, 209, 266}

$$-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

[In] Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]

[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
```

```
EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx &= -\frac{1}{(2+x^2)^2}-\frac{1}{8}\int \frac{16-16x+8x^2-8x^3}{(2+x^2)^2}\,dx\\ &= -\frac{1}{(2+x^2)^2}-\frac{1}{8}\int \frac{8-8x}{2+x^2}\,dx\\ &= -\frac{1}{(2+x^2)^2}-\int \frac{1}{2+x^2}\,dx+\int \frac{x}{2+x^2}\,dx\\ &= -\frac{1}{(2+x^2)^2}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$-\frac{1}{(2+x^2)^2}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Maple [A]
time = 0.19, size = 31, normalized size = 0.89

method	result

default	$-\frac{1}{\left(x^2+2\right)^2}+\frac{\ln\left(x^2+2\right)}{2}-\frac{\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}$
risch	$-\frac{1}{\left(x^2+2\right)^2}+\frac{\ln\left(x^2+2\right)}{2}-\frac{\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}$
meijerg	$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x\left(\frac{3x^2}{2}+5\right)}{4\left(1+\frac{x^2}{2}\right)^2}+\frac{3\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}\right)}{8}-\frac{x^2\left(\frac{9x^2}{2}+6\right)}{24\left(1+\frac{x^2}{2}\right)^2}+\frac{\ln\left(1+\frac{x^2}{2}\right)}{2}-\frac{\sqrt{2}\left(-\frac{x\sqrt{2}\left(\frac{25x^2}{2}+15\right)}{20\left(1+\frac{x^2}{2}\right)^2}+\frac{3\arctan\left(\sqrt{2}\ldots\right.}{2}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x,method=_RETURNVERBOSE)
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 35, normalized size = 1.00

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{1}{x^4+4\,x^2+4}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*log(x^2 + 2)
```

Fricas [A]
time = 0.38, size = 55, normalized size = 1.57

$$-\frac{\sqrt{2}\left(x^4+4\,x^2+4\right)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\left(x^4+4\,x^2+4\right)\log\left(x^2+2\right)+2}{2\left(x^4+4\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")
[Out] -1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*l
og(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)
```

Sympy [A]
time = 0.05, size = 36, normalized size = 1.03

$$\frac{\log\left(x^2+2\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{1}{x^4+4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)
```

```
[Out] log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2 + 4)
```

Giac [A]
time = 4.36, size = 30, normalized size = 0.86

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{1}{\left(x^2+2\right)^2}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)
```

Mupad [B]
time = 2.12, size = 35, normalized size = 1.00

$$\frac{\ln\left(x^2+2\right)}{2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{1}{x^4+4\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - 4*x^2 + 4*x^3 - x^4 + x^5 - 4)/(x^2 + 2)^3,x)
```

```
[Out] log(x^2 + 2)/2 - (2^(1/2)*atan((2^(1/2)*x)/2))/2 - 1/(4*x^2 + x^4 + 4)
```

3.272 $\int \frac{-1-3x+x^2}{-2x+x^2+x^3} \, dx$

Optimal. Leaf size=23

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)$$

```
[Out] -ln(1-x)+1/2*ln(x)+3/2*ln(2+x)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1608, 1642}

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-3x+x^2}{-2x+x^2+x^3} \, dx &= \int \frac{-1-3x+x^2}{x\left(-2+x+x^2\right)} \, dx \\
&= \int \left(\frac{1}{1-x}+\frac{1}{2x}+\frac{3}{2(2+x)}\right) dx \\
&= -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.78

method	result	size
default	$\frac{3\ln(x+2)}{2}-\ln\left(-1+x\right)+\frac{\ln(x)}{2}$	18
norman	$\frac{3\ln(x+2)}{2}-\ln\left(-1+x\right)+\frac{\ln(x)}{2}$	18
risch	$\frac{3\ln(x+2)}{2}-\ln\left(-1+x\right)+\frac{\ln(x)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x-1)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(x+2)-ln(-1+x)+1/2*ln(x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.74

$$\frac{3}{2}\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Fricas [A]
time = 0.40, size = 17, normalized size = 0.74

$$\frac{3}{2}\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.74

$$\frac{\log(x)}{2} - \log(x-1) + \frac{3\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)
[Out] log(x)/2 - log(x - 1) + 3*log(x + 2)/2
```

Giac [A]
time = 3.71, size = 20, normalized size = 0.87

$$\frac{3}{2}\log(|x+2|) - \log(|x-1|) + \frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] 3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))
```

Mupad [B]
time = 2.19, size = 17, normalized size = 0.74

$$\frac{3\ln(x+2)}{2} - \ln(x-1) + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - x^2 + 1)/(x^2 - 2*x + x^3),x)
[Out] (3*log(x + 2))/2 - log(x - 1) + log(x)/2
```

3.273 $\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} \, dx$

Optimal. Leaf size=23

$$\frac{x^2}{2} + \log(x) - \frac{1}{2}\log\left(3 - 2x + x^2\right)$$

[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)

Rubi [A]

time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1608, 1642, 642}

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2 - 2x + 3\right) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]

[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx &= \int \frac{3-x+3x^2-2x^3+x^4}{x\left(3-2x+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+x+\frac{1-x}{3-2x+x^2}\right)dx \\
&= \frac{x^2}{2}+\log(x)+\int \frac{1-x}{3-2x+x^2}\,dx \\
&= \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$
\frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.87

method	result	size
default	$\frac{x^2}{2}+\ln\left(x\right)-\frac{\ln\left(x^2-2x+3\right)}{2}$	20
norman	$\frac{x^2}{2}+\ln\left(x\right)-\frac{\ln\left(x^2-2x+3\right)}{2}$	20
risch	$\frac{x^2}{2}+\ln\left(x\right)-\frac{\ln\left(x^2-2x+3\right)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.83

$$
\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log\left(x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

Fricas [A]
time = 0.38, size = 19, normalized size = 0.83

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 2x + 3\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + \log\left(x\right) - \frac{\log\left(x^2 - 2x + 3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)
[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

Giac [A]
time = 3.54, size = 20, normalized size = 0.87

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 2x + 3\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))
```

Mupad [B]
time = 0.06, size = 19, normalized size = 0.83

$$\ln\left(x\right) - \frac{\ln\left(x^2 - 2x + 3\right)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - x - 2*x^3 + x^4 + 3)/(3*x - 2*x^2 + x^3),x)
[Out] log(x) - log(x^2 - 2*x + 3)/2 + x^2/2
```

3.274 $\int \frac{-1+x+x^3}{(1+x^2)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{x}{2(1+x^2)} - \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\log(1+x^2)$$

```
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1828, 649, 209, 266}

$$-\frac{\text{ArcTan}(x)}{2} - \frac{x}{2(x^2+1)} + \frac{1}{2}\log(x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+x^3}{(1+x^2)^2}\,dx &= -\frac{x}{2(1+x^2)}-\frac{1}{2}\int \frac{1-2x}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)}-\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)}-\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{2}\left(-\frac{x}{1+x^2}-\tan^{-1}(x)+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^3)/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2
```

Maple [A]
time = 0.20, size = 24, normalized size = 0.83

method	result	size
default	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
risch	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}+\frac{\ln(x^2+1)}{2}$	24
meijerg	$\frac{\ln(x^2+1)}{2}-\frac{x}{2x^2+2}-\frac{\arctan(x)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x-1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.47, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2+1)}-\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.38, size = 32, normalized size = 1.10

$$-\frac{(x^2+1)\arctan(x)-(x^2+1)\log(x^2+1)+x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2+2}+\frac{\log(x^2+1)}{2}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x-1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2
```

Giac [A]
time = 2.96, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2+1)}-\frac{1}{2}\arctan(x)+\frac{1}{2}\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 2.13, size = 25, normalized size = 0.86

$$\frac{\ln(x^2+1)}{2}-\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 - 1)/(x^2 + 1)^2,x)
```

```
[Out] log(x^2 + 1)/2 - atan(x)/2 - x/(2*(x^2 + 1))
```

3.275 $\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx$

Optimal. Leaf size=44

$$-\frac{3}{1+x}-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)$$

```
[Out] -3/(1+x)+ln(x)-2*ln(1+x)+ln(x^2-x+1)-2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2
)
```

Rubi [A]
time = 0.17, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 6857, 648, 632, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
+ Log[1 - x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx &= \int \frac{1+2x-x^2+8x^3+x^4}{x(1+x)(1+x^3)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{3}{(1+x)^2}-\frac{2}{1+x}+\frac{2x}{1-x+x^2}\right)dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+2\int \frac{x}{1-x+x^2}\,dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\int \frac{1}{1-x+x^2}\,dx+\int \frac{-1+2x}{1-x+x^2}\,dx \\
&= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)-2\text{Subst}\left(\int \frac{1}{-3-x^2}\right. \\
&= -\frac{3}{1+x}-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.00

$$
-\frac{3}{1+x}+\frac{2\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
[Out] -3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
 + Log[1 - x + x^2]
```

Maple [A]
time = 0.18, size = 42, normalized size = 0.95

method	result	size
risch	$-\frac{3}{1+x}-2\ln\left(1+x\right)+\ln\left(x\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}+\ln\left(x^{2}-x+1\right)$	40
default	$\ln\left(x\right)-\frac{3}{1+x}-2\ln\left(1+x\right)+\ln\left(x^{2}-x+1\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-3/(1+x)-2*ln(1+x)+ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 41, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{3}{x+1}+\log\left(x^{2}-x+1\right)-2\,\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(x + 1) + log(x)
```

Fricas [A]
time = 0.42, size = 58, normalized size = 1.32

$$\frac{2\sqrt{3}\,(x+1)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+3\,(x+1)\log\left(x^{2}-x+1\right)-6\,(x+1)\log\left(x+1\right)+3\,(x+1)\log\left(x\right)-9}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(3)*(x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x + 1)*log(x^2 -
x + 1) - 6*(x + 1)*log(x + 1) + 3*(x + 1)*log(x) - 9)/(x + 1)
```

Sympy [A]
time = 0.08, size = 49, normalized size = 1.11

$$\log(x) - 2\log(x+1) + \log\left(x^2 - x + 1\right) + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/3 - 3/(x + 1)
```

Giac [A]
time = 3.76, size = 43, normalized size = 0.98

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right) - \frac{3}{x+1} + \log\left(x^2 - x + 1\right) - 2\,\log\left(|x+1|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(abs(x + 1)) + log(abs(x))
```

Mupad [B]
time = 0.13, size = 55, normalized size = 1.25

$$\ln(x) - 2\,\ln(x+1) - \frac{3}{x+1} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1 + \frac{\sqrt{3}\,1\mathrm{i}}{3}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1 + \frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 8*x^3 + x^4 + 1)/((x^3 + 1)*(x + x^2)),x)
```

```
[Out] log(x) - 2*log(x + 1) - 3/(x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/3 - 1) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/3 + 1)
```

3.276 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$-\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6857, 209, 648, 632, 210, 642}

$$-\sqrt{5}\,\text{ArcTan}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\text{ArcTan}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right)dx \\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2 \\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$-\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

Maple [A]
time = 0.35, size = 41, normalized size = 0.89

method	result	size
risch	$\frac{\ln(x^2+2x+3)}{2}+\frac{5\arctan\left(\frac{(1+x)\sqrt{2}}{2}\right)\sqrt{2}}{2}-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	39
default	$\frac{\ln(x^2+2x+3)}{2}+\frac{5\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}-\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,method=_RETURNVERBOSE)

[Out] 1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))-arctan(1/5*x*5^(1/2))*5^(1/2)

Maxima [A]
time = 0.48, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

Fricas [A]
time = 0.40, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

Sympy [A]
time = 0.08, size = 51, normalized size = 1.11

$$\frac{\log\left(x^{2}+2x+3\right)}{2}-\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

Giac [A]
time = 4.95, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)+\frac{1}{2}\log\left(x^{2}+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Mupad [B]
time = 0.16, size = 88, normalized size = 1.91

$$\frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2}+\sqrt{5}\,\operatorname{atan}\left(\frac{2000\,\sqrt{5}}{2000\,x+1120}-\frac{224\,\sqrt{5}\,x}{2000\,x+1120}\right)-\frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}+\frac{\sqrt{2}\,\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

3.277 $\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log\left(1+x^2\right)-\log\left(2+x+x^2\right)$$

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

Rubi [A]
time = 0.12, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}=0.114$, Rules used = {6874, 267, 266, 643, 642}

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 +
x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx = \int \left(\frac{6x}{(1+x^2)^2}+\frac{2x}{1+x^2}+\frac{-1-2x}{(2+x+x^2)^2}+\frac{-1-2x}{2+x+x^2}\right)dx$$
$$= 2\int \frac{x}{1+x^2}\,dx + 6\int \frac{x}{(1+x^2)^2}\,dx + \int \frac{-1-2x}{(2+x+x^2)^2}\,dx + \int \frac{-1-2x}{2+x+x^2}\,dx$$
$$= -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log(1+x^2)-\log(2+x+x^2)$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log(1+x^2)-\log(2+x+x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2
*(2 + x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Maple [A]
time = 0.41, size = 34, normalized size = 1.03

method	result	size
default	$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\ln(x^2+1)-\ln(x^2+x+2)$	34
norman	$\frac{-2x^2-3x-5}{(x^2+x+2)(x^2+1)}-\ln(x^2+x+2)+\ln(x^2+1)$	43
risch	$\frac{-2x^2-3x-5}{(x^2+x+2)(x^2+1)}-\ln(x^2+x+2)+\ln(x^2+1)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,method=
_RETURNVERBOSE)
```

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 1.33

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="maxima")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(33) = 66.
time = 0.40, size = 72, normalized size = 2.18

$$-\frac{2\,x^2+(x^4+x^3+3\,x^2+x+2)\log\left(x^2+x+2\right)-(x^4+x^3+3\,x^2+x+2)\log\left(x^2+1\right)+3\,x+5}{x^4+x^3+3\,x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x^3 + 3*x^2
 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.24

$$\frac{-2x^2-3x-5}{x^4+x^3+3x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+
2)**2,x)
```

```
[Out] (-2*x**2 - 3*x - 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1) - log(x*
*2 + x + 2)
```

Giac [A]
time = 5.12, size = 44, normalized size = 1.33

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="giac")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

Mupad [B]
time = 2.16, size = 56, normalized size = 1.70

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}+\mathrm{atan}\left(\frac{\frac{x\,224\mathrm{i}}{11}+\frac{224}{11}\mathrm{i}}{44\,x^2+16\,x+60}-\frac{3}{11}\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6 - 3)/((x^2 + 1)^2*(x + x
^2 + 2)^2),x)
```

```
[Out] atan(((x*224i)/11 + 224i/11)/(16*x + 44*x^2 + 60) - 3i/11)*2i - (3*x + 2*x^
2 + 5)/(x + 3*x^2 + x^3 + x^4 + 2)
```

3.278 $\int \frac{1}{(1+x^2)(4+x^2)} dx$

Optimal. Leaf size=17

$$-\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

```
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {400, 209}

$$\frac{\text{ArcTan}(x)}{3}-\frac{1}{6}\text{ArcTan}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/6*ArcTan[x/2] + ArcTan[x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x^2)(4+x^2)} dx &= \frac{1}{3}\int \frac{1}{1+x^2} dx-\frac{1}{3}\int \frac{1}{4+x^2} dx\\ &= -\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] ArcTan[2/x]/6 + ArcTan[x]/3
```

Maple [A]
time = 0.20, size = 12, normalized size = 0.71

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{6}+\frac{\arctan\left(x\right)}{3}$	12
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{6}+\frac{\arctan\left(x\right)}{3}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A]
time = 0.49, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A]
time = 0.39, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}+\frac{\operatorname{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**2+4),x)
```

```
[Out] -atan(x/2)/6 + atan(x)/3
```

Giac [**A**]
time = 4.11, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

Mupad [**B**]
time = 0.03, size = 11, normalized size = 0.65

$$\frac{\operatorname{atan}(x)}{3}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + 4)),x)
```

```
[Out] atan(x)/3 - atan(x/2)/6
```

3.279 $\int \frac{a+bx^3}{1+x^2} \, dx$

Optimal. Leaf size=24

$$\frac{bx^2}{2}+a\tan^{-1}(x)-\frac{1}{2}b\log\left(1+x^2\right)$$

```
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1824, 649, 209, 266}

$$a\text{ArcTan}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^3}{1+x^2}\,dx &= \int \left(bx+\frac{a-bx}{1+x^2}\right)\,dx\\
&= \frac{bx^2}{2}+\int \frac{a-bx}{1+x^2}\,dx\\
&= \frac{bx^2}{2}+a\int \frac{1}{1+x^2}\,dx-b\int \frac{x}{1+x^2}\,dx\\
&= \frac{bx^2}{2}+a\tan^{-1}(x)-\frac{1}{2}b\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.92

$$a\tan^{-1}(x)+\frac{1}{2}b\left(x^2-\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(1 + x^2),x]
[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2
```

Maple [A]
time = 0.20, size = 21, normalized size = 0.88

method	result	size
default	$\frac{b\,x^2}{2}+a\arctan\left(x\right)-\frac{b\ln\left(x^2+1\right)}{2}$	21
meijerg	$\frac{b\left(x^2-\ln\left(x^2+1\right)\right)}{2}+a\arctan\left(x\right)$	21
risch	$\frac{b\,x^2}{2}+a\arctan\left(x\right)-\frac{b\ln\left(x^2+1\right)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

Maxima [A]
time = 0.49, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2+a\arctan\left(x\right)-\frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Fricas [A]
time = 0.40, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2+a\arctan\left(x\right)-\frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.07, size = 34, normalized size = 1.42

$$\frac{bx^2}{2}+\left(-\frac{ia}{2}-\frac{b}{2}\right)\log\left(x-i\right)+\left(\frac{ia}{2}-\frac{b}{2}\right)\log\left(x+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(x**2+1),x)
```

```
[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

Giac [A]
time = 4.05, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2+a\arctan\left(x\right)-\frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

Mupad [B]
time = 2.13, size = 20, normalized size = 0.83

$$\frac{b\,x^2}{2}-\frac{b\,\ln\left(x^2+1\right)}{2}+a\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(x^2 + 1),x)
```

```
[Out] (b*x^2)/2 - (b*log(x^2 + 1))/2 + a*atan(x)
```

3.280 $\int \frac{x+x^2}{(4+x)(-4+x^2)}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)+\log(4+x)$$

```
[Out] -1/2*arctanh(1/2*x)+ln(4+x)
```

Rubi [A]
time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1607, 1643, 213}

$$\log(x+4)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] -1/2*ArcTanh[x/2] + Log[4 + x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x+x^2}{(4+x)\left(-4+x^2\right)} \, dx = \int \frac{x(1+x)}{(4+x)\left(-4+x^2\right)} \, dx$$
$$= \int \left(\frac{1}{4+x} + \frac{1}{-4+x^2}\right) dx$$
$$= \log(4+x) + \int \frac{1}{-4+x^2} \, dx$$
$$= -\frac{1}{2} \tanh^{-1}\left(\frac{x}{2}\right) + \log(4+x)$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.53

$$\frac{1}{4} \log(2-x) - \frac{1}{4} \log(2+x) + \log(4+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]
```

Maple [A]
time = 0.22, size = 18, normalized size = 1.20

method	result	size
default	$\ln(x+4) - \frac{\ln(x+2)}{4} + \frac{\ln(x-2)}{4}$	18
norman	$\ln(x+4) - \frac{\ln(x+2)}{4} + \frac{\ln(x-2)}{4}$	18
risch	$\ln(x+4) - \frac{\ln(x+2)}{4} + \frac{\ln(x-2)}{4}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(x+4)/(x^2-4),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x+4)-1/4*ln(x+2)+1/4*ln(x-2)
```

Maxima [A]
time = 0.48, size = 17, normalized size = 1.13

$$\log(x+4) - \frac{1}{4} \log(x+2) + \frac{1}{4} \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="maxima")
```

```
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Fricas [A]
time = 0.44, size = 17, normalized size = 1.13

$$\log(x+4) - \frac{1}{4}\log(x+2) + \frac{1}{4}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="fricas")
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 1.13

$$\frac{\log(x-2)}{4} - \frac{\log(x+2)}{4} + \log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(4+x)/(x**2-4),x)
[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

Giac [A]
time = 6.25, size = 20, normalized size = 1.33

$$\log(|x+4|) - \frac{1}{4}\log(|x+2|) + \frac{1}{4}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))
```

Mupad [B]
time = 0.06, size = 19, normalized size = 1.27

$$\ln(x+4) + \frac{\operatorname{atanh}\left(\frac{90}{7\,(21\,x+48)} - \frac{8}{7}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)/((x^2 - 4)*(x + 4)),x)
[Out] log(x + 4) + atanh(90/(7*(21*x + 48)) - 8/7)/2
```

3.281 $\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=20

$$3\tan^{-1}(x)-\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 3*arctan(x)-arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {536, 209}

$$3\text{ArcTan}(x)-\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx &= -\left(2\int \frac{1}{2+x^2}\,dx\right)+3\int \frac{1}{1+x^2}\,dx\\ &= 3\tan^{-1}(x)-\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$3\tan^{-1}(x)-\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Maple [A]
time = 0.21, size = 18, normalized size = 0.90

method	result	size
default	$3\arctan\left(x\right)-\arctan\left(\frac{\sqrt{2}\, x}{2}\right)\sqrt{2}$	18
risch	$3\arctan\left(x\right)-\arctan\left(\frac{\sqrt{2}\, x}{2}\right)\sqrt{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] 3*arctan(x)-arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+3\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Fricas [A]
time = 0.43, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+3\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.95

$$3\operatorname{atan}\left(x\right)-\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A]
time = 3.65, size = 17, normalized size = 0.85

$$-\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+3\ \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

Mupad [B]
time = 0.05, size = 17, normalized size = 0.85

$$3\,\mathrm{atan}(x)-\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 4)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] 3*atan(x) - 2^(1/2)*atan((2^(1/2)*x)/2)
```

3.282 $\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)} dx$

Optimal. Leaf size=37

$$\frac{5}{2(1-x)} + x + 2\tan^{-1}(x) + \frac{1}{2}\log(1-x) + \frac{3}{4}\log\left(1+x^2\right)$$

```
[Out] 5/2/(1-x)+x+2*arctan(x)+1/2*ln(1-x)+3/4*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1643, 649, 209, 266}

$$2\text{ArcTan}(x) + \frac{3}{4}\log\left(x^2+1\right) + x + \frac{5}{2(1-x)} + \frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx &= \int \left(1+\frac{5}{2(-1+x)^2}+\frac{1}{2(-1+x)}+\frac{4+3x}{2(1+x^2)}\right)dx\\ &= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{4+3x}{1+x^2}\,dx\\ &= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{3}{2}\int \frac{x}{1+x^2}\,dx+2\int \frac{1}{1+x^2}\,dx\\ &= \frac{5}{2(1-x)}+x+2\tan^{-1}(x)+\frac{1}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.89

$$\frac{5}{2-2x}+x+2\tan^{-1}(x)+\frac{1}{2}\log(-1+x)+\frac{3}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4
```

Maple [A]
time = 0.18, size = 28, normalized size = 0.76

method	result	size
default	$x-\frac{5}{2(-1+x)}+\frac{\ln(-1+x)}{2}+\frac{3\ln(x^2+1)}{4}+2\arctan(x)$	28
risch	$x-\frac{5}{2(-1+x)}+\frac{\ln(-1+x)}{2}+\frac{3\ln(x^2+1)}{4}+2\arctan(x)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] x-5/2/(-1+x)+1/2*ln(-1+x)+3/4*ln(x^2+1)+2*arctan(x)
```

Maxima [A]
time = 0.48, size = 27, normalized size = 0.73

$$x-\frac{5}{2(x-1)}+2\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Fricas [**A**]
time = 0.39, size = 44, normalized size = 1.19

$$\frac{4\,x^2+8\,(x-1)\arctan(x)+3\,(x-1)\log(x^2+1)+2\,(x-1)\log(x-1)-4\,x-10}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x
 - 1) - 4*x - 10)/(x - 1)
```

Sympy [**A**]
time = 0.06, size = 29, normalized size = 0.78

$$x+\frac{\log(x-1)}{2}+\frac{3\log(x^2+1)}{4}+2\,\mathrm{atan}(x)-\frac{5}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)
```

```
[Out] x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.
time = 3.97, size = 60, normalized size = 1.62

$$\frac{1}{2}\,\pi-2\,\pi\left\lfloor\frac{\pi+4\,\arctan(x)}{4\,\pi}+\frac{1}{2}\right\rfloor+x-\frac{5}{2\,(x-1)}+2\,\arctan(x)+\frac{3}{4}\,\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)+2\,\log(|x-1|)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2*
arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1
```

Mupad [**B**]
time = 2.14, size = 35, normalized size = 0.95

$$x+\frac{\ln(x-1)}{2}-\frac{5}{2\,(x-1)}+\ln(x-\mathrm{i})\,\left(\frac{3}{4}-\mathrm{i}\right)+\ln(x+1\mathrm{i})\,\left(\frac{3}{4}+1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 4*x + x^4 + 5)/((x^2 + 1)*(x - 1)^2),x)
```

```
[Out] x + log(x - 1)/2 + log(x - 1i)*(3/4 - 1i) + log(x + 1i)*(3/4 + 1i) - 5/(2*(
x - 1))
```

3.283 $\int \frac{1+x^4}{2+x^2} \, dx$

Optimal. Leaf size=26

$$-2x+\frac{x^3}{3}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -2*x+1/3*x^3+5/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1168, 209}

$$\frac{5\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{x^3}{3}-2x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(2 + x^2),x]
```

```
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{2+x^2} \, dx &= \int \left(-2+x^2+\frac{5}{2+x^2}\right) \, dx \\
&= -2x+\frac{x^3}{3}+5\int \frac{1}{2+x^2} \, dx \\
&= -2x+\frac{x^3}{3}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-2x+\frac{x^3}{3}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(2 + x^2),x]
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Maple [A]
time = 0.17, size = 22, normalized size = 0.85

method	result	size
default	$-2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}$	22
risch	$-2x+\frac{x^3}{3}+\frac{5\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}$	22
meijerg	$\frac{\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{2}+\sqrt{2}\left(-\frac{x\sqrt{2}\left(-\frac{5x^2}{2}+15\right)}{15}+2\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -2*x+1/3*x^3+5/2*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.81

$$\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="maxima")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Fricas [A]
time = 0.42, size = 21, normalized size = 0.81

$$\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Sympy [A]
time = 0.02, size = 26, normalized size = 1.00

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**2+2),x)
```

```
[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

Giac [A]
time = 3.21, size = 21, normalized size = 0.81

$$\frac{1}{3}\,x^3 + \frac{5}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.81

$$\frac{5\,\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2} - 2\,x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^2 + 2),x)
```

```
[Out] (5*2^(1/2)*atan((2^(1/2)*x)/2))/2 - 2*x + x^3/3
```

3.284 $\int \frac{2+2x+x^4}{x^4+x^5} \, dx$

Optimal. Leaf size=12

$$-\frac{2}{3x^3}+\log(1+x)$$

[Out] -2/3/x^3+ln(1+x)

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1607, 1634}

$$\log(x+1)-\frac{2}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x + x^4)/(x^4 + x^5),x]

[Out] -2/(3*x^3) + Log[1 + x]

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+2x+x^4}{x^4+x^5}\,dx &= \int \frac{2+2x+x^4}{x^4(1+x)}\,dx\\
&= \int \left(\frac{2}{x^4}+\frac{1}{1+x}\right)dx\\
&= -\frac{2}{3x^3}+\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{2}{3x^3} + \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Maple [A]
time = 0.16, size = 11, normalized size = 0.92

method	result	size
default	$-\frac{2}{3x^3} + \ln(1+x)$	11
norman	$-\frac{2}{3x^3} + \ln(1+x)$	11
meijerg	$-\frac{2}{3x^3} + \ln(1+x)$	11
risch	$-\frac{2}{3x^3} + \ln(1+x)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x+2)/(x^5+x^4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/x^3+ln(1+x)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.83

$$-\frac{2}{3\,x^3} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")
```

```
[Out] -2/3/x^3 + log(x + 1)
```

Fricas [A]
time = 0.41, size = 16, normalized size = 1.33

$$\frac{3\,x^3\log(x+1) - 2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.83

$$\log(x+1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x+2)/(x**5+x**4),x)
[Out] log(x + 1) - 2/(3*x**3)
```

Giac [A]
time = 2.44, size = 11, normalized size = 0.92

$$-\frac{2}{3\,x^3} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="giac")
[Out] -2/3/x^3 + log(abs(x + 1))
```

Mupad [B]
time = 2.12, size = 10, normalized size = 0.83

$$\ln(x+1) - \frac{2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4 + 2)/(x^4 + x^5),x)
[Out] log(x + 1) - 2/(3*x^3)
```

3.285 $\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx$

Optimal. Leaf size=21

$$2\log(1-x)-\log(2-x)+\log(1+x)$$

```
[Out] 2*ln(1-x)-ln(2-x)+ln(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2099}

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx &= \int \left(\frac{1}{2-x}+\frac{2}{-1+x}+\frac{1}{1+x}\right)dx\\ &= 2\log(1-x)-\log(2-x)+\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$2\log(1-x)-\log(2-x)+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.86

method	result	size
default	$-\ln(x-2)+2\ln(-1+x)+\ln(1+x)$	18
norman	$-\ln(x-2)+2\ln(-1+x)+\ln(1+x)$	18
risch	$-\ln(x-2)+2\ln(-1+x)+\ln(1+x)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x,method=_RETURNVERBOSE)
[Out] -ln(x-2)+2*ln(-1+x)+ln(1+x)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$\log(x+1)+2\log(x-1)-\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="maxima")
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Fricas [A]
time = 0.43, size = 17, normalized size = 0.81

$$\log(x+1)+2\log(x-1)-\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="fricas")
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.71

$$-\log(x-2)+2\log(x-1)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)
[Out] -log(x - 2) + 2*log(x - 1) + log(x + 1)
```

Giac [A]
time = 3.55, size = 20, normalized size = 0.95

$$\log(|x+1|)+2\log(|x-1|)-\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="giac")
[Out] log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x - 2))
```

Mupad [B]
time = 2.14, size = 21, normalized size = 1.00

$$2\,\ln\left(x-1\right)-2\,\mathrm{atanh}\left(\frac{144}{11\,\left(22\,x-50\right)}+\frac{13}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 2*x^2 + 1)/(x + 2*x^2 - x^3 - 2),x)
[Out] 2*log(x - 1) - 2*atanh(144/(11*(22*x - 50)) + 13/11)
```

3.286 $\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=22

$$\frac{x}{1+x^2}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {28, 1828, 649, 209, 266}

$$\text{ArcTan}(x)+\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx &= \int \frac{2+x+x^3}{(1+x^2)^2}\,dx\\ &= \frac{x}{1+x^2}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx\\ &= \frac{x}{1+x^2}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= \frac{x}{1+x^2}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$\frac{x}{1+x^2}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.95

method	result	size
default	$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
risch	$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+2)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] x/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.49, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1} + \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.38, size = 34, normalized size = 1.55

$$\frac{2\left(x^2+1\right)\arctan\left(x\right) + \left(x^2+1\right)\log\left(x^2+1\right) + 2\,x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{x}{x^2+1} + \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+2)/(x**4+2*x**2+1),x)
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

Giac [A]
time = 3.78, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1} + \arctan\left(x\right) + \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 2.12, size = 20, normalized size = 0.91

$$\frac{\ln\left(x^2+1\right)}{2} + \operatorname{atan}(x) + \frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 2)/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + atan(x) + x/(x^2 + 1)
```

3.287 $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} dx$

Optimal. Leaf size=24

$$-\frac{1}{2(1+x^2)} + \tan^{-1}(x) + \frac{1}{2}\log(1+x^2)$$

```
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {28, 1828, 649, 209, 266}

$$\text{ArcTan}(x) - \frac{1}{2(x^2+1)} + \frac{1}{2}\log(x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx &= \int \frac{1+2x+x^2+x^3}{(1+x^2)^2}\,dx\\ &= -\frac{1}{2(1+x^2)}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx\\ &= -\frac{1}{2(1+x^2)}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1}{2(1+x^2)}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$-\frac{1}{2(1+x^2)}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.88

method	result	size
default	$-\frac{1}{2(x^2+1)}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21
risch	$-\frac{1}{2(x^2+1)}+\arctan(x)+\frac{\ln(x^2+1)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.51, size = 20, normalized size = 0.83

$$-\frac{1}{2\left(x^{2}+1\right)}+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.40, size = 32, normalized size = 1.33

$$\frac{2\left(x^{2}+1\right)\arctan\left(x\right)+\left(x^{2}+1\right)\log\left(x^{2}+1\right)-1}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.79

$$\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}\left(x\right)-\frac{1}{2x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)
[Out] log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)
```

Giac [A]
time = 4.44, size = 20, normalized size = 0.83

$$-\frac{1}{2\left(x^{2}+1\right)}+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 22, normalized size = 0.92

$$\frac{\ln\left(x^2+1\right)}{2}+\mathrm{atan}(x)-\frac{1}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + x^3 + 1)/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + atan(x) - 1/(2*(x^2 + 1))
```

3.288 $\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+2\log\left(1+x^2\right)-2\log\left(2+x^2\right)$$

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {1024, 400, 209, 455, 36, 31}

$$3\text{ArcTan}(x)-\frac{3\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+2\log\left(x^2+1\right)-2\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx &= 3\int \frac{1}{(1+x^2)(2+x^2)}\,dx + 4\int \frac{x}{(1+x^2)(2+x^2)}\,dx \\
&= 2\text{Subst}\left(\int \frac{1}{(1+x)(2+x)}\,dx, x, x^2\right) + 3\int \frac{1}{1+x^2}\,dx - 3\int \frac{1}{2+x^2}\,dx \\
&= 3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - 2\text{Subst}\left(\int \frac{1}{2+x}\right. \\
&= 3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(1+x^2\right) - 2\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.00

$$3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log\left(1+x^2\right) - 2\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Maple [A]
time = 0.21, size = 34, normalized size = 0.94

method	result	size
default	$3\arctan\left(x\right)+2\ln\left(x^{2}+1\right)-2\ln\left(x^{2}+2\right)-\frac{3\arctan\left(\frac{\sqrt{2}\, x}{2}\right)\sqrt{2}}{2}$	34
risch	$3\arctan\left(x\right)+2\ln\left(x^{2}+1\right)-2\ln\left(x^{2}+2\right)-\frac{3\arctan\left(\frac{\sqrt{2}\, x}{2}\right)\sqrt{2}}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*x)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.48, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+3\arctan\left(x\right)-2\log\left(x^{2}+2\right)+2\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Fricas [A]
time = 0.40, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+3\arctan\left(x\right)-2\log\left(x^{2}+2\right)+2\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Sympy [A]
time = 0.08, size = 39, normalized size = 1.08

$$2\log\left(x^{2}+1\right)-2\log\left(x^{2}+2\right)+3\operatorname{atan}\left(x\right)-\frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x**2+1)/(x**2+2),x)
```

```
[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2)
/2
```

Giac [A]
time = 4.63, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+3\arctan\left(x\right)-2\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

Mupad [B]
time = 0.10, size = 56, normalized size = 1.56

$$\ln\left(x-\mathrm{i}\right)\left(2-\frac{3}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(2+\frac{3}{2}\mathrm{i}\right)+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(-2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 1i)*(2 - 3i/2) + log(x + 1i)*(2 + 3i/2) + log(x - 2^(1/2)*1i)*((2^(
1/2)*3i)/4 - 2) - log(x + 2^(1/2)*1i)*((2^(1/2)*3i)/4 + 2)
```

3.289 $\int \frac{2+x}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=37

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

```
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1024, 400, 209, 455, 36, 31}

$$-\frac{1}{3}\text{ArcTan}\left(\frac{x}{2}\right)+\frac{2\text{ArcTan}(x)}{3}+\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx &= 2\int \frac{1}{(1+x^2)(4+x^2)}\,dx+\int \frac{x}{(1+x^2)(4+x^2)}\,dx\\ &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx,x,x^2\right)+\frac{2}{3}\int \frac{1}{1+x^2}\,dx-\frac{2}{3}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-\frac{1}{6}\mathrm{Subst}\left(\int \frac{1}{4+x}\right.\\ &= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A]
time = 0.19, size = 28, normalized size = 0.76

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{3}+\frac{2\arctan\left(x\right)}{3}+\frac{\ln\left(x^{2}+1\right)}{6}-\frac{\ln\left(x^{2}+4\right)}{6}$	28
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{3}+\frac{2\arctan\left(x\right)}{3}+\frac{\ln\left(x^{2}+1\right)}{6}-\frac{\ln\left(x^{2}+4\right)}{6}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A]
time = 0.49, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan\left(x\right)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Fricas [A]
time = 0.39, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan\left(x\right)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A]
time = 0.07, size = 29, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{2\operatorname{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

Giac [A]
time = 6.12, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan\left(x\right)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Mupad [B]
time = 2.14, size = 37, normalized size = 1.00

$$\ln(x-\mathrm{i})\left(\frac{1}{6}-\frac{1}{3}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{6}+\frac{1}{3}\mathrm{i}\right)+\ln(x-2\mathrm{i})\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)+\ln(x+2\mathrm{i})\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^2 + 1)*(x^2 + 4)),x)
```

```
[Out] log(x - 1i)*(1/6 - 1i/3) + log(x + 1i)*(1/6 + 1i/3) - log(x - 2i)*(1/6 - 1i
/6) - log(x + 2i)*(1/6 + 1i/6)
```

3.290 $\int \frac{2-x+x^3}{-7-6x+x^2} \, dx$

Optimal. Leaf size=29

$$6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)$$

```
[Out] 6*x+1/2*x^2+169/4*ln(7-x)-1/4*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1671, 646, 31}

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2-x+x^3}{-7-6x+x^2}\,dx &= \int \left(6+x+\frac{2(22+21x)}{-7-6x+x^2}\right)dx \\
&= 6x+\frac{x^2}{2}+2\int \frac{22+21x}{-7-6x+x^2}\,dx \\
&= 6x+\frac{x^2}{2}-\frac{1}{4}\int \frac{1}{1+x}\,dx+\frac{169}{4}\int \frac{1}{-7+x}\,dx \\
&= 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Maple [A]
time = 0.22, size = 22, normalized size = 0.76

method	result	size
default	$\frac{x^2}{2}+6x-\frac{\ln(1+x)}{4}+\frac{169\ln(x-7)}{4}$	22
norman	$\frac{x^2}{2}+6x-\frac{\ln(1+x)}{4}+\frac{169\ln(x-7)}{4}$	22
risch	$\frac{x^2}{2}+6x-\frac{\ln(1+x)}{4}+\frac{169\ln(x-7)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x+2)/(x^2-6*x-7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+6*x-1/4*ln(1+x)+169/4*ln(x-7)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.72

$$\frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(x+1\right)+\frac{169}{4}\,\log\left(x-7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Fricas [A]
time = 0.40, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(x+1) + \frac{169}{4}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{169\log(x-7)}{4} - \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x+2)/(x**2-6*x-7),x)
[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4
```

Giac [A]
time = 5.07, size = 23, normalized size = 0.79

$$\frac{1}{2}x^2 + 6x - \frac{1}{4}\log(|x+1|) + \frac{169}{4}\log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))
```

Mupad [B]
time = 2.21, size = 21, normalized size = 0.72

$$6x - \frac{\ln(x+1)}{4} + \frac{169\ln(x-7)}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - x + 2)/(6*x - x^2 + 7),x)
[Out] 6*x - log(x + 1)/4 + (169*log(x - 7))/4 + x^2/2
```

3.291 $\int \frac{-1+x^5}{-1+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{x^2}{2}+\frac{x^4}{4}+\log(1+x)$$

[Out] 1/2*x^2+1/4*x^4+ln(1+x)

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1824, 641, 31}

$$\frac{x^4}{4}+\frac{x^2}{2}+\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^5)/(-1 + x^2),x]

[Out] x^2/2 + x^4/4 + Log[1 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1+x^5}{-1+x^2}\,dx = \int \left(x+x^3-\frac{1-x}{-1+x^2}\right)dx$$
$$= \frac{x^2}{2}+\frac{x^4}{4}-\int \frac{1-x}{-1+x^2}\,dx$$
$$= \frac{x^2}{2}+\frac{x^4}{4}-\int \frac{1}{-1-x}\,dx$$
$$= \frac{x^2}{2}+\frac{x^4}{4}+\log(1+x)$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^2}{2}+\frac{x^4}{4}+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-1 + x^2),x]
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Maple [A]
time = 0.19, size = 16, normalized size = 0.84

method	result	size
default	$\frac{x^2}{2}+\frac{x^4}{4}+\ln(1+x)$	16
norman	$\frac{x^2}{2}+\frac{x^4}{4}+\ln(1+x)$	16
risch	$\frac{x^4}{4}+\frac{x^2}{2}+\frac{1}{4}+\ln(1+x)$	17
meijerg	$\operatorname{arctanh}(x)+\frac{x^2(3x^2+6)}{12}+\frac{\ln(-x^2+1)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^2-1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/4*x^4+ln(1+x)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.79

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Fricas [**A**]
time = 0.40, size = 15, normalized size = 0.79

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

Sympy [**A**]
time = 0.02, size = 14, normalized size = 0.74

$$\frac{x^4}{4}+\frac{x^2}{2}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**2-1),x)
```

```
[Out] x**4/4 + x**2/2 + log(x + 1)
```

Giac [**A**]
time = 3.86, size = 16, normalized size = 0.84

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/2*x^2 + log(abs(x + 1))
```

Mupad [**B**]
time = 0.03, size = 15, normalized size = 0.79

$$\ln\left(x+1\right)+\frac{x^2}{2}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 - 1)/(x^2 - 1),x)
```

```
[Out] log(x + 1) + x^2/2 + x^4/4
```

3.292 $\int \frac{5+2x-x^2+x^3}{1+x+x^2} dx$

Optimal. Leaf size=41

$$-2x+\frac{x^2}{2}+\frac{11\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)$$

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1671, 648, 632, 210, 642}

$$\frac{11\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx &= \int \left(-2+x+\frac{7+3x}{1+x+x^2}\right)dx \\
&= -2x+\frac{x^2}{2}+\int \frac{7+3x}{1+x+x^2}\,dx \\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{11}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= -2x+\frac{x^2}{2}+\frac{3}{2}\log\left(1+x+x^2\right)-11\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= -2x+\frac{x^2}{2}+\frac{11\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$-2x+\frac{x^2}{2}+\frac{11\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Maple [A]
time = 0.28, size = 35, normalized size = 0.85

method	result	size

default	$-2x+\frac{x^2}{2}+\frac{3\ln(x^2+x+1)}{2}+\frac{11\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	35
risch	$-2x+\frac{x^2}{2}+\frac{3\ln(4x^2+4x+4)}{2}+\frac{11\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2+2*x+5)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.50, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2+\frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2x+\frac{3}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Fricas [A]
time = 0.43, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2+\frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2x+\frac{3}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Sympy [A]
time = 0.04, size = 46, normalized size = 1.12

$$\frac{x^2}{2}-2x+\frac{3\log\left(x^2+x+1\right)}{2}+\frac{11\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
```

```
[Out] x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

Giac [**A**]
time = 3.36, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

Mupad [**B**]
time = 0.04, size = 36, normalized size = 0.88

$$\frac{3\ln\left(x^2+x+1\right)}{2} - 2x + \frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + x^3 + 5)/(x + x^2 + 1),x)
```

```
[Out] (3*log(x + x^2 + 1))/2 - 2*x + (11*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3
))/3 + x^2/2
```

3.293 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} \, dx$

Optimal. Leaf size=41

$$\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\tan^{-1}(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)$$

```
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(-2+x)+3/4*ln(x^2-4*x+5)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1671, 648, 632, 210, 642}

$$6\text{ArcTan}(2-x)+\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx &= \int \left(\frac{3}{2}+x+\frac{x^2}{2}-\frac{3(6-x)}{10-8x+2x^2}\right)dx\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-3\int \frac{6-x}{10-8x+2x^2}\,dx\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\int \frac{-8+4x}{10-8x+2x^2}\,dx-12\int \frac{1}{10-8x+2x^2}\,dx\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\log\left(5-4x+x^2\right)+24\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,-8+4x\right)\\
&= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\tan^{-1}(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.95

$$\frac{1}{2}\left(3x+x^2+\frac{x^3}{3}+12\tan^{-1}(2-x)+\frac{3}{2}\log\left(5-4x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)/2
```

Maple [A]
time = 0.27, size = 32, normalized size = 0.78

method	result	size
default	$\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\arctan\left(x-2\right)+\frac{3\ln\left(x^2-4x+5\right)}{4}$	32
risch	$\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-6\arctan\left(x-2\right)+\frac{3\ln\left(x^2-4x+5\right)}{4}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x,method=_RETURNVERBOSE)
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)
```

Maxima [A]
time = 0.48, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="fricas")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Sympy [A]
time = 0.04, size = 34, normalized size = 0.83

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} + \frac{3\log(x^2 - 4x + 5)}{4} - 6\,\mathrm{atan}(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)
```

Giac [A]
time = 4.16, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="giac")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

Mupad [B]
time = 2.12, size = 31, normalized size = 0.76

$$\frac{3x}{2} - 6\,\mathrm{atan}(x-2) + \frac{3\ln(x^2 - 4x + 5)}{4} + \frac{x^2}{2} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^3 + x^4 - 3)/(2*x^2 - 8*x + 10),x)
[Out] (3*x)/2 - 6*atan(x - 2) + (3*log(x^2 - 4*x + 5))/4 + x^2/2 + x^3/6
```

3.294 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=30

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

```
[Out] x+7/2*ln(1-x)-25*ln(2-x)+61/2*ln(3-x)
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1626}

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(1+\frac{61}{2(-3+x)}-\frac{25}{-2+x}+\frac{7}{2(-1+x)}\right)dx\\ &= x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.80

$$x+\frac{61}{2}\log(-3+x)-25\log(-2+x)+\frac{7}{2}\log(-1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2
```

Maple [A]
time = 0.22, size = 21, normalized size = 0.70

method	result	size
default	$x+\frac{61\ln(x-3)}{2}-25\ln\left(x-2\right)+\frac{7\ln(-1+x)}{2}$	21
norman	$x+\frac{61\ln(x-3)}{2}-25\ln\left(x-2\right)+\frac{7\ln(-1+x)}{2}$	21
risch	$x+\frac{61\ln(x-3)}{2}-25\ln\left(x-2\right)+\frac{7\ln(-1+x)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+2*x+1)/(x-3)/(x-2)/(-1+x),x,method=_RETURNVERBOSE)
[Out] x+61/2*ln(x-3)-25*ln(x-2)+7/2*ln(-1+x)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.67

$$x+\frac{7}{2}\log\left(x-1\right)-25\log\left(x-2\right)+\frac{61}{2}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Fricas [A]
time = 0.43, size = 20, normalized size = 0.67

$$x+\frac{7}{2}\log\left(x-1\right)-25\log\left(x-2\right)+\frac{61}{2}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.80

$$x+\frac{61\log\left(x-3\right)}{2}-25\log\left(x-2\right)+\frac{7\log\left(x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2
```

Giac [A]
time = 3.07, size = 23, normalized size = 0.77

$$x + \frac{7}{2}\log(|x-1|) - 25\log(|x-2|) + \frac{61}{2}\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))
```

Mupad [B]
time = 2.13, size = 20, normalized size = 0.67

$$x + \frac{7\ln(x-1)}{2} - 25\ln(x-2) + \frac{61\ln(x-3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 + x^3 + 1)/((x - 1)*(x - 2)*(x - 3)),x)
[Out] x + (7*log(x - 1))/2 - 25*log(x - 2) + (61*log(x - 3))/2
```

3.295 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx$

Optimal. Leaf size=35

$$-2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)$$

```
[Out] -2*x+1/2*x^2+13/3*ln(4-x)-22/3*ln(2+x)+20*ln(3+x)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2099}

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx &= \int \left(-2+\frac{13}{3(-4+x)}+x-\frac{22}{3(2+x)}+\frac{20}{3+x}\right)dx\\ &= -2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$-2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
```

```
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.80

method	result	size
default	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+\frac{13\ln(x-4)}{3}+20\ln\left(3+x\right)$	28
norman	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+\frac{13\ln(x-4)}{3}+20\ln\left(3+x\right)$	28
risch	$\frac{x^2}{2}-2x-\frac{22\ln(x+2)}{3}+\frac{13\ln(x-4)}{3}+20\ln\left(3+x\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-2*x-22/3*ln(x+2)+13/3*ln(x-4)+20*ln(3+x)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.77

$$\frac{1}{2}\,x^2-2\,x+20\,\log\left(x+3\right)-\frac{22}{3}\,\log\left(x+2\right)+\frac{13}{3}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Fricas [A]
time = 0.43, size = 27, normalized size = 0.77

$$\frac{1}{2}\,x^2-2\,x+20\,\log\left(x+3\right)-\frac{22}{3}\,\log\left(x+2\right)+\frac{13}{3}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

Sympy [A]
time = 0.05, size = 31, normalized size = 0.89

$$\frac{x^2}{2}-2x+\frac{13\log\left(x-4\right)}{3}-\frac{22\log\left(x+2\right)}{3}+20\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)
```

```
[Out] x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)
```

Giac [A]
time = 2.43, size = 30, normalized size = 0.86

$$\frac{1}{2}\,x^{2}-2\,x+20\,\log\left(|x+3|\right)-\frac{22}{3}\,\log\left(|x+2|\right)+\frac{13}{3}\,\log\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x
- 4))
```

Mupad [B]
time = 0.04, size = 27, normalized size = 0.77

$$20\,\ln\left(x+3\right)-\frac{22\,\ln\left(x+2\right)}{3}-2\,x+\frac{13\,\ln\left(x-4\right)}{3}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 7*x - x^3 + x^4 + 2)/(14*x - x^2 - x^3 + 24),x)
```

```
[Out] 20*log(x + 3) - (22*log(x + 2))/3 - 2*x + (13*log(x - 4))/3 + x^2/2
```

3.296 $\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$

Optimal. Leaf size=34

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)$$

```
[Out] 3/2/(1-x)-5/4*ln(1-x)+2*ln(x)-3/4*ln(1+x)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1626}

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

```
[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx &= \int \left(\frac{3}{2(-1+x)^2}-\frac{5}{4(-1+x)}+\frac{2}{x}-\frac{3}{4(1+x)}\right)dx\\ &= \frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.94

$$-\frac{3}{2(-1+x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
```

```
[Out] -3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Maple [A]
time = 0.20, size = 25, normalized size = 0.74

method	result	size
default	$-\frac{3}{2(-1+x)}-\frac{5\ln(-1+x)}{4}+2\ln(x)-\frac{3\ln(1+x)}{4}$	25
norman	$-\frac{3}{2(-1+x)}-\frac{5\ln(-1+x)}{4}+2\ln(x)-\frac{3\ln(1+x)}{4}$	25
risch	$-\frac{3}{2(-1+x)}-\frac{5\ln(-1+x)}{4}+2\ln(x)-\frac{3\ln(1+x)}{4}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(-1+x)^2/x/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/(-1+x)-5/4*ln(-1+x)+2*ln(x)-3/4*ln(1+x)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.71

$$-\frac{3}{2\,(x-1)}-\frac{3}{4}\,\log\left(x+1\right)-\frac{5}{4}\,\log\left(x-1\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")
```

```
[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)
```

Fricas [A]
time = 0.39, size = 34, normalized size = 1.00

$$-\frac{3\,(x-1)\log\left(x+1\right)+5\,(x-1)\log\left(x-1\right)-8\,(x-1)\log\left(x\right)+6}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log(x) + 6)/(
x - 1)
```

Sympy [A]
time = 0.05, size = 27, normalized size = 0.79

$$2\log\left(x\right)-\frac{5\log\left(x-1\right)}{4}-\frac{3\log\left(x+1\right)}{4}-\frac{3}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(-1+x)**2/x/(1+x),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)
```

Giac [A]
time = 3.16, size = 34, normalized size = 1.00

$$-\frac{3}{2\,(x-1)}+2\,\log\left(\left|-\frac{1}{x-1}-1\right|\right)-\frac{3}{4}\,\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")
```

```
[Out] -3/2/(x - 1) + 2*log(abs(-1/(x - 1) - 1)) - 3/4*log(abs(-2/(x - 1) - 1))
```

Mupad [B]
time = 2.11, size = 26, normalized size = 0.76

$$2\,\ln\left(x\right)-\frac{3\,\ln\left(x+1\right)}{4}-\frac{5\,\ln\left(x-1\right)}{4}-\frac{3}{2\,\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/(x*(x - 1)^2*(x + 1)),x)
```

```
[Out] 2*log(x) - (3*log(x + 1))/4 - (5*log(x - 1))/4 - 3/(2*(x - 1))
```

3.297 $\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx$

Optimal. Leaf size=42

$$\frac{4+x}{4\,(2+x^2)}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

```
[Out] 1/4*(4+x)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1828, 649, 209, 266}

$$\frac{5\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{x+4}{4\,(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
```

```
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2+x^3}{(2+x^2)^2}\,dx &= \frac{4+x}{4(2+x^2)}-\frac{1}{4}\int \frac{-5-4x}{2+x^2}\,dx\\ &= \frac{4+x}{4(2+x^2)}+\frac{5}{4}\int \frac{1}{2+x^2}\,dx+\int \frac{x}{2+x^2}\,dx\\ &= \frac{4+x}{4(2+x^2)}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$\frac{4+x}{4(2+x^2)}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Maple [A]
time = 0.19, size = 35, normalized size = 0.83

method	result	size
default	$\frac{\frac{x}{4}+1}{x^2+2}+\frac{\ln(x^2+2)}{2}+\frac{5\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{8}$	35
risch	$\frac{\frac{x}{4}+1}{x^2+2}+\frac{\ln(x^2+2)}{2}+\frac{5\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{8}$	35
meijerg	$\frac{3\sqrt{2}\left(\frac{\sqrt{2}\,x}{x^2+2}+\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{8}-\frac{x^2}{4\left(1+\frac{x^2}{2}\right)}+\frac{\ln\left(1+\frac{x^2}{2}\right)}{2}+\frac{\sqrt{2}\left(-\frac{x\sqrt{2}}{2\left(1+\frac{x^2}{2}\right)}+\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{4}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+3)/(x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{x+4}{4\,(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

Fricas [A]
time = 0.43, size = 44, normalized size = 1.05

$$\frac{5\sqrt{2}\,(x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+4\,(x^2+2)\log\left(x^2+2\right)+2\,x+8}{8\,(x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2) +
 2*x + 8)/(x^2 + 2)
```

Sympy [A]
time = 0.04, size = 36, normalized size = 0.86

$$\frac{x+4}{4x^2+8}+\frac{\log\left(x^2+2\right)}{2}+\frac{5\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+3)/(x**2+2)**2,x)
```

```
[Out] (x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8
```

Giac [A]
time = 3.88, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{x+4}{4\,(x^2+2)}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

Mupad [B]
time = 2.19, size = 39, normalized size = 0.93

$$\frac{\ln\left(x^2+2\right)}{2}+\frac{5\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}+\frac{x}{4\,\left(x^2+2\right)}+\frac{1}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 3)/(x^2 + 2)^2,x)
[Out] log(x^2 + 2)/2 + (5*2^(1/2)*atan((2^(1/2)*x)/2))/8 + x/(4*(x^2 + 2)) + 1/(x
^2 + 2)
```

3.298 $\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$

Optimal. Leaf size=49

$$-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{1}{4}(-1+x)\right)}{4100}+\frac{1003\log\left(26-10x+x^2\right)}{1025}+\frac{22\log\left(17-2x+x^2\right)}{1025}$$

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+
26)+22/1025*ln(x^2-2*x+17)
```

Rubi [A]
time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {6860, 648, 632, 210, 642}

$$-\frac{15033\text{ArcTan}(5-x)}{1025}-\frac{4607\text{ArcTan}\left(\frac{x-1}{4}\right)}{4100}+\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}$$

Antiderivative was successfully verified.

```
[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \int \left(\frac{5003+2006x}{1025(26-10x+x^2)}+\frac{-4651+44x}{1025(17-2x+x^2)}\right)dx$$

$$= \frac{\int \frac{5003+2006x}{26-10x+x^2}\,dx}{1025}+\frac{\int \frac{-4651+44x}{17-2x+x^2}\,dx}{1025}$$

$$= \frac{22\int \frac{-2+2x}{17-2x+x^2}\,dx}{1025}+\frac{1003\int \frac{-10+2x}{26-10x+x^2}\,dx}{1025}-\frac{4607\int \frac{1}{17-2x+x^2}\,dx}{1025}+\frac{15}{}$$

$$= \frac{1003\log(26-10x+x^2)}{1025}+\frac{22\log(17-2x+x^2)}{1025}+\frac{9214\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{1}{4}(-1+x)\right)}{4100}+\frac{1003\log(26-}{1025}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{1}{4}(-1+x)\right)}{4100}+\frac{1003\log(26-10x+x^2)}{1025}+\frac{22\log(17-2x+x^2)}{1025}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2))
,x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Maple [A]
time = 0.40, size = 38, normalized size = 0.78

method	result	size
default	$\frac{15033\arctan(-5+x)}{1025}-\frac{4607\arctan\left(-\frac{1}{4}+\frac{x}{4}\right)}{4100}+\frac{1003\ln(x^2-10x+26)}{1025}+\frac{22\ln(x^2-2x+17)}{1025}$	38

risch	$\frac{15033\arctan(-5+x)}{1025}-\frac{4607\arctan\left(-\frac{1}{4}+\frac{x}{4}\right)}{4100}+\frac{1003\ln(x^2-10x+26)}{1025}+\frac{22\ln(x^2-2x+17)}{1025}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x,method=_RETURNVERBOS
E)
```

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+
26)+22/1025*ln(x^2-2*x+17)
```

Maxima [A]
time = 0.48, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}\,x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2\,x+17\right)+\frac{1003}{1025}\log\left(x^2-10\,x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="ma
xima")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Fricas [A]
time = 0.44, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}\,x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2\,x+17\right)+\frac{1003}{1025}\log\left(x^2-10\,x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="fr
icas")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Sympy [A]
time = 0.09, size = 46, normalized size = 0.94

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{4607\operatorname{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{15033\operatorname{atan}\left(x-5\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

```
[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 - 4607*atan(
x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

Giac [A]
time = 3.80, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^2-2x+17\right)+\frac{1003}{1025}\log\left(x^2-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="gi
ac")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

Mupad [B]
time = 0.07, size = 41, normalized size = 0.84

$$\ln\left(x-1-4i\right)\left(\frac{22}{1025}+\frac{4607}{8200}i\right)+\ln\left(x-1+4i\right)\left(\frac{22}{1025}-\frac{4607}{8200}i\right)+\ln\left(x-5-i\right)\left(\frac{1003}{1025}-\frac{15033}{2050}i\right)+\ln\left(x-5+1i\right)\left(\frac{1003}{1025}+\frac{15033}{2050}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((70*x - 4*x^2 + 2*x^3 - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x)
```

```
[Out] log(x - (1 + 4i))*(22/1025 + 4607i/8200) + log(x - (1 - 4i))*(22/1025 - 460
7i/8200) + log(x - (5 + 1i))*(1003/1025 - 15033i/2050) + log(x - (5 - 1i))*
(1003/1025 + 15033i/2050)
```

3.299 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$

Optimal. Leaf size=29

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)$$

```
[Out] -11/14*ln(3-x)+3/2*ln(5-x)+2/7*ln(4+x)
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{1626\}$

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
```

```
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx &= \int \left(\frac{3}{2(-5+x)}-\frac{11}{14(-3+x)}+\frac{2}{7(4+x)}\right)dx\\ &= -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
```

[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7

Maple [A]
time = 0.21, size = 20, normalized size = 0.69

method	result	size
default	$\frac{2\ln(x+4)}{7}-\frac{11\ln(x-3)}{14}+\frac{3\ln(-5+x)}{2}$	20
norman	$\frac{2\ln(x+4)}{7}-\frac{11\ln(x-3)}{14}+\frac{3\ln(-5+x)}{2}$	20
risch	$\frac{2\ln(x+4)}{7}-\frac{11\ln(x-3)}{14}+\frac{3\ln(-5+x)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2)/(-5+x)/(x-3)/(x+4),x,method=_RETURNVERBOSE)

[Out] 2/7*ln(x+4)-11/14*ln(x-3)+3/2*ln(-5+x)

Maxima [A]
time = 0.27, size = 19, normalized size = 0.66

$$\frac{2}{7}\log\left(x+4\right)-\frac{11}{14}\log\left(x-3\right)+\frac{3}{2}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")

[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)

Fricas [A]
time = 0.38, size = 19, normalized size = 0.66

$$\frac{2}{7}\log\left(x+4\right)-\frac{11}{14}\log\left(x-3\right)+\frac{3}{2}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="fricas")

[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)

Sympy [A]
time = 0.05, size = 24, normalized size = 0.83

$$\frac{3\log\left(x-5\right)}{2}-\frac{11\log\left(x-3\right)}{14}+\frac{2\log\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)

```
[Out] 3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7
```

Giac [A]
time = 3.24, size = 22, normalized size = 0.76

$$\frac{2}{7}\log\left(|x+4|\right)-\frac{11}{14}\log\left(|x-3|\right)+\frac{3}{2}\log\left(|x-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="giac")
[Out] 2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))
```

Mupad [B]
time = 2.18, size = 19, normalized size = 0.66

$$\frac{2\,\ln\left(x+4\right)}{7}-\frac{11\,\ln\left(x-3\right)}{14}+\frac{3\,\ln\left(x-5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/((x - 3)*(x + 4)*(x - 5)),x)
[Out] (2*log(x + 4))/7 - (11*log(x - 3))/14 + (3*log(x - 5))/2
```

3.300 $\int \frac{x^4}{(-1+x)(2+x^2)}\,dx$

Optimal. Leaf size=46

$$x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)$$

```
[Out] x+1/2*x^2+1/3*ln(1-x)-2/3*ln(x^2+2)-2/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1643, 649, 209, 266}

$$-\frac{2}{3}\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)+\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Log[2 + x^2
])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{(-1+x)\left(2+x^2\right)} \, dx &= \int \left(1+\frac{1}{3(-1+x)}+x-\frac{4(1+x)}{3\left(2+x^2\right)}\right) \, dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1+x}{2+x^2} \, dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1}{2+x^2} \, dx-\frac{4}{3}\int \frac{x}{2+x^2} \, dx \\
&= x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.93

$$
\frac{1}{6}\left(-9+6x+3x^2-4\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+2\log(-1+x)-4\log\left(2+x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2 +
 x^2])/6
```

Maple [A]
time = 0.20, size = 34, normalized size = 0.74

method	result	size
default	$x+\frac{x^2}{2}+\frac{\ln(-1+x)}{3}-\frac{2\ln(x^2+2)}{3}-\frac{2\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{3}$	34
risch	$x+\frac{x^2}{2}+\frac{\ln(-1+x)}{3}-\frac{2\ln(x^2+2)}{3}-\frac{2\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}}{3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-1+x)/(x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+1/3*ln(-1+x)-2/3*ln(x^2+2)-2/3*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 33, normalized size = 0.72

$$
\frac{1}{2}x^2-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+x-\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

Fricas [A]
time = 0.39, size = 33, normalized size = 0.72

$$\frac{1}{2}\,x^{2}-\frac{2}{3}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+x-\frac{2}{3}\,\log\left(x^{2}+2\right)+\frac{1}{3}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 0.89

$$\frac{x^{2}}{2}+x+\frac{\log\left(x-1\right)}{3}-\frac{2\log\left(x^{2}+2\right)}{3}-\frac{2\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-1+x)/(x**2+2),x)
```

```
[Out] x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2)
/3
```

Giac [A]
time = 3.24, size = 34, normalized size = 0.74

$$\frac{1}{2}\,x^{2}-\frac{2}{3}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+x-\frac{2}{3}\,\log\left(x^{2}+2\right)+\frac{1}{3}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 50, normalized size = 1.09

$$x+\frac{\ln\left(x-1\right)}{3}+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\,\left(\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^2 + 2)*(x - 1)),x)
```

```
[Out] x + log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/3 - 2/3) - log(x + 2^(
1/2)*1i)*((2^(1/2)*1i)/3 + 2/3) + x^2/2
```

3.301 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx$

Optimal. Leaf size=16

$$-\frac{3}{1+x}+2\log(1-x)$$

```
[Out] -3/(1+x)+2*ln(1-x)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.042$, Rules used = {2099}

$$2\log(1-x)-\frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx&=\int \left(\frac{2}{-1+x}+\frac{3}{(1+x)^2}\right)dx\\&=-\frac{3}{1+x}+2\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.88

$$-\frac{3}{1+x}+2\log(-1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] -3/(1 + x) + 2*Log[-1 + x]
```

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
default	$-\frac{3}{1+x}+2\ln\left(-1+x\right)$	15
norman	$-\frac{3}{1+x}+2\ln\left(-1+x\right)$	15
risch	$-\frac{3}{1+x}+2\ln\left(-1+x\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+7*x-1)/(x^3+x^2-x-1),x,method=_RETURNVERBOSE)
[Out] -3/(1+x)+2*ln(-1+x)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$-\frac{3}{x+1}+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")
[Out] -3/(x + 1) + 2*log(x - 1)
```

Fricas [A]
time = 0.38, size = 17, normalized size = 1.06

$$\frac{2\,(x+1)\log\left(x-1\right)-3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="fricas")
[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.62

$$2\log\left(x-1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)
[Out] 2*log(x - 1) - 3/(x + 1)
```

Giac [A]
time = 5.42, size = 15, normalized size = 0.94

$$-\frac{3}{x+1} + 2\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="giac")
[Out] -3/(x + 1) + 2*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.88

$$2\ln\left(x-1\right) - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(7*x + 2*x^2 - 1)/(x - x^2 - x^3 + 1),x)
[Out] 2*log(x - 1) - 3/(x + 1)
```

3.302 $\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3}{2(1-x)^2}+\frac{2}{1-x}$$

```
[Out] -3/2/(1-x)^2+2/(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2099}

$$\frac{2}{1-x}-\frac{3}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{-1+3x-3x^2+x^3}\,dx &= \int \left(\frac{3}{(-1+x)^3}+\frac{2}{(-1+x)^2}\right)dx\\ &= -\frac{3}{2(1-x)^2}+\frac{2}{1-x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 0.67

$$\frac{1-4x}{2(-1+x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
[Out] (1 - 4*x)/(2*(-1 + x)^2)
```

Maple [A]
time = 0.01, size = 16, normalized size = 0.76

method	result	size
norman	$\frac{-2x+\frac{1}{2}}{(-1+x)^2}$	12
default	$-\frac{2}{-1+x}-\frac{3}{2(-1+x)^2}$	16
risch	$\frac{-2x+\frac{1}{2}}{x^2-2x+1}$	17
gosper	$-\frac{4x-1}{2(x^2-2x+1)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^3-3*x^2+3*x-1),x,method=_RETURNVERBOSE)
[Out] -2/(-1+x)-3/2/(-1+x)^2
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$-\frac{4x-1}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Fricas [A]
time = 0.39, size = 17, normalized size = 0.81

$$-\frac{4x-1}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

Sympy [A]
time = 0.02, size = 14, normalized size = 0.67

$$\frac{1-4x}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)
```

[Out] (1 - 4*x)/(2*x**2 - 4*x + 2)

Giac [A]
time = 5.93, size = 12, normalized size = 0.57

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="giac")

[Out] -1/2*(4*x - 1)/(x - 1)^2

Mupad [B]
time = 2.09, size = 12, normalized size = 0.57

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/(3*x - 3*x^2 + x^3 - 1),x)

[Out] -(4*x - 1)/(2*(x - 1)^2)

3.303 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{1-x}-\frac{2}{(1+x)^2}$$

```
[Out] 1/(1-x)-2/(1+x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1634}

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx &= \int \left(\frac{1}{(-1+x)^2}+\frac{4}{(1+x)^3}\right)dx\\ &= \frac{1}{1-x}-\frac{2}{(1+x)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{-1+x}-\frac{2}{(1+x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] -(-1 + x)^(-1) - 2/(1 + x)^2
```

Maple [A]
time = 0.23, size = 16, normalized size = 1.07

method	result	size
default	$-\frac{1}{-1+x}-\frac{2}{(1+x)^2}$	16
gosper	$-\frac{x^2+4x-1}{(-1+x)(1+x)^2}$	21
norman	$\frac{-x^2-4x+1}{(1+x)^2(-1+x)}$	22
risch	$\frac{-x^2-4x+1}{(1+x)^2(-1+x)}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x,method=_RETURNVERBOSE)
[Out] -1/(-1+x)-2/(1+x)^2
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.53

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Fricas [A]
time = 0.37, size = 23, normalized size = 1.53

$$-\frac{x^2+4x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 1.13

$$\frac{-x^2-4x+1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)
```

```
[Out] (-x**2 - 4*x + 1)/(x**3 + x**2 - x - 1)
```

Giac [A]
time = 4.35, size = 30, normalized size = 2.00

$$-\frac{1}{x-1}+\frac{\frac{4}{x-1}+1}{2\left(\frac{2}{x-1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2
```

Mupad [B]
time = 2.09, size = 15, normalized size = 1.00

$$-\frac{1}{x-1}-\frac{2}{(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^2 - 5*x + x^3 + 5)/((x - 1)^2*(x + 1)^3),x)
```

```
[Out] - 1/(x - 1) - 2/(x + 1)^2
```

3.304 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx$

Optimal. Leaf size=31

$$-\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)$$

```
[Out] ln(1+x)+ln(x^2+x+1)-2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2099, 648, 632, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(x^2+x+1\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx &= \int \left(\frac{1}{1+x}+\frac{2x}{1+x+x^2}\right)\,dx\\
&= \log(1+x)+2\int \frac{x}{1+x+x^2}\,dx\\
&= \log(1+x)-\int \frac{1}{1+x+x^2}\,dx+\int \frac{1+2x}{1+x+x^2}\,dx\\
&= \log(1+x)+\log\left(1+x+x^2\right)+2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Maple [A]
time = 0.02, size = 29, normalized size = 0.94

method	result	size
default	$\ln(1+x)+\ln(x^2+x+1)-\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	29

risch	$-\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\ln\left(4x^2+4x+4\right)+\ln\left(1+x\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+x)+ln(x^2+x+1)-2/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.48, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\log\left(x^2+x+1\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

Fricas [A]
time = 0.41, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\log\left(x^2+x+1\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

Sympy [A]
time = 0.04, size = 3, normalized size = 0.10

$$\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
```

```
[Out] log(x + 1)
```

Giac [A]
time = 3.24, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\log\left(x^2+x+1\right)+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x +
 1))
```

Mupad [B]
time = 0.11, size = 57, normalized size = 1.84

$$\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)+\ln\left(x+1\right)+\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\ 1\mathrm{i}}{3}-\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\ 1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 3*x^2 + 1)/(2*x + 2*x^2 + x^3 + 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x + 1)
+ (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - (3^(1/2)*log(x + (3^(1/2)*
1i)/2 + 1/2)*1i)/3
```

3.305 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)$$

```
[Out] 1/10*ln(1-2*x)+1/2*ln(x)-1/10*ln(2+x)
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1608, 1642}

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx &= \int \frac{-1+2x+x^2}{x\left(-2+3x+2x^2\right)}\,dx\\
&= \int \left(\frac{1}{2x}-\frac{1}{10(2+x)}+\frac{1}{5(-1+2x)}\right)dx\\
&= \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}+\frac{\ln(x)}{2}$	20
norman	$-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}+\frac{\ln(x)}{2}$	20
risch	$-\frac{\ln(x+2)}{10}+\frac{\ln(2x-1)}{10}+\frac{\ln(x)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x,method=_RETURNVERBOSE)
[Out] -1/10*ln(x+2)+1/10*ln(2*x-1)+1/2*ln(x)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{10}\log\left(2x-1\right)-\frac{1}{10}\log\left(x+2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Fricas [A]
time = 0.40, size = 19, normalized size = 0.76

$$\frac{1}{10}\log\left(2x-1\right)-\frac{1}{10}\log\left(x+2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2} + \frac{\log\left(x - \frac{1}{2}\right)}{10} - \frac{\log(x+2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

```
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

Giac [A]
time = 2.67, size = 22, normalized size = 0.88

$$\frac{1}{10}\log\left(|2x-1|\right) - \frac{1}{10}\log\left(|x+2|\right) + \frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
```

```
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

Mupad [B]
time = 0.06, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{24}{145\left(\frac{29x}{100}-\frac{11}{50}\right)}+\frac{35}{29}\right)}{5} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)
```

```
[Out] atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2
```

3.306 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} dx$

Optimal. Leaf size=30

$$\frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)$$

```
[Out] 2/(1-x)+x+1/2*x^2+ln(1-x)-ln(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2099}

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right)dx\\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.97

$$-\frac{2}{-1+x}+\frac{1}{2}(1+x)^2+\log(1-x)-\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.83

method	result	size
default	$x+\frac{x^2}{2}+\ln(-1+x)-\frac{2}{-1+x}-\ln(1+x)$	25
risch	$x+\frac{x^2}{2}+\ln(-1+x)-\frac{2}{-1+x}-\ln(1+x)$	25
norman	$\frac{\frac{1}{2}x^2+\frac{1}{2}x^3-3}{-1+x}-\ln(1+x)+\ln(-1+x)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+ln(-1+x)-2/(-1+x)-ln(1+x)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2+x-\frac{2}{x-1}-\log(x+1)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

Fricas [A]
time = 0.41, size = 36, normalized size = 1.20

$$\frac{x^3+x^2-2(x-1)\log(x+1)+2(x-1)\log(x-1)-2x-4}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
- 1)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.67

$$\frac{x^2}{2}+x+\log(x-1)-\log(x+1)-\frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

Giac [A]
time = 4.12, size = 26, normalized size = 0.87

$$\frac{1}{2}\,x^2 + x - \frac{2}{x-1} - \log\left(|x+1|\right) + \log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

Mupad [B]
time = 2.11, size = 22, normalized size = 0.73

$$x - \frac{2}{x-1} + \frac{x^2}{2} + \mathrm{atan}(x\,1\mathrm{i})\,\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)
```

```
[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2
```

3.307 $\int \frac{4-x+2x^2}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)$$

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {1607, 1816, 649, 209, 266}

$$-\frac{1}{2}\text{ArcTan}\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{4-x+2x^2}{4x+x^3}\,dx &= \int \frac{4-x+2x^2}{x\,(4+x^2)}\,dx\\ &= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx\\ &= \log(x)+\int \frac{-1+x}{4+x^2}\,dx\\ &= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx\\ &= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

Maple [A]
time = 0.19, size = 18, normalized size = 0.78

method	result	size
default	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^2+4\right)}{2}$	18
risch	$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)+\frac{\ln\left(x^2+4\right)}{2}$	18
meijerg	$\frac{\ln\left(\frac{x^2}{4}+1\right)}{2}+\ln\left(x\right)-\ln\left(2\right)-\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+4)/(x^3+4*x),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Maxima [A]
time = 0.48, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Fricas [A]
time = 0.40, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.74

$$\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

Giac [A]
time = 2.85, size = 18, normalized size = 0.78

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

Mupad [B]
time = 2.12, size = 21, normalized size = 0.91

$$\ln\left(x\right)+\ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 4)/(4*x + x^3),x)
[Out] log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)
```

3.308 $\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx$

Optimal. Leaf size=103

$$\frac{1+x}{8\left(1+x^2\right)^2}-\frac{3(1-x)}{8\left(1+x^2\right)}+\frac{3x}{16\left(1+x^2\right)}+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{8}\log(1-x)-\log(x)+\frac{15}{16}\log\left(1+x^2\right)-\frac{1}{2}$$

```
[Out] 1/8*(1+x)/(x^2+1)^2-3/8*(1-x)/(x^2+1)+3/16*x/(x^2+1)+7/16*arctan(x)+1/8*ln(
1-x)-ln(x)+15/16*ln(x^2+1)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*
3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6860, 653, 205, 209, 649, 266, 648, 632, 210, 642}

$$\frac{7\text{ArcTan}(x)}{16}-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \int \left(\frac{1}{8(-1+x)} - \frac{1}{x} + \frac{1-x}{2\left(1+x^2\right)^3} + \frac{3(1+x)}{4\left(1+x^2\right)^2} + \frac{-1+15x}{8\left(1+x^2\right)} + \frac{}{1}\right.$$

$$= \frac{1}{8}\log(1-x) - \log(x) + \frac{1}{8}\int \frac{-1+15x}{1+x^2}\,dx + \frac{1}{2}\int \frac{1-x}{\left(1+x^2\right)^3}\,dx +$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{1}{8}\log(1-x) - \log(x) - \frac{1}{8}\int \frac{1}{1+x^2}\,dx$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{8}\log(1-$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{7}{16}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.03, size = 93, normalized size = 0.90

$$\frac{1}{48}\left(\frac{6(1+x)}{\left(1+x^2\right)^2} + \frac{9(-2+3x)}{1+x^2} + 21\tan^{-1}(x) - 16\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + 20\log(1-x) - 48\log(x) + 45\log\left(1+x^2\right) - 10\log\left(1+x+x^2\right) - 14\log\left(1-x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

Maple [A]
time = 0.31, size = 73, normalized size = 0.71

method	result
risch	$\frac{\frac{9}{16}x^3-\frac{3}{8}x^2+\frac{11}{16}x-\frac{1}{4}}{\left(x^2+1\right)^2} + \frac{15\ln\left(x^2+1\right)}{16} + \frac{7\arctan(x)}{16} - \ln(x) + \frac{\ln(-1+x)}{8} - \frac{\ln\left(x^2+x+1\right)}{2} - \frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$
default	$-\frac{\ln\left(x^2+x+1\right)}{2} - \frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3} + \frac{\ln(-1+x)}{8} + \frac{\frac{9}{2}x^3-3x^2+\frac{11}{2}x-2}{8\left(x^2+1\right)^2} + \frac{15\ln\left(x^2+1\right)}{16} + \frac{7\arctan(x)}{16} - \ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/8*ln(-1+x)+1/8*(
9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arctan(x)-ln(x)
```

Maxima [A]
time = 0.47, size = 77, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16(x^4+2x^2+1)}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log(x^2+x+1)+\frac{15}{16}\log(x^2+1)+\frac{1}{8}\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

Fricas [A]
time = 0.42, size = 136, normalized size = 1.32

$$\frac{27x^3-16\sqrt{3}(x^4+2x^2+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-18x^2+21(x^4+2x^2+1)\arctan(x)-24(x^4+2x^2+1)\log(x^2+x+1)+45(x^4+2x^2+1)\log(x^2+1)+6(x^4+2x^2+1)\log(x-1)-48(x^4+2x^2+1)\log(x)+33x-12}{48(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

Sympy [A]
time = 0.27, size = 88, normalized size = 0.85

$$-\log(x)+\frac{\log(x-1)}{8}+\frac{15\log(x^2+1)}{16}-\frac{\log(x^2+x+1)}{2}+\frac{7\operatorname{atan}(x)}{16}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

Giac [A]
time = 4.76, size = 74, normalized size = 0.72

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16(x^2+1)^2}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log(x^2+x+1)+\frac{15}{16}\log(x^2+1)+\frac{1}{8}\log(|x-1|)-\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

Mupad [B]
time = 2.20, size = 96, normalized size = 0.93

$$\frac{\ln\left(x-1\right)}{8}-\ln\left(x\right)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{\frac{9\,x^3}{16}-\frac{3\,x^2}{8}+\frac{11\,x}{16}-\frac{1}{4}}{x^4+2\,x^2+1}+\ln\left(x-\mathrm{i}\right)\,\left(\frac{15}{16}-\frac{7}{32}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{15}{16}+\frac{7}{32}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)
```

```
[Out] log(x - 1)/8 + log(x - 1i)*(15/16 - 7i/32) + log(x + 1i)*(15/16 + 7i/32) -
log(x) + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3
)/16 - 1/4)/(2*x^2 + x^4 + 1)
```

3.309 $\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} \, dx$

Optimal. Leaf size=33

$$\frac{2-x}{2(1+x^2)} + \frac{3}{2}\tan^{-1}(x) - \frac{1}{2}\log\left(1+x^2\right)$$

[Out] 1/2*(2-x)/(x^2+1)+3/2*arctan(x)-1/2*ln(x^2+1)

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1828, 649, 209, 266}

$$\frac{3\text{ArcTan}(x)}{2} + \frac{2-x}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]

[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx &= \frac{2-x}{2(1+x^2)}-\frac{1}{2}\int \frac{-3+2x}{1+x^2}\,dx\\ &= \frac{2-x}{2(1+x^2)}+\frac{3}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \frac{2-x}{2(1+x^2)}+\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.91

$$\frac{1}{2}\left(\frac{2-x}{1+x^2}+3\tan^{-1}(x)-\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2
```

Maple [A]
time = 0.18, size = 28, normalized size = 0.85

method	result	size
risch	$\frac{1-\frac{x}{2}}{x^2+1}-\frac{\ln(x^2+1)}{2}+\frac{3\arctan(x)}{2}$	27
default	$-\frac{\frac{x}{2}-1}{x^2+1}-\frac{\ln(x^2+1)}{2}+\frac{3\arctan(x)}{2}$	28
meijerg	$-\frac{x^2}{x^2+1}-\frac{\ln(x^2+1)}{2}-\frac{x}{x^2+1}+\frac{3\arctan(x)}{2}+\frac{x}{2x^2+2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] -(1/2*x-1)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)
```

Maxima [A]
time = 0.50, size = 25, normalized size = 0.76

$$-\frac{x-2}{2(x^2+1)}+\frac{3}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.38, size = 36, normalized size = 1.09

$$\frac{3\left(x^{2}+1\right)\arctan\left(x\right)-\left(x^{2}+1\right)\log\left(x^{2}+1\right)-x+2}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.73

$$-\frac{x-2}{2x^{2}+2}-\frac{\log\left(x^{2}+1\right)}{2}+\frac{3\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)
[Out] -(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2
```

Giac [A]
time = 5.79, size = 25, normalized size = 0.76

$$-\frac{x-2}{2\left(x^{2}+1\right)}+\frac{3}{2}\arctan\left(x\right)-\frac{1}{2}\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{3\operatorname{atan}(x)}{2}-\frac{\ln\left(x^{2}+1\right)}{2}-\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x^2 + 1)^2,x)
[Out] (3*atan(x))/2 - log(x^2 + 1)/2 - x/(2*(x^2 + 1)) + 1/(x^2 + 1)
```

3.310 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} \, dx$

Optimal. Leaf size=33

$$-\frac{1+2x}{2(1+x^2)} - 2\tan^{-1}(x) + \log(x) - \frac{1}{2}\log(1+x^2)$$

```
[Out] 1/2*(-1-2*x)/(x^2+1)-2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1819, 815, 649, 209, 266}

$$-2\text{ArcTan}(x) - \frac{2x+1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -1/2*(1 + 2*x)/(1 + x^2) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2+4x}{x\left(1+x^2\right)}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int \frac{2+x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{-1-2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.18, size = 28, normalized size = 0.85

method	result	size
default	$-\frac{x+\frac{1}{2}}{x^2+1}-\frac{\ln\left(x^2+1\right)}{2}-2\arctan\left(x\right)+\ln\left(x\right)$	28
risch	$\frac{-x-\frac{1}{2}}{x^2+1}+\ln\left(x\right)-\frac{\ln\left(4x^2+4\right)}{2}-2\arctan\left(x\right)$	31
meijerg	$-\frac{2x}{2x^2+2}-2\arctan\left(x\right)+\frac{x^2}{x^2+1}-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{2}+\ln\left(x\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

Maxima [A]
time = 0.48, size = 29, normalized size = 0.88

$$-\frac{2\,x+1}{2\,(x^2+1)}-2\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A]
time = 0.40, size = 44, normalized size = 1.33

$$-\frac{4\,(x^2+1)\arctan\left(x\right)+(x^2+1)\log\left(x^2+1\right)-2\,(x^2+1)\log\left(x\right)+2\,x+1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

Sympy [A]
time = 0.05, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2}+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}-2\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

Giac [A]
time = 5.83, size = 30, normalized size = 0.91

$$-\frac{2\,x+1}{2\,(x^2+1)}-2\,\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B]
time = 2.11, size = 33, normalized size = 1.00

$$\ln(x) - \frac{x+\frac{1}{2}}{x^2+1} + \ln(x-\mathrm{i})\left(-\frac{1}{2}+1\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 +
 1)
```

3.311 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$

Optimal. Leaf size=25

$$x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] x+1/2*x^2-ln(x)+1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1607, 1816, 266}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx &= \int \frac{1-x-x^2+x^3+x^4}{x\left(-1+x^2\right)}\,dx \\
&= \int \left(1-\frac{1}{x}+x+\frac{x}{-1+x^2}\right)dx \\
&= x+\frac{x^2}{2}-\log(x)+\int \frac{x}{-1+x^2}\,dx \\
&= x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Maple [A]
time = 0.21, size = 24, normalized size = 0.96

method	result	size
risch	$x+\frac{x^2}{2}-\ln\left(x\right)+\frac{\ln\left(x^2-1\right)}{2}$	20
default	$x+\frac{x^2}{2}+\frac{\ln\left(-1+x\right)}{2}-\ln\left(x\right)+\frac{\ln\left(1+x\right)}{2}$	24
norman	$x+\frac{x^2}{2}+\frac{\ln\left(-1+x\right)}{2}-\ln\left(x\right)+\frac{\ln\left(1+x\right)}{2}$	24
meijerg	$\frac{\ln\left(-x^2+1\right)}{2}-\ln\left(x\right)-\frac{i\pi}{2}+\frac{x^2}{2}-\frac{i\left(2ix-2i\,\mathrm{arctanh}\left(x\right)\right)}{2}+\mathrm{arctanh}\left(x\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] x+1/2*x^2+1/2*ln(-1+x)-ln(x)+1/2*ln(1+x)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.92

$$\frac{1}{2}\,x^2+x+\frac{1}{2}\,\log\left(x+1\right)+\frac{1}{2}\,\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [**A**]
time = 0.39, size = 19, normalized size = 0.76

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(x^2 - 1\right) - \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

Sympy [**A**]
time = 0.03, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log\left(x\right) + \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

Giac [**A**]
time = 4.65, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(|x + 1|\right) + \frac{1}{2}\log\left(|x - 1|\right) - \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

Mupad [**B**]
time = 0.04, size = 19, normalized size = 0.76

$$x + \frac{\ln\left(x^2 - 1\right)}{2} - \ln\left(x\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)
[Out] x + log(x^2 - 1)/2 - log(x) + x^2/2
```

3.312 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)$$

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6857, 649, 209, 266}

$$6\text{ArcTan}(x)-5\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right)dx\\ &= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx\\ &= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= 6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.00

$$6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Maple [A]
time = 0.20, size = 32, normalized size = 0.89

method	result	size
default	$6\arctan(x)-\frac{\ln(x^2+1)}{2}+\ln(x^2+2)-5\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	32
risch	$6\arctan(x)-\frac{\ln(x^2+1)}{2}+\ln(x^2+2)-5\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.48, size = 31, normalized size = 0.86

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 0.86

$$-5\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+6\,\arctan\left(x\right)+\log\left(x^{2}+2\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Sympy [A]
time = 0.08, size = 36, normalized size = 1.00

$$-\frac{\log\left(x^{2}+1\right)}{2}+\log\left(x^{2}+2\right)+6\operatorname{atan}\left(x\right)-5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A]
time = 3.05, size = 31, normalized size = 0.86

$$-5\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+6\,\arctan\left(x\right)+\log\left(x^{2}+2\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Mupad [B]
time = 0.11, size = 56, normalized size = 1.56

$$\ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}-3\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}+3\mathrm{i}\right)+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\,\left(1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\,\left(-1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x -
 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)
```

3.313 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx$

Optimal. Leaf size=29

$$-\frac{13x}{24(4+x^2)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Rubi [A]
time = 0.07, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6857, 209, 205}

$$\frac{25}{144}\text{ArcTan}\left(\frac{x}{2}\right)+\frac{\text{ArcTan}(x)}{9}-\frac{13x}{24(x^2+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx &= \int \left(\frac{1}{9(1+x^2)} - \frac{13}{3(4+x^2)^2} + \frac{8}{9(4+x^2)}\right)dx \\ &= \frac{1}{9}\int \frac{1}{1+x^2}\,dx + \frac{8}{9}\int \frac{1}{4+x^2}\,dx - \frac{13}{3}\int \frac{1}{(4+x^2)^2}\,dx \\ &= -\frac{13x}{24(4+x^2)} + \frac{4}{9}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x) - \frac{13}{24}\int \frac{1}{4+x^2}\,dx \\ &= -\frac{13x}{24(4+x^2)} + \frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{13x}{24(4+x^2)} + \frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Maple [**A**]
time = 0.19, size = 22, normalized size = 0.76

method	result	size
default	$-\frac{13x}{24(x^2+4)} + \frac{25\arctan\left(\frac{x}{2}\right)}{144} + \frac{\arctan(x)}{9}$	22
risch	$-\frac{13x}{24(x^2+4)} + \frac{25\arctan\left(\frac{x}{2}\right)}{144} + \frac{\arctan(x)}{9}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x,method=_RETURNVERBOSE)
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Maxima [**A**]
time = 0.50, size = 21, normalized size = 0.72

$$-\frac{13\,x}{24\,(x^2+4)} + \frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Fricas [A]
time = 0.38, size = 33, normalized size = 1.14

$$\frac{25\left(x^2+4\right)\arctan\left(\frac{1}{2}x\right)+16\left(x^2+4\right)\arctan\left(x\right)-78x}{144\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

Sympy [A]
time = 0.06, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2+96}+\frac{25\operatorname{atan}\left(\frac{x}{2}\right)}{144}+\frac{\operatorname{atan}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

Giac [A]
time = 3.99, size = 21, normalized size = 0.72

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\arctan\left(\frac{1}{2}x\right)+\frac{1}{9}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.79

$$\frac{25\operatorname{atan}\left(\frac{x}{2}\right)}{144}+\frac{\operatorname{atan}(x)}{9}-\frac{13x}{24\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)
```

```
[Out] (25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))
```

3.314 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)$$

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used = {1608, 1642, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx \\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right) \\
&= -\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.00

$$-\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.78

method	result	size
default	$-\frac{1}{2x}-\frac{\ln(x)}{4}+\frac{5\ln(x^2+x+2)}{8}+\frac{\arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)\sqrt{7}}{28}$	36
risch	$-\frac{1}{2x}+\frac{5\ln(4x^2+4x+8)}{8}+\frac{\arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)\sqrt{7}}{28}-\frac{\ln(x)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x,method=_RETURNVERBOSE)
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(2*x+1)*7^(1/2))*7^(1/2)
```

Maxima [A]
time = 0.49, size = 35, normalized size = 0.76

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

Fricas [A]
time = 0.42, size = 39, normalized size = 0.85

$$\frac{2\sqrt{7}x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+35x\log\left(x^2+x+2\right)-14x\log\left(x\right)-28}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

Sympy [A]
time = 0.06, size = 46, normalized size = 1.00

$$-\frac{\log\left(x\right)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

Giac [A]
time = 4.52, size = 36, normalized size = 0.78

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x+1)\right)-\frac{1}{2\,x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

Mupad [B]
time = 2.17, size = 49, normalized size = 1.07

$$-\frac{\ln\left(x\right)}{4}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{7}\;1\mathrm{i}}{2}\right)\,\left(-\frac{5}{8}+\frac{\sqrt{7}\;1\mathrm{i}}{56}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{7}\;1\mathrm{i}}{2}\right)\,\left(\frac{5}{8}+\frac{\sqrt{7}\;1\mathrm{i}}{56}\right)-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)
```

```
[Out] log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1i
)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)
```

3.315 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} dx$

Optimal. Leaf size=22

$$\frac{x^2}{2} - \frac{2}{7}\tanh^{-1}\left(\frac{1}{7}(1+2x)\right)$$

```
[Out] 1/2*x^2-2/7*arctanh(1/7+2/7*x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1671, 630, 31}

$$\frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \int \left(x+\frac{1}{-12+x+x^2}\right)dx$$
$$= \frac{x^2}{2}+\int \frac{1}{-12+x+x^2}\,dx$$
$$= \frac{x^2}{2}+\frac{1}{7}\int \frac{1}{-3+x}\,dx-\frac{1}{7}\int \frac{1}{4+x}\,dx$$
$$= \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.18

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Maple [A]
time = 0.25, size = 19, normalized size = 0.86

method	result	size
default	$\frac{x^2}{2}-\frac{\ln(x+4)}{7}+\frac{\ln(x-3)}{7}$	19
norman	$\frac{x^2}{2}-\frac{\ln(x+4)}{7}+\frac{\ln(x-3)}{7}$	19
risch	$\frac{x^2}{2}-\frac{\ln(x+4)}{7}+\frac{\ln(x-3)}{7}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/7*ln(x+4)+1/7*ln(x-3)
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.82

$$\frac{1}{2}\,x^2-\frac{1}{7}\,\log\left(x+4\right)+\frac{1}{7}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Fricas [A]
time = 0.39, size = 18, normalized size = 0.82

$$\frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(x+4\right) + \frac{1}{7}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{x^2}{2} + \frac{\log\left(x-3\right)}{7} - \frac{\log\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

Giac [A]
time = 4.65, size = 20, normalized size = 0.91

$$\frac{1}{2}\,x^2 - \frac{1}{7}\,\log\left(|x+4|\right) + \frac{1}{7}\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.64

$$\frac{x^2}{2} - \frac{2\,\mathrm{atanh}\left(\frac{2\,x}{7}+\frac{1}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)
[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7
```

3.316 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} \, dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(3+x)$$

```
[Out] 2*ln(1-x)+ln(x)+3*ln(3+x)
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1608, 1642}

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} \, dx &= \int \frac{-3+5x+6x^2}{x(-3+2x+x^2)} \, dx \\
&= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right) dx \\
&= 2\log(1-x)+\log(x)+3\log(3+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$2\log(1-x)+\log(x)+3\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.94

method	result	size
default	$2\ln(-1+x)+\ln(x)+3\ln(3+x)$	16
norman	$2\ln(-1+x)+\ln(x)+3\ln(3+x)$	16
risch	$2\ln(-1+x)+\ln(x)+3\ln(3+x)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(-1+x)+ln(x)+3*ln(3+x)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.88

$$3\log(x+3)+2\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Fricas [A]
time = 0.40, size = 15, normalized size = 0.88

$$3\log(x+3)+2\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.88

$$\log(x)+2\log(x-1)+3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

Giac [A]
time = 5.37, size = 18, normalized size = 1.06

$$3 \log(|x+3|) + 2 \log(|x-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

Mupad [B]
time = 0.07, size = 15, normalized size = 0.88

$$2 \ln(x-1) + 3 \ln(x+3) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
[Out] 2*log(x - 1) + 3*log(x + 3) + log(x)
```

3.317 $\int \frac{-2+3x+5x^2}{2x^2+x^3} \, dx$

Optimal. Leaf size=14

$$\frac{1}{x}+2\log(x)+3\log(2+x)$$

[Out] 1/x+2*ln(x)+3*ln(2+x)

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1607, 907}

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]

[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+3x+5x^2}{2x^2+x^3} \, dx &= \int \frac{-2+3x+5x^2}{x^2(2+x)} \, dx \\
&= \int \left(-\frac{1}{x^2}+\frac{2}{x}+\frac{3}{2+x}\right) dx \\
&= \frac{1}{x}+2\log(x)+3\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{x}+2\log(x)+3\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Maple [A]
time = 0.19, size = 15, normalized size = 1.07

method	result	size
default	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
norman	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
risch	$\frac{1}{x}+2\ln\left(x\right)+3\ln\left(x+2\right)$	15
meijerg	$3\ln\left(1+\frac{x}{2}\right)+2\ln\left(x\right)-2\ln\left(2\right)+\frac{1}{x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x+2*ln(x)+3*ln(x+2)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 1.00

$$\frac{1}{x}+3\,\log\left(x+2\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

Fricas [A]
time = 0.39, size = 18, normalized size = 1.29

$$\frac{3\,x\log\left(x+2\right)+2\,x\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

Sympy [**A**]
time = 0.03, size = 14, normalized size = 1.00

$$2\log(x) + 3\log(x+2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

Giac [**A**]
time = 3.62, size = 16, normalized size = 1.14

$$\frac{1}{x} + 3\,\log(|x+2|) + 2\,\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

Mupad [**B**]
time = 2.12, size = 14, normalized size = 1.00

$$3\,\ln(x+2) + 2\,\ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)
[Out] 3*log(x + 2) + 2*log(x) + 1/x
```

3.318 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(2+x)-3\log(3+x)$$

```
[Out] ln(1-x)-2*ln(2+x)-3*ln(3+x)
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2099}

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx &= \int \left(\frac{1}{-1+x}-\frac{2}{2+x}-\frac{3}{3+x}\right)dx\\ &= \log(1-x)-2\log(2+x)-3\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(2+x)+\frac{3}{2}\log(3+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
[Out] -2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.95

method	result	size
default	$-2\ln(x+2)+\ln(-1+x)-3\ln(3+x)$	18
norman	$-2\ln(x+2)+\ln(-1+x)-3\ln(3+x)$	18
risch	$-2\ln(x+2)+\ln(-1+x)-3\ln(3+x)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x,method=_RETURNVERBOSE)
[Out] -2*ln(x+2)+ln(-1+x)-3*ln(3+x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.89

$$-3\log(x+3)-2\log(x+2)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Fricas [A]
time = 0.43, size = 17, normalized size = 0.89

$$-3\log(x+3)-2\log(x+2)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.89

$$\log(x-1)-2\log(x+2)-3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

Giac [A]
time = 4.01, size = 20, normalized size = 1.05

$$-3\log(|x+3|)-2\log(|x+2|)+\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

Mupad [B]
time = 2.12, size = 17, normalized size = 0.89

$$\ln(x-1) - 2\ln(x+2) - 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.319 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=23

$$-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)$$

[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1687, 1180, 209, 1261, 640, 31}

$$-\frac{3}{2}\text{ArcTan}\left(\frac{x}{2}\right)+\text{ArcTan}(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]

[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 640

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx+\int \frac{x(1+x^2)}{4+5x^2+x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx,x,x^2\right)-3\int \frac{1}{4+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.78

method	result	size
default	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18

risch	$-\frac{3\arctan\left(\frac{x}{2}\right)}{2}+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Maxima [A]
time = 0.52, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Fricas [A]
time = 0.38, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Sympy [A]
time = 0.07, size = 19, normalized size = 0.83

$$\frac{\log\left(x^{2}+4\right)}{2}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2}+\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

Giac [A]
time = 2.88, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Mupad [B]
time = 0.05, size = 33, normalized size = 1.43

$$-\operatorname{atan}\left(\frac{1305}{4\,\left(144\,x-162\right)}+\frac{9}{8}\right)+\ln\left(x-2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)
```

```
[Out] log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x -
 162)) + 9/8)
```

3.320 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$

Optimal. Leaf size=63

$$\frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{11049\log\left(5+x+x^2\right)}{260015}$$

```
[Out] -3146/80155*ln(7-3*x)-334/323*ln(1+2*x)+4822/4879*ln(2+5*x)+11049/260015*ln
(x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2099, 648, 632, 210, 642}

$$\frac{3988\text{ArcTan}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}+\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \int \left(-\frac{668}{323(1+2x)}-\frac{9438}{80155(-7+3x)}+\frac{24110}{4879(2+5x)}\right.$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}$$

$$= \frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.90

$$\frac{163508\sqrt{19}\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)-418418\log(7-3x)-11023670\log(1+2x)+10536070\log(2+5x)+453009\log\left(5+x+x^2\right)}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

Maple [A]
time = 0.03, size = 51, normalized size = 0.81

method	result
default	$-\frac{334\ln(2x+1)}{323}-\frac{3146\ln(3x-7)}{80155}+\frac{11049\ln(x^2+x+5)}{260015}+\frac{3988\arctan\left(\frac{(2x+1)\sqrt{19}}{19}\right)\sqrt{19}}{260015}+\frac{4822\ln(5x+2)}{4879}$
risch	$\frac{4822\ln(5x+2)}{4879}-\frac{334\ln(2x+1)}{323}+\frac{11049\ln(15904144x^2+15904144x+79520720)}{260015}+\frac{3988\sqrt{19}\arctan\left(\frac{(3988x+1994)\sqrt{19}}{37886}\right)}{260015}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,method=
_RETURNVERBOSE)
```

```
[Out] -334/323*ln(2*x+1)-3146/80155*ln(3*x-7)+11049/260015*ln(x^2+x+5)+3988/26001
5*arctan(1/19*(2*x+1)*19^(1/2))*19^(1/2)+4822/4879*ln(5*x+2)
```

Maxima [A]
time = 0.49, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5x+2)-\frac{3146}{80155}\log(3x-7)-\frac{334}{323}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Fricas [A]
time = 0.39, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5x+2)-\frac{3146}{80155}\log(3x-7)-\frac{334}{323}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Sympy [A]
time = 0.19, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

Giac [A]
time = 3.46, size = 53, normalized size = 0.84

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}\,(2\,x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log\left(|5\,x+2|\right)-\frac{3146}{80155}\log\left(|3\,x-7|\right)-\frac{334}{323}\log\left(|2\,x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

Mupad [B]
time = 2.21, size = 58, normalized size = 0.92

$$\frac{4822\,\ln\left(x+\frac{2}{5}\right)}{4879}-\frac{334\,\ln\left(x+\frac{1}{2}\right)}{323}-\frac{3146\,\ln\left(x-\frac{7}{3}\right)}{80155}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(-\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^
5 + 70),x)
```

```
[Out] (4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/801
55 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/260015
) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/260015)
```

3.321 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

Optimal. Leaf size=69

$$\frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}+\frac{503\tan^{-1}\left(\sqrt{2}\,x\right)}{7986\sqrt{2}}-\frac{59096\log(2-5x)}{99825}+\frac{2843\log\left(1+2x^2\right)}{7986}$$

```
[Out] 5828/9075/(2-5*x)+1/1452*(-313-502*x)/(2*x^2+1)-59096/99825*ln(2-5*x)+2843/
7986*ln(2*x^2+1)+503/15972*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2099, 653, 209, 649, 266}

$$\frac{272\sqrt{2}\,\text{ArcTan}\left(\sqrt{2}\,x\right)}{1331}-\frac{251\text{ArcTan}\left(\sqrt{2}\,x\right)}{726\sqrt{2}}-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
```

```
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx=\int \left(\frac{5828}{1815(-2+5x)^2}-\frac{59096}{19965(-2+5x)}+\frac{-251+}{363\,(1+}\right.$$

$$=\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{2\int \frac{816+2843x}{1+2x^2}\,d}{3993}$$

$$=\frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\,(1+2x^2)}-\frac{59096\log(2-5x}{99825}$$

$$=\frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\,(1+2x^2)}-\frac{251\tan^{-1}\left(\sqrt{2}\,x\right.}{726\sqrt{2}}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.97

$$\frac{-\frac{33(2554+4675x+36458x^2)}{-2+5x-4x^2+10x^3}+12575\sqrt{2}\,\tan^{-1}\left(\sqrt{2}\,x\right)-236384\log(2-5x)+142150\log\left(1+2x^2\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

Maple [A]
time = 0.03, size = 54, normalized size = 0.78

method	result	size
default	$-\frac{5828}{9075(5x-2)}-\frac{59096\ln(5x-2)}{99825}+\frac{-\frac{2761x}{4}-\frac{3443}{8}}{3993x^2+\frac{3993}{2}}+\frac{2843\ln(2x^2+1)}{7986}+\frac{503\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{15972}$	54

risch	$\frac{-\frac{18229}{60500}x^2-\frac{17}{440}x-\frac{1277}{60500}}{x^3-\frac{2}{5}x^2+\frac{1}{2}x-\frac{1}{5}} - \frac{59096\ln(5x-2)}{99825} + \frac{2843\ln\left(\frac{253009}{2}+253009x^2\right)}{7986} + \frac{503\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{15972}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
,method=_RETURNVERBOSE)
```

```
[Out] -5828/9075/(5*x-2)-59096/99825*ln(5*x-2)+1/3993*(-2761/4*x-3443/8)/(x^2+1/2
)+2843/7986*ln(2*x^2+1)+503/15972*arctan(2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 59, normalized size = 0.86

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}\,x\right) - \frac{36458\,x^2+4675\,x+2554}{12100\,(10\,x^3-4\,x^2+5\,x-2)} + \frac{2843}{7986}\log\left(2\,x^2+1\right) - \frac{59096}{99825}\log\left(5\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

Fricas [A]
time = 0.41, size = 103, normalized size = 1.49

$$\frac{12575\,\sqrt{2}\,(10\,x^3-4\,x^2+5\,x-2)\arctan\left(\sqrt{2}\,x\right) - 1203114\,x^2 + 142150\,(10\,x^3-4\,x^2+5\,x-2)\log\left(2\,x^2+1\right) - 236384\,(10\,x^3-4\,x^2+5\,x-2)\log\left(5\,x-2\right) - 154275\,x - 84282}{399300\,(10\,x^3-4\,x^2+5\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

Sympy [A]
time = 0.09, size = 65, normalized size = 0.94

$$\frac{-36458x^2-4675x-2554}{121000x^3-48400x^2+60500x-24200} - \frac{59096\log\left(x-\frac{2}{5}\right)}{99825} + \frac{2843\log\left(x^2+\frac{1}{2}\right)}{7986} + \frac{503\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] (-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

Giac [A]
time = 4.91, size = 59, normalized size = 0.86

$$\frac{503}{15972}\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\,(2\,x^2+1)(5\,x-2)}+\frac{2843}{7986}\,\log\left(2\,x^2+1\right)-\frac{59096}{99825}\,\log\left(|5\,x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

Mupad [B]
time = 2.18, size = 71, normalized size = 1.03

$$-\frac{59096\,\ln\left(x-\frac{2}{5}\right)}{99825}-\frac{\frac{18229\,x^2}{60500}+\frac{17\,x}{440}+\frac{1277}{60500}}{x^3-\frac{2\,x^2}{5}+\frac{x}{2}-\frac{1}{5}}-\ln\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(-\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)+\ln\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x
^5 + 100*x^6 + 4),x)
```

```
[Out] log(x + (2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 + 2843/7986) - ((17*x)/440 +
(18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - log(x - (2^
(1/2)*1i)/2)*((2^(1/2)*503i)/31944 - 2843/7986) - (59096*log(x - 2/5))/9982
5
```

3.322 $\int \frac{9+x^4}{x^2(9+x^2)} \, dx$

Optimal. Leaf size=17

$$-\frac{1}{x}+x-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -1/x+x-10/3*arctan(1/3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1276, 209}

$$-\frac{10}{3}\text{ArcTan}\left(\frac{x}{3}\right)+x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx &= \int \left(1+\frac{1}{x^2}-\frac{10}{9+x^2}\right)dx\\ &= -\frac{1}{x}+x-10\int \frac{1}{9+x^2}\,dx\\ &= -\frac{1}{x}+x-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{x}+x-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Maple [A]
time = 0.20, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
meijerg	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14
risch	$-\frac{1}{x}+x-\frac{10\arctan\left(\frac{x}{3}\right)}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+9)/x^2/(x^2+9),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+x-10/3*arctan(1/3*x)
```

Maxima [A]
time = 0.50, size = 13, normalized size = 0.76

$$x-\frac{1}{x}-\frac{10}{3}\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")
```

```
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

Fricas [A]
time = 0.38, size = 19, normalized size = 1.12

$$\frac{3\,x^2-10\,x\arctan\left(\frac{1}{3}\,x\right)-3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$x-\frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+9)/x**2/(x**2+9),x)
```

```
[Out] x - 10*atan(x/3)/3 - 1/x
```

Giac [A]
time = 4.51, size = 13, normalized size = 0.76

$$x - \frac{1}{x} - \frac{10}{3}\arctan\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")
```

```
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$x - \frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 9)/(x^2*(x^2 + 9)),x)
```

```
[Out] x - (10*atan(x/3))/3 - 1/x
```

3.323 $\int \frac{2x+x^4}{1+x^2} \, dx$

Optimal. Leaf size=19

$$-x+\frac{x^3}{3}+\tan^{-1}(x)+\log\left(1+x^2\right)$$

```
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1607, 1816, 649, 209, 266}

$$\text{ArcTan}(x)+\frac{x^3}{3}+\log\left(x^2+1\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2x+x^4}{1+x^2}\,dx &= \int \frac{x(2+x^3)}{1+x^2}\,dx\\ &= \int \left(-1+x^2+\frac{1+2x}{1+x^2}\right)dx\\ &= -x+\frac{x^3}{3}+\int \frac{1+2x}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+\tan^{-1}(x)+\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$-x+\frac{x^3}{3}+\tan^{-1}(x)+\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Maple [A]
time = 0.20, size = 18, normalized size = 0.95

method	result	size
default	$-x+\frac{x^3}{3}+\arctan(x)+\ln(x^2+1)$	18
risch	$-x+\frac{x^3}{3}+\arctan(x)+\ln(x^2+1)$	18
meijerg	$-\frac{x(-5x^2+15)}{15}+\arctan(x)+\ln(x^2+1)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

Maxima [A]
time = 0.48, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Fricas [A]
time = 0.36, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log(x^2 + 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x)/(x**2+1),x)
[Out] x**3/3 - x + log(x**2 + 1) + atan(x)
```

Giac [A]
time = 4.58, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

Mupad [B]
time = 2.10, size = 17, normalized size = 0.89

$$\ln(x^2 + 1) - x + \operatorname{atan}(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4)/(x^2 + 1),x)
[Out] log(x^2 + 1) - x + atan(x) + x^3/3
```

3.324 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=9

$$\tan^{-1}(x) + \log(1-x)$$

```
[Out] arctan(x)+ln(1-x)
```

Rubi [A]
time = 0.04, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1600, 1607, 1643, 209}

$$\text{ArcTan}(x) + \log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[1 - x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-x+x^3}{(-1+x)^2\,(1+x^2)}\,dx &= \int \frac{x+x^2}{(-1+x)\,(1+x^2)}\,dx\\ &= \int \frac{x(1+x)}{(-1+x)\,(1+x^2)}\,dx\\ &= \int \left(\frac{1}{-1+x}+\frac{1}{1+x^2}\right)dx\\ &= \log(1-x)+\int \frac{1}{1+x^2}\,dx\\ &= \tan^{-1}(x)+\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\tan^{-1}(x)+\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
[Out] ArcTan[x] + Log[1 - x]
```

Maple [A]
time = 0.20, size = 8, normalized size = 0.89

method	result	size
default	$\ln(-1+x)+\arctan(x)$	8
risch	$\ln(-1+x)+\arctan(x)$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(-1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] ln(-1+x)+arctan(x)
```

Maxima [A]
time = 0.49, size = 7, normalized size = 0.78

$$\arctan(x)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
[Out] arctan(x) + log(x - 1)
```

Fricas [A]
time = 0.40, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] arctan(x) + log(x - 1)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.78

$$\log(x - 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(-1+x)**2/(x**2+1),x)
[Out] log(x - 1) + atan(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(9) = 18.
time = 4.32, size = 28, normalized size = 3.11

$$\frac{1}{4}\pi - \pi\left\lfloor\frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \arctan(x) + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="giac")
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x
- 1))
```

Mupad [B]
time = 2.10, size = 19, normalized size = 2.11

$$\ln(x - 1) - \operatorname{atan}\left(\frac{5}{4x + 2} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/((x^2 + 1)*(x - 1)^2),x)
[Out] log(x - 1) - atan(5/(4*x + 2) - 1/2)
```

3.325 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} \, dx$

Optimal. Leaf size=12

$$x + x^2 + \log\left(1 + x + x^2\right)$$

```
[Out] x+x^2+ln(x^2+x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1671, 642}

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{2+5x+3x^2+2x^3}{1+x+x^2} \, dx &= \int \left(1+2x+\frac{1+2x}{1+x+x^2}\right) dx \\ &= x + x^2 + \int \frac{1+2x}{1+x+x^2} \, dx \\ &= x + x^2 + \log\left(1+x+x^2\right) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$x + x^2 + \log\left(1 + x + x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Maple [A]
time = 0.27, size = 13, normalized size = 1.08

method	result	size
default	$x+x^2+\ln\left(x^2+x+1\right)$	13
norman	$x+x^2+\ln\left(x^2+x+1\right)$	13
risch	$x+x^2+\ln\left(x^2+x+1\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+x^2+ln(x^2+x+1)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 1.00

$$x^2+x+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

Fricas [A]
time = 0.38, size = 12, normalized size = 1.00

$$x^2+x+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 1.00

$$x^2+x+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
```

```
[Out] x**2 + x + log(x**2 + x + 1)
```

Giac [A]
time = 3.68, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] x^2 + x + log(x^2 + x + 1)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 1.00

$$x + \ln\left(x^2 + x + 1\right) + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3*x^2 + 2*x^3 + 2)/(x + x^2 + 1),x)
```

```
[Out] x + log(x + x^2 + 1) + x^2
```

3.326 $\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx$

Optimal. Leaf size=65

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)$$

```
[Out] 3/2/x^2-1/x+3*ln(x)-1/10*ln(1+2*x-5^(1/2))*(15-5^(1/2))-1/10*ln(1+2*x+5^(1/
2))*(15+5^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1642, 646, 31}

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10
- ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)}\,dx = \int \left(-\frac{3}{x^3}+\frac{1}{x^2}+\frac{3}{x}+\frac{-1-3x}{-1+x+x^2}\right)dx$$
$$= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\int \frac{-1-3x}{-1+x+x^2}\,dx$$
$$= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\frac{1}{10}\left(-15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx-\frac{1}{10}\left(15+\sqrt{5}\right)\int$$
$$= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.89

$$\frac{1}{10}\left(\frac{15}{x^2}-\frac{10}{x}+\left(-15+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x\right)+30\log(x)-\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15
+ Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Maple [A]
time = 0.24, size = 41, normalized size = 0.63

method	result	si
default	$-\frac{3\ln(x^2+x-1)}{2}-\frac{\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}-\frac{1}{x}+\frac{3}{2x^2}+3\ln(x)$	4
risch	$\frac{-x+\frac{3}{2}}{x^2}+3\ln(x)-\frac{3\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)}{2}+\frac{\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\sqrt{5}}{10}-\frac{3\ln\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{2}-\frac{\ln\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\sqrt{5}}{10}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x,method=_RETURNVERBOSE)
[Out] -3/2*ln(x^2+x-1)-1/5*arctanh(1/5*(2*x+1)*5^(1/2))*5^(1/2)-1/x+3/2/x^2+3*ln(
x)
```

Maxima [A]
time = 0.49, size = 51, normalized size = 0.78

$$\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)-\frac{2x-3}{2x^2}-\frac{3}{2}\log\left(x^2+x-1\right)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x
^2 - 3/2*log(x^2 + x - 1) + 3*log(x)
```

Fricas [A]
time = 0.39, size = 66, normalized size = 1.02

$$\frac{\sqrt{5}\,x^2\log\left(\frac{2\,x^2-\sqrt{5}\,(2\,x+1)+2\,x+3}{x^2+x-1}\right)-15\,x^2\log\left(x^2+x-1\right)+30\,x^2\log\left(x\right)-10\,x+15}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^2*log((2*x^2 - sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1))
- 15*x^2*log(x^2 + x - 1) + 30*x^2*log(x) - 10*x + 15)/x^2
```

Sympy [A]
time = 0.24, size = 99, normalized size = 1.52

$$3\log\left(x\right)+\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}-\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)^2}{101}\right)+\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}+\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)^2}{101}\right)+\frac{3-2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

```
[Out] 3*log(x) + (-3/2 + sqrt(5)/10)*log(x - 405/202 - 35*sqrt(5)/202 + 110*(-3/2
 + sqrt(5)/10)**2/101) + (-3/2 - sqrt(5)/10)*log(x - 405/202 + 35*sqrt(5)/2
02 + 110*(-3/2 - sqrt(5)/10)**2/101) + (3 - 2*x)/(2*x**2)
```

Giac [A]
time = 3.63, size = 55, normalized size = 0.85

$$\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)-\frac{2\,x-3}{2\,x^2}-\frac{3}{2}\,\log\left(\left|x^2+x-1\right|\right)+3\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/2*(2*x
- 3)/x^2 - 3/2*log(abs(x^2 + x - 1)) + 3*log(abs(x))
```

Mupad [B]
time = 0.10, size = 48, normalized size = 0.74

$$3\ln(x)-\frac{x-\frac{3}{2}}{x^2}+\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}-\frac{3}{2}\right)-\ln\left(x+\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x + 5*x^2 - 3*x^3 - 3)/(x^3*(x + x^2 - 1)),x)
```

```
[Out] 3*log(x) - (x - 3/2)/x^2 + log(x - 5^(1/2)/2 + 1/2)*(5^(1/2)/10 - 3/2) - lo
g(x + 5^(1/2)/2 + 1/2)*(5^(1/2)/10 + 3/2)
```

3.327 $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2+2x+x^2}-\tan^{-1}(1+x)+\log\left(2+2x+x^2\right)$$

```
[Out] -1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1674, 648, 631, 210, 642}

$$-\text{ArcTan}(x+1)-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{4+8x+5x^2+2x^3}{\left(2+2x+x^2\right)^2}\,dx &= -\frac{1}{2+2x+x^2}+\frac{1}{4}\int \frac{4+8x}{2+2x+x^2}\,dx\\ &= -\frac{1}{2+2x+x^2}-\int \frac{1}{2+2x+x^2}\,dx+\int \frac{2+2x}{2+2x+x^2}\,dx\\ &= -\frac{1}{2+2x+x^2}+\log\left(2+2x+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right)\\ &= -\frac{1}{2+2x+x^2}-\tan^{-1}(1+x)+\log\left(2+2x+x^2\right)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{2+2x+x^2}-\tan^{-1}(1+x)+\log\left(2+2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Maple [**A**]
time = 0.28, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{x^2+2x+2}-\arctan\left(1+x\right)+\ln\left(x^2+2x+2\right)$	29
risch	$-\frac{1}{x^2+2x+2}-\arctan\left(1+x\right)+\ln\left(x^2+2x+2\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^2+2*x+2)-arctan(1+x)+ln(x^2+2*x+2)
```

Maxima [A]
time = 0.48, size = 28, normalized size = 1.00

$$-\frac{1}{x^2+2\,x+2}-\arctan\left(x+1\right)+\log\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

Fricas [A]
time = 0.37, size = 46, normalized size = 1.64

$$-\frac{\left(x^2+2\,x+2\right)\arctan\left(x+1\right)-\left(x^2+2\,x+2\right)\log\left(x^2+2\,x+2\right)+1}{x^2+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/(
x^2 + 2*x + 2)
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.86

$$\log\left(x^2+2x+2\right)-\operatorname{atan}\left(x+1\right)-\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)
```

```
[Out] log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)
```

Giac [A]
time = 3.68, size = 28, normalized size = 1.00

$$-\frac{1}{x^2+2\,x+2}-\arctan\left(x+1\right)+\log\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

Mupad [B]
time = 0.04, size = 28, normalized size = 1.00

$$\ln\left(x^2+2\,x+2\right)-\mathrm{atan}(x+1)-\frac{1}{x^2+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 5*x^2 + 2*x^3 + 4)/(2*x + x^2 + 2)^2,x)
```

```
[Out] log(2*x + x^2 + 2) - atan(x + 1) - 1/(2*x + x^2 + 2)
```

3.328 $\int \frac{(-1+x)^4x^4}{1+x^2}\,dx$

Optimal. Leaf size=32

$$4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\tan^{-1}(x)$$

```
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1643, 209}

$$-4\text{ArcTan}(x)+\frac{x^7}{7}-\frac{2x^6}{3}+x^5-\frac{4x^3}{3}+4x$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^4x^4}{1+x^2}\,dx &= \int \left(4-4x^2+5x^4-4x^5+x^6-\frac{4}{1+x^2}\right)dx\\ &= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\int \frac{1}{1+x^2}\,dx\\ &= 4x-\frac{4x^3}{3}+x^5-\frac{2x^6}{3}+\frac{x^7}{7}-4\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^4*x^4)/(1 + x^2),x]
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Maple [A]
time = 0.22, size = 27, normalized size = 0.84

method	result
default	$4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4\arctan(x)$
risch	$4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4\arctan(x)$
meijerg	$-\frac{x(-5x^2+15)}{15} - 4\arctan(x) + \frac{x^2(-3x^2+6)}{3} + \frac{2x(21x^4-35x^2+105)}{35} - \frac{x^2(4x^4-6x^2+12)}{6} - \frac{x(-45x^6+63x^4-105x^2+3}{315}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^4*x^4/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Fricas [A]
time = 0.37, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="fricas")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Sympy [A]
time = 0.03, size = 29, normalized size = 0.91

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**4*x**4/(x**2+1),x)
```

```
[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

Giac [A]
time = 4.22, size = 26, normalized size = 0.81

$$\frac{1}{7}\,x^7 - \frac{2}{3}\,x^6 + x^5 - \frac{4}{3}\,x^3 + 4\,x - 4\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

Mupad [B]
time = 0.02, size = 26, normalized size = 0.81

$$4\,x - 4\,\mathrm{atan}(x) - \frac{4\,x^3}{3} + x^5 - \frac{2\,x^6}{3} + \frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x - 1)^4)/(x^2 + 1),x)
```

```
[Out] 4*x - 4*atan(x) - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7
```

3.329 $\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$

Optimal. Leaf size=31

$$\log(1-x)-\frac{1}{2}\log(3-x)+\frac{3}{2}\log(1+x)-2\log(3+x)$$

```
[Out] ln(1-x)-1/2*ln(3-x)+3/2*ln(1+x)-2*ln(3+x)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1607, 1676, 12, 1121, 630, 31, 1144, 213}

$$\frac{5}{4}\log\left(1-x^2\right)-\frac{5}{4}\log\left(9-x^2\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

```
[Out] (-3*ArcTanh[x/3])/2 + ArcTanh[x]/2 + (5*Log[1 - x^2])/4 - (5*Log[9 - x^2])/
4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1676

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx &= \int \frac{x(-20+4x)}{9-10x^2+x^4}\,dx \\
&= \int -\frac{20x}{9-10x^2+x^4}\,dx + \int \frac{4x^2}{9-10x^2+x^4}\,dx \\
&= 4\int \frac{x^2}{9-10x^2+x^4}\,dx - 20\int \frac{x}{9-10x^2+x^4}\,dx \\
&= -\left(\frac{1}{2}\int \frac{1}{-1+x^2}\,dx\right) + \frac{9}{2}\int \frac{1}{-9+x^2}\,dx - 10\mathrm{Subst}\left(\int \frac{1}{9-10x+x^2}\,dx, x, x^2\right) \\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) - \frac{5}{4}\mathrm{Subst}\left(\int \frac{1}{-9+x}\,dx, x, x^2\right) + \frac{5}{4}\mathrm{Subst}\left(\int \frac{1}{-1-\cdots}\right. \\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) + \frac{5}{4}\log\left(1-x^2\right) - \frac{5}{4}\log\left(9-x^2\right)
\end{aligned}
$$

Mathematica [A]

time = 0.01, size = 39, normalized size = 1.26

$$4\left(\frac{1}{4}\log(1-x)-\frac{1}{8}\log(3-x)+\frac{3}{8}\log(1+x)-\frac{1}{2}\log(3+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.77

method	result	size
default	$-\frac{\ln(x-3)}{2}+\ln\left(-1+x\right)+\frac{3\ln(1+x)}{2}-2\ln\left(3+x\right)$	24
norman	$-\frac{\ln(x-3)}{2}+\ln\left(-1+x\right)+\frac{3\ln(1+x)}{2}-2\ln\left(3+x\right)$	24
risch	$-\frac{\ln(x-3)}{2}+\ln\left(-1+x\right)+\frac{3\ln(1+x)}{2}-2\ln\left(3+x\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-20*x)/(x^4-10*x^2+9),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x-3)+ln(-1+x)+3/2*ln(1+x)-2*ln(3+x)
```

Maxima [A]
time = 0.29, size = 23, normalized size = 0.74

$$-2\,\log\left(x+3\right)+\frac{3}{2}\,\log\left(x+1\right)+\log\left(x-1\right)-\frac{1}{2}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

Fricas [A]
time = 0.40, size = 23, normalized size = 0.74

$$-2\,\log\left(x+3\right)+\frac{3}{2}\,\log\left(x+1\right)+\log\left(x-1\right)-\frac{1}{2}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="fricas")
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

<u>**Sympy**</u> [A]
time = 0.08, size = 26, normalized size = 0.84

$$-\frac{\log(x-3)}{2}+\log(x-1)+\frac{3\log(x+1)}{2}-2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)
```

```
[Out] -log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)
```

<u>**Giac**</u> [A]
time = 4.80, size = 27, normalized size = 0.87

$$-2\log(|x+3|)+\frac{3}{2}\log(|x+1|)+\log(|x-1|)-\frac{1}{2}\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="giac")
```

```
[Out] -2*log(abs(x + 3)) + 3/2*log(abs(x + 1)) + log(abs(x - 1)) - 1/2*log(abs(x
- 3))
```

<u>**Mupad**</u> [B]
time = 0.05, size = 23, normalized size = 0.74

$$\ln(x-1)+\frac{3\ln(x+1)}{2}-\frac{\ln(x-3)}{2}-2\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(20*x - 4*x^2)/(x^4 - 10*x^2 + 9),x)
```

```
[Out] log(x - 1) + (3*log(x + 1))/2 - log(x - 3)/2 - 2*log(x + 3)
```

3.330 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{x}+\tan^{-1}(x)+2\log(1-x)-\log\left(1+x^2\right)$$

```
[Out] -1/x+arctan(x)+2*ln(1-x)-ln(x^2+1)
```

Rubi [A]
time = 0.12, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6857, 649, 209, 266}

$$\text{ArcTan}(x)-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
```

```
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{1}{x^2}+\frac{1-2x}{1+x^2}\right)\,dx\\ &= -\frac{1}{x}+2\log(1-x)+\int \frac{1-2x}{1+x^2}\,dx\\ &= -\frac{1}{x}+2\log(1-x)-2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{x}+\tan^{-1}(x)+2\log(1-x)-\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$-\frac{1}{x}+\tan^{-1}(x)+2\log(1-x)-\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Maple [A]
time = 0.21, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{1}{x}+2\ln\left(-1+x\right)-\ln\left(x^2+1\right)+\arctan\left(x\right)$	23
risch	$-\frac{1}{x}+2\ln\left(-1+x\right)-\ln\left(x^2+1\right)+\arctan\left(x\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/x+2*ln(-1+x)-ln(x^2+1)+arctan(x)
```

Maxima [A]
time = 0.48, size = 22, normalized size = 0.92

$$-\frac{1}{x}+\arctan\left(x\right)-\log\left(x^2+1\right)+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A]
time = 0.43, size = 26, normalized size = 1.08

$$\frac{x \arctan\left(x\right) - x \log\left(x^{2} + 1\right) + 2 \, x \log\left(x - 1\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.79

$$2 \log\left(x - 1\right) - \log\left(x^{2} + 1\right) + \operatorname{atan}\left(x\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

Giac [A]
time = 4.49, size = 23, normalized size = 0.96

$$-\frac{1}{x} + \arctan\left(x\right) - \log\left(x^{2} + 1\right) + 2 \, \log\left({\left| x - 1 \right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="giac")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))
```

Mupad [B]
time = 2.13, size = 30, normalized size = 1.25

$$2\,\ln\left(x-1\right)-\frac{1}{x}+\ln\left(x-\mathrm{i}\right)\,\left(-1-\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-1+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^3 - 1)/(x^2*(x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) - log(x - 1i)*(1 + 1i/2) - log(x + 1i)*(1 - 1i/2) - 1/x
```

3.331 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx$

Optimal. Leaf size=23

$$-\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)$$

```
[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1828, 12, 209}

$$\text{ArcTan}(x)+\frac{2}{x^2+1}-\frac{1}{4(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
```

```
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx &= -\frac{1}{4(1+x^2)^2}-\frac{1}{4}\int \frac{-4+16x-4x^2}{(1+x^2)^2}\,dx\\ &= -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\frac{1}{8}\int \frac{8}{1+x^2}\,dx\\ &= -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{4(1+x^2)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Maple [A]
time = 0.20, size = 19, normalized size = 0.83

method	result	size
default	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
risch	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
meijerg	$\frac{x(3x^2+5)}{8(x^2+1)^2}+\arctan(x)-\frac{x(25x^2+15)}{40(x^2+1)^2}-\frac{x^4}{(x^2+1)^2}-\frac{x(-3x^2+3)}{12(x^2+1)^2}-\frac{3x^2(x^2+2)}{4(x^2+1)^2}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Maxima [A]
time = 0.93, size = 24, normalized size = 1.04

$$\frac{8x^2+7}{4(x^4+2x^2+1)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Fricas [A]
time = 0.37, size = 35, normalized size = 1.52

$$\frac{8\,x^2+4\,(x^4+2\,x^2+1)\arctan{(x)}+7}{4\,(x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

Giac [A]
time = 4.89, size = 19, normalized size = 0.83

$$\frac{8\,x^2+7}{4\,(x^2+1)^2}+\arctan{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

Mupad [B]
time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x)+\frac{2\,x^2+\frac{7}{4}}{x^4+2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(x^2 + 1)^3,x)
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

3.332 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)$$

```
[Out] -1/4/(x^2+1)^2+2/(x^2+1)+arctan(x)
```

Rubi [A]

time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2098, 267, 209}

$$\text{ArcTan}(x)+\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx = \int \left(\frac{x}{(1+x^2)^3} - \frac{4x}{(1+x^2)^2} + \frac{1}{1+x^2}\right)dx$$
$$= -\left(4\int \frac{x}{(1+x^2)^2}\,dx\right) + \int \frac{x}{(1+x^2)^3}\,dx + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \tan^{-1}(x)$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.83

method	result	size
default	$\frac{2x^2+\frac{7}{4}}{(x^2+1)^2}+\arctan(x)$	19
risch	$\frac{2x^2+\frac{7}{4}}{x^4+2x^2+1}+\arctan(x)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

Maxima [A]
time = 0.51, size = 24, normalized size = 1.04

$$\frac{8x^2+7}{4(x^4+2x^2+1)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

Fricas [A]
time = 0.37, size = 35, normalized size = 1.52

$$\frac{8\,x^2 + 4\,(x^4 + 2\,x^2 + 1)\arctan(x) + 7}{4\,(x^4 + 2\,x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.87

$$\frac{8x^2 + 7}{4x^4 + 8x^2 + 4} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
```

```
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

Giac [A]
time = 4.45, size = 19, normalized size = 0.83

$$\frac{8\,x^2 + 7}{4\,(x^2 + 1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

Mupad [B]
time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{2\,x^2 + \frac{7}{4}}{x^4 + 2\,x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(3*x^2 + 3*x^4 + x^6 + 1),x)
```

```
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

3.333 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{x}+\log\left(1+x+x^2\right)$$

[Out] -1/x+ln(x^2+x+1)

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1608, 1642, 642}

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]

[Out] -x^(-1) + Log[1 + x + x^2]

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx = \int \frac{1+x+2x^2+2x^3}{x^2(1+x+x^2)}\,dx$$
$$= \int \left(\frac{1}{x^2}+\frac{1+2x}{1+x+x^2}\right)dx$$
$$= -\frac{1}{x}+\int \frac{1+2x}{1+x+x^2}\,dx$$
$$= -\frac{1}{x}+\log\left(1+x+x^2\right)$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{x}+\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
[Out] -x^(-1) + Log[1 + x + x^2]
```

Maple [A]
time = 0.02, size = 14, normalized size = 1.08

method	result	size
default	$-\frac{1}{x}+\ln\left(x^2+x+1\right)$	14
norman	$-\frac{1}{x}+\ln\left(x^2+x+1\right)$	14
risch	$-\frac{1}{x}+\ln\left(x^2+x+1\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x,method=_RETURNVERBOSE)
[Out] -1/x+ln(x^2+x+1)
```

Maxima [A]
time = 0.54, size = 13, normalized size = 1.00

$$-\frac{1}{x}+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="maxima")
```

```
[Out] -1/x + log(x^2 + x + 1)
```

Fricas [A]
time = 0.37, size = 15, normalized size = 1.15

$$\frac{x \log (x^2 + x + 1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="fricas")
[Out] (x*log(x^2 + x + 1) - 1)/x
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$\log \left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
[Out] log(x**2 + x + 1) - 1/x
```

Giac [A]
time = 3.88, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log \left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="giac")
[Out] -1/x + log(x^2 + x + 1)
```

Mupad [B]
time = 2.14, size = 13, normalized size = 1.00

$$\ln \left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*x^2 + 2*x^3 + 1)/(x^2 + x^3 + x^4),x)
[Out] log(x + x^2 + 1) - 1/x
```

3.334 $\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$

Optimal. Leaf size=206

$$\frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}}{}$$

```
[Out] 2*c*d*x/b+1/2*d^2*x^2/b-1/3*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^
(1/3)*x)/b^(5/3)+1/6*a^(1/3)*d*(2*b^(1/3)*c-a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/3*c^2*ln(b*x^3+a)/b+1/3*a^(1/3)*d*(2*b^(1/3
)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^
(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}}+\frac{\sqrt[3]{a}\,d\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{2cdx}{b}+\frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*d
*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)
*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \int \left(\frac{2cd}{b}+\frac{d^2x}{b}-\frac{2acd+ad^2x-bc^2x^2}{b\,(a+bx^3)}\right)dx$$

$$=\frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x-bc^2x^2}{a+bx^3}\,dx}{b}$$

$$=\frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x}{a+bx^3}\,dx}{b}+c^2\int \frac{x^2}{a+bx^3}\,dx$$

$$=\frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{c^2\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}\,cd+a^{4/3}d^2\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}\,cd+a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\ldots$$

$$=\frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{\left(\sqrt[3]{a}\,d\left(\ldots\right.\right.}{\ldots}$$

$$=\frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\ldots}{6\ldots}$$

$$=\frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\ldots}{3b^{5/3}}$$

Mathematica [A]
time = 0.08, size = 193, normalized size = 0.94

$$\frac{12b^{2/3}cdx+3b^{2/3}d^2x^2+2\sqrt{3}\,\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}\,d\left(-2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-\sqrt[3]{a}\,d\left(-2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c +
a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(-2*b^
(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c + a
^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Lo
g[a + b*x^3])/(6*b^(5/3))
```

Maple [A]
time = 0.23, size = 227, normalized size = 1.10

method	result

risch	$\frac{d^2x^2}{2b}+\frac{2cdx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{(\_R^2bc^2-\_Rad^2-2acd)\ln(x-\_R)}{\_R^2}}{3b^2}$
default	$\frac{d(\frac{1}{2}dx^2+2cx)}{b}+\frac{-2acd\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-ad^2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x+c)^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b*(1/2*d*x^2+2*c*x)+(-2*a*c*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-a*d^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^
(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a
/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*c^2*ln(b*x^3+a))/b
```

Maxima [A]
time = 0.50, size = 199, normalized size = 0.97

$$-\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{d^2x^2+4\,cdx}{2\,b}+\frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,acd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) + 2*a*c*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(d^2*x^2 + 4*c*d*x)/b + 1/6*(2*
b*c^2*(a/b)^(2/3) - a*d^2*(a/b)^(1/3) + 2*a*c*d)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)^(1/3)
 - 2*a*c*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.15, size = 4545, normalized size = 22.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b*log(1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*s
qrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2
*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b
^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b
^3 + 3*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (
8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a
*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 + 5*b*c
^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^
5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5
)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqr
t(3) + 1) - 2*c^2/b)*b + 6*c^2 + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b
^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^
3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a
^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
+ 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)
^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt
(3) + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)
^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8
*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*
b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3
)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^
2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/
b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d
^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 +
a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^
(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3
*d + a*d^4)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
```

```
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d
^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c
^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) +
(1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1)
- 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I
*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d
^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b
^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 +
2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*
a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3
) + 1) - 2*c^2/b)*b + 6*c^2 - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2
- (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*
a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*
d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*
(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3
)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)...
```

Sympy [A]
time = 0.70, size = 138, normalized size = 0.67

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c^2 + t\left(18ab^2cd^3 + 9b^3c^4\right) - a^2d^6 + 2abc^3d^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{9t^2b^3 - 18tb^2c^2 + 4acd^3 + 5bc^4}{ad^4 + 8bc^3d}\right)\right)\right) + \frac{2cdx}{b} + \frac{d^2x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3 + 9*b**3*
c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9*_
t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*d))
)) + 2*c*d*x/b + d**2*x**2/(2*b)
```

Giac [A]
time = 6.01, size = 208, normalized size = 1.01

$$\frac{c^2\log\left(|bx^3+a|\right)}{3b} - \frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd - \left(-ab^2\right)^{\frac{2}{3}}d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3} + \frac{bd^2x^2 + 4bcdx}{2b^2} - \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd + \left(-ab^2\right)^{\frac{2}{3}}d^2\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3} + \frac{\left(ab^4d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2ab^4cd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c^2*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c*d - (-a*b
^2)^(2/3)*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 +
```

```
1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/6*(2*(-a*b^2)^(1/3)*b*c*d + (-a*b^2)^(2
/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*d^2*(-a/
b)^(1/3) + 2*a*b^4*c*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

Mupad [B]
time = 0.23, size = 357, normalized size = 1.73

$$\left(\sum_{k=1}^{3}\ln\left(\frac{a\left(b\,c^4+\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k)^2\,b^3\,9-\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k)\,b^2\,c^2\,6+2\,a\,c\,d^3+a\,d^4\,x+2\,b\,c^3\,d\,x-\mathrm{root}(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k)\,b^2\,c\,d\,x\,6\right)}{b}\right)\,\mathrm{root}\left(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k\right)\right)+\frac{d^2\,x^2}{2\,b}+\frac{2\,c\,d\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x)^2)/(a + b*x^3),x)
```

```
[Out] symsum(log((a*(b*c^4 + 9*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*
z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)^2*b^3 - 6*root(2
7*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3
 - b^2*c^6 - a^2*d^6, z, k)*b^2*c^2 + 2*a*c*d^3 + a*d^4*x + 2*b*c^3*d*x - 6
*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*
c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c*d*x))/b)*root(27*b^5*z^3 - 27*b^4*
c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^
6, z, k), k, 1, 3) + (d^2*x^2)/(2*b) + (2*c*d*x)/b
```

3.335 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{5-7x^2}{8(3+2x^2+x^4)}+\frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/8*(-7*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1608, 1677, 1674, 12, 632, 210}

$$\frac{9\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5-7x^2}{8(x^4+2x^2+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
```

```
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx &= \int \frac{x(-1+2x^2+4x^4)}{(3+2x^2+x^4)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+2x+4x^2}{(3+2x+x^2)^2}\,dx, x, x^2\right)\\
&= \frac{5-7x^2}{8\,(3+2x^2+x^4)} + \frac{1}{16}\text{Subst}\left(\int \frac{18}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{5-7x^2}{8\,(3+2x^2+x^4)} + \frac{9}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{5-7x^2}{8\,(3+2x^2+x^4)} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right)\\
&= \frac{5-7x^2}{8\,(3+2x^2+x^4)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{5-7x^2}{8\,(3+2x^2+x^4)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.91

method	result	size
risch	$\frac{-\frac{7x^2}{8}+\frac{5}{8}}{x^4+2x^2+3}+\frac{9\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
default	$\frac{-\frac{7x^2}{4}+\frac{5}{4}}{2x^4+4x^2+6}+\frac{9\sqrt{2}\ \arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(-7/4*x^2+5/4)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
[Out] -1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

Fricas [A]
time = 0.38, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}\,(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}\,(x^2+1)\right)-14\,x^2+10}{16\,(x^4+2\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

Sympy [A]
time = 0.05, size = 44, normalized size = 0.98

$$\frac{5-7x^2}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

Giac [A]
time = 4.09, size = 38, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{7\,x^2-5}{8\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2 +
 3)
```

Mupad [B]
time = 2.19, size = 42, normalized size = 0.93

$$\frac{9\,\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}-\frac{\frac{7\,x^2}{8}-\frac{5}{8}}{x^4+2\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^2
 + x^4 + 3)
```

3.336 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3} \, dx$

Optimal. Leaf size=59

$$\frac{3+4x^2}{16(1+2x^2+2x^4)^2}+\frac{1+2x^2}{2(1+2x^2+2x^4)}+\tan^{-1}(1+2x^2)$$

```
[Out] 1/16*(4*x^2+3)/(2*x^4+2*x^2+1)^2+1/2*(2*x^2+1)/(2*x^4+2*x^2+1)+arctan(2*x^2
+1)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1607, 1677, 1674, 12, 628, 631, 210}

$$\text{ArcTan}(2x^2+1)+\frac{2x^2+1}{2(2x^4+2x^2+1)}+\frac{4x^2+3}{16(2x^4+2x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)
) + ArcTan[1 + 2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x+x^5}{\left(1+2x^2+2x^4\right)^3}\,dx &= \int \frac{x\left(1+x^4\right)}{\left(1+2x^2+2x^4\right)^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{\left(1+2x+2x^2\right)^3}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1}{16}\text{Subst}\left(\int \frac{16}{\left(1+2x+2x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \text{Subst}\left(\int \frac{1}{\left(1+2x+2x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} + \text{Subst}\left(\int \frac{1}{1+2x+2x^2}\,dx, x, x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2} + \frac{1+2x^2}{2\left(1+2x^2+2x^4\right)} + \tan^{-1}\left(1+2x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.75

$$
\frac{11+36x^2+48x^4+32x^6}{16\left(1+2x^2+2x^4\right)^2} + \tan^{-1}\left(1+2x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*x
^2]
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.69

method	result	size
default	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{\left(2x^4+2x^2+1\right)^2}+\arctan\left(2x^2+1\right)$	41
risch	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{\left(2x^4+2x^2+1\right)^2}+\arctan\left(2x^2+1\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x)/(2*x^4+2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) +
2*integrate(x/(2*x^4 + 2*x^2 + 1), x)
```

Fricas [A]
time = 0.39, size = 75, normalized size = 1.27

$$\frac{32\,x^6+48\,x^4+36\,x^2+16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)\arctan\left(2\,x^2+1\right)+11}{16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*arc
tan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)
```

Sympy [A]
time = 0.07, size = 46, normalized size = 0.78

$$\frac{32x^6+48x^4+36x^2+11}{64x^8+128x^6+128x^4+64x^2+16}+\operatorname{atan}\left(2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

```
[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**2
 + 16) + atan(2*x**2 + 1)
```

Giac [A]
time = 5.75, size = 42, normalized size = 0.71

$$\frac{32\,x^6+48\,x^4+36\,x^2+11}{16\,(2\,x^4+2\,x^2+1)^2}+\arctan\left(2\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2 +
 1)
```

Mupad [B]
time = 0.05, size = 47, normalized size = 0.80

$$\operatorname{atan}\left(2\,x^2+1\right)+\frac{\frac{x^6}{2}+\frac{3\,x^4}{4}+\frac{9\,x^2}{16}+\frac{11}{64}}{x^8+2\,x^6+2\,x^4+x^2+\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)
```

```
[Out] atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 + 2
*x^6 + x^8 + 1/4)
```

3.337 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$

Optimal. Leaf size=209

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\,\sqrt{f}\,\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\,\sqrt{f}\,\sqrt{e+\sqrt{e^2-4df}}}-\frac{b\tan\ldots}{}$$

```
[Out] -b*arctanh((2*f*x^2+e)/(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)+1/2*arctan(x*
2^(1/2)*f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2))*(c+(2*a*f-c*e)/(-4*d*f+e^2)^(
1/2))*2^(1/2)/f^(1/2)/(e-(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*f^(
1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2))*(c+(-2*a*f+c*e)/(-4*d*f+e^2)^(1/2))*2^(1
/2)/f^(1/2)/(e+(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1687, 1180, 211, 12, 1121, 632, 212}

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\,\sqrt{f}\,\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\,\sqrt{f}\,\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c +
(c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e
^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(
e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \int \frac{bx}{d+ex^2+fx^4}\,dx + \int \frac{a+cx^2}{d+ex^2+fx^4}\,dx$$
$$= b\int \frac{x}{d+ex^2+fx^4}\,dx + \frac{1}{2}\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{\frac{e}{2}-\frac{1}{2}\sqrt{e^2-4df}+fx^2}\,dx + \frac{1}{2}\Bigg(c$$
$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\Bigg(}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2}}}$$
$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\Bigg(}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2}}}$$
$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\Bigg(}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2}}}$$

Mathematica [A]
time = 0.17, size = 234, normalized size = 1.12

$$\frac{\frac{\sqrt{2}\left(2af+c\left(-e+\sqrt{e^2-4df}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\sqrt{2}\left(-2af+c\left(e+\sqrt{e^2-4df}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}}+b\log\left(-e+\sqrt{e^2-4df}-2fx^2\right)-b\log\left(e+\sqrt{e^2-4df}+2fx^2\right)}{2\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/S
qrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqrt[
2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e +
 Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sq
rt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(2*Sqr
t[e^2 - 4*d*f])
```

Maple [A]
time = 0.06, size = 240, normalized size = 1.15

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(f\_Z^4+e\_Z^2+d\right)}\frac{\left(c\_R^2+\_Rb+a\right)\ln\left(x-\_R\right)}{2\_R^3f+\_Re}\right)}{2}$
default	$4f\left(-\frac{\sqrt{-4df+e^2}\left(-\frac{b\ln\left(2f\,x^2+\sqrt{-4df+e^2}+e\right)}{2}+\frac{\left(c\sqrt{-4df+e^2}-2fa+ce\right)\sqrt{2}\,\arctan\left(\frac{fx\sqrt{2}}{\sqrt{\left(e+\sqrt{-4df}\right.}}\right.}{2\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right.}{4f\left(4df-e^2\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 4*f*(-1/4*(-4*d*f+e^2)^(1/2)/f/(4*d*f-e^2)*(-1/2*b*ln(2*f*x^2+(-4*d*f+e^2)^
(1/2)+e)+1/2*(c*(-4*d*f+e^2)^(1/2)-2*f*a+c*e)*2^(1/2)/((e+(-4*d*f+e^2)^(1/2
))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)))-1/4*(-4*d
*f+e^2)^(1/2)/f/(4*d*f-e^2)*(1/2*b*ln(-2*f*x^2+(-4*d*f+e^2)^(1/2)-e)+1/2*(-
c*(-4*d*f+e^2)^(1/2)-2*f*a+c*e)*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*a
rctanh(f*x*2^(1/2)/((-e+(-4*d*f+e^2)^(1/2))*f)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/(f*x^4 + x^2*e + d), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 28.71, size = 578003, normalized size = 2765.56

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(1/4)^(2/3)*(-I*sqrt(3) + 1)*(3*(4*(e^2 - 4*d*f)^(3/2)*a*c*d*f - (
e^4*f - 8*d*e^2*f^2 + 16*d^2*f^3 + (e^2 - 4*d*f)^(3/2)*e*f)*a^2 + 2*(4*sqrt
(e^2 - 4*d*f)*d^2*f^2 - (sqrt(e^2 - 4*d*f)*e^2*f - 2*(e^2 - 4*d*f)^(3/2)*f)
*d)*b^2 - (8*d^2*e^2*f - 16*d^3*f^2 - (e^4 - (e^2 - 4*d*f)^(3/2)*e)*d)*c^2
- 4*(sqrt(1/2)*d*e^4*f*sqrt(-(c^2*d*e - (4*a*c*d - a^2*e)*f + (c^2*d - a^2*
f)*sqrt(e^2 - 4*d*f))/(d*e^2*f - 4*d^2*f^2)) - 8*sqrt(1/2)*d^2*e^2*f^2*sqrt
(-(c^2*d*e - (4*a*c*d - a^2*e)*f + (c^2*d - a^2*f)*sqrt(e^2 - 4*d*f))/(d*e^
2*f - 4*d^2*f^2)) + 16*sqrt(1/2)*d^3*f^3*sqrt(-(c^2*d*e - (4*a*c*d - a^2*e)
*f + (c^2*d - a^2*f)*sqrt(e^2 - 4*d*f))/(d*e^2*f - 4*d^2*f^2)))*b)/((e^2 -
4*d*f)^(3/2)*d*e^2*f - 4*(e^2 - 4*d*f)^(3/2)*d^2*f^2) + 2*(sqrt(1/2)*(e^2 -
 4*d*f)^(3/2)*sqrt(-(c^2*d*e - (4*a*c*d - a^2*e)*f + (c^2*d - a^2*f)*sqrt(e
^2 - 4*d*f))/(d*e^2*f - 4*d^2*f^2)) + (e^2 - 4*d*f)*b)^2/(e^2 - 4*d*f)^3)/(
27*(sqrt(1/2)*(e^2 - 4*d*f)^(3/2)*d*e^2*f*(-(c^2*d*e - (4*a*c*d - a^2*e)*f
+ (c^2*d - a^2*f)*sqrt(e^ ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(172) = 344.
time = 6.29, size = 1589, normalized size = 7.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
```

```
[Out] -1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*sqrt(-4*d*f + e^2)*b*log(x^2 + 1/2
*(sqrt(-4*d*f + e^2) + e)/f)/((16*d^2*f^2 - 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2
 + f^2*e^2 - 2*f*e^3 + e^4)*f^2) - 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*
sqrt(-4*d*f + e^2)*b*log(x^2 - 1/2*(sqrt(-4*d*f + e^2) - e)/f)/((16*d^2*f^2
 - 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2 + f^2*e^2 - 2*f*e^3 + e^4)*f^2) + 1/4*((
16*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e +
sqrt(-4*d*f + e^2)*f)*d*f^3 - 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*f
 + e^2)*f)*d*f^2*e - 8*d*f^3*e + 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sq
rt(-4*d*f + e^2)*f)*d*f*e - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d
*f + e^2)*f)*f^2*e + 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e + sqrt(-4*d
*f + e^2)*f)*d*f*e^2 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*f^2*e^2 + 1
6*d*f^2*e^2 + 2*(4*d*f - e^2)*f^2*e + 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
```

```
 + sqrt(-4*d*f + e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)
*f*e^3 + 2*f^2*e^3 - 2*(4*d*f - e^2)*f*e^2 - sqrt(2)*sqrt(-4*d*f + e^2)*sqr
t(f*e + sqrt(-4*d*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f
)*e^4 - 2*f*e^4)*a + 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e +
 sqrt(-4*d*f + e^2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-
4*d*f + e^2)*f)*d*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*f
 + e^2)*f)*d*f*e - 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*
f + e^2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sq
rt((sqrt(-4*d*f + e^2) + e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*
d^2*f*e^2 + d*f^2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f)) + 1/4*((16*sqrt(2)*sqrt(
f*e - sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^
2)*f)*d*f^3 + 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f^2
*e - 8*d*f^3*e + 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)
*f)*d*f*e - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2
*e - 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f
*e^2 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2*e^2 - 16*d*f^2*e^2 + 2*
(4*d*f - e^2)*f^2*e + 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f +
 e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f*e^3 + 2*f^2*e
^3 + 2*(4*d*f - e^2)*f*e^2 - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*
d*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*e^4 + 2*f*e^4)
*a + 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e
^2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*
d*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f*e
 - 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*
e - sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqrt(-(sqrt(-4*d*f
 + e^2) - e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d^2*f*e^2 + d*f
^2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f))
```

Mupad [**B**]
time = 3.44, size = 2500, normalized size = 11.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x)
```

```
[Out] symsum(log(a*b^2*f^2 - a^2*c*f^2 + b^3*f^2*x - c^3*d*f - 8*root(16*d*e^4*f*
z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d
^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 +
32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z +
 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a
^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*
f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*e^3*f^2*x + a*c^2*e*f - 16*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
```

^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*d*f^3 - 4*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a^2*f^3*x + 4*root(16
*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 -
 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f
^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*
d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*
d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f
 + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*e^2*f^2 + 16*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*d*f^3*x + 2*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b^2*e*f^2*x + 4*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*d*f^2*x - 2*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*e^2*f*x + 32*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e

```
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*d*e*f^3*x - 4*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*e^2*f^2*x + 4*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^...
```

3.338 $\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$

Optimal. Leaf size=224

$$\frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{2de}{}$$

```
[Out] -2*d*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arcta
n(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(-b*e^2+2*c*d^2)/(-4
*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e^2+(b*e^2-2*c*d^2)/(-4*a*c+
b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1687, 1180, 211, 12, 1121, 632, 212}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
(2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx &= \int \frac{2dex}{a+bx^2+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+bx^2+cx^4}\,dx \\
&= (2de)\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\Big(\\
&= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-}}} \\
&= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-}}} \\
&= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-}}}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 245, normalized size = 1.09

$$
\frac{\frac{\sqrt{2}\left(2cd^2+\left(-b+\sqrt{b^2-4ac}\right)e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(-2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + 2de\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right) - 2de\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{2\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e
*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*c] +
2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.19, size = 253, normalized size = 1.13

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(de\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)+\frac{\left(-\sqrt{-4ac+b^2}\,e^2+e^2b-2cd^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{}{\sqrt{(-b+}}\right.}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{4c(4ac-b^2)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4*(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(d*e*ln(-b-2*c*x^2+(-4*a*c+b^2)^
(1/2))+1/2*(-(-4*a*c+b^2)^(1/2)*e^2+e^2*b-2*c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1
/4*(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(-d*e*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+
1/2*((-4*a*c+b^2)^(1/2)*e^2+e^2*b-2*c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 113.36, size = 540080, normalized size = 2411.07

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/144*((-I*sqrt(3) + 1)*(3*(b^4*c*d^4 + (b^2 - 4*a*c)^(3/2)*b*c*d^4 - 16*(c
^2*e^4 - 8*sqrt(1/2)*c^3*d*e*sqrt(-(b*c*d^4 - 4*a*c*d^2*e^2 + a*b*e^4 - (c*
d^4 - a*e^4)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)))*a^3 + 8*(2*c^3*d^4
- 4*sqrt(b^2 - 4*a*c)*c^2*d^2*e^2 + (c*e^4 - 8*sqrt(1/2)*c^2*d*e*sqrt(-(b*c
*d^4 - 4*a*c*d^2*e^2 + a*b*e^4 - (c*d^4 - a*e^4)*sqrt(b^2 - 4*a*c))/(a*b^2*
c - 4*a^2*c^2)))*b^2)*a^2 - (20*(b^2 - 4*a*c)^(3/2)*c*d^2*e^2 - (b^2 - 4*a*
c)^(3/2)*b*e^4 + (e^4 - 8*sqrt(1/2)*c*d*e*sqrt(-(b*c*d^4 - 4*a*c*d^2*e^2 +
a*b*e^4 - (c*d^4 - a*e^4)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)))*b^4 +
8*(c^2*d^4 - sqrt(b^2 - 4*a*c)*c*d^2*e^2)*b^2)*a)/((b^2 - 4*a*c)^(3/2)*a*b^
2*c - 4*(b^2 - 4*a*c)^(3/2)*a^2*c^2) - 2*(2*b^2*d*e - 8*a*c*d*e + sqrt(1/2)
*(b^2 - 4*a*c)^(3/2)*sqrt(-(b*c*d^4 - 4*a*c*d^2*e^2 + a*b*e^4 - (c*d^4 - a*
e^4)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)))^2/(b^2 - 4*a*c)^3)/(-1/32*(
2*b^3*c*d^5*e - 6*sqrt(b^2 - 4*a*c)*b^2*c*d^5*e + 8*(b^2 - 4*a*c)^(3/2)*c*d
^5*e - 2*sqrt(1/2)*(b^2 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1625 vs. 2(184) = 368.
time = 6.13, size = 1625, normalized size = 7.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*d*e*log(x^2 + 1/2*(b
 + sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c
^2 + b^2*c^2 - 4*a*c^3)*c^2) + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2
 - 4*a*c)*d*e*log(x^2 + 1/2*(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c -
2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c +
16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
```

```
c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*
(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 +
2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(
b^2 - 4*a*c)*a*c^2)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/
c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^
2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c
)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*arctan(2*sq
rt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B]
time = 3.22, size = 3046, normalized size = 13.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^
2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*
a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z
^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*
z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 +
 a^2*e^8, z, k)^3*b^3*c^2*x + 4*c^2*d^3*e^3*x + 4*root(16*a*b^4*c*z^4 - 128
*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^
4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*
a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a
*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4
```

+ c^2*d^8 + a^2*e^8, z, k)^2*b^2*c^2*d^2 + b*c*d^2*e^4 - 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^3*a*b*c^3*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*b^2*c*e^4*x + 2*b*c*d*e^5*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*d*e^3 + 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*b*c^2*d^3*e + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d*e*x + 12*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)

```
*b*c^2*d^2*e^2*x - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^
3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 +
192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e
*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2
+ 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*
b^2*c^2*d*e*x)*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4
- 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^
2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 3
2*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*
c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k), k, 1, 4
)
```

3.339 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

```
[Out] x/b/d+a^2*ln(b*x+a)/b^2/(-a*d+b*c)-c^2*ln(d*x+c)/d^2/(-a*d+b*c)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {84}

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)(c+dx)}\,dx &= \int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx\\ &= \frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.00

$$\frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Maple [A]
time = 0.22, size = 57, normalized size = 1.02

method	result	size
default	$\frac{x}{bd}+\frac{c^2\ln(dx+c)}{d^2(ad-bc)}-\frac{a^2\ln(bx+a)}{b^2(ad-bc)}$	57
norman	$\frac{x}{bd}+\frac{c^2\ln(dx+c)}{d^2(ad-bc)}-\frac{a^2\ln(bx+a)}{b^2(ad-bc)}$	57
risch	$\frac{x}{bd}+\frac{c^2\ln(-dx-c)}{d^2(ad-bc)}-\frac{a^2\ln(bx+a)}{b^2(ad-bc)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)
```

Maxima [A]
time = 0.28, size = 60, normalized size = 1.07

$$\frac{a^2\log(bx+a)}{b^3c-ab^2d}-\frac{c^2\log(dx+c)}{bcd^2-ad^3}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) + x
/(b*d)
```

Fricas [A]
time = 0.38, size = 65, normalized size = 1.16

$$\frac{a^2d^2\log(bx+a)-b^2c^2\log(dx+c)+(b^2cd-abd^2)x}{b^3cd^2-ab^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3*
c*d^2 - a*b^2*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(44) = 88$.
time = 0.61, size = 190, normalized size = 3.39

$$-\frac{a^2\log\left(x+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{b^2(ad-bc)}+\frac{c^2\log\left(x+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2(ad-bc)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] -a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**2
*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b**2
*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*a*b
**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b
**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)
```

Giac [A]
time = 4.56, size = 62, normalized size = 1.11

$$\frac{a^2\log\left(|bx+a|\right)}{b^3c-ab^2d}-\frac{c^2\log\left(|dx+c|\right)}{bcd^2-ad^3}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/(b^3*c - a*b^2*d) - c^2*log(abs(d*x + c))/(b*c*d^2 -
a*d^3) + x/(b*d)
```

Mupad [B]
time = 0.21, size = 61, normalized size = 1.09

$$-\frac{a^2\,d^2\,\ln\left(a+b\,x\right)-b^2\,c^2\,\ln\left(c+d\,x\right)-a\,b\,d^2\,x+b^2\,c\,d\,x}{b^2\,d^2\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)*(c + d*x)),x)
```

```
[Out] -(a^2*d^2*log(a + b*x) - b^2*c^2*log(c + d*x) - a*b*d^2*x + b^2*c*d*x)/(b^2
*d^2*(a*d - b*c))
```

3.340 $\int \frac{x^2}{(c+dx)(a+bx^2)}\,dx$

Optimal. Leaf size=96

$$-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc^2+ad^2)}+\frac{c^2\log(c+dx)}{d\,(bc^2+ad^2)}+\frac{ad\log(a+bx^2)}{2b\,(bc^2+ad^2)}$$

```
[Out] c^2*ln(d*x+c)/d/(a*d^2+b*c^2)+1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-c*arctan(
x*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^2+b*c^2)/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1643, 649, 211, 266}

$$-\frac{\sqrt{a}\,c\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(ad^2+bc^2)}+\frac{ad\log(a+bx^2)}{2b\,(ad^2+bc^2)}+\frac{c^2\log(c+dx)}{d\,(ad^2+bc^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^2
*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a*d
^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(c+dx)\left(a+bx^2\right)}\,dx &= \int \left(\frac{c^2}{\left(bc^2+ad^2\right)(c+dx)}-\frac{a(c-dx)}{\left(bc^2+ad^2\right)\left(a+bx^2\right)}\right)dx\\
&= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}-\frac{a\int \frac{c-dx}{a+bx^2}\,dx}{bc^2+ad^2}\\
&= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}-\frac{(ac)\int \frac{1}{a+bx^2}\,dx}{bc^2+ad^2}+\frac{(ad)\int \frac{x}{a+bx^2}\,dx}{bc^2+ad^2}\\
&= -\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\left(bc^2+ad^2\right)}+\frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)}+\frac{ad\log\left(a+bx^2\right)}{2b\left(bc^2+ad^2\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 0.76

$$\frac{-2\sqrt{a}\,\sqrt{b}\,cd\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+2bc^2\log(c+dx)+ad^2\log\left(a+bx^2\right)}{2b^2c^2d+2abd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x]
+ a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

Maple [A]
time = 0.22, size = 75, normalized size = 0.78

method	result
default	$\frac{c^2\ln(dx+c)}{d(a\,d^2+b\,c^2)}-\frac{a\left(-\frac{d\ln\left(b\,x^2+a\right)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a\,d^2+b\,c^2}$
risch	$\frac{\ln\left(\left(-3a^2bc\,d^3+5a\,b^2c^3d+\sqrt{-ab}\,a^2d^4-5\sqrt{-ab}\,ab\,c^2d^2+2\sqrt{-ab}\,b^2c^4\right)x-5a^2b\,c^2d^2+3\sqrt{-ab}\,a^2c\,d^3-5\sqrt{-ab}\,ab\,c^3d+\right.}{2(a\,d^2+b\,c^2)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] c^2*ln(d*x+c)/d/(a*d^2+b*c^2)-a/(a*d^2+b*c^2)*(-1/2*d*ln(b*x^2+a)/b+c/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [**A**]
time = 0.50, size = 84, normalized size = 0.88

$$\frac{ad\log\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)}+\frac{c^2\log\left(dx+c\right)}{bc^2d+ad^3}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bc^2+ad^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(d*x + c)/(b*c^2*d + a*
d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

Fricas [**A**]
time = 0.46, size = 162, normalized size = 1.69

$$\left[\frac{bcd\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+ad^2\log\left(bx^2+a\right)+2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)},-\frac{2\,bcd\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-ad^2\log\left(bx^2+a\right)-2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a*
d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2*b
*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*log
(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

[Out] Timed out

Giac [A]
time = 4.50, size = 85, normalized size = 0.89

$$\frac{ad\log\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)}+\frac{c^2\log\left(|dx+c|\right)}{bc^2d+ad^3}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bc^2+ad^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*d
 + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

Mupad [B]
time = 1.13, size = 347, normalized size = 3.61

$$\frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\,\left(x\,\left(2\,b^2\,c^2-5\,a\,b\,d^2\right)-5\,a\,b\,c\,d+\frac{2\,b^2\,d\,\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)\,\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}-\frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\,\left(b\,x\,\left(5\,a\,d^2-2\,b\,c^2\right)+5\,a\,b\,c\,d+\frac{d\,\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\,\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{b\,c^2+a\,d^2}\right)}{2\,b^2\,\left(b\,c^2+a\,d^2\right)}\right)\,\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)}{2\,\left(b^3\,c^2+a\,b^2\,d^2\right)}+\frac{c^2\,\ln\left(c+d\,x\right)}{b\,c^2\,d+a\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x)),x)
```

```
[Out] (log(a*c + a*d*x + ((c*(-a*b^3)^(1/2) + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2) -
 5*a*b*c*d + (2*b^2*d*(c*(-a*b^3)^(1/2) + a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c
^2*x))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2))*(c*(-a*b^3)^(
1/2) + a*b*d))/(2*b^3*c^2 + 2*a*b^2*d^2) - (log(a*c + a*d*x + ((c*(-a*b^3)^
(1/2) - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-a*b^3)^(1/2)
- a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x))/(a*d^2 + b*c^2)))/(2*b^2*(a*d^2 +
 b*c^2)))*(c*(-a*b^3)^(1/2) - a*b*d))/(2*(b^3*c^2 + a*b^2*d^2)) + (c^2*log(
c + d*x))/(a*d^3 + b*c^2*d)
```

3.341 $\int \frac{x^2}{(c+dx)(a+bx^3)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt[3]{a}\,d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\,\sqrt[3]{b}\,cd+a^{2/3}d^2\right)}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}-\frac{\sqrt[3]{a}\,c}{}$$

```
[Out] 1/3*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d^3+b
*c^3)-c^2*ln(d*x+c)/(-a*d^3+b*c^3)-1/6*a^(1/3)*d*(b^(1/3)*c+a^(1/3)*d)*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/(-a*d^3+b*c^3)+1/3*c^2*ln(b*x
^3+a)/(-a*d^3+b*c^3)-1/3*a^(1/3)*d*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)
*3^(1/2))/b^(2/3)/(b^(2/3)*c^2+a^(1/3)*b^(1/3)*c*d+a^(2/3)*d^2)*3^(1/2)
```

Rubi [A]

time = 0.34, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$-\frac{\sqrt[3]{a}\,d\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\,\sqrt[3]{b}\,cd+b^{2/3}c^2\right)}-\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^
(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b^(
1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)) -
 (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) +
 (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx = \int \left(-\frac{c^2 d}{\left(bc^3-ad^3\right)(c+dx)} + \frac{acd-ad^2x+bc^2x^2}{\left(bc^3-ad^3\right)\left(a+bx^3\right)}\right)\,dx$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{\int \frac{acd-ad^2x+bc^2x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{\int \frac{acd-ad^2x}{a+bx^3}\,dx}{bc^3-ad^3} + \frac{\left(bc^2\right)\int \frac{x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)} + \frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}\,cd-a^{4/3}d^2\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}\,cd-a^{4/3}d^2 \cdots\right.}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}}{3a^{2/3}\sqrt[3]{b}\left(bc^3-ad^3\right)}$$

$$= \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} + \frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)} + \cdots$$

$$= \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}\left(bc^3-ad^3\right)} - \frac{c^2\log(c+dx)}{bc^3-ad^3} - \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,c+\sqrt[3]{a} \cdots\right.}{\cdots}$$

$$= -\frac{\sqrt[3]{a}\,d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}\,cd+a^{2/3}d^2\right)} + \frac{\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{3b^{2/3}\left(bc^3-ad^3\right)}$$

Mathematica [A]
time = 0.07, size = 228, normalized size = 0.86

$$\frac{2\sqrt{3}\sqrt[3]{a}\,d\left(-\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}\,d\left(\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-6b^{2/3}c^2\log(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\,cd\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-a^{2/3}d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)}{6b^{2/3}\left(bc^3-ad^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3
)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^3))
```

Maple [A]
time = 0.23, size = 245, normalized size = 0.93

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a\,b^2d^3-b^3c^3\right)\_Z^3+3b^2c^2\_Z^2-3bc\_Z\right)}\_R\ln\left(\left(\left(\left(-4a\,b^2d^4-2b^3c^3d\right)\_R^3-3\_R^2b^2c^2d+8\_Rbcd-3d\right)x+\left(-5a\,b^2\right.\right.\right.\right.}{3}$
default	$\frac{-acd\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+a\,d^2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{d^3a-b\,c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*c*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1)))+a*d^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)
*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*c^2*ln(b*x^3+a))/(a*d^3-b*c^3)+c^2/(a*d^3-b*
c^3)*ln(d*x+c)
```

Maxima [A]
time = 0.49, size = 279, normalized size = 1.06

$$-\frac{c^2\log(dx+c)}{bc^3-ad^3}-\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(2bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-acd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -c^2*log(d*x + c)/(b*c^3 - a*d^3) - 1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) - a*c*d*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^3*
(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3))*(a/b)^(1/3)) + 1/6*(2*b*c^2*(a/b)^(2/3)
- a*d^2*(a/b)^(1/3) - a*c*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^
3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)
^(1/3) + a*c*d)*log(x + (a/b)^(1/3))/(b^2*c^3*(a/b)^(2/3) - a*b*d^3*(a/b)^(
2/3))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.28, size = 5975, normalized size = 22.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(b*c^3 - a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3
 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 -
a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*
b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 -
a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*
b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*log(-3/2*(2*(1/2)^
(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c
^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/
((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c
^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/
((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
2*c^2/(b*c^3 - a*d^3))*b*c^2 - 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/
(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 -
 a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a
*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 -
 a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a
*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^
3 - a*d^3))^2 + d*x - 2*c) + 12*c^2*log(d*x + c) - ((b*c^3 - a*d^3)*(2*(1/2
)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2
*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^
3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2
*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^
3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1)
- 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 - 3*sqrt(1/3)*(b*c^3 - a*d^3)*sqrt(-(4*b*c
^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2/3)*(c
^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
- a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
- a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b
*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a
*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3
- 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^
2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3
- 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^
2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)
))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)))*log(3/2*(2*(1/2)^(2/3)*(c^4/(b
*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
```

```
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3
- a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^
3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3
*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2)
+ 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3
*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2)
+ 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2
 + 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2
- c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/
((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(
b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/
((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(
b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*sqrt(-
(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2
/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6
/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((
b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6
/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((
b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*
c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c
^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d
^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3
)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d
^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3
)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 -
a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.47, size = 320, normalized size = 1.21

$$-\frac{c^2 d \log\left(|dx+c|\right)}{bc^3d-ad^4}+\frac{c^2\log\left(|bx^3+a|\right)}{3\left(bc^3-ad^3\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}} d \arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\, b^2c^2-\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}} bcd+\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}} d^2}+\frac{\left(ab^2c^3d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2bd^5\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^2c^4d+a^2bcd^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^3c^6-2\,a^2b^2c^3d^3+a^3bd^6\right)}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}} bcd-\left(-ab^2\right)^{\frac{2}{3}} d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^3c^3-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -c^2*d*log(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*log(abs(b*x^3 + a))/(b
*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)^(1/3)*b*c*d + sqrt(3)*(-a*b
^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/3) -
 a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3*
c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-a*b^2)
^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^2*d^3)
```

Mupad [B]
time = 2.50, size = 570, normalized size = 2.16

$$\left(\sum_{k=1}^{3}\ln\left(-a\,b\,d\,\left(c+d\,x+R^2\,b^2\,c^3\,3+R^3\,b^3\,c^4\,9-R\,b\,c^2\,5-R^2\,a\,b\,d^3\,3-R\,b\,c\,d\,x\,8+R^3\,a\,b^2\,c\,d^3\,45+R^3\,a\,b^2\,d^4\,x\,36+R^2\,b^2\,c^2\,d\,x\,9+R^3\,b^3\,c^3\,d\,x\,18\right)\right)\,R\right)+\frac{c^2\,\ln\left(c+d\,x\right)}{a\,d^3-b\,c^3}$$

where $R=\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x)),x)
```

```
[Out] symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27*b
^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a*b^
2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2 - 3*
root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k
)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9
*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d^3*z^3 -
 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*x + 9*roo
t(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2
*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 -
 9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3
+ 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c + d*x))/(a*d^3
 - b*c^3)
```

3.342 $\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx$

Optimal. Leaf size=417

$$\frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)}-\frac{c\left(\sqrt{b}\,c^2-\sqrt{a}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(bc^4+ad^4)}+\frac{c\left(\sqrt{b}\,c^2-\sqrt{a}\,d^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(bc^4+ad^4)}$$

```
[Out] c^2*d*ln(d*x+c)/(a*d^4+b*c^4)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*c^4)+1/2*d^3*a
rctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(a*d^4+b*c^4)/b^(1/2)+1/4*c*arctan(-1+b^
(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+
b*c^4)*2^(1/2)+1/4*c*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d^2*a^(1/2)+b^(1
/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)+1/8*c*ln(-a^(1/4)*b^(1/4)*x*
2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d
^4+b*c^4)*2^(1/2)-1/8*c*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(
d^2*a^(1/2)+b^(1/2)*c^2)/a^(1/4)/b^(1/4)/(a*d^4+b*c^4)*2^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6857, 1475, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$\frac{\sqrt{a}\,d^3\text{ArcTan}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(ad^4+bc^4)}-\frac{c\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,c^2-\sqrt{a}\,d^2\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(ad^4+bc^4)}+\frac{c\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}\,c^2-\sqrt{a}\,d^2\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(ad^4+bc^4)}-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}+\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(ad^4+bc^4)}-\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(ad^4+bc^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) - (
c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 +
a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*
d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c
^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1475

```
Int[((A_) + (B_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*
(x_)^(n2_))^(p_.), x_Symbol] :> Dist[A, Int[(d + e*x^n)^q*(a + c*x^(2*n))^p
, x], x] + Dist[B, Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ
[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{(c+dx)(a+bx^4)}\,dx = \int \left(\frac{c^2d^2}{(bc^4+ad^4)(c+dx)} + \frac{(c-dx)(-ad^2+bc^2x^2)}{(bc^4+ad^4)(a+bx^4)}\right) dx$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\int \frac{(c-dx)(-ad^2+bc^2x^2)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{c\int \frac{-ad^2+bc^2x^2}{a+bx^4}\,dx}{bc^4+ad^4} - \frac{d\int \frac{x(-ad^2+bc^2x^2)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{d\text{Subst}\left(\int \frac{-ad^2+bc^2x}{a+bx^2}\,dx, x, x^2\right)}{2(bc^4+ad^4)} + \frac{\left(c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\right)\int \frac{\sqrt{a}\,\sqrt{}}{a+b}}{2(bc^4+ad^4)}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{(bc^2d)\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, x^2\right)}{2(bc^4+ad^4)} + \frac{(ad^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x\right.}{2(bc^4+ad^4)}$$

$$= \frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}\,c\left(c^2+\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}-\sqrt{2}\right.}{4\sqrt{2}\,\sqrt[4]{a}\,(bc^4+a}$$

$$= \frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc^4+ad^4)} - \frac{\sqrt[4]{b}\,c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc^4+ad^4)} + \frac{\sqrt[4]{b}\,c\left(\right.}{}$$

Mathematica [A]
time = 0.17, size = 370, normalized size = 0.89

$$\frac{-2\left(\sqrt{2}\,b^{3/4}c^3-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{b}\,cd^2+2a^{3/4}d^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\,b^{3/4}c^3-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{b}\,cd^2-2a^{3/4}d^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\sqrt[4]{b}\,c\left(8\sqrt[4]{a}\,\sqrt[4]{b}\,cd\log(c+dx)+\sqrt{2}\left(\sqrt{b}\,c^2+\sqrt{a}\,d^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\sqrt{2}\,\sqrt{b}\,c^2\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\sqrt{2}\,\sqrt{a}\,d^2\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-2\sqrt[4]{a}\,\sqrt[4]{b}\,cd\log\left(a+bx^4\right)\right)}{8\sqrt[4]{a}\,\sqrt{b}\,(bc^4+ad^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*S
qrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 +
 Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt
[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sq
rt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b*c^4 + a*d^4))
```

Maple [A]
time = 0.23, size = 281, normalized size = 0.67

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^{2}b^{2}d^{4}+a\,b^{3}c^{4}\right)\_Z^{4}+4a\,b^{2}c^{2}d\,\_Z^{3}+2a\,\_Z^{2}d^{2}b\right)}\_R\ln\left(\left(\left(5a^{2}b^{2}d^{6}-3a\,b^{3}c^{4}d^{2}\right)\_R^{4}+10\,\_R^{3}a\,b^{2}c^{2}d^{3}+\left(9ab\,d^{4}+b^{2}\right.\right.\right.}{4}$
default	$\frac{-\frac{c\,d^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d^{3}a\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{c^{3}\sqrt{2}\left(\ln\right.}{}}{d^{4}a+b\,c^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d^4+b*c^4)*(-1/8*c*d^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1
/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/2*d^3*a/(a*b)^(1/2)*a
rctan(x^2*(b/a)^(1/2))+1/8*c^3/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/4*c^2*d*ln(b*x^4+a
))+c^2*d*ln(d*x+c)/(a*d^4+b*c^4)
```

Maxima [A]
time = 0.51, size = 349, normalized size = 0.84

$$\frac{c^{2}d\log\left(dx+c\right)}{bc^{4}+ad^{4}}-\frac{\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c^{2}d+\sqrt{a}\,b^{\frac{3}{2}}c^{3}+abcd^{2}\right)\log\left(\sqrt{b}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{5}{4}}c^{2}d-\sqrt{a}\,b^{\frac{3}{2}}c^{3}-abcd^{2}\right)\log\left(\sqrt{b}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}c^{3}-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}cd^{2}-2\,a^{\frac{3}{2}}bd^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}\,a^{\frac{3}{4}}b^{\frac{7}{4}}c^{3}-\sqrt{2}\,a^{\frac{5}{4}}b^{\frac{5}{4}}cd^{2}+2\,a^{\frac{3}{2}}bd^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8\left(bc^{4}+ad^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] c^2*d*log(d*x + c)/(b*c^4 + a*d^4) - 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*
c^2*d + sqrt(a)*b^(3/2)*c^3 + a*b*c*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*
b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c
^2*d - sqrt(a)*b^(3/2)*c^3 - a*b*c*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b
^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sq
rt(2)*a^(5/4)*b^(5/4)*c*d^2 - 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(b))*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sqrt(2)*a^(5/4)*b^(5/
4)*c*d^2 + 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))
/(b*c^4 + a*d^4)
```

Fricas [C] Result contains complex when optimal does not.
time = 27.98, size = 259898, normalized size = 623.26

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/48*(48*c^2*d*log(d*x + c) - 12*(b*c^4 + a*d^4)*(d^3*sqrt(-a/b)/(b*c^4 + a
*d^4) + c^2*d/(b*c^4 + a*d^4) + sqrt(2*c^4*d^2/(b*c^4 + a*d^4)^2 + (2*c^6*d
^3/(b*c^4 + a*d^4)^3 - c^2*d^3/((b^2*c^4 + a*b*d^4)*(b*c^4 + a*d^4)))*(b*c^
4 + a*d^4)/(d^3*sqrt(-a/b))))*log(-2*(d^3*sqrt(-a/b)/(b*c^4 + a*d^4) + c^2*
d/(b*c^4 + a*d^4) + sqrt(2*c^4*d^2/(b*c^4 + a*d^4)^2 + (2*c^6*d^3/(b*c^4 +
a*d^4)^3 - c^2*d^3/((b^2*c^4 + a*b*d^4)*(b*c^4 + a*d^4)))*(b*c^4 + a*d^4)/(
d^3*sqrt(-a/b))))*a^2*d^6 - a*c^2*d^3 + (a*b^3*c^8 - a^2*b^2*c^4*d^4 - 2*a^
3*b*d^8)*(d^3*sqrt(-a/b)/(b*c^4 + a*d^4) + c^2*d/(b*c^4 + a*d^4) + sqrt(2*c
^4*d^2/(b*c^4 + a*d^4)^2 + (2*c^6*d^3/(b*c^4 + a*d^4)^3 - c^2*d^3/((b^2*c^4
 + a*b*d^4)*(b*c^4 + a*d^4)))*(b*c^4 + a*d^4)/(d^3*sqrt(-a/b))))^3 - 3*(a*b
^2*c^6*d - 3*a^2*b*c^2*d^5)*(d^3*sqrt(-a/b)/(b*c^4 + a*d^4) + c^2*d/(b*c^4
+ a*d^4) + sqrt(2*c^4*d^2/(b*c^4 + a*d^4)^2 + (2*c^6*d^3/(b*c^4 + a*d^4)^3
- c^2*d^3/((b^2*c^4 + a*b*d^4)*(b*c^4 + a*d^4)))*(b*c^4 + a*d^4)/(d^3*sqrt(
-a/b))))^2 - (b*c^5 - 4*a ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.79, size = 401, normalized size = 0.96

$$\frac{c^2d^2\log\left(|dx+c|\right)}{bc^4d+ad^5}-\frac{c^2d\log\left(|bx^4+a|\right)}{4\left(bc^4+ad^4\right)}+\frac{\left(\sqrt{2}\sqrt{ab}\,bd+\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^2d^2+\sqrt{2}\sqrt{ab}\,b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}cd\right)}+\frac{\left(\sqrt{2}\sqrt{ab}\,bd+\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^2d^2+\sqrt{2}\sqrt{ab}\,b^2c^2+2\left(ab^3\right)^{\frac{3}{4}}cd\right)}-\frac{\left(\left(ab^3\right)^{\frac{1}{4}}abcd^2+\left(ab^3\right)^{\frac{3}{4}}c^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,ab^3c^4+\sqrt{2}\,a^2b^2d^4\right)}+\frac{\left(\left(ab^3\right)^{\frac{1}{4}}abcd^2+\left(ab^3\right)^{\frac{3}{4}}c^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}\,ab^3c^4+\sqrt{2}\,a^2b^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] c^2*d^2*log(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*log(abs(b*x^4 + a))
/(b*c^4 + a*d^4) + 1/2*(sqrt(2)*sqrt(a*b)*b*d + (a*b^3)^(1/4)*b*c)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^2*d^2 + sq
rt(2)*sqrt(a*b)*b^2*c^2 - 2*(a*b^3)^(3/4)*c*d) + 1/2*(sqrt(2)*sqrt(a*b)*b*d
 + (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(sqrt(2)*a*b^2*d^2 + sqrt(2)*sqrt(a*b)*b^2*c^2 + 2*(a*b^3)^(3/4)*c*d
) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4) + 1/4*((a
*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4)
```

Mupad [B]
time = 2.45, size = 823, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x)),x)
```

```
[Out] symsum(log(a*b^2*d*(c*d + d^2*x - root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*
z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^3 + 4*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^2*b^2*c^4*x + 36*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 25
6*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*d^4*x - 128*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^4*a*b^3*c^5*d - 5*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 2
56*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^2*d*x + 96*root(256*a^2*
b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1,
 z, k)^3*a*b^2*c^3*d^2 + 384*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 +
 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*c*d^5 + 320*root
(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2
*z^2 + 1, z, k)^4*a^2*b^2*d^6*x + 32*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c
^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*c*d^3 + 160*
root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b
*d^2*z^2 + 1, z, k)^3*a*b^2*c^2*d^3*x - 192*root(256*a^2*b^2*d^4*z^4 + 256*
a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a*b^3*c^4
*d^2*x))*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3
 + 32*a*b*d^2*z^2 + 1, z, k), k, 1, 4) + (c^2*d*log(c + d*x))/(a*d^4 + b*c^
4)
```

3.343 $\int \frac{x}{(1-x)(1+x)^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2(1+x)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2/(1+x)+1/2*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {78, 213}

$$\frac{1}{2(x+1)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1-x)(1+x)^2} \, dx &= \int \left(-\frac{1}{2(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}\right) \, dx \\
&= \frac{1}{2(1+x)}-\frac{1}{2}\int \frac{1}{-1+x^2} \, dx \\
&= \frac{1}{2(1+x)}+\frac{1}{2}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.50

$$\frac{1}{4}\left(\frac{2}{1+x}-\log(1-x)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] (2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

Maple [A]
time = 0.20, size = 21, normalized size = 1.31

method	result	size
default	$-\frac{\ln(-1+x)}{4}+\frac{1}{2x+2}+\frac{\ln(1+x)}{4}$	21
norman	$-\frac{\ln(-1+x)}{4}+\frac{1}{2x+2}+\frac{\ln(1+x)}{4}$	21
risch	$-\frac{\ln(-1+x)}{4}+\frac{1}{2x+2}+\frac{\ln(1+x)}{4}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x)/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(-1+x)+1/2/(1+x)+1/4*ln(1+x)
```

Maxima [A]
time = 0.29, size = 20, normalized size = 1.25

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.37, size = 26, normalized size = 1.62

$$\frac{(x+1)\log(x+1)-(x+1)\log(x-1)+2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A]
time = 0.03, size = 19, normalized size = 1.19

$$-\frac{\log{(x-1)}}{4}+\frac{\log{(x+1)}}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)**2,x)
```

```
[Out] -log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

Giac [A]
time = 3.52, size = 21, normalized size = 1.31

$$\frac{1}{2\,(x+1)}-\frac{1}{4}\,\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))
```

Mupad [B]
time = 0.04, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}(x)}{2}+\frac{1}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x - 1)*(x + 1)^2),x)
```

```
[Out] atanh(x)/2 + 1/(2*(x + 1))
```

3.344 $\int \frac{x^2}{(1-x^2)(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$-\frac{x}{4(1+x^2)}+\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -1/4*x/(x^2+1)+1/4*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {482, 212}

$$\frac{1}{4}\tanh^{-1}(x)-\frac{x}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] -1/4*x/(1 + x^2) + ArcTanh[x]/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1-x^2)(1+x^2)^2} \, dx &= -\frac{x}{4(1+x^2)}+\frac{1}{4}\int \frac{1}{1-x^2} \, dx\\ &= -\frac{x}{4(1+x^2)}+\frac{1}{4}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.42

$$\frac{1}{8}\left(-\frac{2x}{1+x^2}-\log(1-x)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

```
[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

Maple [A]
time = 0.20, size = 24, normalized size = 1.26

method	result	size
default	$-\frac{\ln(-1+x)}{8}-\frac{x}{4(x^2+1)}+\frac{\ln(1+x)}{8}$	24
norman	$-\frac{\ln(-1+x)}{8}-\frac{x}{4(x^2+1)}+\frac{\ln(1+x)}{8}$	24
risch	$-\frac{\ln(-1+x)}{8}-\frac{x}{4(x^2+1)}+\frac{\ln(1+x)}{8}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(-1+x)-1/4*x/(x^2+1)+1/8*ln(1+x)
```

Maxima [A]
time = 0.26, size = 23, normalized size = 1.21

$$-\frac{x}{4\,(x^2+1)}+\frac{1}{8}\,\log\left(x+1\right)-\frac{1}{8}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.
time = 0.40, size = 34, normalized size = 1.79

$$\frac{(x^2+1)\log\left(x+1\right)-(x^2+1)\log\left(x-1\right)-2\,x}{8\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 1.05

$$-\frac{x}{4x^2+4}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
[Out] -x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

Giac [A]
time = 4.08, size = 30, normalized size = 1.58

$$-\frac{1}{4\left(x+\frac{1}{x}\right)}+\frac{1}{16}\log\left(\left|x+\frac{1}{x}+2\right|\right)-\frac{1}{16}\log\left(\left|x+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))
```

Mupad [B]
time = 2.14, size = 17, normalized size = 0.89

$$\frac{\operatorname{atanh}(x)}{4}-\frac{x}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(x^2 + 1)^2),x)
[Out] atanh(x)/4 - x/(4*(x^2 + 1))
```

3.345 $\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{6(1+x^3)}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(1+x)+\frac{1}{72}\log\left(1-x+x^2\right)+\frac{1}{24}\log\left(1-\right.$$

```
[Out] -1/6*x/(x^3+1)-1/12*ln(1-x)-1/36*ln(1+x)+1/72*ln(x^2-x+1)+1/24*ln(x^2+x+1)+
1/36*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {482, 536, 206, 31, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{x}{6(x^3+1)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] -1/6*x/(1 + x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x
)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/7
2 + Log[1 + x + x^2]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{x}{6(1+x^3)} + \frac{1}{6}\int \frac{1+2x^3}{(1-x^3)(1+x^3)}\,dx$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\int \frac{1}{1+x^3}\,dx + \frac{1}{4}\int \frac{1}{1-x^3}\,dx$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{36}\int \frac{1}{1+x}\,dx - \frac{1}{36}\int \frac{2-x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{1}{1-x}\,dx + \frac{1}{12}\int$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{24}\int \frac{}{1-}$$
$$= -\frac{x}{6(1+x^3)} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log(1$$
$$= -\frac{x}{6(1+x^3)} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x)$$

Mathematica [A]
time = 0.04, size = 85, normalized size = 0.88

$$\frac{1}{72}\left(-\frac{12x}{1+x^3} - 2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right) + 6\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - 6\log(1-x) - 2\log(1+x) + \log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] ((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcTa
n[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3*L
og[1 + x + x^2])/72
```

Maple [A]
time = 0.21, size = 90, normalized size = 0.93

method	result
risch	$-\frac{x}{6(x^3+1)} + \frac{\ln(4x^2-4x+4)}{72} - \frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36} - \frac{\ln(-1+x)}{12} + \frac{\ln(x^2+x+1)}{24} + \frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$\frac{\ln(x^2+x+1)}{24} + \frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{\ln(-1+x)}{12} + \frac{1}{18+18x} - \frac{\ln(1+x)}{36} + \frac{-2x-2}{36x^2-36x+36} + \frac{\ln(x^2-x+1)}{72} - \sqrt{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*ln(x^2+x+1)+1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/12*ln(-1+x)+1/1
8/(1+x)-1/36*ln(1+x)+1/36*(-2*x-2)/(x^2-x+1)+1/72*ln(x^2-x+1)-1/36*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 75, normalized size = 0.77

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6(x^3+1)}+\frac{1}{24}\log(x^2+x+1)+\frac{1}{72}\log(x^2-x+1)-\frac{1}{36}\log(x+1)-\frac{1}{12}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

Fricas [A]
time = 0.40, size = 106, normalized size = 1.09

$$\frac{6\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3(x^3+1)\log(x^2+x+1)+(x^3+1)\log(x^2-x+1)-2(x^3+1)\log(x+1)-6(x^3+1)\log(x-1)-12x}{72(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 +
1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 + 1)
*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12*x)
/(x^3 + 1)
```

Sympy [A]
time = 0.17, size = 92, normalized size = 0.95

$$-\frac{x}{6x^3+6}-\frac{\log(x-1)}{12}-\frac{\log(x+1)}{36}+\frac{\log(x^2-x+1)}{72}+\frac{\log(x^2+x+1)}{24}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

```
[Out] -x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + lo
g(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)*a
tan(2*sqrt(3)*x/3 + sqrt(3)/3)/12
```

Giac [A]
time = 4.23, size = 77, normalized size = 0.79

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x}{6(x^3+1)}+\frac{1}{24}\log(x^2+x+1)+\frac{1}{72}\log(x^2-x+1)-\frac{1}{36}\log(|x+1|)-\frac{1}{12}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))
```

Mupad [B]
time = 0.18, size = 103, normalized size = 1.06

$$-\frac{\ln\left(x-1\right)}{12}-\frac{\ln\left(x+1\right)}{36}-\frac{x}{6\,\left(x^3+1\right)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{1}{24}+\frac{\sqrt{3}\,1{}\mathrm{i}}{24}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{24}+\frac{\sqrt{3}\,1{}\mathrm{i}}{24}\right)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{72}+\frac{\sqrt{3}\,1{}\mathrm{i}}{72}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{1}{72}+\frac{\sqrt{3}\,1{}\mathrm{i}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x + 1)/36 - x/
(6*(x^3 + 1)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 - 1/24) - lo
g(x - 1)/12 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 + 1/72) - log(
x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 - 1/72)
```

3.346 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} \, dx$

Optimal. Leaf size=15

$$3\tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)$$

```
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

Rubi [A]

time = 0.07, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6857, 209, 266}

$$3\text{ArcTan}(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx &= \int \left(\frac{3}{1+x^2}+\frac{x}{3+x^2}\right)dx\\
&= 3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{3+x^2}\,dx\\
&= 3\tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$3\tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Maple [A]
time = 0.20, size = 14, normalized size = 0.93

method	result	size
default	$3\arctan\left(x\right)+\frac{\ln\left(x^2+3\right)}{2}$	14
risch	$3\arctan\left(x\right)+\frac{\ln\left(x^2+3\right)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x,method=_RETURNVERBOSE)
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

Maxima [A]
time = 0.48, size = 13, normalized size = 0.87

$$3\,\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.87

$$3\,\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="fricas")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.80

$$\frac{\log(x^2+3)}{2} + 3\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + 3*atan(x)
```

Giac [A]
time = 4.01, size = 13, normalized size = 0.87

$$3\arctan(x) + \frac{1}{2}\log(x^2+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")
```

```
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

Mupad [B]
time = 2.12, size = 13, normalized size = 0.87

$$\frac{\ln(x^2+3)}{2} + 3\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + x^3 + 9)/((x^2 + 1)*(x^2 + 3)),x)
```

```
[Out] log(x^2 + 3)/2 + 3*atan(x)
```

3.347 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx$

Optimal. Leaf size=13

$$\tan^{-1}(x) + \frac{1}{2}\log(3+x^2)$$

```
[Out] arctan(x)+1/2*ln(x^2+3)
```

Rubi [A]

time = 0.06, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6857, 209, 266}

$$\text{ArcTan}(x) + \frac{1}{2}\log(x^2+3)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx &= \int \left(\frac{1}{1+x^2} + \frac{x}{3+x^2}\right) dx \\
&= \int \frac{1}{1+x^2} dx + \int \frac{x}{3+x^2} dx \\
&= \tan^{-1}(x) + \frac{1}{2}\log(3+x^2)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\tan^{-1}(x) + \frac{1}{2}\log\left(3 + x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Maple [A]
time = 0.20, size = 12, normalized size = 0.92

method	result	size
default	$\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	12
risch	$\arctan\left(x\right)+\frac{\ln\left(x^{2}+3\right)}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)+1/2*ln(x^2+3)
```

Maxima [A]
time = 0.48, size = 11, normalized size = 0.85

$$\arctan\left(x\right) + \frac{1}{2}\,\log\left(x^{2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Fricas [A]
time = 0.38, size = 11, normalized size = 0.85

$$\arctan\left(x\right) + \frac{1}{2}\,\log\left(x^{2} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="fricas")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.77

$$\frac{\log\left(x^{2}+3\right)}{2}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + atan(x)
```

Giac [A]
time = 3.73, size = 11, normalized size = 0.85

$$\arctan\left(x\right)+\frac{1}{2}\log\left(x^{2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")
```

```
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

Mupad [B]
time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^{2}+3\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 3)/((x^2 + 1)*(x^2 + 3)),x)
```

```
[Out] log(x^2 + 3)/2 + atan(x)
```

3.348 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$-3\tan^{-1}(x)+\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6857, 649, 209, 266}

$$-3\text{ArcTan}(x)+\sqrt{2}\,\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right)dx\\ &= 2\int \frac{1}{2+x^2}\,dx+3\int \frac{-1+x}{1+x^2}\,dx\\ &= \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\,dx+3\int \frac{x}{1+x^2}\,dx\\ &= -3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Maple [A]
time = 0.20, size = 25, normalized size = 0.86

method	result	size
default	$-3\arctan(x)+\frac{3\ln(x^2+1)}{2}+\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	25
risch	$-3\arctan(x)+\frac{3\ln(x^2+1)}{2}+\arctan\left(\frac{\sqrt{2}\,x}{2}\right)\sqrt{2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.48, size = 24, normalized size = 0.83

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Fricas [A]
time = 0.39, size = 24, normalized size = 0.83

$$\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) - 3\,\arctan\left(x\right) + \frac{3}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Sympy [A]
time = 0.07, size = 29, normalized size = 1.00

$$\frac{3\log\left(x^2+1\right)}{2} - 3\operatorname{atan}\left(x\right) + \sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A]
time = 3.93, size = 24, normalized size = 0.83

$$\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) - 3\,\arctan\left(x\right) + \frac{3}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Mupad [B]
time = 2.15, size = 51, normalized size = 1.76

$$-\sqrt{2}\,\operatorname{atan}\left(\frac{24\,\sqrt{2}}{24\,x-64}+\frac{32\,\sqrt{2}\,x}{24\,x-64}\right)+\ln\left(x-\mathrm{i}\right)\,\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)
[Out] log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1
/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))
```

3.349 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

```
[Out] 1/(2-x)-arctan(-2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {27, 707, 632, 210}

$$\text{ArcTan}(2-x)+\frac{1}{2-x}$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 707

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[-2*b*d*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(d^2*(m
+ 1)*(b^2 - 4*a*c))), x] + Dist[b^2*((m + 2*p + 3)/(d^2*(m + 1)*(b^2 - 4*a*
c))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
```

```
IntegerQ[(m + 2*p + 3)/2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx &= \int \frac{1}{(-2+x)^2(5-4x+x^2)}\,dx \\
&= \frac{1}{2-x} - \int \frac{1}{5-4x+x^2}\,dx \\
&= \frac{1}{2-x} + 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -4+2x\right) \\
&= \frac{1}{2-x} + \tan^{-1}(2-x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{-2+x} + \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A]
time = 0.29, size = 15, normalized size = 1.07

method	result	size
default	$-\frac{1}{x-2} - \arctan(x-2)$	15
risch	$-\frac{1}{x-2} - \arctan(x-2)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x,method=_RETURNVERBOSE)
[Out] -1/(x-2)-arctan(x-2)
```

Maxima [A]
time = 0.50, size = 14, normalized size = 1.00

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Fricas [A]
time = 0.38, size = 17, normalized size = 1.21

$$-\frac{(x-2)\arctan(x-2)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
```

```
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

Giac [A]
time = 3.78, size = 14, normalized size = 1.00

$$-\frac{1}{x-2}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 1.00

$$-\mathrm{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
```

```
[Out] - atan(x - 2) - 1/(x - 2)
```

3.350 $\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{x}+\log(3-x)$$

```
[Out] -1/x+ln(3-x)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {907}

$$\log(3-x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx &= \int \left(\frac{1}{-3+x}+\frac{1}{x^2}\right)dx\\ &= -\frac{1}{x}+\log(3-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{x}+\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
[Out] -x^(-1) + Log[3 - x]
```

Maple [A]
time = 0.21, size = 11, normalized size = 0.92

method	result	size
default	$\ln(x-3)-\frac{1}{x}$	11
norman	$\ln(x-3)-\frac{1}{x}$	11
risch	$\ln(x-3)-\frac{1}{x}$	11
meijerg	$\ln\left(1-\frac{x}{3}\right)-\frac{1}{x}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-3)/(x-3)/x^2,x,method=_RETURNVERBOSE)
[Out] ln(x-3)-1/x
```

Maxima [A]
time = 0.26, size = 10, normalized size = 0.83

$$-\frac{1}{x}+\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")
[Out] -1/x + log(x - 3)
```

Fricas [A]
time = 0.37, size = 12, normalized size = 1.00

$$\frac{x\log(x-3)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")
[Out] (x*log(x - 3) - 1)/x
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.58

$$\log(x-3)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-3)/(-3+x)/x**2,x)
[Out] log(x - 3) - 1/x
```

Giac [A]
time = 4.42, size = 11, normalized size = 0.92

$$-\frac{1}{x} + \log\left(|x - 3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")
[Out] -1/x + log(abs(x - 3))
```

Mupad [B]
time = 0.04, size = 10, normalized size = 0.83

$$\ln\left(x - 3\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 3)/(x^2*(x - 3)),x)
[Out] log(x - 3) - 1/x
```

3.351 $\int \frac{1+x+4x^2}{x+4x^3} \, dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tan^{-1}(2x) + \log(x)$$

[Out] 1/2*arctan(2*x)+ln(x)

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1607, 1816, 209}

$$\frac{1}{2}\text{ArcTan}(2x) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + 4*x^2)/(x + 4*x^3),x]

[Out] ArcTan[2*x]/2 + Log[x]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+4x^2}{x+4x^3}\,dx &= \int \frac{1+x+4x^2}{x(1+4x^2)}\,dx\\ &= \int \left(\frac{1}{x}+\frac{1}{1+4x^2}\right)dx\\ &= \log(x)+\int \frac{1}{1+4x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(2x)+\log(x)\end{aligned}$$

<u>**Mathematica**</u> [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(2x)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

```
[Out] ArcTan[2*x]/2 + Log[x]
```

<u>**Maple**</u> [A]
time = 0.20, size = 10, normalized size = 0.91

method	result	size
default	$\frac{\arctan(2x)}{2}+\ln(x)$	10
risch	$\frac{\arctan(2x)}{2}+\ln(x)$	10
meijerg	$\frac{\arctan(2x)}{2}+\ln(x)+\ln(2)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(4*x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(2*x)+ln(x)
```

<u>**Maxima**</u> [A]
time = 0.49, size = 9, normalized size = 0.82

$$\frac{1}{2}\arctan(2x)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x) + log(x)
```

Fricas [**A**]
time = 0.39, size = 9, normalized size = 0.82

$$\frac{1}{2}\arctan\left(2\,x\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*x) + log(x)
```

Sympy [**A**]
time = 0.04, size = 8, normalized size = 0.73

$$\log\left(x\right)+\frac{\operatorname{atan}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(4*x**3+x),x)
```

```
[Out] log(x) + atan(2*x)/2
```

Giac [**A**]
time = 4.41, size = 10, normalized size = 0.91

$$\frac{1}{2}\arctan\left(2\,x\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*x) + log(abs(x))
```

Mupad [**B**]
time = 2.17, size = 17, normalized size = 1.55

$$\ln\left(x\right)-\frac{\mathrm{atan}\left(\frac{17}{32\,\left(\frac{x}{16}-\frac{1}{8}\right)}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^2 + 1)/(x + 4*x^3),x)
```

```
[Out] log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2
```

3.352 $\int \frac{1-x+3x^2}{-x^2+x^3}\,dx$

Optimal. Leaf size=12

$$\frac{1}{x}+3\log(1-x)$$

[Out] 1/x+3*ln(1-x)

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1607, 907}

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]

[Out] x^(-1) + 3*Log[1 - x]

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+3x^2}{-x^2+x^3}\,dx &= \int \frac{1-x+3x^2}{(-1+x)x^2}\,dx\\ &= \int \left(\frac{3}{-1+x}-\frac{1}{x^2}\right)dx\\ &= \frac{1}{x}+3\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + 3*Log[1 - x]
```

Maple [A]
time = 0.21, size = 11, normalized size = 0.92

method	result	size
default	$\frac{1}{x}+3\ln\left(-1+x\right)$	11
norman	$\frac{1}{x}+3\ln\left(-1+x\right)$	11
risch	$\frac{1}{x}+3\ln\left(-1+x\right)$	11
meijerg	$\frac{1}{x}+3\ln\left(1-x\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x+3*ln(-1+x)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$\frac{1}{x}+3\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x - 1)
```

Fricas [A]
time = 0.39, size = 13, normalized size = 1.08

$$\frac{3\,x\log\left(x-1\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x - 1) + 1)/x
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.67

$$3\log(x-1)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**3-x**2),x)
[Out] 3*log(x - 1) + 1/x
```

Giac [A]
time = 4.87, size = 11, normalized size = 0.92

$$\frac{1}{x}+3\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 10, normalized size = 0.83

$$3\,\ln\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 - x + 1)/(x^2 - x^3),x)
[Out] 3*log(x - 1) + 1/x
```

3.353 $\int \frac{4+3x+x^2}{x+x^2} \, dx$

Optimal. Leaf size=12

$$x + 4\log(x) - 2\log(1+x)$$

```
[Out] x+4*ln(x)-2*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1607, 907}

$$x + 4\log(x) - 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x + x^2)/(x + x^2),x]
```

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+3x+x^2}{x+x^2} \, dx &= \int \frac{4+3x+x^2}{x(1+x)} \, dx \\
&= \int \left(1+\frac{4}{x}-\frac{2}{1+x}\right) dx \\
&= x+4\log(x)-2\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$x + 4\log(x) - 2\log(1 + x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x + x^2)/(x + x^2),x]
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Maple [A]
time = 0.18, size = 13, normalized size = 1.08

method	result	size
default	$x + 4\ln(x) - 2\ln(1 + x)$	13
norman	$x + 4\ln(x) - 2\ln(1 + x)$	13
meijerg	$x + 4\ln(x) - 2\ln(1 + x)$	13
risch	$x + 4\ln(x) - 2\ln(1 + x)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(x^2+x),x,method=_RETURNVERBOSE)
[Out] x+4*ln(x)-2*ln(1+x)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.00

$$x - 2\log(x + 1) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
[Out] x - 2*log(x + 1) + 4*log(x)
```

Fricas [A]
time = 0.38, size = 12, normalized size = 1.00

$$x - 2\log(x + 1) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")
[Out] x - 2*log(x + 1) + 4*log(x)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 1.00

$$x + 4\log(x) - 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(x**2+x),x)
[Out] x + 4*log(x) - 2*log(x + 1)
```

Giac [A]
time = 4.74, size = 14, normalized size = 1.17

$$x - 2\log(|x+1|) + 4\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")
[Out] x - 2*log(abs(x + 1)) + 4*log(abs(x))
```

Mupad [B]
time = 0.05, size = 12, normalized size = 1.00

$$x - 2\ln(x+1) + 4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 + 4)/(x + x^2),x)
[Out] x - 2*log(x + 1) + 4*log(x)
```

3.354 $\int \frac{4+x+3x^2}{x+x^3} \, dx$

Optimal. Leaf size=17

$$\tan^{-1}(x) + 4\log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1607, 1816, 649, 209, 266}

$$\text{ArcTan}(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + 3*x^2)/(x + x^3),x]
```

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x+3x^2}{x+x^3}\,dx &= \int \frac{4+x+3x^2}{x\left(1+x^2\right)}\,dx\\ &= \int \left(\frac{4}{x}+\frac{1-x}{1+x^2}\right)dx\\ &= 4\log(x)+\int \frac{1-x}{1+x^2}\,dx\\ &= 4\log(x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \tan^{-1}(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\tan^{-1}(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + 3*x^2)/(x + x^3),x]
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.20, size = 16, normalized size = 0.94

method	result	size
default	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
meijerg	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16
risch	$\arctan(x)+4\ln(x)-\frac{\ln(x^2+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x+4)/(x^3+x),x,method=_RETURNVERBOSE)
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.48, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Fricas [A]
time = 0.39, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.88

$$4\log(x) - \frac{\log(x^2+1)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x+4)/(x**3+x),x)
[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)
```

Giac [A]
time = 4.17, size = 16, normalized size = 0.94

$$\arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + 4\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))
```

Mupad [B]
time = 2.28, size = 23, normalized size = 1.35

$$4\ln(x) + \ln(x-\mathrm{i})\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 4)/(x + x^3),x)
[Out] 4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)
```

3.355 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx$

Optimal. Leaf size=13

$$-\tan^{-1}(x)+2\log(1+4x)$$

[Out] -arctan(x)+2*ln(1+4*x)

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1643, 210}

$$2\log(4x+1)-\text{ArcTan}(x)$$

Antiderivative was successfully verified.

[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]

[Out] -ArcTan[x] + 2*Log[1 + 4*x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx &= \int \left(\frac{8}{1+4x}+\frac{1}{-1-x^2}\right)dx\\ &= 2\log(1+4x)+\int \frac{1}{-1-x^2}\,dx\\ &= -\tan^{-1}(x)+2\log(1+4x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\tan^{-1}(x)+2\log(1+4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Maple [A]
time = 0.20, size = 14, normalized size = 1.08

method	result	size
default	$-\arctan(x)+2\ln(1+4x)$	14
risch	$-\arctan(x)+2\ln(1+4x)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(x)+2*ln(1+4*x)
```

Maxima [A]
time = 0.49, size = 13, normalized size = 1.00

$$-\arctan(x)+2\log(4x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

Fricas [A]
time = 0.38, size = 13, normalized size = 1.00

$$-\arctan(x)+2\log(4x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -arctan(x) + 2*log(4*x + 1)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.77

$$2\log\left(x+\frac{1}{4}\right)-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)
```

```
[Out] 2*log(x + 1/4) - atan(x)
```

Giac [A]
time = 3.64, size = 14, normalized size = 1.08

$$-\arctan(x) + 2 \log(|4x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")
```

```
[Out] -arctan(x) + 2*log(abs(4*x + 1))
```

Mupad [B]
time = 0.06, size = 19, normalized size = 1.46

$$\operatorname{atan}\left(\frac{4x + 1}{x - 4}\right) + 2 \ln\left(x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)
```

```
[Out] atan((4*x + 1)/(x - 4)) + 2*log(x + 1/4)
```

3.356 $\int \frac{x^2}{(-1+x)(1+2x+x^2)} \, dx$

Optimal. Leaf size=28

$$\frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(1+x)$$

```
[Out] 1/2/(1+x)+1/4*ln(1-x)+3/4*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {27, 90}

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)} \, dx &= \int \frac{x^2}{(-1+x)(1+x)^2} \, dx \\
&= \int \left(\frac{1}{4(-1+x)}-\frac{1}{2(1+x)^2}+\frac{3}{4(1+x)}\right) dx \\
&= \frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{4}\left(\frac{2}{1+x}+\log(-1+x)+3\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

Maple [A]
time = 0.22, size = 21, normalized size = 0.75

method	result	size
default	$\frac{\ln(-1+x)}{4}+\frac{1}{2x+2}+\frac{3\ln(1+x)}{4}$	21
norman	$\frac{\ln(-1+x)}{4}+\frac{1}{2x+2}+\frac{3\ln(1+x)}{4}$	21
risch	$\frac{\ln(-1+x)}{4}+\frac{1}{2x+2}+\frac{3\ln(1+x)}{4}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-1+x)/(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(-1+x)+1/2/(1+x)+3/4*ln(1+x)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.71

$$\frac{1}{2(x+1)}+\frac{3}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A]
time = 0.39, size = 26, normalized size = 0.93

$$\frac{3(x+1)\log(x+1)+(x+1)\log(x-1)+2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.71

$$\frac{\log\left(x-1\right)}{4}+\frac{3\log\left(x+1\right)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x)/(x**2+2*x+1),x)
```

```
[Out] log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

Giac [A]
time = 3.85, size = 22, normalized size = 0.79

$$\frac{1}{2\left(x+1\right)}+\frac{3}{4}\log\left(|x+1|\right)+\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] 1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [B]
time = 0.07, size = 20, normalized size = 0.71

$$\frac{\ln\left(x-1\right)}{4}+\frac{3\,\ln\left(x+1\right)}{4}+\frac{1}{2\,\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)*(2*x + x^2 + 1)),x)
```

```
[Out] log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))
```

3.357 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)$$

```
[Out] -9/32/(1-2*x)+41/128*ln(1-2*x)-25/128*ln(3+2*x)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {907}

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx &= \int \left(-\frac{9}{16(-1+2x)^2}+\frac{41}{64(-1+2x)}-\frac{25}{64(3+2x)}\right)dx\\ &= -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{9}{32(-1+2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Maple [A]
time = 0.21, size = 27, normalized size = 0.84

method	result	size
risch	$\frac{9}{64\left(x-\frac{1}{2}\right)}-\frac{25\ln(3+2x)}{128}+\frac{41\ln(2x-1)}{128}$	25
default	$\frac{9}{32(2x-1)}+\frac{41\ln(2x-1)}{128}-\frac{25\ln(3+2x)}{128}$	27
norman	$\frac{9x}{16(2x-1)}-\frac{25\ln(3+2x)}{128}+\frac{41\ln(2x-1)}{128}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 9/32/(2*x-1)+41/128*ln(2*x-1)-25/128*ln(3+2*x)
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.81

$$\frac{9}{32\,(2\,x-1)}-\frac{25}{128}\,\log\left(2\,x+3\right)+\frac{41}{128}\,\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

Fricas [A]
time = 0.38, size = 37, normalized size = 1.16

$$-\frac{25\,(2\,x-1)\log\left(2\,x+3\right)-41\,(2\,x-1)\log\left(2\,x-1\right)-36}{128\,(2\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
1)
```

Sympy [A]
time = 0.05, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

Giac [A]
time = 3.43, size = 43, normalized size = 1.34

$$\frac{9}{32\,(2\,x-1)}-\frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right)-\frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/
(2*x - 1) - 1))
```

Mupad [B]
time = 2.23, size = 22, normalized size = 0.69

$$\frac{41\,\ln\left(x-\frac{1}{2}\right)}{128}-\frac{25\,\ln\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64\,\left(x-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)
```

```
[Out] (41*log(x - 1/2))/128 - (25*log(x + 3/2))/128 + 9/(64*(x - 1/2))
```

3.358 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$

Optimal. Leaf size=23

$$-3\tan^{-1}(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -3*arctan(x)+2*ln(1-x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1643, 649, 209, 266}

$$-3\text{ArcTan}(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx = \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right)dx$$
$$= 2\log(1-x)+\int \frac{-3+x}{1+x^2}\,dx$$
$$= 2\log(1-x)-3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx$$
$$= -3\tan^{-1}(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.22

$$-3\tan^{-1}(x)+\frac{1}{2}\log\left(2+2(-1+x)+(-1+x)^2\right)+2\log(-1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Maple [A]
time = 0.21, size = 20, normalized size = 0.87

method	result	size
default	$2\ln(-1+x)+\frac{\ln(x^2+1)}{2}-3\arctan(x)$	20
risch	$2\ln(-1+x)+\frac{\ln(9x^2+9)}{2}-3\arctan(x)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-4*x+5)/(-1+x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+1/2*ln(x^2+1)-3*arctan(x)
```

Maxima [A]
time = 0.48, size = 19, normalized size = 0.83

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A]
time = 0.39, size = 19, normalized size = 0.83

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2+1\right) + 2 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.83

$$2\log(x-1) + \frac{\log(x^2+1)}{2} - 3\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Giac [A]
time = 3.91, size = 20, normalized size = 0.87

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2+1\right) + 2 \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

Mupad [B]
time = 0.05, size = 25, normalized size = 1.09

$$2 \ln(x-1) + \ln(x-\mathrm{i}) \left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i}) \left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

3.359 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/(-1+x)+arctan(x)+ln(1-x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1643, 649, 209, 266}

$$\text{ArcTan}(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx = \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right)dx$$
$$= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\,dx$$
$$= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx$$
$$= \frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.92

$$\frac{1}{-1+x}+\tan^{-1}(x)+\log(-1+x)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.22, size = 21, normalized size = 0.88

method	result	size
default	$\ln(-1+x)+\frac{1}{-1+x}-\frac{\ln(x^2+1)}{2}+\arctan(x)$	21
risch	$\ln(-1+x)+\frac{1}{-1+x}-\frac{\ln(x^2+1)}{2}+\arctan(x)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)/(-1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+x)+1/(-1+x)-1/2*ln(x^2+1)+arctan(x)
```

Maxima [A]
time = 0.50, size = 20, normalized size = 0.83

$$\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Fricas [**A**]
time = 0.40, size = 36, normalized size = 1.50

$$\frac{2\,(x-1)\arctan\left(x\right)-(x-1)\log\left(x^2+1\right)+2\,(x-1)\log\left(x-1\right)+2}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

Sympy [**A**]
time = 0.05, size = 20, normalized size = 0.83

$$\log\left(x-1\right)-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}\left(x\right)+\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.
time = 3.18, size = 47, normalized size = 1.96

$$\frac{1}{4}\,\pi-\pi\left\lfloor\frac{\pi+4\,\arctan\left(x\right)}{4\,\pi}+\frac{1}{2}\right\rfloor+\frac{1}{x-1}+\arctan\left(x\right)-\frac{1}{2}\,\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

Mupad [**B**]
time = 2.11, size = 28, normalized size = 1.17

$$\ln\left(x-1\right)+\frac{1}{x-1}+\ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
```

```
[Out] log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1
)
```

3.360 $\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx$

Optimal. Leaf size=49

$$-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

```
[Out] 261/221*arctan(-1+2*x)+1026/221*arctan(-3+x)+56/221*ln(x^2-6*x+10)+109/442*
ln(2*x^2-2*x+1)
```

Rubi [A]
time = 0.10, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6860, 648, 632, 210, 642, 631}

$$-\frac{261}{221}\text{ArcTan}(1-2x)-\frac{1026}{221}\text{ArcTan}(3-x)+\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx &= \int \left(\frac{2(345+56x)}{221\,(10-6x+x^2)}+\frac{2(76+109x)}{221\,(1-2x+2x^2)}\right)dx \\
&= \frac{2}{221}\int \frac{345+56x}{10-6x+x^2}\,dx+\frac{2}{221}\int \frac{76+109x}{1-2x+2x^2}\,dx \\
&= \frac{109}{442}\int \frac{-2+4x}{1-2x+2x^2}\,dx+\frac{56}{221}\int \frac{-6+2x}{10-6x+x^2}\,dx+\frac{261}{221}\int \frac{1}{1-2x+ \\
&= \frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)+\frac{261}{221}\text{Subst}\left(\int \frac{}{-1} \right. \\
&= -\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)+\frac{56}{221}\log\left(10-6x+x^2\right)+\frac{109}{442}\log\left(1-2x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Maple [A]
time = 0.35, size = 40, normalized size = 0.82

method	result	size
default	$\frac{261 \arctan(2x-1)}{221}+\frac{1026 \arctan(x-3)}{221}+\frac{56 \ln(x^2-6x+10)}{221}+\frac{109 \ln(2x^2-2x+1)}{442}$	40
risch	$\frac{56 \ln(x^2-6x+10)}{221}+\frac{1026 \arctan(x-3)}{221}+\frac{109 \ln(4x^2-4x+2)}{442}+\frac{261 \arctan(2x-1)}{221}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 261/221*arctan(2*x-1)+1026/221*arctan(x-3)+56/221*ln(x^2-6*x+10)+109/442*ln
(2*x^2-2*x+1)
```

Maxima [A]
time = 0.49, size = 39, normalized size = 0.80

$$\frac{261}{221} \arctan(2x-1)+\frac{1026}{221} \arctan(x-3)+\frac{109}{442} \log(2x^2-2x+1)+\frac{56}{221} \log(x^2-6x+10)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

Fricas [A]
time = 0.39, size = 39, normalized size = 0.80

$$\frac{261}{221} \arctan(2x-1)+\frac{1026}{221} \arctan(x-3)+\frac{109}{442} \log(2x^2-2x+1)+\frac{56}{221} \log(x^2-6x+10)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

Sympy [A]
time = 0.09, size = 44, normalized size = 0.90

$$\frac{56 \log(x^2-6x+10)}{221}+\frac{109 \log\left(x^2-x+\frac{1}{2}\right)}{442}+\frac{1026 \operatorname{atan}(x-3)}{221}+\frac{261 \operatorname{atan}(2x-1)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

```
[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x - 3
)/221 + 261*atan(2*x - 1)/221
```

Giac [A]
time = 3.70, size = 39, normalized size = 0.80

$$\frac{261}{221} \arctan\left(2 x - 1\right) + \frac{1026}{221} \arctan\left(x - 3\right) + \frac{109}{442} \log\left(2 x^{2} - 2 x + 1\right) + \frac{56}{221} \log\left(x^{2} - 6 x + 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

Mupad [B]
time = 2.16, size = 41, normalized size = 0.84

$$\ln\left(x-3-\mathrm{i}\right)\left(\frac{56}{221}-\frac{513}{221}\mathrm{i}\right)+\ln\left(x-3+1\mathrm{i}\right)\left(\frac{56}{221}+\frac{513}{221}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}-\frac{261}{442}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}+\frac{261}{442}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)
```

```
[Out] log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/22
1) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(10
9/442 + 261i/442)
```

3.361 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=25

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

[Out] 4*ln(1-x)-14*ln(2-x)+11*ln(3-x)

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1626}

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(\frac{11}{-3+x}-\frac{14}{-2+x}+\frac{4}{-1+x}\right)dx\\ &= 4\log(1-x)-14\log(2-x)+11\log(3-x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.76

$$11\log(-3+x)-14\log(-2+x)+4\log(-1+x)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]

Maple [A]
time = 0.22, size = 20, normalized size = 0.80

method	result	size
default	$11\ln(x-3)-14\ln(x-2)+4\ln(-1+x)$	20
norman	$11\ln(x-3)-14\ln(x-2)+4\ln(-1+x)$	20
risch	$11\ln(x-3)-14\ln(x-2)+4\ln(-1+x)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(x-3)/(x-2)/(-1+x),x,method=_RETURNVERBOSE)
[Out] 11*ln(x-3)-14*ln(x-2)+4*ln(-1+x)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.76

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Fricas [A]
time = 0.40, size = 19, normalized size = 0.76

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.76

$$11\log(x-3) - 14\log(x-2) + 4\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
[Out] 11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

Giac [A]
time = 3.38, size = 22, normalized size = 0.88

$$4 \log(|x-1|) - 14 \log(|x-2|) + 11 \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] 4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))
```

Mupad [B]
time = 2.14, size = 19, normalized size = 0.76

$$4\,\ln\left(x-1\right)-14\,\ln\left(x-2\right)+11\,\ln\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

3.362 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

Optimal. Leaf size=60

$$-\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}$$

```
[Out] -79/273/(5+x)+200/3211*ln(3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)+45
1/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6860, 648, 632, 210, 642}

$$\frac{451\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx &= \int \left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481\right.\\ &= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int \frac{-15-481x}{1+x+x^2}\,dx}{2793}\\ &= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int \frac{1}{1+x+x^2}\,d}{5586}\\ &= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x\right.}{5586}\\ &= -\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5-}{24843}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 0.90

$$\frac{-\frac{819546}{5+x}+152438\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+176400\log(3-2x)+311334\log(5+x)-243867\log\left(1+x+x^2\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
- 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

Maple [A]
time = 0.33, size = 48, normalized size = 0.80

method	result
default	$-\frac{481\ln(x^2+x+1)}{5586}+\frac{451\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{8379}+\frac{200\ln(2x-3)}{3211}-\frac{79}{273(5+x)}+\frac{2731\ln(5+x)}{24843}$
risch	$-\frac{79}{273(5+x)}+\frac{2731\ln(5+x)}{24843}+\frac{200\ln(2x-3)}{3211}-\frac{481\ln(203401x^2+203401x+203401)}{5586}+\frac{451\sqrt{3}\,\arctan\left(\frac{2\left(451x+\frac{451}{2}\right)\sqrt{3}}{1353}\right.}{8379}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+16*x)/(5+x)^2/(2*x-3)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+200/3211
*ln(2*x-3)-79/273/(5+x)+2731/24843*ln(5+x)
```

Maxima [A]
time = 0.49, size = 47, normalized size = 0.78

$$\frac{451}{8379}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\,\log\left(x^2+x+1\right)+\frac{200}{3211}\,\log\left(2\,x-3\right)+\frac{2731}{24843}\,\log\left(x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

Fricas [A]
time = 0.39, size = 60, normalized size = 1.00

$$\frac{152438\,\sqrt{3}\,(x+5)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-243867\,(x+5)\log\left(x^2+x+1\right)+176400\,(x+5)\log\left(2\,x-3\right)+311334\,(x+5)\log\left(x+5\right)-819546}{2832102\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

Sympy [A]
time = 0.11, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

Giac [A]
time = 4.41, size = 60, normalized size = 1.00

$$\frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

Mupad [B]
time = 0.13, size = 61, normalized size = 1.02

$$\frac{200\,\ln\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\,\ln\left(x+5\right)}{24843}-\frac{79}{273\,\left(x+5\right)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

```
[Out] (200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log(
x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^(1
/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

3.363 $\int \frac{-1+x^3}{1+x+x^2}\,dx$

Optimal. Leaf size=11

$$-x+\frac{x^2}{2}$$

[Out] -x+1/2*x^2

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1600}

$$\frac{x^2}{2}-x$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(1 + x + x^2),x]

[Out] -x + x^2/2

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^3}{1+x+x^2}\,dx &= \int (-1+x)\,dx\\ &= -x+\frac{x^2}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-x+\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(1 + x + x^2),x]

[Out] -x + x^2/2

Maple [A]
time = 0.32, size = 10, normalized size = 0.91

method	result	size
gosper	$\frac{x(x-2)}{2}$	7
default	$-x+\frac{1}{2}x^2$	10
norman	$-x+\frac{1}{2}x^2$	10
risch	$-x+\frac{1}{2}x^2$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/2*x^2
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x
```

Fricas [A]
time = 0.37, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.45

$$\frac{x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+x+1),x)
```

```
[Out] x**2/2 - x
```

Giac [A]
time = 5.10, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.55

$$\frac{x(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x + x^2 + 1),x)
```

```
[Out] (x*(x - 2))/2
```

3.364 $\int \frac{-3+x^3}{-7-6x+x^2}\,dx$

Optimal. Leaf size=29

$$6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)$$

[Out] 6*x+1/2*x^2+85/2*ln(7-x)+1/2*ln(1+x)

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1671, 646, 31}

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x^3)/(-7 - 6*x + x^2),x]

[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x^3}{-7-6x+x^2}\,dx &= \int \left(6+x+\frac{39+43x}{-7-6x+x^2}\right)dx\\ &= 6x+\frac{x^2}{2}+\int \frac{39+43x}{-7-6x+x^2}\,dx\\ &= 6x+\frac{x^2}{2}+\frac{1}{2}\int \frac{1}{1+x}\,dx+\frac{85}{2}\int \frac{1}{-7+x}\,dx\\ &= 6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Maple [A]
time = 0.23, size = 22, normalized size = 0.76

method	result	size
default	$\frac{x^2}{2}+6x+\frac{\ln(1+x)}{2}+\frac{85\ln(x-7)}{2}$	22
norman	$\frac{x^2}{2}+6x+\frac{\ln(1+x)}{2}+\frac{85\ln(x-7)}{2}$	22
risch	$\frac{x^2}{2}+6x+\frac{\ln(1+x)}{2}+\frac{85\ln(x-7)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3)/(x^2-6*x-7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+6*x+1/2*ln(1+x)+85/2*ln(x-7)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.72

$$\frac{1}{2}\,x^2+6\,x+\frac{1}{2}\,\log\left(x+1\right)+\frac{85}{2}\,\log\left(x-7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Fricas [A]
time = 0.40, size = 21, normalized size = 0.72

$$\frac{1}{2}\,x^{2}+6\,x+\frac{1}{2}\,\log\left(x+1\right)+\frac{85}{2}\,\log\left(x-7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.76

$$\frac{x^{2}}{2}+6x+\frac{85\log\left(x-7\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3)/(x**2-6*x-7),x)
[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2
```

Giac [A]
time = 5.62, size = 23, normalized size = 0.79

$$\frac{1}{2}\,x^{2}+6\,x+\frac{1}{2}\,\log\left(|x+1|\right)+\frac{85}{2}\,\log\left(|x-7|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))
```

Mupad [B]
time = 2.12, size = 21, normalized size = 0.72

$$6\,x+\frac{\ln\left(x+1\right)}{2}+\frac{85\,\ln\left(x-7\right)}{2}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 3)/(6*x - x^2 + 7),x)
[Out] 6*x + log(x + 1)/2 + (85*log(x - 7))/2 + x^2/2
```

3.365 $\int \frac{1+x^3}{(13+4x+x^2)^2}\,dx$

Optimal. Leaf size=45

$$\frac{67+47x}{18(13+4x+x^2)}-\frac{61}{54}\tan^{-1}\left(\frac{2+x}{3}\right)+\frac{1}{2}\log\left(13+4x+x^2\right)$$

```
[Out] 1/18*(67+47*x)/(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)+1/2*ln(x^2+4*x+13)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1674, 648, 632, 210, 642}

$$-\frac{61}{54}\text{ArcTan}\left(\frac{x+2}{3}\right)+\frac{47x+67}{18(x^2+4x+13)}+\frac{1}{2}\log\left(x^2+4x+13\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx &= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{36}\int \frac{-50+36x}{13+4x+x^2}\,dx\\ &= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\int \frac{4+2x}{13+4x+x^2}\,dx-\frac{61}{18}\int \frac{1}{13+4x+x^2}\,dx\\ &= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\log\left(13+4x+x^2\right)+\frac{61}{9}\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,4+2x\right)\\ &= \frac{67+47x}{18\left(13+4x+x^2\right)}-\frac{61}{54}\tan^{-1}\left(\frac{2+x}{3}\right)+\frac{1}{2}\log\left(13+4x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 1.00

$$\frac{67+47x}{18\left(13+4x+x^2\right)}-\frac{61}{54}\tan^{-1}\left(\frac{2+x}{3}\right)+\frac{1}{2}\log\left(13+4x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Maple [A]
time = 0.31, size = 37, normalized size = 0.82

method	result	size

default	$\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}+\frac{\ln(x^2+4x+13)}{2}-\frac{61\arctan\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$	37
risch	$\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}+\frac{\ln(x^2+4x+13)}{2}-\frac{61\arctan\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^2+4*x+13)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(1/3*x+2/3)
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.82

$$\frac{47\,x+67}{18\,(x^2+4\,x+13)}-\frac{61}{54}\,\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)+\frac{1}{2}\,\log\left(x^2+4\,x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

Fricas [A]
time = 0.38, size = 52, normalized size = 1.16

$$-\frac{61\,(x^2+4\,x+13)\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)-27\,(x^2+4\,x+13)\log\left(x^2+4\,x+13\right)-141\,x-201}{54\,(x^2+4\,x+13)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="fricas")
```

```
[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x^
2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.82

$$\frac{47x+67}{18x^2+72x+234}+\frac{\log\left(x^2+4x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**2+4*x+13)**2,x)
```

```
[Out] (47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*atan(x/3 +
 2/3)/54
```

Giac [A]
time = 7.24, size = 37, normalized size = 0.82

$$\frac{47\,x+67}{18\,(x^2+4\,x+13)}-\frac{61}{54}\,\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)+\frac{1}{2}\,\log\left(x^2+4\,x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="giac")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

Mupad [B]
time = 0.04, size = 49, normalized size = 1.09

$$\frac{\ln\left(x^2+4\,x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}+\frac{47\,x}{18\,\left(x^2+4\,x+13\right)}+\frac{67}{18\,\left(x^2+4\,x+13\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(4*x + x^2 + 13)^2,x)
```

```
[Out] log(4*x + x^2 + 13)/2 - (61*atan(x/3 + 2/3))/54 + (47*x)/(18*(4*x + x^2 + 1
3)) + 67/(18*(4*x + x^2 + 13))
```

3.366 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx$

Optimal. Leaf size=32

$$\frac{1}{4+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-2\log(x)+\log\left(4+x^2\right)$$

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

Rubi [A]
time = 0.18, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {6857, 209, 267, 649, 266}

$$\frac{1}{2}\text{ArcTan}\left(\frac{x}{2}\right)+2\text{ArcTan}(x)+\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^
2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x\left(1+x^2\right)\left(4+x^2\right)^2}\,dx = \int \left(-\frac{2}{x}+\frac{2}{1+x^2}-\frac{2x}{\left(4+x^2\right)^2}+\frac{1+2x}{4+x^2}\right)dx$$

$$= -2\log(x)+2\int \frac{1}{1+x^2}\,dx-2\int \frac{x}{\left(4+x^2\right)^2}\,dx+\int \frac{1+2x}{4+x^2}$$

$$= \frac{1}{4+x^2}+2\tan^{-1}(x)-2\log(x)+2\int \frac{x}{4+x^2}\,dx+\int \frac{1}{4+x}$$

$$= \frac{1}{4+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-2\log(x)+\log\left(4+x\right.$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{1}{4+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-2\log(x)+\log\left(4+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 +
 x^2)^2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Maple [A]
time = 0.21, size = 29, normalized size = 0.91

method	result	size
default	$\frac{1}{x^2+4}+\frac{\arctan\left(\frac{x}{2}\right)}{2}+2\arctan\left(x\right)-2\ln\left(x\right)+\ln\left(x^2+4\right)$	29
risch	$\frac{1}{x^2+4}+\frac{\arctan\left(\frac{x}{2}\right)}{2}+2\arctan\left(x\right)-2\ln\left(x\right)+\ln\left(x^2+4\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

Maxima [A]
time = 0.50, size = 28, normalized size = 0.88

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="maxima")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(x)
```

Fricas [A]
time = 0.38, size = 52, normalized size = 1.62

$$\frac{(x^2+4)\arctan\left(\frac{1}{2}x\right)+4(x^2+4)\arctan(x)+2(x^2+4)\log\left(x^2+4\right)-4(x^2+4)\log(x)+2}{2(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="fricas")
```

```
[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2
+ 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)
```

Sympy [A]
time = 0.11, size = 29, normalized size = 0.91

$$-2\log(x)+\log\left(x^2+4\right)+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}+2\operatorname{atan}(x)+\frac{1}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,x
)
```

```
[Out] -2*log(x) + log(x**2 + 4) + atan(x/2)/2 + 2*atan(x) + 1/(x**2 + 4)
```

Giac [A]
time = 6.75, size = 29, normalized size = 0.91

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="giac")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x)
)
```

Mupad [B]
time = 0.07, size = 44, normalized size = 1.38

$$\frac{1}{x^2+4}-2\ln\left(x\right)-2\,\mathrm{atan}\left(\frac{328000}{7\,\left(36288\,x-19584\right)}+\frac{34}{63}\right)+\ln\left(x-2\mathrm{i}\right)\,\left(1-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\,\left(1+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5 - 32)/(x*(x^2 + 1)*(x^2 + 4)^2
),x)
```

```
[Out] log(x - 2i)*(1 - 1i/4) + log(x + 2i)*(1 + 1i/4) - 2*atan(328000/(7*(36288*x
 - 19584)) + 34/63) - 2*log(x) + 1/(x^2 + 4)
```

3.367 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$

Optimal. Leaf size=148

$$\frac{x^2}{2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\log\left(\sqrt{7}-\sqrt{2}\,\sqrt[4]{7}\,x+x^2\right)}{4\sqrt{2}\,7^{3/4}}+\frac{\log\left(\sqrt{7}+\sqrt{\ldots}\right.}{4\sqrt{\ldots}}$$

```
[Out] 1/2*x^2-1/2*arctanh(x^2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1
/2)+1/28*arctan(1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(1/2)-1/56*ln(x^2-7^(1/4
)*x*2^(1/2)+7^(1/2))*7^(1/4)*2^(1/2)+1/56*ln(x^2+7^(1/4)*x*2^(1/2)+7^(1/2))
*7^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1804, 1417, 217, 1179, 642, 1176, 631, 210, 1598, 1492, 281, 327, 213}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}\,7^{3/4}}+\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\,\sqrt[4]{7}\,x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\,\sqrt[4]{7}\,x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (S
qrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqr
t[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x
 + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1417

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /;
FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1492

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1804

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx = \int \left(\frac{-1+x^4}{-7+6x^4+x^8}+\frac{x(7x^4+x^8)}{-7+6x^4+x^8}\right)dx$$
$$= \int \frac{-1+x^4}{-7+6x^4+x^8}\,dx + \int \frac{x(7x^4+x^8)}{-7+6x^4+x^8}\,dx$$
$$= \int \frac{1}{7+x^4}\,dx + \int \frac{x^5(7+x^4)}{-7+6x^4+x^8}\,dx$$
$$= \frac{\int \frac{\sqrt{7}-x^2}{7+x^4}\,dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7}+x^2}{7+x^4}\,dx}{2\sqrt{7}} + \int \frac{x^5}{-1+x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, x^2\right) - \frac{\int \frac{\sqrt{2}\sqrt[4]{7}+2x}{-\sqrt{7}-\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}\,7^{3/4}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{7}-2x}{-\sqrt{7}+\sqrt{2}\sqrt[4]{7}\,x}}{4\sqrt{2}\,7^{3/4}}$$
$$= \frac{x^2}{2} - \frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}\,x+x^2\right)}{4\sqrt{2}\,7^{3/4}} + \frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}\,x+x^2\right)}{4\sqrt{2}\,7^{3/4}} + \frac{1}{2}\text{Subst}\Big(\int$$
$$= \frac{x^2}{2} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}} - \frac{1}{2}\tanh^{-1}\left(x^2\right) - \frac{\log\left(\sqrt{7}-\right.}{4}$$

Mathematica [A]

time = 0.05, size = 159, normalized size = 1.07

$$\frac{1}{56}\left(28x^2-2\sqrt{2}\sqrt[4]{7}\tan^{-1}\left(1-\frac{\sqrt{2}\,x}{\sqrt[4]{7}}\right)+2\sqrt{2}\sqrt[4]{7}\tan^{-1}\left(1+\frac{\sqrt{2}\,x}{\sqrt[4]{7}}\right)+14\log(1-x)+14\log(1+x)-14\log\left(1+x^2\right)-\sqrt{2}\sqrt[4]{7}\log\left(7-\sqrt{2}\,7^{3/4}x+\sqrt{7}\,x^2\right)+\sqrt{2}\sqrt[4]{7}\log\left(7+\sqrt{2}\,7^{3/4}x+\sqrt{7}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^(
1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14*L
og[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] + Sq
rt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56
```

Maple [A]

time = 0.03, size = 99, normalized size = 0.67

method	result
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(343\_Z^4+1\right)}\_R\ln\left(x+7\_R\right)\right)}{4}-\frac{\ln\left(x^2+1\right)}{4}+\frac{\ln\left(x^2-1\right)}{4}$

default	$\frac{x^2}{2}+\frac{7^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+7^{\frac{1}{4}}x\sqrt{2}+\sqrt{7}}{x^2-7^{\frac{1}{4}}x\sqrt{2}+\sqrt{7}}\right)+2\arctan\left(1+\frac{x\sqrt{2}\,7^{\frac{3}{4}}}{7}\right)+2\arctan\left(-1+\frac{x\sqrt{2}\,7^{\frac{3}{4}}}{7}\right)\right)}{56}+\frac{\ln(-1+x)}{4}-\frac{\ln(x^2+1)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/56*7^(1/4)*2^(1/2)*(ln((x^2+7^(1/4)*x*2^(1/2)+7^(1/2))/(x^2-7^(1/
4)*x*2^(1/2)+7^(1/2)))+2*arctan(1+1/7*x*2^(1/2)*7^(3/4))+2*arctan(-1+1/7*x*
2^(1/2)*7^(3/4)))+1/4*ln(-1+x)-1/4*ln(x^2+1)+1/4*ln(1+x)
```

Maxima [A]
time = 0.50, size = 132, normalized size = 0.89

$$\frac{1}{2}x^2+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x+7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x-7^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2+7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)-\frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2-7^{\frac{1}{4}}\sqrt{2}x+\sqrt{7}\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x+1\right)+\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*s
qrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*
sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1
/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 +
1) + 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A]
time = 0.41, size = 179, normalized size = 1.21

$$-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot 343^{\frac{1}{4}}\sqrt{2}x+\frac{1}{49}\cdot 343^{\frac{1}{4}}\sqrt{2}\sqrt{343^{\frac{3}{4}}\sqrt{2}x+49x^2+49\sqrt{7}}-1\right)-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot 343^{\frac{1}{4}}\sqrt{2}x+\frac{1}{686}\cdot 343^{\frac{1}{4}}\sqrt{2}\sqrt{-196\cdot 343^{\frac{3}{4}}\sqrt{2}x+9604x^2+9604\sqrt{7}}+1\right)+\frac{1}{2744}\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(196\cdot 343^{\frac{3}{4}}\sqrt{2}x+9604x^2+9604\sqrt{7}\right)-\frac{1}{2744}\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(-196\cdot 343^{\frac{3}{4}}\sqrt{2}x+9604x^2+9604\sqrt{7}\right)+\frac{1}{2}x^2-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="fricas")
```

```
[Out] -1/686*343^(3/4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*s
qrt(2)*sqrt(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1) - 1/686*343^(3/
4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/686*343^(1/4)*sqrt(2)*sqrt(-
196*343^(3/4)*sqrt(2)*x + 9604*x^2 + 9604*sqrt(7)) + 1) + 1/2744*343^(3/4)*
sqrt(2)*log(196*343^(3/4)*sqrt(2)*x + 9604*x^2 + 9604*sqrt(7)) - 1/2744*343
^(3/4)*sqrt(2)*log(-196*343^(3/4)*sqrt(2)*x + 9604*x^2 + 9604*sqrt(7)) + 1/
2*x^2 - 1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Sympy [A]
time = 0.20, size = 146, normalized size = 0.99

$$\frac{x^2}{2}+\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4}-\frac{\sqrt{2}\cdot\sqrt[4]{7}\log\left(x^2-\sqrt{2}\cdot\sqrt[4]{7}x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\cdot\sqrt[4]{7}\log\left(x^2+\sqrt{2}\cdot\sqrt[4]{7}x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{28}+\frac{\sqrt{2}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

```
[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - sq
rt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/
4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + s
qrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28
```

Giac [A]
time = 7.94, size = 122, normalized size = 0.82

$$\frac{1}{2}x^2 + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x + 7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x - 7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2 + 7^{\frac{1}{4}}\sqrt{2}x + \sqrt{7}\right) - \frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2 - 7^{\frac{1}{4}}\sqrt{2}x + \sqrt{7}\right) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{4}\log\left(|x+1|\right) + \frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2))
) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/
56*28^(1/4)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*28^(1/4)*log(x^2
- 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(abs(x + 1)) + 1
/4*log(abs(x - 1))
```

Mupad [B]
time = 2.19, size = 124, normalized size = 0.84

$$\frac{\mathrm{atan}(x^2\,1\mathrm{i})\,1\mathrm{i}}{2} + \frac{x^2}{2} + \sqrt{2}\,7^{1/4}\,\mathrm{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\,\left(\frac{89653248}{2401}+\frac{89653248}{2401}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}} + \frac{\sqrt{2}\,7^{3/4}\,x\,\left(-\frac{524288}{343}+\frac{524288}{343}\mathrm{i}\right)}{-\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}\right)\,\left(\frac{1}{28}+\frac{1}{28}\mathrm{i}\right) + \sqrt{2}\,7^{1/4}\,\mathrm{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\,\left(\frac{89653248}{2401}-\frac{89653248}{2401}\mathrm{i}\right)}{\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}} + \frac{\sqrt{2}\,7^{3/4}\,x\,\left(-\frac{524288}{343}-\frac{524288}{343}\mathrm{i}\right)}{\frac{1048576}{49}+\frac{\sqrt{7}\,179306496\mathrm{i}}{2401}}\right)\,\left(-\frac{1}{28}+\frac{1}{28}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 7*x^5 + x^9 - 1)/(6*x^4 + x^8 - 7),x)
```

```
[Out] (atan(x^2*1i)*1i)/2 + x^2/2 + 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(8965
3248/2401 + 89653248i/2401))/((7^(1/2)*179306496i)/2401 - 1048576/49) - (2^
(1/2)*7^(3/4)*x*(524288/343 - 524288i/343))/((7^(1/2)*179306496i)/2401 - 10
48576/49))*(1/28 + 1i/28) - 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(896532
48/2401 - 89653248i/2401))/((7^(1/2)*179306496i)/2401 + 1048576/49) - (2^(1
/2)*7^(3/4)*x*(524288/343 + 524288i/343))/((7^(1/2)*179306496i)/2401 + 1048
576/49))*(1/28 - 1i/28)
```

3.368 $\int \frac{1+x^3+x^6}{x+x^5} \, dx$

Optimal. Leaf size=112

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}+\log(x)+\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,x\right.}{4\sqrt{2}}$$

```
[Out] 1/2*x^2-1/2*arctan(x^2)+ln(x)-1/4*ln(x^4+1)+1/4*arctan(-1+x*2^(1/2))*2^(1/2
)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+
x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {1607, 1847, 303, 1176, 631, 210, 1179, 642, 1848, 1262, 649, 209, 266}

$$-\frac{\text{ArcTan}(x^2)}{2}-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{1}{4}\log\left(x^4+1\right)+\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt
[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1
+ Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1848

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3+x^6}{x+x^5}\,dx &= \int \frac{1+x^3+x^6}{x\left(1+x^4\right)}\,dx \\
&= \int \left(\frac{x^2}{1+x^4}+\frac{1+x^6}{x\left(1+x^4\right)}\right)\,dx \\
&= \int \frac{x^2}{1+x^4}\,dx+\int \frac{1+x^6}{x\left(1+x^4\right)}\,dx \\
&= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx+\int \left(\frac{1}{x}+x+\frac{x\left(-1-x^2\right)}{1+x^4}\right)\,dx \\
&= \frac{x^2}{2}+\log(x)+\frac{1}{4}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}}+ \\
&= \frac{x^2}{2}+\log(x)+\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{1}{2}\text{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx \right. \\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}+\log(x)+\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}- \\
&= \frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}+\log(x)+
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 101, normalized size = 0.90

$$\frac{1}{8}\left(4x^2-2\left(-2+\sqrt{2}\right)\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\left(2+\sqrt{2}\right)\tan^{-1}\left(1+\sqrt{2}\,x\right)+8\log(x)+\sqrt{2}\,\log\left(1-\sqrt{2}\,x+x^2\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,x+x^2\right)-2\log\left(1+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1
+ Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1
+ Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

Maple [A]
time = 0.20, size = 74, normalized size = 0.66

method	result
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+4\_Z^3+8\_Z^2+4\_Z+1\right)}\_R\ln\left(-\_R^3-5\_R^2-10\_R+3x-5\right)\right)}{4}+\ln\left(x\right)$
default	$\frac{x^2}{2}-\frac{\arctan\left(x^2\right)}{2}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8}-\frac{\ln\left(x^4+1\right)}{4}+\ln\left(x\right)$
meijerg	$\frac{x^2}{2}-\frac{\arctan\left(x^2\right)}{2}+\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,\left(x^4\right)^{\frac{1}{4}}\right.}{8\left(x^4\right)^{\frac{3}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+1)/(x^5+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/2*arctan(x^2)+1/8*2^(1/2)*(ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x)
)+2*arctan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))-1/4*ln(x^4+1)+ln(x)
```

Maxima [A]
time = 0.49, size = 99, normalized size = 0.88

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{4}\sqrt{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\left(\sqrt{2}+1\right)\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{8}\sqrt{2}\left(\sqrt{2}-1\right)\log\left(x^2-\sqrt{2}\,x+1\right)+\frac{1}{2}x^2+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2)
*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2) +
 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(2)*
x + 1) + 1/2*x^2 + log(x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.16, size = 515, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/4*(2*sqrt(1/4*I) + I + 1)*log((2*sqrt(1/4*I) + I + 1)^3 - 5*(2*
sqrt(1/4*I) + I + 1)^2 + 3*x + 20*sqrt(1/4*I) + 10*I + 5) - 1/4*(2*sqrt(-1/
4*I) - I + 1)*log(-(2*sqrt(1/4*I) + I + 1)^3 - (2*sqrt(1/4*I) + I + 2)*(2*s
qrt(-1/4*I) - I + 1)^2 + 4*(2*sqrt(1/4*I) + I + 1)^2 - ((2*sqrt(1/4*I) + I
+ 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 3*x - 16*sqrt(
1/4*I) - 8*I - 9) + 1/4*(sqrt(1/4*I) + sqrt(-1/4*I) - 2*sqrt(-3/16*(2*sqrt(
1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) -
3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(
2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I
+ 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(
-1/4*I) - I + 1) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4
*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + s
qrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)
 + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*sqrt(1/4*I) - I + 2) + 1/4*(sqrt(1/4*I)
 + sqrt(-1/4*I) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*
I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sq
rt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*
I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I + 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1
)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) - 2*sqrt(-3/16*(2*s
qrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1
) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1
/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1) + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*
sqrt(1/4*I) - I + 2) + log(x)
```

Sympy [A]
time = 0.52, size = 61, normalized size = 0.54

$$\frac{x^2}{2} + \log(x) + \operatorname{RootSum}\left(256t^4 + 256t^3 + 128t^2 + 16t + 1, \left(t \mapsto t\log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, La
mbda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/21
9 + x - 344/219)))
```

Giac [A]
time = 8.67, size = 92, normalized size = 0.82

$$\frac{1}{2}x^2 + \frac{1}{4}\left(\sqrt{2} + 2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2} - 2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\log\left(x^2 + \sqrt{2}x + 1\right) + \frac{1}{8}\sqrt{2}\log\left(x^2 - \sqrt{2}x + 1\right) - \frac{1}{4}\log\left(x^4 + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqrt
(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2
)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log(ab
s(x))
```

Mupad [B]
time = 2.23, size = 170, normalized size = 1.52

$$\ln(x)+\left(\sum_{k=1}^{4}\ln\left(\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\left(8\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)+x+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)x\,96+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)^2x\,240+\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)^3x\,320-16\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)^2+8\right)\right)\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\right)+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^6 + 1)/(x + x^5),x)
```

```
[Out] log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*(8*root(z^
4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z/1
6 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*x +
 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z^3 +
 z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1/256,
 z, k), k, 1, 4) + x^2/2
```

3.369 $\int \frac{1+x^2}{-x+x^2} \, dx$

Optimal. Leaf size=14

$$x + 2\log(1-x) - \log(x)$$

[Out] x+2*ln(1-x)-ln(x)

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 908}

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(-x + x^2),x]

[Out] x + 2*Log[1 - x] - Log[x]

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{-x+x^2} \, dx &= \int \frac{1+x^2}{(-1+x)x} \, dx \\ &= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}\right) \, dx \\ &= x+2\log(1-x)-\log(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Maple [A]
time = 0.20, size = 13, normalized size = 0.93

method	result	size
default	$x+2\ln\left(-1+x\right)-\ln\left(x\right)$	13
norman	$x+2\ln\left(-1+x\right)-\ln\left(x\right)$	13
risch	$x+2\ln\left(-1+x\right)-\ln\left(x\right)$	13
meijerg	$2\ln\left(1-x\right)-\ln\left(x\right)-i\pi+x$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x),x,method=_RETURNVERBOSE)
```

```
[Out] x+2*ln(-1+x)-ln(x)
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$x+2\,\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Fricas [A]
time = 0.40, size = 12, normalized size = 0.86

$$x+2\,\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.71

$$x-\log\left(x\right)+2\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x),x)
[Out] x - log(x) + 2*log(x - 1)
```

Giac [A]
time = 9.07, size = 14, normalized size = 1.00

$$x + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B]
time = 0.04, size = 12, normalized size = 0.86

$$x + 2 \ln(x - 1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/(x - x^2),x)
[Out] x + 2*log(x - 1) - log(x)
```

3.370 $\int \frac{1+x^3}{-x+x^3}\,dx$

Optimal. Leaf size=12

$$x+\log(1-x)-\log(x)$$

```
[Out] x+ln(1-x)-ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 1816}

$$x+\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{-x+x^3}\,dx &= \int \frac{1+x^3}{x\left(-1+x^2\right)}\,dx\\
&= \int \left(1+\frac{1}{-1+x}-\frac{1}{x}\right)\,dx\\
&= x+\log(1-x)-\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$x+\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

Maple [A]
time = 0.20, size = 11, normalized size = 0.92

method	result	size
default	$x+\ln \left(-1+x\right)-\ln \left(x\right)$	11
norman	$x+\ln \left(-1+x\right)-\ln \left(x\right)$	11
risch	$x+\ln \left(-1+x\right)-\ln \left(x\right)$	11
meijerg	$\frac{\ln\left(-x^{2}+1\right)}{2}-\ln \left(x\right)-\frac{i \pi}{2}-\frac{i \left(2 i x-2 i \,\mathrm{arctanh}\left(x\right)\right)}{2}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x),x,method=_RETURNVERBOSE)
```

```
[Out] x+ln(-1+x)-ln(x)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$x + \log\left(x - 1\right) - \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="maxima")
```

```
[Out] x + log(x - 1) - log(x)
```

Fricas [A]
time = 0.40, size = 10, normalized size = 0.83

$$x + \log\left(x - 1\right) - \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] x + log(x - 1) - log(x)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$x - \log\left(x\right) + \log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x),x)
[Out] x - log(x) + log(x - 1)
```

Giac [A]
time = 8.83, size = 12, normalized size = 1.00

$$x + \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="giac")
[Out] x + log(abs(x - 1)) - log(abs(x))
```

Mupad [B]
time = 2.14, size = 10, normalized size = 0.83

$$x - 2\,\mathrm{atanh}(2\,x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/(x - x^3),x)
[Out] x - 2*atanh(2*x - 1)
```

3.371 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal. Leaf size=17

$$\frac{1}{x}+x+2\log(1-x)-\log(x)$$

[Out] 1/x+x+2*ln(1-x)-ln(x)

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 1634}

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(-x^2 + x^3),x]

[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{-x^2+x^3} \, dx &= \int \frac{1+x^3}{(-1+x)x^2} \, dx\\ &= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right) dx\\ &= \frac{1}{x}+x+2\log(1-x)-\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{x}+x+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Maple [A]
time = 0.20, size = 16, normalized size = 0.94

method	result	size
default	$x+2\ln\left(-1+x\right)+\frac{1}{x}-\ln\left(x\right)$	16
risch	$x+2\ln\left(-1+x\right)+\frac{1}{x}-\ln\left(x\right)$	16
norman	$\frac{x^{2}+1}{x}-\ln\left(x\right)+2\ln\left(-1+x\right)$	21
meijerg	$2\ln\left(1-x\right)-\ln\left(x\right)-i\pi+\frac{1}{x}+x$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x+2*ln(-1+x)+1/x-ln(x)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.88

$$x+\frac{1}{x}+2\,\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

Fricas [A]
time = 0.40, size = 21, normalized size = 1.24

$$\frac{x^{2}+2\,x\log\left(x-1\right)-x\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.82

$$x - \log(x) + 2\log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x**2),x)
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

Giac [A]
time = 7.41, size = 17, normalized size = 1.00

$$x + \frac{1}{x} + 2\log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$x + 2\ln(x - 1) - \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/(x^2 - x^3),x)
[Out] x + 2*log(x - 1) - log(x) + 1/x
```

3.372 $\int \frac{-1+x^5}{-x+x^3} \, dx$

Optimal. Leaf size=17

$$x+\frac{x^3}{3}+\log(x)-\log(1+x)$$

[Out] x+1/3*x^3+ln(x)-ln(1+x)

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 1816}

$$\frac{x^3}{3}+x+\log(x)-\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^5)/(-x + x^3),x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^5}{-x+x^3} \, dx &= \int \frac{-1+x^5}{x\left(-1+x^2\right)} \, dx \\ &= \int \left(1+\frac{1}{-1-x}+\frac{1}{x}+x^2\right) \, dx \\ &= x+\frac{x^3}{3}+\log(x)-\log(1+x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$x+\frac{x^3}{3}+\log(x)-\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-x + x^3),x]
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Maple [A]
time = 0.20, size = 16, normalized size = 0.94

method	result	size
default	$x+\frac{x^{3}}{3}+\ln\left(x\right)-\ln\left(1+x\right)$	16
norman	$x+\frac{x^{3}}{3}+\ln\left(x\right)-\ln\left(1+x\right)$	16
risch	$x+\frac{x^{3}}{3}+\ln\left(x\right)-\ln\left(1+x\right)$	16
meijerg	$-\frac{\ln\left(-x^{2}+1\right)}{2}+\ln\left(x\right)+\frac{i\pi}{2}+\frac{i\left(-\frac{2ix\left(5x^{2}+15\right)}{15}+2i\,\mathrm{arctanh}\left(x\right)\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^3-x),x,method=_RETURNVERBOSE)
[Out] x+1/3*x^3+ln(x)-ln(1+x)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.88

$$\frac{1}{3}\,x^{3}+x-\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="maxima")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Fricas [A]
time = 0.41, size = 15, normalized size = 0.88

$$\frac{1}{3}\,x^{3}+x-\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="fricas")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.82

$$\frac{x^{3}}{3}+x+\log\left(x\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**3-x),x)
```

```
[Out] x**3/3 + x + log(x) - log(x + 1)
```

Giac [A]
time = 5.46, size = 17, normalized size = 1.00

$$\frac{1}{3}x^3 + x - \log\left(|x+1|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$x - 2\,\mathrm{atanh}(2\,x+1) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5 - 1)/(x - x^3),x)
```

```
[Out] x - 2*atanh(2*x + 1) + x^3/3
```

3.373 $\int \frac{1+x^4}{x^3+x^5} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)$$

```
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

Rubi [A]

time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 1266, 908}

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \int \frac{1+x^4}{x^3\left(1+x^2\right)}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^2(1+x)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{1+x}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)$$

<u>**Mathematica**</u> [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

<u>**Maple**</u> [A]
time = 0.22, size = 17, normalized size = 0.94

method	result	size
default	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17
norman	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17
meijerg	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17
risch	$-\frac{1}{2x^2}-\ln(x)+\ln(x^2+1)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^5+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

<u>**Maxima**</u> [A]
time = 0.50, size = 16, normalized size = 0.89

$$-\frac{1}{2\,x^2}+\log\left(x^2+1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")
[Out] -1/2/x^2 + log(x^2 + 1) - log(x)
```

Fricas [A]
time = 0.40, size = 25, normalized size = 1.39

$$\frac{2\,x^2\log\left(x^2+1\right)-2\,x^2\log\left(x\right)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="fricas")
[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.83

$$-\log\left(x\right)+\log\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**5+x**3),x)
[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)
```

Giac [A]
time = 5.48, size = 23, normalized size = 1.28

$$\frac{x^2-1}{2\,x^2}+\log\left(x^2+1\right)-\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)
```

Mupad [B]
time = 0.05, size = 16, normalized size = 0.89

$$\ln\left(x^2+1\right)-\ln\left(x\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^3 + x^5),x)
[Out] log(x^2 + 1) - log(x) - 1/(2*x^2)
```

3.374 $\int \frac{1+x^2}{x+2x^2+x^3} \, dx$

Optimal. Leaf size=10

$$\frac{2}{1+x}+\log(x)$$

```
[Out] 2/(1+x)+ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1608, 27, 908}

$$\frac{2}{x+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

```
[Out] 2/(1 + x) + Log[x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \int \frac{1+x^2}{x\,(1+2x+x^2)}\,dx$$
$$= \int \frac{1+x^2}{x(1+x)^2}\,dx$$
$$= \int \left(\frac{1}{x} - \frac{2}{(1+x)^2}\right)\,dx$$
$$= \frac{2}{1+x} + \log(x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{1+x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]
[Out] 2/(1 + x) + Log[x]
```

Maple [A]
time = 0.02, size = 11, normalized size = 1.10

method	result	size
default	$\frac{2}{1+x}+\ln\left(x\right)$	11
norman	$\frac{2}{1+x}+\ln\left(x\right)$	11
risch	$\frac{2}{1+x}+\ln\left(x\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+2*x^2+x),x,method=_RETURNVERBOSE)
[Out] 2/(1+x)+ln(x)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 1.00

$$\frac{2}{x+1} + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")
```

```
[Out] 2/(x + 1) + log(x)
```

Fricas [A]
time = 0.40, size = 14, normalized size = 1.40

$$\frac{(x+1)\log(x)+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="fricas")
[Out] ((x + 1)*log(x) + 2)/(x + 1)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.70

$$\log(x)+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+2*x**2+x),x)
[Out] log(x) + 2/(x + 1)
```

Giac [A]
time = 5.47, size = 11, normalized size = 1.10

$$\frac{2}{x+1}+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")
[Out] 2/(x + 1) + log(abs(x))
```

Mupad [B]
time = 0.03, size = 10, normalized size = 1.00

$$\ln(x)+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + 2*x^2 + x^3),x)
[Out] log(x) + 2/(x + 1)
```

3.375 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal. Leaf size=42

$$19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

```
[Out] 19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1608, 1642}

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx &= \int \frac{1+x^5}{x\left(-10-3x+x^2\right)}\,dx\\ &= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx\\ &= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$19x + \frac{3x^2}{2} + \frac{x^3}{3} + \frac{3126}{35}\log(5-x) - \frac{\log(x)}{10} - \frac{31}{14}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.74

method	result	size
default	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(x+2)}{14}+\frac{3126\ln(-5+x)}{35}-\frac{\ln(x)}{10}$	31
norman	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(x+2)}{14}+\frac{3126\ln(-5+x)}{35}-\frac{\ln(x)}{10}$	31
risch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(x+2)}{14}+\frac{3126\ln(-5+x)}{35}-\frac{\ln(x)}{10}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(x+2)+3126/35*ln(-5+x)-1/10*ln(x)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 0.71

$$\frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log\left(x+2\right) + \frac{3126}{35}\,\log\left(x-5\right) - \frac{1}{10}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Fricas [A]
time = 0.41, size = 30, normalized size = 0.71

$$\frac{1}{3}\,x^3 + \frac{3}{2}\,x^2 + 19\,x - \frac{31}{14}\,\log\left(x+2\right) + \frac{3126}{35}\,\log\left(x-5\right) - \frac{1}{10}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)

Sympy [A]
time = 0.06, size = 36, normalized size = 0.86

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\log(x)}{10}+\frac{3126\log(x-5)}{35}-\frac{31\log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)

[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4

Giac [A]
time = 5.42, size = 33, normalized size = 0.79

$$\frac{1}{3}x^3+\frac{3}{2}x^2+19x-\frac{31}{14}\log(|x+2|)+\frac{3126}{35}\log(|x-5|)-\frac{1}{10}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))

Mupad [B]
time = 0.05, size = 30, normalized size = 0.71

$$19x-\frac{31\ln(x+2)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}+\frac{3x^2}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)

[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
x^3/3

3.376 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

```
[Out] 1/2*ln(x^2+2*x+3)+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)-arctan(1/5*x*5^(1/2
))*5^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6857, 209, 648, 632, 210, 642}

$$-\sqrt{5}\,\text{ArcTan}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\text{ArcTan}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right) dx\\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2\right.\\
&= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

Maple [A]
time = 0.31, size = 41, normalized size = 0.89

method	result	size
risch	$\frac{\ln(x^2+2x+3)}{2} + \frac{5\arctan\left(\frac{(1+x)\sqrt{2}}{2}\right)\sqrt{2}}{2} - \arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	39
default	$\frac{\ln(x^2+2x+3)}{2} + \frac{5\sqrt{2}\ \arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2} - \arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$	41

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,method=_RETURNVERBOSE)

[Out] 1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))-arctan(1/5*x*5^(1/2))*5^(1/2)

Maxima [A]
time = 0.48, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

Fricas [A]
time = 0.42, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

Sympy [A]
time = 0.08, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

Giac [A]
time = 5.09, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)+\frac{1}{2}\log\left(x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

Mupad [B]
time = 0.00, size = 88, normalized size = 1.91

$$\frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2}+\sqrt{5}\operatorname{atan}\left(\frac{2000\sqrt{5}}{2000\,x+1120}-\frac{224\sqrt{5}\,x}{2000\,x+1120}\right)-\frac{\sqrt{2}\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)5\mathrm{i}}{4}+\frac{\sqrt{2}\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)5\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

3.377 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$

Optimal. Leaf size=19

$$-\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

[Out] -1/8*ln(3+x)+1/8*ln(1+3*x)

Rubi [A]

time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6820, 630, 31}

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]

[Out] -1/8*Log[3 + x] + Log[1 + 3*x]/8

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx &= \int \frac{1}{3+10x+3x^2} dx \\
&= \frac{3}{8}\int \frac{1}{1+3x} dx - \frac{3}{8}\int \frac{1}{9+3x} dx \\
&= -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$-\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
[Out] -1/8*Log[3 + x] + Log[1 + 3*x]/8
```

Maple [A]
time = 0.18, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16
norman	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16
risch	$-\frac{\ln(3+x)}{8}+\frac{\ln(1+3x)}{8}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(3+10*x/(x^2+1)),x,method=_RETURNVERBOSE)
[Out] -1/8*ln(3+x)+1/8*ln(1+3*x)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.79

$$\frac{1}{8}\,\log\left(3\,x+1\right)-\frac{1}{8}\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Fricas [A]
time = 0.44, size = 15, normalized size = 0.79

$$\frac{1}{8}\,\log\left(3\,x+1\right)-\frac{1}{8}\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.74

$$\frac{\log\left(x+\frac{1}{3}\right)}{8}-\frac{\log\left(x+3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
```

```
[Out] log(x + 1/3)/8 - log(x + 3)/8
```

Giac [A]
time = 5.70, size = 17, normalized size = 0.89

$$\frac{1}{8}\log\left(|3\,x+1|\right)-\frac{1}{8}\log\left(|x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

Mupad [B]
time = 0.08, size = 8, normalized size = 0.42

$$-\frac{\operatorname{atanh}\left(\frac{3\,x}{4}+\frac{5}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)
```

```
[Out] -atanh((3*x)/4 + 5/4)/4
```

3.378 $\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=40

$$\frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1400, 715, 646, 31}

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567}\log(3x+2) + \frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
```

```
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^4}{2+13x+15x^2}\,dx\\
&= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\,(2+13x+15x^2)}\right)dx\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.00

$$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(13 + 2/x + 15*x),x]
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.78

method	result	size
default	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Fricas [A]
time = 0.42, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A]
time = 0.04, size = 34, normalized size = 0.85

$$\frac{x^{3}}{45}-\frac{13x^{2}}{450}+\frac{139x}{3375}+\frac{\log\left(x+\frac{1}{5}\right)}{4375}-\frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(13+2/x+15*x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Giac [A]
time = 3.80, size = 32, normalized size = 0.80

$$\frac{1}{45}\,x^{3}-\frac{13}{450}\,x^{2}+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(\left|5\,x+1\right|\right)-\frac{16}{567}\,\log\left(\left|3\,x+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

Mupad [B]
time = 2.09, size = 26, normalized size = 0.65

$$\frac{139\,x}{3375}-\frac{16\,\ln\left(x+\frac{2}{3}\right)}{567}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{13\,x^2}{450}+\frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(15*x + 2/x + 13),x)
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

3.379 $\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=33

$$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1400, 715, 646, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(13 + 2/x + 15*x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
```

```
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^3}{2+13x+15x^2}\,dx \\
&= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\,(2+13x+15x^2)}\right)dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx \\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.00, size = 33, normalized size = 1.00

$$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(13 + 2/x + 15*x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [**A**]
time = 0.02, size = 26, normalized size = 0.79

method	result	size
default	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26
norman	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26
risch	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A]
time = 0.41, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A]
time = 0.04, size = 27, normalized size = 0.82

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(13+2/x+15*x),x)
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Giac [A]
time = 4.82, size = 27, normalized size = 0.82

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(|5\,x+1|\right)+\frac{8}{189}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.64

$$\frac{8\,\ln\left(x+\frac{2}{3}\right)}{189}-\frac{13\,x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15*x + 2/x + 13),x)
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

3.380 $\int \frac{x}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1400, 717, 646, 31}

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

[In] Int[x/(13 + 2/x + 15*x),x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
```

```
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^2}{2+13x+15x^2}\,dx\\ &= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx\\ &= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx\\ &= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(13 + 2/x + 15*x),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.81

method	result	size
default	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A]
time = 0.40, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(5\,x + 1\right) - \frac{4}{63}\,\log\left(3\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x + \frac{1}{5}\right)}{175} - \frac{4\log\left(x + \frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Giac [A]
time = 3.60, size = 22, normalized size = 0.85

$$\frac{1}{15}\,x + \frac{1}{175}\,\log\left(\left|5\,x + 1\right|\right) - \frac{4}{63}\,\log\left(\left|3\,x + 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

Mupad [B]
time = 2.12, size = 16, normalized size = 0.62

$$\frac{x}{15} - \frac{4\,\ln\left(x + \frac{2}{3}\right)}{63} + \frac{\ln\left(x + \frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15*x + 2/x + 13),x)
```

```
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

3.381 $\int \frac{1}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1364, 646, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

[In] Int[(13 + 2/x + 15*x)^(-1),x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1364

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[(b
 + a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n
] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x}{2+13x+15x^2}\,dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(13 + 2/x + 15*x)^(-1),x]
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.86

method	result	size
default	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log\left(5\,x+1\right)+\frac{2}{21}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="maxima")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A]
time = 0.39, size = 17, normalized size = 0.81

$$-\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="fricas")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x)
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Giac [A]
time = 3.30, size = 19, normalized size = 0.90

$$-\frac{1}{35}\,\log\left(|5\,x+1|\right)+\frac{2}{21}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="giac")
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

Mupad [B]
time = 2.11, size = 13, normalized size = 0.62

$$\frac{2\,\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x + 2/x + 13),x)
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

3.382 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

[Out] -1/7*ln(2+3*x)+1/7*ln(1+5*x)

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1400, 630, 31}

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(13 + 2/x + 15*x)),x]

[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx = \int \frac{1}{2+13x+15x^2}\,dx$$
$$= \frac{15}{7}\int \frac{1}{3+15x}\,dx - \frac{15}{7}\int \frac{1}{10+15x}\,dx$$
$$= -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.86

method	result	size
default	$\frac{\ln(1+5x)}{7}-\frac{\ln(2+3x)}{7}$	18
norman	$\frac{\ln(1+5x)}{7}-\frac{\ln(2+3x)}{7}$	18
risch	$\frac{\ln(1+5x)}{7}-\frac{\ln(2+3x)}{7}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*ln(1+5*x)-1/7*ln(2+3*x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.81

$$\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A]
time = 0.39, size = 17, normalized size = 0.81

$$\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x)
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Giac [A]
time = 3.38, size = 19, normalized size = 0.90

$$\frac{1}{7}\,\log\left(|5\,x+1|\right)-\frac{1}{7}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

Mupad [B]
time = 0.08, size = 8, normalized size = 0.38

$$-\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(15*x + 2/x + 13)),x)
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

3.383 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {1400, 719, 29, 646, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(13 + 2/x + 15*x)),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x\,(2+13x+15x^2)}\,dx\\
&= \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\
&= \frac{\log(x)}{2} + \frac{45}{14}\int \frac{1}{10+15x}\,dx - \frac{75}{7}\int \frac{1}{3+15x}\,dx\\
&= \frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2} + \frac{3}{14}\log(2+3x) - \frac{5}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(13 + 2/x + 15*x)),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.81

method	result	size
default	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.89

$$\frac{\log\left(x\right)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(13+2/x+15*x),x)
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Giac [A]
time = 4.67, size = 24, normalized size = 0.89

$$-\frac{5}{7}\log\left(|5x+1|\right)+\frac{3}{14}\log\left(|3x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B]
time = 0.09, size = 17, normalized size = 0.63

$$\frac{3\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(15*x + 2/x + 13)),x)
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

3.384 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1400, 723, 814}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(13 + 2/x + 15*x)),x]

[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx\\ &= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(13 + 2/x + 15*x)),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A]
time = 0.02, size = 27, normalized size = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(13+2/x+15*x),x,method=_RETURNVERBOSE)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.76

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")
```

[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)

Fricas [A]
time = 0.42, size = 30, normalized size = 0.88

$$\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")

[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x

Sympy [A]
time = 0.06, size = 31, normalized size = 0.91

$$-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(13+2/x+15*x),x)

[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)

Giac [A]
time = 3.92, size = 29, normalized size = 0.85

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")

[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))

Mupad [B]
time = 0.03, size = 22, normalized size = 0.65

$$\frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln\left(x\right)}{4}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(15*x + 2/x + 13)),x)

[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)

3.385 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1400, 723, 814}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx\\ &= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\ &= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A]
time = 0.02, size = 32, normalized size = 0.78

method	result	size
risch	$\frac{\frac{13x}{4}-\frac{1}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	32
norman	$\frac{-\frac{1}{4}x+\frac{13}{4}x^2}{x^3}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.76

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(5\,x+1\right)+\frac{27}{56}\,\log\left(3\,x+2\right)+\frac{139}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Fricas [A]
time = 0.37, size = 39, normalized size = 0.95

$$-\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A]
time = 0.06, size = 36, normalized size = 0.88

$$\frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(13+2/x+15*x),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Giac [A]
time = 4.92, size = 34, normalized size = 0.83

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(\left|5\,x+1\right|\right)+\frac{27}{56}\,\log\left(\left|3\,x+2\right|\right)+\frac{139}{8}\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.63

$$\frac{27\,\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\,\ln\left(x\right)}{8}+\frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(15*x + 2/x + 13)),x)
```

```
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

3.386 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1400, 723, 814}

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1400

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx \\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.77

method	result	size
risch	$\frac{-\frac{139}{8}x^2+\frac{13}{8}x-\frac{1}{6}}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	37
norman	$\frac{-\frac{1}{6}x+\frac{13}{8}x^2-\frac{139}{8}x^3}{x^4}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(13+2/x+15*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.75

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(5\,x+1\right)-\frac{81}{112}\,\log\left(3\,x+2\right)-\frac{1417}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Fricas [A]
time = 0.39, size = 44, normalized size = 0.92

$$\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A]
time = 0.07, size = 41, normalized size = 0.85

$$-\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(13+2/x+15*x),x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

Giac [A]
time = 4.69, size = 39, normalized size = 0.81

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(\left|5\,x+1\right|\right)-\frac{81}{112}\,\log\left(\left|3\,x+2\right|\right)-\frac{1417}{16}\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.67

$$\frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(15*x + 2/x + 13)),x)
```

```
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

3.387 $\int \frac{x^2}{2-(1+x^2)^4} \, dx$

Optimal. Leaf size=157

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{}}}\right.}{4\ 2^{3/4}}$$

```
[Out] 1/8*I*arctan(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)-1/8*I*arcta
n(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(-1+2^(1
/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(1+2^(1/4))^(1/2))*(1+2
^(1/4))^(1/2)*2^(1/4)
```

Rubi [A]
time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6872, 212, 209, 1997, 211}

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\,\text{ArcTan}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\,\text{ArcTan}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\,\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\,\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) - ((I/4)*
Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) - (Sqrt[1 + 2^(1
/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh
[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx = \int \left(\frac{-\sqrt[4]{2}+\sqrt{2}}{8\left(-1+\sqrt[4]{2}-x^2\right)}+\frac{-\sqrt[4]{2}-\sqrt{2}}{8\left(1+\sqrt[4]{2}+x^2\right)}+\frac{-\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{2}+}{8\left(\sqrt[4]{2}+i\left(\right.\right.}\right.$$

$$= -\frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}-x^2}\,dx}{4\ 2^{3/4}} - \frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i(1+x^2)}\,dx}{4\ 2^{3/4}} - \frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i}}{4\ 2^{3/4}}$$

$$= -\frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} + \frac{\sqrt{-1+\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{(1}{}$$

$$= \frac{i\sqrt{1-i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}} - \frac{\sqrt{1+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[-1+4\#1^2+6\#1^4+4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{1+3\#1^2+3\#1^4+\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] -1/8*RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 +
 3*#1^2 + 3*#1^4 + #1^6) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 54, normalized size = 0.34

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+4\_Z^{6}+6\_Z^{4}+4\_Z^{2}-1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{\_R^{7}+3\_R^{5}+3\_R^{3}+\_R}\right)}{8}$	54
risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+4\_Z^{6}+6\_Z^{4}+4\_Z^{2}-1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{\_R^{7}+3\_R^{5}+3\_R^{3}+\_R}\right)}{8}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^4
+4*_Z^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. 2(97) = 194.
time = 1.15, size = 1506, normalized size = 9.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^
(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log
(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(
2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(
3/4) - sqrt(2)) + 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(
2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4
) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) -
 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) +
```

```
 sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(1/2*sqrt
(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
 sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 -
 (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) + 4*sqrt(-
3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) -
3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2
^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2
^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4)
+ sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4)
- sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4
) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2
))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2
+ 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2
+ 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)
*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(
2))*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sq
rt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16
*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4)
 + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4
*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(
2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4)
 - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*
(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*s
qrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) - sqr
t(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2))
 - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) - sqrt(2))*l
og(1/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))
^2 - ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2
) - 6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) - sqrt(2))*log(-1
/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))^2 -
 ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2) -
6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*lo
g(4*((2^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4
) - 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*log(-4*((2
^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4) - 1/2
56*sqrt(2)) + 3*x)
```

Sympy [A]

time = 0.10, size = 41, normalized size = 0.26

$$-\mathrm{RootSum}\left(1073741824t^8-65536t^4+1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}-\frac{262144t^5}{3}-\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(x**2+1)**4),x)
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")
[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)
```

Mupad [B]
time = 2.75, size = 144, normalized size = 0.92

$$\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56x-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(4096x-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2\left(262144x+\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2x\,67108864\right)\right)+256\right)\right)-1\right)\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 + 1)^4 - 2),x)
[Out] symsum(log(- root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(56*x
 - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 + z^2/1048576 - 1/1073741824, z, k)*(4096*x - root(z^8 - z^4/16384 +
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x + 67108864*root(z^8 - z^4/16
384 + z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 + z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

3.388 $\int \frac{x^2}{2-(1-x^2)^4}\,dx$

Optimal. Leaf size=157

$$-\frac{\sqrt{-1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{4\ 2^{3/4}}$$

```
[Out] -1/8*I*arctanh(x/(1-I*2^(1/4))^(1/2))*(1-I*2^(1/4))^(1/2)*2^(1/4)+1/8*I*arc
tanh(x/(1+I*2^(1/4))^(1/2))*(1+I*2^(1/4))^(1/2)*2^(1/4)-1/8*arctan(x/(-1+2^
(1/4))^(1/2))*(-1+2^(1/4))^(1/2)*2^(1/4)+1/8*arctanh(x/(1+2^(1/4))^(1/2))*(
1+2^(1/4))^(1/2)*2^(1/4)
```

Rubi [A]

time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6872, 212, 209, 1997, 214}

$$-\frac{\sqrt{\sqrt[4]{2}-1}\,\text{ArcTan}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -1/4*(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/2^(3/4) - ((I/4)*Sqr
t[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) + ((I/4)*Sqrt[1 +
I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) + (Sqrt[1 + 2^(1/4)]*Arc
Tanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2-(1-x^2)^4}\,dx = \int \left(\frac{\sqrt[4]{2}+\sqrt{2}}{8\left(1+\sqrt[4]{2}-x^2\right)}+\frac{\sqrt[4]{2}-\sqrt{2}}{8\left(-1+\sqrt[4]{2}+x^2\right)}+\frac{\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{2}-\cdots}{8\left(\sqrt[4]{2}+i\cdots\right.}\right.$$

$$=\frac{\left(1-\sqrt[4]{2}\right)\int\frac{1}{-1+\sqrt[4]{2}+x^2}\,dx}{4\ 2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int\frac{1}{\sqrt[4]{2}+i(1-x^2)}\,dx}{4\ 2^{3/4}}+\frac{\left(1+i\sqrt[4]{2}\right)\int\frac{1}{\sqrt[4]{2}-i(\cdots}}{4\ 2^{3/4}}$$

$$=-\frac{\sqrt{-1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\cdots$$

$$=-\frac{\sqrt{-1+\sqrt[4]{2}}\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[-1-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -1/8*RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1
+ 3*#1^2 - 3*#1^4 + #1^6) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 56, normalized size = 0.36

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56
risch	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^4
-4*_Z^2-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(97) = 194$.
time = 1.20, size = 1546, normalized size = 9.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2
^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*lo
g(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt
(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^
(3/4) + sqrt(2)) + 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sq
rt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2)
 + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sq
rt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(-1/2*s
```

```
qrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4
) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4)
 - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^
2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) + 4*((s
qrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4
) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) +
 sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)
*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) +
 sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) -
 sqrt(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(
3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(
3/4) - sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2
^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - s
qrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)
) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))
*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*
sqrt(2))*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4
) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x)
+ 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(
2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*l
og(-1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sq
rt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(
2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) +
sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + s
qrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - s
qrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(
2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) + sq
rt(2))*log(1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sq
rt(2) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) +
4*sqrt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) + sqrt(2)
)*log(-1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sqrt(2
) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) + 4*sq
rt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) + 1/256*sqr
t(2))*log(4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/25
6*2^(3/4) + 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) + 1/256*sqrt(2))*lo
g(-4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/256*2^(3/
4) + 1/256*sqrt(2)) + 3*x)
```

Sympy [**A**]

time = 0.10, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8-65536t^4-1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}+\frac{262144t^5}{3}+\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(-x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)
```

Mupad [B]
time = 2.80, size = 142, normalized size = 0.90

$$\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x+\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(4096\,x+\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2\left(262144\,x-\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2x\,67108864\right)\right)+256\right)\right)-1\right)\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)^4 - 2),x)
```

```
[Out] symsum(log(- root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(56*x
 + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 - z^2/1048576 - 1/1073741824, z, k)*(4096*x + root(z^8 - z^4/16384 -
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x - 67108864*root(z^8 - z^4/16
384 - z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 - z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

3.389 $\int \frac{x^2}{2+(1+x^2)^4}\,dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1-}\cdots}{\cdots}$$

```
[Out] 1/8*(-1)^(1/4)*arctan(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4)-
1/8*(-1)^(3/4)*2^(1/4)*arctan(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))^(1
/2)-1/8*(-1)^(1/4)*arctan(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*2^(1
/4)+1/8*I*arctan(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*((1+I)
/(1+I+2^(3/4)))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6872, 210, 209, 1997, 211}

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\text{ArcTan}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\text{ArcTan}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\text{ArcTan}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\,\text{ArcTan}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)
) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/(4
*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4)]]
)/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4
))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2+(1+x^2)^4}\,dx = \int \left(\frac{-\sqrt[4]{-2}+i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}-x^2\right)}+\frac{-\sqrt[4]{-2}-i\sqrt{2}}{8\left(1+\sqrt[4]{-2}+x^2\right)}+\frac{-\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1+x^2\right)\right)}+\frac{-\sqrt[4]{-}}{8\left(\sqrt[4]{-2}\right.}\right.$$

$$= \frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}+i\left(1+x^2\right)}\,dx+\frac{1}{8}\left(-\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{1+\sqrt[4]{-2}+x^2}$$

$$= \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-}}}\right.}{4\,2^{3/4}}$$

$$= \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tan^{-1}\left(\frac{x}{\sqrt{1+i}}\right.}{4\,2^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[3+4\#1^2+6\#1^4+4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{1+3\#1^2+3\#1^4+\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 + 3*#1^
2 + 3*#1^4 + #1^6) & ]/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 54, normalized size = 0.29

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8}$	54
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7+3\_R^5+3\_R^3+\_R}}{8}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8+4*_Z^6+6*_Z^4+
4*_Z^2+3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2271 vs. 2(118) = 236.
time = 1.21, size = 2271, normalized size = 12.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
 128*sqrt(1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2))) - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 -
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12
```

```
288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(
2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) - sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(
-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/
8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2
)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I
*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-
I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqr
t(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))
- sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(sqrt(-
12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*
I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192
*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(
-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/819
2*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqr
t(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(
2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*
(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*
sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1
/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) +
128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I
*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2))
 + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/2
56*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/
2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))
```

```
)*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)
) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + 163
84*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(1/256*I*s
qrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/
8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))
^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
- sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I*sqrt(2) +
 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2
) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8
192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(
2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt...
```

Sympy [A]
time = 0.08, size = 39, normalized size = 0.21

$$\operatorname{RootSum}\left(1073741824t^8+65536t^4+1024t^2+3,\left(t\mapsto t\log\left(67108864t^7-262144t^5+4096t^3+40t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(x**2+1)**4),x)
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

Mupad [B]
time = 2.78, size = 142, normalized size = 0.76

$$\sum_{k=1}^{8}\ln\left(\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x+\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(4096\,x-\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2\left(786432\,x-\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2x\,67108864\right)\right)-768\right)\right)-3\right)\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)^4 + 2),x)
[Out] symsum(log(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(40*x +
 root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 + z^2/1048576 + 3/1073741824, z, k)*(4096*x - root(z^8 + z^4/16384 + z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x - 67108864*root(z^8 + z^4/1638
4 + z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 + z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

3.390 $\int \frac{x^2}{2+(1-x^2)^4}\,dx$

Optimal. Leaf size=188

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{}}{}$$

```
[Out] -1/8*(-1)^(1/4)*arctanh(x/(1-(-2)^(1/4))^(1/2))*(1-(-2)^(1/4))^(1/2)*2^(1/4
)+1/8*(-1)^(3/4)*2^(1/4)*arctanh(x/(1+I*(-2)^(1/4))^(1/2))*(1+I*(-2)^(1/4))
^(1/2)+1/8*(-1)^(1/4)*arctanh(x/(1+(-2)^(1/4))^(1/2))*(1+(-2)^(1/4))^(1/2)*
2^(1/4)-1/8*I*arctanh(x*((1+I)/(1+I+2^(3/4)))^(1/2))*((-2)^(1/4)+2^(1/2))*(
(1+I)/(1+I+2^(3/4)))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6872, 212, 213, 1997, 214}

$$-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\tanh^{-1}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] -1/4*((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/2^(3
/4) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]])
/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^(1/
4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^
(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{2+(1-x^2)^4}\,dx &= \int \left(\frac{\sqrt[4]{-2}+i\sqrt{2}}{8\left(1+\sqrt[4]{-2}-x^2\right)}+\frac{\sqrt[4]{-2}-i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}+x^2\right)}+\frac{\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{-}}{8\left(\sqrt[4]{-2}\right.}\right.\\ &= \frac{1}{8}\left(\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}-i\left(1-x^2\right)}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{-1+\sqrt[4]{-2}+x^2}\\ &= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{}{\sqrt{1}}\right.}{4\ 2^{3/4}}\\ &= -\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{}{\sqrt{}}\right.}{4\ 2^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[3-4\#1^2+6\#1^4-4\#1^6+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+3\#1^2-3\#1^4+\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1 + 3*#1
^2 - 3*#1^4 + #1^6) & ]/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 56, normalized size = 0.30

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}\right)}{8}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8-4*_Z^6+6*_Z^4-
4*_Z^2+3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2259 vs. 2(118) = 236.
time = 1.25, size = 2259, normalized size = 12.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2
)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 +
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-122
```

88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2
) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
sqrt(2))) + sqrt(2)(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1
/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/81
92*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) +
sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*
(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + s
qrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(sqrt(-1228
8*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqr
t(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*s
qrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*
sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/25
6*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sq
rt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt
(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*
sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*s

```
qrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32
*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(
1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sq
rt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sq
rt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/
8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2
))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sq
rt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sq
rt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*
sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*...
```

Sympy [A]
time = 0.08, size = 39, normalized size = 0.21

$$\operatorname{RootSum}\left(1073741824t^8 + 65536t^4 - 1024t^2 + 3, \left(t \mapsto t\log\left(67108864t^7 + 262144t^5 + 4096t^3 - 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(-x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

Mupad [B]
time = 2.74, size = 142, normalized size = 0.76

$$\sum_{k=1}^{8}\ln\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x-\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(4096\,x+\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2\left(786432\,x+\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2 x\,67108864\right)\right)-768\right)\right)-3\right)\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(40*x -
 root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 - z^2/1048576 + 3/1073741824, z, k)*(4096*x + root(z^8 + z^4/16384 - z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x + 67108864*root(z^8 + z^4/1638
4 - z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 - z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

3.391 $\int \frac{1-x^2}{a+b(1-x^2)^4}\,dx$

Optimal. Leaf size=663

$$-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\,\sqrt{-a}\,\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}}-$$

```
[Out] -1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^(3
/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)+(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1
/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(1/4)+(
(-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/
2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/
((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)-b
^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)
^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2)
)^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*x*2^(1/2)*(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$,
Rules used = {6872, 2015, 1180, 211, 214, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(-\sqrt{2}\,\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(\sqrt{2}\,\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -1/4*ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(Sqrt[-a]*Sqrt[(-a)^(1/
4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqr
t[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*
b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt
[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b
]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + Arc
Tanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) +
```

```
b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt
[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*
x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqr
t[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*
Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = \int \left(-\frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2\right)} - \frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2\right)}\right) dx$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}-b(1-x^2)^2}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}+b(1-x^2)^2}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(1+\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} - \frac{\int \cdots}{4\sqrt{2}\sqrt{-a}\sqrt{\cdots}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{\cdots}}}\cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\cdots}}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{\cdots}}}}{\sqrt{\sqrt{\sqrt{-a}\cdots}}}\cdots\right.}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order

3 in optimal.
time = 0.03, size = 63, normalized size = 0.10

$$-\frac{\text{RootSum}\left[a+b-4b\#1^2+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 69, normalized size = 0.10

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69
risch	$\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z^
6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.90, size = 322185, normalized size = 485.95

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(1/2)*sqrt(1/6)*sqrt((6*sqrt(1/2)*sqrt(1/6)*(a^2 + a*b)*sqrt(-((a^4
*b + 2*a^3*b^2 + a^2*b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((a^2*b*sqrt(-((2
*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(
a*b)) + a^2*b^4*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) + a*b^2*sqrt(-((2*a*sqrt(-1
/(a*b)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^
2*b^4*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) - 3*b*sqrt(-1/(a*b)) - 1)^2/(a^2*b*sq
rt(-1/(a*b)) + a*b^2*sqrt(-1/(a*b)))^2 + 3*(2*(b^2*sqrt(-((2*a*sqrt(-1/(a*b
)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^2*b^4
*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) + b*sqrt(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)
/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^2*b^4*sqrt(-1/(a*b)
))))*a - 3*b*sqrt(-1/(a*b)) - 1)/(a^3*b^2*sqrt(-1/(a*b)) + a^2*b^3*sqrt(-1/
(a*b))))/(27*(a^5*b^2*(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(
a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^2*b^4*sqrt(-1/(a*b))))^(3/2)*sqrt(-1/(
a*b)) + 2*a^4*b^3*(-((2*a ...
```

Sympy [A]
time = 2.16, size = 133, normalized size = 0.20

$$-\operatorname{RootSum}\left(t^{8}\cdot\left(16777216a^{5}b^{3}+16777216a^{4}b^{4}\right)+1048576t^{6}a^{3}b^{3}+24576t^{4}a^{2}b^{2}+256t^{2}ab+1,\left(t\mapsto t\log\left(-6291456t^{7}a^{4}b^{3}-6291456t^{7}a^{3}b^{4}+65536t^{5}a^{3}b^{2}-327680t^{5}a^{2}b^{3}-512t^{3}a^{2}b-5632t^{3}ab^{2}-32tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Mupad [B]
time = 2.70, size = 328, normalized size = 0.49

$$\sum_{k=1}^{8}\ln\left(a\,b^5\,\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2\,a\,b\,64+1\right)\,\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^4\,a^2\,b^2\,4096+\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2\,a\,b\,128-\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^5\,a^3\,b^2\,x\,32768+1\right)\right)\,\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)

[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
 z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

3.392 $\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx$

Optimal. Leaf size=663

$$-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}}+$$

```
[Out] -1/4*arctan(b^(1/8)*x/((-a)^(1/4)-b^(1/4))^(1/2))/b^(3/8)/(-a)^(1/2)/((-a)^
(1/4)-b^(1/4))^(1/2)+1/4*arctanh(b^(1/8)*x/((-a)^(1/4)+b^(1/4))^(1/2))/b^(3
/8)/(-a)^(1/2)/((-a)^(1/4)+b^(1/4))^(1/2)-1/8*arctan((-b^(1/8)*x*2^(1/2)+(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1
/2))^(1/2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a
)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)+1/8*arctan((b^(1/8)*x*2^(1/2)+(b^(1/4)+(
(-a)^(1/2)+b^(1/2))^(1/2))^(1/2))/(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/
2))*(-b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/
((-a)^(1/2)+b^(1/2))^(1/2)+1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)-b
^(1/8)*x*2^(1/2)*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)
^(1/2)+b^(1/2))^(1/2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2)
)^(1/2)-1/16*ln(b^(1/4)*x^2+((-a)^(1/2)+b^(1/2))^(1/2)+b^(1/8)*x*2^(1/2)*(b
^(1/4)+((-a)^(1/2)+b^(1/2))^(1/2))^(1/2))*(b^(1/4)+((-a)^(1/2)+b^(1/2))^(1/
2))^(1/2)/b^(3/8)*2^(1/2)/(-a)^(1/2)/((-a)^(1/2)+b^(1/2))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6872, 2015, 1180, 211, 214, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -1/4*ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(Sqrt[-a]*Sqrt[(-a)^(1/
4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqr
t[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*
b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt
[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b
]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + Arc
Tanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) +
```

```
b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt
[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*
x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (
Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqr
t[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*
Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = -\int \frac{-1+x^2}{a+b\left(-1+x^2\right)^4}\,dx$$

$$= -\int \left(-\frac{\sqrt{b}\,(-1+x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2\right)} - \frac{\sqrt{b}\,(-1+x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right.\right.}\right.$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}-b(-1+x^2)^2}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}+b(-1+x^2)^2}\,dx}{2\sqrt{-a}}$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(-1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\cdots}}}{\cdots}}{\frac{\sqrt{\sqrt{-a}\cdots}}{\sqrt[4]{b}}\cdots}}{4\sqrt{2}\sqrt{-a}\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\cdots}}\right.\cdots}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}\cdots}}}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\cdots}}}{\sqrt{\sqrt{\sqrt{-\cdots}}}}\right.}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 63, normalized size = 0.10

$$-\frac{\text{RootSum}\left[a+b-4b\#1^2+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 69, normalized size = 0.10

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6\_Z^4b-4\_Z^2b+a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^7-3\_R^5+3\_R^3-\_R}}{8b}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(x^2-1)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z^
6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.91, size = 322185, normalized size = 485.95

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(1/2)*sqrt(1/6)*sqrt((6*sqrt(1/2)*sqrt(1/6)*(a^2 + a*b)*sqrt(-((a^4
*b + 2*a^3*b^2 + a^2*b^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((a^2*b*sqrt(-((2
*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(
a*b)) + a^2*b^4*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) + a*b^2*sqrt(-((2*a*sqrt(-1
/(a*b)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^
2*b^4*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) - 3*b*sqrt(-1/(a*b)) - 1)^2/(a^2*b*sq
rt(-1/(a*b)) + a*b^2*sqrt(-1/(a*b)))^2 + 3*(2*(b^2*sqrt(-((2*a*sqrt(-1/(a*b
)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^2*b^4
*sqrt(-1/(a*b))))*sqrt(-1/(a*b)) + b*sqrt(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)
/(a^4*b^2*sqrt(-1/(a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^2*b^4*sqrt(-1/(a*b)
))))*a - 3*b*sqrt(-1/(a*b)) - 1)/(a^3*b^2*sqrt(-1/(a*b)) + a^2*b^3*sqrt(-1/
(a*b))))/(27*(a^5*b^2*(-((2*a*sqrt(-1/(a*b)) + 1)*b - a)/(a^4*b^2*sqrt(-1/(
a*b)) + 2*a^3*b^3*sqrt(-1/(a*b)) + a^2*b^4*sqrt(-1/(a*b))))^(3/2)*sqrt(-1/(
a*b)) + 2*a^4*b^3*(-((2*a ...
```

Sympy [A]
time = 2.15, size = 133, normalized size = 0.20

$$-\operatorname{RootSum}\left(t^{8}\cdot\left(16777216a^{5}b^{3}+16777216a^{4}b^{4}\right)+1048576t^{6}a^{3}b^{3}+24576t^{4}a^{2}b^{2}+256t^{2}ab+1,\left(t\mapsto t\log\left(-6291456t^{7}a^{4}b^{3}-6291456t^{7}a^{3}b^{4}+65536t^{5}a^{3}b^{2}-327680t^{5}a^{2}b^{3}-512t^{3}a^{2}b-5632t^{3}ab^{2}-32tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Mupad [B]
time = 0.00, size = 328, normalized size = 0.49

$$\sum_{k=1}^{8}\ln\left(a\,b^5\,\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2\,a\,b\,64+1\right)\,\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^4\,a^2\,b^2\,4096+\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2\,a\,b\,128-\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^5\,a^3\,b^2\,x\,32768+1\right)\right)\,\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

```
[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
 z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

3.393 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} \, dx$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}}$$

```
[Out] 1/3*arctan(x*(a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(a^(1/3)+b^(1/3))^(1/
2)+1/3*arctan(x*(-(-1)^(1/3)*a^(1/3)+b^(1/3))^(1/2)/b^(1/6))/b^(5/6)/(-(-1)
^(1/3)*a^(1/3)+b^(1/3))^(1/2)+1/3*arctan(x*((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/
2)/b^(1/6))/b^(5/6)/((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/2)
```

Rubi [F]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a*x^6 + b*(
1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} \, dx &= \int \left(\frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{2x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{}{b+3bx^2}\right. \\
&= 2\int \frac{x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} \, dx+\int \frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} \, dx+ \\
&= 2\int \frac{x^2}{ax^6+b(1+x^2)^3} \, dx+\int \frac{1}{ax^6+b(1+x^2)^3} \, dx+\int \frac{x^4}{ax^6+b(1+x^2)^3} \, dx
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 95, normalized size = 0.57

$$\frac{1}{6}\text{RootSum}\left[b+3b\#1^2+3b\#1^4+a\#1^6+b\#1^6\&,\frac{\log(x-\#1)+2\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{b\#1+2b\#1^3+a\#1^5+b\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log[
x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) & ]/6
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.30, size = 67, normalized size = 0.40

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{6}+3\_Z^{4}b+3\_Z^{2}b+b\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(x-\_R\right)}{\_R^{5}a+\_R^{5}b+2\_R^{3}b+\_Rb}\right)}{6}$	67
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^{6}+3\_Z^{4}b+3\_Z^{2}b+b\right)}\frac{\left(\_R^{4}+2\_R^{2}+1\right)\ln\left(x-\_R\right)}{\_R^{5}a+\_R^{5}b+2\_R^{3}b+\_Rb}\right)}{6}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(x-_R),_R=RootOf((a
+b)*_Z^6+3*_Z^4*b+3*_Z^2*b+b))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.12, size = 5653, normalized size = 33.65

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-
1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b
+ b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9331
2/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)
```

^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*log(1/6*sqrt(1/2)*s
qrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 +
b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/933
12*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*b + x) - 1/36*sqrt(1/2)*sqrt((-I*sq
rt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/
31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a +
b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/
((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*
b^5))^(1/3) - 72/(a*b + b^2))*log(-1/6*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(
a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/
3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*
(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 7
2/(a*b + b^2))*b + x) + 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3
+ b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)
*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) -
1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*
b + b^2)) + 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*s
qrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1
/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a
+ b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104
/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2
*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(
a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/
3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*
(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 7
2/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b +
b^2))*log(1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1
/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2
)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqr
t(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1
/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) +
3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)
*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*
b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5)
)^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3
) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4
) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(
I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)

```
) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^
2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x)
 - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^
2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/466
56/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*
(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*
b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/
3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3
 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)
*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) -
1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*
b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b
 + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) -
1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3)
+ 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/4665
6/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 2073
6*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)...
```

Sympy [**A**]
time = 1.08, size = 42, normalized size = 0.25

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656ab^{5}+46656b^{6}\right)+3888t^{4}b^{4}+108t^{2}b^{2}+1,\left(t\mapsto t\log\left(6tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

```
[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b**
2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

Mupad [**B**]
time = 3.08, size = 504, normalized size = 3.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)
```

```
[Out] symsum(log(-3*a^3*(a + b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*
b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 46656
*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*a*b
^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3 - 60
*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z,
k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b
^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b
^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 46656*b^6*z^
6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^5*z^6 + 4
6656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*root(46656
*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*a*b^2*
x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2
+ 1, z, k)^5*a*b^4*x - 1))*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*
z^4 + 108*b^2*z^2 + 1, z, k), k, 1, 6)
```

3.394 $\int \frac{(d+ex)^3}{a+cx^4}\,dx$

Optimal. Leaf size=320

$$\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}-\frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{3/4}c^{3/4}}$$

```
[Out] 1/4*e^3*ln(c*x^4+a)/c+3/2*d^2*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
-1/8*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)+d
^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*c^(1/2))*(-3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*d
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c
^(3/4)*2^(1/2)+1/4*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e^2*a^(1/2)+d^2
*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$-\frac{d\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{3d^2e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}+\frac{e^3\log\left(a+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d
^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a + c*x^
4])/(4*c)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3}{a+cx^4}\,dx &= \int \left(\frac{d^3+3de^2x^2}{a+cx^4}+\frac{x(3d^2e+e^3x^2)}{a+cx^4}\right)dx \\
&= \int \frac{d^3+3de^2x^2}{a+cx^4}\,dx+\int \frac{x(3d^2e+e^3x^2)}{a+cx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{3d^2e+e^3x}{a+cx^2}\,dx,x,x^2\right)+\frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-3e^2\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c}+\frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}\right.\right.}{} \\
&= \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)+\frac{1}{2}e^3\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)+\frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+\right.\right.}{} \\
&= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}-\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\right.}{} \\
&= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}-\frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right.}{2\sqrt{}}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 322, normalized size = 1.01

$$
\frac{-2\sqrt[4]{a}\,\sqrt[4]{c}\,d\left(\sqrt{2}\,\sqrt{c}\,d^2+6\sqrt[4]{a}\,\sqrt[4]{c}\,de+3\sqrt{2}\,\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\,\sqrt[4]{c}\,d\left(\sqrt{2}\,\sqrt{c}\,d^2-6\sqrt[4]{a}\,\sqrt[4]{c}\,de+3\sqrt{2}\,\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-\sqrt{2}\,\sqrt[4]{c}\left(\sqrt[4]{a}\,\sqrt{c}\,d^3-3a^{3/4}de^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\sqrt{2}\,\sqrt[4]{c}\left(\sqrt[4]{a}\,\sqrt{c}\,d^3-3a^{3/4}de^2\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+2ae^3\log\left(a+cx^4\right)}{8ac}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt
[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1/4
)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d
^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8*a*c)
```

Maple [A]
time = 0.21, size = 250, normalized size = 0.78

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^3e^3+3\_R^2de^2+3\_Rd^2e+d^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{3d^2e\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d^3*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(
x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2
*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+3/2*d^2*e/(a*c)^(1/2)*arctan(x^2*(c/a)^(1
/2))+3/8*d*e^2/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(
1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*e^3*ln(c*x^4+a)/c
```

Maxima [A]
time = 0.50, size = 306, normalized size = 0.96

$$\frac{\sqrt{2}\left(cd^3-3\sqrt{a}\sqrt{c}\,de^2+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^3\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(cd^3-3\sqrt{a}\sqrt{c}\,de^2-\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^3\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{5}{4}}d^3-6\sqrt{a}\,cd^2e+3\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+6\sqrt{a}\,cd^2e+3\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(c*d^3 - 3*sqrt(a)*sqrt(c)*d*e^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^3)
*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) -
 1/8*sqrt(2)*(c*d^3 - 3*sqrt(a)*sqrt(c)*d*e^2 - sqrt(2)*a^(3/4)*c^(1/4)*e^3
)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
+ 1/4*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 - 6*sqrt(a)*c*d^2*e + 3*sqrt(2)*a^(3/4)*
c^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 1/4*(sqrt(2
```

```
)*a^(1/4)*c^(5/4)*d^3 + 6*sqrt(a)*c*d^2*e + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
```

Fricas [C] Result contains complex when optimal does not.
time = 20.59, size = 141845, normalized size = 443.27

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/3456*(2*((-I*sqrt(3) + 1)*((sqrt(6)*a*c*sqrt(-1/(a*c))*sqrt(-(6*a*c*d^4*e
^2*sqrt(-1/(a*c)) + c*d^6 - 9*a*d^2*e^4)/(a^2*c^2*sqrt(-1/(a*c)))) + 3*sqrt
(6)*(a*e^3*sqrt(-1/(a*c)) - d^2*e))^2/(a*c) - 3*sqrt(6)*(sqrt(6)*c*d^6 - 2*
sqrt(6)*a^2*c*e^3*sqrt(-1/(a*c))*sqrt(-(6*a*c*d^4*e^2*sqrt(-1/(a*c)) + c*d^
6 - 9*a*d^2*e^4)/(a^2*c^2*sqrt(-1/(a*c)))) + 6*sqrt(6)*a*c*d^2*e*sqrt(-(6*a
*c*d^4*e^2*sqrt(-1/(a*c)) + c*d^6 - 9*a*d^2*e^4)/(a^2*c^2*sqrt(-1/(a*c))))
- 3*sqrt(6)*(a^2*e^6*sqrt(-1/(a*c)) + (5*c*d^4*e^2*sqrt(-1/(a*c)) + d^2*e^4
)*a))/(a^2*c^2*sqrt(-1/(a*c)))))/(-1/2304*(sqrt(6)*c*d^6 - 2*sqrt(6)*a^2*c*e
^3*sqrt(-1/(a*c))*sqrt(-(6*a*c*d^4*e^2*sqrt(-1/(a*c)) + c*d^6 - 9*a*d^2*e^4
)/(a^2*c^2*sqrt(-1/(a*c)))) + 6*sqrt(6)*a*c*d^2*e*sqrt(-(6*a*c*d^4*e^2*sqrt
(-1/(a*c)) + c*d^6 - 9*a*d^2*e^4)/(a^2*c^2*sqrt(-1/(a*c)))) - 3*sqrt(6)*(a^
2*e^6*sqrt(-1/(a*c)) + (5*c*d^4*e^2*sqrt(-1/(a*c)) + d^2*e^4)*a))*(sqrt(6)*
a*c*sqrt(-1/(a*c))*sqrt(-(6*a*c*d^4*e^2*sqrt(-1/(a*c)) + c*d^6 - 9*a*d^2*e^
4)/(a^2*c^2*sqrt(-1/(a*c) ...
```

Sympy [A]
time = 27.77, size = 384, normalized size = 1.20

$$\operatorname{RootSum}\left(256t^4a^3c^4 - 256t^3a^3c^3e^3 + t^2\cdot\left(96a^3c^2e^6 + 480a^2c^3d^4e^2\right) + t\left(-16a^3ce^9 + 192a^2c^2d^4e^5 - 48ac^3d^8e\right) + a^3e^{12} + 3a^2cd^4e^8 + 3ac^2d^8e^4 + c^3d^{12}, \left(t \mapsto t\log\left(x + \frac{1728t^3a^4c^3e^6 + 960t^3a^3c^4d^4e^2 - 1296t^2a^4c^2e^9 - 2016t^2a^3c^3d^4e^5 + 48t^2a^2c^4d^8e + 324ta^4ce^{12} + 4716ta^3c^2d^4e^8 + 1452ta^2c^3d^8e^4 + 4tac^4d^{12} - 27a^4e^{15} + 1119a^3cd^4e^{11} - 609a^2c^2d^8e^7 - 91ac^3d^{12}e^3}{729a^3cd^3e^{12} - 1053a^2c^2d^7e^8 - 117ac^3d^{11}e^4 + c^4d^{15}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c**
2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d**
4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**
8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 + 960
*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c*
*3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a*
*3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 - 27*a*
*4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**
12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**
11*e**4 + c**4*d**15))))
```

Giac [A]
time = 3.10, size = 311, normalized size = 0.97

$$\frac{e^{3} \log\left(\left|cx^{4} + a\right|\right)}{4\,c} + \frac{\sqrt{2}\left(3\,\sqrt{2}\,\sqrt{ac}\,c^{2}d^{2}e + \left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3} + 3\,\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^{3}} + \frac{\sqrt{2}\left(3\,\sqrt{2}\,\sqrt{ac}\,c^{2}d^{2}e + \left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3} + 3\,\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^{3}} + \frac{\sqrt{2}\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3} - 3\,\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\log\left(x^{2} + \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^{3}} - \frac{\sqrt{2}\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3} - 3\,\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\log\left(x^{2} - \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*e^3*log(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e
+ (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c
)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3
)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d
*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B]
time = 2.84, size = 894, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4),x)
```

```
[Out] symsum(log(-2*c*d^2*(5*a*d*e^6 - 3*c*d^5*e^2 + 3*a*e^7*x + 8*root(256*a^3*c
^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2
 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*
e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*d + 2*root(256*a
^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6
*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*
d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*c^2*d^4*x - 5*c*d^4*
e^3*x - 24*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2
*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a
^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)
^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*
d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z
 - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12
, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*
c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8
*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*
e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^
2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d
^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^
3*e^12, z, k), k, 1, 4)
```

3.395 $\int \frac{(d+ex)^2}{a+cx^4} \, dx$

Optimal. Leaf size=291

$$\frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}-\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}-$$

```
[Out] d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)-1/8*ln(-a^(1/4)*c^(1/4)*x*2
^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1
/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2
*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^
(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{de\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqr
t[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right)dx$$

$$= (2de)\int \frac{x}{a+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+cx^4}\,dx$$

$$= (de)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int}{2c}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{}{\frac{\sqrt{a}}{\sqrt{c}}+}}{4c}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}\,d^2-\right.}{}$$

$$= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}}{2\sqrt{2}\,a^{3/4}}$$

Mathematica [A]
time = 0.07, size = 243, normalized size = 0.84

$$\frac{-2\left(\sqrt{2}\sqrt{c}\,d^2+4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt{c}\,d^2-4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)}{8a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c
^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
)/(8*a^(3/4)*c^(3/4))
```

Maple [A]
time = 0.20, size = 230, normalized size = 0.79

method	result

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d^2\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{\sqrt{ac}}+ -$ e^2

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/8*d^2*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+d*e/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*e^2/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))

Maxima [A]
time = 0.51, size = 273, normalized size = 0.94

$$\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2-4\sqrt{a}\sqrt{c}\,de+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+4\sqrt{a}\sqrt{c}\,de+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 - 4*sqrt(a)*sqrt(c)*d*e + sqrt(2)*a^(3/4)*c^(1/4)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 4*sqrt(a)*sqrt(c)*d*e + sqrt(2)*a^(3/4)*c^(1/4)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))

Fricas [C] Result contains complex when optimal does not.
time = 3.04, size = 86139, normalized size = 296.01

too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")

```
[Out] 1/288*((-I*sqrt(3) + 1)*((a*c*sqrt(-1/(a*c))*sqrt(-(2*a*c*d^2*e^2*sqrt(-1/(
a*c)) + c*d^4 - a*e^4)/(a^2*c^2*sqrt(-1/(a*c)))) - 2*d*e)^2/(a*c) - 3*(c*d^
4 - (e^4 + 2*(3*d^2*e^2*sqrt(-1/(a*c)) - 2*d*e*sqrt(-(2*a*c*d^2*e^2*sqrt(-1
/(a*c)) + c*d^4 - a*e^4)/(a^2*c^2*sqrt(-1/(a*c)))))*c)*a)/(a^2*c^2*sqrt(-1/
(a*c))))/(-1/384*(c*d^4 - (e^4 + 2*(3*d^2*e^2*sqrt(-1/(a*c)) - 2*d*e*sqrt(-
(2*a*c*d^2*e^2*sqrt(-1/(a*c)) + c*d^4 - a*e^4)/(a^2*c^2*sqrt(-1/(a*c)))))*c
)*a)*(a*c*sqrt(-1/(a*c))*sqrt(-(2*a*c*d^2*e^2*sqrt(-1/(a*c)) + c*d^4 - a*e^
4)/(a^2*c^2*sqrt(-1/(a*c)))) - 2*d*e)/(a^2*c^2) + 1/128*(2*c*d^5*e*sqrt(-1/
(a*c)) - 2*a*d*e^5*sqrt(-1/(a*c)) - 4*d^3*e^3 + a^2*c^2*sqrt(-1/(a*c))*(-(2
*a*c*d^2*e^2*sqrt(-1/(a*c)) + c*d^4 - a*e^4)/(a^2*c^2*sqrt(-1/(a*c))))^(3/2
))/(a^2*c^2*sqrt(-1/(a*c))) + 1/1728*(a*c*sqrt(-1/(a*c))*sqrt(-(2*a*c*d^2*e
^2*sqrt(-1/(a*c)) + c*d^4 - a*e^4)/(a^2*c^2*sqrt(-1/(a*c)))) - 2*d*e)^3/(a^
3*c^3*(-1/(a*c))^(3/2)) + 1/1728*sqrt(-27*c^3*d^12*(-1/(a*c))^(3/2) + 27*a^
3*e^12*(-1/(a*c))^(3/2) - ...
```

Sympy [A]
time = 1.71, size = 277, normalized size = 0.95

$$\mathrm{RootSum}\left(256t^4a^3c^3+192t^2a^2c^2d^2e^2+t\left(32a^2cde^5-32ac^2d^5e\right)+a^2e^8+2acd^4e^4+c^2d^8,\ \left(t\mapsto t\log\left(x+\frac{64t^3a^4c^2e^6+448t^3a^3c^3d^4e^2-160t^2a^3c^2d^3e^5+32t^2a^2c^3d^7e+60ta^3cd^2e^8+256ta^2c^2d^6e^4+4tac^3d^{10}+6a^3de^{11}-24a^2cd^5e^7-30ac^2d^9e^3}{a^3e^{12}-33a^2cd^4e^8-33ac^2d^8e^4+c^3d^{12}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*c
*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e**2
- 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*
d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11
 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*
e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

Giac [A]
time = 3.57, size = 285, normalized size = 0.98

$$\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}\,c^2de+(ac^3)^{\frac14}c^2d^2+(ac^3)^{\frac34}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{4ac^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}\,c^2de+(ac^3)^{\frac14}c^2d^2+(ac^3)^{\frac34}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{4ac^3}+\frac{\sqrt{2}\left((ac^3)^{\frac14}c^2d^2-(ac^3)^{\frac34}e^2\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8ac^3}-\frac{\sqrt{2}\left((ac^3)^{\frac14}c^2d^2-(ac^3)^{\frac34}e^2\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^
(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c
^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a
*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*
```

```
c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^3)
```

Mupad [B]
time = 2.66, size = 556, normalized size = 1.91

$$\sum_{k=1}^{4} \ln\left(3\,c^2\,d^4\,e^2-a\,c\,e^6+4\,c^2\,d^3\,e^3\,x-\mathrm{root}\left(256\,a^3\,c^3\,z^4+192\,a^2\,c^2\,d^2\,e^2\,z^2+32\,a^2\,c\,d\,e^5\,z-32\,a\,c^2\,d^5\,e\,z+2\,a\,c\,d^4\,e^4+c^2\,d^8+a^2\,e^8,z,k\right)\,c^3\,d^4\,x\,4-\mathrm{root}\left(256\,a^3\,c^3\,z^4+192\,a^2\,c^2\,d^2\,e^2\,z^2+32\,a^2\,c\,d\,e^5\,z-32\,a\,c^2\,d^5\,e\,z+2\,a\,c\,d^4\,e^4+c^2\,d^8+a^2\,e^8,z,k\right)^2\,a\,c^3\,d^2\,16+\mathrm{root}\left(256\,a^3\,c^3\,z^4+192\,a^2\,c^2\,d^2\,e^2\,z^2+32\,a^2\,c\,d\,e^5\,z-32\,a\,c^2\,d^5\,e\,z+2\,a\,c\,d^4\,e^4+c^2\,d^8+a^2\,e^8,z,k\right)\,a\,c^2\,e^4\,x\,4-\mathrm{root}\left(256\,a^3\,c^3\,z^4+192\,a^2\,c^2\,d^2\,e^2\,z^2+32\,a^2\,c\,d\,e^5\,z-32\,a\,c^2\,d^5\,e\,z+2\,a\,c\,d^4\,e^4+c^2\,d^8+a^2\,e^8,z,k\right)\,a\,c^2\,d\,e^3\,16+\mathrm{root}\left(256\,a^3\,c^3\,z^4+192\,a^2\,c^2\,d^2\,e^2\,z^2+32\,a^2\,c\,d\,e^5\,z-32\,a\,c^2\,d^5\,e\,z+2\,a\,c\,d^4\,e^4+c^2\,d^8+a^2\,e^8,z,k\right)^2\,a\,c^3\,d\,e\,x\,32\right)\,\mathrm{root}\left(256\,a^3\,c^3\,z^4+192\,a^2\,c^2\,d^2\,e^2\,z^2+32\,a^2\,c\,d\,e^5\,z-32\,a\,c^2\,d^5\,e\,z+2\,a\,c\,d^4\,e^4+c^2\,d^8+a^2\,e^8,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4),x)
```

```
[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*z
^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*
d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 + 19
2*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4
 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192*a^2*
c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d
^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8
+ a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^
2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2
*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 +
 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k), k, 1, 4)
```

3.396 $\int \frac{d+ex}{a+cx^4} dx$

Optimal. Leaf size=219

$$\frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{d\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

```
[Out] 1/4*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*d*ar
ctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*d*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*d*ln(a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/2*e*
arctan(x^2*c^(1/2)/a^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$-\frac{d\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{d\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1
/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{d+ex}{a+cx^4}\,dx = \int \left(\frac{d}{a+cx^4}+\frac{ex}{a+cx^4}\right)dx$$
$$= d\int \frac{1}{a+cx^4}\,dx + e\int \frac{x}{a+cx^4}\,dx$$
$$= \frac{d\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{d\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)$$
$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{d\int \frac{\frac{\sqrt{2}\sqrt{}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}-\sqrt{}}}{4\sqrt{2}\,a}$$
$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$
$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{d\log\left(\sqrt{a}-\right.}{4}$$

Mathematica [A]
time = 0.04, size = 184, normalized size = 0.84

$$\frac{-2\left(\sqrt{2}\sqrt[4]{c}\,d+2\sqrt[4]{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}\,d-2\sqrt[4]{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{c}\,d\left(-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)}{8a^{3/4}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/
4)*Sqrt[c])
```

Maple [A]
time = 0.19, size = 124, normalized size = 0.57

method	result	size
risch	$\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^4+a\right)} \frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$	32

default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(x^2\sqrt{\frac{c}{a}}\right)}{2\sqrt{ac}}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^
2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*e/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))
```

Maxima [A]
time = 0.52, size = 209, normalized size = 0.95

$$\frac{\sqrt{2}\,d\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,d\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d-2\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d+2\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/
4)*c^(1/4)) - 1/8*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d - 2*sqrt(a)*e)*a
rctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt
(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4
)*d + 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))
```

Fricas [C] Result contains complex when optimal does not.
time = 1.40, size = 41851, normalized size = 191.10

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(e*sqrt(-1/(a*c)) + sqrt(-d^2/(a^2*c*sqrt(-1/(a*c)))))*log(2*(e*sqrt(-
1/(a*c)) + sqrt(-d^2/(a^2*c*sqrt(-1/(a*c)))))^3*a^3*c*e^2 - (e*sqrt(-1/(a*c
)) + sqrt(-d^2/(a^2*c*sqrt(-1/(a*c)))))^2*a^2*c*d^2*e + 5*a*d^2*e^3 + (a*c*
d^4 + 2*a^2*e^4)*(e*sqrt(-1/(a*c)) + sqrt(-d^2/(a^2*c*sqrt(-1/(a*c))))) - (
c*d^5 - 4*a*d*e^4)*x) + 1/24*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((a*c*sqrt(-d^
2/(a^2*c*sqrt(-1/(a*c))))*sqrt(-1/(a*c)) - e)^2/(a*c) + 3*((e^2*sqrt(-1/(a*
c)) - 2*e*sqrt(-d^2/(a^2*c*sqrt(-1/(a*c)))))*a - d^2)/(a^2*c*sqrt(-1/(a*c))
))/(9*(a*c*sqrt(-d^2/(a^2*c*sqrt(-1/(a*c))))*sqrt(-1/(a*c)) - e)*((e^2*sqrt
```

```
(-1/(a*c)) - 2*e*sqrt(-d^2/(a^2*c*sqrt(-1/(a*c)))))*a - d^2)/(a^2*c) + 27*(
a^2*c^2*(-d^2/(a^2*c*sqrt(-1/(a*c))))^(3/2)*sqrt(-1/(a*c)) - a*c*e^2*sqrt(-
d^2/(a^2*c*sqrt(-1/(a*c))))*sqrt(-1/(a*c)) + c*d^2*e*sqrt(-1/(a*c)) - e^3)/
(a^2*c^2*sqrt(-1/(a*c))) + 2*(a*c*sqrt(-d^2/(a^2*c*sqrt(-1/(a*c))))*sqrt(-1
/(a*c)) - e)^3/(a^3*c^3*(-1/(a*c))^(3/2)) + sqrt(-108*c^3*d^6*(-1/(a*c))^(3
/2) - 36*(43*a^2*c^3*(-1/ ...
```

Sympy [A]
time = 0.44, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3c^2+32t^2a^2ce^2-16tacd^2e+ae^4+cd^4,\ \left(t\mapsto t\log\left(x+\frac{-128t^3a^3ce^2-16t^2a^2cd^2e-8ta^2e^4-4tacd^4+5ad^2e^3}{4ade^4-cd^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*e
**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a**2
*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4 - c
*d**5))))
```

Giac [A]
time = 3.69, size = 215, normalized size = 0.98

$$\frac{\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ac}\,ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2}-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{ac}\,ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)
```

Mupad [B]
time = 2.32, size = 160, normalized size = 0.73

$$\begin{cases} -\frac{2\,d+3\,e\,x}{6\,c\,x^3} & \text{if}\ \ a=0\\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,e+\sqrt{2}\,c^{1/4}\,d\right)}{4\,a^{3/4}\,\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,e-2\,\sqrt{2}\,c^{1/4}\,d\right)}{8\,a^{3/4}\,\sqrt{c}}+\frac{\sqrt{2}\,d\,\ln\left(\frac{\sqrt{a}+\sqrt{c}\,x^2+\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}{\sqrt{a}+\sqrt{c}\,x^2-\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}\right)}{8\,a^{3/4}\,c^{1/4}} & \text{if}\ \ a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4),x)
```

```
[Out] piecewise(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a ~= 0, (atan((2^(1/2)*c^(1/4)*
x)/a^(1/4) - 1)*(2*a^(1/4)*e + 2^(1/2)*c^(1/4)*d))/(4*a^(3/4)*c^(1/2)) - (a
tan((2^(1/2)*c^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*e - 2*2^(1/2)*c^(1/4)*d))/(
8*a^(3/4)*c^(1/2)) + (2^(1/2)*d*log((a^(1/2) + c^(1/2)*x^2 + 2^(1/2)*a^(1/4
)*c^(1/4)*x)/(a^(1/2) + c^(1/2)*x^2 - 2^(1/2)*a^(1/4)*c^(1/4)*x)))/(8*a^(3/
4)*c^(1/4)))
```

3.397 $\int \frac{1}{a+cx^4}\,dx$

Optimal. Leaf size=185

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(3/4)*c^(1/4)) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
 + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4
)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\,\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\,\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{}{-\frac{\sqrt{a}}{\sqrt{c}}}}{4}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Mathematica [A]
time = 0.01, size = 134, normalized size = 0.72

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

Maple [A]
time = 0.18, size = 102, normalized size = 0.55

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1))
```

Maxima [A]
time = 0.52, size = 169, normalized size = 0.91

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
```

```
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

Fricas [A]
time = 0.38, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}+\sqrt{a^2\sqrt{-\frac{1}{a^3c}}+x^2}\,a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Giac [A]
time = 3.63, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

Mupad [B]
time = 0.08, size = 33, normalized size = 0.18

$$-\frac{\mathrm{atan}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)+\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{2\,{(-a)}^{3/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

3.398 $\int \frac{1}{(d+ex)(a+cx^4)}\,dx$

Optimal. Leaf size=416

$$-\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^4+ae^4)}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,(cd^4+ae^4)}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(\right.}{2\sqrt{2}\,a^{3/4}\,(cd^4+a}$$

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)-1/4*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)-1/2*d^2*e*arc
tan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)/a^(1/2)-1/8*c^(1/4)*d*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(
3/4)/(a*e^4+c*d^4)*2^(1/2)+1/8*c^(1/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/
4*c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/
a^(3/4)/(a*e^4+c*d^4)*2^(1/2)+1/4*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1
/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)*2^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6857, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$-\frac{\sqrt[4]{c}\,d\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{2\sqrt{2}\,a^{3/4}\,(ae^4+cd^4)}+\frac{\sqrt[4]{c}\,d\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{2\sqrt{2}\,a^{3/4}\,(ae^4+cd^4)}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\,(ae^4+cd^4)}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\,(ae^4+cd^4)}-\frac{\sqrt{c}\,d^2e\,\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(ae^4+cd^4)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^4+cd^4\right)}+\frac{e^3\log(d+ex)}{ae^4+cd^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)
) - (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 + Sq
rt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^4 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx &= \int \left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)}+\frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)}\right)dx \\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{a+cx^4}\,dx}{cd^4+ae^4} \\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\int \left(\frac{d^3+de^2x^2}{a+cx^4}+\frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\right)dx}{cd^4+ae^4} \\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\int \frac{d^3+de^2x^2}{a+cx^4}\,dx}{cd^4+ae^4}+\frac{c\int \frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\,dx}{cd^4+ae^4} \\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}+\frac{c\text{Subst}\left(\int \frac{-d^2e-e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)}+\frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}\,-}{a+cx^4}}{2\left(cd^4+ae^4\right)} \\
&= \frac{e^3\log(d+ex)}{cd^4+ae^4}-\frac{\left(cd^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)}-\frac{\left(ce^3\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} \\
&= -\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^4+ae^4\right)}+\frac{e^3\log(d+ex)}{cd^4+ae^4}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}\,-\right.}{4\sqrt{2}\,a^{3/4}\left(cd^4+\right.} \\
&= -\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^4+ae^4\right)}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}}{}
\end{aligned}
$$

Mathematica [A]

time = 0.11, size = 404, normalized size = 0.97

$$
\frac{-2\sqrt[4]{c}\,d\left(\sqrt{2}\,\sqrt{c}\,d^2-2\sqrt[4]{a}\,\sqrt[4]{c}\,de+\sqrt{2}\,\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{c}\,d\left(\sqrt{2}\,\sqrt{c}\,d^2+2\sqrt[4]{a}\,\sqrt[4]{c}\,de+\sqrt{2}\,\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+8a^{3/4}e^3\log(d+ex)-\sqrt{2}\,c^{3/4}d^3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\sqrt{2}\,\sqrt{a}\,\sqrt[4]{c}\,de^2\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\sqrt{2}\,c^{3/4}d^3\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{c}\,de^2\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-2a^{3/4}e^3\log\left(a+cx^4\right)}{8a^{3/4}\left(cd^4+ae^4\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] (-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqrt[
c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(1/4)
*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(
3/4)*d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*S
```

```
qrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
 - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4))
```

Maple [A]
time = 0.23, size = 289, normalized size = 0.69

method	result
risch	$\frac{e^3\ln(ex+d)}{e^4a+d^4c}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^4e^4+a^3c\,d^4\right)\_Z^4+4a^3e^3\_Z^3+6\_Z^2a^2e^2+4ae\_Z\right)}\_R\ln\left(\left(\left(5e^6a^3-3a^2d^4e^2c\right)\_R^3+\left(15a^2e^5-3\right.\right.\right.\right.$
default	$\frac{e^3\ln(ex+d)}{e^4a+d^4c}+\frac{c\left(\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{d^2e\arctan\left(x^2\sqrt{\right.}}{2\sqrt{ac}}\right.}{e^4a+d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)+c/(a*e^4+c*d^4)*(1/8*d^3*(a/c)^(1/4)/a*2^(1/2)*
(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c
)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-
1))-1/2*d^2*e/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*d*e^2/c/(a/c)^(1/4)*2
^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(
1/4)*x-1))-1/4*e^3*ln(c*x^4+a)/c)
```

Maxima [A]
time = 0.51, size = 341, normalized size = 0.82

$$\frac{c\left(\frac{\sqrt{2}\left(cd^3-\sqrt{a}\sqrt{c}\,de^2-\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^3\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(cd^3-\sqrt{a}\sqrt{c}\,de^2+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^3\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+2\sqrt{a}\,cd^2e+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{5}{4}}d^3-2\sqrt{a}\,cd^2e+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}\right)}{8\left(cd^4+ae^4\right)}+\frac{e^3\log\left(xe+d\right)}{cd^4+ae^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*c*(sqrt(2)*(c*d^3 - sqrt(a)*sqrt(c)*d*e^2 - sqrt(2)*a^(3/4)*c^(1/4)*e^3
)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
- sqrt(2)*(c*d^3 - sqrt(a)*sqrt(c)*d*e^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^3)*log
(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + 2*(
sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 2*sqrt(a)*c*d^2*e + sqrt(2)*a^(3/4)*c^(3/4)*d
*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(
a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(sqrt(2)*a^(1/4)*c
^(5/4)*d^3 - 2*sqrt(a)*c*d^2*e + sqrt(2)*a^(3/4)*c^(3/4)*d*e^2)*arctan(1/2*
```

```
sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c*d^4 + a*e^4) + e^3*log(x*e + d)/(c*
d^4 + a*e^4)
```

Fricas [C] Result contains complex when optimal does not.
time = 138.08, size = 352864, normalized size = 848.23

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/48*(48*e^3*log(e*x + d) + 2*(c*d^4 + a*e^4)*(2*(1/2)^(2/3)*(-I*sqrt(3) +
1)*((c*d^2*e - (a^2*e^4*sqrt(-(2*a*c*d^4*e^2*sqrt(-c/a) - c^2*d^6 + a*c*d^2
*e^4)/(a^2*c^2*d^8*sqrt(-c/a) + 2*a^3*c*d^4*e^4*sqrt(-c/a) + a^4*e^8*sqrt(-
c/a))) + (c*d^4*sqrt(-(2*a*c*d^4*e^2*sqrt(-c/a) - c^2*d^6 + a*c*d^2*e^4)/(a
^2*c^2*d^8*sqrt(-c/a) + 2*a^3*c*d^4*e^4*sqrt(-c/a) + a^4*e^8*sqrt(-c/a))) -
 3*e^3)*a)*sqrt(-c/a))^2*a/((a*c*d^4 + a^2*e^4)^2*c) - 3*(2*a*c*d^2*e*sqrt(
-(2*a*c*d^4*e^2*sqrt(-c/a) - c^2*d^6 + a*c*d^2*e^4)/(a^2*c^2*d^8*sqrt(-c/a)
 + 2*a^3*c*d^4*e^4*sqrt(-c/a) + a^4*e^8*sqrt(-c/a))) - c*d^2 + (2*a^2*e^3*s
qrt(-(2*a*c*d^4*e^2*sqrt(-c/a) - c^2*d^6 + a*c*d^2*e^4)/(a^2*c^2*d^8*sqrt(-
c/a) + 2*a^3*c*d^4*e^4*sqrt(-c/a) + a^4*e^8*sqrt(-c/a))) - 3*a*e^2)*sqrt(-c
/a))/((a^2*c*d^4 + a^3*e^4)*sqrt(-c/a)))/(9*(2*a*c*d^2*e*sqrt(-(2*a*c*d^4*e
^2*sqrt(-c/a) - c^2*d^6 + a*c*d^2*e^4)/(a^2*c^2*d^8*sqrt(-c/a) + 2*a^3*c*d^
4*e^4*sqrt(-c/a) + a^4*e^8*sqrt(-c/a))) - c*d^2 + (2*a^2*e^3*sqrt(-(2*a*c*d
^4*e^2*sqrt(-c/a) - c^2*d ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.91, size = 371, normalized size = 0.89

$$\frac{\left(ac^3\right)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^2d^2-2\left(ac^3\right)^{\frac{1}{4}}acde+\sqrt{2}\,\sqrt{ac}\,ace^2\right)}+\frac{\left(ac^3\right)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^2d^2+2\left(ac^3\right)^{\frac{1}{4}}acde+\sqrt{2}\,\sqrt{ac}\,ace^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}\,ac^3d^4+\sqrt{2}\,a^2c^2e^4\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}\,ac^3d^4+\sqrt{2}\,a^2c^2e^4\right)}-\frac{e^3\log\left(|cx^4+a|\right)}{4\left(cd^4+ae^4\right)}+\frac{e^4\log\left(|xe+d|\right)}{cd^4e+ae^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(sqrt(2)*a*c^2*d^2 - 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(a*c)*a*c
*e^2) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(
a*c)*a*c*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 +
 sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^
4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*
(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4) - 1/4*e^
3*log(abs(c*x^4 + a))/(c*d^4 + a*e^4) + e^4*log(abs(x*e + d))/(c*d^4*e + a*
e^5)
```

Mupad [B]
time = 0.42, size = 874, normalized size = 2.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)),x)
```

```
[Out] symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a^3
*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z
+ 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*
a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 25
6*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^
3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 2
56*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d*e^5 - 128*roo
t(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +
16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*z^4 + 256*a^4*e^
4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^4*
e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(256*a^3*c*d^4*z^4 +
256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k), k
, 1, 4) + (e^3*log(d + e*x))/(a*e^4 + c*d^4)
```

3.399 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{e^3}{(cd^4+ae^4)(d+ex)}-\frac{\sqrt{c}\,de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2(cd^4-3ae^4)+\sqrt{a}\,e^2(3cd^4-ae^4)\right)\text{ta}}{2\sqrt{2}\,a^{3/4}(cd^4+ae^4)^2}$$

```
[Out] -e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-c*d^3*e^3*
ln(c*x^4+a)/(a*e^4+c*d^4)^2-d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*
c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(
1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4
)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(-1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2)
)/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(3/4)/
(a*e^4+c*d^4)^2*2^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6857, 1890, 1262, 649, 211, 266, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e^2(3cd^4-ae^4)+\sqrt{c}\,d^2(cd^4-3ae^4)\right)}{2\sqrt{2}\,a^{3/4}(ae^4+cd^4)^2}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e^2(3cd^4-ae^4)+\sqrt{c}\,d^2(cd^4-3ae^4)\right)}{2\sqrt{2}\,a^{3/4}(ae^4+cd^4)^2}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2(cd^4-3ae^4)-\sqrt{a}\,e^2(3cd^4-ae^4)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^4+cd^4)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2(cd^4-3ae^4)-\sqrt{a}\,e^2(3cd^4-ae^4)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^4+cd^4)^2}-\frac{\sqrt{c}\,de\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(cd^4-ae^4)}{\sqrt{a}\,(ae^4+cd^4)^2}-\frac{e^3}{(d+ex)(ae^4+cd^4)}-\frac{cd^3e^3\log\left(a+cx^4\right)}{(ae^4+cd^4)^2}+\frac{4cd^3e^3\log(d+ex)}{(ae^4+cd^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2*(
c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d
^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Lo
g[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - S
qrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c
*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^
3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx = \int \left(\frac{e^4}{(cd^4+ae^4)\,(d+ex)^2} + \frac{4cd^3e^4}{(cd^4+ae^4)^2\,(d+ex)} + \frac{c(d^2(cd^4-3ae^4)-2de(c\cdots}{(c\cdots}\right.$$

$$= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\int \frac{d^2(cd^4-3ae^4)-2de(cd^4-ae^4)x+e^2(3c\cdots}{a+cx^4}}{(cd^4+ae^4)^2}\cdots$$

$$= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\int \left(\frac{x(-2de(cd^4-ae^4)-4cd^3e^3x^2)}{a+cx^4} + \frac{d\cdots}{\cdots}\right.}{(cd^4+a\cdots}$$

$$= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\int \frac{x(-2de(cd^4-ae^4)-4cd^3e^3x^2)}{a+cx^4}\,dx}{(cd^4+ae^4)^2} + \cdots$$

$$= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} + \frac{c\text{Subst}\left(\int \frac{-2de(cd^4-ae^4)-4cd^3e^3x}{a+cx^2}\,dx\cdots\right.}{2\,(cd^4+ae^4)^2}\cdots$$

$$= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)} + \frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2} - \frac{(2c^2d^3e^3)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x\cdots\right.}{(cd^4+ae^4)^2}\cdots$$

$$= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)} - \frac{\sqrt{c}\,de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^4+ae^4)^2} + \frac{4cd^3e^3\log(d+e\cdots}{(cd^4+ae^4)^2}\cdots$$

$$= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)} - \frac{\sqrt{c}\,de(cd^4-ae^4)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}\left(3cd^4e^2-a\cdots\right.}{\cdots}$$

Mathematica [A]

time = 0.45, size = 524, normalized size = 0.95

$$\frac{-\frac{8e^3(cd^4+ae^4)}{d+ex} + \frac{2\sqrt[4]{c}\left(-\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\left(\sqrt{2}\,cd^4-4\sqrt[4]{a}\,c^{3/4}d^3e+4\sqrt{2}\,\sqrt{a}\,\sqrt{c}\,d^2e^2-4a^{3/4}\sqrt[4]{c}\,de^3+\sqrt{2}\,ae^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt[4]{c}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\sqrt{2}\,cd^4+4\sqrt[4]{a}\,c^{3/4}d^3e+4\sqrt{2}\,\sqrt{a}\,\sqrt{c}\,d^2e^2+4a^{3/4}\sqrt[4]{c}\,de^3+\sqrt{2}\,ae^4\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} + 32cd^3e^3\log(d+ex) - \frac{\sqrt{2}\,\sqrt[4]{c}\left(c^{3/2}d^6-3\sqrt{a}\,cd^4e^2-3a\sqrt{c}\,d^2e^4+a^{3/2}e^6\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{3/4}} + \frac{\sqrt{2}\,\sqrt[4]{c}\left(c^{3/2}d^6-3\sqrt{a}\,cd^4e^2-3a\sqrt{c}\,d^2e^4+a^{3/2}e^6\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{3/4}} - 8cd^3e^3\log(a+cx^4)}{8\,(cd^4+ae^4)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]*
e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d
^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[2]*c
*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4*a^(3
/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
```

/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log[a + c*x^4])/(8*(c*d^4 + a*e^4)^2)

Maple [A]

time = 0.24, size = 354, normalized size = 0.64

method	result
default	$-\frac{e^3}{(e^4a+d^4c)(ex+d)}+\frac{4c\,d^3e^3\ln(ex+d)}{(e^4a+d^4c)^2}-\frac{c\left(\frac{\left(3a\,d^2e^4-c\,d^6\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\,a\right.}{8a}\right.}{}$
risch	$-\frac{e^3}{(e^4a+d^4c)(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5e^8+2a^4c\,d^4e^4+a^3c^2d^8\right)\_Z^4+16a^3c\,d^3e^3\_Z^3+20a^2c\,d^2e^2\_Z^2+8acde\_Z+c\right)}\_R\ln\left(\left(\left(5a^5e^{14}\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] -e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-c/(a*e^4+c*d^4)^2*(1/8*(3*a*d^2*e^4-c*d^6)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-2*a*d*e^5+2*c*d^5*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(a*e^6-3*c*d^4*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+d^3*e^3*ln(c*x^4+a))

Maxima [A]

time = 0.49, size = 540, normalized size = 0.98

$$\frac{4cd^3e^3\log(xe+d)}{c^2d^8+2acd^4e^4+a^2e^8}+\frac{c\left(\frac{\sqrt{2}\left(c^2d^6-3\sqrt{a}c^{\frac{3}{2}}d^4e^2-4\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3-3acd^2e^4+a^{\frac{3}{2}}\sqrt{c}e^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(c^2d^6-3\sqrt{a}c^{\frac{3}{2}}d^4e^2+4\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3-3acd^2e^4+a^{\frac{3}{2}}\sqrt{c}e^6\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{9}{4}}d^6+4\sqrt{a}c^2d^5e+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{7}{4}}d^4e^2-3\sqrt{2}a^{\frac{5}{4}}c^{\frac{5}{4}}d^2e^4-4a^{\frac{3}{2}}cde^5-\sqrt{2}a^{\frac{7}{4}}c^{\frac{3}{4}}e^6\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{9}{4}}d^6-4\sqrt{a}c^2d^5e+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{7}{4}}d^4e^2-3\sqrt{2}a^{\frac{5}{4}}c^{\frac{5}{4}}d^2e^4+4a^{\frac{3}{2}}cde^5-\sqrt{2}a^{\frac{7}{4}}c^{\frac{3}{4}}e^6\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}\right)}{8\left(c^2d^8+2acd^4e^4+a^2e^8\right)}-\frac{e^3}{cd^5+ade^4+\left(cd^4e+ae^5\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")

[Out] 4*c*d^3*e^3*log(x*e + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/8*c*(sqrt(2)*(c^2*d^6 - 3*sqrt(a)*c^(3/2)*d^4*e^2 - 4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - sqrt(2)*(c^2*d^6 - 3*sqrt(a)*c^(3/2

```
)*d^4*e^2 + 4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 - 3*a*c*d^2*e^4 + a^(3/2)*sqr
t(c)*e^6)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c
^(5/4)) + 2*(sqrt(2)*a^(1/4)*c^(9/4)*d^6 + 4*sqrt(a)*c^2*d^5*e + 3*sqrt(2)*
a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)*d^2*e^4 - 4*a^(3/2)*c*d
*e^5 - sqrt(2)*a^(7/4)*c^(3/4)*e^6)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)) + 2*(sqrt(2)*a^(1/4)*c^(9/4)*d^6 - 4*sqrt(a)*c^2*d^5*e + 3*sqrt(2)*
a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)*d^2*e^4 + 4*a^(3/2)*c*d
*e^5 - sqrt(2)*a^(7/4)*c^(3/4)*e^6)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5 + a*d*e^4 + (c*d^
4*e + a*e^5)*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.10, size = 646, normalized size = 1.17

$$-\frac{cd^3e^3\log\left(|cx^4+a|\right)}{c^2d^8+2\,acd^4e^4+a^2e^8}+\frac{4\,cd^3e^4\log\left(|xe+d|\right)}{c^2d^8e+2\,acd^4e^5+a^2e^9}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^4-4\left(ac^3\right)^{\frac{1}{4}}ac^2d^3e+4\sqrt{2}\sqrt{ac}\,ac^2d^2e^2+\sqrt{2}\,a^2c^2e^4-4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^4+4\left(ac^3\right)^{\frac{1}{4}}ac^2d^3e+4\sqrt{2}\sqrt{ac}\,ac^2d^2e^2+\sqrt{2}\,a^2c^2e^4+4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)}+\frac{\left(\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}c^3d^6-3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}cd^4e^2-3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^4+\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}ae^6\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}-\frac{\left(\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}c^3d^6-3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}cd^4e^2-3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^4+\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}ae^6\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}-\frac{cd^4e^3+ae^7}{\left(cd^4+ae^4\right)^2(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -c*d^3*e^3*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 4*c*d^
3*e^4*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/2*((a*c^3
)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 - 4*(a*c^3)^(1/4)*a*c^2*d^3*e + 4*
sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + sqrt(2)*a^2*c^2*e^4 - 4*(a*c^3)^(3/4)*a*d
```

```
*e^3) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 4*(a*c^3)^(1/
4)*a*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + sqrt(2)*a^2*c^2*e^4 +
4*(a*c^3)^(3/4)*a*d*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3*sqrt(2)*(
a*c^3)^(3/4)*c*d^4*e^2 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 + sqrt(2)*(a
*c^3)^(3/4)*a*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^8
+ 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3
*sqrt(2)*(a*c^3)^(3/4)*c*d^4*e^2 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 +
sqrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - (c*d^4*e^3 + a*e^7)/((c*d^4
+ a*e^4)^2*(x*e + d))
```

Mupad [B]
time = 2.78, size = 2436, normalized size = 4.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)^2),x)

[Out] symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3
*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2
*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e^4*
z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*
a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*root(51
2*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^
3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^8*e
^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4
 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)
^3*a^3*c^5*d^4*e^9 - 128*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 +
 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*
d*e*z + c, z, k)^4*a^2*c^7*d^13*e^2 + 128*root(512*a^4*c*d^4*e^4*z^4 + 256*
a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*
e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^9*e^6 + 640*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d^5*e^10 + 32*r
oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d^2
*e^7 - 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^
4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k
)^3*a*c^7*d^12*e + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 25
6*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e
*z + c, z, k)*c^6*d^5*e^4*x - 4*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^
8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 +
32*a*c*d*e*z + c, z, k)^2*c^7*d^10*e*x + 64*root(512*a^4*c*d^4*e^4*z^4 + 25
```

6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a*c^6*d^7*e^4 + 384*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*d*e^14 + 320*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*e^15*x + 248*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a*c^6*d^6*e^5*x - 64*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a*c^7*d^11*e^2*x + 32*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d*e^8*x + 316*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^2*e^9*x + 640*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^7*e^6*x + 704*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^3*c^5*d^3*e^10*x - 192*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^2*c^7*d^12*e^3*x - 64*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^8*e^7*x + 448*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d^4*e^11*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k), k, 1, 4) - e^3/(c*d^5 + a*d*e^4 + a*e^5*x + c*d^4*e*x) + (4*c*d^3*e^3*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)

3.400 $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$

Optimal. Leaf size=680

$$-\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)}-\frac{\sqrt{c}\,e(3c^2d^8-12acd^4e^4+a^2e^8)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^4+ae^4\right)^3}-\frac{c^{3/4}d(c^2 \cdots}{}$$

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)+2*c*d^
2*e^3*(-3*a*e^4+5*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^3-1/2*c*d^2*e^3*(-3*a*e^4+
5*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/2*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^
8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(3/4)*
d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^8-12*a*c*d^4*e^
4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*
d^4)^3*2^(1/2)+1/8*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*
c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*arctan(-1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*
d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(3/4)*d*arctan(
1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c^2*d^8-12*a*c*d^4*e^4+3*a^2*e^8+2*d^2*e^2*(-
5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)
```

Rubi [A]

time = 0.67, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6857, 1890, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 211, 266}

$$-\frac{\sqrt{c}\,e\,\text{ArcTan}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\left(a^2e^8-12acd^4e^4+3c^2d^8\right)}{2\sqrt{a}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)}{2\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\sqrt{c}d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{e^3}{2(d+ex)^2\left(ae^4+cd^4\right)}-\frac{4cd^3e^3}{(d+ex)\left(ae^4+cd^4\right)^2}-\frac{cd^2e^3\left(5cd^4-3ae^4\right)\log\left(a+cx^4\right)}{2\left(ae^4+cd^4\right)^3}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(ae^4+cd^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] -1/2*e^3/((c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(
d + e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[
c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 - 12*
a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^
2*e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[d + e
*x])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 -
 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/
4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c
```

```
*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[a
+ c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx = \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)^3} + \frac{4cd^3e^4}{(cd^4+ae^4)^2(d+ex)^2} + \frac{2cd^2e^4(5cd^4-3ae^4)}{(cd^4+ae^4)^3(d+ex)} + \cdots\right.$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d \cdots}{(cd^4+ae^4)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d \cdots}{(cd^4+ae^4)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d \cdots}{(cd^4+ae^4)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d \cdots}{(cd^4+ae^4)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d \cdots}{(cd^4+ae^4)^3}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}\,e(3c^2d^8-12acd^4e^4+\cdots}{2\sqrt{a}\,(cd^4+\cdots}$$

$$= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}\,e(3c^2d^8-12acd^4e^4+\cdots}{2\sqrt{a}\,(cd^4+\cdots}$$

Mathematica [A]
time = 0.64, size = 738, normalized size = 1.09

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] (-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 + a*e^4)*(d
 + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*
Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 + 24*a^(5/4)*c*d^4*e
^5 - 10*Sqrt[2]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 - 2*a
^(9/4)*e^9)*(d + e*x)^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[c]
*(Sqrt[2]*c^(9/4)*d^9 + 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7
```

```
*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 - 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt[2]*a^(
3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 + 2*a^(9/4)*e^9)*(d + e*
x)^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 16*a^(3/4)*c*d^2*e^3*(5*c*d^
4 - 3*a*e^4)*(d + e*x)^2*Log[d + e*x] - Sqrt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt
[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e
^8)*(d + e*x)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sq
rt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*
a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2] + 4*a^(3/4)*c*d^2*e^3*(-5*c*d^4 + 3*a*e^4)*(d
+ e*x)^2*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4)^3*(d + e*x)^2)
```

Maple [A]

time = 0.27, size = 446, normalized size = 0.66

method	result
default	$-\frac{e^3}{2(e^4a+d^4c)(ex+d)^2}-\frac{4c\,d^3e^3}{(e^4a+d^4c)^2(ex+d)}-\frac{2d^2e^3c(3e^4a-5d^4c)\ln(ex+d)}{(e^4a+d^4c)^3}+\frac{c\left(\frac{\left(3a^2d\,e^8-12ac\,d^5e^4+c^2d^9\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2}{x^2}\right.\right.}{}\right.}{}$
risch	$\frac{-\frac{4d^3e^4cx}{a^2e^8+2ac\,d^4e^4+c^2d^8}-\frac{\left(e^4a+9d^4c\right)e^3}{2\left(a^2e^8+2ac\,d^4e^4+c^2d^8\right)}}{(ex+d)^2}-\frac{6d^2e^7c\ln(ex+d)a}{a^3e^{12}+3a^2c\,d^4e^8+3a\,c^2d^8e^4+c^3d^{12}}+\frac{10d^6e^3c^2\ln(ex+d)}{a^3e^{12}+3a^2c\,d^4e^8+3a\,c^2d^8e^4+c^3d^{12}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-2*d^2*
e^3*c*(3*a*e^4-5*c*d^4)/(a*e^4+c*d^4)^3*ln(e*x+d)+c/(a*e^4+c*d^4)^3*(1/8*(3
*a^2*d*e^8-12*a*c*d^5*e^4+c^2*d^9)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-a^2*e^9+
12*a*c*d^4*e^5-3*c^2*d^8*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(-10*a*
c*d^3*e^6+6*c^2*d^7*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*(6*a*c*d^2*e^7-10*c^
2*d^6*e^3)/c*ln(c*x^4+a))
```

Maxima [A]

time = 0.50, size = 779, normalized size = 1.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*c*(sqrt(2)*(c^3*d^9 - 6*sqrt(a)*c^(5/2)*d^7*e^2 - 10*sqrt(2)*a^(3/4)*c^
(9/4)*d^6*e^3 - 12*a*c^2*d^5*e^4 + 10*a^(3/2)*c^(3/2)*d^3*e^6 + 6*sqrt(2)*a
^(7/4)*c^(5/4)*d^2*e^7 + 3*a^2*c*d*e^8)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - sqrt(2)*(c^3*d^9 - 6*sqrt(a)*c^(5/2
)*d^7*e^2 + 10*sqrt(2)*a^(3/4)*c^(9/4)*d^6*e^3 - 12*a*c^2*d^5*e^4 + 10*a^(3
/2)*c^(3/2)*d^3*e^6 - 6*sqrt(2)*a^(7/4)*c^(5/4)*d^2*e^7 + 3*a^2*c*d*e^8)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + 2*
(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(a)*c^3*d^8*e + 6*sqrt(2)*a^(3/4)*c^(
11/4)*d^7*e^2 - 12*sqrt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 24*a^(3/2)*c^2*d^4*e^5
 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt(2)*a^(9/4)*c^(5/4)*d*e^8 + 2
*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(sqrt(2)
*a^(1/4)*c^(13/4)*d^9 - 6*sqrt(a)*c^3*d^8*e + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^
7*e^2 - 12*sqrt(2)*a^(5/4)*c^(9/4)*d^5*e^4 + 24*a^(3/2)*c^2*d^4*e^5 - 10*sq
rt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt(2)*a^(9/4)*c^(5/4)*d*e^8 - 2*a^(5/2)
*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqr
t(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^3*d^12 + 3*a*c^2
*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*
log(x*e + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) - 1/
2*(8*c*d^3*x*e^4 + 9*c*d^4*e^3 + a*e^7)/(c^2*d^10 + 2*a*c*d^6*e^4 + a^2*d^2
*e^8 + (c^2*d^8*e^2 + 2*a*c*d^4*e^6 + a^2*e^10)*x^2 + 2*(c^2*d^9*e + 2*a*c*
d^5*e^5 + a^2*d*e^9)*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.03, size = 901, normalized size = 1.32

$$\frac{\left(\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3}+2\,ac^{2}e^{3}-3\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\left(ac^{3}d^{6}-3\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{5}e+9\,\sqrt{ac}\,ac^{2}d^{4}e^{2}+9\,a^{2}c^{2}d^{2}e^{4}-8\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}ad^{3}e^{3}-3\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}a^{2}cde^{5}+\sqrt{ac}\,a^{2}ce^{6}\right)}+\frac{\left(\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3}-2\,ac^{2}e^{3}-3\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\left(ac^{3}d^{6}+3\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{5}e-9\,\sqrt{ac}\,ac^{2}d^{4}e^{2}+9\,a^{2}c^{2}d^{2}e^{4}+8\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}ad^{3}e^{3}+3\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}a^{2}cde^{5}+\sqrt{ac}\,a^{2}ce^{6}\right)}+\frac{\left(\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{9}-6\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}cd^{7}e^{2}-12\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{5}e^{4}+10\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}ad^{3}e^{6}+3\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}a^{2}cde^{8}\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^{4}d^{12}+3\,a^{2}c^{3}d^{8}e^{4}+3\,a^{3}c^{2}d^{4}e^{8}+a^{4}ce^{12}\right)}-\frac{\left(\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{9}-6\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}cd^{7}e^{2}-12\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}d^{5}e^{4}+10\,\sqrt{2}\left(ac^{3}\right)^{\frac{3}{4}}ad^{3}e^{6}+3\,\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}a^{2}cde^{8}\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^{4}d^{12}+3\,a^{2}c^{3}d^{8}e^{4}+3\,a^{3}c^{2}d^{4}e^{8}+a^{4}ce^{12}\right)}-\frac{\left(5\,c^{2}d^{6}e^{3}-3\,acd^{2}e^{7}\right)\log\left(|cx^{4}+a|\right)}{2\left(c^{3}d^{12}+3\,ac^{2}d^{8}e^{4}+3\,a^{2}cd^{4}e^{8}+a^{3}e^{12}\right)}+\frac{2\left(5\,c^{2}d^{6}e^{4}-3\,acd^{2}e^{8}\right)\log\left(|xe+d|\right)}{c^{3}d^{12}e+3\,ac^{2}d^{8}e^{5}+3\,a^{2}cd^{4}e^{9}+a^{3}e^{13}}-\frac{9\,c^{2}d^{8}e^{3}+10\,acd^{4}e^{7}+a^{2}e^{11}+8\left(c^{2}d^{7}e^{4}+acd^{3}e^{8}\right)x}{2\left(cd^{4}+ae^{4}\right)^{3}\left(xe+d\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="giac")

[Out] 1/4*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 + 2*a*c^2*e^3 - 3*sqrt(2)*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d^6 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e + 9*sqrt(a*c)*a*c^2*d^4*e^2 + 9*a^2*c^2*d^2*e^4 - 8*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^3 - 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^5 + sqrt(a*c)*a^2*c*e^6) + 1/4*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 - 2*a*c^2*e^3 - 3*sqrt(2)*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d^6 + 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e - 9*sqrt(a*c)*a*c^2*d^4*e^2 + 9*a^2*c^2*d^2*e^4 + 8*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^3 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^5 + sqrt(a*c)*a^2*c*e^6) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 - 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 - 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12) - 1/2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^4 - 3*a*c*d^2*e^8)*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3*e^13) - 1/2*(9*c^2*d^8*e^3 + 10*a*c*d^4*e^7 + a^2*e^11 + 8*(c^2*d^7*e^4 + a*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(x*e + d)^2)

Mupad [B]
time = 3.67, size = 1955, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)*(d + e*x)^3),x)

[Out] symsum(log((c^7*d^5*e^6 + a*c^6*d*e^10)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((208*a*c^7*d^7*e^7 - 48*a^2*c^6*d^3*e^11)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((144*a*c^8*d^13*e^4 + 16*a^4*c^5*d*e^16 + 2608*a^2*c^7*d^9*e^8 - 592*a^3*c^6*d^5*e^12)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 +

4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((896*a^4*c^6*d^7*e^13 - 1120*a^3*c^7*d^11*e^9 - 1024*a^2*c^8*d^15*e^5 + 976*a^5*c^5*d^3*e^17 + 16*a*c^9*d^19*e)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((384*a^7*c^4*d*e^22 - 128*a^2*c^9*d^21*e^2 - 128*a^3*c^8*d^17*e^6 + 768*a^4*c^7*d^13*e^10 + 1792*a^5*c^6*d^9*e^14 + 1408*a^6*c^5*d^5*e^18)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + (x*(320*a^7*c^4*e^23 - 192*a^2*c^9*d^20*e^3 - 448*a^3*c^8*d^16*e^7 + 128*a^4*c^7*d^12*e^11 + 1152*a^5*c^6*d^8*e^15 + 1088*a^6*c^5*d^4*e^19))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(80*a*c^9*d^18*e^2 - 1536*a^2*c^8*d^14*e^6 - 2016*a^3*c^7*d^10*e^10 + 896*a^4*c^6*d^6*e^14 + 1296*a^5*c^5*d^2*e^18))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(36*a^4*c^5*e^17 - 4*c^9*d^16*e + 792*a*c^8*d^12*e^5 + 1632*a^2*c^7*d^8*e^9 - 152*a^3*c^6*d^4*e^13))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(40*c^8*d^10*e^4 - 16*a*c^7*d^6*e^8 + 72*a^2*c^6*d^2*e^12))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(a*c^6*e^11 + c^7*d^4*e^7))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8))*root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k), k, 1, 4) - ((a*e^7 + 9*c*d^4*e^3)/(2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (4*c*d^3*e^4*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))/(d^2 + e^2*x^2 + 2*d*e*x) + (log(d + e*x)*(10*c^2*d^6*e^3 - 6*a*c*d^2*e^7))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)

3.401 $\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{3d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d(\sqrt{}}{}$$

```
[Out] 1/4*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)+3/4*d^2*e*arctan
(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/3
2*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(
1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(
e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+3/16*d*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {1868, 27, 12, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{3d\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d^2e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] -1/4*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*(a + c*x^4)) + (3*d
^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) - (3*d*(Sqrt[c]*d^2
 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)
*c^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
c^(3/4)) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} - \frac{\int \frac{-3d^3-6d^2ex-3de^2x^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} - \frac{\int -\frac{3d(d+ex)^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} + \frac{(3d)\int \frac{(d+ex)^2}{a+cx^4}\,dx}{4a}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} + \frac{(3d)\int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right)dx}{4a}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} + \frac{(3d)\int \frac{d^2+e^2x^2}{a+cx^4}\,dx}{4a} + \frac{(3d^2e)\int \frac{x}{a+cx^4}\,dx}{2a}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} + \frac{(3d^2e)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a} + \frac{\left(3d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{\left(3d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{\cdots}{\frac{\sqrt{a}}{\sqrt{c}}\cdots}}{16ac}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{\cdots}\right.}{16\sqrt{2}\cdots}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{4ac\,(a+cx^4)} + \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(\cdots\right.}{8\sqrt{2}\,a^{7/4}c^{3/\cdots}}$$

<u>Mathematica</u> [A]
time = 0.27, size = 347, normalized size = 0.99

$$\frac{-\frac{8a(ae^3-cdx(d^2+3dex+3e^2x^2))}{a+cx^4}-6\sqrt[4]{a}\sqrt[4]{c}\,d\left(\sqrt{2}\sqrt{c}\,d^2+4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+6\sqrt[4]{a}\sqrt[4]{c}\,d\left(\sqrt{2}\sqrt{c}\,d^2-4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+3\sqrt{2}\sqrt[4]{c}\left(-\sqrt[4]{a}\sqrt{c}\,d^3+a^{3/4}de^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+3\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}\,d^3-a^{3/4}de^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{32a^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4)
*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e
^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*
Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt
```

```
[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) + a^(3/
4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2
]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)
```

Maple [A]
time = 0.21, size = 317, normalized size = 0.91

method	result
risch	$\frac{\frac{3d\,e^2x^3}{4a}+\frac{3e\,x^2d^2}{4a}+\frac{d^3x}{4a}-\frac{e^3}{4c}}{c\,x^4+a}+\frac{3d\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e^2\_R^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$
default	$d^3\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+3d^2e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*
2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+3*d^2*e*(1/4*x^2/a
/(c*x^4+a)+1/4/a/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2)))+3*d*e^2*(1/4*x^3/a/(c
*x^4+a)+1/32/a/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(
1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+1/4*e^3*x^4/a/(c*x^4+a)
```

Maxima [A]
time = 0.50, size = 329, normalized size = 0.94

$$\frac{3\,d\left(\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2-4\sqrt{a}\sqrt{c}\,de+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+4\sqrt{a}\sqrt{c}\,de+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}\right)}{32\,a}+\frac{3\,cdx^3e^2+3\,cd^2x^2e+cd^3x-ae^3}{4\left(ac^2x^4+a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 3/32*d*(sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/
4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4
)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 - 4*sqrt(a)*sqrt(c)*d*e + sqrt(2)*a^(3/
4)*c^(1/4)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(sqrt(2)
*a^(1/4)*c^(3/4)*d^2 + 4*sqrt(a)*sqrt(c)*d*e + sqrt(2)*a^(3/4)*c^(1/4)*e^2)
```

```
*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sq
rt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a + 1/4*(3*c*d*x^3*e^2 + 3
*c*d^2*x^2*e + c*d^3*x - a*e^3)/(a*c^2*x^4 + a^2*c)
```

Fricas [C] Result contains complex when optimal does not.
time = 15.92, size = 91191, normalized size = 261.29

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1024*(768*c*d*e^2*x^3 + 768*c*d^2*e*x^2 + 256*c*d^3*x - 256*a*e^3 + 2*(a*
c^2*x^4 + a^2*c)*((-I*sqrt(3) + 1)*((a^2*c*sqrt(-1/(a*c))*sqrt(-(2*a*c*d^4*
e^2*sqrt(-1/(a*c)) + c*d^6 - a*d^2*e^4)/(a^4*c^2*sqrt(-1/(a*c)))) - 2*d^2*e
)^2/(a^3*c) - 3*(c*d^6 + 4*a^2*c*d^2*e*sqrt(-(2*a*c*d^4*e^2*sqrt(-1/(a*c))
+ c*d^6 - a*d^2*e^4)/(a^4*c^2*sqrt(-1/(a*c)))) - (6*c*d^4*e^2*sqrt(-1/(a*c)
) + d^2*e^4)*a)/(a^4*c^2*sqrt(-1/(a*c))))/(-9/8192*(c*d^6 + 4*a^2*c*d^2*e*s
qrt(-(2*a*c*d^4*e^2*sqrt(-1/(a*c)) + c*d^6 - a*d^2*e^4)/(a^4*c^2*sqrt(-1/(a
*c)))) - (6*c*d^4*e^2*sqrt(-1/(a*c)) + d^2*e^4)*a)*(a^2*c*sqrt(-1/(a*c))*sq
rt(-(2*a*c*d^4*e^2*sqrt(-1/(a*c)) + c*d^6 - a*d^2*e^4)/(a^4*c^2*sqrt(-1/(a*
c)))) - 2*d^2*e)/(a^5*c^2) + 27/8192*(2*c*d^8*e*sqrt(-1/(a*c)) - 2*a*d^4*e^
5*sqrt(-1/(a*c)) - 4*d^6*e^3 + a^5*c^2*sqrt(-1/(a*c))*(-(2*a*c*d^4*e^2*sqrt
(-1/(a*c)) + c*d^6 - a*d^2*e^4)/(a^4*c^2*sqrt(-1/(a*c))))^(3/2))/(a^5*c^2*s
qrt(-1/(a*c))) + 1/4096*(a^2*c*sqrt(-1/(a*c))*sqrt(-(2*a*c*d^4*e^2*sqrt(-1/
(a*c)) + c*d^6 - a*d^2*e^ ...
```

Sympy [A]
time = 184.58, size = 350, normalized size = 1.00

$$\operatorname{RootSum}\left(65536t^4a^7c^3+27648t^2a^4c^2d^4e^2+t\left(3456a^3cd^4e^5-3456a^2c^2d^8e\right)+81a^2d^4e^8+162acd^8e^4+81c^2d^{12},\left(t\mapsto t\log\left(x+\frac{4096t^3a^7c^2e^6+28672t^3a^6c^3d^4e^2-7680t^2a^5c^2d^4e^5+1536t^2a^4c^3d^8e+2160ta^4cd^4e^8+9216ta^3c^2d^8e^4+144ta^2c^3d^{12}+162a^3d^4e^{11}-648a^2cd^8e^7-810ac^2d^{12}e^3}{27a^3d^3e^{12}-891a^2cd^7e^8-891ac^2d^{11}e^4+27c^3d^{15}}\right)\right)\right)+\frac{-ae^3+cd^3x+3cd^2ex^2+3cde^2x^3}{4a^2c+4ac^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 + _t*(3456*
a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**4*e**8 + 162*a*c*d**
8*e**4 + 81*c**2*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 +
28672*_t**3*a**6*c**3*d**4*e**2 - 7680*_t**2*a**5*c**2*d**4*e**5 + 1536*_t*
*2*a**4*c**3*d**8*e + 2160*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**8*e**
4 + 144*_t*a**2*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e**7 - 8
10*a*c**2*d**12*e**3)/(27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 891*a*c*
*2*d**11*e**4 + 27*c**3*d**15)))) + (-a*e**3 + c*d**3*x + 3*c*d**2*e*x**2 +
 3*c*d*e**2*x**3)/(4*a**2*c + 4*a*c**2*x**4)
```

Giac [A]
time = 3.46, size = 342, normalized size = 0.98

$$\frac{3\,cdx^{3}e^{2}+3\,cd^{2}x^{2}e+cd^{3}x-ae^{3}}{4\,(cx^{4}+a)ac}+\frac{3\,\sqrt{2}\left(2\,\sqrt{2}\,\sqrt{ac}\,c^{2}d^{2}e+(ac^{3})^{\frac{1}{4}}c^{2}d^{3}+(ac^{3})^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}c^{3}}+\frac{3\,\sqrt{2}\left(2\,\sqrt{2}\,\sqrt{ac}\,c^{2}d^{2}e+(ac^{3})^{\frac{1}{4}}c^{2}d^{3}+(ac^{3})^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^{2}c^{3}}+\frac{3\,\sqrt{2}\left((ac^{3})^{\frac{1}{4}}c^{2}d^{3}-(ac^{3})^{\frac{3}{4}}de^{2}\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^{2}c^{3}}-\frac{3\,\sqrt{2}\left((ac^{3})^{\frac{1}{4}}c^{2}d^{3}-(ac^{3})^{\frac{3}{4}}de^{2}\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d*x^3*e^2 + 3*c*d^2*x^2*e + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) + 3
/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3
)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/
(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2
*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4
)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/32*sqrt
(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B]
time = 0.43, size = 670, normalized size = 1.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*root(6
5536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*
a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a
^3*c^2*d - 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3
*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81
*c^2*d^12, z, k)*a*c^2*d^4*x + 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^
4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 +
 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*root(65536*a^7*c^3*z
^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*
z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*e*x - 1
92*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*
z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12,
z, k)*a^2*c*d*e^3))/(64*a^3))*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^
2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*
a^2*d^4*e^8 + 81*c^2*d^12, z, k), k, 1, 4) + ((d^3*x)/(4*a) - e^3/(4*c) + (
3*d^2*e*x^2)/(4*a) + (3*d*e^2*x^3)/(4*a))/(a + c*x^4)
```

3.402 $\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=322

$$\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}+\frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}-\frac{\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(\right.}{8\sqrt{2}\,a^{7/4}c^{3/4}}$$

```
[Out] 1/4*x*(e*x+d)^2/a/(c*x^4+a)+1/2*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(
1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+
3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/1
6*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/
c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2
*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e^2+3\sqrt{c}\,d^2\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e^2+3\sqrt{c}\,d^2\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{de\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}-\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((
3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx &= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-4dex-e^2x^2}{a+cx^4}\,dx}{4a} \\
&= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \left(-\frac{4dex}{a+cx^4}+\frac{-3d^2-e^2x^2}{a+cx^4}\right)dx}{4a} \\
&= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-e^2x^2}{a+cx^4}\,dx}{4a} + \frac{(de)\int \frac{x}{a+cx^4}\,dx}{a} \\
&= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{(de)\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \cdots \\
&= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} + \cdots \\
&= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\cdots\right.}{16\sqrt{2}\,a^{7/4}c^{3/4}} \cdots \\
&= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}} + \cdots
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 321, normalized size = 1.00

$$\frac{\frac{8ax(d+ex)^2}{a+cx^4}-\frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}\,d^2+8\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}\,d^2-8\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{\sqrt{2}\left(-3\sqrt[4]{a}\sqrt{c}\,d^2+a^{3/4}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}}+\frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{c}\,d^2-a^{3/4}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^
(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d*e
+ Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (
Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2 - a^(
3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/
(32*a^2)
```

Maple [A]
time = 0.21, size = 293, normalized size = 0.91

method	result
risch	$\frac{\frac{e^2 x^3}{4a}+\frac{de\,x^2}{2a}+\frac{d^2 x}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e^2\_R^2+4\_R de+3d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d^2\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+2de\Bigg($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*
2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+2*d*e*(1/4*x^2/a/(
c*x^4+a)+1/4/a/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2)))+e^2*(1/4*x^3/a/(c*x^4+a
)+1/32/a/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/
(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+
2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.48, size = 316, normalized size = 0.98

$$\frac{x^3e^2+2\,dx^2e+d^2x}{4\,(acx^4+a^2)}+\frac{\frac{\sqrt{2}\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2-8\sqrt{a}\sqrt{c}\,de+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+8\sqrt{a}\sqrt{c}\,de+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^3*e^2 + 2*d*x^2*e + d^2*x)/(a*c*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(c
)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2
```

```
- sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(3*sqrt(2)*a^(
1/4)*c^(3/4)*d^2 - 8*sqrt(a)*sqrt(c)*d*e + sqrt(2)*a^(3/4)*c^(1/4)*e^2)*arc
tan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(3/4)*
d^2 + 8*sqrt(a)*sqrt(c)*d*e + sqrt(2)*a^(3/4)*c^(1/4)*e^2)*arctan(1/2*sqrt(
2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*
sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 3.17, size = 90963, normalized size = 282.49

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9216*(2304*e^2*x^3 + 4608*d*e*x^2 + 2304*d^2*x + 2*(a*c*x^4 + a^2)*((-I*s
qrt(3) + 1)*((a^2*c*sqrt(-1/(a*c))*sqrt(-(6*a*c*d^2*e^2*sqrt(-1/(a*c)) + 9*
c*d^4 - a*e^4)/(a^4*c^2*sqrt(-1/(a*c)))) - 4*d*e)^2/(a^3*c) - 3*(8*a^2*c*d*
e*sqrt(-(6*a*c*d^2*e^2*sqrt(-1/(a*c)) + 9*c*d^4 - a*e^4)/(a^4*c^2*sqrt(-1/(
a*c)))) + 9*c*d^4 - (22*c*d^2*e^2*sqrt(-1/(a*c)) + e^4)*a)/(a^4*c^2*sqrt(-1
/(a*c))))/(-1/24576*(8*a^2*c*d*e*sqrt(-(6*a*c*d^2*e^2*sqrt(-1/(a*c)) + 9*c*
d^4 - a*e^4)/(a^4*c^2*sqrt(-1/(a*c)))) + 9*c*d^4 - (22*c*d^2*e^2*sqrt(-1/(a
*c)) + e^4)*a)*(a^2*c*sqrt(-1/(a*c))*sqrt(-(6*a*c*d^2*e^2*sqrt(-1/(a*c)) +
9*c*d^4 - a*e^4)/(a^4*c^2*sqrt(-1/(a*c)))) - 4*d*e)/(a^5*c^2) + 1/8192*(a^5
*c^2*sqrt(-1/(a*c))*(-(6*a*c*d^2*e^2*sqrt(-1/(a*c)) + 9*c*d^4 - a*e^4)/(a^4
*c^2*sqrt(-1/(a*c))))^(3/2) - 4*a^2*c*d^2*e^2*sqrt(-1/(a*c))*sqrt(-(6*a*c*d
^2*e^2*sqrt(-1/(a*c)) + 9*c*d^4 - a*e^4)/(a^4*c^2*sqrt(-1/(a*c)))) + 36*c*d
^5*e*sqrt(-1/(a*c)) - 4*a*d*e^5*sqrt(-1/(a*c)) - 40*d^3*e^3)/(a^5*c^2*sqrt(
-1/(a*c))) + 1/110592*(a^ ...
```

Sympy [A]
time = 3.81, size = 318, normalized size = 0.99

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 11264t^2a^4c^2d^2e^2 + t\left(256a^3cde^5 - 2304a^2c^2d^5e\right) + a^2e^8 + 82acd^4e^4 + 81c^2d^8, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7c^2e^6 + 356352t^3a^6c^3d^4e^2 - 23552t^2a^5c^2d^3e^5 + 27648t^2a^4c^3d^7e + 912ta^4cd^2e^8 + 43584ta^3c^2d^6e^4 + 3888ta^2c^3d^{10} + 12a^3de^{11} - 1088a^2cd^5e^7 - 7020ac^2d^9e^3}{a^3e^{12} - 649a^2cd^4e^8 - 5841ac^2d^8e^4 + 729c^3d^{12}}\right)\right)\right) + \frac{d^2x + 2dex^2 + e^2x^3}{4a^2 + 4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*a
**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81*c
**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**3*a
**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**
3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t
*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d*
```

```
*9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 + 729*c
**3*d**12)))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x**4)
```

Giac [A]
time = 3.51, size = 323, normalized size = 1.00

$$\frac{x^3e^2+2\,dx^2e+d^2x}{4\,(cx^4+a)a}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ac}\,c^2de+3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(4\,\sqrt{2}\,\sqrt{ac}\,c^2de+3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}-\frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e^2 + 2*d*x^2*e + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2)
*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)
*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1
/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d
^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
^3)
```

Mupad [B]
time = 2.48, size = 391, normalized size = 1.21

$$\frac{\frac{d^2x}{4a}+\frac{e^2x^3}{4a}+\frac{dex^2}{2a}}{cx^4+a}+\left(\sum_{k=1}^{4}\ln\left(\frac{39\,c^2\,d^4\,e^2-a\,c\,e^6}{64\,a^3}-\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z,k\right)\left(\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z,k\right)\left(12\,c^3\,d^2-16\,c^3\,d\,e\,x\right)+\frac{x\left(18\,a\,c^3\,d^4-2\,a^2\,c^2\,e^4\right)}{8\,a^3}+\frac{2\,c^2\,d\,e^3}{a}\right)+\frac{5\,c^2\,d^3\,e^3\,x}{8\,a^3}\right)\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^2,x)
```

```
[Out] ((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(lo
g((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^4*
c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4
 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*
e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^
2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4 - 2*a
^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))*root(6
5536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a
^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), k, 1, 4)
```

3.403 $\int \frac{d+ex}{(a+cx^4)^2} \, dx$

Optimal. Leaf size=241

$$\frac{x(d+ex)}{4a\left(a+cx^4\right)}+\frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3d\log\left(\sqrt{a}-\sqrt{\right.}}{16\sqrt{}}$$

```
[Out] 1/4*x*(e*x+d)/a/(c*x^4+a)+3/16*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/
4)/c^(1/4)*2^(1/2)+3/16*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/
4)*2^(1/2)-3/32*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4
)/c^(1/4)*2^(1/2)+3/32*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/
a^(7/4)/c^(1/4)*2^(1/2)+1/4*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/c^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$-\frac{3d\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3d\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{3d\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{(a+cx^4)^2}\,dx &= \frac{x(d+ex)}{4a(a+cx^4)} - \frac{\int \frac{-3d-2ex}{a+cx^4}\,dx}{4a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} - \frac{\int \left(-\frac{3d}{a+cx^4} - \frac{2ex}{a+cx^4}\right)\,dx}{4a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d)\int \frac{1}{a+cx^4}\,dx}{4a} + \frac{e\int \frac{x}{a+cx^4}\,dx}{2a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{(3d)\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{e\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x\right.}{4a} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{c}}}}{16a^{3/2}\sqrt{c}} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\right.}{} \\
&= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4} - \frac{2\left(3\sqrt{2}\sqrt[4]{c}\,d+4\sqrt[4]{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{2\left(3\sqrt{2}\sqrt[4]{c}\,d-4\sqrt[4]{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - \frac{3\sqrt{2}\,d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\,d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)*
e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/4)
*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (3*Sqr
```

```
t[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3
*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))
/(32*a^(7/4))
```

Maple [A]
time = 0.20, size = 163, normalized size = 0.68

method	result
risch	$\frac{\frac{e\,x^2}{4a}+\frac{dx}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2e\_R+3d\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+e\left(\frac{}{4a(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2
)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+e*(1/4*x^2/a/(c*x^4+
a)+1/4/a/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2)))
```

Maxima [A]
time = 0.49, size = 241, normalized size = 1.00

$$\frac{x^2e+dx}{4\left(acx^4+a^2\right)}+\frac{\frac{3\sqrt{2}\,d\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{3\sqrt{2}\,d\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d-4\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d+4\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^2*e + d*x)/(a*c*x^4 + a^2) + 1/32*(3*sqrt(2)*d*log(sqrt(c)*x^2 + sqr
t(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 3*sqrt(2)*d*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(3*sqrt
(2)*a^(1/4)*c^(1/4)*d - 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*
c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d + 4*sqrt(a)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(c))*c^(1/4)))/a
```

Fricas [C] Result contains complex when optimal does not.
time = 1.57, size = 43065, normalized size = 178.69

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(48*e*x^2 - 12*(a*c*x^4 + a^2)*(2*e*sqrt(-1/(a*c))/a + 3*sqrt(-d^2/(a
^4*c*sqrt(-1/(a*c)))))*log(8*a^6*c*e^2*(2*e*sqrt(-1/(a*c))/a + 3*sqrt(-d^2/
(a^4*c*sqrt(-1/(a*c)))))^3 - 18*a^4*c*d^2*e*(2*e*sqrt(-1/(a*c))/a + 3*sqrt(
-d^2/(a^4*c*sqrt(-1/(a*c)))))^2 + 360*a*d^2*e^3 - 3*(81*c*d^5 - 64*a*d*e^4)
*x + (81*a^2*c*d^4 + 32*a^3*e^4)*(2*e*sqrt(-1/(a*c))/a + 3*sqrt(-d^2/(a^4*c
*sqrt(-1/(a*c)))))) + 2*(a*c*x^4 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((3
*a^2*c*sqrt(-1/(a*c))*sqrt(-d^2/(a^4*c*sqrt(-1/(a*c)))) - 2*e)^2/(a^3*c) +
3*(4*a*e^2*sqrt(-1/(a*c)) - 12*a^2*e*sqrt(-d^2/(a^4*c*sqrt(-1/(a*c)))) - 9*
d^2)/(a^4*c*sqrt(-1/(a*c))))/(9*(3*a^2*c*sqrt(-1/(a*c))*sqrt(-d^2/(a^4*c*sq
rt(-1/(a*c)))) - 2*e)*(4*a*e^2*sqrt(-1/(a*c)) - 12*a^2*e*sqrt(-d^2/(a^4*c*s
qrt(-1/(a*c)))) - 9*d^2)/(a^5*c) + 27*(27*a^5*c^2*sqrt(-1/(a*c))*(-d^2/(a^4
*c*sqrt(-1/(a*c))))^(3/2) - 12*a^2*c*e^2*sqrt(-1/(a*c))*sqrt(-d^2/(a^4*c*sq
rt(-1/(a*c)))) + 18*c*d^2*e*sqrt(-1/(a*c)) - 8*e^3)/(a^5*c^2*sqrt(-1/(a*c))
) + 2*(3*a^2*c*sqrt(-1/(a ...
```

Sympy [A]
time = 0.58, size = 155, normalized size = 0.64

$$\operatorname{RootSum}\left(65536t^4a^7c^2 + 2048t^2a^4ce^2 - 1152ta^2cd^2e + 16ae^4 + 81cd^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6ce^2 - 4608t^2a^4cd^2e - 512ta^3e^4 - 1296ta^2cd^4 + 360ad^2e^3}{192ade^4 - 243cd^5}\right)\right)\right) + \frac{dx + ex^2}{4a^2 + 4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d**
2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e**
2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 + 360
*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a
*c*x**4)
```

Giac [A]
time = 4.93, size = 241, normalized size = 1.00

$$\frac{3\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,d\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,d\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c} + \frac{x^2e + dx}{4\,(cx^4 + a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\,\sqrt{ac}\,ce + 3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2} + \frac{\sqrt{2}\left(2\sqrt{2}\,\sqrt{ac}\,ce + 3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqr
t(a/c))/(a^2*c) + 1/4*(x^2*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e
```

```
+ 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a^2*c^2)
```

Mupad [B]
time = 0.27, size = 282, normalized size = 1.17

$$\left(\sum_{k=1}^{4}\ln\left(\frac{c^2\,\left(3\,d\,e^2+2\,e^3\,x-\mathrm{root}(65536\,a^7\,c^2\,z^4+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k)^2\,a^3\,c\,d\,192+\mathrm{root}(65536\,a^7\,c^2\,z^4+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k)^2\,a^3\,c\,e\,x\,128-\mathrm{root}(65536\,a^7\,c^2\,z^4+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k)\,a\,c\,d^2\,x\,36\right)}{a^3\,16}\right)\,\mathrm{root}\left(65536\,a^7\,c^2\,z^4+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k\right)\right)+\frac{\frac{e\,x^2}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((c^2*(3*d*e^2 + 2*e^3*x - 192*root(65536*a^7*c^2*z^4 + 2048*a^4*
c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 128
*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^
4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^
2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x))/(16*a^3
))*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*
d^4 + 16*a*e^4, z, k), k, 1, 4) + ((e*x^2)/(4*a) + (d*x)/(4*a))/(a + c*x^4)
```

3.404 $\int \frac{1}{(a+cx^4)^2} dx$

Optimal. Leaf size=202

$$\frac{x}{4a(a+cx^4)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}$$

```
[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)
*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3
/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1
/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*
2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {205, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{x}{4a(a+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}-\sqrt{2}}}{16\sqrt{2}\,a}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{}\right.}{16\sqrt{}}$$

Mathematica [A]
time = 0.07, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

Maple [A]
time = 0.19, size = 118, normalized size = 0.58

method	result	size
risch	$\frac{x}{4a(c\,x^4+a)} + \frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$	46

default	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))
```

Maxima [A]
time = 0.48, size = 189, normalized size = 0.94

$$\frac{x}{4\,(acx^4+a^2)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

Fricas [A]
time = 0.41, size = 173, normalized size = 0.86

$$\frac{12\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)+4\,x}{16\,(acx^4+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

Sympy [A]
time = 0.13, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4}+\operatorname{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

Giac [A]
time = 4.35, size = 194, normalized size = 0.96

$$\frac{x}{4\,(cx^4+a)a}+\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\,\left(2\,x+\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\,\left(2\,x-\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}-\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

Mupad [B]
time = 0.09, size = 58, normalized size = 0.29

$$\frac{x}{4\,a\,\left(c\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

3.405 $\int \frac{1}{(d+ex)(a+cx^4)^2} \, dx$

Optimal. Leaf size=855

$$\frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)}-\frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)}-\frac{\sqrt[4]{c}\,de^4\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)}{2\sqrt{2}\,a^{3/4}\,(c}$$

```
[Out] 1/4*(a*e^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)+e^7*ln(e*
x+d)/(a*e^4+c*d^4)^2-1/4*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)^2-1/4*d^2*e*arctan(x
^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)-1/2*d^2*e^5*arctan(x^2*c^
(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^2/a^(1/2)-1/8*c^(1/4)*d*e^4*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)
/(a*e^4+c*d^4)^2*2^(1/2)+1/8*c^(1/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2
)+1/4*c^(1/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c
^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/4*c^(1/4)*d*e^4*arctan(1+c^(1/4)*
x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2*2^(1/2
)-1/32*c^(1/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a
^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/32*c^(1/4)*d*ln(a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7
/4)/(a*e^4+c*d^4)*2^(1/2)+1/16*c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)*2^(1/2)+1/16*c^(1/4)*d
*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a
*e^4+c*d^4)*2^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6874, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

$$\frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\,\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)e^4}{4\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)e^4}{4\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e}{4a^{3/2}\left(cd^4+ae^4\right)}+\frac{ae^3+cx\left(d^3-exd^2+e^2x^2d\right)}{4a\left(cd^4+ae^4\right)\left(cx^4+a\right)}-\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}-\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)}{16\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)}{16\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4))
 - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^
4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a
*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2)
```

```
+ (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 +
a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4
*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16
*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)
^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
```

```
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} - \frac{ce^4(-d^3+d^2ex-}{\left(cd^4+ae^4\right)^2}\right.$$

$$= \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} + \frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{\left(a+cx^4\right)^2}\,dx}{cd^4+ae^4}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \left(\frac{-d^3-de^2x^2}{a+cx^4} + \frac{x(d^2e+e^3x}{a+cx^4}\right.}{\left(cd^4+ae^4\right)^2}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{-d^3-de^2x^2}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int}{(c}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\mathrm{Subst}\left(\int \frac{d^2e+e^3x}{a+cx^2}\,dx, x, x\right.}{2\left(cd^4+ae^4\right)^2}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(cd^2e^5\right)\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x\right.}{2\left(cd^4+ae^4\right)^2}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.}{4a^{3/2}\left(cd^4+ae^4\right.}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.}{4a^{3/2}\left(cd^4+ae^4\right.}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.}{4a^{3/2}\left(cd^4+ae^4\right.}$$

Mathematica [A]

time = 0.27, size = 558, normalized size = 0.65

$$\frac{\frac{8(cd^4+ae^4)(ae^3+cdx(d^2-dex+e^2x^2))}{a(a+cx^4)}-\frac{2\sqrt[4]{c}\,d\left(3\sqrt{2}c^{3/2}d^6-4\sqrt[4]{a}c^{5/4}d^5e+\sqrt{2}\sqrt{a}cd^4e^2+7\sqrt{2}a\sqrt{c}d^2e^4-12a^{5/4}\sqrt[4]{c}de^5+5\sqrt{2}a^{3/2}e^6\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt[4]{c}\,d\left(3\sqrt{2}c^{3/2}d^6+4\sqrt[4]{a}c^{5/4}d^5e+\sqrt{2}\sqrt{a}cd^4e^2+7\sqrt{2}a\sqrt{c}d^2e^4+12a^{5/4}\sqrt[4]{c}de^5+5\sqrt{2}a^{3/2}e^6\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+32e^7\log(d+ex)+\frac{\sqrt{2}\sqrt[4]{c}\left(-3c^{3/2}d^7+\sqrt{a}cd^5e^2-7a\sqrt{c}d^3e^4+5a^{3/2}de^6\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(3c^{3/2}d^7-\sqrt{a}cd^5e^2+7a\sqrt{c}d^3e^4-5a^{3/2}de^6\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{7/4}}-8e^7\log(a+cx^4)}{32(cd^4+ae^4)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)
) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]
*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*e^5
 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4)
+ (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*S
qrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 +
 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) +
32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*d^5*e^2
- 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c
*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/(32*(c*d^4 +
 a*e^4)^2)
```

Maple [A]

time = 0.25, size = 430, normalized size = 0.50

method	result
default	$\frac{e^7\ln(ex+d)}{(e^4a+d^4c)^2}+\frac{c\left(\frac{\frac{de^2(e^4a+d^4c)x^3}{4a}-\frac{d^2e(e^4a+d^4c)x^2}{4a}+\frac{d^3(e^4a+d^4c)x}{4a}+\frac{e^3(e^4a+d^4c)}{4c}}{cx^4+a}+\frac{(7ad^3e^4+3cd^7)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{}}\right.\right.}{}\right.}{}$
risch	$\frac{\frac{de^2cx^3}{4a(e^4a+d^4c)}-\frac{d^2ecx^2}{4a(e^4a+d^4c)}+\frac{d^3cx}{4a(e^4a+d^4c)}+\frac{e^3}{4e^4a+4d^4c}}{cx^4+a}+\Big(\sum_{\_R=\mathrm{RootOf}\left((a^9e^8+2a^8cd^4e^4+a^7c^2d^8)\_Z^4+16a^7e^7\_Z^3+(96a^5e^6+20a^4cd\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^7*ln(e*x+d)/(a*e^4+c*d^4)^2+c/(a*e^4+c*d^4)^2*((1/4*d*e^2*(a*e^4+c*d^4)/a
*x^3-1/4*d^2*e*(a*e^4+c*d^4)/a*x^2+1/4*d^3*(a*e^4+c*d^4)/a*x+1/4*e^3*(a*e^4
+c*d^4)/c)/(c*x^4+a)+1/4/a*(1/8*(7*a*d^3*e^4+3*c*d^7)*(a/c)^(1/4)/a*2^(1/2)
*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/
```

```
c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x
-1))+1/2*(-6*a*d^2*e^5-2*c*d^6*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(
5*a*d*e^6+c*d^5*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)
^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-a*e^7/c*ln(c*x^4+a)))
```

Maxima [A]
time = 0.50, size = 582, normalized size = 0.68

$$\frac{c\left(\frac{\sqrt{2}\left(3c^2d^7-\sqrt{a}c^{\frac{3}{2}}d^5e^2+7acd^3e^4-5a^{\frac{3}{2}}\sqrt{c}de^6-4\sqrt{2}a^{\frac{7}{4}}c^{\frac{1}{4}}e^7\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(3c^2d^7-\sqrt{a}c^{\frac{3}{2}}d^5e^2+7acd^3e^4-5a^{\frac{3}{2}}\sqrt{c}de^6+4\sqrt{2}a^{\frac{7}{4}}c^{\frac{1}{4}}e^7\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{9}{4}}d^7+4\sqrt{a}c^2d^6e+\sqrt{2}a^{\frac{3}{4}}c^{\frac{7}{4}}d^5e^2+7\sqrt{2}a^{\frac{5}{4}}c^{\frac{5}{4}}d^3e^4+12a^{\frac{3}{2}}cd^2e^5+5\sqrt{2}a^{\frac{7}{4}}c^{\frac{3}{4}}de^6\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{9}{4}}d^7-4\sqrt{a}c^2d^6e+\sqrt{2}a^{\frac{3}{4}}c^{\frac{7}{4}}d^5e^2+7\sqrt{2}a^{\frac{5}{4}}c^{\frac{5}{4}}d^3e^4-12a^{\frac{3}{2}}cd^2e^5+5\sqrt{2}a^{\frac{7}{4}}c^{\frac{3}{4}}de^6\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}\right)}{32\left(ac^2d^8+2a^2cd^4e^4+a^3e^8\right)}+\frac{e^7\log\left(xe+d\right)}{c^2d^8+2acd^4e^4+a^2e^8}+\frac{cdx^3e^2-cd^2x^2e+cd^3x+ae^3}{4\left(a^2cd^4+\left(ac^2d^4+a^2ce^4\right)x^4+a^3e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*c*(sqrt(2)*(3*c^2*d^7 - sqrt(a)*c^(3/2)*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^
(3/2)*sqrt(c)*d*e^6 - 4*sqrt(2)*a^(7/4)*c^(1/4)*e^7)*log(sqrt(c)*x^2 + sqrt
(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - sqrt(2)*(3*c^2*d^7 - s
qrt(a)*c^(3/2)*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^(3/2)*sqrt(c)*d*e^6 + 4*sqrt(2
)*a^(7/4)*c^(1/4)*e^7)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + 4*sqrt(a)*c^2*d^6
*e + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 +
12*a^(3/2)*c*d^2*e^5 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6)*arctan(1/2*sqrt(2)*
(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 - 4*sqrt(a)*
c^2*d^6*e + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3
*e^4 - 12*a^(3/2)*c*d^2*e^5 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6)*arctan(1/2*s
qrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3
/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8)
 + e^7*log(x*e + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/4*(c*d*x^3*e^2
- c*d^2*x^2*e + c*d^3*x + a*e^3)/(a^2*c*d^4 + (a*c^2*d^4 + a^2*c*e^4)*x^4 +
 a^3*e^4)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.31, size = 771, normalized size = 0.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3
/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqr
t(2)*a^2*c^3*d^4 - 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*
c^2*d^2*e^2 + sqrt(2)*a^3*c^2*e^4 - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqr
t(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c
^3*d^4 + 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^
2 + sqrt(2)*a^3*c^2*e^4 + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c
^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sqrt(2)*(a*c^3)^(1/
4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/3
2*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sq
rt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^
2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a
^4*c^2*e^8) - 1/4*e^7*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^
8) + e^8*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/4*(a*c
*d^4*e^3 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 +
a^2*e^7 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)
```

Mupad [B]
time = 2.99, size = 1591, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)),x)
```

```
[Out] e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + symsum(log((81*c^
5*d^5*e^6 + 64*a*c^4*d*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)
) + root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z
^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*
a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a
```

^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 32768*a^7*c^6*d^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 - 16384*a^7*c^6*d^8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 13312*a^5*c^6*d^9*e^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - 4096*a^5*c^6*d^8*e^6 + 65536*a^6*c^5*d^4*e^10))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (8704*a^5*c^4*d*e^12 + 3584*a^3*c^6*d^9*e^4 + 15360*a^4*c^5*d^5*e^8)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(15360*a^5*c^4*e^13 - 576*a^2*c^7*d^12*e + 1920*a^3*c^6*d^8*e^5 + 18880*a^4*c^5*d^4*e^9))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (192*a*c^6*d^9*e^3 + 704*a^3*c^4*d*e^11 + 1536*a^2*c^5*d^5*e^7)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(1280*a^3*c^4*e^12 + 256*a*c^6*d^8*e^4 + 2240*a^2*c^5*d^4*e^8))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (81*c^5*d^4*e^7*x)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)))*root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k), k, 1, 4) + (e^7*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4) + (c*d^3*x)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*x^4)) - (c*d^2*e*x^2)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*x^4)) + (c*d*e^2*x^3)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*x^4))

3.406 $\int \frac{1}{(d+ex)^2(a+cx^4)^2}\,dx$

Optimal. Leaf size=1141

$$-\frac{e^7}{(cd^4+ae^4)^2(d+ex)}+\frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2(3cd^4-ae^4)\,x^2))}{4a\,(cd^4+ae^4)^2\,(a+cx^4)}-\frac{\sqrt{c}\,de^5(3cd\ldots}{\sqrt{\ldots}}$$

```
[Out] -e^7/(a*e^4+c*d^4)^2/(e*x+d)+1/4*c*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d
*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+
8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)
^3-1/2*d*e*(-a*e^4+c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^
4+c*d^4)^2-d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^
4+c*d^4)^3/a^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c
^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(7
/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/
2)+x^2*c^(1/2))*(-e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/
2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2
)/a^(1/4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^
(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(e^2*(-a*e^4+3*c*d^4)*a^(1/2)+3*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(7/4)/(
a*e^4+c*d^4)^2*2^(1/2)-1/8*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^(1/2
))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(1/4)*e^4*ln(a^(1/4)*c^(1/4)*x*2^(
1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*
d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*arctan(-1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4
)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*e^4*arctan(1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2)+d^2*(-3*a*e^4+5*c*d^4)*c^
(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)
```

Rubi [A]
time = 1.17, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6874, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*
a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4
 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]
```

```
*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*
Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)
*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
+ a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(
7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(
Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a
*e^4)^3) - (2*c*d^3*e^7*Log[a + c*x^4])/(c*d^4 + a*e^4)^3
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
```

```
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2 \left(a+cx^4\right)^2}\, dx = \int \left(\frac{e^8}{\left(cd^4+ae^4\right)^2 (d+ex)^2} + \frac{8cd^3e^8}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c(d^2(cd^4-3ae^4)-2de(c}{(c}\right.$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{8cd^3e^7 \log(d+ex)}{\left(cd^4+ae^4\right)^3} + \frac{(ce^4)\int \frac{d^2(5cd^4-3ae^4)-2de(3cd^4-ae^4)}{a+c}}{(cd^4+ae}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{4a\left(cd^4+ae^4\right)^2 (a+cx^4)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{4a\left(cd^4+ae^4\right)^2 (a+cx^4)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{4a\left(cd^4+ae^4\right)^2 (a+cx^4)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{4a\left(cd^4+ae^4\right)^2 (a+cx^4)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{4a\left(cd^4+ae^4\right)^2 (a+cx^4)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{4a\left(cd^4+ae^4\right)^2 (a+cx^4)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2 (d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{4a\left(cd^4+ae^4\right)^2 (a+cx^4)}$$

Mathematica [A]
time = 0.57, size = 807, normalized size = 0.71

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] ((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^4*x*(d^2 -
 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))
```

```
/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(5/2)*d^10 + 8*a^(1/4)*c^(9/4)*
d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 48*a
^(5/4)*c^(5/4)*d^5*e^5 - 30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2*Sqrt
[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/2)*d^10
 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 14*Sqrt[2]*a*c
^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*Sqrt[2]*a^(3/2)*c*d^4*e^6
- 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 - 5*Sqrt[2]*a^(
5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 256*c*d^3*e^7
*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 +
14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^
(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4)
 + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 + 14*a*c^(3/2)*
d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^(5/2)*e^10)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 64*c*d^3*e
^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^3)
```

Maple [A]
time = 0.27, size = 534, normalized size = 0.47

method	result
default	$-\frac{e^7}{(e^4a+d^4c)^2(ex+d)}+\frac{8cd^3e^7\ln(ex+d)}{(e^4a+d^4c)^3}-\frac{c\left(\frac{\frac{e^2\left(a^2e^8-2acd^4e^4-3c^2d^8\right)x^3}{4a}-\frac{de\left(a^2e^8-c^2d^8\right)x^2}{2a}+\frac{d^2\left(3a^2e^8+2acd^4e^4-c^2d^8\right)x}{4a}-d^3e}{cx^4+a}\right.}{}$
risch	$\frac{-\frac{ce^3\left(5e^4a-3d^4c\right)x^4}{4a\left(e^4a+d^4c\right)^2}+\frac{de^2cx^3}{4a\left(e^4a+d^4c\right)}-\frac{d^2ecx^2}{4a\left(e^4a+d^4c\right)}+\frac{d^3cx}{4a\left(e^4a+d^4c\right)}-\frac{e^3\left(e^4a-d^4c\right)}{\left(e^4a+d^4c\right)^2}}{(cx^4+a)(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{10}e^{12}+3a^9cd^4e^8+3a^8c^2d^8e^4+a\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e^7/(a*e^4+c*d^4)^2/(e*x+d)+8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-c/(a*e^4
+c*d^4)^3*((1/4*e^2*(a^2*e^8-2*a*c*d^4*e^4-3*c^2*d^8)/a*x^3-1/2*d*e*(a^2*e^
8-c^2*d^8)/a*x^2+1/4*d^2*(3*a^2*e^8+2*a*c*d^4*e^4-c^2*d^8)/a*x-d^3*e^3*(a*e
^4+c*d^4))/(c*x^4+a)+1/4/a*(1/8*(21*a^2*d^2*e^8-14*a*c*d^6*e^4-3*c^2*d^10)*
(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(
2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-12*a^2*d*e^9+24*a*c*d^5*e^5+4*c^2*d^9*e)/(a
*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(5*a^2*e^10-30*a*c*d^4*e^6-3*c^2*d^8*
e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2
```

+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+8*a*d^3*e^7*ln(c*x^4+a)))

Maxima [A]
time = 0.52, size = 919, normalized size = 0.81

$$\frac{8\,cd^3e^7\log\left(xe+d\right)}{c^3d^{12}+3\,ac^2d^8e^4+3\,a^2cd^4e^8+a^3e^{12}}+\frac{c\left(\frac{\sqrt{2}\left(3\,c^3d^{10}-3\,\sqrt{a}\,c^{\frac{5}{2}}d^8e^2+14\,ac^2d^6e^4-30\,a^{\frac{3}{2}}c^{\frac{3}{2}}d^4e^6-32\,\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{5}{4}}d^3e^7-21\,a^2cd^2e^8+5\,a^{\frac{5}{2}}\sqrt{c}\,e^{10}\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{\sqrt{2}\left(3\,c^3d^{10}-3\,\sqrt{a}\,c^{\frac{5}{2}}d^8e^2+14\,ac^2d^6e^4-30\,a^{\frac{3}{2}}c^{\frac{3}{2}}d^4e^6+32\,\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{5}{4}}d^3e^7-21\,a^2cd^2e^8+5\,a^{\frac{5}{2}}\sqrt{c}\,e^{10}\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{2\left(3\,\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{13}{4}}d^{10}+8\,\sqrt{a}\,c^3d^9e+3\,\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{11}{4}}d^8e^2+14\,\sqrt{2}\,a^{\frac{5}{4}}c^{\frac{9}{4}}d^6e^4+48\,a^{\frac{3}{2}}c^2d^5e^5+30\,\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{7}{4}}d^4e^6-21\,\sqrt{2}\,a^{\frac{9}{4}}c^{\frac{5}{4}}d^2e^8-24\,a^{\frac{5}{2}}cde^9-5\,\sqrt{2}\,a^{\frac{11}{4}}c^{\frac{3}{4}}e^{10}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}+\frac{2\left(3\,\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{13}{4}}d^{10}-8\,\sqrt{a}\,c^3d^9e+3\,\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{11}{4}}d^8e^2+14\,\sqrt{2}\,a^{\frac{5}{4}}c^{\frac{9}{4}}d^6e^4-48\,a^{\frac{3}{2}}c^2d^5e^5+30\,\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{7}{4}}d^4e^6-21\,\sqrt{2}\,a^{\frac{9}{4}}c^{\frac{5}{4}}d^2e^8+24\,a^{\frac{5}{2}}cde^9-5\,\sqrt{2}\,a^{\frac{11}{4}}c^{\frac{3}{4}}e^{10}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}\right)}{32\left(ac^3d^{12}+3\,a^2c^2d^8e^4+3\,a^3cd^4e^8+a^4e^{12}\right)}+\frac{4\,acd^4e^3+\left(3\,c^2d^4e^3-5\,ace^7\right)x^4+\left(c^2d^5e^2+acde^6\right)x^3-\left(c^2d^6e+acd^2e^5\right)x^2-4\,a^2e^7+\left(c^2d^7+acd^3e^4\right)x}{4\left(a^2c^2d^9+2\,a^3cd^5e^4+\left(ac^3d^8e+2\,a^2c^2d^4e^5+a^3ce^9\right)x^5+a^4de^8+\left(ac^3d^9+2\,a^2c^2d^5e^4+a^3cde^8\right)x^4+\left(a^2c^2d^8e+2\,a^3cd^4e^5+a^4e^9\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 8*c*d^3*e^7*log(x*e + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 1/32*c*(sqrt(2)*(3*c^3*d^10 - 3*sqrt(a)*c^(5/2)*d^8*e^2 + 14*a*c^2*d^6*e^4 - 30*a^(3/2)*c^(3/2)*d^4*e^6 - 32*sqrt(2)*a^(7/4)*c^(5/4)*d^3*e^7 - 21*a^2*c*d^2*e^8 + 5*a^(5/2)*sqrt(c)*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - sqrt(2)*(3*c^3*d^10 - 3*sqrt(a)*c^(5/2)*d^8*e^2 + 14*a*c^2*d^6*e^4 - 30*a^(3/2)*c^(3/2)*d^4*e^6 + 32*sqrt(2)*a^(7/4)*c^(5/4)*d^3*e^7 - 21*a^2*c*d^2*e^8 + 5*a^(5/2)*sqrt(c)*e^10)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 8*sqrt(a)*c^3*d^9*e + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 48*a^(3/2)*c^2*d^5*e^5 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 24*a^(5/2)*c*d*e^9 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 - 8*sqrt(a)*c^3*d^9*e + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 - 48*a^(3/2)*c^2*d^5*e^5 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 + 24*a^(5/2)*c*d*e^9 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^3*d^12 + 3*a^2*c^2*d^8*e^4 + 3*a^3*c*d^4*e^8 + a^4*e^12) + 1/4*(4*a*c*d^4*e^3 + (3*c^2*d^4*e^3 - 5*a*c*e^7)*x^4 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 - 4*a^2*e^7 + (c^2*d^7 + a*c*d^3*e^4)*x)/(a^2*c^2*d^9 + 2*a^3*c*d^5*e^4 + (a*c^3*d^8*e + 2*a^2*c^2*d^4*e^5 + a^3*c*e^9)*x^5 + a^4*d*e^8 + (a*c^3*d^9 + 2*a^2*c^2*d^5*e^4 + a^3*c*d*e^8)*x^4 + (a^2*c^2*d^8*e + 2*a^3*c*d^4*e^5 + a^4*e^9)*x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 15.45, size = 1104, normalized size = 0.97

$$-\frac{2\,cd^3e^7\log\left(|cx^4+a|\right)}{c^3d^{12}+3\,ac^2d^8e^4+3\,a^2cd^4e^8+a^3e^{12}}+\frac{8\,cd^3e^8\log\left(|xe+d|\right)}{c^3d^{12}e+3\,ac^2d^8e^5+3\,a^2cd^4e^9+a^3e^{13}}+\frac{\left(5\sqrt{2}\sqrt{ac}\,c^2d^3e+3\,(ac^3)^{\frac{1}{4}}c^2d^4+3\sqrt{2}\,ac^2de^3+6\,(ac^3)^{\frac{3}{4}}d^2e^2-5\,(ac^3)^{\frac{1}{4}}ace^4\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^3d^6-6\,(ac^3)^{\frac{1}{4}}a^2c^2d^5e+9\sqrt{2}\sqrt{ac}\,a^2c^2d^4e^2+9\sqrt{2}\,a^3c^2d^2e^4-16\,(ac^3)^{\frac{3}{4}}a^2d^3e^3-6\,(ac^3)^{\frac{1}{4}}a^3cde^5+\sqrt{2}\sqrt{ac}\,a^3ce^6\right)}+\frac{\left(5\sqrt{2}\sqrt{ac}\,c^2d^3e+3\,(ac^3)^{\frac{1}{4}}c^2d^4-3\sqrt{2}\,ac^2de^3+6\,(ac^3)^{\frac{3}{4}}d^2e^2-5\,(ac^3)^{\frac{1}{4}}ace^4\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^3d^6+6\,(ac^3)^{\frac{1}{4}}a^2c^2d^5e+9\sqrt{2}\sqrt{ac}\,a^2c^2d^4e^2+9\sqrt{2}\,a^3c^2d^2e^4+16\,(ac^3)^{\frac{3}{4}}a^2d^3e^3+6\,(ac^3)^{\frac{1}{4}}a^3cde^5+\sqrt{2}\sqrt{ac}\,a^3ce^6\right)}+\frac{\left(3\sqrt{2}(ac^3)^{\frac{1}{4}}c^4d^{10}-3\sqrt{2}(ac^3)^{\frac{3}{4}}c^2d^8e^2+14\sqrt{2}(ac^3)^{\frac{1}{4}}ac^3d^6e^4-30\sqrt{2}(ac^3)^{\frac{3}{4}}acd^4e^6-21\sqrt{2}(ac^3)^{\frac{1}{4}}a^2c^2d^2e^8+5\sqrt{2}(ac^3)^{\frac{3}{4}}a^2e^{10}\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\left(a^2c^5d^{12}+3\,a^3c^4d^8e^4+3\,a^4c^3d^4e^8+a^5c^2e^{12}\right)}-\frac{\left(3\sqrt{2}(ac^3)^{\frac{1}{4}}c^4d^{10}-3\sqrt{2}(ac^3)^{\frac{3}{4}}c^2d^8e^2+14\sqrt{2}(ac^3)^{\frac{1}{4}}ac^3d^6e^4-30\sqrt{2}(ac^3)^{\frac{3}{4}}acd^4e^6-21\sqrt{2}(ac^3)^{\frac{1}{4}}a^2c^2d^2e^8+5\sqrt{2}(ac^3)^{\frac{3}{4}}a^2e^{10}\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\left(a^2c^5d^{12}+3\,a^3c^4d^8e^4+3\,a^4c^3d^4e^8+a^5c^2e^{12}\right)}+\frac{3\,c^2d^4x^4e^3+c^2d^5x^3e^2-c^2d^6x^2e+c^2d^7x-5\,acx^4e^7+acdx^3e^6-acd^2x^2e^5+acd^3xe^4+4\,acd^4e^3-4\,a^2e^7}{4\left(ac^2d^8+2\,a^2cd^4e^4+a^3e^8\right)\left(cx^5e+cdx^4+axe+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*d^3*e^7*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*
e^8 + a^3*e^12) + 8*c*d^3*e^8*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e
^5 + 3*a^2*c*d^4*e^9 + a^3*e^13) + 1/8*(5*sqrt(2)*sqrt(a*c)*c^2*d^3*e + 3*(
a*c^3)^(1/4)*c^2*d^4 + 3*sqrt(2)*a*c^2*d*e^3 + 6*(a*c^3)^(3/4)*d^2*e^2 - 5*
(a*c^3)^(1/4)*a*c*e^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(sqrt(2)*a^2*c^3*d^6 - 6*(a*c^3)^(1/4)*a^2*c^2*d^5*e + 9*sqrt(2)*sq
rt(a*c)*a^2*c^2*d^4*e^2 + 9*sqrt(2)*a^3*c^2*d^2*e^4 - 16*(a*c^3)^(3/4)*a^2*
d^3*e^3 - 6*(a*c^3)^(1/4)*a^3*c*d*e^5 + sqrt(2)*sqrt(a*c)*a^3*c*e^6) + 1/8*
(5*sqrt(2)*sqrt(a*c)*c^2*d^3*e + 3*(a*c^3)^(1/4)*c^2*d^4 - 3*sqrt(2)*a*c^2*
d*e^3 + 6*(a*c^3)^(3/4)*d^2*e^2 - 5*(a*c^3)^(1/4)*a*c*e^4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6 + 6*(a*c^3
)^(1/4)*a^2*c^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^2*c^2*d^4*e^2 + 9*sqrt(2)*a^3
*c^2*d^2*e^4 + 16*(a*c^3)^(3/4)*a^2*d^3*e^3 + 6*(a*c^3)^(1/4)*a^3*c*d*e^5 +
 sqrt(2)*sqrt(a*c)*a^3*c*e^6) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 - 3*
sqrt(2)*(a*c^3)^(3/4)*c^2*d^8*e^2 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4
- 30*sqrt(2)*(a*c^3)^(3/4)*a*c*d^4*e^6 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d
^2*e^8 + 5*sqrt(2)*(a*c^3)^(3/4)*a^2*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4)
+ sqrt(a/c))/(a^2*c^5*d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^
2*e^12) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 - 3*sqrt(2)*(a*c^3)^(3/4)*
c^2*d^8*e^2 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 30*sqrt(2)*(a*c^3)^(
3/4)*a*c*d^4*e^6 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^2*e^8 + 5*sqrt(2)*(a*
c^3)^(3/4)*a^2*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*
d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2*e^12) + 1/4*(3*c^2*d
^4*x^4*e^3 + c^2*d^5*x^3*e^2 - c^2*d^6*x^2*e + c^2*d^7*x - 5*a*c*x^4*e^7 +
a*c*d*x^3*e^6 - a*c*d^2*x^2*e^5 + a*c*d^3*x*e^4 + 4*a*c*d^4*e^3 - 4*a^2*e^7
)/((a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8)*(c*x^5*e + c*d*x^4 + a*x*e + a*d
))
```

Mupad [B]
time = 4.10, size = 2246, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^2),x)
```

```
[Out] symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 655
36*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 1812
48*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 1
9200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e^3 +
 2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^15)/(
256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^
6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^
4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3
 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*
e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((4096*a^3*c^8*
d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 144384*a^6*c^5*
d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12
*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2
*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*
d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a
^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*(roo
t(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^1
2*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e
^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e
^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^11*c^4*d*e^22 - 32768*a^6*
c^9*d^21*e^2 - 32768*a^7*c^8*d^17*e^6 + 196608*a^8*c^7*d^13*e^10 + 458752*a
^9*c^6*d^9*e^14 + 360448*a^10*c^5*d^5*e^18)/(256*(a^8*e^16 + a^4*c^4*d^16 +
 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(81920*a^
11*c^4*e^23 - 49152*a^6*c^9*d^20*e^3 - 114688*a^7*c^8*d^16*e^7 + 32768*a^8*
c^7*d^12*e^11 + 294912*a^9*c^6*d^8*e^15 + 278528*a^10*c^5*d^4*e^19))/(256*(
a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2
*d^8*e^8))) + (5120*a^9*c^4*e^21 - 3072*a^4*c^9*d^20*e + 17408*a^5*c^8*d^16
*e^5 + 337920*a^6*c^7*d^12*e^9 + 616448*a^7*c^6*d^8*e^13 + 304128*a^8*c^5*d
^4*e^17)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*
e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(356352*a^6*c^7*d^11*e^10 - 32768*a^5*c^8*d^
15*e^6 - 10240*a^4*c^9*d^19*e^2 + 770048*a^7*c^6*d^7*e^14 + 391168*a^8*c^5*
d^3*e^18))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^1
2*e^4 + 6*a^6*c^2*d^8*e^8))) + (x*(768*a^3*c^8*d^14*e^5 - 576*a^2*c^9*d^18*
e + 105088*a^4*c^7*d^10*e^9 + 221952*a^5*c^6*d^6*e^13 + 183744*a^6*c^5*d^2*
e^17))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^
4 + 6*a^6*c^2*d^8*e^8))) + (x*(200*a*c^8*d^13*e^4 + 19400*a^4*c^5*d*e^16 +
7512*a^2*c^7*d^9*e^8 + 2136*a^3*c^6*d^5*e^12))/(256*(a^8*e^16 + a^4*c^4*d^1
```

```
6 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (81*c^7*
d^9*e^6 - 254*a*c^6*d^5*e^10 + 625*a^2*c^5*d*e^14)/(256*(a^8*e^16 + a^4*c^4
*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(6
25*a^2*c^5*e^15 + 81*c^7*d^8*e^7 - 894*a*c^6*d^4*e^11))/(256*(a^8*e^16 + a^
4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)))*r
oot(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d
^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2
*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d
*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k), k, 1, 4) + ((x^4*(3*c^2*d^4*e^3 -
 5*a*c*e^7))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (a*e^7 - c*d^4*e^3
)/(a*e^4 + c*d^4)^2 + (c*d^3*x)/(4*a*(a*e^4 + c*d^4)) - (c*d^2*e*x^2)/(4*a*
(a*e^4 + c*d^4)) + (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)))/(a*d + a*e*x + c*d*
x^4 + c*e*x^5) + (8*c*d^3*e^7*log(d + e*x))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*
d^8*e^4 + 3*a^2*c*d^4*e^8)
```

3.407 $\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=1384

$$-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex)^2}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}+\frac{c(2ad^2e^3(5cd^4-3ae^4)+x(d(c^2d^8-12acd^4e^4+3a^2e^8)-}{4a\left(cd^4+ae^4\right.}$$

```
[Out] -1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)+1/4*
c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-e
*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2))/
a/(a*e^4+c*d^4)^3/(c*x^4+a)+12*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(e*x+d)/(a*e^4+
c*d^4)^4-3*c*d^2*e^7*(-a*e^4+3*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/4*e*(a^
2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)
/(a*e^4+c*d^4)^3-1/2*e^5*(a^2*e^8-26*a*c*d^4*e^4+21*c^2*d^8)*arctan(x^2*c^(
1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/32*c^(3/4)*d*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8-2*
d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)
+1/32*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c^2*d^
8-36*a*c*d^4*e^4+9*a^2*e^8-2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^
(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^
(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(
1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c^(3/4)*d*arctan(1+c^(1/
4)*x*2^(1/2)/a^(1/4))*(3*c^2*d^8-36*a*c*d^4*e^4+9*a^2*e^8+2*d^2*e^2*(-5*a*e
^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/8*c^(3/4)*d*
e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d
^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^4*2^(1/2)-1/8*c^(3/4)*d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(-3*a^2*e^8+30*a*c*d^4*e^4-15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c
*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(3/4)*d*e^4*ar
ctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4*e^4+15*c^2*d^8+4*d
^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+
1/4*c^(3/4)*d*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*a^2*e^8-30*a*c*d^4
*e^4+15*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^
4+c*d^4)^4*2^(1/2)
```

Rubi [A]

time = 1.38, antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6874, 1868, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^2),x]

[Out] -1/2*e^7/((c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3
*(d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*
d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3
*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt
[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4
+ a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (
c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e
^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*
c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)
*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c
*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(
3*c*d^4 - a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 -
36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) +
(c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*
e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sq
rt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8
))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(c*d
^4 + a*e^4)^4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
```

```
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

```
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3 (a+cx^4)^2}\,dx = \int \Bigg(\frac{e^8}{(cd^4+ae^4)^2 (d+ex)^3} + \frac{8cd^3e^8}{(cd^4+ae^4)^3 (d+ex)^2} + \frac{12cd^2e^8(3cd^4-ae^4)}{(cd^4+ae^4)^4 (d+ex)} +$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{12cd^2e^7(3cd^4-ae^4)\log(d}{(cd^4+ae^4)^4}$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

$$= -\frac{e^7}{2\,(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

Mathematica [A]
time = 0.90, size = 996, normalized size = 0.72

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^2),x]
[Out] ((-16*e^7*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (256*c*d^3*e^7*(c*d^4 + a*e^4))/
(d + e*x) + (8*c*(c*d^4 + a*e^4)*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) +
```

c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)) - (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d^13 - 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2]*a*c^(9/4)*d^9*e^4 - 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 + 84*a^(9/4)*c*d^4*e^9 - 30*Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 - 4*a^(13/4)*e^13)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d^13 + 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2]*a*c^(9/4)*d^9*e^4 + 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 - 84*a^(9/4)*c*d^4*e^9 - 30*Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 + 4*a^(13/4)*e^13)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 384*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 49*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 49*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 96*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^4)

Maple [A]

time = 0.33, size = 680, normalized size = 0.49

method	result
default	$-\frac{e^7}{2(e^4a+d^4c)^2(ex+d)^2}-\frac{8c\,d^3e^7}{(e^4a+d^4c)^3(ex+d)}-\frac{12e^7d^2c(e^4a-3d^4c)\ln(ex+d)}{(e^4a+d^4c)^4}+\frac{c\left(-\frac{d^3e^2c\left(5a^2e^8+2ac\,d^4e^4-3c^2d^8\right)x^3}{2a}-\frac{e\left(a^3e^1\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)-12*e^7*d^2*c*(a*e^4-3*c*d^4)/(a*e^4+c*d^4)^4*ln(e*x+d)+c/(a*e^4+c*d^4)^4*((-1/2*d^3*e^2*c*(5*a^2*e^8+2*a*c*d^4*e^4-3*c^2*d^8)/a*x^3-1/4*e*(a^3*e^12-11*a^2*c*d^4*e^8-9*a*c^2*d^8*e^4+3*c^3*d^12)/a*x^2+1/4*d*(3*a^3*e^12-9*a^2*c*d^4*e^8-11*a*c^2*d^8*e^4+c^3*d^12)/a*x-1/2*d^2*e^3*(3*a^2*e^8-2*a*c*d^4*e^4-5*c^2*d^8))/(c*x^4+a)+3/4/a*(1/8*(7*a^3*d*e^12-49*a^2*c*d^5*e^8+9*a*c^2*d^9*e^4+c^3*d^13)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2

))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-2*a^3*e^13+42*a^2*c*d^4*e^9-22*
a*c^2*d^8*e^5-2*c^3*d^12*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(-30*a^
2*c*d^3*e^10+36*a*c^2*d^7*e^6+2*c^3*d^11*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^
2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4
*(16*a^2*c*d^2*e^11-48*a*c^2*d^6*e^7)/c*ln(c*x^4+a)))

Maxima [A]
time = 0.56, size = 1321, normalized size = 0.95

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")

[Out] 3/32*c*(sqrt(2)*(c^4*d^13 - 2*sqrt(a)*c^(7/2)*d^11*e^2 + 9*a*c^3*d^9*e^4 -
36*a^(3/2)*c^(5/2)*d^7*e^6 - 48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 49*a^2*c^
2*d^5*e^8 + 30*a^(5/2)*c^(3/2)*d^3*e^10 + 16*sqrt(2)*a^(11/4)*c^(5/4)*d^2*e
^11 + 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a)
)/(a^(3/4)*c^(5/4)) - sqrt(2)*(c^4*d^13 - 2*sqrt(a)*c^(7/2)*d^11*e^2 + 9*a*
c^3*d^9*e^4 - 36*a^(3/2)*c^(5/2)*d^7*e^6 + 48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e
^7 - 49*a^2*c^2*d^5*e^8 + 30*a^(5/2)*c^(3/2)*d^3*e^10 - 16*sqrt(2)*a^(11/4)
*c^(5/4)*d^2*e^11 + 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + 2*(sqrt(2)*a^(1/4)*c^(17/4)*d^13 + 4*sq
rt(a)*c^4*d^12*e + 2*sqrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/4)*
c^(13/4)*d^9*e^4 + 44*a^(3/2)*c^3*d^8*e^5 + 36*sqrt(2)*a^(7/4)*c^(11/4)*d^7
*e^6 - 49*sqrt(2)*a^(9/4)*c^(9/4)*d^5*e^8 - 84*a^(5/2)*c^2*d^4*e^9 - 30*sqr
t(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(13/4)*c^(5/4)*d*e^12 + 4*a^(7
/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(sqrt(2)*a^(
1/4)*c^(17/4)*d^13 - 4*sqrt(a)*c^4*d^12*e + 2*sqrt(2)*a^(3/4)*c^(15/4)*d^11
*e^2 + 9*sqrt(2)*a^(5/4)*c^(13/4)*d^9*e^4 - 44*a^(3/2)*c^3*d^8*e^5 + 36*sqr
t(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9/4)*c^(9/4)*d^5*e^8 + 84*a^(
5/2)*c^2*d^4*e^9 - 30*sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(13/4
)*c^(5/4)*d*e^12 - 4*a^(7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*
c^(5/4)))/(a*c^4*d^16 + 4*a^2*c^3*d^12*e^4 + 6*a^3*c^2*d^8*e^8 + 4*a^4*c*d^
4*e^12 + a^5*e^16) + 12*(3*c^2*d^6*e^7 - a*c*d^2*e^11)*log(x*e + d)/(c^4*d^
16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) +
1/4*(10*a*c^2*d^8*e^3 - 40*a^2*c*d^4*e^7 + 6*(c^3*d^7*e^4 - 7*a*c^2*d^3*e^8
)*x^5 + 3*(3*c^3*d^8*e^3 - 14*a*c^2*d^4*e^7 - a^2*c*e^11)*x^4 + (c^3*d^9*e^
2 + 2*a*c^2*d^5*e^6 + a^2*c*d*e^10)*x^3 - 2*a^3*e^11 - (c^3*d^10*e + 2*a*c^
2*d^6*e^5 + a^2*c*d^2*e^9)*x^2 + (c^3*d^11 + 8*a*c^2*d^7*e^4 - 41*a^2*c*d^3
*e^8)*x)/(a^2*c^3*d^14 + 3*a^3*c^2*d^10*e^4 + 3*a^4*c*d^6*e^8 + a^5*d^2*e^1

```
2 + (a*c^4*d^12*e^2 + 3*a^2*c^3*d^8*e^6 + 3*a^3*c^2*d^4*e^10 + a^4*c*e^14)*
x^6 + 2*(a*c^4*d^13*e + 3*a^2*c^3*d^9*e^5 + 3*a^3*c^2*d^5*e^9 + a^4*c*d*e^1
3)*x^5 + (a*c^4*d^14 + 3*a^2*c^3*d^10*e^4 + 3*a^3*c^2*d^6*e^8 + a^4*c*d^2*e
^12)*x^4 + (a^2*c^3*d^12*e^2 + 3*a^3*c^2*d^8*e^6 + 3*a^4*c*d^4*e^10 + a^5*e
^14)*x^2 + 2*(a^2*c^3*d^13*e + 3*a^3*c^2*d^9*e^5 + 3*a^4*c*d^5*e^9 + a^5*d*
e^13)*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.98, size = 1488, normalized size = 1.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/8*(2*sqrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 + 4*sqrt(2)*a*c^
3*d^2*e^3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqr
t(2)*sqrt(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*a^2*c^4*d^8 - 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2
)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 - 40*(a*c^3)^(3/4)
*a^2*c*d^5*e^3 - 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^
3*c^2*d^2*e^6 + sqrt(2)*a^4*c^2*e^8 - 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/8*(2*s
qrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 - 4*sqrt(2)*a*c^3*d^2*e^
3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqrt(2)*sqr
t(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(sqrt(2)*a^2*c^4*d^8 + 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2)*sqrt(a
```

```
*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + 40*(a*c^3)^(3/4)*a^2*c*d
^5*e^3 + 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^3*c^2*d^
2*e^6 + sqrt(2)*a^4*c^2*e^8 + 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/32*(sqrt(2)*(a
*c^3)^(1/4)*c^4*d^13 - 2*sqrt(2)*(a*c^3)^(3/4)*c^2*d^11*e^2 + 9*sqrt(2)*(a*
c^3)^(1/4)*a*c^3*d^9*e^4 - 36*sqrt(2)*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*sqrt(2
)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 30*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10 + 7
*sqrt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d
^4*e^12 + a^6*c*e^16) - 3/32*(sqrt(2)*(a*c^3)^(1/4)*c^4*d^13 - 2*sqrt(2)*(a
*c^3)^(3/4)*c^2*d^11*e^2 + 9*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^9*e^4 - 36*sqrt(
2)*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 3
0*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10 + 7*sqrt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^
12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d^4*e^12 + a^6*c*e^16) - 3*(3*c^2*d^
6*e^7 - a*c*d^2*e^11)*log(abs(c*x^4 + a))/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*
a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) + 12*(3*c^2*d^6*e^8 - a*c*d^
2*e^12)*log(abs(x*e + d))/(c^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2*d^8*e^
9 + 4*a^3*c*d^4*e^13 + a^4*e^17) + 1/4*(10*a*c^3*d^12*e^3 - 30*a^2*c^2*d^8*
e^7 - 42*a^3*c*d^4*e^11 + 6*(c^4*d^11*e^4 - 6*a*c^3*d^7*e^8 - 7*a^2*c^2*d^3
*e^12)*x^5 + 3*(3*c^4*d^12*e^3 - 11*a*c^3*d^8*e^7 - 15*a^2*c^2*d^4*e^11 - a
^3*c*e^15)*x^4 - 2*a^4*e^15 + (c^4*d^13*e^2 + 3*a*c^3*d^9*e^6 + 3*a^2*c^2*d
^5*e^10 + a^3*c*d*e^14)*x^3 - (c^4*d^14*e + 3*a*c^3*d^10*e^5 + 3*a^2*c^2*d^
6*e^9 + a^3*c*d^2*e^13)*x^2 + (c^4*d^15 + 9*a*c^3*d^11*e^4 - 33*a^2*c^2*d^7
*e^8 - 41*a^3*c*d^3*e^12)*x)/((c*d^4 + a*e^4)^4*(c*x^4 + a)*(x*e + d)^2*a)
```

Mupad [B]
time = 5.04, size = 2500, normalized size = 1.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^3),x)
```

```
[Out] symsum(log(root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 2
62144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 -
 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d
^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c
^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((
108*a*c^10*d^19*e^3 + 3888*a^2*c^9*d^15*e^7 - 99576*a^3*c^8*d^11*e^11 + 591
408*a^4*c^7*d^7*e^15 - 79380*a^5*c^6*d^3*e^19)/(256*(a^10*e^24 + a^4*c^6*d^
24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c
^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 39
3216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16
*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^
6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*
```

a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((6912*a^8*c^5*d*e^24 + 4608*a^3*c^10*d^21*e^4 + 154368*a^4*c^9*d^17*e^8 - 331776*a^5*c^8*d^13*e^12 + 5976576*a^6*c^7*d^9*e^16 - 612864*a^7*c^6*d^5*e^20)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((18432*a^5*c^10*d^23*e^5 - 3072*a^4*c^11*d^27*e + 1170432*a^6*c^9*d^19*e^9 + 2863104*a^7*c^8*d^15*e^13 + 1797120*a^8*c^7*d^11*e^17 - 423936*a^9*c^6*d^7*e^21 - 506880*a^10*c^5*d^3*e^25)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((98304*a^13*c^4*d*e^30 - 32768*a^6*c^11*d^29*e^2 - 98304*a^7*c^10*d^25*e^6 + 98304*a^8*c^9*d^21*e^10 + 819200*a^9*c^8*d^17*e^14 + 1474560*a^10*c^7*d^13*e^18 + 1277952*a^11*c^6*d^9*e^22 + 557056*a^12*c^5*d^5*e^26)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + (x*(81920*a^13*c^4*e^31 - 49152*a^6*c^11*d^28*e^3 - 212992*a^7*c^10*d^24*e^7 - 245760*a^8*c^9*d^20*e^11 + 245760*a^9*c^8*d^16*e^15 + 901120*a^10*c^7*d^12*e^19 + 933888*a^11*c^6*d^8*e^23 + 442368*a^12*c^5*d^4*e^27))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) - (x*(12288*a^4*c^11*d^26*e^2 + 98304*a^5*c^10*d^22*e^6 - 1413120*a^6*c^9*d^18*e^10 - 4030464*a^7*c^8*d^14*e^14 - 2813952*a^8*c^7*d^10*e^18 + 393216*a^9*c^6*d^6*e^22 + 675840*a^10*c^5*d^2*e^26))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(20736*a^8*c^5*e^25 - 576*a^2*c^11*d^24*e - 576*a^3*c^10*d^20*e^5 + 484992*a^4*c^9*d^16*e^9 + 2468736*a^5*c^8*d^12*e^13 + 4093632*a^6*c^7*d^8*e^17 - 228672*a^7*c^6*d^4*e^21))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(216*a*c^10*d^18*e^4 + 25056*a^2*c^9*d^14*e^8 - 2160*a^3*c^8*d^10*e^12 + 59616*a^4*c^7*d^6*e^16 + 86616*a^5*c^6*d^2*e^20))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (81*c^9*d^13*e^6 - 2430*a*c^8*d^9*e^10 + 1296*a^3*c^6*d*e^18 + 3969*a^2*c^7*d^5*e^14)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + (x*(1296*a^3*c^6*e^19 + 81*c^9*d^12*e^7 - 6318*a*c^8*d^8*e

```
^11 + 5265*a^2*c^7*d^4*e^15))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)))*root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e
^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*
e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712
*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 5
8752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4
, z, k), k, 1, 4) - ((a^2*e^11 - 5*c^2*d^8*e^3 + 20*a*c*d^4*e^7)/(2*(a*e^4
+ c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^5*(c^3*d^7*e^4 - 7*a*c
^2*d^3*e^8))/(2*a*(a^3*e^12 + c^3*d^12 + 3*a*c^...
```

3.408 $\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$

Optimal. Leaf size=394

$$\frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}+\frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{3d\left(7\sqrt{c}\,d^2+5\sqrt{a}\,e^2\right.}{64\sqrt{2}}$$

```
[Out] 1/32*x*(15*d*e^2*x^2+18*d^2*e*x+7*d^3)/a^2/(c*x^4+a)+1/8*(-a*e^3+c*x*(3*d*e
^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^2+9/16*d^2*e*arctan(x^2*c^(1/2)/a^(1/2
))/a^(5/2)/c^(1/2)-3/256*d*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2
))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/256*d*ln(a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^
(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*
a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+3/128*d*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+7*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{3d\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\,e^2+7\sqrt{c}\,d^2\right)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3d\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\,e^2+7\sqrt{c}\,d^2\right)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{9d^2e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2(a+cx^4)}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a*e^3 - c*x
*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2) + (9*d^2*e*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/
4)) + (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(
11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} - \frac{\int \frac{-7d^3-18d^2ex-15de^2x^2}{(a+cx^4)^2}\,dx}{8a}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} + \frac{\int \frac{21d^3+36d^2ex+15de^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} + \frac{\int \left(\frac{36d^2ex}{a+cx^4} + \frac{21d^3+15de^2x^2}{a+cx^4}\right.}{32a^2}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} + \frac{\int \frac{21d^3+15de^2x^2}{a+cx^4}\,dx}{32a^2} + \frac{(9d^2}{}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} + \frac{(9d^2e)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx\right.}{16a^2}$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} -$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} -$$

$$= \frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac\,(a+cx^4)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} -$$

<u>Mathematica</u> [A]
time = 0.25, size = 388, normalized size = 0.98

$$\frac{\frac{8adx(7d^2+18dex+15e^2x^2)}{a+cx^4} - \frac{32a^2(ae^3-cdx(d^2+3dex+3e^2x^2))}{c(a+cx^4)^2} - \frac{6\sqrt[4]{a}\,d\left(7\sqrt{2}\sqrt{c}\,d^2+24\sqrt[4]{a}\sqrt[4]{c}\,de+5\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{6\sqrt[4]{a}\,d\left(7\sqrt{2}\sqrt{c}\,d^2-24\sqrt[4]{a}\sqrt[4]{c}\,de+5\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{3\sqrt{2}\left(-7\sqrt[4]{a}\sqrt{c}\,d^3+5a^{3/4}de^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}} + \frac{3\sqrt{2}\left(7\sqrt[4]{a}\sqrt{c}\,d^3-5a^{3/4}de^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 - c
*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sqrt
[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d
^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)
```

```
*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (
3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)
```

Maple [A]
time = 0.21, size = 403, normalized size = 1.02

method	result
risch	$\frac{\frac{15 d\,e^{2} c\,x^{7}}{32 a^{2}}+\frac{9 c\,d^{2} e\,x^{6}}{16 a^{2}}+\frac{7 c\,d^{3} x^{5}}{32 a^{2}}+\frac{27 d\,e^{2} x^{3}}{32 a}+\frac{15 e\,x^{2} d^{2}}{16 a}+\frac{11 d^{3} x}{32 a}-\frac{e^{3}}{8 c}}{\left(c\,x^{4}+a\right)^{2}}+\frac{3 d\left(\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^{4}+a\right)}\frac{\left(5 e^{2}\,\_R^{2}+12 d e\,\_R+7 d^{2}\right)\ln\left(x-\_\_\right.}{\_R^{3}}\right.}{128 a^{2} c}$
default	$d^{3}\left(\frac{x}{8 a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{7 x}{32 a\left(c\,x^{4}+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256 a^{2}}}{a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*(1/8*x/a/(c*x^4+a)^2+7/8/a*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1
/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4
)*x-1))))+3*d^2*e*(1/8*x^2/a/(c*x^4+a)^2+3/4/a*(1/4*x^2/a/(c*x^4+a)+1/4/a/(
a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))))+3*d*e^2*(1/8*x^3/a/(c*x^4+a)^2+5/8/a*(
1/4*x^3/a/(c*x^4+a)+1/32/a/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))+e^3*(1/8*x^4/a/(c*x^
4+a)^2+1/8/a^2*x^4/(c*x^4+a))
```

Maxima [A]
time = 0.48, size = 389, normalized size = 0.99

$$\frac{15\,c^{2} d x^{7} e^{2}+18\,c^{2} d^{2} x^{6} e+7\,c^{2} d^{3} x^{5}+27\,a c d x^{3} e^{2}+30\,a c d^{2} x^{2} e+11\,a c d^{3} x-4\,a^{2} e^{3}}{32\left(a^{2} c^{3} x^{8}+2\,a^{3} c^{2} x^{4}+a^{4} c\right)}+\frac{3\,d\left(\frac{\sqrt{2}\left(7\sqrt{c}\,d^{2}-5\sqrt{a}\,e^{2}\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7\sqrt{c}\,d^{2}-5\sqrt{a}\,e^{2}\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{3}{4}} d^{2}-24\sqrt{a}\sqrt{c}\,d e+5\sqrt{2}\,a^{\frac{3}{4}} c^{\frac{1}{4}} e^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{3}{4}} d^{2}+24\sqrt{a}\sqrt{c}\,d e+5\sqrt{2}\,a^{\frac{3}{4}} c^{\frac{1}{4}} e^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}\right)}{256\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(15*c^2*d*x^7*e^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3*e^
2 + 30*a*c*d^2*x^2*e + 11*a*c*d^3*x - 4*a^2*e^3)/(a^2*c^3*x^8 + 2*a^3*c^2*x
^4 + a^4*c) + 3/256*d*(sqrt(2)*(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*
x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(7*s
```

```
qrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 - 24*sqrt(a)*s
qrt(c)*d*e + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(c))*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 24*sqrt(a)*sqrt(c)*d*e
 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)
*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(
3/4)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 16.51, size = 95566, normalized size = 242.55

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/65536*(30720*c^2*d*e^2*x^7 + 36864*c^2*d^2*e*x^6 + 14336*c^2*d^3*x^5 + 55
296*a*c*d*e^2*x^3 + 61440*a*c*d^2*e*x^2 + 22528*a*c*d^3*x - 8192*a^2*e^3 +
2*(a^2*c^3*x^8 + 2*a^3*c^2*x^4 + a^4*c)*((-I*sqrt(3) + 1)*((a^3*c*sqrt(-1/(
a*c))*sqrt(-(70*a*c*d^4*e^2*sqrt(-1/(a*c)) + 49*c*d^6 - 25*a*d^2*e^4)/(a^6*
c^2*sqrt(-1/(a*c)))) - 12*d^2*e)^2/(a^5*c) - 3*(24*a^3*c*d^2*e*sqrt(-(70*a*
c*d^4*e^2*sqrt(-1/(a*c)) + 49*c*d^6 - 25*a*d^2*e^4)/(a^6*c^2*sqrt(-1/(a*c))
)) + 49*c*d^6 - (214*c*d^4*e^2*sqrt(-1/(a*c)) + 25*d^2*e^4)*a)/(a^6*c^2*sqr
t(-1/(a*c))))/(-9/4194304*(24*a^3*c*d^2*e*sqrt(-(70*a*c*d^4*e^2*sqrt(-1/(a*
c)) + 49*c*d^6 - 25*a*d^2*e^4)/(a^6*c^2*sqrt(-1/(a*c)))) + 49*c*d^6 - (214*
c*d^4*e^2*sqrt(-1/(a*c)) + 25*d^2*e^4)*a)*(a^3*c*sqrt(-1/(a*c))*sqrt(-(70*a
*c*d^4*e^2*sqrt(-1/(a*c)) + 49*c*d^6 - 25*a*d^2*e^4)/(a^6*c^2*sqrt(-1/(a*c)
))) - 12*d^2*e)/(a^8*c^2) + 27/4194304*(a^8*c^2*sqrt(-1/(a*c))*(-(70*a*c*d^
4*e^2*sqrt(-1/(a*c)) + 49*c*d^6 - 25*a*d^2*e^4)/(a^6*c^2*sqrt(-1/(a*c))))^(
3/2) - 4*a^3*c*d^4*e^2*sq ...
```

Sympy [A]
time = 130.25, size = 413, normalized size = 1.05

$$\operatorname{RootSum}\left(268435456t^4a^{11}c^3 + 63111168t^2a^6c^2d^4e^2 + t\left(4147200a^4cd^4e^5 - 8128512a^3c^2d^8e\right) + 50625a^2d^4e^8 + 245106acd^8e^4 + 194481c^2d^{12}, \left(t \mapsto t\log\left(x + \frac{262144000t^3a^{10}c^2e^6 + 3714056192t^3a^9c^3d^4e^2 - 539688960t^2a^7c^2d^4e^5 + 202309632t^2a^6c^3d^8e + 77328000ta^5cd^4e^8 + 660699648ta^4c^2d^8e^4 + 19361664ta^3c^3d^{12} + 3037500a^3d^4e^{11} - 26360640a^2cd^8e^7 - 60566940ac^2d^{12}e^3}{421875a^3d^3e^{12} - 29598075a^2cd^7e^8 - 58012227ac^2d^{11}e^4 + 3176523c^3d^{15}}\right)\right)\right) + \frac{-4a^2e^3 + 11acd^3x + 30acd^2ex^2 + 27acde^2x^3 + 7c^2d^3x^5 + 18c^2d^2ex^6 + 15c^2de^2x^7}{32a^4c + 64a^3c^2x^4 + 32a^2c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 + _
t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4*e
**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (2621
44000*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 539688
960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000
*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*
```

```
c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 60566940
*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*e**8 - 5
8012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15)))) + (-4*a**2*e**3 + 11*a*c
*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x**5 + 18*c
**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*
a**2*c**3*x**8)
```

Giac [**A**]
time = 4.18, size = 389, normalized size = 0.99

$$\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}\,c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3}+\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}\,c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3}+\frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}-\frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}+\frac{15\,c^2dx^7e^2+18\,c^2d^2x^6e+7\,c^2d^3x^5+27\,acdx^3e^2+30\,acd^2x^2e+11\,acd^3x-4\,a^2e^3}{32\left(cx^4+a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5
*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3
)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^
3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*lo
g(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2*d*x^7*e
^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3*e^2 + 30*a*c*d^2*x^2*e
 + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)
```

Mupad [**B**]
time = 0.48, size = 721, normalized size = 1.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^3*x)/(32*a) - e^3/(8*c) + (7*c*d^3*x^5)/(32*a^2) + (15*d^2*e*x^2)/(1
6*a) + (27*d*e^2*x^3)/(32*a) + (9*c*d^2*e*x^6)/(16*a^2) + (15*c*d*e^2*x^7)/
(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c*d^2*(6867*c*d^5*e^2
 - 1125*a*d*e^6 + 7992*c*d^4*e^3*x - 114688*root(268435456*a^11*c^3*z^4 + 6
3111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e
^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^
5*c^2*d + 9600*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 -
 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 5
0625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*e^4*x - 18816*root(26843545
6*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4
147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^
2*d^12, z, k)*a^2*c^2*d^4*x + 196608*root(268435456*a^11*c^3*z^4 + 63111168
```

```
*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z +
245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^5*c^2*e
*x - 46080*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 812
8512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625
*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*d*e^3))/(32768*a^6))*root(26843
5456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z
+ 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481
*c^2*d^12, z, k), k, 1, 4)
```

3.409 $\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$

Optimal. Leaf size=360

$$\frac{x(d+ex)^2}{8a\,(a+cx^4)^2}+\frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)}+\frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}-\frac{(21\sqrt{c}\,d^2+5\sqrt{a}\,e^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}c^{3/4}}$$

```
[Out] 1/8*x*(e*x+d)^2/a/(c*x^4+a)^2+1/32*x*(5*e^2*x^2+12*d*e*x+7*d^2)/a^2/(c*x^4+
a)+3/8*d*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)-1/256*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/
4)/c^(3/4)*2^(1/2)+1/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*
(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^(1/2)+1/128*arctan(-1+c^
(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/c^(3/4)*2^
(1/2)+1/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/
2))/a^(11/4)/c^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(5\sqrt{a}\,e^2+21\sqrt{c}\,d^2)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(5\sqrt{a}\,e^2+21\sqrt{c}\,d^2)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3de\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}-\frac{(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)}+\frac{x(d+ex)^2}{8a\,(a+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(3
2*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*
Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(1
1/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} - \frac{\int \frac{-7d^2-12dex-5e^2x^2}{(a+cx^4)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)} + \frac{\int \frac{21d^2+24dex+5e^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)} + \frac{\int \left(\frac{24dex}{a+cx^4} + \frac{21d^2+5e^2x^2}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)} + \frac{\int \frac{21d^2+5e^2x^2}{a+cx^4}\,dx}{32a^2} + \frac{(3de)\int \frac{x}{a+cx^4}\,dx}{4a^2}$$

$$= \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)} + \frac{(3de)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{8a^2} + \frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}} - \right.}{}$$

$$= \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}} - 5e^2\right)\int}{128\sqrt{2}}$$

$$= \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}} - 5e^2\right)\log}{1}$$

$$= \frac{x(d+ex)^2}{8a\,(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2\,(a+cx^4)} + \frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{(21\sqrt{c}\,d^2 + 5\sqrt{a}\,e^2)}{64\sqrt{2}}$$

Mathematica [A]

time = 0.21, size = 358, normalized size = 0.99

$$\frac{\frac{32a^2x(d+ex)^2}{(a+cx^4)^2}+\frac{8ax(7d^2+12dex+5e^2x^2)}{a+cx^4}-\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}\,d^2+48\sqrt[4]{a}\sqrt[4]{c}\,de+5\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}\,d^2-48\sqrt[4]{a}\sqrt[4]{c}\,de+5\sqrt{2}\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{\sqrt{2}\left(-21\sqrt[4]{a}\sqrt{c}\,d^2+5a^{3/4}e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}}+\frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{c}\,d^2-5a^{3/4}e^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x^
2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)*d
*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4
) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]
*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(
-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5*a^(3/4
)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(25
6*a^3)
```

Maple [A]
time = 0.21, size = 360, normalized size = 1.00

method	result
risch	$\frac{\frac{5c\,e^2x^7}{32a^2}+\frac{3cde\,x^6}{8a^2}+\frac{7c\,d^2x^5}{32a^2}+\frac{9e^2x^3}{32a}+\frac{5de\,x^2}{8a}+\frac{11d^2x}{32a}}{(c\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(5e^2\_R^2+24de\_R+21d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2c}$
default	$d^2\left(\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a\left(c\,x^4+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(1/8*x/a/(c*x^4+a)^2+7/8/a*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1
/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4
)*x-1))))+2*d*e*(1/8*x^2/a/(c*x^4+a)^2+3/4/a*(1/4*x^2/a/(c*x^4+a)+1/4/a/(a*
c)^(1/2)*arctan(x^2*(c/a)^(1/2))))+e^2*(1/8*x^3/a/(c*x^4+a)^2+5/8/a*(1/4*x^
3/a/(c*x^4+a)+1/32/a/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(
a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)
^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))))
```

Maxima [A]
time = 0.49, size = 362, normalized size = 1.01

$$\frac{5\,cx^{7}e^{2}+12\,cdx^{6}e+7\,cd^{2}x^{5}+9\,ax^{3}e^{2}+20\,adx^{2}e+11\,ad^{2}x}{32\,(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4})}+\frac{\frac{\sqrt{2}\,(21\,\sqrt{c}\,d^{2}-5\,\sqrt{a}\,e^{2})\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\,(21\,\sqrt{c}\,d^{2}-5\,\sqrt{a}\,e^{2})\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\,\left(21\,\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^{2}-48\,\sqrt{a}\,\sqrt{c}\,de+5\,\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^{2}\right)\arctan\left(\frac{\sqrt{2}\,\left(2\,\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\,\left(21\,\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^{2}+48\,\sqrt{a}\,\sqrt{c}\,de+5\,\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^{2}\right)\arctan\left(\frac{\sqrt{2}\,\left(2\,\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\,\sqrt{c}}\,c^{\frac{3}{4}}}}{256\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*c*x^7*e^2 + 12*c*d*x^6*e + 7*c*d^2*x^5 + 9*a*x^3*e^2 + 20*a*d*x^2*e
 + 11*a*d^2*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(
c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(
a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c
)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(21*sqrt
(2)*a^(1/4)*c^(3/4)*d^2 - 48*sqrt(a)*sqrt(c)*d*e + 5*sqrt(2)*a^(3/4)*c^(1/4
)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(21*sqrt(2)*a^(1/
4)*c^(3/4)*d^2 + 48*sqrt(a)*sqrt(c)*d*e + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 8.57, size = 91420, normalized size = 253.94

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/589824*(92160*c*e^2*x^7 + 221184*c*d*e*x^6 + 129024*c*d^2*x^5 + 165888*a*
e^2*x^3 + 368640*a*d*e*x^2 + 202752*a*d^2*x + 2*(a^2*c^2*x^8 + 2*a^3*c*x^4
+ a^4)*((-I*sqrt(3) + 1)*((a^3*c*sqrt(-1/(a*c))*sqrt(-(210*a*c*d^2*e^2*sqrt
(-1/(a*c)) + 441*c*d^4 - 25*a*e^4)/(a^6*c^2*sqrt(-1/(a*c)))) - 24*d*e)^2/(a
^5*c) - 3*(48*a^3*c*d*e*sqrt(-(210*a*c*d^2*e^2*sqrt(-1/(a*c)) + 441*c*d^4 -
 25*a*e^4)/(a^6*c^2*sqrt(-1/(a*c)))) + 441*c*d^4 - (786*c*d^2*e^2*sqrt(-1/(
a*c)) + 25*e^4)*a)/(a^6*c^2*sqrt(-1/(a*c))))/(-1/12582912*(48*a^3*c*d*e*sqr
t(-(210*a*c*d^2*e^2*sqrt(-1/(a*c)) + 441*c*d^4 - 25*a*e^4)/(a^6*c^2*sqrt(-1
/(a*c)))) + 441*c*d^4 - (786*c*d^2*e^2*sqrt(-1/(a*c)) + 25*e^4)*a)*(a^3*c*s
qrt(-1/(a*c))*sqrt(-(210*a*c*d^2*e^2*sqrt(-1/(a*c)) + 441*c*d^4 - 25*a*e^4)
/(a^6*c^2*sqrt(-1/(a*c)))) - 24*d*e)/(a^8*c^2) + 1/4194304*(a^8*c^2*sqrt(-1
/(a*c))*(-(210*a*c*d^2*e^2*sqrt(-1/(a*c)) + 441*c*d^4 - 25*a*e^4)/(a^6*c^2*
sqrt(-1/(a*c))))^(3/2) - 156*a^3*c*d^2*e^2*sqrt(-1/(a*c))*sqrt(-(210*a*c*d^
2*e^2*sqrt(-1/(a*c)) + 44 ...
```

Sympy [A]
time = 14.20, size = 374, normalized size = 1.04

$$\operatorname{RootSum}\left(268435456t^{4}a^{11}c^{3}+25755648t^{2}a^{6}c^{2}d^{2}e^{2}+t(307200a^{4}cde^{5}-5419008a^{3}c^{2}d^{5}e)+625a^{2}e^{8}+111906acd^{4}e^{4}+194481c^{2}d^{8},\left(t\mapsto t\log\left(x+\frac{262144000t^{3}a^{10}c^{2}e^{6}+46110081024t^{3}a^{9}c^{3}d^{4}e^{2}-1645608960t^{2}a^{7}c^{2}d^{3}e^{5}+3641573376t^{2}a^{6}c^{3}d^{7}e+32688000ta^{5}cd^{2}e^{8}+3128219136ta^{4}c^{2}d^{6}e^{4}+522764928ta^{3}c^{3}d^{10}+225000a^{3}de^{11}-43338240a^{2}cd^{5}e^{7}-523431720ac^{2}d^{9}e^{3}}{15625a^{3}e^{12}-21357225a^{2}cd^{4}e^{8}-376741449ac^{2}d^{8}e^{4}+85766121c^{3}d^{12}}\right)\right)\right)+\frac{11ad^{2}x+20adex^{2}+9ae^{2}x^{3}+7cd^{2}x^{5}+12cdex^{6}+5ce^{2}x^{7}}{32a^{4}+64a^{3}cx^{4}+32a^{2}c^{2}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 + _
t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 11190
6*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t**3*
a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t**2*
a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*
c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d*
*10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c**2*d*
*9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*a*c**2*d
**8*e**4 + 85766121*c**3*d**12)))) + (11*a*d**2*x + 20*a*d*e*x**2 + 9*a*e**
2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4 + 64*a**3*
c*x**4 + 32*a**2*c**2*x**8)
```

Giac [A]
time = 5.86, size = 356, normalized size = 0.99

$$\frac{5\,cx^7e^2+12\,cdx^6e+7\,cd^2x^5+9\,ax^3e^2+20\,adx^2e+11\,ad^2x}{32\,(cx^4+a)^2a^2}+\frac{\sqrt{2}\left(24\,\sqrt{2}\,\sqrt{ac}\,c^2de+21\,(ac^3)^{\frac{1}{4}}\,c^2d^2+5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,(\frac{a}{c})^{\frac{1}{4}}\right)}{2\,(\frac{a}{c})^{\frac{1}{4}}}\right)}{128\,a^3c^3}+\frac{\sqrt{2}\left(24\,\sqrt{2}\,\sqrt{ac}\,c^2de+21\,(ac^3)^{\frac{1}{4}}\,c^2d^2+5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,(\frac{a}{c})^{\frac{1}{4}}\right)}{2\,(\frac{a}{c})^{\frac{1}{4}}}\right)}{128\,a^3c^3}+\frac{\sqrt{2}\left(21\,(ac^3)^{\frac{1}{4}}\,c^2d^2-5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2+\sqrt{2}\,x(\frac{a}{c})^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}-\frac{\sqrt{2}\left(21\,(ac^3)^{\frac{1}{4}}\,c^2d^2-5\,(ac^3)^{\frac{3}{4}}\,e^2\right)\log\left(x^2-\sqrt{2}\,x(\frac{a}{c})^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*c*x^7*e^2 + 12*c*d*x^6*e + 7*c*d^2*x^5 + 9*a*x^3*e^2 + 20*a*d*x^2*e
 + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^
2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*sqrt
(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/256*
sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*
x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*
d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3)
```

Mupad [B]
time = 0.47, size = 676, normalized size = 1.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^2
*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^8
+ 2*a*c*x^4) + symsum(log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*root(268
```

```
435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*
z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^
8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*root(268435456*a^11*c^3*z^
4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d
*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*e^4
*x + 56448*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 541
9008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c
^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*root(268435456*a^11*c^3*z
^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*
d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*d*
e^3 - 393216*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5
419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481
*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6))*root(268435456
*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 30
7200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z,
k), k, 1, 4)
```

3.410 $\int \frac{d+ex}{(a+cx^4)^3}\,dx$

Optimal. Leaf size=266

$$\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}+\frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)}+\frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}$$

```
[Out] 1/8*x*(e*x+d)/a/(c*x^4+a)^2+1/32*x*(6*e*x+7*d)/a^2/(c*x^4+a)+21/128*d*arcta
n(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*d*arctan(1+
c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*d*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*d*ln(a
^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+3/16
*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/c^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1869, 1890, 217, 1179, 642, 1176, 631, 210, 281, 211}

$$-\frac{21d\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{3e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{21d\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a + c*x^4))
+ (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (21*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/
4)*c^(1/4)) + (21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{x(d+ex)}{8a(a+cx^4)^2} - \frac{\int \frac{-7d-6ex}{(a+cx^4)^2}\,dx}{8a}$$

$$= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{\int \frac{21d+12ex}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{\int \left(\frac{21d}{a+cx^4} + \frac{12ex}{a+cx^4}\right)dx}{32a^2}$$

$$= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{(21d)\int \frac{1}{a+cx^4}\,dx}{32a^2} + \frac{(3e)\int \frac{x}{a+cx^4}\,dx}{8a^2}$$

$$= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{(21d)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(21d)\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(3e}{}$$

$$= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} +$$

$$= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}$$

$$= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21}{}$$

Mathematica [A]
time = 0.14, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(d+ex)}{(a+cx^4)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6\left(7\sqrt{2}\sqrt[4]{c}\,d+8\sqrt[4]{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6\left(7\sqrt{2}\sqrt[4]{c}\,d-8\sqrt[4]{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - \frac{21\sqrt{2}\,d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}\,d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a +
c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [A]
time = 0.20, size = 207, normalized size = 0.78

method	result
risch	$\frac{\frac{3ce\,x^{6}}{16a^{2}}+\frac{7cd\,x^{5}}{32a^{2}}+\frac{5e\,x^{2}}{16a}+\frac{11dx}{32a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(4e\_R+7d\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}c}$
default	$d\left(\frac{x}{8a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(c\,x^{4}+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/8*x/a/(c*x^4+a)^2+7/8/a*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2
)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a
/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*
x-1))))+e*(1/8*x^2/a/(c*x^4+a)^2+3/4/a*(1/4*x^2/a/(c*x^4+a)+1/4/a/(a*c)^(1/
2)*arctan(x^2*(c/a)^(1/2))))
```

Maxima [A]
time = 0.48, size = 273, normalized size = 1.03

$$\frac{6\,cx^{6}e+7\,cdx^{5}+10\,ax^{2}e+11\,adx}{32\,\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)}+\frac{3\left(\frac{7\sqrt{2}\,d\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{7\sqrt{2}\,d\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d-8\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}d+8\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}\right)}{256\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*c*x^6*e + 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*c
*x^4 + a^4) + 3/256*(7*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - 7*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1
```

```
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(1/4)*d
 - 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(7*sqrt(
2)*a^(1/4)*c^(1/4)*d + 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(1/4)))/a^2
```

Fricas [C] Result contains complex when optimal does not.
time = 2.67, size = 43180, normalized size = 162.33

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/65536*(12288*c*e*x^6 + 14336*c*d*x^5 + 20480*a*e*x^2 + 22528*a*d*x - 1536
*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(7*sqrt(-d^2/(a^6*c*sqrt(-1/(a*c)))) + 4
*e*sqrt(-1/(a*c))/a^2)*log(864*a^9*c*e^2*(7*sqrt(-d^2/(a^6*c*sqrt(-1/(a*c))
)) + 4*e*sqrt(-1/(a*c))/a^2)^3 - 5292*a^6*c*d^2*e*(7*sqrt(-d^2/(a^6*c*sqrt(
-1/(a*c)))) + 4*e*sqrt(-1/(a*c))/a^2)^2 + 423360*a*d^2*e^3 - 189*(2401*c*d^
5 - 1024*a*d*e^4)*x + 27*(2401*a^3*c*d^4 + 512*a^4*e^4)*(7*sqrt(-d^2/(a^6*c
*sqrt(-1/(a*c)))) + 4*e*sqrt(-1/(a*c))/a^2)) + 2*(a^2*c^2*x^8 + 2*a^3*c*x^4
 + a^4)*((-I*sqrt(3) + 1)*((7*a^3*c*sqrt(-1/(a*c))*sqrt(-d^2/(a^6*c*sqrt(-1
/(a*c)))) - 4*e)^2/(a^5*c) - 3*(56*a^3*e*sqrt(-d^2/(a^6*c*sqrt(-1/(a*c))))
- 16*a*e^2*sqrt(-1/(a*c)) + 49*d^2)/(a^6*c*sqrt(-1/(a*c)))))/(-9/4194304*(7*
a^3*c*sqrt(-1/(a*c))*sqrt(-d^2/(a^6*c*sqrt(-1/(a*c)))) - 4*e)*(56*a^3*e*sqr
t(-d^2/(a^6*c*sqrt(-1/(a*c)))) - 16*a*e^2*sqrt(-1/(a*c)) + 49*d^2)/(a^8*c)
+ 27/4194304*(343*a^8*c^2*sqrt(-1/(a*c))*(-d^2/(a^6*c*sqrt(-1/(a*c))))^(3/2
) - 112*a^3*c*e^2*sqrt(-1 ...
```

Sympy [A]
time = 1.13, size = 192, normalized size = 0.72

$$\operatorname{RootSum}\left(268435456t^4a^{11}c^2 + 4718592t^2a^6ce^2 - 2709504ta^3cd^2e + 20736ae^4 + 194481cd^4, \left(t \mapsto t\log\left(x + \frac{-67108864t^3a^9ce^2 - 9633792t^2a^6cd^2e - 589824ta^4e^4 - 2765952ta^3cd^4 + 423360ad^2e^3}{193536ade^4 - 453789cd^5}\right)\right)\right) + \frac{11adx + 10aex^2 + 7cdx^5 + 6cex^6}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_t
*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e**4
- 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 453789*c*
d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64
*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

Giac [A]
time = 3.02, size = 260, normalized size = 0.98

$$\frac{21\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}-\frac{21\,\sqrt{2}\,\left(ac^3\right)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}+\frac{3\,\sqrt{2}\,\left(4\,\sqrt{2}\,\sqrt{ac}\,ce+7\,\left(ac^3\right)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\,\left(2\,x+\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2}+\frac{3\,\sqrt{2}\,\left(4\,\sqrt{2}\,\sqrt{ac}\,ce+7\,\left(ac^3\right)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\,\left(2\,x-\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2}+\frac{6\,cx^6e+7\,cdx^5+10\,ax^2e+11\,adx}{32\,\left(cx^4+a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))
/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1
/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32*(6*c*
x^6*e + 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

Mupad [B]
time = 0.30, size = 315, normalized size = 1.18

$$\frac{\frac{5\,e\,x^2}{16\,a}+\frac{11\,d\,x}{32\,a}+\frac{7\,c\,d\,x^5}{32\,a^2}+\frac{3\,c\,e\,x^6}{16\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\left(\sum_{k=1}^{4}\ln\left(\frac{c^2\,\left(63\,d\,e^2+36\,e^3\,x-\mathrm{root}\left(268435456\,a^{11}\,c^2\,z^4+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+194481\,c\,d^4+20736\,a\,e^4,z,k\right)^2\,a^5\,c\,d\,7168-\mathrm{root}\left(268435456\,a^{11}\,c^2\,z^4+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+194481\,c\,d^4+20736\,a\,e^4,z,k\right)\,a^2\,c\,d^2\,x\,1176+\mathrm{root}\left(268435456\,a^{11}\,c^2\,z^4+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+194481\,c\,d^4+20736\,a\,e^4,z,k\right)^2\,a^5\,c\,e\,x\,4096\right)\,3}{a^6\,2048}\right)\,\mathrm{root}\left(268435456\,a^{11}\,c^2\,z^4+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+194481\,c\,d^4+20736\,a\,e^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^3,x)
```

```
[Out] ((5*e*x^2)/(16*a) + (11*d*x)/(32*a) + (7*c*d*x^5)/(32*a^2) + (3*c*e*x^6)/(1
6*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c^2*(63*d*e^2 + 36*e^3*
x - 7168*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*
c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*d - 1176*root(2684354
56*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*
d^4 + 20736*a*e^4, z, k)*a^2*c*d^2*x + 4096*root(268435456*a^11*c^2*z^4 + 4
718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4,
z, k)^2*a^5*c*e*x))/(2048*a^6))*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c
*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k), k, 1,
 4)
```

3.411 $\int \frac{1}{(a+cx^4)^3} \, dx$

Optimal. Leaf size=219

$$\frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\ldots}\right.}{128\sqrt{2}\,a\ldots}$$

[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)+21/128*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(11/4)/c^(1/4)*2^(1/2)-21/256*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)+21/256*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(11/4)/c^(1/4)*2^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {205, 217, 1179, 642, 1176, 631, 210}

$$-\frac{21\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{x}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-3),x]

[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4))

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{x}{8a\,(a+cx^4)^2} + \frac{7\int \frac{1}{(a+cx^4)^2}\,dx}{8a}$$
$$= \frac{x}{8a\,(a+cx^4)^2} + \frac{7x}{32a^2\,(a+cx^4)} + \frac{21\int \frac{1}{a+cx^4}\,dx}{32a^2}$$
$$= \frac{x}{8a\,(a+cx^4)^2} + \frac{7x}{32a^2\,(a+cx^4)} + \frac{21\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{21\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{64a^{5/2}}$$
$$= \frac{x}{8a\,(a+cx^4)^2} + \frac{7x}{32a^2\,(a+cx^4)} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,d}{128a^{5/2}\sqrt{c}}$$
$$= \frac{x}{8a\,(a+cx^4)^2} + \frac{7x}{32a^2\,(a+cx^4)} - \frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21\log\left(\sqrt{a}\right.}{1}$$
$$= \frac{x}{8a\,(a+cx^4)^2} + \frac{7x}{32a^2\,(a+cx^4)} - \frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}} + \frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}$$

Mathematica [A]
time = 0.06, size = 200, normalized size = 0.91

$$\frac{\frac{32a^{7/4}x}{(a+cx^4)^2}+\frac{56a^{3/4}x}{a+cx^4}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}-\frac{21\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3),x]
```

```
[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Maple [A]
time = 0.20, size = 139, normalized size = 0.63

method	result	size

risch	$\frac{\frac{7c\,x^{5}}{32a^{2}}+\frac{11x}{32a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{21\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}c}$	57
default	$\frac{x}{8a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(c\,x^{4}+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/8/a*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*(
ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)))
```

Maxima [A]
time = 0.49, size = 212, normalized size = 0.97

$$\frac{7\,cx^{5}+11\,ax}{32\,\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)}+\frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{256\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

Fricas [A]
time = 0.43, size = 232, normalized size = 1.06

$$\frac{28\,cx^{5}+84\,\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\arctan\left(-a^{8}cx\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}+\sqrt{a^{6}\sqrt{-\frac{1}{a^{11}c}}+x^{2}}\,a^{8}c\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}\right)+21\,\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(a^{3}\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}+x\right)-21\,\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(-a^{3}\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}+x\right)+44\,ax}{128\,\left(a^{2}c^{2}x^{8}+2\,a^{3}cx^{4}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(28*c*x^5 + 84*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*
arctan(-a^8*c*x*(-1/(a^11*c))^(3/4) + sqrt(a^6*sqrt(-1/(a^11*c)) + x^2)*a^8
*c*(-1/(a^11*c))^(3/4)) + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c)
)^(1/4)*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 +
a^4)*(-1/(a^11*c))^(1/4)*log(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c
^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [A]
time = 0.21, size = 63, normalized size = 0.29

$$\frac{11ax+7cx^5}{32a^4+64a^3cx^4+32a^2c^2x^8}+\operatorname{RootSum}\left(268435456t^4a^{11}c+194481,\left(t\mapsto t\log\left(\frac{128ta^3}{21}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSu
m(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))
```

Giac [A]
time = 3.77, size = 204, normalized size = 0.93

$$\frac{21\sqrt{2}\,(ac^3)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{128\,a^3c}+\frac{21\sqrt{2}\,(ac^3)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{128\,a^3c}+\frac{21\sqrt{2}\,(ac^3)^{\frac14}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}-\frac{21\sqrt{2}\,(ac^3)^{\frac14}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c}+\frac{7\,cx^5+11\,ax}{32\,(cx^4+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(1/4
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*
c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c
*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

Mupad [B]
time = 0.10, size = 80, normalized size = 0.37

$$\frac{\frac{11\,x}{32\,a}+\frac{7\,c\,x^5}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{64\,{\left(-a\right)}^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{64\,{\left(-a\right)}^{11/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^3,x)
```

```
[Out] ((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atan
((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x)/
(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

3.412 $\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx$

Optimal. Leaf size=1352

$$\frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)}+\frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2}+\frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}-\frac{\sqrt{c}\,d^2e^9\tan^{-}}{2\sqrt{a}\,\left(cd^4\right.}$$

```
[Out] 1/32*c*x*(5*d*e^2*x^2-6*d^2*e*x+7*d^3)/a^2/(a*e^4+c*d^4)/(c*x^4+a)+1/8*(a*e
^3+c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)/(c*x^4+a)^2+1/4*e^4*(a*e^3+
c*x*(d*e^2*x^2-d^2*e*x+d^3))/a/(a*e^4+c*d^4)^2/(c*x^4+a)+e^11*ln(e*x+d)/(a*
e^4+c*d^4)^3-1/4*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^3-1/4*d^2*e^5*arctan(x^2*c^
(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^2-3/16*d^2*e*arctan(x^2*c^(1/2
)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)-1/2*d^2*e^9*arctan(x^2*c^(1/2)/a^(
1/2))*c^(1/2)/(a*e^4+c*d^4)^3/a^(1/2)-1/8*c^(1/4)*d*e^8*ln(-a^(1/4)*c^(1/4)
*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c
*d^4)^3*2^(1/2)+1/8*c^(1/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(
1/4)*d*e^8*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a
^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(1/4)*d*e^8*arctan(1+c^(1/4)*x*2^(1/2)
/a^(1/4))*(e^2*a^(1/2)+d^2*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^3*2^(1/2)-1/32*c^
(1/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2
)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/32*c^(1/4)*d*e^4*ln(a^(1
/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*a^(1/2)+3*d^2*c^(1/2))/a^(
7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*c^(1/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)
/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/16*
c^(1/4)*d*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*a^(1/2)+3*d^2*c^(1/2
))/a^(7/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/256*c^(1/4)*d*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+
c*d^4)*2^(1/2)+1/256*c^(1/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)+1/128*
c^(1/4)*d*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2
))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)+1/128*c^(1/4)*d*arctan(1+c^(1/4)*x*2^(1/2
)/a^(1/4))*(5*e^2*a^(1/2)+21*d^2*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)*2^(1/2)
```

Rubi [A]
time = 0.99, antiderivative size = 1352, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6874, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^3),x]
[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)
) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x
^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^
4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4
*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (
c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e
^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[
c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx = \int \left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^3} - \frac{ce^4(-d^3+d^2ex-}{\left(cd^4+ae^4\right)^2(}\right.$$

$$= \frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^8\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^4\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{\left(a+cx^4\right)^2}\,dx}{\left(cd^4+ae^4\right)^2}$$

$$= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-d^2ex+de^2x^2))}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)} + \frac{e^{11}\log(d+}{(cd^4+ae}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-}{4a\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-}{4a\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-}{4a\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-}{4a\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-}{4a\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-}{4a\left(cd^4+ae^4\right)}$$

$$= \frac{cx(7d^3-6d^2ex+5de^2x^2)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4(ae^3+cx(d^3-}{4a\left(cd^4+ae^4\right)}$$

Mathematica [A]
time = 0.50, size = 835, normalized size = 0.62

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(a + c*x^4)^3),x]

[Out] ((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e^2*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4)) - (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 + 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 + 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^11*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^2 - 66*a*c^(3/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 + 45*a^(5/2)*d*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) + (Sqrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c^2*d^9*e^2 + 66*a*c^(3/2)*d^7*e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2*Sqrt[c]*d^3*e^8 - 45*a^(5/2)*d*e^10)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 64*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^3)

Maple [A]
time = 0.29, size = 677, normalized size = 0.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)

[Out] e^11*ln(e*x+d)/(a*e^4+c*d^4)^3+c/(a*e^4+c*d^4)^3*((1/32*d*e^2*c*(13*a^2*e^8+18*a*c*d^4*e^4+5*c^2*d^8)/a^2*x^7-1/16*c*d^2*e*(7*a^2*e^8+10*a*c*d^4*e^4+3*c^2*d^8)/a^2*x^6+1/32*c*d^3*(15*a^2*e^8+22*a*c*d^4*e^4+7*c^2*d^8)/a^2*x^5+(1/4*a*e^11+1/4*e^7*c*d^4)*x^4+1/32*d*e^2*(17*a^2*e^8+26*a*c*d^4*e^4+9*c^2*d^8)/a*x^3-1/16*d^2*e*(9*a^2*e^8+14*a*c*d^4*e^4+5*c^2*d^8)/a*x^2+1/32*d^3*(19*a^2*e^8+30*a*c*d^4*e^4+11*c^2*d^8)/a*x+1/8*e^3*(3*a^2*e^8+4*a*c*d^4*e^4+c^2*d^8)/c)/(c*x^4+a)^2+1/32/a^2*(1/8*(77*a^2*d^3*e^8+66*a*c*d^7*e^4+21*c^2*d^11)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-60*a^2*d^2*e^9-40*a*c*d^6*e^5-12*c^2*d^10*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(45*a^2*d*e^10+18*a*c*d^5*e^6+5*c^2*d^9*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-8*a^2*e^11/c*ln(c*x^4+a)))

Maxima [A]

time = 0.53, size = 971, normalized size = 0.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*c*(sqrt(2)*(21*c^3*d^11 - 5*sqrt(a)*c^(5/2)*d^9*e^2 + 66*a*c^2*d^7*e^
4 - 18*a^(3/2)*c^(3/2)*d^5*e^6 + 77*a^2*c*d^3*e^8 - 45*a^(5/2)*sqrt(c)*d*e^
10 - 32*sqrt(2)*a^(11/4)*c^(1/4)*e^11)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^
(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - sqrt(2)*(21*c^3*d^11 - 5*sqrt(a)*c^(
5/2)*d^9*e^2 + 66*a*c^2*d^7*e^4 - 18*a^(3/2)*c^(3/2)*d^5*e^6 + 77*a^2*c*d^3
*e^8 - 45*a^(5/2)*sqrt(c)*d*e^10 + 32*sqrt(2)*a^(11/4)*c^(1/4)*e^11)*log(sq
rt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + 2*(21*
sqrt(2)*a^(1/4)*c^(13/4)*d^11 + 24*sqrt(a)*c^3*d^10*e + 5*sqrt(2)*a^(3/4)*c
^(11/4)*d^9*e^2 + 66*sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4 + 80*a^(3/2)*c^2*d^6*e
^5 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*sqrt(2)*a^(9/4)*c^(5/4)*d^3*e^
8 + 120*a^(5/2)*c*d^2*e^9 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10)*arctan(1/2*
sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(21*sqrt(2)*a^(1/4)*c^(13/4)*d^11 -
 24*sqrt(a)*c^3*d^10*e + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*sqrt(2)*a^
(5/4)*c^(9/4)*d^7*e^4 - 80*a^(3/2)*c^2*d^6*e^5 + 18*sqrt(2)*a^(7/4)*c^(7/4)
*d^5*e^6 + 77*sqrt(2)*a^(9/4)*c^(5/4)*d^3*e^8 - 120*a^(5/2)*c*d^2*e^9 + 45*
sqrt(2)*a^(11/4)*c^(3/4)*d*e^10)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*
a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5
/4)))/(a^2*c^3*d^12 + 3*a^3*c^2*d^8*e^4 + 3*a^4*c*d^4*e^8 + a^5*e^12) + e^1
1*log(x*e + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) +
1/32*((5*c^3*d^5*e^2 + 13*a*c^2*d*e^6)*x^7 + 8*a^2*c*x^4*e^7 + 4*a^2*c*d^4*
e^3 - 2*(3*c^3*d^6*e + 7*a*c^2*d^2*e^5)*x^6 + (7*c^3*d^7 + 15*a*c^2*d^3*e^4
)*x^5 + (9*a*c^2*d^5*e^2 + 17*a^2*c*d*e^6)*x^3 + 12*a^3*e^7 - 2*(5*a*c^2*d^
6*e + 9*a^2*c*d^2*e^5)*x^2 + (11*a*c^2*d^7 + 19*a^2*c*d^3*e^4)*x)/(a^4*c^2*
d^8 + 2*a^5*c*d^4*e^4 + (a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8)*x^8
 + a^6*e^8 + 2*(a^3*c^3*d^8 + 2*a^4*c^2*d^4*e^4 + a^5*c*e^8)*x^4)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.06, size = 1259, normalized size = 0.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(51*sqrt(2)*sqrt(a*c)*c^2*d^4*e + 21*(a*c^3)^(1/4)*c^2*d^5 - 75*sqrt(2
)*a*c^2*d^2*e^3 + 122*(a*c^3)^(3/4)*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c*d*e^4)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3
*d^6 - 6*(a*c^3)^(1/4)*a^3*c^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2
+ 9*sqrt(2)*a^4*c^2*d^2*e^4 - 16*(a*c^3)^(3/4)*a^3*d^3*e^3 - 6*(a*c^3)^(1/4
)*a^4*c*d*e^5 + sqrt(2)*sqrt(a*c)*a^4*c*e^6) + 1/64*(51*sqrt(2)*sqrt(a*c)*c
^2*d^4*e + 21*(a*c^3)^(1/4)*c^2*d^5 + 75*sqrt(2)*a*c^2*d^2*e^3 + 122*(a*c^3
)^(3/4)*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c*d*e^4)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^6 + 6*(a*c^3)^(1/4)*a^3*c
^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2 + 9*sqrt(2)*a^4*c^2*d^2*e^4
+ 16*(a*c^3)^(3/4)*a^3*d^3*e^3 + 6*(a*c^3)^(1/4)*a^4*c*d*e^5 + sqrt(2)*sqrt
(a*c)*a^4*c*e^6) + 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*c^4*d^11 - 5*sqrt(2)*(a*
c^3)^(3/4)*c^2*d^9*e^2 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*sqrt(2
)*(a*c^3)^(3/4)*a*c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 45
*sqrt(2)*(a*c^3)^(3/4)*a^2*d*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(a^3*c^5*d^12 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12)
- 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*c^4*d^11 - 5*sqrt(2)*(a*c^3)^(3/4)*c^2*d^
9*e^2 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*sqrt(2)*(a*c^3)^(3/4)*a
*c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 45*sqrt(2)*(a*c^3)^
(3/4)*a^2*d*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^5*d^1
2 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12) - 1/4*e^11*log(ab
s(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + e
^12*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3
*e^13) + 1/32*(4*a^2*c^2*d^8*e^3 + 16*a^3*c*d^4*e^7 + (5*c^4*d^9*e^2 + 18*a
*c^3*d^5*e^6 + 13*a^2*c^2*d*e^10)*x^7 - 2*(3*c^4*d^10*e + 10*a*c^3*d^6*e^5
+ 7*a^2*c^2*d^2*e^9)*x^6 + (7*c^4*d^11 + 22*a*c^3*d^7*e^4 + 15*a^2*c^2*d^3*
e^8)*x^5 + 8*(a^2*c^2*d^4*e^7 + a^3*c*e^11)*x^4 + 12*a^4*e^11 + (9*a*c^3*d^
9*e^2 + 26*a^2*c^2*d^5*e^6 + 17*a^3*c*d*e^10)*x^3 - 2*(5*a*c^3*d^10*e + 14*
a^2*c^2*d^6*e^5 + 9*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 30*a^2*c^2*d^7*e^
4 + 19*a^3*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(c*x^4 + a)^2*a^2)
```

Mupad [B]

time = 4.41, size = 2500, normalized size = 1.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)),x)
```

```
[Out] symsum(log((194481*c^7*d^13*e^6 + 871362*a*c^6*d^9*e^10 + 425984*a^3*c^4*d*
e^18 + 1148881*a^2*c^5*d^5*e^14)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^1
1*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + root(805306368*a
^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^1
2*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*
d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 96522
24*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*
a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*
c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^
4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*
e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a
^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*
d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*
d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 +
268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*
z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c
*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*
e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*d^8*
e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 2684
35456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2
+ 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4
*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4
+ 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*((402653184*a^15*c^4*d*e^22 - 134
217728*a^10*c^9*d^21*e^2 - 134217728*a^11*c^8*d^17*e^6 + 805306368*a^12*c^7
*d^13*e^10 + 1879048192*a^13*c^6*d^9*e^14 + 1476395008*a^14*c^5*d^5*e^18)/(
1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4
+ 6*a^10*c^2*d^8*e^8)) + (x*(335544320*a^15*c^4*e^23 - 201326592*a^10*c^9*d
^20*e^3 - 469762048*a^11*c^8*d^16*e^7 + 134217728*a^12*c^7*d^12*e^11 + 1207
959552*a^13*c^6*d^8*e^15 + 1140850688*a^14*c^5*d^4*e^19))/(1048576*(a^12*e^
16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8
*e^8))) + (211288064*a^12*c^4*d*e^21 - 11010048*a^7*c^9*d^21*e + 20447232*a
^8*c^8*d^17*e^5 + 204472320*a^9*c^7*d^13*e^9 + 514850816*a^10*c^6*d^9*e^13
+ 553123840*a^11*c^5*d^5*e^17)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*
c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(251658240*a^12
*c^4*e^22 - 28311552*a^7*c^9*d^20*e^2 - 67108864*a^8*c^8*d^16*e^6 + 1887436
8*a^9*c^7*d^12*e^10 + 377487360*a^10*c^6*d^8*e^14 + 571473920*a^11*c^5*d^4*
e^18))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d
^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (36962304*a^9*c^4*d*e^20 + 11010048*a^5*c
```

^8*d^17*e^4 + 57999360*a^6*c^7*d^13*e^8 + 138805248*a^7*c^6*d^9*e^12 + 1413 61152*a^8*c^5*d^5*e^16)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e ^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(62914560*a^9*c^4*e^21 - 1806336*a^4*c^9*d^20*e + 2670592*a^5*c^8*d^16*e^5 + 43032576*a^6*c^7*d^1 2*e^9 + 143179776*a^7*c^6*d^8*e^13 + 171732992*a^8*c^5*d^4*e^17))/(1048576* (a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10 *c^2*d^8*e^8))) + (4030464*a^6*c^4*d*e^19 + 576576*a^2*c^8*d^17*e^3 + 50618 24*a^3*c^7*d^13*e^7 + 15959232*a^4*c^6*d^9*e^11 + 17863744*a^5*c^5*d^5*e^15)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e ^4 + 6*a^10*c^2*d^8*e^8)) + (x*(5242880*a^6*c^4*e^20 + 755136*a^2*c^8*d^16* e^4 + 6023488*a^3*c^7*d^12*e^8 + 19579200*a^4*c^6*d^8*e^12 + 22240704*a^5*c ^5*d^4*e^16))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^ 9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (x*(194481*c^7*d^12*e^7 + 871362*a *c^6*d^8*e^11 + 970321*a^2*c^5*d^4*e^15))/(1048576*(a^12*e^16 + a^8*c^4*d^1 6 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)))*root(805 306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11 *c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 4305715 2*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k), k, 1, 4) + ((3*a*e^7 + c*d^4*e^3)/(8*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^5*(7*c^3 *d^7 + 15*a*c^2*d^3*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x ^2*(5*c^2*d^6*e + 9*a*c*d^2*e^5))/(16*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (c*e^7*x^4)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2*d^7 + 19*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(3*c^3*d ^6*e + 7*a*c^2*d^2*e^5))/(16*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^ 2*x^3*(9*c^2*d^5 + 17*a*c*d*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^2*x^7*(5*c^3*d^5 + 13*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a *c*d^4*e^4)))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (e^...

3.413 $\int \frac{1}{(d+ex)^2(a+cx^4)^3} dx$

Optimal. Leaf size=1830

result too large to display

```
[Out] -e^11/(a*e^4+c*d^4)^3/(e*x+d)+1/32*c*x*(7*d^2*(-3*a*e^4+c*d^4)-12*d*e*(-a*e
^4+c*d^4)*x+5*e^2*(-a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^2/(c*x^4+a)+1/8*c
*(4*a*d^3*e^3+x*(d^2*(-3*a*e^4+c*d^4)-2*d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*
c*d^4)*x^2))/a/(a*e^4+c*d^4)^2/(c*x^4+a)^2+1/4*c*e^4*(8*a*d^3*e^3+x*(d^2*(-
3*a*e^4+5*c*d^4)-2*d*e*(-a*e^4+3*c*d^4)*x+e^2*(-a*e^4+7*c*d^4)*x^2))/a/(a*e
^4+c*d^4)^3/(c*x^4+a)+12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^11*
ln(c*x^4+a)/(a*e^4+c*d^4)^4-1/2*d*e^5*(-a*e^4+3*c*d^4)*arctan(x^2*c^(1/2)/a
^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4)^3-3/8*d*e*(-a*e^4+c*d^4)*arctan(x^2*c
^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^4+c*d^4)^2-d*e^9*(-a*e^4+5*c*d^4)*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^4/a^(1/2)-1/8*c^(1/4)*e^8*ln(
-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)*d^6+a^(3/2)*e^6-
11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1
/8*c^(1/4)*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(9*c^(3/2)
*d^6+a^(3/2)*e^6-11*c*d^4*e^2*a^(1/2)-3*a*d^2*e^4*c^(1/2))/a^(3/4)/(a*e^4+c
*d^4)^4*2^(1/2)-1/256*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^(1/2))/a^(
11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/256*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^
(1/2)+x^2*c^(1/2))*(-5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)
*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(-1+c^(1/4)*
x*2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*
c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)+1/128*c^(1/4)*arctan(1+c^(1/4)*x*
2^(1/2)/a^(1/4))*(5*e^2*(-a*e^4+3*c*d^4)*a^(1/2)+21*d^2*(-3*a*e^4+c*d^4)*c^
(1/2))/a^(11/4)/(a*e^4+c*d^4)^2*2^(1/2)-1/32*c^(1/4)*e^4*ln(-a^(1/4)*c^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/2)+3*d^2*(-3*a
*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(1/4)*e^4*ln(
a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e^2*(-a*e^4+7*c*d^4)*a^(1/
2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c
^(1/4)*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/
2)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/16*c
^(1/4)*e^4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+7*c*d^4)*a^(1/2
)+3*d^2*(-3*a*e^4+5*c*d^4)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/4*c^(
1/4)*e^8*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2
)+3*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/4*c^(1/
4)*e^8*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e^2*(-a*e^4+11*c*d^4)*a^(1/2)+3
*d^2*(-a*e^4+3*c*d^4)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^4*2^(1/2)
```

Rubi [A]

time = 1.95, antiderivative size = 1830, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6874, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + c*x^4)^3),x]

[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^11*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)

```
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*L
og[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^3}\,dx = \int \left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)^2} + \frac{12cd^3e^{12}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{c(d^2(cd^4-3ae^4)-2de(c}{(c}\right.$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{12cd^3e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^4} + \frac{(ce^8)\int \frac{3d^2(3cd^4-ae^4)-2de(5cd^4-a}{}}{(cd^4+}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c(4ad^3e^3+x(d^2(cd^4-3ae^4)-2de(cd^4-ae^4)\,x+e^2}{8a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)^2}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx(7d^2(cd^4-3ae^4)-12de(cd^4-ae^4)\,x+5e^2(3cd^4-}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

Mathematica [A]
time = 1.01, size = 1115, normalized size = 0.61

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] ((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^2*d^8*x*(7
*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2 - 24*d*e*x + 33*e^2
*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e^2*x^2 - 13*e^3*x^3)))/(a^2*(a
 + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^2) +
 a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2) - (6
*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*S
qrt[a]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(9/4)*d^
9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 - 2
40*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^
3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(7*Sqrt[2]*c
^(7/2)*d^14 + 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^12*e^2 +
 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 + 27*Sqrt[2]*a^
(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9/4)*c^(5/4)*d^
5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^12 -
80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072*c*d^3*e^11*Log[d + e*x] - (3*Sqrt[2]*c^
(1/4)*(7*c^(7/2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27
*a^(3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77
*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(11/4) + (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5*Sqr
t[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^8*e^6 + 77*a^2
*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^
(7/2)*e^14)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4
) - 768*c*d^3*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^4)
```

Maple [A]
time = 0.33, size = 829, normalized size = 0.45

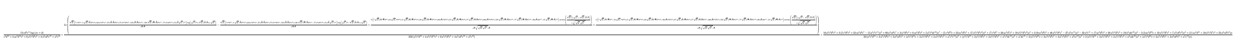
method	result
default	$-\frac{e^{11}}{\left(e^4 a+d^4 c\right)^3 (ex+d)}+\frac{12c\, d^3 e^{11} \ln(ex+d)}{\left(e^4 a+d^4 c\right)^4}-\frac{c\left(\frac{c\, e^2\left(13a^3 e^{12}-53a^2 c\, d^4 e^8-81a\, c^2 d^8 e^4-15c^3 d^{12}\right)x^7}{32a^2}-\frac{cde\left(7a^3 e^{12}-5a^2 c\, d^4 e^8-15a\, c^2 \right.}{8a^2}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

[Out] -e^11/(a*e^4+c*d^4)^3/(e*x+d)+12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-c/(a*
e^4+c*d^4)^4*((1/32*c*e^2*(13*a^3*e^12-53*a^2*c*d^4*e^8-81*a*c^2*d^8*e^4-15
*c^3*d^12)/a^2*x^7-1/8*c*d*e*(7*a^3*e^12-5*a^2*c*d^4*e^8-15*a*c^2*d^8*e^4-3
*c^3*d^12)/a^2*x^6+1/32*c*d^2*(45*a^3*e^12+19*a^2*c*d^4*e^8-33*a*c^2*d^8*e^
4-7*c^3*d^12)/a^2*x^5+(-2*a*c*d^3*e^11-2*c^2*d^7*e^7)*x^4+1/32*e^2*(17*a^3*
e^12-57*a^2*c*d^4*e^8-101*a*c^2*d^8*e^4-27*c^3*d^12)/a*x^3-1/8*d*e*(9*a^3*e
^12-3*a^2*c*d^4*e^8-17*a*c^2*d^8*e^4-5*c^3*d^12)/a*x^2+1/32*d^2*(57*a^3*e^1
2+39*a^2*c*d^4*e^8-29*a*c^2*d^8*e^4-11*c^3*d^12)/a*x-5/2*a^2*d^3*e^11-3*a*d
^7*e^7*c-1/2*d^11*e^3*c^2)/(c*x^4+a)^2+3/32/a^2*(1/8*(77*a^3*d^2*e^12-77*a^
2*c*d^6*e^8-33*a*c^2*d^10*e^4-7*c^3*d^14)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2
*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-4
0*a^3*d*e^13+120*a^2*c*d^5*e^9+40*a*c^2*d^9*e^5+8*c^3*d^13*e)/(a*c)^(1/2)*a
rctan(x^2*(c/a)^(1/2))+1/8*(15*a^3*e^14-135*a^2*c*d^4*e^10-27*a*c^2*d^8*e^6
-5*c^3*d^12*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+32*a^2*d^3*e^11*ln(c*x^4+a)))

Maxima [A]
time = 0.59, size = 1487, normalized size = 0.81



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")

[Out] 12*c*d^3*e^11*log(x*e + d)/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8
 + 4*a^3*c*d^4*e^12 + a^4*e^16) + 3/256*c*(sqrt(2)*(7*c^4*d^14 - 5*sqrt(a)*
c^(7/2)*d^12*e^2 + 33*a*c^3*d^10*e^4 - 27*a^(3/2)*c^(5/2)*d^8*e^6 + 77*a^2*
c^2*d^6*e^8 - 135*a^(5/2)*c^(3/2)*d^4*e^10 - 128*sqrt(2)*a^(11/4)*c^(5/4)*d
^3*e^11 - 77*a^3*c*d^2*e^12 + 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - sqrt(2)*(7*c^4*d^14
- 5*sqrt(a)*c^(7/2)*d^12*e^2 + 33*a*c^3*d^10*e^4 - 27*a^(3/2)*c^(5/2)*d^8*e
^6 + 77*a^2*c^2*d^6*e^8 - 135*a^(5/2)*c^(3/2)*d^4*e^10 + 128*sqrt(2)*a^(11/
4)*c^(5/4)*d^3*e^11 - 77*a^3*c*d^2*e^12 + 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt
(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + 2*(7*sqr
t(2)*a^(1/4)*c^(17/4)*d^14 + 16*sqrt(a)*c^4*d^13*e + 5*sqrt(2)*a^(3/4)*c^(1
5/4)*d^12*e^2 + 33*sqrt(2)*a^(5/4)*c^(13/4)*d^10*e^4 + 80*a^(3/2)*c^3*d^9*e
^5 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqrt(2)*a^(9/4)*c^(9/4)*d^6*e
^8 + 240*a^(5/2)*c^2*d^5*e^9 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*s
qrt(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 80*a^(7/2)*c*d*e^13 - 15*sqrt(2)*a^(15/4
)*c^(3/4)*e^14)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(7*sqrt(
2)*a^(1/4)*c^(17/4)*d^14 - 16*sqrt(a)*c^4*d^13*e + 5*sqrt(2)*a^(3/4)*c^(15/
4)*d^12*e^2 + 33*sqrt(2)*a^(5/4)*c^(13/4)*d^10*e^4 - 80*a^(3/2)*c^3*d^9*e^5

```
 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqrt(2)*a^(9/4)*c^(9/4)*d^6*e^8
 - 240*a^(5/2)*c^2*d^5*e^9 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*sqr
t(2)*a^(13/4)*c^(5/4)*d^2*e^12 + 80*a^(7/2)*c*d*e^13 - 15*sqrt(2)*a^(15/4)*
c^(3/4)*e^14)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a^2*c^4*d^16
 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6*e^16) +
1/32*(16*a^2*c^2*d^8*e^3 + 3*(5*c^4*d^8*e^3 + 22*a*c^3*d^4*e^7 - 15*a^2*c^2
*e^11)*x^8 + 80*a^3*c*d^4*e^7 + 3*(c^4*d^9*e^2 + 6*a*c^3*d^5*e^6 + 5*a^2*c^
2*d*e^10)*x^7 - (5*c^4*d^10*e + 22*a*c^3*d^6*e^5 + 17*a^2*c^2*d^2*e^9)*x^6
+ (7*c^4*d^11 + 26*a*c^3*d^7*e^4 + 19*a^2*c^2*d^3*e^8)*x^5 + 3*(9*a*c^3*d^8
*e^3 + 46*a^2*c^2*d^4*e^7 - 27*a^3*c*e^11)*x^4 - 32*a^4*e^11 + (7*a*c^3*d^9
*e^2 + 26*a^2*c^2*d^5*e^6 + 19*a^3*c*d*e^10)*x^3 - 3*(3*a*c^3*d^10*e + 10*a
^2*c^2*d^6*e^5 + 7*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 34*a^2*c^2*d^7*e^4
 + 23*a^3*c*d^3*e^8)*x)/(a^4*c^3*d^13 + 3*a^5*c^2*d^9*e^4 + 3*a^6*c*d^5*e^8
 + (a^2*c^5*d^12*e + 3*a^3*c^4*d^8*e^5 + 3*a^4*c^3*d^4*e^9 + a^5*c^2*e^13)*
x^9 + (a^2*c^5*d^13 + 3*a^3*c^4*d^9*e^4 + 3*a^4*c^3*d^5*e^8 + a^5*c^2*d*e^1
2)*x^8 + a^7*d*e^12 + 2*(a^3*c^4*d^12*e + 3*a^4*c^3*d^8*e^5 + 3*a^5*c^2*d^4
*e^9 + a^6*c*e^13)*x^5 + 2*(a^3*c^4*d^13 + 3*a^4*c^3*d^9*e^4 + 3*a^5*c^2*d^
5*e^8 + a^6*c*d*e^12)*x^4 + (a^4*c^3*d^12*e + 3*a^5*c^2*d^8*e^5 + 3*a^6*c*d
^4*e^9 + a^7*e^13)*x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 81.89, size = 1729, normalized size = 0.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] -3*c*d^3*e^11*log(abs(c*x^4 + a))/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*
d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) + 12*c*d^3*e^12*log(abs(x*e + d))/(c
^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2*d^8*e^9 + 4*a^3*c*d^4*e^13 + a^4*e
^17) + 3/64*(20*sqrt(2)*sqrt(a*c)*c^3*d^5*e + 7*(a*c^3)^(1/4)*c^3*d^6 - 32*
sqrt(2)*a*c^3*d^3*e^3 + 53*(a*c^3)^(3/4)*c*d^4*e^2 + 3*(a*c^3)^(1/4)*a*c^2*
d^2*e^4 + 20*sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 - 15*(a*c^3)^(3/4)*a*e^6)*arctan
(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^4*d^8
- 8*(a*c^3)^(1/4)*a^3*c^3*d^7*e + 16*sqrt(2)*sqrt(a*c)*a^3*c^3*d^6*e^2 + 34
*sqrt(2)*a^4*c^3*d^4*e^4 - 40*(a*c^3)^(3/4)*a^3*c*d^5*e^3 - 40*(a*c^3)^(1/4
)*a^4*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^4*c^2*d^2*e^6 + sqrt(2)*a^5*c^2*
e^8 - 8*(a*c^3)^(3/4)*a^4*d*e^7) + 3/64*(20*sqrt(2)*sqrt(a*c)*c^3*d^5*e + 7
*(a*c^3)^(1/4)*c^3*d^6 + 32*sqrt(2)*a*c^3*d^3*e^3 + 53*(a*c^3)^(3/4)*c*d^4*
e^2 + 3*(a*c^3)^(1/4)*a*c^2*d^2*e^4 + 20*sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 - 15
*(a*c^3)^(3/4)*a*e^6)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(sqrt(2)*a^3*c^4*d^8 + 8*(a*c^3)^(1/4)*a^3*c^3*d^7*e + 16*sqrt(2)*sq
rt(a*c)*a^3*c^3*d^6*e^2 + 34*sqrt(2)*a^4*c^3*d^4*e^4 + 40*(a*c^3)^(3/4)*a^3
*c*d^5*e^3 + 40*(a*c^3)^(1/4)*a^4*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^4*c^
2*d^2*e^6 + sqrt(2)*a^5*c^2*e^8 + 8*(a*c^3)^(3/4)*a^4*d*e^7) + 3/256*(7*sqr
t(2)*(a*c^3)^(1/4)*c^5*d^14 - 5*sqrt(2)*(a*c^3)^(3/4)*c^3*d^12*e^2 + 33*sqr
t(2)*(a*c^3)^(1/4)*a*c^4*d^10*e^4 - 27*sqrt(2)*(a*c^3)^(3/4)*a*c^2*d^8*e^6
+ 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^6*e^8 - 135*sqrt(2)*(a*c^3)^(3/4)*a^2*
c*d^4*e^10 - 77*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^2*e^12 + 15*sqrt(2)*(a*c^3)
^(3/4)*a^3*e^14)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^16
 + 4*a^4*c^5*d^12*e^4 + 6*a^5*c^4*d^8*e^8 + 4*a^6*c^3*d^4*e^12 + a^7*c^2*e^
16) - 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^14 - 5*sqrt(2)*(a*c^3)^(3/4)*c^3
*d^12*e^2 + 33*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^10*e^4 - 27*sqrt(2)*(a*c^3)^(3
/4)*a*c^2*d^8*e^6 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^6*e^8 - 135*sqrt(2)*
(a*c^3)^(3/4)*a^2*c*d^4*e^10 - 77*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^2*e^12 +
15*sqrt(2)*(a*c^3)^(3/4)*a^3*e^14)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(a^3*c^6*d^16 + 4*a^4*c^5*d^12*e^4 + 6*a^5*c^4*d^8*e^8 + 4*a^6*c^3*d^4
*e^12 + a^7*c^2*e^16) + 1/32*(16*a^2*c^3*d^12*e^3 + 96*a^3*c^2*d^8*e^7 + 48
*a^4*c*d^4*e^11 + 3*(5*c^5*d^12*e^3 + 27*a*c^4*d^8*e^7 + 7*a^2*c^3*d^4*e^11
 - 15*a^3*c^2*e^15)*x^8 + 3*(c^5*d^13*e^2 + 7*a*c^4*d^9*e^6 + 11*a^2*c^3*d^
5*e^10 + 5*a^3*c^2*d*e^14)*x^7 - (5*c^5*d^14*e + 27*a*c^4*d^10*e^5 + 39*a^2
*c^3*d^6*e^9 + 17*a^3*c^2*d^2*e^13)*x^6 + (7*c^5*d^15 + 33*a*c^4*d^11*e^4 +
 45*a^2*c^3*d^7*e^8 + 19*a^3*c^2*d^3*e^12)*x^5 - 32*a^5*e^15 + 3*(9*a*c^4*d
^12*e^3 + 55*a^2*c^3*d^8*e^7 + 19*a^3*c^2*d^4*e^11 - 27*a^4*c*e^15)*x^4 + (
7*a*c^4*d^13*e^2 + 33*a^2*c^3*d^9*e^6 + 45*a^3*c^2*d^5*e^10 + 19*a^4*c*d*e^
14)*x^3 - 3*(3*a*c^4*d^14*e + 13*a^2*c^3*d^10*e^5 + 17*a^3*c^2*d^6*e^9 + 7*
a^4*c*d^2*e^13)*x^2 + (11*a*c^4*d^15 + 45*a^2*c^3*d^11*e^4 + 57*a^3*c^2*d^7
*e^8 + 23*a^4*c*d^3*e^12)*x)/((c*d^4 + a*e^4)^4*(c*x^4 + a)^2*(x*e + d)*a^2
)
```

Mupad [B]

time = 5.75, size = 2500, normalized size = 1.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log((194481*c^9*d^17*e^6 + 1527012*a*c^8*d^13*e^10 + 4100625*a^4*c^5
*d*e^22 + 1926342*a^2*c^7*d^9*e^14 - 3102300*a^3*c^6*d^5*e^18)/(1048576*(a^
14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c
^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612
736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^
14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 +
 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*
a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5
*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*
d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c
^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e
^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 322122547
2*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^
10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5
419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 +
4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c^2*d^8*e^8
*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 +
268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d
^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2
 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3
*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^
2*c*e^8 + 194481*c^3*d^8, z, k)*((23592960*a^14*c^4*e^29 - 11010048*a^7*c^1
1*d^28*e + 33030144*a^8*c^10*d^24*e^5 + 504889344*a^9*c^9*d^20*e^9 + 310326
0672*a^10*c^8*d^16*e^13 + 6799491072*a^11*c^7*d^12*e^17 + 6101139456*a^12*c
^6*d^8*e^21 + 1967652864*a^13*c^5*d^4*e^25)/(1048576*(a^14*e^24 + a^8*c^6*d
^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^
11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612736*a^13*c^2*d^8*e^
8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 +
 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*
d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^
2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^
3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a
^2*c*e^8 + 194481*c^3*d^8, z, k)*((402653184*a^17*c^4*d*e^30 - 134217728*a^
10*c^11*d^29*e^2 - 402653184*a^11*c^10*d^25*e^6 + 402653184*a^12*c^9*d^21*e
^10 + 3355443200*a^13*c^8*d^17*e^14 + 6039797760*a^14*c^7*d^13*e^18 + 52344
91392*a^15*c^6*d^9*e^22 + 2281701376*a^16*c^5*d^5*e^26)/(1048576*(a^14*e^24
 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16
*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(335544320*a^17*
```

c^4*e^31 - 201326592*a^10*c^11*d^28*e^3 - 872415232*a^11*c^10*d^24*e^7 - 10
06632960*a^12*c^9*d^20*e^11 + 1006632960*a^13*c^8*d^16*e^15 + 3690987520*a^
14*c^7*d^12*e^19 + 3825205248*a^15*c^6*d^8*e^23 + 1811939328*a^16*c^5*d^4*e
^27))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^
20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^1
6))) + (x*(2554331136*a^10*c^8*d^15*e^14 - 144703488*a^8*c^10*d^23*e^6 - 15
4140672*a^9*c^9*d^19*e^10 - 34603008*a^7*c^11*d^27*e^2 + 7659847680*a^11*c^
7*d^11*e^18 + 7556038656*a^12*c^6*d^7*e^22 + 2494562304*a^13*c^5*d^3*e^26))
/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^
4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)))
+ (12681216*a^5*c^10*d^23*e^4 + 127107072*a^6*c^9*d^19*e^8 + 674168832*a^7*
c^8*d^15*e^12 + 1018626048*a^8*c^7*d^11*e^16 - 446201856*a^9*c^6*d^7*e^20 +
906854400*a^10*c^5*d^3*e^24)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c
*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^
12 + 15*a^12*c^2*d^8*e^16)) + (x*(516096*a^5*c^10*d^22*e^5 - 1806336*a^4*c^
11*d^26*e + 90427392*a^6*c^9*d^18*e^9 + 896090112*a^7*c^8*d^14*e^13 + 19609
06752*a^8*c^7*d^10*e^17 + 1732829184*a^9*c^6*d^6*e^21 + 1183887360*a^10*c^5
*d^2*e^25))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*
c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d
^8*e^16))) + (387072*a^2*c^10*d^22*e^3 + 8004096*a^3*c^9*d^18*e^7 + 4937932
8*a^4*c^8*d^14*e^11 + 49572864*a^5*c^7*d^10*e^15 - 156930048*a^6*c^6*d^6*e^
19 + 125452800*a^7*c^5*d^2*e^23)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^1
3*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12
*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(126360000*a^7*c^5*d*e^24 + 561600*a^2*
c^10*d^21*e^4 + 9609408*a^3*c^9*d^17*e^8 + 75731328*a^4*c^8*d^13*e^12 + 114
991488*a^5*c^7*d^9*e^16 - 80136000*a^6*c^6*d^5*e^20))/(1048576*(a^14*e^24 +
a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e
^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16))) + (x*(4100625*a^4*c^5*
e^23 + 194481*c^9*d^16*e^7 + 1527012*a*c^8*d^12...

3.414 $\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx$

Optimal. Leaf size=2204

result too large to display

```
[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+1
/32*c*x*(7*d*(3*a^2*e^8-12*a*c*d^4*e^4+c^2*d^8)-6*e*(a^2*e^8-12*a*c*d^4*e^4
+3*c^2*d^8)*x+10*c*d^3*e^2*(-5*a*e^4+3*c*d^4)*x^2)/a^2/(a*e^4+c*d^4)^3/(c*x
^4+a)+1/8*c*(2*a*d^2*e^3*(-3*a*e^4+5*c*d^4)+x*(d*(3*a^2*e^8-12*a*c*d^4*e^4+
c^2*d^8)-e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*x+2*c*d^3*e^2*(-5*a*e^4+3*c*d
^4)*x^2))/a/(a*e^4+c*d^4)^3/(c*x^4+a)^2+1/4*c*e^4*(12*a*d^2*e^3*(-a*e^4+3*c
*d^4)+x*(3*d*(a^2*e^8-10*a*c*d^4*e^4+5*c^2*d^8)-e*(a^2*e^8-26*a*c*d^4*e^4+2
1*c^2*d^8)*x+4*c*d^3*e^2*(-5*a*e^4+7*c*d^4)*x^2))/a/(a*e^4+c*d^4)^4/(c*x^4+
a)+6*c*d^2*e^11*(-3*a*e^4+13*c*d^4)*ln(e*x+d)/(a*e^4+c*d^4)^5-3/2*c*d^2*e^1
1*(-3*a*e^4+13*c*d^4)*ln(c*x^4+a)/(a*e^4+c*d^4)^5-1/4*e^5*(a^2*e^8-26*a*c*d
^4*e^4+21*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^4+c*d^4
)^4-3/16*e*(a^2*e^8-12*a*c*d^4*e^4+3*c^2*d^8)*arctan(x^2*c^(1/2)/a^(1/2))*c
^(1/2)/a^(5/2)/(a*e^4+c*d^4)^3-1/2*e^9*(a^2*e^8-40*a*c*d^4*e^4+55*c^2*d^8)*
arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^4+c*d^4)^5/a^(1/2)+1/256*c^(3/4)*d
*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^
4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a
*e^4+c*d^4)^3*2^(1/2)-1/256*c^(3/4)*d*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+
x^2*c^(1/2))*(-63*a^2*e^8+252*a*c*d^4*e^4-21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3
*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(1/2)+1/128*c^(3/4)*d*a
rctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*c*d^4*e^4+21*c^2*d^8+
10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4)/(a*e^4+c*d^4)^3*2^(
1/2)+1/128*c^(3/4)*d*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(63*a^2*e^8-252*a*
c*d^4*e^4+21*c^2*d^8+10*d^2*e^2*(-5*a*e^4+3*c*d^4)*a^(1/2)*c^(1/2))/a^(11/4
)/(a*e^4+c*d^4)^3*2^(1/2)+1/32*c^(3/4)*d*e^4*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+
a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*
e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-1/32*c^(3/4)*
d*e^4*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-9*a^2*e^8+90*a*c*
d^4*e^4-45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a
*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a*e^4+7*c*d^4)*a^(1/2
)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)+1/16*c^(3/4)*d*e^4*arctan(1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(9*a^2*e^8-90*a*c*d^4*e^4+45*c^2*d^8+4*d^2*e^2*(-5*a
*e^4+7*c*d^4)*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^4+c*d^4)^4*2^(1/2)-3/8*c^(3/4)*
d*e^8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(15*c^2*d^8-16*a*c
*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*
e^4+c*d^4)^5*2^(1/2)+3/8*c^(3/4)*d*e^8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8-2*d^2*e^2*(-5*a*e^4+11*c*d
^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4*c^(3/4)*d*e^8*arct
an(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e^4+a^2*e^8+2*d^2*e
```

```
^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*d^4)^5*2^(1/2)+3/4
*c^(3/4)*d*e^8*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(15*c^2*d^8-16*a*c*d^4*e
^4+a^2*e^8+2*d^2*e^2*(-5*a*e^4+11*c*d^4)*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^4+c*
d^4)^5*2^(1/2)
```

Rubi [A]

time = 2.24, antiderivative size = 2204, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6874, 1868, 1869, 1890, 281, 211, 1182, 1176, 631, 210, 1179, 642, 1262, 649, 266}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] -1/2*e^11/((c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^4
)^4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*
c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^
2))/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a
*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c
*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4
+ a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d
*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 +
a^2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*
(a + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^
2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)
*(c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*A
rcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d
*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11
*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4
 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(1
0*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^
4 + 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/
4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2
*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4
*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4
*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4
)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*
a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(
```

```
13*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*
c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 -
5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7
*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e
^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c
^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*
e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*
c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^
2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d
^4 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4)
 - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3*c*
d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^5)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
```

```
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1890

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3(a+cx^4)^3}\,dx = \int \left(\frac{e^{12}}{(cd^4+ae^4)^3(d+ex)^3} + \frac{12cd^3e^{12}}{(cd^4+ae^4)^4(d+ex)^2} + \frac{6cd^2e^{12}(13cd^4-3ae^4)}{(cd^4+ae^4)^5(d+ex)} + \right.$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{6cd^2e^{11}(13cd^4-3ae^4)\log(}{(cd^4+ae^4)^5}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4)+x}{}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3}{}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3}{}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3}{}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3}{}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3}{}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3}{}$$

$$= -\frac{e^{11}}{2(cd^4+ae^4)^3(d+ex)^2} - \frac{12cd^3e^{11}}{(cd^4+ae^4)^4(d+ex)} + \frac{cx(7d(c^2d^8-12acd^4e^4+3}{}$$

Mathematica [A]
time = 1.87, size = 1338, normalized size = 0.61

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]

[Out] ((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2 + 45*d*e*x - 14*e^2*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2*x^2) + a*c^2*d^7*e^4*x*(43*d^2 - 114*d*e*x + 204*e^2*x^2) + a^2*c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 274*d*e^2*x^2 - 210*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 - 176*a^(5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 - 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 - 40*a^(17/4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 + 176*a^(5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 + 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/4)*e^17)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 1536*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 - 702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) + (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 - 702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 384*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^5)

Maple [A]
time = 0.35, size = 1008, normalized size = 0.46

method	result

default	$-\frac{e^{11}}{2(e^4a+d^4c)^3(ex+d)^2}-\frac{12c\,d^3e^{11}}{(e^4a+d^4c)^4(ex+d)}-\frac{6e^{11}d^2c(3e^4a-13d^4c)\ln(ex+d)}{(e^4a+d^4c)^5}+\frac{c\left(-\frac{3c^2d^3e^2(35a^3e^{12}+a^2c\,d^4e^8-39a\,c^2d^8e^4-5}{16a^2}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)-6
*e^11*d^2*c*(3*a*e^4-13*c*d^4)/(a*e^4+c*d^4)^5*ln(e*x+d)+c/(a*e^4+c*d^4)^5*
((-3/16*c^2*d^3*e^2*(35*a^3*e^12+a^2*c*d^4*e^8-39*a*c^2*d^8*e^4-5*c^3*d^12)
/a^2*x^7-1/16*c*e*(7*a^4*e^16-130*a^3*c*d^4*e^12-80*a^2*c^2*d^8*e^8+66*a*c^
3*d^12*e^4+9*c^4*d^16)/a^2*x^6+1/32*c*d*(45*a^4*e^16-258*a^3*c*d^4*e^12-260
*a^2*c^2*d^8*e^8+50*a*c^3*d^12*e^4+7*c^4*d^16)/a^2*x^5+(-3*a^2*c*d^2*e^15+6
*a*c^2*d^6*e^11+9*c^3*d^10*e^7)*x^4-1/16*c*d^3*e^2*(125*a^3*e^12+31*a^2*c*d
^4*e^8-121*a*c^2*d^8*e^4-27*c^3*d^12)/a*x^3-3/16*e*(3*a^4*e^16-50*a^3*c*d^4
*e^12-40*a^2*c^2*d^8*e^8+18*a*c^3*d^12*e^4+5*c^4*d^16)/a*x^2+1/32*d*(57*a^4
*e^16-282*a^3*c*d^4*e^12-340*a^2*c^2*d^8*e^8+10*a*c^3*d^12*e^4+11*c^4*d^16)
/a*x-15/4*a^3*d^2*e^15+23/4*a^2*d^6*e^11*c+43/4*a*d^10*e^7*c^2+5/4*d^14*e^3
*c^3)/(c*x^4+a)^2+3/32/a^2*(1/8*(77*a^4*d*e^16-770*a^3*c*d^5*e^12+220*a^2*c
^2*d^9*e^8+50*a*c^3*d^13*e^4+7*c^4*d^17)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/
c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*
arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-20
*a^4*e^17+600*a^3*c*d^4*e^13-480*a^2*c^2*d^8*e^9-88*a*c^3*d^12*e^5-12*c^4*d
^16*e)/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+1/8*(-390*a^3*c*d^3*e^14+702*a^2
*c^2*d^7*e^10+78*a*c^3*d^11*e^6+10*c^4*d^15*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln(
(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1
/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+
1/4*(192*a^3*c*d^2*e^15-832*a^2*c^2*d^6*e^11)/c*ln(c*x^4+a)))
```

Maxima [A]
time = 0.56, size = 2077, normalized size = 0.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 3/256*c*(sqrt(2)*(7*c^5*d^17 - 10*sqrt(a)*c^(9/2)*d^15*e^2 + 50*a*c^4*d^13*
e^4 - 78*a^(3/2)*c^(7/2)*d^11*e^6 + 220*a^2*c^3*d^9*e^8 - 702*a^(5/2)*c^(5/
```

2)*d^7*e^10 - 832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 770*a^3*c^2*d^5*e^12 + 390*a^(7/2)*c^(3/2)*d^3*e^14 + 192*sqrt(2)*a^(15/4)*c^(5/4)*d^2*e^15 + 77*a^4*c*d*e^16)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - sqrt(2)*(7*c^5*d^17 - 10*sqrt(a)*c^(9/2)*d^15*e^2 + 50*a*c^4*d^13*e^4 - 78*a^(3/2)*c^(7/2)*d^11*e^6 + 220*a^2*c^3*d^9*e^8 - 702*a^(5/2)*c^(5/2)*d^7*e^10 + 832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 770*a^3*c^2*d^5*e^12 + 390*a^(7/2)*c^(3/2)*d^3*e^14 - 192*sqrt(2)*a^(15/4)*c^(5/4)*d^2*e^15 + 77*a^4*c*d*e^16)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 24*sqrt(a)*c^5*d^16*e + 10*sqrt(2)*a^(3/4)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 176*a^(3/2)*c^4*d^12*e^5 + 78*sqrt(2)*a^(7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 + 960*a^(5/2)*c^3*d^8*e^9 + 702*sqrt(2)*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 - 1200*a^(7/2)*c^2*d^4*e^13 - 390*sqrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 + 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 - 24*sqrt(a)*c^5*d^16*e + 10*sqrt(2)*a^(3/4)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 - 176*a^(3/2)*c^4*d^12*e^5 + 78*sqrt(2)*a^(7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 - 960*a^(5/2)*c^3*d^8*e^9 + 702*sqrt(2)*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 + 1200*a^(7/2)*c^2*d^4*e^13 - 390*sqrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 - 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a^2*c^5*d^20 + 5*a^3*c^4*d^16*e^4 + 10*a^4*c^3*d^12*e^8 + 10*a^5*c^2*d^8*e^12 + 5*a^6*c*d^4*e^16 + a^7*e^20) + 6*(13*c^2*d^6*e^11 - 3*a*c*d^2*e^15)*log(x*e + d)/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^12*e^8 + 10*a^3*c^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 1/32*(40*a^2*c^3*d^12*e^3 + 304*a^3*c^2*d^8*e^7 + 6*(5*c^5*d^11*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^12)*x^9 - 520*a^4*c*d^4*e^11 + 6*(7*c^5*d^12*e^3 + 49*a*c^4*d^8*e^7 - 91*a^2*c^3*d^4*e^11 - 5*a^3*c^2*e^15)*x^8 + (c^5*d^13*e^2 + 19*a*c^4*d^9*e^6 + 35*a^2*c^3*d^5*e^10 + 17*a^3*c^2*d*e^14)*x^7 - 4*(c^5*d^14*e + 7*a*c^4*d^10*e^5 + 11*a^2*c^3*d^6*e^9 + 5*a^3*c^2*d^2*e^13)*x^6 + (7*c^5*d^15 + 97*a*c^4*d^11*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*c^2*d^3*e^12)*x^5 - 16*a^5*e^15 + 2*(39*a*c^4*d^12*e^3 + 293*a^2*c^3*d^8*e^7 - 539*a^3*c^2*d^4*e^11 - 25*a^4*c*e^15)*x^4 + (5*a*c^4*d^13*e^2 + 31*a^2*c^3*d^9*e^6 + 47*a^3*c^2*d^5*e^10 + 21*a^4*c*d*e^14)*x^3 - 8*(a*c^4*d^14*e + 5*a^2*c^3*d^10*e^5 + 7*a^3*c^2*d^6*e^9 + 3*a^4*c*d^2*e^13)*x^2 + (11*a*c^4*d^15 + 79*a^2*c^3*d^11*e^4 + 269*a^3*c^2*d^7*e^8 - 567*a^4*c*d^3*e^12)*x)/(a^4*c^4*d^18 + 4*a^5*c^3*d^14*e^4 + 6*a^6*c^2*d^10*e^8 + 4*a^7*c*d^6*e^12 + (a^2*c^6*d^16*e^2 + 4*a^3*c^5*d^12*e^6 + 6*a^4*c^4*d^8*e^10 + 4*a^5*c^3*d^4*e^14 + a^6*c^2*e^18)*x^10 + a^8*d^2*e^16 + 2*(a^2*c^6*d^17*e + 4*a^3*c^5*d^13*e^5 + 6*a^4*c^4*d^9*e^9 + 4*a^5*c^3*d^5*e^13 + a^6*c^2*d*e^17)*x^9 + (a^2*c^6*d^18 + 4*a^3*c^5*d^14*e^4 + 6*a^4*c^4*d^10*e^8 + 4*a^5*c^3*d^6*e^12 + a^6*c^2*d^2*e^16)*x^8 + 2*(a^3*c^5*d^16*e^2 + 4*a^4*c^4*d^12*e^6 + 6*a^5*c^3*d^8*e^10

```
 + 4*a^6*c^2*d^4*e^14 + a^7*c*e^18)*x^6 + 4*(a^3*c^5*d^17*e + 4*a^4*c^4*d^1
3*e^5 + 6*a^5*c^3*d^9*e^9 + 4*a^6*c^2*d^5*e^13 + a^7*c*d*e^17)*x^5 + 2*(a^3
*c^5*d^18 + 4*a^4*c^4*d^14*e^4 + 6*a^5*c^3*d^10*e^8 + 4*a^6*c^2*d^6*e^12 +
a^7*c*d^2*e^16)*x^4 + (a^4*c^4*d^16*e^2 + 4*a^5*c^3*d^12*e^6 + 6*a^6*c^2*d^
8*e^10 + 4*a^7*c*d^4*e^14 + a^8*e^18)*x^2 + 2*(a^4*c^4*d^17*e + 4*a^5*c^3*d
^13*e^5 + 6*a^6*c^2*d^9*e^9 + 4*a^7*c*d^5*e^13 + a^8*d*e^17)*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.04, size = 2119, normalized size = 0.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/64*(23*sqrt(2)*a*c^4*d^6*e - 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 + 30*sqrt(2)*
sqrt(a*c)*a*c^3*d^4*e^3 - 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^2
*e^5 + 65*(a*c^3)^(3/4)*a*c*d^3*e^4 + 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 + 20*
sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e^
2 + 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e + 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90*
sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 + 148*(a*c^3
)^(3/4)*a^4*c*d^5*e^5 + 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt(
a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 + 10*(a*c^3)^(3/4)*a^5*d*e^9) -
 3/64*(23*sqrt(2)*a*c^4*d^6*e + 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 - 30*sqrt(2)
*sqrt(a*c)*a*c^3*d^4*e^3 + 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^
```

```
2*e^5 - 65*(a*c^3)^(3/4)*a*c*d^3*e^4 - 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 - 20
*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e
^2 - 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e - 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90
*sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 - 148*(a*c^
3)^(3/4)*a^4*c*d^5*e^5 - 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt
(a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 - 10*(a*c^3)^(3/4)*a^5*d*e^9)
+ 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3)^(3/4)*c^3*d^
15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)*(a*c^3)^(3/4)
*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 702*sqrt(2)*(
a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 +
390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(1/4)*a^4*c*d*e
^16)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^20 + 5*a^4*c^5
*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7*c^2*d^4*e^16
+ a^8*c*e^20) - 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3
)^(3/4)*c^3*d^15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)
*(a*c^3)^(3/4)*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 -
 702*sqrt(2)*(a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c
^2*d^5*e^12 + 390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(
1/4)*a^4*c*d*e^16)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^
20 + 5*a^4*c^5*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7
*c^2*d^4*e^16 + a^8*c*e^20) - 3/2*(13*c^2*d^6*e^11 - 3*a*c*d^2*e^15)*log(ab
s(c*x^4 + a))/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^12*e^8 + 10*a^3*c
^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 6*(13*c^2*d^6*e^12 - 3*a*c*d^2
*e^16)*log(abs(x*e + d))/(c^5*d^20*e + 5*a*c^4*d^16*e^5 + 10*a^2*c^3*d^12*e
^9 + 10*a^3*c^2*d^8*e^13 + 5*a^4*c*d^4*e^17 + a^5*e^21) + 1/32*(30*c^5*d^11
*x^9*e^4 + 42*c^5*d^12*x^8*e^3 + c^5*d^13*x^7*e^2 - 4*c^5*d^14*x^6*e + 7*c^
5*d^15*x^5 + 204*a*c^4*d^7*x^9*e^8 + 294*a*c^4*d^8*x^8*e^7 + 19*a*c^4*d^9*x
^7*e^6 - 28*a*c^4*d^10*x^6*e^5 + 97*a*c^4*d^11*x^5*e^4 + 78*a*c^4*d^12*x^4*
e^3 + 5*a*c^4*d^13*x^3*e^2 - 8*a*c^4*d^14*x^2*e + 11*a*c^4*d^15*x - 594*a^2
*c^3*d^3*x^9*e^12 - 546*a^2*c^3*d^4*x^8*e^11 + 35*a^2*c^3*d^5*x^7*e^10 - 44
*a^2*c^3*d^6*x^6*e^9 + 461*a^2*c^3*d^7*x^5*e^8 + 586*a^2*c^3*d^8*x^4*e^7 +
31*a^2*c^3*d^9*x^3*e^6 - 40*a^2*c^3*d^10*x^2*e^5 + 79*a^2*c^3*d^11*x*e^4 +
40*a^2*c^3*d^12*e^3 - 30*a^3*c^2*x^8*e^15 + 17*a^3*c^2*d*x^7*e^14 - 20*a^3*
c^2*d^2*x^6*e^13 - 1165*a^3*c^2*d^3*x^5*e^12 - 1078*a^3*c^2*d^4*x^4*e^11 +
47*a^3*c^2*d^5*x^3*e^10 - 56*a^3*c^2*d^6*x^2*e^9 + 269*a^3*c^2*d^7*x*e^8 +
304*a^3*c^2*d^8*e^7 - 50*a^4*c*x^4*e^15 + 21*a^4*c*d*x^3*e^14 - 24*a^4*c*d^
2*x^2*e^13 - 567*a^4*c*d^3*x*e^12 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15)/((a^2
*c^4*d^16 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6
*e^16)*(c*x^5*e + c*d*x^4 + a*x*e + a*d)^2)
```

Mupad [B]
time = 7.78, size = 2500, normalized size = 1.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^3),x)
```

```
[Out] symsum(log(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^1
2*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^3
6*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16
*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^
5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268
435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13
*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8
*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^3
2*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e
^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 81
28512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*
a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8
*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^
6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462
847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^
5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z
+ 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^
23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(268435
4560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 322122547
20*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^
17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^1
9*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c
^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 +
 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 79
1941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 155544
4924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 8156676096
0*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^
14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z
^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2
+ 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 559382
63040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*
a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^2
4*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548
810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z -
977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d
^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(2684354560*a^12*c^9*d^36*e^4
*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*
z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 +
 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 1
2079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268
435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d
```

^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*((44040192*a^8*c^12*d^31*e^5 - 11010048*a^7*c^13*d^35*e + 994050048*a^9*c^11*d^27*e^9 + 13683916800*a^10*c^10*d^23*e^13 + 42936041472*a^11*c^9*d^19*e^17 + 52628029440*a^12*c^8*d^15*e^21 + 23429382144*a^13*c^7*d^11*e^25 - 2132803584*a^14*c^6*d^7*e^29 - 3125280768*a^15*c^5*d^3*e^33)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6...

3.415 $\int \frac{-1+x}{1-x+x^2} dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 - x + x^2),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{1-x+x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.03

$$-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 - x + x^2),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A]
time = 0.29, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$\frac{\ln\left(4x^2-4x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Fricas [A]
time = 0.40, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A]
time = 0.04, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-x+1),x)
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A]
time = 4.16, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.94

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^2 - x + 1),x)
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.416 $\int \frac{-1+x^2}{1+x^3}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

```
[Out] 1/2*ln(x^2-x+1)+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.385$, Rules used = {1886, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1886

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{1+x^3}\,dx &= \int \frac{-1+x}{1-x+x^2}\,dx\\ &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.03

$$-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^3),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Maple [A]
time = 0.19, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29

risch	$\frac{\ln(4x^2-4x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	31
meijerg	$\frac{\ln(x^3+1)}{3}-\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}-\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 28, normalized size = 0.88

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Fricas [A]
time = 0.40, size = 28, normalized size = 0.88

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A]
time = 0.04, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3+1),x)
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A]
time = 4.27, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Mupad [B]
time = 0.03, size = 30, normalized size = 0.94

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3 + 1),x)
```

```
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.417 $\int \frac{-4+3x}{4-2x+x^2}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(4-2x+x^2\right)$$

```
[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x}{4-2x+x^2}\,dx &= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx\\ &= \frac{3}{2}\log\left(4-2x+x^2\right) + 2\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.97

$$-\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A]
time = 0.35, size = 29, normalized size = 0.91

method	result	size
risch	$\frac{3\ln(x^2-2x+4)}{2} - \frac{\sqrt{3}\ \arctan\left(\frac{(-1+x)\sqrt{3}}{3}\right)}{3}$	27
default	$\frac{3\ln(x^2-2x+4)}{2} - \frac{\sqrt{3}\ \arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4+3*x)/(x^2-2*x+4),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\,(x-1)\right) + \frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Fricas [A]
time = 0.40, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A]
time = 0.03, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x**2-2*x+4),x)
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A]
time = 4.64, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.94

$$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4)/(x^2 - 2*x + 4),x)
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.418 $\int \frac{-8+2x+3x^2}{8+x^3} \, dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(4-2x+x^2\right)$$

[Out] 3/2*ln(x^2-2*x+4)+1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1886, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Antiderivative was successfully verified.

[In] Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]

[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1886

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-8+2x+3x^2}{8+x^3}\,dx &= \frac{1}{2}\int \frac{-8+6x}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\log\left(4-2x+x^2\right) + 2\mathrm{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 0.97

$$-\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Maple [A]
time = 0.20, size = 29, normalized size = 0.91

method	result
risch	$\frac{3\ln(x^2-2x+4)}{2} - \frac{\sqrt{3}\,\arctan\left(\frac{(-1+x)\sqrt{3}}{3}\right)}{3}$

default	$\frac{3\ln(x^2-2x+4)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$
meijerg	$-\frac{2x\ln\left(1+\frac{(x^3)^{\frac{1}{3}}}{2}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x\ln\left(1-\frac{(x^3)^{\frac{1}{3}}}{2}+\frac{(x^3)^{\frac{2}{3}}}{4}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{2x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{4-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}+\ln\left(1+\frac{x^3}{8}\right)-\frac{x^2\ln\left(1+\frac{(x^3)^{\frac{1}{3}}}{2}\right)}{3(x^3)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x-8)/(x^3+8),x,method=_RETURNVERBOSE)
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(x-1)\right)+\frac{3}{2}\log\left(x^2-2\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Fricas [A]
time = 0.42, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(x-1)\right)+\frac{3}{2}\log\left(x^2-2\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A]
time = 0.04, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x-8)/(x**3+8),x)
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A]
time = 4.45, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Mupad [B]
time = 0.03, size = 30, normalized size = 0.94

$$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 - 8)/(x^3 + 8),x)
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

3.419 $\int \frac{2+x}{-1+2x+x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

```
[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {646, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{-1+2x+x^2}\,dx &= -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx\\ &= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)-\left(-2+\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

Maple [A]
time = 0.25, size = 29, normalized size = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1+x-\sqrt{2}\right)}{2}+\frac{\ln\left(1+x-\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+x+\sqrt{2}\right)}{2}-\frac{\ln\left(1+x+\sqrt{2}\right)\sqrt{2}}{4}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+2*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 35, normalized size = 0.78

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

Fricas [A]
time = 0.42, size = 45, normalized size = 1.00

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)+\frac{1}{2}\log\left(x^2+2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

Sympy [A]
time = 0.03, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+2*x-1),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

Giac [A]
time = 3.91, size = 44, normalized size = 0.98

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^{2}+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

Mupad [B]
time = 2.32, size = 34, normalized size = 0.76

$$\ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(2*x + x^2 - 1),x)
```

```
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

3.420 $\int \frac{-4+x^2}{2-5x+x^3} \, dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

```
[Out] 1/4*ln(1+x+2^(1/2))*(2-2^(1/2))+1/4*ln(1+x-2^(1/2))*(2+2^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2080, 646, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2080

```
Int[(u_.)*(P_)*(Q_)^(q_), x_Symbol] :> Module[{gcd = PolyGCD[P, Q, x]}, Int
[u*gcd^(q + 1)*PolynomialQuotient[P, gcd, x]*PolynomialQuotient[Q, gcd, x]^
q, x] /; NeQ[gcd, 1]] /; ILtQ[q, 0] && PolyQ[P, x] && PolyQ[Q, x]
```

Rubi steps

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \int \frac{2+x}{-1+2x+x^2}\,dx$$
$$= -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right) + \frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx$$
$$= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right) + \frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right) - \left(-2+\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

Maple [A]
time = 0.01, size = 29, normalized size = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1+x-\sqrt{2}\right)}{2}+\frac{\ln\left(1+x-\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln\left(1+x+\sqrt{2}\right)}{2}-\frac{\ln\left(1+x+\sqrt{2}\right)\sqrt{2}}{4}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)/(x^3-5*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

Maxima [A]
time = 0.50, size = 35, normalized size = 0.78

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right) + \frac{1}{2}\log\left(x^2+2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

Fricas [A]
time = 0.41, size = 45, normalized size = 1.00

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

Sympy [A]
time = 0.04, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(x**3-5*x+2),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

Giac [A]
time = 5.68, size = 44, normalized size = 0.98

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.76

$$\ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 4)/(x^3 - 5*x + 2),x)
```

```
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

3.421 $\int \frac{2}{-1+4x^2}\,dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(2x)$$

```
[Out] -arctanh(2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 213}

$$-\tanh^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[2/(-1 + 4*x^2),x]
```

```
[Out] -ArcTanh[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2}{-1+4x^2}\,dx &= 2\int \frac{1}{-1+4x^2}\,dx\\ &= -\tanh^{-1}(2x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(6) = 12.
time = 0.00, size = 23, normalized size = 3.83

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(-1 + 4*x^2),x]
```

```
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.20, size = 18, normalized size = 3.00

method	result	size
meijerg	$-\operatorname{arctanh}(2x)$	7
default	$-\frac{\ln(2x+1)}{2}+\frac{\ln(2x-1)}{2}$	18
norman	$-\frac{\ln(2x+1)}{2}+\frac{\ln(2x-1)}{2}$	18
risch	$-\frac{\ln(2x+1)}{2}+\frac{\ln(2x-1)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(2*x+1)+1/2*ln(2*x-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.28, size = 17, normalized size = 2.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.41, size = 17, normalized size = 2.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.03, size = 15, normalized size = 2.50

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x**2-1),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.
time = 3.80, size = 15, normalized size = 2.50

$$-\frac{1}{2}\log\left(\left|x+\frac{1}{2}\right|\right)+\frac{1}{2}\log\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))
```

Mupad [B]
time = 2.27, size = 6, normalized size = 1.00

$$-\mathrm{atanh}(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*x^2 - 1),x)
```

```
[Out] -atanh(2*x)
```

3.422 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

[Out] 1/2*ln(1-2*x)-1/2*ln(1+2*x)

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]

[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2

Rubi steps

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) \, dx=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.10

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]

[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)

Maple [A]
time = 0.20, size = 18, normalized size = 0.86

method	result	size
default	$-\frac{\ln(2x+1)}{2}+\frac{\ln(2x-1)}{2}$	18
norman	$-\frac{\ln(2x+1)}{2}+\frac{\ln(2x-1)}{2}$	18

meijerg	$\frac{\ln(1-2x)}{2}-\frac{\ln(2x+1)}{2}$	18
risch	$-\frac{\ln(2x+1)}{2}+\frac{\ln(2x-1)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-1)-1/(2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(2*x+1)+1/2*ln(2*x-1)
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.81

$$-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Fricas [A]
time = 0.39, size = 17, normalized size = 0.81

$$-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

Giac [A]
time = 3.39, size = 19, normalized size = 0.90

$$-\frac{1}{2}\,\log\left(|2\,x+1|\right)+\frac{1}{2}\,\log\left(|2\,x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))
```

Mupad [B]
time = 0.15, size = 6, normalized size = 0.29

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x - 1) - 1/(2*x + 1),x)
```

```
[Out] -atanh(2*x)
```

3.423 $\int \frac{x}{(1-x^2)^5} \, dx$

Optimal. Leaf size=13

$$\frac{1}{8(1-x^2)^4}$$

```
[Out] 1/8/(-x^2+1)^4
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{1}{8(1-x^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(1-x^2)^5} \, dx = \frac{1}{8(1-x^2)^4}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8(-1+x^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

Maple [A]
time = 0.19, size = 10, normalized size = 0.77

method	result	size
gosper	$\frac{1}{8(x^2-1)^4}$	10
default	$\frac{1}{8(x^2-1)^4}$	10
norman	$\frac{1}{8(x^2-1)^4}$	10
risch	$\frac{1}{8(x^2-1)^4}$	10
derivativedivides	$\frac{1}{8(-x^2+1)^4}$	12
meijerg	$\frac{x^2(-x^6+4x^4-6x^2+4)}{8(-x^2+1)^4}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^5,x,method=_RETURNVERBOSE)
[Out] 1/8/(x^2-1)^4
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="maxima")
[Out] 1/8/(x^2 - 1)^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.
time = 0.38, size = 24, normalized size = 1.85

$$\frac{1}{8(x^8-4x^6+6x^4-4x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="fricas")
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.
time = 0.04, size = 22, normalized size = 1.69

$$\frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**5,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

Giac [A]
time = 3.75, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/8/(x^2 - 1)^4
```

Mupad [B]
time = 2.32, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x^2 - 1)^5,x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

3.424 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\right.$

Optimal. Leaf size=13

$$\frac{1}{8\left(1-x^2\right)^4}$$

[Out] 1/8/(-x^2+1)^4

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

Antiderivative was successfully verified.

```
[In] Int[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-
1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(25
6*(1 + x)^2),x]
```

```
[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x))
+ 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 + x)
)
```

Rubi steps

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+\right.$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8\left(-1+x^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(
256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) -
 5/(256*(1 + x)^2),x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.
time = 0.36, size = 58, normalized size = 4.46

method	result
gosper	$\frac{1}{8(1+x)^{4}(-1+x)^{4}}$
norman	$\frac{1}{8(1+x)^{4}(-1+x)^{4}}$
risch	$\frac{1}{8(1+x)^{4}(-1+x)^{4}}$
default	$\frac{1}{128(-1+x)^{4}}-\frac{1}{64(-1+x)^{3}}+\frac{5}{256(-1+x)^{2}}-\frac{5}{256(-1+x)}+\frac{1}{128(1+x)^{4}}+\frac{1}{64(1+x)^{3}}+\frac{5}{256(1+x)^{2}}+\frac{5}{256(1+x)}$
meijerg	$\frac{x(-x^{3}+4x^{2}-6x+4)}{128(1-x)^{4}}+\frac{x(x^{2}-3x+3)}{64(1-x)^{3}}+\frac{5x(2-x)}{256(1-x)^{2}}+\frac{5x}{256(1-x)}-\frac{x(x^{3}+4x^{2}+6x+4)}{128(1+x)^{4}}-\frac{x(x^{2}+3x+3)}{64(1+x)^{3}}-\frac{5x(x+2)}{256(1+x)^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5
-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/(-1+x)^4-1/64/(-1+x)^3+5/256/(-1+x)^2-5/256/(-1+x)+1/128/(1+x)^4+1/64
/(1+x)^3+5/256/(1+x)^2+5/256/(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(9) = 18$.
time = 0.27, size = 57, normalized size = 4.38

$$\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^{2}}+\frac{5}{256\,(x-1)^{2}}+\frac{1}{64\,(x+1)^{3}}-\frac{1}{64\,(x-1)^{3}}+\frac{1}{128\,(x+1)^{4}}+\frac{1}{128\,(x-1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.
time = 0.38, size = 24, normalized size = 1.85

$$\frac{1}{8\,(x^{8}-4\,x^{6}+6\,x^{4}-4\,x^{2}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.
time = 0.16, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1/
32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(9) = 18$.
time = 3.13, size = 57, normalized size = 4.38

$$\frac{5}{256\,(x+1)} - \frac{5}{256\,(x-1)} + \frac{5}{256\,(x+1)^2} + \frac{5}{256\,(x-1)^2} + \frac{1}{64\,(x+1)^3} - \frac{1}{64\,(x-1)^3} + \frac{1}{128\,(x+1)^4} + \frac{1}{128\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.69

$$\frac{1}{8\,(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5/(256*(x - 1)^2) - 5/(256*(x + 1)^2) - 5/(128*(x - 1)^3) - 5/(128*(x +
 1)^3) + 3/(64*(x - 1)^4) - 3/(64*(x + 1)^4) - 1/(32*(x - 1)^5) - 1/(32*(x
+ 1)^5),x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

3.425 $\int \frac{1+x^6}{-1+x^6}\,dx$

Optimal. Leaf size=69

$$x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {396, 216, 648, 632, 210, 642, 212}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^6}{-1+x^6}\,dx &= x+2\int \frac{1}{-1+x^6}\,dx\\
&= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1
\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1
\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(6x-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

Maple [A]
time = 0.21, size = 67, normalized size = 0.97

method	result
risch	$x-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}+\frac{\ln(-1-}{3}$
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(-1+x)}{3}-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\right.\right.}{6(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^6-1),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/3*ln(-1+x)-1/3*
ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.47, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Fricas [A]
time = 0.37, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Sympy [A]
time = 0.12, size = 85, normalized size = 1.23

$$x+\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^{2}-x+1\right)}{6}-\frac{\log\left(x^{2}+x+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**6-1),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

Giac [A]
time = 3.89, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+x-\frac{1}{6}\log\left(x^{2}+x+1\right)+\frac{1}{6}\log\left(x^{2}-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

Mupad [B]
time = 0.10, size = 94, normalized size = 1.36

$$x+\frac{\operatorname{atan}(x\,1\mathrm{i})\,2\mathrm{i}}{3}-\operatorname{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x^6 - 1),x)
```

```
[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

3.426 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

Optimal. Leaf size=69

$$x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] x-2/3*arctanh(x)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1607, 1598, 396, 216, 648, 632, 210, 642, 212}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
```

```
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx = \int \frac{x^3\left(\frac{1}{x^3}+x^3\right)}{-1+x^6}\,dx$$
$$= \int \frac{1+x^6}{-1+x^6}\,dx$$
$$= x+2\int \frac{1}{-1+x^6}\,dx$$
$$= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx$$
$$= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int$$
$$= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-\right.$$
$$= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x\right.$$

Mathematica [A]
time = 0.00, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(6x-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

Maple [A]
time = 0.03, size = 67, normalized size = 0.97

method	result
risch	$x-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}+\frac{\ln(-1-}{3}$
default	$x-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(-1+x)}{3}-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^3+x^3)/(-1/x^3+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/3*ln(-1+x)-1/3*
ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Fricas [A]
time = 0.39, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

Sympy [A]
time = 0.16, size = 85, normalized size = 1.23

$$x+\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

Giac [A]
time = 4.42, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 94, normalized size = 1.36

$$x+\frac{\mathrm{atan}(x\,1\mathrm{i})\,2\mathrm{i}}{3}-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\,\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\,\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/x^3 + x^3)/(1/x^3 - x^3),x)
```

```
[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

3.427 $\int \frac{-x+x^3}{6+2x}\,dx$

Optimal. Leaf size=24

$$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\log(3+x)$$

```
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1607, 786}

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/(6 + 2*x),x]
```

```
[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x+x^3}{6+2x}\,dx &= \int \frac{x\left(-1+x^2\right)}{6+2x}\,dx\\ &= \int \left(4-\frac{3x}{2}+\frac{x^2}{2}-\frac{12}{3+x}\right)dx\\ &= 4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.29

$$\frac{1}{2}\left(\frac{93}{2}+8x-\frac{3x^2}{2}+\frac{x^3}{3}-24\log(3+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/(6 + 2*x),x]
[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

Maple [A]
time = 0.19, size = 21, normalized size = 0.88

method	result	size
default	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
risch	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(3+x\right)$	21
norman	$4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\ln\left(6+2x\right)$	23
meijerg	$\frac{3x\left(\frac{4}{9}x^2-2x+12\right)}{8}-12\ln\left(1+\frac{x}{3}\right)-\frac{x}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(6+2*x),x,method=_RETURNVERBOSE)
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.83

$$\frac{1}{6}\,x^3-\frac{3}{4}\,x^2+4\,x-12\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="maxima")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Fricas [A]
time = 0.38, size = 20, normalized size = 0.83

$$\frac{1}{6}\,x^3-\frac{3}{4}\,x^2+4\,x-12\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="fricas")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Sympy [A]
time = 0.02, size = 20, normalized size = 0.83

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(6+2*x),x)
```

```
[Out] x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

Giac [A]
time = 4.25, size = 21, normalized size = 0.88

$$\frac{1}{6}\,x^{3} - \frac{3}{4}\,x^{2} + 4\,x - 12\,\log\left(|x + 3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.83

$$4\,x - 12\,\ln\left(x + 3\right) - \frac{3\,x^2}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/(2*x + 6),x)
```

```
[Out] 4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6
```

3.428 $\int \frac{x+x^3}{-1+x}\,dx$

Optimal. Leaf size=26

$$2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)$$

```
[Out] 2*x+1/2*x^2+1/3*x^3+2*ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 786}

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x+x^3}{-1+x}\,dx &= \int \frac{x(1+x^2)}{-1+x}\,dx\\ &= \int \left(2+\frac{2}{-1+x}+x+x^2\right)dx\\ &= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(-17+12x+3x^2+2x^3+12\log(-1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)/(-1 + x),x]
```

```
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Maple [A]
time = 0.20, size = 21, normalized size = 0.81

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(-1+x\right)$	21
norman	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(-1+x\right)$	21
risch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln\left(-1+x\right)$	21
meijerg	$\frac{x\left(4x^2+6x+12\right)}{12}+2\ln\left(1-x\right)+x$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)/(-1+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(-1+x)
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.77

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Fricas [A]
time = 0.38, size = 20, normalized size = 0.77

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.73

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

Giac [A]
time = 3.75, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.77

$$2x + 2\ln(x-1) + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3)/(x - 1),x)
```

```
[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

3.429 $\int (ac+(bc+d)x)\,dx$

Optimal. Leaf size=17

$$acx+\frac{1}{2}(bc+d)x^2$$

[Out] a*c*x+1/2*(b*c+d)*x^2

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$acx+\frac{1}{2}x^2(bc+d)$$

Antiderivative was successfully verified.

[In] Int[a*c + (b*c + d)*x,x]

[Out] a*c*x + ((b*c + d)*x^2)/2

Rubi steps

$$\int (ac+(bc+d)x)\,dx=acx+\frac{1}{2}(bc+d)x^2$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.29

$$acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[a*c + (b*c + d)*x,x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2

Maple [A]
time = 0.01, size = 19, normalized size = 1.12

method	result	size
gosper	$\frac{x(bcx+2ac+dx)}{2}$	16
norman	$\left(\frac{bc}{2}+\frac{d}{2}\right)x^2+acx$	18

default	$\frac{1}{2}bc\,x^2 + acx + \frac{1}{2}d\,x^2$	19
risch	$\frac{1}{2}bc\,x^2 + acx + \frac{1}{2}d\,x^2$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c+(b*c+d)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*c*x^2+a*c*x+1/2*d*x^2
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}\,(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="maxima")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

Fricas [A]
time = 0.35, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.88

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

Giac [A]
time = 4.37, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}\,(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

Mupad [B]
time = 0.02, size = 17, normalized size = 1.00

$$\left(\frac{d}{2}+\frac{b\,c}{2}\right)\,x^2+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c + x*(d + b*c),x)
```

```
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

3.430 $\int (dx + c(a + bx))\,dx$

Optimal. Leaf size=24

$$\frac{dx^2}{2} + \frac{c(a+bx)^2}{2b}$$

```
[Out] 1/2*d*x^2+1/2*c*(b*x+a)^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{c(a+bx)^2}{2b} + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d*x + c*(a + b*x),x]
[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi steps

$$\int (dx + c(a+bx))\,dx = \frac{dx^2}{2} + \frac{c(a+bx)^2}{2b}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.92

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d*x + c*(a + b*x),x]
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

Maple [A]
time = 0.01, size = 19, normalized size = 0.79

method	result	size
gosper	$\frac{x(bcx+2ac+dx)}{2}$	16
norman	$\left(\frac{bc}{2}+\frac{d}{2}\right)x^2+acx$	18

default	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19
risch	$\frac{1}{2}bc\,x^2+acx+\frac{1}{2}d\,x^2$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*c*x^2+a*c*x+1/2*d*x^2
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.83

$$\frac{1}{2}\,dx^2+\frac{1}{2}\,\left(bx^2+2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

Fricas [A]
time = 0.34, size = 18, normalized size = 0.75

$$\frac{1}{2}x^2cb+\frac{1}{2}x^2d+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.62

$$acx+x^2\left(\frac{bc}{2}+\frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

Giac [A]
time = 4.61, size = 20, normalized size = 0.83

$$\frac{1}{2}\,dx^2+\frac{1}{2}\,\left(bx^2+2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

Mupad [B]
time = 0.02, size = 17, normalized size = 0.71

$$\left(\frac{d}{2}+\frac{b\,c}{2}\right)\,x^2+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x + c*(a + b*x),x)
```

```
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

3.431 $\int \frac{4+4x}{x^2(1+x^2)} \, dx$

Optimal. Leaf size=22

$$-\frac{4}{x} - 4\tan^{-1}(x) + 4\log(x) - 2\log\left(1+x^2\right)$$

```
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {815, 649, 209, 266}

$$-4\text{ArcTan}(x) - 2\log\left(x^2+1\right) - \frac{4}{x} + 4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx &= \int \left(\frac{4}{x^2}+\frac{4}{x}-\frac{4(1+x)}{1+x^2}\right)dx\\ &= -\frac{4}{x}+4\log(x)-4\int \frac{1+x}{1+x^2}\,dx\\ &= -\frac{4}{x}+4\log(x)-4\int \frac{1}{1+x^2}\,dx-4\int \frac{x}{1+x^2}\,dx\\ &= -\frac{4}{x}-4\tan^{-1}(x)+4\log(x)-2\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.09

$$4\left(-\frac{1}{x}-\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]
[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2]/2)
```

Maple [A]
time = 0.20, size = 23, normalized size = 1.05

method	result	size
default	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^2+1\right)$	23
meijerg	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^2+1\right)$	23
risch	$-\frac{4}{x}-4\arctan\left(x\right)+4\ln\left(x\right)-2\ln\left(x^2+1\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+4*x)/x^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

Maxima [A]
time = 0.50, size = 22, normalized size = 1.00

$$-\frac{4}{x}-4\arctan\left(x\right)-2\log\left(x^2+1\right)+4\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)
```

Fricas [A]
time = 0.40, size = 25, normalized size = 1.14

$$-\frac{2\left(2\,x\arctan\left(x\right)+x\log\left(x^{2}+1\right)-2\,x\log\left(x\right)+2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.91

$$4\log\left(x\right)-2\log\left(x^{2}+1\right)-4\operatorname{atan}\left(x\right)-\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x**2/(x**2+1),x)
[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

Giac [A]
time = 3.42, size = 23, normalized size = 1.05

$$-\frac{4}{x}-4\,\arctan\left(x\right)-2\,\log\left(x^{2}+1\right)+4\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))
```

Mupad [B]
time = 0.04, size = 28, normalized size = 1.27

$$4\,\ln\left(x\right)-\frac{4}{x}+\ln\left(x-\mathrm{i}\right)\,\left(-2+2\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-2-2\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4)/(x^2*(x^2 + 1)),x)
[Out] 4*log(x) - log(x + 1i)*(2 + 2i) - log(x - 1i)*(2 - 2i) - 4/x
```

3.432 $\int \frac{24+8x}{x(-4+x^2)}\,dx$

Optimal. Leaf size=17

$$5\log(2-x)-6\log(x)+\log(2+x)$$

[Out] 5*ln(2-x)-6*ln(x)+ln(2+x)

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{815\}$

$$5\log(2-x)-6\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(24 + 8*x)/(x*(-4 + x^2)),x]

[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{24+8x}{x\,(-4+x^2)}\,dx &= \int \left(\frac{5}{-2+x}-\frac{6}{x}+\frac{1}{2+x}\right)dx\\ &= 5\log(2-x)-6\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.59

$$8\left(\frac{5}{8}\log(2-x)-\frac{3\log(x)}{4}+\frac{1}{8}\log(2+x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]

[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)

Maple [A]
time = 0.21, size = 16, normalized size = 0.94

method	result	size
default	$\ln(x+2)+5\ln(x-2)-6\ln(x)$	16
norman	$\ln(x+2)+5\ln(x-2)-6\ln(x)$	16
risch	$\ln(x+2)+5\ln(x-2)-6\ln(x)$	16
meijerg	$3\ln\left(1-\frac{x^2}{4}\right)-6\ln(x)+6\ln(2)-3i\pi-4\,\mathrm{arctanh}\left(\frac{x}{2}\right)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24+8*x)/x/(x^2-4),x,method=_RETURNVERBOSE)
[Out] ln(x+2)+5*ln(x-2)-6*ln(x)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.88

$$\log(x+2)+5\,\log(x-2)-6\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Fricas [A]
time = 0.37, size = 15, normalized size = 0.88

$$\log(x+2)+5\,\log(x-2)-6\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="fricas")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.88

$$-6\log(x)+5\log(x-2)+\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x**2-4),x)
[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)
```

Giac [A]
time = 3.72, size = 18, normalized size = 1.06

$$\log\left(|x+2|\right) + 5\,\log\left(|x-2|\right) - 6\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))
```

Mupad [B]
time = 0.06, size = 15, normalized size = 0.88

$$5\,\ln\left(x-2\right) + \ln\left(x+2\right) - 6\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 24)/(x*(x^2 - 4)),x)
[Out] 5*log(x - 2) + log(x + 2) - 6*log(x)
```

3.433 $\int \frac{-1+x^2}{-2x+x^3}\,dx$

Optimal. Leaf size=19

$$\frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)$$

[Out] 1/2*ln(x)+1/4*ln(-x^2+2)

Rubi [A]

time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 457, 78}

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(-2*x + x^3),x]

[Out] Log[x]/2 + Log[2 - x^2]/4

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{-2x+x^3}\,dx &= \int \frac{-1+x^2}{x\left(-2+x^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{(-2+x)x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{2(-2+x)}+\frac{1}{2x}\right)\,dx, x, x^2\right)\\ &= \frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Maple [**A**]
time = 0.21, size = 14, normalized size = 0.74

method	result	size
default	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
norman	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
risch	$\frac{\ln(x)}{2}+\frac{\ln(x^2-2)}{4}$	14
meijerg	$\frac{\ln\left(1-\frac{x^2}{2}\right)}{4}+\frac{\ln(x)}{2}-\frac{\ln(2)}{4}+\frac{i\pi}{4}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3-2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)+1/4*ln(x^2-2)
```

Maxima [**A**]
time = 0.28, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Fricas [A]
time = 0.38, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.63

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3-2*x),x)
```

```
[Out] log(x)/2 + log(x**2 - 2)/4
```

Giac [A]
time = 3.63, size = 16, normalized size = 0.84

$$\frac{1}{4}\log\left(x^2\right)+\frac{1}{4}\log\left(\left|x^2-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2) + 1/4*log(abs(x^2 - 2))
```

Mupad [B]
time = 2.34, size = 13, normalized size = 0.68

$$\frac{\ln\left(x^2-2\right)}{4}+\frac{\ln\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(2*x - x^3),x)
```

```
[Out] log(x^2 - 2)/4 + log(x)/2
```

3.434 $\int \frac{1+x^2}{3x+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{3}\log\left(3x+x^3\right)$$

```
[Out] 1/3*ln(x^3+3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1601}

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[3*x + x^3]/3
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\log\left(3x+x^3\right)$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.42

$$\frac{\log(x)}{3}+\frac{1}{3}\log\left(3+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[x]/3 + Log[3 + x^2]/3
```

Maple [A]
time = 0.20, size = 11, normalized size = 0.92

method	result	size
default	$\frac{\ln\left(x\left(x^2+3\right)\right)}{3}$	11
risch	$\frac{\ln\left(x^3+3x\right)}{3}$	11
norman	$\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x^2+3\right)}{3}$	14
meijerg	$\frac{\ln\left(1+\frac{x^2}{3}\right)}{3}+\frac{\ln\left(x\right)}{3}-\frac{\ln\left(3\right)}{6}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x*(x^2+3))
```

Maxima [A]
time = 0.26, size = 10, normalized size = 0.83

$$\frac{1}{3}\log\left(x^3+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x)
```

Fricas [A]
time = 0.38, size = 10, normalized size = 0.83

$$\frac{1}{3}\log\left(x^3+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.67

$$\frac{\log\left(x^3+3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+3*x),x)
```

```
[Out] log(x**3 + 3*x)/3
```

Giac [A]
time = 3.21, size = 13, normalized size = 1.08

$$\frac{1}{3}\log\left(3\left|\frac{1}{3}x^3+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(3*abs(1/3*x^3 + x))
```

Mupad [B]
time = 2.27, size = 10, normalized size = 0.83

$$\frac{\ln\left(x^3+3\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(3*x + x^3),x)
```

```
[Out] log(3*x + x^3)/3
```

3.435 $\int \frac{a+3bx^2}{ax+bx^3}\,dx$

Optimal. Leaf size=10

$$\log\left(ax+bx^3\right)$$

[Out] ln(b*x^3+a*x)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1601}

$$\log\left(ax+bx^3\right)$$

Antiderivative was successfully verified.

[In] Int[(a + 3*b*x^2)/(a*x + b*x^3),x]

[Out] Log[a*x + b*x^3]

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\log\left(ax+bx^3\right)$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.10

$$\log(x)+\log\left(a+bx^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]

[Out] Log[x] + Log[a + b*x^2]

Maple [A]
time = 0.20, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\ln\left(b\,x^{3}+ax\right)$	11
default	$\ln\left(x(b\,x^{2}+a)\right)$	11
risch	$\ln\left(b\,x^{3}+ax\right)$	11
norman	$\ln\left(x\right)+\ln\left(b\,x^{2}+a\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b*x^2+a)/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] ln(x*(b*x^2+a))
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$\log\left(bx^3+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="maxima")
[Out] log(b*x^3 + a*x)
```

Fricas [A]
time = 0.39, size = 10, normalized size = 1.00

$$\log\left(bx^3+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="fricas")
[Out] log(b*x^3 + a*x)
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.80

$$\log\left(ax+bx^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x**2+a)/(b*x**3+a*x),x)
[Out] log(a*x + b*x**3)
```

Giac [A]
time = 3.52, size = 11, normalized size = 1.10

$$\log\left(\left|bx^3+ax\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] log(abs(b*x^3 + a*x))
```

Mupad [B]
time = 0.06, size = 10, normalized size = 1.00

$$\ln\left(b\,x^3+a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 3*b*x^2)/(a*x + b*x^3),x)
```

```
[Out] log(a*x + b*x^3)
```

3.436 $\int \frac{-2+4x}{-x+x^3}\,dx$

Optimal. Leaf size=17

$$\log(1-x)+2\log(x)-3\log(1+x)$$

```
[Out] ln(1-x)+2*ln(x)-3*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1607, 815}

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+4x}{-x+x^3}\,dx &= \int \frac{-2+4x}{x\left(-1+x^2\right)}\,dx \\
&= \int \left(\frac{1}{-1+x}+\frac{2}{x}-\frac{3}{1+x}\right)dx \\
&= \log(1-x)+2\log(x)-3\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\log(1-x)+2\log(x)-3\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Maple [A]
time = 0.21, size = 16, normalized size = 0.94

method	result	size
default	$\ln(-1+x)+2\ln(x)-3\ln(1+x)$	16
norman	$\ln(-1+x)+2\ln(x)-3\ln(1+x)$	16
risch	$\ln(-1+x)+2\ln(x)-3\ln(1+x)$	16
meijerg	$-\ln(-x^2+1)+2\ln(x)+i\pi-4\,\mathrm{arctanh}(x)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+4*x)/(x^3-x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+x)+2*ln(x)-3*ln(1+x)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.88

$$-3\log(x+1)+\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")
```

```
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Fricas [A]
time = 0.41, size = 15, normalized size = 0.88

$$-3\log(x+1)+\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="fricas")
```

```
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.88

$$2\log(x)+\log(x-1)-3\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x**3-x),x)
```

```
[Out] 2*log(x) + log(x - 1) - 3*log(x + 1)
```

Giac [A]
time = 3.73, size = 18, normalized size = 1.06

$$-3 \log(|x+1|) + \log(|x-1|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="giac")
```

```
[Out] -3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))
```

Mupad [B]
time = 0.06, size = 15, normalized size = 0.88

$$\ln(x-1) - 3 \ln(x+1) + 2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 2)/(x - x^3),x)
```

```
[Out] log(x - 1) - 3*log(x + 1) + 2*log(x)
```

3.437 $\int \frac{4+x}{4x+x^3} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)$$

[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1607, 815, 649, 209, 266}

$$\frac{1}{2}\text{ArcTan}\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(4 + x)/(4*x + x^3),x]

[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x}{4x+x^3}\,dx &= \int \frac{4+x}{x(4+x^2)}\,dx\\ &= \int \left(\frac{1}{x}+\frac{1-x}{4+x^2}\right)dx\\ &= \log(x)+\int \frac{1-x}{4+x^2}\,dx\\ &= \log(x)+\int \frac{1}{4+x^2}\,dx-\int \frac{x}{4+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(4*x + x^3),x]
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Maple [A]
time = 0.21, size = 18, normalized size = 0.78

method	result	size
default	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^2+4\right)}{2}$	18
risch	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^2+4\right)}{2}$	18
meijerg	$-\frac{\ln\left(\frac{x^2}{4}+1\right)}{2}+\ln\left(x\right)-\ln\left(2\right)+\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+4)/(x^3+4*x),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.74

$$\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{2}\,\log\left(x^2+4\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="maxima")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Fricas [A]
time = 0.41, size = 17, normalized size = 0.74

$$\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{2}\,\log\left(x^2+4\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="fricas")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.74

$$\log\left(x\right) - \frac{\log\left(x^2+4\right)}{2} + \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x**3+4*x),x)
[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

Giac [A]
time = 3.53, size = 18, normalized size = 0.78

$$\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right) - \frac{1}{2}\,\log\left(x^2+4\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="giac")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))
```

Mupad [B]
time = 0.05, size = 21, normalized size = 0.91

$$\ln\left(x\right) + \ln\left(x-2\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+2\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/(4*x + x^3),x)
[Out] log(x) - log(x + 2i)*(1/2 - 1i/4) - log(x - 2i)*(1/2 + 1i/4)
```

3.438 $\int \frac{-x+2x^3}{1-x^2+x^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(1-x^2+x^4\right)$$

```
[Out] 1/2*ln(x^4-x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{1601\}$

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\ln(x^4-x^2+1)}{2}$	14
norman	$\frac{\ln(x^4-x^2+1)}{2}$	14
risch	$\frac{\ln(x^4-x^2+1)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-x)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^4-x^2+1)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Fricas [A]
time = 0.39, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
[Out] log(x**4 - x**2 + 1)/2
```

Giac [A]
time = 3.63, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^4 - x^2 + 1)
```

Mupad [B]
time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
[Out] log(x^4 - x^2 + 1)/2
```

3.439 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1608, 814}

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+x}{2x+3x^2+x^3}\,dx &= \int \frac{-3+x}{x\left(2+3x+x^2\right)}\,dx\\
&= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)dx\\
&= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.86

method	result	size
default	$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(x+2)}{2}$	18
norman	$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(x+2)}{2}$	18
risch	$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(x+2)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)/(x^3+3*x^2+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(x+2)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.81

$$-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
```

```
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Fricas [A]
time = 0.38, size = 17, normalized size = 0.81

$$-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
```

```
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.95

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
```

```
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Giac [A]
time = 3.82, size = 20, normalized size = 0.95

$$-\frac{5}{2}\log(|x+2|)+4\log(|x+1|)-\frac{3}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
```

```
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

Mupad [B]
time = 0.07, size = 17, normalized size = 0.81

$$4\ln(x+1)-\frac{5\ln(x+2)}{2}-\frac{3\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)
```

```
[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

3.440 $\int \frac{2+4x}{x^2+2x^3+x^4} \, dx$

Optimal. Leaf size=10

$$-\frac{2}{x(1+x)}$$

[Out] -2/x/(1+x)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1608, 27, 75}

$$-\frac{2}{x(x+1)}$$

Antiderivative was successfully verified.

[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]

[Out] -2/(x*(1 + x))

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx = \int \frac{2+4x}{x^2\left(1+2x+x^2\right)}\,dx$$
$$= \int \frac{2+4x}{x^2(1+x)^2}\,dx$$
$$= -\frac{2}{x(1+x)}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.90

$$-\frac{2}{x+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

```
[Out] -2/(x + x^2)
```

Maple [A]
time = 0.01, size = 14, normalized size = 1.40

method	result	size
gosper	$-\frac{2}{x(1+x)}$	11
norman	$-\frac{2}{x(1+x)}$	11
risch	$-\frac{2}{x(1+x)}$	11
default	$\frac{2}{1+x}-\frac{2}{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+4*x)/(x^4+2*x^3+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(1+x)-2/x
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")
```

```
[Out] -2/(x^2 + x)
```

Fricas [A]
time = 0.39, size = 9, normalized size = 0.90

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="fricas")
[Out] -2/(x^2 + x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.70

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x**4+2*x**3+x**2),x)
[Out] -2/(x**2 + x)
```

Giac [A]
time = 3.57, size = 9, normalized size = 0.90

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")
[Out] -2/(x^2 + x)
```

Mupad [B]
time = 2.20, size = 10, normalized size = 1.00

$$-\frac{2}{x\,\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)/(x^2 + 2*x^3 + x^4),x)
[Out] -2/(x*(x + 1))
```

3.441 $\int \frac{1+x}{-6x+x^2+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(3+x)$$

```
[Out] 3/10*ln(2-x)-1/6*ln(x)-2/15*ln(3+x)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1608, 814}

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-6*x + x^2 + x^3),x]
```

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{-6x+x^2+x^3}\,dx &= \int \frac{1+x}{x\,(-6+x+x^2)}\,dx\\ &= \int \left(\frac{3}{10(-2+x)}-\frac{1}{6x}-\frac{2}{15(3+x)}\right)dx\\ &= \frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-6*x + x^2 + x^3),x]
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.72

method	result	size
default	$\frac{3\ln(x-2)}{10}-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}$	18
norman	$\frac{3\ln(x-2)}{10}-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}$	18
risch	$\frac{3\ln(x-2)}{10}-\frac{\ln(x)}{6}-\frac{2\ln(3+x)}{15}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^3+x^2-6*x),x,method=_RETURNVERBOSE)
[Out] 3/10*ln(x-2)-1/6*ln(x)-2/15*ln(3+x)
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.68

$$-\frac{2}{15}\log(x+3)+\frac{3}{10}\log(x-2)-\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Fricas [A]
time = 0.42, size = 17, normalized size = 0.68

$$-\frac{2}{15}\log(x+3)+\frac{3}{10}\log(x-2)-\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="fricas")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.80

$$-\frac{\log(x)}{6}+\frac{3\log(x-2)}{10}-\frac{2\log(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3+x**2-6*x),x)
[Out] -log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15
```

Giac [A]
time = 3.60, size = 20, normalized size = 0.80

$$-\frac{2}{15}\log(|x+3|)+\frac{3}{10}\log(|x-2|)-\frac{1}{6}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="giac")
[Out] -2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))
```

Mupad [B]
time = 0.11, size = 17, normalized size = 0.68

$$\frac{3\ln(x-2)}{10}-\frac{2\ln(x+3)}{15}-\frac{\ln(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^2 - 6*x + x^3),x)
[Out] (3*log(x - 2))/10 - (2*log(x + 3))/15 - log(x)/6
```

3.442 $\int \frac{4x^2+x^3}{x+x^3}\,dx$

Optimal. Leaf size=14

$$x-\tan^{-1}(x)+2\log\left(1+x^2\right)$$

[Out] x-arctan(x)+2*ln(x^2+1)

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.353$, Rules used = {1607, 1598, 788, 649, 209, 266}

$$-\text{ArcTan}(x)+2\log\left(x^2+1\right)+x$$

Antiderivative was successfully verified.

[In] Int[(4*x^2 + x^3)/(x + x^3),x]

[Out] x - ArcTan[x] + 2*Log[1 + x^2]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{4x^2+x^3}{x+x^3}\,dx &= \int \frac{4x^2+x^3}{x\,(1+x^2)}\,dx\\ &= \int \frac{x(4+x)}{1+x^2}\,dx\\ &= x+\int \frac{-1+4x}{1+x^2}\,dx\\ &= x+4\int \frac{x}{1+x^2}\,dx-\int \frac{1}{1+x^2}\,dx\\ &= x-\tan^{-1}(x)+2\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$x-\tan^{-1}(x)+2\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + x^3)/(x + x^3),x]
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Maple [A]
time = 0.21, size = 15, normalized size = 1.07

method	result	size
default	$x-\arctan\left(x\right)+2\ln\left(x^{2}+1\right)$	15
meijerg	$x-\arctan\left(x\right)+2\ln\left(x^{2}+1\right)$	15
risch	$x-\arctan\left(x\right)+2\ln\left(x^{2}+1\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4*x^2)/(x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] x-arctan(x)+2*ln(x^2+1)
```

Maxima [A]
time = 0.49, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Fricas [A]
time = 0.39, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.86

$$x + 2\log(x^2 + 1) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*x**2)/(x**3+x),x)
[Out] x + 2*log(x**2 + 1) - atan(x)
```

Giac [A]
time = 3.66, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

Mupad [B]
time = 2.22, size = 14, normalized size = 1.00

$$x + 2 \ln(x^2 + 1) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + x^3)/(x + x^3),x)
[Out] x + 2*log(x^2 + 1) - atan(x)
```

3.443 $\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{4\left(x^2+x^4\right)^2}$$

[Out] -1/4/(x^4+x^2)^2

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1602}

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] -1/4*1/(x^2 + x^4)^2

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+2x^3}{\left(x^2+x^4\right)^3}\,dx = -\frac{1}{4\left(x^2+x^4\right)^2}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4\left(1+x^2\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]

```
[Out] -1/4*1/(x^4*(1 + x^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.
time = 0.22, size = 30, normalized size = 2.31

method	result	size
gosper	$-\frac{1}{4x^4(x^2+1)^2}$	13
norman	$-\frac{1}{4x^4(x^2+1)^2}$	13
risch	$-\frac{1}{4x^4(x^2+1)^2}$	13
default	$-\frac{1}{2(x^2+1)} - \frac{1}{4(x^2+1)^2} - \frac{1}{4x^4} + \frac{1}{2x^2}$	30
meijerg	$\frac{x^2(5x^2+6)}{2(x^2+1)^2} - \frac{3}{4} + \frac{1}{2x^2} - \frac{x^2(7x^2+8)}{4(x^2+1)^2} - \frac{1}{4x^4}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+x)/(x^4+x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/(x^2+1)-1/4/(x^2+1)^2-1/4/x^4+1/2/x^2
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.85

$$-\frac{1}{4\,(x^4+x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")
[Out] -1/4/(x^4 + x^2)^2
```

Fricas [A]
time = 0.39, size = 16, normalized size = 1.23

$$-\frac{1}{4\,(x^8+2\,x^6+x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 1.31

$$-\frac{1}{4x^8+8x^6+4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+x)/(x**4+x**2)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

Giac [A]
time = 3.90, size = 11, normalized size = 0.85

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

Mupad [B]
time = 2.24, size = 20, normalized size = 1.54

$$-\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*x^3)/(x^2 + x^4)^3,x)
```

```
[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)
```

3.444 $\int \frac{ax^2+bx^3}{cx^2+dx^3} dx$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

[Out] b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1607, 1598, 45}

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]

[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx = \int \frac{x^2(a+bx)}{cx^2+dx^3}\,dx$$
$$= \int \frac{a+bx}{c+dx}\,dx$$
$$= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx$$
$$= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.96

$$\frac{bx}{d}+\frac{(-bc+ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Maple [A]
time = 0.20, size = 26, normalized size = 1.00

method	result	size
default	$\frac{xb}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
norman	$\frac{xb}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
risch	$\frac{xb}{d}+\frac{\ln(dx+c)a}{d}-\frac{\ln(dx+c)bc}{d^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/(d*x^3+c*x^2),x,method=_RETURNVERBOSE)
[Out] 1/d*x*b+(a*d-b*c)/d^2*ln(d*x+c)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 1.00

$$\frac{bx}{d}-\frac{(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Fricas [A]
time = 0.37, size = 25, normalized size = 0.96

$$\frac{bdx - (bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")
```

```
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Giac [A]
time = 3.62, size = 27, normalized size = 1.04

$$\frac{bx}{d} - \frac{(bc - ad)\log(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

Mupad [B]
time = 2.23, size = 25, normalized size = 0.96

$$\frac{\ln(c + d\,x)\,\left(a\,d - b\,c\right)}{d^2} + \frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)
```

```
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

3.445 $\int \frac{x+x^2}{-2x-x^2+x^3} \, dx$

Optimal. Leaf size=6

$$\log(2-x)$$

```
[Out] ln(2-x)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1600, 31}

$$\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[2 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x+x^2}{-2x-x^2+x^3} \, dx &= \int \frac{1}{-2+x} \, dx \\ &= \log(2-x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 0.67

$$\log(-2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
```

```
[Out] Log[-2 + x]
```

Maple [A]
time = 0.01, size = 5, normalized size = 0.83

method	result	size
default	$\ln(x-2)$	5
norman	$\ln(x-2)$	5
risch	$\ln(x-2)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(x^3-x^2-2*x),x,method=_RETURNVERBOSE)
[Out] ln(x-2)
```

Maxima [A]
time = 0.26, size = 4, normalized size = 0.67

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")
[Out] log(x - 2)
```

Fricas [A]
time = 0.38, size = 4, normalized size = 0.67

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="fricas")
[Out] log(x - 2)
```

Sympy [A]
time = 0.01, size = 3, normalized size = 0.50

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(x**3-x**2-2*x),x)
[Out] log(x - 2)
```

Giac [A]
time = 4.44, size = 5, normalized size = 0.83

$$\log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")
```

```
[Out] log(abs(x - 2))
```

Mupad [B]
time = 0.02, size = 4, normalized size = 0.67

$$\ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2)/(2*x + x^2 - x^3),x)
```

```
[Out] log(x - 2)
```

3.446 $\int \frac{1-5x^2}{x^3(1+x^2)}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)$$

```
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 78}

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
```

```
[Out] -1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1-5x}{x^2(1+x)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{6}{x}+\frac{6}{1+x}\right)\,dx,x,x^2\right)\\ &= -\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
[Out] -1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]
```

Maple [A]
time = 0.21, size = 19, normalized size = 0.95

method	result	size
default	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19
norman	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19
meijerg	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19
risch	$-\frac{1}{2x^2}-6\ln\left(x\right)+3\ln\left(x^2+1\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5*x^2+1)/x^3/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 1.00

$$-\frac{1}{2\,x^2}+3\,\log\left(x^2+1\right)-3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")
[Out] -1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

Fricas [A]
time = 0.38, size = 25, normalized size = 1.25

$$\frac{6\,x^2\log\left(x^2+1\right)-12\,x^2\log\left(x\right)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.95

$$-6\log\left(x\right)+3\log\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x**2+1)/x**3/(x**2+1),x)
[Out] -6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)
```

Giac [A]
time = 4.07, size = 27, normalized size = 1.35

$$\frac{6\,x^2-1}{2\,x^2}+3\,\log\left(x^2+1\right)-3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="giac")
[Out] 1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.90

$$3\,\ln\left(x^2+1\right)-6\,\ln\left(x\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x^2 - 1)/(x^3*(x^2 + 1)),x)
[Out] 3*log(x^2 + 1) - 6*log(x) - 1/(2*x^2)
```

3.447 $\int \frac{2x}{(-1+x)(5+x^2)} dx$

Optimal. Leaf size=38

$$\frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(5+x^2\right)$$

```
[Out] 1/3*ln(1-x)-1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 815, 649, 209, 266}

$$\frac{1}{3}\sqrt{5}\,\text{ArcTan}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x)/((-1 + x)*(5 + x^2)),x]
```

```
[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x}{(-1+x)(5+x^2)}\,dx &= 2\int \frac{x}{(-1+x)(5+x^2)}\,dx\\
&= 2\int \left(\frac{1}{6(-1+x)}+\frac{5-x}{6(5+x^2)}\right)dx\\
&= \frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{5-x}{5+x^2}\,dx\\
&= \frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{x}{5+x^2}\,dx+\frac{5}{3}\int \frac{1}{5+x^2}\,dx\\
&= \frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(5+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.05

$$2\left(\frac{1}{6}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{6}\log(1-x)-\frac{1}{12}\log\left(5+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x)/((-1 + x)*(5 + x^2)),x]
[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)
```

Maple [A]
time = 0.22, size = 28, normalized size = 0.74

method	result	size
default	$-\frac{\ln(x^2+5)}{6}+\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{3}+\frac{\ln(-1+x)}{3}$	28
risch	$-\frac{\ln(x^2+5)}{6}+\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{3}+\frac{\ln(-1+x)}{3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x/(-1+x)/(x^2+5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(x^2+5)+1/3*arctan(1/5*x*5^(1/2))*5^(1/2)+1/3*ln(-1+x)
```

Maxima [A]
time = 0.48, size = 27, normalized size = 0.71

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

Fricas [A]
time = 0.40, size = 27, normalized size = 0.71

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="fricas")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

Sympy [A]
time = 0.05, size = 31, normalized size = 0.82

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+5\right)}{6}+\frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x**2+5),x)
[Out] log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

Giac [A]
time = 4.00, size = 28, normalized size = 0.74

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="giac")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))
```

Mupad [B]
time = 0.16, size = 44, normalized size = 1.16

$$\frac{\ln(x-1)}{3}-\ln\left(x-\sqrt{5}\ 1\mathrm{i}\right)\left(\frac{1}{6}+\frac{\sqrt{5}\ 1\mathrm{i}}{6}\right)+\ln\left(x+\sqrt{5}\ 1\mathrm{i}\right)\left(-\frac{1}{6}+\frac{\sqrt{5}\ 1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x)/((x^2 + 5)*(x - 1)),x)
```

```
[Out] log(x - 1)/3 - log(x - 5^(1/2)*1i)*((5^(1/2)*1i)/6 + 1/6) + log(x + 5^(1/2)
*1i)*((5^(1/2)*1i)/6 - 1/6)
```

3.448 $\int \frac{2+x^2}{2+x} \, dx$

Optimal. Leaf size=17

$$-2x + \frac{x^2}{2} + 6\log(2+x)$$

```
[Out] -2*x+1/2*x^2+6*ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{711\}$

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{2+x} \, dx &= \int \left(-2+x+\frac{6}{2+x}\right) dx \\ &= -2x + \frac{x^2}{2} + 6\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.06

$$-6 - 2x + \frac{x^2}{2} + 6\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/(2 + x),x]
```

```
[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]
```

Maple [A]
time = 0.22, size = 16, normalized size = 0.94

method	result	size
default	$-2x+\frac{x^2}{2}+6\ln\left(x+2\right)$	16
norman	$-2x+\frac{x^2}{2}+6\ln\left(x+2\right)$	16
risch	$-2x+\frac{x^2}{2}+6\ln\left(x+2\right)$	16
meijerg	$6\ln\left(1+\frac{x}{2}\right)-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x+2),x,method=_RETURNVERBOSE)
[Out] -2*x+1/2*x^2+6*ln(x+2)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2-2\,x+6\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Fricas [A]
time = 0.38, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2-2\,x+6\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

Sympy [A]
time = 0.01, size = 14, normalized size = 0.82

$$\frac{x^2}{2}-2x+6\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(2+x),x)
```

```
[Out] x**2/2 - 2*x + 6*log(x + 2)
```

Giac [A]
time = 3.76, size = 16, normalized size = 0.94

$$\frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(abs(x + 2))
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$6\,\ln\left(x+2\right) - 2\,x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/(x + 2),x)
```

```
[Out] 6*log(x + 2) - 2*x + x^2/2
```

3.449 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$

Optimal. Leaf size=31

$$-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{13}\log(3-x)-\frac{1}{26}\log\left(4+x^2\right)$$

```
[Out] -3/26*arctan(1/2*x)+1/13*ln(3-x)-1/26*ln(x^2+4)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {720, 31, 649, 209, 266}

$$-\frac{3}{26}\text{ArcTan}\left(\frac{x}{2}\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
```

```
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(-3+x)(4+x^2)}\,dx &= \frac{1}{13}\int \frac{1}{-3+x}\,dx + \frac{1}{13}\int \frac{-3-x}{4+x^2}\,dx \\
&= \frac{1}{13}\log(3-x) - \frac{1}{13}\int \frac{x}{4+x^2}\,dx - \frac{3}{13}\int \frac{1}{4+x^2}\,dx \\
&= -\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{13}\log(3-x) - \frac{1}{26}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 1.16

$$-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{26}\log\left(13+6(-3+x)+(-3+x)^2\right) + \frac{1}{13}\log(-3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-3 + x)*(4 + x^2)),x]
[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/13
```

Maple [A]
time = 0.22, size = 22, normalized size = 0.71

method	result	size
default	$\frac{\ln(x-3)}{13}-\frac{\ln(x^2+4)}{26}-\frac{3\arctan(\frac{x}{2})}{26}$	22
risch	$\frac{\ln(x-3)}{13}-\frac{\ln(9x^2+36)}{26}-\frac{3\arctan(\frac{x}{2})}{26}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-3)/(x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/13*ln(x-3)-1/26*ln(x^2+4)-3/26*arctan(1/2*x)
```

Maxima [A]
time = 0.50, size = 21, normalized size = 0.68

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right) - \frac{1}{26}\log\left(x^2+4\right) + \frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Fricas [A]
time = 0.41, size = 21, normalized size = 0.68

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

Sympy [A]
time = 0.05, size = 22, normalized size = 0.71

$$\frac{\log\left(x-3\right)}{13}-\frac{\log\left(x^2+4\right)}{26}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x**2+4),x)
```

```
[Out] log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

Giac [A]
time = 3.92, size = 22, normalized size = 0.71

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(x-3\right)}{13}+\ln\left(x-2\mathrm{i}\right)\left(-\frac{1}{26}+\frac{3}{52}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\left(-\frac{1}{26}-\frac{3}{52}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 4)*(x - 3)),x)
```

```
[Out] log(x - 3)/13 - log(x - 2i)*(1/26 - 3i/52) - log(x + 2i)*(1/26 + 3i/52)
```

3.450 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$

Optimal. Leaf size=19

$$-\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

```
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 78}

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{-2+3x}{x(5+2x)}\,dx,x,x^6\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{5x}+\frac{19}{5(5+2x)}\right)\,dx,x,x^6\right)\\
&= -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$-\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Maple [A]
time = 0.20, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{2\ln(x)}{5}+\frac{19\ln(2x^6+5)}{60}$	16
norman	$-\frac{2\ln(x)}{5}+\frac{19\ln(2x^6+5)}{60}$	16
risch	$-\frac{2\ln(x)}{5}+\frac{19\ln(2x^6+5)}{60}$	16
meijerg	$\frac{19\ln\left(1+\frac{2x^6}{5}\right)}{60}-\frac{2\ln(x)}{5}-\frac{\ln(2)}{15}+\frac{\ln(5)}{15}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-2)/x/(2*x^6+5),x,method=_RETURNVERBOSE)
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.89

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{1}{15}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Fricas [A]
time = 0.39, size = 15, normalized size = 0.79

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2}{5}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")
```

```
[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.89

$$-\frac{2\log(x)}{5}+\frac{19\log(2x^6+5)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-2)/x/(2*x**6+5),x)
```

```
[Out] -2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

Giac [A]
time = 3.85, size = 17, normalized size = 0.89

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{1}{15}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")
```

```
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Mupad [B]
time = 0.09, size = 13, normalized size = 0.68

$$\frac{19\ln\left(x^6+\frac{5}{2}\right)}{60}-\frac{2\ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)
```

```
[Out] (19*log(x^6 + 5/2))/60 - (2*log(x))/5
```

3.451 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(5+x)$$

[Out] ln(2-x)+ln(5+x)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.062$, Rules used = {78}

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[2 - x] + Log[5 + x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(-2+x)(5+x)}\,dx &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right)dx\\ &= \log(2-x)+\log(5+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.82

$$\log(-2+x)+\log(5+x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[-2 + x] + Log[5 + x]

Maple [A]
time = 0.21, size = 9, normalized size = 0.82

method	result	size
default	$\ln\left(\left(x-2\right)\left(5+x\right)\right)$	9
norman	$\ln\left(5+x\right)+\ln\left(x-2\right)$	10
risch	$\ln\left(x^{2}+3x-10\right)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(x-2)/(5+x),x,method=_RETURNVERBOSE)
[Out] ln((x-2)*(5+x))
```

Maxima [A]
time = 0.27, size = 9, normalized size = 0.82

$$\log\left(x+5\right)+\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
[Out] log(x + 5) + log(x - 2)
```

Fricas [A]
time = 0.39, size = 9, normalized size = 0.82

$$\log\left(x^{2}+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
[Out] log(x^2 + 3*x - 10)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.73

$$\log\left(x^{2}+3x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
[Out] log(x**2 + 3*x - 10)
```

Giac [A]
time = 4.83, size = 11, normalized size = 1.00

$$\log\left(|x+5|\right)+\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

```
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

Mupad [B]
time = 2.22, size = 9, normalized size = 0.82

$$\ln\left(x^2+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
```

```
[Out] log(3*x + x^2 - 10)
```

3.452 $\int \frac{x^4}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=18

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1136, 1180, 209}

$$-\frac{8}{3}\text{ArcTan}\left(\frac{x}{2}\right)+\frac{\text{ArcTan}(x)}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x-\int \frac{4+5x^2}{4+5x^2+x^4}\,dx$$
$$= x+\frac{1}{3}\int \frac{1}{1+x^2}\,dx-\frac{16}{3}\int \frac{1}{4+x^2}\,dx$$
$$= x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$x+\frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.72

method	result	size
default	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13
risch	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A]
time = 0.50, size = 12, normalized size = 0.67

$$x-\frac{8}{3}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A]
time = 0.40, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A]
time = 0.06, size = 14, normalized size = 0.78

$$x - \frac{8\operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Giac [A]
time = 6.42, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Mupad [B]
time = 2.22, size = 12, normalized size = 0.67

$$x - \frac{8\operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(5*x^2 + x^4 + 4),x)
```

```
[Out] x - (8*atan(x/2))/3 + atan(x)/3
```

3.453 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)$$

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {90}

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx &= \int \left(\frac{1}{8(1+x)}-\frac{1}{(2+x)^2}+\frac{2}{2+x}-\frac{1}{2(3+x)^3}-\frac{5}{4(3+x)^2}-\frac{17}{8(3+x)}\right) dx\\ &= \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{2+x}+\frac{2}{(3+x)^2}+\frac{10}{3+x}+\log(-1-x)+16\log(2+x)-17\log(3+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

Maple [A]
time = 0.22, size = 39, normalized size = 0.85

method	result	size
default	$\frac{1}{x+2}+\frac{1}{4(3+x)^{2}}+\frac{5}{4(3+x)}+\frac{\ln(1+x)}{8}+2\ln\left(x+2\right)-\frac{17\ln(3+x)}{8}$	39
norman	$\frac{\frac{9}{4}x^{2}+\frac{25}{2}x+17}{(x+2)(3+x)^{2}}+\frac{\ln(1+x)}{8}-\frac{17\ln(3+x)}{8}+2\ln\left(x+2\right)$	41
risch	$\frac{\frac{9}{4}x^{2}+\frac{25}{2}x+17}{(x+2)(3+x)^{2}}+\frac{\ln(1+x)}{8}-\frac{17\ln(3+x)}{8}+2\ln\left(x+2\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x+2)^2/(3+x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(x+2)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(x+2)-17/8*ln(3+x)
```

Maxima [A]
time = 0.27, size = 46, normalized size = 1.00

$$\frac{9\,x^{2}+50\,x+68}{4\,(x^{3}+8\,x^{2}+21\,x+18)}-\frac{17}{8}\,\log\left(x+3\right)+2\,\log\left(x+2\right)+\frac{1}{8}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(38) = 76.
time = 0.41, size = 83, normalized size = 1.80

$$\frac{18\,x^{2}-17\,(x^{3}+8\,x^{2}+21\,x+18)\log\left(x+3\right)+16\,(x^{3}+8\,x^{2}+21\,x+18)\log\left(x+2\right)+(x^{3}+8\,x^{2}+21\,x+18)\log\left(x+1\right)+100\,x+136}{8\,(x^{3}+8\,x^{2}+21\,x+18)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A]
time = 0.08, size = 46, normalized size = 1.00

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72}+\frac{\log(x+1)}{8}+2\log(x+2)-\frac{17\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

Giac [A]
time = 7.59, size = 52, normalized size = 1.13

$$\frac{1}{x+2}-\frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2}+\frac{1}{8}\log\left(\left|-\frac{1}{x+2}+1\right|\right)-\frac{17}{8}\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

Mupad [B]
time = 0.04, size = 45, normalized size = 0.98

$$\frac{\ln(x+1)}{8}+2\ln(x+2)-\frac{17\ln(x+3)}{8}+\frac{\frac{9x^2}{4}+\frac{25x}{2}+17}{x^3+8x^2+21x+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 1
7)/(21*x + 8*x^2 + x^3 + 18)
```

3.454 $\int \frac{x}{-1+x^2}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {266}

$$\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2),x]
```

```
[Out] Log[1 - x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(-1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2),x]
```

```
[Out] Log[-1 + x^2]/2
```

Maple [A]
time = 0.20, size = 14, normalized size = 1.17

method	result	size
derivativedivides	$\frac{\ln(x^2-1)}{2}$	9
risch	$\frac{\ln(x^2-1)}{2}$	9
meijerg	$\frac{\ln(-x^2+1)}{2}$	11
default	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
norman	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(-1+x)+1/2*ln(1+x)
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1)
```

Fricas [A]
time = 0.38, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.58

$$\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1),x)
```

```
[Out] log(x**2 - 1)/2
```

Giac [A]
time = 5.33, size = 9, normalized size = 0.75

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.04, size = 8, normalized size = 0.67

$$\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 1),x)
```

```
[Out] log(x^2 - 1)/2
```

3.455 $\int \frac{1}{\left(-1+x^2\right)^2}\, dx$

Optimal. Leaf size=21

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

[Out] 1/2*x/(-x^2+1)+1/2*arctanh(x)

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {205, 213}

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)^(-2),x]

[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^2\right)^2}\, dx &= \frac{x}{2\left(1-x^2\right)}-\frac{1}{2}\int \frac{1}{-1+x^2}\, dx\\ &= \frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{-1+x^2}-\log(1-x)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(-2),x]
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A]
time = 0.21, size = 28, normalized size = 1.33

method	result	size
meijerg	$-\frac{i\left(\frac{2ix}{-2x^2+2}+i\,\mathrm{arctanh}(x)\right)}{2}$	23
norman	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
risch	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
default	$-\frac{1}{4(-1+x)}-\frac{\ln(-1+x)}{4}-\frac{1}{4(1+x)}+\frac{\ln(1+x)}{4}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2,x,method=_RETURNVERBOSE)
[Out] -1/4/(-1+x)-1/4*ln(-1+x)-1/4/(1+x)+1/4*ln(1+x)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.10

$$-\frac{x}{2\,(x^2-1)}+\frac{1}{4}\,\log\left(x+1\right)-\frac{1}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.
time = 0.39, size = 34, normalized size = 1.62

$$\frac{(x^2-1)\log\left(x+1\right)-(x^2-1)\log\left(x-1\right)-2\,x}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2}-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2,x)
```

```
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Giac [A]
time = 4.60, size = 25, normalized size = 1.19

$$-\frac{x}{2(x^2-1)}+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B]
time = 2.22, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}(x)}{2}-\frac{x}{2\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^2,x)
```

```
[Out] atanh(x)/2 - x/(2*(x^2 - 1))
```

3.456 $\int \frac{x^2}{(1+x^2)^2} dx$

Optimal. Leaf size=19

$$-\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {294, 209}

$$\frac{\text{ArcTan}(x)}{2}-\frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+x^2)^2}\,dx &= -\frac{x}{2(1+x^2)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Maple [A]
time = 0.20, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A]
time = 0.49, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A]
time = 0.39, size = 21, normalized size = 1.11

$$\frac{(x^2+1)\arctan(x)-x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Giac [A]
time = 4.95, size = 15, normalized size = 0.79

$$-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + 1)^2,x)
```

```
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

3.457 $\int \frac{1}{2+3x}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(2+3x)$$

[Out] 1/3*ln(2+3*x)

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)^(-1),x]

[Out] Log[2 + 3*x]/3

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\log(2+3x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3}\log(2+3x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^(-1),x]

[Out] Log[2 + 3*x]/3

Maple [A]
time = 0.20, size = 9, normalized size = 0.90

method	result	size
default	$\frac{\ln(2+3x)}{3}$	9
norman	$\frac{\ln(2+3x)}{3}$	9
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
risch	$\frac{\ln(2+3x)}{3}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(2+3*x)
```

Maxima [A]
time = 0.26, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(3*x + 2)
```

Fricas [A]
time = 0.38, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log\left(3x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

Giac [A]
time = 5.18, size = 9, normalized size = 0.90

$$\frac{1}{3}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2))
```

Mupad [B]
time = 0.07, size = 6, normalized size = 0.60

$$\frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + 2),x)
```

```
[Out] log(x + 2/3)/3
```

3.458 $\int \frac{1}{a^2+x^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

[Out] arctan(x/a)/a

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {209}

$$\frac{\text{ArcTan}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + x^2)^(-1),x]

[Out] ArcTan[x/a]/a

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + x^2)^(-1),x]

[Out] ArcTan[x/a]/a

Maple [A]
time = 0.21, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x/a)/a
```

Maxima [A]
time = 0.48, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a)/a
```

Fricas [A]
time = 0.38, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a)/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.03, size = 20, normalized size = 2.00

$$\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

Giac [A]
time = 3.44, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a)/a
```

Mupad [B]
time = 2.25, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + x^2),x)
```

```
[Out] atan(x/a)/a
```

3.459 $\int \frac{1}{a+bx^2} dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.22, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.41, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.
time = 0.06, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [**A**]
time = 4.68, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [**B**]
time = 2.24, size = 16, normalized size = 0.67

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.460 $\int \frac{1}{2-x+x^2} \, dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-x+x^2} \, dx &= -\left(2\text{Subst}\left(\int \frac{1}{-7-x^2} \, dx, x, -1+2x\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{-1+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)^(-1),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A]
time = 0.39, size = 17, normalized size = 0.89

method	result	size
default	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17
risch	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-x+2),x,method=_RETURNVERBOSE)
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Maxima [A]
time = 0.48, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\,\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Fricas [A]
time = 0.39, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\,\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A]
time = 0.03, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-x+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

Giac [A]
time = 4.61, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2\,x-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.84

$$\frac{2\,\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,\left(2\,x-1\right)}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x + 2),x)
```

```
[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

3.461 $\int x^2(4-x^2)^2\,dx$

Optimal. Leaf size=22

$$\frac{16x^3}{3}-\frac{8x^5}{5}+\frac{x^7}{7}$$

```
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {276}

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(4 - x^2)^2,x]
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(4-x^2\right)^2\,dx&=\int\left(16x^2-8x^4+x^6\right)\,dx\\&=\frac{16x^3}{3}-\frac{8x^5}{5}+\frac{x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{16x^3}{3}-\frac{8x^5}{5}+\frac{x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(4 - x^2)^2,x]
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Maple [A]
time = 0.21, size = 17, normalized size = 0.77

method	result	size
default	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
norman	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
risch	$\frac{16}{3}x^3-\frac{8}{5}x^5+\frac{1}{7}x^7$	17
gosper	$\frac{x^3(15x^4-168x^2+560)}{105}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+4)^2,x,method=_RETURNVERBOSE)
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7-\frac{8}{5}x^5+\frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Fricas [A]
time = 0.39, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7-\frac{8}{5}x^5+\frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Sympy [A]
time = 0.01, size = 17, normalized size = 0.77

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+4)**2,x)
```

```
[Out] x**7/7 - 8*x**5/5 + 16*x**3/3
```

Giac [A]
time = 4.61, size = 16, normalized size = 0.73

$$\frac{1}{7}\,x^{7}-\frac{8}{5}\,x^{5}+\frac{16}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Mupad [B]
time = 0.02, size = 17, normalized size = 0.77

$$\frac{x^3\,\left(15\,x^4-168\,x^2+560\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 - 4)^2,x)
```

```
[Out] (x^3*(15*x^4 - 168*x^2 + 560))/105
```

3.462 $\int x(1-x^3)^2\,dx$

Optimal. Leaf size=22

$$\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

```
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {276}

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^2,x]
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(1-x^3\right)^2\,dx&=\int\left(x-2x^4+x^7\right)\,dx\\&=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^2,x]
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Maple [A]
time = 0.22, size = 17, normalized size = 0.77

method	result	size
default	$\frac{1}{2}x^{2}-\frac{2}{5}x^{5}+\frac{1}{8}x^{8}$	17
norman	$\frac{1}{2}x^{2}-\frac{2}{5}x^{5}+\frac{1}{8}x^{8}$	17
risch	$\frac{1}{2}x^{2}-\frac{2}{5}x^{5}+\frac{1}{8}x^{8}$	17
gosper	$\frac{x^{2}\left(5x^{6}-16x^{3}+20\right)}{40}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.73

$$\frac{1}{8}\,x^{8}-\frac{2}{5}\,x^{5}+\frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Fricas [A]
time = 0.36, size = 16, normalized size = 0.73

$$\frac{1}{8}\,x^{8}-\frac{2}{5}\,x^{5}+\frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.68

$$\frac{x^{8}}{8}-\frac{2x^{5}}{5}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**2,x)
```

```
[Out] x**8/8 - 2*x**5/5 + x**2/2
```

Giac [A]
time = 4.29, size = 16, normalized size = 0.73

$$\frac{1}{8}\,x^8-\frac{2}{5}\,x^5+\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{x^2\,\left(5\,x^6-16\,x^3+20\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3 - 1)^2,x)
```

```
[Out] (x^2*(5*x^6 - 16*x^3 + 20))/40
```

3.463 $\int \frac{-4+5x^2+x^3}{x^2} \, dx$

Optimal. Leaf size=16

$$\frac{4}{x}+5x+\frac{x^2}{2}$$

[Out] 4/x+5*x+1/2*x^2

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 5*x^2 + x^3)/x^2,x]

[Out] 4/x + 5*x + x^2/2

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{-4+5x^2+x^3}{x^2} \, dx &= \int \left(5-\frac{4}{x^2}+x\right) \, dx \\ &= \frac{4}{x}+5x+\frac{x^2}{2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{4}{x}+5x+\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 5*x^2 + x^3)/x^2,x]

[Out] 4/x + 5*x + x^2/2

Maple [A]
time = 0.01, size = 15, normalized size = 0.94

method	result	size
default	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
risch	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
gosper	$\frac{x^3+10x^2+8}{2x}$	16
norman	$\frac{\frac{1}{2}x^3+5x^2+4}{x}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5*x^2-4)/x^2,x,method=_RETURNVERBOSE)
[Out] 4/x+5*x+1/2*x^2
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x^2+5\,x+\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")
[Out] 1/2*x^2 + 5*x + 4/x
```

Fricas [A]
time = 0.37, size = 15, normalized size = 0.94

$$\frac{x^3+10\,x^2+8}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")
[Out] 1/2*(x^3 + 10*x^2 + 8)/x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.62

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5*x**2-4)/x**2,x)
```

```
[Out] x**2/2 + 5*x + 4/x
```

Giac [A]
time = 4.54, size = 14, normalized size = 0.88

$$\frac{1}{2}\,x^{2}+5\,x+\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")
[Out] 1/2*x^2 + 5*x + 4/x
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.94

$$\frac{x^{3}+10\,x^{2}+8}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + x^3 - 4)/x^2,x)
[Out] (10*x^2 + x^3 + 8)/(2*x)
```

3.464 $\int \frac{-1+x}{3-4x+3x^2} \, dx$

Optimal. Leaf size=37

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}+\frac{1}{6}\log\left(3-4x+3x^2\right)$$

```
[Out] 1/6*ln(3*x^2-4*x+3)+1/15*arctan(1/5*(2-3*x)*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}+\frac{1}{6}\log\left(3x^2-4x+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

```
[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{-1+x}{3-4x+3x^2}\,dx = \frac{1}{6}\int \frac{-4+6x}{3-4x+3x^2}\,dx - \frac{1}{3}\int \frac{1}{3-4x+3x^2}\,dx$$
$$= \frac{1}{6}\log\left(3-4x+3x^2\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{-20-x^2}\,dx, x, -4+6x\right)$$
$$= \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}} + \frac{1}{6}\log\left(3-4x+3x^2\right)$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{-2+3x}{\sqrt{5}}\right)}{3\sqrt{5}} + \frac{1}{6}\log\left(3-4x+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(3 - 4*x + 3*x^2),x]
[Out] -1/3*ArcTan[(-2 + 3*x)/Sqrt[5]]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6
```

Maple [A]
time = 0.46, size = 31, normalized size = 0.84

method	result	size
default	$\frac{\ln(3x^2-4x+3)}{6} - \frac{\sqrt{5}\,\arctan\left(\frac{(6x-4)\sqrt{5}}{10}\right)}{15}$	31
risch	$\frac{\ln(9x^2-12x+9)}{6} - \frac{\sqrt{5}\,\arctan\left(\frac{(3x-2)\sqrt{5}}{5}\right)}{15}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(3*x^2-4*x+3),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))
```

Maxima [A]
time = 0.49, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right) + \frac{1}{6}\,\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="maxima")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Fricas [A]
time = 0.40, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="fricas")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Sympy [A]
time = 0.04, size = 39, normalized size = 1.05

$$\frac{\log\left(x^2-\frac{4x}{3}+1\right)}{6}-\frac{\sqrt{5}\operatorname{atan}\left(\frac{3\sqrt{5}\,x}{5}-\frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x**2-4*x+3),x)
[Out] log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/5)/15
```

Giac [A]
time = 3.81, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="giac")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

Mupad [B]
time = 2.22, size = 30, normalized size = 0.81

$$\frac{\ln\left(x^2-\frac{4\,x}{3}+1\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{3\,\sqrt{5}\,x}{5}-\frac{2\,\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(3*x^2 - 4*x + 3),x)
[Out] log(x^2 - (4*x)/3 + 1)/6 - (5^(1/2)*atan((3*5^(1/2)*x)/5 - (2*5^(1/2))/5))/
15
```

3.465 $\int (2+x^3)^2 \, dx$

Optimal. Leaf size=14

$$4x+x^4+\frac{x^7}{7}$$

[Out] 4*x+x^4+1/7*x^7

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {200}

$$\frac{x^7}{7}+x^4+4x$$

Antiderivative was successfully verified.

[In] Int[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (2+x^3)^2 \, dx &= \int (4+4x^3+x^6) \, dx\\ &= 4x+x^4+\frac{x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$4x+x^4+\frac{x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^3)^2,x]

[Out] 4*x + x^4 + x^7/7

Maple [A]
time = 0.19, size = 13, normalized size = 0.93

method	result	size
gosper	$4x + x^4 + \frac{1}{7}x^7$	13
default	$4x + x^4 + \frac{1}{7}x^7$	13
norman	$4x + x^4 + \frac{1}{7}x^7$	13
risch	$4x + x^4 + \frac{1}{7}x^7$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*x+x^4+1/7*x^7
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$\frac{1}{7}\,x^7 + x^4 + 4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

Fricas [A]
time = 0.38, size = 12, normalized size = 0.86

$$\frac{1}{7}\,x^7 + x^4 + 4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

Sympy [A]
time = 0.00, size = 10, normalized size = 0.71

$$\frac{x^7}{7} + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)**2,x)
```

```
[Out] x**7/7 + x**4 + 4*x
```

Giac [A]
time = 3.92, size = 12, normalized size = 0.86

$$\frac{1}{7}\,x^7 + x^4 + 4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="giac")
```

```
[Out] 1/7*x^7 + x^4 + 4*x
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.93

$$\frac{x\,\left(x^6+7\,x^3+28\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 2)^2,x)
```

```
[Out] (x*(7*x^3 + x^6 + 28))/7
```

3.466 $\int \frac{-4+x^2}{2+x} \, dx$

Optimal. Leaf size=11

$$-2x + \frac{x^2}{2}$$

```
[Out] -2*x+1/2*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{641\}$

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\begin{aligned} \int \frac{-4 + x^2}{2 + x} \, dx &= \int (-2 + x) \, dx \\ &= -2x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-2x + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 + x),x]
```

[Out] -2*x + x^2/2

Maple [A]
time = 0.20, size = 10, normalized size = 0.91

method	result	size
gosper	$\frac{x(x-4)}{2}$	7
meijerg	$-\frac{x\left(-\frac{3x}{2}+6\right)}{3}$	9
default	$-2x+\frac{1}{2}x^2$	10
norman	$-2x+\frac{1}{2}x^2$	10
risch	$-2x+\frac{1}{2}x^2$	10

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-4)/(x+2),x,method=_RETURNVERBOSE)

[Out] -2*x+1/2*x^2

Maxima [A]
time = 0.28, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x),x, algorithm="maxima")

[Out] 1/2*x^2 - 2*x

Fricas [A]
time = 0.37, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x),x, algorithm="fricas")

[Out] 1/2*x^2 - 2*x

Sympy [A]
time = 0.01, size = 7, normalized size = 0.64

$$\frac{x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(2+x),x)
```

```
[Out] x**2/2 - 2*x
```

Giac [**A**]
time = 4.30, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x
```

Mupad [**B**]
time = 0.02, size = 6, normalized size = 0.55

$$\frac{x\,(x-4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 4)/(x + 2),x)
```

```
[Out] (x*(x - 4))/2
```

3.467 $\int \frac{1}{(2+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$\frac{2}{5}\tan^{-1}(x)+\frac{1}{5}\log(2+x)-\frac{1}{10}\log\left(1+x^2\right)$$

```
[Out] 2/5*arctan(x)+1/5*ln(2+x)-1/10*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {720, 31, 649, 209, 266}

$$\frac{2\text{ArcTan}(x)}{5}-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)*(1 + x^2)),x]
```

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
```

```
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+x)(1+x^2)}\,dx &= \frac{1}{5}\int \frac{1}{2+x}\,dx+\frac{1}{5}\int \frac{2-x}{1+x^2}\,dx\\ &= \frac{1}{5}\log(2+x)-\frac{1}{5}\int \frac{x}{1+x^2}\,dx+\frac{2}{5}\int \frac{1}{1+x^2}\,dx\\ &= \frac{2}{5}\tan^{-1}(x)+\frac{1}{5}\log(2+x)-\frac{1}{10}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{2}{5}\tan^{-1}(x)+\frac{1}{5}\log(2+x)-\frac{1}{10}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)*(1 + x^2)),x]
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Maple [A]
time = 0.21, size = 20, normalized size = 0.80

method	result	size
default	$\frac{2\arctan(x)}{5}+\frac{\ln(x+2)}{5}-\frac{\ln(x^2+1)}{10}$	20
risch	$\frac{\ln(x+2)}{5}-\frac{\ln(4x^2+4)}{10}+\frac{2\arctan(x)}{5}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/5*arctan(x)+1/5*ln(x+2)-1/10*ln(x^2+1)
```

Maxima [A]
time = 0.50, size = 19, normalized size = 0.76

$$\frac{2}{5}\arctan\left(x\right)-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Fricas [A]
time = 0.39, size = 19, normalized size = 0.76

$$\frac{2}{5}\arctan\left(x\right)-\frac{1}{10}\log\left(x^{2}+1\right)+\frac{1}{5}\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.80

$$\frac{\log\left(x+2\right)}{5}-\frac{\log\left(x^{2}+1\right)}{10}+\frac{2\,\mathrm{atan}\left(x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x**2+1),x)
```

```
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

Giac [A]
time = 3.56, size = 20, normalized size = 0.80

$$\frac{2}{5}\arctan\left(x\right)-\frac{1}{10}\log\left(x^{2}+1\right)+\frac{1}{5}\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))
```

Mupad [B]
time = 0.05, size = 25, normalized size = 1.00

$$\frac{\ln\left(x+2\right)}{5}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{10}-\frac{1}{5}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{10}+\frac{1}{5}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + 2)),x)
```

```
[Out] log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)
```

3.468 $\int \frac{1}{(1+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {720, 31, 649, 209, 266}

$$\frac{\text{ArcTan}(x)}{2}-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
```

```
c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\left(1+x^2\right)} \, dx &= \frac{1}{2} \int \frac{1}{1+x} \, dx + \frac{1}{2} \int \frac{1-x}{1+x^2} \, dx \\ &= \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1}{1+x^2} \, dx - \frac{1}{2} \int \frac{x}{1+x^2} \, dx \\ &= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x) - \frac{1}{4} \log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x) - \frac{1}{4} \log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A]
time = 0.21, size = 20, normalized size = 0.80

method	result	size
default	$\frac{\arctan(x)}{2} + \frac{\ln(1+x)}{2} - \frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2} + \frac{\ln(1+x)}{2} - \frac{\ln(x^2+1)}{4}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A]
time = 0.48, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log\left(x^2+1\right) + \frac{1}{2} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A]
time = 0.39, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+1),x)
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Giac [A]
time = 4.69, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

Mupad [B]
time = 2.23, size = 25, normalized size = 1.00

$$\frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4} - \frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4} + \frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + 1)),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.469 $\int \frac{x}{(1+x)(1+x^2)} \, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {815, 649, 209, 266}

$$\frac{\text{ArcTan}(x)}{2}+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{(1+x)(1+x^2)} dx = \int \left(-\frac{1}{2(1+x)} + \frac{1+x}{2(1+x^2)}\right) dx$$
$$= -\frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{1+x}{1+x^2} dx$$
$$= -\frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{1}{1+x^2} dx + \frac{1}{2}\int \frac{x}{1+x^2} dx$$
$$= \frac{1}{2}\tan^{-1}(x) - \frac{1}{2}\log(1+x) + \frac{1}{4}\log\left(1+x^2\right)$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x) - \frac{1}{2}\log(1+x) + \frac{1}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Maple [A]
time = 0.22, size = 20, normalized size = 0.80

method	result	size
default	$\frac{\arctan(x)}{2} - \frac{\ln(1+x)}{2} + \frac{\ln(x^2+1)}{4}$	20
risch	$\frac{\arctan(x)}{2} - \frac{\ln(1+x)}{2} + \frac{\ln(x^2+1)}{4}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)-1/2*ln(1+x)+1/4*ln(x^2+1)
```

Maxima [A]
time = 0.48, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Fricas [A]
time = 0.39, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x^{2}+1\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.76

$$-\frac{\log\left(x+1\right)}{2}+\frac{\log\left(x^{2}+1\right)}{4}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x**2+1),x)
```

```
[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

Giac [A]
time = 4.30, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x^{2}+1\right)-\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))
```

Mupad [B]
time = 2.21, size = 25, normalized size = 1.00

$$-\frac{\ln\left(x+1\right)}{2}+\ln\left(x-\mathrm{i}\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x + 1)),x)
```

```
[Out] log(x - 1i)*(1/4 - 1i/4) - log(x + 1)/2 + log(x + 1i)*(1/4 + 1i/4)
```

3.470 $\int \frac{2x+x^2}{(1+x)^2}\,dx$

Optimal. Leaf size=9

$$\frac{x^2}{1+x}$$

```
[Out] x^2/(1+x)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {697}

$$x+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Rule 697

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{2x+x^2}{(1+x)^2}\,dx &= \int \left(1-\frac{1}{(1+x)^2}\right)dx\\ &= x+\frac{1}{1+x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 0.78

$$x+\frac{1}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Maple [A]
time = 0.20, size = 8, normalized size = 0.89

method	result	size
default	$x+\frac{1}{1+x}$	8
risch	$x+\frac{1}{1+x}$	8
gosper	$\frac{x^2}{1+x}$	10
norman	$\frac{x^2}{1+x}$	10
meijerg	$\frac{x(3x+6)}{3+3x}-\frac{2x}{1+x}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/(1+x)
```

Maxima [A]
time = 0.26, size = 7, normalized size = 0.78

$$x+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] x + 1/(x + 1)
```

Fricas [A]
time = 0.39, size = 12, normalized size = 1.33

$$\frac{x^2+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] (x^2 + x + 1)/(x + 1)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.56

$$x+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(1+x)**2,x)
```

```
[Out] x + 1/(x + 1)
```

Giac [A]
time = 5.36, size = 8, normalized size = 0.89

$$x+\frac{1}{x+1}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] x + 1/(x + 1) + 1
```

Mupad [B]
time = 0.02, size = 7, normalized size = 0.78

$$x+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(x + 1)^2,x)
```

```
[Out] x + 1/(x + 1)
```

3.471 $\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] arctan(1/2*x)-3/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1180, 209}

$$\text{ArcTan}\left(\frac{x}{2}\right)-\frac{3\text{ArcTan}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx &= -\left(3\int \frac{1}{1+2x^2}\,dx\right)+4\int \frac{1}{8+2x^2}\,dx\\ &= \tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
default	$\arctan\left(\frac{x}{2}\right)-\frac{3\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{2}$	17
risch	$\arctan\left(\frac{x}{2}\right)-\frac{3\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-10)/(2*x^4+9*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(1/2*x)-3/2*arctan(2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}\,x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Fricas [A]
time = 0.40, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}\,x\right)+\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="fricas")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.91

$$\operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-10)/(2*x**4+9*x**2+4),x)
[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

Giac [A]
time = 9.61, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}\,x\right) + \arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="giac")
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

Mupad [B]
time = 0.05, size = 16, normalized size = 0.73

$$\operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\,\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 10)/(9*x^2 + 2*x^4 + 4),x)
[Out] atan(x/2) - (3*2^(1/2)*atan(2^(1/2)*x))/2
```

3.472 $\int \frac{31+5x}{11-4x+3x^2}\,dx$

Optimal. Leaf size=37

$$-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}+\frac{5}{6}\log\left(11-4x+3x^2\right)$$

```
[Out] 5/6*ln(3*x^2-4*x+11)-103/87*arctan(1/29*(2-3*x)*29^(1/2))*29^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {648, 632, 210, 642}

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\text{ArcTan}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In] Int[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{31+5x}{11-4x+3x^2}\,dx &= \frac{5}{6}\int \frac{-4+6x}{11-4x+3x^2}\,dx+\frac{103}{3}\int \frac{1}{11-4x+3x^2}\,dx\\ &= \frac{5}{6}\log\left(11-4x+3x^2\right)-\frac{206}{3}\text{Subst}\left(\int \frac{1}{-116-x^2}\,dx,x,-4+6x\right)\\ &= -\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}+\frac{5}{6}\log\left(11-4x+3x^2\right)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 37, normalized size = 1.00

$$\frac{103\tan^{-1}\left(\frac{-2+3x}{\sqrt{29}}\right)}{3\sqrt{29}}+\frac{5}{6}\log\left(11-4x+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Maple [**A**]
time = 0.41, size = 31, normalized size = 0.84

method	result	size
default	$\frac{5\ln\left(3x^2-4x+11\right)}{6}+\frac{103\sqrt{29}\,\arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)}{87}$	31
risch	$\frac{5\ln\left(9x^2-12x+33\right)}{6}+\frac{103\sqrt{29}\,\arctan\left(\frac{(3x-2)\sqrt{29}}{29}\right)}{87}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((31+5*x)/(3*x^2-4*x+11),x,method=_RETURNVERBOSE)
[Out] 5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2))
```

Maxima [A]
time = 0.48, size = 30, normalized size = 0.81

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}\,(3\,x-2)\right)+\frac{5}{6}\log\left(3\,x^2-4\,x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="maxima")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Fricas [A]
time = 0.42, size = 30, normalized size = 0.81

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}\,(3\,x-2)\right)+\frac{5}{6}\log\left(3\,x^2-4\,x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="fricas")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Sympy [A]
time = 0.04, size = 44, normalized size = 1.19

$$\frac{5\log\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\operatorname{atan}\left(\frac{3\sqrt{29}\,x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x**2-4*x+11),x)
[Out] 5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29 - 2*sqrt(2
9)/29)/87
```

Giac [A]
time = 5.38, size = 30, normalized size = 0.81

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}\,(3\,x-2)\right)+\frac{5}{6}\log\left(3\,x^2-4\,x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="giac")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

Mupad [B]
time = 0.04, size = 30, normalized size = 0.81

$$\frac{5\ln\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\,\mathrm{atan}\left(\frac{3\sqrt{29}\,x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 31)/(3*x^2 - 4*x + 11),x)
[Out] (5*log(x^2 - (4*x)/3 + 11/3))/6 + (103*29^(1/2)*atan((3*29^(1/2)*x)/29 - (2
*29^(1/2))/29))/87
```

3.473 $\int \frac{-2+x^2+x^3}{x^4} \, dx$

Optimal. Leaf size=15

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

```
[Out] 2/3/x^3-1/x+ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {14}

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2 + x^3)/x^4,x]
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x^2+x^3}{x^4} \, dx &= \int \left(-\frac{2}{x^4}+\frac{1}{x^2}+\frac{1}{x}\right) \, dx \\ &= \frac{2}{3x^3}-\frac{1}{x}+\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2 + x^3)/x^4,x]
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.93

method	result	size
default	$\frac{2}{3x^3}-\frac{1}{x}+\ln(x)$	14
norman	$\frac{\frac{2}{3}-x^2}{x^3}+\ln(x)$	15
risch	$\frac{\frac{2}{3}-x^2}{x^3}+\ln(x)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/x^3-1/x+ln(x)
```

Maple [A]

Maxima [A]
time = 0.26, size = 15, normalized size = 1.00

$$-\frac{3\,x^2-2}{3\,x^3}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)
```

Fricas [A]
time = 0.38, size = 19, normalized size = 1.27

$$\frac{3\,x^3\log(x)-3\,x^2+2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

Sympy [A]
time = 0.02, size = 14, normalized size = 0.93

$$\log(x)+\frac{2-3x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-2)/x**4,x)
```

```
[Out] log(x) + (2 - 3*x**2)/(3*x**3)
```

Giac [A]
time = 4.87, size = 16, normalized size = 1.07

$$-\frac{3\,x^2-2}{3\,x^3}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 - 2)/x^3 + log(abs(x))
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.87

$$\ln\left(x\right)-\frac{x^2-\frac{2}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 - 2)/x^4,x)
```

```
[Out] log(x) - (x^2 - 2/3)/x^3
```

3.474 $\int \frac{1+x+x^3}{x^2} dx$

Optimal. Leaf size=15

$$-\frac{1}{x}+\frac{x^2}{2}+\log(x)$$

[Out] -1/x+1/2*x^2+ln(x)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {14}

$$\frac{x^2}{2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^3}{x^2}\,dx &= \int \left(\frac{1}{x^2}+\frac{1}{x}+x\right)\,dx\\ &= -\frac{1}{x}+\frac{x^2}{2}+\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{x}+\frac{x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
default	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln\left(x\right)$	14
risch	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln\left(x\right)$	14
norman	$\frac{-1+\frac{x^{3}}{2}}{x}+\ln\left(x\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+1/2*x^2+ln(x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$\frac{1}{2}\,x^{2}-\frac{1}{x}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/x + log(x)
```

Fricas [A]
time = 0.37, size = 15, normalized size = 1.00

$$\frac{x^{3}+2\,x\log\left(x\right)-2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 2*x*log(x) - 2)/x
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^{2}}{2}+\log\left(x\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/x**2,x)
```

```
[Out] x**2/2 + log(x) - 1/x
```

Giac [**A**]
time = 4.18, size = 14, normalized size = 0.93

$$\frac{1}{2}x^2 - \frac{1}{x} + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/x + log(abs(x))
```

Mupad [**B**]
time = 0.02, size = 13, normalized size = 0.87

$$\ln\left(x\right) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 1)/x^2,x)
```

```
[Out] log(x) - 1/x + x^2/2
```

3.475 $\int \frac{-2+x^2}{x(2+x^2)}\,dx$

Optimal. Leaf size=11

$$-\log(x)+\log\left(2+x^2\right)$$

[Out] -ln(x)+ln(x^2+2)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 78}

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]

[Out] -Log[x] + Log[2 + x^2]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-2+x}{x(2+x)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right)\,dx,x,x^2\right)\\
&= -\log(x)+\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\log(x)+\log\left(2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
[Out] -Log[x] + Log[2 + x^2]
```

Maple [A]
time = 0.20, size = 12, normalized size = 1.09

method	result	size
default	$-\ln\left(x\right)+\ln\left(x^{2}+2\right)$	12
norman	$-\ln\left(x\right)+\ln\left(x^{2}+2\right)$	12
risch	$-\ln\left(x\right)+\ln\left(x^{2}+2\right)$	12
meijerg	$\ln\left(1+\frac{x^{2}}{2}\right)-\ln\left(x\right)+\frac{\ln\left(2\right)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/x/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -ln(x)+ln(x^2+2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 1.18

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Fricas [A]
time = 0.37, size = 11, normalized size = 1.00

$$\log\left(x^2+2\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
[Out] log(x^2 + 2) - log(x)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$-\log(x) + \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/x/(x**2+2),x)
[Out] -log(x) + log(x**2 + 2)
```

Giac [A]
time = 2.85, size = 13, normalized size = 1.18

$$\log(x^2 + 2) - \frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Mupad [B]
time = 0.06, size = 11, normalized size = 1.00

$$\ln(x^2 + 2) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 2)/(x*(x^2 + 2)),x)
[Out] log(x^2 + 2) - log(x)
```

3.476 $\int(-3+x)\left(-7+4x^2\right)\,dx$

Optimal. Leaf size=22

$$21x-4x^3+\frac{1}{16}\left(7-4x^2\right)^2$$

```
[Out] 21*x-4*x^3+1/16*(-4*x^2+7)^2
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{655\}$

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int(-3+x)\left(-7+4x^2\right)\,dx&=\frac{1}{16}\left(7-4x^2\right)^2-3\int\left(-7+4x^2\right)\,dx\\&=21x-4x^3+\frac{1}{16}\left(7-4x^2\right)^2\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.86

$$21x-\frac{7x^2}{2}-4x^3+x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

Maple [A]
time = 0.04, size = 18, normalized size = 0.82

method	result	size
gosper	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
default	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
norman	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18
risch	$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)*(4*x^2-7),x,method=_RETURNVERBOSE)
```

```
[Out] x^4-4*x^3-7/2*x^2+21*x
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Fricas [A]
time = 0.37, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="fricas")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Sympy [A]
time = 0.00, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x**2-7),x)
```

```
[Out] x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

Giac [A]
time = 3.42, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="giac")
```

```
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7\,x^2}{2} + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 7)*(x - 3),x)
```

```
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

3.477 $\int (-2+7x)^3 \, dx$

Optimal. Leaf size=11

$$\frac{1}{28}(2-7x)^4$$

```
[Out] 1/28*(2-7*x)^4
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{1}{28}(2-7x)^4$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 7*x)^3,x]
```

```
[Out] (2 - 7*x)^4/28
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (-2+7x)^3 \, dx = \frac{1}{28}(2-7x)^4$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{28}(-2+7x)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 7*x)^3,x]
```

```
[Out] (-2 + 7*x)^4/28
```

Maple [A]
time = 0.20, size = 10, normalized size = 0.91

method	result	size
default	$\frac{(-2+7x)^4}{28}$	10
gosper	$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$	20
norman	$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$	20
risch	$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x + \frac{4}{7}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+7*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*(-2+7*x)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 0.27, size = 19, normalized size = 1.73

$$\frac{343}{4}\,x^4 - 98\,x^3 + 42\,x^2 - 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="maxima")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 0.37, size = 19, normalized size = 1.73

$$\frac{343}{4}\,x^4 - 98\,x^3 + 42\,x^2 - 8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="fricas")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.01, size = 19, normalized size = 1.73

$$\frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)**3,x)
```

[Out] 343*x***4/4 - 98*x**3 + 42*x**2 - 8*x

Giac [**A**]
time = 3.95, size = 9, normalized size = 0.82

$$\frac{1}{28}\,(7\,x-2)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="giac")
```

```
[Out] 1/28*(7*x - 2)^4
```

Mupad [**B**]
time = 0.14, size = 9, normalized size = 0.82

$$\frac{(7\,x-2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x - 2)^3,x)
```

```
[Out] (7*x - 2)^4/28
```

3.478 $\int \frac{-7+4x^2}{3+2x}\,dx$

Optimal. Leaf size=13

$$-3x+x^2+\log(3+2x)$$

[Out] -3*x+x^2+ln(3+2*x)

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {711}

$$x^2-3x+\log(2x+3)$$

Antiderivative was successfully verified.

[In] Int[(-7 + 4*x^2)/(3 + 2*x),x]

[Out] -3*x + x^2 + Log[3 + 2*x]

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-7+4x^2}{3+2x}\,dx &= \int \left(-3+2x+\frac{2}{3+2x}\right)dx\\ &= -3x+x^2+\log(3+2x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.23

$$-\frac{27}{4}-3x+x^2+\log(3+2x)$$

Antiderivative was successfully verified.

[In] Integrate[(-7 + 4*x^2)/(3 + 2*x),x]

[Out] -27/4 - 3*x + x^2 + Log[3 + 2*x]

Maple [A]
time = 0.20, size = 14, normalized size = 1.08

method	result	size
default	$-3x + x^2 + \ln(3+2x)$	14
norman	$-3x + x^2 + \ln(3+2x)$	14
risch	$-3x + x^2 + \ln(3+2x)$	14
meijerg	$\ln\left(1+\frac{2x}{3}\right) - \frac{x(-2x+6)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-7)/(3+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -3*x+x^2+ln(3+2*x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 1.00

$$x^2 - 3\,x + \log\left(2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")
```

```
[Out] x^2 - 3*x + log(2*x + 3)
```

Fricas [A]
time = 0.38, size = 13, normalized size = 1.00

$$x^2 - 3\,x + \log\left(2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")
```

```
[Out] x^2 - 3*x + log(2*x + 3)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.92

$$x^2 - 3x + \log\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-7)/(3+2*x),x)
```

```
[Out] x**2 - 3*x + log(2*x + 3)
```

Giac [A]
time = 4.21, size = 14, normalized size = 1.08

$$x^2 - 3\,x + \log\left(|2\,x + 3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")
```

```
[Out] x^2 - 3*x + log(abs(2*x + 3))
```

Mupad [B]
time = 2.21, size = 11, normalized size = 0.85

$$\ln\left(x+\frac{3}{2}\right)-3\,x+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 7)/(2*x + 3),x)
```

```
[Out] log(x + 3/2) - 3*x + x^2
```

3.479 $\int \frac{1+x}{(-1+x)x^2}\,dx$

Optimal. Leaf size=16

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

[Out] 1/x+2*ln(1-x)-2*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {78}

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((-1 + x)*x^2),x]

[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-1+x)x^2}\,dx &= \int \left(\frac{2}{-1+x}-\frac{1}{x^2}-\frac{2}{x}\right)dx\\ &= \frac{1}{x}+2\log(1-x)-2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*x^2),x]
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Maple [A]
time = 0.22, size = 15, normalized size = 0.94

method	result	size
default	$2\ln\left(-1+x\right)+\frac{1}{x}-2\ln\left(x\right)$	15
norman	$2\ln\left(-1+x\right)+\frac{1}{x}-2\ln\left(x\right)$	15
risch	$2\ln\left(-1+x\right)+\frac{1}{x}-2\ln\left(x\right)$	15
meijerg	$2\ln\left(1-x\right)-2\ln\left(x\right)-2i\pi+\frac{1}{x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-1+x)/x^2,x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+1/x-2*ln(x)
```

Maxima [A]
time = 0.26, size = 14, normalized size = 0.88

$$\frac{1}{x}+2\,\log\left(x-1\right)-2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")
[Out] 1/x + 2*log(x - 1) - 2*log(x)
```

Fricas [A]
time = 0.38, size = 18, normalized size = 1.12

$$\frac{2\,x\log\left(x-1\right)-2\,x\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")
[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.88

$$-2\log\left(x\right)+2\log\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x**2,x)
```

```
[Out] -2*log(x) + 2*log(x - 1) + 1/x
```

Giac [A]
time = 3.94, size = 16, normalized size = 1.00

$$\frac{1}{x} + 2 \log(|x-1|) - 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")
```

```
[Out] 1/x + 2*log(abs(x - 1)) - 2*log(abs(x))
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{1}{x} - 4\,\mathrm{atanh}(2\,x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^2*(x - 1)),x)
```

```
[Out] 1/x - 4*atanh(2*x - 1)
```

3.480 $\int \frac{1}{4x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=27

$$\frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\tanh^{-1}(1+x)$$

```
[Out] 1/2*(1+x)/(1-(1+x)^2)+1/2*arctanh(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1120, 205, 212}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{2\,(1-(1+x)^2)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)$$
$$= \frac{1+x}{2\,(1-(1+x)^2)} + \frac{1}{2}\tanh^{-1}(1+x)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)} - \log(x) + \log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.89

method	result	size
default	$-\frac{1}{4(x+2)}+\frac{\ln(x+2)}{4}-\frac{1}{4x}-\frac{\ln(x)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(x+2)}-\frac{\ln(x)}{4}+\frac{\ln(x+2)}{4}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^3+4*x^2),x,method=_RETURNVERBOSE)
[Out] -1/4/(x+2)+1/4*ln(x+2)-1/4/x-1/4*ln(x)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.93

$$-\frac{x+1}{2\,(x^2+2\,x)} + \frac{1}{4}\,\log\left(x+2\right) - \frac{1}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Fricas [A]
time = 0.38, size = 39, normalized size = 1.44

$$\frac{(x^2+2\,x)\log\left(x+2\right)-(x^2+2\,x)\log\left(x\right)-2\,x-2}{4\,(x^2+2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A]
time = 0.04, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x}-\frac{\log\left(x\right)}{4}+\frac{\log\left(x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**3+4*x**2),x)
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Giac [A]
time = 4.27, size = 27, normalized size = 1.00

$$-\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\log\left(|x+2|\right)-\frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

Mupad [B]
time = 2.23, size = 23, normalized size = 0.85

$$\frac{\mathrm{atanh}(x+1)}{2}-\frac{\frac{x}{2}+\frac{1}{2}}{x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 4*x^3 + x^4),x)
[Out] atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)
```

3.481 $\int \frac{1+x^2}{1+x} \, dx$

Optimal. Leaf size=17

$$-x+\frac{x^2}{2}+2\log(1+x)$$

```
[Out] -x+1/2*x^2+2*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {711}

$$\frac{x^2}{2}-x+2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x),x]
```

```
[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+x} \, dx &= \int \left(-1+x+\frac{2}{1+x}\right) \, dx\\ &= -x+\frac{x^2}{2}+2\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.06

$$\frac{1}{2}\left(-3-2x+x^2+4\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x),x]
```

```
[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

Maple [A]
time = 0.20, size = 16, normalized size = 0.94

method	result	size
default	$-x+\frac{x^{2}}{2}+2\ln\left(1+x\right)$	16
norman	$-x+\frac{x^{2}}{2}+2\ln\left(1+x\right)$	16
meijerg	$-\frac{x(6-3x)}{6}+2\ln\left(1+x\right)$	16
risch	$-x+\frac{x^{2}}{2}+2\ln\left(1+x\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/2*x^2+2*ln(1+x)
```

Maxima [A]
time = 0.30, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^{2}-x+2\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Fricas [A]
time = 0.39, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^{2}-x+2\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{x^{2}}{2}-x+2\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(1+x),x)
```

```
[Out] x**2/2 - x + 2*log(x + 1)
```

Giac [A]
time = 4.16, size = 16, normalized size = 0.94

$$\frac{1}{2}\,x^2 - x + 2\,\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="giac")
[Out] 1/2*x^2 - x + 2*log(abs(x + 1))
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$2\,\ln\left(x + 1\right) - x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + 1),x)
[Out] 2*log(x + 1) - x + x^2/2
```

3.482 $\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx$

Optimal. Leaf size=18

$$\frac{1}{x}-3x+\frac{x^2}{2}+3\log(x)$$

[Out] 1/x-3*x+1/2*x^2+3*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = $\{14\}$

$$\frac{x^2}{2}-3x+\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx &= \int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)dx\\ &= \frac{1}{x}-3x+\frac{x^2}{2}+3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{x}-3x+\frac{x^2}{2}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.94

method	result	size
default	$\frac{1}{x}-3x+\frac{x^{2}}{2}+3\ln\left(x\right)$	17
risch	$\frac{1}{x}-3x+\frac{x^{2}}{2}+3\ln\left(x\right)$	17
norman	$\frac{1-3x^{2}+\frac{1}{2}x^{3}}{x}+3\ln\left(x\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.89

$$\frac{1}{2}\,x^{2}-3\,x+\frac{1}{x}+3\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)
```

Fricas [A]
time = 0.37, size = 20, normalized size = 1.11

$$\frac{x^{3}-6\,x^{2}+6\,x\log\left(x\right)+2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")
[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

Sympy [A]
time = 0.02, size = 15, normalized size = 0.83

$$\frac{x^{2}}{2}-3x+3\log\left(x\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/x**2,x)
[Out] x**2/2 - 3*x + 3*log(x) + 1/x
```

Giac [A]
time = 3.38, size = 17, normalized size = 0.94

$$\frac{1}{2}\,x^{2} - 3\,x + \frac{1}{x} + 3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(abs(x))
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.89

$$3\,\ln\left(x\right) - 3\,x + \frac{1}{x} + \frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 3*x^2 + x^3 - 1)/x^2,x)
```

```
[Out] 3*log(x) - 3*x + 1/x + x^2/2
```

3.483 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx$

Optimal. Leaf size=18

$$-7x+\frac{3x^2}{2}+\frac{x^3}{3}$$

```
[Out] -7*x+3/2*x^2+1/3*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.034$, Rules used = $\{45\}$

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

```
[In] Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right)\,dx &= \int \left(-7+3x+x^2\right)\,dx\\ &= -7x+\frac{3x^2}{2}+\frac{x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-7x+\frac{3x^2}{2}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Maple [A]
time = 0.02, size = 28, normalized size = 1.56

method	result	size
norman	$-7x+\frac{3}{2}x^{2}+\frac{1}{3}x^{3}$	15
risch	$-7x+\frac{3}{2}x^{2}+\frac{1}{3}x^{3}$	15
default	$\frac{x^{3}}{3}+\frac{3x^{2}}{2}+\left(\frac{3}{2}-\frac{\sqrt{37}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{37}}{2}\right)x$	28
gosper	$-\frac{x\left(2x^{2}+9x-42\right)\left(-2x-3+\sqrt{37}\right)\left(2x+3+\sqrt{37}\right)}{24\left(x^{2}+3x-7\right)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+3/2*x^2+(3/2-1/2*37^(1/2))*(3/2+1/2*37^(1/2))*x
```

Maxima [A]
time = 0.49, size = 14, normalized size = 0.78

$$\frac{1}{3}\,x^{3}+\frac{3}{2}\,x^{2}-7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Fricas [A]
time = 0.37, size = 14, normalized size = 0.78

$$\frac{1}{3}\,x^{3}+\frac{3}{2}\,x^{2}-7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Sympy [A]
time = 0.01, size = 14, normalized size = 0.78

$$\frac{x^{3}}{3}+\frac{3x^{2}}{2}-7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)
[Out] x**3/3 + 3*x**2/2 - 7*x
```

Giac [A]
time = 4.16, size = 14, normalized size = 0.78

$$\frac{1}{3}\,x^{3}+\frac{3}{2}\,x^{2}-7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(2\,x^2+9\,x-42\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 37^(1/2)/2 + 3/2)*(x + 37^(1/2)/2 + 3/2),x)
[Out] (x*(9*x + 2*x^2 - 42))/6
```

3.484 $\int \frac{4+3x^2+2x^3}{(1+x)^4} dx$

Optimal. Leaf size=23

$$-\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)$$

```
[Out] -5/3/(1+x)^3+3/(1+x)+2*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1864}

$$\frac{3}{x+1}-\frac{5}{3(x+1)^3}+2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{4+3x^2+2x^3}{(1+x)^4} dx &= \int \left(\frac{5}{(1+x)^4}-\frac{3}{(1+x)^2}+\frac{2}{1+x}\right) dx\\ &= -\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]
```

```
[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Maple [A]
time = 0.22, size = 22, normalized size = 0.96

method	result	size
default	$-\frac{5}{3(1+x)^3}+\frac{3}{1+x}+2\ln(1+x)$	22
norman	$\frac{3x^2+6x+\frac{4}{3}}{(1+x)^3}+2\ln(1+x)$	24
risch	$\frac{3x^2+6x+\frac{4}{3}}{(1+x)^3}+2\ln(1+x)$	24
meijerg	$\frac{4x(x^2+3x+3)}{3(1+x)^3}-\frac{x(22x^2+30x+12)}{6(1+x)^3}+2\ln(1+x)+\frac{x^3}{(1+x)^3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+4)/(1+x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -5/3/(1+x)^3+3/(1+x)+2*ln(1+x)
```

Maxima [A]
time = 0.27, size = 34, normalized size = 1.48

$$\frac{9\,x^2+18\,x+4}{3\,(x^3+3\,x^2+3\,x+1)}+2\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.
time = 0.39, size = 46, normalized size = 2.00

$$\frac{9\,x^2+6\,(x^3+3\,x^2+3\,x+1)\log(x+1)+18\,x+4}{3\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^3 + 3*x^2
+ 3*x + 1)
```

Sympy [A]
time = 0.03, size = 31, normalized size = 1.35

$$\frac{9x^2+18x+4}{3x^3+9x^2+9x+3}+2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+4)/(1+x)**4,x)
```

```
[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)
```

Giac [A]
time = 4.53, size = 25, normalized size = 1.09

$$\frac{9\,x^2+18\,x+4}{3\,{\left(x+1\right)}^3}+2\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))
```

Mupad [B]
time = 0.03, size = 23, normalized size = 1.00

$$2\,\ln\left(x+1\right)+\frac{3\,x^2+6\,x+\frac{4}{3}}{{\left(x+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2*x^3 + 4)/(x + 1)^4,x)
```

```
[Out] 2*log(x + 1) + (6*x + 3*x^2 + 4/3)/(x + 1)^3
```

3.485 $\int \frac{x}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2(1+x)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2/(1+x)+1/2*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {815, 209}

$$\frac{\text{ArcTan}(x)}{2}+\frac{1}{2(x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)^2\,(1+x^2)}\,dx &= \int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\,(1+x^2)}\right)dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{1}{2}\left(\frac{1}{1+x}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] ((1 + x)^(-1) + ArcTan[x])/2
```

Maple [A]
time = 0.22, size = 13, normalized size = 0.81

method	result	size
default	$\frac{1}{2x+2}+\frac{\arctan(x)}{2}$	13
risch	$\frac{1}{2x+2}+\frac{\arctan(x)}{2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(1+x)+1/2*arctan(x)
```

Maxima [A]
time = 0.51, size = 12, normalized size = 0.75

$$\frac{1}{2\left(x+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/2*arctan(x)
```

Fricas [A]
time = 0.39, size = 15, normalized size = 0.94

$$\frac{\left(x+1\right)\arctan\left(x\right)+1}{2\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}(x)}{2} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2/(x**2+1),x)
```

```
[Out] atan(x)/2 + 1/(2*x + 2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.
time = 4.15, size = 32, normalized size = 2.00

$$-\frac{1}{8}\pi - \frac{1}{2}\pi\left\lfloor -\frac{\pi - 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1/
2*arctan(x)
```

Mupad [B]
time = 2.22, size = 12, normalized size = 0.75

$$\frac{\operatorname{atan}(x)}{2} + \frac{1}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x + 1)^2),x)
```

```
[Out] atan(x)/2 + 1/(2*(x + 1))
```

3.486 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x} \, dx$

Optimal. Leaf size=29

$$-20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47\log(2+x)$$

```
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1864}

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{7-2x+3x^2-x^3+x^4}{2+x} \, dx &= \int \left(-20+9x-3x^2+x^3+\frac{47}{2+x}\right) dx \\ &= -20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.03

$$-70 - 20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]

Maple [A]
time = 0.21, size = 26, normalized size = 0.90

method	result	size
default	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln(x+2)$	26
norman	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln(x+2)$	26
risch	$-20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\ln(x+2)$	26
meijerg	$47\ln\left(1+\frac{x}{2}\right)-\frac{2x\left(-\frac{15}{8}x^3+5x^2-15x+60\right)}{15}-\frac{x(x^2-3x+12)}{3}-x\left(-\frac{3x}{2}+6\right)-2x$	50

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^3+3*x^2-2*x+7)/(x+2),x,method=_RETURNVERBOSE)

[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(x+2)

Maxima [A]
time = 0.26, size = 25, normalized size = 0.86

$$\frac{1}{4}\,x^4-x^3+\frac{9}{2}\,x^2-20\,x+47\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)

Fricas [A]
time = 0.40, size = 25, normalized size = 0.86

$$\frac{1}{4}\,x^4-x^3+\frac{9}{2}\,x^2-20\,x+47\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="fricas")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)

Sympy [A]
time = 0.02, size = 24, normalized size = 0.83

$$\frac{x^4}{4}-x^3+\frac{9x^2}{2}-20x+47\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)
```

```
[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)
```

Giac [A]
time = 3.87, size = 26, normalized size = 0.90

$$\frac{1}{4}\,x^4 - x^3 + \frac{9}{2}\,x^2 - 20\,x + 47\,\log\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.86

$$47\,\ln\left(x + 2\right) - 20\,x + \frac{9\,x^2}{2} - x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 2*x - x^3 + x^4 + 7)/(x + 2),x)
```

```
[Out] 47*log(x + 2) - 20*x + (9*x^2)/2 - x^3 + x^4/4
```

3.487 $\int \frac{-1+x^3}{-1+x} \, dx$

Optimal. Leaf size=16

$$x+\frac{x^2}{2}+\frac{x^3}{3}$$

```
[Out] x+1/2*x^2+1/3*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1600}

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{-1+x} \, dx &= \int \left(1+x+x^2\right) \, dx \\ &= x+\frac{x^2}{2}+\frac{x^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$x+\frac{x^2}{2}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

Maple [A]
time = 0.20, size = 13, normalized size = 0.81

method	result	size
default	$x+\frac{1}{2}x^2+\frac{1}{3}x^3$	13
norman	$x+\frac{1}{2}x^2+\frac{1}{3}x^3$	13
risch	$x+\frac{1}{2}x^2+\frac{1}{3}x^3$	13
gosper	$\frac{x(2x^2+3x+6)}{6}$	14
meijerg	$\frac{x(4x^2+6x+12)}{12}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(-1+x),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+1/3*x^3
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Fricas [A]
time = 0.37, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.62

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + x
```

Giac [A]
time = 3.42, size = 12, normalized size = 0.75

$$\frac{1}{3}\,x^{3} + \frac{1}{2}\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.81

$$\frac{x\,\left(2\,x^2+3\,x+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x - 1),x)
```

```
[Out] (x*(3*x + 2*x^2 + 6))/6
```

3.488 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\tan^{-1}(x)$$

```
[Out] -1/(1-x)^2+1/(-1+x)+arctan(x)
```

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {815, 209}

$$\text{ArcTan}(x)+\frac{1}{x-1}-\frac{1}{(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x}{(-1+x)^3\,(1+x^2)}\,dx &= \int \left(\frac{2}{(-1+x)^3}-\frac{1}{(-1+x)^2}+\frac{1}{1+x^2}\right)dx\\ &= -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{-2+x+(-1+x)^2\tan^{-1}(x)}{(-1+x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

Maple [A]
time = 0.23, size = 16, normalized size = 0.94

method	result	size
risch	$\frac{x-2}{(-1+x)^2}+\arctan(x)$	13
default	$-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\arctan(x)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+2)/(-1+x)^3/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-1+x)^2+1/(-1+x)+arctan(x)
```

Maxima [A]
time = 0.51, size = 17, normalized size = 1.00

$$\frac{x-2}{x^2-2\,x+1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] (x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

Fricas [A]
time = 0.39, size = 25, normalized size = 1.47

$$\frac{(x^2-2\,x+1)\arctan(x)+x-2}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.82

$$\frac{x-2}{x^2-2x+1}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)**3/(x**2+1),x)
```

```
[Out] (x - 2)/(x**2 - 2*x + 1) + atan(x)
```

Giac [A]
time = 3.31, size = 12, normalized size = 0.71

$$\frac{x-2}{(x-1)^2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="giac")
```

```
[Out] (x - 2)/(x - 1)^2 + arctan(x)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 1.00

$$\operatorname{atan}(x)+\frac{x-2}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)
```

```
[Out] atan(x) + (x - 2)/(x^2 - 2*x + 1)
```

3.489 $\int \frac{1}{bx+c(d+ex)^2} dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

```
[Out] -2*arctanh((b+2*c*e*(e*x+d))/b^(1/2)/(4*c*d*e+b)^(1/2))/b^(1/2)/(4*c*d*e+b)
^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2006, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2006

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{bx+c(d+ex)^2}\,dx &= \int \frac{1}{cd^2+(b+2cde)x+ce^2x^2}\,dx \\
&= -\left(2\text{Subst}\left(\int \frac{1}{b(b+4cde)-x^2}\,dx, x, b+2cde+2ce^2x\right)\right) \\
&= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$
-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Maple [A]
time = 0.35, size = 43, normalized size = 0.91

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{2c\,e^2x+2cde+b}{\sqrt{4bcde+b^2}}\right)}{\sqrt{4bcde+b^2}}$	43
risch	$\frac{\ln\left(-2c\,e^2x-2cde+\sqrt{b\,(4cde+b)}-b\right)}{\sqrt{b\,(4cde+b)}}-\frac{\ln\left(2c\,e^2x+2cde+\sqrt{b\,(4cde+b)}+b\right)}{\sqrt{b\,(4cde+b)}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/2
))
```

Maxima [A]
time = 0.27, size = 71, normalized size = 1.51

$$
\frac{\log\left(\frac{2\,cxe^2+2\,cde+b-\sqrt{(4\,cde+b)b}}{2\,cxe^2+2\,cde+b+\sqrt{(4\,cde+b)b}}\right)}{\sqrt{(4\,cde+b)b}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] log((2*c*x*e^2 + 2*c*d*e + b - sqrt((4*c*d*e + b)*b))/(2*c*x*e^2 + 2*c*d*e
+ b + sqrt((4*c*d*e + b)*b)))/sqrt((4*c*d*e + b)*b)
```

Fricas [A]
time = 0.40, size = 190, normalized size = 4.04

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^2+2\,c^2d^2e^2+4\,bcde+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^2}}, \frac{2\,\sqrt{-4\,bcde-b^2}\,\arctan\left(\frac{\sqrt{-4\,bcde-b^2}\,(2\,ce^2x+2\,cde+b)}{4\,bcde+b^2}\right)}{4\,bcde+b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b*
c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c*
d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*arc
tan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))/(4*
b*c*d*e + b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(49) = 98$.
time = 0.14, size = 151, normalized size = 3.21

$$\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{-b^2\sqrt{\frac{1}{b\,(b+4cde)}}-4bcde\sqrt{\frac{1}{b\,(b+4cde)}}+b+2cde}{2ce^2}\right)-\sqrt{\frac{1}{b\,(b+4cde)}}\log\left(x+\frac{b^2\sqrt{\frac{1}{b\,(b+4cde)}}+4bcde\sqrt{\frac{1}{b\,(b+4cde)}}+b+2cde}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)**2),x)
```

```
[Out] sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c*
d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b + 4
*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*(b
+ 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))
```

Giac [A]
time = 3.12, size = 48, normalized size = 1.02

$$\frac{2\,\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde-b^2}}\right)}{\sqrt{-4\,bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e
- b^2)
```

Mupad [B]
time = 0.10, size = 42, normalized size = 0.89

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x\,e^2+2\,c\,d\,e+b}{\sqrt{b}\,\sqrt{b+4\,c\,d\,e}}\right)}{\sqrt{b}\,\sqrt{b+4\,c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(d + e*x)^2 + b*x),x)
```

```
[Out] -(2*atanh((b + 2*c*d*e + 2*c*e^2*x)/(b^(1/2)*(b + 4*c*d*e)^(1/2))))/(b^(1/2
)*(b + 4*c*d*e)^(1/2))
```

3.490 $\int \frac{1}{a+bx+c(d+ex)^2} dx$

Optimal. Leaf size=57

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}$$

```
[Out] -2*arctanh((b+2*c*e*(e*x+d))/(-4*a*c*e^2+4*b*c*d*e+b^2)^(1/2))/(-4*a*c*e^2+
4*b*c*d*e+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2006, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/Sqrt[
b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2006

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx=\int \frac{1}{a+cd^2+(b+2cde)x+ce^2x^2}\,dx$$
$$=-\left(2\text{Subst}\left(\int \frac{1}{b^2+4bcde-4ace^2-x^2}\,dx,x,b+2cde+2ce^2x\right)\right)$$
$$=-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-b^2-4bcde+4ace^2}}\right)}{\sqrt{-b^2-4bcde+4ace^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[-
b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

Maple [A]
time = 0.34, size = 61, normalized size = 1.07

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,e^2x+2cde+b}{\sqrt{4ac\,e^2-4bcde-b^2}}\right)}{\sqrt{4ac\,e^2-4bcde-b^2}}$	61
risch	$-\frac{\ln\left(2c\,e^2x+2cde+\sqrt{-4ac\,e^2+4bcde+b^2}+b\right)}{\sqrt{-4ac\,e^2+4bcde+b^2}}+\frac{\ln\left(-2c\,e^2x-2cde+\sqrt{-4ac\,e^2+4bcde+b^2}-b\right)}{\sqrt{-4ac\,e^2+4bcde+b^2}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+c*(e*x+d)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-4
*b*c*d*e-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*%e*b*c*d>0)', see `assume?` for m
ore det
```

Fricas [A]
time = 0.38, size = 240, normalized size = 4.21

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^2+4\,bcde+2\,(c^2d^2-ac)e^2+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde-4\,ace^2+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x+a}\right)}{\sqrt{4\,bcde-4\,ace^2+b^2}}, -\frac{2\,\sqrt{-4\,bcde+4\,ace^2-b^2}\,\arctan\left(-\frac{\sqrt{-4\,bcde+4\,ace^2-b^2}\,(2\,ce^2x+2\,cde+b)}{4\,bcde-4\,ace^2+b^2}\right)}{4\,bcde-4\,ace^2+b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*
e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e +
 b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2
+ b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e + 4*a
*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/(4*b
*c*d*e - 4*a*c*e^2 + b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(63) = 126$.
time = 0.22, size = 294, normalized size = 5.16

$$-\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+2cde}{2ce^2}\right)+\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+2cde}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

```
[Out] -sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sqrt(-1/(4*a
*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)
) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*
e**2)) + sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (4*a*c*e**2*sqrt(
-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b
*c*d*e)) - 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e
)/(2*c*e**2))
```

Giac [A]
time = 4.18, size = 60, normalized size = 1.05

$$\frac{2\,\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde+4\,ace^2-b^2}}\right)}{\sqrt{-4\,bcde+4\,ace^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqrt
(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

Mupad [B]
time = 2.23, size = 82, normalized size = 1.44

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,d\,e}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}+\frac{2\,c\,e^2\,x}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}\right)}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*(d + e*x)^2 + b*x),x)
```

```
[Out] (2*atan((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2) + (2*c*e^2*x)/(4*
a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)))/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)
```

3.491 $\int \frac{x^2}{1+(-1+x^2)^2} \, dx$

Optimal. Leaf size=188

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)+$$

```
[Out] -1/4*arctan((-2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^
(1/2)+1/4*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/
2))^(1/2)+1/4*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)-1/4
*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2014, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -1/2*(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1
 + Sqrt[2])]]) + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x
)/Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]
/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 2014

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] &&  !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int \frac{x^2}{1+(-1+x^2)^2}\,dx = \int \frac{x^2}{2-2x^2+x^4}\,dx$$
$$= -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right) + \frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,dx + \cdots$$
$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}} - \frac{1}{2}\cdots$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}-\cdots\right.}{4\sqrt{2\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

Maple [A]
time = 0.05, size = 168, normalized size = 0.89

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^2+2\right)}\frac{\_R^2\ln\left(x-\_R\right)}{\_R^3-\_R}\right)}{4}$
default	$\frac{\sqrt{2+2\sqrt{2}}\,\left(\sqrt{2}-1\right)\left(\frac{\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{2}+\frac{\sqrt{2+2\sqrt{2}}\,\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{\sqrt{-2+2\sqrt{2}}}\right)}{4}-\frac{\sqrt{2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(x^2-1)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(1/2*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/
2))+(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/2))^(1/2)*arctan((2*x-(2+2*2^(1/2))^(1/2
))/(-2+2*2^(1/2))^(1/2)))-1/4*(2+2*2^(1/2))^(1/2)*(2^(1/2)-1)*(1/2*ln(x^2+2
^(1/2)+x*(2+2*2^(1/2))^(1/2))-(2+2*2^(1/2))^(1/2)/(-2+2*2^(1/2))^(1/2)*arct
an((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

Fricas [A]
time = 0.41, size = 240, normalized size = 1.28

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+x^2+\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+x^2+\sqrt{2}\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{2\sqrt{2}+4}-\sqrt{2}-1\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{2\sqrt{2}+4}+\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(1/2*2^(3/4)*x*sqrt(2*sqr
t(2) + 4) + x^2 + sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + x^2 + sqrt(2)) - 1/4*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

Sympy [A]
time = 0.28, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(x**2-1)**2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

Giac [A]
time = 4.57, size = 147, normalized size = 0.78

$$\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2)
)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2
^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

Mupad [B]
time = 2.30, size = 101, normalized size = 0.54

$$\operatorname{atanh}\left(32x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32x\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^2 + 1),x)
```

```
[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*
(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((-
 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32
- 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

3.492 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

Optimal. Leaf size=60

$$\frac{2}{(3+x+x^4)^3}-\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}$$

```
[Out] 2/(x^4+x+3)^3-3*x/(x^4+x+3)^3+5*x^2/(x^4+x+3)^3+x^4/(x^4+x+3)^3-5*x^6/(x^4+
x+3)^3
```

Rubi [A]
time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2127, 1602}

$$\frac{x^4}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*
x^9)/(3 + x + x^4)^4),x]
```

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(
3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2127

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = -\frac{5x^6}{(3+x+x^4)^3}+\frac{1}{6}\int \frac{-90+216x}{}$$
$$= \frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}-$$
$$= \frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-$$
$$= -\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^3}+$$
$$= \frac{2}{(3+x+x^4)^3}-\frac{3x}{(3+x+x^4)^3}+$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.45

$$\frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7
 - 30*x^9)/(3 + x + x^4)^4),x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.47

method	result	size
default	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
risch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x
,method=_RETURNVERBOSE)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Maxima [A]
time = 0.29, size = 65, normalized size = 1.08

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [A]
time = 0.38, size = 65, normalized size = 1.08

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [A]
time = 0.10, size = 60, normalized size = 1.00

$$\frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-15)/
(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Giac [A]
time = 4.14, size = 30, normalized size = 0.50

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{\left(x^4+x+3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="giac")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

Mupad [B]
time = 2.34, size = 27, normalized size = 0.45

$$\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{{\left(x^4+x+3\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x^2 - 36*x + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9 +
15)/(x + x^4 + 3)^4,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

3.493 $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4}\right.$

Optimal. Leaf size=27

$$\frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3

Rubi [F]
time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx$$

Verification is not applicable to the result.

[In] Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]

[Out] -19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^(-2) - (621*Defer[Int][(3 + x + x^4)^(-4), x])/4 + 684*Defer[Int][x/(3 + x + x^4)^4, x] + 360*Defer[Int][x^2/(3 + x + x^4)^4, x] + 44*Defer[Int][(3 + x + x^4)^(-3), x] - 320*Defer[Int][x/(3 + x + x^4)^3, x] - 75*Defer[Int][x^2/(3 + x + x^4)^3, x] + 30*Defer[Int][x/(3 + x + x^4)^2, x]

Rubi steps

$$\begin{aligned}\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right) dx &= 3\int \frac{-47+228x+}{(3+x}\\ &= -\frac{19}{4(3+x+x^4)^3}+\\ &= -\frac{19}{4(3+x+x^4)^3}+\\ &= -\frac{19}{4(3+x+x^4)^3}+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*
x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 1.
time = 0.06, size = 250, normalized size = 9.26

method	result
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
default	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+\_Z+3)}\frac{(377432\_R^2}{}\right.}{19507}$
risch	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+\_Z+3)}\frac{(377432\_R^2}{}\right.}{19507}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x
+3)^3+30*x/(x^4+x+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (377432/195075*x^7-1404328/195075*x^6+234517/195075*x^5+660506/195075*x^4-2
08792/195075*x^3-13339729/390150*x^2+89881/13005*x+121303/21675)/(x^4+x+3)^
2+1/195075*sum((377432*_R^2-2808656*_R+703551)/(4*_R^3+1)*ln(x-_R),_R=RootO
f(_Z^4+_Z+3))+30*(-16/765*x^3+64/765*x^2-1/765*x-4/255)/(x^4+x+3)+2/51*sum(
(-16*_R^2+128*_R-3)/(4*_R^3+1)*ln(x-_R),_R=RootOf(_Z^4+_Z+3))+3*(-255032/58
5225*x^11+914728/585225*x^10-226867/585225*x^9-701338/585225*x^8+236024/585
225*x^7+13501313/1170450*x^6-2360372/585225*x^5-1873778/585225*x^4+10935781
/1170450*x^3+3415123/130050*x^2-62987/7225*x-76253/21675)/(x^4+x+3)^3+1/195
075*sum((-255032*_R^2+1829456*_R-680601)/(4*_R^3+1)*ln(x-_R),_R=RootOf(_Z^4
+_Z+3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.
time = 0.29, size = 65, normalized size = 2.41

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.
time = 0.38, size = 65, normalized size = 2.41

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.
time = 0.15, size = 60, normalized size = 2.22

$$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{x^{12}+3x^{9}+9x^{8}+3x^{6}+18x^{5}+27x^{4}+x^{3}+9x^{2}+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320*
x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(30) = 60$.
time = 5.46, size = 197, normalized size = 7.30

$$\frac{1}{195075}\,x\left(\frac{377432\,x^{2}-2808656\,x+703551}{x^{4}+x+3}-\frac{255032\,x^{2}-1829456\,x+680601}{x^{4}+x+3}-\frac{7650\,(16\,x^{2}-128\,x+3)}{x^{4}+x+3}\right)-\frac{2\,(16\,x^{3}-64\,x^{2}+x+12)}{51\,(x^{4}+x+3)}+\frac{754864\,x^{7}-2808656\,x^{6}+469034\,x^{5}+1321012\,x^{4}-417584\,x^{3}-13339729\,x^{2}+2696430\,x+2183454}{390150\,(x^{4}+x+3)^{2}}-\frac{510064\,x^{11}-1829456\,x^{10}+453734\,x^{9}+1402676\,x^{8}-472048\,x^{7}-13501313\,x^{6}+4720744\,x^{5}+3747556\,x^{4}-10935781\,x^{3}-30736107\,x^{2}+10203894\,x+4117662}{390150\,(x^{4}+x+3)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")
```

```
[Out] 1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2 -
 1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x + 3
)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^7 -
```

```
 2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 26964
30*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10 + 45
3734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 3747556*
x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x + 3)^3
```

Mupad [B]
time = 0.05, size = 27, normalized size = 1.00

$$\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{{\left(x^4+x+3\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8*
x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

3.494 $\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx$

Optimal. Leaf size=27

$$\frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Rubi [**F**]
time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x
+ 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] 7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3 + x + x^4
)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4)^3) - (10*x^6)/(3 +
 x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)/(3 + x + x^4)^2 + (144*Defer[
Int][(3 + x + x^4)^(-4), x])/11 + (828*Defer[Int][x/(3 + x + x^4)^4, x])/11
 + 18*Defer[Int][x^2/(3 + x + x^4)^4, x] - 4*Defer[Int][(3 + x + x^4)^(-3),
 x] - 20*Defer[Int][x/(3 + x + x^4)^3, x]
```

Rubi steps

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx = -\left(3\int \frac{(1+4x^3)(2-3x+5 \ldots}{(3+x+ \ldots}\right.$$

$$= -\frac{10x^6}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x \ldots}$$

$$= \frac{3x^4}{2(3+x+x^4)^3}-\frac{10x^6}{(3+x+x \ldots}$$

$$= -\frac{5x^3}{(3+x+x^4)^3}+\frac{3x^4}{2(3+x+ \ldots}$$

$$= -\frac{12x^2}{(3+x+x^4)^3}-\frac{5x^3}{(3+x+x \ldots}$$

$$= -\frac{63x}{22(3+x+x^4)^3}-\frac{12x \ldots}{(3+x+ \ldots}$$

$$= \frac{7}{2(3+x+x^4)^3}-\frac{63x}{22(3+x+ \ldots}$$

$$= \frac{7}{2(3+x+x^4)^3}-\frac{63x}{22(3+x+ \ldots}$$

$$= \frac{7}{2(3+x+x^4)^3}-\frac{63x}{22(3+x+ \ldots}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2
- 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(27) = 54$.
time = 0.04, size = 112, normalized size = 4.15

method	result

norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
risch	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
default	$-\frac{-\frac{34568}{195075}x^7+\frac{73672}{195075}x^6+\frac{15392}{195075}x^5-\frac{60494}{195075}x^4-\frac{68792}{195075}x^3-\frac{583927}{195075}x^2+\frac{3356}{13005}x-\frac{2069}{43350}}{(x^4+x+3)^2}+\frac{-\frac{34568}{195075}x^{11}+\frac{73672}{195075}x^{10}+\frac{15392}{195075}x^9-\frac{95062}{195075}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)
/(x^4+x+3)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(-34568/195075*x^7+73672/195075*x^6+15392/195075*x^5-60494/195075*x^4-6879
2/195075*x^3-583927/195075*x^2+3356/13005*x-2069/43350)/(x^4+x+3)^2+3*(-345
68/585225*x^11+73672/585225*x^10+15392/585225*x^9-95062/585225*x^8-98824/58
5225*x^7-1322894/585225*x^6+36022/585225*x^5-129019/1170450*x^4-790303/5852
25*x^3-80674/65025*x^2-10951/14450*x+26831/43350)/(x^4+x+3)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.
time = 0.29, size = 65, normalized size = 2.41

$$-\frac{5x^6-x^4-5x^2+3x-2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.
time = 0.38, size = 65, normalized size = 2.41

$$-\frac{5x^6-x^4-5x^2+3x-2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.
time = 0.12, size = 60, normalized size = 2.22

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**4
+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(30) = 60$.
time = 4.04, size = 111, normalized size = 4.11

$$\frac{69136\,x^7 - 147344\,x^6 - 30784\,x^5 + 120988\,x^4 + 137584\,x^3 + 1167854\,x^2 - 100680\,x + 18621}{390150\,(x^4 + x + 3)^2} - \frac{69136\,x^{11} - 147344\,x^{10} - 30784\,x^9 + 190124\,x^8 + 197648\,x^7 + 2645788\,x^6 - 72044\,x^5 + 129019\,x^4 + 1580606\,x^3 + 1452132\,x^2 + 887031\,x - 724437}{390150\,(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="giac")
```

```
[Out] 1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 11
67854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 1473
44*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5 + 1
29019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3)^3
```

Mupad [B]
time = 0.04, size = 27, normalized size = 1.00

$$\frac{-5\,x^6 + x^4 + 5\,x^2 - 3\,x + 2}{{\left(x^4 + x + 3\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 - 3
*x + x^4 - 5*x^6 + 2))/(x + x^4 + 3)^4,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```