Computer algebra independent integration tests

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11-MIT

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [321]. This is test number [211].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.2.1 (February 10, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2023 (March, 2023) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-41 on Linux via sagemath 9.8.
7. Sympy 1.11.1 (March 20, 2022) Using Python 3.10.9 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

In addition of the above systems, ChatGPT 3.5 was used in this file only for testing purposes only. This was done by using ChatGPT directly in the openAI website.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.38 (319)	0.62 (2)
Fricas	96.57 (310)	3.43 (11)
Maple	95.33 (306)	4.67 (15)
Rubi	94.08 (302)	5.92 (19)
Maxima	92.52 (297)	7.48 (24)
Giac	91.59 (294)	8.41 (27)
Mupad	90.03 (289)	9.97 (32)
Sympy	82.24 (264)	17.76 (57)
ChatGPT	14.33 (46)	85.67 (275)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

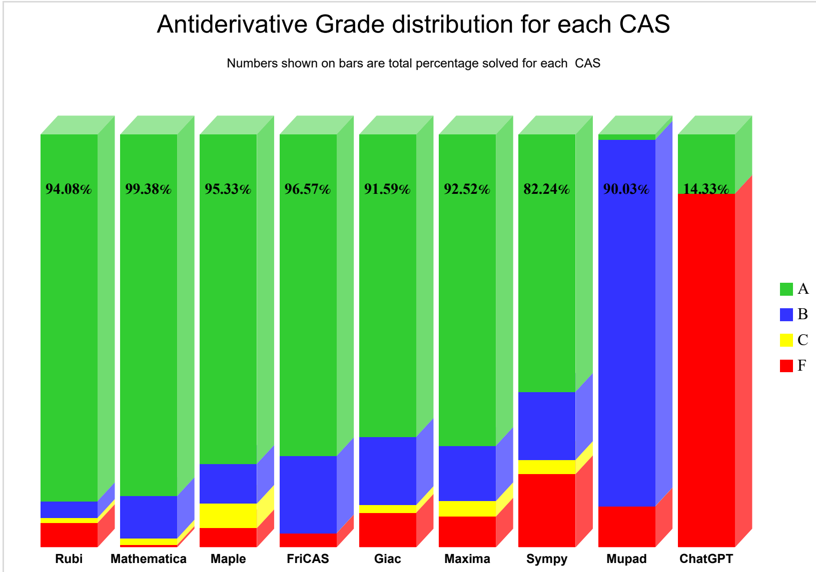
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

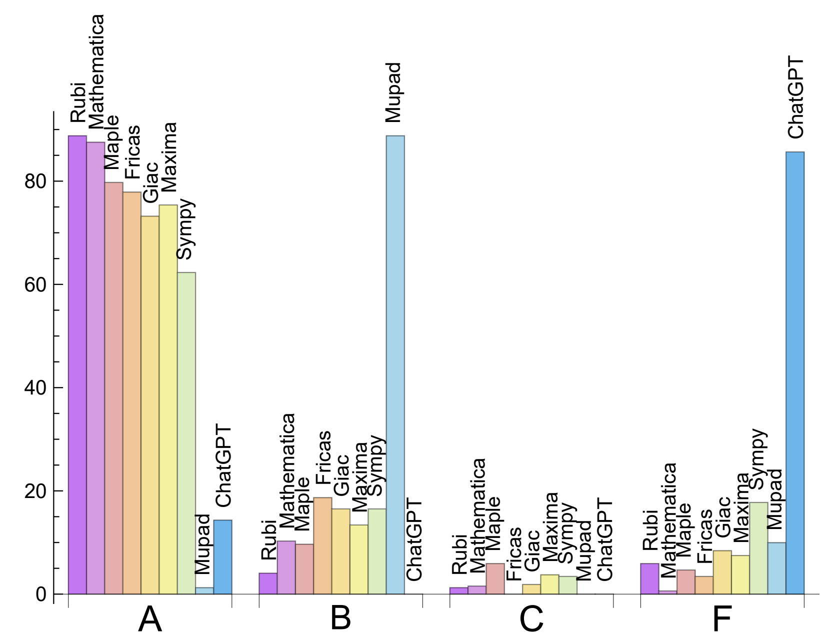
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.79	4.05	1.25	5.92
Mathematica	87.54	10.28	1.56	0.62
Maple	79.75	9.66	5.92	4.67
Fricas	77.88	18.69	0.00	3.43
Maxima	75.39	13.40	3.74	7.48
Giac	73.21	16.51	1.87	8.41
Sympy	62.31	16.51	3.43	17.76
ChatGPT	14.33	0.00	0.00	85.67
Mupad	N/A	88.79	0.00	9.97

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	19	100.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	15	86.67 %	13.33 %	0.00 %
Fricas	11	45.45 %	18.18 %	36.36 %
Giac	27	92.59 %	3.70 %	3.70 %
Maxima	24	91.67 %	4.17 %	4.17 %
Sympy	57	87.72 %	12.28 %	0.00 %
Mupad	32	100.00 %	0.00 %	0.00 %
ChatGPT	275	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

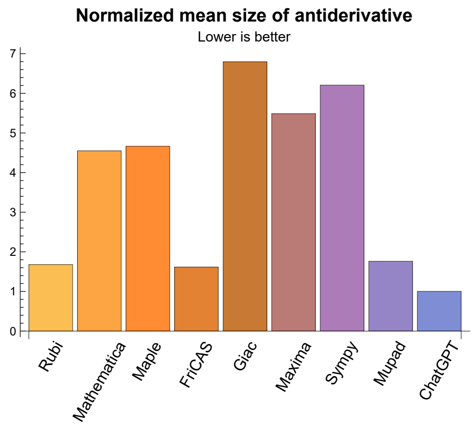
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

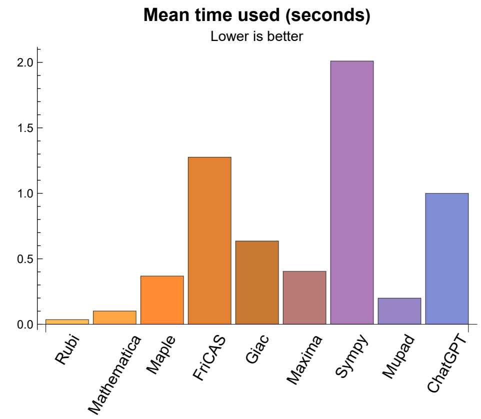
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	27.09	1.68	17.00	1.00
Mathematica	0.10	97.68	4.55	18.00	1.00
Maple	0.37	91.94	4.67	17.00	1.00
Maxima	0.40	103.22	5.49	15.00	0.93
Fricas	1.28	25.96	1.62	17.00	1.00
Sympy	2.01	133.08	6.21	15.00	0.92
Giac	0.64	111.91	6.80	16.00	0.95
Mupad	0.20	26.91	1.76	13.00	0.86
ChatGPT	N/A	9.43	1.00	9.00	0.88

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
7
6
5
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
ChatGPT


Mean time used (seconds)
Lower is better
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
ChatGPT

1.4 list of integrals that has no closed form antiderivative

$\{194, 225, 226, 236\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

ChatGPT {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{58, 270\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

ChatGPT {}

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

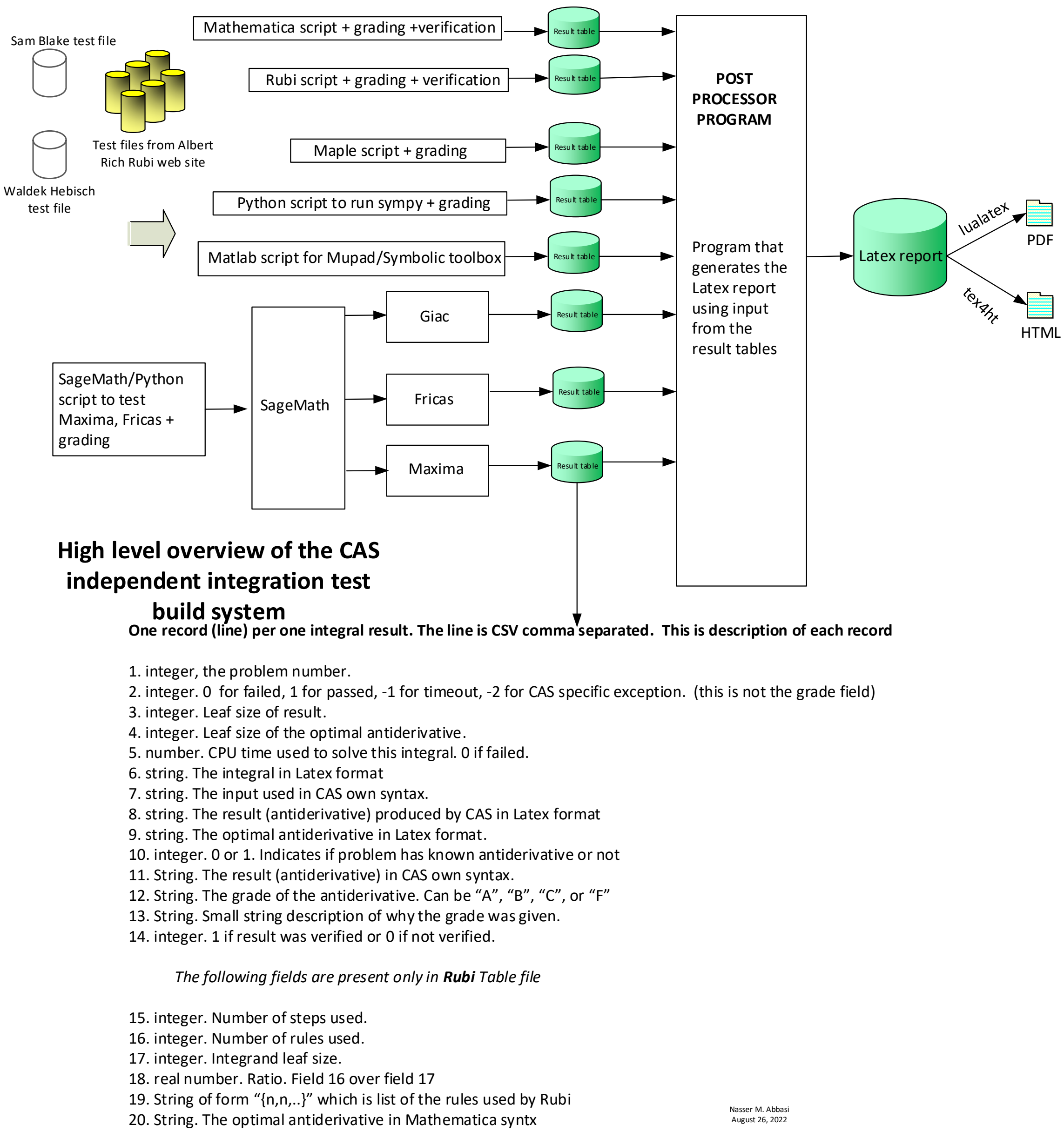
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 247, 248, 249, 251, 252, 253, 255, 256, 259, 260, 261, 262, 263, 264, 267, 268, 269, 270, 271, 273, 274, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 314, 315, 316, 318, 319, 321 }

B grade: { 46, 78, 118, 119, 196, 207, 242, 244, 254, 266, 272, 276, 287 }

C grade: { 246, 258, 288, 294 }

F grade: { 20, 77, 140, 147, 180, 211, 243, 245, 250, 257, 265, 275, 277, 293, 298, 307, 313, 317, 320 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 56, 59, 60, 61, 62, 63, 65, 66, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 98, 99, 100, 101, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321 }

B grade: { 20, 36, 43, 55, 57, 64, 67, 71, 78, 89, 94, 97, 105, 121, 129, 159, 168, 170, 183, 193, 196, 197, 207, 213, 220, 227, 238, 247, 258, 280, 290, 310, 315 }

C grade: { 49, 58, 102, 147, 209 }

F grade: { 266, 294 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 176, 177, 178, 179, 181, 182, 183, 184, 185, 187, 189, 190, 192, 193, 194, 195, 197, 198, 199, 200, 201, 202, 203, 204, 206, 209, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 235, 236, 237, 239, 240, 241, 242, 243, 245, 247, 248, 249, 251, 252, 254, 255, 256, 260, 261, 262, 264, 267, 269, 271, 272, 273, 274, 275, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 289, 290, 292, 295, 296, 297, 298, 299, 300, 301, 305, 306, 310, 311, 314, 316, 321 }

B grade: { 9, 19, 43, 46, 52, 64, 71, 84, 94, 105, 118, 124, 135, 180, 186, 191, 196, 208, 210, 211, 220, 227, 234, 238, 244, 253, 304, 308, 309, 312, 319 }

C grade: { 20, 81, 93, 116, 131, 205, 207, 246, 257, 258, 263, 268, 276, 278, 288, 293, 303, 313, 317 }

F grade: { 173, 175, 188, 250, 259, 265, 266, 270, 291, 294, 302, 307, 315, 318, 320 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 24, 25, 28, 31, 33, 34, 35, 37, 38, 39, 40, 41, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 148, 149, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 185, 186, 189, 190, 192, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 236, 237, 241, 242, 243, 245, 247, 248, 249, 250, 251, 252, 253, 260, 262, 263, 264, 265, 267, 269, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 291, 292, 295, 296, 298, 300, 304, 306, 308, 310, 311, 312, 314, 316, 317, 318, 319, 321 }

B grade: { 7, 19, 26, 27, 29, 30, 32, 36, 43, 64, 71, 74, 78, 89, 90, 93, 94, 105, 113, 128, 145, 156, 180, 184, 187, 193, 196, 197, 207, 208, 227, 238, 239, 244, 254, 255, 258, 261, 290, 293, 297, 301, 303 }

C grade: { 23, 119, 147, 150, 167, 173, 246, 256, 277, 289, 302, 309 }

F grade: { 20, 42, 46, 52, 58, 59, 116, 170, 188, 191, 235, 240, 257, 259, 266, 268, 270, 294, 299, 305, 307, 313, 315, 320 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 44, 45, 47, 48, 49, 50, 51, 54, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 136, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 171, 172, 173, 174, 175, 176, 177, 179, 181, 183, 184, 185, 187, 188, 189, 191, 192, 194, 195, 198, 201, 202, 203, 204, 206, 208, 209, 210, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 229, 230, 231, 232, 233, 235, 236, 237, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251, 252, 254, 256, 258, 260, 261, 262, 263, 264, 265, 267, 269, 271, 272, 273, 274, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 293, 295, 296, 298, 299, 300, 301, 303, 305, 306, 307, 308, 309, 311, 314, 315, 316, 317, 319, 320, 321 }

B grade: { 7, 10, 20, 27, 30, 36, 43, 52, 53, 55, 64, 71, 78, 80, 89, 94, 121, 129, 135, 137, 140, 149, 156, 159, 168, 169, 170, 178, 180, 182, 186, 190, 193, 196, 197, 200, 207, 211, 213, 224, 228, 234, 238, 239, 243, 247, 253, 255, 257, 275, 280, 290, 292, 294, 297, 304, 310, 312, 313, 318 }

C grade: { }

F grade: { 11, 46, 105, 199, 205, 227, 259, 266, 268, 270, 302 }

2.1.6 Sympy

A grade: { 4, 5, 6, 7, 9, 13, 15, 17, 19, 22, 23, 25, 26, 27, 28, 33, 34, 35, 40, 41, 44, 45, 47, 48, 50, 54, 55, 56, 57, 60, 61, 62, 63, 65, 66, 67, 68, 69, 72, 73, 75, 76, 77, 79, 80, 82, 83, 84, 85, 86, 88, 90, 91, 92, 95, 96, 97, 99, 100, 101, 102, 104, 106, 107, 108, 109, 110, 111, 114, 116, 117, 120, 123, 124, 125, 126, 127, 129, 130, 131, 133, 134, 135, 136, 138, 139, 140, 141, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 164, 165, 166, 167, 168, 171, 174, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 190, 192, 194, 195, 198, 199, 200, 201, 202, 203, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 229, 230, 231, 232, 236, 237, 240, 242, 245, 247, 248, 249, 251, 253, 254, 255, 256, 261, 262, 263, 264, 265, 269, 273, 274, 279, 281, 282, 283, 285, 286, 287, 289, 293, 295, 296, 298, 300, 301, 305, 306, 308, 309, 310, 311, 312, 314, 316, 321 }

B grade: { 1, 2, 14, 16, 18, 29, 30, 31, 36, 38, 39, 43, 51, 53, 64, 70, 71, 78, 87, 89, 93, 94, 103, 105, 118, 119, 121, 137, 142, 159, 189, 193, 197, 204, 211, 227, 228, 234, 238, 239, 244, 260, 268, 272, 280, 284, 288, 292, 294, 302, 303, 304, 315 }

C grade: { 3, 8, 24, 98, 112, 161, 223, 241, 252, 267, 278 }

F grade: { 10, 11, 12, 20, 21, 32, 37, 42, 46, 49, 52, 58, 59, 74, 81, 113, 115, 122, 128, 132, 147, 163, 169, 170, 172, 173, 175, 184, 188, 191, 196, 207, 219, 233, 235, 243, 246, 250, 257, 258, 259, 266, 270, 271, 275, 276, 277, 290, 291, 297, 299, 307, 313, 317, 318, 319, 320 }

2.1.7 Giac

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 31, 33, 34, 35, 37, 39, 40, 41, 42, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 118, 120, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 171, 173, 174, 176, 177, 178, 179, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 195, 198, 199, 200, 202, 203, 204, 206, 210, 212, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 231, 233, 234, 235, 236, 237, 241, 242, 243, 245, 248, 249, 251, 253, 254, 256, 260, 261, 262, 263, 264, 267, 269, 272, 273, 274, 276, 277, 279, 280, 281, 282, 283, 284, 286, 288, 289, 291, 292, 295, 296, 299, 300, 301, 303, 305, 306, 307, 308, 310, 311, 312, 314, 316, 317, 318, 319, 321 }

B grade: { 4, 16, 19, 20, 29, 30, 36, 43, 46, 55, 64, 71, 77, 78, 89, 94, 105, 113, 117, 121, 129, 137, 145, 168, 170, 180, 181, 187, 193, 196, 197, 207, 208, 209, 213, 217, 227, 228, 232, 238, 239, 244, 247, 255, 257, 258, 265, 285, 290, 293, 298, 304, 313 }

C grade: { 119, 150, 246, 287, 302, 309 }

F grade: { 11, 32, 38, 58, 59, 74, 116, 128, 172, 175, 201, 205, 211, 240, 250, 252, 259, 266, 268, 270, 271, 275, 278, 294, 297, 315, 320 }

2.1.8 Mupad

A grade: { 194, 225, 226, 236 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 227, 228, 229, 230, 231, 232, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 260, 261, 263, 264, 267, 269, 272, 273, 274, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 316, 317, 319, 321 }

C grade: { }

F grade: { 11, 42, 52, 63, 81, 93, 115, 116, 172, 191, 205, 223, 233, 250, 258, 259, 262, 265, 266, 268, 270, 271, 275, 278, 291, 294, 297, 307, 309, 315, 318, 320 }

2.1.9 ChatGPT

A grade: { 17, 19, 28, 30, 36, 50, 54, 55, 57, 69, 75, 76, 83, 87, 91, 92, 99, 104, 106, 121, 125, 136, 137, 138, 139, 144, 151, 157, 158, 160, 162, 176, 177, 178, 183, 184, 186, 192, 206, 213, 230, 233, 237, 279, 281, 295 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 84, 85, 86, 88, 89, 90, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 159, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 182, 185, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	112	13	14	13
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.48	0.52	0.56	0.52
time (sec)	N/A	0.130	0.025	0.681	0.340	0.591	1.309	0.402	0.363	1.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	24	23	13	65	13	14	20
N.S.	1	1.00	1.82	1.41	1.35	0.76	3.82	0.76	0.82	1.18
time (sec)	N/A	0.025	0.016	0.200	0.353	0.607	0.744	0.379	0.069	1.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	32	21	23	22	22	144	22	19	15
N.S.	1	1.52	1.00	1.10	1.05	1.05	6.86	1.05	0.90	0.71
time (sec)	N/A	0.003	0.036	0.120	0.323	0.544	1.111	0.400	0.073	1.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	32	21	23	20	60	31	15
N.S.	1	1.00	0.81	1.19	0.78	0.85	0.74	2.22	1.15	0.56
time (sec)	N/A	0.007	0.016	0.044	0.337	0.582	0.052	0.407	0.234	1.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	20	20	21	37	21	24	20
N.S.	1	1.00	0.96	0.74	0.74	0.78	1.37	0.78	0.89	0.74
time (sec)	N/A	0.006	0.005	0.119	0.322	0.606	0.227	0.414	0.106	1.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	15	14	11	11	8	11	11	19
N.S.	1	1.00	1.25	1.17	0.92	0.92	0.67	0.92	0.92	1.58
time (sec)	N/A	0.005	0.009	0.027	0.373	0.582	0.027	0.393	0.081	1.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	33	50	69	46	46	30	6
N.S.	1	1.00	1.03	1.10	1.67	2.30	1.53	1.53	1.00	0.20
time (sec)	N/A	0.019	0.018	0.250	0.347	0.617	0.068	0.390	0.173	1.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	24	10	10	14
N.S.	1	1.00	1.00	0.79	0.71	0.71	1.71	0.71	0.71	1.00
time (sec)	N/A	0.004	0.013	0.244	0.475	0.564	0.679	0.427	0.194	1.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	29	15	11	10	13	11	13
N.S.	1	1.00	1.00	2.23	1.15	0.85	0.77	1.00	0.85	1.00
time (sec)	N/A	0.003	0.002	0.063	0.360	0.556	0.033	0.420	0.089	1.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	98	40	49	80	180	0	80	65	13
N.S.	1	1.38	0.56	0.69	1.13	2.54	0.00	1.13	0.92	0.18
time (sec)	N/A	0.047	0.025	0.086	0.560	0.622	0.000	0.441	0.153	1.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	70	56	14	0	0	-1	10
N.S.	1	1.00	1.00	0.79	0.63	0.16	0.00	0.00	-0.01	0.11
time (sec)	N/A	0.046	0.006	0.079	0.502	0.572	0.000	0.000	0.000	1.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	36	35	35	0	36	35	0
N.S.	1	1.00	0.96	0.68	0.66	0.66	0.00	0.68	0.66	0.00
time (sec)	N/A	0.014	0.014	0.067	0.441	0.572	0.000	0.419	0.098	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	0.00
time (sec)	N/A	0.066	0.015	0.015	0.389	0.576	0.051	0.447	0.357	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	46	29	36	28	31	66	28	25	0
N.S.	1	1.59	1.00	1.24	0.97	1.07	2.28	0.97	0.86	0.00
time (sec)	N/A	0.007	0.079	0.055	0.392	0.696	113.789	0.430	0.470	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	16	36
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	0.84	1.89
time (sec)	N/A	0.001	0.005	0.057	0.447	0.549	0.038	0.483	0.057	1.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	37	26	25	25	26	29	13	33
N.S.	1	1.00	1.76	1.24	1.19	1.19	1.24	1.38	0.62	1.57
time (sec)	N/A	0.003	0.003	0.021	0.361	0.578	0.108	0.423	0.071	1.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	11	8	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73	1.00
time (sec)	N/A	0.004	0.005	0.024	0.351	0.568	0.072	0.483	0.107	1.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	16	15	11	15	16	14	10
N.S.	1	1.00	1.00	2.00	1.88	1.38	1.88	2.00	1.75	1.25
time (sec)	N/A	0.013	0.026	0.047	0.331	0.604	0.140	0.470	0.137	1.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	25	65	10	8	79	10	10
N.S.	1	1.00	0.91	2.27	5.91	0.91	0.73	7.18	0.91	0.91
time (sec)	N/A	0.140	0.042	0.172	0.454	0.625	0.096	0.429	0.119	1.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	B	C	F	B	F	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	93	57	0	251	0	81	233	14
N.S.	1	0.00	3.32	2.04	0.00	8.96	0.00	2.89	8.32	0.50
time (sec)	N/A	0.091	0.619	0.185	0.000	0.979	0.000	0.503	0.312	1.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	20	18	37	0	37	21	15
N.S.	1	1.00	0.88	0.77	0.69	1.42	0.00	1.42	0.81	0.58
time (sec)	N/A	0.012	0.048	0.025	0.450	0.594	0.000	0.483	0.175	1.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	14	17	11	11	10	11	11	0
N.S.	1	1.00	0.64	0.77	0.50	0.50	0.45	0.50	0.50	0.00
time (sec)	N/A	0.012	0.013	0.016	0.334	0.560	0.024	0.499	0.072	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	12	6	3	6	6	7
N.S.	1	1.00	1.00	1.17	2.00	1.00	0.50	1.00	1.00	1.17
time (sec)	N/A	0.004	0.007	0.012	0.380	0.538	0.031	0.478	0.154	1.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	72	61	38	52	124	42	41	29
N.S.	1	0.96	1.36	1.15	0.72	0.98	2.34	0.79	0.77	0.55
time (sec)	N/A	0.008	0.049	0.088	0.413	0.579	2.756	0.459	0.098	1.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13	11
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81	0.69
time (sec)	N/A	0.007	0.001	0.012	0.334	0.556	0.022	0.463	0.022	1.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	21	6	5	6	6	16
N.S.	1	1.00	1.00	1.17	3.50	1.00	0.83	1.00	1.00	2.67
time (sec)	N/A	0.004	0.004	0.009	0.387	0.571	0.034	0.502	0.054	1.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	16	18	20	4	14	10
N.S.	1	1.00	1.00	1.25	4.00	4.50	5.00	1.00	3.50	2.50
time (sec)	N/A	0.002	0.055	0.124	0.427	0.578	0.621	0.479	0.180	1.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	18	16	15	15	12	16	15	15
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88	0.88
time (sec)	N/A	0.005	0.002	0.057	0.330	0.555	0.016	0.430	0.032	1.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	31	301	29	162	39	39	8
N.S.	1	1.00	1.82	1.82	17.71	1.71	9.53	2.29	2.29	0.47
time (sec)	N/A	0.055	0.039	0.100	0.439	0.589	11.965	0.464	0.374	1.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	10	20	15	10	4	4
N.S.	1	1.00	1.00	1.25	2.50	5.00	3.75	2.50	1.00	1.00
time (sec)	N/A	0.004	0.004	0.086	0.419	0.539	0.271	0.406	0.013	1.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	14	13	20	29	13	13	13
N.S.	1	1.00	1.59	0.82	0.76	1.18	1.71	0.76	0.76	0.76
time (sec)	N/A	0.015	0.014	0.905	0.351	0.578	0.014	0.474	0.222	1.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	0	15	0
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.00	1.15	0.00
time (sec)	N/A	0.030	0.105	0.490	0.550	0.591	0.000	0.000	0.304	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	24	25	36	22	22	15
N.S.	1	1.00	0.88	0.71	0.71	0.74	1.06	0.65	0.65	0.44
time (sec)	N/A	0.011	0.003	0.121	0.337	0.611	0.014	0.442	0.041	1.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	15	19	12	15	16	11
N.S.	1	1.00	0.84	0.84	0.79	1.00	0.63	0.79	0.84	0.58
time (sec)	N/A	0.002	0.002	0.016	0.429	0.561	0.045	0.463	0.064	1.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	10	13	15	13	13	7
N.S.	1	1.00	1.00	0.82	0.59	0.76	0.88	0.76	0.76	0.41
time (sec)	N/A	0.002	0.002	0.040	0.346	0.604	0.155	0.418	0.094	1.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	33	7	15	17	15	17	11	6
N.S.	1	1.00	11.00	2.33	5.00	5.67	5.00	5.67	3.67	2.00
time (sec)	N/A	0.001	0.003	0.046	0.346	0.598	0.062	0.434	0.038	1.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	11	8	7	21	0	20	16	10
N.S.	1	1.00	0.46	0.33	0.29	0.88	0.00	0.83	0.67	0.42
time (sec)	N/A	0.011	0.019	0.219	0.467	0.612	0.000	0.467	0.215	1.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	22	0	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	1.57	0.00	0.86	0.86
time (sec)	N/A	0.002	0.089	0.076	0.354	0.583	0.498	0.000	0.177	1.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	30	66	23	27	19
N.S.	1	1.00	1.00	0.77	0.74	0.97	2.13	0.74	0.87	0.61
time (sec)	N/A	0.020	0.020	0.621	0.389	0.619	1.336	0.481	0.277	1.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5	8
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00	1.60
time (sec)	N/A	0.003	0.006	0.019	0.359	0.588	0.104	0.443	0.131	1.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	16	8	14	11	11	10	11	11	10
N.S.	1	2.00	1.00	1.75	1.38	1.38	1.25	1.38	1.38	1.25
time (sec)	N/A	0.004	0.008	0.010	0.375	0.578	0.022	0.490	0.086	1.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	F	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	43	0	33	0	16	-1	27
N.S.	1	1.00	1.17	1.05	0.00	0.80	0.00	0.39	-0.02	0.66
time (sec)	N/A	0.013	0.078	0.545	0.000	0.656	0.000	0.499	0.000	1.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	25	20	15	15	12	16	6	19
N.S.	1	1.00	4.17	3.33	2.50	2.50	2.00	2.67	1.00	3.17
time (sec)	N/A	0.002	0.004	0.055	0.331	0.564	0.043	0.423	0.090	1.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	0.00
time (sec)	N/A	0.019	0.007	0.000	0.398	0.573	0.054	0.455	0.002	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	29	35	47	28	36	35	26	25	26	28
N.S.	1	1.21	1.62	0.97	1.24	1.21	0.90	0.86	0.90	0.97
time (sec)	N/A	0.006	0.037	0.176	0.426	0.560	0.268	0.484	0.085	1.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	B	F(-2)	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	801	12	602	0	0	0	601	2500	0
N.S.	1	66.75	1.00	50.17	0.00	0.00	0.00	50.08	208.33	0.00
time (sec)	N/A	0.332	0.003	1.491	0.000	0.000	0.000	0.485	4.667	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	7	7	7	7	7	0
N.S.	1	1.00	1.00	1.10	0.70	0.70	0.70	0.70	0.70	0.00
time (sec)	N/A	0.020	0.009	0.033	0.349	0.571	0.916	0.403	0.049	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	27	32	46	29	23	21	29
N.S.	1	1.00	0.86	0.96	1.14	1.64	1.04	0.82	0.75	1.04
time (sec)	N/A	0.018	0.009	0.020	0.377	0.613	0.078	0.483	0.048	1.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	C	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	72	39	43	32	0	29	43	18
N.S.	1	1.00	1.89	1.03	1.13	0.84	0.00	0.76	1.13	0.47
time (sec)	N/A	0.009	0.060	0.087	0.465	0.589	0.000	0.483	0.094	1.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	25	15	14	15	15	14	14
N.S.	1	1.00	0.90	1.25	0.75	0.70	0.75	0.75	0.70	0.70
time (sec)	N/A	0.004	0.007	0.057	0.350	0.630	0.045	0.402	0.056	1.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	105	13	9	13
N.S.	1	1.00	0.71	0.67	0.62	0.67	5.00	0.62	0.43	0.62
time (sec)	N/A	0.004	0.003	0.045	0.408	0.593	2.266	0.421	0.029	1.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	F	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	32	21	41	0	37	0	21	-1	19
N.S.	1	1.60	1.05	2.05	0.00	1.85	0.00	1.05	-0.05	0.95
time (sec)	N/A	0.102	0.062	0.064	0.000	0.567	0.000	0.411	0.000	1.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	186	138	16	53	0
N.S.	1	1.00	0.81	0.74	0.70	6.89	5.11	0.59	1.96	0.00
time (sec)	N/A	0.006	0.016	0.052	0.471	0.573	83.214	0.443	0.732	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74	0.74
time (sec)	N/A	0.002	0.032	0.181	0.451	0.578	0.201	0.457	0.072	1.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	10	11	21	10	29	11	9
N.S.	1	1.00	2.30	1.00	1.10	2.10	1.00	2.90	1.10	0.90
time (sec)	N/A	0.009	0.044	0.124	0.418	0.581	0.288	0.437	0.088	1.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	7	8	7	7	8	9
N.S.	1	1.00	1.00	1.12	0.88	1.00	0.88	0.88	1.00	1.12
time (sec)	N/A	0.001	0.002	0.010	0.368	0.577	0.026	0.490	0.021	1.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	10	10	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.80	1.00	1.00	1.00
time (sec)	N/A	0.004	0.006	0.030	0.332	0.571	0.318	0.452	0.067	1.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	C	A	F	A	F	F(-2)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	53	0	64	0	0	64	51
N.S.	1	1.00	1.17	1.00	0.00	1.21	0.00	0.00	1.21	0.96
time (sec)	N/A	0.028	0.075	0.025	0.000	0.578	0.000	0.000	1.588	1.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	18	0	18	0	0	179	21
N.S.	1	1.00	1.00	0.39	0.00	0.39	0.00	0.00	3.89	0.46
time (sec)	N/A	0.232	0.115	0.471	0.000	0.641	0.000	0.000	0.259	1.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	32	44	27	28	32	22
N.S.	1	1.00	0.95	0.81	0.86	1.19	0.73	0.76	0.86	0.59
time (sec)	N/A	0.025	0.007	0.020	0.398	0.557	0.036	0.414	0.089	1.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5	0
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00	0.00
time (sec)	N/A	0.018	0.015	0.134	0.376	0.596	4.355	0.475	0.214	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	13	10	16	13	17
N.S.	1	1.00	1.00	1.06	1.00	0.76	0.59	0.94	0.76	1.00
time (sec)	N/A	0.006	0.002	0.067	0.352	0.559	0.029	0.440	0.108	1.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	12	15	-1	39
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.71	0.88	-0.06	2.29
time (sec)	N/A	0.019	0.004	0.097	0.503	0.638	0.086	0.473	0.000	1.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	567	502	501	501	561	501	12	20
N.S.	1	1.00	24.65	21.83	21.78	21.78	24.39	21.78	0.52	0.87
time (sec)	N/A	0.003	0.004	0.091	0.378	0.565	0.098	0.437	0.125	1.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	12	12	11	11	14
N.S.	1	1.00	1.00	0.92	0.85	0.92	0.92	0.85	0.85	1.08
time (sec)	N/A	0.010	0.005	0.061	0.407	0.591	0.338	0.466	0.155	1.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12	6
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75	0.38
time (sec)	N/A	0.004	0.018	0.054	0.446	0.580	0.503	0.426	0.039	1.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	54	22	24	28	26	36	21	14
N.S.	1	1.00	2.16	0.88	0.96	1.12	1.04	1.44	0.84	0.56
time (sec)	N/A	0.003	0.017	0.094	0.444	0.569	0.064	0.491	0.128	1.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	9	9	10	9	9	23
N.S.	1	1.00	1.00	0.83	0.75	0.75	0.83	0.75	0.75	1.92
time (sec)	N/A	0.006	0.014	0.038	0.437	0.565	0.291	0.510	0.106	1.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	7	8	8	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.58	0.67	0.67	0.67
time (sec)	N/A	0.003	0.003	0.069	0.432	0.571	0.051	0.409	0.095	1.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	13	13	9	8	15	22	8	13	7
N.S.	1	1.62	1.62	1.12	1.00	1.88	2.75	1.00	1.62	0.88
time (sec)	N/A	0.009	0.010	0.099	0.399	0.575	0.476	0.519	0.202	1.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	39	27	26	80	22	49	30	20
N.S.	1	1.00	3.55	2.45	2.36	7.27	2.00	4.45	2.73	1.82
time (sec)	N/A	0.008	0.008	0.163	0.437	0.586	0.265	0.463	0.114	1.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	35	19	19	41	19	19	5
N.S.	1	1.00	0.74	1.30	0.70	0.70	1.52	0.70	0.70	0.19
time (sec)	N/A	0.007	0.011	0.120	0.392	0.567	0.101	0.406	0.086	1.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	16	11	9	10	11	10	0
N.S.	1	1.00	1.00	1.60	1.10	0.90	1.00	1.10	1.00	0.00
time (sec)	N/A	0.022	0.030	0.019	0.443	0.577	0.702	0.586	0.123	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	0	15	0
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.00	1.15	0.00
time (sec)	N/A	0.028	0.012	0.365	0.487	0.592	0.000	0.000	0.002	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	5	7	14	16	5
N.S.	1	1.00	1.00	1.07	1.00	0.36	0.50	1.00	1.14	0.36
time (sec)	N/A	0.003	0.006	0.014	0.375	0.569	0.024	0.525	0.076	1.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	2	2
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.000	0.000	0.009	0.379	0.567	0.016	1.023	0.008	1.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	A	A	A	A	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	0	7	8	7	7	5	20	7	19
N.S.	1	0.00	1.00	1.14	1.00	1.00	0.71	2.86	1.00	2.71
time (sec)	N/A	0.100	0.451	0.116	0.432	0.583	0.513	0.890	0.275	1.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	23	23	8	19	19	15	19	19	19
N.S.	1	2.09	2.09	0.73	1.73	1.73	1.36	1.73	1.73	1.73
time (sec)	N/A	0.002	0.000	0.008	0.342	0.554	0.006	0.485	0.049	1.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	23	22	36	27	23	25	25
N.S.	1	1.00	1.23	0.66	0.63	1.03	0.77	0.66	0.71	0.71
time (sec)	N/A	0.003	0.062	0.200	0.437	0.585	0.143	0.508	0.032	1.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	20	19	45	15	19	45	17
N.S.	1	1.00	1.07	0.74	0.70	1.67	0.56	0.70	1.67	0.63
time (sec)	N/A	0.004	0.011	0.103	0.335	0.615	0.051	0.427	0.197	1.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	13	5	0	6	-1	3
N.S.	1	1.00	1.00	2.40	0.87	0.33	0.00	0.40	-0.07	0.20
time (sec)	N/A	0.021	0.008	0.315	0.345	0.635	0.000	0.437	0.000	1.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	5	7	6	43
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.56	0.78	0.67	4.78
time (sec)	N/A	0.008	0.000	0.010	0.407	0.552	0.011	0.474	0.016	1.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75	1.00
time (sec)	N/A	0.001	0.002	0.007	0.334	0.565	0.030	0.441	0.016	1.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	12	6	16	9	9	8	10	7	11
N.S.	1	2.00	1.00	2.67	1.50	1.50	1.33	1.67	1.17	1.83
time (sec)	N/A	0.006	0.008	0.014	0.332	0.578	0.022	0.461	0.084	1.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	24	14	19	10	19	14	10	18	13
N.S.	1	1.71	1.00	1.36	0.71	1.36	1.00	0.71	1.29	0.93
time (sec)	N/A	0.017	0.006	0.055	0.347	0.596	0.016	0.453	0.092	1.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80	0.80
time (sec)	N/A	0.007	0.013	0.065	0.333	0.596	0.118	0.530	0.166	1.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	9	6	6	7	6	6	6
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.17	1.00	1.00	1.00
time (sec)	N/A	0.003	0.003	0.023	0.477	0.579	0.014	0.483	0.029	1.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	52	31	35	21	21	48	21	28	9
N.S.	1	1.73	1.03	1.17	0.70	0.70	1.60	0.70	0.93	0.30
time (sec)	N/A	0.017	0.007	0.013	0.344	0.569	0.178	0.441	0.022	1.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	17	21	19	19	18	10
N.S.	1	1.00	2.38	1.50	2.12	2.62	2.38	2.38	2.25	1.25
time (sec)	N/A	0.006	0.004	0.074	0.374	0.621	0.035	0.497	0.228	1.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	21	6	5	6	6	15
N.S.	1	1.00	1.00	1.17	3.50	1.00	0.83	1.00	1.00	2.50
time (sec)	N/A	0.003	0.004	0.008	0.380	0.571	0.047	0.509	0.002	1.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	14	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.67	0.78	0.78	0.78
time (sec)	N/A	0.005	0.002	0.059	0.336	0.575	0.019	0.443	0.029	1.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	5	4	4	3	4	4	4
N.S.	1	1.00	1.00	0.62	0.50	0.50	0.38	0.50	0.50	0.50
time (sec)	N/A	0.002	0.003	0.142	0.469	0.558	0.037	0.519	0.094	1.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	111	89	28	105	26	-1	0
N.S.	1	1.00	1.73	7.40	5.93	1.87	7.00	1.73	-0.07	0.00
time (sec)	N/A	0.012	0.008	0.187	0.380	0.636	0.775	0.438	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	22	17	17	15	18	6	33
N.S.	1	1.00	2.88	2.75	2.12	2.12	1.88	2.25	0.75	4.12
time (sec)	N/A	0.002	0.003	0.070	0.371	0.563	0.033	0.541	0.071	1.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	23	22	36	27	23	25	25
N.S.	1	1.00	1.23	0.66	0.63	1.03	0.77	0.66	0.71	0.71
time (sec)	N/A	0.003	0.001	0.086	0.506	0.581	0.178	0.540	0.002	1.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	14	14	14	13	19
N.S.	1	1.00	1.00	0.82	0.82	0.82	0.82	0.82	0.76	1.12
time (sec)	N/A	0.014	0.010	3.464	0.322	0.592	0.084	0.515	0.498	1.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	10	10	16
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.73	0.91	0.91	1.45
time (sec)	N/A	0.005	0.008	0.038	0.325	0.568	0.264	0.477	0.023	1.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	18	14	24	39	18	18	14
N.S.	1	1.00	1.00	0.82	0.64	1.09	1.77	0.82	0.82	0.64
time (sec)	N/A	0.007	0.014	0.135	0.420	0.568	0.619	0.458	0.124	1.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	8	10	8	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.80	1.00	0.80	1.00
time (sec)	N/A	0.001	0.002	0.009	0.386	0.567	0.036	0.461	0.017	1.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	26	21	21	34	21	21	0
N.S.	1	1.00	0.83	0.74	0.60	0.60	0.97	0.60	0.60	0.00
time (sec)	N/A	0.014	0.013	0.020	0.375	0.589	0.170	0.432	0.212	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	9	9	7	9	11	28
N.S.	1	1.00	1.00	1.00	0.90	0.90	0.70	0.90	1.10	2.80
time (sec)	N/A	0.006	0.020	0.021	0.418	0.577	0.044	0.513	0.203	1.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	C	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	11	24	14	14	31	14	23	11
N.S.	1	1.00	0.32	0.71	0.41	0.41	0.91	0.41	0.68	0.32
time (sec)	N/A	0.015	0.002	0.008	0.416	0.570	0.071	0.484	0.115	1.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	13	26	11	13	12
N.S.	1	1.00	1.00	0.80	0.73	0.87	1.73	0.73	0.87	0.80
time (sec)	N/A	0.005	0.008	0.083	0.404	0.595	0.142	0.497	0.039	1.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00	1.00
time (sec)	N/A	0.002	0.003	0.026	0.437	0.590	0.013	0.430	0.123	1.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	B	F(-2)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	12138	10077	10076	0	12024	10076	15	14
N.S.	1	1.00	527.74	438.13	438.09	0.00	522.78	438.09	0.65	0.61
time (sec)	N/A	0.013	0.019	8.487	2.628	0.000	3.221	1.680	6.241	1.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	22	12	22	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.38	0.75	1.38	0.88	0.88
time (sec)	N/A	0.002	0.002	0.007	0.343	0.594	0.053	0.471	0.065	1.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	16	12	11	11	10	12	11	20
N.S.	1	1.00	1.07	0.80	0.73	0.73	0.67	0.80	0.73	1.33
time (sec)	N/A	0.003	0.002	0.069	0.374	0.584	0.018	0.481	0.024	1.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	15	14	16
N.S.	1	1.00	1.00	0.76	0.71	0.62	0.71	0.71	0.67	0.76
time (sec)	N/A	0.005	0.003	0.029	0.420	0.588	0.100	0.495	0.019	1.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	12	15	8	14
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.79	0.42	0.74
time (sec)	N/A	0.002	0.003	0.083	0.353	0.577	0.045	0.453	0.090	1.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	28	16	18	17	17	19	27	15	0
N.S.	1	1.47	0.84	0.95	0.89	0.89	1.00	1.42	0.79	0.00
time (sec)	N/A	0.160	0.021	0.148	0.563	0.620	1.393	0.515	0.267	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	23	22	32
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.88	1.28
time (sec)	N/A	0.022	0.005	0.033	0.489	0.635	0.087	0.439	0.031	1.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	13	22	68	18	26	25
N.S.	1	1.00	1.00	0.77	0.59	1.00	3.09	0.82	1.18	1.14
time (sec)	N/A	0.006	0.012	0.141	0.453	0.576	1.019	0.459	0.137	1.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	74	28	0	103	15	8
N.S.	1	1.00	1.00	0.84	3.89	1.47	0.00	5.42	0.79	0.42
time (sec)	N/A	0.012	0.006	0.115	0.460	0.637	0.000	0.451	0.081	1.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	20	19	18	15	22	18	21
N.S.	1	1.00	0.96	0.80	0.76	0.72	0.60	0.88	0.72	0.84
time (sec)	N/A	0.010	0.003	0.018	0.340	0.562	0.047	0.427	0.062	1.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	17	16	11	0	12	-1	32
N.S.	1	1.00	1.00	1.55	1.45	1.00	0.00	1.09	-0.09	2.91
time (sec)	N/A	0.014	0.030	0.087	0.421	0.569	0.000	0.451	0.000	1.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	57	0	26	42	0	-1	27
N.S.	1	1.00	1.00	1.54	0.00	0.70	1.14	0.00	-0.03	0.73
time (sec)	N/A	0.007	0.029	0.034	0.000	0.581	0.142	0.000	0.000	1.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	30	20	16	19	23	32	40	32	0
N.S.	1	1.50	1.00	0.80	0.95	1.15	1.60	2.00	1.60	0.00
time (sec)	N/A	0.014	0.009	0.038	0.476	0.595	0.253	0.495	0.201	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	29	8	28	6	5	29	6	6	12
N.S.	1	5.80	1.60	5.60	1.20	1.00	5.80	1.20	1.20	2.40
time (sec)	N/A	0.015	0.004	0.153	0.337	0.588	0.019	0.494	0.110	1.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	A	C	A	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	37	16	20	19	12	31	22	14	9
N.S.	1	2.31	1.00	1.25	1.19	0.75	1.94	1.38	0.88	0.56
time (sec)	N/A	0.004	0.013	0.081	0.327	0.556	0.706	0.504	0.036	1.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	0.00
time (sec)	N/A	0.005	0.002	0.030	0.414	0.596	0.112	0.451	0.040	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	33	7	6	17	15	25	11	6
N.S.	1	1.00	11.00	2.33	2.00	5.67	5.00	8.33	3.67	2.00
time (sec)	N/A	0.001	0.003	0.019	0.340	0.608	0.058	0.458	0.002	1.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	0	3	3	15
N.S.	1	1.00	1.00	1.00	0.75	0.75	0.00	0.75	0.75	3.75
time (sec)	N/A	0.004	0.004	0.422	0.352	0.591	0.000	0.454	0.146	1.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17	18
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61	0.64
time (sec)	N/A	0.006	0.002	0.014	0.362	0.564	0.036	0.409	0.036	1.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	59	18	18	20	18	18	19
N.S.	1	1.00	1.00	2.68	0.82	0.82	0.91	0.82	0.82	0.86
time (sec)	N/A	0.009	0.010	0.090	0.545	0.578	0.095	0.441	0.058	1.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	1.00	1.00
time (sec)	N/A	0.002	0.000	0.025	0.365	0.571	0.030	0.425	0.051	1.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	28	28	21	20	27	22	21	22	20
N.S.	1	1.27	1.27	0.95	0.91	1.23	1.00	0.95	1.00	0.91
time (sec)	N/A	0.006	0.006	0.071	0.378	0.567	0.040	0.468	0.159	1.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	17	16	15	19	16	18	5
N.S.	1	1.00	1.20	1.13	1.07	1.00	1.27	1.07	1.20	0.33
time (sec)	N/A	0.008	0.006	0.030	0.366	0.592	0.081	0.434	0.302	1.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	0	15	0
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.00	1.15	0.00
time (sec)	N/A	0.030	0.011	0.352	0.598	0.605	0.000	0.000	0.002	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	16	5	4	14	3	25	4	4
N.S.	1	1.00	2.67	0.83	0.67	2.33	0.50	4.17	0.67	0.67
time (sec)	N/A	0.001	0.010	0.084	0.438	0.567	0.065	0.507	0.084	1.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	11	7	7	7	9
N.S.	1	1.00	1.00	1.14	1.00	1.57	1.00	1.00	1.00	1.29
time (sec)	N/A	0.016	0.003	0.018	0.390	0.579	0.020	0.496	0.106	1.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	47	25	25	27	25	25	22
N.S.	1	1.00	0.89	1.34	0.71	0.71	0.77	0.71	0.71	0.63
time (sec)	N/A	0.024	0.009	0.170	0.374	0.628	5.226	0.462	0.188	1.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	32	116	39	47	0	48	44	46
N.S.	1	1.00	0.45	1.63	0.55	0.66	0.00	0.68	0.62	0.65
time (sec)	N/A	0.031	0.031	0.153	0.497	0.617	0.000	0.488	0.255	1.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	13	12	9	9	8	10	9	11
N.S.	1	1.00	1.08	1.00	0.75	0.75	0.67	0.83	0.75	0.92
time (sec)	N/A	0.016	0.012	0.019	0.357	0.592	0.034	0.462	0.108	1.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	12	12	19	12	12	25
N.S.	1	1.00	1.29	1.07	0.86	0.86	1.36	0.86	0.86	1.79
time (sec)	N/A	0.006	0.003	0.028	0.428	0.581	0.023	0.511	0.031	1.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	20	7	11	7	7	7	13
N.S.	1	1.00	1.00	4.00	1.40	2.20	1.40	1.40	1.40	2.60
time (sec)	N/A	0.009	0.008	0.112	0.362	0.608	0.035	0.468	0.205	1.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	11	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.85
time (sec)	N/A	0.002	0.002	0.335	0.366	0.588	0.031	0.452	0.033	1.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	3	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	1.00	1.00
time (sec)	N/A	0.002	0.004	0.023	0.405	0.587	0.059	0.484	0.022	1.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	8	11	10	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.53	0.73	0.67	0.73
time (sec)	N/A	0.001	0.000	0.008	0.340	0.546	0.006	0.445	0.018	1.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	10	9	9	8	9	7	9
N.S.	1	1.00	0.86	0.71	0.64	0.64	0.57	0.64	0.50	0.64
time (sec)	N/A	0.001	0.002	0.011	0.334	0.562	0.027	0.485	0.024	1.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	A	A	B	A	A	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	11	9	8	20	14	12	8	26
N.S.	1	0.00	1.00	0.82	0.73	1.82	1.27	1.09	0.73	2.36
time (sec)	N/A	0.034	0.020	0.125	0.478	0.556	0.121	0.431	0.150	1.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7	0
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78	0.00
time (sec)	N/A	0.015	0.003	0.024	0.328	0.575	0.063	0.452	0.219	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	19	16	20	36	16	10	16
N.S.	1	1.00	1.00	1.36	1.14	1.43	2.57	1.14	0.71	1.14
time (sec)	N/A	0.007	0.002	0.024	0.500	0.581	0.386	0.474	0.104	1.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	34	33	25	37	49	25	28
N.S.	1	1.00	0.87	0.76	0.73	0.56	0.82	1.09	0.56	0.62
time (sec)	N/A	0.012	0.025	0.045	0.481	22.015	0.235	0.506	0.592	1.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75	1.00
time (sec)	N/A	0.001	0.002	0.006	0.367	0.557	0.021	0.441	0.002	1.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	21	5	5	21	5	13
N.S.	1	1.00	1.00	1.00	3.50	0.83	0.83	3.50	0.83	2.17
time (sec)	N/A	0.004	0.003	0.073	0.378	0.586	0.139	0.496	0.048	1.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	21	13	13	12	13	13	13
N.S.	1	1.00	0.62	0.81	0.50	0.50	0.46	0.50	0.50	0.50
time (sec)	N/A	0.012	0.016	0.026	0.408	0.572	0.041	0.447	0.114	1.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	C	A	C	A	F	A	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	0	27	10	473	12	0	12	9	22
N.S.	1	0.00	2.70	1.00	47.30	1.20	0.00	1.20	0.90	2.20
time (sec)	N/A	0.118	0.334	0.198	0.471	0.609	0.000	0.419	0.068	1.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	35	34	32	51	34	34	29
N.S.	1	1.00	0.89	0.62	0.61	0.57	0.91	0.61	0.61	0.52
time (sec)	N/A	0.056	0.022	0.021	0.350	0.587	11.200	0.471	0.031	1.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	21	8	7	7	0
N.S.	1	1.00	1.00	0.89	0.78	2.33	0.89	0.78	0.78	0.00
time (sec)	N/A	0.011	1.365	0.205	0.377	0.591	0.372	0.499	0.174	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	C	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	27	10	19	29	14	15
N.S.	1	1.00	1.00	1.07	1.80	0.67	1.27	1.93	0.93	1.00
time (sec)	N/A	0.050	0.030	0.144	0.445	0.574	2.121	0.509	0.063	1.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	23	22	19	37	19	20	19
N.S.	1	1.00	0.74	0.85	0.81	0.70	1.37	0.70	0.74	0.70
time (sec)	N/A	0.007	0.012	0.084	0.477	0.571	0.163	0.421	0.051	1.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	26	18	20	0
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.30	0.90	1.00	0.00
time (sec)	N/A	0.013	0.005	0.023	0.435	0.570	0.037	0.475	0.077	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	25	20	19	19	34	38	30	0
N.S.	1	1.00	0.60	0.48	0.45	0.45	0.81	0.90	0.71	0.00
time (sec)	N/A	0.016	0.009	0.019	0.388	0.574	0.566	0.485	0.113	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	9	5	8	26	8
N.S.	1	1.00	1.00	1.14	1.00	1.29	0.71	1.14	3.71	1.14
time (sec)	N/A	0.002	0.003	0.097	0.322	0.615	0.013	0.470	0.165	1.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	19	19	20	25	19
N.S.	1	1.00	1.00	0.74	0.70	0.70	0.70	0.74	0.93	0.70
time (sec)	N/A	0.009	0.004	0.024	0.457	0.580	0.060	0.421	0.105	1.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	30	20	16	30	39	36	26	42	8
N.S.	1	1.50	1.00	0.80	1.50	1.95	1.80	1.30	2.10	0.40
time (sec)	N/A	0.013	0.010	0.033	0.489	0.573	0.210	0.467	0.127	1.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.69	0.69
time (sec)	N/A	0.002	0.008	0.105	0.331	0.559	0.072	0.482	0.039	1.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	9	8	13	9	13
N.S.	1	1.00	1.00	1.08	1.00	0.69	0.62	1.00	0.69	1.00
time (sec)	N/A	0.004	0.002	0.011	0.362	0.579	0.042	0.453	0.087	1.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	9	4	3	8	7	5	5	5
N.S.	1	1.00	3.00	1.33	1.00	2.67	2.33	1.67	1.67	1.67
time (sec)	N/A	0.002	0.005	0.064	0.372	0.557	0.217	0.471	0.021	1.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	14	17	11	11	10	11	11	16
N.S.	1	1.00	0.64	0.77	0.50	0.50	0.45	0.50	0.50	0.73
time (sec)	N/A	0.012	0.002	0.017	0.334	0.568	0.029	0.492	0.002	1.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	7	14	17	8	8	9
N.S.	1	1.00	1.00	1.10	0.70	1.40	1.70	0.80	0.80	0.90
time (sec)	N/A	0.004	0.009	0.096	0.430	0.582	0.658	0.489	0.091	1.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	11	10	15	11	11
N.S.	1	1.00	1.00	0.92	1.15	0.85	0.77	1.15	0.85	0.85
time (sec)	N/A	0.004	0.002	0.056	0.330	0.581	0.026	0.443	0.092	1.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	14	22	0	22	17	29
N.S.	1	1.00	1.00	0.86	0.67	1.05	0.00	1.05	0.81	1.38
time (sec)	N/A	0.003	0.026	0.013	0.363	0.588	0.000	0.438	0.065	1.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	23	22	21	32	21	22	10
N.S.	1	1.00	1.00	0.68	0.65	0.62	0.94	0.62	0.65	0.29
time (sec)	N/A	0.009	0.031	0.014	0.469	0.576	0.337	0.480	0.081	1.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16	23
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73	1.05
time (sec)	N/A	0.006	0.010	0.028	0.334	0.595	0.112	0.512	0.165	1.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17	12
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89	0.63
time (sec)	N/A	0.003	0.006	0.025	0.467	0.557	0.045	0.476	0.086	1.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	11	9	8	9	9	10
N.S.	1	1.00	1.00	1.00	1.10	0.90	0.80	0.90	0.90	1.00
time (sec)	N/A	0.014	0.021	0.043	0.354	0.546	0.035	0.479	0.053	1.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	16	5	25	6	30
N.S.	1	1.00	5.00	0.88	0.75	2.00	0.62	3.12	0.75	3.75
time (sec)	N/A	0.002	0.024	0.172	0.428	0.571	0.586	0.443	0.010	1.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	19	12	23	0	10	12	18
N.S.	1	1.00	1.00	1.58	1.00	1.92	0.00	0.83	1.00	1.50
time (sec)	N/A	0.009	0.009	0.024	0.426	0.574	0.000	0.474	0.045	1.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	F	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	0	24	0	22	16	0
N.S.	1	1.00	3.33	1.42	0.00	2.00	0.00	1.83	1.33	0.00
time (sec)	N/A	0.004	0.008	0.144	0.000	0.577	0.000	0.498	0.032	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	27	15	14	15	14	14	16
N.S.	1	1.00	1.00	1.50	0.83	0.78	0.83	0.78	0.78	0.89
time (sec)	N/A	0.003	0.007	0.057	0.327	0.560	0.046	0.482	0.037	1.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	38	35	26	43	0	0	-1	32
N.S.	1	1.00	0.73	0.67	0.50	0.83	0.00	0.00	-0.02	0.62
time (sec)	N/A	0.035	0.036	0.161	0.446	0.618	0.000	0.000	0.000	1.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F	C	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	0	137	33	0	33	45	36
N.S.	1	1.00	0.95	0.00	3.51	0.85	0.00	0.85	1.15	0.92
time (sec)	N/A	0.125	0.330	0.000	0.473	0.580	0.000	0.456	0.306	1.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	19	19	19
N.S.	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70	0.70
time (sec)	N/A	0.006	0.022	0.057	0.330	0.577	0.104	0.451	0.027	1.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	0	7	13	0	0	7	0
N.S.	1	1.00	1.00	0.00	1.00	1.86	0.00	0.00	1.00	0.00
time (sec)	N/A	0.028	0.016	0.000	0.350	0.611	0.000	0.000	0.764	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.009	0.008	0.020	0.321	0.566	0.026	0.460	0.099	1.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	0.75
time (sec)	N/A	0.002	0.004	0.027	0.372	0.580	0.023	0.482	0.089	1.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	10	16	10	6	6	6
N.S.	1	1.00	1.00	0.88	1.25	2.00	1.25	0.75	0.75	0.75
time (sec)	N/A	0.007	0.001	0.044	0.395	0.584	0.286	0.468	0.119	1.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	14	14	17	14	14	74
N.S.	1	1.00	1.00	0.79	0.74	0.74	0.89	0.74	0.74	3.89
time (sec)	N/A	0.004	0.005	0.076	0.504	0.573	0.042	0.460	0.039	1.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	B	B	B	A	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	0	5	16	52	34	5	45	5	13
N.S.	1	0.00	1.00	3.20	10.40	6.80	1.00	9.00	1.00	2.60
time (sec)	N/A	0.016	0.010	0.709	0.380	0.594	0.166	0.456	0.057	1.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	7	77	6	30
N.S.	1	1.00	1.00	1.00	0.86	0.86	1.00	11.00	0.86	4.29
time (sec)	N/A	0.018	0.028	0.173	0.325	0.602	0.060	0.497	0.108	1.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	44	5	4	4	0
N.S.	1	1.00	1.00	1.25	1.00	11.00	1.25	1.00	1.00	0.00
time (sec)	N/A	0.013	6.248	7.477	0.389	0.616	1.113	0.436	0.123	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	10	10	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.80	1.00	1.00	1.00
time (sec)	N/A	0.004	0.007	0.025	0.357	0.568	0.188	0.461	0.002	1.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	42	6	0	6	6	6
N.S.	1	1.00	1.00	1.17	7.00	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.013	0.003	0.134	0.420	0.594	0.000	0.441	0.087	1.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5	0
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83	0.00
time (sec)	N/A	0.005	0.005	0.042	0.430	0.579	0.074	0.444	0.116	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	20	12	16	3	12	6	6
N.S.	1	1.00	1.00	3.33	2.00	2.67	0.50	2.00	1.00	1.00
time (sec)	N/A	0.003	0.003	0.020	0.374	0.569	0.057	0.493	0.053	1.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	12	7	8	36	9	8	21	26	28
N.S.	1	1.71	1.00	1.14	5.14	1.29	1.14	3.00	3.71	4.00
time (sec)	N/A	0.060	0.005	0.215	0.388	0.624	1.557	0.441	0.154	1.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F(-1)	F	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	0	0	6	0	6	6	5
N.S.	1	1.00	1.00	0.00	0.00	0.60	0.00	0.60	0.60	0.50
time (sec)	N/A	0.005	0.019	180.000	0.000	0.584	0.000	0.524	0.169	1.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	219	3	3	20
N.S.	1	1.00	1.00	1.33	1.00	1.00	73.00	1.00	1.00	6.67
time (sec)	N/A	0.013	0.007	0.145	0.464	0.599	10.361	0.503	0.072	1.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	22	8	9	9	10
N.S.	1	1.00	1.00	0.91	0.82	2.00	0.73	0.82	0.82	0.91
time (sec)	N/A	0.001	0.015	0.065	0.419	0.557	0.450	0.510	0.034	1.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	F	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	46	0	18	0	15	-1	14
N.S.	1	1.00	1.00	2.30	0.00	0.90	0.00	0.75	-0.05	0.70
time (sec)	N/A	0.004	0.056	0.078	0.000	0.578	0.000	0.480	0.000	1.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	12	13	11	11	7	12	12	12
N.S.	1	1.00	1.71	1.86	1.57	1.57	1.00	1.71	1.71	1.71
time (sec)	N/A	0.038	0.014	0.094	0.359	0.578	0.048	0.424	0.181	1.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	9	4	17	19	14	16	3	3
N.S.	1	1.00	3.00	1.33	5.67	6.33	4.67	5.33	1.00	1.00
time (sec)	N/A	0.005	0.005	0.064	0.396	0.595	0.046	0.478	0.096	1.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	N/A	A	A	A	A	A	A	A	A	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	0	0	0	0	5	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.83	0.00	0.00	-0.17	0.00
time (sec)	N/A	0.083	0.019	0.000	0.000	0.585	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5	0
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00	0.00
time (sec)	N/A	0.026	0.013	0.105	0.383	0.567	0.039	0.511	0.110	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	15	15	12	11	27	0	11	5	0
N.S.	1	2.50	2.50	2.00	1.83	4.50	0.00	1.83	0.83	0.00
time (sec)	N/A	0.010	0.012	0.036	0.365	0.586	0.000	0.474	0.119	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2	10
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00	5.00
time (sec)	N/A	0.001	0.002	0.071	0.353	0.557	0.027	0.460	0.077	1.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	13	12	13	24	14	10	14	13	9
N.S.	1	1.08	1.00	1.08	2.00	1.17	0.83	1.17	1.08	0.75
time (sec)	N/A	0.003	0.003	0.032	0.361	0.579	0.098	0.457	0.113	1.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	29	0	60	29	29	29
N.S.	1	1.00	1.00	0.77	0.74	0.00	1.54	0.74	0.74	0.74
time (sec)	N/A	0.032	0.021	1.137	0.327	0.000	0.289	0.475	0.798	1.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	41	5	4	4	0
N.S.	1	1.00	1.00	1.25	1.00	10.25	1.25	1.00	1.00	0.00
time (sec)	N/A	0.014	3.403	9.075	0.329	0.612	1.130	0.504	0.111	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	29	17	22	0	17	8
N.S.	1	1.00	1.00	0.75	1.21	0.71	0.92	0.00	0.71	0.33
time (sec)	N/A	0.016	0.004	0.032	0.372	0.605	0.449	0.000	0.130	1.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73	0.00
time (sec)	N/A	0.007	0.012	0.021	0.356	0.612	0.100	0.492	0.002	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	14	12	12	22
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.88	0.75	0.75	1.38
time (sec)	N/A	0.002	0.010	0.073	0.460	0.585	0.062	0.427	0.090	1.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	114	22	22	38
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.80	0.73	0.73	1.27
time (sec)	N/A	0.020	0.015	0.657	0.339	0.630	1.046	0.469	0.246	1.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	34	34	185	30	10	29	0	-1	0
N.S.	1	1.48	1.48	8.04	1.30	0.43	1.26	0.00	-0.04	0.00
time (sec)	N/A	0.002	0.004	0.035	0.388	0.592	0.439	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	9	4	4	3	4	4	4
N.S.	1	1.00	1.00	3.00	1.33	1.33	1.00	1.33	1.33	1.33
time (sec)	N/A	0.000	0.000	0.013	0.342	0.541	0.004	0.477	0.006	1.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	B	C	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	264	131	16	2527	123	0	134	15	7
N.S.	1	24.00	11.91	1.45	229.73	11.18	0.00	12.18	1.36	0.64
time (sec)	N/A	2.746	0.144	0.430	0.521	0.872	0.000	0.462	1.428	1.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	11	43	34	7	8	33	8	34
N.S.	1	1.00	1.38	5.38	4.25	0.88	1.00	4.12	1.00	4.25
time (sec)	N/A	0.010	0.008	0.153	0.325	0.586	0.157	0.458	0.116	1.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	C	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	29	27	33	21	22	44	46	1
N.S.	1	1.00	1.21	1.12	1.38	0.88	0.92	1.83	1.92	0.04
time (sec)	N/A	0.009	0.046	0.116	0.418	0.639	8.965	0.483	0.276	1.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	29	8	8	10	8	8	8
N.S.	1	1.00	1.00	2.42	0.67	0.67	0.83	0.67	0.67	0.67
time (sec)	N/A	0.002	0.010	0.076	0.432	0.584	0.061	0.472	0.187	1.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	B	A	B	B	F	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	0	5	12	5	11	7	0	5	44
N.S.	1	0.00	1.00	2.40	1.00	2.20	1.40	0.00	1.00	8.80
time (sec)	N/A	0.041	0.009	0.020	0.408	0.598	0.055	0.000	0.232	1.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	13	11	8	13	11	10
N.S.	1	1.00	1.00	1.73	1.18	1.00	0.73	1.18	1.00	0.91
time (sec)	N/A	0.008	0.003	0.069	0.348	0.589	0.027	0.463	0.122	1.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	16	3	2	14	2	25	2	10
N.S.	1	1.00	8.00	1.50	1.00	7.00	1.00	12.50	1.00	5.00
time (sec)	N/A	0.001	0.011	0.093	0.462	0.571	0.054	0.447	0.025	1.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	18	9	10	18	9	10	10	9	9
N.S.	1	2.00	1.00	1.11	2.00	1.00	1.11	1.11	1.00	1.00
time (sec)	N/A	0.025	0.008	0.032	0.335	0.589	0.036	0.462	0.136	1.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	9	9	7	9	9	9
N.S.	1	1.00	1.00	1.20	0.90	0.90	0.70	0.90	0.90	0.90
time (sec)	N/A	0.004	0.009	0.018	0.353	0.576	0.018	0.452	0.044	1.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	13	7
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.65	0.35
time (sec)	N/A	0.015	0.007	0.029	0.341	0.570	0.092	0.466	0.170	1.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	36	22	20	15	107	20	16
N.S.	1	1.00	1.00	1.44	0.88	0.80	0.60	4.28	0.80	0.64
time (sec)	N/A	0.004	0.005	0.056	0.323	0.572	0.045	0.427	0.102	1.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6	16
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75	2.00
time (sec)	N/A	0.003	0.005	0.150	0.429	0.558	0.033	0.431	0.050	1.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	13	5	12	0	19	12	5
N.S.	1	1.00	1.00	0.65	0.25	0.60	0.00	0.95	0.60	0.25
time (sec)	N/A	0.038	0.004	0.076	0.467	0.582	0.000	0.541	0.220	1.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	25	80	8	19	10	19	16	13
N.S.	1	1.00	2.08	6.67	0.67	1.58	0.83	1.58	1.33	1.08
time (sec)	N/A	0.073	0.017	0.211	0.338	0.611	1.197	0.478	0.248	1.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	15	16	16	20
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00	1.25
time (sec)	N/A	0.003	0.002	0.040	0.403	0.588	0.039	0.468	0.105	1.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	13	9	12
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.60	0.80
time (sec)	N/A	0.002	0.002	0.131	0.333	0.572	0.020	0.488	0.037	1.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	23	23	22	19	37	40	-1	19
N.S.	1	1.00	0.82	0.82	0.79	0.68	1.32	1.43	-0.04	0.68
time (sec)	N/A	0.008	0.005	0.008	0.418	0.633	0.835	0.459	0.000	1.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	13	47	17	13	24	0
N.S.	1	1.00	1.00	1.62	1.00	3.62	1.31	1.00	1.85	0.00
time (sec)	N/A	0.014	1.120	10.307	0.341	0.610	0.823	0.464	0.264	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	N/A	A	A	A	A	A	A	A	A	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	9	0	0	-1	11
N.S.	1	0.00	0.00	0.00	0.00	0.75	0.00	0.00	-0.08	0.92
time (sec)	N/A	0.005	0.795	0.000	0.000	0.596	0.000	0.000	0.000	1.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	N/A	A	A	A	A	A	A	A	A	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	14	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.82	0.00	0.00	-0.06	0.00
time (sec)	N/A	0.012	3.118	0.000	0.000	0.580	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	B	F(-2)	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	11128	10107	10106	0	11171	10106	15	20
N.S.	1	1.00	483.83	439.43	439.39	0.00	485.70	439.39	0.65	0.87
time (sec)	N/A	0.013	0.021	6.801	2.616	0.000	2.591	1.991	6.029	1.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	19	19	10	9	19	29	19	15	19
N.S.	1	1.73	1.73	0.91	0.82	1.73	2.64	1.73	1.36	1.73
time (sec)	N/A	0.035	0.010	3.334	0.318	0.645	0.296	0.477	0.180	1.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	5	5	8
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	1.00	1.00	1.60
time (sec)	N/A	0.036	0.007	0.022	0.361	0.581	0.056	0.478	0.221	1.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74	0.74
time (sec)	N/A	0.002	0.001	0.075	0.416	0.580	0.072	0.524	0.002	1.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	22	21	21	20	35	21	23
N.S.	1	1.00	1.00	0.79	0.75	0.75	0.71	1.25	0.75	0.82
time (sec)	N/A	0.015	0.020	0.023	0.350	0.590	0.580	0.497	0.111	1.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	72	5	0
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	14.40	1.00	0.00
time (sec)	N/A	0.012	0.025	0.121	0.337	0.616	0.049	0.496	0.089	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	12	9	5	0	5	-1	5
N.S.	1	1.00	1.00	0.21	0.16	0.09	0.00	0.09	-0.02	0.09
time (sec)	N/A	0.030	0.005	0.045	0.562	0.572	0.000	0.427	0.000	1.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	21	22	36	10	29	42	10	10	7
N.S.	1	1.50	1.57	2.57	0.71	2.07	3.00	0.71	0.71	0.50
time (sec)	N/A	0.015	0.004	0.069	0.339	0.570	0.214	0.498	0.151	1.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	F	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	30	0	33	0	33	51	15
N.S.	1	1.00	0.76	0.67	0.00	0.73	0.00	0.73	1.13	0.33
time (sec)	N/A	0.019	0.153	0.025	0.000	0.567	0.000	0.455	0.159	1.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	N/A	A	A	A	A	A	A	A	A	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	7	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.88	0.00	0.00	-0.12	0.00
time (sec)	N/A	0.004	0.726	0.000	0.000	0.601	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80	0.80
time (sec)	N/A	0.010	0.002	0.084	0.327	0.598	0.120	0.503	0.066	1.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	22	17	17	15	18	6	8
N.S.	1	1.00	2.88	2.75	2.12	2.12	1.88	2.25	0.75	1.00
time (sec)	N/A	0.002	0.003	0.067	0.369	0.563	0.032	0.533	0.002	1.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	10	20	14	10	2	9
N.S.	1	1.00	1.00	1.50	5.00	10.00	7.00	5.00	1.00	4.50
time (sec)	N/A	0.004	0.006	0.090	0.343	0.548	0.319	0.460	0.009	1.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	F	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	0	7	7	0	7	9
N.S.	1	1.00	1.00	1.00	0.00	0.78	0.78	0.00	0.78	1.00
time (sec)	N/A	0.022	0.007	0.031	0.000	0.570	0.042	0.000	0.118	1.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	24	23	21	306	30	10	11
N.S.	1	1.00	0.74	0.69	0.66	0.60	8.74	0.86	0.29	0.31
time (sec)	N/A	0.006	0.013	0.076	0.380	0.576	0.601	0.499	0.192	1.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	193	20	21	20	20	19	21	20	33
N.S.	1	8.04	0.83	0.88	0.83	0.83	0.79	0.88	0.83	1.38
time (sec)	N/A	0.429	0.004	0.018	0.416	0.559	0.029	0.413	0.123	1.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	A	A	B	F	A	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	5	0	5	6	5	65	0	5	5	7
N.S.	1	0.00	1.00	1.20	1.00	13.00	0.00	1.00	1.00	1.40
time (sec)	N/A	0.012	0.082	10.263	0.608	0.607	0.000	0.441	0.373	1.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	B	B	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	33	12	68	79	14	80	55	14	13
N.S.	1	2.75	1.00	5.67	6.58	1.17	6.67	4.58	1.17	1.08
time (sec)	N/A	0.054	0.015	5.123	0.428	0.600	0.094	0.492	0.330	1.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	A	A	A	A	A	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	0	18	19	16	11	10	11	11	15
N.S.	1	0.00	1.80	1.90	1.60	1.10	1.00	1.10	1.10	1.50
time (sec)	N/A	0.019	0.007	0.087	0.391	0.581	0.193	0.526	0.167	1.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	C	A	C	C	A	F	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	42	3	54	14	3	0	40	3	13
N.S.	1	14.00	1.00	18.00	4.67	1.00	0.00	13.33	1.00	4.33
time (sec)	N/A	0.058	0.000	0.391	0.447	0.558	0.000	0.504	0.361	1.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	16	5	25	6	12
N.S.	1	1.00	5.00	0.88	0.75	2.00	0.62	3.12	0.75	1.50
time (sec)	N/A	0.002	0.005	0.086	0.433	0.580	0.204	0.481	0.081	1.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	15	14	13	31	63	46	26	13
N.S.	1	1.00	0.44	0.41	0.38	0.91	1.85	1.35	0.76	0.38
time (sec)	N/A	0.009	0.017	0.063	0.452	0.619	0.260	0.501	0.131	1.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	19	14	20	18	20
N.S.	1	1.00	1.00	0.89	1.00	1.06	0.78	1.11	1.00	1.11
time (sec)	N/A	0.006	0.003	0.028	0.334	0.584	0.046	0.476	0.047	1.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	F	A	A	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	0	56	0	51	69	0	0	-1	37
N.S.	1	0.00	1.00	0.00	0.91	1.23	0.00	0.00	-0.02	0.66
time (sec)	N/A	0.017	0.018	0.000	0.532	0.615	0.000	0.000	0.000	1.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	17	29
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.85	1.45
time (sec)	N/A	0.016	0.001	0.012	0.315	0.544	0.005	0.500	0.030	1.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	13	12	32	11	65	0	11	9
N.S.	1	1.00	0.62	0.57	1.52	0.52	3.10	0.00	0.52	0.43
time (sec)	N/A	0.026	6.144	0.133	0.460	0.573	1.201	0.000	0.096	1.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	25	33	36	25	39	29	18	38	25
N.S.	1	1.47	1.94	2.12	1.47	2.29	1.71	1.06	2.24	1.47
time (sec)	N/A	0.018	0.019	0.216	0.381	0.598	0.016	0.584	0.207	1.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	25	8	7	61	10	5	6	6	11
N.S.	1	3.12	1.00	0.88	7.62	1.25	0.62	0.75	0.75	1.38
time (sec)	N/A	0.040	0.018	0.056	0.345	0.584	0.143	0.464	0.107	1.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	23	14	39	42	19	37	14	22
N.S.	1	1.00	1.35	0.82	2.29	2.47	1.12	2.18	0.82	1.29
time (sec)	N/A	0.016	0.013	8.172	0.374	0.577	0.186	0.445	0.061	1.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	22	23	31	22	24	51	22	0
N.S.	1	1.00	0.67	0.70	0.94	0.67	0.73	1.55	0.67	0.00
time (sec)	N/A	0.016	0.009	0.058	0.452	0.565	0.062	0.456	0.138	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	C	F	B	F	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	6	0	6	112	0	15	0	35	35	36
N.S.	1	0.00	1.00	18.67	0.00	2.50	0.00	5.83	5.83	6.00
time (sec)	N/A	0.182	0.024	0.395	0.000	0.612	0.000	0.430	0.499	1.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	C	B	C	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	81	21	28	45	16	0	5161	-1	12
N.S.	1	9.00	2.33	3.11	5.00	1.78	0.00	573.44	-0.11	1.33
time (sec)	N/A	0.049	0.469	1.365	0.439	0.610	0.000	1.681	0.000	1.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	-1	27
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01	0.40
time (sec)	N/A	0.035	0.025	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	63	66	62	394	62	78	25
N.S.	1	1.00	1.00	0.75	0.79	0.74	4.69	0.74	0.93	0.30
time (sec)	N/A	0.128	0.080	1.046	0.409	0.661	4.334	0.713	0.342	1.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	25	58	16	19	23	32	11
N.S.	1	1.00	1.00	1.25	2.90	0.80	0.95	1.15	1.60	0.55
time (sec)	N/A	0.019	0.025	0.223	0.438	0.620	0.059	0.410	0.582	1.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	89	53	62	83	119	50	-1	46
N.S.	1	1.00	1.41	0.84	0.98	1.32	1.89	0.79	-0.02	0.73
time (sec)	N/A	0.013	0.096	0.103	0.607	0.597	1.012	0.459	0.000	1.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	14	17	20	17	14	17
N.S.	1	1.00	1.00	1.24	0.67	0.81	0.95	0.81	0.67	0.81
time (sec)	N/A	0.006	0.006	0.141	0.335	0.602	0.399	0.493	0.138	1.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	41	69	103	39	36	39	39	41
N.S.	1	1.00	0.54	0.91	1.36	0.51	0.47	0.51	0.51	0.54
time (sec)	N/A	0.156	0.281	0.049	0.328	0.563	0.032	0.434	0.063	1.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	F	A	A	A	B	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	38	0	34	34	34	163	-1	11
N.S.	1	0.00	1.00	0.00	0.89	0.89	0.89	4.29	-0.03	0.29
time (sec)	N/A	0.007	0.011	0.000	0.421	0.626	0.097	0.460	0.000	1.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	F	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	52	115	0	0	0	24	0	0	-1	0
N.S.	1	2.21	0.00	0.00	0.00	0.46	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.082	0.022	0.000	0.000	0.593	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	33	47	46	29	138	61	46	27
N.S.	1	1.00	0.52	0.73	0.72	0.45	2.16	0.95	0.72	0.42
time (sec)	N/A	0.008	0.013	0.096	0.440	0.575	1.906	0.428	0.026	1.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	43	77	70	47	0	8	56	0	-1	0
N.S.	1	1.79	1.63	1.09	0.00	0.19	1.30	0.00	-0.02	0.00
time (sec)	N/A	0.008	0.017	0.168	0.000	0.604	0.802	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	58	31	30	48	34	30	31	17
N.S.	1	1.00	1.41	0.76	0.73	1.17	0.83	0.73	0.76	0.41
time (sec)	N/A	0.007	0.066	0.120	0.427	0.572	0.091	0.404	0.044	1.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	142	0	0	14	0	0	-1	0
N.S.	1	1.00	1.43	0.00	0.00	0.14	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.027	0.093	0.000	0.000	0.584	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	29	36	0	0	-1	0
N.S.	1	1.00	1.00	0.92	0.81	1.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.055	0.052	0.033	0.383	0.583	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	93	47	48	41	45	85	41	55	97
N.S.	1	2.11	1.07	1.09	0.93	1.02	1.93	0.93	1.25	2.20
time (sec)	N/A	0.420	0.024	0.118	0.362	0.579	1.188	0.451	0.255	1.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	33	59	58	29	68	42	30	0
N.S.	1	1.00	0.59	1.05	1.04	0.52	1.21	0.75	0.54	0.00
time (sec)	N/A	0.042	0.023	0.123	0.352	0.567	0.972	0.412	0.134	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	26	28	37	26	23	28	9
N.S.	1	1.00	0.81	0.84	0.90	1.19	0.84	0.74	0.90	0.29
time (sec)	N/A	0.003	0.008	0.081	0.437	0.577	0.047	0.414	0.123	1.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	A	A	B	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	108	0	132	96	115	262	0	0	-1	40
N.S.	1	0.00	1.22	0.89	1.06	2.43	0.00	0.00	-0.01	0.37
time (sec)	N/A	0.354	0.039	0.431	0.577	0.715	0.000	0.000	0.000	1.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	C	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	100	29	17	8	8	0	32	8	0
N.S.	1	3.12	0.91	0.53	0.25	0.25	0.00	1.00	0.25	0.00
time (sec)	N/A	0.164	0.015	1.599	0.388	0.637	0.000	1.147	0.261	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	A	C	A	F	A	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	21	22	27	13	0	13	19	32
N.S.	1	0.00	1.00	1.05	1.29	0.62	0.00	0.62	0.90	1.52
time (sec)	N/A	0.067	0.031	0.077	0.415	0.580	0.000	0.588	0.026	1.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	114	13	117	114	31	0	-1	14
N.S.	1	1.00	1.12	0.13	1.15	1.12	0.30	0.00	-0.01	0.14
time (sec)	N/A	0.013	0.199	0.111	0.476	0.600	1.918	0.000	0.000	1.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	9	4	4	3	4	4	4
N.S.	1	1.00	1.00	3.00	1.33	1.33	1.00	1.33	1.33	1.33
time (sec)	N/A	0.000	0.000	0.010	0.354	0.546	0.003	0.434	0.002	1.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	9	4	3	8	7	5	5	5
N.S.	1	1.00	3.00	1.33	1.00	2.67	2.33	1.67	1.67	1.67
time (sec)	N/A	0.002	0.005	0.051	0.324	0.561	0.180	0.408	0.002	1.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	7	6	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	1.00	0.86	0.86	0.86
time (sec)	N/A	0.041	0.014	0.031	0.333	0.563	0.048	0.444	0.144	1.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	20	22	22	45
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.67	0.73	0.73	1.50
time (sec)	N/A	0.018	0.000	0.050	0.336	0.548	0.012	0.483	0.018	1.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	27	26	26	27	23	26	26
N.S.	1	1.00	1.00	0.75	0.72	0.72	0.75	0.64	0.72	0.72
time (sec)	N/A	0.019	0.002	0.010	0.334	0.557	0.006	0.436	0.022	1.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	10	20	20	17	16	16	15	16	8	14
N.S.	1	2.00	2.00	1.70	1.60	1.60	1.50	1.60	0.80	1.40
time (sec)	N/A	0.013	0.006	0.109	0.327	0.581	0.057	0.560	0.057	1.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	24	14	24	17	28	14	9
N.S.	1	1.00	1.00	2.00	1.17	2.00	1.42	2.33	1.17	0.75
time (sec)	N/A	0.019	0.030	0.910	0.462	0.599	0.022	0.420	0.194	1.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	7	7	7	19
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.88	0.88	0.88	2.38
time (sec)	N/A	0.006	0.003	0.026	0.386	0.564	0.050	0.403	0.255	1.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	B	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1	3	1	2	1	1	0	26	1	41
N.S.	1	3.00	1.00	2.00	1.00	1.00	0.00	26.00	1.00	41.00
time (sec)	N/A	0.026	0.000	0.100	0.413	0.565	0.011	0.472	0.005	1.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	C	A	C	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	1	50	1	55	1	1	58	1	1	35
N.S.	1	50.00	1.00	55.00	1.00	1.00	58.00	1.00	1.00	35.00
time (sec)	N/A	0.041	0.000	0.160	0.396	0.552	0.021	0.446	0.154	1.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	81	8	8	8	9	0
N.S.	1	1.00	1.00	1.00	9.00	0.89	0.89	0.89	1.00	0.00
time (sec)	N/A	0.015	0.017	0.198	0.398	0.593	0.175	0.484	0.188	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	B	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	100	61	79	114	0	73	238	28
N.S.	1	1.00	2.04	1.24	1.61	2.33	0.00	1.49	4.86	0.57
time (sec)	N/A	0.012	0.069	0.231	0.433	0.577	0.000	0.569	0.542	1.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F(-1)	A	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	11	11	0	15	-1	13
N.S.	1	1.00	1.00	0.00	0.85	0.85	0.00	1.15	-0.08	1.00
time (sec)	N/A	0.016	1.000	180.000	0.328	188.881	0.000	0.416	0.000	1.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	17	17	14	13	131	34	13	13	25
N.S.	1	1.89	1.89	1.56	1.44	14.56	3.78	1.44	1.44	2.78
time (sec)	N/A	0.009	0.018	1.555	0.326	1.844	0.129	0.473	0.198	1.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	C	B	A	A	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	21	30	91	16	17	119	22	21
N.S.	1	0.00	1.11	1.58	4.79	0.84	0.89	6.26	1.16	1.11
time (sec)	N/A	0.037	0.037	0.202	0.342	0.606	0.067	0.536	0.164	1.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	C	F	F	F	B	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	87	0	0	0	30	56	0	-1	27
N.S.	1	9.67	0.00	0.00	0.00	3.33	6.22	0.00	-0.11	3.00
time (sec)	N/A	0.033	0.026	0.000	0.000	0.613	19.090	0.000	0.000	1.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	24	24	26	24	24	24
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.81	0.75	0.75	0.75
time (sec)	N/A	0.003	0.000	0.017	0.352	0.556	0.007	0.618	0.023	1.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	14	16	10	13
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.00	1.14	0.71	0.93
time (sec)	N/A	0.005	0.004	0.073	0.350	0.580	0.030	0.501	0.128	1.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	28	80	65	0	0	-1	13
N.S.	1	1.00	0.87	0.72	2.05	1.67	0.00	0.00	-0.03	0.33
time (sec)	N/A	0.032	0.007	0.028	0.356	0.616	0.000	0.000	0.000	1.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	A	A	A	A	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	24	33	28	32	27	72	28	35
N.S.	1	0.00	1.00	1.38	1.17	1.33	1.12	3.00	1.17	1.46
time (sec)	N/A	0.243	0.022	0.095	0.388	0.576	0.046	0.479	0.170	1.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	F	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	20	29	0	34	0	22	17	29
N.S.	1	1.00	0.77	1.12	0.00	1.31	0.00	0.85	0.65	1.12
time (sec)	N/A	0.008	0.014	0.186	0.000	0.568	0.000	0.433	0.048	1.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	38	32	35	83	32	38	34
N.S.	1	1.00	0.95	0.86	0.73	0.80	1.89	0.73	0.86	0.77
time (sec)	N/A	0.017	0.010	0.199	0.327	0.586	0.183	0.430	0.151	1.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	54	17	15	16	11	4
N.S.	1	1.00	1.00	1.09	4.91	1.55	1.36	1.45	1.00	0.36
time (sec)	N/A	0.049	0.009	0.110	0.427	0.639	112.515	0.491	0.079	1.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F	C	F(-2)	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	41	0	83	41	9	18
N.S.	1	1.00	1.00	0.00	3.15	0.00	6.38	3.15	0.69	1.38
time (sec)	N/A	0.025	0.010	0.000	0.440	0.000	0.248	0.414	0.221	1.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	C	B	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	67	94	22	83	12	35	12
N.S.	1	1.00	1.00	5.58	7.83	1.83	6.92	1.00	2.92	1.00
time (sec)	N/A	0.012	0.011	0.281	0.452	0.613	1.773	0.485	0.348	1.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	87	19	85	82	85	85	0
N.S.	1	1.00	1.00	4.58	1.00	4.47	4.32	4.47	4.47	0.00
time (sec)	N/A	0.039	0.031	0.117	0.351	0.568	0.498	0.476	0.309	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	F(-1)	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	86	108	86	67	0	71	100	66	66	15
N.S.	1	1.26	1.00	0.78	0.00	0.83	1.16	0.77	0.77	0.17
time (sec)	N/A	0.226	0.011	0.210	0.000	0.846	71.463	0.442	0.244	1.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	19	22	22	36
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.73	0.85	0.85	1.38
time (sec)	N/A	0.025	0.015	0.095	0.464	0.564	0.024	0.707	0.035	1.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	F	F	A	F	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	25	0	0	12	0	12	-1	9
N.S.	1	0.00	1.09	0.00	0.00	0.52	0.00	0.52	-0.04	0.39
time (sec)	N/A	0.006	0.035	0.000	0.000	0.588	0.000	0.444	0.000	1.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	32	10	10	8	10	10	0
N.S.	1	1.00	1.00	2.67	0.83	0.83	0.67	0.83	0.83	0.00
time (sec)	N/A	0.001	0.028	0.089	0.344	0.559	0.280	0.436	0.079	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	C	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	34	46	15	27	42	-1	0
N.S.	1	1.00	0.96	1.48	2.00	0.65	1.17	1.83	-0.04	0.00
time (sec)	N/A	0.010	0.021	0.116	0.446	0.609	1.482	0.471	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	24	8	12	17	7	8	7	19
N.S.	1	1.00	2.67	0.89	1.33	1.89	0.78	0.89	0.78	2.11
time (sec)	N/A	0.006	0.010	0.092	0.332	0.595	0.099	0.600	0.062	1.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	23	34	24	20	22	18	6
N.S.	1	1.00	0.91	1.05	1.55	1.09	0.91	1.00	0.82	0.27
time (sec)	N/A	0.007	0.003	0.037	0.342	0.597	0.038	0.499	0.037	1.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	34	40	17	31	38	9
N.S.	1	1.00	1.00	1.86	1.55	1.82	0.77	1.41	1.73	0.41
time (sec)	N/A	0.005	0.021	0.070	0.335	0.566	0.895	0.547	0.083	1.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	C	F	B	F	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	21	52	0	84	0	136	15	0
N.S.	1	0.00	1.00	2.48	0.00	4.00	0.00	6.48	0.71	0.00
time (sec)	N/A	0.027	0.050	1.803	0.000	0.617	0.000	0.546	0.359	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	17	15	21	17	17
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.88	1.24	1.00	1.00
time (sec)	N/A	0.073	0.025	0.127	0.359	0.589	0.058	0.440	0.206	1.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	B	F	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	107	0	0	37	4362	0	-1	0
N.S.	1	1.31	2.38	0.00	0.00	0.82	96.93	0.00	-0.02	0.00
time (sec)	N/A	0.042	0.301	0.000	0.000	0.723	3.773	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	44	44	25
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81	0.46
time (sec)	N/A	0.011	0.005	0.080	0.341	0.555	0.012	0.454	0.098	1.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	C	A	A	F	A	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	18	34	23	23	0	23	18	7
N.S.	1	0.00	1.00	1.89	1.28	1.28	0.00	1.28	1.00	0.39
time (sec)	N/A	0.004	0.006	0.249	0.419	0.575	0.000	0.728	0.187	1.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	F	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	77	0	56	108	0	57	-1	0
N.S.	1	1.00	1.26	0.00	0.92	1.77	0.00	0.93	-0.02	0.00
time (sec)	N/A	0.058	0.130	0.000	0.435	0.636	0.000	0.620	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	B	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	64	78	64	228	52	55	0	61	66	0
N.S.	1	1.22	1.00	3.56	0.81	0.86	0.00	0.95	1.03	0.00
time (sec)	N/A	1.269	0.125	0.616	0.358	0.764	0.000	45.320	1.547	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	F	A	F	F	A	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	105	0	105	0	0	148	0	0	-1	0
N.S.	1	0.00	1.00	0.00	0.00	1.41	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.014	0.289	0.000	0.000	2.380	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	ChatGPT
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	15	19	23	31
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.60	0.76	0.92	1.24
time (sec)	N/A	0.011	0.005	0.044	0.358	0.577	0.070	0.686	0.098	1.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [319] had the largest ratio of [47]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.00	11	0.182
2	A	4	3	1.00	9	0.333
3	A	2	1	1.52	13	0.077
4	A	4	3	1.00	8	0.375
5	A	2	2	1.00	5	0.400
6	A	4	4	1.00	9	0.444
7	A	4	3	1.00	9	0.333
8	A	3	3	1.00	13	0.231
9	A	4	4	1.00	11	0.364
10	A	11	8	1.38	6	1.333
11	A	8	4	1.00	12	0.333
12	A	4	3	1.00	21	0.143
13	A	3	2	1.00	8	0.250
14	A	3	2	1.59	17	0.118
15	A	2	2	1.00	7	0.286
16	A	3	2	1.00	12	0.167
17	A	1	1	1.00	7	0.143
18	A	1	1	1.00	13	0.077
19	A	2	2	1.00	17	0.118
20	F	0	0	N/A	0.	N/A
21	A	3	3	1.00	14	0.214
22	A	2	2	1.00	9	0.222
23	A	4	2	1.00	11	0.182
24	A	5	4	0.96	17	0.235
25	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	2	1.00	9	0.222
27	A	3	3	1.00	14	0.214
28	A	2	1	1.00	11	0.091
29	A	4	3	1.00	27	0.111
30	A	2	2	1.00	4	0.500
31	A	3	2	1.00	9	0.222
32	A	3	3	1.00	11	0.273
33	A	4	2	1.00	4	0.500
34	A	3	3	1.00	12	0.250
35	A	1	1	1.00	3	0.333
36	A	1	1	1.00	2	0.500
37	A	2	1	1.00	15	0.067
38	A	1	1	1.00	13	0.077
39	A	6	2	1.00	11	0.182
40	A	4	2	1.00	8	0.250
41	A	4	4	2.00	7	0.571
42	A	4	4	1.00	15	0.267
43	A	3	3	1.00	12	0.250
44	A	3	2	1.00	8	0.250
45	A	3	3	1.21	13	0.231
46	B	102	2	66.75	9	0.222
47	A	3	3	1.00	9	0.333
48	A	3	2	1.00	34	0.059
49	A	3	3	1.00	15	0.200
50	A	3	2	1.00	9	0.222
51	A	1	1	1.00	8	0.125
52	A	3	2	1.60	21	0.095
53	A	3	2	1.00	15	0.133
54	A	2	2	1.00	11	0.182
55	A	2	2	1.00	14	0.143
56	A	1	1	1.00	4	0.250
57	A	1	1	1.00	6	0.167
58	A	7	6	1.00	23	0.261
59	A	4	4	1.00	21	0.190
60	A	2	1	1.00	34	0.029

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	12	0.250
62	A	5	5	1.00	9	0.556
63	A	2	2	1.00	15	0.133
64	A	2	1	1.00	9	0.111
65	A	2	0	1.00	7	0.000
66	A	4	4	1.00	15	0.267
67	A	2	2	1.00	11	0.182
68	A	3	3	1.00	9	0.333
69	A	2	2	1.00	9	0.222
70	A	2	2	1.62	11	0.182
71	A	2	2	1.00	6	0.333
72	A	3	2	1.00	13	0.154
73	A	5	4	1.00	10	0.400
74	A	3	3	1.00	11	0.273
75	A	3	1	1.00	11	0.091
76	A	1	1	1.00	3	0.333
77	F	0	0	N/A	0.	N/A
78	B	1	0	2.09	13	0.000
79	A	2	2	1.00	11	0.182
80	A	2	1	1.00	15	0.067
81	A	3	3	1.00	13	0.231
82	A	2	1	1.00	27	0.037
83	A	1	1	1.00	2	0.500
84	A	4	4	2.00	11	0.364
85	A	3	3	1.71	9	0.333
86	A	3	3	1.00	15	0.200
87	A	2	2	1.00	4	0.500
88	A	5	3	1.73	7	0.429
89	A	3	3	1.00	5	0.600
90	A	4	2	1.00	9	0.222
91	A	3	2	1.00	11	0.182
92	A	2	2	1.00	10	0.200
93	A	4	5	1.00	6	0.833
94	A	2	2	1.00	11	0.182
95	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.00	9	0.222
97	A	1	1	1.00	8	0.125
98	A	3	3	1.00	13	0.231
99	A	1	1	1.00	4	0.250
100	A	4	3	1.00	6	0.500
101	A	1	1	1.00	16	0.062
102	A	4	3	1.00	13	0.231
103	A	1	1	1.00	9	0.111
104	A	2	1	1.00	6	0.167
105	A	2	1	1.00	9	0.111
106	A	2	2	1.00	2	1.000
107	A	2	1	1.00	9	0.111
108	A	3	3	1.00	4	0.750
109	A	3	2	1.00	10	0.200
110	A	13	8	1.47	18	0.444
111	A	3	3	1.00	4	0.750
112	A	4	4	1.00	13	0.308
113	A	2	2	1.00	8	0.250
114	A	3	2	1.00	19	0.105
115	A	3	3	1.00	12	0.250
116	A	3	2	1.00	15	0.133
117	A	1	1	1.50	6	0.167
118	B	7	2	5.80	11	0.182
119	B	2	1	2.31	11	0.091
120	A	2	2	1.00	12	0.167
121	A	1	1	1.00	2	0.500
122	A	2	2	1.00	7	0.286
123	A	2	2	1.00	6	0.333
124	A	5	5	1.00	12	0.417
125	A	1	1	1.00	3	0.333
126	A	2	1	1.27	12	0.083
127	A	3	2	1.00	9	0.222
128	A	3	3	1.00	11	0.273
129	A	1	1	1.00	9	0.111
130	A	3	1	1.00	14	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
131	A	5	5	1.00	10	0.500
132	A	6	3	1.00	11	0.273
133	A	3	2	1.00	15	0.133
134	A	3	2	1.00	4	0.500
135	A	3	2	1.00	7	0.286
136	A	1	1	1.00	14	0.071
137	A	1	1	1.00	2	0.500
138	A	1	0	1.00	7	0.000
139	A	1	1	1.00	6	0.167
140	F	0	0	N/A	0.	N/A
141	A	2	1	1.00	11	0.091
142	A	3	3	1.00	8	0.375
143	A	5	4	1.00	10	0.400
144	A	1	1	1.00	2	0.500
145	A	1	1	1.00	11	0.091
146	A	2	2	1.00	11	0.182
147	F	0	0	N/A	0.	N/A
148	A	3	1	1.00	25	0.040
149	A	3	3	1.00	9	0.333
150	A	11	4	1.00	21	0.190
151	A	3	2	1.00	13	0.154
152	A	3	3	1.00	12	0.250
153	A	4	3	1.00	11	0.273
154	A	2	1	1.00	6	0.167
155	A	5	4	1.00	15	0.267
156	A	1	1	1.50	6	0.167
157	A	1	1	1.00	13	0.077
158	A	1	1	1.00	6	0.167
159	A	1	1	1.00	2	0.500
160	A	2	2	1.00	9	0.222
161	A	3	3	1.00	13	0.231
162	A	4	4	1.00	11	0.364
163	A	2	2	1.00	2	1.000
164	A	2	2	1.00	12	0.167
165	A	3	3	1.00	6	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	3	1.00	7	0.429
167	A	1	1	1.00	12	0.083
168	A	3	3	1.00	11	0.273
169	A	3	3	1.00	8	0.375
170	A	1	1	1.00	8	0.125
171	A	3	2	1.00	9	0.222
172	A	7	3	1.00	18	0.167
173	A	5	5	1.00	25	0.200
174	A	1	1	1.00	10	0.100
175	A	4	3	1.00	13	0.231
176	A	2	2	1.00	11	0.182
177	A	1	1	1.00	4	0.250
178	A	2	1	1.00	7	0.143
179	A	2	2	1.00	11	0.182
180	F	0	0	N/A	0.	N/A
181	A	1	1	1.00	15	0.067
182	A	3	3	1.00	10	0.300
183	A	1	1	1.00	6	0.167
184	A	3	3	1.00	13	0.231
185	A	1	1	1.00	10	0.100
186	A	2	2	1.00	4	0.500
187	A	6	4	1.71	25	0.160
188	A	4	4	1.00	13	0.308
189	A	2	2	1.00	13	0.154
190	A	1	1	1.00	9	0.111
191	A	2	2	1.00	15	0.133
192	A	2	2	1.00	14	0.143
193	A	2	2	1.00	5	0.400
194	A	0	0	0.00	0	0.000
195	A	1	1	1.00	13	0.077
196	B	5	2	2.50	17	0.118
197	A	1	1	1.00	9	0.111
198	A	2	2	1.08	4	0.500
199	A	6	4	1.00	21	0.190
200	A	3	2	1.00	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	2	2	1.00	13	0.154
202	A	3	3	1.00	6	0.500
203	A	3	3	1.00	11	0.273
204	A	5	2	1.00	11	0.182
205	A	1	1	1.48	9	0.111
206	A	1	1	1.00	1	1.000
207	B	14	5	24.00	21	0.238
208	A	1	1	1.00	20	0.050
209	A	3	3	1.00	11	0.273
210	A	2	2	1.00	9	0.222
211	F	0	0	N/A	0.	N/A
212	A	3	2	1.00	18	0.111
213	A	1	1	1.00	9	0.111
214	A	2	4	2.00	12	0.333
215	A	4	4	1.00	7	0.571
216	A	2	2	1.00	9	0.222
217	A	2	2	1.00	12	0.167
218	A	3	3	1.00	11	0.273
219	A	4	3	1.00	15	0.200
220	A	2	1	1.00	21	0.048
221	A	3	3	1.00	6	0.500
222	A	1	1	1.00	18	0.056
223	A	2	2	1.00	6	0.333
224	A	4	3	1.00	8	0.375
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	2	1	1.00	9	0.111
228	A	5	5	1.73	15	0.333
229	A	5	4	1.00	12	0.333
230	A	2	2	1.00	11	0.182
231	A	3	3	1.00	11	0.273
232	A	1	1	1.00	11	0.091
233	A	8	4	1.00	9	0.444
234	A	2	2	1.50	12	0.167
235	A	5	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	0	0	0.00	0	0.000
237	A	3	3	1.00	8	0.375
238	A	2	2	1.00	11	0.182
239	A	2	2	1.00	4	0.500
240	A	6	3	1.00	17	0.176
241	A	3	2	1.00	13	0.154
242	B	5	4	8.04	23	0.174
243	F	0	0	N/A	0.	N/A
244	B	9	3	2.75	19	0.158
245	F	0	0	N/A	0.	N/A
246	C	11	5	14.00	37	0.135
247	A	2	2	1.00	11	0.182
248	A	2	2	1.00	8	0.250
249	A	4	4	1.00	8	0.500
250	F	0	0	N/A	0.	N/A
251	A	2	1	1.00	14	0.071
252	A	6	4	1.00	22	0.182
253	A	3	2	1.47	9	0.222
254	B	6	6	3.12	11	0.546
255	A	3	2	1.00	9	0.222
256	A	3	3	1.00	9	0.333
257	F	0	0	N/A	0.	N/A
258	C	7	5	9.00	19	0.263
259	A	5	5	1.00	12	0.417
260	A	29	4	1.00	13	0.308
261	A	1	1	1.00	21	0.048
262	A	6	3	1.00	34	0.088
263	A	1	1	1.00	7	0.143
264	A	25	5	1.00	20	0.250
265	F	0	0	N/A	0.	N/A
266	B	3	3	2.21	22	0.136
267	A	2	1	1.00	13	0.077
268	A	3	2	1.79	6	0.333
269	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	2	2	1.00	12	0.167
271	A	8	7	1.00	16	0.438
272	B	53	5	2.11	11	0.454
273	A	4	3	1.00	29	0.103
274	A	3	2	1.00	7	0.286
275	F	0	0	N/A	0.	N/A
276	B	4	3	3.12	39	0.077
277	F	0	0	N/A	0.	N/A
278	A	3	2	1.00	15	0.133
279	A	1	1	1.00	1	1.000
280	A	1	1	1.00	2	0.500
281	A	4	4	1.00	19	0.210
282	A	2	1	1.00	29	0.034
283	A	3	2	1.00	17	0.118
284	A	6	4	2.00	13	0.308
285	A	13	4	1.00	25	0.160
286	A	1	1	1.00	13	0.077
287	B	8	4	3.00	43	0.093
288	C	12	3	50.00	19	0.158
289	A	1	1	1.00	11	0.091
290	A	7	6	1.00	31	0.194
291	A	4	3	1.00	15	0.200
292	A	3	2	1.89	17	0.118
293	F	0	0	N/A	0.	N/A
294	C	4	5	9.67	16	0.312
295	A	1	0	1.00	20	0.000
296	A	2	1	1.00	17	0.059
297	A	4	4	1.00	4	1.000
298	F	0	0	N/A	0.	N/A
299	A	2	2	1.00	13	0.154
300	A	3	2	1.00	6	0.333
301	A	4	3	1.00	17	0.176
302	A	8	4	1.00	25	0.160
303	A	3	4	1.00	8	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	A	1	1	1.00	40	0.025
305	A	13	4	1.26	7	0.571
306	A	3	2	1.00	18	0.111
307	F	0	0	N/A	0.	N/A
308	A	1	1	1.00	16	0.062
309	A	3	2	1.00	13	0.154
310	A	2	2	1.00	8	0.250
311	A	3	2	1.00	4	0.500
312	A	4	4	1.00	9	0.444
313	F	0	0	N/A	0.	N/A
314	A	3	2	1.00	25	0.080
315	A	4	2	1.31	17	0.118
316	A	3	1	1.00	17	0.059
317	F	0	0	N/A	0.	N/A
318	A	6	5	1.00	14	0.357
319	A	10	4	1.22	47	0.085
320	F	0	0	N/A	0.	N/A
321	A	4	3	1.00	16	0.188

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Rubi [A]
time = 0.13, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2718}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx \\
&= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx \\
&= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A]
time = 0.68, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
parallelrisch	$\frac{1}{16}+\frac{\cos(6x)}{24}-\frac{\cos(4x)}{16}-\frac{\cos(2x)}{8}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A]
time = 0.34, size = 19, normalized size = 0.76

$$\frac{1}{24}\cos\left(6\,x\right)-\frac{1}{16}\cos\left(4\,x\right)-\frac{1}{8}\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A]
time = 0.59, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos\left(x\right)^{6}-\frac{5}{2}\cos\left(x\right)^{4}+\cos\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(19) = 38.
time = 1.31, size = 112, normalized size = 4.48

$$\frac{x\sin\left(x\right)\sin\left(2x\right)\sin\left(3x\right)}{4}+\frac{x\sin\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{4}+\frac{x\sin\left(2x\right)\cos\left(x\right)\cos\left(3x\right)}{4}-\frac{x\sin\left(3x\right)\cos\left(x\right)\cos\left(2x\right)}{4}-\frac{\sin\left(x\right)\sin\left(2x\right)\cos\left(3x\right)}{3}+\frac{\sin\left(x\right)\sin\left(3x\right)\cos\left(2x\right)}{8}-\frac{\cos\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - sin(x)*sin(2*x)*cos(3*x)/3
 + sin(x)*sin(3*x)*cos(2*x)/8 - cos(x)*cos(2*x)*cos(3*x)/24
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.52

$$-\frac{4}{3}\sin(x)^6+\frac{3}{2}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

Mupad [B]
time = 0.36, size = 14, normalized size = 0.56

$$-\frac{\sin(x)^4\left(8\sin(x)^2-9\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*sin(x),x)
```

```
[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 0.52

$$-\frac{\sin(x)\left(\cos(x)-\frac{\cos(3x)}{6}\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -1/4*sin(x)*(cos(x)-1/6*cos(3*x))
```

3.2 $\int \cos(x)\sin^3(2x)\,dx$

Optimal. Leaf size=17

$$-\frac{8}{5}\cos^5(x)+\frac{8\cos^7(x)}{7}$$

```
[Out] -8/5*cos(x)^5+8/7*cos(x)^7
```

Rubi [A]

time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{4372, 2645, 14\}$

$$\frac{8\cos^7(x)}{7}-\frac{8\cos^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[2*x]^3,x]
```

```
[Out] (-8*Cos[x]^5)/5 + (8*Cos[x]^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 8\int \cos^4(x)\sin^3(x)\,dx\\ &= -\left(8\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \cos(x)\right)\right)\\ &= -\left(8\text{Subst}\left(\int \left(x^4-x^6\right)\,dx, x, \cos(x)\right)\right)\\ &= -\frac{8}{5}\cos^5(x)+\frac{8\cos^7(x)}{7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.82

$$-\frac{3\cos(x)}{8}-\frac{1}{8}\cos(3x)+\frac{1}{40}\cos(5x)+\frac{1}{56}\cos(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[2*x]^3,x]
```

```
[Out] (-3*Cos[x])/8 - Cos[3*x]/8 + Cos[5*x]/40 + Cos[7*x]/56
```

Maple [A]
time = 0.20, size = 24, normalized size = 1.41

method	result	size
default	$-\frac{3\cos(x)}{8}-\frac{\cos(3x)}{8}+\frac{\cos(5x)}{40}+\frac{\cos(7x)}{56}$	24
risch	$-\frac{3\cos(x)}{8}-\frac{\cos(3x)}{8}+\frac{\cos(5x)}{40}+\frac{\cos(7x)}{56}$	24
parallelrisch	$\frac{8}{21}-\frac{3\cos(x)}{8}-\frac{\cos(3x)}{8}+\frac{\cos(5x)}{40}+\frac{\cos(7x)}{56}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^3*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*cos(x)-1/8*cos(3*x)+1/40*cos(5*x)+1/56*cos(7*x)
```

Maxima [A]
time = 0.35, size = 23, normalized size = 1.35

$$\frac{1}{56}\cos\left(7\,x\right)+\frac{1}{40}\cos\left(5\,x\right)-\frac{1}{8}\cos\left(3\,x\right)-\frac{3}{8}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^3*cos(x),x, algorithm="maxima")
```

```
[Out] 1/56*cos(7*x) + 1/40*cos(5*x) - 1/8*cos(3*x) - 3/8*cos(x)
```

Fricas [A]
time = 0.61, size = 13, normalized size = 0.76

$$\frac{8}{7}\cos\left(x\right)^{7}-\frac{8}{5}\cos\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^3*cos(x),x, algorithm="fricas")
```

```
[Out] 8/7*cos(x)^7 - 8/5*cos(x)^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(15) = 30$.
time = 0.74, size = 65, normalized size = 3.82

$$-\frac{9\sin\left(x\right)\sin^{3}\left(2x\right)}{35}-\frac{8\sin\left(x\right)\sin\left(2x\right)\cos^{2}\left(2x\right)}{35}-\frac{22\sin^{2}\left(2x\right)\cos\left(x\right)\cos\left(2x\right)}{35}-\frac{16\cos\left(x\right)\cos^{3}\left(2x\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)**3*cos(x),x)
```

```
[Out] -9*sin(x)*sin(2*x)**3/35 - 8*sin(x)*sin(2*x)*cos(2*x)**2/35 - 22*sin(2*x)**
2*cos(x)*cos(2*x)/35 - 16*cos(x)*cos(2*x)**3/35
```

Giac [A]
time = 0.38, size = 13, normalized size = 0.76

$$\frac{8}{7}\cos\left(x\right)^{7}-\frac{8}{5}\cos\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^3*cos(x),x, algorithm="giac")
```

```
[Out] 8/7*cos(x)^7 - 8/5*cos(x)^5
```

Mupad [B]
time = 0.07, size = 14, normalized size = 0.82

$$\frac{8\cos\left(x\right)^{5}\left(5\cos\left(x\right)^{2}-7\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^3*cos(x),x)
```

```
[Out] (8*cos(x)^5*(5*cos(x)^2 - 7))/35
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 20, normalized size = 1.18

$$\frac{3(\sin^2(2x))}{16} - \frac{\ln(1 - (\sin^4(x)))}{8}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(2*x)^3*cos(x),x)
```

```
[Out] 3/16*sin(2*x)^2-1/8*ln(1-sin(x)^4)
```

3.3 $\int \sqrt[3]{-1+x}(1+x)^2\,dx$

Optimal. Leaf size=21

$$\frac{3}{70}(-1+x)^{4/3}\left(37+26x+7x^2\right)$$

```
[Out] 3/70*(x-1)^(4/3)*(7*x^2+26*x+37)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.52, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{10}(x-1)^{10/3}+\frac{12}{7}(x-1)^{7/3}+3(x-1)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(1/3)*(1 + x)^2,x]
```

```
[Out] 3*(-1 + x)^(4/3) + (12*(-1 + x)^(7/3))/7 + (3*(-1 + x)^(10/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(4\sqrt[3]{-1+x}+4(-1+x)^{4/3}+(-1+x)^{7/3}\right)\,dx\\ &= 3(-1+x)^{4/3}+\frac{12}{7}(-1+x)^{7/3}+\frac{3}{10}(-1+x)^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 21, normalized size = 1.00

$$\frac{3}{70}(-1+x)^{4/3}\left(37+26x+7x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(1/3)*(1 + x)^2,x]
```

```
[Out] (3*(-1 + x)^(4/3)*(37 + 26*x + 7*x^2))/70
```

Maple [A]
time = 0.12, size = 23, normalized size = 1.10

method	result	size
gosper	$\frac{3(x-1)^{\frac{4}{3}}(7x^2+26x+37)}{70}$	18
trager	$\left(\frac{3}{10}x^3+\frac{57}{70}x^2+\frac{33}{70}x-\frac{111}{70}\right)(x-1)^{\frac{1}{3}}$	22
derivativedivides	$\frac{3(x-1)^{\frac{10}{3}}}{10}+\frac{12(x-1)^{\frac{7}{3}}}{7}+3(x-1)^{\frac{4}{3}}$	23
default	$\frac{3(x-1)^{\frac{10}{3}}}{10}+\frac{12(x-1)^{\frac{7}{3}}}{7}+3(x-1)^{\frac{4}{3}}$	23
risch	$\frac{3(x-1)^{\frac{1}{3}}(7x^3+19x^2+11x-37)}{70}$	23
meijerg	$\frac{\operatorname{signum}(x-1)^{\frac{1}{3}}x\,{}_2F_1\left(-\frac{1}{3},1;2;x\right)}{(-\operatorname{signum}(x-1))^{\frac{1}{3}}}+\frac{\operatorname{signum}(x-1)^{\frac{1}{3}}x^3\,{}_2F_1\left(-\frac{1}{3},3;4;x\right)}{3(-\operatorname{signum}(x-1))^{\frac{1}{3}}}+\frac{\operatorname{signum}(x-1)^{\frac{1}{3}}x^2\,{}_2F_1\left(-\frac{1}{3},2;3;x\right)}{(-\operatorname{signum}(x-1))^{\frac{1}{3}}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^2*(x-1)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/10*(x-1)^(10/3)+12/7*(x-1)^(7/3)+3*(x-1)^(4/3)
```

Maxima [A]
time = 0.32, size = 22, normalized size = 1.05

$$\frac{3}{10}\,(x-1)^{\frac{10}{3}}+\frac{12}{7}\,(x-1)^{\frac{7}{3}}+3\,(x-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)^2*(x-1)^(1/3),x, algorithm="maxima")
[Out] 3/10*(x - 1)^(10/3) + 12/7*(x - 1)^(7/3) + 3*(x - 1)^(4/3)
```

Fricas [A]
time = 0.54, size = 22, normalized size = 1.05

$$\frac{3}{70}\left(7\,x^3+19\,x^2+11\,x-37\right)(x-1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)^2*(x-1)^(1/3),x, algorithm="fricas")
[Out] 3/70*(7*x^3 + 19*x^2 + 11*x - 37)*(x - 1)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.11, size = 144, normalized size = 6.86

$$\begin{cases} \frac{3\sqrt[3]{x-1}(x+1)^3}{10}-\frac{3\sqrt[3]{x-1}(x+1)^2}{35}-\frac{9\sqrt[3]{x-1}(x+1)}{35}-\frac{54\sqrt[3]{x-1}}{35} & \text{for } |x+1|>2 \\ -\frac{3\sqrt[3]{1-x}(x+1)^3e^{-\frac{2i\pi}{3}}}{10}+\frac{3\sqrt[3]{1-x}(x+1)^2e^{-\frac{2i\pi}{3}}}{35}+\frac{9\sqrt[3]{1-x}(x+1)e^{-\frac{2i\pi}{3}}}{35}+\frac{54\sqrt[3]{1-x}e^{-\frac{2i\pi}{3}}}{35} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)**2*(x-1)**(1/3),x)
```

```
[Out] Piecewise((3*(x - 1)**(1/3)*(x + 1)**3/10 - 3*(x - 1)**(1/3)*(x + 1)**2/35
- 9*(x - 1)**(1/3)*(x + 1)/35 - 54*(x - 1)**(1/3)/35, Abs(x + 1) > 2), (-3*
(1 - x)**(1/3)*(x + 1)**3*exp(-2*I*pi/3)/10 + 3*(1 - x)**(1/3)*(x + 1)**2*e
xp(-2*I*pi/3)/35 + 9*(1 - x)**(1/3)*(x + 1)*exp(-2*I*pi/3)/35 + 54*(1 - x)*
*(1/3)*exp(-2*I*pi/3)/35, True))
```

Giac [**A**]
time = 0.40, size = 22, normalized size = 1.05

$$\frac{3}{10}\,(x-1)^{\frac{10}{3}}+\frac{12}{7}\,(x-1)^{\frac{7}{3}}+3\,(x-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)^2*(x-1)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(x - 1)^(10/3) + 12/7*(x - 1)^(7/3) + 3*(x - 1)^(4/3)
```

Mupad [**B**]
time = 0.07, size = 19, normalized size = 0.90

$$\frac{3\,(x-1)^{4/3}\,\left(40\,x+7\,(x-1)^2+30\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/3)*(x + 1)^2,x)
```

```
[Out] (3*(x - 1)^(4/3)*(40*x + 7*(x - 1)^2 + 30))/70
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 15, normalized size = 0.71

$$\frac{3(x-1)^{\frac{5}{3}}}{8}+\frac{52(x-1)^{\frac{4}{3}}}{15}$$

Warning: Unable to verify antiderivative.

```
[In] int((x+1)^2*(x-1)^(1/3),x)
```

```
[Out] 3/8*(x-1)^(5/3)+52/15*(x-1)^(4/3)
```

3.4 $\int x \log \left(1+\frac{1}{x}\right) dx$

Optimal. Leaf size=27

$$\frac{x}{2}+\frac{1}{2}x^2\log\left(1+\frac{1}{x}\right)-\frac{1}{2}\log(1+x)$$

```
[Out] 1/2*x+1/2*x^2*ln(1+1/x)-1/2*ln(x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {2505, 199, 45}

$$\frac{1}{2}x^2\log\left(\frac{1}{x}+1\right)+\frac{x}{2}-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[1 + x^(-1)],x]
```

```
[Out] x/2 + (x^2*Log[1 + x^(-1)])/2 - Log[1 + x]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}x^2\log\left(1+\frac{1}{x}\right)+\frac{1}{2}\int\frac{1}{1+\frac{1}{x}}\,dx\\ &= \frac{1}{2}x^2\log\left(1+\frac{1}{x}\right)+\frac{1}{2}\int\frac{x}{1+x}\,dx\\ &= \frac{1}{2}x^2\log\left(1+\frac{1}{x}\right)+\frac{1}{2}\int\left(1+\frac{1}{-1-x}\right)dx\\ &= \frac{x}{2}+\frac{1}{2}x^2\log\left(1+\frac{1}{x}\right)-\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.81

$$\frac{1}{2}\left(x+x^2\log\left(1+\frac{1}{x}\right)-\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[1 + x^(-1)],x]
[Out] (x + x^2*Log[1 + x^(-1)] - Log[1 + x])/2
```

Maple [A]
time = 0.04, size = 32, normalized size = 1.19

method	result	size
risch	$\frac{x}{2}+\frac{x^2\ln\left(1+\frac{1}{x}\right)}{2}-\frac{\ln(x+1)}{2}$	22
parts	$\frac{x}{2}+\frac{x^2\ln\left(1+\frac{1}{x}\right)}{2}-\frac{\ln(x+1)}{2}$	22
derivativedivides	$\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{x}{2}-\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)\left(\frac{1}{x}-1\right)x^2}{2}$	32
default	$\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{x}{2}-\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)\left(\frac{1}{x}-1\right)x^2}{2}$	32
parallelrisch	$\frac{\ln\left(\frac{x+1}{x}\right)x^2}{2}-\frac{1}{2}-\frac{\ln(x)}{2}+\frac{x}{2}-\frac{\ln\left(\frac{x+1}{x}\right)}{2}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(1+1/x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1/x)+1/2*x-1/2*ln(1+1/x)*(1+1/x)*(1/x-1)*x^2
```

Maxima [A]
time = 0.34, size = 21, normalized size = 0.78

$$\frac{1}{2}\,x^2\log\left(\frac{1}{x}+1\right)+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+1/x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(1/x + 1) + 1/2*x - 1/2*log(x + 1)
```

Fricas [A]
time = 0.58, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x^2\log\left(\frac{x+1}{x}\right)+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+1/x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x + 1)/x) + 1/2*x - 1/2*log(x + 1)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.74

$$\frac{x^2\log\left(1+\frac{1}{x}\right)}{2}+\frac{x}{2}-\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(1+1/x),x)
```

```
[Out] x**2*log(1 + 1/x)/2 + x/2 - log(x + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(21) = 42.
time = 0.41, size = 60, normalized size = 2.22

$$\frac{1}{2\left(\frac{x+1}{x}-1\right)}+\frac{\log\left(\frac{x+1}{x}\right)}{2\left(\frac{x+1}{x}-1\right)^2}-\frac{1}{2}\,\log\left(\frac{|x+1|}{|x|}\right)+\frac{1}{2}\,\log\left(\left|\frac{x+1}{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+1/x),x, algorithm="giac")
```

```
[Out] 1/2/((x + 1)/x - 1) + 1/2*log((x + 1)/x)/((x + 1)/x - 1)^2 - 1/2*log(abs(x
+ 1)/abs(x)) + 1/2*log(abs((x + 1)/x - 1))
```

Mupad [B]
time = 0.23, size = 31, normalized size = 1.15

$$\frac{x}{2}-\frac{\ln\left(x\,\left(x+1\right)\right)}{4}-\frac{\ln\left(\frac{1}{x}+1\right)}{4}+\frac{x^2\,\ln\left(\frac{1}{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(1/x + 1),x)
```

```
[Out] x/2 - log(x*(x + 1))/4 - log(1/x + 1)/4 + (x^2*log(1/x + 1))/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 15, normalized size = 0.56

$$-x\ln\left(1+\frac{1}{x}\right)+x-\ln\left(x\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(x*ln(1+1/x),x)
```

```
[Out] -x*ln(1+1/x)+x-ln(x)
```

3.5 $\int \sin^2(\log(x))\,dx$

Optimal. Leaf size=27

$$\frac{2x}{5}-\frac{2}{5}x\cos(\log(x))\sin(\log(x))+\frac{1}{5}x\sin^2(\log(x))$$

```
[Out] 2/5*x-2/5*x*cos(ln(x))*sin(ln(x))+1/5*x*sin(ln(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4565, 8}

$$\frac{2x}{5}+\frac{1}{5}x\sin^2(\log(x))-\frac{2}{5}x\sin(\log(x))\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]]^2,x]
```

```
[Out] (2*x)/5 - (2*x*Cos[Log[x]]*Sin[Log[x]])/5 + (x*Sin[Log[x]]^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4565

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Dist[b^2*d^2
*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1)), Int[Sin[d*(a + b*Log[c*x^n])]^(p -
2), x], x] - Simp[b*d*n*p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x
^n])]^(p - 1)/(b^2*d^2*n^2*p^2 + 1)), x]) /; FreeQ[{a, b, c, d, n}, x] && I
GtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\frac{2}{5}x\cos(\log(x))\sin(\log(x))+\frac{1}{5}x\sin^2(\log(x))+\frac{2\int 1\,dx}{5}\\ &= \frac{2x}{5}-\frac{2}{5}x\cos(\log(x))\sin(\log(x))+\frac{1}{5}x\sin^2(\log(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{x}{2}-\frac{1}{10}x\cos(2\log(x))-\frac{1}{5}x\sin(2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]]^2,x]
[Out] x/2 - (x*Cos[2*Log[x]])/10 - (x*Sin[2*Log[x]])/5
```

Maple [A]
time = 0.12, size = 20, normalized size = 0.74

method	result	size
parallelrisch	$-\frac{x(-5+\cos(2\ln(x))+2\sin(2\ln(x)))}{10}$	18
default	$\frac{(\sin(\ln(x))-2\cos(\ln(x)))x\sin(\ln(x))}{5}+\frac{2x}{5}$	20
risch	$\frac{x}{2}+\left(-\frac{1}{20}+\frac{i}{10}\right)x\,x^{2i}+\left(-\frac{1}{20}-\frac{i}{10}\right)x\,x^{-2i}$	27
norman	$\frac{\frac{2x}{5}-\frac{4x\tan\left(\frac{\ln(x)}{2}\right)}{5}+\frac{8x\left(\tan^2\left(\frac{\ln(x)}{2}\right)\right)}{5}+\frac{4x\left(\tan^3\left(\frac{\ln(x)}{2}\right)\right)}{5}+\frac{2x\left(\tan^4\left(\frac{\ln(x)}{2}\right)\right)}{5}}{\left(1+\tan^2\left(\frac{\ln(x)}{2}\right)\right)^2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/5*(sin(ln(x))-2*cos(ln(x)))*x*sin(ln(x))+2/5*x
```

Maxima [A]
time = 0.32, size = 20, normalized size = 0.74

$$-\frac{1}{10}\,x\cos\left(2\,\log\left(x\right)\right)-\frac{1}{5}\,x\sin\left(2\,\log\left(x\right)\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2,x, algorithm="maxima")
[Out] -1/10*x*cos(2*log(x)) - 1/5*x*sin(2*log(x)) + 1/2*x
```

Fricas [A]
time = 0.61, size = 21, normalized size = 0.78

$$-\frac{1}{5}\,x\cos\left(\log\left(x\right)\right)^2-\frac{2}{5}\,x\cos\left(\log\left(x\right)\right)\sin\left(\log\left(x\right)\right)+\frac{3}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2,x, algorithm="fricas")
[Out] -1/5*x*cos(log(x))^2 - 2/5*x*cos(log(x))*sin(log(x)) + 3/5*x
```

Sympy [A]
time = 0.23, size = 37, normalized size = 1.37

$$\frac{3x\sin^2\left(\log\left(x\right)\right)}{5}-\frac{2x\sin\left(\log\left(x\right)\right)\cos\left(\log\left(x\right)\right)}{5}+\frac{2x\cos^2\left(\log\left(x\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x))**2,x)
```

```
[Out] 3*x*sin(log(x))**2/5 - 2*x*sin(log(x))*cos(log(x))/5 + 2*x*cos(log(x))**2/5
```

Giac [A]
time = 0.41, size = 21, normalized size = 0.78

$$-\frac{1}{5}\,x\cos\left(\log\left(x\right)\right)^{2}-\frac{2}{5}\,x\cos\left(\log\left(x\right)\right)\sin\left(\log\left(x\right)\right)+\frac{3}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2,x, algorithm="giac")
```

```
[Out] -1/5*x*cos(log(x))^2 - 2/5*x*cos(log(x))*sin(log(x)) + 3/5*x
```

Mupad [B]
time = 0.11, size = 24, normalized size = 0.89

$$\frac{x}{2}+\frac{x\,\left(2\sin\left(\ln\left(x\right)\right)^{2}-1\right)}{10}-\frac{x\,\sin\left(2\,\ln\left(x\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(log(x))^2,x)
```

```
[Out] x/2 + (x*(2*sin(log(x))^2 - 1))/10 - (x*sin(2*log(x)))/5
```

Chatgpt [F] Failed to verify
time = 1.00, size = 20, normalized size = 0.74

$$-\sin\left(\ln\left(x\right)\right)\cos\left(\ln\left(x\right)\right)+\frac{\ln\left(x\right)}{2}+\frac{\sin\left(2\ln\left(x\right)\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(ln(x))^2,x)
```

```
[Out] -sin(ln(x))*cos(ln(x))+1/2*ln(x)+1/4*sin(2*ln(x))
```

3.6 $\int \frac{1}{1+3e^x} dx$

Optimal. Leaf size=12

$$x - \log(1 + 3e^x)$$

[Out] x-ln(1+3*exp(x))

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2320, 36, 29, 31}

$$x - \log(3e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 3*E^x)^(-1),x]

[Out] x - Log[1 + 3*E^x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{x(1+3x)}\,dx, x, e^x\right)\\ &= -\left(3\text{Subst}\left(\int \frac{1}{1+3x}\,dx, x, e^x\right)\right) + \text{Subst}\left(\int \frac{1}{x}\,dx, x, e^x\right)\\ &= x - \log\left(1+3e^x\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.25

$$\log\left(e^x\right) - \log\left(1+3e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*E^x)^(-1),x]
[Out] Log[E^x] - Log[1 + 3*E^x]
```

Maple [A]
time = 0.03, size = 14, normalized size = 1.17

method	result	size
risch	$x-\ln\left(\frac{1}{3}+e^{x}\right)$	10
parallelrisch	$x-\ln\left(\frac{1}{3}+e^{x}\right)$	10
norman	$x-\ln\left(1+3\,e^{x}\right)$	12
derivativedivides	$-\ln\left(1+3\,e^{x}\right)+\ln\left(e^{x}\right)$	14
default	$-\ln\left(1+3\,e^{x}\right)+\ln\left(e^{x}\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+3*exp(x)),x,method=_RETURNVERBOSE)
[Out] -ln(1+3*exp(x))+ln(exp(x))
```

Maxima [A]
time = 0.37, size = 11, normalized size = 0.92

$$x - \log\left(3\,e^x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*exp(x)),x, algorithm="maxima")
[Out] x - log(3*e^x + 1)
```

Fricas [A]
time = 0.58, size = 11, normalized size = 0.92

$$x - \log\left(3 e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*exp(x)),x, algorithm="fricas")
[Out] x - log(3*e^x + 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$x - \log\left(e^{x} + \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*exp(x)),x)
[Out] x - log(exp(x) + 1/3)
```

Giac [A]
time = 0.39, size = 11, normalized size = 0.92

$$x - \log\left(3 e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+3*exp(x)),x, algorithm="giac")
[Out] x - log(3*e^x + 1)
```

Mupad [B]
time = 0.08, size = 11, normalized size = 0.92

$$x - \ln\left(3 \mathrm{e}^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*exp(x) + 1),x)
[Out] x - log(3*exp(x) + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 19, normalized size = 1.58

$$\frac{\ln\left(1 + 3 \mathrm{e}^{x}\right)}{3} - \frac{\ln\left(-2 + 3 \mathrm{e}^{x}\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1+3*exp(x)),x)
[Out] 1/3*ln(1+3*exp(x))-1/3*ln(-2+3*exp(x))
```

3.7 $\int \csc^3(x)\sec^5(x)\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2}\cot^2(x)+3\log(\tan(x))+\frac{3\tan^2(x)}{2}+\frac{\tan^4(x)}{4}$$

```
[Out] -1/2*cot(x)^2+3*ln(tan(x))+3/2*tan(x)^2+1/4*tan(x)^4
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2700, 272, 45}

$$\frac{\tan^4(x)}{4}+\frac{3\tan^2(x)}{2}-\frac{\cot^2(x)}{2}+3\log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3*Sec[x]^5,x]
```

```
[Out] -1/2*Cot[x]^2 + 3*Log[Tan[x]] + (3*Tan[x]^2)/2 + Tan[x]^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, \tan^2(x)\right) \\
&= -\frac{1}{2}\cot^2(x)+3\log(\tan(x))+\frac{3\tan^2(x)}{2}+\frac{\tan^4(x)}{4}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.03

$$-\frac{1}{2}\csc^2(x)-3\log(\cos(x))+3\log(\sin(x))+\sec^2(x)+\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3*Sec[x]^5,x]
```

```
[Out] -1/2*Csc[x]^2 - 3*Log[Cos[x]] + 3*Log[Sin[x]] + Sec[x]^2 + Sec[x]^4/4
```

Maple [A]
time = 0.25, size = 33, normalized size = 1.10

method	result
default	$\frac{1}{4\sin(x)^2\cos(x)^4}+\frac{3}{4\sin(x)^2\cos(x)^2}-\frac{3}{2\sin(x)^2}+3\ln(\tan(x))$
parallelrisch	$-3\ln(1+\csc(x)-\cot(x))-3\ln(\csc(x)-\cot(x)-1)+3\ln(-\cot(x)+\csc(x))+\frac{(8(\text{se}}{}$
norman	$\frac{-\frac{1}{8}-10\left(\tan^6\left(\frac{x}{2}\right)\right)+\frac{57\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}+\frac{57\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{8}}{\tan\left(\frac{x}{2}\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)^4}+3\ln\left(\tan\left(\frac{x}{2}\right)\right)-3\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-3\ln($
risch	$\frac{6\,e^{10ix}+12\,e^{8ix}-4\,e^{6ix}+12\,e^{4ix}+6\,e^{2ix}}{(e^{2ix}+1)^4(e^{2ix}-1)^2}+3\ln(e^{2ix}-1)-3\ln(e^{2ix}+1)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^3/cos(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/sin(x)^2/cos(x)^4+3/4/sin(x)^2/cos(x)^2-3/2/sin(x)^2+3*ln(tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(24) = 48.
time = 0.35, size = 50, normalized size = 1.67

$$-\frac{6\sin(x)^4-9\sin(x)^2+2}{4\left(\sin(x)^6-2\sin(x)^4+\sin(x)^2\right)}-\frac{3}{2}\log\left(\sin(x)^2-1\right)+\frac{3}{2}\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^3/cos(x)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(6*sin(x)^4 - 9*sin(x)^2 + 2)/(sin(x)^6 - 2*sin(x)^4 + sin(x)^2) - 3/2
*log(sin(x)^2 - 1) + 3/2*log(sin(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(24) = 48$.
time = 0.62, size = 69, normalized size = 2.30

$$\frac{6 \cos(x)^4 - 3 \cos(x)^2 - 6 \left(\cos(x)^6 - \cos(x)^4\right) \log\left(\cos(x)^2\right) + 6 \left(\cos(x)^6 - \cos(x)^4\right) \log\left(-\frac{1}{4} \cos(x)^2 + \frac{1}{4}\right) - 1}{4 \left(\cos(x)^6 - \cos(x)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^3/cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(x)^4 - 3*cos(x)^2 - 6*(cos(x)^6 - cos(x)^4)*log(cos(x)^2) + 6*(c
os(x)^6 - cos(x)^4)*log(-1/4*cos(x)^2 + 1/4) - 1)/(cos(x)^6 - cos(x)^4)
```

Sympy [A]
time = 0.07, size = 46, normalized size = 1.53

$$\frac{3 \log\left(\cos^2(x) - 1\right)}{2} - 3 \log\left(\cos(x)\right) - \frac{-6 \cos^4(x) + 3 \cos^2(x) + 1}{4 \cos^6(x) - 4 \cos^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**3/cos(x)**5,x)
```

```
[Out] 3*log(cos(x)**2 - 1)/2 - 3*log(cos(x)) - (-6*cos(x)**4 + 3*cos(x)**2 + 1)/(
4*cos(x)**6 - 4*cos(x)**4)
```

Giac [A]
time = 0.39, size = 46, normalized size = 1.53

$$\frac{6 \cos(x)^4 - 3 \cos(x)^2 - 1}{4 \left(\cos(x)^2 - 1\right) \cos(x)^4} + \frac{3}{2} \log\left(-\cos(x)^2 + 1\right) - 3 \log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^3/cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/4*(6*cos(x)^4 - 3*cos(x)^2 - 1)/((cos(x)^2 - 1)*cos(x)^4) + 3/2*log(-cos(
x)^2 + 1) - 3*log(abs(cos(x)))
```

Mupad [B]
time = 0.17, size = 30, normalized size = 1.00

$$3 \ln\left(\tan(x)\right) + \frac{\frac{3}{4 \cos(x)^2} + \frac{1}{4 \cos(x)^4}}{\sin(x)^2} - \frac{3}{2 \sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^5*sin(x)^3),x)
```

```
[Out] 3*log(tan(x)) + (3/(4*cos(x)^2) + 1/(4*cos(x)^4))/sin(x)^2 - 3/(2*sin(x)^2)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 6, normalized size = 0.20

$$\frac{8}{15\cos{(x)}^3}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/sin(x)^3/cos(x)^5,x)
```

```
[Out] 8/15/cos(x)^3
```

3.8 $\int \frac{1}{x\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2}\arctan\left(\sqrt{-1+x^4}\right)$$

```
[Out] 1/2*arctan((x^4-1)^(1/2))
```

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 65, 209}

$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^4]),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^4]]/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx, x, x^4\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^4}\right)\\ &= \frac{1}{2}\arctan\left(\sqrt{-1+x^4}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{2}\arctan\left(\sqrt{-1+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^4]),x]
[Out] ArcTan[Sqrt[-1 + x^4]]/2
```

Maple [A]
time = 0.24, size = 11, normalized size = 0.79

method	result	size
default	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
elliptic	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
pseudoelliptic	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
trager	$\frac{\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathit{RootOf}\left(\_Z^2+1\right)+\sqrt{x^4-1}}{x^2}\right)}{2}$	28
meijerg	$\frac{\sqrt{-\mathrm{signum}(x^4-1)}\left((-2\ln(2)+4\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^4+1}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{\mathrm{signum}(x^4-1)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(1/(x^4-1)^(1/2))
```

Maxima [A]
time = 0.48, size = 10, normalized size = 0.71

$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(sqrt(x^4 - 1))
```

Fricas [A]
time = 0.56, size = 10, normalized size = 0.71

$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(sqrt(x^4 - 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.68, size = 24, normalized size = 1.71

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^2}\right)}{2} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(x**(-2))/2, 1/Abs(x**4) > 1), (-asin(x**(-2))/2, True))
```

Giac [A]
time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(x^4 - 1))
```

Mupad [B]
time = 0.19, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\sqrt{x^4-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 - 1)^(1/2)),x)
```

```
[Out] atan((x^4 - 1)^(1/2))/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 14, normalized size = 1.00

$$\frac{\ln\left(\frac{\sqrt{x^4-1}}{x^2}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/x/(x^4-1)^(1/2),x)
```

```
[Out] 1/2*ln((x^4-1)^(1/2)/x^2)
```

3.9 $\int \frac{1}{x(1+x^5)}\,dx$

Optimal. Leaf size=13

$$\log(x) - \frac{1}{5}\log\left(1+x^5\right)$$

[Out] ln(x)-1/5*ln(x^5+1)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {272, 36, 29, 31}

$$\log(x) - \frac{1}{5}\log\left(x^5+1\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + x^5)),x]

[Out] Log[x] - Log[1 + x^5]/5

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^5\right)\\ &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^5\right) - \frac{1}{5}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^5\right)\\ &= \log(x) - \frac{1}{5}\log\left(1+x^5\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\log(x) - \frac{1}{5}\log\left(1+x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^5)),x]
```

```
[Out] Log[x] - Log[1 + x^5]/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 0.06, size = 29, normalized size = 2.23

method	result	size
meijerg	$\ln\left(x\right)-\frac{\ln\left(x^{5}+1\right)}{5}$	12
risch	$\ln\left(x\right)-\frac{\ln\left(x^{5}+1\right)}{5}$	12
default	$-\frac{\ln\left(x+1\right)}{5}+\ln\left(x\right)-\frac{\ln\left(x^{4}-x^{3}+x^{2}-x+1\right)}{5}$	29
norman	$-\frac{\ln\left(x+1\right)}{5}+\ln\left(x\right)-\frac{\ln\left(x^{4}-x^{3}+x^{2}-x+1\right)}{5}$	29
parallelrisch	$-\frac{\ln\left(x+1\right)}{5}+\ln\left(x\right)-\frac{\ln\left(x^{4}-x^{3}+x^{2}-x+1\right)}{5}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^5+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln(x+1)+ln(x)-1/5*ln(x^4-x^3+x^2-x+1)
```

Maxima [A]
time = 0.36, size = 15, normalized size = 1.15

$$-\frac{1}{5}\log\left(x^5+1\right)+\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^5+1),x, algorithm="maxima")
[Out] -1/5*log(x^5 + 1) + 1/5*log(x^5)
```

Fricas [A]
time = 0.56, size = 11, normalized size = 0.85

$$-\frac{1}{5}\log\left(x^5+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^5+1),x, algorithm="fricas")
[Out] -1/5*log(x^5 + 1) + log(x)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$\log\left(x\right)-\frac{\log\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**5+1),x)
[Out] log(x) - log(x**5 + 1)/5
```

Giac [A]
time = 0.42, size = 13, normalized size = 1.00

$$-\frac{1}{5}\log\left(\left|x^5+1\right|\right)+\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^5+1),x, algorithm="giac")
[Out] -1/5*log(abs(x^5 + 1)) + log(abs(x))
```

Mupad [B]
time = 0.09, size = 11, normalized size = 0.85

$$\ln\left(x\right)-\frac{\ln\left(x^5+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^5 + 1)),x)
[Out] log(x) - log(x^5 + 1)/5
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 1.00

$$\frac{\ln(x)}{5} - \frac{\ln(x^5 + 1)}{10}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/x/(x^5+1),x)
```

```
[Out] 1/5*ln(x)-1/10*ln(x^5+1)
```

3.10 $\int \sqrt{\tan(x)}\,dx$

Optimal. Leaf size=71

$$-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(x)}}{1+\tan(x)}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tan(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*tan(x)^(1/
2))*2^(1/2)-1/2*arctanh(2^(1/2)*tan(x)^(1/2)/(1+tan(x)))*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 8, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x
]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 +
Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{2}}\\
&= \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}\\
&= -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 0.56

$$\frac{\left(\arctan\left(\sqrt[4]{-\tan^2(x)}\right)-\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(x)}\right)\right)\sqrt[4]{-\tan(x)}}{\sqrt[4]{\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[x]],x]
```

```
[Out] ((ArcTan[(-Tan[x]^2)^(1/4)] - ArcTanh[(-Tan[x]^2)^(1/4)])*(-Tan[x])^(1/4))/
Tan[x]^(1/4)
```

Maple [A]
time = 0.09, size = 49, normalized size = 0.69

method	result	size
lookup	$\frac{\left(\sqrt{\tan}(x)\right)\cos(x)\sqrt{2}\,\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,\left(\sqrt{\tan}(x)\right)\cos(x)+\sin(x)\right)}{2}$	49
default	$\frac{\left(\sqrt{\tan}(x)\right)\cos(x)\sqrt{2}\,\arccos(\cos(x)-\sin(x))}{2\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}\,\ln\left(\cos(x)+\sqrt{2}\,\left(\sqrt{\tan}(x)\right)\cos(x)+\sin(x)\right)}{2}$	49
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan(x)-\sqrt{2}\,\left(\sqrt{\tan}(x)\right)+1}{\tan(x)+\sqrt{2}\,\left(\sqrt{\tan}(x)\right)+1}\right)+2\arctan\left(1+\sqrt{2}\,\left(\sqrt{\tan}(x)\right)\right)+2\arctan\left(-1+\sqrt{2}\,\left(\sqrt{\tan}(x)\right)\right)\right)}{4}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(x)-sin(x))
-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(x))
```

Maxima [A]
time = 0.56, size = 80, normalized size = 1.13

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(53) = 106$.
time = 0.62, size = 180, normalized size = 2.54

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)+\cos(x)+\sin(x)}{\cos(x)}}-\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}-1\right)-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{-\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)-\cos(x)-\sin(x)}{\cos(x)}}-\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}+1\right)-\frac{1}{4}\sqrt{2}\log\left(\frac{4\left(\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)+\cos(x)+\sin(x)\right)}{\cos(x)}\right)+\frac{1}{4}\sqrt{2}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)-\cos(x)-\sin(x)\right)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt((sqrt(2)*sqrt(sin(x)/cos(x))*cos(x) + cos(x) +
 sin(x))/cos(x)) - sqrt(2)*sqrt(sin(x)/cos(x)) - 1) - sqrt(2)*arctan(sqrt(2
)*sqrt(-(sqrt(2)*sqrt(sin(x)/cos(x))*cos(x) - cos(x) - sin(x))/cos(x)) - sq
rt(2)*sqrt(sin(x)/cos(x)) + 1) - 1/4*sqrt(2)*log(4*(sqrt(2)*sqrt(sin(x)/cos
(x))*cos(x) + cos(x) + sin(x))/cos(x)) + 1/4*sqrt(2)*log(-4*(sqrt(2)*sqrt(s
in(x)/cos(x))*cos(x) - cos(x) - sin(x))/cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)), x)
```

Giac [A]
time = 0.44, size = 80, normalized size = 1.13

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(x)}+\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

Mupad [B]
time = 0.15, size = 65, normalized size = 0.92

$$\frac{\sqrt{2}\left(\ln\left(\sqrt{2}\sqrt{\tan(x)}-\tan(x)-1\right)-\ln\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{4}+\frac{\sqrt{2}\left(\mathrm{atan}\left(\sqrt{2}\sqrt{\tan(x)}-1\right)+\mathrm{atan}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(1/2),x)
```

```
[Out] (2^(1/2)*(log(2^(1/2)*tan(x)^(1/2) - tan(x) - 1) - log(tan(x) + 2^(1/2)*tan
(x)^(1/2) + 1)))/4 + (2^(1/2)*(atan(2^(1/2)*tan(x)^(1/2) - 1) + atan(2^(1/2
)*tan(x)^(1/2) + 1)))/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 0.18

$$\frac{4\left(\tan^{\frac{3}{2}}(x)\right)}{3}+\frac{2\left(\tan^{\frac{5}{2}}(x)\right)}{5}$$

Warning: Unable to verify antiderivative.

```
[In] int(tan(x)^(1/2),x)
```

```
[Out] 4/3*tan(x)^(3/2)+2/5*tan(x)^(5/2)
```

3.11 $\int \frac{\log(1+x)}{1+x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)\log(1+x)+\frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)\log(1+x)-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(\right.$$

```
[Out] -1/2*I*ln((1/2-1/2*I)*(I-x))*ln(x+1)+1/2*I*ln((-1/2-1/2*I)*(I+x))*ln(x+1)-1
/2*I*polylog(2,(1/2-1/2*I)*(x+1))+1/2*I*polylog(2,(1/2+1/2*I)*(x+1))
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2456, 2441, 2440, 2438}

$$-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(x+1)\right)+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(x+1)\right)-\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-x+i)\right)\log(x+1)+\frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(x+i)\right)\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x]/(1 + x^2),x]
```

```
[Out] (-1/2*I)*Log[(1/2 - I/2)*(I - x)]*Log[1 + x] + (I/2)*Log[(-1/2 - I/2)*(I +
x)]*Log[1 + x] - (I/2)*PolyLog[2, (1/2 - I/2)*(1 + x)] + (I/2)*PolyLog[2, (
1/2 + I/2)*(1 + x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
```

```
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{i\log(1+x)}{2(i-x)}+\frac{i\log(1+x)}{2(i+x)}\right)dx\\
&= \frac{1}{2}i\int \frac{\log(1+x)}{i-x}\,dx+\frac{1}{2}i\int \frac{\log(1+x)}{i+x}\,dx\\
&= -\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)\log(1+x)+\frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)\log(1+x)+\frac{1}{2}i\int \frac{\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)}{1+x}\,dx-\frac{1}{2}i\int \frac{\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)}{1+x}\,dx\\
&= -\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)\log(1+x)+\frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)\log(1+x)-\frac{1}{2}i\text{Subst}\left(\int \frac{\log\left(1-\left(\frac{1}{2}+\frac{i}{2}\right)x\right)}{x}\,dx,x,1+x\right)+\frac{1}{2}i\text{Subst}\left(\int \frac{\log\left(1-\left(\frac{1}{2}-\frac{i}{2}\right)x\right)}{x}\,dx,x,1+x\right)\\
&= -\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)\log(1+x)+\frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)\log(1+x)-\frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(1+x)\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(1+x)\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 89, normalized size = 1.00

$$-\frac{1}{2}i\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-x)\right)\log(1+x)+\frac{1}{2}i\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+x)\right)\log(1+x)-\frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(1+x)\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x]/(1 + x^2),x]
```

```
[Out] (-1/2*I)*Log[(1/2 - I/2)*(I - x)]*Log[1 + x] + (I/2)*Log[(-1/2 - I/2)*(I +
x)]*Log[1 + x] - (I/2)*PolyLog[2, (1/2 - I/2)*(1 + x)] + (I/2)*PolyLog[2, (
1/2 + I/2)*(1 + x)]
```

Maple [A]
time = 0.08, size = 70, normalized size = 0.79

method	result
derivativedivides	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$
default	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$
risch	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$
parts	$-\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\ln(x+1)\ln\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}-\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}+\frac{i(x+1)}{2}\right)}{2}+\frac{i\,dilog\left(\frac{1}{2}-\frac{x}{2}-\frac{i(x+1)}{2}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+1)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*ln(x+1)*ln(1/2-1/2*x+1/2*I*(x+1))+1/2*I*ln(x+1)*ln(1/2-1/2*x-1/2*I*(
x+1))-1/2*I*dilog(1/2-1/2*x+1/2*I*(x+1))+1/2*I*dilog(1/2-1/2*x-1/2*I*(x+1))
```

Maxima [A]
time = 0.50, size = 56, normalized size = 0.63

$$\frac{1}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2},\frac{1}{2}x+\frac{1}{2}\right)\log\left(x^{2}+1\right)-\frac{1}{2}\arctan\left(x\right)\log\left(\frac{1}{2}x^{2}+x+\frac{1}{2}\right)+\arctan\left(x\right)\log\left(x+1\right)+\frac{1}{2}i\operatorname{Li}_{2}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)x+\frac{1}{2}i+\frac{1}{2}\right)-\frac{1}{2}i\operatorname{Li}_{2}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)x-\frac{1}{2}i+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+1)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(1/2*x + 1/2, 1/2*x + 1/2)*log(x^2 + 1) - 1/2*arctan(x)*log(1/2*
x^2 + x + 1/2) + arctan(x)*log(x + 1) + 1/2*I*dilog((1/2*I - 1/2)*x + 1/2*I
 + 1/2) - 1/2*I*dilog(-(1/2*I + 1/2)*x - 1/2*I + 1/2)
```

Fricas [F]
time = 0.57, size = 14, normalized size = 0.16

$$\operatorname{integral}\left(\frac{\log\left(x+1\right)}{x^{2}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+1)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log(x + 1)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(x+1\right)}{x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+1)/(x**2+1),x)
```

```
[Out] Integral(log(x + 1)/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+1)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(log(x + 1)/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\ln\left(x+1\right)}{x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + 1)/(x^2 + 1),x)
```

```
[Out] int(log(x + 1)/(x^2 + 1), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 10, normalized size = 0.11

$$\frac{\ln\left(x^2+1\right)^3}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(x+1)/(x^2+1),x)
```

```
[Out] 1/2*ln(x^2+1)^3
```

3.12 $\int \frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=53

$$6\sqrt[6]{x}+3\sqrt[3]{x}+2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+x+6\log\left(1-\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)+3*x^(1/3)+2*x^(1/2)+3/2*x^(2/3)+6/5*x^(5/6)+x+6*ln(1-x^(1/6))
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {1598, 272, 45}

$$\frac{6x^{5/6}}{5}+\frac{3x^{2/3}}{2}+x+2\sqrt{x}+3\sqrt[3]{x}+6\sqrt[6]{x}+6\log\left(1-\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-x^(1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + 3*x^(1/3) + 2*Sqrt[x] + (3*x^(2/3))/2 + (6*x^(5/6))/5 + x + 6*L
og[1 - x^(1/6)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{\sqrt[6]{x}}{-1+\sqrt[6]{x}}\,dx \\ &= 6\text{Subst}\left(\int \frac{x^6}{-1+x}\,dx, x, \sqrt[6]{x}\right) \\ &= 6\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x+x^2+x^3+x^4+x^5\right)\,dx, x, \sqrt[6]{x}\right) \\ &= 6\sqrt[6]{x}+3\sqrt[3]{x}+2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+x+6\log\left(1-\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.96

$$6\sqrt[6]{x}+3\sqrt[3]{x}+2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+x+6\log\left(-1+\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-x^(1/3) + Sqrt[x]),x]
[Out] 6*x^(1/6) + 3*x^(1/3) + 2*Sqrt[x] + (3*x^(2/3))/2 + (6*x^(5/6))/5 + x + 6*L
og[-1 + x^(1/6)]
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.68

method	result	size
derivativedivides	$x+\frac{6x^{\frac{5}{6}}}{5}+\frac{3x^{\frac{2}{3}}}{2}+2\sqrt{x}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+6\ln\left(x^{\frac{1}{6}}-1\right)$	36
default	$x+\frac{6x^{\frac{5}{6}}}{5}+\frac{3x^{\frac{2}{3}}}{2}+2\sqrt{x}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+6\ln\left(x^{\frac{1}{6}}-1\right)$	36
meijerg	$\frac{x^{\frac{1}{6}}\left(70x^{\frac{5}{6}}+84x^{\frac{2}{3}}+105\sqrt{x}+140x^{\frac{1}{3}}+210x^{\frac{1}{6}}+420\right)}{70}+6\ln\left(1-x^{\frac{1}{6}}\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2)-x^(1/3)),x,method=_RETURNVERBOSE)
[Out] x+6/5*x^(5/6)+3/2*x^(2/3)+2*x^(1/2)+3*x^(1/3)+6*x^(1/6)+6*ln(x^(1/6)-1)
```

Maxima [A]
time = 0.44, size = 35, normalized size = 0.66

$$x+\frac{6}{5}\,x^{\frac{5}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}+2\,\sqrt{x}+3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}+6\,\log\left(x^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/2)-x^(1/3)),x, algorithm="maxima")
```

```
[Out] x + 6/5*x^(5/6) + 3/2*x^(2/3) + 2*sqrt(x) + 3*x^(1/3) + 6*x^(1/6) + 6*log(x
^(1/6) - 1)
```

Fricas [A]
time = 0.57, size = 35, normalized size = 0.66

$$x+\frac{6}{5}x^{\frac{5}{6}}+\frac{3}{2}x^{\frac{2}{3}}+2\sqrt{x}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+6\log\left(x^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/2)-x^(1/3)),x, algorithm="fricas")
```

```
[Out] x + 6/5*x^(5/6) + 3/2*x^(2/3) + 2*sqrt(x) + 3*x^(1/3) + 6*x^(1/6) + 6*log(x
^(1/6) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{-\sqrt[3]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/2)-x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)/(-x**(1/3) + sqrt(x)), x)
```

Giac [A]
time = 0.42, size = 36, normalized size = 0.68

$$x+\frac{6}{5}x^{\frac{5}{6}}+\frac{3}{2}x^{\frac{2}{3}}+2\sqrt{x}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+6\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/2)-x^(1/3)),x, algorithm="giac")
```

```
[Out] x + 6/5*x^(5/6) + 3/2*x^(2/3) + 2*sqrt(x) + 3*x^(1/3) + 6*x^(1/6) + 6*log(a
bs(x^(1/6) - 1))
```

Mupad [B]
time = 0.10, size = 35, normalized size = 0.66

$$x+6\ln\left(x^{1/6}-1\right)+2\sqrt{x}+3x^{1/3}+\frac{3x^{2/3}}{2}+6x^{1/6}+\frac{6x^{5/6}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2) - x^(1/3)),x)
```

```
[Out] x + 6*log(x^(1/6) - 1) + 2*x^(1/2) + 3*x^(1/3) + (3*x^(2/3))/2 + 6*x^(1/6)
+ (6*x^(5/6))/5
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(x^(1/2)/(x^(1/2)-x^(1/3)),x)
```

```
[Out] not solved
```

3.13 $\int x^x(1+\log(x))\,dx$

Optimal. Leaf size=3

$$x^x$$

```
[Out] x^x
```

Rubi [A]
time = 0.07, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6874, 2633}

$$x^x$$

Antiderivative was successfully verified.

```
[In] Int[x^x*(1 + Log[x]),x]
```

```
[Out] x^x
```

Rule 2633

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(x^x+x^x\log(x)\right)\,dx\\ &= \int x^x\,dx+\int x^x\log(x)\,dx\\ &= x^x\end{aligned}$$

Mathematica [A]
time = 0.02, size = 3, normalized size = 1.00

$$x^x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^x*(1 + Log[x]),x]
```

```
[Out] x^x
```

Maple [A]
time = 0.02, size = 4, normalized size = 1.33

method	result	size
derivativedivides	x^x	4
default	x^x	4
risch	x^x	4
parallelrisch	x^x	4
norman	$e^{x \ln(x)}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^x*(1+ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x^x
```

Maxima [A]
time = 0.39, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^x*(1+log(x)),x, algorithm="maxima")
```

```
[Out] x^x
```

Fricas [A]
time = 0.58, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^x*(1+log(x)),x, algorithm="fricas")
```

```
[Out] x^x
```

Sympy [A]
time = 0.05, size = 2, normalized size = 0.67

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**x*(1+ln(x)),x)
```

```
[Out] x**x
```

Giac [A]
time = 0.45, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^x*(1+log(x)),x, algorithm="giac")
```

```
[Out] x^x
```

Mupad [B]
time = 0.36, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^x*(log(x) + 1),x)
```

```
[Out] x^x
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(x^x*(1+ln(x)),x)
```

```
[Out] not solved
```

3.14 $\int x^{13/2}\sqrt{1+x^{5/2}}\,dx$

Optimal. Leaf size=29

$$\frac{4}{525}\left(1+x^{5/2}\right)^{3/2}\left(8-12x^{5/2}+15x^5\right)$$

```
[Out] 4/525*(1+x^(5/2))^(3/2)*(8-12*x^(5/2)+15*x^5)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.59, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {272, 45}

$$\frac{4}{35}\left(x^{5/2}+1\right)^{7/2}-\frac{8}{25}\left(x^{5/2}+1\right)^{5/2}+\frac{4}{15}\left(x^{5/2}+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)*Sqrt[1 + x^(5/2)],x]
```

```
[Out] (4*(1 + x^(5/2))^(3/2))/15 - (8*(1 + x^(5/2))^(5/2))/25 + (4*(1 + x^(5/2))^
(7/2))/35
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{2}{5}\text{Subst}\left(\int x^2\sqrt{1+x}\,dx,x,x^{5/2}\right)\\
&= \frac{2}{5}\text{Subst}\left(\int \left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx,x,x^{5/2}\right)\\
&= \frac{4}{15}\left(1+x^{5/2}\right)^{3/2}-\frac{8}{25}\left(1+x^{5/2}\right)^{5/2}+\frac{4}{35}\left(1+x^{5/2}\right)^{7/2}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 29, normalized size = 1.00

$$\frac{4}{525}\left(1+x^{5/2}\right)^{3/2}\left(8-12x^{5/2}+15x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)*Sqrt[1 + x^(5/2)],x]
[Out] (4*(1 + x^(5/2))^(3/2)*(8 - 12*x^(5/2) + 15*x^5))/525
```

Maple [A]
time = 0.06, size = 36, normalized size = 1.24

method	result	size
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(1+x^{\frac{5}{2}}\right)^{\frac{3}{2}}\left(8-12x^{\frac{5}{2}}+15x^{5}\right)}{105}}{5\sqrt{\pi}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(1+x^(5/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/5/Pi^(1/2)*(32/105*Pi^(1/2)-4/105*Pi^(1/2)*(1+x^(5/2))^(3/2)*(8-12*x^(5/
2)+15*x^5))
```

Maxima [A]
time = 0.39, size = 28, normalized size = 0.97

$$\frac{4}{35}\left(x^{\frac{5}{2}}+1\right)^{\frac{7}{2}}-\frac{8}{25}\left(x^{\frac{5}{2}}+1\right)^{\frac{5}{2}}+\frac{4}{15}\left(x^{\frac{5}{2}}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(1+x^(5/2))^(1/2),x, algorithm="maxima")
[Out] 4/35*(x^(5/2) + 1)^(7/2) - 8/25*(x^(5/2) + 1)^(5/2) + 4/15*(x^(5/2) + 1)^(3
/2)
```

Fricas [A]
time = 0.70, size = 31, normalized size = 1.07

$$\frac{4}{525}\left(3\,x^{5}+\left(15\,x^{7}-4\,x^{2}\right)\sqrt{x}+8\right)\sqrt{x^{\frac{5}{2}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(1+x^(5/2))^(1/2),x, algorithm="fricas")
[Out] 4/525*(3*x^5 + (15*x^7 - 4*x^2)*sqrt(x) + 8)*sqrt(x^(5/2) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 113.79, size = 66, normalized size = 2.28

$$\frac{4x^{\frac{15}{2}}\sqrt{x^{\frac{5}{2}}+1}}{35}-\frac{16x^{\frac{5}{2}}\sqrt{x^{\frac{5}{2}}+1}}{525}+\frac{4x^{5}\sqrt{x^{\frac{5}{2}}+1}}{175}+\frac{32\sqrt{x^{\frac{5}{2}}+1}}{525}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)*(1+x**(5/2))**(1/2),x)
```

```
[Out] 4*x**(15/2)*sqrt(x**(5/2) + 1)/35 - 16*x**(5/2)*sqrt(x**(5/2) + 1)/525 + 4*
x**5*sqrt(x**(5/2) + 1)/175 + 32*sqrt(x**(5/2) + 1)/525
```

Giac [A]
time = 0.43, size = 28, normalized size = 0.97

$$\frac{4}{35}\left(x^{\frac{5}{2}}+1\right)^{\frac{7}{2}}-\frac{8}{25}\left(x^{\frac{5}{2}}+1\right)^{\frac{5}{2}}+\frac{4}{15}\left(x^{\frac{5}{2}}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)*(1+x^(5/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*(x^(5/2) + 1)^(7/2) - 8/25*(x^(5/2) + 1)^(5/2) + 4/15*(x^(5/2) + 1)^(3
/2)
```

Mupad [B]
time = 0.47, size = 25, normalized size = 0.86

$$-\frac{4\left(x^{5/2}+1\right)^{3/2}\left(42\,x^{5/2}-15\left(x^{5/2}+1\right)^2+7\right)}{525}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)*(x^(5/2) + 1)^(1/2),x)
```

```
[Out] -(4*(x^(5/2) + 1)^(3/2)*(42*x^(5/2) - 15*(x^(5/2) + 1)^2 + 7))/525
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(x^(13/2)*(1+x^(5/2))^(1/2),x)
```

```
[Out] not solved
```

3.15 $\int \frac{1}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

[Out] x/(2*x^2+2)+1/2*arctan(x)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {205, 209}

$$\frac{\arctan(x)}{2} + \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)^(-2),x]

[Out] x/(2*(1 + x^2)) + ArcTan[x]/2

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2} \, dx \\ &= \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{1+x^2} + \arctan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-2),x]
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A]
time = 0.06, size = 16, normalized size = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	16
meijerg	$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}$	17
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(i+x)x^2+i\ln(x-i)-i\ln(i+x)-2x}{4(x^2+1)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)
```

Maxima [A]
time = 0.45, size = 15, normalized size = 0.79

$$\frac{x}{2\,(x^2+1)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A]
time = 0.55, size = 19, normalized size = 1.00

$$\frac{(x^2+1)\arctan\left(x\right)+x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2,x)
[Out] x/(2*x**2 + 2) + atan(x)/2
```

Giac [A]
time = 0.48, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.84

$$\frac{\operatorname{atan}(x)}{2}+\frac{x}{2\ \left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^2,x)
[Out] atan(x)/2 + x/(2*(x^2 + 1))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 36, normalized size = 1.89

$$\frac{\ln\left(\frac{\frac{x}{x^2+1}-1}{\frac{x}{x^2+1}+1}\right)}{2}+\frac{1}{x^2+1}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^2+1)^2,x)
[Out] 1/2*ln((x/(x^2+1)-1)/(x/(x^2+1)+1))+1/(x^2+1)
```

3.16 $\int \frac{1}{36-13x^2+x^4}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{15}\text{arctanh}\left(\frac{x}{3}\right)+\frac{1}{10}\text{arctanh}\left(\frac{x}{2}\right)$$

```
[Out] -1/15*arctanh(1/3*x)+1/10*arctanh(1/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {1107, 213}

$$\frac{1}{10}\text{arctanh}\left(\frac{x}{2}\right)-\frac{1}{15}\text{arctanh}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(36 - 13*x^2 + x^4)^(-1),x]
```

```
[Out] -1/15*ArcTanh[x/3] + ArcTanh[x/2]/10
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{1}{5}\int\frac{1}{-9+x^2}\,dx-\frac{1}{5}\int\frac{1}{-4+x^2}\,dx\\&=-\frac{1}{15}\text{arctanh}\left(\frac{x}{3}\right)+\frac{1}{10}\text{arctanh}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.76

$$-\frac{1}{20}\log(2-x)+\frac{1}{30}\log(3-x)+\frac{1}{20}\log(2+x)-\frac{1}{30}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(36 - 13*x^2 + x^4)^(-1),x]
```

```
[Out] -1/20*Log[2 - x] + Log[3 - x]/30 + Log[2 + x]/20 - Log[3 + x]/30
```

Maple [A]
time = 0.02, size = 26, normalized size = 1.24

method	result	size
default	$-\frac{\ln(x-2)}{20}-\frac{\ln(x+3)}{30}+\frac{\ln(2+x)}{20}+\frac{\ln(-3+x)}{30}$	26
norman	$-\frac{\ln(x-2)}{20}-\frac{\ln(x+3)}{30}+\frac{\ln(2+x)}{20}+\frac{\ln(-3+x)}{30}$	26
risch	$-\frac{\ln(x-2)}{20}-\frac{\ln(x+3)}{30}+\frac{\ln(2+x)}{20}+\frac{\ln(-3+x)}{30}$	26
parallelrisch	$-\frac{\ln(x-2)}{20}-\frac{\ln(x+3)}{30}+\frac{\ln(2+x)}{20}+\frac{\ln(-3+x)}{30}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-13*x^2+36),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*ln(x-2)-1/30*ln(x+3)+1/20*ln(2+x)+1/30*ln(-3+x)
```

Maxima [A]
time = 0.36, size = 25, normalized size = 1.19

$$-\frac{1}{30}\log\left(x+3\right)+\frac{1}{20}\log\left(x+2\right)-\frac{1}{20}\log\left(x-2\right)+\frac{1}{30}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-13*x^2+36),x, algorithm="maxima")
```

```
[Out] -1/30*log(x + 3) + 1/20*log(x + 2) - 1/20*log(x - 2) + 1/30*log(x - 3)
```

Fricas [A]
time = 0.58, size = 25, normalized size = 1.19

$$-\frac{1}{30}\log\left(x+3\right)+\frac{1}{20}\log\left(x+2\right)-\frac{1}{20}\log\left(x-2\right)+\frac{1}{30}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-13*x^2+36),x, algorithm="fricas")
```

```
[Out] -1/30*log(x + 3) + 1/20*log(x + 2) - 1/20*log(x - 2) + 1/30*log(x - 3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.11, size = 26, normalized size = 1.24

$$\frac{\log\left(x-3\right)}{30}-\frac{\log\left(x-2\right)}{20}+\frac{\log\left(x+2\right)}{20}-\frac{\log\left(x+3\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-13*x**2+36),x)
```

```
[Out] log(x - 3)/30 - log(x - 2)/20 + log(x + 2)/20 - log(x + 3)/30
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.
time = 0.42, size = 29, normalized size = 1.38

$$-\frac{1}{30}\log\left(|x+3|\right)+\frac{1}{20}\log\left(|x+2|\right)-\frac{1}{20}\log\left(|x-2|\right)+\frac{1}{30}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-13*x^2+36),x, algorithm="giac")
```

```
[Out] -1/30*log(abs(x + 3)) + 1/20*log(abs(x + 2)) - 1/20*log(abs(x - 2)) + 1/30*
log(abs(x - 3))
```

Mupad [B]
time = 0.07, size = 13, normalized size = 0.62

$$\frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{10}-\frac{\operatorname{atanh}\left(\frac{x}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 13*x^2 + 36),x)
```

```
[Out] atanh(x/2)/10 - atanh(x/3)/15
```

Chatgpt [F] Failed to verify
time = 1.00, size = 33, normalized size = 1.57

$$\frac{\ln\left(\frac{\frac{x}{3}-1}{\frac{x}{3}+1}\right)}{6}-\frac{\ln\left(\frac{\frac{x}{2}-1}{\frac{x}{2}+1}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^4-13*x^2+36),x)
```

```
[Out] 1/6*ln((1/3*x-1)/(1/3*x+1))-1/2*ln((1/2*x-1)/(1/2*x+1))
```

3.17 $\int \frac{\log(\log(x))}{x}\,dx$

Optimal. Leaf size=11

$$-\log(x)+\log(x)\log(\log(x))$$

```
[Out] -ln(x)+ln(x)*ln(ln(x))
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2601}

$$\log(x)\log(\log(x))-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]/x,x]
```

```
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{Integral} = -\log(x)+\log(x)\log(\log(x))$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\log(x)+\log(x)\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]/x,x]
```

```
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Maple [A]
time = 0.02, size = 12, normalized size = 1.09

method	result	size

derivativedivides	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
default	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
norman	$-\ln(x)+\ln(x)\ln(\ln(x))$	12
risch	$-\ln(x)+\ln(x)\ln(\ln(x))$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+ln(x)*ln(ln(x))
```

Maxima [A]
time = 0.35, size = 11, normalized size = 1.00

$$\log(x)\log(\log(x))-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Fricas [A]
time = 0.57, size = 11, normalized size = 1.00

$$\log(x)\log(\log(x))-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Sympy [A]
time = 0.07, size = 10, normalized size = 0.91

$$\log(x)\log(\log(x))-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))/x,x)
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Giac [A]
time = 0.48, size = 11, normalized size = 1.00

$$\log(x)\log(\log(x))-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="giac")
[Out] log(x)*log(log(x)) - log(x)
```

Mupad [B]
time = 0.11, size = 8, normalized size = 0.73

$$\ln(x)\ (\ln(\ln(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))/x,x)
[Out] log(x)*(log(log(x)) - 1)
```

Chatgpt [A]
time = 1.00, size = 11, normalized size = 1.00

$$\ln(\ln(x))\ln(x) - \ln(x)$$

Antiderivative was successfully verified.

```
[In] int(ln(ln(x))/x,x)
[Out] ln(ln(x))*ln(x)-ln(x)
```

3.18 $\int \frac{1+\cot(x)}{1-\cot(x)}\,dx$

Optimal. Leaf size=8

$$\log(\cos(x)-\sin(x))$$

[Out] ln(cos(x)-sin(x))

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3611}

$$\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Cot[x])/(1 - Cot[x]),x]

[Out] Log[Cos[x] - Sin[x]]

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\text{Integral} = \log(\cos(x)-\sin(x))$$

Mathematica [A]
time = 0.03, size = 8, normalized size = 1.00

$$\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cot[x])/(1 - Cot[x]),x]

[Out] Log[Cos[x] - Sin[x]]

Maple [A]
time = 0.05, size = 16, normalized size = 2.00

method	result	size

parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec^2(x)}}\right)+\ln(\tan(x)-1)$	14
risch	$-ix+\ln\left(e^{2ix}-i\right)$	15
derivativedivides	$-\frac{\ln(1+\cot^2(x))}{2}+\ln(-1+\cot(x))$	16
default	$-\frac{\ln(1+\cot^2(x))}{2}+\ln(-1+\cot(x))$	16
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x)-1)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(x))/(1-cot(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1+cot(x)^2)+ln(-1+cot(x))
```

Maxima [A]
time = 0.33, size = 15, normalized size = 1.88

$$-\frac{1}{2}\log\left(\tan(x)^2+1\right)+\log(\tan(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x))/(1-cot(x)),x, algorithm="maxima")
[Out] -1/2*log(tan(x)^2 + 1) + log(tan(x) - 1)
```

Fricas [A]
time = 0.60, size = 11, normalized size = 1.38

$$\frac{1}{2}\log(-\sin(2x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x))/(1-cot(x)),x, algorithm="fricas")
[Out] 1/2*log(-sin(2*x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.
time = 0.14, size = 15, normalized size = 1.88

$$\log(\tan(x)-1)-\frac{\log(\tan^2(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x))/(1-cot(x)),x)
```

```
[Out] log(tan(x) - 1) - log(tan(x)**2 + 1)/2
```

Giac [A]
time = 0.47, size = 16, normalized size = 2.00

$$-\frac{1}{2}\log\left(\tan\left(x\right)^{2}+1\right)+\log\left(\left|\tan\left(x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x))/(1-cot(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(tan(x)^2 + 1) + log(abs(tan(x) - 1))
```

Mupad [B]
time = 0.14, size = 14, normalized size = 1.75

$$-x\,1\mathrm{i}+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cot(x) + 1)/(cot(x) - 1),x)
```

```
[Out] log(exp(x*2i) - 1i) - x*1i
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 1.25

$$-\ln\left(\sin\left(x\right)-\cos\left(x\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((1+cot(x))/(1-cot(x)),x)
```

```
[Out] -ln(sin(x)-cos(x))
```

3.19 $\int \frac{\cos(x)+x\sin(x)}{x(x+\cos(x))}\,dx$

Optimal. Leaf size=11

$$-\log\left(1+\frac{\cos(x)}{x}\right)$$

```
[Out] -ln(1+cos(x)/x)
```

Rubi [A]
time = 0.14, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6844, 31}

$$-\log\left(\frac{\cos(x)}{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + x*Sin[x])/(x*(x + Cos[x])),x]
```

```
[Out] -Log[1 + Cos[x]/x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6844

```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Dist[(-c)*q, Subst[I
nt[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{a
, b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Integ
erQ[m]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos(x)}{x}\right)\\ &= -\log\left(1+\frac{\cos(x)}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 10, normalized size = 0.91

$$\log(x)-\log(x+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + x*Sin[x])/(x*(x + Cos[x])),x]
```

```
[Out] Log[x] - Log[x + Cos[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.
time = 0.17, size = 25, normalized size = 2.27

method	result	size
parallelrisch	$-\ln\left(\frac{x+\cos(x)}{1+\cos(x)}\right)+\ln(x)+\ln\left(\frac{1}{1+\cos(x)}\right)$	25
risch	$ix+\ln(x)-\ln(2x\,\mathrm{e}^{ix}+\mathrm{e}^{2ix}+1)$	26
norman	$-\ln\left(x\left(\tan^2\left(\frac{x}{2}\right)\right)-\left(\tan^2\left(\frac{x}{2}\right)\right)+x+1\right)+\ln(x)+\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+x*sin(x))/x/(x+cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln((x+cos(x))/(1+cos(x)))+ln(x)+ln(1/(1+cos(x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(11) = 22$.
time = 0.45, size = 65, normalized size = 5.91

$$-\frac{1}{2}\log\left(4x^2\cos(x)^2+4x^2\sin(x)^2+4x\sin(2x)\sin(x)+2(2x\cos(x)+1)\cos(2x)+\cos(2x)^2+4x\cos(x)+\sin(2x)^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(4*x^2*cos(x)^2 + 4*x^2*sin(x)^2 + 4*x*sin(2*x)*sin(x) + 2*(2*x*cos
(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*x*cos(x) + sin(2*x)^2 + 1) + log(x)
```

Fricas [A]
time = 0.62, size = 10, normalized size = 0.91

$$-\log(x+\cos(x))+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x, algorithm="fricas")
```

```
[Out] -log(x + cos(x)) + log(x)
```

Sympy [A]
time = 0.10, size = 8, normalized size = 0.73

$$\log(x)-\log(x+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x)
```

```
[Out] log(x) - log(x + cos(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(11) = 22$.
time = 0.43, size = 79, normalized size = 7.18

$$-\frac{1}{2}\log\left(\frac{4\left(x^{2}\tan\left(\frac{1}{2}x\right)^{4}-2x\tan\left(\frac{1}{2}x\right)^{4}+2x^{2}\tan\left(\frac{1}{2}x\right)^{2}+\tan\left(\frac{1}{2}x\right)^{4}+x^{2}-2\tan\left(\frac{1}{2}x\right)^{2}+2x+1\right)}{\tan\left(\frac{1}{2}x\right)^{4}+2\tan\left(\frac{1}{2}x\right)^{2}+1}\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+x*sin(x))/x/(x+cos(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(4*(x^2*tan(1/2*x)^4 - 2*x*tan(1/2*x)^4 + 2*x^2*tan(1/2*x)^2 + tan(
1/2*x)^4 + x^2 - 2*tan(1/2*x)^2 + 2*x + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 +
 1)) + log(abs(x))
```

Mupad [B]
time = 0.12, size = 10, normalized size = 0.91

$$\ln\left(x\right)-\ln\left(x+\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + x*sin(x))/(x*(x + cos(x))),x)
```

```
[Out] log(x) - log(x + cos(x))
```

Chatgpt [A] valid for real x
time = 1.00, size = 10, normalized size = 0.91

$$\ln\left(x\right)-\ln\left(x+\cos\left(x\right)\right)$$

Antiderivative was successfully verified.

```
[In] int((cos(x)+x*sin(x))/x/(x+cos(x)),x)
```

```
[Out] ln(x)-ln(x+cos(x))
```

3.20 $\int \frac{1}{\sec(x)+\sin(x)}\,dx$

Optimal. Leaf size=28

$$\arctan(\cos(x)+\sin(x))-\frac{\operatorname{arctanh}\left(\frac{\cos(x)-\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] arctan(cos(x)+sin(x))-1/3*3^(1/2)*arctanh(1/3*(cos(x)-sin(x))*3^(1/2))

Rubi [F]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sec(x)+\sin(x)}\,dx$$

Verification is not applicable to the result.

[In] Int[(Sec[x] + Sin[x])^(-1),x]

[Out] Defer[Int][(Sec[x] + Sin[x])^(-1), x]

Rubi steps

$$\text{Integral} = \int \frac{1}{\sec(x)+\sin(x)}\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(28) = 56.
time = 0.62, size = 93, normalized size = 3.32

$$\arctan\left(1-\left(-1+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right)+\arctan\left(1+\left(1+\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right)+\frac{-\log\left(\sec^2\left(\frac{x}{2}\right)\left(\sqrt{3}+\cos(x)-\sin(x)\right)\right)+\log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{3}-\cos(x)+\sin(x)\right)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[x] + Sin[x])^(-1),x]

[Out] ArcTan[1 - (-1 + Sqrt[3])*Tan[x/2]] + ArcTan[1 + (1 + Sqrt[3])*Tan[x/2]] + (-Log[Sec[x/2]^2*(Sqrt[3] + Cos[x] - Sin[x])] + Log[-(Sec[x/2]^2*(Sqrt[3] - Cos[x] + Sin[x]))])/(2*Sqrt[3])

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 57, normalized size = 2.04

method	result
default	$\sum_{\_R=RootOf\left(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1\right)}\frac{\left(-\_R^2+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3-3\_R^2+2\_R+1}$
risch	$-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{1}{2}-\frac{i}{2}+\frac{i\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)}{2}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{1}{2}-\frac{i}{2}+\frac{i\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{1}{2}-\frac{i}{2}-\frac{i\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)}{2}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{1}{2}-\frac{i}{2}-\frac{i\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\sqrt{3}}{6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] sum((-_R^2+1)/(2*_R^3-3*_R^2+2*_R+1)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^4-2*_Z^
3+2*_Z^2+2*_Z+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+sec(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sec(x) + sin(x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(25) = 50$.
time = 0.98, size = 251, normalized size = 8.96

$$-\frac{1}{12}\sqrt{3}\log\left(-4\left(\sqrt{3}+\cos(x)\right)\sin(x)+4\sqrt{3}\cos(x)+8\right)+\frac{1}{12}\sqrt{3}\log\left(4\left(\sqrt{3}-\cos(x)\right)\sin(x)-4\sqrt{3}\cos(x)+8\right)+\frac{1}{2}\arctan\left(\frac{\sqrt{3}\cos(x)-\sqrt{3}\sin(x)+\sqrt{-4\left(\sqrt{3}+\cos(x)\right)\sin(x)+4\sqrt{3}\cos(x)+8}+2}{\cos(x)+\sin(x)}\right)-\frac{1}{2}\arctan\left(-\frac{\sqrt{3}\cos(x)-\sqrt{3}\sin(x)-\sqrt{-4\left(\sqrt{3}+\cos(x)\right)\sin(x)+4\sqrt{3}\cos(x)+8}+2}{\cos(x)+\sin(x)}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{3}\cos(x)-\sqrt{3}\sin(x)+2\sqrt{\left(\sqrt{3}-\cos(x)\right)\sin(x)-\sqrt{3}\cos(x)+2}-2}{\cos(x)+\sin(x)}\right)+\frac{1}{2}\arctan\left(-\frac{\sqrt{3}\cos(x)-\sqrt{3}\sin(x)-2\sqrt{\left(\sqrt{3}-\cos(x)\right)\sin(x)-\sqrt{3}\cos(x)+2}-2}{\cos(x)+\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+sec(x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(-4*(sqrt(3) + cos(x))*sin(x) + 4*sqrt(3)*cos(x) + 8) + 1/
12*sqrt(3)*log(4*(sqrt(3) - cos(x))*sin(x) - 4*sqrt(3)*cos(x) + 8) + 1/2*ar
ctan((sqrt(3)*cos(x) - sqrt(3)*sin(x) + sqrt(-4*(sqrt(3) + cos(x))*sin(x) +
 4*sqrt(3)*cos(x) + 8) + 2)/(cos(x) + sin(x))) - 1/2*arctan(-(sqrt(3)*cos(x
) - sqrt(3)*sin(x) - sqrt(-4*(sqrt(3) + cos(x))*sin(x) + 4*sqrt(3)*cos(x) +
 8) + 2)/(cos(x) + sin(x))) - 1/2*arctan((sqrt(3)*cos(x) - sqrt(3)*sin(x) +
 2*sqrt((sqrt(3) - cos(x))*sin(x) - sqrt(3)*cos(x) + 2) - 2)/(cos(x) + sin(
x))) + 1/2*arctan(-(sqrt(3)*cos(x) - sqrt(3)*sin(x) - 2*sqrt((sqrt(3) - cos
(x))*sin(x) - sqrt(3)*cos(x) + 2) - 2)/(cos(x) + sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sin(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+sec(x)),x)
```

```
[Out] Integral(1/(sin(x) + sec(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(25) = 50$.
time = 0.50, size = 81, normalized size = 2.89

$$\frac{1}{2}\pi + \frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3} + \tan\left(\frac{1}{2}x\right) - 1\right)^2 + \tan\left(\frac{1}{2}x\right)^2\right) - \frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3} - \tan\left(\frac{1}{2}x\right) + 1\right)^2 + \tan\left(\frac{1}{2}x\right)^2\right) + \arctan\left(\left(\sqrt{3} + 1\right)\tan\left(\frac{1}{2}x\right) + 1\right) + \arctan\left(-\left(\sqrt{3} - 1\right)\tan\left(\frac{1}{2}x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+sec(x)),x, algorithm="giac")
```

```
[Out] 1/2*pi + 1/6*sqrt(3)*log((sqrt(3) + tan(1/2*x) - 1)^2 + tan(1/2*x)^2) - 1/6
*sqrt(3)*log((sqrt(3) - tan(1/2*x) + 1)^2 + tan(1/2*x)^2) + arctan((sqrt(3)
 + 1)*tan(1/2*x) + 1) + arctan(-(sqrt(3) - 1)*tan(1/2*x) + 1)
```

Mupad [B]
time = 0.31, size = 233, normalized size = 8.32

$$-\mathrm{atan}\left(\frac{96\tan\left(\frac{x}{2}\right)}{64\tan\left(\frac{x}{2}\right) + 64 - \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}} + \frac{\sqrt{3}\,32\mathrm{i}}{64\tan\left(\frac{x}{2}\right) + 64 - \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}} + \frac{32}{64\tan\left(\frac{x}{2}\right) + 64 - \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}} + \frac{\sqrt{3}\tan\left(\frac{x}{2}\right)\,32\mathrm{i}}{64\tan\left(\frac{x}{2}\right) + 64 - \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}}\right)\left(-1 + \frac{\sqrt{3}\,1\mathrm{i}}{3}\right) - \mathrm{atan}\left(-\frac{96\tan\left(\frac{x}{2}\right)}{64\tan\left(\frac{x}{2}\right) + 64 + \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}} + \frac{\sqrt{3}\,32\mathrm{i}}{64\tan\left(\frac{x}{2}\right) + 64 + \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}} - \frac{32}{64\tan\left(\frac{x}{2}\right) + 64 + \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}} + \frac{\sqrt{3}\tan\left(\frac{x}{2}\right)\,32\mathrm{i}}{64\tan\left(\frac{x}{2}\right) + 64 + \sqrt{3}\tan\left(\frac{x}{2}\right)\,64\mathrm{i}}\right)\left(1 + \frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + 1/cos(x)),x)
```

```
[Out] - atan((96*tan(x/2))/(64*tan(x/2) - 3^(1/2)*tan(x/2)*64i + 64) + (3^(1/2)*3
2i)/(64*tan(x/2) - 3^(1/2)*tan(x/2)*64i + 64) + 32/(64*tan(x/2) - 3^(1/2)*t
an(x/2)*64i + 64) + (3^(1/2)*tan(x/2)*32i)/(64*tan(x/2) - 3^(1/2)*tan(x/2)*
64i + 64))*((3^(1/2)*1i)/3 - 1) - atan((3^(1/2)*32i)/(64*tan(x/2) + 3^(1/2)
*tan(x/2)*64i + 64) - (96*tan(x/2))/(64*tan(x/2) + 3^(1/2)*tan(x/2)*64i + 6
4) - 32/(64*tan(x/2) + 3^(1/2)*tan(x/2)*64i + 64) + (3^(1/2)*tan(x/2)*32i)/
(64*tan(x/2) + 3^(1/2)*tan(x/2)*64i + 64))*((3^(1/2)*1i)/3 + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 14, normalized size = 0.50

$$-\frac{\ln\left(\frac{2-\tan(x)}{\tan(x)}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(sin(x)+sec(x)),x)
```

```
[Out] -1/2*ln((2-tan(x))/tan(x))
```

3.21 $\int \frac{1}{\sqrt{1+e^x+e^{2x}}}\,dx$

Optimal. Leaf size=26

$$-\operatorname{arctanh}\left(\frac{2+e^x}{2\sqrt{1+e^x+e^{2x}}}\right)$$

```
[Out] -arctanh(1/2*(2+exp(x))/(1+exp(x)+exp(2*x))^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 738, 212}

$$-\operatorname{arctanh}\left(\frac{e^x+2}{2\sqrt{e^x+e^{2x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + E^x + E^(2*x)],x]
```

```
[Out] -ArcTanh[(2 + E^x)/(2*Sqrt[1 + E^x + E^(2*x)])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{x\sqrt{1+x+x^2}}\,dx, x, e^x\right)\\ &= -\left(2\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{2+e^x}{\sqrt{1+e^x+e^{2x}}}\right)\right)\\ &= -\text{arctanh}\left(\frac{2+e^x}{2\sqrt{1+e^x+e^{2x}}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.88

$$2\text{arctanh}\left(e^x-\sqrt{1+e^x+e^{2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + E^x + E^(2*x)],x]
[Out] 2*ArcTanh[E^x - Sqrt[1 + E^x + E^(2*x)]]
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.77

method	result	size
default	$-\operatorname{arctanh}\left(\frac{2+e^x}{2\sqrt{1+e^x+e^{2x}}}\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+exp(x)+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctanh(1/2*(2+exp(x))/(1+exp(x)+exp(x)^2)^(1/2))
```

Maxima [A]
time = 0.45, size = 18, normalized size = 0.69

$$-\operatorname{arsinh}\left(\frac{2}{3}\sqrt{3}e^{(-x)}+\frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="maxima")
[Out] -arcsinh(2/3*sqrt(3)*e^(-x) + 1/3*sqrt(3))
```

Fricas [A]
time = 0.59, size = 37, normalized size = 1.42

$$-\log\left(\sqrt{e^{(2x)}+e^x+1}-e^x+1\right)+\log\left(\sqrt{e^{(2x)}+e^x+1}-e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(e^(2*x) + e^x + 1) - e^x + 1) + log(sqrt(e^(2*x) + e^x + 1) - e^x
 - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e^{2x} + e^{x} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)+exp(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(exp(2*x) + exp(x) + 1), x)
```

Giac [**A**]
time = 0.48, size = 37, normalized size = 1.42

$$-\log\left(\sqrt{e^{(2\,x)} + e^{x} + 1} - e^{x} + 1\right) + \log\left(-\sqrt{e^{(2\,x)} + e^{x} + 1} + e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(e^(2*x) + e^x + 1) - e^x + 1) + log(-sqrt(e^(2*x) + e^x + 1) + e^
x + 1)
```

Mupad [**B**]
time = 0.18, size = 21, normalized size = 0.81

$$x - \ln\left(\frac{\mathrm{e}^{x}}{2} + \sqrt{\mathrm{e}^{2\,x} + \mathrm{e}^{x} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(2*x) + exp(x) + 1)^(1/2),x)
```

```
[Out] x - log(exp(x)/2 + (exp(2*x) + exp(x) + 1)^(1/2) + 1)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 15, normalized size = 0.58

$$\frac{\mathrm{e}^{x} + \frac{1}{2}}{\sqrt{1 + \left(\mathrm{e}^{x} + \frac{1}{2}\right)^{2}}}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1+exp(x)+exp(2*x))^(1/2),x)
```

```
[Out] (exp(x)+1/2)/(1+(exp(x)+1/2)^2)^(1/2)
```

3.22 $\int e^{x^2} x^3 \, dx$

Optimal. Leaf size=22

$$-\frac{e^{x^2}}{2}+\frac{1}{2}e^{x^2}x^2$$

```
[Out] -1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2243, 2240}

$$\frac{1}{2}e^{x^2}x^2-\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x^3,x]
```

```
[Out] -1/2*E^x^2 + (E^x^2*x^2)/2
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}e^{x^2}x^2-\int e^{x^2}x\,dx\\ &= -\frac{e^{x^2}}{2}+\frac{1}{2}e^{x^2}x^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.64

$$\frac{1}{2}e^{x^2}\left(-1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x^3,x]
[Out] (E^x^2*(-1 + x^2))/2
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
gosper	$\frac{(x^2-1)\mathrm{e}^{x^2}}{2}$	12
risch	$\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{x^2}$	13
meijerg	$\frac{1}{2}-\frac{(-2x^2+2)\mathrm{e}^{x^2}}{4}$	16
derivativedivides	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
default	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
norman	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
parallelrisch	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
parts	$\frac{\sqrt{\pi}\,\mathrm{erfi}(x)x^3}{2}-\frac{3\sqrt{\pi}\left(\frac{x^3\,\mathrm{erfi}(x)}{3}-\frac{2\left(-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}\right)}{3\sqrt{\pi}}\right)}{2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Maxima [A]
time = 0.33, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(x^2),x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Fricas [A]
time = 0.56, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(x^2),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.45

$$\frac{(x^2-1)\,e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*exp(x**2),x)
[Out] (x**2 - 1)*exp(x**2)/2
```

Giac [A]
time = 0.50, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(x^2),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Mupad [B]
time = 0.07, size = 11, normalized size = 0.50

$$\frac{\mathrm{e}^{x^2}\,(x^2-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(x^2),x)
[Out] (exp(x^2)*(x^2 - 1))/2
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(x^3*exp(x^2),x)
[Out] not solved
```

3.23 $\int \left(-\frac{1}{\log^2(x)}+\frac{1}{\log(x)}\right) dx$

Optimal. Leaf size=6

$$\frac{x}{\log(x)}$$

[Out] x/ln(x)

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2334, 2335}

$$\frac{x}{\log(x)}$$

Antiderivative was successfully verified.

[In] Int[-Log[x]^(-2) + Log[x]^(-1),x]

[Out] x/Log[x]

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\int \frac{1}{\log^2(x)}\,dx+\int \frac{1}{\log(x)}\,dx\\
&= \frac{x}{\log(x)}+\text{LogIntegral}(x)-\int \frac{1}{\log(x)}\,dx\\
&= \frac{x}{\log(x)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$\frac{x}{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[-Log[x]^(-2) + Log[x]^(-1),x]
```

```
[Out] x/Log[x]
```

Maple [A]
time = 0.01, size = 7, normalized size = 1.17

method	result	size
default	$\frac{x}{\ln(x)}$	7
norman	$\frac{x}{\ln(x)}$	7
risch	$\frac{x}{\ln(x)}$	7
parallelrisch	$\frac{x}{\ln(x)}$	7
parts	$\frac{x}{\ln(x)}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x)-1/ln(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/ln(x)
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 12, normalized size = 2.00

$$\mathrm{Ei}(\log{(x)}) - \Gamma(-1, -\log{(x)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x)-1/log(x)^2,x, algorithm="maxima")
```

```
[Out] Ei(log(x)) - gamma(-1, -log(x))
```

Fricas [A]
time = 0.54, size = 6, normalized size = 1.00

$$\frac{x}{\log{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x)-1/log(x)^2,x, algorithm="fricas")
```

```
[Out] x/log(x)
```

Sympy [A]
time = 0.03, size = 3, normalized size = 0.50

$$\frac{x}{\log{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x)-1/ln(x)**2,x)
```

```
[Out] x/log(x)
```

Giac [A]
time = 0.48, size = 6, normalized size = 1.00

$$\frac{x}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x)-1/log(x)^2,x, algorithm="giac")
```

```
[Out] x/log(x)
```

Mupad [B]
time = 0.15, size = 6, normalized size = 1.00

$$\frac{x}{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(x) - 1/log(x)^2,x)
```

```
[Out] x/log(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 7, normalized size = 1.17

$$-\frac{x}{\ln(x)}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/ln(x)-1/ln(x)^2,x)
```

```
[Out] -x/ln(x)
```

3.24 $\int \sqrt{2-x}\sqrt{-1+x}\,dx$

Optimal. Leaf size=53

$$\frac{1}{4}\sqrt{2-x}\sqrt{-1+x}-\frac{1}{2}(2-x)^{3/2}\sqrt{-1+x}+\frac{1}{4}\arcsin\left(\sqrt{-1+x}\right)$$

```
[Out] 1/4*(x-1)^(1/2)*(2-x)^(1/2)-1/2*(2-x)^(3/2)*(x-1)^(1/2)+1/4*arcsin((x-1)^(1
/2))
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {52, 55, 633, 222}

$$-\frac{1}{8}\arcsin(3-2x)-\frac{1}{2}\sqrt{x-1}(2-x)^{3/2}+\frac{1}{4}\sqrt{x-1}\sqrt{2-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - x]*Sqrt[-1 + x],x]
```

```
[Out] (Sqrt[2 - x]*Sqrt[-1 + x])/4 - ((2 - x)^(3/2)*Sqrt[-1 + x])/2 - ArcSin[3 -
2*x]/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\frac{1}{2}(2-x)^{3/2}\sqrt{-1+x} + \frac{1}{4}\int \frac{\sqrt{2-x}}{\sqrt{-1+x}}\,dx \\
&= \frac{1}{4}\sqrt{2-x}\sqrt{-1+x} - \frac{1}{2}(2-x)^{3/2}\sqrt{-1+x} + \frac{1}{8}\int \frac{1}{\sqrt{2-x}\sqrt{-1+x}}\,dx \\
&= \frac{1}{4}\sqrt{2-x}\sqrt{-1+x} - \frac{1}{2}(2-x)^{3/2}\sqrt{-1+x} + \frac{1}{8}\int \frac{1}{\sqrt{-2+3x-x^2}}\,dx \\
&= \frac{1}{4}\sqrt{2-x}\sqrt{-1+x} - \frac{1}{2}(2-x)^{3/2}\sqrt{-1+x} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 3-2x\right) \\
&= \frac{1}{4}\sqrt{2-x}\sqrt{-1+x} - \frac{1}{2}(2-x)^{3/2}\sqrt{-1+x} - \frac{1}{8}\arcsin(3-2x)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 1.36

$$\frac{\sqrt{-2+3x-x^2}\left(\sqrt{-1+x}(6-7x+2x^2) - \sqrt{-2+x}\,\text{arctanh}\left(\frac{1}{\sqrt{\frac{-2+x}{-1+x}}}\right)\right)}{4(-2+x)\sqrt{-1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - x]*Sqrt[-1 + x],x]
```

```
[Out] (Sqrt[-2 + 3*x - x^2]*(Sqrt[-1 + x]*(6 - 7*x + 2*x^2) - Sqrt[-2 + x]*ArcTan
h[1/Sqrt[(-2 + x)/(-1 + x)]]))/(4*(-2 + x)*Sqrt[-1 + x])
```

Maple [A]
time = 0.09, size = 61, normalized size = 1.15

method	result	size
default	$-\frac{(2-x)^{\frac{3}{2}}\sqrt{x-1}}{2}+\frac{\sqrt{x-1}\,\sqrt{2-x}}{4}+\frac{\sqrt{(2-x)(x-1)}\,\arcsin(-3+2x)}{8\sqrt{2-x}\,\sqrt{x-1}}$	61
risch	$-\frac{(-3+2x)\sqrt{x-1}\,(x-2)\sqrt{(2-x)(x-1)}}{4\sqrt{-(x-1)(x-2)}\,\sqrt{2-x}}+\frac{\sqrt{(2-x)(x-1)}\,\arcsin(-3+2x)}{8\sqrt{2-x}\,\sqrt{x-1}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)*(2-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2-x)^(3/2)*(x-1)^(1/2)+1/4*(x-1)^(1/2)*(2-x)^(1/2)+1/8*((2-x)*(x-1))^
(1/2)/(2-x)^(1/2)/(x-1)^(1/2)*arcsin(-3+2*x)
```

Maxima [A]
time = 0.41, size = 38, normalized size = 0.72

$$\frac{1}{2}\sqrt{-x^2+3x-2}x-\frac{3}{4}\sqrt{-x^2+3x-2}+\frac{1}{8}\arcsin(2x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)^(1/2)*(2-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 3*x - 2)*x - 3/4*sqrt(-x^2 + 3*x - 2) + 1/8*arcsin(2*x - 3)
```

Fricas [A]
time = 0.58, size = 52, normalized size = 0.98

$$\frac{1}{4}(2x-3)\sqrt{x-1}\sqrt{-x+2}-\frac{1}{8}\arctan\left(\frac{(2x-3)\sqrt{x-1}\sqrt{-x+2}}{2(x^2-3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)^(1/2)*(2-x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 3)*sqrt(x - 1)*sqrt(-x + 2) - 1/8*arctan(1/2*(2*x - 3)*sqrt(x -
1)*sqrt(-x + 2)/(x^2 - 3*x + 2))
```

Sympy [C] Result contains complex when optimal does not.
time = 2.76, size = 124, normalized size = 2.34

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\sqrt{x-1}\right)}{4}+\frac{i(x-1)^{\frac{5}{2}}}{2\sqrt{x-2}}-\frac{3i(x-1)^{\frac{3}{2}}}{4\sqrt{x-2}}+\frac{i\sqrt{x-1}}{4\sqrt{x-2}} & \text{for } |x-1|>1 \\ \frac{\operatorname{asin}\left(\sqrt{x-1}\right)}{4}-\frac{(x-1)^{\frac{5}{2}}}{2\sqrt{2-x}}+\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{2-x}}-\frac{\sqrt{x-1}}{4\sqrt{2-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)**(1/2)*(2-x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(x - 1))/4 + I*(x - 1)**(5/2)/(2*sqrt(x - 2)) - 3*I
*(x - 1)**(3/2)/(4*sqrt(x - 2)) + I*sqrt(x - 1)/(4*sqrt(x - 2)), Abs(x - 1)
 > 1), (asin(sqrt(x - 1))/4 - (x - 1)**(5/2)/(2*sqrt(2 - x)) + 3*(x - 1)**(
3/2)/(4*sqrt(2 - x)) - sqrt(x - 1)/(4*sqrt(2 - x)), True))
```

Giac [A]
time = 0.46, size = 42, normalized size = 0.79

$$\frac{1}{4}(2x+1)\sqrt{x-1}\sqrt{-x+2}-\sqrt{x-1}\sqrt{-x+2}+\frac{1}{4}\arcsin\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)^(1/2)*(2-x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + 1)*sqrt(x - 1)*sqrt(-x + 2) - sqrt(x - 1)*sqrt(-x + 2) + 1/4*arc
sin(sqrt(x - 1))
```

Mupad [B]
time = 0.10, size = 41, normalized size = 0.77

$$\left(\frac{x}{2}-\frac{3}{4}\right)\sqrt{x-1}\,\sqrt{2-x}-\frac{\ln\left(x-\frac{3}{2}-\sqrt{x-1}\,\sqrt{2-x}\,1\mathrm{i}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)*(2 - x)^(1/2),x)
```

```
[Out] (x/2 - 3/4)*(x - 1)^(1/2)*(2 - x)^(1/2) - (log(x - (x - 1)^(1/2)*(2 - x)^(1
/2)*1i - 3/2)*1i)/8
```

Chatgpt [F] Failed to verify
time = 1.00, size = 29, normalized size = 0.55

$$\frac{(2-x)^{\frac{3}{2}}\sqrt{x-1}}{3}-\frac{(2-x)^{\frac{5}{2}}\sqrt{x-1}}{10}$$

Warning: Unable to verify antiderivative.

```
[In] int((x-1)^(1/2)*(2-x)^(1/2),x)
```

```
[Out] 1/3*(2-x)^(3/2)*(x-1)^(1/2)-1/10*(2-x)^(5/2)*(x-1)^(1/2)
```

3.25 $\int \frac{-1+x^6}{-1-x+x^3+x^4} \, dx$

Optimal. Leaf size=16

$$x-\frac{x^2}{2}+\frac{x^3}{3}$$

```
[Out] x-1/2*x^2+1/3*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1600}

$$\frac{x^3}{3}-\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)/(-1 - x + x^3 + x^4),x]
```

```
[Out] x - x^2/2 + x^3/3
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \int \left(1-x+x^2\right) \, dx \\ &= x-\frac{x^2}{2}+\frac{x^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$x-\frac{x^2}{2}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)/(-1 - x + x^3 + x^4),x]
```

```
[Out] x - x^2/2 + x^3/3
```

Maple [A]
time = 0.01, size = 13, normalized size = 0.81

method	result	size
default	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
norman	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
risch	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parallelrisch	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parts	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
gosper	$\frac{x(2x^{2}-3x+6)}{6}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-1)/(x^4+x^3-x-1),x,method=_RETURNVERBOSE)
[Out] x-1/2*x^2+1/3*x^3
```

Maxima [A]
time = 0.33, size = 12, normalized size = 0.75

$$\frac{1}{3}x^{3}-\frac{1}{2}x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+x^3-x-1),x, algorithm="maxima")
[Out] 1/3*x^3 - 1/2*x^2 + x
```

Fricas [A]
time = 0.56, size = 12, normalized size = 0.75

$$\frac{1}{3}x^{3}-\frac{1}{2}x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+x^3-x-1),x, algorithm="fricas")
[Out] 1/3*x^3 - 1/2*x^2 + x
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.62

$$\frac{x^{3}}{3}-\frac{x^{2}}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-1)/(x**4+x**3-x-1),x)
```

```
[Out] x**3/3 - x**2/2 + x
```

Giac [A]
time = 0.46, size = 12, normalized size = 0.75

$$\frac{1}{3}\,x^3 - \frac{1}{2}\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-1)/(x^4+x^3-x-1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 1/2*x^2 + x
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.81

$$\frac{x\,\left(2\,x^2-3\,x+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 - 1)/(x - x^3 - x^4 + 1),x)
```

```
[Out] (x*(2*x^2 - 3*x + 6))/6
```

Chatgpt [F] Failed to verify
time = 1.00, size = 11, normalized size = 0.69

$$\frac{1}{3}x^3 + \frac{1}{2}x^2$$

Warning: Unable to verify antiderivative.

```
[In] int((x^6-1)/(x^4+x^3-x-1),x)
```

```
[Out] 1/3*x^3+1/2*x^2
```

3.26 $\int \left(2\log(x)+\log^2(x)\right)\, dx$

Optimal. Leaf size=6

$$x\log^2(x)$$

```
[Out] x*ln(x)^2
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2332, 2333}

$$x\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[2*Log[x] + Log[x]^2,x]
```

```
[Out] x*Log[x]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\int \log(x)\,dx+\int \log^2(x)\,dx\\
&= -2x+2x\log(x)+x\log^2(x)-2\int \log(x)\,dx\\
&= x\log^2(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[2*Log[x] + Log[x]^2,x]
[Out] x*Log[x]^2
```

Maple [A]
time = 0.01, size = 7, normalized size = 1.17

method	result	size
default	$x \ln(x)^2$	7
norman	$x \ln(x)^2$	7
risch	$x \ln(x)^2$	7
parallelrisch	$x \ln(x)^2$	7
parts	$x \ln(x)^2$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*ln(x)+ln(x)^2,x,method=_RETURNVERBOSE)
[Out] x*ln(x)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(6) = 12.
time = 0.39, size = 21, normalized size = 3.50

$$(\log(x)^2 - 2 \log(x) + 2)x + 2x \log(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*log(x)+log(x)^2,x, algorithm="maxima")
[Out] (log(x)^2 - 2*log(x) + 2)*x + 2*x*log(x) - 2*x
```

Fricas [A]
time = 0.57, size = 6, normalized size = 1.00

$$x \log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*log(x)+log(x)^2,x, algorithm="fricas")
[Out] x*log(x)^2
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.83

$$x \log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*ln(x)+ln(x)**2,x)
```

```
[Out] x*log(x)**2
```

Giac [**A**]
time = 0.50, size = 6, normalized size = 1.00

$$x \log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*log(x)+log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2
```

Mupad [**B**]
time = 0.05, size = 6, normalized size = 1.00

$$x \ln(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*log(x) + log(x)^2,x)
```

```
[Out] x*log(x)^2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 16, normalized size = 2.67

$$\ln(x)(2\ln(x) + x) - 2x\ln(x) + x$$

Warning: Unable to verify antiderivative.

```
[In] int(2*ln(x)+ln(x)^2,x)
```

```
[Out] ln(x)*(2*ln(x)+x)-2*x*ln(x)+x
```

3.27 $\int \frac{2x}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=4

$$\arcsin\left(x^2\right)$$

[Out] arcsin(x^2)

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {12, 281, 222}

$$\arcsin\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[(2*x)/Sqrt[1 - x^4],x]

[Out] ArcSin[x^2]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\int \frac{x}{\sqrt{1-x^4}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^2\right)\\
&= \arcsin\left(x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 4, normalized size = 1.00

$$\arcsin\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x)/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]
```

Maple [A]
time = 0.12, size = 5, normalized size = 1.25

method	result	size
default	$\arcsin\left(x^{2}\right)$	5
meijerg	$\arcsin\left(x^{2}\right)$	5
elliptic	$\arcsin\left(x^{2}\right)$	5
pseudoelliptic	$\arcsin\left(x^{2}\right)$	5
trager	$\mathit{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathit{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{4}+1}+x^{2}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(x^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(4) = 8.
time = 0.43, size = 16, normalized size = 4.00

$$-\arctan\left(\frac{\sqrt{-x^{4}+1}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(-x^4 + 1)/x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(4) = 8.
time = 0.58, size = 18, normalized size = 4.50

$$-2\arctan\left(\frac{\sqrt{-x^{4}+1}-1}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

Sympy [A]
time = 0.62, size = 20, normalized size = 5.00

$$2\left(\begin{cases} -\frac{i\,\mathrm{acosh}\,(x^2)}{2} & \text{for } |x^4| > 1 \\ \frac{\mathrm{asin}\,(x^2)}{2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-x**4+1)**(1/2),x)
```

```
[Out] 2*Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

Giac [A]
time = 0.48, size = 4, normalized size = 1.00

$$\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x^2)
```

Mupad [B]
time = 0.18, size = 14, normalized size = 3.50

$$\mathrm{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x)/(1 - x^4)^(1/2),x)
```

```
[Out] atan(x^2/(1 - x^4)^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 2.50

$$\frac{\pi}{2} - \arcsin\left(x^2\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(2*x/(-x^4+1)^(1/2),x)
```

```
[Out] 1/2*Pi-arcsin(x^2)
```

3.28 $\int \frac{1+x^2}{1+x} \, dx$

Optimal. Leaf size=17

$$-x+\frac{x^2}{2}+2\log(1+x)$$

[Out] -x+1/2*x^2+2*ln(x+1)

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {711}

$$\frac{x^2}{2}-x+2\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + x),x]

[Out] -x + x^2/2 + 2*Log[1 + x]

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(-1+x+\frac{2}{1+x}\right) \, dx\\ &= -x+\frac{x^2}{2}+2\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.06

$$\frac{1}{2}\left(-3-2x+x^2+4\log(1+x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + x),x]

[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2

Maple [A]
time = 0.06, size = 16, normalized size = 0.94

method	result	size
default	$-x+\frac{x^2}{2}+2\ln(x+1)$	16
norman	$-x+\frac{x^2}{2}+2\ln(x+1)$	16
meijerg	$-\frac{x(-3x+6)}{6}+2\ln(x+1)$	16
risch	$-x+\frac{x^2}{2}+2\ln(x+1)$	16
parallelrisch	$-x+\frac{x^2}{2}+2\ln(x+1)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x+1),x,method=_RETURNVERBOSE)
[Out] -x+1/2*x^2+2*ln(x+1)
```

Maxima [A]
time = 0.33, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x+1),x, algorithm="maxima")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Fricas [A]
time = 0.56, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x+1),x, algorithm="fricas")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.71

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x+1),x)
```

```
[Out] x**2/2 - x + 2*log(x + 1)
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.94

$$\frac{1}{2}x^2 - x + 2\log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + 2*log(abs(x + 1))
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$2\ln(x + 1) - x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + 1),x)
```

```
[Out] 2*log(x + 1) - x + x^2/2
```

Chatgpt [A] valid for real x
time = 1.00, size = 15, normalized size = 0.88

$$\frac{x^2}{2} - x + 2\ln(x + 1)$$

Antiderivative was successfully verified.

```
[In] int((x^2+1)/(x+1),x)
```

```
[Out] 1/2*x^2-x+2*ln(x+1)
```

3.29 $\int \frac{-2-2\sin(x)+\sin^2(x)+\sin^3(x)}{1+2\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=17

$$-x-\cos(x)+\frac{\cos(x)}{1+\sin(x)}$$

[Out] -x-cos(x)+cos(x)/(1+sin(x))

Rubi [A]
time = 0.06, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {4486, 2727, 2718}

$$-x-\cos(x)+\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

[In] Int[(-2 - 2*Sin[x] + Sin[x]^2 + Sin[x]^3)/(1 + 2*Sin[x] + Sin[x]^2),x]

[Out] -x - Cos[x] + Cos[x]/(1 + Sin[x])

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(-1+\frac{1}{-1-\sin(x)}+\sin(x)\right)\,dx\\
&= -x+\int \frac{1}{-1-\sin(x)}\,dx+\int \sin(x)\,dx\\
&= -x-\cos(x)+\frac{\cos(x)}{1+\sin(x)}
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 31, normalized size = 1.82

$$-x-\cos(x)-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 2*Sin[x] + Sin[x]^2 + Sin[x]^3)/(1 + 2*Sin[x] + Sin[x]^2),x
]
```

```
[Out] -x - Cos[x] - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [**A**]
time = 0.10, size = 31, normalized size = 1.82

method	result
parallelrisch	$-x+\tan(x)\left(-1+\sin(x)\right)$
default	$-\frac{2}{1+\tan^2\left(\frac{x}{2}\right)}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\frac{2}{1+\tan\left(\frac{x}{2}\right)}$
risch	$-x-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+\frac{2}{i+e^{ix}}$
norman	$\frac{-2\tan\left(\frac{x}{2}\right)-2\left(\tan^2\left(\frac{x}{2}\right)\right)-2\left(\tan^3\left(\frac{x}{2}\right)\right)-2\left(\tan^4\left(\frac{x}{2}\right)\right)+2\left(\tan^8\left(\frac{x}{2}\right)\right)+2\left(\tan^7\left(\frac{x}{2}\right)\right)+2\left(\tan^6\left(\frac{x}{2}\right)\right)+2\left(\tan^5\left(\frac{x}{2}\right)\right)-x-3x\tan\left(\frac{x}{2}\right)-6x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3(1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^3+sin(x)^2-2*sin(x)-2)/(1+sin(x)^2+2*sin(x)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/(1+tan(1/2*x)^2)-2*arctan(tan(1/2*x))+2/(1+tan(1/2*x))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(17)=34$.
time = 0.44, size = 301, normalized size = 17.71

$$-\frac{4\left(\frac{12\sin(x)}{\cos(x)+1}+\frac{11\sin(x)^2}{(\cos(x)+1)^2}+\frac{9\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{4\sin(x)^2}{(\cos(x)+1)^2}+\frac{4\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^5}{(\cos(x)+1)^5}+1\right)}+\frac{2\left(\frac{9\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+4\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}+\frac{4\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+2\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}+\frac{4\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^3+sin(x)^2-2*sin(x)-2)/(1+sin(x)^2+2*sin(x)),x, algorithm
="maxima")
```

```
[Out] -4/3*(12*sin(x)/(cos(x) + 1) + 11*sin(x)^2/(cos(x) + 1)^2 + 9*sin(x)^3/(cos
(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 5)/(3*sin(x)/(cos(x) + 1) + 4*sin(
x)^2/(cos(x) + 1)^2 + 4*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4
 + sin(x)^5/(cos(x) + 1)^5 + 1) + 2/3*(9*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(
cos(x) + 1)^2 + 4)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + sin
```

```
(x)^3/(cos(x) + 1)^3 + 1) + 4/3*(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x)
 + 1)^2 + 2)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/
(cos(x) + 1)^3 + 1) + 4/3*(3*sin(x)/(cos(x) + 1) + 1)/(3*sin(x)/(cos(x) + 1
) + 3*sin(x)^2/(cos(x) + 1)^2 + sin(x)^3/(cos(x) + 1)^3 + 1) - 2*arctan(sin
(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.59, size = 29, normalized size = 1.71

$$-\frac{x\cos\left(x\right)+\cos\left(x\right)^{2}+\left(x+\cos\left(x\right)+1\right)\sin\left(x\right)+x-1}{\cos\left(x\right)+\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^3+sin(x)^2-2*sin(x)-2)/(1+sin(x)^2+2*sin(x)),x, algorithm
="fricas")
```

```
[Out] -(x*cos(x) + cos(x)^2 + (x + cos(x) + 1)*sin(x) + x - 1)/(cos(x) + sin(x) +
 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(12) = 24$.
time = 11.96, size = 162, normalized size = 9.53

$$-\frac{x\tan^{3}\left(\frac{x}{2}\right)}{\tan^{3}\left(\frac{x}{2}\right)+\tan^{2}\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1}-\frac{x\tan^{2}\left(\frac{x}{2}\right)}{\tan^{3}\left(\frac{x}{2}\right)+\tan^{2}\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1}-\frac{x\tan\left(\frac{x}{2}\right)}{\tan^{3}\left(\frac{x}{2}\right)+\tan^{2}\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1}-\frac{x}{\tan^{3}\left(\frac{x}{2}\right)+\tan^{2}\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1}+\frac{2\tan^{2}\left(\frac{x}{2}\right)}{\tan^{3}\left(\frac{x}{2}\right)+\tan^{2}\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1}-\frac{2\tan\left(\frac{x}{2}\right)}{\tan^{3}\left(\frac{x}{2}\right)+\tan^{2}\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)**3+sin(x)**2-2*sin(x)-2)/(1+sin(x)**2+2*sin(x)),x)
```

```
[Out] -x*tan(x/2)**3/(tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) - x*tan(x/2)**2/(
tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) - x*tan(x/2)/(tan(x/2)**3 + tan(x
/2)**2 + tan(x/2) + 1) - x/(tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) + 2*t
an(x/2)**2/(tan(x/2)**3 + tan(x/2)**2 + tan(x/2) + 1) - 2*tan(x/2)/(tan(x/2
)**3 + tan(x/2)**2 + tan(x/2) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.
time = 0.46, size = 39, normalized size = 2.29

$$-x+\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}-\tan\left(\frac{1}{2}\,x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)^{3}+\tan\left(\frac{1}{2}\,x\right)^{2}+\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^3+sin(x)^2-2*sin(x)-2)/(1+sin(x)^2+2*sin(x)),x, algorithm
="giac")
```

```
[Out] -x + 2*(tan(1/2*x)^2 - tan(1/2*x))/(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x
) + 1)
```

Mupad [B]
time = 0.37, size = 39, normalized size = 2.29

$$-x-\frac{2\tan\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)^2}{\left(\tan\left(\frac{x}{2}\right)^2+1\right)\left(\tan\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*sin(x) - sin(x)^2 - sin(x)^3 + 2)/(2*sin(x) + sin(x)^2 + 1),x)
[Out] - x - (2*tan(x/2) - 2*tan(x/2)^2)/((tan(x/2)^2 + 1)*(tan(x/2) + 1))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 8, normalized size = 0.47

$$-\cos(x)-x$$

Warning: Unable to verify antiderivative.

```
[In] int((sin(x)^3+sin(x)^2-2*sin(x)-2)/(1+sin(x)^2+2*sin(x)),x)
[Out] -cos(x)-x
```

3.30 $\int \operatorname{csch}^2(x) \, dx$

Optimal. Leaf size=4

$$-\coth(x)$$

```
[Out] -coth(x)
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3852, 8\}$

$$-\coth(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^2,x]
```

```
[Out] -Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= -\left(i \text{Subst}\left(\int 1 \, dx, x, -i \coth(x)\right)\right) \\ &= -\coth(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$-\coth(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^2,x]
```

```
[Out] -Coth[x]
```

Maple [A]
time = 0.09, size = 5, normalized size = 1.25

method	result	size
default	$-\coth(x)$	5
parallelrisch	$-\coth(x)$	5
risch	$-\frac{2}{e^{2x}-1}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -coth(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(4) = 8.
time = 0.42, size = 10, normalized size = 2.50

$$\frac{2}{e^{(-2x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(x)^2,x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(4) = 8.
time = 0.54, size = 20, normalized size = 5.00

$$-\frac{2}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(x)^2,x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.
time = 0.27, size = 15, normalized size = 3.75

$$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(x)**2,x)
```

```
[Out] -tanh(x/2)/2 - 1/(2*tanh(x/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(4) = 8$.
time = 0.41, size = 10, normalized size = 2.50

$$-\frac{2}{e^{(2x)} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sinh(x)^2,x, algorithm="giac")
[Out] -2/(e^(2*x) - 1)
```

Mupad [B]
time = 0.01, size = 4, normalized size = 1.00

$$-\coth(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sinh(x)^2,x)
[Out] -coth(x)
```

Chatgpt [A] valid for real x
time = 1.00, size = 4, normalized size = 1.00

$$-\coth\left(x\right)$$

Antiderivative was successfully verified.

```
[In] int(1/sinh(x)^2,x)
[Out] -coth(x)
```

3.31 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}$$

```
[Out] 1/5*tan(x)^5+1/3*tan(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2687, 14}

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(x)\right)\\
&= \frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.59

$$-\frac{2\tan(x)}{15}-\frac{1}{15}\sec^2(x)\tan(x)+\frac{1}{5}\sec^4(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^2,x]
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A]
time = 0.90, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$\frac{(\tan^5(x))}{5}+\frac{(\tan^3(x))}{3}$	14
default	$\frac{(\tan^5(x))}{5}+\frac{(\tan^3(x))}{3}$	14
risch	$-\frac{4i(15\,e^{6ix}-5\,e^{4ix}+5\,e^{2ix}+1)}{15(e^{2ix}+1)^5}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/5*tan(x)^5+1/3*tan(x)^3
```

Maxima [A]
time = 0.35, size = 13, normalized size = 0.76

$$\frac{1}{5}\,\tan\left(x\right)^{5}+\frac{1}{3}\,\tan\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Fricas [A]
time = 0.58, size = 20, normalized size = 1.18

$$-\frac{\left(2\,\cos\left(x\right)^{4}+\cos\left(x\right)^{2}-3\right)\sin\left(x\right)}{15\,\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12)=24$.
time = 0.01, size = 29, normalized size = 1.71

$$-\frac{2\sin\left(x\right)}{15\cos\left(x\right)}-\frac{\sin\left(x\right)}{15\cos^{3}\left(x\right)}+\frac{\sin\left(x\right)}{5\cos^{5}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Giac [A]
time = 0.47, size = 13, normalized size = 0.76

$$\frac{1}{5}\tan\left(x\right)^{5}+\frac{1}{3}\tan\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Mupad [B]
time = 0.22, size = 13, normalized size = 0.76

$$\frac{\tan(x)^5}{5}+\frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/cos(x)^4,x)
```

```
[Out] tan(x)^3/3 + tan(x)^5/5
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 0.76

$$\frac{(\sec^5(x))}{6}-\frac{(\sec^3(x))}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(sec(x)^4*tan(x)^2,x)
```

```
[Out] 1/6*sec(x)^5-1/4*sec(x)^3
```

3.32 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

Optimal. Leaf size=13

$$2\sqrt{\cos(x)\cot(x)}\tan(x)$$

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4482, 4485, 2669}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \sqrt{\cos(x)\cot(x)}\,dx\\
&= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= 2\sqrt{\cos(x)\cot(x)}\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 13, normalized size = 1.00

$$2\sqrt{\cos(x)\cot(x)}\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x] - Sin[x]],x]
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Maple [A]
time = 0.49, size = 12, normalized size = 0.92

method	result	size
default	$2\sqrt{\cos(x)\cot(x)}\,\tan(x)$	12
risch	$-\frac{i\sqrt{2}\sqrt{\frac{i\left(e^{2ix}+1\right)^{2}e^{-ix}}{e^{2ix}-1}}\,\left(e^{2ix}-1\right)}{e^{2ix}+1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(11) = 22.
time = 0.55, size = 188, normalized size = 14.46

$$\frac{\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)-\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)+\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)}{\left(\cos(x)^{2}+\sin(x)^{2}+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^{2}+\sin(x)^{2}-2\cos(x)+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

Fricas [A]
time = 0.59, size = 19, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sin(x) + \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(1/2),x)
[Out] Integral(sqrt(-sin(x) + csc(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(csc(x) - sin(x)), x)
```

Mupad [**B**]
time = 0.30, size = 15, normalized size = 1.15

$$\frac{2\,|\cos(x)|}{\cos(x)\,\sqrt{\frac{1}{\sin(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(1/2),x)
[Out] (2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((csc(x)-sin(x))^(1/2),x)
[Out] not solved
```

3.33 $\int \cos^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}+\frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)$$

```
[Out] 5/16*x+5/16*cos(x)*sin(x)+5/24*cos(x)^3*sin(x)+1/6*cos(x)^5*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{6}\int\cos^4(x)\,dx\\
&= \frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{8}\int\cos^2(x)\,dx\\
&= \frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5\int 1\,dx}{16}\\
&= \frac{5x}{16}+\frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 0.88

$$\frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6,x]
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Maple [A]
time = 0.12, size = 24, normalized size = 0.71

method	result
risch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
parallelrisch	$\frac{5x}{16}+\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}+\frac{15\sin(2x)}{64}$
default	$\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{5\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}+\frac{15\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{15\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{5\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}-\frac{11\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x,method=_RETURNVERBOSE)
[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x
```

Maxima [A]
time = 0.34, size = 24, normalized size = 0.71

$$-\frac{1}{48}\sin\left(2x\right)^3 + \frac{5}{16}x + \frac{3}{64}\sin\left(4x\right) + \frac{1}{4}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="maxima")
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A]
time = 0.61, size = 25, normalized size = 0.74

$$\frac{1}{48}\left(8\cos\left(x\right)^5 + 10\cos\left(x\right)^3 + 15\cos\left(x\right)\right)\sin\left(x\right) + \frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A]
time = 0.01, size = 36, normalized size = 1.06

$$\frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

Giac [A]
time = 0.44, size = 22, normalized size = 0.65

$$\frac{5}{16}\,x + \frac{1}{192}\,\sin(6\,x) + \frac{3}{64}\,\sin(4\,x) + \frac{15}{64}\,\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.65

$$\frac{5\,x}{16} + \frac{15\,\sin(2\,x)}{64} + \frac{3\,\sin(4\,x)}{64} + \frac{\sin(6\,x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x)
```

```
[Out] (5*x)/16 + (15*sin(2*x))/64 + (3*sin(4*x))/64 + sin(6*x)/192
```

Chatgpt [F] Failed to verify
time = 1.00, size = 15, normalized size = 0.44

$$\sin(x) - \frac{2(\sin^3(x))}{3} + \frac{(\sin^5(x))}{5}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)^6,x)
```

```
[Out] sin(x)-2/3*sin(x)^3+1/5*sin(x)^5
```

3.34 $\int \frac{1}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=19

$$\frac{x}{2\left(1+x^2\right)}+\frac{\arctan(x)}{2}$$

[Out] x/(2*x^2+2)+1/2*arctan(x)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 205, 209}

$$\frac{\arctan(x)}{2}+\frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2 + x^4)^(-1),x]

[Out] x/(2*(1 + x^2)) + ArcTan[x]/2

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{1}{(1+x^2)^2}\,dx\\ &= \frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{1+x^2} + \arctan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^4)^(-1),x]
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.84

method	result	size
default	$\frac{x}{2x^2+2} + \frac{\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2} + \frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2 - i\ln(i+x)x^2 + i\ln(x-i) - i\ln(i+x) - 2x}{4(x^2+1)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)
```

Maxima [A]
time = 0.43, size = 15, normalized size = 0.79

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [**A**]
time = 0.56, size = 19, normalized size = 1.00

$$\frac{(x^2+1)\arctan(x)+x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [**A**]
time = 0.05, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+2*x**2+1),x)
[Out] x/(2*x**2 + 2) + atan(x)/2
```

Giac [**A**]
time = 0.46, size = 15, normalized size = 0.79

$$\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+2*x^2+1),x, algorithm="giac")
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [**B**]
time = 0.06, size = 16, normalized size = 0.84

$$\frac{\operatorname{atan}(x)}{2}+\frac{x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 + x^4 + 1),x)
[Out] atan(x)/2 + x/(2*(x^2 + 1))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 11, normalized size = 0.58

$$-\frac{1}{2x^2+2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^4+2*x^2+1),x)
[Out] -1/(2*x^2+2)
```

3.35 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{4564\}$

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4564

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\text{Integral} = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A]
time = 0.04, size = 14, normalized size = 0.82

method	result	size
parallelrisch	$\frac{x(\cos(\ln(x))+\sin(\ln(x)))}{2}$	11
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A]
time = 0.35, size = 10, normalized size = 0.59

$$\frac{1}{2}\,x(\cos\left(\log\left(x\right)\right)+\sin\left(\log\left(x\right)\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A]
time = 0.60, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A]
time = 0.16, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}+\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Giac [**A**]
time = 0.42, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Mupad [**B**]
time = 0.09, size = 13, normalized size = 0.76

$$\frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4}+\ln\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(log(x)),x)
```

```
[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 7, normalized size = 0.41

$$\frac{\sin\left(\ln\left(x\right)\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(ln(x)),x)
```

```
[Out] sin(ln(x))/x
```

3.36 $\int \sec(x)\,dx$

Optimal. Leaf size=3

$$\operatorname{arctanh}(\sin(x))$$

[Out] arctanh(sin(x))

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\operatorname{arctanh}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x],x]

[Out] ArcTanh[Sin[x]]

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{Integral} = \operatorname{arctanh}(\sin(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(3) = 6.
time = 0.00, size = 33, normalized size = 11.00

$$-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x],x]

[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]

Maple [A]
time = 0.05, size = 7, normalized size = 2.33

method	result	size

default	$\ln(\sec(x)+\tan(x))$	7
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
risch	$\ln(i+e^{ix})-\ln(e^{ix}-i)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sec(x)+tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.
time = 0.35, size = 15, normalized size = 5.00

$$\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.
time = 0.60, size = 17, normalized size = 5.67

$$\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.
time = 0.06, size = 15, normalized size = 5.00

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.
time = 0.43, size = 17, normalized size = 5.67

$$\frac{1}{2}\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Mupad [**B**]
time = 0.04, size = 11, normalized size = 3.67

$$\ln\left(\frac{1}{\cos\left(x\right)}\right)+\ln\left(\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x)
```

```
[Out] log(1/cos(x)) + log(sin(x) + 1)
```

Chatgpt [**A**] valid for real x
time = 1.00, size = 6, normalized size = 2.00

$$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)$$

Antiderivative was successfully verified.

```
[In] int(1/cos(x),x)
```

```
[Out] ln(sec(x)+tan(x))
```

3.37 $\int \frac{1}{9\cos^2(x)+4\sin^2(x)}\,dx$

Optimal. Leaf size=24

$$\frac{x}{6}-\frac{1}{6}\arctan\left(\frac{\cos(x)\sin(x)}{2+\cos^2(x)}\right)$$

```
[Out] 1/6*x-1/6*arctan(cos(x)*sin(x)/(2+cos(x)^2))
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {209}

$$\frac{x}{6}-\frac{1}{6}\arctan\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9*Cos[x]^2 + 4*Sin[x]^2)^(-1),x]
```

```
[Out] x/6 - ArcTan[(Cos[x]*Sin[x])/(2 + Cos[x]^2)]/6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{9+4x^2}\,dx, x, \tan(x)\right)\\ &= \frac{x}{6}-\frac{1}{6}\arctan\left(\frac{\cos(x)\sin(x)}{2+\cos^2(x)}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 0.46

$$\frac{1}{6}\arctan\left(\frac{2\tan(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9*Cos[x]^2 + 4*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(2*Tan[x])/3]/6
```

Maple [A]
time = 0.22, size = 8, normalized size = 0.33

method	result	size
default	$\frac{\arctan\left(\frac{2\tan(x)}{3}\right)}{6}$	8
risch	$-\frac{i\ln\left(e^{2ix}+\frac{1}{5}\right)}{12}+\frac{i\ln\left(e^{2ix}+5\right)}{12}$	24
parallelrisch	$-\frac{i\left(\ln\left(\frac{-2i\sin(x)-3\cos(x)}{1+\cos(x)}\right)-\ln\left(\frac{2i\sin(x)-3\cos(x)}{1+\cos(x)}\right)\right)}{12}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*cos(x)^2+4*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(2/3*tan(x))
```

Maxima [A]
time = 0.47, size = 7, normalized size = 0.29

$$\frac{1}{6}\arctan\left(\frac{2}{3}\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*cos(x)^2+4*sin(x)^2),x, algorithm="maxima")
[Out] 1/6*arctan(2/3*tan(x))
```

Fricas [A]
time = 0.61, size = 21, normalized size = 0.88

$$-\frac{1}{12}\arctan\left(\frac{13\cos\left(x\right)^{2}-4}{12\cos\left(x\right)\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*cos(x)^2+4*sin(x)^2),x, algorithm="fricas")
[Out] -1/12*arctan(1/12*(13*cos(x)^2 - 4)/(cos(x)*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{4\sin^{2}\left(x\right)+9\cos^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*cos(x)**2+4*sin(x)**2),x)
```

```
[Out] Integral(1/(4*sin(x)**2 + 9*cos(x)**2), x)
```

Giac [A]
time = 0.47, size = 20, normalized size = 0.83

$$\frac{1}{6}\,x - \frac{1}{6}\,\arctan\left(\frac{\sin\left(2\,x\right)}{\cos\left(2\,x\right) + 5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*cos(x)^2+4*sin(x)^2),x, algorithm="giac")
[Out] 1/6*x - 1/6*arctan(sin(2*x)/(cos(2*x) + 5))
```

Mupad [B]
time = 0.22, size = 16, normalized size = 0.67

$$\frac{x}{6} - \frac{\mathrm{atan}(\tan(x))}{6} + \frac{\mathrm{atan}\left(\frac{2\tan(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*cos(x)^2 + 4*sin(x)^2),x)
[Out] x/6 - atan(tan(x))/6 + atan((2*tan(x))/3)/6
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 0.42

$$\frac{\arctan\left(\frac{\sqrt{5}\,\sin(x)}{3}\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(9*cos(x)^2+4*sin(x)^2),x)
[Out] 1/3*arctan(1/3*5^(1/2)*sin(x))
```

3.38 $\int \frac{1}{x^2(1+x^4)^{3/4}}\,dx$

Optimal. Leaf size=14

$$-\frac{\sqrt[4]{1+x^4}}{x}$$

```
[Out] -(x^4+1)^(1/4)/x
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$-\frac{\sqrt[4]{x^4+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^4)^(3/4)),x]
[Out] -((1 + x^4)^(1/4)/x)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{Integral} = -\frac{\sqrt[4]{1+x^4}}{x}$$

Mathematica [A]
time = 0.09, size = 14, normalized size = 1.00

$$-\frac{\sqrt[4]{1+x^4}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x^4)^(3/4)),x]
[Out] -((1 + x^4)^(1/4)/x)
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{(x^4+1)^{\frac{1}{4}}}{x}$	13
trager	$-\frac{(x^4+1)^{\frac{1}{4}}}{x}$	13
meijerg	$-\frac{(x^4+1)^{\frac{1}{4}}}{x}$	13
risch	$-\frac{(x^4+1)^{\frac{1}{4}}}{x}$	13
pseudoelliptic	$-\frac{(x^4+1)^{\frac{1}{4}}}{x}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^4+1)^(3/4),x,method=_RETURNVERBOSE)
[Out] -(x^4+1)^(1/4)/x
```

Maxima [A]
time = 0.35, size = 12, normalized size = 0.86

$$-\frac{(x^4+1)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="maxima")
[Out] -(x^4 + 1)^(1/4)/x
```

Fricas [A]
time = 0.58, size = 12, normalized size = 0.86

$$-\frac{(x^4+1)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="fricas")
[Out] -(x^4 + 1)^(1/4)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.50, size = 22, normalized size = 1.57

$$\frac{\sqrt[4]{1+\frac{1}{x^4}}\Gamma\left(-\frac{1}{4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**4+1)**(3/4),x)
```

```
[Out] (1 + x**(-4))**(1/4)*gamma(-1/4)/(4*gamma(3/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^4+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)^(3/4)*x^2), x)
```

Mupad [**B**]
time = 0.18, size = 12, normalized size = 0.86

$$-\frac{\left(x^4+1\right)^{1/4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 + 1)^(3/4)),x)
```

```
[Out] -(x^4 + 1)^(1/4)/x
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 12, normalized size = 0.86

$$\frac{x^2}{2\sqrt{x^4+1}}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/x^2/(x^4+1)^(3/4),x)
```

```
[Out] 1/2*x^2/(x^4+1)^(1/2)
```

3.39 $\int \cos(x)\cos(3x)\cos(5x)\,dx$

Optimal. Leaf size=31

$$\frac{\sin(x)}{4}+\frac{1}{12}\sin(3x)+\frac{1}{28}\sin(7x)+\frac{1}{36}\sin(9x)$$

```
[Out] 1/4*sin(x)+1/12*sin(3*x)+1/28*sin(7*x)+1/36*sin(9*x)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4440, 2717}

$$\frac{\sin(x)}{4}+\frac{1}{12}\sin(3x)+\frac{1}{28}\sin(7x)+\frac{1}{36}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[x]/4 + Sin[3*x]/12 + Sin[7*x]/28 + Sin[9*x]/36
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{\cos(x)}{4}+\frac{1}{4}\cos(3x)+\frac{1}{4}\cos(7x)+\frac{1}{4}\cos(9x)\right)dx\\
&= \frac{1}{4}\int \cos(x)\,dx+\frac{1}{4}\int \cos(3x)\,dx+\frac{1}{4}\int \cos(7x)\,dx+\frac{1}{4}\int \cos(9x)\,dx\\
&= \frac{\sin(x)}{4}+\frac{1}{12}\sin(3x)+\frac{1}{28}\sin(7x)+\frac{1}{36}\sin(9x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{\sin(x)}{4}+\frac{1}{12}\sin(3x)+\frac{1}{28}\sin(7x)+\frac{1}{36}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[3*x]*Cos[5*x],x]
[Out] Sin[x]/4 + Sin[3*x]/12 + Sin[7*x]/28 + Sin[9*x]/36
```

Maple [A]
time = 0.62, size = 24, normalized size = 0.77

method	result	size
default	$\frac{\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{\sin(7x)}{28}+\frac{\sin(9x)}{36}$	24
risch	$\frac{\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{\sin(7x)}{28}+\frac{\sin(9x)}{36}$	24
parallelrisch	$\frac{\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{\sin(7x)}{28}+\frac{\sin(9x)}{36}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(3*x)*cos(5*x),x,method=_RETURNVERBOSE)
[Out] 1/4*sin(x)+1/12*sin(3*x)+1/28*sin(7*x)+1/36*sin(9*x)
```

Maxima [A]
time = 0.39, size = 23, normalized size = 0.74

$$\frac{1}{36}\sin\left(9\,x\right)+\frac{1}{28}\sin\left(7\,x\right)+\frac{1}{12}\sin\left(3\,x\right)+\frac{1}{4}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x)*cos(5*x),x, algorithm="maxima")
[Out] 1/36*sin(9*x) + 1/28*sin(7*x) + 1/12*sin(3*x) + 1/4*sin(x)
```

Fricas [A]
time = 0.62, size = 30, normalized size = 0.97

$$\frac{1}{63}\left(448\,\cos\left(x\right)^{8}-640\,\cos\left(x\right)^{6}+240\,\cos\left(x\right)^{4}+5\,\cos\left(x\right)^{2}+10\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x)*cos(5*x),x, algorithm="fricas")
[Out] 1/63*(448*cos(x)^8 - 640*cos(x)^6 + 240*cos(x)^4 + 5*cos(x)^2 + 10)*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.
time = 1.34, size = 66, normalized size = 2.13

$$-\frac{10\sin(x)\sin(3x)\sin(5x)}{63} - \frac{11\sin(x)\cos(3x)\cos(5x)}{63} - \frac{17\sin(3x)\cos(x)\cos(5x)}{63} + \frac{25\sin(5x)\cos(x)\cos(3x)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x)*cos(5*x),x)
```

```
[Out] -10*sin(x)*sin(3*x)*sin(5*x)/63 - 11*sin(x)*cos(3*x)*cos(5*x)/63 - 17*sin(3
*x)*cos(x)*cos(5*x)/63 + 25*sin(5*x)*cos(x)*cos(3*x)/63
```

Giac [A]
time = 0.48, size = 23, normalized size = 0.74

$$\frac{1}{36}\sin(9x) + \frac{1}{28}\sin(7x) + \frac{1}{12}\sin(3x) + \frac{1}{4}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x)*cos(5*x),x, algorithm="giac")
```

```
[Out] 1/36*sin(9*x) + 1/28*sin(7*x) + 1/12*sin(3*x) + 1/4*sin(x)
```

Mupad [B]
time = 0.28, size = 27, normalized size = 0.87

$$\frac{64\sin(x)^9}{9} - \frac{128\sin(x)^7}{7} + 16\sin(x)^5 - \frac{17\sin(x)^3}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(5*x)*cos(x),x)
```

```
[Out] sin(x) - (17*sin(x)^3)/3 + 16*sin(x)^5 - (128*sin(x)^7)/7 + (64*sin(x)^9)/9
```

Chatgpt [F] Failed to verify
time = 1.00, size = 19, normalized size = 0.61

$$\frac{\sin(8x)}{48} + \frac{\sin(6x)}{12} + \frac{\sin(2x)}{24}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)*cos(3*x)*cos(5*x),x)
```

```
[Out] 1/48*sin(8*x)+1/12*sin(6*x)+1/24*sin(2*x)
```

3.40 $\int \left(\frac{1}{\log(x)}+\log(\log(x))\right) \, dx$

Optimal. Leaf size=5

$$x\log(\log(x))$$

[Out] x*ln(ln(x))

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2335, 2600}

$$x\log(\log(x))$$

Antiderivative was successfully verified.

[In] Int[Log[x]^(-1) + Log[Log[x]],x]

[Out] x*Log[Log[x]]

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2600

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \frac{1}{\log(x)} \, dx+\int \log(\log(x)) \, dx\\
&= x\log(\log(x))+\text{LogIntegral}(x)-\int \frac{1}{\log(x)} \, dx\\
&= x\log(\log(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$x\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^(-1) + Log[Log[x]],x]
```

```
[Out] x*Log[Log[x]]
```

Maple [A]
time = 0.02, size = 6, normalized size = 1.20

method	result	size
default	$x\ln(\ln(x))$	6
norman	$x\ln(\ln(x))$	6
risch	$x\ln(\ln(x))$	6
parallelrisch	$x\ln(\ln(x))$	6
parts	$x\ln(\ln(x))$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x)+ln(ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(ln(x))
```

Maxima [A]
time = 0.36, size = 5, normalized size = 1.00

$$x\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x)+log(log(x)),x, algorithm="maxima")
```

```
[Out] x*log(log(x))
```

Fricas [A]
time = 0.59, size = 5, normalized size = 1.00

$$x\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x)+log(log(x)),x, algorithm="fricas")
```

```
[Out] x*log(log(x))
```

Sympy [A]
time = 0.10, size = 5, normalized size = 1.00

$$x\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x)+ln(ln(x)),x)
```

```
[Out] x*log(log(x))
```

Giac [**A**]
time = 0.44, size = 5, normalized size = 1.00

$$x \log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x)+log(log(x)),x, algorithm="giac")
```

```
[Out] x*log(log(x))
```

Mupad [**B**]
time = 0.13, size = 5, normalized size = 1.00

$$x \ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x)) + 1/log(x),x)
```

```
[Out] x*log(log(x))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 8, normalized size = 1.60

$$\text{hyperbolicCosineIntegral}(x) + x \ln(\ln(x))$$

Warning: Unable to verify antiderivative.

```
[In] int(1/ln(x)+ln(ln(x)),x)
```

```
[Out] Li(x)+x*ln(ln(x))
```

3.41 $\int \frac{1}{2+e^x} dx$

Optimal. Leaf size=8

$$-\text{arctanh}(1+e^x)$$

[Out] -arctanh(1+exp(x))

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 2.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2320, 36, 29, 31}

$$\frac{x}{2} - \frac{1}{2}\log\left(e^x + 2\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + E^x)^(-1),x]

[Out] x/2 - Log[2 + E^x]/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{1}{x(2+x)}\,dx, x, e^x\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, e^x\right) \\
&= \frac{x}{2} - \frac{1}{2}\log\left(2+e^x\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$-\operatorname{arctanh}(1+e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + E^x)^(-1),x]
```

```
[Out] -ArcTanh[1 + E^x]
```

Maple [A]
time = 0.01, size = 14, normalized size = 1.75

method	result	size
norman	$\frac{x}{2}-\frac{\ln(2+{\rm e}^{x})}{2}$	12
risch	$\frac{x}{2}-\frac{\ln(2+{\rm e}^{x})}{2}$	12
parallelrisch	$\frac{x}{2}-\frac{\ln(2+{\rm e}^{x})}{2}$	12
derivativedivides	$-\frac{\ln(2+{\rm e}^{x})}{2}+\frac{\ln({\rm e}^{x})}{2}$	14
default	$-\frac{\ln(2+{\rm e}^{x})}{2}+\frac{\ln({\rm e}^{x})}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(2+exp(x))+1/2*ln(exp(x))
```

Maxima [A]
time = 0.38, size = 11, normalized size = 1.38

$$\frac{1}{2}\,x - \frac{1}{2}\,\log\left(e^x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*log(e^x + 2)
```

Fricas [A]
time = 0.58, size = 11, normalized size = 1.38

$$\frac{1}{2}x - \frac{1}{2}\log\left(e^{x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+exp(x)),x, algorithm="fricas")
[Out] 1/2*x - 1/2*log(e^x + 2)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 1.25

$$\frac{x}{2} - \frac{\log\left(e^{x}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+exp(x)),x)
[Out] x/2 - log(exp(x) + 2)/2
```

Giac [A]
time = 0.49, size = 11, normalized size = 1.38

$$\frac{1}{2}x - \frac{1}{2}\log\left(e^{x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+exp(x)),x, algorithm="giac")
[Out] 1/2*x - 1/2*log(e^x + 2)
```

Mupad [B]
time = 0.09, size = 11, normalized size = 1.38

$$\frac{x}{2} - \frac{\ln\left(\mathrm{e}^{x}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x) + 2),x)
[Out] x/2 - log(exp(x) + 2)/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 1.25

$$\ln\left(2+\mathrm{e}^{x}\right)\mathrm{e}^{-x}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(2+exp(x)),x)
[Out] ln(2+exp(x))*exp(-x)
```

3.42 $\int \sqrt{\frac{x}{1-x^3}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\arcsin\left(x^{3/2}\right)}{3\sqrt{x}}$$

```
[Out] 2/3*(x/(-x^3+1))^(1/2)*(-x^3+1)^(1/2)*arcsin(x^(3/2))/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6851, 335, 281, 222}

$$\frac{2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\arcsin\left(x^{3/2}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 - x^3)],x]
```

```
[Out] (2*Sqrt[x/(1 - x^3)]*Sqrt[1 - x^3]*ArcSin[x^(3/2)])/(3*Sqrt[x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{\left(\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\right)\int\frac{\sqrt{x}}{\sqrt{1-x^3}}\,dx}{\sqrt{x}}\\ &= \frac{\left(2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^6}}\,dx,x,\sqrt{x}\right)}{\sqrt{x}}\\ &= \frac{\left(2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}\,dx,x,x^{3/2}\right)}{3\sqrt{x}}\\ &= \frac{2\sqrt{\frac{x}{1-x^3}}\sqrt{1-x^3}\,\arcsin\left(x^{3/2}\right)}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.17

$$\frac{2\sqrt{-\frac{x}{-1+x^3}}\sqrt{-1+x^3}\,\log\left(x^{3/2}+\sqrt{-1+x^3}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 - x^3)],x]
```

```
[Out] (2*Sqrt[-(x/(-1 + x^3))]*Sqrt[-1 + x^3]*Log[x^(3/2) + Sqrt[-1 + x^3]])/(3*S
qrt[x])
```

Maple [A]
time = 0.54, size = 43, normalized size = 1.05

method	result
default	$\frac{2\sqrt{-\frac{x}{x^3-1}}\,(x^3-1)\,\operatorname{arctanh}\left(\frac{\sqrt{x^4-x}}{x^2}\right)}{3\sqrt{(x^3-1)x}}$
trager	$\frac{\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(-2\sqrt{-\frac{x}{x^3-1}}\,x^4-2\mathit{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{-\frac{x}{x^3-1}}\,x+\mathit{RootOf}\left(\_Z^2+1\right)\right)}{3}$
elliptic	$\frac{2\sqrt{-\frac{x}{x^3-1}}\,\sqrt{(x^3-1)x}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x-1)}}\,(x-1)^2\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x-1)}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x-1)}}\left(F\left(\sqrt{\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x-1)}},\cdots\right.\right.}{x\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{x(x-1)\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x+\frac{1}{2}-\cdots\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(-x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-x/(x^3-1))^(1/2)*(x^3-1)/((x^3-1)*x)^(1/2)*arctanh((x^4-x)^(1/2)/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(-x^3+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x/(x^3 - 1)), x)
```

Fricas [A]
time = 0.66, size = 33, normalized size = 0.80

$$\frac{1}{3}\arctan\left(\frac{2\left(x^{4}-x\right)\sqrt{-\frac{x}{x^{3}-1}}}{2\,x^{3}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(-x^3+1))^(1/2),x, algorithm="fricas")
[Out] 1/3*arctan(2*(x^4 - x)*sqrt(-x/(x^3 - 1))/(2*x^3 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x}{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(-x**3+1))**(1/2),x)
[Out] Integral(sqrt(x/(1 - x**3)), x)
```

Giac [A]
time = 0.50, size = 16, normalized size = 0.39

$$\frac{2}{3}\arctan\left(\sqrt{\frac{1}{x^{3}}-1}\right)\operatorname{sgn}\left(x^{3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(-x^3+1))^(1/2),x, algorithm="giac")
[Out] 2/3*arctan(sqrt(1/x^3 - 1))*sgn(x^3 - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{-\frac{x}{x^{3}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(x^3 - 1))^(1/2),x)
```

```
[Out] int((-x/(x^3 - 1))^(1/2), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 27, normalized size = 0.66

$$-\frac{2\sqrt{-x^3+1}\,\sqrt{x}}{3}+\frac{2\arcsin\left(\sqrt{-x^3+1}\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int((x/(-x^3+1))^(1/2),x)
```

```
[Out] -2/3*(-x^3+1)^(1/2)*x^(1/2)+2/3*arcsin((-x^3+1)^(1/2))
```

3.43 $\int \frac{4x}{1-x^4} dx$

Optimal. Leaf size=6

$$2\text{arctanh}\left(x^2\right)$$

```
[Out] 2*arctanh(x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {12, 281, 212}

$$2\text{arctanh}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x)/(1 - x^4),x]
```

```
[Out] 2*ArcTanh[x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 4\int \frac{x}{1-x^4}\,dx \\
&= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x^2\right) \\
&= 2\text{arctanh}\left(x^2\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.
time = 0.00, size = 25, normalized size = 4.17

$$-4\left(\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{4}\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x)/(1 - x^4),x]
```

```
[Out] -4*(Log[1 - x^2]/4 - Log[1 + x^2]/4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(6) = 12$.
time = 0.06, size = 20, normalized size = 3.33

method	result	size
meijerg	$2\,\mathrm{arctanh}\left(x^2\right)$	7
risch	$\ln\left(x^2+1\right)-\ln\left(x^2-1\right)$	16
default	$-\ln\left(x-1\right)-\ln\left(x+1\right)+\ln\left(x^2+1\right)$	20
norman	$-\ln\left(x-1\right)-\ln\left(x+1\right)+\ln\left(x^2+1\right)$	20
parallelrisch	$-\ln\left(x-1\right)-\ln\left(x+1\right)+\ln\left(x^2+1\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x-1)-ln(x+1)+ln(x^2+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.
time = 0.33, size = 15, normalized size = 2.50

$$\log\left(x^2+1\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] log(x^2 + 1) - log(x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.
time = 0.56, size = 15, normalized size = 2.50

$$\log\left(x^2+1\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] log(x^2 + 1) - log(x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.
time = 0.04, size = 12, normalized size = 2.00

$$-\log\left(x^2-1\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x/(-x**4+1),x)
```

```
[Out] -log(x**2 - 1) + log(x**2 + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.42, size = 16, normalized size = 2.67

$$\log\left(x^2+1\right)-\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x/(-x^4+1),x, algorithm="giac")
```

```
[Out] log(x^2 + 1) - log(abs(x^2 - 1))
```

Mupad [B]
time = 0.09, size = 6, normalized size = 1.00

$$2\,\mathrm{atanh}\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x)/(x^4 - 1),x)
```

```
[Out] 2*atanh(x^2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 19, normalized size = 3.17

$$\ln\left(1-x\right)+\ln\left(x+1\right)-\frac{\ln\left(x^2+1\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(4*x/(-x^4+1),x)
```

```
[Out] ln(1-x)+ln(x+1)-1/2*ln(x^2+1)
```

3.44 $\int x^x(1+\log(x))\,dx$

Optimal. Leaf size=3

$$x^x$$

```
[Out] x^x
```

Rubi [A]
time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6874, 2633}

$$x^x$$

Antiderivative was successfully verified.

```
[In] Int[x^x*(1 + Log[x]),x]
```

```
[Out] x^x
```

Rule 2633

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int (x^x + x^x\log(x))\,dx\\ &= \int x^x\,dx + \int x^x\log(x)\,dx\\ &= x^x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$x^x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^x*(1 + Log[x]),x]
```

```
[Out] x^x
```

Maple [A]
time = 0.00, size = 4, normalized size = 1.33

method	result	size
derivativedivides	x^x	4
default	x^x	4
risch	x^x	4
parallelrisch	x^x	4
norman	$e^{x \ln(x)}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^x*(1+ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x^x
```

Maxima [A]
time = 0.40, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^x*(1+log(x)),x, algorithm="maxima")
```

```
[Out] x^x
```

Fricas [A]
time = 0.57, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^x*(1+log(x)),x, algorithm="fricas")
```

```
[Out] x^x
```

Sympy [A]
time = 0.05, size = 2, normalized size = 0.67

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**x*(1+ln(x)),x)
```

```
[Out] x**x
```

Giac [A]
time = 0.45, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^x*(1+log(x)),x, algorithm="giac")
```

```
[Out] x^x
```

Mupad [B]
time = 0.00, size = 3, normalized size = 1.00

$$x^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^x*(log(x) + 1),x)
```

```
[Out] x^x
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(x^x*(1+ln(x)),x)
```

```
[Out] not solved
```

3.45 $\int \sqrt{6x-x^2}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\left((-3+x)\sqrt{-((-6+x)x)}-9\arcsin\left(1-\frac{x}{3}\right)\right)$$

```
[Out] 1/2*(-3+x)*(-(-6+x)*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {626, 633, 222}

$$-\frac{9}{2}\arcsin\left(1-\frac{x}{3}\right)-\frac{1}{2}\sqrt{6x-x^2}(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6*x - x^2],x]
```

```
[Out] -1/2*((3 - x)*Sqrt[6*x - x^2]) - (9*ArcSin[1 - x/3])/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\frac{1}{2}(3-x)\sqrt{6x-x^2}+\frac{9}{2}\int\frac{1}{\sqrt{6x-x^2}}\,dx\\
&= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{3}{4}\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx,x,6-2x\right)\\
&= -\frac{1}{2}(3-x)\sqrt{6x-x^2}-\frac{9}{2}\arcsin\left(1-\frac{x}{3}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 1.62

$$\frac{1}{2}\sqrt{-((-6+x)x)}\left(-3+x+\frac{18\log\left(\sqrt{-6+x}-\sqrt{x}\right)}{\sqrt{-6+x}\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6*x - x^2],x]
```

```
[Out] (Sqrt[-((-6 + x)*x)]*(-3 + x + (18*Log[Sqrt[-6 + x] - Sqrt[x]])/(Sqrt[-6 +
x]*Sqrt[x])))/2
```

Maple [A]
time = 0.18, size = 28, normalized size = 0.97

method	result	siz
risch	$-\frac{(-3+x)(-6+x)x}{2\sqrt{-(-6+x)x}}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$	27
default	$-\frac{(-2x+6)\sqrt{-x^2+6x}}{4}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$	28
pseudoelliptic	$-9\arctan\left(\frac{\sqrt{-(-6+x)x}}{x}\right)+\frac{(-3+x)\sqrt{-(-6+x)x}}{2}$	30
meijerg	$-\frac{18i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{6}\,(3-x)\sqrt{-\frac{x}{6}+1}}{36}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)}{2}\right)}{\sqrt{\pi}}$	47
trager	$\left(-\frac{3}{2}+\frac{x}{2}\right)\sqrt{-x^2+6x}+\frac{9\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathit{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+6x}+3\mathit{RootOf}\left(\_Z^2+1\right)\right)}{2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*x+6)*(-x^2+6*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

Maxima [A]
time = 0.43, size = 36, normalized size = 1.24

$$\frac{1}{2}\sqrt{-x^2+6\,x}x-\frac{3}{2}\sqrt{-x^2+6\,x}-\frac{9}{2}\arcsin\left(-\frac{1}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)
```

Fricas [A]
time = 0.56, size = 35, normalized size = 1.21

$$\frac{1}{2}\sqrt{-x^2+6\,x}(x-3)-9\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/x)
```

Sympy [A]
time = 0.27, size = 26, normalized size = 0.90

$$\left(\frac{x}{2}-\frac{3}{2}\right)\sqrt{-x^2+6x}+\frac{9\,\mathrm{asin}\left(\frac{x}{3}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+6*x)**(1/2),x)
[Out] (x/2 - 3/2)*sqrt(-x**2 + 6*x) + 9*asin(x/3 - 1)/2
```

Giac [A]
time = 0.48, size = 25, normalized size = 0.86

$$\frac{1}{2}\sqrt{-x^2+6\,x}(x-3)+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+6*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

Mupad [B]
time = 0.09, size = 26, normalized size = 0.90

$$\frac{9\,\mathrm{asin}\left(\frac{x}{3}-1\right)}{2}+\left(\frac{x}{2}-\frac{3}{2}\right)\,\sqrt{6\,x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - x^2)^(1/2),x)
[Out] (9*asin(x/3 - 1))/2 + (x/2 - 3/2)*(6*x - x^2)^(1/2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 28, normalized size = 0.97

$$\frac{3(x-3)\sqrt{-x^2+6x}}{2}+9\pi-9\arccos\left(-1+\frac{2x}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((-x^2+6*x)^(1/2),x)
```

```
[Out] 3/2*(x-3)*(-x^2+6*x)^(1/2)+9*Pi-9*arccos(-1+2/3*x)
```

3.46 $\int \sin^{99}(x)\sin(101x)\,dx$

Optimal. Leaf size=12

$$\frac{1}{100}\sin^{100}(x)\sin(100x)$$

[Out] 1/100*sin(x)^100*sin(100*x)

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 801 vs. $2(12) = 24$.
time = 0.33, antiderivative size = 801, normalized size of antiderivative = 66.75, number of steps used = 102, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4439, 2717}

Too large to display

Antiderivative was successfully verified.

[In] Int[Sin[x]^99*Sin[101*x],x]

```
[Out] -1/126765060022822940149670320537 6*Sin[2*x] + (99*Sin[4*x])/253530120045645
8802993406410752 - (1617*Sin[6*x])/1267650600228229401496703205376 + (15684
9*Sin[8*x])/5070602400912917605986812821504 - (470547*Sin[10*x])/7922816251
426433759354395033 60 + (2980131*Sin[12*x])/3169126500570573503741758013 44 -
 (20009451*Sin[14*x])/158456325028528675187087900672 + (1860878943*Sin[16*x
])/1267650600228229401496703205376 - (4755579521*Sin[18*x])/316912650057057
350374175801344 + (432757736411*Sin[20*x])/3169126500570573503741758013440
- (354074511609*Sin[22*x])/316912650057057350374175801344 + (10504210511067
*Sin[24*x])/1267650600228229401496703205376 - (8888178124749*Sin[26*x])/158
456325028528675187087900672 + (110467356693309*Sin[28*x])/31691265005705735
0374175801344 - (1583365445937429*Sin[30*x])/7922816251426433759354395033 60
 + (26917212580936293*Sin[32*x])/2535301200456458802993406410752 - (3325067
4364686009*Sin[34*x])/633825300114114700748351602688 + (306645108029882083*
Sin[36*x])/1267650600228229401496703205376 - (661707864696061337*Sin[38*x])
/633825300114114700748351602688 + (5359833704038096829 7*Sin[40*x])/12676506
0022822940149670320537 60 - (255230176382766515 7*Sin[42*x])/1584563250285286
75187087900672 + (18330167212944140673*Sin[44*x])/31691265005705735037417 58
01344 - (31081587882818325489*Sin[46*x])/158456325028528675187087900672 + (
79776075565900368755 1*Sin[48*x])/1267650600228229401496703205376 - (1515745
4357521070063469*Sin[50*x])/79228162514264337593543950336 00 + (349787408250
4862322339*Sin[52*x])/633825300114114700748351602688 - (4793383001951107626
909*Sin[54*x])/316912650057057350374175801344 + (49988137020347265252051*Si
n[56*x])/1267650600228229401496703205376 - (15513559764935358181671*Sin[58*
x])/158456325028528675187087900672 + (36715424777013681029954 7*Sin[60*x])/1
584563250285286751870879006720 - (82905797883579279745059*Sin[62*x])/158456
325028528675187087900672 + (572050005396697030240907 1*Sin[64*x])/5070602400
912917605986812821504 - (29469242702254089436652 79*Sin[66*x])/1267650600228
```

```
229401496703205376 + (1161434859441778818973962 9*Sin[68*x])/253530120045645
8802993406410752 - (547533576593981443230582 51*Sin[70*x])/63382530011411470
07483516026880 + (790881832857973195777508 07*Sin[72*x])/5070602400912917605
986812821504 - (213751846718371133993921 1*Sin[74*x])/7922816251426433759354
3950336 + (708756123329335865348264 7*Sin[76*x])/1584563250285286751870879 00
672 - (563370251877164405789646 3*Sin[78*x])/79228162514264337593543950336 +
 (343655853645070287531684243*Sin[80*x])/316912650057057350374175801344 0 -
(251455502667124600632939 69*Sin[82*x])/158456325028528675187087900672 + (70
647022177906435415921151*Sin[84*x])/316912650057057350374175801344 - (47645
666119983409931667753*Sin[86*x])/158456325028528675187087900672 + (24689117
898536857873682381 1*Sin[88*x])/633825300114114700748351602688 - (1920264725
44175561239751853*Sin[90*x])/396140812571321687967719751680 + (918387477385
18746679881321*Sin[92*x])/158456325028528675187087900672 - (527584295519150
24688442461*Sin[94*x])/79228162514264337593543950336 + (932065588750498769 4
95816811*Sin[96*x])/126765060022822940149670320537 6 - (247282707219520081 70
2971807*Sin[98*x])/316912650057057350374175801344 + (126114180681955241668 5
1562157*Sin[100*x])/1584563250285286751870879006720 0 - (247282707219520081 7
02971807*Sin[102*x])/316912650057057350374175801344 + (932065588750498769 49
5816811*Sin[104*x])/126765060022822940149670320537 6 - (527584295519150246 88
442461*Sin[106*x])/79228162514264337593543950336 + (918387477385187466798 81
321*Sin[108*x])/158456325028528675187087900672 - (192026472544175561239751 8
53*Sin[110*x])/396140812571321687967719751680 + (246891178985368578736823 81
1*Sin[112*x])/633825300114114700748351602688 - (47645666119983409931667753*
Sin[114*x])/158456325028528675187087900672 + (70647022177906435415921151*Si
n[116*x])/316912650057057350374175801344 - (25145550266712460063293969*Sin[
118*x])/158456325028528675187087900672 + (343655853645070287531684243*Sin[1
20*x])/3169126500570573503741758013440 - (563370251877164405789646 3*Sin[122
*x])/79228162514264337593543950336 + (708756123329335865348264 7*Sin[124*x])
/158456325028528675187087900672 - (213751846718371133993921 1*Sin[126*x])/79
228162514264337593543950336 + (790881832857973195777508 07*Sin[128*x])/50706
02400912917605986812821504 - (547533576593981443230582 51*Sin[130*x])/633825
30011411470074835160268 80 + (116143485944177881897396 29*Sin[132*x])/2535301
200456458802993406410752 - (294692427022540894366527 9*Sin[134*x])/126765060
0228229401496703205376 + (572050005396697030240907 1*Sin[136*x])/50706024009
12917605986812821504 - (829057978835792797450 59*Sin[138*x])/158456325028528
675187087900672 + (367154247770136810299547*Sin[140*x])/158456325028528675 1
870879006720 - (155135597649353581816 71*Sin[142*x])/15845632502852867518708
7900672 + (499881370203472652520 51*Sin[144*x])/126765060022822940149670320 5
376 - (47933830019511076269 09*Sin[146*x])/316912650057057350374175801344 +
(349787408250486232233 9*Sin[148*x])/63382530011...
```

<u>Rule 2717</u>

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

Integral = Rest of rules removed due to large latex content

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{100}\sin^{100}(x)\sin(100x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^99*Sin[101*x],x]
```

```
[Out] (Sin[x]^100*Sin[100*x])/100
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(10) = 20.
time = 1.49, size = 602, normalized size = 50.17

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(101*x)*sin(x)^99,x)
```

```
[Out] 790881832857973195777750807/5070602400912917605986812821504*sin(128*x)-31081
587882818325489/158456325028528675187087900672*sin(154*x)+1/126765060022822
940149670320537600*sin(200*x)-354074511609/316912650057057350374175801344*s
in(178*x)+2980131/316912650057057350374175801344*sin(12*x)-20009451/1584563
25028528675187087900672*sin(14*x)+1860878943/126765060022822940149670320537
6*sin(16*x)-4755579521/316912650057057350374175801344*sin(18*x)-15833654459
37429/792281625142643375935439503360*sin(30*x)-354074511609/316912650057057
350374175801344*sin(22*x)+156849/5070602400912917605986812821504*sin(8*x)-4
70547/792281625142643375935439503360*sin(10*x)-1617/12676506002282294014967
03205376*sin(6*x)+432757736411/3169126500570573503741758013440*sin(20*x)+99
/2535301200456458802993406410752*sin(4*x)+12611418068195524166851562157/158
45632502852867518708790067200*sin(100*x)-1/1267650600228229401496703205376*
sin(2*x)-2137518467183711339939211/79228162514264337593543950336*sin(126*x)
```

+53598337040380968297/1267650600228229401496703205376*sin(40*x)-5475335765
9398144323058251/633825300114114700748351602688*sin(130*x)+708756123329335
8653482647/158456325028528675187087900672*sin(124*x)+3497874082504862322339
/633825300114114700748351602688*sin(148*x)+156849/5070602400912917605986812
821504*sin(192*x)+918387477385187466798813211/158456325028528675187087900672
*sin(108*x)-1920264725441755612397518531/39614081257132168796771975168*sin(
110*x)-2472827072195200817029718071/316912650057057350374175801344*sin(102*x
)+53598337040380968297/1267650600228229401496703205376*sin(160*x)+36715424
7770136810299547/158456325028528675187087900672*sin(140*x)+432757736411/31
6912650057057350374175801344*sin(180*x)+7977607556590036875511/1267650600228229401496703205376*sin(48*x)-3108158788281832548 9/158456325028528675187087
900672*sin(46*x)-527584295519150246884424611/79228162514264337593543950336*s
in(106*x)-8888178124749/158456325028528675187087900672*sin(174*x)+306645108
029882083/1267650600228229401496703205376*sin(36*x)+26917212580936293/25353
012004564588029934064107520*sin(32*x)+93206558875049876949581681 1/1267650600
228229401496703205376*sin(96*x)+306645108029882083/1267650600228229401496703205376*sin(164*x)+110467356693309/316912650057057350374175801344*sin(28*x)
-1583365445937429/79228162514264337593543950336 0*sin(170*x)-155135597649353
58181671/158456325028528675187087900672*sin(142*x)-8888178124749/1584563250
28528675187087900672*sin(26*x)-82905797883579279745059/158456325028528675187087900672*sin(138*x)-563370251877164405789646 3/7922816251426433759354395033 6*sin(78*x)-661707864696061337/633825300114114700748351602688*sin(38*x)-25
52301763827665157/158456325028528675187087900672*sin(42*x)-3325067436468600
9/633825300114114700748351602688*sin(34*x)-2472827072195200817029718071/3169
12650057057350374175801344*sin(98*x)-476456661199834099316677533/15845632502
8528675187087900672*sin(86*x)-2514555026671246006329396 9/158456325028528675
187087900672*sin(118*x)-1920264725441755612397518531/396140812571321687967719751680*sin(90*x)+706470221779064354159211 51/316912650057057350374175801344
*sin(84*x)-4755579521/316912650057057350374175801344*sin(182*x)+10504210511
067/1267650600228229401496703205376*sin(24*x)+246891178985368578736823811/6
33825300114114700748351602688*sin(88*x)+2980131/316912650057057350374175801
344*sin(188*x)+706470221779064354159211 51/316912650057057350374175801344*si
n(116*x)-20009451/158456325028528675187087900672*sin(186*x)-470547/79228162
5142643375935439503360*sin(190*x)-213751846718371133993921 1/792281625142643
37593543950336*sin(74*x)+932065588750498769495816811/1267650600228229401496
703205376*sin(104*x)+790881832857973195777508 07/5070602400912917605986812821504*sin(72*x)-5475335765939814432305825 1/6338253001141147007483516026880*s
in(70*x)-661707864696061337/633825300114114700748351602688*sin(162*x)+18608
78943/1267650600228229401496703205376*sin(184*x)-15513559764935358181671/15
8456325028528675187087900672*sin(58*x)+572050005396697030240907 1/5070602400
912917605986812821504*sin(64*x)+499881370203472652520511/1267650600228229401
496703205376*sin(56*x)+18330167212944140673/316912650057057350374175801344*
sin(156*x)-479338300195110762690 9/316912650057057350374175801344*sin(54*x)-
82905797883579279745059/158456325028528675187087900672*sin(62*x)-2946924270
225408943665279/1267650600228229401496703205376*sin(66*x)+10504210511067/12

```
676506002282294014967032053 76*sin(176*x)+3497874082504862322339/63382530011
4114700748351602688*sin(52*x)-4764566611998340993166 7753/15845632502852867 5
187087900672*sin(114*x)+99/2535301200456458802993406410752*sin(196*x)+49988
13702034726525205 1/1267650600228229401496703205376*sin(144*x)+1161434859441
7788189739629/2535301200456458802993406410752*sin(68*x)-2946924270225408943
665279/1267650600228229401496703205376*sin(134*x)-1515745435752107006 3469/7
9228162514264337593543950336 00*sin(50*x)+18330167212944140673/3169126500570
573503741758013 44*sin(44*x)+79776075565900368 7551/126765060022822940149670 3
205376*sin(152*x)-33250674364686009/63382530011...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(101*x)*sin(x)^99,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please ju
mp to an outer pointer, quit program and enlarge thememory limits before ex
ecuting the program again.
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(101*x)*sin(x)^99,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(101*x)*sin(x)**99,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(10) = 20.
time = 0.49, size = 601, normalized size = 50.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(101*x)*sin(x)^99,x, algorithm="giac")

[Out] 1/126765060022822940149670320537600*sin(200*x) - 1/126765060022822940149670
3205376*sin(198*x) + 99/2535301200456458802993406410752*sin(196*x) - 1617/1
267650600228229401496703205376*sin(194*x) + 156849/507060240091291760598681
2821504*sin(192*x) - 470547/792281625142643375935439503360*sin(190*x) + 298
0131/316912650057057350374175801344*sin(188*x) - 20009451/15845632502852867
5187087900672*sin(186*x) + 1860878943/1267650600228229401496703205376*sin(1
84*x) - 4755579521/316912650057057350374175801344*sin(182*x) + 432757736411
/3169126500570573503741758013440*sin(180*x) - 354074511609/3169126500570573
50374175801344*sin(178*x) + 10504210511067/1267650600228229401496703205376*
sin(176*x) - 8888178124749/158456325028528675187087900672*sin(174*x) + 1104
67356693309/316912650057057350374175801344*sin(172*x) - 1583365445937429/79
2281625142643375935439503360*sin(170*x) + 26917212580936293/253530120045645
8802993406410752*sin(168*x) - 33250674364686009/633825300114114700748351602
688*sin(166*x) + 306645108029882083/1267650600228229401496703205376*sin(164
*x) - 661707864696061337/633825300114114700748351602688*sin(162*x) + 535983
37040380968297/12676506002282294014967032053760*sin(160*x) - 25523017638276
65157/158456325028528675187087900672*sin(158*x) + 18330167212944140673/3169
12650057057350374175801344*sin(156*x) - 31081587882818325489/15845632502852
8675187087900672*sin(154*x) + 797760755659003687551/12676506002282294014967
03205376*sin(152*x) - 15157454357521070063469/79228162514264337593543950336
00*sin(150*x) + 349787408250486232339/633825300114114700748351602688*sin(1
48*x) - 4793383001951107626909/316912650057057350374175801344*sin(146*x) +
49988137020347265252051/1267650600228229401496703205376*sin(144*x) - 155135
59764935358181671/158456325028528675187087900672*sin(142*x) + 3671542477701
36810299547/1584563250285286751870879006720*sin(140*x) - 829057978835792797
45059/158456325028528675187087900672*sin(138*x) + 5720500053966970302409071
/5070602400912917605986812821504*sin(136*x) - 2946924270225408943665279/126
7650600228229401496703205376*sin(134*x) + 11614348594417788189739629/253530
1200456458802993406410752*sin(132*x) - 54753357659398144323058251/633825300
1141147007483516026880*sin(130*x) + 79088183285797319577750807/507060240091
2917605986812821504*sin(128*x) - 2137518467183711339939211/7922816251426433
7593543950336*sin(126*x) + 7087561233293358653482647/1584563250285286751870
87900672*sin(124*x) - 5633702518771644057896463/792281625142643375935439503
36*sin(122*x) + 343655853645070287531684243/3169126500570573503741758013440
*sin(120*x) - 25145550266712460063293969/158456325028528675187087900672*sin
(118*x) + 70647022177906435415921151/316912650057057350374175801344*sin(116
*x) - 47645666119983409931667753/158456325028528675187087900672*sin(114*x)
+ 246891178985368578736823811/633825300114114700748351602688*sin(112*x) - 1
92026472544175561239751853/3961408125713216879677197516800*sin(110*x) + 9183
8747738518746679881321/158456325028528675187087900672*sin(108*x) - 52758429
551915024688442461/79228162514264337593543950336*sin(106*x) + 9320655887504

```
98769495816811/1267650600228229401496703205376*sin(104*x) - 247282707219520
081702971807/3169126500570573503741758013444*sin(102*x) + 126114180681955241
66851562157/15845632502852867518708790067200*sin(100*x) - 24728270721952008
1702971807/3169126500570573503741758013444*sin(98*x) + 9320655887504987694 95
816811/1267650600228229401496703205376*sin(96*x) - 527584295519150246884424
61/79228162514264337593543950336*sin(94*x) + 918387477385187466798813 21/158
456325028528675187087900672*sin(92*x) - 192026472544175561239751853/3961408
125713216879677197516800*sin(90*x) + 246891178985368578736823811/63382530011
4114700748351602688*sin(88*x) - 476456661199834099316677 53/1584563250285286
75187087900672*sin(86*x) + 706470221779064354159211 51/316912650057057350374
175801344*sin(84*x) - 251455502667124600632939 69/158456325028528675187087 90
0672*sin(82*x) + 343655853645070287531684243/3169126500570573503741758013 44
0*sin(80*x) - 563370251877164405789646 3/79228162514264337593543950336*sin(7
8*x) + 708756123329335865348264 7/158456325028528675187087900672*sin(76*x) -
 21375184671837113399392 11/79228162514264337593543950336*sin(74*x) + 790881
83285797319577750807/5070602400912917605986812821504*sin(72*x) - 5475335765
9398144323058251/63382530011411470074835160268 80*sin(70*x) + 11614348594417
788189739629/25353012004564588029934064107 52*sin(68*x) - 294692427022540894
3665279/1267650600228229401496703205376*sin(66*x) + 572050005396697030240 90
71/5070602400912917605986812821504*sin(64*x) - 829057978835792797450 59/1584
563250285286751870879006 72*sin(62*x) + 367154247770136810299547/15845632502
852867518708790067 20*sin(60*x) - 155135597649353581816 71/15845632502852867 5
187087900672*sin(58*x) + 499881370203472652520 51/126765060022822940149670 32
05376*sin(56*x) - 479338300195110762690 9/316912650057057350374175801344*sin
(54*x) + 349787408250486232233 9/633825300114114700748351602688*sin(52*x) -
151574543575210700634 69/7922816251426433759354 3...
```

Mupad [B]
time = 4.67, size = 2500, normalized size = 208.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(101*x)*sin(x)^99,x)
```

```
[Out] (99*sin(4*x))/2535301200456458802993406410752 - sin(2*x)/12676506002282294 0
1496703205376 - (1617*sin(6*x))/1267650600228229401496703205376 + (156849*s
in(8*x))/5070602400912917605986812821504 - (470547*sin(10*x))/7922816251426
43375935439503360 + (2980131*sin(12*x))/316912650057057350374175801344 - (2
0009451*sin(14*x))/158456325028528675187087900672 + (1860878943*sin(16*x))/
1267650600228229401496703205376 - (4755579521*sin(18*x))/316912650057057350
374175801344 + (432757736411*sin(20*x))/3169126500570573503741758013440 - (
354074511609*sin(22*x))/316912650057057350374175801344 + (10504210511067*si
n(24*x))/1267650600228229401496703205376 - (8888178124749*sin(26*x))/158456
325028528675187087900672 + (110467356693309*sin(28*x))/31691265005705735037
```

4175801344 - (1583365445937429*sin(30*x))/79228162514264337593543950336 0 +
(26917212580936293*sin(32*x))/2535301200456458802993406410752 - (3325067436
4686009*sin(34*x))/633825300114114700748351602688 + (306645108029882083*sin
(36*x))/1267650600228229401496703205376 - (661707864696061337*sin(38*x))/63
3825300114114700748351602688 + (5359833704038096829 7*sin(40*x))/12676506002
282294014967032053760 - (2552301763827665157*sin(42*x))/1584563250285286751
87087900672 + (18330167212944140673*sin(44*x))/3169126500570573503741758013
44 - (31081587882818325489*sin(46*x))/158456325028528675187087900672 + (797
76075565900368755 1*sin(48*x))/1267650600228229401496703205376 - (1515745435
7521070063469*sin(50*x))/79228162514264337593543950336 00 + (349787408250486
2322339*sin(52*x))/633825300114114700748351602688 - (4793383001951107626909
*sin(54*x))/316912650057057350374175801344 + (4998813702034726525205 1*sin(5
6*x))/1267650600228229401496703205376 - (15513559764935358181671*sin(58*x))
/158456325028528675187087900672 + (36715424777013681029954 7*sin(60*x))/1584
563250285286751870879006720 - (82905797883579279745059*sin(62*x))/158456325
028528675187087900672 + (57205000539669703024090 71*sin(64*x))/5070602400912
917605986812821504 - (294692427022540894366527 9*sin(66*x))/1267650600228229
401496703205376 + (11614348594417788189739629*sin(68*x))/253530120045645880
2993406410752 - (54753357659398144323058251*sin(70*x))/6338253001141147007 4
83516026880 + (79088183285797319577750807*sin(72*x))/5070602400912917605986
812821504 - (21375184671837113399392 11*sin(74*x))/79228162514264337593543 95
0336 + (70875612332933586534826 47*sin(76*x))/158456325028528675187087900672
 - (56337025187716440578964 63*sin(78*x))/79228162514264337593543950336 + (3
43655853645070287531684243*sin(80*x))/3169126500570573503741758013440 - (25
145550266712460063293969*sin(82*x))/158456325028528675187087900672 + (70647
022177906435415921151*sin(84*x))/316912650057057350374175801344 - (47645666
119983409931667753*sin(86*x))/158456325028528675187087900672 + (24689117898
536857873682381 1*sin(88*x))/633825300114114700748351602688 - (1920264725441
75561239751853*sin(90*x))/3961408125713216879677197516 80 + (918387477385187
46679881321*sin(92*x))/158456325028528675187087900672 - (527584295519150246
88442461*sin(94*x))/79228162514264337593543950336 + (932065588750498769495 8
16811*sin(96*x))/1267650600228229401496703205376 - (2472827072195200817029 7
1807*sin(98*x))/316912650057057350374175801344 + (126114180681955241668515 6
2157*sin(100*x))/1584563250285286751870879006720 0 - (247282707219520081702 9
71807*sin(102*x))/316912650057057350374175801344 + (93206558875049876949581
6811*sin(104*x))/1267650600228229401496703205376 - (52758429551915024688442
461*sin(106*x))/79228162514264337593543950336 + (91838747738518746679881321
*sin(108*x))/158456325028528675187087900672 - (192026472544175561239751853*
sin(110*x))/39614081257132168796771975168 0 + (246891178985368578736823811*s
in(112*x))/633825300114114700748351602688 - (47645666119983409931667753*sin
(114*x))/158456325028528675187087900672 + (70647022177906435415921151*sin(1
16*x))/316912650057057350374175801344 - (25145550266712460063293969*sin(118
*x))/158456325028528675187087900672 + (343655853645070287531684243*sin(120*
x))/3169126500570573503741758013440 - (56337025187716440578964 63*sin(122*x)
)/79228162514264337593543950336 + (70875612332933586534826 47*sin(124*x))/15

```
8456325028528675187087900672 - (213751846718371133993921l*sin(126*x))/79228
162514264337593543950336 + (7908818328579731957775080 7*sin(128*x))/50706024
009129176059868128215 04 - (5475335765939814432305825 1*sin(130*x))/633825300
114114700748351602688 0 + (116143485944177881897396 29*sin(132*x))/2535301200
456458802993406410752 - (2946924270225408943665279*sin(134*x))/126765060022
822940149670320537 6 + (5720500053966970302409071*sin(136*x))/50706024009129
176059868128215 04 - (829057978835792797450 59*sin(138*x))/15845632502852867 5
187087900672 + (3671542477701368102995 47*sin(140*x))/1584563250285286751870
879006720 - (1551355976493535818167 1*sin(142*x))/1584563250285286751870879 0
0672 + (4998813702034726525205 1*sin(144*x))/1267650600228229401496703205376
 - (4793383001951107626909*sin(146*x))/3169126500570573503741758013 44 + (34
97874082504862322339*sin(148*x))/63382530011411...
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(sin(101*x)*sin(x)^99,x)
```

```
[Out] not solved
```

3.47 $\int e^{e^{x^2}} x \, dx$

Optimal. Leaf size=10

$$\frac{\text{ExpIntegralEi}\left(e^{x^2}\right)}{2}$$

[Out] 1/2*Ei(exp(x^2))

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6847, 2320, 2209}

$$\frac{\text{ExpIntegralEi}\left(e^{x^2}\right)}{2}$$

Antiderivative was successfully verified.

[In] Int[E^E^x^2*x,x]

[Out] ExpIntegralEi[E^x^2]/2

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int e^{e^x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{e^x}{x}\,dx, x, e^{x^2}\right)\\ &= \frac{\text{ExpIntegralEi}\left(e^{x^2}\right)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{\text{ExpIntegralEi}\left(e^{x^2}\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^E^x^2*x,x]
```

```
[Out] ExpIntegralEi[E^x^2]/2
```

Maple [A]
time = 0.03, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$-\frac{\text{expIntegral}_1\left(-e^{x^2}\right)}{2}$	11
default	$-\frac{\text{expIntegral}_1\left(-e^{x^2}\right)}{2}$	11
risch	$-\frac{\text{expIntegral}_1\left(-e^{x^2}\right)}{2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(exp(x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*Ei(1,-exp(x^2))
```

Maxima [A]
time = 0.35, size = 7, normalized size = 0.70

$$\frac{1}{2}\,\text{Ei}\left(e^{(x^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*exp(exp(x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*Ei(e^(x^2))
```

Fricas [A]
time = 0.57, size = 7, normalized size = 0.70

$$\frac{1}{2}\,\mathrm{Ei}\left(e^{\left(x^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*exp(exp(x^2)),x, algorithm="fricas")
[Out] 1/2*Ei(e^(x^2))
```

Sympy [A]
time = 0.92, size = 7, normalized size = 0.70

$$\frac{\mathrm{Ei}\left(e^{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*exp(exp(x**2)),x)
[Out] Ei(exp(x**2))/2
```

Giac [A]
time = 0.40, size = 7, normalized size = 0.70

$$\frac{1}{2}\,\mathrm{Ei}\left(e^{\left(x^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*exp(exp(x^2)),x, algorithm="giac")
[Out] 1/2*Ei(e^(x^2))
```

Mupad [B]
time = 0.05, size = 7, normalized size = 0.70

$$\frac{\mathrm{ei}\left(\mathrm{e}^{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(exp(x^2)),x)
[Out] ei(exp(x^2))/2
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(x*exp(exp(x^2)),x)
```

```
[Out] not solved
```

3.48 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=28

$$\frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)$$

```
[Out] 8/3/(x+1)^3-6/(x+1)^2+6/(x+1)+ln(x+1)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1694, 45}

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{(-2+x)^3}{x^4}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \left(-\frac{8}{x^4}+\frac{12}{x^3}-\frac{6}{x^2}+\frac{1}{x}\right)dx, x, 1+x\right)\\
&= \frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.86

$$\frac{2(4+9x+9x^2)}{3(1+x)^3}+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

Maple [A]
time = 0.02, size = 27, normalized size = 0.96

method	result	size
norman	$\frac{6x+6x^2+\frac{8}{3}}{(x+1)^3}+\ln(x+1)$	22
default	$\frac{8}{3(x+1)^3}-\frac{6}{(x+1)^2}+\frac{6}{x+1}+\ln(x+1)$	27
risch	$\frac{6x+6x^2+\frac{8}{3}}{x^3+3x^2+3x+1}+\ln(x+1)$	32
parallelrisch	$\frac{3\ln(x+1)x^3+8+9\ln(x+1)x^2+9\ln(x+1)x+18x^2+3\ln(x+1)+18x}{3x^3+9x^2+9x+3}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
[Out] 8/3/(x+1)^3-6/(x+1)^2+6/(x+1)+ln(x+1)
```

Maxima [A]
time = 0.38, size = 32, normalized size = 1.14

$$\frac{2\,(9\,x^2+9\,x+4)}{3\,(x^3+3\,x^2+3\,x+1)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Fricas [A]
time = 0.61, size = 46, normalized size = 1.64

$$\frac{18\,x^2+3\,(x^3+3\,x^2+3\,x+1)\log\left(x+1\right)+18\,x+8}{3\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2
 + 3*x + 1)
```

Sympy [A]
time = 0.08, size = 29, normalized size = 1.04

$$\frac{18x^2 + 18x + 8}{3x^3 + 9x^2 + 9x + 3} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

Giac [A]
time = 0.48, size = 23, normalized size = 0.82

$$\frac{2\,(9\,x^2 + 9\,x + 4)}{3\,(x + 1)^3} + \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B]
time = 0.05, size = 21, normalized size = 0.75

$$\ln(x + 1) + \frac{6\,x^2 + 6\,x + \frac{8}{3}}{(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

```
[Out] log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

Chatgpt [F] Failed to verify
time = 1.00, size = 29, normalized size = 1.04

$$\frac{x^2}{2} - x + 2\ln(x + 1) - \frac{1}{x + 1} + \frac{1}{2\,(x + 1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

```
[Out] 1/2*x^2-x+2*ln(x+1)-1/(x+1)+1/2/(x+1)^2
```

3.49 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{1+x}}(1+x)-2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

```
[Out] ((1-x)/(x+1))^(1/2)*(x+1)-2*arctan(((1-x)/(x+1))^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1979, 294, 210}

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\left(4\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\ &= \sqrt{\frac{1-x}{1+x}}(1+x) + 2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\\ &= \sqrt{\frac{1-x}{1+x}}(1+x) - 2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 72, normalized size = 1.89

$$\frac{\sqrt{\frac{1-x}{1+x}}\left(\sqrt{1-x}(1+x) + 2i\sqrt{1+x}\log\left(\sqrt{1-x} - i\sqrt{1+x}\right)\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*(Sqrt[1 - x]*(1 + x) + (2*I)*Sqrt[1 + x]*Log[Sqrt[1
- x] - I*Sqrt[1 + x]]))/Sqrt[1 - x]
```

Maple [A]
time = 0.09, size = 39, normalized size = 1.03

method	result
default	$\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\left(\sqrt{-x^2+1}+\arcsin(x)\right)}{\sqrt{-(x-1)(x+1)}}$
risch	$(x+1)\sqrt{-\frac{x-1}{x+1}} - \frac{\arcsin(x)\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-(x-1)(x+1)}}{x-1}$
trager	$(x+1)\sqrt{-\frac{x-1}{x+1}} + \mathit{RootOf}\left(\_Z^2+1\right)\ln\left(\mathit{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}\,x + \mathit{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x}{x}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

Maxima [A]
time = 0.46, size = 43, normalized size = 1.13

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1} - 1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

Fricas [A]
time = 0.59, size = 32, normalized size = 0.84

$$(x+1)\sqrt{-\frac{x-1}{x+1}} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1-x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(x+1))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

Giac [A]
time = 0.48, size = 29, normalized size = 0.76

$$\frac{1}{2}\pi\mathrm{sgn}(x+1) + \arcsin(x)\,\mathrm{sgn}(x+1) + \sqrt{-x^2+1}\mathrm{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

Mupad [B]
time = 0.09, size = 43, normalized size = 1.13

$$-2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2),x)
```

```
[Out] - 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)/
(x + 1) - 1)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 18, normalized size = 0.47

$$-\arccos\left(\sqrt{x}\right)+\frac{\sqrt{x\left(1-x\right)}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(((1-x)/(x+1))^(1/2),x)
```

```
[Out] -arccos(x^(1/2))+1/2*(x*(1-x))^(1/2)
```

3.50 $\int \frac{1}{-1+\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$2\sqrt{x}+2\log\left(1-\sqrt{x}\right)$$

```
[Out] 2*x^(1/2)+2*ln(1-x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {196, 45}

$$2\sqrt{x}+2\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + 2*Log[1 - Sqrt[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 2\text{Subst}\left(\int \frac{x}{-1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{x}+2\log\left(1-\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.90

$$2\sqrt{x}+2\log\left(-1+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + 2*Log[-1 + Sqrt[x]]
```

Maple [A]
time = 0.06, size = 25, normalized size = 1.25

method	result	size
derivativedivides	$2\sqrt{x}+2\ln\left(\sqrt{x}-1\right)$	15
meijerg	$2\sqrt{x}+2\ln\left(1-\sqrt{x}\right)$	17
trager	$2\sqrt{x}+\ln\left(2\sqrt{x}-1-x\right)$	18
default	$\ln\left(x-1\right)+2\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(1+\sqrt{x}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)-1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x-1)+2*x^(1/2)+ln(x^(1/2)-1)-ln(1+x^(1/2))
```

Maxima [A]
time = 0.35, size = 15, normalized size = 0.75

$$2\sqrt{x}+2\log\left(\sqrt{x}-1\right)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)-1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) + 2*log(sqrt(x) - 1) - 2
```

Fricas [A]
time = 0.63, size = 14, normalized size = 0.70

$$2\sqrt{x}+2\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)-1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) + 2*log(sqrt(x) - 1)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.75

$$2\sqrt{x}+2\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)-1),x)
[Out] 2*sqrt(x) + 2*log(sqrt(x) - 1)
```

Giac [**A**]
time = 0.40, size = 15, normalized size = 0.75

$$2\sqrt{x} + 2\log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)-1),x, algorithm="giac")
[Out] 2*sqrt(x) + 2*log(abs(sqrt(x) - 1))
```

Mupad [**B**]
time = 0.06, size = 14, normalized size = 0.70

$$2\ln\left(\sqrt{x} - 1\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - 1),x)
[Out] 2*log(x^(1/2) - 1) + 2*x^(1/2)
```

Chatgpt [**A**]
time = 1.00, size = 14, normalized size = 0.70

$$2\sqrt{x} + 2\ln\left(\sqrt{x} - 1\right)$$

Antiderivative was successfully verified.

```
[In] int(1/(x^(1/2)-1),x)
[Out] 2*x^(1/2)+2*ln(x^(1/2)-1)
```

3.51 $\int \sqrt[4]{x} \log(x) \, dx$

Optimal. Leaf size=21

$$-\frac{16x^{5/4}}{25} + \frac{4}{5}x^{5/4}\log(x)$$

[Out] -16/25*x^(5/4)+4/5*x^(5/4)*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$\frac{4}{5}x^{5/4}\log(x) - \frac{16x^{5/4}}{25}$$

Antiderivative was successfully verified.

[In] Int[x^(1/4)*Log[x],x]

[Out] (-16*x^(5/4))/25 + (4*x^(5/4)*Log[x])/5

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{Integral} = -\frac{16x^{5/4}}{25} + \frac{4}{5}x^{5/4}\log(x)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 0.71

$$\frac{4}{25}x^{5/4}(-4 + 5\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[x^(1/4)*Log[x],x]

[Out] (4*x^(5/4)*(-4 + 5*Log[x]))/25

Maple [A]
time = 0.04, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$-\frac{16x^{\frac{5}{4}}}{25}+\frac{4x^{\frac{5}{4}}\ln(x)}{5}$	14
default	$-\frac{16x^{\frac{5}{4}}}{25}+\frac{4x^{\frac{5}{4}}\ln(x)}{5}$	14
risch	$-\frac{16x^{\frac{5}{4}}}{25}+\frac{4x^{\frac{5}{4}}\ln(x)}{5}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/4)*ln(x),x,method=_RETURNVERBOSE)
[Out] -16/25*x^(5/4)+4/5*x^(5/4)*ln(x)
```

Maxima [A]
time = 0.41, size = 13, normalized size = 0.62

$$\frac{4}{5}x^{\frac{5}{4}}\log(x)-\frac{16}{25}x^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/4)*log(x),x, algorithm="maxima")
[Out] 4/5*x^(5/4)*log(x) - 16/25*x^(5/4)
```

Fricas [A]
time = 0.59, size = 14, normalized size = 0.67

$$\frac{4}{25}(5x\log(x)-4x)x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/4)*log(x),x, algorithm="fricas")
[Out] 4/25*(5*x*log(x) - 4*x)*x^(1/4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(19) = 38.
time = 2.27, size = 105, normalized size = 5.00

$$\begin{cases} -\frac{4x^{\frac{5}{4}}\log\left(\frac{1}{x}\right)}{5}+\frac{4x^{\frac{5}{4}}\log(x)}{5}-\frac{32x^{\frac{5}{4}}}{25} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \frac{4x^{\frac{5}{4}}\log(x)}{5}-\frac{16x^{\frac{5}{4}}}{25} & \text{for } |x|<1 \\ -\frac{4x^{\frac{5}{4}}\log\left(\frac{1}{x}\right)}{5}-\frac{16x^{\frac{5}{4}}}{25} & \text{for } \frac{1}{|x|}<1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{9}{4},\frac{9}{4} \\ \frac{5}{4},\frac{5}{4} & 0 \end{matrix}\middle| x\right)+G_{3,3}^{0,3}\left(\begin{matrix} \frac{9}{4},\frac{9}{4},1 & \\ & \frac{5}{4},\frac{5}{4},0 \end{matrix}\middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/4)*ln(x),x)
```

```
[Out] Piecewise((-4*x**(5/4)*log(1/x)/5 + 4*x**(5/4)*log(x)/5 - 32*x**(5/4)/25, (
Abs(x) < 1) & (1/Abs(x) < 1)), (4*x**(5/4)*log(x)/5 - 16*x**(5/4)/25, Abs(x
) < 1), (-4*x**(5/4)*log(1/x)/5 - 16*x**(5/4)/25, 1/Abs(x) < 1), (-meijerg(
((1,), (9/4, 9/4)), ((5/4, 5/4), (0,)), x) + meijerg(((9/4, 9/4, 1), ()), (
(), (5/4, 5/4, 0)), x), True))
```

Giac [**A**]
time = 0.42, size = 13, normalized size = 0.62

$$\frac{4}{5} x^{\frac{5}{4}} \log \left(x\right) - \frac{16}{25} x^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/4)*log(x),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4)*log(x) - 16/25*x^(5/4)
```

Mupad [**B**]
time = 0.03, size = 9, normalized size = 0.43

$$\frac{4 x^{5/4} \left(\ln\left(x\right) - \frac{4}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/4)*log(x),x)
```

```
[Out] (4*x^(5/4)*(log(x) - 4/5))/5
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 0.62

$$\frac{4 \ln\left(x\right) x^{\frac{5}{4}}}{5} - \frac{8 x^{\frac{5}{4}}}{15}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(1/4)*ln(x),x)
```

```
[Out] 4/5*ln(x)*x^(5/4)-8/15*x^(5/4)
```

3.52 $\int \frac{1}{(1+\sqrt{x})\sqrt{x-x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2(1-\sqrt{x})}{\sqrt{1-x}}$$

```
[Out] -2*(1-x^(1/2))/(1-x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 32, normalized size of antiderivative = 1.60, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2081, 665}

$$-\frac{2(1-x)\sqrt{x}}{\left(\sqrt{x}+1\right)\sqrt{x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[x])*Sqrt[x - x^2]),x]
```

```
[Out] (-2*(1 - x)*Sqrt[x])/((1 + Sqrt[x])*Sqrt[x - x^2])
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{x^2-x^4}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\left(2\sqrt{1-x}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)}{\sqrt{x-x^2}}\\
&= -\frac{2(1-x)\sqrt{x}}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 21, normalized size = 1.05

$$\frac{2(-\sqrt{x}+x)}{\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + Sqrt[x])*Sqrt[x - x^2]),x]
[Out] (2*(-Sqrt[x] + x))/Sqrt[-((-1 + x)*x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.
time = 0.06, size = 41, normalized size = 2.05

method	result	size
default	$\frac{2\sqrt{-(x-1)x}}{\sqrt{x}\,(x-1)}-\frac{2\sqrt{-(x-1)^{2}-x+1}}{x-1}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(1/2))/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(-(x-1)*x)^(1/2)/x^(1/2)/(x-1)-2/(x-1)*(-(x-1)^2-x+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))/(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + x)*(sqrt(x) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.
time = 0.57, size = 37, normalized size = 1.85

$$-\frac{2\left(\sqrt{-x^2+x}\,x-\sqrt{-x^2+x}\sqrt{x}\right)}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))/(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(-x^2 + x)*x - sqrt(-x^2 + x)*sqrt(x))/(x^2 - x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x\,(x-1)}\,\left(\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/2))/(-x**2+x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x*(x - 1))*(sqrt(x) + 1)), x)
```

Giac [**A**]
time = 0.41, size = 21, normalized size = 1.05

$$\frac{4}{\frac{\sqrt{-x+1}-1}{\sqrt{x}}-1}+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))/(-x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/((sqrt(-x + 1) - 1)/sqrt(x) - 1) + 4
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x-x^2}\,\left(\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - x^2)^(1/2)*(x^(1/2) + 1)),x)
```

```
[Out] int(1/((x - x^2)^(1/2)*(x^(1/2) + 1)), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 19, normalized size = 0.95

$$\arctan\left(\sqrt{-x^2+x}\right)-\ln\left(\sqrt{x}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^(1/2)+1)/(-x^2+x)^(1/2),x)
```

```
[Out] arctan((-x^2+x)^(1/2))-ln(x^(1/2)+1)
```

3.53 $\int \frac{1}{\left(1+\sqrt[4]{x}\right)^{10}\sqrt{x}}\,dx$

Optimal. Leaf size=27

$$\frac{4}{9\left(1+\sqrt[4]{x}\right)^9}-\frac{1}{2\left(1+\sqrt[4]{x}\right)^8}$$

```
[Out] 4/9/(x^(1/4)+1)^9-1/2/(x^(1/4)+1)^8
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {272, 45}

$$\frac{4}{9\left(\sqrt[4]{x}+1\right)^9}-\frac{1}{2\left(\sqrt[4]{x}+1\right)^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^(1/4))^10*Sqrt[x]),x]
```

```
[Out] 4/(9*(1 + x^(1/4))^9) - 1/(2*(1 + x^(1/4))^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 4\text{Subst}\left(\int \frac{x}{(1+x)^{10}}\,dx,x,\sqrt[4]{x}\right)\\
&= 4\text{Subst}\left(\int \left(-\frac{1}{(1+x)^{10}}+\frac{1}{(1+x)^9}\right)\,dx,x,\sqrt[4]{x}\right)\\
&= \frac{4}{9\left(1+\sqrt[4]{x}\right)^9}-\frac{1}{2\left(1+\sqrt[4]{x}\right)^8}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.81

$$\frac{-1-9\sqrt[4]{x}}{18\left(1+\sqrt[4]{x}\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^(1/4))^10*Sqrt[x]),x]
```

```
[Out] (-1 - 9*x^(1/4))/(18*(1 + x^(1/4))^9)
```

Maple [A]
time = 0.05, size = 20, normalized size = 0.74

method	result	size
derivativedivides	$\frac{4}{9\left(x^{\frac{1}{4}}+1\right)^{9}}-\frac{1}{2\left(x^{\frac{1}{4}}+1\right)^{8}}$	20
default	$\frac{4}{9\left(x^{\frac{1}{4}}+1\right)^{9}}-\frac{1}{2\left(x^{\frac{1}{4}}+1\right)^{8}}$	20
meijerg	$\frac{\sqrt{x}\left(x^{\frac{7}{4}}+9x^{\frac{3}{2}}+36x^{\frac{5}{4}}+84x+126x^{\frac{3}{4}}+126\sqrt{x}+84x^{\frac{1}{4}}+36\right)}{18\left(x^{\frac{1}{4}}+1\right)^{9}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x^(1/4)+1)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 4/9/(x^(1/4)+1)^9-1/2/(x^(1/4)+1)^8
```

Maxima [A]
time = 0.47, size = 19, normalized size = 0.70

$$-\frac{1}{2\left(x^{\frac{1}{4}}+1\right)^{8}}+\frac{4}{9\left(x^{\frac{1}{4}}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x^(1/4)+1)^10,x, algorithm="maxima")
```

```
[Out] -1/2/(x^(1/4) + 1)^8 + 4/9/(x^(1/4) + 1)^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(19) = 38.
time = 0.57, size = 186, normalized size = 6.89

$$-\frac{9\,x^{7}+4209\,x^{6}+71109\,x^{5}+227277\,x^{4}+184587\,x^{3}+36099\,x^{2}-16\left(5\,x^{6}+648\,x^{5}+6813\,x^{4}+15288\,x^{3}+8847\,x^{2}+1152\,x+15\right)x^{\frac{3}{4}}+4\left(99\,x^{6}+5544\,x^{5}+38027\,x^{4}+60552\,x^{3}+24777\,x^{2}+2064\,x+9\right)\sqrt{x}-32\left(45\,x^{6}+1311\,x^{5}+6066\,x^{4}+6894\,x^{3}+1969\,x^{2}+99\,x\right)x^{\frac{1}{4}}+999\,x-1}{18\left(x^{9}-9\,x^{8}+36\,x^{7}-84\,x^{6}+126\,x^{5}-126\,x^{4}+84\,x^{3}-36\,x^{2}+9\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x^(1/4)+1)^10,x, algorithm="fricas")
```

```
[Out] -1/18*(9*x^7 + 4209*x^6 + 71109*x^5 + 227277*x^4 + 184587*x^3 + 36099*x^2 -
 16*(5*x^6 + 648*x^5 + 6813*x^4 + 15288*x^3 + 8847*x^2 + 1152*x + 15)*x^(3/
4) + 4*(99*x^6 + 5544*x^5 + 38027*x^4 + 60552*x^3 + 24777*x^2 + 2064*x + 9)
*sqrt(x) - 32*(45*x^6 + 1311*x^5 + 6066*x^4 + 6894*x^3 + 1969*x^2 + 99*x)*x
^(1/4) + 999*x - 1)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 + 84
*x^3 - 36*x^2 + 9*x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(22) = 44$.
time = 83.21, size = 138, normalized size = 5.11

$$-\frac{\sqrt[4]{x}}{162x^{\frac{9}{4}} + 1512x^{\frac{7}{4}} + 2268x^{\frac{5}{4}} + 648x^{\frac{3}{4}} + 18\sqrt[4]{x} + 18x^{\frac{5}{2}} + 2268x^{\frac{3}{2}} + 162\sqrt{x} + 648x^2 + 1512x} - \frac{9\sqrt{x}}{162x^{\frac{9}{4}} + 1512x^{\frac{7}{4}} + 2268x^{\frac{5}{4}} + 648x^{\frac{3}{4}} + 18\sqrt[4]{x} + 18x^{\frac{5}{2}} + 2268x^{\frac{3}{2}} + 162\sqrt{x} + 648x^2 + 1512x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x**(1/4)+1)**10,x)
```

```
[Out] -x**(1/4)/(162*x**(9/4) + 1512*x**(7/4) + 2268*x**(5/4) + 648*x**(3/4) + 18
*x**(1/4) + 18*x**(5/2) + 2268*x**(3/2) + 162*sqrt(x) + 648*x**2 + 1512*x)
- 9*sqrt(x)/(162*x**(9/4) + 1512*x**(7/4) + 2268*x**(5/4) + 648*x**(3/4) +
18*x**(1/4) + 18*x**(5/2) + 2268*x**(3/2) + 162*sqrt(x) + 648*x**2 + 1512*x
)
```

Giac [A]
time = 0.44, size = 16, normalized size = 0.59

$$-\frac{9\,x^{\frac{1}{4}}+1}{18\,{\left(x^{\frac{1}{4}}+1\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x^(1/4)+1)^10,x, algorithm="giac")
```

```
[Out] -1/18*(9*x^(1/4) + 1)/(x^(1/4) + 1)^9
```

Mupad [B]
time = 0.73, size = 53, normalized size = 1.96

$$-\frac{9\,x^{1/4}+1}{2268\,x+\sqrt{x}\,\left(1512\,x+648\right)+x^{3/4}\,\left(648\,x+1512\right)+x^{1/4}\,\left(18\,x^2+2268\,x+162\right)+162\,x^2+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/4) + 1)^10),x)
```

```
[Out] -(9*x^(1/4) + 1)/(2268*x + x^(1/2)*(1512*x + 648) + x^(3/4)*(648*x + 1512)
+ x^(1/4)*(2268*x + 18*x^2 + 162) + 162*x^2 + 18)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(1/x^(1/2)/(x^(1/4)+1)^10,x)
```

```
[Out] not solved
```

3.54 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}$$

```
[Out] 1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Rubi [A]

time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 222}

$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\,dx\\&=\frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 37, normalized size = 1.61

$$\frac{1}{2}x\sqrt{1-x^2}-\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.18, size = 18, normalized size = 0.78

method	result	size
default	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(\mathit{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Maxima [A]
time = 0.45, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.58, size = 31, normalized size = 1.35

$$\frac{1}{2}\sqrt{-x^2+1}x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [**A**]
time = 0.20, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Giac [**A**]
time = 0.46, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [**B**]
time = 0.07, size = 17, normalized size = 0.74

$$\frac{\operatorname{asin}(x)}{2}+\frac{x\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2),x)
```

```
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

Chatgpt [**A**]
time = 1.00, size = 17, normalized size = 0.74

$$\frac{\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{2}$$

Antiderivative was successfully verified.

```
[In] int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

3.55 $\int \frac{1}{\sqrt{1-4x-x^2}}\,dx$

Optimal. Leaf size=10

$$\arcsin\left(\frac{2+x}{\sqrt{5}}\right)$$

[Out] arcsin(1/5*(2+x)*5^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\arcsin\left(\frac{x+2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - 4*x - x^2],x]

[Out] ArcSin[(2 + x)/Sqrt[5]]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{20}}}\,dx, x, -4-2x\right)}{2\sqrt{5}}\\ &= \arcsin\left(\frac{2+x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.
time = 0.04, size = 23, normalized size = 2.30

$$2\arctan\left(\frac{x}{-1+\sqrt{1-4x-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - 4*x - x^2],x]
```

```
[Out] 2*ArcTan[x/(-1 + Sqrt[1 - 4*x - x^2])]
```

Maple [A]
time = 0.12, size = 10, normalized size = 1.00

method	result	size
default	$\arcsin\left(\frac{(2+x)\sqrt{5}}{5}\right)$	10
trager	$\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathit{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-4x+1}-2\mathit{RootOf}\left(\_Z^2+1\right)\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(1/5*(2+x)*5^(1/2))
```

Maxima [A]
time = 0.42, size = 11, normalized size = 1.10

$$-\arcsin\left(-\frac{1}{5}\sqrt{5}(x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/5*sqrt(5)*(x + 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.
time = 0.58, size = 21, normalized size = 2.10

$$-2\arctan\left(\frac{\sqrt{-x^2-4x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 - 4*x + 1) - 1)/x)
```

Sympy [A]
time = 0.29, size = 10, normalized size = 1.00

$$\operatorname{asin}\left(\frac{\sqrt{5}(x+2)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+1)**(1/2),x)
```

```
[Out] asin(sqrt(5)*(x + 2)/5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(9) = 18$.
time = 0.44, size = 29, normalized size = 2.90

$$\frac{1}{2}\sqrt{-x^2-4x+1}(x+2)+\frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}(x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 - 4*x + 1)*(x + 2) + 5/2*arcsin(1/5*sqrt(5)*(x + 2))
```

Mupad [B]
time = 0.09, size = 11, normalized size = 1.10

$$\mathrm{asin}\left(\frac{\sqrt{20}\,(2x+4)}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^2 - 4*x)^(1/2),x)
```

```
[Out] asin((20^(1/2)*(2*x + 4))/20)
```

Chatgpt [A] valid for real x
time = 1.00, size = 9, normalized size = 0.90

$$\arcsin\left(\frac{(2+x)\sqrt{5}}{5}\right)$$

Antiderivative was successfully verified.

```
[In] int(1/(-x^2-4*x+1)^(1/2),x)
```

```
[Out] arcsin(1/5*(2+x)*5^(1/2))
```

3.56 $\int \log\left(\frac{1}{x}\right)\,dx$

Optimal. Leaf size=8

$$x + x\log\left(\frac{1}{x}\right)$$

```
[Out] x+x*ln(1/x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2332}

$$x + x\log\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^(-1)],x]
```

```
[Out] x + x*Log[x^(-1)]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{Integral} = x + x\log\left(\frac{1}{x}\right)$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$x + x\log\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^(-1)],x]
```

```
[Out] x + x*Log[x^(-1)]
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size
derivativedivides	$x + x\ln\left(\frac{1}{x}\right)$	9
default	$x + x\ln\left(\frac{1}{x}\right)$	9
norman	$x + x\ln\left(\frac{1}{x}\right)$	9
risch	$x + x\ln\left(\frac{1}{x}\right)$	9
parallelrisch	$x + x\ln\left(\frac{1}{x}\right)$	9
parts	$x + x\ln\left(\frac{1}{x}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/x),x,method=_RETURNVERBOSE)
```

```
[Out] x+x*ln(1/x)
```

Maxima [A]
time = 0.37, size = 7, normalized size = 0.88

$$-x\log(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x),x, algorithm="maxima")
```

```
[Out] -x*log(x) + x
```

Fricas [A]
time = 0.58, size = 8, normalized size = 1.00

$$x\log\left(\frac{1}{x}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x),x, algorithm="fricas")
```

```
[Out] x*log(1/x) + x
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.88

$$x\log\left(\frac{1}{x}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/x),x)
```

```
[Out] x*log(1/x) + x
```

Giac [A]
time = 0.49, size = 7, normalized size = 0.88

$$-x\log(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x),x, algorithm="giac")
```

```
[Out] -x*log(x) + x
```

Mupad [B]
time = 0.02, size = 8, normalized size = 1.00

$$x\left(\ln\left(\frac{1}{x}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/x),x)
```

```
[Out] x*(log(1/x) + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 1.12

$$x\ln\left(\frac{1}{x}\right) + \ln(x)$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(1/x),x)
```

```
[Out] x*ln(1/x)+ln(x)
```

3.57 $\int \frac{1}{1+\sin(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{1+\sin(x)}$$

```
[Out] -cos(x)/(1+sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2727}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{Integral} = -\frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.
time = 0.01, size = 23, normalized size = 2.30

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$-\frac{2}{1+\tan(\frac{x}{2})}$	11
norman	$-\frac{2}{1+\tan(\frac{x}{2})}$	11
parallelrisch	$-\frac{2}{1+\tan(\frac{x}{2})}$	11
risch	$-\frac{2}{i+e^{ix}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(1+tan(1/2*x))
```

Maxima [A]
time = 0.33, size = 15, normalized size = 1.50

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

Fricas [A]
time = 0.57, size = 18, normalized size = 1.80

$$-\frac{\cos(x)-\sin(x)+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A]
time = 0.32, size = 8, normalized size = 0.80

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

Giac [A]
time = 0.45, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="giac")
[Out] -2/(tan(1/2*x) + 1)
```

Mupad [B]
time = 0.07, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + 1),x)
[Out] -2/(tan(x/2) + 1)
```

Chatgpt [A]
time = 1.00, size = 10, normalized size = 1.00

$$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] int(1/(1+sin(x)),x)
[Out] -2/(1+tan(1/2*x))
```

3.58 $\int \frac{\sqrt{x}}{\sqrt{2012-x}+\sqrt{x}} \, dx$

Optimal. Leaf size=53

$$-\frac{1}{2}\sqrt{2012-x}\sqrt{x}+\frac{x}{2}+503\text{arctanh}\left(\frac{\sqrt{2012-x}\sqrt{x}}{1006}\right)+503\log(1006-x)$$

```
[Out] -1/2*(2012-x)^(1/2)*x^(1/2)+1/2*x+503*arctanh(1/1006*(2012-x)^(1/2)*x^(1/2)
)+503*ln(1006-x)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2130, 103, 12, 94, 212, 45}

$$503\text{arctanh}\left(\frac{\sqrt{2012-x}\sqrt{x}}{1006}\right)+\frac{x}{2}-\frac{1}{2}\sqrt{2012-x}\sqrt{x}+503\log(1006-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[2012 - x] + Sqrt[x]),x]
```

```
[Out] -1/2*(Sqrt[2012 - x]*Sqrt[x]) + x/2 + 503*ArcTanh[(Sqrt[2012 - x]*Sqrt[x])/
1006] + 503*Log[1006 - x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
```

```
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2130

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[e, Int[(u*Sqrt[a + b*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2
)*x), x], x] - Dist[f, Int[(u*Sqrt[c + d*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^
2)*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a*e^2 - c*f^2, 0] && N
eQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \frac{\sqrt{2012-x}\sqrt{x}}{2012-2x}\,dx - \int \frac{x}{2012-2x}\,dx \\
&= -\frac{1}{2}\sqrt{2012-x}\sqrt{x} + \frac{1}{2}\int \frac{2024072}{(2012-2x)\sqrt{2012-x}\sqrt{x}}\,dx - \int \left(-\frac{1}{2} - \frac{503}{-1006+x}\right)dx \\
&= -\frac{1}{2}\sqrt{2012-x}\sqrt{x} + \frac{x}{2} + 503\log(1006-x) + 1012036\int \frac{1}{(2012-2x)\sqrt{2012-x}\sqrt{x}}\,dx \\
&= -\frac{1}{2}\sqrt{2012-x}\sqrt{x} + \frac{x}{2} + 503\log(1006-x) + 2024072\text{Subst}\left(\int \frac{1}{4048144-4x^2}\,dx, x, \sqrt{2012-x}\sqrt{x}\right) \\
&= -\frac{1}{2}\sqrt{2012-x}\sqrt{x} + \frac{x}{2} + 503\text{arctanh}\left(\frac{\sqrt{2012-x}\sqrt{x}}{1006}\right) + 503\log(1006-x)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 62, normalized size = 1.17

$$\frac{1}{2}\left(x - \sqrt{-((-2012+x)x)} - 2012\log\left(\sqrt{2012-x} - i\sqrt{x}\right) + 2012\log\left((1006+1006i) - ix + \sqrt{-((-2012+x)x)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x]/(Sqrt[2012 - x] + Sqrt[x]),x]
[Out] (x - Sqrt[-((-2012 + x)*x)] - 2012*Log[Sqrt[2012 - x] - I*Sqrt[x]] + 2012*L
og[(1006 + 1006*I) - I*x + Sqrt[-((-2012 + x)*x)]])/2
```

Maple [A]
time = 0.02, size = 53, normalized size = 1.00

method	result	size
default	$\frac{x}{2}+503\ln\left(x-1006\right)-\frac{\sqrt{x}\,\sqrt{2012-x}\,\left(\sqrt{-x\left(-2012+x\right)}-1006\,\operatorname{arctanh}\left(\frac{1006}{\sqrt{-x\left(-2012+x\right)}}\right)\right)}{2\sqrt{-x\left(-2012+x\right)}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+503*ln(x-1006)-1/2*x^(1/2)*(2012-x)^(1/2)*((-x*(-2012+x))^(1/2)-1006*
arctanh(1006/(-x*(-2012+x))^(1/2)))/(-x*(-2012+x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(sqrt(x) + sqrt(-x + 2012)), x)
```

Fricas [A]
time = 0.58, size = 64, normalized size = 1.21

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\sqrt{-x+2012}+503\,\log\left(x-1006\right)+503\,\log\left(\frac{x+\sqrt{x}\sqrt{-x+2012}}{x}\right)-503\,\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2012}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*sqrt(x)*sqrt(-x + 2012) + 503*log(x - 1006) + 503*log((x + sqrt
(x)*sqrt(-x + 2012))/x) - 503*log(-(x - sqrt(x)*sqrt(-x + 2012))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\sqrt{x}+\sqrt{2012-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/((2012-x)**(1/2)+x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)/(sqrt(x) + sqrt(2012 - x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> unable to parse Giac output: Warni
ng, choosing root of [1,0,-4024,0,%%%{4,[2]%%%}+%%%{-8048,[1]%%%}+%%%{40481
44,[0]%%%}] at parameters values [1917.38357631]Warning, choosing root of [
1,0,-4024,0,%%%{
```

Mupad [B]
time = 1.59, size = 64, normalized size = 1.21

$$\frac{x}{2}+1006\,\mathrm{atanh}\left(\frac{2\,\sqrt{503}\,\sqrt{x}-\sqrt{x}\,\sqrt{2012-x}}{x+2\,\sqrt{503}\,\sqrt{2012-x}-2012}\right)+503\,\ln\left(x-1006\right)-\frac{\sqrt{x}\,\sqrt{2012-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((2012 - x)^(1/2) + x^(1/2)),x)
```

```
[Out] x/2 + 1006*atanh((2*503^(1/2)*x^(1/2) - x^(1/2)*(2012 - x)^(1/2))/(x + 2*50
3^(1/2)*(2012 - x)^(1/2) - 2012)) + 503*log(x - 1006) - (x^(1/2)*(2012 - x)
^(1/2))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 51, normalized size = 0.96

$$\frac{2\sqrt{x}\left(2012-x-2\sqrt{x}\,\sqrt{2012-x}+2012\ln\left(\sqrt{2012-x}+\sqrt{x}\right)-2x\ln\left(\sqrt{2012-x}+\sqrt{x}\right)\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(1/2)/((2012-x)^(1/2)+x^(1/2)),x)
```

```
[Out] 2/3*x^(1/2)*(2012-x-2*x^(1/2)*(2012-x)^(1/2)+2012*ln((2012-x)^(1/2)+x^(1/2)
)-2*x*ln((2012-x)^(1/2)+x^(1/2)))
```

3.59 $\int \frac{-1+x}{(1+x)\sqrt{x+x^2+x^3}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{x}\sqrt{1+x+x^2}\arctan\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x+x^2+x^3}}$$

```
[Out] -2*x^(1/2)*(x^2+x+1)^(1/2)*arctan(x^(1/2)/(x^2+x+1)^(1/2))/(x^3+x^2+x)^(1/2
)
```

Rubi [A]
time = 0.23, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2081, 6865, 1712, 209}

$$-\frac{2\sqrt{x}\sqrt{x^2+x+1}\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)}{\sqrt{x^3+x^2+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*Sqrt[x + x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + x + x^2]*ArcTan[Sqrt[x]/Sqrt[1 + x + x^2]])/Sqrt[x + x
^2 + x^3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \frac{\left(\sqrt{x}\sqrt{1+x+x^2}\right)\int \frac{-1+x}{\sqrt{x}(1+x)\sqrt{1+x+x^2}}\,dx}{\sqrt{x+x^2+x^3}} \\
&= \frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\text{Subst}\left(\int \frac{-1+x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{x+x^2+x^3}} \\
&= -\frac{\left(2\sqrt{x}\sqrt{1+x+x^2}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x+x^2+x^3}} \\
&= -\frac{2\sqrt{x}\sqrt{1+x+x^2}\arctan\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x+x^2+x^3}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 46, normalized size = 1.00

$$
-\frac{2\sqrt{x}\sqrt{1+x+x^2}\arctan\left(\frac{\sqrt{x}}{\sqrt{1+x+x^2}}\right)}{\sqrt{x\left(1+x+x^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*Sqrt[x + x^2 + x^3]),x]
[Out] (-2*Sqrt[x]*Sqrt[1 + x + x^2]*ArcTan[Sqrt[x]/Sqrt[1 + x + x^2]])/Sqrt[x*(1
+ x + x^2)]
```

Maple [A]
time = 0.47, size = 18, normalized size = 0.39

method	result
default	$2\arctan\left(\frac{\sqrt{(x^2+x+1)x}}{x}\right)$
pseudoelliptic	$2\arctan\left(\frac{\sqrt{(x^2+x+1)x}}{x}\right)$
trager	$\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathit{RootOf}\left(\_Z^2+1\right)x^2+2\sqrt{x^3+x^2+x}-\mathit{RootOf}\left(\_Z^2+1\right)}{(x+1)^2}\right)$

elliptic	$\frac{2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}},\frac{\sqrt{3}\,\sqrt{i\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}\right)}{3\sqrt{x^3+x^2+x}} - \frac{4\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+}{\frac{1}{2}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x+1)/(x^3+x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arctan(((x^2+x+1)*x)^(1/2)/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+1)/(x^3+x^2+x)^(1/2),x, algorithm="maxima")
[Out] integrate((x - 1)/(sqrt(x^3 + x^2 + x)*(x + 1)), x)
```

Fricas [A]
time = 0.64, size = 18, normalized size = 0.39

$$\arctan\left(\frac{x^2+1}{2\sqrt{x^3+x^2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+1)/(x^3+x^2+x)^(1/2),x, algorithm="fricas")
[Out] arctan(1/2*(x^2 + 1)/sqrt(x^3 + x^2 + x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{\sqrt{x\left(x^2+x+1\right)}\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+1)/(x**3+x**2+x)**(1/2),x)
[Out] Integral((x - 1)/(sqrt(x*(x**2 + x + 1))*(x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+1)/(x^3+x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - 1)/(sqrt(x^3 + x^2 + x)*(x + 1)), x)
```

Mupad [B]
time = 0.26, size = 179, normalized size = 3.89

$$\frac{\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(\sqrt{3}+1i\right)\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-2\,\Pi\left(\frac{1}{2}-\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{\frac{x}{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\,1i}{\sqrt{x^3+x^2-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x + 1)*(x + x^2 + x^3)^(1/2)),x)
```

```
[Out] ((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 1/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 1/2))^(1/2)
*(3^(1/2) + 1i)*(ellipticF(asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 + 1/2)) - 2*ellipticPi(1/2 - (3^(1/2)*1i)/2
, asin((x/((3^(1/2)*1i)/2 - 1/2))^(1/2)), -((3^(1/2)*1i)/2 - 1/2)/((3^(1/2)
*1i)/2 + 1/2)))*1i)/(x^2 + x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2))^(1/2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 21, normalized size = 0.46

$$\frac{\ln\left(x+1\right)}{2}-\frac{\ln\left(x^2+1\right)}{2}-\arctan\left(\sqrt{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((x-1)/(x+1)/(x^3+x^2+x)^(1/2),x)
```

```
[Out] 1/2*ln(x+1)-1/2*ln(x^2+1)-arctan(x^(1/2))
```

3.60 $\int \frac{1+4x+6x^2+4x^3+x^4}{-1+3x-3x^2+x^3} \, dx$

Optimal. Leaf size=37

$$-\frac{8}{(1-x)^2}+\frac{32}{1-x}+7x+\frac{x^2}{2}+24\log(1-x)$$

```
[Out] -8/(1-x)^2+32/(1-x)+7*x+1/2*x^2+24*ln(1-x)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2099}

$$\frac{x^2}{2}+7x+\frac{32}{1-x}-\frac{8}{(1-x)^2}+24\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 6*x^2 + 4*x^3 + x^4)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] -8/(1 - x)^2 + 32/(1 - x) + 7*x + x^2/2 + 24*Log[1 - x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \int \left(7+\frac{16}{(-1+x)^3}+\frac{32}{(-1+x)^2}+\frac{24}{-1+x}+x\right) dx \\ &= -\frac{8}{(1-x)^2}+\frac{32}{1-x}+7x+\frac{x^2}{2}+24\log(1-x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.95

$$-\frac{8}{(-1+x)^2}-\frac{32}{-1+x}+8(-1+x)+\frac{1}{2}(-1+x)^2+24\log(-1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 6*x^2 + 4*x^3 + x^4)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] -8/(-1 + x)^2 - 32/(-1 + x) + 8*(-1 + x) + (-1 + x)^2/2 + 24*Log[-1 + x]
```

Maple [A]
time = 0.02, size = 30, normalized size = 0.81

method	result	size
norman	$\frac{-52x+6x^3+\frac{1}{2}x^4+\frac{75}{2}}{(x-1)^2}+24\ln(x-1)$	29
default	$\frac{x^2}{2}+7x-\frac{32}{x-1}-\frac{8}{(x-1)^2}+24\ln(x-1)$	30
risch	$\frac{x^2}{2}+7x+\frac{-32x+24}{x^2-2x+1}+24\ln(x-1)$	32
parallelrisch	$\frac{x^4+48\ln(x-1)x^2+12x^3+75-96\ln(x-1)x+48\ln(x-1)-104x}{2x^2-4x+2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+7*x-32/(x-1)-8/(x-1)^2+24*ln(x-1)
```

Maxima [A]
time = 0.40, size = 32, normalized size = 0.86

$$\frac{1}{2}x^2+7x-\frac{8(4x-3)}{x^2-2x+1}+24\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
[Out] 1/2*x^2 + 7*x - 8*(4*x - 3)/(x^2 - 2*x + 1) + 24*log(x - 1)
```

Fricas [A]
time = 0.56, size = 44, normalized size = 1.19

$$\frac{x^4+12x^3-27x^2+48(x^2-2x+1)\log(x-1)-50x+48}{2(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
[Out] 1/2*(x^4 + 12*x^3 - 27*x^2 + 48*(x^2 - 2*x + 1)*log(x - 1) - 50*x + 48)/(x^
2 - 2*x + 1)
```

Sympy [A]
time = 0.04, size = 27, normalized size = 0.73

$$\frac{x^2}{2}+7x+\frac{24-32x}{x^2-2x+1}+24\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+4*x**3+6*x**2+4*x+1)/(x**3-3*x**2+3*x-1),x)
```

```
[Out] x**2/2 + 7*x + (24 - 32*x)/(x**2 - 2*x + 1) + 24*log(x - 1)
```

Giac [A]
time = 0.41, size = 28, normalized size = 0.76

$$\frac{1}{2}\,x^2 + 7\,x - \frac{8\,(4\,x - 3)}{(x - 1)^2} + 24\,\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 7*x - 8*(4*x - 3)/(x - 1)^2 + 24*log(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 32, normalized size = 0.86

$$7\,x + 24\,\ln\left(x - 1\right) - \frac{32\,x - 24}{x^2 - 2\,x + 1} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 6*x^2 + 4*x^3 + x^4 + 1)/(3*x - 3*x^2 + x^3 - 1),x)
```

```
[Out] 7*x + 24*log(x - 1) - (32*x - 24)/(x^2 - 2*x + 1) + x^2/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 22, normalized size = 0.59

$$\frac{3\ln\left(x - 1\right)}{2} - \frac{\ln\left(x^2 + 1\right)}{2} + \frac{\arctan\left(x\right)}{2} - \frac{x}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((x^4+4*x^3+6*x^2+4*x+1)/(x^3-3*x^2+3*x-1),x)
```

```
[Out] 3/2*ln(x-1)-1/2*ln(x^2+1)+1/2*arctan(x)-1/2*x
```

3.61 $\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx$

Optimal. Leaf size=5

$$\log(x)\sin(x)$$

[Out] ln(x)*sin(x)

Rubi [A]

time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2717, 2634, 3380}

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Log[x] + Sin[x]/x,x]

[Out] Log[x]*Sin[x]

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \cos(x)\log(x)\,dx+\int \frac{\sin(x)}{x}\,dx\\
&= \log(x)\sin(x)+\text{Si}(x)-\int \frac{\sin(x)}{x}\,dx\\
&= \log(x)\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[x] + Sin[x]/x,x]
[Out] Log[x]*Sin[x]
```

Maple [A]
time = 0.13, size = 6, normalized size = 1.20

method	result	size
risch	$\ln\left(x\right)\sin\left(x\right)$	6
parallelrisch	$\ln\left(x\right)\sin\left(x\right)$	6
norman	$\frac{2\ln(x)\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(x)+sin(x)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*sin(x)
```

Maxima [A]
time = 0.38, size = 5, normalized size = 1.00

$$\log\left(x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="maxima")
[Out] log(x)*sin(x)
```

Fricas [A]
time = 0.60, size = 5, normalized size = 1.00

$$\log\left(x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="fricas")
[Out] log(x)*sin(x)
```

Sympy [A]
time = 4.35, size = 5, normalized size = 1.00

$$\log\left(x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(x)+sin(x)/x,x)
```

```
[Out] log(x)*sin(x)
```

Giac [A]
time = 0.47, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="giac")
```

```
[Out] log(x)*sin(x)
```

Mupad [B]
time = 0.21, size = 5, normalized size = 1.00

$$\ln(x)\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*log(x) + sin(x)/x,x)
```

```
[Out] log(x)*sin(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)*ln(x)+sin(x)/x,x)
```

```
[Out] not solved
```

3.62 $\int \frac{1}{-x+x^3} dx$

Optimal. Leaf size=17

$$-\log(x) + \frac{1}{2}\log\left(1 - x^2\right)$$

```
[Out] -ln(x)+1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1607, 272, 36, 31, 29}

$$\frac{1}{2}\log\left(1 - x^2\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{1}{x\left(-1+x^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)\\ &= -\log(x) + \frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\log(x) + \frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - x^2]/2
```

Maple [A]
time = 0.07, size = 18, normalized size = 1.06

method	result	size
risch	$-\ln(x) + \frac{\ln(x^2-1)}{2}$	14
default	$\frac{\ln(x+1)}{2} - \ln(x) + \frac{\ln(x-1)}{2}$	18
norman	$\frac{\ln(x+1)}{2} - \ln(x) + \frac{\ln(x-1)}{2}$	18
parallelrisch	$\frac{\ln(x+1)}{2} - \ln(x) + \frac{\ln(x-1)}{2}$	18
meijerg	$-\ln(x) - \frac{i\pi}{2} + \frac{\ln(-x^2+1)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x+1)-ln(x)+1/2*ln(x-1)
```

Maxima [A]
time = 0.35, size = 17, normalized size = 1.00

$$\frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [**A**]
time = 0.56, size = 13, normalized size = 0.76

$$\frac{1}{2}\log\left(x^2-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1) - log(x)
```

Sympy [**A**]
time = 0.03, size = 10, normalized size = 0.59

$$-\log\left(x\right)+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1)/2
```

Giac [**A**]
time = 0.44, size = 16, normalized size = 0.94

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(x^2 - 1))
```

Mupad [**B**]
time = 0.11, size = 13, normalized size = 0.76

$$\frac{\ln\left(x^2-1\right)}{2}-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x - x^3),x)
```

```
[Out] log(x^2 - 1)/2 - log(x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 17, normalized size = 1.00

$$-\frac{\ln(x)}{2}+\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^3-x),x)
```

```
[Out] -1/2*ln(x)+1/2*ln(x-1)+1/2*ln(x+1)
```

3.63 $\int \frac{x \arcsin(x)}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=17

$$x-\sqrt{1-x^2}\arcsin(x)$$

```
[Out] x-(-x^2+1)^(1/2)*arcsin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4767, 8}

$$x-\sqrt{1-x^2}\arcsin(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] x - Sqrt[1 - x^2]*ArcSin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\sqrt{1-x^2}\arcsin(x)+\int 1\,dx\\ &= x-\sqrt{1-x^2}\arcsin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$x-\sqrt{1-x^2}\arcsin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] x - Sqrt[1 - x^2]*ArcSin[x]
```

Maple [A]
time = 0.10, size = 16, normalized size = 0.94

method	result	size
default	$x-\sqrt{-x^{2}+1}\, \arcsin\left(x\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] x-(-x^2+1)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 15, normalized size = 0.88

$$-\sqrt{-x^{2}+1}\arcsin\left(x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)*arcsin(x) + x
```

Fricas [A]
time = 0.64, size = 15, normalized size = 0.88

$$-\sqrt{-x^{2}+1}\arcsin\left(x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 1)*arcsin(x) + x
```

Sympy [A]
time = 0.09, size = 12, normalized size = 0.71

$$x-\sqrt{1-x^{2}}\operatorname{asin}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x)/(-x**2+1)**(1/2),x)
[Out] x - sqrt(1 - x**2)*asin(x)
```

Giac [A]
time = 0.47, size = 15, normalized size = 0.88

$$-\sqrt{-x^{2}+1}\arcsin\left(x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*arcsin(x) + x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x\,\mathrm{asin}(x)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asin(x))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x*asin(x))/(1 - x^2)^(1/2), x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 39, normalized size = 2.29

$$-\frac{x\sqrt{-x^2+1}\,\arcsin\left(x\right)}{2}+\frac{\ln\left(x\right)\sqrt{-x^2+1}}{2}+\frac{\sqrt{-x^2+1}}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*arcsin(x)/(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*x*(-x^2+1)^(1/2)*arcsin(x)+1/2*ln(x)*(-x^2+1)^(1/2)+1/4*(-x^2+1)^(1/2)
```

3.64 $\int (1-x)^{99} x \, dx$

Optimal. Leaf size=23

$$-\frac{1}{100}(1-x)^{100}+\frac{1}{101}(1-x)^{101}$$

```
[Out] -1/100*(1-x)^100+1/101*(1-x)^101
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{1}{101}(1-x)^{101}-\frac{1}{100}(1-x)^{100}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^99*x,x]
```

```
[Out] -1/100*(1 - x)^100 + (1 - x)^101/101
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left((1-x)^{99}-(1-x)^{100}\right) dx\\ &= -\frac{1}{100}(1-x)^{100}+\frac{1}{101}(1-x)^{101}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 567 vs. 2(23) = 46.
time = 0.00, size = 567, normalized size = 24.65

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^99*x,x]
```

```
[Out] x^2/2 - 33*x^3 + (4851*x^4)/4 - (156849*x^5)/5 + 627396*x^6 - 10217592*x^7
+ 140066157*x^8 - 1654114616*x^9 + (85600431378*x^10)/5 - 157366449604*x^11
```

```
 + 1298273209233*x^12 - 9696194317908*x^13 + 66026466069564*x^14 - (2062057
324941768*x^15)/5 + (4750096337812287*x^16)/2 - 12666923567499432*x^17 + 62
806829355518017*x^18 - 290505891817783026*x^19 + (12572449429225165403*x^20
)/10 - 5104603527655330314*x^21 + 19490304378320352108*x^22 - 7013281368430
8016488*x^23 + 238292173768273828749*x^24 - (19146258135816088501224*x^25)/
25 + 2331916055003241548226*x^26 - 6736646381120475583764*x^27 + 1848876300
7525700846649*x^28 - 48264408157576669898532*x^29 + (599857644244167183024 6
12*x^30)/5 - 284248449886557530554488*x^31 + (2570079734390957672096829*x^3
2)/4 - 1386787891870780679371896*x^33 + (5720500053966970302409071*x^34)/2
- (28206275157871771317939099*x^35)/5 + (42585944846198556695711973*x^36)/4
 - 19237666204653402059452899*x^37 + 33300287699283081927474024*x^38 - 5524
6631151825154632274992*x^39 + (439428796464188236515924114*x^40)/5 - 134109
601422466453670901168*x^41 + 196374773511468735732390996*x^42 - 27601627269
5076305811040776*x^43 + 372502480574415750374856978*x^44 - (241404708341249
2769871166152*x^45)/5 + 601126348833940887359223192*x^46 - 7190778546335084
84642475024*x^47 + 826548729646668720118931889*x^48 - 913043842041304917057
126672*x^49 + (24233705307512968006891237086*x^50)/25 - 989130828878080326 8
11887228*x^51 + 970109082168886474373197089*x^52 - 914479445566527094599669
324*x^53 + 828502745555998906218503832*x^54 - (360675809300364532415533915 2
*x^55)/5 + 603511770853123192467791538*x^56 - 485119509585285628395162576*x
^57 + 374593512943317843600698196*x^58 - 277798460089394796889723848*x^59 +
 (989058310490690095822896114*x^60)/5 - 135208860450519457389515112*x^61 +
88685381585824590330757224*x^62 - 55800482090690569716307824*x^63 + (134663
66343257381441617029 3*x^64)/4 - (97339302505596701018770224*x^65)/5 + (2156
950453249017806665931 1*x^66)/2 - 5720500053966970302409071*x^67 + (11614348
594417788189739629*x^68)/4 - 1409398564020847755666003*x^69 + (326885717369
5411601376612*x^70)/5 - 289586448945460019391192*x^71 + 1223847492567122700
99849*x^72 - 49303368020068535591064*x^73 + 18914430223915181446722*x^74 -
(172561788070239874568724*x^75)/25 + 2393282266977011062653*x^76 - 78740022
6364730912388*x^77 + 245464847895078057708*x^78 - 72392559119475593544*x^79
 + (201631839342385547403*x^80)/10 - 5293662917568490696*x^81 + 13072765131
80023617*x^82 - 302950588656028082*x^83 + (131419332012806607*x^84)/2 - (66
501348729372018*x^85)/5 + 2503926751715004*x^86 - 436790467844808*x^87 + 70
297408804833*x^88 - 10386185673864*x^89 + (7002807007378*x^90)/5 - 17120086
2756*x^91 + 18815553757*x^92 - 1840869492*x^93 + 158372676*x^94 - (58975224
*x^95)/5 + (2980131*x^96)/4 - 38808*x^97 + (3201*x^98)/2 - 49*x^99 + (99*x^
100)/100 - x^101/101
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(19) = 38$.

time = 0.09, size = 502, normalized size = 21.83

method	result	size
gosper	Expression too large to display	501
default	Expression too large to display	502

risch	Expression too large to display	502
parallelrisch	Expression too large to display	502

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1-x)^99,x,method=_RETURNVERBOSE)
```

```
[Out] -157366449604*x^11-9696194317908*x^13-2062057324941768/5*x^15-1266692356749
9432*x^17-1654114616*x^9-10217592*x^7+4851/4*x^4-156849/5*x^5+627396*x^6-33
*x^3+1/2*x^2-2895864489454600193911 92*x^71+12238474925671227009984 9*x^72-49
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(15) = 30.

time = 0.38, size = 501, normalized size = 21.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^99,x, algorithm="maxima")
```

```
[Out] -1/101*x^101 + 99/100*x^100 - 49*x^99 + 3201/2*x^98 - 38808*x^97 + 2980131/
4*x^96 - 58975224/5*x^95 + 158372676*x^94 - 1840869492*x^93 + 18815553757*x
^92 - 171200862756*x^91 + 7002807007378/5*x^90 - 10386185673864*x^89 + 7029
7408804833*x^88 - 436790467844808*x^87 + 2503926751715004*x^86 - 6650134872
9372018/5*x^85 + 131419332012806607/2*x^84 - 302950588656028082*x^83 + 1307
276513180023617*x^82 - 5293662917568490696*x^81 + 201631839342385547403/10*
x^80 - 72392559119475593544*x^79 + 245464847895078057708*x^78 - 78740022636
4730912388*x^77 + 2393282266977011062653*x^76 - 172561788070239874568724/25
*x^75 + 18914430223915181446722*x^74 - 49303368020068535591064*x^73 + 12238
4749256712270099849*x^72 - 289586448945460019391192*x^71 + 3268857173695411
601376612/5*x^70 - 1409398564020847755666003*x^69 + 11614348594417788189739
629/4*x^68 - 5720500053966970302409071*x^67 + 21569504532490178066659311/2*
x^66 - 97339302505596701018770224/5*x^65 + 134663663432573814416170293/4*x^
64 - 55800482090690569716307824*x^63 + 88685381585824590330757224*x^62 - 13
5208860450519457389515112*x^61 + 989058310490690095822896114/5*x^60 - 27779
8460089394796889723848*x^59 + 374593512943317843600698196*x^58 - 4851195095
85285628395162576*x^57 + 603511770853123192467791538*x^56 - 360675809300364
5324155339152/5*x^55 + 828502745555998906218503832*x^54 - 91447944556652709
4599669324*x^53 + 970109082168886474373197089*x^52 - 989130828878080326811 8
87228*x^51 + 24233705307512968006891237086/25*x^50 - 913043842041304917057 1
26672*x^49 + 826548729646668720118931889*x^48 - 719077854633508484642475024
*x^47 + 601126348833940887359223192*x^46 - 2414047083412492769871166152/5*x
^45 + 372502480574415750374856978*x^44 - 276016272695076305811040776*x^43 +
 196374773511468735732390996*x^42 - 134109601422466453670901168*x^41 + 4394
287964641882365159241 14/5*x^40 - 55246631151825154632274992*x^39 + 33300287
699283081927474024*x^38 - 19237666204653402059452899*x^37 + 425859448461985
56695711973/4*x^36 - 28206275157871771317939099/5*x^35 + 572050005396697030
2409071/2*x^34 - 1386787891870780679371896*x^33 + 2570079734390957672096829
/4*x^32 - 284248449886557530554488*x^31 + 599857644244167183024612/5*x^30 -
 48264408157576669898532*x^29 + 18488763007525700846649*x^28 - 673664638112
0475583764*x^27 + 2331916055003241548226*x^26 - 19146258135816088501224/25*
x^25 + 238292173768273828749*x^24 - 70132813684308016488*x^23 + 19490304378
320352108*x^22 - 5104603527655330314*x^21 + 12572449429225165403/10*x^20 -
290505891817783026*x^19 + 62806829355518017*x^18 - 12666923567499432*x^17 +
 4750096337812287/2*x^16 - 2062057324941768/5*x^15 + 66026466069564*x^14 -
9696194317908*x^13 + 1298273209233*x^12 - 157366449604*x^11 + 85600431378/5
*x^10 - 1654114616*x^9 + 140066157*x^8 - 10217592*x^7 + 627396*x^6 - 156849
/5*x^5 + 4851/4*x^4 - 33*x^3 + 1/2*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(15) = 30$.
time = 0.56, size = 501, normalized size = 21.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^99,x, algorithm="fricas")
```

```
[Out] -1/101*x^101 + 99/100*x^100 - 49*x^99 + 3201/2*x^98 - 38808*x^97 + 2980131/
4*x^96 - 58975224/5*x^95 + 158372676*x^94 - 1840869492*x^93 + 18815553757*x
^92 - 171200862756*x^91 + 7002807007378/5*x^90 - 10386185673864*x^89 + 7029
7408804833*x^88 - 436790467844808*x^87 + 2503926751715004*x^86 - 6650134872
9372018/5*x^85 + 131419332012806607/2*x^84 - 302950588656028082*x^83 + 1307
276513180023617*x^82 - 5293662917568490696*x^81 + 201631839342385547403/10*
x^80 - 72392559119475593544*x^79 + 245464847895078057708*x^78 - 78740022636
4730912388*x^77 + 2393282266977011062653*x^76 - 172561788070239874568724/25
*x^75 + 18914430223915181446722*x^74 - 49303368020068535591064*x^73 + 12238
4749256712270099849*x^72 - 289586448945460019391192*x^71 + 3268857173695411
601376612/5*x^70 - 1409398564020847755666003*x^69 + 11614348594417788189739
629/4*x^68 - 5720500053966970302409071*x^67 + 21569504532490178066659311/2*
x^66 - 97339302505596701018770224/5*x^65 + 134663663432573814416170293/4*x^
64 - 55800482090690569716307824*x^63 + 88685381585824590330757224*x^62 - 13
5208860450519457389515112*x^61 + 989058310490690095822896114/5*x^60 - 27779
8460089394796889723848*x^59 + 374593512943317843600698196*x^58 - 4851195095
85285628395162576*x^57 + 603511770853123192467791538*x^56 - 360675809300364
5324155339152/5*x^55 + 828502745555998906218503832*x^54 - 91447944556652709
4599669324*x^53 + 970109082168886474373197089*x^52 - 9891308288780803268118
87228*x^51 + 24233705307512968006891237086/25*x^50 - 9130438420413049170571
26672*x^49 + 826548729646668720118931889*x^48 - 719077854633508484642475024
*x^47 + 601126348833940887359223192*x^46 - 2414047083412492769871166152/5*x
^45 + 372502480574415750374856978*x^44 - 276016272695076305811040776*x^43 +
 196374773511468735732390996*x^42 - 134109601422466453670901168*x^41 + 4394
28796464188236515924114/5*x^40 - 55246631151825154632274992*x^39 + 33300287
699283081927474024*x^38 - 19237666204653402059452899*x^37 + 425859448461985
56695711973/4*x^36 - 28206275157871771317939099/5*x^35 + 572050005396697030
2409071/2*x^34 - 1386787891870780679371896*x^33 + 2570079734390957672096829
/4*x^32 - 2842484449886557530554488*x^31 + 599857644244167183024612/5*x^30 -
 48264408157576669898532*x^29 + 18488763007525700846649*x^28 - 673664638112
0475583764*x^27 + 2331916055003241548226*x^26 - 19146258135816088501224/25*
x^25 + 238292173768273828749*x^24 - 70132813684308016488*x^23 + 19490304378
320352108*x^22 - 5104603527655330314*x^21 + 12572449429225165403/10*x^20 -
290505891817783026*x^19 + 62806829355518017*x^18 - 12666923567499432*x^17 +
 4750096337812287/2*x^16 - 2062057324941768/5*x^15 + 66026466069564*x^14 -
9696194317908*x^13 + 1298273209233*x^12 - 157366449604*x^11 + 85600431378/5
```

```
*x^10 - 1654114616*x^9 + 140066157*x^8 - 10217592*x^7 + 627396*x^6 - 156849
/5*x^5 + 4851/4*x^4 - 33*x^3 + 1/2*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(12) = 24$.
time = 0.10, size = 561, normalized size = 24.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)**99,x)
```

```
[Out] -x**101/101 + 99*x**100/100 - 49*x**99 + 3201*x**98/2 - 38808*x**97 + 29801
31*x**96/4 - 58975224*x**95/5 + 158372676*x**94 - 1840869492*x**93 + 188155
53757*x**92 - 171200862756*x**91 + 7002807007378*x**90/5 - 10386185673864*x
**89 + 70297408804833*x**88 - 436790467844808*x**87 + 2503926751715004*x**8
6 - 66501348729372018*x**85/5 + 131419332012806607*x**84/2 - 30295058865602
8082*x**83 + 1307276513180023617*x**82 - 5293662917568490696*x**81 + 201631
839342385547403*x**80/10 - 72392559119475593544*x**79 + 245464847895078057
08*x**78 - 787400226364730912388*x**77 + 2393282266977011062653*x**76 - 172
561788070239874568724*x**75/25 + 18914430223915181446722*x**74 - 4930336802
0068535591064*x**73 + 122384749256712270099849*x**72 - 28958644894546001939
1192*x**71 + 3268857173695411601376612*x**70/5 - 1409398564020847755666003*
x**69 + 11614348594417788189739629*x**68/4 - 5720500053966970302409071*x**6
7 + 21569504532490178066659311*x**66/2 - 97339302505596701018770224*x**65/5
 + 134663663432573814416170293*x**64/4 - 55800482090690569716307824*x**63 +
 88685381585824590330757224*x**62 - 135208860450519457389515112*x**61 + 989
058310490690095822896114*x**60/5 - 277798460089394796889723848*x**59 + 3745
93512943317843600698196*x**58 - 485119509585285628395162576*x**57 + 6035117
70853123192467791538*x**56 - 3606758093003645324155339152*x**55/5 + 8285027
45555998906218503832*x**54 - 914479445566527094599669324*x**53 + 9701090821
68886474373197089*x**52 - 989130828878080326811887228*x**51 + 2423370530751
2968006891237086*x**50/25 - 913043842041304917057126672*x**49 + 82654872964
6668720118931889*x**48 - 719077854633508484642475024*x**47 + 60112634883394
0887359223192*x**46 - 2414047083412492769871166152*x**45/5 + 37250248057441
5750374856978*x**44 - 276016272695076305811040776*x**43 + 19637477351146873
5732390996*x**42 - 134109601422466453670901168*x**41 + 43942879646418823651
5924114*x**40/5 - 55246631151825154632274992*x**39 + 3330028769928308192747
4024*x**38 - 19237666204653402059452899*x**37 + 42585944846198556695711973*
x**36/4 - 28206275157871771317939099*x**35/5 + 5720500053966970302409071*x*
*34/2 - 1386787891870780679371896*x**33 + 2570079734390957672096829*x**32/4
 - 284248449886557530554488*x**31 + 599857644244167183024612*x**30/5 - 4826
4408157576669898532*x**29 + 18488763007525700846649*x**28 - 673664638112047
5583764*x**27 + 2331916055003241548226*x**26 - 19146258135816088501224*x**2
5/25 + 238292173768273828749*x**24 - 70132813684308016488*x**23 + 194903043
```

```
78320352108*x**22 - 5104603527655330314*x**21 + 12572449429225165403*x**20/
10 - 290505891817783026*x**19 + 62806829355518017*x**18 - 12666923567499432
*x**17 + 4750096337812287*x**16/2 - 2062057324941768*x**15/5 + 660264660695
64*x**14 - 9696194317908*x**13 + 1298273209233*x**12 - 157366449604*x**11 +
 85600431378*x**10/5 - 1654114616*x**9 + 140066157*x**8 - 10217592*x**7 + 6
27396*x**6 - 156849*x**5/5 + 4851*x**4/4 - 33*x**3 + x**2/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(15) = 30.
time = 0.44, size = 501, normalized size = 21.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^99,x, algorithm="giac")
```

```
[Out] -1/101*x^101 + 99/100*x^100 - 49*x^99 + 3201/2*x^98 - 38808*x^97 + 2980131/
4*x^96 - 58975224/5*x^95 + 158372676*x^94 - 1840869492*x^93 + 18815553757*x
^92 - 171200862756*x^91 + 7002807007378/5*x^90 - 10386185673864*x^89 + 7029
7408804833*x^88 - 436790467844808*x^87 + 2503926751715004*x^86 - 6650134872
9372018/5*x^85 + 131419332012806607/2*x^84 - 302950588656028082*x^83 + 1307
276513180023617*x^82 - 5293662917568490696*x^81 + 201631839342385547403/10*
x^80 - 72392559119475593544*x^79 + 245464847895078057708*x^78 - 78740022636
4730912388*x^77 + 2393282266977011062653*x^76 - 172561788070239874568724/25
*x^75 + 18914430223915181446722*x^74 - 49303368020068535591064*x^73 + 12238
4749256712270099849*x^72 - 289586448945460019391192*x^71 + 3268857173695411
601376612/5*x^70 - 1409398564020847755666003*x^69 + 11614348594417788189739
629/4*x^68 - 5720500053966970302409071*x^67 + 21569504532490178066659311/2*
x^66 - 97339302505596701018770224/5*x^65 + 134663663432573814416170293/4*x^
64 - 55800482090690569716307824*x^63 + 88685381585824590330757224*x^62 - 13
5208860450519457389515112*x^61 + 989058310490690095822896114/5*x^60 - 27779
8460089394796889723848*x^59 + 374593512943317843600698196*x^58 - 4851195095
85285628395162576*x^57 + 603511770853123192467791538*x^56 - 360675809300364
5324155339152/5*x^55 + 828502745555998906218503832*x^54 - 91447944556652709
4599669324*x^53 + 970109082168886474373197089*x^52 - 9891308288780803268118
87228*x^51 + 24233705307512968006891237086/25*x^50 - 9130438420413049170571
26672*x^49 + 826548729646668720118931889*x^48 - 719077854633508484642475024
*x^47 + 601126348833940887359223192*x^46 - 2414047083412492769871166152/5*x
^45 + 372502480574415750374856978*x^44 - 276016272695076305811040776*x^43 +
 196374773511468735732390996*x^42 - 134109601422466453670901168*x^41 + 4394
28796464188236515924114/5*x^40 - 55246631151825154632274992*x^39 + 33300287
699283081927474024*x^38 - 19237666204653402059452899*x^37 + 425859448461985
56695711973/4*x^36 - 28206275157871771317939099/5*x^35 + 572050005396697030
2409071/2*x^34 - 1386787891870780679371896*x^33 + 2570079734390957672096829
/4*x^32 - 284248449886557530554488*x^31 + 599857644244167183024612/5*x^30 -
```

```
 48264408157576669898532*x^29 + 18488763007525700846649*x^28 - 673664638112
0475583764*x^27 + 2331916055003241548226*x^26 - 19146258135816088501224/25*
x^25 + 238292173768273828749*x^24 - 70132813684308016488*x^23 + 19490304378
320352108*x^22 - 5104603527655330314*x^21 + 12572449429225165403/10*x^20 -
290505891817783026*x^19 + 62806829355518017*x^18 - 12666923567499432*x^17 +
 4750096337812287/2*x^16 - 2062057324941768/5*x^15 + 66026466069564*x^14 -
9696194317908*x^13 + 1298273209233*x^12 - 157366449604*x^11 + 85600431378/5
*x^10 - 1654114616*x^9 + 140066157*x^8 - 10217592*x^7 + 627396*x^6 - 156849
/5*x^5 + 4851/4*x^4 - 33*x^3 + 1/2*x^2
```

Mupad [B]
time = 0.12, size = 12, normalized size = 0.52

$$-\frac{(100\,x+1)\,\left(x-1\right)^{100}}{10100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*(x - 1)^99,x)
[Out] -((100*x + 1)*(x - 1)^100)/10100
```

Chatgpt [F] Failed to verify
time = 1.00, size = 20, normalized size = 0.87

$$\frac{(1-x)^{101}}{10100}-\frac{x(1-x)^{100}}{100}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*(1-x)^99,x)
[Out] 1/10100*(1-x)^101-1/100*x*(1-x)^100
```

3.65 $\int \csc(x)\sin(4x)\,dx$

Optimal. Leaf size=13

$$4\sin(x)-\frac{8\sin^3(x)}{3}$$

```
[Out] 4*sin(x)-8/3*sin(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$4\sin(x)-\frac{8\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sin[4*x],x]
[Out] 4*Sin[x] - (8*Sin[x]^3)/3
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \left(4-8x^2\right)\,dx,x,\sin(x)\right)\\ &= 4\sin(x)-\frac{8\sin^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$4\sin(x)-\frac{8\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sin[4*x],x]
[Out] 4*Sin[x] - (8*Sin[x]^3)/3
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.92

method	result	size

derivativedivides	$4\sin(x) - \frac{8(\sin^3(x))}{3}$	12
default	$4\sin(x) - \frac{8(\sin^3(x))}{3}$	12
risch	$2\sin(x) + \frac{2\sin(3x)}{3}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)/sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*sin(x)-8/3*sin(x)^3
```

Maxima [A]
time = 0.41, size = 11, normalized size = 0.85

$$-\frac{8}{3}\sin(x)^3 + 4\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x),x, algorithm="maxima")
```

```
[Out] -8/3*sin(x)^3 + 4*sin(x)
```

Fricas [A]
time = 0.59, size = 12, normalized size = 0.92

$$\frac{4}{3}\left(2\cos(x)^2 + 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x),x, algorithm="fricas")
```

```
[Out] 4/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A]
time = 0.34, size = 12, normalized size = 0.92

$$-\frac{8\sin^3(x)}{3} + 4\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x),x)
```

```
[Out] -8*sin(x)**3/3 + 4*sin(x)
```

Giac [A]
time = 0.47, size = 11, normalized size = 0.85

$$\frac{2}{3}\sin(3x) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x),x, algorithm="giac")
```

```
[Out] 2/3*sin(3*x) + 2*sin(x)
```

Mupad [B]
time = 0.15, size = 11, normalized size = 0.85

$$4\,\sin\left(x\right)-\frac{8\sin\left(x\right)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)/sin(x),x)
```

```
[Out] 4*sin(x) - (8*sin(x)^3)/3
```

Chatgpt [F] Failed to verify
time = 1.00, size = 14, normalized size = 1.08

$$-2\ln\left(\sin\left(x\right)\right)-\frac{\cos\left(x\right)}{\sin\left(x\right)}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(4*x)/sin(x),x)
```

```
[Out] -2*ln(sin(x))-cos(x)/sin(x)
```

3.66 $\int \frac{1}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=16

$$6\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)-6*arctan(x^(1/6))
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {348, 52, 65, 209}

$$6\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 6*ArcTan[x^(1/6)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
```

```
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 3\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[6]{x} - 3\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x)}\,dx, x, \sqrt[3]{x}\right)\\ &= 6\sqrt[6]{x} - 6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x} - 6\arctan\left(\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$6\sqrt[6]{x} - 6\arctan\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^(1/3))*Sqrt[x]),x]
[Out] 6*x^(1/6) - 6*ArcTan[x^(1/6)]
```

Maple [A]
time = 0.05, size = 13, normalized size = 0.81

method	result	size
derivativedivides	$6x^{\frac{1}{6}} - 6\arctan\left(x^{\frac{1}{6}}\right)$	13
default	$6x^{\frac{1}{6}} - 6\arctan\left(x^{\frac{1}{6}}\right)$	13
meijerg	$6x^{\frac{1}{6}} - 6\arctan\left(x^{\frac{1}{6}}\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(1+x^(1/3)),x,method=_RETURNVERBOSE)
[Out] 6*x^(1/6)-6*arctan(x^(1/6))
```

Maxima [A]
time = 0.45, size = 12, normalized size = 0.75

$$6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")
[Out] 6*x^(1/6) - 6*arctan(x^(1/6))
```

Fricas [A]
time = 0.58, size = 12, normalized size = 0.75

$$6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/3)),x, algorithm="fricas")
[Out] 6*x^(1/6) - 6*arctan(x^(1/6))
```

Sympy [A]
time = 0.50, size = 15, normalized size = 0.94

$$6\sqrt[6]{x} + 6\,\mathrm{atan}\left(\frac{1}{\sqrt[6]{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x**(1/3)),x)
[Out] 6*x**(1/6) + 6*atan(x**(-1/6))
```

Giac [A]
time = 0.43, size = 12, normalized size = 0.75

$$6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x^(1/3)),x, algorithm="giac")
[Out] 6*x^(1/6) - 6*arctan(x^(1/6))
```

Mupad [B]
time = 0.04, size = 12, normalized size = 0.75

$$6\,x^{1/6} - 6\,\mathrm{atan}\left(x^{1/6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/3) + 1)),x)
[Out] 6*x^(1/6) - 6*atan(x^(1/6))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 6, normalized size = 0.38

$$3\arctan\left(x^{\frac{1}{6}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/x^(1/2)/(1+x^(1/3)),x)
```

```
[Out] 3*arctan(x^(1/6))
```

3.67 $\int \frac{1}{\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+2x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(2^(1/2)*x/(2*x^2-1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {223, 212}

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{2x^2-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + 2*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + 2*x^2]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{-1+2x^2}}\right)\\ &= \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{-1+2x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.
time = 0.02, size = 54, normalized size = 2.16

$$\frac{-\log\left(\sqrt{2}-\frac{2x}{\sqrt{-1+2x^2}}\right)+\log\left(\sqrt{2}+\frac{2x}{\sqrt{-1+2x^2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + 2*x^2],x]
```

```
[Out] (-Log[Sqrt[2] - (2*x)/Sqrt[-1 + 2*x^2]] + Log[Sqrt[2] + (2*x)/Sqrt[-1 + 2*x
^2]])/(2*Sqrt[2])
```

Maple [A]
time = 0.09, size = 22, normalized size = 0.88

method	result	size
default	$\frac{\ln\left(\sqrt{2}\,x+\sqrt{2x^2-1}\right)\sqrt{2}}{2}$	22
pseudoelliptic	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2x^2-1}\,\sqrt{2}}{2x}\right)}{2}$	24
trager	$-\frac{\mathit{RootOf}\left(\_Z^2-2\right)\ln\left(-\mathit{RootOf}\left(\_Z^2-2\right)\sqrt{2x^2-1}+2x\right)}{2}$	31
meijerg	$\frac{\sqrt{-\operatorname{signum}(2x^2-1)}\,\sqrt{2}\,\arcsin\left(\sqrt{2}\,x\right)}{2\sqrt{\operatorname{signum}(2x^2-1)}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(2^(1/2)*x+(2*x^2-1)^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.44, size = 24, normalized size = 0.96

$$\frac{1}{2}\,\sqrt{2}\log\left(2\,\sqrt{2}\sqrt{2\,x^2-1}+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(2*sqrt(2)*sqrt(2*x^2 - 1) + 4*x)
```

Fricas [A]
time = 0.57, size = 28, normalized size = 1.12

$$\frac{1}{4}\,\sqrt{2}\log\left(2\,\sqrt{2}\sqrt{2\,x^2-1}x+4\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(2*x^2 - 1)*x + 4*x^2 - 1)
```

Sympy [A]
time = 0.06, size = 26, normalized size = 1.04

$$\frac{\sqrt{2}\log\left(2x+\sqrt{2}\sqrt{2x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-1)**(1/2),x)
[Out] sqrt(2)*log(2*x + sqrt(2)*sqrt(2*x**2 - 1))/2
```

Giac [A]
time = 0.49, size = 36, normalized size = 1.44

$$\frac{1}{2}\sqrt{2x^2-1}x+\frac{1}{4}\sqrt{2}\log\left(\left|-\sqrt{2}x+\sqrt{2x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(2*x^2 - 1)*x + 1/4*sqrt(2)*log(abs(-sqrt(2)*x + sqrt(2*x^2 - 1)))
```

Mupad [B]
time = 0.13, size = 21, normalized size = 0.84

$$\frac{\sqrt{2}\,\ln\left(\sqrt{2}\,x+\sqrt{2\,x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2 - 1)^(1/2),x)
[Out] (2^(1/2)*log(2^(1/2)*x + (2*x^2 - 1)^(1/2)))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 14, normalized size = 0.56

$$-\frac{\sqrt{2}}{2\sqrt{-2x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(2*x^2-1)^(1/2),x)
[Out] -1/2*2^(1/2)/(-2*x^2+1)^(1/2)
```

3.68 $\int \frac{1}{\sqrt{-1+e^x}}\,dx$

Optimal. Leaf size=12

$$2\arctan\left(\sqrt{-1+e^x}\right)$$

```
[Out] 2*arctan((exp(x)-1)^(1/2))
```

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2320, 65, 209}

$$2\arctan\left(\sqrt{e^x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + E^x],x]
```

```
[Out] 2*ArcTan[Sqrt[-1 + E^x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx, x, e^x\right)\\ &= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+e^x}\right)\\ &= 2\arctan\left(\sqrt{-1+e^x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$2\arctan\left(\sqrt{-1+e^x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + E^x],x]
```

```
[Out] 2*ArcTan[Sqrt[-1 + E^x]]
```

Maple [A]
time = 0.04, size = 10, normalized size = 0.83

method	result	size
derivativedivides	$2\arctan\left(\sqrt{e^x-1}\right)$	10
default	$2\arctan\left(\sqrt{e^x-1}\right)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x)-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan((exp(x)-1)^(1/2))
```

Maxima [A]
time = 0.44, size = 9, normalized size = 0.75

$$2\arctan\left(\sqrt{e^x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(exp(x)-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(e^x - 1))
```

Fricas [A]
time = 0.56, size = 9, normalized size = 0.75

$$2\arctan\left(\sqrt{e^x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(exp(x)-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(e^x - 1))
```

Sympy [A]
time = 0.29, size = 10, normalized size = 0.83

$$2\,\mathrm{atan}\left(\sqrt{e^{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(exp(x)-1)**(1/2),x)
```

```
[Out] 2*atan(sqrt(exp(x) - 1))
```

Giac [A]
time = 0.51, size = 9, normalized size = 0.75

$$2\arctan\left(\sqrt{e^{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(exp(x)-1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(e^x - 1))
```

Mupad [B]
time = 0.11, size = 9, normalized size = 0.75

$$2\,\mathrm{atan}\left(\sqrt{\mathrm{e}^{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x) - 1)^(1/2),x)
```

```
[Out] 2*atan((exp(x) - 1)^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 23, normalized size = 1.92

$$2\sqrt{\mathrm{e}^{x}-1}+2\ln\left(\sqrt{\mathrm{e}^{x}-1}+\sqrt{\mathrm{e}^{x}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(exp(x)-1)^(1/2),x)
```

```
[Out] 2*(exp(x)-1)^(1/2)+2*ln((exp(x)-1)^(1/2)+exp(x)^(1/2))
```

3.69 $\int \frac{x}{4+x^4} \, dx$

Optimal. Leaf size=12

$$\frac{1}{4} \arctan \left(\frac{x^2}{2}\right)$$

```
[Out] 1/4*arctan(1/2*x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {281, 209}

$$\frac{1}{4} \arctan \left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4 + x^4),x]
```

```
[Out] ArcTan[x^2/2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+x^2} \, dx, x, x^2\right) \\ &= \frac{1}{4} \arctan \left(\frac{x^2}{2}\right) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{4} \arctan \left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4 + x^4),x]
```

```
[Out] ArcTan[x^2/2]/4
```

Maple [A]
time = 0.07, size = 9, normalized size = 0.75

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{2}\right)}{4}$	9
meijerg	$\frac{\arctan\left(\frac{x^2}{2}\right)}{4}$	9
risch	$\frac{\arctan\left(\frac{x^2}{2}\right)}{4}$	9
parallelrisch	$-\frac{i\ln(x-1-i)}{8}+\frac{i\ln(x-1+i)}{8}+\frac{i\ln(x+1-i)}{8}-\frac{i\ln(x+1+i)}{8}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctan(1/2*x^2)
```

Maxima [A]
time = 0.43, size = 8, normalized size = 0.67

$$\frac{1}{4}\arctan\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+4),x, algorithm="maxima")
```

```
[Out] 1/4*arctan(1/2*x^2)
```

Fricas [A]
time = 0.57, size = 8, normalized size = 0.67

$$\frac{1}{4}\arctan\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+4),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(1/2*x^2)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.58

$$\frac{\operatorname{atan}\left(\frac{x^2}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+4),x)
```

```
[Out] atan(x**2/2)/4
```

Giac [A]
time = 0.41, size = 8, normalized size = 0.67

$$\frac{1}{4}\arctan\left(\frac{1}{2}x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+4),x, algorithm="giac")
```

```
[Out] 1/4*arctan(1/2*x^2)
```

Mupad [B]
time = 0.09, size = 8, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{x^2}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 4),x)
```

```
[Out] atan(x^2/2)/4
```

Chatgpt [A]
time = 1.00, size = 8, normalized size = 0.67

$$\frac{\arctan\left(\frac{x^2}{2}\right)}{4}$$

Antiderivative was successfully verified.

```
[In] int(x/(x^4+4),x)
```

```
[Out] 1/4*arctan(1/2*x^2)
```

3.70 $\int \frac{2}{(\cos(x)-\sin(x))^2}\,dx$

Optimal. Leaf size=8

$$\frac{2}{-1+\cot(x)}$$

```
[Out] 2/(-1+cot(x))
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.62, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 3154}

$$\frac{2\sin(x)}{\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[2/(Cos[x] - Sin[x])^2,x]
```

```
[Out] (2*Sin[x])/(Cos[x] - Sin[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 2\int \frac{1}{(\cos(x)-\sin(x))^2}\,dx\\ &= \frac{2\sin(x)}{\cos(x)-\sin(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.62

$$\frac{2\sin(x)}{\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(Cos[x] - Sin[x])^2,x]
```

```
[Out] (2*Sin[x])/(Cos[x] - Sin[x])
```

Maple [A]
time = 0.10, size = 9, normalized size = 1.12

method	result	size
default	$-\frac{2}{\tan(x)-1}$	9
risch	$\frac{2}{e^{2ix}-i}$	13
parallelrisch	$\frac{2\sin(x)}{\cos(x)-\sin(x)}$	14
norman	$-\frac{4\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(cos(x)-sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(tan(x)-1)
```

Maxima [A]
time = 0.40, size = 8, normalized size = 1.00

$$-\frac{2}{\tan(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(cos(x)-sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2/(tan(x) - 1)
```

Fricas [A]
time = 0.57, size = 15, normalized size = 1.88

$$\frac{\cos(x)+\sin(x)}{\cos(x)-\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(cos(x)-sin(x))^2,x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x))/(cos(x) - sin(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(5)=10$.
time = 0.48, size = 22, normalized size = 2.75

$$-\frac{4\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(cos(x)-sin(x))**2,x)
```

```
[Out] -4*tan(x/2)/(tan(x/2)**2 + 2*tan(x/2) - 1)
```

Giac [A]
time = 0.52, size = 8, normalized size = 1.00

$$-\frac{2}{\tan(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(cos(x)-sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/(tan(x) - 1)
```

Mupad [B]
time = 0.20, size = 13, normalized size = 1.62

$$\frac{2 \sin(x)}{\cos(x) - \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(cos(x) - sin(x))^2,x)
```

```
[Out] (2*sin(x))/(cos(x) - sin(x))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 7, normalized size = 0.88

$$-2 \arctan(\cos(x) - 1)$$

Warning: Unable to verify antiderivative.

```
[In] int(2/(cos(x)-sin(x))^2,x)
```

```
[Out] -2*arctan(cos(x)-1)
```

3.71 $\int x \coth(x)\mathbf{csch}(x)\,dx$

Optimal. Leaf size=11

$$-\text{arctanh}(\cosh(x)) - x\text{csch}(x)$$

```
[Out] -arctanh(cosh(x))-x*csch(x)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5527, 3855}

$$-\text{arctanh}(\cosh(x)) - x\text{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[x]*Csch[x],x]
```

```
[Out] -ArcTanh[Cosh[x]] - x*Csch[x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 5527

```
Int[Coth[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csch[(a_.) + (b_.)*(x_)^(n_.)]^(p_
.)*(x_)^(m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csch[a + b*x^n]^p/(b*n*p
)), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csch[a + b*x^n]^p, x], x]
/; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && E
qQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -x\text{csch}(x) + \int \text{csch}(x)\,dx\\ &= -\text{arctanh}(\cosh(x)) - x\text{csch}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(11) = 22.
time = 0.01, size = 39, normalized size = 3.55

$$-\frac{1}{2}x\coth\left(\frac{x}{2}\right) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{1}{2}x\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[x]*Csch[x],x]
```

```
[Out] -1/2*(x*Coth[x/2]) - Log[Cosh[x/2]] + Log[Sinh[x/2]] + (x*Tanh[x/2])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.
time = 0.16, size = 27, normalized size = 2.45

method	result	size
risch	$-\frac{2x\,\mathrm{e}^{x}}{\mathrm{e}^{2x}-1}+\ln\left(\mathrm{e}^{x}-1\right)-\ln\left(1+\mathrm{e}^{x}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)/sinh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x*exp(x)/(exp(2*x)-1)+ln(exp(x)-1)-ln(1+exp(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.
time = 0.44, size = 26, normalized size = 2.36

$$-\frac{2\,xe^{x}}{e^{(2\,x)}-1}-\log\left(e^{x}+1\right)+\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)/sinh(x)^2,x, algorithm="maxima")
```

```
[Out] -2*x*e^x/(e^(2*x) - 1) - log(e^x + 1) + log(e^x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(11) = 22$.
time = 0.59, size = 80, normalized size = 7.27

$$-\frac{2\,x\cosh\left(x\right)+\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2\,x\sinh\left(x\right)}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)/sinh(x)^2,x, algorithm="fricas")
```

```
[Out] -(2*x*cosh(x) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x)
 + sinh(x) + 1) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(
x) + sinh(x) - 1) + 2*x*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.27, size = 22, normalized size = 2.00

$$\frac{x\tanh\left(\frac{x}{2}\right)}{2}-\frac{x}{2\tanh\left(\frac{x}{2}\right)}+\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)/sinh(x)**2,x)
```

```
[Out] x*tanh(x/2)/2 - x/(2*tanh(x/2)) + log(tanh(x/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(11) = 22$.
time = 0.46, size = 49, normalized size = 4.45

$$-\frac{2\,xe^{x} + e^{(2\,x)}\log\left(e^{x}+1\right) - e^{(2\,x)}\log\left(e^{x}-1\right) - \log\left(e^{x}+1\right) + \log\left(e^{x}-1\right)}{e^{(2\,x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x)/sinh(x)^2,x, algorithm="giac")
```

```
[Out] -(2*x*e^x + e^(2*x)*log(e^x + 1) - e^(2*x)*log(e^x - 1) - log(e^x + 1) + lo
g(e^x - 1))/(e^(2*x) - 1)
```

Mupad [B]
time = 0.11, size = 30, normalized size = 2.73

$$\ln\left(2-2\,\mathrm{e}^{x}\right)-\ln\left(-2\,\mathrm{e}^{x}-2\right)-\frac{2\,x\,\mathrm{e}^{x}}{\mathrm{e}^{2\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cosh(x))/sinh(x)^2,x)
```

```
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) - (2*x*exp(x))/(exp(2*x) - 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 20, normalized size = 1.82

$$\sqrt{x^2+1}+2\ln\left(\sqrt{x^2+1}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(x*cosh(x)/sinh(x)^2,x)
```

```
[Out] (x^2+1)^(1/2)+2*ln((x^2+1)^(1/2)+1)
```

3.72 $\int x^5\sqrt{1+x^3}\,dx$

Optimal. Leaf size=27

$$-\frac{2}{9}\left(1+x^3\right)^{3/2}+\frac{2}{15}\left(1+x^3\right)^{5/2}$$

```
[Out] -2/9*(x^3+1)^(3/2)+2/15*(x^3+1)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{2}{15}\left(x^3+1\right)^{5/2}-\frac{2}{9}\left(x^3+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 + x^3],x]
```

```
[Out] (-2*(1 + x^3)^(3/2))/9 + (2*(1 + x^3)^(5/2))/15
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{3}\text{Subst}\left(\int x\sqrt{1+x}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\sqrt{1+x}+(1+x)^{3/2}\right)\,dx,x,x^3\right)\\ &= -\frac{2}{9}\left(1+x^3\right)^{3/2}+\frac{2}{15}\left(1+x^3\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.74

$$\frac{2}{45}\left(1+x^3\right)^{3/2}\left(-2+3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 + x^3],x]
```

```
[Out] (2*(1 + x^3)^(3/2)*(-2 + 3*x^3))/45
```

Maple [A]
time = 0.12, size = 35, normalized size = 1.30

method	result	size
pseudoelliptic	$\frac{2\left(x^{3}+1\right)^{\frac{3}{2}}\left(3 x^{3}-2\right)}{45}$	17
risch	$\frac{2\left(3 x^{6}+x^{3}-2\right) \sqrt{x^{3}+1}}{45}$	20
trager	$\left(\frac{2}{15} x^{6}+\frac{2}{45} x^{3}-\frac{4}{45}\right) \sqrt{x^{3}+1}$	21
gosper	$\frac{2(x+1)\left(x^{2}-x+1\right)\left(3 x^{3}-2\right) \sqrt{x^{3}+1}}{45}$	28
meijerg	$-\frac{-\frac{8 \sqrt{\pi}}{15}+\frac{4 \sqrt{\pi}\left(x^{3}+1\right)^{\frac{3}{2}}\left(-3 x^{3}+2\right)}{15}}{6 \sqrt{\pi}}$	31
default	$\frac{2 x^{6} \sqrt{x^{3}+1}}{15}+\frac{2 x^{3} \sqrt{x^{3}+1}}{45}-\frac{4 \sqrt{x^{3}+1}}{45}$	35
elliptic	$\frac{2 x^{6} \sqrt{x^{3}+1}}{15}+\frac{2 x^{3} \sqrt{x^{3}+1}}{45}-\frac{4 \sqrt{x^{3}+1}}{45}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*x^6*(x^3+1)^(1/2)+2/45*x^3*(x^3+1)^(1/2)-4/45*(x^3+1)^(1/2)
```

Maxima [A]
time = 0.39, size = 19, normalized size = 0.70

$$\frac{2}{15}\left(x^{3}+1\right)^{\frac{5}{2}}-\frac{2}{9}\left(x^{3}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(x^3 + 1)^(5/2) - 2/9*(x^3 + 1)^(3/2)
```

Fricas [A]
time = 0.57, size = 19, normalized size = 0.70

$$\frac{2}{45}\left(3 x^{6}+x^{3}-2\right) \sqrt{x^{3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*x^6 + x^3 - 2)*sqrt(x^3 + 1)
```

Sympy [A]
time = 0.10, size = 41, normalized size = 1.52

$$\frac{2x^6\sqrt{x^3+1}}{15}+\frac{2x^3\sqrt{x^3+1}}{45}-\frac{4\sqrt{x^3+1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**3+1)**(1/2),x)
[Out] 2*x**6*sqrt(x**3 + 1)/15 + 2*x**3*sqrt(x**3 + 1)/45 - 4*sqrt(x**3 + 1)/45
```

Giac [A]
time = 0.41, size = 19, normalized size = 0.70

$$\frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}}-\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/15*(x^3 + 1)^(5/2) - 2/9*(x^3 + 1)^(3/2)
```

Mupad [B]
time = 0.09, size = 19, normalized size = 0.70

$$\frac{2\left(x^3+1\right)^{5/2}}{15}-\frac{2\left(x^3+1\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^3 + 1)^(1/2),x)
[Out] (2*(x^3 + 1)^(5/2))/15 - (2*(x^3 + 1)^(3/2))/9
```

Chatgpt [F] Failed to verify
time = 1.00, size = 5, normalized size = 0.19

$$\frac{2x^{\frac{21}{2}}}{21}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^5*(x^3+1)^(1/2),x)
[Out] 2/21*x^(21/2)
```

3.73 $\int \frac{-1+x^7}{\log(x)}\,dx$

Optimal. Leaf size=10

$$\text{ExpIntegralEi}(8\log(x)) - \text{LogIntegral}(x)$$

```
[Out] Ei(8*ln(x))-Li(x)
```

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2367, 2335, 2346, 2209}

$$\text{ExpIntegralEi}(8\log(x)) - \text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^7)/Log[x],x]
```

```
[Out] ExpIntegralEi[8*Log[x]] - LogIntegral[x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(-\frac{1}{\log(x)} + \frac{x^7}{\log(x)}\right) dx \\ &= -\int \frac{1}{\log(x)} dx + \int \frac{x^7}{\log(x)} dx \\ &= -\text{LogIntegral}(x) + \text{Subst}\left(\int \frac{e^{8x}}{x} dx, x, \log(x)\right) \\ &= \text{ExpIntegralEi}(8\log(x)) - \text{LogIntegral}(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 10, normalized size = 1.00

$$\text{ExpIntegralEi}(8\log(x)) - \text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^7)/Log[x],x]
[Out] ExpIntegralEi[8*Log[x]] - LogIntegral[x]
```

Maple [A]
time = 0.02, size = 16, normalized size = 1.60

method	result	size
default	$-\text{expIntegral}_1(-8\ln(x)) + \text{expIntegral}_1(-\ln(x))$	16
risch	$-\text{expIntegral}_1(-8\ln(x)) + \text{expIntegral}_1(-\ln(x))$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7-1)/ln(x),x,method=_RETURNVERBOSE)
[Out] -Ei(1,-8*ln(x))+Ei(1,-ln(x))
```

Maxima [A]
time = 0.44, size = 11, normalized size = 1.10

$$\text{Ei}(8\log(x)) - \text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7-1)/log(x),x, algorithm="maxima")
[Out] Ei(8*log(x)) - Ei(log(x))
```

Fricas [A]
time = 0.58, size = 9, normalized size = 0.90

$$\text{log\_integral}\left(x^8\right) - \text{log\_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7-1)/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x^8) - log_integral(x)
```

Sympy [A]
time = 0.70, size = 10, normalized size = 1.00

$$-\operatorname{Ei}(\log(x)) + \operatorname{Ei}(8\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**7-1)/ln(x),x)
```

```
[Out] -Ei(log(x)) + Ei(8*log(x))
```

Giac [A]
time = 0.59, size = 11, normalized size = 1.10

$$\operatorname{Ei}(8\log(x)) - \operatorname{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7-1)/log(x),x, algorithm="giac")
```

```
[Out] Ei(8*log(x)) - Ei(log(x))
```

Mupad [B]
time = 0.12, size = 10, normalized size = 1.00

$$\operatorname{ei}(8\ln(x)) - \operatorname{logint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7 - 1)/log(x),x)
```

```
[Out] ei(8*log(x)) - logint(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((x^7-1)/ln(x),x)
```

```
[Out] not solved
```

3.74 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

Optimal. Leaf size=13

$$2\sqrt{\cos(x)\cot(x)}\tan(x)$$

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A]

time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4482, 4485, 2669}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \sqrt{\cos(x)\cot(x)}\,dx \\
&= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}} \\
&= 2\sqrt{\cos(x)\cot(x)}\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$2\sqrt{\cos(x)\cot(x)}\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Maple [A]
time = 0.36, size = 12, normalized size = 0.92

method	result	size
default	$2\sqrt{\cos\left(x\right)\cot\left(x\right)}\,\tan\left(x\right)$	12
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-ix}}{\mathrm{e}^{2ix}-1}}\,\left(\mathrm{e}^{2ix}-1\right)}{\mathrm{e}^{2ix}+1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(11) = 22$.
time = 0.49, size = 188, normalized size = 14.46

$$\frac{\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)-\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\right)-\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)+\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\right)}{\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)^{\frac{1}{4}}\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

Fricas [A]
time = 0.59, size = 19, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}}\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sin(x) + \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(1/2),x)
[Out] Integral(sqrt(-sin(x) + csc(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(csc(x) - sin(x)), x)
```

Mupad [B]
time = 0.00, size = 15, normalized size = 1.15

$$\frac{2\,|\cos(x)|}{\cos(x)\,\sqrt{\frac{1}{\sin(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(1/2),x)
[Out] (2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((csc(x)-sin(x))^(1/2),x)
[Out] not solved
```

3.75 $\int \left(-2 \log (2 x)+\log \left(x^2\right)\right) \, dx$

Optimal. Leaf size=14

$$-2 x \log (2 x)+x \log \left(x^2\right)$$

```
[Out] -2*x*ln(2*x)+x*ln(x^2)
```

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2332}

$$x \log \left(x^2\right)-2 x \log (2 x)$$

Antiderivative was successfully verified.

```
[In] Int[-2*Log[2*x] + Log[x^2],x]
```

```
[Out] -2*x*Log[2*x] + x*Log[x^2]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= -\left(2 \int \log (2 x) \, dx\right)+\int \log \left(x^2\right) \, dx\\ &= -2 x \log (2 x)+x \log \left(x^2\right) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 14, normalized size = 1.00

$$-2 x \log (2 x)+x \log \left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*Log[2*x] + Log[x^2],x]
```

```
[Out] -2*x*Log[2*x] + x*Log[x^2]
```

Maple [A]

time = 0.01, size = 15, normalized size = 1.07

method	result	size

default	$-2x\ln(2x)+x\ln(x^2)$	15
norman	$-2x\ln(2x)+x\ln(x^2)$	15
risch	$-2x\ln(2x)+x\ln(x^2)$	15
parallelrisch	$-2x\ln(2x)+x\ln(x^2)$	15
parts	$-2x\ln(2x)+x\ln(x^2)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2)-2*ln(2*x),x,method=_RETURNVERBOSE)
[Out] -2*x*ln(2*x)+x*ln(x^2)
```

Maxima [A]
time = 0.38, size = 14, normalized size = 1.00

$$x\log(x^2)-2\,x\log(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)-2*log(2*x),x, algorithm="maxima")
[Out] x*log(x^2) - 2*x*log(2*x)
```

Fricas [A]
time = 0.57, size = 5, normalized size = 0.36

$$-2\,x\log(2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)-2*log(2*x),x, algorithm="fricas")
[Out] -2*x*log(2)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.50

$$-2x\log(2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2)-2*ln(2*x),x)
[Out] -2*x*log(2)
```

Giac [A]
time = 0.52, size = 14, normalized size = 1.00

$$x\log(x^2)-2\,x\log(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)-2*log(2*x),x, algorithm="giac")
```

```
[Out] x*log(x^2) - 2*x*log(2*x)
```

Mupad [B]
time = 0.08, size = 16, normalized size = 1.14

$$-x\left(2\,\ln\left(2\,x\right)-\ln\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2) - 2*log(2*x),x)
```

```
[Out] -x*(2*log(2*x) - log(x^2))
```

Chatgpt [A] valid for positive x
time = 1.00, size = 5, normalized size = 0.36

$$-2\ln\left(2\right)x$$

Antiderivative was successfully verified.

```
[In] int(ln(x^2)-2*ln(2*x),x)
```

```
[Out] -2*ln(2)*x
```

3.76 $\int e^x \, dx$

Optimal. Leaf size=3

$$e^x$$

[Out] exp(x)

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$e^x$$

Antiderivative was successfully verified.

[In] Int[E^x,x]

[Out] E^x

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{Integral} = e^x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$e^x$$

Antiderivative was successfully verified.

[In] Integrate[E^x,x]

[Out] E^x

Maple [A]
time = 0.01, size = 3, normalized size = 1.00

method	result	size

gosper	e^x	3
lookup	e^x	3
derivativedivides	e^x	3
default	e^x	3
norman	e^x	3
risch	e^x	3
parallelrisch	e^x	3
meijerg	$e^x - 1$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)
```

Maxima [A]
time = 0.38, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="maxima")
```

```
[Out] e^x
```

Fricas [A]
time = 0.57, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="fricas")
```

```
[Out] e^x
```

Sympy [A]
time = 0.02, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
```

```
[Out] exp(x)
```

Giac [**A**]
time = 1.02, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="giac")
```

```
[Out] e^x
```

Mupad [**B**]
time = 0.01, size = 2, normalized size = 0.67

$$\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

Chatgpt [**A**]
time = 1.00, size = 2, normalized size = 0.67

$$\mathrm{e}^x$$

Antiderivative was successfully verified.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

3.77 $\int \frac{\cos(x)\log(x)-\frac{\sin(x)}{x}}{\log^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{\sin(x)}{\log(x)}$$

[Out] sin(x)/ln(x)

Rubi [F]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(x)\log(x) - \frac{\sin(x)}{x}}{\log^2(x)}\,dx$$

Verification is not applicable to the result.

[In] Int[(Cos[x]*Log[x] - Sin[x]/x)/Log[x]^2,x]

[Out] Defer[Int][Cos[x]/Log[x], x] - Defer[Int][Sin[x]/(x*Log[x]^2), x]

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(\frac{\cos(x)}{\log(x)} - \frac{\sin(x)}{x\log^2(x)}\right)dx\\ &= \int \frac{\cos(x)}{\log(x)}\,dx - \int \frac{\sin(x)}{x\log^2(x)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.45, size = 7, normalized size = 1.00

$$\frac{\sin(x)}{\log(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]*Log[x] - Sin[x]/x)/Log[x]^2,x]

[Out] Sin[x]/Log[x]

Maple [A]
time = 0.12, size = 8, normalized size = 1.14

method	result	size

risch	$\frac{\sin(x)}{\ln(x)}$	8
parallelrisch	$\frac{\sin(x)}{\ln(x)}$	8
norman	$\frac{2\tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))\ln(x)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*ln(x)-sin(x)/x)/ln(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)/ln(x)
```

Maxima [A]
time = 0.43, size = 7, normalized size = 1.00

$$\frac{\sin(x)}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*log(x)-sin(x)/x)/log(x)^2,x, algorithm="maxima")
```

```
[Out] sin(x)/log(x)
```

Fricas [A]
time = 0.58, size = 7, normalized size = 1.00

$$\frac{\sin(x)}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*log(x)-sin(x)/x)/log(x)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/log(x)
```

Sympy [A]
time = 0.51, size = 5, normalized size = 0.71

$$\frac{\sin(x)}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*ln(x)-sin(x)/x)/ln(x)**2,x)
```

```
[Out] sin(x)/log(x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.
time = 0.89, size = 20, normalized size = 2.86

$$\frac{2\tan\left(\frac{1}{2}x\right)}{\log\left(x\right)\tan\left(\frac{1}{2}x\right)^{2}+\log\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*log(x)-sin(x)/x)/log(x)^2,x, algorithm="giac")
```

```
[Out] 2*tan(1/2*x)/(log(x)*tan(1/2*x)^2 + log(x))
```

Mupad [**B**]
time = 0.28, size = 7, normalized size = 1.00

$$\frac{\sin\left(x\right)}{\ln\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*log(x) - sin(x)/x)/log(x)^2,x)
```

```
[Out] sin(x)/log(x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 19, normalized size = 2.71

$$-\frac{\cos\left(x\right)}{\ln\left(x\right)}+\frac{\sin\left(x\right)}{x\ln\left(x\right)}$$

Warning: Unable to verify antiderivative.

```
[In] int((cos(x)*ln(x)-sin(x)/x)/ln(x)^2,x)
```

```
[Out] -cos(x)/ln(x)+sin(x)/x/ln(x)
```

3.78 $\int (-1+3x-3x^2+x^3)\,dx$

Optimal. Leaf size=11

$$\frac{1}{4}(1-x)^4$$

[Out] 1/4*(1-x)^4

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(11)=22$.
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 2.09, number of steps used = 1, number of rules used = 0, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\frac{x^4}{4}-x^3+\frac{3x^2}{2}-x$$

Antiderivative was successfully verified.

[In] Int[-1 + 3*x - 3*x^2 + x^3,x]

[Out] -x + (3*x^2)/2 - x^3 + x^4/4

Rubi steps

$$\text{Integral}=-x+\frac{3x^2}{2}-x^3+\frac{x^4}{4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(11)=22$.
time = 0.00, size = 23, normalized size = 2.09

$$-x+\frac{3x^2}{2}-x^3+\frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[-1 + 3*x - 3*x^2 + x^3,x]

[Out] -x + (3*x^2)/2 - x^3 + x^4/4

Maple [A]
time = 0.01, size = 8, normalized size = 0.73

method	result	size
default	$\frac{(x-1)^4}{4}$	8
gosper	$\frac{x(x^3-4x^2+6x-4)}{4}$	17

norman	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20
risch	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20
parallelrisch	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20
parts	$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3-3*x^2+3*x-1,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(x-1)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.34, size = 19, normalized size = 1.73

$$\frac{1}{4}\,x^4 - x^3 + \frac{3}{2}\,x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3-3*x^2+3*x-1,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - x^3 + 3/2*x^2 - x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.55, size = 19, normalized size = 1.73

$$\frac{1}{4}\,x^4 - x^3 + \frac{3}{2}\,x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3-3*x^2+3*x-1,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - x^3 + 3/2*x^2 - x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.01, size = 15, normalized size = 1.36

$$\frac{x^4}{4} - x^3 + \frac{3x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3-3*x**2+3*x-1,x)
```

```
[Out] x**4/4 - x**3 + 3*x**2/2 - x
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.48, size = 19, normalized size = 1.73

$$\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3-3*x^2+3*x-1,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - x^3 + 3/2*x^2 - x
```

Mupad [**B**]
time = 0.05, size = 19, normalized size = 1.73

$$\frac{x^4}{4} - x^3 + \frac{3x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x - 3*x^2 + x^3 - 1,x)
```

```
[Out] (3*x^2)/2 - x - x^3 + x^4/4
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 19, normalized size = 1.73

$$-x + \frac{3}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4$$

Warning: Unable to verify antiderivative.

```
[In] int(x^3-3*x^2+3*x-1,x)
```

```
[Out] -x+3/2*x^2-1/3*x^3+1/4*x^4
```

3.79 $\int \sqrt{12-3x^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+2\sqrt{3}\arcsin\left(\frac{x}{2}\right)$$

```
[Out] 1/2*3^(1/2)*x*(-x^2+4)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 222}

$$2\sqrt{3}\arcsin\left(\frac{x}{2}\right)+\frac{1}{2}\sqrt{3}\sqrt{4-x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[12 - 3*x^2],x]
```

```
[Out] (Sqrt[3]*x*Sqrt[4 - x^2])/2 + 2*Sqrt[3]*ArcSin[x/2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+6\int\frac{1}{\sqrt{12-3x^2}}\,dx\\&=\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+2\sqrt{3}\arcsin\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.23

$$\frac{1}{2}\sqrt{3}\left(x\sqrt{4-x^2}-8\arctan\left(\frac{\sqrt{4-x^2}}{2+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[12 - 3*x^2],x]
[Out] (Sqrt[3]*(x*Sqrt[4 - x^2] - 8*ArcTan[Sqrt[4 - x^2]/(2 + x)]))/2
```

Maple [A]
time = 0.20, size = 23, normalized size = 0.66

method	result	size
default	$\frac{x\sqrt{-3x^2+12}}{2}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	23
risch	$-\frac{3x\left(x^2-4\right)}{2\sqrt{-3x^2+12}}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	28
meijerg	$\frac{i\sqrt{3}\left(-i\sqrt{\pi}\,x\sqrt{-\frac{x^2}{4}+1}-2i\sqrt{\pi}\,\arcsin\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}}$	37
pseudoelliptic	$\frac{x\sqrt{-3x^2+12}}{2}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{-3x^2+12}\,\sqrt{3}}{3x}\right)$	37
trager	$\frac{x\sqrt{-3x^2+12}}{2}+2\mathit{RootOf}\left(\_Z^2+3\right)\ln\left(\mathit{RootOf}\left(\_Z^2+3\right)\sqrt{-3x^2+12}+3x\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(-3*x^2+12)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Maxima [A]
time = 0.44, size = 22, normalized size = 0.63

$$\frac{1}{2}\sqrt{-3\,x^2+12}x+2\sqrt{3}\arcsin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+12)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-3*x^2 + 12)*x + 2*sqrt(3)*arcsin(1/2*x)
```

Fricas [A]
time = 0.59, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-3\,x^2+12}x-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+12}}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+12)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-3*x^2 + 12)*x - 2*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 12)/x)
```

Sympy [A]
time = 0.14, size = 27, normalized size = 0.77

$$\frac{\sqrt{3}x\sqrt{4-x^2}}{2}+2\sqrt{3}\operatorname{asin}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*x*sqrt(4 - x**2)/2 + 2*sqrt(3)*asin(x/2)
```

Giac [A]
time = 0.51, size = 23, normalized size = 0.66

$$\frac{1}{2}\sqrt{3}\left(\sqrt{-x^2+4}x+4\arcsin\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*(sqrt(-x^2 + 4)*x + 4*arcsin(1/2*x))
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.71

$$2\sqrt{3}\operatorname{asin}\left(\frac{x}{2}\right)+\frac{\sqrt{3}\,x\,\sqrt{4-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12 - 3*x^2)^(1/2),x)
```

```
[Out] 2*3^(1/2)*asin(x/2) + (3^(1/2)*x*(4 - x^2)^(1/2))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 25, normalized size = 0.71

$$\frac{2x\sqrt{-3x^2+12}}{3}+\frac{8\sqrt{12}\arcsin\left(\frac{\sqrt{3}\,x}{6}\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int((-3*x^2+12)^(1/2),x)
```

```
[Out] 2/3*x*(-3*x^2+12)^(1/2)+8/3*12^(1/2)*arcsin(1/6*3^(1/2)*x)
```

3.80 $\int \left((-3+x)^7+x-\sin(3-x)\right) \, dx$

Optimal. Leaf size=27

$$\frac{1}{8}(3-x)^8+\frac{x^2}{2}-\cos(3-x)$$

[Out] 1/8*(3-x)^8+1/2*x^2-cos(-3+x)

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2718}

$$\frac{x^2}{2}+\frac{1}{8}(3-x)^8-\cos(3-x)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x)^7 + x - Sin[3 - x],x]

[Out] (3 - x)^8/8 + x^2/2 - Cos[3 - x]

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \frac{1}{8}(3-x)^8+\frac{x^2}{2}-\int \sin(3-x) \, dx \\ &= \frac{1}{8}(3-x)^8+\frac{x^2}{2}-\cos(3-x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.07

$$\frac{1}{8}(-3+x)^8+\frac{x^2}{2}-\cos(3)\cos(x)-\sin(3)\sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x)^7 + x - Sin[3 - x],x]

[Out] (-3 + x)^8/8 + x^2/2 - Cos[3]*Cos[x] - Sin[3]*Sin[x]

Maple [A]
time = 0.10, size = 20, normalized size = 0.74

method	result
default	$\frac{x^2}{2}+\frac{(-3+x)^8}{8}-\cos(-3+x)$
derivativedivides	$-9+3x+\frac{(-3+x)^2}{2}+\frac{(-3+x)^8}{8}-\cos(-3+x)$
parts	$-2187x+2552x^2-1701x^3+\frac{2835x^4}{4}-189x^5+\frac{63x^6}{2}-3x^7+\frac{x^8}{8}-\cos(-3+x)$
risch	$2552x^2+\frac{x^8}{8}-3x^7+\frac{63x^6}{2}-189x^5+\frac{2835x^4}{4}-1701x^3-2187x+\frac{6561}{8}-\cos(-3+x)$
parallelrisch	$\frac{x^8}{8}-3x^7+\frac{63x^6}{2}-189x^5+\frac{2835x^4}{4}-1701x^3+2552x^2-2187x-1-\cos(-3+x)$
norman	$\frac{-2187x+2552x^2-1701x^3+\frac{2835x^4}{4}-189x^5+\frac{63x^6}{2}-3x^7+\frac{x^8}{8}-2187x\left(\tan^2\left(-\frac{3}{2}+\frac{x}{2}\right)\right)+2552x^2\left(\tan^2\left(-\frac{3}{2}+\frac{x}{2}\right)\right)-1701x^3(}{1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+(-3+x)^7+sin(-3+x),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/8*(-3+x)^8-cos(-3+x)
```

Maxima [A]
time = 0.33, size = 19, normalized size = 0.70

$$\frac{1}{8}(x-3)^8+\frac{1}{2}x^2-\cos(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-3+x)^7+sin(-3+x),x, algorithm="maxima")
[Out] 1/8*(x - 3)^8 + 1/2*x^2 - cos(x - 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.
time = 0.62, size = 45, normalized size = 1.67

$$\frac{1}{8}x^8-3x^7+\frac{63}{2}x^6-189x^5+\frac{2835}{4}x^4-1701x^3+2552x^2-2187x-\cos(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-3+x)^7+sin(-3+x),x, algorithm="fricas")
[Out] 1/8*x^8 - 3*x^7 + 63/2*x^6 - 189*x^5 + 2835/4*x^4 - 1701*x^3 + 2552*x^2 - 2
187*x - cos(x - 3)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.56

$$\frac{x^2}{2}+\frac{(x-3)^8}{8}-\cos(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-3+x)**7+sin(-3+x),x)
```

```
[Out] x**2/2 + (x - 3)**8/8 - cos(x - 3)
```

Giac [A]
time = 0.43, size = 19, normalized size = 0.70

$$\frac{1}{8}(x-3)^8+\frac{1}{2}x^2-\cos(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-3+x)^7+sin(-3+x),x, algorithm="giac")
```

```
[Out] 1/8*(x - 3)^8 + 1/2*x^2 - cos(x - 3)
```

Mupad [B]
time = 0.20, size = 45, normalized size = 1.67

$$2552x^2-\cos(x-3)-2187x-1701x^3+\frac{2835x^4}{4}-189x^5+\frac{63x^6}{2}-3x^7+\frac{x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + sin(x - 3) + (x - 3)^7,x)
```

```
[Out] 2552*x^2 - cos(x - 3) - 2187*x - 1701*x^3 + (2835*x^4)/4 - 189*x^5 + (63*x^
6)/2 - 3*x^7 + x^8/8
```

Chatgpt [F] Failed to verify
time = 1.00, size = 17, normalized size = 0.63

$$\frac{(x-3)^8}{8}+\frac{x^2}{2}+\cos(x-3)$$

Warning: Unable to verify antiderivative.

```
[In] int(x+(x-3)^7+sin(x-3),x)
```

```
[Out] 1/8*(x-3)^8+1/2*x^2+cos(x-3)
```

3.81 $\int \sin(x)\sqrt{1+\tan^2(x)}\,dx$

Optimal. Leaf size=15

$$-\cos(x)\log(\cos(x))\sqrt{\sec^2(x)}$$

[Out] -cos(x)*ln(cos(x))*(sec(x)^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3738, 4210, 3556}

$$-\cos(x)\sqrt{\sec^2(x)}\log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sqrt[1 + Tan[x]^2],x]

[Out] -(Cos[x]*Log[Cos[x]]*Sqrt[Sec[x]^2])

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 4210

```
Int[(u_.)*((b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sec[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sec[e + f*x]^
n)^FracPart[p]/(Sec[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sec
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \sqrt{\sec^2(x)}\sin(x)\,dx\\ &= \left(\cos(x)\sqrt{\sec^2(x)}\right)\int \tan(x)\,dx\\ &= -\cos(x)\log(\cos(x))\sqrt{\sec^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\cos(x)\log(\cos(x))\sqrt{\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sqrt[1 + Tan[x]^2],x]
[Out] -(Cos[x]*Log[Cos[x]]*Sqrt[Sec[x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.32, size = 36, normalized size = 2.40

method	result	size
default	$-\operatorname{csgn}(\sec(x))\left(\ln(\csc(x)-\cot(x)-1)-\ln\left(\frac{2}{1+\cos(x)}\right)+\ln(1+\csc(x)-\cot(x))\right)$	36
risch	$2i\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,x\cos(x)-2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{2ix}+1)\cos(x)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(1+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -csgn(sec(x))*(ln(csc(x)-cot(x)-1)-ln(2/(1+cos(x)))+ln(1+csc(x)-cot(x)))
```

Maxima [A]
time = 0.35, size = 13, normalized size = 0.87

$$-\sqrt{\frac{1}{\cos(x)^2}}\cos(x)\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+tan(x)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(cos(x)^(-2))*cos(x)*log(cos(x))
```

Fricas [A]
time = 0.64, size = 5, normalized size = 0.33

$$\log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+tan(x)^2)^(1/2),x, algorithm="fricas")
[Out] log(-cos(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan^2(x) + 1} \sin(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+tan(x)**2)**(1/2),x)
[Out] Integral(sqrt(tan(x)**2 + 1)*sin(x), x)
```

Giac [**A**]
time = 0.44, size = 6, normalized size = 0.40

$$-\log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+tan(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(abs(cos(x)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \sin(x) \sqrt{\tan(x)^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(tan(x)^2 + 1)^(1/2),x)
[Out] int(sin(x)*(tan(x)^2 + 1)^(1/2), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 3, normalized size = 0.20

$$\operatorname{arcsinh}(\tan(x))$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)*(1+tan(x)^2)^(1/2),x)
[Out] arcsinh(tan(x))
```

3.82 $\int \frac{-1+x^2-x^3+x^5}{-1+x-x^3+x^4} dx$

Optimal. Leaf size=9

$$\frac{1}{2}(1+x)^2$$

[Out] 1/2*(x+1)^2

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{1600\}$

$$\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2 - x^3 + x^5)/(-1 + x - x^3 + x^4),x]

[Out] x + x^2/2

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int (1+x)\,dx\\ &= x+\frac{x^2}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$x+\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2 - x^3 + x^5)/(-1 + x - x^3 + x^4),x]

[Out] x + x^2/2

Maple [A]
time = 0.01, size = 8, normalized size = 0.89

method	result	size
gosper	$\frac{x(2+x)}{2}$	7
default	$\frac{1}{2}x^2+x$	8
norman	$\frac{1}{2}x^2+x$	8
risch	$\frac{1}{2}x^2+x$	8
parallelrisch	$\frac{1}{2}x^2+x$	8
parts	$\frac{1}{2}x^2+x$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+x
```

Maxima [A]
time = 0.41, size = 7, normalized size = 0.78

$$\frac{1}{2}\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x
```

Fricas [A]
time = 0.55, size = 7, normalized size = 0.78

$$\frac{1}{2}\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.56

$$\frac{x^2}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-x**3+x**2-1)/(x**4-x**3+x-1),x)
```

```
[Out] x**2/2 + x
```

Giac [A]
time = 0.47, size = 7, normalized size = 0.78

$$\frac{1}{2}\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x, algorithm="giac")
[Out] 1/2*x^2 + x
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.67

$$\frac{x\,(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^3 + x^5 - 1)/(x - x^3 + x^4 - 1),x)
[Out] (x*(x + 2))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 43, normalized size = 4.78

$$\frac{\ln\left(x^2-x+1\right)}{4} - \frac{\ln\left(x+1\right)}{4} + \frac{\ln\left(x^2+x+1\right)}{6} + \frac{\ln\left(-x^4+x^3-x+1\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((x^5-x^3+x^2-1)/(x^4-x^3+x-1),x)
[Out] 1/4*ln(x^2-x+1)-1/4*ln(x+1)+1/6*ln(x^2+x+1)+1/2*ln(-x^4+x^3-x+1)
```

3.83 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$-x + x\log(x)$$

[Out] -x+x*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2332}

$$x\log(x) - x$$

Antiderivative was successfully verified.

[In] Int[Log[x],x]

[Out] -x + x*Log[x]

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{Integral} = -x + x\log(x)$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-x + x\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x],x]

[Out] -x + x*Log[x]

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size

lookup	$-x + x\ln(x)$	9
default	$-x + x\ln(x)$	9
norman	$-x + x\ln(x)$	9
risch	$-x + x\ln(x)$	9
parallelrisch	$-x + x\ln(x)$	9
parts	$-x + x\ln(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -x+x*ln(x)
```

Maxima [A]
time = 0.33, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A]
time = 0.56, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.62

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

Giac [A]
time = 0.44, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="giac")
[Out] x*log(x) - x
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.75

$$x\,(\ln{(x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x),x)
[Out] x*(log(x) - 1)
```

Chatgpt [A]
time = 1.00, size = 8, normalized size = 1.00

$$x\ln{(x)} - x$$

Antiderivative was successfully verified.

```
[In] int(ln(x),x)
[Out] x*ln(x)-x
```

3.84 $\int \frac{1}{1-e^{-x}} \, dx$

Optimal. Leaf size=6

$$\log(-1+e^x)$$

[Out] ln(exp(x)-1)

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 2.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2320, 36, 31, 29}

$$x + \log\left(1 - e^{-x}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - E^(-x))^(-1),x]

[Out] x + Log[1 - E^(-x)]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, e^{-x}\right)\\
&= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, e^{-x}\right) - \text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{-x}\right)\\
&= x + \log\left(1-e^{-x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$\log\left(-1+e^{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - E^(-x))^(-1),x]
```

```
[Out] Log[-1 + E^x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.01, size = 16, normalized size = 2.67

method	result	size
norman	$x+\ln\left(-1+\mathrm{e}^{-x}\right)$	10
risch	$x+\ln\left(-1+\mathrm{e}^{-x}\right)$	10
parallelrisch	$x+\ln\left(-1+\mathrm{e}^{-x}\right)$	10
derivativedivides	$\ln\left(-1+\mathrm{e}^{-x}\right)-\ln\left(\mathrm{e}^{-x}\right)$	16
default	$\ln\left(-1+\mathrm{e}^{-x}\right)-\ln\left(\mathrm{e}^{-x}\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-exp(-x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+exp(-x))-ln(exp(-x))
```

Maxima [A]
time = 0.33, size = 9, normalized size = 1.50

$$x+\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-exp(-x)),x, algorithm="maxima")
```

```
[Out] x + log(e^(-x) - 1)
```

Fricas [A]
time = 0.58, size = 9, normalized size = 1.50

$$x + \log\left(e^{(-x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-exp(-x)),x, algorithm="fricas")
[Out] x + log(e^(-x) - 1)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 1.33

$$x + \log\left(-1 + e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-exp(-x)),x)
[Out] x + log(-1 + exp(-x))
```

Giac [A]
time = 0.46, size = 10, normalized size = 1.67

$$x + \log\left(\left|e^{(-x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-exp(-x)),x, algorithm="giac")
[Out] x + log(abs(e^(-x) - 1))
```

Mupad [B]
time = 0.08, size = 7, normalized size = 1.17

$$\ln\left(1 - \mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(exp(-x) - 1),x)
[Out] log(1 - exp(x))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 11, normalized size = 1.83

$$-\ln\left(1 - \mathrm{e}^{-x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1-exp(-x)),x)
[Out] -ln(1-exp(-x))
```

3.85 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{1}{32}(4x-\sin(4x))$$

```
[Out] 1/8*x-1/32*sin(4*x)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.71, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\frac{1}{4}\cos^3(x)\sin(x)+\frac{1}{4}\int \cos^2(x)\,dx\\
&= \frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)+\frac{\int 1\,dx}{8}\\
&= \frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{x}{8} - \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A]
time = 0.06, size = 19, normalized size = 1.36

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
parallelrisch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$-\frac{(\cos^3(x))\sin(x)}{4}+\frac{\cos(x)\sin(x)}{8}+\frac{x}{8}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*cos(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(x)^3*sin(x)+1/8*cos(x)*sin(x)+1/8*x
```

Maxima [A]
time = 0.35, size = 10, normalized size = 0.71

$$\frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A]
time = 0.60, size = 19, normalized size = 1.36

$$-\frac{1}{8}\left(2\,\cos\left(x\right)^{3} - \cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*cos(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A]
time = 0.02, size = 14, normalized size = 1.00

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*cos(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Giac [A]
time = 0.45, size = 10, normalized size = 0.71

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Mupad [B]
time = 0.09, size = 18, normalized size = 1.29

$$\frac{\cos(x)\sin(x)^3}{4} - \frac{\cos(x)\sin(x)}{8} + \frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
```

```
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 0.93

$$-\frac{(\cos^3(x))}{12} + \frac{(\cos^5(x))}{20}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)^2*cos(x)^2,x)
```

```
[Out] -1/12*cos(x)^3+1/20*cos(x)^5
```

3.86 $\int \frac{\pi \sin(\pi\sqrt{x})}{\sqrt{x}} \, dx$

Optimal. Leaf size=10

$$-2\cos\left(\pi\sqrt{x}\right)$$

[Out] -2*cos(Pi*x^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {12, 3460, 2718}

$$-2\cos\left(\pi\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(Pi*Sin[Pi*Sqrt[x]])/Sqrt[x],x]

[Out] -2*Cos[Pi*Sqrt[x]]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \pi \int \frac{\sin\left(\pi\sqrt{x}\right)}{\sqrt{x}} \, dx \\
&= (2\pi)\text{Subst}\left(\int \sin(\pi x) \, dx, x, \sqrt{x}\right) \\
&= -2\cos\left(\pi\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-2\cos\left(\pi\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi*Sin[Pi*Sqrt[x]])/Sqrt[x],x]
[Out] -2*Cos[Pi*Sqrt[x]]
```

Maple [A]
time = 0.06, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-2\cos\left(\pi\sqrt{x}\right)$	9
default	$-2\cos\left(\pi\sqrt{x}\right)$	9
meijerg	$2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\pi\sqrt{x}\right)}{\sqrt{\pi}}\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi*sin(Pi*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*cos(Pi*x^(1/2))
```

Maxima [A]
time = 0.33, size = 8, normalized size = 0.80

$$-2\cos\left(\pi\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*sin(pi*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] -2*cos(pi*sqrt(x))
```

Fricas [A]
time = 0.60, size = 8, normalized size = 0.80

$$-2\cos\left(\pi\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*sin(pi*x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -2*cos(pi*sqrt(x))
```

Sympy [A]
time = 0.12, size = 10, normalized size = 1.00

$$-2\cos\left(\pi\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*sin(pi*x**(1/2))/x**(1/2),x)
[Out] -2*cos(pi*sqrt(x))
```

Giac [A]
time = 0.53, size = 8, normalized size = 0.80

$$-2\,\cos\left(\pi\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*sin(pi*x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] -2*cos(pi*sqrt(x))
```

Mupad [B]
time = 0.17, size = 8, normalized size = 0.80

$$-2\,\cos\left(\Pi\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((Pi*sin(Pi*x^(1/2)))/x^(1/2),x)
[Out] -2*cos(Pi*x^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 8, normalized size = 0.80

$$-\cos\left(\pi\sqrt{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(Pi*sin(Pi*x^(1/2))/x^(1/2),x)
[Out] -cos(Pi*x^(1/2))
```

3.87 $\int \tan^2(x)\,dx$

Optimal. Leaf size=6

$$-x+\tan(x)$$

```
[Out] -x+tan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$\tan(x)-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \tan(x)-\int 1\,dx\\ &= -x+\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$-x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Maple [A]
time = 0.02, size = 9, normalized size = 1.50

method	result	size
norman	$-x+\tan(x)$	7
parallelrisch	$-x+\tan(x)$	7
derivativedivides	$\tan(x)-\arctan(\tan(x))$	9
default	$\tan(x)-\arctan(\tan(x))$	9
risch	$-x+\frac{2i}{e^{2ix}+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x,method=_RETURNVERBOSE)
[Out] tan(x)-arctan(tan(x))
```

Maxima [A]
time = 0.48, size = 6, normalized size = 1.00

$$-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
[Out] -x + tan(x)
```

Fricas [A]
time = 0.58, size = 6, normalized size = 1.00

$$-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
[Out] -x + tan(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.
time = 0.01, size = 7, normalized size = 1.17

$$-x+\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2,x)
```

```
[Out] -x + sin(x)/cos(x)
```

Giac [A]
time = 0.48, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="giac")
```

```
[Out] -x + tan(x)
```

Mupad [B]
time = 0.03, size = 6, normalized size = 1.00

$$\tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
```

```
[Out] tan(x) - x
```

Chatgpt [A]
time = 1.00, size = 6, normalized size = 1.00

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] int(tan(x)^2,x)
```

```
[Out] tan(x)-x
```

3.88 $\int e^{\sqrt[4]{x}}\,dx$

Optimal. Leaf size=30

$$4e^{\sqrt[4]{x}}\left(-6+6\sqrt[4]{x}-3\sqrt{x}+x^{3/4}\right)$$

[Out] 4*exp(x^(1/4))*(-6+6*x^(1/4)-3*x^(1/2)+x^(3/4))

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.73, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2238, 2207, 2225}

$$4e^{\sqrt[4]{x}}x^{3/4}-12e^{\sqrt[4]{x}}\sqrt{x}+24e^{\sqrt[4]{x}}\sqrt[4]{x}-24e^{\sqrt[4]{x}}$$

Antiderivative was successfully verified.

[In] Int[E^x^(1/4),x]

[Out] -24*E^x^(1/4) + 24*E^x^(1/4)*x^(1/4) - 12*E^x^(1/4)*Sqrt[x] + 4*E^x^(1/4)*x^(3/4)

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2238

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 4\text{Subst}\left(\int e^x x^3\,dx, x, \sqrt[4]{x}\right)\\
&= 4e^{\sqrt[4]{x}}x^{3/4} - 12\text{Subst}\left(\int e^x x^2\,dx, x, \sqrt[4]{x}\right)\\
&= -12e^{\sqrt[4]{x}}\sqrt{x} + 4e^{\sqrt[4]{x}}x^{3/4} + 24\text{Subst}\left(\int e^x x\,dx, x, \sqrt[4]{x}\right)\\
&= 24e^{\sqrt[4]{x}}\sqrt[4]{x} - 12e^{\sqrt[4]{x}}\sqrt{x} + 4e^{\sqrt[4]{x}}x^{3/4} - 24\text{Subst}\left(\int e^x\,dx, x, \sqrt[4]{x}\right)\\
&= -24e^{\sqrt[4]{x}} + 24e^{\sqrt[4]{x}}\sqrt[4]{x} - 12e^{\sqrt[4]{x}}\sqrt{x} + 4e^{\sqrt[4]{x}}x^{3/4}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.03

$$e^{\sqrt[4]{x}}\left(-24 + 24\sqrt[4]{x} - 12\sqrt{x} + 4x^{3/4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^(1/4),x]
```

```
[Out] E^x^(1/4)*(-24 + 24*x^(1/4) - 12*Sqrt[x] + 4*x^(3/4))
```

Maple [A]
time = 0.01, size = 35, normalized size = 1.17

method	result	size
meijerg	$24-\left(-4x^{\frac{3}{4}}+12\sqrt{x}-24x^{\frac{1}{4}}+24\right)\mathrm{e}^{x^{\frac{1}{4}}}$	26
derivativedivides	$4\,\mathrm{e}^{x^{\frac{1}{4}}}x^{\frac{3}{4}}-12\sqrt{x}\,\mathrm{e}^{x^{\frac{1}{4}}}+24x^{\frac{1}{4}}\mathrm{e}^{x^{\frac{1}{4}}}-24\,\mathrm{e}^{x^{\frac{1}{4}}}$	35
default	$4\,\mathrm{e}^{x^{\frac{1}{4}}}x^{\frac{3}{4}}-12\sqrt{x}\,\mathrm{e}^{x^{\frac{1}{4}}}+24x^{\frac{1}{4}}\mathrm{e}^{x^{\frac{1}{4}}}-24\,\mathrm{e}^{x^{\frac{1}{4}}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/4)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*exp(x^(1/4))*x^(3/4)-12*x^(1/2)*exp(x^(1/4))+24*x^(1/4)*exp(x^(1/4))-24*e
xp(x^(1/4))
```

Maxima [A]
time = 0.34, size = 21, normalized size = 0.70

$$4\left(x^{\frac{3}{4}} - 3\sqrt{x} + 6\,x^{\frac{1}{4}} - 6\right)e^{\left(x^{\frac{1}{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/4)),x, algorithm="maxima")
[Out] 4*(x^(3/4) - 3*sqrt(x) + 6*x^(1/4) - 6)*e^(x^(1/4))
```

Fricas [A]
time = 0.57, size = 21, normalized size = 0.70

$$4\left(x^{\frac{3}{4}} - 3\sqrt{x} + 6x^{\frac{1}{4}} - 6\right)e^{\left(x^{\frac{1}{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/4)),x, algorithm="fricas")
[Out] 4*(x^(3/4) - 3*sqrt(x) + 6*x^(1/4) - 6)*e^(x^(1/4))
```

Sympy [A]
time = 0.18, size = 48, normalized size = 1.60

$$4x^{\frac{3}{4}}e^{\sqrt[4]{x}} + 24\sqrt[4]{x}e^{\sqrt[4]{x}} - 12\sqrt{x}e^{\sqrt[4]{x}} - 24e^{\sqrt[4]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/4)),x)
[Out] 4*x**(3/4)*exp(x**(1/4)) + 24*x**(1/4)*exp(x**(1/4)) - 12*sqrt(x)*exp(x**(1
/4)) - 24*exp(x**(1/4))
```

Giac [A]
time = 0.44, size = 21, normalized size = 0.70

$$4\left(x^{\frac{3}{4}} - 3\sqrt{x} + 6x^{\frac{1}{4}} - 6\right)e^{\left(x^{\frac{1}{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/4)),x, algorithm="giac")
[Out] 4*(x^(3/4) - 3*sqrt(x) + 6*x^(1/4) - 6)*e^(x^(1/4))
```

Mupad [B]
time = 0.02, size = 28, normalized size = 0.93

$$-4\,x\,\mathrm{e}^{x^{1/4}}\left(\frac{6}{x} + \frac{3}{\sqrt{x}} - \frac{1}{x^{1/4}} - \frac{6}{x^{3/4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/4)),x)
[Out] -4*x*exp(x^(1/4))*(6/x + 3/x^(1/2) - 1/x^(1/4) - 6/x^(3/4))
```

<u>**Chatgpt**</u> [**F**] Failed to verify
time = 1.00, size = 9, normalized size = 0.30

$$\frac{4x^{\frac{3}{4}} e^{x^{\frac{1}{4}}}}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(x^(1/4)),x)
```

```
[Out] 4/3*x^(3/4)*exp(x^(1/4))
```

3.89 $\int \cos(x)\cot(x)\,dx$

Optimal. Leaf size=8

$$-\text{arctanh}(\cos(x)) + \cos(x)$$

```
[Out] -arctanh(cos(x))+cos(x)
```

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2672, 327, 212}

$$\cos(x) - \text{arctanh}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right)\\
&= \cos(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)\\
&= -\text{arctanh}(\cos(x)) + \cos(x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.
time = 0.00, size = 19, normalized size = 2.38

$$\cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[x],x]
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A]
time = 0.07, size = 12, normalized size = 1.50

method	result	size
default	$\cos(x) + \ln(-\cot(x) + \csc(x))$	12
risch	$\frac{e^{ix}}{2} + \frac{e^{-ix}}{2} - \ln(e^{ix} + 1) + \ln(e^{ix} - 1)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+ln(-cot(x)+csc(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.
time = 0.37, size = 17, normalized size = 2.12

$$\cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(x),x, algorithm="maxima")
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.
time = 0.62, size = 21, normalized size = 2.62

$$\cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(x),x, algorithm="fricas")
[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.03, size = 19, normalized size = 2.38

$$\frac{\log(\cos(x)-1)}{2} - \frac{\log(\cos(x)+1)}{2} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.50, size = 19, normalized size = 2.38

$$\cos(x) - \frac{1}{2}\log(\cos(x)+1) + \frac{1}{2}\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

Mupad [B]
time = 0.23, size = 18, normalized size = 2.25

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(x),x)
```

```
[Out] log(tan(x/2)) + 2/(tan(x/2)^2 + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 1.25

$$\ln(\sin(x)) - \frac{(\sin^2(x))}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)*cot(x),x)
```

```
[Out] ln(sin(x))-1/2*sin(x)^2
```

3.90 $\int \left(2 \log(x)+\log^2(x)\right) \, dx$

Optimal. Leaf size=6

$$x \log^2(x)$$

```
[Out] x*ln(x)^2
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2332, 2333}

$$x \log^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[2*Log[x] + Log[x]^2,x]
```

```
[Out] x*Log[x]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2 \int \log(x) \, dx+\int \log^2(x) \, dx\\
&= -2 x+2 x \log(x)+x \log^2(x)-2 \int \log(x) \, dx\\
&= x \log^2(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x \log^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[2*Log[x] + Log[x]^2,x]

[Out] x*Log[x]^2

Maple [A]
time = 0.01, size = 7, normalized size = 1.17

method	result	size
default	$x \ln(x)^2$	7
norman	$x \ln(x)^2$	7
risch	$x \ln(x)^2$	7
parallelrisch	$x \ln(x)^2$	7
parts	$x \ln(x)^2$	7

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*ln(x)+ln(x)^2,x,method=_RETURNVERBOSE)

[Out] x*ln(x)^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(6) = 12.
time = 0.38, size = 21, normalized size = 3.50

$$\left(\log(x)^2 - 2\log(x) + 2\right)x + 2x\log(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*log(x)+log(x)^2,x, algorithm="maxima")

[Out] (log(x)^2 - 2*log(x) + 2)*x + 2*x*log(x) - 2*x

Fricas [A]
time = 0.57, size = 6, normalized size = 1.00

$$x\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*log(x)+log(x)^2,x, algorithm="fricas")

[Out] x*log(x)^2

Sympy [A]
time = 0.05, size = 5, normalized size = 0.83

$$x\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*ln(x)+ln(x)**2,x)
```

```
[Out] x*log(x)**2
```

Giac [A]
time = 0.51, size = 6, normalized size = 1.00

$$x \log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*log(x)+log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2
```

Mupad [B]
time = 0.00, size = 6, normalized size = 1.00

$$x \ln(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*log(x) + log(x)^2,x)
```

```
[Out] x*log(x)^2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 15, normalized size = 2.50

$$x \ln(x)^2 - x \ln(x) - x$$

Warning: Unable to verify antiderivative.

```
[In] int(2*ln(x)+ln(x)^2,x)
```

```
[Out] x*ln(x)^2-x*ln(x)-x
```

3.91 $\int \frac{x^3}{1+x^2} \, dx$

Optimal. Leaf size=18

$$\frac{x^2}{2}-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {272, 45}

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Maple [A]
time = 0.06, size = 15, normalized size = 0.83

method	result	size
default	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
norman	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
meijerg	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
risch	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15
parallelrisch	$\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.34, size = 14, normalized size = 0.78

$$\frac{1}{2}x^2-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.58, size = 14, normalized size = 0.78

$$\frac{1}{2}x^2-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.67

$$\frac{x^2}{2} - \frac{\log(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+1),x)
[Out] x**2/2 - log(x**2 + 1)/2
```

Giac [A]
time = 0.44, size = 14, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.78

$$\frac{x^2}{2} - \frac{\ln(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2 + 1),x)
[Out] x^2/2 - log(x^2 + 1)/2
```

Chatgpt [A]
time = 1.00, size = 14, normalized size = 0.78

$$\frac{x^2}{2} - \frac{\ln(x^2+1)}{2}$$

Antiderivative was successfully verified.

```
[In] int(x^3/(x^2+1),x)
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

3.92 $\int \frac{1}{2-2x+x^2} \, dx$

Optimal. Leaf size=8

$$-\arctan(1-x)$$

[Out] arctan(x-1)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {631, 210}

$$-\arctan(1-x)$$

Antiderivative was successfully verified.

[In] Int[(2 - 2*x + x^2)^(-1),x]

[Out] -ArcTan[1 - x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1-x\right) \\ &= -\arctan(1-x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\arctan(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x + x^2)^(-1),x]
```

```
[Out] -ArcTan[1 - x]
```

Maple [A]
time = 0.14, size = 5, normalized size = 0.62

method	result	size
default	$\arctan(x-1)$	5
risch	$\arctan(x-1)$	5
parallelrisch	$\frac{i\ln(x-1+i)}{2}-\frac{i\ln(x-1-i)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x-1)
```

Maxima [A]
time = 0.47, size = 4, normalized size = 0.50

$$\arctan(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+2),x, algorithm="maxima")
```

```
[Out] arctan(x - 1)
```

Fricas [A]
time = 0.56, size = 4, normalized size = 0.50

$$\arctan(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+2),x, algorithm="fricas")
```

```
[Out] arctan(x - 1)
```

Sympy [A]
time = 0.04, size = 3, normalized size = 0.38

$$\operatorname{atan}(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+2),x)
```

```
[Out] atan(x - 1)
```

Giac [A]
time = 0.52, size = 4, normalized size = 0.50

$$\arctan\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+2),x, algorithm="giac")
```

```
[Out] arctan(x - 1)
```

Mupad [B]
time = 0.09, size = 4, normalized size = 0.50

$$\operatorname{atan}(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x + 2),x)
```

```
[Out] atan(x - 1)
```

Chatgpt [A]
time = 1.00, size = 4, normalized size = 0.50

$$\arctan\left(x-1\right)$$

Antiderivative was successfully verified.

```
[In] int(1/(x^2-2*x+2),x)
```

```
[Out] arctan(x-1)
```

3.93 $\int \log(\sin(x)) \sin(x) \, dx$

Optimal. Leaf size=15

$$-\operatorname{arctanh}(\cos(x)) + \cos(x) - \cos(x) \log(\sin(x))$$

```
[Out] -arctanh(cos(x))+cos(x)-cos(x)*ln(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {2718, 2634, 2672, 327, 212}

$$-\operatorname{arctanh}(\cos(x)) + \cos(x) - \cos(x) \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sin[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Cos[x] - Cos[x]*Log[Sin[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= -\cos(x)\log(\sin(x)) + \int \cos(x)\cot(x)\,dx \\
&= -\cos(x)\log(\sin(x)) - \text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right) \\
&= \cos(x) - \cos(x)\log(\sin(x)) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right) \\
&= -\text{arctanh}(\cos(x)) + \cos(x) - \cos(x)\log(\sin(x))
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.73

$$\cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \cos(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sin[x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]] - Cos[x]*Log[Sin[x]]
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 111, normalized size = 7.40

method	result
parallelrisch	$-\cos(x)\ln(\sin(x)) + \cos(x) + \ln(-\cot(x) + \csc(x)) + 1$
norman	$\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)+2}{1+\tan^2\left(\frac{x}{2}\right)} + \ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$
default	$-\frac{e^{ix}\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)}{2} + \frac{e^{ix}}{2} + \ln\left(e^{ix}-1\right) - \ln\left(e^{ix}+1\right) - \frac{e^{-ix}\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)}{2} + \frac{e^{-ix}}{2} + \frac{\ln(2)\left(e^{ix}+e\right.}{2}$
risch	$\frac{e^{ix}}{2} - \frac{e^{-ix}\ln\left(e^{2ix}-1\right)}{2} - \frac{e^{ix}\ln\left(e^{2ix}-1\right)}{2} + \frac{e^{ix}\ln(2)}{2} + \frac{e^{-ix}\ln(2)}{2} + \ln\left(e^{ix}\right)\cos(x) + \frac{e^{-ix}}{2} - \frac{ie^{ix}\pi\,\text{csgn}(\sin(x)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*x))+1/2*exp(I*x)+ln(exp(I*x)-1)-ln
(exp(I*x)+1)-1/2*exp(-I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*x))+1/2/exp(I*x)+1/2*
ln(2)*(exp(I*x)+1/exp(I*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(15) = 30$.
time = 0.38, size = 89, normalized size = 5.93

$$-\frac{2\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}+\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*log(sin(x)),x, algorithm="maxima")
```

```
[Out] -2*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)))/(sin(x)^2/(co
s(x) + 1)^2 + 1) + 2/(sin(x)^2/(cos(x) + 1)^2 + 1) - log(sin(x)^2/(cos(x) +
 1)^2 + 1) + log(sin(x)^2/(cos(x) + 1)^2)
```

Fricas [A]
time = 0.64, size = 28, normalized size = 1.87

$$-\cos(x)\log(\sin(x))+\cos(x)-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*log(sin(x)),x, algorithm="fricas")
```

```
[Out] -cos(x)*log(sin(x)) + cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos
(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(15) = 30$.
time = 0.77, size = 105, normalized size = 7.00

$$\frac{2\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{2\log(2)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{2}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*ln(sin(x)),x)
```

```
[Out] 2*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**2/(tan(x/2)**2 + 1) + log(tan(x
/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 + 1) + log(tan(x/2)**2 + 1)/(tan(x/2)*
*2 + 1) + 2*log(2)*tan(x/2)**2/(tan(x/2)**2 + 1) + 2/(tan(x/2)**2 + 1)
```

Giac [A]
time = 0.44, size = 26, normalized size = 1.73

$$-\cos(x)\log(\sin(x))+\cos(x)-\frac{1}{2}\log(\cos(x)+1)+\frac{1}{2}\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] -cos(x)*log(sin(x)) + cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \ln(\sin(x)) \sin(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*sin(x),x)
```

```
[Out] int(log(sin(x))*sin(x), x)
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Antiderivative was successfully verified.

```
[In] int(sin(x)*ln(sin(x)),x)
```

```
[Out] not solved
```

3.94 $\int \frac{x}{1-x^4} \, dx$

Optimal. Leaf size=8

$$\frac{\operatorname{arctanh}(x^2)}{2}$$

```
[Out] 1/2*arctanh(x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 212}

$$\frac{\operatorname{arctanh}(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^4),x]
```

```
[Out] ArcTanh[x^2]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, x^2\right) \\ &= \frac{\operatorname{arctanh}(x^2)}{2} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.
time = 0.00, size = 23, normalized size = 2.88

$$-\frac{1}{4}\log\left(1-x^2\right)+\frac{1}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^4),x]
```

```
[Out] -1/4*Log[1 - x^2] + Log[1 + x^2]/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(6) = 12$.
time = 0.07, size = 22, normalized size = 2.75

method	result	size
meijerg	$\frac{\operatorname{arctanh}(x^2)}{2}$	7
risch	$\frac{\ln(x^2+1)}{4} - \frac{\ln(x^2-1)}{4}$	18
default	$-\frac{\ln(x-1)}{4} - \frac{\ln(x+1)}{4} + \frac{\ln(x^2+1)}{4}$	22
norman	$-\frac{\ln(x-1)}{4} - \frac{\ln(x+1)}{4} + \frac{\ln(x^2+1)}{4}$	22
parallelrisch	$-\frac{\ln(x-1)}{4} - \frac{\ln(x+1)}{4} + \frac{\ln(x^2+1)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(x-1)-1/4*ln(x+1)+1/4*ln(x^2+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.37, size = 17, normalized size = 2.12

$$\frac{1}{4}\log\left(x^2+1\right) - \frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.56, size = 17, normalized size = 2.12

$$\frac{1}{4}\log\left(x^2+1\right) - \frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.03, size = 15, normalized size = 1.88

$$-\frac{\log(x^2-1)}{4}+\frac{\log(x^2+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+1),x)
```

```
[Out] -log(x**2 - 1)/4 + log(x**2 + 1)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.
time = 0.54, size = 18, normalized size = 2.25

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{4}\log\left(|x^2-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2 + 1) - 1/4*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.07, size = 6, normalized size = 0.75

$$\frac{\operatorname{atanh}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x^4 - 1),x)
```

```
[Out] atanh(x^2)/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 33, normalized size = 4.12

$$\frac{\ln(x^2+1)}{4}+\frac{\ln\left((x^2+1)^2-4x^2\right)}{8}-\frac{\arctan(x^2-1)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(x/(-x^4+1),x)
```

```
[Out] 1/4*ln(x^2+1)+1/8*ln((x^2+1)^2-4*x^2)-1/4*arctan(x^2-1)
```

3.95 $\int \sqrt{12-3x^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+2\sqrt{3}\arcsin\left(\frac{x}{2}\right)$$

```
[Out] 1/2*3^(1/2)*x*(-x^2+4)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 222}

$$2\sqrt{3}\arcsin\left(\frac{x}{2}\right)+\frac{1}{2}\sqrt{3}\sqrt{4-x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[12 - 3*x^2],x]
```

```
[Out] (Sqrt[3]*x*Sqrt[4 - x^2])/2 + 2*Sqrt[3]*ArcSin[x/2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+6\int\frac{1}{\sqrt{12-3x^2}}\,dx\\&=\frac{1}{2}\sqrt{3}x\sqrt{4-x^2}+2\sqrt{3}\arcsin\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.23

$$\frac{1}{2}\sqrt{3}\left(x\sqrt{4-x^2}-8\arctan\left(\frac{\sqrt{4-x^2}}{2+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[12 - 3*x^2],x]
[Out] (Sqrt[3]*(x*Sqrt[4 - x^2] - 8*ArcTan[Sqrt[4 - x^2]/(2 + x)]))/2
```

Maple [A]
time = 0.09, size = 23, normalized size = 0.66

method	result	size
default	$\frac{x\sqrt{-3x^2+12}}{2}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	23
risch	$-\frac{3x(x^2-4)}{2\sqrt{-3x^2+12}}+2\sqrt{3}\,\arcsin\left(\frac{x}{2}\right)$	28
meijerg	$\frac{i\sqrt{3}\left(-i\sqrt{\pi}\,x\sqrt{-\frac{x^2}{4}+1}-2i\sqrt{\pi}\,\arcsin\left(\frac{x}{2}\right)\right)}{\sqrt{\pi}}$	37
pseudoelliptic	$\frac{x\sqrt{-3x^2+12}}{2}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{-3x^2+12}\,\sqrt{3}}{3x}\right)$	37
trager	$\frac{x\sqrt{-3x^2+12}}{2}+2 RootOf\left(\_Z^2+3\right)\ln\left(RootOf\left(\_Z^2+3\right)\sqrt{-3x^2+12}+3x\right)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(-3*x^2+12)^(1/2)+2*3^(1/2)*arcsin(1/2*x)
```

Maxima [A]
time = 0.51, size = 22, normalized size = 0.63

$$\frac{1}{2}\sqrt{-3\,x^2+12}x+2\,\sqrt{3}\arcsin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+12)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-3*x^2 + 12)*x + 2*sqrt(3)*arcsin(1/2*x)
```

Fricas [A]
time = 0.58, size = 36, normalized size = 1.03

$$\frac{1}{2}\sqrt{-3\,x^2+12}x-2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+12}}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+12)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-3*x^2 + 12)*x - 2*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-3*x^2 + 12)/x)
```

Sympy [A]
time = 0.18, size = 27, normalized size = 0.77

$$\frac{\sqrt{3}x\sqrt{4-x^2}}{2}+2\sqrt{3}\,\mathrm{asin}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+12)**(1/2),x)
```

```
[Out] sqrt(3)*x*sqrt(4 - x**2)/2 + 2*sqrt(3)*asin(x/2)
```

Giac [A]
time = 0.54, size = 23, normalized size = 0.66

$$\frac{1}{2}\sqrt{3}\left(\sqrt{-x^2+4}x+4\arcsin\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+12)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*(sqrt(-x^2 + 4)*x + 4*arcsin(1/2*x))
```

Mupad [B]
time = 0.00, size = 25, normalized size = 0.71

$$2\sqrt{3}\,\mathrm{asin}\left(\frac{x}{2}\right)+\frac{\sqrt{3}\,x\,\sqrt{4-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12 - 3*x^2)^(1/2),x)
```

```
[Out] 2*3^(1/2)*asin(x/2) + (3^(1/2)*x*(4 - x^2)^(1/2))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 25, normalized size = 0.71

$$\frac{2\sqrt{3}\arcsin\left(\frac{\sqrt{3}\,x}{2}\right)}{3}+\frac{x\sqrt{-3x^2+12}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((-3*x^2+12)^(1/2),x)
```

```
[Out] 2/3*3^(1/2)*arcsin(1/2*3^(1/2)*x)+1/2*x*(-3*x^2+12)^(1/2)
```

3.96 $\int \sec^5(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{5}\sec^5(x)+\frac{\sec^7(x)}{7}$$

```
[Out] -1/5*sec(x)^5+1/7*sec(x)^7
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2686, 14}

$$\frac{\sec^7(x)}{7}-\frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5*Tan[x]^3,x]
```

```
[Out] -1/5*Sec[x]^5 + Sec[x]^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int x^4\left(-1+x^2\right)\,dx,x,\sec(x)\right)\\
&= \text{Subst}\left(\int \left(-x^4+x^6\right)\,dx,x,\sec(x)\right)\\
&= -\frac{1}{5}\sec^5(x)+\frac{\sec^7(x)}{7}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{5}\sec^5(x)+\frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5*Tan[x]^3,x]
```

```
[Out] -1/5*Sec[x]^5 + Sec[x]^7/7
```

Maple [A]
time = 3.46, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{\left(\sec^5(x)\right)}{5}+\frac{\left(\sec^7(x)\right)}{7}$	14
default	$-\frac{\left(\sec^5(x)\right)}{5}+\frac{\left(\sec^7(x)\right)}{7}$	14
risch	$-\frac{32\left(7\,\mathrm{e}^{9ix}-6\,\mathrm{e}^{7ix}+7\,\mathrm{e}^{5ix}\right)}{35\left(\mathrm{e}^{2ix}+1\right)^{7}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5*tan(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*sec(x)^5+1/7*sec(x)^7
```

Maxima [A]
time = 0.32, size = 14, normalized size = 0.82

$$-\frac{7\,\cos\left(x\right)^{2}-5}{35\,\cos\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/35*(7*cos(x)^2 - 5)/cos(x)^7
```

Fricas [A]
time = 0.59, size = 14, normalized size = 0.82

$$-\frac{7\,\cos\left(x\right)^{2}-5}{35\,\cos\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*cos(x)^2 - 5)/cos(x)^7
```

Sympy [A]
time = 0.08, size = 14, normalized size = 0.82

$$\frac{5-7\cos^{2}\left(x\right)}{35\cos^{7}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5*tan(x)**3,x)
```

```
[Out] (5 - 7*cos(x)**2)/(35*cos(x)**7)
```

Giac [A]
time = 0.52, size = 14, normalized size = 0.82

$$-\frac{7\cos(x)^2-5}{35\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/35*(7*cos(x)^2 - 5)/cos(x)^7
```

Mupad [B]
time = 0.50, size = 13, normalized size = 0.76

$$\frac{1}{7\cos(x)^7}-\frac{1}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/cos(x)^5,x)
```

```
[Out] 1/(7*cos(x)^7) - 1/(5*cos(x)^5)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 19, normalized size = 1.12

$$\frac{(\sec^{10}(x))}{10}-\frac{(\sec^{8}(x))}{4}+\frac{(\sec^{6}(x))}{6}$$

Warning: Unable to verify antiderivative.

```
[In] int(sec(x)^5*tan(x)^3,x)
```

```
[Out] 1/10*sec(x)^10-1/4*sec(x)^8+1/6*sec(x)^6
```

3.97 $\int \frac{1}{1-\sin(x)} \, dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

[Out] cos(x)/(1-sin(x))

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sin[x])^(-1),x]

[Out] Cos[x]/(1 - Sin[x])

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{Integral} = \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.01, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sin[x])^(-1),x]

[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])

Maple [A]
time = 0.04, size = 11, normalized size = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
norman	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
parallelrisch	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(tan(1/2*x)-1)
```

Maxima [A]
time = 0.32, size = 15, normalized size = 1.36

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A]
time = 0.57, size = 17, normalized size = 1.55

$$\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A]
time = 0.26, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

Giac [A]
time = 0.48, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 16, normalized size = 1.45

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1-sin(x)),x)
```

```
[Out] 1/2*ln(tan(1/2*x))+1/4*tan(1/2*x)^2
```

3.98 $\int \frac{1}{x\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(1/2*(x^2-2)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 65, 209}

$$\frac{\arctan\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-2 + x^2]),x]
```

```
[Out] ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/Sqrt[2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{Integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-2+xx}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-2+x^2}\right)$$
$$= \frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-2 + x^2]),x]
[Out] ArcTan[Sqrt[-2 + x^2]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.14, size = 18, normalized size = 0.82

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}}{\sqrt{x^2-2}}\right)}{2}$	18
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{x^2-2}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}$	19
trager	$\frac{\mathit{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\mathit{RootOf}\left(\_Z^2+2\right)+\sqrt{x^2-2}}{x}\right)}{2}$	28
meijerg	$\frac{\sqrt{2}\,\sqrt{-\mathrm{signum}\left(-1+\frac{x^2}{2}\right)}\left(\left(-3\ln(2)+2\ln(x)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1-\frac{x^2}{2}}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(-1+\frac{x^2}{2}\right)}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*2^(1/2)*arctan(1/(x^2-2)^(1/2)*2^(1/2))
```

Maxima [A]
time = 0.42, size = 14, normalized size = 0.64

$$-\frac{1}{2}\sqrt{2}\arcsin\left(\frac{\sqrt{2}}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arcsin(sqrt(2)/abs(x))
```

Fricas [A]
time = 0.57, size = 24, normalized size = 1.09

$$\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt(x^2 - 2))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.62, size = 39, normalized size = 1.77

$$\begin{cases}\frac{\sqrt{2}i\operatorname{acosh}\left(\frac{\sqrt{2}}{x}\right)}{2} & \text{for}\ \frac{1}{|x^2|}>\frac{1}{2}\\ -\frac{\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{x}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-2)**(1/2),x)
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/x)/2, 1/Abs(x**2) > 1/2), (-sqrt(2)*asin
(sqrt(2)/x)/2, True))
```

Giac [A]
time = 0.46, size = 18, normalized size = 0.82

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2))
```

Mupad [B]
time = 0.12, size = 18, normalized size = 0.82

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x^2-2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 2)^(1/2)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x^2 - 2)^(1/2))/2))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 14, normalized size = 0.64

$$-\frac{\sqrt{2}\,\sqrt{-x^2+3}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/x/(x^2-2)^(1/2),x)
```

```
[Out] -1/2*2^(1/2)*(-x^2+3)^(1/2)
```

3.99 $\int \log(x^2)\,dx$

Optimal. Leaf size=10

$$-2x+x\log(x^2)$$

```
[Out] -2*x+x*ln(x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2332}

$$x\log(x^2)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^2],x]
```

```
[Out] -2*x + x*Log[x^2]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{Integral} = -2x+x\log(x^2)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-2x+x\log(x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^2],x]
```

```
[Out] -2*x + x*Log[x^2]
```

Maple [A]
time = 0.01, size = 11, normalized size = 1.10

method	result	size

default	$-2x + x\ln(x^2)$	11
norman	$-2x + x\ln(x^2)$	11
risch	$-2x + x\ln(x^2)$	11
parallelrisch	$-2x + x\ln(x^2)$	11
parts	$-2x + x\ln(x^2)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2),x,method=_RETURNVERBOSE)
[Out] -2*x+x*ln(x^2)
```

Maxima [A]
time = 0.39, size = 10, normalized size = 1.00

$$x\log\left(x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2),x, algorithm="maxima")
[Out] x*log(x^2) - 2*x
```

Fricas [A]
time = 0.57, size = 10, normalized size = 1.00

$$x\log\left(x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2),x, algorithm="fricas")
[Out] x*log(x^2) - 2*x
```

Sympy [A]
time = 0.04, size = 8, normalized size = 0.80

$$x\log\left(x^2\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2),x)
[Out] x*log(x**2) - 2*x
```

Giac [A]
time = 0.46, size = 10, normalized size = 1.00

$$x\log\left(x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2),x, algorithm="giac")
```

```
[Out] x*log(x^2) - 2*x
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.80

$$x\left(\ln\left(x^2\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2),x)
```

```
[Out] x*(log(x^2) - 2)
```

Chatgpt [A]
time = 1.00, size = 10, normalized size = 1.00

$$x\ln\left(x^2\right)-2x$$

Antiderivative was successfully verified.

```
[In] int(ln(x^2),x)
```

```
[Out] x*ln(x^2)-2*x
```

3.100 $\int \sin\left(\sqrt[3]{x}\right)\,dx$

Optimal. Leaf size=35

$$6\cos\left(\sqrt[3]{x}\right)-3x^{2/3}\cos\left(\sqrt[3]{x}\right)+6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)$$

```
[Out] 6*cos(x^(1/3))-3*x^(2/3)*cos(x^(1/3))+6*x^(1/3)*sin(x^(1/3))
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3442, 3377, 2718}

$$-3x^{2/3}\cos\left(\sqrt[3]{x}\right)+6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)+6\cos\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(1/3)],x]
```

```
[Out] 6*Cos[x^(1/3)] - 3*x^(2/3)*Cos[x^(1/3)] + 6*x^(1/3)*Sin[x^(1/3)]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 3\text{Subst}\left(\int x^2\sin(x)\,dx,x,\sqrt[3]{x}\right)\\
&= -3x^{2/3}\cos\left(\sqrt[3]{x}\right)+6\text{Subst}\left(\int x\cos(x)\,dx,x,\sqrt[3]{x}\right)\\
&= -3x^{2/3}\cos\left(\sqrt[3]{x}\right)+6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)-6\text{Subst}\left(\int \sin(x)\,dx,x,\sqrt[3]{x}\right)\\
&= 6\cos\left(\sqrt[3]{x}\right)-3x^{2/3}\cos\left(\sqrt[3]{x}\right)+6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.83

$$-3(-2+x^{2/3})\cos\left(\sqrt[3]{x}\right)+6\sqrt[3]{x}\sin\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(1/3)],x]
[Out] -3*(-2 + x^(2/3))*Cos[x^(1/3)] + 6*x^(1/3)*Sin[x^(1/3)]
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.74

method	result	size
derivativedivides	$6\cos\left(x^{\frac{1}{3}}\right)-3x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right)+6x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$	26
default	$6\cos\left(x^{\frac{1}{3}}\right)-3x^{\frac{2}{3}}\cos\left(x^{\frac{1}{3}}\right)+6x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$	26
meijerg	$12\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^{\frac{2}{3}}}{2}+1\right)\cos\left(x^{\frac{1}{3}}\right)}{2\sqrt{\pi}}+\frac{x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)}{2\sqrt{\pi}}\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3)),x,method=_RETURNVERBOSE)
[Out] 6*cos(x^(1/3))-3*x^(2/3)*cos(x^(1/3))+6*x^(1/3)*sin(x^(1/3))
```

Maxima [A]
time = 0.37, size = 21, normalized size = 0.60

$$-3\left(x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)+6\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3)),x, algorithm="maxima")
[Out] -3*(x^(2/3) - 2)*cos(x^(1/3)) + 6*x^(1/3)*sin(x^(1/3))
```

Fricas [A]
time = 0.59, size = 21, normalized size = 0.60

$$-3\left(x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)+6\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*(x^(2/3) - 2)*cos(x^(1/3)) + 6*x^(1/3)*sin(x^(1/3))
```

Sympy [A]
time = 0.17, size = 34, normalized size = 0.97

$$-3x^{\frac{2}{3}} \cos\left(\sqrt[3]{x}\right) + 6\sqrt[3]{x} \sin\left(\sqrt[3]{x}\right) + 6\cos\left(\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/3)),x)
[Out] -3*x**(2/3)*cos(x**(1/3)) + 6*x**(1/3)*sin(x**(1/3)) + 6*cos(x**(1/3))
```

Giac [A]
time = 0.43, size = 21, normalized size = 0.60

$$-3\left(x^{\frac{2}{3}} - 2\right)\cos\left(x^{\frac{1}{3}}\right) + 6\,x^{\frac{1}{3}}\sin\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/3)),x, algorithm="giac")
[Out] -3*(x^(2/3) - 2)*cos(x^(1/3)) + 6*x^(1/3)*sin(x^(1/3))
```

Mupad [B]
time = 0.21, size = 21, normalized size = 0.60

$$6\,x^{1/3}\,\sin\left(x^{1/3}\right) - 3\,\cos\left(x^{1/3}\right)\,\left(x^{2/3} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/3)),x)
[Out] 6*x^(1/3)*sin(x^(1/3)) - 3*cos(x^(1/3))*(x^(2/3) - 2)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Antiderivative was successfully verified.

```
[In] int(sin(x^(1/3)),x)
[Out] not solved
```

3.101 $\int e^{1+x-x^2}(1-2x)\,dx$

Optimal. Leaf size=10

$$e^{1+x-x^2}$$

```
[Out] exp(-x^2+x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2268}

$$e^{-x^2+x+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(1 + x - x^2)*(1 - 2*x),x]
[Out] E^(1 + x - x^2)
```

Rule 2268

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[b*e - 2*c*d, 0]
```

Rubi steps

$$\text{Integral} = e^{1+x-x^2}$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 1.00

$$e^{1+x-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(1 + x - x^2)*(1 - 2*x),x]
[Out] E^(1 + x - x^2)
```

Maple [A]
time = 0.02, size = 10, normalized size = 1.00

method	result	size

gosper	e^{-x^2+x+1}	10
derivativedivides	e^{-x^2+x+1}	10
default	e^{-x^2+x+1}	10
norman	e^{-x^2+x+1}	10
risch	e^{-x^2+x+1}	10
parallelrisch	e^{-x^2+x+1}	10
parts	$-\sqrt{\pi}\,\mathrm{e}^{\frac{5}{4}}\,\mathrm{erf}\left(x-\frac{1}{2}\right)x+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{5}{4}}\,\mathrm{erf}\left(x-\frac{1}{2}\right)}{2}+\frac{\mathrm{e}^{\frac{5}{4}}\left(2\,\mathrm{erf}\left(x-\frac{1}{2}\right)x\sqrt{\pi}-\mathrm{erf}\left(x-\frac{1}{2}\right)\sqrt{\pi}+2\,\mathrm{e}^{-\left(x-\frac{1}{2}\right)^{2}}\right)}{2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x^2+x+1)*(1-2*x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(-x^2+x+1)
```

Maxima [A]
time = 0.42, size = 9, normalized size = 0.90

$$e^{\left(-x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x^2+x+1)*(1-2*x),x, algorithm="maxima")
```

```
[Out] e^(-x^2 + x + 1)
```

Fricas [A]
time = 0.58, size = 9, normalized size = 0.90

$$e^{\left(-x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x^2+x+1)*(1-2*x),x, algorithm="fricas")
```

```
[Out] e^(-x^2 + x + 1)
```

Sympy [A]
time = 0.04, size = 7, normalized size = 0.70

$$e^{-x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x**2+x+1)*(1-2*x),x)
```

```
[Out] exp(-x**2 + x + 1)
```

Giac [**A**]
time = 0.51, size = 9, normalized size = 0.90

$$e^{\left(-x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x^2+x+1)*(1-2*x),x, algorithm="giac")
```

```
[Out] e^(-x^2 + x + 1)
```

Mupad [**B**]
time = 0.20, size = 11, normalized size = 1.10

$$\mathrm{e}\,\mathrm{e}^{-x^2}\,\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(x - x^2 + 1)*(2*x - 1),x)
```

```
[Out] exp(1)*exp(-x^2)*exp(x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 28, normalized size = 2.80

$$-\frac{(1-2x)\,\mathrm{e}^{-x^2+x+1}}{2}-\frac{\mathrm{e}^{-x^2+x+1}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((1-2*x)*exp(-x^2+x+1),x)
```

```
[Out] -1/2*(1-2*x)*exp(-x^2+x+1)-1/2*exp(-x^2+x+1)
```

3.102 $\int e^{\sqrt{x}}\sqrt{x}\,dx$

Optimal. Leaf size=34

$$4e^{\sqrt{x}}-4e^{\sqrt{x}}\sqrt{x}+2e^{\sqrt{x}}x$$

```
[Out] 4*exp(x^(1/2))-4*x^(1/2)*exp(x^(1/2))+2*exp(x^(1/2))*x
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2247, 2207, 2225}

$$2e^{\sqrt{x}}x-4e^{\sqrt{x}}\sqrt{x}+4e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sqrt[x]*Sqrt[x],x]
```

```
[Out] 4*E^Sqrt[x] - 4*E^Sqrt[x]*Sqrt[x] + 2*E^Sqrt[x]*x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2247

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k/d, Subst[Int[x^(k*(m + 1
) - 1)*F^(a + b*x^(k*n)), x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c
, d, m, n}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[0, (m + 1)/n, 5] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 2\text{Subst}\left(\int e^x x^2\,dx, x, \sqrt{x}\right)\\ &= 2e^{\sqrt{x}}x - 4\text{Subst}\left(\int e^x x\,dx, x, \sqrt{x}\right)\\ &= -4e^{\sqrt{x}}\sqrt{x} + 2e^{\sqrt{x}}x + 4\text{Subst}\left(\int e^x\,dx, x, \sqrt{x}\right)\\ &= 4e^{\sqrt{x}} - 4e^{\sqrt{x}}\sqrt{x} + 2e^{\sqrt{x}}x\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.00, size = 11, normalized size = 0.32

$$2\Gamma\left(3, -\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sqrt[x]*Sqrt[x],x]
[Out] 2*Gamma[3, -Sqrt[x]]
```

Maple [A]
time = 0.01, size = 24, normalized size = 0.71

method	result	size
meijerg	$-4+\frac{2\left(3x-6\sqrt{x}+6\right)\mathrm{e}^{\sqrt{x}}}{3}$	19
derivativedivides	$4\,\mathrm{e}^{\sqrt{x}}-4\sqrt{x}\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}x$	24
default	$4\,\mathrm{e}^{\sqrt{x}}-4\sqrt{x}\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}x$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*exp(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 4*exp(x^(1/2))-4*x^(1/2)*exp(x^(1/2))+2*exp(x^(1/2))*x
```

Maxima [A]
time = 0.42, size = 14, normalized size = 0.41

$$2\left(x - 2\sqrt{x} + 2\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(x - 2*sqrt(x) + 2)*e^sqrt(x)
```

Fricas [A]
time = 0.57, size = 14, normalized size = 0.41

$$2\left(x-2\sqrt{x}+2\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(x - 2*sqrt(x) + 2)*e^sqrt(x)
```

Sympy [A]
time = 0.07, size = 31, normalized size = 0.91

$$-4\sqrt{x}e^{\sqrt{x}}+2xe^{\sqrt{x}}+4e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*exp(x**(1/2)),x)
[Out] -4*sqrt(x)*exp(sqrt(x)) + 2*x*exp(sqrt(x)) + 4*exp(sqrt(x))
```

Giac [A]
time = 0.48, size = 14, normalized size = 0.41

$$2\left(x-2\sqrt{x}+2\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(x - 2*sqrt(x) + 2)*e^sqrt(x)
```

Mupad [B]
time = 0.12, size = 23, normalized size = 0.68

$$4\,\mathrm{e}^{\sqrt{x}}+2\,x\,\mathrm{e}^{\sqrt{x}}-4\,\sqrt{x}\,\mathrm{e}^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*exp(x^(1/2)),x)
[Out] 4*exp(x^(1/2)) + 2*x*exp(x^(1/2)) - 4*x^(1/2)*exp(x^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 11, normalized size = 0.32

$$2\left(\sqrt{x}-1\right)\mathrm{e}^{\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(1/2)*exp(x^(1/2)),x)
[Out] 2*(x^(1/2)-1)*exp(x^(1/2))
```

3.103 $\int \cos(3x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

```
[Out] 1/2*cos(x)-1/10*cos(5*x)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {4369}

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\text{Integral}=\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Maple [A]
time = 0.08, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
risch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
parallelrisch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}-\frac{2}{5}$	13
norman	$\frac{-\frac{4\left(\tan^2(x)\right)}{5}-\frac{4\left(\tan^2\left(\frac{3x}{2}\right)\right)}{5}+\frac{12\tan(x)\tan\left(\frac{3x}{2}\right)}{5}}{(1+\tan^2(x))\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cos(x)-1/10*cos(5*x)
```

Maxima [A]
time = 0.40, size = 11, normalized size = 0.73

$$-\frac{1}{10}\cos(5x)+\frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

Fricas [A]
time = 0.59, size = 13, normalized size = 0.87

$$-\frac{8}{5}\cos(x)^5+2\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 2*cos(x)^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.14, size = 26, normalized size = 1.73

$$\frac{3\sin(2x)\sin(3x)}{5}+\frac{2\cos(2x)\cos(3x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x),x)
```

```
[Out] 3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5
```

Giac [**A**]
time = 0.50, size = 11, normalized size = 0.73

$$-\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x),x, algorithm="giac")
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

Mupad [**B**]
time = 0.04, size = 13, normalized size = 0.87

$$2\cos(x)^3 - \frac{8\cos(x)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(2*x),x)
[Out] 2*cos(x)^3 - (8*cos(x)^5)/5
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 12, normalized size = 0.80

$$\frac{(3(\sin^2(x)) - 1)\cos(x)}{5}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(2*x)*cos(3*x),x)
[Out] 1/5*(3*sin(x)^2-1)*cos(x)
```

3.104 $\int (1+2\sin(x))\,dx$

Optimal. Leaf size=6

$$x-2\cos(x)$$

[Out] x-2*cos(x)

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2718}

$$x-2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[1 + 2*Sin[x],x]
```

```
[Out] x - 2*Cos[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= x+2\int \sin(x)\,dx\\ &= x-2\cos(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x-2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + 2*Sin[x],x]
```

```
[Out] x - 2*Cos[x]
```

Maple [A]
time = 0.03, size = 7, normalized size = 1.17

method	result	size

default	$x - 2\cos(x)$	7
risch	$x - 2\cos(x)$	7
parts	$x - 2\cos(x)$	7
parallelrisch	$-2 - 2\cos(x) + x$	8
norman	$\frac{x + x(\tan^2(\frac{x}{2})) - 4}{1 + \tan^2(\frac{x}{2})}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+2*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] x-2*cos(x)
```

Maxima [A]
time = 0.44, size = 6, normalized size = 1.00

$$x - 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+2*sin(x),x, algorithm="maxima")
```

```
[Out] x - 2*cos(x)
```

Fricas [A]
time = 0.59, size = 6, normalized size = 1.00

$$x - 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+2*sin(x),x, algorithm="fricas")
```

```
[Out] x - 2*cos(x)
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.83

$$x - 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+2*sin(x),x)
```

```
[Out] x - 2*cos(x)
```

Giac [A]
time = 0.43, size = 6, normalized size = 1.00

$$x - 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+2*sin(x),x, algorithm="giac")
```

```
[Out] x - 2*cos(x)
```

Mupad [B]
time = 0.12, size = 6, normalized size = 1.00

$$x - 2\ \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*sin(x) + 1,x)
```

```
[Out] x - 2*cos(x)
```

Chatgpt [A]
time = 1.00, size = 6, normalized size = 1.00

$$x - 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] int(1+2*sin(x),x)
```

```
[Out] x-2*cos(x)
```

3.105 $\int (1-x)^{2014} x \, dx$

Optimal. Leaf size=23

$$-\frac{(1-x)^{2015}}{2015}+\frac{(1-x)^{2016}}{2016}$$

```
[Out] -1/2015*(1-x)^2015+1/2016*(1-x)^2016
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{(1-x)^{2016}}{2016}-\frac{(1-x)^{2015}}{2015}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^2014*x,x]
```

```
[Out] -1/2015*(1 - x)^2015 + (1 - x)^2016/2016
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left((1-x)^{2014}-(1-x)^{2015}\right) dx\\ &= -\frac{(1-x)^{2015}}{2015}+\frac{(1-x)^{2016}}{2016}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12138 vs. 2(23) = 46.
time = 0.02, size = 12138, normalized size = 527.74

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^2014*x,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10076 vs. $2(19) = 38$.
time = 8.49, size = 10077, normalized size = 438.13

method	result	size
gosper	Expression too large to display	10076
default	Expression too large to display	10077
risch	Expression too large to display	10077
parallelrisch	Expression too large to display	10077

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1-x)^2014,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10076 vs. $2(15) = 30$.
time = 2.63, size = 10076, normalized size = 438.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^2014,x, algorithm="maxima")
```

```
[Out] 1/2016*x^2016 - 2014/2015*x^2015 + 2013/2*x^2014 - 2026084/3*x^2013 + 13588
26667/4*x^2012 - 136629987582*x^2011 + 1373131035323509/30*x^2010 - 9195398
5549170536/7*x^2009 + 26377651026988133103/8*x^2008 - 661749155854244491587
4/9*x^2007 + 2949929948352647316611117/2*x^2006 - 134422910799097740606580l6
4/5*x^2005 + 538766892328182148445244548 23/12*x^2004 - 69176270279062348945
1562620638*x^2003 + 2571973087266166342850029070691063/26*x^2002 - 39588591
24827482426775656940394 0400/3*x^2001 + 131961937837090040663501674882660163
383/80*x^2000 - 19396459391350520940299292765173 3959950*x^1999 + 3875411615
5980707530148947755981227393 5075/18*x^1998 - 2262922530915969121458419278 66
1196219696900*x^1997 + 9033584475959102925277719721919224105426 59825/4*x^19
96 - 2145475773979272745021812337899023630563 0977010*x^1995 + 3889161470317
16864628102563094663281706298266 0525/2*x^1994 - 168502105628944447818705199
67004418713049353335 3400*x^1993 + 111885369941148961252482398045233357 98487
0743212666075/8*x^1992 - 111381857678631284360055355209228691542106081304 48
65230*x^1991 + 1704998775459180790734328767331702095659848281669175783 71/2*
x^1990 - 73476903655441985945968330316081626303723736209989757096414 0/117*x
^1989 + 17835414835995795079540559282458474093945703402107243496405 35/4*x^1
988 - 3055082784747576760477564985383679639490542834878909087548189 0*x^1987
 + 121347857443982563149967869360779278103528716985047520932146317 35/6*x^19
86 - 12950292252359328049213699468709059079824445040862703694844269958 4*x^1
```

985 + 796494572340791669224338603159427771066474545536265964567956747579715
5/992*x^1984 - 144744165624118561261874947183971562119263975542371901322485
6987762340/3*x^1983 + 281257709056525543089735253629024316307074166614499962
135634978911861255*x^1982 - 15919182276095284925068629695414983283231118143
72651235402040612137475080*x^1981 + 157599864333388078093318512094129305678
2727079281496851166395104490693223777/18*x^1980 - 4683033812006962024438255
993532176273777170532620461943334529407905781402420*x^1979 + 24376417138393
9582809061001601068990795547409624002219219883316766423225728865*x^1978 - 3
707089565584711138545323444480070738322008193622357674343613479298216558155
6720/3*x^1977 + 31742564123997639240757908526821868866411524435554862316447
839037129021749036585525/52*x^1976 - 29405041309901956341188419072725681104
134131163443548404833409099760693787449612268*x^1975 + 41461097606839703910
04500755905823687091180545350679873368223491236430795491757775525/3*x^1974
- 6341296477833618458583972079875122380745913656825226355055630034376410525
6910301716600*x^1973 + 5684106108463777496364455720240876368514714260586800
253709270268295316442312451914078575/2*x^1972 - 112033702559129484368221675
0899064746395773848203857184049716971541571645243688771886809300/9*x^1971 +
53310709461735581641917056651063408558603844279169439438453156589650480105
62527907902040665*x^1970 - 223337786925829485322981480991438350221372508041
201117237722302034525923462936129127215408800*x^1969 + 73254775197564099825
039778493724892741313585299056307138160140869975075852658475240572087079675
/8*x^1968 - 257307331378089265618294271999354580767215753792536830264152936
9583213434895514150007213443740700/7*x^1967 + 14453316646020616414051419806
56394934784635673375952175816948183474989874139452259042547417732285*x^196
6 - 55687764419711927947641849414937962138091803555309959228824227239335771
1099559188691208229092310776*x^1965 + 4206566960232898345464498815975895477
5468201354159354767262170532890818036890454346362487193259926655/2*x^1964 -
10127108905672074866716134495513810074049503443438355901587873018991904643
082375127060735008804049547340/13*x^1963 + 25473567322183688080684516663278
0310468643604667004332685617284023675830425396294703455768279984362513795/9
*x^1962 - 10091646975497330900852886078425142186921518650235743476881976089
90183198035912371247439438789779967687120*x^1961 + 988981146158764563555753
732295712954085089773602729130230773532270173159028459930987702066483515289
307875639/28*x^1960 - 36417548129421635926211441537962552722093013183415290
98880465874988579919304712880932120180334504864257742540/3*x^1959 + 4098019
576899248565078726672328787585605458199813607487910274234291924411836319599
0927194823828420896033237685*x^1958 - 1359291901349214321338405479553195610
468798911828009406507555490127093365391727648159674832991084850660654090760
*x^1957 + 26587742641045533203015605155492614764514029472007644011689592607
9723975200408973392729321685058600107994356274635/6*x^1956 - 14201918924419
578912795063139046092470298515642771168727535379857992410473234775706432614
88664366848376947188162908*x^1955 + 447589392099830459773388933347137149207
859948603354912059082666379128770710327827503268426948174879427703941735119
15*x^1954 - 387119963732950144741800058205015258867531310395125880366224394
277687979653854440427570625898672156578726484398554499520/279*x^1953 + 1354

```
225754277894499676937345006632528266873746430900140410227088281348549726611
191833845296592767838728398462456726127035/32*x...
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^2014,x, algorithm="fricas")
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12024 vs. $2(12) = 24$.
time = 3.22, size = 12024, normalized size = 522.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)**2014,x)
```

```
[Out] x**2016/2016 - 2014*x**2015/2015 + 2013*x**2014/2 - 2026084*x**2013/3 + 135
8826667*x**2012/4 - 136629987582*x**2011 + 137313103532509*x**2010/30 - 91
953985549170536*x**2009/7 + 26377651026988133103*x**2008/8 - 66174915585424
44915874*x**2007/9 + 2949929948352647316611117*x**2006/2 - 13442291079909774
0606580164*x**2005/5 + 53876689232818214844524454823*x**2004/12 - 691762702
790623489451562620638*x**2003 + 2571973087266166342850029070691063*x**2002/
26 - 39588591248274824267756569403940400*x**2001/3 + 1319619378370900406635
01674882660163383*x**2000/80 - 193964593913505209402992927651733959950*x**1
999 + 387541161559807075301489477559812273935075*x**1998/18 - 2262922530915
969121458419278661196219696900*x**1997 + 9033584475959102925277719721919224
10542659825*x**1996/4 - 21454757739792727450218123378990236305630977010*x**
1995 + 3889161470317168646281025630946632817062982660525*x**1994/2 - 168502
105628944447818705199670044187130493533353400*x**1993 + 1118853699411489612
52482398045233357984870743212666075*x**1992/8 - 111381857678631284360055355
2092286915421060813044865230*x**1991 + 170499877545918079073432876733170209
565984828166917578371*x**1990/2 - 73476903655441985945968330316081626303723
7362099897570964140*x**1989/117 + 17835414835995795079540559282458474093945
70340210724349640535*x**1988/4 - 305508278474757676047756498538367963949054
28348789090875481890*x**1987 + 12134785744398256314996786936077927810352871
698504752093214631735*x**1986/6 - 12950292252359328049213699468709059079824
4450408627036948442699584*x**1985 + 796494572340791669224338603159427771066
4745455362659645679567475797155*x**1984/992 - 14474416562411856126187494718
39715621192639755423719013224856987762340*x**1983/3 + 281257709056525543089
73525362902431630707416661449962135634978911861255*x**1982 - 15919182276095
```

284925068629695414983283231118143726512354020406121374750800*x**1981 + 15759
98643333880780933185120941293056782727079281496851166395104490693223777*x**
1980/18 - 468303381200696202443825599353217627377717053262046194333345294079
05781402420*x**1979 + 24376417138393958280906100160106899079554740962400221
921988331676642322572886 5*x**1978 - 37070895655847111385453234444800707383 2
200819362235767434361347929821655815567 20*x**1977/3 + 317425641239976392407
579085268218688664115244355548623164478390371290217490365855 25*x**1976/52 -
294050413099019563411884190727256811041341311634435484048334090997606937 87
449612268*x**1975 + 4146109760683970391004500755905823687091180545350679873
36822349123643079549175777552 5*x**1974/3 - 634129647783361845858397207987 51
22380745913656825226355055630034376410525691030171660 0*x**1973 + 5684106108
463777496364455720240876368514714260586800253709270268295316442312451914078
575*x**1972/2 - 112033702559129484368221675089906474639577384820385718404 97
169715415716452436887718868093 00*x**1971/9 + 5331070946173558164191705665 10
63408558603844279169439438453156589650480105625279079020406 65*x**1970 - 223
33778692582948532298148099143835022137250804120111723772230203452592346293 6
129127215408800*x**1969 + 73254775197564099825039778493724892741313585299 05
630713816014086997507585265847524057208707967 5*x**1968/8 - 2573073313780892
65618294271999354580767215753792536830264152936958321343489551415000721344 3
740700*x**1967/7 + 144533166460206164140514198065639493478463567337595217 58
169481834749898741394522590424547417732285*x**1966 - 556877644197119279476 4
18494149379621380918035553099592288242272393357711099559188691208229092310 7
76*x**1965 + 420656696023289834546449881597589547754682013541593547672621 70
532890818036890454346362487193259926655*x**1964/2 - 1012710890567207486671 6
134495513810074049503443438355901587873018991904643082375127060735008804049
547340*x**1963/13 + 2547356732218368808068451666327803104686436046670043326
85617284023675830425396294703455768279984362513795*x**1962/9 - 100916469754
973309008528860784251421869215186502357434768819760899018319803591237124743
94387897799676871 20*x**1961 + 98898114615876456355575373229571295408508977 3
6027291302307735322701731590284599309877020664835152893078756 39*x**1960/28
- 364175481294216359262114415379625527220930131834152909888046587498857991 9
30471288093212018033450486425774254 0*x**1959/3 + 4098019576899248565078726 6
7232878758560545819981360748791027423429192441183631959909271948238284208 96
033237685*x**1958 - 135929190134921432133840547955319561046879891182800940 6
507555490127093365391727648159674832991084850660654090760*x**1957 + 2658774
264104553320301560515549261476451402947200764401168959260797239752004089733
927293216850586001079943562746 35*x**1956/6 - 14201918924419578912795063139 0
460924702985156427711687275353798579924104732347757064326148866436684837694
7188162908*x**1955 + 4475893920998304597733889333471371492078599486033549 12
059082666379128770710327827503268426948174879427703941735119 15*x**1954 - 38
71199637329501447418000582050152588675313103951258803662243942776879796538 5
44404275706258986721565787264843985544995 20*x**1953/279 + 13542257542778944
99676937345006632528266873746430900140410227088...

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 10076 vs.

$2(15) = 30$.
time = 1.68, size = 10076, normalized size = 438.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^2014,x, algorithm="giac")
```

```
[Out] 1/2016*x^2016 - 2014/2015*x^2015 + 2013/2*x^2014 - 2026084/3*x^2013 + 13588
26667/4*x^2012 - 136629987582*x^2011 + 1373131035323509/30*x^2010 - 9195398
5549170536/7*x^2009 + 26377651026988133103/8*x^2008 - 661749155854244491587
4/9*x^2007 + 2949929948352647316611117/2*x^2006 - 13442291079909774060658016
4/5*x^2005 + 5387668923281821484452445482 3/12*x^2004 - 691762702790623489 45
1562620638*x^2003 + 25719730872661663428500290706910 63/26*x^2002 - 39588591
2482748242677565694039404 00/3*x^2001 + 1319619378370900406635016748826601 63
383/80*x^2000 - 1939645939135052094029929276517339599 50*x^1999 + 3875411615
5980707530148947755981227393507 5/18*x^1998 - 2262922530915969121458419278 66
1196219696900*x^1997 + 903358447595910292527771972191922410542659825/4*x^19
96 - 2145475773979272745021812337899023630563097701 0*x^1995 + 3889161470317
168646281025630946632817062982660525/2*x^1994 - 1685021056289444478187051 99
6700441871304935333534 00*x^1993 + 1118853699411489612524823980452333579848 7
0743212666075/8*x^1992 - 1113818576786312843600553552092286915421060813044 8
65230*x^1991 + 1704998775459180790734328767331702095659848281669175783 71/2*
x^1990 - 73476903655441985945968330316081626303723736209989757096414 0/117*x
^1989 + 1783541483599579507954055928245847409394570340210724349640535/4*x^1
988 - 30550827847475767604775649853836796394905428348789090875481890*x^1987
 + 1213478574439825631499678693607792781035287169850475209321463173 5/6*x^19
86 - 1295029225235932804921369946870905907982444504086270369484426995 84*x^1
985 + 79649457234079166922433860315942777106647454553626596456795674757971 5
5/992*x^1984 - 1447441656241185612618749471839715621192639755423719013224 85
6987762340/3*x^1983 + 2812577090565255430897352536290243163070741666144996 2
135634978911861255*x^1982 - 1591918227609528492506862969541498328323111814 3
726512354020406121374750 80*x^1981 + 1575998643333880780933185120941293056 78
2727079281496851166395104490693223777/18*x^1980 - 468303381200696202443825 5
993532176273777170532620461943334529407905781402420*x^1979 + 24376417138393
9582809061001601068990795547409624002219219883316766423225728865*x^1978 - 3
707089565584711138545323444480070738322008193622357674343613479298216558155
6720/3*x^1977 + 3174256412399763924075790852682186886641152443555486231644 7
8390371290217490365855 25/52*x^1976 - 2940504130990195634118841907272568110 4
1341311634435484048334090997606937874496122 68*x^1975 + 4146109760683970391 0
0450075590582368709118054535067987336822349123643079549175777552 5/3*x^1974
- 6341296477833618458583972079875122380745913656825226355055630034376410525
6910301716600*x^1973 + 5684106108463777496364455720240876368514714260586800
253709270268295316442312451914078575/2*x^1972 - 1120337025591294843682216 75
0899064746395773848203857184049716971541571645243688771886809300/9*x^1971 +
```

```
 53310709461735581641917056651063408558603844279169439438453156589650480105
62527907902040665*x^1970 - 223337786925829485322981480991438350221372508041
201117237722302034525923462936129127215408800*x^1969 + 73254775197564099825
039778493724892741313585299056307138160140869975075852658475240572087079675
/8*x^1968 - 257307331378089265618294271999354580767215753792536830264152936
9583213434895514150007213443740700/7*x^1967 + 14453316646020616414051419806
563949347846356733759521758169481834749898741394522590424547417732285*x^196
6 - 55687764419711927947641849414937962138091803555309959228824227239335771
1099559188691208229092310776*x^1965 + 4206566960232898345464498815975895477
5468201354159354767262170532890818036890454346362487193259926655/2*x^1964 -
 10127108905672074866716134495513810074049503443438355901587873018991904643
082375127060735008804049547340/13*x^1963 + 25473567322183688080684516663278
0310468643604667004332685617284023675830425396294703455768279984362513795/9
*x^1962 - 10091646975497330900852886078425142186921518650235743476881976089
90183198035912371247439438789779967687120*x^1961 + 988981146158764563555753
732295712954085089773602729130230773532270173159028459930987702066483515289
307875639/28*x^1960 - 36417548129421635926211441537962552722093013183415290
98880465874988579919304712880932120180334504864257742540/3*x^1959 + 4098019
576899248565078726672328787585605458199813607487910274234291924411836319599
0927194823828420896033237685*x^1958 - 1359291901349214321338405479553195610
468798911828009406507555490127093365391727648159674832991084850660654090760
*x^1957 + 26587742641045533203015605155492614764514029472007644011689592607
9723975200408973392729321685058600107994356274635/6*x^1956 - 14201918924419
578912795063139046092470298515642771168727535379857992410473234775706432614
88664366848376947188162908*x^1955 + 447589392099830459773388933347137149207
859948603354912059082666379128770710327827503268426948174879427703941735119
15*x^1954 - 387119963732950144741800058205015258867531310395125880366224394
277687979653854440427570625898672156578726484398554499520/279*x^1953 + 1354
225754277894499676937345006632528266873746430900140410227088281348549726611
191833845296592767838728398462456726127035/32*x...
```

Mupad [B]
time = 6.24, size = 15, normalized size = 0.65

$$\frac{(x-1)^{2015}}{2015} + \frac{(x-1)^{2016}}{2016}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x - 1)^2014,x)
```

```
[Out] (x - 1)^2015/2015 + (x - 1)^2016/2016
```

Chatgpt [F] Failed to verify
time = 1.00, size = 14, normalized size = 0.61

$$-\frac{(1-x)^{2015}\,(2015x+1)}{2015}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*(1-x)^2014,x)
```

```
[Out] -1/2015*(1-x)^2015*(2015*x+1)
```

3.106 $\int \operatorname{arcsinh}(x)\,dx$

Optimal. Leaf size=16

$$-\sqrt{1+x^2}+x\operatorname{arcsinh}(x)$$

```
[Out] x*arcsinh(x)-(x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}=1.000$, Rules used = {5772, 267}

$$x\operatorname{arcsinh}(x)-\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSinh[x],x]
```

```
[Out] -Sqrt[1 + x^2] + x*ArcSinh[x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= x\operatorname{arcsinh}(x)-\int \frac{x}{\sqrt{1+x^2}}\,dx\\ &= -\sqrt{1+x^2}+x\operatorname{arcsinh}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-\sqrt{1+x^2}+x\operatorname{arcsinh}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSinh[x],x]
```

```
[Out] -Sqrt[1 + x^2] + x*ArcSinh[x]
```

Maple [A]
time = 0.01, size = 15, normalized size = 0.94

method	result	size
lookup	$x\ \mathrm{arcsinh}\left(x\right)-\sqrt{x^{2}+1}$	15
default	$x\ \mathrm{arcsinh}\left(x\right)-\sqrt{x^{2}+1}$	15
parts	$x\ \mathrm{arcsinh}\left(x\right)-\sqrt{x^{2}+1}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsinh(x),x,method=_RETURNVERBOSE)
[Out] x*arcsinh(x)-(x^2+1)^(1/2)
```

Maxima [A]
time = 0.34, size = 14, normalized size = 0.88

$$x\,\mathrm{arsinh}\left(x\right)-\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x),x, algorithm="maxima")
[Out] x*arcsinh(x) - sqrt(x^2 + 1)
```

Fricas [A]
time = 0.59, size = 22, normalized size = 1.38

$$x\log\left(x+\sqrt{x^{2}+1}\right)-\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x),x, algorithm="fricas")
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.75

$$x\,\mathrm{asinh}\left(x\right)-\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asinh(x),x)
[Out] x*asinh(x) - sqrt(x**2 + 1)
```

Giac [**A**]
time = 0.47, size = 22, normalized size = 1.38

$$x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsinh(x),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Mupad [**B**]
time = 0.06, size = 14, normalized size = 0.88

$$x\,\mathrm{asinh}(x)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asinh(x),x)
[Out] x*asinh(x) - (x^2 + 1)^(1/2)
```

Chatgpt [**A**]
time = 1.00, size = 14, normalized size = 0.88

$$x\ \mathrm{arcsinh}\left(x\right)-\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] int(arcsinh(x),x)
[Out] x*arcsinh(x)-(x^2+1)^(1/2)
```

3.107 $\int \frac{x^2}{-1+x} \, dx$

Optimal. Leaf size=15

$$x + \frac{x^2}{2} + \log(1-x)$$

[Out] x+1/2*x^2+ln(1-x)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\frac{x^2}{2} + x + \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[x^2/(-1 + x),x]

[Out] x + x^2/2 + Log[1 - x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(1 + \frac{1}{-1+x} + x\right) dx \\
&= x + \frac{x^2}{2} + \log(1-x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.07

$$-\frac{3}{2} + x + \frac{x^2}{2} + \log(-1+x)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-1 + x),x]

[Out] -3/2 + x + x^2/2 + Log[-1 + x]

Maple [A]
time = 0.07, size = 12, normalized size = 0.80

method	result	size
default	$\frac{x^2}{2}+x+\ln\left(x-1\right)$	12
norman	$\frac{x^2}{2}+x+\ln\left(x-1\right)$	12
risch	$\frac{x^2}{2}+x+\ln\left(x-1\right)$	12
parallelrisch	$\frac{x^2}{2}+x+\ln\left(x-1\right)$	12
meijerg	$\frac{x(3x+6)}{6}+\ln\left(1-x\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x-1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+x+ln(x-1)
```

Maxima [A]
time = 0.37, size = 11, normalized size = 0.73

$$\frac{1}{2}\,x^2+x+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x-1),x, algorithm="maxima")
[Out] 1/2*x^2 + x + log(x - 1)
```

Fricas [A]
time = 0.58, size = 11, normalized size = 0.73

$$\frac{1}{2}\,x^2+x+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x-1),x, algorithm="fricas")
[Out] 1/2*x^2 + x + log(x - 1)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^2}{2}+x+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x-1),x)
```

```
[Out] x**2/2 + x + log(x - 1)
```

Giac [A]
time = 0.48, size = 12, normalized size = 0.80

$$\frac{1}{2}x^2 + x + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x-1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + log(abs(x - 1))
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.73

$$x + \ln(x - 1) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x - 1),x)
```

```
[Out] x + log(x - 1) + x^2/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 20, normalized size = 1.33

$$-\frac{x^3}{3} - \frac{x^2}{2} - x - \ln(x - 1)$$

Warning: Unable to verify antiderivative.

```
[In] int(x^2/(x-1),x)
```

```
[Out] -1/3*x^3-1/2*x^2-x-ln(x-1)
```

3.108 $\int x \arctan(x)\,dx$

Optimal. Leaf size=21

$$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{1}{2}x^2\arctan(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4946, 327, 209}

$$\frac{1}{2}x^2\arctan(x)+\frac{\arctan(x)}{2}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx \\ &= -\frac{x}{2} + \frac{1}{2}x^2\arctan(x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\ &= -\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2\arctan(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{1}{2}x^2\arctan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^2+3)\arctan(x)}{6}$	16
parallelrisch	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
parts	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
risch	$-\frac{ix^2\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A]
time = 0.42, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan\left(x\right) - \frac{1}{2}\,x + \frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A]
time = 0.59, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^2+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A]
time = 0.10, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Giac [A]
time = 0.50, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.67

$$\operatorname{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x),x)
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 16, normalized size = 0.76

$$\frac{x^2\arctan\left(x\right)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*arctan(x),x)
```

```
[Out] 1/2*x^2*arctan(x)-1/4*ln(x^2+1)
```

3.109 $\int \frac{1}{-2014-15x+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{91}\log(53-x)-\frac{1}{91}\log(38+x)$$

```
[Out] 1/91*ln(53-x)-1/91*ln(38+x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {630, 31}

$$\frac{1}{91}\log(53-x)-\frac{1}{91}\log(x+38)$$

Antiderivative was successfully verified.

```
[In] Int[(-2014 - 15*x + x^2)^(-1),x]
```

```
[Out] Log[53 - x]/91 - Log[38 + x]/91
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{91}\int \frac{1}{-53+x}\,dx-\frac{1}{91}\int \frac{1}{38+x}\,dx\\ &= \frac{1}{91}\log(53-x)-\frac{1}{91}\log(38+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{91}\log(53-x)-\frac{1}{91}\log(38+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2014 - 15*x + x^2)^(-1),x]
[Out] Log[53 - x]/91 - Log[38 + x]/91
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.74

method	result	size
default	$\frac{\ln(x-53)}{91}-\frac{\ln(38+x)}{91}$	14
norman	$\frac{\ln(x-53)}{91}-\frac{\ln(38+x)}{91}$	14
risch	$\frac{\ln(x-53)}{91}-\frac{\ln(38+x)}{91}$	14
parallelrisch	$\frac{\ln(x-53)}{91}-\frac{\ln(38+x)}{91}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-15*x-2014),x,method=_RETURNVERBOSE)
[Out] 1/91*ln(x-53)-1/91*ln(38+x)
```

Maxima [A]
time = 0.35, size = 13, normalized size = 0.68

$$-\frac{1}{91}\log\left(x+38\right)+\frac{1}{91}\log\left(x-53\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-15*x-2014),x, algorithm="maxima")
[Out] -1/91*log(x + 38) + 1/91*log(x - 53)
```

Fricas [A]
time = 0.58, size = 13, normalized size = 0.68

$$-\frac{1}{91}\log\left(x+38\right)+\frac{1}{91}\log\left(x-53\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-15*x-2014),x, algorithm="fricas")
[Out] -1/91*log(x + 38) + 1/91*log(x - 53)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.63

$$\frac{\log\left(x-53\right)}{91}-\frac{\log\left(x+38\right)}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-15*x-2014),x)
```

```
[Out] log(x - 53)/91 - log(x + 38)/91
```

Giac [**A**]
time = 0.45, size = 15, normalized size = 0.79

$$-\frac{1}{91}\,\log\left(|x+38|\right)+\frac{1}{91}\,\log\left(|x-53|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-15*x-2014),x, algorithm="giac")
```

```
[Out] -1/91*log(abs(x + 38)) + 1/91*log(abs(x - 53))
```

Mupad [**B**]
time = 0.09, size = 8, normalized size = 0.42

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,x}{91}-\frac{15}{91}\right)}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(15*x - x^2 + 2014),x)
```

```
[Out] -(2*atanh((2*x)/91 - 15/91))/91
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 14, normalized size = 0.74

$$\frac{2\sqrt{4013}\,\arctan\left(\frac{2\left(x-\frac{15}{2}\right)\sqrt{4013}}{4013}\right)}{4013}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^2-15*x-2014),x)
```

```
[Out] 2/4013*4013^(1/2)*arctan(2/4013*(x-15/2)*4013^(1/2))
```

3.110 $\int e^x(-2(1+x)\arctan(x)+\log(1+x^2))\,dx$

Optimal. Leaf size=19

$$-2e^x x\arctan(x)+e^x\log\left(1+x^2\right)$$

[Out] -2*exp(x)*x*arctan(x)+exp(x)*ln(x^2+1)

Rubi [A]
time = 0.16, antiderivative size = 28, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.444$, Rules used = {6874, 2207, 2225, 5315, 6857, 2209, 2634, 12}

$$2e^x\arctan(x)-2e^x(x+1)\arctan(x)+e^x\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

[In] Int[E^x*(-2*(1 + x)*ArcTan[x] + Log[1 + x^2]),x]

[Out] 2*E^x*ArcTan[x] - 2*E^x*(1 + x)*ArcTan[x] + E^x*Log[1 + x^2]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
```

```
]] /; InverseFunctionFreeQ[u, x]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int \left(-2e^x(1+x)\arctan(x)+e^x\log\left(1+x^2\right)\right)\,dx \\
&= -\left(2\int e^x(1+x)\arctan(x)\,dx\right)+\int e^x\log\left(1+x^2\right)\,dx \\
&= 2e^x\arctan(x)-2e^x(1+x)\arctan(x)+e^x\log\left(1+x^2\right)+2\int\frac{e^x x}{1+x^2}\,dx-\int\frac{2e^x x}{1+x^2}\,dx \\
&= 2e^x\arctan(x)-2e^x(1+x)\arctan(x)+e^x\log\left(1+x^2\right)-2\int\frac{e^x x}{1+x^2}\,dx+2\int\left(-\frac{e^x}{2(i-x)}+\frac{e^x}{2(i+x)}\right)dx \\
&= 2e^x\arctan(x)-2e^x(1+x)\arctan(x)+e^x\log\left(1+x^2\right)-2\int\left(-\frac{e^x}{2(i-x)}+\frac{e^x}{2(i+x)}\right)dx-\int\frac{e^x}{i-x}\,dx+\int\frac{e^x}{i+x}\,dx \\
&= 2e^x\arctan(x)-2e^x(1+x)\arctan(x)+e^i\,\text{ExpIntegralEi}(-i+x)+e^{-i}\,\text{ExpIntegralEi}(i+x)+e^x\log\left(1+x^2\right)+\int\frac{e^x}{i-x}\,dx-\int\frac{e^x}{i+x}\,dx \\
&= 2e^x\arctan(x)-2e^x(1+x)\arctan(x)+e^x\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 0.84

$$e^x\left(-2x\arctan(x)+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*(-2*(1 + x)*ArcTan[x] + Log[1 + x^2]),x]
[Out] E^x*(-2*x*ArcTan[x] + Log[1 + x^2])
```

Maple [A]
time = 0.15, size = 18, normalized size = 0.95

method	result
parallelrisch	$-2\,\mathrm{e}^{x} x \arctan\left(x\right)+\mathrm{e}^{x} \ln\left(x^{2}+1\right)$
risch	$\frac{i \pi \,\mathrm{csgn}\left(i \left(x-i\right)\right) \mathrm{csgn}\left(i \left(x-i\right) \left(i+x\right)\right)^{2} \mathrm{e}^{x}}{2}+\mathrm{e}^{x} \ln\left(x-i\right)+\frac{i \pi \,\mathrm{csgn}\left(i \left(i+x\right)\right) \mathrm{csgn}\left(i \left(x-i\right) \left(i+x\right)\right)^{2} \mathrm{e}^{x}}{2}-\frac{i \pi \mathrm{csgn}\left(i \left(x-i\right) \left(i+x\right)\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(ln(x^2+1)-2*(x+1)*arctan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*exp(x)*x*arctan(x)+exp(x)*ln(x^2+1)
```

Maxima [A]
time = 0.56, size = 17, normalized size = 0.89

$$-2\,x \arctan\left(x\right) e^{x} + e^{x} \log\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(log(x^2+1)-2*(x+1)*arctan(x)),x, algorithm="maxima")
```

```
[Out] -2*x*arctan(x)*e^x + e^x*log(x^2 + 1)
```

Fricas [A]
time = 0.62, size = 17, normalized size = 0.89

$$-2\,x \arctan\left(x\right) e^{x} + e^{x} \log\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(log(x^2+1)-2*(x+1)*arctan(x)),x, algorithm="fricas")
```

```
[Out] -2*x*arctan(x)*e^x + e^x*log(x^2 + 1)
```

Sympy [A]
time = 1.39, size = 19, normalized size = 1.00

$$-2xe^{x} \operatorname{atan}{\left(x \right)} + e^{x} \log{\left(x^{2} + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(ln(x**2+1)-2*(x+1)*atan(x)),x)
```

```
[Out] -2*x*exp(x)*atan(x) + exp(x)*log(x**2 + 1)
```

Giac [A]
time = 0.52, size = 27, normalized size = 1.42

$$-\pi x e^{x} \mathrm{sgn}\left(x\right) + 2\,x \arctan\left(\frac{1}{x}\right) e^{x} + e^{x} \log\left(x^{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(log(x^2+1)-2*(x+1)*arctan(x)),x, algorithm="giac")
```

```
[Out] -pi*x*e^x*sgn(x) + 2*x*arctan(1/x)*e^x + e^x*log(x^2 + 1)
```

Mupad [B]
time = 0.27, size = 15, normalized size = 0.79

$$e^x \left(\ln\left(x^2+1\right)-2\,x\,\mathrm{atan}(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(log(x^2 + 1) - 2*atan(x)*(x + 1)),x)
```

```
[Out] exp(x)*(log(x^2 + 1) - 2*x*atan(x))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(exp(x)*(ln(x^2+1)-2*(x+1)*arctan(x)),x)
```

```
[Out] not solved
```

3.111 $\int \arcsin(x)^2 \, dx$

Optimal. Leaf size=25

$$-2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2$$

```
[Out] -2*x+2*(-x^2+1)^(1/2)*arcsin(x)+x*arcsin(x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4715, 4767, 8}

$$2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2 - 2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= x\arcsin(x)^2 - 2\int \frac{x\arcsin(x)}{\sqrt{1-x^2}}\,dx \\
&= 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2 - 2\int 1\,dx \\
&= -2x + 2\sqrt{1-x^2}\arcsin(x) + x\arcsin(x)^2
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-2x+2\sqrt{1-x^2}\arcsin(x)+x\arcsin(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]^2,x]
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.96

method	result	size
default	$-2x+2\sqrt{-x^2+1}\,\arcsin\left(x\right)+x\arcsin\left(x\right)^2$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)^2,x,method=_RETURNVERBOSE)
[Out] -2*x+2*(-x^2+1)^(1/2)*arcsin(x)+x*arcsin(x)^2
```

Maxima [A]
time = 0.49, size = 23, normalized size = 0.92

$$x\arcsin\left(x\right)^2+2\sqrt{-x^2+1}\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Fricas [A]
time = 0.63, size = 23, normalized size = 0.92

$$x\arcsin\left(x\right)^2+2\sqrt{-x^2+1}\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A]
time = 0.09, size = 22, normalized size = 0.88

$$x\operatorname{asin}^2\left(x\right)-2x+2\sqrt{1-x^2}\operatorname{asin}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)**2,x)
```

```
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

Giac [A]
time = 0.44, size = 23, normalized size = 0.92

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Mupad [B]
time = 0.03, size = 22, normalized size = 0.88

$$2\operatorname{asin}(x)\sqrt{1-x^2} + x\left(\operatorname{asin}(x)^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)^2,x)
```

```
[Out] 2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 32, normalized size = 1.28

$$x \arcsin(x) - \sqrt{-x^2+1}\arcsin(x)^2 + \frac{x\sqrt{-x^2+1}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(arcsin(x)^2,x)
```

```
[Out] x*arcsin(x)-(-x^2+1)^(1/2)*arcsin(x)^2+1/2*x*(-x^2+1)^(1/2)
```

3.112 $\int \frac{\sqrt{-1+x^2}}{x}\,dx$

Optimal. Leaf size=22

$$\sqrt{-1+x^2}-\arctan\left(\sqrt{-1+x^2}\right)$$

```
[Out] (x^2-1)^(1/2)-arctan((x^2-1)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 52, 65, 209}

$$\sqrt{x^2-1}-\arctan\left(\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/x,x]
```

```
[Out] Sqrt[-1 + x^2] - ArcTan[Sqrt[-1 + x^2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, x^2\right)\\
&= \sqrt{-1+x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, x^2\right)\\
&= \sqrt{-1+x^2} - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^2}\right)\\
&= \sqrt{-1+x^2} - \arctan\left(\sqrt{-1+x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$\sqrt{-1+x^2} - \arctan\left(\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/x,x]
[Out] Sqrt[-1 + x^2] - ArcTan[Sqrt[-1 + x^2]]
```

Maple [A]
time = 0.14, size = 17, normalized size = 0.77

method	result	size
default	$\sqrt{x^2-1}+\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)$	17
pseudoelliptic	$\sqrt{x^2-1}-\arctan\left(\sqrt{x^2-1}\right)$	19
trager	$\sqrt{x^2-1}+\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathit{RootOf}\left(\_Z^2+1\right)+\sqrt{x^2-1}}{x}\right)$	37
meijerg	$-\frac{\sqrt{\mathrm{signum}(x^2-1)}\left(-2(2-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-x^2+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{-\mathrm{signum}(x^2-1)}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (x^2-1)^(1/2)+arctan(1/(x^2-1)^(1/2))
```

Maxima [A]
time = 0.45, size = 13, normalized size = 0.59

$$\sqrt{x^2-1}+\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1) + arcsin(1/abs(x))
```

Fricas [A]
time = 0.58, size = 22, normalized size = 1.00

$$\sqrt{x^2-1}-2\arctan\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - 1) - 2*arctan(-x + sqrt(x^2 - 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 68, normalized size = 3.09

$$\begin{cases} -\frac{ix}{\sqrt{-1+\frac{1}{x^2}}}-i\operatorname{acosh}\left(\frac{1}{x}\right)+\frac{i}{x\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|}>1 \\ \frac{x}{\sqrt{1-\frac{1}{x^2}}}+\operatorname{asin}\left(\frac{1}{x}\right)-\frac{1}{x\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/x,x)
```

```
[Out] Piecewise((-I*x/sqrt(-1 + x**(-2)) - I*acosh(1/x) + I/(x*sqrt(-1 + x**(-2))
), 1/Abs(x**2) > 1), (x/sqrt(1 - 1/x**2) + asin(1/x) - 1/(x*sqrt(1 - 1/x**2
)), True))
```

Giac [A]
time = 0.46, size = 18, normalized size = 0.82

$$\sqrt{x^2-1}-\arctan\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 1) - arctan(sqrt(x^2 - 1))
```

Mupad [B]
time = 0.14, size = 26, normalized size = 1.18

$$\sqrt{x^2-1}-\ln\left(\frac{\sqrt{x^2-1}+1\mathrm{i}}{x}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/x,x)
```

```
[Out] (x^2 - 1)^(1/2) - log(((x^2 - 1)^(1/2) + 1i)/x)*1i
```

Chatgpt [F] Failed to verify
time = 1.00, size = 25, normalized size = 1.14

$$\ln\left(x+\sqrt{-x^2+1}\right)-x\sqrt{-x^2+1}$$

Warning: Unable to verify antiderivative.

```
[In] int((x^2-1)^(1/2)/x,x)
```

```
[Out] ln(x+(-x^2+1)^(1/2))-x*(-x^2+1)^(1/2)
```

3.113 $\int x \sec^2(4x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{16}\log(\cos(4x))+\frac{1}{4}x\tan(4x)$$

```
[Out] 1/16*ln(cos(4*x))+1/4*x*tan(4*x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4269, 3556}

$$\frac{1}{4}x\tan(4x)+\frac{1}{16}\log(\cos(4x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[4*x]^2,x]
```

```
[Out] Log[Cos[4*x]]/16 + (x*Tan[4*x])/4
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{1}{4}x\tan(4x)-\frac{1}{4}\int\tan(4x)\,dx\\&=\frac{1}{16}\log(\cos(4x))+\frac{1}{4}x\tan(4x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{16}\log(\cos(4x))+\frac{1}{4}x\tan(4x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[4*x]^2,x]

[Out] Log[Cos[4*x]]/16 + (x*Tan[4*x])/4

Maple [A]
time = 0.12, size = 16, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\ln(\cos(4x))}{16}+\frac{x\tan(4x)}{4}$	16
default	$\frac{\ln(\cos(4x))}{16}+\frac{x\tan(4x)}{4}$	16
risch	$-\frac{ix}{2}+\frac{ix}{2\,e^{8ix}+2}+\frac{\ln(e^{8ix}+1)}{16}$	29
norman	$-\frac{x\tan(2x)}{2(\tan^2(2x)-1)}+\frac{\ln(\tan(2x)-1)}{16}+\frac{\ln(\tan(2x)+1)}{16}-\frac{\ln(1+\tan^2(2x))}{16}$	48
parallelrisch	$\frac{\ln(\tan(2x)-1)\cos(4x)+\ln(\tan(2x)+1)\cos(4x)-\ln(\sec^2(2x))\cos(4x)+4x\sin(4x)}{16\cos(4x)}$	54

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sec(4*x)^2,x,method=_RETURNVERBOSE)

[Out] 1/16*ln(cos(4*x))+1/4*x*tan(4*x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(15) = 30.
time = 0.46, size = 74, normalized size = 3.89

$$\frac{\left(\cos\left(8\,x\right)^2+\sin\left(8\,x\right)^2+2\,\cos\left(8\,x\right)+1\right)\log\left(\cos\left(8\,x\right)^2+\sin\left(8\,x\right)^2+2\,\cos\left(8\,x\right)+1\right)+16\,x\sin\left(8\,x\right)}{32\left(\cos\left(8\,x\right)^2+\sin\left(8\,x\right)^2+2\,\cos\left(8\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(4*x)^2,x, algorithm="maxima")

[Out] 1/32*((cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1)*log(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1) + 16*x*sin(8*x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1)

Fricas [A]
time = 0.64, size = 28, normalized size = 1.47

$$\frac{\cos\left(4\,x\right)\log\left(-\cos\left(4\,x\right)\right)+4\,x\sin\left(4\,x\right)}{16\,\cos\left(4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(4*x)^2,x, algorithm="fricas")

[Out] 1/16*(cos(4*x)*log(-cos(4*x)) + 4*x*sin(4*x))/cos(4*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^2(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(4*x)**2,x)
[Out] Integral(x*sec(4*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(15) = 30.
time = 0.45, size = 103, normalized size = 5.42

$$\frac{\log\left(\frac{4\left(\tan(2x)^4-2\tan(2x)^2+1\right)}{\tan(2x)^4+2\tan(2x)^2+1}\right)\tan\left(2x\right)^2-16\,x\tan\left(2x\right)-\log\left(\frac{4\left(\tan(2x)^4-2\tan(2x)^2+1\right)}{\tan(2x)^4+2\tan(2x)^2+1}\right)}{32\left(\tan\left(2x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(4*x)^2,x, algorithm="giac")
[Out] 1/32*(log(4*(tan(2*x)^4 - 2*tan(2*x)^2 + 1)/(tan(2*x)^4 + 2*tan(2*x)^2 + 1)
)*tan(2*x)^2 - 16*x*tan(2*x) - log(4*(tan(2*x)^4 - 2*tan(2*x)^2 + 1)/(tan(2
*x)^4 + 2*tan(2*x)^2 + 1)))/(tan(2*x)^2 - 1)
```

Mupad [B]
time = 0.08, size = 15, normalized size = 0.79

$$\frac{\ln\left(\cos\left(4x\right)\right)}{16}+\frac{x\tan(4x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(4*x)^2,x)
[Out] log(cos(4*x))/16 + (x*tan(4*x))/4
```

Chatgpt [F] Failed to verify
time = 1.00, size = 8, normalized size = 0.42

$$\frac{1}{4\sin\left(4x\right)}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*sec(4*x)^2,x)
[Out] 1/4/sin(4*x)
```

3.114 $\int \frac{2}{6-11x+6x^2-x^3}\,dx$

Optimal. Leaf size=25

$$-\log(1-x)+2\log(2-x)-\log(3-x)$$

[Out] -ln(1-x)+2*ln(2-x)-ln(3-x)

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {12, 2083}

$$-\log(1-x)+2\log(2-x)-\log(3-x)$$

Antiderivative was successfully verified.

[In] Int[2/(6 - 11*x + 6*x^2 - x^3),x]

[Out] -Log[1 - x] + 2*Log[2 - x] - Log[3 - x]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\int \frac{1}{6-11x+6x^2-x^3}\,dx \\
&= 2\int \left(-\frac{1}{2(-3+x)}+\frac{1}{-2+x}-\frac{1}{2(-1+x)}\right)dx \\
&= -\log(1-x)+2\log(2-x)-\log(3-x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.96

$$-2\left(-\log(2-x)+\frac{1}{2}\log\left(3-4x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(6 - 11*x + 6*x^2 - x^3),x]
[Out] -2*(-Log[2 - x] + Log[3 - 4*x + x^2]/2)
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.80

method	result	size
risch	$2\ln(x-2)-\ln(x^2-4x+3)$	19
default	$2\ln(x-2)-\ln(-3+x)-\ln(x-1)$	20
norman	$2\ln(x-2)-\ln(-3+x)-\ln(x-1)$	20
parallelrisch	$2\ln(x-2)-\ln(-3+x)-\ln(x-1)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(-x^3+6*x^2-11*x+6),x,method=_RETURNVERBOSE)
[Out] 2*ln(x-2)-ln(-3+x)-ln(x-1)
```

Maxima [A]
time = 0.34, size = 19, normalized size = 0.76

$$-\log(x-1)+2\log(x-2)-\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-x^3+6*x^2-11*x+6),x, algorithm="maxima")
[Out] -log(x - 1) + 2*log(x - 2) - log(x - 3)
```

Fricas [A]
time = 0.56, size = 18, normalized size = 0.72

$$-\log\left(x^2-4x+3\right)+2\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-x^3+6*x^2-11*x+6),x, algorithm="fricas")
[Out] -log(x^2 - 4*x + 3) + 2*log(x - 2)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.60

$$2\log(x-2)-\log\left(x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-x**3+6*x**2-11*x+6),x)
```

```
[Out] 2*log(x - 2) - log(x**2 - 4*x + 3)
```

Giac [**A**]
time = 0.43, size = 22, normalized size = 0.88

$$-\log(|x-1|)+2\log(|x-2|)-\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(-x^3+6*x^2-11*x+6),x, algorithm="giac")
```

```
[Out] -log(abs(x - 1)) + 2*log(abs(x - 2)) - log(abs(x - 3))
```

Mupad [**B**]
time = 0.06, size = 18, normalized size = 0.72

$$2\ln(x-2)-\ln\left(x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(11*x - 6*x^2 + x^3 - 6),x)
```

```
[Out] 2*log(x - 2) - log(x^2 - 4*x + 3)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 21, normalized size = 0.84

$$\frac{\ln(x-1)}{8}-\frac{\ln(x-2)}{8}+\frac{\arctan(2x-11)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(2/(-x^3+6*x^2-11*x+6),x)
```

```
[Out] 1/8*ln(x-1)-1/8*ln(x-2)+1/4*arctan(2*x-11)
```

3.115 $\int \frac{1}{1-\log(1-x)}\,dx$

Optimal. Leaf size=11

$$e\,\text{ExpIntegralEi}(-1+\log(1-x))$$

```
[Out] exp(1)*Ei(-1+ln(1-x))
```

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2436, 2336, 2209}

$$e\,\text{ExpIntegralEi}(\log(1-x)-1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Log[1 - x])^(-1),x]
```

```
[Out] E*ExpIntegralEi[-1 + Log[1 - x]]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\text{Subst}\left(\int \frac{1}{1-\log(x)}\,dx, x, 1-x\right)\\ &= -\text{Subst}\left(\int \frac{e^x}{1-x}\,dx, x, \log(1-x)\right)\\ &= e\,\text{ExpIntegralEi}(-1+\log(1-x))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 11, normalized size = 1.00

$$e\,\text{ExpIntegralEi}(-1+\log(1-x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Log[1 - x])^(-1),x]
[Out] E*ExpIntegralEi[-1 + Log[1 - x]]
```

Maple [A]
time = 0.09, size = 17, normalized size = 1.55

method	result	size
derivativedivides	$-\mathrm{e}\ \text{expIntegral}_1\left(1-\ln\left(1-x\right)\right)$	17
default	$-\mathrm{e}\ \text{expIntegral}_1\left(1-\ln\left(1-x\right)\right)$	17
risch	$-\mathrm{e}\ \text{expIntegral}_1\left(1-\ln\left(1-x\right)\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-ln(1-x)),x,method=_RETURNVERBOSE)
[Out] -exp(1)*Ei(1,1-ln(1-x))
```

Maxima [A]
time = 0.42, size = 16, normalized size = 1.45

$$-eE_1(-\log\left(-x+1\right)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-log(1-x)),x, algorithm="maxima")
[Out] -e*exp_integral_e(1, -log(-x + 1) + 1)
```

Fricas [A]
time = 0.57, size = 11, normalized size = 1.00

$$e\,\text{log\_integral}\left(-(x-1)e^{(-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-log(1-x)),x, algorithm="fricas")
[Out] e*log_integral(-(x - 1)*e^(-1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\log\left(1-x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-ln(1-x)),x)
```

```
[Out] -Integral(1/(log(1 - x) - 1), x)
```

Giac [A]
time = 0.45, size = 12, normalized size = 1.09

$$\mathrm{Ei}(\log(-x+1)-1)\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-log(1-x)),x, algorithm="giac")
```

```
[Out] Ei(log(-x + 1) - 1)*e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.09

$$-\int \frac{1}{\ln(1-x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(log(1 - x) - 1),x)
```

```
[Out] -int(1/(log(1 - x) - 1), x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 32, normalized size = 2.91

$$2\ln(1-\ln(1-x))\,\mathrm{e}^{\ln(1-x)-1}-\mathrm{cosineIntegral}(\ln(1-x))$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1-ln(1-x)),x)
```

```
[Out] 2*ln(1-ln(1-x))*exp(ln(1-x)-1)-Ci(ln(1-x))
```

3.116 $\int \sqrt{x+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}$$

```
[Out] -1/(x+(x^2+1)^(1/2))^(1/2)+1/3*(x+(x^2+1)^(1/2))^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2142, 14}

$$\frac{1}{3}\left(\sqrt{x^2+1}+x\right)^{3/2}-\frac{1}{\sqrt{\sqrt{x^2+1}+x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x^{3/2}}\,dx,x,x+\sqrt{1+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}+\sqrt{x}\right)\,dx,x,x+\sqrt{1+x^2}\right)\\ &= -\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 1.00

$$-\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\frac{1}{3}\left(x+\sqrt{1+x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x^2]],x]
```

```
[Out] -(1/Sqrt[x + Sqrt[1 + x^2]]) + (x + Sqrt[1 + x^2])^(3/2)/3
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.03, size = 57, normalized size = 1.54

method	result	size
meijerg	$\frac{\frac{16\sqrt{\pi}\,\sqrt{2}\,x^{\frac{3}{2}}\left(-\frac{1}{x^2}+1\right)\cosh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3}+\frac{16\sqrt{\pi}\,\sqrt{2}\,\sqrt{x}\,\sqrt{\frac{1}{x^2}+1}\,\sinh\left(\frac{\operatorname{arcsinh}\left(\frac{1}{x}\right)}{2}\right)}{3}}{8\sqrt{\pi}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/Pi^(1/2)*(16/3*Pi^(1/2)*2^(1/2)*x^(3/2)*(-1/x^2+1)*cosh(1/2*arcsinh(1/x
))+16/3*Pi^(1/2)*2^(1/2)*x^(1/2)*(1/x^2+1)^(1/2)*sinh(1/2*arcsinh(1/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x^2 + 1)), x)
```

Fricas [A]
time = 0.58, size = 26, normalized size = 0.70

$$\frac{2}{3}\left(2\,x-\sqrt{x^2+1}\right)\sqrt{x+\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x - sqrt(x^2 + 1))*sqrt(x + sqrt(x^2 + 1))
```

Sympy [**A**]
time = 0.14, size = 42, normalized size = 1.14

$$\frac{4x\sqrt{x+\sqrt{x^2+1}}}{3}-\frac{2\sqrt{x+\sqrt{x^2+1}}\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 4*x*sqrt(x + sqrt(x**2 + 1))/3 - 2*sqrt(x + sqrt(x**2 + 1))*sqrt(x**2 + 1)/
3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(x^2 + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{x+\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x^2 + 1)^(1/2))^(1/2),x)
```

```
[Out] int((x + (x^2 + 1)^(1/2))^(1/2), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 27, normalized size = 0.73

$$\frac{(2+x)\sqrt{x+\sqrt{x^2+1}}}{8}+\frac{\operatorname{arcsinh}\left(\sqrt{x^2+1}\right)}{8}$$

Warning: Unable to verify antiderivative.

```
[In] int((x+(x^2+1)^(1/2))^(1/2),x)
```

```
[Out] 1/8*(2+x)*(x+(x^2+1)^(1/2))^(1/2)+1/8*arcsinh((x^2+1)^(1/2))
```

3.117 $\int \frac{1}{2+\cos(x)}\,dx$

Optimal. Leaf size=20

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan(1/3*tan(1/2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.50, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2736}

$$\frac{x}{\sqrt{3}}-\frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + Cos[x])^(-1),x]
```

```
[Out] x/Sqrt[3] - (2*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\text{Integral} = \frac{x}{\sqrt{3}}-\frac{2\arctan\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.04, size = 16, normalized size = 0.80

method	result	size
default	$\frac{2 \arctan\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	16
risch	$\frac{i\sqrt{3}\,\ln\left(e^{ix}+2+\sqrt{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(e^{ix}+2-\sqrt{3}\right)}{3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+cos(x)),x,method=_RETURNVERBOSE)
[Out] 2/3*arctan(1/3*tan(1/2*x)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.48, size = 19, normalized size = 0.95

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin\left(x\right)}{3\left(\cos\left(x\right)+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cos(x)),x, algorithm="maxima")
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.59, size = 23, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos\left(x\right)+\sqrt{3}}{3\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cos(x)),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

Sympy [A]
time = 0.25, size = 32, normalized size = 1.60

$$\frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cos(x)),x)
```

```
[Out] 2*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.
time = 0.49, size = 40, normalized size = 2.00

$$\frac{1}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\sin(x)}{\sqrt{3}\cos(x)+\sqrt{3}-\cos(x)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cos(x)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos(x) + sqrt
(3) - cos(x) + 1)))
```

Mupad [B]
time = 0.20, size = 32, normalized size = 1.60

$$\frac{2\sqrt{3}\left(\frac{x}{2}-\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)\right)\right)}{3}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 2),x)
```

```
[Out] (2*3^(1/2)*(x/2 - atan(tan(x/2))))/3 + (2*3^(1/2)*atan((3^(1/2)*tan(x/2))/3
))/3
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(1/(2+cos(x)),x)
```

```
[Out] not solved
```

3.118 $\int (\cos^4(x) - \sin^4(x))\, dx$

Optimal. Leaf size=5

$$\cos(x)\sin(x)$$

```
[Out] cos(x)*sin(x)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 29 vs. $2(5) = 10$.
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 5.80, number of steps used = 7, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2715, 8}

$$\frac{1}{4}\sin(x)\cos^3(x) + \frac{1}{4}\sin^3(x)\cos(x) + \frac{3}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4 - Sin[x]^4,x]
```

```
[Out] (3*Cos[x]*Sin[x])/4 + (Cos[x]^3*Sin[x])/4 + (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \cos^4(x)\, dx - \int \sin^4(x)\, dx \\
&= \frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\int \cos^2(x)\, dx - \frac{3}{4}\int \sin^2(x)\, dx \\
&= \frac{3}{4}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\cos(x)\sin^3(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.60

$$\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4 - Sin[x]^4,x]
```

```
[Out] Sin[2*x]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(5) = 10$.
time = 0.15, size = 28, normalized size = 5.60

method	result	size
risch	$\frac{\sin(2x)}{2}$	7
parallelrisch	$\frac{\sin(2x)}{2}$	7
default	$\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}$	28
parts	$\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}$	28
norman	$\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)-2\left(\tan^5\left(\frac{x}{2}\right)\right)-2\left(\tan^7\left(\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4-sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+1/4*(sin(x)^3+3/2*sin(x))*cos(x)
```

Maxima [A]
time = 0.34, size = 6, normalized size = 1.20

$$\frac{1}{2}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4-sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*x)
```

Fricas [A]
time = 0.59, size = 5, normalized size = 1.00

$$\cos\left(x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4-sin(x)^4,x, algorithm="fricas")
```

```
[Out] cos(x)*sin(x)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(5) = 10$.
time = 0.02, size = 29, normalized size = 5.80

$$\frac{\sin^3(x)\cos(x)}{4} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4-sin(x)**4,x)
```

```
[Out] sin(x)**3*cos(x)/4 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/4
```

Giac [**A**]
time = 0.49, size = 6, normalized size = 1.20

$$\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4-sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/2*sin(2*x)
```

Mupad [**B**]
time = 0.11, size = 6, normalized size = 1.20

$$\frac{\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4 - sin(x)^4,x)
```

```
[Out] sin(2*x)/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 12, normalized size = 2.40

$$2x^2 + \frac{\sin(2x)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)^4-sin(x)^4,x)
```

```
[Out] 2*x^2+1/2*sin(2*x)
```

3.119 $\int \frac{x}{\sqrt{2+4x}}\,dx$

Optimal. Leaf size=16

$$\frac{1}{6}(-1+x)\sqrt{2+4x}$$

```
[Out] 1/6*(x-1)*(2+4*x)^(1/2)
```

Rubi [**B**] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 2.31, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{(2x+1)^{3/2}}{6\sqrt{2}} - \frac{\sqrt{2x+1}}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 + 4*x],x]
```

```
[Out] -1/2*Sqrt[1 + 2*x]/Sqrt[2] + (1 + 2*x)^(3/2)/(6*Sqrt[2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(-\frac{1}{2\sqrt{2+4x}} + \frac{1}{4}\sqrt{2+4x}\right) dx \\
&= -\frac{\sqrt{1+2x}}{2\sqrt{2}} + \frac{(1+2x)^{3/2}}{6\sqrt{2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{1}{3}(-1+x)\sqrt{\frac{1}{2}+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 + 4*x],x]
```

```
[Out] ((-1 + x)*Sqrt[1/2 + x])/3
```

Maple [A]
time = 0.08, size = 20, normalized size = 1.25

method	result	size
pseudoelliptic	$\frac{(x-1)\sqrt{2+4x}}{6}$	13
trager	$\frac{\left(\frac{x}{3}-\frac{1}{3}\right)\sqrt{2+4x}}{2}$	15
gosper	$\frac{(2x+1)(x-1)}{3\sqrt{2+4x}}$	18
risch	$\frac{(2x+1)(x-1)}{3\sqrt{2+4x}}$	18
derivativedivides	$\frac{(2+4x)^{\frac{3}{2}}}{24}-\frac{\sqrt{2+4x}}{4}$	20
default	$\frac{(2+4x)^{\frac{3}{2}}}{24}-\frac{\sqrt{2+4x}}{4}$	20
meijerg	$\frac{\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(-8x+8)\sqrt{2x+1}}{6}\right)}{8\sqrt{\pi}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/24*(2+4*x)^(3/2)-1/4*(2+4*x)^(1/2)
```

Maxima [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.33, size = 19, normalized size = 1.19

$$\frac{1}{24}\,(4\,x+2)^{\frac{3}{2}}-\frac{1}{4}\,\sqrt{4\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+4*x)^(1/2),x, algorithm="maxima")
[Out] 1/24*(4*x + 2)^(3/2) - 1/4*sqrt(4*x + 2)
```

Fricas [A]
time = 0.56, size = 12, normalized size = 0.75

$$\frac{1}{6}\,\sqrt{4\,x+2}(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+4*x)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(4*x + 2)*(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.71, size = 31, normalized size = 1.94

$$\frac{\sqrt{2}x\sqrt{2x+1}}{6} - \frac{\sqrt{2}\sqrt{2x+1}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+4*x)**(1/2),x)
```

```
[Out] sqrt(2)*x*sqrt(2*x + 1)/6 - sqrt(2)*sqrt(2*x + 1)/6
```

Giac [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.50, size = 22, normalized size = 1.38

$$\frac{1}{12}\sqrt{2}\left((2x+1)^{\frac{3}{2}} - 3\sqrt{2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*((2*x + 1)^(3/2) - 3*sqrt(2*x + 1))
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.88

$$\frac{\sqrt{4x+2}\,(4x-4)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x + 2)^(1/2),x)
```

```
[Out] ((4*x + 2)^(1/2)*(4*x - 4))/24
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 0.56

$$\frac{\sqrt{2+4x}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(x/(2+4*x)^(1/2),x)
```

```
[Out] 1/2*(2+4*x)^(1/2)
```

3.120 $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\sin\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3461, 2717}

$$2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 2\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Sin[Sqrt[x]]
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right)$	7
default	$2\sin\left(\sqrt{x}\right)$	7
meijerg	$2\sin\left(\sqrt{x}\right)$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*sin(x^(1/2))
```

Maxima [A]
time = 0.41, size = 6, normalized size = 0.75

$$2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sin(sqrt(x))
```

Fricas [A]
time = 0.60, size = 6, normalized size = 0.75

$$2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*sin(sqrt(x))
```

Sympy [A]
time = 0.11, size = 7, normalized size = 0.88

$$2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))/x**(1/2),x)
[Out] 2*sin(sqrt(x))
```

Giac [A]
time = 0.45, size = 6, normalized size = 0.75

$$2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*sin(sqrt(x))
```

Mupad [B]
time = 0.04, size = 6, normalized size = 0.75

$$2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))/x^(1/2),x)
[Out] 2*sin(x^(1/2))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(cos(x^(1/2))/x^(1/2),x)
[Out] not solved
```

3.121 $\int \sec(x)\,dx$

Optimal. Leaf size=3

$$\operatorname{arctanh}(\sin(x))$$

[Out] arctanh(sin(x))

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\operatorname{arctanh}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x],x]

[Out] ArcTanh[Sin[x]]

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{Integral} = \operatorname{arctanh}(\sin(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(3) = 6.
time = 0.00, size = 33, normalized size = 11.00

$$-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x],x]

[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]

Maple [A]
time = 0.02, size = 7, normalized size = 2.33

method	result	size

lookup	$\ln(\sec(x)+\tan(x))$	7
default	$\ln(\sec(x)+\tan(x))$	7
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
risch	$\ln(i+e^{ix})-\ln(e^{ix}-i)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sec(x)+tan(x))
```

Maxima [A]
time = 0.34, size = 6, normalized size = 2.00

$$\log(\sec(x)+\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x),x, algorithm="maxima")
```

```
[Out] log(sec(x) + tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.
time = 0.61, size = 17, normalized size = 5.67

$$\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.
time = 0.06, size = 15, normalized size = 5.00

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(3) = 6$.
time = 0.46, size = 25, normalized size = 8.33

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right) - \frac{1}{4}\log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x),x, algorithm="giac")
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2))
```

Mupad [B]
time = 0.00, size = 11, normalized size = 3.67

$$\ln\left(\frac{1}{\cos(x)}\right) + \ln(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x)
[Out] log(1/cos(x)) + log(sin(x) + 1)
```

Chatgpt [A]
time = 1.00, size = 6, normalized size = 2.00

$$\ln(\sec(x) + \tan(x))$$

Antiderivative was successfully verified.

```
[In] int(sec(x),x)
[Out] ln(sec(x)+tan(x))
```

3.122 $\int e^{\sin(x)} \cos(x) \, dx$

Optimal. Leaf size=4

$$e^{\sin(x)}$$

```
[Out] exp(sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4419, 2225}

$$e^{\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sin[x]*Cos[x],x]
```

```
[Out] E^Sin[x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \text{Subst}\left(\int e^x \, dx, x, \sin(x)\right) \\ &= e^{\sin(x)} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$e^{\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Cos[x],x]
```

```
[Out] E^Sin[x]
```

Maple [A]
time = 0.42, size = 4, normalized size = 1.00

method	result	size
derivativedivides	$e^{\sin(x)}$	4
default	$e^{\sin(x)}$	4
risch	$e^{\sin(x)}$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))/tan(x)/csc(x),x,method=_RETURNVERBOSE)
[Out] exp(sin(x))
```

Maxima [A]
time = 0.35, size = 3, normalized size = 0.75

$$e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))/tan(x)/csc(x),x, algorithm="maxima")
[Out] e^sin(x)
```

Fricas [A]
time = 0.59, size = 3, normalized size = 0.75

$$e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))/tan(x)/csc(x),x, algorithm="fricas")
[Out] e^sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{\sin(x)}}{\tan(x)\csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))/tan(x)/csc(x),x)
[Out] Integral(exp(sin(x))/(tan(x)*csc(x)), x)
```

Giac [A]
time = 0.45, size = 3, normalized size = 0.75

$$e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))/tan(x)/csc(x),x, algorithm="giac")
```

```
[Out] e^sin(x)
```

Mupad [B]
time = 0.15, size = 3, normalized size = 0.75

$$\mathrm{e}^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(sin(x))*sin(x))/tan(x),x)
```

```
[Out] exp(sin(x))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 15, normalized size = 3.75

$$\frac{\cos(x)\,\mathrm{e}^{\sin(x)}}{2} - \frac{\sin(x)\,\mathrm{e}^{\sin(x)}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(sin(x))/tan(x)/csc(x),x)
```

```
[Out] 1/2*cos(x)*exp(sin(x))-1/2*sin(x)*exp(sin(x))
```

3.123 $\int x \log^2(x) \, dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2342, 2341}

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \frac{1}{2}x^2 \log^2(x) - \int x \log(x) \, dx \\ &= \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parallelrisch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
parts	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A]
time = 0.36, size = 17, normalized size = 0.61

$$\frac{1}{4}\left(2\log\left(x\right)^2-2\log\left(x\right)+1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A]
time = 0.56, size = 22, normalized size = 0.79

$$\frac{1}{2}x^2\log\left(x\right)^2-\frac{1}{2}x^2\log\left(x\right)+\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.79

$$\frac{x^2\log\left(x\right)^2}{2}-\frac{x^2\log\left(x\right)}{2}+\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Giac [A]
time = 0.41, size = 22, normalized size = 0.79

$$\frac{1}{2}\,x^2\log\left(x\right)^2 - \frac{1}{2}\,x^2\log\left(x\right) + \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Mupad [B]
time = 0.04, size = 17, normalized size = 0.61

$$\frac{x^2\left(2\ln\left(x\right)^2 - 2\ln\left(x\right) + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x)^2,x)
```

```
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

Chatgpt [F] Failed to verify
time = 1.00, size = 18, normalized size = 0.64

$$\frac{\ln\left(x\right)^2 x^2}{2} - 2x\ln\left(x\right) + 2x$$

Warning: Unable to verify antiderivative.

```
[In] int(x*ln(x)^2,x)
```

```
[Out] 1/2*ln(x)^2*x^2-2*x*ln(x)+2*x
```

3.124 $\int \frac{1}{5+4\sqrt{x}+x} \, dx$

Optimal. Leaf size=22

$$-4\arctan\left(2+\sqrt{x}\right)+\log\left(5+4\sqrt{x}+x\right)$$

[Out] -4*arctan(2+x^(1/2))+ln(5+4*x^(1/2)+x)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1355, 648, 632, 210, 642}

$$\log\left(x+4\sqrt{x}+5\right)-4\arctan\left(\sqrt{x}+2\right)$$

Antiderivative was successfully verified.

[In] Int[(5 + 4*Sqrt[x] + x)^(-1),x]

[Out] -4*ArcTan[2 + Sqrt[x]] + Log[5 + 4*Sqrt[x] + x]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int \frac{x}{5+4x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\left(4\text{Subst}\left(\int \frac{1}{5+4x+x^2}\,dx, x, \sqrt{x}\right)\right) + \text{Subst}\left(\int \frac{4+2x}{5+4x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \log\left(5+4\sqrt{x}+x\right) + 8\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, 4+2\sqrt{x}\right)\\
&= -4\arctan\left(2+\sqrt{x}\right) + \log\left(5+4\sqrt{x}+x\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-4\arctan\left(2+\sqrt{x}\right) + \log\left(5+4\sqrt{x}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Sqrt[x] + x)^(-1),x]
[Out] -4*ArcTan[2 + Sqrt[x]] + Log[5 + 4*Sqrt[x] + x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(18) = 36$.
time = 0.09, size = 59, normalized size = 2.68

method	result
derivativedivides	$-4\arctan\left(2+\sqrt{x}\right)+\ln\left(5+4\sqrt{x}+x\right)$
default	$2\arctan\left(\frac{x}{4}-\frac{3}{4}\right)+\frac{\ln(x^2-6x+25)}{2}-\frac{\ln(x+5-4\sqrt{x})}{2}-2\arctan\left(\sqrt{x}-2\right)+\frac{\ln(5+4\sqrt{x}+x)}{2}-2\,\text{arc}$
trager	$\mathit{RootOf}\left(\_Z^2-2\_Z+5\right)\ln\left(5+4\sqrt{x}+x\right)-\ln\left(79\mathit{RootOf}\left(\_Z^2-2\_Z+5\right)^2x-79\mathit{Ro}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*x^(1/2)+x),x,method=_RETURNVERBOSE)
[Out] 2*arctan(1/4*x-3/4)+1/2*ln(x^2-6*x+25)-1/2*ln(x+5-4*x^(1/2))-2*arctan(x^(1/
2)-2)+1/2*ln(5+4*x^(1/2)+x)-2*arctan(2+x^(1/2))
```

Maxima [A]
time = 0.55, size = 18, normalized size = 0.82

$$-4\arctan\left(\sqrt{x}+2\right)+\log\left(x+4\sqrt{x}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*x^(1/2)+x),x, algorithm="maxima")
[Out] -4*arctan(sqrt(x) + 2) + log(x + 4*sqrt(x) + 5)
```

Fricas [A]
time = 0.58, size = 18, normalized size = 0.82

$$-4\arctan\left(\sqrt{x}+2\right)+\log\left(x+4\sqrt{x}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*x^(1/2)+x),x, algorithm="fricas")
[Out] -4*arctan(sqrt(x) + 2) + log(x + 4*sqrt(x) + 5)
```

Sympy [A]
time = 0.10, size = 20, normalized size = 0.91

$$\log\left(4\sqrt{x}+x+5\right)-4\operatorname{atan}\left(\sqrt{x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*x**(1/2)+x),x)
[Out] log(4*sqrt(x) + x + 5) - 4*atan(sqrt(x) + 2)
```

Giac [A]
time = 0.44, size = 18, normalized size = 0.82

$$-4\arctan\left(\sqrt{x}+2\right)+\log\left(x+4\sqrt{x}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*x^(1/2)+x),x, algorithm="giac")
[Out] -4*arctan(sqrt(x) + 2) + log(x + 4*sqrt(x) + 5)
```

Mupad [B]
time = 0.06, size = 18, normalized size = 0.82

$$\ln\left(x+4\sqrt{x}+5\right)-4\operatorname{atan}\left(\sqrt{x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 4*x^(1/2) + 5),x)
```

```
[Out] log(x + 4*x^(1/2) + 5) - 4*atan(x^(1/2) + 2)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 19, normalized size = 0.86

$$2\ln\left(2\sqrt{x}+5\right)-2\ln\left(\sqrt{x}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(5+4*x^(1/2)+x),x)
```

```
[Out] 2*ln(2*x^(1/2)+5)-2*ln(x^(1/2)+1)
```

3.125 $\int 2015^x \, dx$

Optimal. Leaf size=8

$$\frac{2015^x}{\log(2015)}$$

```
[Out] 2015^x/ln(2015)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$\frac{2015^x}{\log(2015)}$$

Antiderivative was successfully verified.

```
[In] Int[2015^x,x]
```

```
[Out] 2015^x/Log[2015]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\text{Integral} = \frac{2015^x}{\log(2015)}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{2015^x}{\log(2015)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2015^x,x]
```

```
[Out] 2015^x/Log[2015]
```

Maple [A]
time = 0.02, size = 9, normalized size = 1.12

method	result	size
gosper	$\frac{2015^x}{\ln(2015)}$	9
derivativedivides	$\frac{2015^x}{\ln(2015)}$	9
default	$\frac{2015^x}{\ln(2015)}$	9
parallelrisch	$\frac{2015^x}{\ln(2015)}$	9
norman	$\frac{e^{x\ln(2015)}}{\ln(2015)}$	11
meijerg	$-\frac{1-e^{x\ln(2015)}}{\ln(2015)}$	16
risch	$\frac{31^x 5^x 13^x}{\ln(5)+\ln(13)+\ln(31)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2015^x,x,method=_RETURNVERBOSE)
[Out] 2015^x/ln(2015)
```

Maxima [A]
time = 0.37, size = 8, normalized size = 1.00

$$\frac{2015^x}{\log(2015)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2015^x,x, algorithm="maxima")
[Out] 2015^x/log(2015)
```

Fricas [A]
time = 0.57, size = 8, normalized size = 1.00

$$\frac{2015^x}{\log(2015)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2015^x,x, algorithm="fricas")
[Out] 2015^x/log(2015)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.62

$$\frac{2015^x}{\log(2015)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2015**x,x)
```

```
[Out] 2015**x/log(2015)
```

Giac [A]
time = 0.42, size = 8, normalized size = 1.00

$$\frac{2015^x}{\log(2015)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2015^x,x, algorithm="giac")
```

```
[Out] 2015^x/log(2015)
```

Mupad [B]
time = 0.05, size = 8, normalized size = 1.00

$$\frac{2015^x}{\ln(2015)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2015^x,x)
```

```
[Out] 2015^x/log(2015)
```

Chatgpt [A]
time = 1.00, size = 8, normalized size = 1.00

$$\frac{2015^x}{\ln(2015)}$$

Antiderivative was successfully verified.

```
[In] int(2015^x,x)
```

```
[Out] 1/ln(2015)*2015^x
```

3.126 $\int \frac{x}{(-3+x)(5+x)^2} \, dx$

Optimal. Leaf size=22

$$-\frac{5}{8(5+x)}-\frac{3}{32}\text{arctanh}\left(\frac{1+x}{4}\right)$$

```
[Out] -5/(8*x+40)-3/32*arctanh(1/4*x+1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{78\}$

$$-\frac{5}{8(x+5)}+\frac{3}{64}\log(3-x)-\frac{3}{64}\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-3 + x)*(5 + x)^2),x]
```

```
[Out] -5/(8*(5 + x)) + (3*Log[3 - x])/64 - (3*Log[5 + x])/64
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{3}{64(-3+x)}+\frac{5}{8(5+x)^2}-\frac{3}{64(5+x)}\right) dx\\
&= -\frac{5}{8(5+x)}+\frac{3}{64}\log(3-x)-\frac{3}{64}\log(5+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.27

$$-\frac{5}{8(5+x)}+\frac{3}{64}\log(3-x)-\frac{3}{64}\log(5+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-3 + x)*(5 + x)^2),x]
[Out] -5/(8*(5 + x)) + (3*Log[3 - x])/64 - (3*Log[5 + x])/64
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.95

method	result	size
default	$-\frac{5}{8(x+5)}-\frac{3\ln(x+5)}{64}+\frac{3\ln(-3+x)}{64}$	21
norman	$-\frac{5}{8(x+5)}-\frac{3\ln(x+5)}{64}+\frac{3\ln(-3+x)}{64}$	21
risch	$-\frac{5}{8(x+5)}-\frac{3\ln(x+5)}{64}+\frac{3\ln(-3+x)}{64}$	21
parallelrisch	$\frac{3\ln(-3+x)x-3\ln(x+5)x-40+15\ln(-3+x)-15\ln(x+5)}{64x+320}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3+x)/(x+5)^2,x,method=_RETURNVERBOSE)
[Out] -5/8/(x+5)-3/64*ln(x+5)+3/64*ln(-3+x)
```

Maxima [A]
time = 0.38, size = 20, normalized size = 0.91

$$-\frac{5}{8\,(x+5)}-\frac{3}{64}\,\log\left(x+5\right)+\frac{3}{64}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3+x)/(x+5)^2,x, algorithm="maxima")
[Out] -5/8/(x + 5) - 3/64*log(x + 5) + 3/64*log(x - 3)
```

Fricas [A]
time = 0.57, size = 27, normalized size = 1.23

$$-\frac{3\,(x+5)\log\left(x+5\right)-3\,(x+5)\log\left(x-3\right)+40}{64\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3+x)/(x+5)^2,x, algorithm="fricas")
[Out] -1/64*(3*(x + 5)*log(x + 5) - 3*(x + 5)*log(x - 3) + 40)/(x + 5)
```

Sympy [A]
time = 0.04, size = 22, normalized size = 1.00

$$\frac{3\log\left(x-3\right)}{64}-\frac{3\log\left(x+5\right)}{64}-\frac{5}{8x+40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3+x)/(x+5)**2,x)
```

```
[Out] 3*log(x - 3)/64 - 3*log(x + 5)/64 - 5/(8*x + 40)
```

Giac [A]
time = 0.47, size = 21, normalized size = 0.95

$$-\frac{5}{8\,(x+5)}+\frac{3}{64}\,\log\left(\left|-\frac{8}{x+5}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3+x)/(x+5)^2,x, algorithm="giac")
```

```
[Out] -5/8/(x + 5) + 3/64*log(abs(-8/(x + 5) + 1))
```

Mupad [B]
time = 0.16, size = 22, normalized size = 1.00

$$-\frac{3\,\ln\left(\frac{x+5}{x-3}\right)}{64}-\frac{5}{8\,\left(x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 3)*(x + 5)^2),x)
```

```
[Out] - (3*log((x + 5)/(x - 3)))/64 - 5/(8*(x + 5))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 20, normalized size = 0.91

$$-\frac{\ln\left(x+5\right)}{16}-\frac{9}{16\,(x+5)}-\frac{\ln\left(x-3\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(x/(x-3)/(x+5)^2,x)
```

```
[Out] -1/16*ln(x+5)-9/16/(x+5)-1/4*ln(x-3)
```

3.127 $\int \frac{\log(1+\log(x))}{x} dx$

Optimal. Leaf size=15

$$-\log(x) + (1 + \log(x))\log(1 + \log(x))$$

```
[Out] -ln(x)+(1+ln(x))*ln(1+ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2436, 2332}

$$(\log(x) + 1)\log(\log(x) + 1) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + Log[x]]/x,x]
```

```
[Out] -Log[x] + (1 + Log[x])*Log[1 + Log[x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \log(1+x)\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \log(x)\,dx, x, 1+\log(x)\right)\\
&= -\log(x) + (1+\log(x))\log(1+\log(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.20

$$-\log(x) + \log(1 + \log(x)) + \log(x)\log(1 + \log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + Log[x]]/x,x]
[Out] -Log[x] + Log[1 + Log[x]] + Log[x]*Log[1 + Log[x]]
```

Maple [A]
time = 0.03, size = 17, normalized size = 1.13

method	result	size
derivativedivides	$(1+\ln(x))\ln(1+\ln(x))-1-\ln(x)$	17
default	$(1+\ln(x))\ln(1+\ln(x))-1-\ln(x)$	17
norman	$-\ln(x)+\ln(1+\ln(x))\ln(x)+\ln(1+\ln(x))$	19
risch	$-\ln(x)+\ln(1+\ln(x))\ln(x)+\ln(1+\ln(x))$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+ln(x))/x,x,method=_RETURNVERBOSE)
[Out] (1+ln(x))*ln(1+ln(x))-1-ln(x)
```

Maxima [A]
time = 0.37, size = 16, normalized size = 1.07

$$(\log(x)+1)\log(\log(x)+1)-\log(x)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+log(x))/x,x, algorithm="maxima")
[Out] (log(x) + 1)*log(log(x) + 1) - log(x) - 1
```

Fricas [A]
time = 0.59, size = 15, normalized size = 1.00

$$(\log(x)+1)\log(\log(x)+1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+log(x))/x,x, algorithm="fricas")
[Out] (log(x) + 1)*log(log(x) + 1) - log(x)
```

Sympy [A]
time = 0.08, size = 19, normalized size = 1.27

$$\log(x)\log(\log(x)+1)-\log(x)+\log(\log(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+ln(x))/x,x)
```

```
[Out] log(x)*log(log(x) + 1) - log(x) + log(log(x) + 1)
```

Giac [A]
time = 0.43, size = 16, normalized size = 1.07

$$(\log(x)+1)\log(\log(x)+1)-\log(x)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+log(x))/x,x, algorithm="giac")
[Out] (log(x) + 1)*log(log(x) + 1) - log(x) - 1
```

Mupad [B]
time = 0.30, size = 18, normalized size = 1.20

$$\ln(\ln(x)+1)-\ln(x)+\ln(\ln(x)+1)\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x) + 1)/x,x)
[Out] log(log(x) + 1) - log(x) + log(log(x) + 1)*log(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 5, normalized size = 0.33

$$\text{hyperbolicCosineIntegral}(1+\ln(x))$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(1+ln(x))/x,x)
[Out] Li(1+ln(x))
```

3.128 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

Optimal. Leaf size=13

$$2\sqrt{\cos(x)\cot(x)}\tan(x)$$

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4482, 4485, 2669}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \sqrt{\cos(x)\cot(x)}\,dx\\ &= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\ &= 2\sqrt{\cos(x)\cot(x)}\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$2\sqrt{\cos(x)\cot(x)}\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Maple [A]
time = 0.35, size = 12, normalized size = 0.92

method	result	size
default	$2\sqrt{\cos\left(x\right)\cot\left(x\right)}\,\tan\left(x\right)$	12
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left(e^{2ix}+1\right)^{2}e^{-ix}}{e^{2ix}-1}}\,\left(e^{2ix}-1\right)}{e^{2ix}+1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(11) = 22$.
time = 0.60, size = 188, normalized size = 14.46

$$\frac{\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)-\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\right)-\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)+\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)-1\right)\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)\right)}{\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)^{\frac{1}{4}}\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

Fricas [A]
time = 0.60, size = 19, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}}\,\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sin(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(1/2),x)
[Out] Integral(sqrt(-sin(x) + csc(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(csc(x) - sin(x)), x)
```

Mupad [B]
time = 0.00, size = 15, normalized size = 1.15

$$\frac{2\,|\cos(x)|}{\cos(x)\,\sqrt{\frac{1}{\sin(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(1/2),x)
[Out] (2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((csc(x)-sin(x))^(1/2),x)
[Out] not solved
```

3.129 $\int \frac{1}{\sqrt{25+x^2}}\,dx$

Optimal. Leaf size=6

$$\operatorname{arcsinh}\left(\frac{x}{5}\right)$$

```
[Out] arcsinh(1/5*x)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\operatorname{arcsinh}\left(\frac{x}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[25 + x^2],x]
```

```
[Out] ArcSinh[x/5]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{Integral} = \operatorname{arcsinh}\left(\frac{x}{5}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.
time = 0.01, size = 16, normalized size = 2.67

$$-\log\left(-x+\sqrt{25+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[25 + x^2],x]
```

```
[Out] -Log[-x + Sqrt[25 + x^2]]
```

Maple [A]
time = 0.08, size = 5, normalized size = 0.83

method	result	size

default	$\operatorname{arcsinh}\left(\frac{x}{5}\right)$	5
meijerg	$\operatorname{arcsinh}\left(\frac{x}{5}\right)$	5
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{x^2+25}}{x}\right)$	13
trager	$-\ln\left(x-\sqrt{x^2+25}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+25)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(1/5*x)
```

Maxima [A]
time = 0.44, size = 4, normalized size = 0.67

$$\operatorname{arsinh}\left(\frac{1}{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+25)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/5*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.
time = 0.57, size = 14, normalized size = 2.33

$$-\log\left(-x+\sqrt{x^2+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+25)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 25))
```

Sympy [A]
time = 0.06, size = 3, normalized size = 0.50

$$\operatorname{asinh}\left(\frac{x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+25)**(1/2),x)
```

```
[Out] asinh(x/5)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(4) = 8$.
time = 0.51, size = 25, normalized size = 4.17

$$\frac{1}{2}\sqrt{x^2+25}x - \frac{25}{2}\log\left(-x+\sqrt{x^2+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+25)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 25)*x - 25/2*log(-x + sqrt(x^2 + 25))
```

Mupad [**B**]
time = 0.08, size = 4, normalized size = 0.67

$$\operatorname{asinh}\left(\frac{x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 25)^(1/2),x)
```

```
[Out] asinh(x/5)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 4, normalized size = 0.67

$$\arctan\left(\frac{x}{5}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^2+25)^(1/2),x)
```

```
[Out] arctan(1/5*x)
```

3.130 $\int \frac{-1+\log^2(x)}{x\log^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{1}{\log(x)}+\log(x)$$

[Out] 1/ln(x)+ln(x)

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {14}

$$\log(x)+\frac{1}{\log(x)}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Log[x]^2)/(x*Log[x]^2),x]

[Out] Log[x]^(-1) + Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\text{Subst}\left(\int\frac{-1+x^2}{x^2}\,dx,x,\log(x)\right)\\&=\text{Subst}\left(\int\left(1-\frac{1}{x^2}\right)\,dx,x,\log(x)\right)\\&=\frac{1}{\log(x)}+\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{1}{\log(x)}+\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Log[x]^2)/(x*Log[x]^2),x]

```
[Out] Log[x]^(-1) + Log[x]
```

Maple [A]
time = 0.02, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$\frac{1}{\ln(x)}+\ln(x)$	8
default	$\frac{1}{\ln(x)}+\ln(x)$	8
risch	$\frac{1}{\ln(x)}+\ln(x)$	8
parts	$\frac{1}{\ln(x)}+\ln(x)$	8
norman	$\frac{1+\ln(x)^2}{\ln(x)}$	12
parallelrisch	$\frac{1+\ln(x)^2}{\ln(x)}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)^2-1)/x/ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/ln(x)+ln(x)
```

Maxima [A]
time = 0.39, size = 7, normalized size = 1.00

$$\frac{1}{\log(x)}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^2-1)/x/log(x)^2,x, algorithm="maxima")
[Out] 1/log(x) + log(x)
```

Fricas [A]
time = 0.58, size = 11, normalized size = 1.57

$$\frac{\log(x)^2+1}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^2-1)/x/log(x)^2,x, algorithm="fricas")
[Out] (log(x)^2 + 1)/log(x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.00

$$\log(x)+\frac{1}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(x)**2-1)/x/ln(x)**2,x)
```

```
[Out] log(x) + 1/log(x)
```

Giac [A]
time = 0.50, size = 7, normalized size = 1.00

$$\frac{1}{\log(x)} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^2-1)/x/log(x)^2,x, algorithm="giac")
```

```
[Out] 1/log(x) + log(x)
```

Mupad [B]
time = 0.11, size = 7, normalized size = 1.00

$$\ln(x) + \frac{1}{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^2 - 1)/(x*log(x)^2),x)
```

```
[Out] log(x) + 1/log(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 1.29

$$\ln(x) - \frac{1}{\ln(x)}$$

Warning: Unable to verify antiderivative.

```
[In] int((ln(x)^2-1)/x/ln(x)^2,x)
```

```
[Out] ln(x)-1/ln(x)
```

3.131 $\int e^{3x} \arctan(e^x)\, dx$

Optimal. Leaf size=35

$$-\frac{e^{2x}}{6}+\frac{1}{3}e^{3x}\arctan(e^x)+\frac{1}{6}\log\left(1+e^{2x}\right)$$

[Out] -1/6*exp(2*x)+1/3*exp(3*x)*arctan(exp(x))+1/6*ln(1+exp(2*x))

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2225, 5315, 12, 2280, 45}

$$\frac{1}{3}e^{3x}\arctan(e^x)-\frac{e^{2x}}{6}+\frac{1}{6}\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

[In] Int[E^(3*x)*ArcTan[E^x],x]

[Out] -1/6*E^(2*x) + (E^(3*x)*ArcTan[E^x])/3 + Log[1 + E^(2*x)]/6

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{1}{3}e^{3x}\arctan\left(e^x\right)-\int\frac{e^{4x}}{3\left(1+e^{2x}\right)}\,dx\\
&= \frac{1}{3}e^{3x}\arctan\left(e^x\right)-\frac{1}{3}\int\frac{e^{4x}}{1+e^{2x}}\,dx\\
&= \frac{1}{3}e^{3x}\arctan\left(e^x\right)-\frac{1}{6}\text{Subst}\left(\int\frac{x}{1+x}\,dx,x,e^{2x}\right)\\
&= \frac{1}{3}e^{3x}\arctan\left(e^x\right)-\frac{1}{6}\text{Subst}\left(\int\left(1+\frac{1}{-1-x}\right)\,dx,x,e^{2x}\right)\\
&= -\frac{e^{2x}}{6}+\frac{1}{3}e^{3x}\arctan\left(e^x\right)+\frac{1}{6}\log\left(1+e^{2x}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 31, normalized size = 0.89

$$\frac{1}{6}\left(-e^{2x}+2e^{3x}\arctan\left(e^x\right)+\log\left(1+e^{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*ArcTan[E^x],x]
```

```
[Out] (-E^(2*x) + 2*E^(3*x)*ArcTan[E^x] + Log[1 + E^(2*x)])/6
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.17, size = 47, normalized size = 1.34

method	result	size
risch	$-\frac{i e^{3x}\ln(1+ie^x)}{6}+\frac{i e^{3x}\ln(1-ie^x)}{6}-\frac{e^{2x}}{6}+\frac{\ln(1+e^{2x})}{6}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*arctan(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I*exp(3*x)*ln(1+I*exp(x))+1/6*I*exp(3*x)*ln(1-I*exp(x))-1/6*exp(2*x)+1
/6*ln(1+exp(2*x))
```

Maxima [A]
time = 0.37, size = 25, normalized size = 0.71

$$\frac{1}{3}\arctan\left(e^{x}\right)e^{(3\,x)} - \frac{1}{6}\,e^{(2\,x)} + \frac{1}{6}\,\log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*arctan(exp(x)),x, algorithm="maxima")
[Out] 1/3*arctan(e^x)*e^(3*x) - 1/6*e^(2*x) + 1/6*log(e^(2*x) + 1)
```

Fricas [A]
time = 0.63, size = 25, normalized size = 0.71

$$\frac{1}{3}\arctan\left(e^{x}\right)e^{(3\,x)} - \frac{1}{6}\,e^{(2\,x)} + \frac{1}{6}\,\log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*arctan(exp(x)),x, algorithm="fricas")
[Out] 1/3*arctan(e^x)*e^(3*x) - 1/6*e^(2*x) + 1/6*log(e^(2*x) + 1)
```

Sympy [A]
time = 5.23, size = 27, normalized size = 0.77

$$\frac{e^{3x}\operatorname{atan}\left(e^{x}\right)}{3} - \frac{e^{2x}}{6} + \frac{\log\left(e^{2x} + 1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*atan(exp(x)),x)
[Out] exp(3*x)*atan(exp(x))/3 - exp(2*x)/6 + log(exp(2*x) + 1)/6
```

Giac [A]
time = 0.46, size = 25, normalized size = 0.71

$$\frac{1}{3}\arctan\left(e^{x}\right)e^{(3\,x)} - \frac{1}{6}\,e^{(2\,x)} + \frac{1}{6}\,\log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*arctan(exp(x)),x, algorithm="giac")
[Out] 1/3*arctan(e^x)*e^(3*x) - 1/6*e^(2*x) + 1/6*log(e^(2*x) + 1)
```

Mupad [B]
time = 0.19, size = 25, normalized size = 0.71

$$\frac{\ln\left(\mathrm{e}^{2\,x} + 1\right)}{6} - \frac{\mathrm{e}^{2\,x}}{6} + \frac{\mathrm{atan}(\mathrm{e}^{x})\,\mathrm{e}^{3\,x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(exp(x))*exp(3*x),x)
```

```
[Out] log(exp(2*x) + 1)/6 - exp(2*x)/6 + (atan(exp(x))*exp(3*x))/3
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 22, normalized size = 0.63

$$\frac{e^{3x}(3x\arctan\left(e^{x}\right)-\ln\left(1+e^{2x}\right))}{6}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(3*x)*arctan(exp(x)),x)
```

```
[Out] 1/6*exp(3*x)*(3*x*arctan(exp(x))-ln(1+exp(2*x)))
```

3.132 $\int \frac{1}{\cos^4(x)+\sin^4(x)}\,dx$

Optimal. Leaf size=71

$$\sqrt{2}x+\frac{\arctan\left(\frac{1-2\cos^2(x)}{1+\sqrt{2}-2\cos(x)\sin(x)}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{1-2\cos^2(x)}{1+\sqrt{2}+2\cos(x)\sin(x)}\right)}{\sqrt{2}}$$

```
[Out] 2^(1/2)*x+1/2*arctan((1-2*cos(x)^2)/(1+2^(1/2)-2*cos(x)*sin(x)))*2^(1/2)-1/
2*arctan((1-2*cos(x)^2)/(1+2^(1/2)+2*cos(x)*sin(x)))*2^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1176, 631, 210}

$$\frac{\arctan\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{2}+1}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{2}+1}\right)}{\sqrt{2}}+\sqrt{2}x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^4 + Sin[x]^4)^(-1),x]
```

```
[Out] Sqrt[2]*x + ArcTan[(1 - 2*Cos[x]^2)/(1 + Sqrt[2] - 2*Cos[x]*Sin[x])]/Sqrt[2
] - ArcTan[(1 - 2*Cos[x]^2)/(1 + Sqrt[2] + 2*Cos[x]*Sin[x])]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \tan(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \tan(x)\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\tan(x)\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\tan(x)\right)}{\sqrt{2}} \\
&= \sqrt{2}x + \frac{\arctan\left(\frac{1-2\cos^2(x)}{1+\sqrt{2}-2\cos(x)\sin(x)}\right)}{\sqrt{2}} - \frac{\arctan\left(\frac{1-2\cos^2(x)}{1+\sqrt{2}+2\cos(x)\sin(x)}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.45

$$
\frac{-\arctan\left(1-\sqrt{2}\tan(x)\right)+\arctan\left(1+\sqrt{2}\tan(x)\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^4 + Sin[x]^4)^(-1),x]
[Out] (-ArcTan[1 - Sqrt[2]*Tan[x]] + ArcTan[1 + Sqrt[2]*Tan[x]])/Sqrt[2]
```

Maple [A]
time = 0.15, size = 116, normalized size = 1.63

method	result
risch	$\frac{i\sqrt{2}\,\ln\left(e^{4ix}+2\sqrt{2}+3\right)}{4} - \frac{i\sqrt{2}\,\ln\left(e^{4ix}-2\sqrt{2}+3\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan^2(x)+\tan(x)\sqrt{2}+1}{\tan^2(x)-\tan(x)\sqrt{2}+1}\right)+2\arctan\left(\tan(x)\sqrt{2}+1\right)+2\arctan\left(\tan(x)\sqrt{2}-1\right)\right)}{8} + \frac{\sqrt{2}\left(\ln\left(\frac{\tan^2(x)-\tan(x)\sqrt{2}+1}{\tan^2(x)+\tan(x)\sqrt{2}+1}\right)+2\arctan\left(\tan\right.\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4+cos(x)^4),x,method=_RETURNVERBOSE)
[Out] 1/8*2^(1/2)*(ln((tan(x)^2+tan(x)*2^(1/2)+1)/(tan(x)^2-tan(x)*2^(1/2)+1))+2*
arctan(tan(x)*2^(1/2)+1)+2*arctan(tan(x)*2^(1/2)-1))+1/8*2^(1/2)*(ln((tan(x
)^2-tan(x)*2^(1/2)+1)/(tan(x)^2+tan(x)*2^(1/2)+1))+2*arctan(tan(x)*2^(1/2)+
1)+2*arctan(tan(x)*2^(1/2)-1))
```

Maxima [A]
time = 0.50, size = 39, normalized size = 0.55

$$
\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,\tan\left(x\right)\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,\tan\left(x\right)\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)^4+cos(x)^4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*tan(x))) + 1/2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*tan(x)))
```

Fricas [A]
time = 0.62, size = 47, normalized size = 0.66

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{6\sqrt{2}\cos(x)^4-6\sqrt{2}\cos(x)^2+\sqrt{2}}{4\left(2\cos(x)^3-\cos(x)\right)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)^4+cos(x)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*(6*sqrt(2)*cos(x)^4 - 6*sqrt(2)*cos(x)^2 + sqrt(2))
/((2*cos(x)^3 - cos(x))*sin(x)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)**4+cos(x)**4),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 48, normalized size = 0.68

$$\frac{1}{2}\sqrt{2}\left(2x+\arctan\left(-\frac{\sqrt{2}\sin(4x)-\sin(4x)}{\sqrt{2}\cos(4x)+\sqrt{2}-\cos(4x)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)^4+cos(x)^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(2*x + arctan(-(sqrt(2)*sin(4*x) - sin(4*x))/(sqrt(2)*cos(4*x)
+ sqrt(2) - cos(4*x) + 1)))
```

Mupad [B]
time = 0.25, size = 44, normalized size = 0.62

$$\sqrt{2}\left(x-\mathrm{atan}(\tan(x))\right)+\frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)^3}{2}+\frac{\sqrt{2}\tan(x)}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4 + sin(x)^4),x)
```

```
[Out] 2^(1/2)*(x - atan(tan(x))) + (2^(1/2)*(atan((2^(1/2)*tan(x))/2) + atan((2^(
1/2)*tan(x)^3)/2 + (2^(1/2)*tan(x))/2)))/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 46, normalized size = 0.65

$$\frac{x}{2x^2+2}+\frac{\sqrt{2}\,\arctan\left(\frac{(\cos(x)-\sin(x))\sqrt{2}}{2}\right)}{16}-\frac{\sqrt{2}\,\arctan\left(\frac{(\cos(x)+\sin(x))\sqrt{2}}{2}\right)}{16}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(sin(x)^4+cos(x)^4),x)
```

```
[Out] x/(2*x^2+2)+1/16*2^(1/2)*arctan(1/2*(cos(x)-sin(x))*2^(1/2))-1/16*2^(1/2)*a
rctan(1/2*(cos(x)+sin(x))*2^(1/2))
```

3.133 $\int \frac{1+e^x}{1-e^x} \, dx$

Optimal. Leaf size=12

$$x - 2\log(1 - e^x)$$

[Out] x-2*ln(1-exp(x))

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2320, 78}

$$x - 2\log(1 - e^x)$$

Antiderivative was successfully verified.

[In] Int[(1 + E^x)/(1 - E^x),x]

[Out] x - 2*Log[1 - E^x]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{1+x}{(1-x)x} \, dx, x, e^x\right) \\
&= \text{Subst}\left(\int \left(-\frac{2}{-1+x} + \frac{1}{x}\right) \, dx, x, e^x\right) \\
&= x - 2\log(1 - e^x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.08

$$\log(e^x) - 2\log(-1 + e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)/(1 - E^x),x]
[Out] Log[E^x] - 2*Log[-1 + E^x]
```

Maple [A]
time = 0.02, size = 12, normalized size = 1.00

method	result	size
norman	$x - 2\ln(e^x - 1)$	10
risch	$x - 2\ln(e^x - 1)$	10
parallelrisch	$x - 2\ln(e^x - 1)$	10
derivativedivides	$\ln(e^x) - 2\ln(e^x - 1)$	12
default	$\ln(e^x) - 2\ln(e^x - 1)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+exp(x))/(1-exp(x)),x,method=_RETURNVERBOSE)
[Out] ln(exp(x))-2*ln(exp(x)-1)
```

Maxima [A]
time = 0.36, size = 9, normalized size = 0.75

$$x - 2\log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="maxima")
[Out] x - 2*log(e^x - 1)
```

Fricas [A]
time = 0.59, size = 9, normalized size = 0.75

$$x - 2\log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="fricas")
[Out] x - 2*log(e^x - 1)
```

Sympy [**A**]
time = 0.03, size = 8, normalized size = 0.67

$$x - 2\log\left(e^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x)
[Out] x - 2*log(exp(x) - 1)
```

Giac [**A**]
time = 0.46, size = 10, normalized size = 0.83

$$x - 2\,\log\left(|e^{x} - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="giac")
[Out] x - 2*log(abs(e^x - 1))
```

Mupad [**B**]
time = 0.11, size = 9, normalized size = 0.75

$$x - 2\,\ln\left(\mathrm{e}^{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x) + 1)/(exp(x) - 1),x)
[Out] x - 2*log(exp(x) - 1)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 11, normalized size = 0.92

$$-\ln\left(1 - \mathrm{e}^{x}\right)^{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((exp(x)+1)/(1-exp(x)),x)
[Out] -ln(1-exp(x))^2
```

3.134 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x-\tan(x)+\frac{\tan^3(x)}{3}$$

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4,x]
```

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{\tan^3(x)}{3}-\int \tan^2(x)\,dx\\&=-\tan(x)+\frac{\tan^3(x)}{3}+\int 1\,dx\\&=x-\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.29

$$x-\frac{4\tan(x)}{3}+\frac{1}{3}\sec^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4,x]
```

```
[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A]
time = 0.03, size = 15, normalized size = 1.07

method	result	size
norman	$x-\tan\left(x\right)+\frac{\left(\tan^{3}\left(x\right)\right)}{3}$	13
parallelrisch	$x-\tan\left(x\right)+\frac{\left(\tan^{3}\left(x\right)\right)}{3}$	13
derivativedivides	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
default	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
risch	$x-\frac{4i\left(3\,{\mathrm e}^{4ix}+3\,{\mathrm e}^{2ix}+2\right)}{3\left({\mathrm e}^{2ix}+1\right)^{3}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tan(x)^3-tan(x)+arctan(tan(x))
```

Maxima [A]
time = 0.43, size = 12, normalized size = 0.86

$$\frac{1}{3}\,\tan\left(x\right)^{3}+x-\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Fricas [A]
time = 0.58, size = 12, normalized size = 0.86

$$\frac{1}{3}\,\tan\left(x\right)^{3}+x-\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A]
time = 0.02, size = 19, normalized size = 1.36

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4,x)
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

Giac [A]
time = 0.51, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="giac")
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.86

$$\frac{\tan(x)^3}{3} - \tan(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
[Out] x - tan(x) + tan(x)^3/3
```

Chatgpt [F] Failed to verify
time = 1.00, size = 25, normalized size = 1.79

$$\frac{(\tan^4(x))}{3} + \frac{\ln(\sec(x) + \tan(x))}{6} - \frac{\ln(\sec(x) - \tan(x))}{6}$$

Warning: Unable to verify antiderivative.

```
[In] int(tan(x)^4,x)
[Out] 1/3*tan(x)^4+1/6*ln(sec(x)+tan(x))-1/6*ln(sec(x)-tan(x))
```

3.135 $\int \sin(x)\tan^2(x)\,dx$

Optimal. Leaf size=5

$$\cos(x)+\sec(x)$$

```
[Out] cos(x)+sec(x)
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2670, 14}

$$\cos(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[x]^2,x]
```

```
[Out] Cos[x] + Sec[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(x)\right)\\ &= \cos(x)+\sec(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\cos(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[x]^2,x]
```

```
[Out] Cos[x] + Sec[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.
time = 0.11, size = 20, normalized size = 4.00

method	result	size
default	$\frac{\sin^4(x)}{\cos(x)} + (2 + \sin^2(x))\cos(x)$	20
risch	$\frac{e^{3ix}+7\cos(x)+5i\sin(x)}{2\,e^{2ix}+2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)^4/cos(x)+(2+sin(x)^2)*cos(x)
```

Maxima [A]
time = 0.36, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/cos(x) + cos(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.61, size = 11, normalized size = 2.20

$$\frac{\cos(x)^2 + 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] (cos(x)^2 + 1)/cos(x)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)**2,x)
```

```
[Out] cos(x) + 1/cos(x)
```

Giac [**A**]
time = 0.47, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/cos(x) + cos(x)
```

Mupad [**B**]
time = 0.21, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^2,x)
```

```
[Out] cos(x) + 1/cos(x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 2.60

$$\frac{(\tan^3(x))}{3} + \frac{(\tan^5(x))}{5}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)*tan(x)^2,x)
```

```
[Out] 1/3*tan(x)^3+1/5*tan(x)^5
```

3.136 $\int \frac{1+x}{3+2x+x^2} dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(3+2x+x^2\right)$$

```
[Out] 1/2*ln(x^2+2*x+3)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {642}

$$\frac{1}{2}\log\left(x^2+2x+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(3 + 2*x + x^2),x]
[Out] Log[3 + 2*x + x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{Integral} = \frac{1}{2}\log\left(3+2x+x^2\right)$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{2}\log\left(3+2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(3 + 2*x + x^2),x]
[Out] Log[3 + 2*x + x^2]/2
```

Maple [A]
time = 0.34, size = 12, normalized size = 0.92

method	result	size
default	$\frac{\ln(x^2+2x+3)}{2}$	12
norman	$\frac{\ln(x^2+2x+3)}{2}$	12
risch	$\frac{\ln(x^2+2x+3)}{2}$	12
parallelrisch	$\frac{\ln(x^2+2x+3)}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^2+2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2+2*x+3)
```

Maxima [A]
time = 0.37, size = 11, normalized size = 0.85

$$\frac{1}{2}\log\left(x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 + 2*x + 3)
```

Fricas [A]
time = 0.59, size = 11, normalized size = 0.85

$$\frac{1}{2}\log\left(x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + 2*x + 3)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$\frac{\log\left(x^2+2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2
```

Giac [**A**]
time = 0.45, size = 11, normalized size = 0.85

$$\frac{1}{2}\log\left(x^{2}+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2 + 2*x + 3)
```

Mupad [**B**]
time = 0.03, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^{2}+2\,x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(2*x + x^2 + 3),x)
```

```
[Out] log(2*x + x^2 + 3)/2
```

Chatgpt [**A**]
time = 1.00, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^{2}+2x+3\right)}{2}$$

Antiderivative was successfully verified.

```
[In] int((x+1)/(x^2+2*x+3),x)
```

```
[Out] 1/2*ln(x^2+2*x+3)
```

3.137 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

```
[Out] ln(cosh(x))
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{Integral} = \log(\cosh(x))$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Maple [A]
time = 0.02, size = 4, normalized size = 1.33

method	result	size

lookup	$\ln(\cosh(x))$	4
derivativedivides	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x+\ln(1+e^{2x})$	12
parallelrisch	$-\ln(1-\tanh(x))-x$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x,method=_RETURNVERBOSE)
[Out] ln(cosh(x))
```

Maxima [A]
time = 0.41, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="maxima")
[Out] log(cosh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.
time = 0.59, size = 18, normalized size = 6.00

$$-x+\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="fricas")
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.
time = 0.06, size = 7, normalized size = 2.33

$$x-\log(\tanh(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x)
[Out] x - log(tanh(x) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.
time = 0.48, size = 11, normalized size = 3.67

$$-x+\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + log(e^(2*x) + 1)
```

Mupad [B]
time = 0.02, size = 3, normalized size = 1.00

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x)
```

```
[Out] log(cosh(x))
```

Chatgpt [A]
time = 1.00, size = 3, normalized size = 1.00

$$\ln(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] int(tanh(x),x)
```

```
[Out] ln(cosh(x))
```

3.138 $\int (-x + x^3) \, dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2} + \frac{x^4}{4}$$

```
[Out] -1/2*x^2+1/4*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^4}{4} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[-x + x^3,x]
```

```
[Out] -1/2*x^2 + x^4/4
```

Rubi steps

$$\text{Integral} = -\frac{x^2}{2} + \frac{x^4}{4}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{x^2}{2} + \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-x + x^3,x]
```

```
[Out] -1/2*x^2 + x^4/4
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{x^2(x^2-2)}{4}$	11
default	$-\frac{1}{2}x^2 + \frac{1}{4}x^4$	12
norman	$-\frac{1}{2}x^2 + \frac{1}{4}x^4$	12

risch	$-\frac{1}{2}x^2 + \frac{1}{4}x^4$	12
parallelrisch	$-\frac{1}{2}x^2 + \frac{1}{4}x^4$	12
parts	$-\frac{1}{2}x^2 + \frac{1}{4}x^4$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3-x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2+1/4*x^4
```

Maxima [A]
time = 0.34, size = 11, normalized size = 0.73

$$\frac{1}{4}x^4 - \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3-x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/2*x^2
```

Fricas [A]
time = 0.55, size = 11, normalized size = 0.73

$$\frac{1}{4}x^4 - \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3-x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - 1/2*x^2
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.53

$$\frac{x^4}{4} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3-x,x)
```

```
[Out] x**4/4 - x**2/2
```

Giac [A]
time = 0.44, size = 11, normalized size = 0.73

$$\frac{1}{4}x^4 - \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3-x,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/2*x^2
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^2\,\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3 - x,x)
```

```
[Out] (x^2*(x^2 - 2))/4
```

Chatgpt [A]
time = 1.00, size = 11, normalized size = 0.73

$$\frac{1}{4}x^4 - \frac{1}{2}x^2$$

Antiderivative was successfully verified.

```
[In] int(x^3-x,x)
```

```
[Out] 1/4*x^4-1/2*x^2
```

3.139 $\int \log\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=14

$$-\frac{x}{2}+x\log\left(\sqrt{x}\right)$$

```
[Out] -1/2*x+1/2*x*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2332}

$$x\log\left(\sqrt{x}\right)-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x]],x]
```

```
[Out] -1/2*x + x*Log[Sqrt[x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{Integral}=-\frac{x}{2}+x\log\left(\sqrt{x}\right)$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 0.86

$$\frac{1}{2}(-x+x\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x]],x]
```

```
[Out] (-x + x*Log[x])/2
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.71

method	result	size

lookup	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
default	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
norman	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
risch	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
parallelrisch	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10
parts	$-\frac{x}{2}+\frac{x\ln(x)}{2}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/2*x*ln(x)
```

Maxima [A]
time = 0.33, size = 9, normalized size = 0.64

$$\frac{1}{2}\,x\log\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x*log(x) - 1/2*x
```

Fricas [A]
time = 0.56, size = 9, normalized size = 0.64

$$\frac{1}{2}\,x\log\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(x) - 1/2*x
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.57

$$\frac{x\log\left(x\right)}{2}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln(x),x)
```

```
[Out] x*log(x)/2 - x/2
```

Giac [A]
time = 0.48, size = 9, normalized size = 0.64

$$\frac{1}{2}\,x\log\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="giac")
[Out] 1/2*x*log(x) - 1/2*x
```

Mupad [B]
time = 0.02, size = 7, normalized size = 0.50

$$\frac{x\,\left(\ln\left(x\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/2,x)
[Out] (x*(log(x) - 1))/2
```

Chatgpt [A]
time = 1.00, size = 9, normalized size = 0.64

$$\frac{x\ln\left(x\right)}{2}-\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] int(1/2*ln(x),x)
[Out] 1/2*x*ln(x)-1/4*x
```

3.140 $\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx$

Optimal. Leaf size=11

$$e^{e^{-x}+e^x}$$

```
[Out] exp(exp(x)+exp(-x))
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(-e^{e^{-x}+e^x-x}+e^{e^{-x}+e^x+x}\right) dx$$

Verification is not applicable to the result.

```
[In] Int[-E^(E^(-x) + E^x - x) + E^(E^(-x) + E^x + x),x]
```

```
[Out] Defer[Subst][Defer[Int][E^(x^(-1) + x), x], x, E^x] - Defer[Subst][Defer[In
t][E^(x^(-1) + x)/x^2, x], x, E^x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\int e^{e^{-x}+e^x-x}\,dx+\int e^{e^{-x}+e^x+x}\,dx\\
&= \text{Subst}\left(\int e^{\frac{1}{x}+x}\,dx,x,e^x\right)-\text{Subst}\left(\int \frac{e^{\frac{1}{x}+x}}{x^2}\,dx,x,e^x\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$e^{e^{-x}+e^x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-E^(E^(-x) + E^x - x) + E^(E^(-x) + E^x + x),x]
```

```
[Out] E^(E^(-x) + E^x)
```

Maple [A]
time = 0.12, size = 9, normalized size = 0.82

method	result	size

risch	$e^{e^x+e^{-x}}$	9
norman	$e^{e^x+e^{-x}-x}e^x$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(x)+exp(-x)+x)-exp(exp(x)+exp(-x)-x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(exp(x)+exp(-x))
```

Maxima [A]
time = 0.48, size = 8, normalized size = 0.73

$$e^{\left(e^{(-x)}+e^x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x)+exp(-x)+x)-exp(exp(x)+exp(-x)-x),x, algorithm="maxima"
)
```

```
[Out] e^(e^(-x) + e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.
time = 0.56, size = 20, normalized size = 1.82

$$e^{\left(\left(xe^x+e^{(2x)}+1\right)e^{(-x)}-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x)+exp(-x)+x)-exp(exp(x)+exp(-x)-x),x, algorithm="fricas"
)
```

```
[Out] e^((x*e^x + e^(2*x) + 1)*e^(-x) - x)
```

Sympy [A]
time = 0.12, size = 14, normalized size = 1.27

$$e^x e^{-x+e^x+e^{-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x)+exp(-x)+x)-exp(exp(x)+exp(-x)-x),x)
```

```
[Out] exp(x)*exp(-x + exp(x) + exp(-x))
```

Giac [A]
time = 0.43, size = 12, normalized size = 1.09

$$e^{\left(\left(e^{(2x)}+1\right)e^{(-x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x)+exp(-x)+x)-exp(exp(x)+exp(-x)-x),x, algorithm="giac")
```

```
[Out] e^((e^(2*x) + 1)*e^(-x))
```

Mupad [B]
time = 0.15, size = 8, normalized size = 0.73

$$e^{e^{-x}+e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x + exp(-x) + exp(x)) - exp(exp(-x) - x + exp(x)),x)
```

```
[Out] exp(exp(-x) + exp(x))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 26, normalized size = 2.36

$$\frac{e^{e^{x}+e^{-x}+x}}{2}-\frac{e^{e^{x}+e^{-x}-x}}{2}+x$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(exp(x)+exp(-x)+x)-exp(exp(x)+exp(-x)-x),x)
```

```
[Out] 1/2*exp(exp(x)+exp(-x)+x)-1/2*exp(exp(x)+exp(-x)-x)+x
```

3.141 $\int \frac{\log(\log(x))}{x\log(x)}\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\log(x))$$

```
[Out] 1/2*ln(ln(x))^2
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2338}

$$\frac{1}{2}\log^2(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \log(x)\right)\\ &= \frac{1}{2}\log^2(\log(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2}\log^2(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]^2/2
```

Maple [A]
time = 0.02, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln(\ln(x))^2}{2}$	8
default	$\frac{\ln(\ln(x))^2}{2}$	8
norman	$\frac{\ln(\ln(x))^2}{2}$	8
risch	$\frac{\ln(\ln(x))^2}{2}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))/x/ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(ln(x))^2
```

Maxima [A]
time = 0.33, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\log\left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x/log(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(log(x))^2
```

Fricas [A]
time = 0.57, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\log\left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x/log(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(log(x))^2
```

Sympy [A]
time = 0.06, size = 7, normalized size = 0.78

$$\frac{\log\left(\log\left(x\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))/x/ln(x),x)
```

```
[Out] log(log(x))**2/2
```

Giac [**A**]
time = 0.45, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\log\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x/log(x),x, algorithm="giac")
```

```
[Out] 1/2*log(log(x))^2
```

Mupad [**B**]
time = 0.22, size = 7, normalized size = 0.78

$$\frac{\ln\left(\ln\left(x\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))/(x*log(x)),x)
```

```
[Out] log(log(x))^2/2
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(ln(ln(x))/x/ln(x),x)
```

```
[Out] not solved
```

3.142 $\int \frac{1}{1+\tan^2(x)} dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3738, 2715, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \cos^2(x)\,dx\\ &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.36

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
derivativedivides	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
default	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
parallelrisch	$\frac{x(\tan^2(x))+x+\tan(x)}{2+2(\tan^2(x))}$	21
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2(x)\right)}{2}+\frac{\tan(x)}{2}}{1+\tan^2(x)}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(1+tan(x)^2)*tan(x)+1/2*arctan(tan(x))
```

Maxima [A]
time = 0.50, size = 16, normalized size = 1.14

$$\frac{1}{2}\,x+\frac{\tan\left(x\right)}{2\,\left(\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Fricas [A]
time = 0.58, size = 20, normalized size = 1.43

$$\frac{x\tan\left(x\right)^{2}+x+\tan\left(x\right)}{2\,\left(\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="fricas")
[Out] 1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.
time = 0.39, size = 36, normalized size = 2.57

$$\frac{x\tan^2(x)}{2\tan^2(x)+2} + \frac{x}{2\tan^2(x)+2} + \frac{\tan(x)}{2\tan^2(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**2),x)
[Out] x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2 +
 2)
```

Giac [**A**]
time = 0.47, size = 16, normalized size = 1.14

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="giac")
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Mupad [**B**]
time = 0.10, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^2 + 1),x)
[Out] x/2 + sin(2*x)/4
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 16, normalized size = 1.14

$$\frac{x}{2} + \frac{\sin(2x)}{4} + \frac{(\cos^2(x))}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1+tan(x)^2),x)
[Out] 1/2*x+1/4*sin(2*x)+1/2*cos(x)^2
```

3.143 $\int \arcsin\left(\frac{\sqrt[3]{x}}{3}\right)\,dx$

Optimal. Leaf size=45

$$9\sqrt{9-x^{2/3}}-\frac{1}{3}\left(9-x^{2/3}\right)^{3/2}+x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)$$

```
[Out] 9*(9-x^(2/3))^(1/2)-1/3*(9-x^(2/3))^(3/2)+x*arcsin(1/3*x^(1/3))
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4924, 12, 272, 45}

$$x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\frac{1}{3}\left(9-x^{2/3}\right)^{3/2}+9\sqrt{9-x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x^(1/3)/3],x]
[Out] 9*Sqrt[9 - x^(2/3)] - (9 - x^(2/3))^(3/2)/3 + x*ArcSin[x^(1/3)/3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\int\frac{\sqrt[3]{x}}{3\sqrt{9-x^{2/3}}}\,dx\\
&= x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\frac{1}{3}\int\frac{\sqrt[3]{x}}{\sqrt{9-x^{2/3}}}\,dx\\
&= x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{x}{\sqrt{9-x}}\,dx,x,x^{2/3}\right)\\
&= x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)-\frac{1}{2}\text{Subst}\left(\int\left(\frac{9}{\sqrt{9-x}}-\sqrt{9-x}\right)\,dx,x,x^{2/3}\right)\\
&= 9\sqrt{9-x^{2/3}}-\frac{1}{3}\left(9-x^{2/3}\right)^{3/2}+x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.87

$$-\frac{1}{3}\left(-18-x^{2/3}\right)\sqrt{9-x^{2/3}}+x\arcsin\left(\frac{\sqrt[3]{x}}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x^(1/3)/3],x]
```

```
[Out] -1/3*((-18 - x^(2/3))*Sqrt[9 - x^(2/3)]) + x*ArcSin[x^(1/3)/3]
```

Maple [A]
time = 0.04, size = 34, normalized size = 0.76

method	result	size
derivativedivides	$x\arcsin\left(\frac{x^{\frac{1}{3}}}{3}\right)+x^{\frac{2}{3}}\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}+18\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}$	34
default	$x\arcsin\left(\frac{x^{\frac{1}{3}}}{3}\right)+x^{\frac{2}{3}}\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}+18\sqrt{-\frac{x^{\frac{2}{3}}}{9}+1}$	34
parts	$x\arcsin\left(\frac{x^{\frac{1}{3}}}{3}\right)+\frac{x^{\frac{2}{3}}\sqrt{9-x^{\frac{2}{3}}}}{3}+6\sqrt{9-x^{\frac{2}{3}}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(1/3*x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arcsin(1/3*x^(1/3))+x^(2/3)*(-1/9*x^(2/3)+1)^(1/2)+18*(-1/9*x^(2/3)+1)^(1
/2)
```

Maxima [A]
time = 0.48, size = 33, normalized size = 0.73

$$x\arcsin\left(\frac{1}{3}x^{\frac{1}{3}}\right)+x^{\frac{2}{3}}\sqrt{-\frac{1}{9}x^{\frac{2}{3}}+1}+18\sqrt{-\frac{1}{9}x^{\frac{2}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(1/3*x^(1/3)),x, algorithm="maxima")
```

```
[Out] x*arcsin(1/3*x^(1/3)) + x^(2/3)*sqrt(-1/9*x^(2/3) + 1) + 18*sqrt(-1/9*x^(2/
3) + 1)
```

Fricas [A]
time = 22.01, size = 25, normalized size = 0.56

$$x\arcsin\left(\frac{1}{3}x^{\frac{1}{3}}\right)+\frac{1}{3}\left(x^{\frac{2}{3}}+18\right)\sqrt{-x^{\frac{2}{3}}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(1/3*x^(1/3)),x, algorithm="fricas")
```

```
[Out] x*arcsin(1/3*x^(1/3)) + 1/3*(x^(2/3) + 18)*sqrt(-x^(2/3) + 9)
```

Sympy [A]
time = 0.24, size = 37, normalized size = 0.82

$$\frac{x^{\frac{2}{3}}\sqrt{9-x^{\frac{2}{3}}}}{3}+x\operatorname{asin}\left(\frac{\sqrt[3]{x}}{3}\right)+6\sqrt{9-x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(1/3*x**(1/3)),x)
```

```
[Out] x**(2/3)*sqrt(9 - x**(2/3))/3 + x*asin(x**(1/3)/3) + 6*sqrt(9 - x**(2/3))
```

Giac [A]
time = 0.51, size = 49, normalized size = 1.09

$$x^{\frac{1}{3}}\left(x^{\frac{2}{3}}-9\right)\arcsin\left(\frac{1}{3}x^{\frac{1}{3}}\right)-9\left(-\frac{1}{9}x^{\frac{2}{3}}+1\right)^{\frac{3}{2}}+9x^{\frac{1}{3}}\arcsin\left(\frac{1}{3}x^{\frac{1}{3}}\right)+27\sqrt{-\frac{1}{9}x^{\frac{2}{3}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(1/3*x^(1/3)),x, algorithm="giac")
```

```
[Out] x^(1/3)*(x^(2/3) - 9)*arcsin(1/3*x^(1/3)) - 9*(-1/9*x^(2/3) + 1)^(3/2) + 9*
x^(1/3)*arcsin(1/3*x^(1/3)) + 27*sqrt(-1/9*x^(2/3) + 1)
```

Mupad [B]
time = 0.59, size = 25, normalized size = 0.56

$$x\,\mathrm{asin}\left(\frac{x^{1/3}}{3}\right)+\frac{\sqrt{9-x^{2/3}}\,\left(x^{2/3}+18\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x^(1/3)/3),x)
[Out] x*asin(x^(1/3)/3) + ((9 - x^(2/3))^(1/2)*(x^(2/3) + 18))/3
```

Chatgpt [F] Failed to verify
time = 1.00, size = 28, normalized size = 0.62

$$-\frac{x^{\frac{1}{3}}\arcsin\left(\frac{x^{\frac{1}{3}}}{3}\right)}{2}+\frac{\sqrt{9x^{\frac{2}{3}}-9x^{\frac{1}{3}}+1}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(arcsin(1/3*x^(1/3)),x)
[Out] -1/2*x^(1/3)*arcsin(1/3*x^(1/3))+1/2*(9*x^(2/3)-9*x^(1/3)+1)^(1/2)
```

3.144 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$-x+x\log(x)$$

[Out] -x+x*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2332\}$

$$x\log(x)-x$$

Antiderivative was successfully verified.

[In] Int[Log[x],x]

[Out] -x + x*Log[x]

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{Integral} = -x+x\log(x)$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-x+x\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x],x]

[Out] -x + x*Log[x]

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size

lookup	$-x + x\ln(x)$	9
default	$-x + x\ln(x)$	9
norman	$-x + x\ln(x)$	9
risch	$-x + x\ln(x)$	9
parallelrisch	$-x + x\ln(x)$	9
parts	$-x + x\ln(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -x+x*ln(x)
```

Maxima [A]
time = 0.37, size = 8, normalized size = 1.00

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A]
time = 0.56, size = 8, normalized size = 1.00

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.62

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

Giac [A]
time = 0.44, size = 8, normalized size = 1.00

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="giac")
```

```
[Out] x*log(x) - x
```

Mupad [**B**]
time = 0.00, size = 6, normalized size = 0.75

$$x(\ln(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x),x)
```

```
[Out] x*(log(x) - 1)
```

Chatgpt [**A**]
time = 1.00, size = 8, normalized size = 1.00

$$x\ln(x) - x$$

Antiderivative was successfully verified.

```
[In] int(ln(x),x)
```

```
[Out] x*ln(x)-x
```

3.145 $\int e^x(\cos(x)-\sin(x))\,dx$

Optimal. Leaf size=6

$$e^x \cos(x)$$

```
[Out] exp(x)*cos(x)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{2326\}$

$$e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*(Cos[x] - Sin[x]),x]
```

```
[Out] E^x*Cos[x]
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{Integral} = e^x \cos(x)$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*(Cos[x] - Sin[x]),x]
```

```
[Out] E^x*Cos[x]
```

Maple [A]
time = 0.07, size = 6, normalized size = 1.00

method	result	size

default	$e^x \cos(x)$	6
parallelrisch	$e^x \cos(x)$	6
parts	$e^x \cos(x)$	6
risch	$\frac{e^{(1+i)x}}{2} + \frac{e^{(1-i)x}}{2}$	18
norman	$\frac{-e^x(\tan^2(\frac{x}{2}))+e^x}{1+\tan^2(\frac{x}{2})}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(cos(x)-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)*cos(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(5) = 10$.
time = 0.38, size = 21, normalized size = 3.50

$$\frac{1}{2}(\cos(x) + \sin(x))e^x + \frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(cos(x)-sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x + 1/2*(cos(x) - sin(x))*e^x
```

Fricas [A]
time = 0.59, size = 5, normalized size = 0.83

$$\cos(x)\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(cos(x)-sin(x)),x, algorithm="fricas")
```

```
[Out] cos(x)*e^x
```

Sympy [A]
time = 0.14, size = 5, normalized size = 0.83

$$e^x \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(cos(x)-sin(x)),x)
```

```
[Out] exp(x)*cos(x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(5) = 10.
time = 0.50, size = 21, normalized size = 3.50

$$\frac{1}{2}(\cos(x) + \sin(x))e^x + \frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(cos(x)-sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x + 1/2*(cos(x) - sin(x))*e^x
```

Mupad [**B**]
time = 0.05, size = 5, normalized size = 0.83

$$e^x \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(cos(x) - sin(x)),x)
```

```
[Out] exp(x)*cos(x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 2.17

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(x)*(cos(x)-sin(x)),x)
```

```
[Out] 1/2*exp(x)*sin(x)-1/2*exp(x)*cos(x)
```

3.146 $\int e^{-x^2} x^3 \, dx$

Optimal. Leaf size=26

$$-\frac{e^{-x^2}}{2} - \frac{1}{2}e^{-x^2}x^2$$

```
[Out] -1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2243, 2240}

$$-\frac{1}{2}e^{-x^2}x^2 - \frac{e^{-x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^x^2,x]
```

```
[Out] -1/2*1/E^x^2 - x^2/(2*E^x^2)
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\frac{1}{2}e^{-x^2}x^2 + \int e^{-x^2}x \, dx \\ &= -\frac{e^{-x^2}}{2} - \frac{1}{2}e^{-x^2}x^2\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 0.62

$$-\frac{1}{2}e^{-x^2}\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^x^2,x]
```

```
[Out] -1/2*(1 + x^2)/E^x^2
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.81

method	result	size
gosper	$-\frac{(x^2+1)\mathrm{e}^{-x^2}}{2}$	14
risch	$\left(-\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{-x^2}$	15
meijerg	$\frac{1}{2}-\frac{(2x^2+2)\mathrm{e}^{-x^2}}{4}$	18
default	$-\frac{x^2\mathrm{e}^{-x^2}}{2}-\frac{\mathrm{e}^{-x^2}}{2}$	21
norman	$-\frac{x^2\mathrm{e}^{-x^2}}{2}-\frac{\mathrm{e}^{-x^2}}{2}$	21
parallelrisch	$-\frac{x^2\mathrm{e}^{-x^2}}{2}-\frac{\mathrm{e}^{-x^2}}{2}$	21
parts	$\frac{\sqrt{\pi}\,\mathrm{erf}(x)x^3}{2}-\frac{3\sqrt{\pi}\left(\frac{\mathrm{erf}(x)x^3}{3}-\frac{2\left(-\frac{x^2\mathrm{e}^{-x^2}}{2}-\frac{\mathrm{e}^{-x^2}}{2}\right)}{3\sqrt{\pi}}\right)}{2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(-x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

Maxima [A]
time = 0.41, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(-x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Fricas [A]
time = 0.57, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(-x^2),x, algorithm="fricas")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.46

$$\frac{\left(-x^2-1\right)e^{-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*exp(-x**2),x)
[Out] (-x**2 - 1)*exp(-x**2)/2
```

Giac [A]
time = 0.45, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(-x^2),x, algorithm="giac")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Mupad [B]
time = 0.11, size = 13, normalized size = 0.50

$$-\frac{e^{-x^2}\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(-x^2),x)
[Out] -(exp(-x^2)*(x^2 + 1))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 0.50

$$-\frac{e^{-x^2}\left(x^2+\frac{1}{2}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^3*exp(-x^2),x)
[Out] -1/2*exp(-x^2)*(x^2+1/2)
```

3.147 $\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx$

Optimal. Leaf size=10

$$\left(e^{x^2}+x\right)\cos(x)$$

```
[Out] (x+exp(x^2))*cos(x)
```

Rubi [F]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\left(1+2e^{x^2}x\right)\cos(x)-\left(e^{x^2}+x\right)\sin(x)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + 2*E^x^2*x)*Cos[x] - (E^x^2 + x)*Sin[x],x]
```

```
[Out] x*Cos[x] - (I/4)*E^(1/4)*Sqrt[Pi]*Erfi[(-I + 2*x)/2] + (I/4)*E^(1/4)*Sqrt[P
i]*Erfi[(I + 2*x)/2] + 2*Defer[Int][E^x^2*x*Cos[x], x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(1+2e^{x^2}x\right)\cos(x)\,dx-\int \left(e^{x^2}+x\right)\sin(x)\,dx\\
&= \int \left(\cos(x)+2e^{x^2}x\cos(x)\right)\,dx-\int \left(e^{x^2}\sin(x)+x\sin(x)\right)\,dx\\
&= 2\int e^{x^2}x\cos(x)\,dx+\int \cos(x)\,dx-\int e^{x^2}\sin(x)\,dx-\int x\sin(x)\,dx\\
&= x\cos(x)+\sin(x)+2\int e^{x^2}x\cos(x)\,dx-\int \left(\frac{1}{2}ie^{-ix+x^2}-\frac{1}{2}ie^{ix+x^2}\right)\,dx-\int \cos(x)\,dx\\
&= x\cos(x)-\frac{1}{2}i\int e^{-ix+x^2}\,dx+\frac{1}{2}i\int e^{ix+x^2}\,dx+2\int e^{x^2}x\cos(x)\,dx\\
&= x\cos(x)+2\int e^{x^2}x\cos(x)\,dx-\frac{1}{2}\left(i\sqrt[4]{e}\right)\int e^{\frac{1}{4}(-i+2x)^2}\,dx+\frac{1}{2}\left(i\sqrt[4]{e}\right)\int e^{\frac{1}{4}(i+2x)^2}\,dx\\
&= x\cos(x)-\frac{1}{4}i\sqrt[4]{e}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-i+2x)\right)+\frac{1}{4}i\sqrt[4]{e}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(i+2x)\right)+2\int e^{x^2}x\cos(x)\,dx
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 27, normalized size = 2.70

$$\frac{1}{2}e^{-ix}\left(1+e^{2ix}\right)\left(e^{x^2}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*E^x^2*x)*Cos[x] - (E^x^2 + x)*Sin[x],x]
```

```
[Out] ((1 + E^((2*I)*x))*(E^x^2 + x))/(2*E^(I*x))
```

Maple [A]
time = 0.20, size = 10, normalized size = 1.00

method	result	size
parallelrisch	$\left(x+{\mathrm e}^{x^{2}}\right) \cos \left(x \right)$	10
risch	$\frac{{\mathrm e}^{x \left(x -i\right)}}{2}+\frac{{\mathrm e}^{x \left(i+x \right)}}{2}+x \cos \left(x \right)$	24
norman	$\frac{x -x \left(\tan^{2}\left(\frac{x}{2}\right)\right)-{\mathrm e}^{x^{2}} \left(\tan^{2}\left(\frac{x}{2}\right)\right)+{\mathrm e}^{x^{2}}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x*exp(x^2))*cos(x)-(x+exp(x^2))*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] (x+exp(x^2))*cos(x)
```

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.47, size = 473, normalized size = 47.30

$$-\frac{1}{4}\sqrt{\pi}\left(\operatorname{erf}\left(ix+\frac{1}{2}\right)-\operatorname{erf}\left(ix-\frac{1}{2}\right)\right)e^{\frac{1}{4}}+x\cos\left(x\right)+\frac{2\left(16x^4+8x^2+1\right)^{\frac{1}{4}}\left(e^{\left(x^2+ix-\frac{1}{4}\right)}+e^{\left(x^2-ix-\frac{1}{4}\right)}+e^{\left(\overline{x}^2+i\overline{x}-\frac{1}{4}\right)}+e^{\left(\overline{x}^2-i\overline{x}-\frac{1}{4}\right)}\right)e^{\frac{1}{4}}-\left(\left(\left(2\left(i\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)}-1\right)-i\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)}-1\right)-i\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)-1\right)+i\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)-1\right)\right)x-\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)}-1\right)-\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)}-1\right)-\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)-1\right)-\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)-1\right)\right)\cos\left(\frac{1}{2}\arctan\left(4x,-4x^2+1\right)\right)-\left(2\left(\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)}-1\right)+\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)}-1\right)+\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)-1\right)+\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)-1\right)\right)x+i\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)}-1\right)-i\sqrt{\pi}\left(\overline{\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)}-1\right)-i\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2+ix+\frac{1}{4}}\right)-1\right)+i\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{-x^2-ix+\frac{1}{4}}\right)-1\right)\right)\sin\left(\frac{1}{2}\arctan\left(4x,-4x^2+1\right)\right)\right)e^{\frac{1}{4}}}{8\left(16x^4+8x^2+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x*exp(x^2))*cos(x)-(x+exp(x^2))*sin(x),x, algorithm="maxima"
)
```

```
[Out] -1/4*sqrt(pi)*(erf(I*x + 1/2) - erf(I*x - 1/2))*e^(1/4) + x*cos(x) + 1/8*(2
*(16*x^4 + 8*x^2 + 1)^(1/4)*(e^(x^2 + I*x - 1/4) + e^(x^2 - I*x - 1/4) + e^
(conjugate(x)^2 + I*conjugate(x) - 1/4) + e^(conjugate(x)^2 - I*conjugate(x
) - 1/4))*e^(1/4) - ((2*(I*sqrt(pi)*(conjugate(erf(sqrt(-x^2 + I*x + 1/4)))
 - 1) - I*sqrt(pi)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1) - I*sqrt(pi
)*(erf(sqrt(-x^2 + I*x + 1/4)) - 1) + I*sqrt(pi)*(erf(sqrt(-x^2 - I*x + 1/4
)) - 1))*x - sqrt(pi)*(conjugate(erf(sqrt(-x^2 + I*x + 1/4))) - 1) - sqrt(p
i)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1) - sqrt(pi)*(erf(sqrt(-x^2 +
 I*x + 1/4)) - 1) - sqrt(pi)*(erf(sqrt(-x^2 - I*x + 1/4)) - 1))*cos(1/2*arc
tan2(4*x, -4*x^2 + 1)) - (2*(sqrt(pi)*(conjugate(erf(sqrt(-x^2 + I*x + 1/4)
)) - 1) + sqrt(pi)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1) + sqrt(pi)*
(erf(sqrt(-x^2 + I*x + 1/4)) - 1) + sqrt(pi)*(erf(sqrt(-x^2 - I*x + 1/4)) -
 1))*x + I*sqrt(pi)*(conjugate(erf(sqrt(-x^2 + I*x + 1/4))) - 1) - I*sqrt(p
i)*(conjugate(erf(sqrt(-x^2 - I*x + 1/4))) - 1) - I*sqrt(pi)*(erf(sqrt(-x^2
 + I*x + 1/4)) - 1) + I*sqrt(pi)*(erf(sqrt(-x^2 - I*x + 1/4)) - 1))*sin(1/2
*arctan2(4*x, -4*x^2 + 1)))*e^(1/4))/(16*x^4 + 8*x^2 + 1)^(1/4)
```

Fricas [A]
time = 0.61, size = 12, normalized size = 1.20

$$x\cos(x) + \cos(x)\,e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x*exp(x^2))*cos(x)-(x+exp(x^2))*sin(x),x, algorithm="fricas"
)
[Out] x*cos(x) + cos(x)*e^(x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(x + e^{x^2}\right)\sin(x) + \left(2xe^{x^2} + 1\right)\cos(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x*exp(x**2))*cos(x)-(x+exp(x**2))*sin(x),x)
[Out] Integral(-(x + exp(x**2))*sin(x) + (2*x*exp(x**2) + 1)*cos(x), x)
```

Giac [A]
time = 0.42, size = 12, normalized size = 1.20

$$x\cos(x) + \cos(x)\,e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x*exp(x^2))*cos(x)-(x+exp(x^2))*sin(x),x, algorithm="giac")
[Out] x*cos(x) + cos(x)*e^(x^2)
```

Mupad [B]
time = 0.07, size = 9, normalized size = 0.90

$$\cos(x)\,\left(x + \mathrm{e}^{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(2*x*exp(x^2) + 1) - sin(x)*(x + exp(x^2)),x)
[Out] cos(x)*(x + exp(x^2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 22, normalized size = 2.20

$$\frac{\mathrm{e}^{x^2}\sin(x)}{2} + \frac{\cos(x)\,\left(2x\,\mathrm{e}^{x^2} - 1\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((1+2*x*exp(x^2))*cos(x)-(x+exp(x^2))*sin(x),x)
[Out] 1/2*exp(x^2)*sin(x)+1/2*cos(x)*(2*x*exp(x^2)-1)
```

3.148 $\int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right) \left(1+\sqrt[3]{x}+\sqrt{x}\right) \, dx$

Optimal. Leaf size=56

$$2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+3x+\frac{6x^{7/6}}{7}+\frac{3x^{4/3}}{4}+\frac{2x^{3/2}}{3}$$

```
[Out] 2*x^(1/2)+3/2*x^(2/3)+6/5*x^(5/6)+3*x+6/7*x^(7/6)+3/4*x^(4/3)+2/3*x^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {6874}

$$\frac{2x^{3/2}}{3}+\frac{3x^{4/3}}{4}+\frac{6x^{7/6}}{7}+\frac{6x^{5/6}}{5}+\frac{3x^{2/3}}{2}+3x+2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 1/Sqrt[x] + x^(-1/3))*(1 + x^(1/3) + Sqrt[x]),x]
```

```
[Out] 2*Sqrt[x] + (3*x^(2/3))/2 + (6*x^(5/6))/5 + 3*x + (6*x^(7/6))/7 + (3*x^(4/3
))/4 + (2*x^(3/2))/3
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 6\text{Subst}\left(\int x^2\left(1+x+x^3\right)\left(1+x^2+x^3\right)\,dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(x^2+x^3+x^4+3x^5+x^6+x^7+x^8\right)\,dx, x, \sqrt[6]{x}\right)\\
&= 2\sqrt{x}+\frac{3x^{2/3}}{2}+\frac{6x^{5/6}}{5}+3x+\frac{6x^{7/6}}{7}+\frac{3x^{4/3}}{4}+\frac{2x^{3/2}}{3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.89

$$\frac{1}{420}\left(840\sqrt{x}+630x^{2/3}+504x^{5/6}+1260x+360x^{7/6}+315x^{4/3}+280x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 1/Sqrt[x] + x^(-1/3))*(1 + x^(1/3) + Sqrt[x]),x]
```

```
[Out] (840*Sqrt[x] + 630*x^(2/3) + 504*x^(5/6) + 1260*x + 360*x^(7/6) + 315*x^(4/
3) + 280*x^(3/2))/420
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.62

$$2\sqrt{x}+\frac{3x^{\frac{2}{3}}}{2}+\frac{6x^{\frac{5}{6}}}{5}+3x+\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{4}{3}}}{4}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2)+x^(1/3))*(1+1/x^(1/2)+1/x^(1/3)),x)
```

```
[Out] 2*x^(1/2)+3/2*x^(2/3)+6/5*x^(5/6)+3*x+6/7*x^(7/6)+3/4*x^(4/3)+2/3*x^(3/2)
```

Maxima [A]
time = 0.35, size = 34, normalized size = 0.61

$$\frac{2}{3}\,x^{\frac{3}{2}}+\frac{3}{4}\,x^{\frac{4}{3}}+\frac{6}{7}\,x^{\frac{7}{6}}+3\,x+\frac{6}{5}\,x^{\frac{5}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}+2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2)+x^(1/3))*(1+1/x^(1/2)+1/x^(1/3)),x, algorithm="maxima"
)
```

```
[Out] 2/3*x^(3/2) + 3/4*x^(4/3) + 6/7*x^(7/6) + 3*x + 6/5*x^(5/6) + 3/2*x^(2/3) +
 2*sqrt(x)
```

Fricas [A]
time = 0.59, size = 32, normalized size = 0.57

$$\frac{2}{3}\,(x+3)\sqrt{x}+\frac{3}{4}\,x^{\frac{4}{3}}+\frac{6}{7}\,x^{\frac{7}{6}}+3\,x+\frac{6}{5}\,x^{\frac{5}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2)+x^(1/3))*(1+1/x^(1/2)+1/x^(1/3)),x, algorithm="fricas"
)
```

```
[Out] 2/3*(x + 3)*sqrt(x) + 3/4*x^(4/3) + 6/7*x^(7/6) + 3*x + 6/5*x^(5/6) + 3/2*x
^(2/3)
```

Sympy [A]
time = 11.20, size = 51, normalized size = 0.91

$$\frac{6x^{\frac{7}{6}}}{7}+\frac{6x^{\frac{5}{6}}}{5}+\frac{3x^{\frac{4}{3}}}{4}+\frac{3x^{\frac{2}{3}}}{2}+\frac{2x^{\frac{3}{2}}}{3}+2\sqrt{x}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2)+x**(1/3))*(1+1/x**(1/2)+1/x**(1/3)),x)
```

```
[Out] 6*x**(7/6)/7 + 6*x**(5/6)/5 + 3*x**(4/3)/4 + 3*x**(2/3)/2 + 2*x**(3/2)/3 +
2*sqrt(x) + 3*x
```

Giac [A]
time = 0.47, size = 34, normalized size = 0.61

$$\frac{2}{3}\,x^{\frac{3}{2}}+\frac{3}{4}\,x^{\frac{4}{3}}+\frac{6}{7}\,x^{\frac{7}{6}}+3\,x+\frac{6}{5}\,x^{\frac{5}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}+2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2)+x^(1/3))*(1+1/x^(1/2)+1/x^(1/3)),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) + 3/4*x^(4/3) + 6/7*x^(7/6) + 3*x + 6/5*x^(5/6) + 3/2*x^(2/3) +
 2*sqrt(x)
```

Mupad [B]
time = 0.03, size = 34, normalized size = 0.61

$$3\,x+2\,\sqrt{x}+\frac{3\,x^{2/3}}{2}+\frac{2\,x^{3/2}}{3}+\frac{3\,x^{4/3}}{4}+\frac{6\,x^{5/6}}{5}+\frac{6\,x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^(1/2) + 1/x^(1/3) + 1)*(x^(1/2) + x^(1/3) + 1),x)
```

```
[Out] 3*x + 2*x^(1/2) + (3*x^(2/3))/2 + (2*x^(3/2))/3 + (3*x^(4/3))/4 + (6*x^(5/6
))/5 + (6*x^(7/6))/7
```

Chatgpt [F] Failed to verify
time = 1.00, size = 29, normalized size = 0.52

$$x+4\sqrt{x}+6x^{\frac{2}{3}}+\ln\left(x\right)-\frac{4}{\sqrt{x}}-\frac{3}{x^{\frac{2}{3}}}-\frac{3}{x^{\frac{1}{3}}}$$

Warning: Unable to verify antiderivative.

```
[In] int((1+x^(1/2)+x^(1/3))*(1+1/x^(1/2)+1/x^(1/3)),x)
```

```
[Out] x+4*x^(1/2)+6*x^(2/3)+ln(x)-4/x^(1/2)-3/x^(2/3)-3/x^(1/3)
```

3.149 $\int \cos(x)\cos(\sin(x))\sin(\sin(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin^2(\sin(x))$$

```
[Out] 1/2*sin(sin(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4419, 2644, 30}

$$\frac{1}{2}\sin^2(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Sin[x]]*Sin[Sin[x]],x]
```

```
[Out] Sin[Sin[x]]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \cos(x)\sin(x)\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int x\,dx, x, \sin(\sin(x))\right)\\
&= \frac{1}{2}\sin^2(\sin(x))
\end{aligned}$$

Mathematica [A]
time = 1.37, size = 9, normalized size = 1.00

$$-\frac{1}{2}\cos^2(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]]*Sin[Sin[x]],x]
```

```
[Out] -1/2*Cos[Sin[x]]^2
```

Maple [A]
time = 0.20, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$\frac{(\sin^2(\sin(x)))}{2}$	8
default	$\frac{(\sin^2(\sin(x)))}{2}$	8
risch	$-\frac{\cos(2\sin(x))}{4}$	8
parallelrisch	$-\frac{3}{4}-\frac{\cos(2\sin(x))}{4}$	10
norman	$\frac{2\left(\tan^2\left(\frac{\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)\right)+2\left(\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2\left(\frac{\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)\right)}{\left(1+\tan^2\left(\frac{\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)\right)^2\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(sin(x))*cos(sin(x))*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(sin(x))^2
```

Maxima [A]
time = 0.38, size = 7, normalized size = 0.78

$$-\frac{1}{2}\cos\left(\sin\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(sin(x))*cos(sin(x))*cos(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(sin(x))^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.
time = 0.59, size = 21, normalized size = 2.33

$$-\frac{1}{2}\cos\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(sin(x))*cos(sin(x))*cos(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))^2
```

Sympy [A]
time = 0.37, size = 8, normalized size = 0.89

$$-\frac{\cos^2(\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(sin(x))*cos(sin(x))*cos(x),x)
```

```
[Out] -cos(sin(x))**2/2
```

Giac [A]
time = 0.50, size = 7, normalized size = 0.78

$$-\frac{1}{2}\cos(\sin(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(sin(x))*cos(sin(x))*cos(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(sin(x))^2
```

Mupad [B]
time = 0.17, size = 7, normalized size = 0.78

$$\frac{\sin(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(sin(x))*sin(sin(x))*cos(x),x)
```

```
[Out] sin(sin(x))^2/2
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(sin(sin(x))*cos(sin(x))*cos(x),x)
```

```
[Out] not solved
```

3.150 $\int \left(\frac{-\cos(x)+\sin(x)}{x}+\frac{\cos(x)+\sin(x)}{x^2}\right) dx$

Optimal. Leaf size=15

$$-\frac{\cos(x)}{x}-\frac{\sin(x)}{x}$$

[Out] -cos(x)/x-sin(x)/x

Rubi [A]
time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {14, 3383, 3380, 3378}

$$-\frac{\sin(x)}{x}-\frac{\cos(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sin[x])/x + (Cos[x] + Sin[x])/x^2,x]

[Out] -(Cos[x]/x) - Sin[x]/x

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{-\cos(x)+\sin(x)}{x}\,dx + \int \frac{\cos(x)+\sin(x)}{x^2}\,dx \\ &= \int \left(\frac{\cos(x)}{x^2}+\frac{\sin(x)}{x^2}\right)dx + \int \left(-\frac{\cos(x)}{x}+\frac{\sin(x)}{x}\right)dx \\ &= \int \frac{\cos(x)}{x^2}\,dx - \int \frac{\cos(x)}{x}\,dx + \int \frac{\sin(x)}{x^2}\,dx + \int \frac{\sin(x)}{x}\,dx \\ &= -\frac{\cos(x)}{x} - \text{CosIntegral}(x) - \frac{\sin(x)}{x} + \text{Si}(x) + \int \frac{\cos(x)}{x}\,dx - \int \frac{\sin(x)}{x}\,dx \\ &= -\frac{\cos(x)}{x} - \frac{\sin(x)}{x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 15, normalized size = 1.00

$$-\frac{\cos(x)}{x} - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sin[x])/x + (Cos[x] + Sin[x])/x^2,x]
[Out] -(Cos[x]/x) - Sin[x]/x
```

Maple [A]
time = 0.14, size = 16, normalized size = 1.07

method	result
parallelrisch	$\frac{-\cos(x)-\sin(x)}{x}$
default	$-\frac{\cos(x)}{x}-\frac{\sin(x)}{x}$
parts	$-\frac{\cos(x)}{x}-\frac{\sin(x)}{x}$
risch	$\frac{i(2i\cos(x)+2i\sin(x))}{2x}$
norman	$\frac{-1+\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)x}$
meijerg	$\frac{\sqrt{\pi}\left(-\frac{4\cos(x)}{x\sqrt{\pi}}-\frac{4\ \text{sinIntegral}(x)}{\sqrt{\pi}}\right)}{4}+\frac{\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}}-\frac{4\sin(x)}{\sqrt{\pi}\,x}+\frac{4\ \text{cosineIntegral}(x)}{\sqrt{\pi}}\right)}{4}+\text{sinInte}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/x^2+(sin(x)-cos(x))/x,x,method=_RETURNVERBOSE)
[Out] -cos(x)/x-sin(x)/x
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.44, size = 27, normalized size = 1.80

$$-\left(\frac{1}{2}i+\frac{1}{2}\right)\operatorname{Ei}(i\,x)+\left(\frac{1}{2}i-\frac{1}{2}\right)\operatorname{Ei}(-i\,x)-\left(\frac{1}{2}i-\frac{1}{2}\right)\Gamma(-1,i\,x)+\left(\frac{1}{2}i+\frac{1}{2}\right)\Gamma(-1,-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/x^2+(sin(x)-cos(x))/x,x, algorithm="maxima")
```

```
[Out] -(1/2*I + 1/2)*Ei(I*x) + (1/2*I - 1/2)*Ei(-I*x) - (1/2*I - 1/2)*gamma(-1, I
*x) + (1/2*I + 1/2)*gamma(-1, -I*x)
```

Fricas [A]
time = 0.57, size = 10, normalized size = 0.67

$$-\frac{\cos(x)+\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/x^2+(sin(x)-cos(x))/x,x, algorithm="fricas")
```

```
[Out] -(cos(x) + sin(x))/x
```

Sympy [A]
time = 2.12, size = 19, normalized size = 1.27

$$-\log(x)+\frac{\log(x^2)}{2}-\frac{\sin(x)}{x}-\frac{\cos(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/x**2+(sin(x)-cos(x))/x,x)
```

```
[Out] -log(x) + log(x**2)/2 - sin(x)/x - cos(x)/x
```

Giac [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.51, size = 29, normalized size = 1.93

$$\frac{x\operatorname{Ci}(x)-x\operatorname{Si}(x)-\cos(x)-\sin(x)}{x}-\operatorname{Ci}(x)+\operatorname{Si}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/x^2+(sin(x)-cos(x))/x,x, algorithm="giac")
```

```
[Out] (x*cos_integral(x) - x*sin_integral(x) - cos(x) - sin(x))/x - cos_integral(
x) + sin_integral(x)
```

Mupad [**B**]
time = 0.06, size = 14, normalized size = 0.93

$$-\frac{\sqrt{2}\,\sin\left(x+\frac{\pi}{4}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/x^2 - (cos(x) - sin(x))/x,x)
```

```
[Out] -(2^(1/2)*sin(x + pi/4))/x
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 15, normalized size = 1.00

$$-\frac{\cos\left(x\right)}{x}-\frac{\sin\left(x\right)}{x^{2}}$$

Warning: Unable to verify antiderivative.

```
[In] int((cos(x)+sin(x))/x^2+(sin(x)-cos(x))/x,x)
```

```
[Out] -cos(x)/x-sin(x)/x^2
```

3.151 $\int x^3\sqrt{1+x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}$$

```
[Out] -1/3*(x^2+1)^(3/2)+1/5*(x^2+1)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 + x^2],x]
```

```
[Out] -1/3*(1 + x^2)^(3/2) + (1 + x^2)^(5/2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int x\sqrt{1+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\sqrt{1+x}+(1+x)^{3/2}\right)\,dx,x,x^2\right)\\ &= -\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.74

$$\frac{1}{15}\left(1+x^2\right)^{3/2}\left(-2+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 + x^2],x]
```

```
[Out] ((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

Maple [A]
time = 0.08, size = 23, normalized size = 0.85

method	result	size
gosper	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}} \left(3 x^{2}-2\right)}{15}$	17
pseudoelliptic	$\frac{\left(x^{2}+1\right)^{\frac{3}{2}} \left(3 x^{2}-2\right)}{15}$	17
risch	$\frac{\left(3 x^{4}+x^{2}-2\right) \sqrt{x^{2}+1}}{15}$	20
trager	$\left(\frac{1}{5} x^{4}+\frac{1}{15} x^{2}-\frac{2}{15}\right) \sqrt{x^{2}+1}$	21
default	$\frac{x^{2} \left(x^{2}+1\right)^{\frac{3}{2}}}{5}-\frac{2 \left(x^{2}+1\right)^{\frac{3}{2}}}{15}$	23
meijerg	$-\frac{-\frac{8 \sqrt{\pi}}{15}+\frac{4 \sqrt{\pi}\, \left(x^{2}+1\right)^{\frac{3}{2}} \left(-3 x^{2}+2\right)}{15}}{4 \sqrt{\pi}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^2*(x^2+1)^(3/2)-2/15*(x^2+1)^(3/2)
```

Maxima [A]
time = 0.48, size = 22, normalized size = 0.81

$$\frac{1}{5} \left(x^{2}+1\right)^{\frac{3}{2}} x^{2}-\frac{2}{15} \left(x^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

Fricas [A]
time = 0.57, size = 19, normalized size = 0.70

$$\frac{1}{15} \left(3 \, x^{4}+x^{2}-2\right) \sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)
```

Sympy [A]
time = 0.16, size = 37, normalized size = 1.37

$$\frac{x^4\sqrt{x^2+1}}{5}+\frac{x^2\sqrt{x^2+1}}{15}-\frac{2\sqrt{x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)**(1/2),x)
[Out] x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15
```

Giac [A]
time = 0.42, size = 19, normalized size = 0.70

$$\frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}}-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

Mupad [B]
time = 0.05, size = 20, normalized size = 0.74

$$\sqrt{x^2+1}\left(\frac{x^4}{5}+\frac{x^2}{15}-\frac{2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 + 1)^(1/2),x)
[Out] (x^2 + 1)^(1/2)*(x^2/15 + x^4/5 - 2/15)
```

Chatgpt [A]
time = 1.00, size = 19, normalized size = 0.70

$$\frac{\left(x^2+1\right)^{\frac{5}{2}}}{5}-\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Antiderivative was successfully verified.

```
[In] int(x^3*(x^2+1)^(1/2),x)
[Out] 1/5*(x^2+1)^(5/2)-1/3*(x^2+1)^(3/2)
```

3.152 $\int \frac{x}{1+x^2+x^4} \, dx$

Optimal. Leaf size=20

$$\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1121, 632, 210}

$$\frac{\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2} \, dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1+2x^2\right) \\
&= \frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\arctan\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2 + x^4),x]
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.95

method	result	size
default	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
risch	$\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.44, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Fricas [A]
time = 0.57, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Sympy [A]
time = 0.04, size = 26, normalized size = 1.30

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

Giac [A]
time = 0.48, size = 18, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x^2+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Mupad [B]
time = 0.08, size = 20, normalized size = 1.00

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^4 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/3
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Antiderivative was successfully verified.

```
[In] int(x/(x^4+x^2+1),x)
[Out] not solved
```

3.153 $\int e^{e^{2016x}+6048x}\,dx$

Optimal. Leaf size=42

$$\frac{e^{e^{2016x}}}{1008}-\frac{e^{e^{2016x}+2016x}}{1008}+\frac{e^{e^{2016x}+4032x}}{2016}$$

```
[Out] 1/1008*exp(exp(1)^(2016*x))-1/1008*exp(exp(1)^(2016*x)+2016*x)+1/2016*exp(e
xp(1)^(2016*x)+4032*x)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2320, 2207, 2225}

$$\frac{e^{e^{2016x}}}{1008}-\frac{e^{2016x+e^{2016x}}}{1008}+\frac{e^{4032x+e^{2016x}}}{2016}$$

Antiderivative was successfully verified.

```
[In] Int[E^(E^(2016*x) + 6048*x),x]
```

```
[Out] E^E^(2016*x)/1008 - E^(E^(2016*x) + 2016*x)/1008 + E^(E^(2016*x) + 4032*x)/
2016
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{\text{Subst}\left(\int e^x x^2\,dx, x, e^{2016x}\right)}{2016}\\ &= \frac{e^{e^{2016x}+4032x}}{2016} - \frac{\text{Subst}\left(\int e^x x\,dx, x, e^{2016x}\right)}{1008}\\ &= -\frac{e^{e^{2016x}+2016x}}{1008} + \frac{e^{e^{2016x}+4032x}}{2016} + \frac{\text{Subst}\left(\int e^x\,dx, x, e^{2016x}\right)}{1008}\\ &= \frac{e^{e^{2016x}}}{1008} - \frac{e^{e^{2016x}+2016x}}{1008} + \frac{e^{e^{2016x}+4032x}}{2016}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.60

$$\frac{e^{e^{2016x}}\left(2-2e^{2016x}+e^{4032x}\right)}{2016}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(E^(2016*x) + 6048*x),x]
[Out] (E^E^(2016*x)*(2 - 2*E^(2016*x) + E^(4032*x)))/2016
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.48

method	result	size
risch	$\frac{\left(e^{4032x}-2\,e^{2016x}+2\right)e^{e^{2016x}}}{2016}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(2016*x)+6048*x),x,method=_RETURNVERBOSE)
[Out] 1/2016*(exp(4032*x)-2*exp(2016*x)+2)*exp(exp(2016*x))
```

Maxima [A]
time = 0.39, size = 19, normalized size = 0.45

$$\frac{1}{2016}\left(e^{(4032\,x)} - 2\,e^{(2016\,x)} + 2\right)e^{\left(e^{(2016\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(2016*x)+6048*x),x, algorithm="maxima")
[Out] 1/2016*(e^(4032*x) - 2*e^(2016*x) + 2)*e^(e^(2016*x))
```

Fricas [A]
time = 0.57, size = 19, normalized size = 0.45

$$\frac{1}{2016}\left(e^{(4032\,x)}-2\,e^{(2016\,x)}+2\right)e^{\left(e^{(2016\,x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(2016*x)+6048*x),x, algorithm="fricas")
```

```
[Out] 1/2016*(e^(4032*x) - 2*e^(2016*x) + 2)*e^(e^(2016*x))
```

Sympy [A]
time = 0.57, size = 34, normalized size = 0.81

$$\frac{e^{4032x}e^{e^{2016x}}}{2016}-\frac{e^{2016x}e^{e^{2016x}}}{1008}+\frac{e^{e^{2016x}}}{1008}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(2016*x)+6048*x),x)
```

```
[Out] exp(4032*x)*exp(exp(2016*x))/2016 - exp(2016*x)*exp(exp(2016*x))/1008 + exp
(exp(2016*x))/1008
```

Giac [A]
time = 0.49, size = 38, normalized size = 0.90

$$\frac{1}{2016}\left(e^{\left(10080\,x+e^{(2016\,x)}\right)}-2\,e^{\left(8064\,x+e^{(2016\,x)}\right)}+2\,e^{\left(6048\,x+e^{(2016\,x)}\right)}\right)e^{(-6048\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(2016*x)+6048*x),x, algorithm="giac")
```

```
[Out] 1/2016*(e^(10080*x + e^(2016*x)) - 2*e^(8064*x + e^(2016*x)) + 2*e^(6048*x
+ e^(2016*x)))*e^(-6048*x)
```

Mupad [B]
time = 0.11, size = 30, normalized size = 0.71

$$\frac{\mathrm{e}^{\mathrm{e}^{2016\,x}}}{1008}-\frac{\mathrm{e}^{2016\,x}\,\mathrm{e}^{\mathrm{e}^{2016\,x}}}{1008}+\frac{\mathrm{e}^{4032\,x}\,\mathrm{e}^{\mathrm{e}^{2016\,x}}}{2016}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(6048*x + exp(2016*x)),x)
```

```
[Out] exp(exp(2016*x))/1008 - (exp(2016*x)*exp(exp(2016*x)))/1008 + (exp(4032*x)*
exp(exp(2016*x)))/2016
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Antiderivative was successfully verified.

```
[In] int(exp(exp(2016*x)+6048*x),x)
```

```
[Out] not solved
```

3.154 $\int (1-\cot(x))\,dx$

Optimal. Leaf size=7

$$x-\log(\sin(x))$$

[Out] x-ln(sin(x))

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$x-\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[1 - Cot[x],x]

[Out] x - Log[Sin[x]]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= x-\int \cot(x)\,dx\\ &= x-\log(\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$x-\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[1 - Cot[x],x]

[Out] x - Log[Sin[x]]

Maple [A]
time = 0.10, size = 8, normalized size = 1.14

method	result	size

default	$x-\ln(\sin(x))$	8
parts	$x-\ln(\sin(x))$	8
risch	$x+ix-\ln(e^{2ix}-1)$	17
norman	$x-\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	19
parallelrisch	$-\ln\left(-\frac{\cot(x)}{2}+\frac{\csc(x)}{2}\right)+\ln\left(\frac{1}{1+\cos(x)}\right)+x$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-cos(x)/sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] x-ln(sin(x))
```

Maxima [A]
time = 0.32, size = 7, normalized size = 1.00

$$x-\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-cos(x)/sin(x),x, algorithm="maxima")
```

```
[Out] x - log(sin(x))
```

Fricas [A]
time = 0.62, size = 9, normalized size = 1.29

$$x-\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-cos(x)/sin(x),x, algorithm="fricas")
```

```
[Out] x - log(1/2*sin(x))
```

Sympy [A]
time = 0.01, size = 5, normalized size = 0.71

$$x-\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-cos(x)/sin(x),x)
```

```
[Out] x - log(sin(x))
```

Giac [A]
time = 0.47, size = 8, normalized size = 1.14

$$x - \log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-cos(x)/sin(x),x, algorithm="giac")
[Out] x - log(abs(sin(x)))
```

Mupad [B]
time = 0.17, size = 26, normalized size = 3.71

$$-\ln(\tan(x)) + \ln(\tan(x) - \mathrm{i})\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \ln(\tan(x) + 1\mathrm{i})\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - cos(x)/sin(x),x)
[Out] log(tan(x) - 1i)*(1/2 - 1i/2) - log(tan(x)) + log(tan(x) + 1i)*(1/2 + 1i/2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 8, normalized size = 1.14

$$\ln\left(\frac{\sin(x)}{\cos(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1-cos(x)/sin(x),x)
[Out] ln(sin(x)/cos(x))
```

3.155 $\int \frac{1}{1-x+x^2-x^3} dx$

Optimal. Leaf size=27

$$\frac{\arctan(x)}{2} - \frac{1}{2}\log(1-x) + \frac{1}{4}\log\left(1+x^2\right)$$

[Out] 1/2*arctan(x)-1/2*ln(1-x)+1/4*ln(x^2+1)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2083, 649, 209, 266}

$$\frac{\arctan(x)}{2} + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x + x^2 - x^3)^(-1),x]

[Out] ArcTan[x]/2 - Log[1 - x]/2 + Log[1 + x^2]/4

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(-\frac{1}{2(-1+x)} + \frac{1+x}{2(1+x^2)} \right) dx \\
&= -\frac{1}{2}\log(1-x) + \frac{1}{2}\int \frac{1+x}{1+x^2}\,dx \\
&= -\frac{1}{2}\log(1-x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= \frac{\arctan(x)}{2} - \frac{1}{2}\log(1-x) + \frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{\arctan(x)}{2} - \frac{1}{2}\log(1-x) + \frac{1}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + x^2 - x^3)^(-1),x]
[Out] ArcTan[x]/2 - Log[1 - x]/2 + Log[1 + x^2]/4
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.74

method	result	size
default	$\frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{2} - \frac{\ln(x-1)}{2}$	20
risch	$\frac{\ln(x^2+1)}{4} + \frac{\arctan(x)}{2} - \frac{\ln(x-1)}{2}$	20
parallelrisch	$-\frac{\ln(x-1)}{2} + \frac{\ln(x-i)}{4} - \frac{i\ln(x-i)}{4} + \frac{\ln(i+x)}{4} + \frac{i\ln(i+x)}{4}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x^2+1)+1/2*arctan(x)-1/2*ln(x-1)
```

Maxima [A]
time = 0.46, size = 19, normalized size = 0.70

$$\frac{1}{2}\arctan\left(x\right) + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x - 1)
```

Fricas [A]
time = 0.58, size = 19, normalized size = 0.70

$$\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x^{2}+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x^2-x+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x - 1)
```

Sympy [A]
time = 0.06, size = 19, normalized size = 0.70

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x^{2}+1\right)}{4}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+x**2-x+1),x)
[Out] -log(x - 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

Giac [A]
time = 0.42, size = 20, normalized size = 0.74

$$\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x^{2}+1\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x^2-x+1),x, algorithm="giac")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.11, size = 25, normalized size = 0.93

$$-\frac{\ln\left(x-1\right)}{2}+\ln\left(x-\mathrm{i}\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x - x^2 + x^3 - 1),x)
[Out] log(x - 1i)*(1/4 - 1i/4) - log(x - 1)/2 + log(x + 1i)*(1/4 + 1i/4)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 19, normalized size = 0.70

$$\frac{\ln\left(x-1\right)}{2}-\ln\left(x^{2}+1\right)-\frac{\arctan\left(x\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(-x^3+x^2-x+1),x)
```

```
[Out] 1/2*ln(x-1)-ln(x^2+1)-1/2*arctan(x)
```

3.156 $\int \frac{1}{2+\cosh(x)} dx$

Optimal. Leaf size=20

$$\frac{2\coth^{-1}\left(\sqrt{3}\coth\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

```
[Out] 2/3*arccoth(3^(1/2)*coth(1/2*x))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.50, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2736}

$$\frac{x}{\sqrt{3}} - \frac{2\text{arctanh}\left(\frac{\sinh(x)}{\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + Cosh[x])^(-1),x]
```

```
[Out] x/Sqrt[3] - (2*ArcTanh[Sinh[x]/(2 + Sqrt[3] + Cosh[x])])/Sqrt[3]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\text{Integral} = \frac{x}{\sqrt{3}} - \frac{2\text{arctanh}\left(\frac{\sinh(x)}{2+\sqrt{3}+\cosh(x)}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{2\text{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + Cosh[x])^(-1),x]
```

```
[Out] (2*ArcTanh[Tanh[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.80

method	result	size
default	$\frac{2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tanh\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{\sqrt{3}\,\ln\left(e^{x}+2-\sqrt{3}\right)}{3}-\frac{\sqrt{3}\,\ln\left(e^{x}+2+\sqrt{3}\right)}{3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*3^(1/2)*arctanh(1/3*tanh(1/2*x)*3^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.49, size = 30, normalized size = 1.50

$$-\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}-2}{\sqrt{3}+e^{(-x)}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cosh(x)),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(-(sqrt(3) - e^(-x) - 2)/(sqrt(3) + e^(-x) + 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.
time = 0.57, size = 39, normalized size = 1.95

$$\frac{1}{3}\sqrt{3}\log\left(-\frac{2\left(\sqrt{3}-2\right)\cosh\left(x\right)-\left(2\sqrt{3}-3\right)\sinh\left(x\right)+\sqrt{3}-2}{\cosh\left(x\right)+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log(-(2*(sqrt(3) - 2)*cosh(x) - (2*sqrt(3) - 3)*sinh(x) + sqrt(
3) - 2)/(cosh(x) + 2))
```

Sympy [A]
time = 0.21, size = 36, normalized size = 1.80

$$-\frac{\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3}+\frac{\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cosh(x)),x)
```

```
[Out] -sqrt(3)*log(tanh(x/2) - sqrt(3))/3 + sqrt(3)*log(tanh(x/2) + sqrt(3))/3
```

Giac [A]
time = 0.47, size = 26, normalized size = 1.30

$$\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{x}-2}{\sqrt{3}+e^{x}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+cosh(x)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*log(-(sqrt(3) - e^x - 2)/(sqrt(3) + e^x + 2))
```

Mupad [B]
time = 0.13, size = 42, normalized size = 2.10

$$\frac{\sqrt{3}\left(\ln\left(-2\,\mathrm{e}^{x}-\frac{\sqrt{3}\,\left(4\,\mathrm{e}^{x}+2\right)}{3}\right)-\ln\left(\frac{\sqrt{3}\,\left(4\,\mathrm{e}^{x}+2\right)}{3}-2\,\mathrm{e}^{x}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x) + 2),x)
```

```
[Out] (3^(1/2)*(log(- 2*exp(x) - (3^(1/2)*(4*exp(x) + 2))/3) - log((3^(1/2)*(4*ex
p(x) + 2))/3 - 2*exp(x))))/3
```

Chatgpt [F] Failed to verify
time = 1.00, size = 8, normalized size = 0.40

$$\ln\left(\mathrm{e}^{x}+\mathrm{e}^{-x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(2+cosh(x)),x)
```

```
[Out] ln(exp(x)+exp(-x))
```

3.157 $\int \frac{x^2}{\sqrt{2+x^3}} \, dx$

Optimal. Leaf size=13

$$\frac{2\sqrt{2+x^3}}{3}$$

[Out] 2/3*(x^3+2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{2\sqrt{x^3+2}}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[2 + x^3],x]

[Out] (2*Sqrt[2 + x^3])/3

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{Integral} = \frac{2\sqrt{2+x^3}}{3}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{2\sqrt{2+x^3}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[2 + x^3],x]

[Out] (2*Sqrt[2 + x^3])/3

Maple [A]
time = 0.10, size = 10, normalized size = 0.77

method	result	size
gosper	$\frac{2\sqrt{x^3+2}}{3}$	10
derivativedivides	$\frac{2\sqrt{x^3+2}}{3}$	10
default	$\frac{2\sqrt{x^3+2}}{3}$	10
trager	$\frac{2\sqrt{x^3+2}}{3}$	10
risch	$\frac{2\sqrt{x^3+2}}{3}$	10
elliptic	$\frac{2\sqrt{x^3+2}}{3}$	10
pseudoelliptic	$\frac{2\sqrt{x^3+2}}{3}$	10
meijerg	$\frac{\sqrt{2}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^3}{2}}\right)}{3\sqrt{\pi}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(x^3+2)^(1/2)
```

Maxima [A]
time = 0.33, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^3 + 2)
```

Fricas [A]
time = 0.56, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^3 + 2)
```

Sympy [A]
time = 0.07, size = 10, normalized size = 0.77

$$\frac{2\sqrt{x^3+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**3+2)**(1/2),x)
```

```
[Out] 2*sqrt(x**3 + 2)/3
```

Giac [A]
time = 0.48, size = 9, normalized size = 0.69

$$\frac{2}{3}\sqrt{x^3+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^3+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x^3 + 2)
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.69

$$\frac{2\sqrt{x^3+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + 2)^(1/2),x)
```

```
[Out] (2*(x^3 + 2)^(1/2))/3
```

Chatgpt [A]
time = 1.00, size = 9, normalized size = 0.69

$$\frac{2\sqrt{x^3+2}}{3}$$

Antiderivative was successfully verified.

```
[In] int(x^2/(x^3+2)^(1/2),x)
```

```
[Out] 2/3*(x^3+2)^(1/2)
```

3.158 $\int \frac{\log(x)}{x^2} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{x}-\frac{\log(x)}{x}$$

```
[Out] -1/x-ln(x)/x
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2341}

$$-\frac{1}{x}-\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/x^2,x]
[Out] -x^(-1) - Log[x]/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{Integral} = -\frac{1}{x}-\frac{\log(x)}{x}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{x}-\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/x^2,x]
[Out] -x^(-1) - Log[x]/x
```

Maple [A]
time = 0.01, size = 14, normalized size = 1.08

method	result	size
norman	$\frac{-1-\ln(x)}{x}$	11
parallelrisch	$\frac{-1-\ln(x)}{x}$	11
default	$-\frac{1}{x}-\frac{\ln(x)}{x}$	14
risch	$-\frac{1}{x}-\frac{\ln(x)}{x}$	14
parts	$-\frac{1}{x}-\frac{\ln(x)}{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x-ln(x)/x
```

Maxima [A]
time = 0.36, size = 13, normalized size = 1.00

$$-\frac{\log(x)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2,x, algorithm="maxima")
```

```
[Out] -log(x)/x - 1/x
```

Fricas [A]
time = 0.58, size = 9, normalized size = 0.69

$$-\frac{\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2,x, algorithm="fricas")
```

```
[Out] -(log(x) + 1)/x
```

Sympy [A]
time = 0.04, size = 8, normalized size = 0.62

$$-\frac{\log(x)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**2,x)
```

```
[Out] -log(x)/x - 1/x
```

Giac [A]
time = 0.45, size = 13, normalized size = 1.00

$$-\frac{\log(x)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2,x, algorithm="giac")
```

```
[Out] -log(x)/x - 1/x
```

Mupad [B]
time = 0.09, size = 9, normalized size = 0.69

$$-\frac{\ln(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/x^2,x)
```

```
[Out] -(log(x) + 1)/x
```

Chatgpt [A]
time = 1.00, size = 13, normalized size = 1.00

$$-\frac{\ln(x)}{x} - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] int(ln(x)/x^2,x)
```

```
[Out] -ln(x)/x-1/x
```

3.159 $\int \operatorname{sech}(x)\,dx$

Optimal. Leaf size=3

$$\arctan(\sinh(x))$$

[Out] arctan(sinh(x))

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\arctan(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Sech[x],x]

[Out] ArcTan[Sinh[x]]

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{Integral} = \arctan(\sinh(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6.
time = 0.01, size = 9, normalized size = 3.00

$$2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x],x]

[Out] 2*ArcTan[Tanh[x/2]]

Maple [A]
time = 0.06, size = 4, normalized size = 1.33

method	result	size

lookup	$\arctan(\sinh(x))$	4
default	$\arctan(\sinh(x))$	4
risch	$i\ln(e^x + i) - i\ln(e^x - i)$	20
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{x}{2}\right) - i\right) - \ln\left(\tanh\left(\frac{x}{2}\right) + i\right)\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sinh(x))
```

Maxima [A]
time = 0.37, size = 3, normalized size = 1.00

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.
time = 0.56, size = 8, normalized size = 2.67

$$2 \arctan(\cosh(x) + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.
time = 0.22, size = 7, normalized size = 2.33

$$2 \operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x)
```

```
[Out] 2*atan(tanh(x/2))
```

Giac [A]
time = 0.47, size = 5, normalized size = 1.67

$$2 \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

Mupad [B]
time = 0.02, size = 5, normalized size = 1.67

$$2\,\mathrm{atan}(\mathrm{e}^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x),x)
```

```
[Out] 2*atan(exp(x))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 5, normalized size = 1.67

$$2\arctan\left(\cosh\left(x\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(sech(x),x)
```

```
[Out] 2*arctan(cosh(x))
```

3.160 $\int e^{x^2} x^3 \, dx$

Optimal. Leaf size=22

$$-\frac{e^{x^2}}{2}+\frac{1}{2}e^{x^2}x^2$$

[Out] -1/2*exp(x^2)+1/2*exp(x^2)*x^2

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2243, 2240}

$$\frac{1}{2}e^{x^2}x^2-\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x^3,x]

[Out] -1/2*E^x^2 + (E^x^2*x^2)/2

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}e^{x^2}x^2-\int e^{x^2}x\,dx\\ &= -\frac{e^{x^2}}{2}+\frac{1}{2}e^{x^2}x^2\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 0.64

$$\frac{1}{2}e^{x^2}\left(-1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x^3,x]
[Out] (E^x^2*(-1 + x^2))/2
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
gosper	$\frac{(x^2-1)\mathrm{e}^{x^2}}{2}$	12
risch	$\left(\frac{x^2}{2}-\frac{1}{2}\right)\mathrm{e}^{x^2}$	13
meijerg	$\frac{1}{2}-\frac{(-2x^2+2)\mathrm{e}^{x^2}}{4}$	16
derivativedivides	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
default	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
norman	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
parallelrisch	$-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}$	17
parts	$\frac{\sqrt{\pi}\,\mathrm{erfi}(x)x^3}{2}-\frac{3\sqrt{\pi}\left(\frac{x^3\,\mathrm{erfi}(x)}{3}-\frac{2\left(-\frac{\mathrm{e}^{x^2}}{2}+\frac{\mathrm{e}^{x^2}x^2}{2}\right)}{3\sqrt{\pi}}\right)}{2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Maxima [A]
time = 0.33, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(x^2),x, algorithm="maxima")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Fricas [A]
time = 0.57, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^{2}-1\right)e^{\left(x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(x^2),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.45

$$\frac{\left(x^{2}-1\right)e^{x^{2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*exp(x**2),x)
[Out] (x**2 - 1)*exp(x**2)/2
```

Giac [A]
time = 0.49, size = 11, normalized size = 0.50

$$\frac{1}{2}\left(x^{2}-1\right)e^{\left(x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*exp(x^2),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Mupad [B]
time = 0.00, size = 11, normalized size = 0.50

$$\frac{e^{x^{2}}\left(x^{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(x^2),x)
[Out] (exp(x^2)*(x^2 - 1))/2
```

Chatgpt [A]
time = 1.00, size = 16, normalized size = 0.73

$$\frac{x^{2}e^{x^{2}}}{2}-\frac{e^{x^{2}}}{2}$$

Antiderivative was successfully verified.

```
[In] int(x^3*exp(x^2),x)
[Out] 1/2*x^2*exp(x^2)-1/2*exp(x^2)
```

3.161 $\int \frac{1}{x\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=10

$$\arctan\left(\sqrt{-1+x^2}\right)$$

```
[Out] arctan((x^2-1)^(1/2))
```

Rubi [A]

time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 65, 209}

$$\arctan\left(\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^2]),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x^2}\right)\\
&= \arctan\left(\sqrt{-1+x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\arctan\left(\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^2]),x]
```

```
[Out] ArcTan[Sqrt[-1 + x^2]]
```

Maple [A]
time = 0.10, size = 11, normalized size = 1.10

method	result	size
pseudoelliptic	$\arctan\left(\sqrt{x^2-1}\right)$	9
default	$-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)$	11
trager	$\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathit{RootOf}\left(\_Z^2+1\right)+\sqrt{x^2-1}}{x}\right)$	27
meijerg	$\frac{\sqrt{-\mathrm{signum}(x^2-1)}\left((-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)\right)}{2\sqrt{\pi}\,\sqrt{\mathrm{signum}(x^2-1)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(1/(x^2-1)^(1/2))
```

Maxima [A]
time = 0.43, size = 7, normalized size = 0.70

$$-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x))
```

Fricas [A]
time = 0.58, size = 14, normalized size = 1.40

$$2\arctan\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 - 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 17, normalized size = 1.70

$$\begin{cases} i \operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ -\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-1)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(1/x), 1/Abs(x**2) > 1), (-asin(1/x), True))
```

Giac [A]
time = 0.49, size = 8, normalized size = 0.80

$$\arctan\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x^2 - 1))
```

Mupad [B]
time = 0.09, size = 8, normalized size = 0.80

$$\operatorname{atan}\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 1)^(1/2)),x)
```

```
[Out] atan((x^2 - 1)^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 0.90

$$-\frac{1}{2\left(x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/x/(x^2-1)^(1/2),x)
```

```
[Out] -1/2/(x^2-1)
```

3.162 $\int \frac{1}{x(1+x^2)}\,dx$

Optimal. Leaf size=13

$$\log(x) - \frac{1}{2}\log\left(1 + x^2\right)$$

```
[Out] ln(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {272, 36, 29, 31}

$$\log(x) - \frac{1}{2}\log\left(x^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)),x]
```

```
[Out] Log[x] - Log[1 + x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)\\ &= \log(x) - \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)),x]
```

```
[Out] Log[x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.92

method	result	size
default	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
norman	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
meijerg	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
risch	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12
parallelrisch	$\ln(x) - \frac{\ln(x^2+1)}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.33, size = 15, normalized size = 1.15

$$-\frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A]
time = 0.58, size = 11, normalized size = 0.85

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1),x, algorithm="fricas")
[Out] -1/2*log(x^2 + 1) + log(x)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.77

$$\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1),x)
[Out] log(x) - log(x**2 + 1)/2
```

Giac [A]
time = 0.44, size = 15, normalized size = 1.15

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1),x, algorithm="giac")
[Out] -1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B]
time = 0.09, size = 11, normalized size = 0.85

$$\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)),x)
[Out] log(x) - log(x^2 + 1)/2
```

Chatgpt [A]
time = 1.00, size = 11, normalized size = 0.85

$$\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$$

Antiderivative was successfully verified.

```
[In] int(1/x/(x^2+1),x)
[Out] ln(x)-1/2*ln(x^2+1)
```

3.163 $\int \text{arccosh}(x)\,dx$

Optimal. Leaf size=21

$$-\sqrt{-1+x}\sqrt{1+x}+x\text{arccosh}(x)$$

```
[Out] -(x-1)^(1/2)*(x+1)^(1/2)+x*arccosh(x)
```

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5879, 75}

$$x\text{arccosh}(x)-\sqrt{x-1}\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[x],x]
```

```
[Out] -(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5879

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcCosh[c*x])^(n - 1)/(Sqrt[
1 + c*x]*Sqrt[-1 + c*x])), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= x\text{arccosh}(x)-\int\frac{x}{\sqrt{-1+x}\sqrt{1+x}}\,dx\\ &= -\sqrt{-1+x}\sqrt{1+x}+x\text{arccosh}(x)\end{aligned}$$

Mathematica [A]

time = 0.03, size = 21, normalized size = 1.00

$$-\sqrt{-1+x}\sqrt{1+x}+x\text{arccosh}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[x],x]
[Out] -(Sqrt[-1 + x]*Sqrt[1 + x]) + x*ArcCosh[x]
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.86

method	result	size
lookup	$-\sqrt{x-1}\,\sqrt{x+1}+x\,\mathrm{arccosh}(x)$	18
default	$-\sqrt{x-1}\,\sqrt{x+1}+x\,\mathrm{arccosh}(x)$	18
parts	$-\sqrt{x-1}\,\sqrt{x+1}+x\,\mathrm{arccosh}(x)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x),x,method=_RETURNVERBOSE)
[Out] -(x-1)^(1/2)*(x+1)^(1/2)+x*arccosh(x)
```

Maxima [A]
time = 0.36, size = 14, normalized size = 0.67

$$x\,\mathrm{arcosh}\,(x)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x),x, algorithm="maxima")
[Out] x*arccosh(x) - sqrt(x^2 - 1)
```

Fricas [A]
time = 0.59, size = 22, normalized size = 1.05

$$x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x),x, algorithm="fricas")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \mathrm{acosh}\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x),x)
```

```
[Out] Integral(acosh(x), x)
```

Giac [A]
time = 0.44, size = 22, normalized size = 1.05

$$x \log \left(x + \sqrt{x^2 - 1} \right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.81

$$x \operatorname{acosh}(x) - \sqrt{x - 1}\, \sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x),x)
[Out] x*acosh(x) - (x - 1)^(1/2)*(x + 1)^(1/2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 29, normalized size = 1.38

$$x \operatorname{arccosh}(x) - \frac{\ln \left(\frac{\sqrt{x^2-1}-1}{\sqrt{x^2-1}+1} \right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(arccosh(x),x)
[Out] x*arccosh(x)-1/2*ln(((x^2-1)^(1/2)-1)/((x^2-1)^(1/2)+1))
```

3.164 $\int e^{-3-5x-2x^2} \, dx$

Optimal. Leaf size=34

$$\frac{\sqrt[8]{e}\sqrt{\pi}\operatorname{erf}\left(\frac{5+4x}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*exp(1/8)*Pi^(1/2)*erf(1/4*(5+4*x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2266, 2236}

$$\frac{\sqrt[8]{e}\sqrt{\pi}\operatorname{erf}\left(\frac{4x+5}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-3 - 5*x - 2*x^2),x]
```

```
[Out] (E^(1/8)*Sqrt[Pi]*Erf[(5 + 4*x)/(2*Sqrt[2])])/(2*Sqrt[2])
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \sqrt[8]{e}\int e^{-\frac{1}{8}(-5-4x)^2} \, dx \\
&= \frac{\sqrt[8]{e}\sqrt{\pi}\operatorname{erf}\left(\frac{5+4x}{2\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 34, normalized size = 1.00

$$\frac{\sqrt[8]{e}\sqrt{\pi}\operatorname{erf}\left(\frac{5+4x}{2\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-3 - 5*x - 2*x^2),x]
[Out] (E^(1/8)*Sqrt[Pi]*Erf[(5 + 4*x)/(2*Sqrt[2])])/(2*Sqrt[2])
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.68

method	result	size
default	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{1}{8}}\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,x+\frac{5\sqrt{2}}{4}\right)}{4}$	23
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{1}{8}}\sqrt{2}\,\mathrm{erf}\left(\sqrt{2}\,x+\frac{5\sqrt{2}}{4}\right)}{4}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*x^2-5*x-3),x,method=_RETURNVERBOSE)
[Out] 1/4*Pi^(1/2)*exp(1/8)*2^(1/2)*erf(2^(1/2)*x+5/4*2^(1/2))
```

Maxima [A]
time = 0.47, size = 22, normalized size = 0.65

$$\frac{1}{4}\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}x+\frac{5}{4}\sqrt{2}\right)e^{\frac{1}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*x^2-5*x-3),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(sqrt(2)*x + 5/4*sqrt(2))*e^(1/8)
```

Fricas [A]
time = 0.58, size = 21, normalized size = 0.62

$$\frac{1}{4}\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{4}\sqrt{2}(4\,x+5)\right)e^{\frac{1}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*x^2-5*x-3),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(1/4*sqrt(2)*(4*x + 5))*e^(1/8)
```

Sympy [A]
time = 0.34, size = 32, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt{\pi}e^{\frac{1}{8}}\,\mathrm{erf}\left(\sqrt{2}x+\frac{5\sqrt{2}}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*x**2-5*x-3),x)
```

```
[Out] sqrt(2)*sqrt(pi)*exp(1/8)*erf(sqrt(2)*x + 5*sqrt(2)/4)/4
```

Giac [A]
time = 0.48, size = 21, normalized size = 0.62

$$\frac{1}{4}\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{4}\sqrt{2}(4x+5)\right)e^{\frac{1}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*x^2-5*x-3),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(pi)*erf(1/4*sqrt(2)*(4*x + 5))*e^(1/8)
```

Mupad [B]
time = 0.08, size = 22, normalized size = 0.65

$$\frac{\sqrt{2}\sqrt{\pi}\,e^{1/8}\operatorname{erf}\left(\sqrt{2}\,x+\frac{5\sqrt{2}}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(- 5*x - 2*x^2 - 3),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*exp(1/8)*erf(2^(1/2)*x + (5*2^(1/2))/4))/4
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 0.29

$$-\frac{e^{-2\left(x+\frac{5}{4}\right)^2}}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(-2*x^2-5*x-3),x)
```

```
[Out] -1/4*exp(-2*(x+5/4)^2)
```

3.165 $\int \sin\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

```
[Out] -2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3442, 3377, 2717}

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int x\sin(x)\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\text{Subst}\left(\int \cos(x)\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A]
time = 0.03, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
default	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
meijerg	$4\sqrt{\pi}\,\left(-\frac{\sqrt{x}\,\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Maxima [A]
time = 0.33, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A]
time = 0.59, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A]
time = 0.11, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A]
time = 0.51, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B]
time = 0.16, size = 16, normalized size = 0.73

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
```

```
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 23, normalized size = 1.05

$$2\sqrt{x}\ \mathrm{sinIntegral}\left(2\sqrt{x}\right)-2\sqrt{x}\cos\left(2\sqrt{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x^(1/2)),x)
```

```
[Out] 2*x^(1/2)*Si(2*x^(1/2))-2*x^(1/2)*cos(2*x^(1/2))
```

3.166 $\int \frac{1}{\left(\frac{1}{x}+x\right)^2} \, dx$

Optimal. Leaf size=19

$$-\frac{x}{2\left(1+x^2\right)}+\frac{\arctan(x)}{2}$$

```
[Out] -x/(2*x^2+2)+1/2*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1607, 294, 209}

$$\frac{\arctan(x)}{2}-\frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + x)^(-2),x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{x^2}{(1+x^2)^2}\,dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\ &= -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + x)^(-2),x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)} + \frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)} + \frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2 - i\ln(i+x)x^2 + i\ln(x-i) - i\ln(i+x) + 2x}{4(x^2+1)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)
```

Maxima [A]
time = 0.47, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+1/x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A]
time = 0.56, size = 21, normalized size = 1.11

$$\frac{(x^2+1)\arctan(x)-x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+1/x)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+1/x)**2,x)
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Giac [A]
time = 0.48, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+1/x)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B]
time = 0.09, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\ (x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1/x)^2,x)
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 12, normalized size = 0.63

$$\arctan(x)-\frac{1}{x^2+1}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x+1/x)^2,x)
[Out] arctan(x)-1/(x^2+1)
```

3.167 $\int \frac{e^{-x}(2+x)}{x^3} \, dx$

Optimal. Leaf size=10

$$-\frac{e^{-x}}{x^2}$$

```
[Out] -1/exp(x)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2228}

$$-\frac{e^{-x}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(E^x*x^3),x]
```

```
[Out] -(1/(E^x*x^2))
```

Rule 2228

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[g*u^(m + 1)*(F^(c*v)/(b*c*e*Log
[F])), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\text{Integral} = -\frac{e^{-x}}{x^2}$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 1.00

$$-\frac{e^{-x}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(E^x*x^3),x]
```

```
[Out] -(1/(E^x*x^2))
```

Maple [A]
time = 0.04, size = 10, normalized size = 1.00

method	result	size
gosper	$-\frac{e^{-x}}{x^2}$	10
derivativedivides	$-\frac{e^{-x}}{x^2}$	10
default	$-\frac{e^{-x}}{x^2}$	10
norman	$-\frac{e^{-x}}{x^2}$	10
risch	$-\frac{e^{-x}}{x^2}$	10
parallelrisch	$-\frac{e^{-x}}{x^2}$	10
meijerg	$-\frac{1}{x^2}+\frac{1}{x}-\frac{1}{2}+\frac{9x^2-12x+6}{6x^2}-\frac{(3-3x)e^{-x}}{3x^2}+\frac{-2x+2}{2x}-\frac{e^{-x}}{x}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*exp(-x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^2*exp(-x)
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 11, normalized size = 1.10

$$-\Gamma(-1,x)-2\,\Gamma(-2,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*exp(-x)/x^3,x, algorithm="maxima")
```

```
[Out] -gamma(-1, x) - 2*gamma(-2, x)
```

Fricas [A]
time = 0.55, size = 9, normalized size = 0.90

$$-\frac{e^{(-x)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*exp(-x)/x^3,x, algorithm="fricas")
```

```
[Out] -e^(-x)/x^2
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.80

$$-\frac{e^{-x}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*exp(-x)/x**3,x)
```

```
[Out] -exp(-x)/x**2
```

Giac [A]
time = 0.48, size = 9, normalized size = 0.90

$$-\frac{e^{(-x)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*exp(-x)/x^3,x, algorithm="giac")
```

```
[Out] -e^(-x)/x^2
```

Mupad [B]
time = 0.05, size = 9, normalized size = 0.90

$$-\frac{\mathrm{e}^{-x}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(-x)*(x + 2))/x^3,x)
```

```
[Out] -exp(-x)/x^2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 1.00

$$-\frac{\mathrm{e}^{x}(x+4)}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] int((2+x)*exp(-x)/x^3,x)
```

```
[Out] -1/x^2*exp(x)*(x+4)
```

3.168 $\int \frac{1}{\sqrt{(1-x)x}} \, dx$

Optimal. Leaf size=8

$$-\arcsin(1-2x)$$

[Out] arcsin(-1+2*x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1976, 633, 222}

$$-\arcsin(1-2x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(1 - x)*x],x]

[Out] -ArcSin[1 - 2*x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \frac{1}{\sqrt{x-x^2}} \, dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, 1-2x\right) \\
&= -\arcsin(1-2x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.
time = 0.02, size = 40, normalized size = 5.00

$$-\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x)*x],x]
[Out] (-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

Maple [A]
time = 0.17, size = 7, normalized size = 0.88

method	result	size
default	$\arcsin(-1+2x)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-(x-1)x}}{x}\right)$	16
trager	$\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(-2\mathit{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}+\mathit{RootOf}\left(\_Z^2+1\right)\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1-x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(-1+2*x)
```

Maxima [A]
time = 0.43, size = 6, normalized size = 0.75

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(1-x))^(1/2),x, algorithm="maxima")
[Out] arcsin(2*x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.
time = 0.57, size = 16, normalized size = 2.00

$$-2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(1-x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [**A**]
time = 0.59, size = 5, normalized size = 0.62

$$\operatorname{asin}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(1-x))**(1/2),x)
[Out] asin(2*x - 1)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.
time = 0.44, size = 25, normalized size = 3.12

$$\frac{1}{4}\sqrt{-x^2 + x}(2x - 1) + \frac{1}{8}\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(1-x))^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

Mupad [**B**]
time = 0.01, size = 6, normalized size = 0.75

$$\operatorname{asin}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 1))^(1/2),x)
[Out] asin(2*x - 1)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 30, normalized size = 3.75

$$2\sqrt{x(1-x)} + 4\sqrt{x(1-x)+1} - 4\sqrt{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x*(1-x))^(1/2),x)
[Out] 2*(x*(1-x))^(1/2)+4*(x*(1-x)+1)^(1/2)-4*2^(1/2)
```

3.169 $\int e^{-x} \tanh(x) \, dx$

Optimal. Leaf size=12

$$e^{-x} + 2 \arctan(e^x)$$

[Out] exp(-x)+2*arctan(exp(x))

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2320, 464, 209}

$$2 \arctan(e^x) + e^{-x}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/E^x,x]

[Out] E^(-x) + 2*ArcTan[E^x]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{-1+x^2}{x^2(1+x^2)}\,dx, x, e^x\right)\\ &= e^{-x} + 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\ &= e^{-x} + 2\arctan(e^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$e^{-x} + 2\arctan(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/E^x,x]
```

```
[Out] E^(-x) + 2*ArcTan[E^x]
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.58

method	result	size
default	$\frac{2}{\tanh(\frac{x}{2})+1} + 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	19
risch	$e^{-x} + i\ln(e^x + i) - i\ln(e^x - i)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(tanh(1/2*x)+1)+2*arctan(tanh(1/2*x))
```

Maxima [A]
time = 0.43, size = 12, normalized size = 1.00

$$-2\arctan\left(e^{(-x)}\right) + e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/exp(x),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x)) + e^(-x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.57, size = 23, normalized size = 1.92

$$\frac{2(\cosh(x) + \sinh(x))\arctan(\cosh(x) + \sinh(x)) + 1}{\cosh(x) + \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/exp(x),x, algorithm="fricas")
```

```
[Out] (2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-x}\tanh{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/exp(x),x)
```

```
[Out] Integral(exp(-x)*tanh(x), x)
```

Giac [**A**]
time = 0.47, size = 10, normalized size = 0.83

$$2\arctan\left(e^{x}\right)+e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/exp(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x) + e^(-x)
```

Mupad [**B**]
time = 0.05, size = 12, normalized size = 1.00

$$\mathrm{e}^{-x}-2\,\mathrm{atan}\left(\mathrm{e}^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x)*tanh(x),x)
```

```
[Out] exp(-x) - 2*atan(exp(-x))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 18, normalized size = 1.50

$$x-\ln\left(\mathrm{e}^{x}+1\right)+\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(tanh(x)/exp(x),x)
```

```
[Out] x-ln(exp(x)+1)+1/2*ln(1+exp(2*x))
```

3.170 $\int \sqrt{1+\sin(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

```
[Out] -2*cos(x)/(1+sin(x))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2725}

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sin[x]],x]
```

```
[Out] (-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{Integral} = -\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(12) = 24.
time = 0.01, size = 40, normalized size = 3.33

$$\frac{2\left(-\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{1+\sin(x)}}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.14, size = 17, normalized size = 1.42

method	result	size
default	$\frac{2(-1+\sin(x))\sqrt{1+\sin(x)}}{\cos(x)}$	17
risch	$-\frac{i\sqrt{2}\,\sqrt{2+2\sin(x)}\,(e^{ix}-i)\,(i+e^{ix})}{e^{2ix}+2ie^{ix}-1}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(-1+sin(x))*(1+sin(x))^(1/2)/cos(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 0.58, size = 24, normalized size = 2.00

$$-\frac{2\left(\cos\left(x\right)-\sin\left(x\right)+1\right)\sqrt{\sin\left(x\right)+1}}{\cos\left(x\right)+\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="fricas")
[Out] -2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))**(1/2),x)
[Out] Integral(sqrt(sin(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.50, size = 22, normalized size = 1.83

$$2\sqrt{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x)
```

Mupad [B]
time = 0.03, size = 16, normalized size = 1.33

$$\frac{2\left(\sin\left(x\right)-1\right)\sqrt{\sin\left(x\right)+1}}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + 1)^(1/2),x)
```

```
[Out] (2*(sin(x) - 1)*(sin(x) + 1)^(1/2))/cos(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((1+sin(x))^(1/2),x)
```

```
[Out] not solved
```

3.171 $\int \frac{1}{1+\sqrt{x}} \, dx$

Optimal. Leaf size=18

$$2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)$$

```
[Out] 2*x^(1/2)-2*ln(1+x^(1/2))
```

Rubi [A]

time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {196, 45}

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int \frac{x}{1+x} \, dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) \, dx, x, \sqrt{x}\right) \\
&= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 18, normalized size = 1.00

$$2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Maple [A]
time = 0.06, size = 27, normalized size = 1.50

method	result	size
derivativedivides	$2\sqrt{x}-2\ln\left(1+\sqrt{x}\right)$	15
meijerg	$2\sqrt{x}-2\ln\left(1+\sqrt{x}\right)$	15
trager	$2\sqrt{x}-\ln\left(2\sqrt{x}+1+x\right)$	18
default	$2\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(1+\sqrt{x}\right)-\ln\left(x-1\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)+ln(x^(1/2)-1)-ln(1+x^(1/2))-ln(x-1)
```

Maxima [A]
time = 0.33, size = 15, normalized size = 0.83

$$2\sqrt{x}-2\log\left(\sqrt{x}+1\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1) + 2
```

Fricas [A]
time = 0.56, size = 14, normalized size = 0.78

$$2\sqrt{x}-2\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.83

$$2\sqrt{x}-2\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Giac [**A**]
time = 0.48, size = 14, normalized size = 0.78

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Mupad [**B**]
time = 0.04, size = 14, normalized size = 0.78

$$2\sqrt{x} - 2\ln\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 1),x)
```

```
[Out] 2*x^(1/2) - 2*log(x^(1/2) + 1)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 16, normalized size = 0.89

$$\sqrt{x} + \arctan\left(\sqrt{x}\right) - \ln\left(\sqrt{x} + 1\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^(1/2)+1),x)
```

```
[Out] x^(1/2)+arctan(x^(1/2))-ln(x^(1/2)+1)
```

3.172 $\int e^{-x^2} \sin^2\left(\frac{\pi}{4}+x\right) \, dx$

Optimal. Leaf size=52

$$\frac{1}{4}\sqrt{\pi}\text{erf}(x)-\frac{\sqrt{\pi}\text{erfi}(1-ix)}{8e}-\frac{\sqrt{\pi}\text{erfi}(1+ix)}{8e}$$

```
[Out] 1/4*Pi^(1/2)*erf(x)+1/8*Pi^(1/2)*erfi(-1+I*x)*exp(-1)-1/8*Pi^(1/2)*erfi(1+I
*x)*exp(-1)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4560, 2236, 2266}

$$\frac{1}{4}\sqrt{\pi}\text{erf}(x)-\frac{\sqrt{\pi}\text{erfi}(1-ix)}{8e}-\frac{\sqrt{\pi}\text{erfi}(1+ix)}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Pi/4 + x]^2/E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erf[x])/4 - (Sqrt[Pi]*Erfi[1 - I*x])/(8*E) - (Sqrt[Pi]*Erfi[1 + I
*x])/(8*E)
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 4560

```
Int[(F_)^(u_)*Sin[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sin[v]^n
, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v,
 x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int \left(\frac{e^{-x^2}}{2} + \frac{1}{4} i e^{-2ix-x^2} - \frac{1}{4} i e^{2ix-x^2}\right) dx \\
&= \frac{1}{4} i \int e^{-2ix-x^2} dx - \frac{1}{4} i \int e^{2ix-x^2} dx + \frac{1}{2} \int e^{-x^2} dx \\
&= \frac{1}{4}\sqrt{\pi}\text{erf}(x) + \frac{i \int e^{-\frac{1}{4}(-2i-2x)^2} dx}{4e} - \frac{i \int e^{-\frac{1}{4}(2i-2x)^2} dx}{4e} \\
&= \frac{1}{4}\sqrt{\pi}\text{erf}(x) - \frac{\sqrt{\pi}\text{erfi}(1-ix)}{8e} - \frac{\sqrt{\pi}\text{erfi}(1+ix)}{8e}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 0.73

$$
\frac{\sqrt{\pi}(2e\text{erf}(x) - \text{erfi}(1-ix) - \text{erfi}(1+ix))}{8e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Pi/4 + x]^2/E^x^2,x]
[Out] (Sqrt[Pi]*(2*E*Erf[x] - Erfi[1 - I*x] - Erfi[1 + I*x]))/(8*E)
```

Maple [A]
time = 0.16, size = 35, normalized size = 0.67

method	result	size
risch	$\frac{i\sqrt{\pi}\,e^{-1}\,\text{erf}(i+x)}{8} - \frac{i\sqrt{\pi}\,e^{-1}\,\text{erf}(x-i)}{8} + \frac{\sqrt{\pi}\,\text{erf}(x)}{4}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x+1/4*Pi)^2/exp(x^2),x,method=_RETURNVERBOSE)
[Out] 1/8*I*Pi^(1/2)*exp(-1)*erf(I+x)-1/8*I*Pi^(1/2)*exp(-1)*erf(x-I)+1/4*Pi^(1/2
)*erf(x)
```

Maxima [A]
time = 0.45, size = 26, normalized size = 0.50

$$
\frac{1}{8}\sqrt{\pi}(2\,\text{erf}(x)\,e + i\,\text{erf}(x+i) - i\,\text{erf}(x-i))e^{(-1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+1/4*pi)^2/exp(x^2),x, algorithm="maxima")
[Out] 1/8*sqrt(pi)*(2*erf(x)*e + I*erf(x + I) - I*erf(x - I))*e^(-1)
```

Fricas [A]
time = 0.62, size = 43, normalized size = 0.83

$$\frac{1}{8}\sqrt{\pi}\left(\left(\operatorname{erf}\left(-x+i\right)e^{\left(\frac{1}{2}i\pi-1\right)}+2\operatorname{erf}\left(x\right)\right)e^{\left(\frac{1}{2}i\pi+1\right)}-\operatorname{erf}\left(x+i\right)\right)e^{\left(-\frac{1}{2}i\pi-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+1/4*pi)^2/exp(x^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(pi)*((erf(-x + I)*e^(1/2*I*pi - 1) + 2*erf(x))*e^(1/2*I*pi + 1) -
erf(x + I))*e^(-1/2*I*pi - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{-x^2}\sin^2\left(x+\frac{\pi}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+1/4*pi)**2/exp(x**2),x)
```

```
[Out] Integral(exp(-x**2)*sin(x + pi/4)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+1/4*pi)^2/exp(x^2),x, algorithm="giac")
```

```
[Out] integrate(e^(-x^2)*sin(1/4*pi + x)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{e}^{-x^2}\sin\left(\frac{\Pi}{4}+x\right)^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x^2)*sin(Pi/4 + x)^2,x)
```

```
[Out] int(exp(-x^2)*sin(Pi/4 + x)^2, x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 32, normalized size = 0.62

$$\frac{\sqrt{\pi}\operatorname{erf}\left(x\right)}{2}-\frac{\sqrt{2}\sqrt{\frac{\pi}{1-2x\sqrt{2}+\frac{x^2}{16}}}}{8}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x+1/4*Pi)^2/exp(x^2),x)
[Out] 1/2*Pi^(1/2)*erf(x)-1/8*2^(1/2)*(Pi/(1-2*x*2^(1/2)+1/16*x^2))^(1/2)
```

3.173 $\int 3e^{-2x^3-x^6}x^2(1+x^3)^2\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}e^{-2x^3-x^6}(1+x^3)+\frac{1}{4}e\sqrt{\pi}\text{erf}(1+x^3)$$

```
[Out] -1/2*exp(-x^6-2*x^3)*(x^3+1)+1/4*exp(1)*Pi^(1/2)*erf(x^3+1)
```

Rubi [A]
time = 0.12, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {12, 6847, 2269, 2266, 2236}

$$\frac{1}{4}e\sqrt{\pi}\text{erf}(x^3+1)-\frac{1}{2}e^{-x^6-2x^3}(x^3+1)$$

Antiderivative was successfully verified.

```
[In] Int[3*E^(-2*x^3 - x^6)*x^2*(1 + x^3)^2,x]
```

```
[Out] -1/2*(E^(-2*x^3 - x^6)*(1 + x^3)) + (E*Sqrt[Pi]*Erf[1 + x^3])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2269

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] -
 Dist[(m - 1)*(e^2/(2*c*Log[F])), Int[(d + e*x)^(m - 2)*F^(a + b*x + c*x^2)
, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && GtQ[m, 1
]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 3\int e^{-2x^3-x^6}x^2\left(1+x^3\right)^2\,dx\\
&= \text{Subst}\left(\int e^{-2x-x^2}(1+x)^2\,dx,x,x^3\right)\\
&= -\frac{1}{2}e^{-2x^3-x^6}\left(1+x^3\right)+\frac{1}{2}\text{Subst}\left(\int e^{-2x-x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{2}e^{-2x^3-x^6}\left(1+x^3\right)+\frac{1}{2}e\text{Subst}\left(\int e^{-\frac{1}{4}(-2-2x)^2}\,dx,x,x^3\right)\\
&= -\frac{1}{2}e^{-2x^3-x^6}\left(1+x^3\right)+\frac{1}{4}e\sqrt{\pi}\text{erf}\left(1+x^3\right)
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 37, normalized size = 0.95

$$\frac{1}{4}\left(-2e^{-x^3\left(2+x^3\right)}\left(1+x^3\right)+e\sqrt{\pi}\text{erf}\left(1+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*E^(-2*x^3 - x^6)*x^2*(1 + x^3)^2,x]
[Out] ((-2*(1 + x^3))/E^(x^3*(2 + x^3)) + E*Sqrt[Pi]*Erf[1 + x^3])/4
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int 3x^2\left(x^3+1\right)^2\mathrm{e}^{-x^6-2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*(x^3+1)^2*exp(-x^6-2*x^3),x)
[Out] int(3*x^2*(x^3+1)^2*exp(-x^6-2*x^3),x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.47, size = 137, normalized size = 3.51

$$\frac{1}{2}\sqrt{\pi}\,\text{erf}\left(x^3+1\right)e+\frac{1}{2}i\left(\frac{i\left(x^3+1\right)^3\Gamma\left(\frac{3}{2},\left(x^3+1\right)^2\right)}{\left(\left(x^3+1\right)^2\right)^{\frac{3}{2}}}-\frac{i\sqrt{\pi}\left(x^3+1\right)\left(\text{erf}\left(\sqrt{\left(x^3+1\right)^2}\right)-1\right)}{\sqrt{\left(x^3+1\right)^2}}-2i\,e^{\left(-\left(x^3+1\right)^2\right)}\right)e+i\left(\frac{i\sqrt{\pi}\left(x^3+1\right)\left(\text{erf}\left(\sqrt{\left(x^3+1\right)^2}\right)-1\right)}{\sqrt{\left(x^3+1\right)^2}}+i\,e^{\left(-\left(x^3+1\right)^2\right)}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*(x^3+1)^2*exp(-x^6-2*x^3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(pi)*erf(x^3 + 1)*e + 1/2*I*(I*(x^3 + 1)^3*gamma(3/2, (x^3 + 1)^2)/
((x^3 + 1)^2)^(3/2) - I*sqrt(pi)*(x^3 + 1)*(erf(sqrt((x^3 + 1)^2)) - 1)/sqr
t((x^3 + 1)^2) - 2*I*e^(-(x^3 + 1)^2))*e + I*(I*sqrt(pi)*(x^3 + 1)*(erf(sqr
t((x^3 + 1)^2)) - 1)/sqrt((x^3 + 1)^2) + I*e^(-(x^3 + 1)^2))*e
```

Fricas [A]
time = 0.58, size = 33, normalized size = 0.85

$$\frac{1}{4}\sqrt{\pi}\operatorname{erf}\left(x^3+1\right)e-\frac{1}{2}\left(x^3+1\right)e^{\left(-x^6-2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*(x^3+1)^2*exp(-x^6-2*x^3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(pi)*erf(x^3 + 1)*e - 1/2*(x^3 + 1)*e^(-x^6 - 2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$3\left(\int x^2e^{-2x^3}e^{-x^6}\,dx+\int 2x^5e^{-2x^3}e^{-x^6}\,dx+\int x^8e^{-2x^3}e^{-x^6}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x**2*(x**3+1)**2*exp(-x**6-2*x**3),x)
```

```
[Out] 3*(Integral(x**2*exp(-2*x**3)*exp(-x**6), x) + Integral(2*x**5*exp(-2*x**3)
*exp(-x**6), x) + Integral(x**8*exp(-2*x**3)*exp(-x**6), x))
```

Giac [A]
time = 0.46, size = 33, normalized size = 0.85

$$\frac{1}{4}\sqrt{\pi}\operatorname{erf}\left(x^3+1\right)e-\frac{1}{2}\left(x^3+1\right)e^{\left(-x^6-2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*(x^3+1)^2*exp(-x^6-2*x^3),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(x^3 + 1)*e - 1/2*(x^3 + 1)*e^(-x^6 - 2*x^3)
```

Mupad [B]
time = 0.31, size = 45, normalized size = 1.15

$$\frac{\sqrt{\pi}\,\mathrm{e}\,\mathrm{erf}\left(x^3+1\right)}{4}-\frac{x^3\,\mathrm{e}^{-x^6-2\,x^3}}{2}-\frac{\mathrm{e}^{-x^6-2\,x^3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*exp(- 2*x^3 - x^6)*(x^3 + 1)^2,x)
```

```
[Out] (pi^(1/2)*exp(1)*erf(x^3 + 1))/4 - (x^3*exp(- 2*x^3 - x^6))/2 - exp(- 2*x^3
 - x^6)/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 36, normalized size = 0.92

$$-\frac{(x^3+1)^2 e^{-x^6-2x^3}}{2} - e^{-x^6-2x^3}$$

Warning: Unable to verify antiderivative.

```
[In] int(3*x^2*(x^3+1)^2*exp(-x^6-2*x^3),x)
```

```
[Out] -1/2*(x^3+1)^2*exp(-x^6-2*x^3)-exp(-x^6-2*x^3)
```

3.174 $\int e^{2x} \cos(3x)\, dx$

Optimal. Leaf size=27

$$\frac{2}{13}e^{2x}\cos(3x) + \frac{3}{13}e^{2x}\sin(3x)$$

```
[Out] 2/13*exp(2*x)*cos(3*x)+3/13*exp(2*x)*sin(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{4518\}$

$$\frac{3}{13}e^{2x}\sin(3x) + \frac{2}{13}e^{2x}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Cos[3*x],x]
[Out] (2*E^(2*x)*Cos[3*x])/13 + (3*E^(2*x)*Sin[3*x])/13
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{Integral} = \frac{2}{13}e^{2x}\cos(3x) + \frac{3}{13}e^{2x}\sin(3x)$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{2x}(2\cos(3x) + 3\sin(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[3*x],x]
[Out] (E^(2*x)*(2*Cos[3*x] + 3*Sin[3*x]))/13
```

Maple [A]
time = 0.06, size = 22, normalized size = 0.81

method	result	size
parallelrisch	$\frac{e^{2x}(2\cos(3x)+3\sin(3x))}{13}$	20
default	$\frac{2\,e^{2x}\cos(3x)}{13}+\frac{3\,e^{2x}\sin(3x)}{13}$	22
risch	$\frac{e^{(2+3i)x}}{13}-\frac{3ie^{(2+3i)x}}{26}+\frac{e^{(2-3i)x}}{13}+\frac{3ie^{(2-3i)x}}{26}$	36
norman	$\frac{\frac{6\,e^{2x}\tan\left(\frac{3x}{2}\right)}{13}-\frac{2\,e^{2x}\left(\tan^2\left(\frac{3x}{2}\right)\right)}{13}+\frac{2\,e^{2x}}{13}}{1+\tan^2\left(\frac{3x}{2}\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
[Out] 2/13*exp(2*x)*cos(3*x)+3/13*exp(2*x)*sin(3*x)
```

Maxima [A]
time = 0.33, size = 19, normalized size = 0.70

$$\frac{1}{13}\left(2\,\cos\left(3\,x\right)+3\,\sin\left(3\,x\right)\right)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(3*x),x, algorithm="maxima")
[Out] 1/13*(2*cos(3*x) + 3*sin(3*x))*e^(2*x)
```

Fricas [A]
time = 0.58, size = 21, normalized size = 0.78

$$\frac{2}{13}\,\cos\left(3\,x\right)e^{(2\,x)}+\frac{3}{13}\,e^{(2\,x)}\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(3*x),x, algorithm="fricas")
[Out] 2/13*cos(3*x)*e^(2*x) + 3/13*e^(2*x)*sin(3*x)
```

Sympy [A]
time = 0.10, size = 26, normalized size = 0.96

$$\frac{3e^{2x}\sin\left(3x\right)}{13}+\frac{2e^{2x}\cos\left(3x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(3*x),x)
[Out] 3*exp(2*x)*sin(3*x)/13 + 2*exp(2*x)*cos(3*x)/13
```

Giac [**A**]
time = 0.45, size = 19, normalized size = 0.70

$$\frac{1}{13}\left(2 \cos\left(3 x\right)+3 \sin\left(3 x\right)\right)e^{\left(2 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(3*x),x, algorithm="giac")
[Out] 1/13*(2*cos(3*x) + 3*sin(3*x))*e^(2*x)
```

Mupad [**B**]
time = 0.03, size = 19, normalized size = 0.70

$$\frac{e^{2 x}\left(2 \cos\left(3 x\right)+3 \sin\left(3 x\right)\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*exp(2*x),x)
[Out] (exp(2*x)*(2*cos(3*x) + 3*sin(3*x)))/13
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 19, normalized size = 0.70

$$\frac{e^{2x}(13\cos\left(3x\right)-6\sin\left(3x\right))}{13}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(2*x)*cos(3*x),x)
[Out] 1/13*exp(2*x)*(13*cos(3*x)-6*sin(3*x))
```

3.175 $\int \cos^{\cos(x)}(x)(1+\log(\cos(x)))\sin(x)\,dx$

Optimal. Leaf size=7

$$-\cos^{\cos(x)}(x)$$

```
[Out] -cos(x)^cos(x)
```

Rubi [A]
time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4420, 6874, 2633}

$$-\cos^{\cos(x)}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^Cos[x]*(1 + Log[Cos[x]])*Sin[x],x]
```

```
[Out] -Cos[x]^Cos[x]
```

Rule 2633

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int x^x(1+\log(x))\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \left(x^x+x^x\log(x)\right)\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int x^x\,dx, x, \cos(x)\right)-\text{Subst}\left(\int x^x\log(x)\,dx, x, \cos(x)\right)\\
&= -\cos^{\cos(x)}(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 7, normalized size = 1.00

$$-\cos^{\cos(x)}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^Cos[x]*(1 + Log[Cos[x]])*Sin[x],x]
```

```
[Out] -Cos[x]^Cos[x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\cos^{1+\cos(x)}(x)\right)\tan(x)\left(1+\ln(\cos(x))\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(1+cos(x))*tan(x)*(1+ln(cos(x))),x)
```

```
[Out] int(cos(x)^(1+cos(x))*tan(x)*(1+ln(cos(x))),x)
```

Maxima [A]
time = 0.35, size = 7, normalized size = 1.00

$$-\cos(x)^{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(1+cos(x))*tan(x)*(1+log(cos(x))),x, algorithm="maxima")
```

```
[Out] -cos(x)^cos(x)
```

Fricas [A]
time = 0.61, size = 13, normalized size = 1.86

$$-\frac{\cos(x)^{\cos(x)+1}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(1+cos(x))*tan(x)*(1+log(cos(x))),x, algorithm="fricas")
```

```
[Out] -cos(x)^(cos(x) + 1)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (\log(\cos(x))+1)\cos^{\cos(x)+1}(x)\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**(1+cos(x))*tan(x)*(1+ln(cos(x))),x)
```

```
[Out] Integral((log(cos(x)) + 1)*cos(x)**(cos(x) + 1)*tan(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(1+cos(x))*tan(x)*(1+log(cos(x))),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^(cos(x) + 1)*(log(cos(x)) + 1)*tan(x), x)
```

Mupad [B]
time = 0.76, size = 7, normalized size = 1.00

$$-\cos(x)^{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(cos(x) + 1)*tan(x)*(log(cos(x)) + 1),x)
```

```
[Out] -cos(x)^cos(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)^(1+cos(x))*tan(x)*(1+ln(cos(x))),x)
```

```
[Out] not solved
```

3.176 $\int \frac{e^x}{2+e^x}\,dx$

Optimal. Leaf size=6

$$\log(2+e^x)$$

[Out] ln(2+exp(x))

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2278, 31}

$$\log(e^x+2)$$

Antiderivative was successfully verified.

[In] Int[E^x/(2 + E^x),x]

[Out] Log[2 + E^x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{2+x}\,dx, x, e^x\right)\\ &= \log(2+e^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$\log(2+e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(2 + E^x),x]

```
[Out] Log[2 + E^x]
```

Maple [A]
time = 0.02, size = 6, normalized size = 1.00

method	result	size
derivativedivides	$\ln(2 + e^x)$	6
default	$\ln(2 + e^x)$	6
norman	$\ln(2 + e^x)$	6
risch	$\ln(2 + e^x)$	6
parallelrisch	$\ln(2 + e^x)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(2+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(2+exp(x))
```

Maxima [A]
time = 0.32, size = 5, normalized size = 0.83

$$\log(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+exp(x)),x, algorithm="maxima")
```

```
[Out] log(e^x + 2)
```

Fricas [A]
time = 0.57, size = 5, normalized size = 0.83

$$\log(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+exp(x)),x, algorithm="fricas")
```

```
[Out] log(e^x + 2)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.83

$$\log(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+exp(x)),x)
```

```
[Out] log(exp(x) + 2)
```

Giac [A]
time = 0.46, size = 5, normalized size = 0.83

$$\log(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+exp(x)),x, algorithm="giac")
```

```
[Out] log(e^x + 2)
```

Mupad [B]
time = 0.10, size = 5, normalized size = 0.83

$$\ln(e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(x) + 2),x)
```

```
[Out] log(exp(x) + 2)
```

Chatgpt [A]
time = 1.00, size = 5, normalized size = 0.83

$$\ln(2 + e^x)$$

Antiderivative was successfully verified.

```
[In] int(exp(x)/(2+exp(x)),x)
```

```
[Out] ln(2+exp(x))
```

3.177 $\int \sin(2018x)\,dx$

Optimal. Leaf size=8

$$-\frac{\cos(2018x)}{2018}$$

[Out] -1/2018*cos(2018*x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2718}

$$-\frac{\cos(2018x)}{2018}$$

Antiderivative was successfully verified.

[In] Int[Sin[2018*x],x]

[Out] -1/2018*Cos[2018*x]

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{Integral} = -\frac{\cos(2018x)}{2018}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{\cos(2018x)}{2018}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[2018*x],x]

[Out] -1/2018*Cos[2018*x]

Maple [A]
time = 0.03, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\cos(2018x)}{2018}$	7
default	$-\frac{\cos(2018x)}{2018}$	7
risch	$-\frac{\cos(2018x)}{2018}$	7
parallelrisch	$-\frac{\cos(2018x)}{2018}-\frac{1}{2018}$	9
norman	$-\frac{1}{1009(1+\tan^2(1009x))}$	13
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2018x)}{\sqrt{\pi}}\right)}{2018}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2018*x),x,method=_RETURNVERBOSE)
[Out] -1/2018*cos(2018*x)
```

Maxima [A]
time = 0.37, size = 6, normalized size = 0.75

$$-\frac{1}{2018}\cos(2018\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2018*x),x, algorithm="maxima")
[Out] -1/2018*cos(2018*x)
```

Fricas [A]
time = 0.58, size = 6, normalized size = 0.75

$$-\frac{1}{2018}\cos(2018\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2018*x),x, algorithm="fricas")
[Out] -1/2018*cos(2018*x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.88

$$-\frac{\cos(2018x)}{2018}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2018*x),x)
```

```
[Out] -cos(2018*x)/2018
```

Giac [A]
time = 0.48, size = 6, normalized size = 0.75

$$-\frac{1}{2018}\cos(2018x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2018*x),x, algorithm="giac")
```

```
[Out] -1/2018*cos(2018*x)
```

Mupad [B]
time = 0.09, size = 6, normalized size = 0.75

$$-\frac{\cos(2018x)}{2018}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2018*x),x)
```

```
[Out] -cos(2018*x)/2018
```

Chatgpt [A]
time = 1.00, size = 6, normalized size = 0.75

$$-\frac{\cos(2018x)}{2018}$$

Antiderivative was successfully verified.

```
[In] int(sin(2018*x),x)
```

```
[Out] -1/2018*cos(2018*x)
```

3.178 $\int \frac{1}{\cot(x)+\tan(x)}\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

[Out] 1/2*sin(x)^2

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {267}

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Tan[x])^(-1),x]

[Out] -1/2*Cos[x]^2

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{x}{(1+x^2)^2}\,dx, x, \tan(x)\right)\\ &= -\frac{1}{2}\cos^2(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[x] + Tan[x])^(-1),x]

[Out] -1/2*Cos[x]^2

Maple [A]
time = 0.04, size = 7, normalized size = 0.88

method	result	size
default	$-\frac{(\cos^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
parallelrisch	$-\frac{\cos(2x)}{4}-\frac{1}{4}$	9
norman	$-\frac{1}{2(1+\tan^2(x))}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)+cot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(x)^2
```

Maxima [A]
time = 0.39, size = 10, normalized size = 1.25

$$-\frac{1}{2\left(\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)+cot(x)),x, algorithm="maxima")
```

```
[Out] -1/2/(tan(x)^2 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.58, size = 16, normalized size = 2.00

$$\frac{\tan\left(x\right)^{2}-1}{4\left(\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)+cot(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(tan(x)^2 - 1)/(tan(x)^2 + 1)
```

Sympy [A]
time = 0.29, size = 10, normalized size = 1.25

$$-\frac{1}{2\tan^{2}\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)+cot(x)),x)
```

```
[Out] -1/(2*tan(x)**2 + 2)
```

Giac [**A**]
time = 0.47, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)+cot(x)),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

Mupad [**B**]
time = 0.12, size = 6, normalized size = 0.75

$$\frac{\sin\left(x\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + tan(x)),x)
```

```
[Out] sin(x)^2/2
```

Chatgpt [**A**] valid for real x
time = 1.00, size = 6, normalized size = 0.75

$$-\frac{\cos\left(2x\right)}{4}$$

Antiderivative was successfully verified.

```
[In] int(1/(tan(x)+cot(x)),x)
```

```
[Out] -1/4*cos(2*x)
```

3.179 $\int \frac{x^5}{2+x^{12}} \, dx$

Optimal. Leaf size=19

$$\frac{\arctan\left(\frac{x^6}{\sqrt{2}}\right)}{6\sqrt{2}}$$

[Out] 1/12*arctan(1/2*x^6*2^(1/2))*2^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 209}

$$\frac{\arctan\left(\frac{x^6}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(2 + x^12),x]

[Out] ArcTan[x^6/Sqrt[2]]/(6*Sqrt[2])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{2+x^2} \, dx, x, x^6\right) \\ &= \frac{\arctan\left(\frac{x^6}{\sqrt{2}}\right)}{6\sqrt{2}} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{\arctan\left(\frac{x^6}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(2 + x^12),x]
[Out] ArcTan[x^6/Sqrt[2]]/(6*Sqrt[2])
```

Maple [**A**]
time = 0.08, size = 15, normalized size = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^{6}\sqrt{2}}{2}\right)\sqrt{2}}{12}$	15
meijerg	$\frac{\arctan\left(\frac{x^{6}\sqrt{2}}{2}\right)\sqrt{2}}{12}$	15
risch	$\frac{\arctan\left(\frac{x^{6}\sqrt{2}}{2}\right)\sqrt{2}}{12}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^12+2),x,method=_RETURNVERBOSE)
[Out] 1/12*arctan(1/2*x^6*2^(1/2))*2^(1/2)
```

Maxima [**A**]
time = 0.50, size = 14, normalized size = 0.74

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x^{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^12+2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*x^6)
```

Fricas [**A**]
time = 0.57, size = 14, normalized size = 0.74

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x^{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^12+2),x, algorithm="fricas")
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*x^6)
```

Sympy [**A**]
time = 0.04, size = 17, normalized size = 0.89

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^{6}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**12+2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x**6/2)/12
```

Giac [A]
time = 0.46, size = 14, normalized size = 0.74

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^12+2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*x^6)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.74

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^6}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^12 + 2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x^6)/2))/12
```

Chatgpt [F] Failed to verify
time = 1.00, size = 74, normalized size = 3.89

$$\frac{\arctan\left(\frac{x^6\sqrt{2}}{2}\right)}{3}-\frac{\ln\left(x^4-x^2\sqrt{2}+2\right)}{12}+\frac{\ln\left(x^4+x^2\sqrt{2}+2\right)}{12}-\frac{\ln\left(x^2-x\sqrt{2}+\sqrt{2}\right)}{24}+\frac{\ln\left(x^2+x\sqrt{2}+\sqrt{2}\right)}{24}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^5/(x^12+2),x)
```

```
[Out] 1/3*arctan(1/2*x^6*2^(1/2))-1/12*ln(x^4-x^2*2^(1/2)+2)+1/12*ln(x^4+x^2*2^(1
/2)+2)-1/24*ln(x^2-x*2^(1/2)+2^(1/2))+1/24*ln(x^2+x*2^(1/2)+2^(1/2))
```

3.180 $\int (\cos(x)\cosh(x) + \sin(x)\sinh(x))\,dx$

Optimal. Leaf size=5

$$\cosh(x)\sin(x)$$

[Out] cosh(x)*sin(x)

Rubi [F]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (\cos(x)\cosh(x) + \sin(x)\sinh(x))\,dx$$

Verification is not applicable to the result.

[In] Int[Cos[x]*Cosh[x] + Sin[x]*Sinh[x],x]

[Out] Defer[Int][Cos[x]*Cosh[x], x] + Defer[Int][Sin[x]*Sinh[x], x]

Rubi steps

$$\text{Integral} = \int \cos(x)\cosh(x)\,dx + \int \sin(x)\sinh(x)\,dx$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\cosh(x)\sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cosh[x] + Sin[x]*Sinh[x],x]

[Out] Cosh[x]*Sin[x]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.71, size = 16, normalized size = 3.20

method	result
default	$\frac{e^{x}\sin(x)}{2} + \frac{e^{-x}\sin(x)}{2}$
parts	$\frac{e^{x}\sin(x)}{2} + \frac{e^{-x}\sin(x)}{2}$
risch	$\frac{i e^{(1-i)x}}{4} + \frac{i e^{(-1-i)x}}{4} - \frac{i e^{(1+i)x}}{4} - \frac{i e^{(-1+i)x}}{4}$

meijerg	$\pi^{\frac{3}{2}}\left(\frac{e^x\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^x\sin(x)}{4\pi^{\frac{3}{2}}}-\frac{e^{-x}\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{-x}\sin(x)}{4\pi^{\frac{3}{2}}}\right)+\pi^{\frac{3}{2}}\left(-\frac{e^x\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^x\sin(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{-x}\cos(x)}{4\pi^{\frac{3}{2}}}+\frac{e^{-x}\sin(x)}{4\pi^{\frac{3}{2}}}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cosh(x)+sin(x)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(x)*sin(x)+1/2*exp(-x)*sin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(5) = 10$.
time = 0.38, size = 52, normalized size = 10.40

$$\frac{1}{4}\left(\left(e^{(2x)}-1\right)\cos(x)+\left(e^{(2x)}+1\right)\sin(x)\right)e^{(-x)}-\frac{1}{4}\left(\left(e^{(2x)}-1\right)\cos(x)-\left(e^{(2x)}+1\right)\sin(x)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/4*((e^(2*x) - 1)*cos(x) + (e^(2*x) + 1)*sin(x))*e^(-x) - 1/4*((e^(2*x) -
1)*cos(x) - (e^(2*x) + 1)*sin(x))*e^(-x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(5) = 10$.
time = 0.59, size = 34, normalized size = 6.80

$$\frac{2\cosh(x)\sin(x)\sinh(x)+\sin(x)\sinh(x)^2+\left(\cosh(x)^2+1\right)\sin(x)}{2\left(\cosh(x)+\sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)*sin(x)*sinh(x) + sin(x)*sinh(x)^2 + (cosh(x)^2 + 1)*sin(x))/
(cosh(x) + sinh(x))
```

Sympy [A]
time = 0.17, size = 5, normalized size = 1.00

$$\sin(x)\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x)
```

```
[Out] sin(x)*cosh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(5) = 10$.
time = 0.46, size = 45, normalized size = 9.00

$$\frac{1}{4}\left(\cos(x)+\sin(x)\right)e^{(-x)}-\frac{1}{4}\left(\cos(x)-\sin(x)\right)e^{(-x)}+\frac{1}{4}\left(\cos(x)+\sin(x)\right)e^{x}-\frac{1}{4}\left(\cos(x)-\sin(x)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cosh(x)+sin(x)*sinh(x),x, algorithm="giac")
```

```
[Out] 1/4*(cos(x) + sin(x))*e^(-x) - 1/4*(cos(x) - sin(x))*e^(-x) + 1/4*(cos(x) +
 sin(x))*e^x - 1/4*(cos(x) - sin(x))*e^x
```

Mupad [B]
time = 0.06, size = 5, normalized size = 1.00

$$\cosh(x)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cosh(x) + sin(x)*sinh(x),x)
```

```
[Out] cosh(x)*sin(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 2.60

$$\frac{\sinh\left(2x\right)}{4}+\frac{\cosh\left(2x\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)*cosh(x)+sin(x)*sinh(x),x)
```

```
[Out] 1/4*sinh(2*x)+1/2*cosh(2*x)
```

3.181 $\int \frac{e^x+\cos(x)}{e^x+\sin(x)}\,dx$

Optimal. Leaf size=7

$$\log\left(e^x+\sin(x)\right)$$

```
[Out] ln(exp(x)+sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6816}

$$\log\left(e^x+\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + Cos[x])/(E^x + Sin[x]),x]
```

```
[Out] Log[E^x + Sin[x]]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\text{Integral} = \log\left(e^x+\sin(x)\right)$$

Mathematica [A]
time = 0.03, size = 7, normalized size = 1.00

$$\log\left(e^x+\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + Cos[x])/(E^x + Sin[x]),x]
```

```
[Out] Log[E^x + Sin[x]]
```

Maple [A]
time = 0.17, size = 7, normalized size = 1.00

method	result	size

derivativedivides	$\ln(e^x + \sin(x))$	7
default	$\ln(e^x + \sin(x))$	7
risch	$-ix + \ln\left(e^{2ix} + 2ie^{(1+i)x} - 1\right)$	23
parallelrisch	$-\ln\left(\frac{1}{1+\cos(x)}\right) + \ln\left(\frac{e^x+\sin(x)}{1+\cos(x)}\right)$	24
norman	$-\ln\left(1 + \tan^2\left(\frac{x}{2}\right)\right) + \ln\left(e^x\left(\tan^2\left(\frac{x}{2}\right)\right) + e^x + 2\tan\left(\frac{x}{2}\right)\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+cos(x))/(exp(x)+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(x)+sin(x))
```

Maxima [A]
time = 0.32, size = 6, normalized size = 0.86

$$\log(e^x + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] log(e^x + sin(x))
```

Fricas [A]
time = 0.60, size = 6, normalized size = 0.86

$$\log(e^x + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] log(e^x + sin(x))
```

Sympy [A]
time = 0.06, size = 7, normalized size = 1.00

$$\log(e^x + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x)
```

```
[Out] log(exp(x) + sin(x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(6) = 12$.
time = 0.50, size = 77, normalized size = 11.00

$$\frac{1}{2}\log\left(\frac{4\left(e^{(2x)}\tan\left(\frac{1}{2}x\right)^4+4e^x\tan\left(\frac{1}{2}x\right)^3+2e^{(2x)}\tan\left(\frac{1}{2}x\right)^2+4e^x\tan\left(\frac{1}{2}x\right)+4\tan\left(\frac{1}{2}x\right)^2+e^{(2x)}\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+cos(x))/(exp(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(4*(e^(2*x)*tan(1/2*x)^4 + 4*e^x*tan(1/2*x)^3 + 2*e^(2*x)*tan(1/2*x)
^2 + 4*e^x*tan(1/2*x) + 4*tan(1/2*x)^2 + e^(2*x))/(tan(1/2*x)^4 + 2*tan(1/2
*x)^2 + 1))
```

Mupad [**B**]
time = 0.11, size = 6, normalized size = 0.86

$$\ln\left(\mathrm{e}^x+\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + exp(x))/(exp(x) + sin(x)),x)
```

```
[Out] log(exp(x) + sin(x))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 30, normalized size = 4.29

$$x+\ln\left(\mathrm{e}^x+\sin\left(x\right)\right)+\frac{\ln\left(\frac{1-\mathrm{e}^{2ix}}{1+\mathrm{e}^{2ix}}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((exp(x)+cos(x))/(exp(x)+sin(x)),x)
```

```
[Out] x+ln(exp(x)+sin(x))+1/2*ln((1-exp(2*I*x))/(1+exp(2*I*x)))
```

3.182 $\int \cos(x) \sin(\cos(\sin(x))) \sin(\sin(x)) \, dx$

Optimal. Leaf size=4

$$\cos(\cos(\sin(x)))$$

```
[Out] cos(cos(sin(x)))
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4419, 4420, 2718}

$$\cos(\cos(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[Cos[Sin[x]]]*Sin[Sin[x]],x]
```

```
[Out] Cos[Cos[Sin[x]]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \sin(x) \sin(\cos(x)) \, dx, x, \sin(x)\right) \\
&= -\text{Subst}\left(\int \sin(x) \, dx, x, \cos(\sin(x))\right) \\
&= \cos(\cos(\sin(x)))
\end{aligned}$$

Mathematica [A]
time = 6.25, size = 4, normalized size = 1.00

$$\cos(\cos(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[Cos[Sin[x]]]*Sin[Sin[x]],x]
[Out] Cos[Cos[Sin[x]]]
```

Maple [A]
time = 7.48, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$\cos(\cos(\sin(x)))$	5
default	$\cos(\cos(\sin(x)))$	5
risch	$\cos(\cos(\sin(x)))$	5
parallelrisch	$-1+\cos(\cos(\sin(x)))$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(cos(sin(x)))*sin(sin(x))*cos(x),x,method=_RETURNVERBOSE)
[Out] cos(cos(sin(x)))
```

Maxima [A]
time = 0.39, size = 4, normalized size = 1.00

$$\cos(\cos(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(cos(sin(x)))*sin(sin(x))*cos(x),x, algorithm="maxima")
[Out] cos(cos(sin(x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(4) = 8.
time = 0.62, size = 44, normalized size = 11.00

$$\cos\left(\frac{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2-1}{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(cos(sin(x)))*sin(sin(x))*cos(x),x, algorithm="fricas")
```

```
[Out] cos((tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))^2 - 1)/(tan(tan(1/2*x)/(tan(1/2*x)^
2 + 1))^2 + 1))
```

Sympy [A]
time = 1.11, size = 5, normalized size = 1.25

$$\cos(\cos(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(cos(sin(x)))*sin(sin(x))*cos(x),x)
[Out] cos(cos(sin(x)))
```

Giac [A]
time = 0.44, size = 4, normalized size = 1.00

$$\cos(\cos(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(cos(sin(x)))*sin(sin(x))*cos(x),x, algorithm="giac")
[Out] cos(cos(sin(x)))
```

Mupad [B]
time = 0.12, size = 4, normalized size = 1.00

$$\cos(\cos(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(sin(x))*sin(cos(sin(x)))*cos(x),x)
[Out] cos(cos(sin(x)))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(sin(cos(sin(x)))*sin(sin(x))*cos(x),x)
[Out] not solved
```

3.183 $\int \frac{1}{1+\sin(x)} dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{1+\sin(x)}$$

```
[Out] -cos(x)/(1+sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2727}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{Integral} = -\frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.
time = 0.01, size = 23, normalized size = 2.30

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.02, size = 11, normalized size = 1.10

method	result	size
default	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
parallelrisch	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
risch	$-\frac{2}{i+e^{ix}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(1+tan(1/2*x))
```

Maxima [A]
time = 0.36, size = 15, normalized size = 1.50

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

Fricas [A]
time = 0.57, size = 18, normalized size = 1.80

$$-\frac{\cos(x)-\sin(x)+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A]
time = 0.19, size = 8, normalized size = 0.80

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x)
```

[Out] -2/(tan(x/2) + 1)

Giac [A]
time = 0.46, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sin(x)),x, algorithm="giac")

[Out] -2/(tan(1/2*x) + 1)

Mupad [B]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x) + 1),x)

[Out] -2/(tan(x/2) + 1)

Chatgpt [A]
time = 1.00, size = 10, normalized size = 1.00

$$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

[In] int(1/(1+sin(x)),x)

[Out] -2/(1+tan(1/2*x))

3.184 $\int \frac{\cos(x)}{1-\cos(2x)} \, dx$

Optimal. Leaf size=6

$$-\frac{\csc(x)}{2}$$

[Out] -1/2*csc(x)

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4441, 12, 30}

$$-\frac{\csc(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(1 - Cos[2*x]),x]

[Out] -1/2*Csc[x]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{1}{2x^2} \, dx, x, \sin(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2} \, dx, x, \sin(x)\right) \\
&= -\frac{\csc(x)}{2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$-\frac{\csc(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(1 - Cos[2*x]),x]
```

```
[Out] -1/2*Csc[x]
```

Maple [A]
time = 0.13, size = 7, normalized size = 1.17

method	result	size
default	$-\frac{1}{2\sin(x)}$	7
risch	$-\frac{i e^{ix}}{e^{2ix}-1}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1-cos(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/sin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(4) = 8$.
time = 0.42, size = 42, normalized size = 7.00

$$-\frac{\cos(x)\sin(2x) - \cos(2x)\sin(x) + \sin(x)}{\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x)),x, algorithm="maxima")
```

```
[Out] -(cos(x)*sin(2*x) - cos(2*x)*sin(x) + sin(x))/(cos(2*x)^2 + sin(2*x)^2 - 2*
cos(2*x) + 1)
```

Fricas [A]
time = 0.59, size = 6, normalized size = 1.00

$$-\frac{1}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x)),x, algorithm="fricas")
```

```
[Out] -1/2/sin(x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos(x)}{\cos(2x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x)),x)
[Out] -Integral(cos(x)/(cos(2*x) - 1), x)
```

Giac [**A**]
time = 0.44, size = 6, normalized size = 1.00

$$-\frac{1}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x)),x, algorithm="giac")
[Out] -1/2/sin(x)
```

Mupad [**B**]
time = 0.09, size = 6, normalized size = 1.00

$$-\frac{1}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(cos(2*x) - 1),x)
[Out] -1/(2*sin(x))
```

Chatgpt [**A**] valid for real x
time = 1.00, size = 6, normalized size = 1.00

$$-\frac{1}{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In] int(cos(x)/(1-cos(2*x)),x)
[Out] -1/2/sin(x)
```

3.185 $\int e^x \left(\frac{1}{x}+\log(x)\right)\,dx$

Optimal. Leaf size=6

$$e^x \log(x)$$

```
[Out] exp(x)*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2326}

$$e^x \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*(x^(-1) + Log[x]),x]
[Out] E^x*Log[x]
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{Integral} = e^x \log(x)$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$e^x \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*(x^(-1) + Log[x]),x]
[Out] E^x*Log[x]
```

Maple [A]
time = 0.04, size = 6, normalized size = 1.00

method	result	size

norman	$e^x \ln(x)$	6
risch	$e^x \ln(x)$	6
parallelrisch	$e^x \ln(x)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1/x+ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)*ln(x)
```

Maxima [A]
time = 0.43, size = 5, normalized size = 0.83

$$e^x \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1/x+log(x)),x, algorithm="maxima")
```

```
[Out] e^x*log(x)
```

Fricas [A]
time = 0.58, size = 5, normalized size = 0.83

$$e^x \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1/x+log(x)),x, algorithm="fricas")
```

```
[Out] e^x*log(x)
```

Sympy [A]
time = 0.07, size = 5, normalized size = 0.83

$$e^x \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1/x+ln(x)),x)
```

```
[Out] exp(x)*log(x)
```

Giac [A]
time = 0.44, size = 5, normalized size = 0.83

$$e^x \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1/x+log(x)),x, algorithm="giac")
```

```
[Out] e^x*log(x)
```

Mupad [**B**]
time = 0.12, size = 5, normalized size = 0.83

$$e^x \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(log(x) + 1/x),x)
```

```
[Out] exp(x)*log(x)
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Antiderivative was successfully verified.

```
[In] int(exp(x)*(1/x+ln(x)),x)
```

```
[Out] not solved
```

3.186 $\int \tanh^2(x)\,dx$

Optimal. Leaf size=6

$$x - \tanh(x)$$

[Out] x-tanh(x)

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$x - \tanh(x)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2,x]

[Out] x - Tanh[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\tanh(x) + \int 1\,dx\\ &= x - \tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x - \tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^2,x]

[Out] x - Tanh[x]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(6) = 12$.
time = 0.02, size = 20, normalized size = 3.33

method	result	size
parallelrisch	$x - \tanh(x)$	7
risch	$x + \frac{2}{1+e^{2x}}$	13
derivativedivides	$-\tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(\tanh(x)+1)}{2}$	20
default	$-\tanh(x) - \frac{\ln(\tanh(x)-1)}{2} + \frac{\ln(\tanh(x)+1)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2,x,method=_RETURNVERBOSE)
[Out] -tanh(x)-1/2*ln(tanh(x)-1)+1/2*ln(tanh(x)+1)
```

Maxima [A]
time = 0.37, size = 12, normalized size = 2.00

$$x - \frac{2}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2,x, algorithm="maxima")
[Out] x - 2/(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.57, size = 16, normalized size = 2.67

$$\frac{(x+1)\cosh(x) - \sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2,x, algorithm="fricas")
[Out] ((x + 1)*cosh(x) - sinh(x))/cosh(x)
```

Sympy [A]
time = 0.06, size = 3, normalized size = 0.50

$$x - \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2,x)
```

[Out] x - tanh(x)

Giac [A]
time = 0.49, size = 12, normalized size = 2.00

$$x + \frac{2}{e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2,x, algorithm="giac")
[Out] x + 2/(e^(2*x) + 1)
```

Mupad [B]
time = 0.05, size = 6, normalized size = 1.00

$$x - \tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2,x)
[Out] x - tanh(x)
```

Chatgpt [A]
time = 1.00, size = 6, normalized size = 1.00

$$x - \tanh\left(x\right)$$

Antiderivative was successfully verified.

```
[In] int(tanh(x)^2,x)
[Out] x-tanh(x)
```

3.187 $\int \frac{-\sin^2(x)+\sin(2x)}{-\cos^2(x)+\cos(2x)}\,dx$

Optimal. Leaf size=7

$$x - 2\log(\sin(x))$$

```
[Out] x-2*ln(sin(x))
```

Rubi [A]
time = 0.06, antiderivative size = 12, normalized size of antiderivative = 1.71, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {815, 649, 209, 266}

$$x - 2\log(\tan(x)) - 2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Sin[x]^2 + Sin[2*x])/(-Cos[x]^2 + Cos[2*x]),x]
```

```
[Out] x - 2*Log[Cos[x]] - 2*Log[Tan[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{-2+x}{x(1+x^2)}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{2}{x}+\frac{1+2x}{1+x^2}\right)\,dx, x, \tan(x)\right)\\
&= -2\log(\tan(x)) + \text{Subst}\left(\int \frac{1+2x}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -2\log(\tan(x)) + 2\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(x)\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= x - 2\log(\cos(x)) - 2\log(\tan(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$x - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sin[x]^2 + Sin[2*x])/(-Cos[x]^2 + Cos[2*x]),x]
[Out] x - 2*Log[Sin[x]]
```

Maple [A]
time = 0.22, size = 8, normalized size = 1.14

method	result	size
default	$x - 2\ln(\sin(x))$	8
risch	$2ix - 2\ln(e^{2ix} - 1) + x$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(2*x)-sin(x)^2)/(cos(2*x)-cos(x)^2),x,method=_RETURNVERBOSE)
[Out] x-2*ln(sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(7) = 14$.
time = 0.39, size = 36, normalized size = 5.14

$$x - \log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) - \log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(2*x)-sin(x)^2)/(cos(2*x)-cos(x)^2),x, algorithm="maxima")
[Out] x - log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*c
os(x) + 1)
```

Fricas [A]
time = 0.62, size = 9, normalized size = 1.29

$$x - 2 \log\left(\frac{1}{2} \sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(2*x)-sin(x)^2)/(cos(2*x)-cos(x)^2),x, algorithm="fricas")
[Out] x - 2*log(1/2*sin(x))
```

Sympy [A]
time = 1.56, size = 8, normalized size = 1.14

$$x - \log\left(\cos^{2}\left(x\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(2*x)-sin(x)**2)/(cos(2*x)-cos(x)**2),x)
[Out] x - log(cos(x)**2 - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.
time = 0.44, size = 21, normalized size = 3.00

$$x + 2 \log\left(\tan\left(\frac{1}{2} x\right)^{2} + 1\right) - 2 \log\left(\left|\tan\left(\frac{1}{2} x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(2*x)-sin(x)^2)/(cos(2*x)-cos(x)^2),x, algorithm="giac")
[Out] x + 2*log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x)))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 3.71

$$-2 \ln\left(\tan(x)\right) + \ln\left(\tan(x) - \mathrm{i}\right) \left(1 - \frac{1}{2}\mathrm{i}\right) + \ln\left(\tan(x) + 1\mathrm{i}\right) \left(1 + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(2*x) - sin(x)^2)/(cos(2*x) - cos(x)^2),x)
[Out] log(tan(x) - 1i)*(1 - 1i/2) - 2*log(tan(x)) + log(tan(x) + 1i)*(1 + 1i/2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 28, normalized size = 4.00

$$-\frac{\ln\left(\frac{1+\cos(x)}{1-\cos(x)}\right)}{4} - \frac{\ln\left(1 - 2(\sin^{2}\left(x\right))\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((sin(2*x)-sin(x)^2)/(cos(2*x)-cos(x)^2),x)
[Out] -1/4*ln((1+cos(x))/(1-cos(x)))-1/2*ln(1-2*sin(x)^2)
```

3.188 $\int \frac{1}{x^{9/25}+x^{41/25}} dx$

Optimal. Leaf size=10

$$\frac{25}{16}\arctan\left(x^{16/25}\right)$$

```
[Out] 25/16*arctan(x^(16/25))
```

Rubi [A]

time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1607, 348, 281, 209}

$$\frac{25}{16}\arctan\left(x^{16/25}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(9/25) + x^(41/25))^(-1),x]
```

```
[Out] (25*ArcTan[x^(16/25)])/16
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{1}{x^{9/25}\left(1+x^{32/25}\right)}\,dx \\ &= 25\text{Subst}\left(\int \frac{x^{15}}{1+x^{32}}\,dx, x, \sqrt[25]{x}\right) \\ &= \frac{25}{16}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^{16/25}\right) \\ &= \frac{25}{16}\arctan\left(x^{16/25}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{25}{16}\arctan\left(x^{16/25}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(9/25) + x^(41/25))^(-1),x]
```

```
[Out] (25*ArcTan[x^(16/25)])/16
```

Maple [F(-1)]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(41/25)+x^(9/25)),x)
```

```
[Out] int(1/(x^(41/25)+x^(9/25)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(41/25)+x^(9/25)),x, algorithm="maxima")
```

```
[Out] 25/16*x^(16/25) - integrate(x^(23/25)/(x^(32/25) + 1), x)
```

Fricas [A]
time = 0.58, size = 6, normalized size = 0.60

$$\frac{25}{16}\arctan\left(x^{\frac{16}{25}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(41/25)+x^(9/25)),x, algorithm="fricas")
```

```
[Out] 25/16*arctan(x^(16/25))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(41/25)+x**(9/25)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 6, normalized size = 0.60

$$\frac{25}{16}\arctan\left(x^{\frac{16}{25}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(41/25)+x^(9/25)),x, algorithm="giac")
```

```
[Out] 25/16*arctan(x^(16/25))
```

Mupad [**B**]
time = 0.17, size = 6, normalized size = 0.60

$$\frac{25\,\mathrm{atan}\left(x^{16/25}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/25) + x^(41/25)),x)
```

```
[Out] (25*atan(x^(16/25)))/16
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 5, normalized size = 0.50

$$-\frac{25}{28x^{\frac{28}{25}}}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^(41/25)+x^(9/25)),x)
```

```
[Out] -25/28/x^(28/25)
```

3.189 $\int \frac{\cos(x)}{2-\cos^2(x)} \, dx$

Optimal. Leaf size=3

$$\arctan(\sin(x))$$

```
[Out] arctan(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3265, 209}

$$\arctan(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(2 - Cos[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sin(x)\right) \\ &= \arctan(\sin(x)) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(2 - Cos[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Maple [A]
time = 0.14, size = 4, normalized size = 1.33

method	result	size
default	$\arctan\left(\sin\left(x\right)\right)$	4
parallelrisch	$-\frac{i\left(-\ln\left(-\frac{2i\left(\sin\left(x\right)+i\right)}{1+\cos\left(x\right)}\right)+\ln\left(\frac{2+2i\sin\left(x\right)}{1+\cos\left(x\right)}\right)\right)}{2}$	37
risch	$\frac{i\ln\left({\mathrm e}^{2ix}-2\,{\mathrm e}^{ix}-1\right)}{2}-\frac{i\ln\left({\mathrm e}^{2ix}+2\,{\mathrm e}^{ix}-1\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(2-cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sin(x))
```

Maxima [A]
time = 0.46, size = 3, normalized size = 1.00

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2-cos(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(sin(x))
```

Fricas [A]
time = 0.60, size = 3, normalized size = 1.00

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2-cos(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(3) = 6$.
time = 10.36, size = 219, normalized size = 73.00

$$\frac{1607521\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857}+\frac{1136689\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857}-\frac{195025\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857}-\frac{275807\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{470832\sqrt{2}+665857}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2-cos(x)**2),x)
```

```
[Out] 1607521*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor(
(x/2 - pi/2)/pi))/(470832*sqrt(2) + 665857) + 1136689*sqrt(2)*sqrt(3 - 2*sq
rt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(47
0832*sqrt(2) + 665857) - 195025*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/
sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(470832*sqrt(2) + 665857)
 - 275807*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floo
r((x/2 - pi/2)/pi))/(470832*sqrt(2) + 665857)
```

Giac [**A**]
time = 0.50, size = 3, normalized size = 1.00

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2-cos(x)^2),x, algorithm="giac")
```

```
[Out] arctan(sin(x))
```

Mupad [**B**]
time = 0.07, size = 3, normalized size = 1.00

$$\mathrm{atan}(\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(cos(x)^2 - 2),x)
```

```
[Out] atan(sin(x))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 20, normalized size = 6.67

$$-\frac{\ln\left(\frac{\cos(x)-\sqrt{2}}{\cos(x)+\sqrt{2}}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)/(2-cos(x)^2),x)
```

```
[Out] -1/2*ln((cos(x)-2^(1/2))/(cos(x)+2^(1/2)))
```

3.190 $\int \frac{1}{\left(1+x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=11

$$\frac{x}{\sqrt{1+x^2}}$$

```
[Out] x/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {197}

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-3/2),x]
[Out] x/Sqrt[1 + x^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\text{Integral} = \frac{x}{\sqrt{1+x^2}}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$\frac{x}{\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-3/2),x]
[Out] x/Sqrt[1 + x^2]
```

Maple [A]
time = 0.06, size = 10, normalized size = 0.91

method	result	size
gosper	$\frac{x}{\sqrt{x^2+1}}$	10
default	$\frac{x}{\sqrt{x^2+1}}$	10
trager	$\frac{x}{\sqrt{x^2+1}}$	10
meijerg	$\frac{x}{\sqrt{x^2+1}}$	10
risch	$\frac{x}{\sqrt{x^2+1}}$	10
pseudoelliptic	$\frac{x}{\sqrt{x^2+1}}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/(x^2+1)^(1/2)
```

Maxima [A]
time = 0.42, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.
time = 0.56, size = 22, normalized size = 2.00

$$\frac{x^2+\sqrt{x^2+1}x+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (x^2 + sqrt(x^2 + 1)*x + 1)/(x^2 + 1)
```

Sympy [A]
time = 0.45, size = 8, normalized size = 0.73

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(3/2),x)
```

```
[Out] x/sqrt(x**2 + 1)
```

Giac [**A**]
time = 0.51, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] x/sqrt(x^2 + 1)
```

Mupad [**B**]
time = 0.03, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(3/2),x)
```

```
[Out] x/(x^2 + 1)^(1/2)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 10, normalized size = 0.91

$$\ln\left(x + \sqrt{x^2 + 1}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^2+1)^(3/2),x)
```

```
[Out] ln(x+(x^2+1)^(1/2))
```

3.191 $\int \frac{1}{\sqrt{x^{3/2}-x^2}}\,dx$

Optimal. Leaf size=20

$$4\arctan\left(\frac{x}{\sqrt{x^{3/2}-x^2}}\right)$$

```
[Out] 4*arctan(x/(x^(3/2)-x^2)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2033, 209}

$$4\arctan\left(\frac{x}{\sqrt{x^{3/2}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(3/2) - x^2],x]
```

```
[Out] 4*ArcTan[x/Sqrt[x^(3/2) - x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 4\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{x^{3/2}-x^2}}\right)\\ &= 4\arctan\left(\frac{x}{\sqrt{x^{3/2}-x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 20, normalized size = 1.00

$$4\arctan\left(\frac{x}{\sqrt{x^{3/2}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(3/2) - x^2],x]
```

```
[Out] 4*ArcTan[x/Sqrt[x^(3/2) - x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.
time = 0.08, size = 46, normalized size = 2.30

method	result	size
meijerg	$4 \arcsin \left(x^{\frac{1}{4}}\right)$	7
derivativedivides	$\frac{2\sqrt{x}\,\sqrt{-\sqrt{x}\,(\sqrt{x}-1)}\,\arcsin(2\sqrt{x}-1)}{\sqrt{x^{\frac{3}{2}}-x^{2}}}$	37
default	$\frac{2x(\sqrt{x}-1)\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{x-\sqrt{x}}\right)}{\sqrt{x^{\frac{3}{2}}-x^{2}}\,\sqrt{\sqrt{x}\,(\sqrt{x}-1)}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(x^(3/2)-x^2)^(1/2)*x*(x^(1/2)-1)/(x^(1/2)*(x^(1/2)-1))^(1/2)*ln(x^(1/2)-
1/2+(x-x^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^2 + x^(3/2)), x)
```

Fricas [A]
time = 0.58, size = 18, normalized size = 0.90

$$-4\,\arctan\left(\frac{\sqrt{-x^{2}+x^{\frac{3}{2}}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*arctan(sqrt(-x^2 + x^(3/2))/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{\frac{3}{2}} - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(3/2)-x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**(3/2) - x**2), x)
```

Giac [**A**]
time = 0.48, size = 15, normalized size = 0.75

$$\left(\pi + 2 \arcsin\left(2\sqrt{x} - 1\right)\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (pi + 2*arcsin(2*sqrt(x) - 1))*sgn(x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x^{3/2} - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2) - x^2)^(1/2),x)
```

```
[Out] int(1/(x^(3/2) - x^2)^(1/2), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 14, normalized size = 0.70

$$\frac{4\ln\left(\frac{3-2\sqrt{x}}{\sqrt{x}}\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^(3/2)-x^2)^(1/2),x)
```

```
[Out] 4/3*ln((3-2*x^(1/2))/x^(1/2))
```

3.192 $\int \frac{-1+x}{x+x^2 \log(x)} \, dx$

Optimal. Leaf size=7

$$\log \left(\frac{1}{x}+\log (x)\right)$$

[Out] ln(1/x+ln(x))

Rubi [A]
time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2641, 6816}

$$\log \left(\frac{1}{x}+\log (x)\right)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(x + x^2*Log[x]),x]

[Out] Log[x^(-1) + Log[x]]

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \int \frac{-1+x}{x^2 \left(\frac{1}{x}+\log (x)\right)} \, dx \\ &= \log \left(\frac{1}{x}+\log (x)\right) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.71

$$-\log (x)+\log (1+x \log (x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(x + x^2*Log[x]),x]
[Out] -Log[x] + Log[1 + x*Log[x]]
```

Maple [A]
time = 0.09, size = 13, normalized size = 1.86

method	result	size
risch	$\ln\left(\frac{1}{x}+\ln\left(x\right)\right)$	8
default	$-\ln\left(x\right)+\ln\left(x\ln\left(x\right)+1\right)$	13
norman	$-\ln\left(x\right)+\ln\left(x\ln\left(x\right)+1\right)$	13
parallelrisch	$-\ln\left(x\right)+\ln\left(x\ln\left(x\right)+1\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x+x^2*ln(x)),x,method=_RETURNVERBOSE)
[Out] -ln(x)+ln(x*ln(x)+1)
```

Maxima [A]
time = 0.36, size = 11, normalized size = 1.57

$$\log\left(\frac{x\log\left(x\right)+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+x^2*log(x)),x, algorithm="maxima")
[Out] log((x*log(x) + 1)/x)
```

Fricas [A]
time = 0.58, size = 11, normalized size = 1.57

$$\log\left(\frac{x\log\left(x\right)+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+x^2*log(x)),x, algorithm="fricas")
[Out] log((x*log(x) + 1)/x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 1.00

$$\log\left(\log\left(x\right)+\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+x**2*ln(x)),x)
```

```
[Out] log(log(x) + 1/x)
```

Giac [**A**]
time = 0.42, size = 12, normalized size = 1.71

$$\log\left(x\log\left(x\right)+1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-1)/(x+x^2*log(x)),x, algorithm="giac")
```

```
[Out] log(x*log(x) + 1) - log(x)
```

Mupad [**B**]
time = 0.18, size = 12, normalized size = 1.71

$$\ln\left(x\ \ln\left(x\right)+1\right)-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x + x^2*log(x)),x)
```

```
[Out] log(x*log(x) + 1) - log(x)
```

Chatgpt [**A**] valid for real x
time = 1.00, size = 12, normalized size = 1.71

$$-\ln\left(x\right)+\ln\left(1+x\ln\left(x\right)\right)$$

Antiderivative was successfully verified.

```
[In] int((x-1)/(x+x^2*ln(x)),x)
```

```
[Out] -ln(x)+ln(1+x*ln(x))
```

3.193 $\int \csc(x)\sec(x)\,dx$

Optimal. Leaf size=3

$$\log(\tan(x))$$

```
[Out] ln(tan(x))
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = $\{2700, 29\}$

$$\log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sec[x],x]
```

```
[Out] Log[Tan[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(x)\right)\\ &= \log(\tan(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6.
time = 0.01, size = 9, normalized size = 3.00

$$-\log(\cos(x)) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sec[x],x]
```

```
[Out] -Log[Cos[x]] + Log[Sin[x]]
```

Maple [A]
time = 0.06, size = 4, normalized size = 1.33

method	result	size
default	$\ln(\tan(x))$	4
risch	$\ln(e^{2ix}-1)-\ln(e^{2ix}+1)$	20
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	25
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sec(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.
time = 0.40, size = 17, normalized size = 5.67

$$-\frac{1}{2}\log\left(\sin(x)^2-1\right)+\frac{1}{2}\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sec(x),x, algorithm="maxima")
```

```
[Out] -1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(3) = 6$.
time = 0.59, size = 19, normalized size = 6.33

$$-\frac{1}{2}\log\left(\cos(x)^2\right)+\frac{1}{2}\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sec(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(cos(x)^2) + 1/2*log(-1/4*cos(x)^2 + 1/4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.
time = 0.05, size = 14, normalized size = 4.67

$$-\frac{\log\left(\sin^2(x)-1\right)}{2}+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sec(x),x)
```

```
[Out] -log(sin(x)**2 - 1)/2 + log(sin(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(3) = 6$.
time = 0.48, size = 16, normalized size = 5.33

$$-\frac{1}{2} \log \left(-\sin \left(x\right)^{2}+1\right)+\log \left(\left|\sin \left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sec(x),x, algorithm="giac")
```

```
[Out] -1/2*log(-sin(x)^2 + 1) + log(abs(sin(x)))
```

Mupad [B]
time = 0.10, size = 3, normalized size = 1.00

$$\ln\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*sin(x)),x)
```

```
[Out] log(tan(x))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 3, normalized size = 1.00

$$\frac{\pi}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(csc(x)*sec(x),x)
```

```
[Out] 1/2*Pi
```

3.194 $\int \tan(\cos(x))\,dx$

Optimal. Leaf size=6

$$\text{Int}(\tan(\cos(x)),x)$$

```
[Out] CannotIntegrate(tan(cos(x)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \tan(\cos(x))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Tan[Cos[x]],x]
[Out] Defer[Int][Tan[Cos[x]], x]
```

Rubi steps

$$\text{Integral} = \int \tan(\cos(x))\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \tan(\cos(x))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Tan[Cos[x]],x]
[Out] Integrate[Tan[Cos[x]], x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan\left(\cos\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(cos(x)),x)
[Out] int(tan(cos(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(cos(x)),x, algorithm="maxima")
[Out] integrate(tan(cos(x)), x)
```

Fricas [A]
time = 0.58, size = 5, normalized size = 0.83

$$\mathrm{integral}(\tan\left(\cos\left(x\right)\right),x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(cos(x)),x, algorithm="fricas")
[Out] integral(tan(cos(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int\tan\left(\cos\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(cos(x)),x)
[Out] Integral(tan(cos(x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(cos(x)),x, algorithm="giac")
[Out] integrate(tan(cos(x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.17

$$\int \tan(\cos\left(x\right))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(cos(x)),x)
```

```
[Out] int(tan(cos(x)), x)
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(tan(cos(x)),x)
```

```
[Out] not solved
```

3.195 $\int \frac{1+x}{x(x+\log(x))} \, dx$

Optimal. Leaf size=5

$$\log(x+\log(x))$$

[Out] ln(x+ln(x))

Rubi [A]
time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6816}

$$\log(x+\log(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(x*(x + Log[x])),x]

[Out] Log[x + Log[x]]

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\text{Integral} = \log(x+\log(x))$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\log(x+\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/(x*(x + Log[x])),x]

[Out] Log[x + Log[x]]

Maple [A]
time = 0.10, size = 6, normalized size = 1.20

method	result	size

default	$\ln(x+\ln(x))$	6
norman	$\ln(x+\ln(x))$	6
risch	$\ln(x+\ln(x))$	6
parallelrisch	$\ln(x+\ln(x))$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/x/(x+ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x+ln(x))
```

Maxima [A]
time = 0.38, size = 5, normalized size = 1.00

$$\log(x+\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] log(x + log(x))
```

Fricas [A]
time = 0.57, size = 5, normalized size = 1.00

$$\log(x+\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/x/(x+log(x)),x, algorithm="fricas")
```

```
[Out] log(x + log(x))
```

Sympy [A]
time = 0.04, size = 5, normalized size = 1.00

$$\log(x+\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/x/(x+ln(x)),x)
```

```
[Out] log(x + log(x))
```

Giac [A]
time = 0.51, size = 5, normalized size = 1.00

$$\log(x+\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/x/(x+log(x)),x, algorithm="giac")
```

```
[Out] log(x + log(x))
```

Mupad [B]
time = 0.11, size = 5, normalized size = 1.00

$$\ln(x + \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x*(x + log(x))),x)
```

```
[Out] log(x + log(x))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((x+1)/x/(x+ln(x)),x)
```

```
[Out] not solved
```

3.196 $\int \left(e^{-e^x+x}+e^{e^x+x}\right) dx$

Optimal. Leaf size=6

$$2\sinh\left(e^x\right)$$

[Out] 2*sinh(exp(x))

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 2.50, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2320, 2225}

$$e^{e^x}-e^{-e^x}$$

Antiderivative was successfully verified.

[In] Int[E^(-E^x + x) + E^(E^x + x),x]

[Out] -E^(-E^x) + E^E^x

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int e^{-e^x+x}\,dx+\int e^{e^x+x}\,dx\\
&= \text{Subst}\left(\int e^{-x}\,dx,x,e^x\right)+\text{Subst}\left(\int e^x\,dx,x,e^x\right)\\
&= -e^{-e^x}+e^{e^x}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.
time = 0.01, size = 15, normalized size = 2.50

$$-e^{-e^x}+e^{e^x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-E^x + x) + E^(E^x + x),x]
```

```
[Out] -E^(-E^x) + E^E^x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.04, size = 12, normalized size = 2.00

method	result	size
default	$-\mathrm{e}^{-\mathrm{e}^{x}}+\mathrm{e}^{\mathrm{e}^{x}}$	12
parts	$-\mathrm{e}^{-\mathrm{e}^{x}}+\mathrm{e}^{\mathrm{e}^{x}}$	12
risch	$\left(\mathrm{e}^{x+\mathrm{e}^{x}}-\mathrm{e}^{x-\mathrm{e}^{x}}\right)\mathrm{e}^{-x}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x+exp(x))+exp(x-exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/exp(exp(x))+exp(exp(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.37, size = 11, normalized size = 1.83

$$-e^{\left(-e^{x}\right)}+e^{\left(e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x))+exp(x-exp(x)),x, algorithm="maxima")
```

```
[Out] -e^(-e^x) + e^(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(5) = 10$.
time = 0.59, size = 27, normalized size = 4.50

$$-\left(e^{(2x)}-e^{\left(2x+2e^{x}\right)}\right)e^{\left(-2x-e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x))+exp(x-exp(x)),x, algorithm="fricas")
```

```
[Out] -(e^(2*x) - e^(2*x + 2*e^x))*e^(-2*x - e^x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x))+exp(x-exp(x)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.
time = 0.47, size = 11, normalized size = 1.83

$$-e^{(-e^x)} + e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x))+exp(x-exp(x)),x, algorithm="giac")
```

```
[Out] -e^(-e^x) + e^(e^x)
```

Mupad [**B**]
time = 0.12, size = 5, normalized size = 0.83

$$2\sinh(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x + exp(x)) + exp(x - exp(x)),x)
```

```
[Out] 2*sinh(exp(x))
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(exp(x+exp(x))+exp(x-exp(x)),x)
```

```
[Out] not solved
```

3.197 $\int \frac{1}{1-x^2} dx$

Optimal. Leaf size=2

$$\operatorname{arctanh}(x)$$

```
[Out] arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {212}

$$\operatorname{arctanh}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\text{Integral} = \operatorname{arctanh}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(2) = 4.
time = 0.00, size = 19, normalized size = 9.50

$$-\frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^(-1),x]
```

```
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

Maple [A]
time = 0.07, size = 3, normalized size = 1.50

method	result	size

default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
risch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14
parallelrisch	$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 0.35, size = 13, normalized size = 6.50

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 0.56, size = 13, normalized size = 6.50

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.
time = 0.03, size = 12, normalized size = 6.00

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.
time = 0.46, size = 15, normalized size = 7.50

$$\frac{1}{2}\log\left(|x+1|\right) - \frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.08, size = 2, normalized size = 1.00

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2 - 1),x)
[Out] atanh(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 5.00

$$\ln\left(\frac{x+\frac{1}{2}}{x-\frac{1}{2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(-x^2+1),x)
[Out] ln((x+1/2)/(x-1/2))
```

3.198 $\int 2^{\log(x)}\,dx$

Optimal. Leaf size=12

$$\frac{2^{\log(x)}x}{1+\log(2)}$$

[Out] 2^ln(x)*x/(1+ln(2))

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2306, 30}

$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Log[x],x]

[Out] x^(1 + Log[2])/(1 + Log[2])

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2306

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int x^{\log(2)}\,dx\\ &= \frac{x^{1+\log(2)}}{1+\log(2)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{2^{\log(x)}x}{1+\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[x],x]
[Out] (2^Log[x]*x)/(1 + Log[2])
```

Maple [A]
time = 0.03, size = 13, normalized size = 1.08

method	result	size
gosper	$\frac{2^{\ln(x)} x}{1+\ln(2)}$	13
risch	$\frac{x\, x^{\ln(2)}}{1+\ln(2)}$	13
parallelrisch	$\frac{2^{\ln(x)} x}{1+\ln(2)}$	13
norman	$\frac{x\, e^{\ln(x)\ln(2)}}{1+\ln(2)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(x),x,method=_RETURNVERBOSE)
[Out] 2^ln(x)*x/(1+ln(2))
```

Maxima [A]
time = 0.36, size = 24, normalized size = 2.00

$$\frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(x)}}{\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="maxima")
[Out] 2^((1/log(2) + 1)*log(x))/((1/log(2) + 1)*log(2))
```

Fricas [A]
time = 0.58, size = 14, normalized size = 1.17

$$\frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="fricas")
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

Sympy [A]
time = 0.10, size = 10, normalized size = 0.83

$$\frac{2^{\log(x)}x}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(x),x)
[Out] 2**log(x)*x/(log(2) + 1)
```

Giac [A]
time = 0.46, size = 14, normalized size = 1.17

$$\frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="giac")
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

Mupad [B]
time = 0.11, size = 13, normalized size = 1.08

$$\frac{x^{\ln(2)+1}}{\ln(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^log(x),x)
[Out] x^(log(2) + 1)/(log(2) + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 0.75

$$\frac{x^{\ln(2)}}{\ln(2)}$$

Warning: Unable to verify antiderivative.

```
[In] int(2^ln(x),x)
[Out] x^ln(2)/ln(2)
```

3.199 $\int (\cos(3x)+\sin(2x)(\cos(3x)-\sin(2019x)))\,dx$

Optimal. Leaf size=39

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)+\frac{1}{3}\sin(3x)-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}$$

```
[Out] 1/2*cos(x)-1/10*cos(5*x)+1/3*sin(3*x)-1/4034*sin(2017*x)+1/4042*sin(2021*x)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2717, 4486, 4369, 4367}

$$\frac{1}{3}\sin(3x)-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}+\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x] + Sin[2*x]*(Cos[3*x] - Sin[2019*x]),x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10 + Sin[3*x]/3 - Sin[2017*x]/4034 + Sin[2021*x]/4042
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int \cos(3x)\,dx + \int \sin(2x)(\cos(3x) - \sin(2019x))\,dx \\
&= \frac{1}{3}\sin(3x) + \int (\cos(3x)\sin(2x) - \sin(2x)\sin(2019x))\,dx \\
&= \frac{1}{3}\sin(3x) + \int \cos(3x)\sin(2x)\,dx - \int \sin(2x)\sin(2019x)\,dx \\
&= \frac{\cos(x)}{2} - \frac{1}{10}\cos(5x) + \frac{1}{3}\sin(3x) - \frac{\sin(2017x)}{4034} + \frac{\sin(2021x)}{4042}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{\cos(x)}{2} - \frac{1}{10}\cos(5x) + \frac{1}{3}\sin(3x) - \frac{\sin(2017x)}{4034} + \frac{\sin(2021x)}{4042}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x] + Sin[2*x]*(Cos[3*x] - Sin[2019*x]),x]
[Out] Cos[x]/2 - Cos[5*x]/10 + Sin[3*x]/3 - Sin[2017*x]/4034 + Sin[2021*x]/4042
```

Maple [A]
time = 1.14, size = 30, normalized size = 0.77

method	result	size
default	$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10} + \frac{\sin(3x)}{3} - \frac{\sin(2017x)}{4034} + \frac{\sin(2021x)}{4042}$	30
risch	$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10} + \frac{\sin(3x)}{3} - \frac{\sin(2017x)}{4034} + \frac{\sin(2021x)}{4042}$	30
parts	$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10} + \frac{\sin(3x)}{3} - \frac{\sin(2017x)}{4034} + \frac{\sin(2021x)}{4042}$	30
parallelrisch	$-\frac{2}{5} - \frac{\sin(2017x)}{4034} + \frac{\sin(2021x)}{4042} - \frac{\cos(5x)}{10} + \frac{\sin(3x)}{3} + \frac{\cos(x)}{2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)+sin(2*x)*(-sin(2019*x)+cos(3*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*cos(x)-1/10*cos(5*x)+1/3*sin(3*x)-1/4034*sin(2017*x)+1/4042*sin(2021*x)
```

Maxima [A]
time = 0.33, size = 29, normalized size = 0.74

$$-\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x) + \frac{1}{4042}\sin(2021x) - \frac{1}{4034}\sin(2017x) + \frac{1}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)+sin(2*x)*(-sin(2019*x)+cos(3*x)),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x) + 1/4042*sin(2021*x) - 1/4034*sin(2017*x) + 1/3
*sin(3*x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)+sin(2*x)*(-sin(2019*x)+cos(3*x)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 0.29, size = 60, normalized size = 1.54

$$\frac{3\sin(2x)\sin(3x)}{5}+\frac{2019\sin(2x)\cos(2019x)}{4076357}+\frac{\sin(3x)}{3}-\frac{2\sin(2019x)\cos(2x)}{4076357}+\frac{2\cos(2x)\cos(3x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)+sin(2*x)*(-sin(2019*x)+cos(3*x)),x)
```

```
[Out] 3*sin(2*x)*sin(3*x)/5 + 2019*sin(2*x)*cos(2019*x)/4076357 + sin(3*x)/3 - 2*
sin(2019*x)*cos(2*x)/4076357 + 2*cos(2*x)*cos(3*x)/5
```

Giac [A]
time = 0.47, size = 29, normalized size = 0.74

$$-\frac{1}{10}\cos(5x)+\frac{1}{2}\cos(x)+\frac{1}{4042}\sin(2021x)-\frac{1}{4034}\sin(2017x)+\frac{1}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)+sin(2*x)*(-sin(2019*x)+cos(3*x)),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x) + 1/4042*sin(2021*x) - 1/4034*sin(2017*x) + 1/3
*sin(3*x)
```

Mupad [B]
time = 0.80, size = 29, normalized size = 0.74

$$\frac{\sin(3x)}{3}-\frac{\cos(5x)}{10}-\frac{\sin(2017x)}{4034}+\frac{\sin(2021x)}{4042}+\frac{\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x) + sin(2*x)*(cos(3*x) - sin(2019*x)),x)
```

```
[Out] sin(3*x)/3 - cos(5*x)/10 - sin(2017*x)/4034 + sin(2021*x)/4042 + cos(x)/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 29, normalized size = 0.74

$$\frac{\sin(3x)}{3} - \frac{\sin(2017x)}{4} + \frac{\sin(2021x)}{4} + \frac{\sin(x)}{10} + \frac{\sin(5x)}{50}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(3*x)+sin(2*x)*(-sin(2019*x)+cos(3*x)),x)
```

```
[Out] 1/3*sin(3*x)-1/4*sin(2017*x)+1/4*sin(2021*x)+1/10*sin(x)+1/50*sin(5*x)
```

3.200 $\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$

Optimal. Leaf size=4

$$\sin(\sin(\sin(x)))$$

[Out] sin(sin(sin(x)))

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4419, 2717}

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]

[Out] Sin[Sin[Sin[x]]]

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \cos(x)\cos(\sin(x))\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \cos(x)\,dx, x, \sin(\sin(x))\right)\\
&= \sin(\sin(\sin(x)))
\end{aligned}$$

Mathematica [A]
time = 3.40, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
[Out] Sin[Sin[Sin[x]]]
```

Maple [A]
time = 9.08, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$\sin(\sin(\sin(x)))$	5
default	$\sin(\sin(\sin(x)))$	5
risch	$\sin(\sin(\sin(x)))$	5
parallelrisch	$\sin(\sin(\sin(x)))$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x,method=_RETURNVERBOSE)
[Out] sin(sin(sin(x)))
```

Maxima [A]
time = 0.33, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="maxima")
[Out] sin(sin(sin(x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(4) = 8$.
time = 0.61, size = 41, normalized size = 10.25

$$\sin\left(\frac{2\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)}{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="fricas")
[Out] sin(2*tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))/(tan(tan(1/2*x)/(tan(1/2*x)^2 + 1)
)^2 + 1))
```

Sympy [A]
time = 1.13, size = 5, normalized size = 1.25

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

```
[Out] sin(sin(sin(x)))
```

Giac [A]
time = 0.50, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="giac")
```

```
[Out] sin(sin(sin(x)))
```

Mupad [B]
time = 0.11, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(sin(x))*cos(sin(sin(x)))*cos(x),x)
```

```
[Out] sin(sin(sin(x)))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

```
[Out] not solved
```

3.201 $\int \frac{e^{-\frac{2019}{4x^2}}}{x^2}\,dx$

Optimal. Leaf size=24

$$-\sqrt{\frac{\pi}{2019}}\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)$$

```
[Out] -1/2019*2019^(1/2)*Pi^(1/2)*erf(1/2*2019^(1/2)/x)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2242, 2236}

$$-\sqrt{\frac{\pi}{2019}}\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(2019/(4*x^2))*x^2),x]
```

```
[Out] -(Sqrt[Pi/2019]*Erf[Sqrt[2019]/(2*x)])
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2242

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[1/(d*(m + 1)), Subst[Int[F^(a + b*x^2), x], x, (c + d
*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int e^{-\frac{2019x^2}{4}}\,dx, x, \frac{1}{x}\right)\\
&= -\sqrt{\frac{\pi}{2019}}\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\sqrt{\frac{\pi}{2019}}\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(2019/(4*x^2))*x^2),x]
```

```
[Out] -(Sqrt[Pi/2019]*Erf[Sqrt[2019]/(2*x)])
```

Maple [A]
time = 0.03, size = 18, normalized size = 0.75

method	result	size
derivativedivides	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18
default	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18
meijerg	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18
risch	$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2019/4/x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2019*2019^(1/2)*Pi^(1/2)*erf(1/2*2019^(1/2)/x)
```

Maxima [A]
time = 0.37, size = 29, normalized size = 1.21

$$-\frac{\sqrt{2019}\sqrt{\pi}\sqrt{x^2}\left(\operatorname{erf}\left(\frac{1}{2}\,\sqrt{2019}\sqrt{\frac{1}{x^2}}\right)-1\right)}{2019\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2019/4/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2019*sqrt(2019)*sqrt(pi)*sqrt(x^2)*(erf(1/2*sqrt(2019)*sqrt(x^(-2))) - 1
)/x
```

Fricas [A]
time = 0.60, size = 17, normalized size = 0.71

$$-\frac{1}{2019}\,\sqrt{2019}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{2019}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2019/4/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2019*sqrt(2019)*sqrt(pi)*erf(1/2*sqrt(2019)/x)
```

Sympy [A]
time = 0.45, size = 22, normalized size = 0.92

$$-\frac{\sqrt{2019}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{2019}}{2x}\right)}{2019}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2019/4/x**2)/x**2,x)
[Out] -sqrt(2019)*sqrt(pi)*erf(sqrt(2019)/(2*x))/2019
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2019/4/x^2)/x^2,x, algorithm="giac")
[Out] integrate(e^(-2019/4/x^2)/x^2, x)
```

Mupad [B]
time = 0.13, size = 17, normalized size = 0.71

$$-\frac{\sqrt{2019}\,\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{2019}}{2\,x}\right)}{2019}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2019/(4*x^2))/x^2,x)
[Out] -(2019^(1/2)*pi^(1/2)*erf(2019^(1/2)/(2*x)))/2019
```

Chatgpt [F] Failed to verify
time = 1.00, size = 8, normalized size = 0.33

$$\frac{2\,\mathrm{e}^{-\frac{2019}{4x^4}}}{2019}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(-2019/4/x^2)/x^2,x)
[Out] 2/2019*exp(-2019/4/x^4)
```

3.202 $\int \sin\left(\sqrt{x}\right) \, dx$

Optimal. Leaf size=22

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

```
[Out] -2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3442, 3377, 2717}

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int x\sin(x)\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\text{Subst}\left(\int \cos(x)\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
default	$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\,\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2*x^(1/2)*cos(x^(1/2))+2*sin(x^(1/2))
```

Maxima [A]
time = 0.36, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A]
time = 0.61, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A]
time = 0.10, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A]
time = 0.49, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B]
time = 0.00, size = 16, normalized size = 0.73

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(sin(x^(1/2)),x)
[Out] not solved
```

3.203 $\int \frac{\sqrt{x}}{1+x} \, dx$

Optimal. Leaf size=16

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

```
[Out] 2*x^(1/2)-2*arctan(x^(1/2))
```

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {52, 65, 209}

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x),x]
```

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 2\sqrt{x} - \int \frac{1}{\sqrt{x}(1+x)}\,dx \\ &= 2\sqrt{x} - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\ &= 2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x),x]
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Maple [A]
time = 0.07, size = 13, normalized size = 0.81

method	result	size
derivativedivides	$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$	13
default	$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$	13
meijerg	$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$	13
risch	$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$	13
trager	$2\sqrt{x} + \mathit{RootOf}\left(\_Z^2 + 1\right)\ln\left(-\frac{2\mathit{RootOf}\left(\_Z^2+1\right)\sqrt{x}-x+1}{x+1}\right)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x+1),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)-2*arctan(x^(1/2))
```

Maxima [A]
time = 0.46, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x+1),x, algorithm="maxima")
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Fricas [A]
time = 0.58, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x+1),x, algorithm="fricas")
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Sympy [A]
time = 0.06, size = 14, normalized size = 0.88

$$2\sqrt{x} - 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x+1),x)
[Out] 2*sqrt(x) - 2*atan(sqrt(x))
```

Giac [A]
time = 0.43, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x+1),x, algorithm="giac")
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Mupad [B]
time = 0.09, size = 12, normalized size = 0.75

$$2\sqrt{x} - 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + 1),x)
[Out] 2*x^(1/2) - 2*atan(x^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 22, normalized size = 1.38

$$\frac{\sqrt{x}\,\ln\left(\frac{x+1}{|x+1|}\right)}{4} + \frac{\sqrt{x}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(1/2)/(x+1),x)
[Out] 1/4*x^(1/2)*ln((x+1)/abs(x+1))+1/2*x^(1/2)
```

3.204 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4440, 2717}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)\,dx\\
&= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\
&= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A]
time = 0.66, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
parallelrisch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A]
time = 0.34, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x+\frac{1}{24}\,\sin\left(6\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A]
time = 0.63, size = 25, normalized size = 0.83

$$\frac{1}{12}\,\left(16\,\cos\left(x\right)^{5}-10\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44.
time = 1.05, size = 114, normalized size = 3.80

$$-\frac{x\sin\left(x\right)\sin\left(2x\right)\cos\left(3x\right)}{4}+\frac{x\sin\left(x\right)\sin\left(3x\right)\cos\left(2x\right)}{4}+\frac{x\sin\left(2x\right)\sin\left(3x\right)\cos\left(x\right)}{4}+\frac{x\cos\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{4}+\frac{\sin\left(x\right)\sin\left(2x\right)\sin\left(3x\right)}{6}+\frac{\sin\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{8}+\frac{5\sin\left(3x\right)\cos\left(x\right)\cos\left(2x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
6 + sin(x)*cos(2*x)*cos(3*x)/8 + 5*sin(3*x)*cos(x)*cos(2*x)/24
```

Giac [A]
time = 0.47, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Mupad [B]
time = 0.25, size = 22, normalized size = 0.73

$$\frac{x}{4} + \frac{\sin\left(2\,x\right)}{8} + \frac{\sin\left(4\,x\right)}{16} + \frac{\sin\left(6\,x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

Chatgpt [F] Failed to verify
time = 1.00, size = 38, normalized size = 1.27

$$-\frac{2\sin\left(x\right)\sin\left(2x\right)\sin\left(3x\right)}{23} - \frac{\cos\left(x\right)\sin\left(3x\right)\left(3\cos\left(2x\right)\sin\left(x\right) - 2\sin\left(2x\right)\cos\left(x\right)\right)}{23}$$

Warning: Unable to verify antiderivative.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -2/23*sin(x)*sin(2*x)*sin(3*x)-1/23*cos(x)*sin(3*x)*(3*cos(2*x)*sin(x)-2*si
n(2*x)*cos(x))
```

3.205 $\int e^{-x^{2n}}\,dx$

Optimal. Leaf size=23

$$-\frac{x\,\text{ExpIntegralE}\left(1-\frac{1}{2n},x^{2n}\right)}{2n}$$

[Out] -1/2*x*Ei(1-1/2/n,x^(2*n))/n

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.48, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2239}

$$-\frac{x\left(x^{2n}\right)^{-\frac{1}{2}/n}\Gamma\left(\frac{1}{2n},x^{2n}\right)}{2n}$$

Antiderivative was successfully verified.

[In] Int[E^(-x^(2*n)),x]

[Out] -1/2*(x*Gamma[1/(2*n), x^(2*n)])/(n*(x^(2*n))^(1/(2*n)))

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rubi steps

$$\text{Integral}=-\frac{x\left(x^{2n}\right)^{-\frac{1}{2}/n}\Gamma\left(\frac{1}{2n},x^{2n}\right)}{2n}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.48

$$-\frac{x\left(x^{2n}\right)^{-\frac{1}{2}/n}\Gamma\left(\frac{1}{2n},x^{2n}\right)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-x^(2*n)),x]

[Out] -1/2*(x*Gamma[1/(2*n), x^(2*n)])/(n*(x^(2*n))^(1/(2*n)))

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 185, normalized size = 8.04

method	result
meijerg	$\frac{\frac{4n^2 x^{-2n+1}\left(2n\,x^{2n}+2n+1\right)\left(x^{2n}\right)^{-\frac{2n+1}{4n}} \mathrm{e}^{-\frac{x^{2n}}{2}} M_{\frac{1}{2n}-\frac{2n+1}{4n},\frac{2n+1}{4n}+\frac{1}{2}}\left(x^{2n}\right)}{(2n+1)(4n+1)}+\frac{2n\,x^{-2n+1}(2n+1)\left(x^{2n}\right)^{-\frac{2n+1}{4n}} \mathrm{e}^{-\frac{x^{2n}}{2}} M_{\frac{1}{2n}-\frac{2n+1}{4n}+1,\frac{2n-}{4n}}}{4n+1}}{2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/n*(4*n^2*x^(-2*n+1)*(2*n*x^(2*n)+2*n+1)/(2*n+1)/(4*n+1)*(x^(2*n))^(-1/4
*(2*n+1)/n)*exp(-1/2*x^(2*n))*WhittakerM(1/2/n-1/4*(2*n+1)/n,1/4*(2*n+1)/n+
1/2,x^(2*n))+2*n*x^(-2*n+1)*(2*n+1)/(4*n+1)*(x^(2*n))^(-1/4*(2*n+1)/n)*exp(
-1/2*x^(2*n))*WhittakerM(1/2/n-1/4*(2*n+1)/n+1,1/4*(2*n+1)/n+1/2,x^(2*n)))
```

Maxima [A]
time = 0.39, size = 30, normalized size = 1.30

$$-\frac{x\Gamma\left(\frac{1}{2\,n},x^{2\,n}\right)}{2\,n\left(x^{2\,n}\right)^{\frac{1}{2\,n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x^(2*n)),x, algorithm="maxima")
```

```
[Out] -1/2*x*gamma(1/2/n, x^(2*n))/(n*(x^(2*n))^(1/2/n))
```

Fricas [F]
time = 0.59, size = 10, normalized size = 0.43

$$\mathrm{integral}\left(e^{\left(-x^{2\,n}\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(e^(-x^(2*n)), x)
```

Sympy [A]
time = 0.44, size = 29, normalized size = 1.26

$$\frac{\Gamma\left(\frac{1}{2n}\right)\gamma\left(\frac{1}{2n},x^{2n}\right)}{4n^2\Gamma\left(1+\frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x**(2*n)),x)
```

```
[Out] gamma(1/(2*n))*lowergamma(1/(2*n), x**(2*n))/(4*n**2*gamma(1 + 1/(2*n)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x^(2*n)),x, algorithm="giac")
[Out] integrate(e^(-x^(2*n)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int e^{-x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x^(2*n)),x)
[Out] int(exp(-x^(2*n)), x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(exp(-x^(2*n)),x)
[Out] not solved
```

3.206 $\int e\,dx$

Optimal. Leaf size=3

$$ex$$

```
[Out] exp(1)*x
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$ex$$

Antiderivative was successfully verified.

```
[In] Int[E,x]
[Out] E*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{Integral} = ex$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$ex$$

Antiderivative was successfully verified.

```
[In] Integrate[E,x]
[Out] E*x
```

Maple [A]
time = 0.01, size = 9, normalized size = 3.00

method	result	size
parallelrisch	$x^{\frac{1}{\ln(x)}}x$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/ln(x))*x
```

Maxima [A]
time = 0.34, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/log(x)),x, algorithm="maxima")
```

```
[Out] x*e
```

Fricas [A]
time = 0.54, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/log(x)),x, algorithm="fricas")
```

```
[Out] x*e
```

Sympy [A]
time = 0.00, size = 3, normalized size = 1.00

$$ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/ln(x)),x)
```

```
[Out] E*x
```

Giac [A]
time = 0.48, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/log(x)),x, algorithm="giac")
```

```
[Out] x*e
```

Mupad [**B**]
time = 0.01, size = 4, normalized size = 1.33

$$x\,\mathrm{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/log(x)),x)
```

```
[Out] x*exp(1)
```

Chatgpt [**A**] valid for real x
time = 1.00, size = 4, normalized size = 1.33

$$\mathrm{e}x$$

Antiderivative was successfully verified.

```
[In] int(x^(1/ln(x)),x)
```

```
[Out] exp(1)*x
```

3.207 $\int \frac{\sin(19x)+\sin(20x)}{\cos(19x)+\cos(20x)} \, dx$

Optimal. Leaf size=11

$$-\frac{2}{39}\log\left(\cos\left(\frac{39x}{2}\right)\right)$$

```
[Out] -2/39*ln(cos(39/2*x))
```

Rubi [**B**] Leaf count is larger than twice the leaf count of optimal. 264 vs. $2(11) = 22$.
time = 2.75, antiderivative size = 264, normalized size of antiderivative = 24.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = $\{4486, 2099, 1601, 12, 2125\}$

$$-\frac{1}{780}\log\left(-64\cos^6(x)+32\cos^5(x)+80\cos^4(x)-32\cos^3(x)-24\cos^2(x)+6\cos(x)+1\right)-\frac{1}{780}\log(1-2\cos(x))+\frac{19}{780}\log(\cos(x)+1)-\frac{1}{780}\log\left(4096\cos^{12}(x)+2048\cos^{11}(x)-12288\cos^{10}(x)-6144\cos^9(x)+13568\cos^8(x)+6784\cos^7(x)-6592\cos^6(x)-3296\cos^5(x)+1264\cos^4(x)+632\cos^3(x)-48\cos^2(x)-24\cos(x)+1\right)-\frac{1}{20}\log\left(524288\cos^{20}(x)+262144\cos^{19}(x)-2621440\cos^{18}(x)-1245184\cos^{17}(x)+5570560\cos^{16}(x)+2490368\cos^{15}(x)-6553600\cos^{14}(x)-2723840\cos^{13}(x)+4659200\cos^{12}(x)+1770496\cos^{11}(x)-2050048\cos^{10}(x)-695552\cos^9(x)+549120\cos^8(x)+160512\cos^7(x)-84480\cos^6(x)-20064\cos^5(x)+6600\cos^4(x)+1140\cos^3(x)-200\cos^2(x)-19\cos(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[19*x] + Sin[20*x])/(Cos[19*x] + Cos[20*x]),x]
```

```
[Out] -1/780*Log[1 - 2*Cos[x]] + (19*Log[1 + Cos[x]])/780 - Log[1 + 6*Cos[x] - 24
*Cos[x]^2 - 32*Cos[x]^3 + 80*Cos[x]^4 + 32*Cos[x]^5 - 64*Cos[x]^6]/780 - Lo
g[1 - 24*Cos[x] - 48*Cos[x]^2 + 632*Cos[x]^3 + 1264*Cos[x]^4 - 3296*Cos[x]^
5 - 6592*Cos[x]^6 + 6784*Cos[x]^7 + 13568*Cos[x]^8 - 6144*Cos[x]^9 - 12288*
Cos[x]^10 + 2048*Cos[x]^11 + 4096*Cos[x]^12]/780 - Log[1 - 19*Cos[x] - 200*
Cos[x]^2 + 1140*Cos[x]^3 + 6600*Cos[x]^4 - 20064*Cos[x]^5 - 84480*Cos[x]^6
+ 160512*Cos[x]^7 + 549120*Cos[x]^8 - 695552*Cos[x]^9 - 2050048*Cos[x]^10 +
 1770496*Cos[x]^11 + 4659200*Cos[x]^12 - 2723840*Cos[x]^13 - 6553600*Cos[x]
^14 + 2490368*Cos[x]^15 + 5570560*Cos[x]^16 - 1245184*Cos[x]^17 - 2621440*C
os[x]^18 + 262144*Cos[x]^19 + 524288*Cos[x]^20]/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2125

```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Dist[1/(n*Coeff[Qn,
x, n]), Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]
/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{\sin(19x)}{\cos(19x)+\cos(20x)}+\frac{\sin(20x)}{\cos(19x)+\cos(20x)}\right) dx \\
&= \int \frac{\sin(19x)}{\cos(19x)+\cos(20x)}\,dx+\int \frac{\sin(20x)}{\cos(19x)+\cos(20x)}\,dx \\
&= -\text{Subst}\left(\int \frac{4x(-5+330x^2-6336x^4+54912x^6-256256x^8+698880x^{10}-1146880x^{12}+1114112x^{14}-589824x^{16}+131072x^{18})}{1-19x-200x^2+1140x^3+6600x^4-20064x^5-84480x^6+160512x^7+549120x^8-695552x^9-2050048x^{10}+1770496x^{11}+4659200x^{12}-2723840x^{13}-6553600x^{14}+2490368x^{15}+5570560x^{16}-1245184x^{17}-2621440x^{18}+262144x^{19}+524288x^{20}}\,dx,x,\cos(x)\right)-\text{Subst}\left(\int \frac{-1+180x^2-5280x^4+59136x^6-329472x^8+1025024x^{10}-1863680x^{12}+1966080x^{14}-1114112x^{16}+262144x^{18}}{1-19x-200x^2+1140x^3+6600x^4-20064x^5-84480x^6+160512x^7+549120x^8-695552x^9-2050048x^{10}+1770496x^{11}+4659200x^{12}-2723840x^{13}-6553600x^{14}+2490368x^{15}+5570560x^{16}-1245184x^{17}-2621440x^{18}+262144x^{19}+524288x^{20}}\,dx,x,\cos(x)\right) \\
&= -\left(4\text{Subst}\left(\int \frac{x(-5+330x^2-6336x^4+54912x^6-256256x^8+698880x^{10}-1146880x^{12}+1114112x^{14}-589824x^{16}+131072x^{18})}{1-19x-200x^2+1140x^3+6600x^4-20064x^5-84480x^6+160512x^7+549120x^8-695552x^9-2050048x^{10}+1770496x^{11}+4659200x^{12}-2723840x^{13}-6553600x^{14}+2490368x^{15}+5570560x^{16}-1245184x^{17}-2621440x^{18}+262144x^{19}+524288x^{20}}\,dx,x,\cos(x)\right)\right)-\text{Subst}\left(\int \left(-\frac{19}{39(1+x)}+\frac{2}{39(-1+2x)}+\frac{2(-3+24x+48x^2-160x^3-80x^4+192x^5)}{39(-1-6x+24x^2+32x^3-80x^4-32x^5+64x^6)}+\frac{8(-3-12x+237x^2+632x^3-2060x^4-4944x^5+5936x^6+13568x^7-6912x^8-15360x^9+2816x^{10}+6144x^{11})}{39(1-24x-48x^2+632x^3+1264x^4-3296x^5-6592x^6+6784x^7+13568x^8-6144x^9-12288x^{10}+2048x^{11}+4096x^{12})}\right)dx,x,\cos(x)\right) \\
&= \text{Rest of rules removed due to large latex content}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 131 vs. 2(11) = 22.
time = 0.14, size = 131, normalized size = 11.91

$$-\frac{2}{39}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{2}{39}\log(1-2\cos(x)+2\cos(2x)-2\cos(3x)+2\cos(4x)-2\cos(5x)+2\cos(6x)-2\cos(7x)+2\cos(8x)-2\cos(9x)+2\cos(10x)-2\cos(11x)+2\cos(12x)-2\cos(13x)+2\cos(14x)-2\cos(15x)+2\cos(16x)-2\cos(17x)+2\cos(18x)-2\cos(19x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[19*x] + Sin[20*x])/(Cos[19*x] + Cos[20*x]),x]
```

```
[Out] (-2*Log[Cos[x/2]])/39 - (2*Log[1 - 2*Cos[x] + 2*Cos[2*x] - 2*Cos[3*x] + 2*C
os[4*x] - 2*Cos[5*x] + 2*Cos[6*x] - 2*Cos[7*x] + 2*Cos[8*x] - 2*Cos[9*x] +
2*Cos[10*x] - 2*Cos[11*x] + 2*Cos[12*x] - 2*Cos[13*x] + 2*Cos[14*x] - 2*Cos
[15*x] + 2*Cos[16*x] - 2*Cos[17*x] + 2*Cos[18*x] - 2*Cos[19*x]])/39
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 16, normalized size = 1.45

method	result	size
risch	$ix-\frac{2\ln(e^{39ix}+1)}{39}$	16
parallelrisch	$\ln\left(\left(\sec^2(10x)\right)^{\frac{1}{39}}\right)+\ln\left(\left(\sec^2\left(\frac{19x}{2}\right)\right)^{\frac{1}{39}}\right)+\ln\left(\frac{1}{\left(\tan\left(\frac{19x}{2}\right)\tan(10x)-1\right)^{\frac{2}{39}}}\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(20*x)+sin(19*x))/(cos(20*x)+cos(19*x)),x,method=_RETURNVERBOSE)
```

```
[Out] I*x-2/39*ln(exp(39*I*x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(7) = 14$.
time = 0.52, size = 2527, normalized size = 229.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(20*x)+sin(19*x))/(cos(20*x)+cos(19*x)),x, algorithm="maxima"
)
```

```
[Out] -1/39*log(2*(cos(23*x) - cos(21*x) - cos(20*x) + cos(18*x) + cos(17*x) - co
s(15*x) - cos(14*x) + cos(12*x) - cos(10*x) - cos(9*x) + cos(7*x) + cos(6*x
) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(24*x) + cos(24*x)^2 - 2*(cos(21*x
) + cos(20*x) - cos(18*x) - cos(17*x) + cos(15*x) + cos(14*x) - cos(12*x) +
 cos(10*x) + cos(9*x) - cos(7*x) - cos(6*x) + cos(4*x) + cos(3*x) - cos(x)
- 1)*cos(23*x) + cos(23*x)^2 + 2*(cos(20*x) - cos(18*x) - cos(17*x) + cos(1
5*x) + cos(14*x) - cos(12*x) + cos(10*x) + cos(9*x) - cos(7*x) - cos(6*x) +
 cos(4*x) + cos(3*x) - cos(x) - 1)*cos(21*x) + cos(21*x)^2 - 2*(cos(18*x) +
 cos(17*x) - cos(15*x) - cos(14*x) + cos(12*x) - cos(10*x) - cos(9*x) + cos
(7*x) + cos(6*x) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(20*x) + cos(20*x)^
2 + 2*(cos(17*x) - cos(15*x) - cos(14*x) + cos(12*x) - cos(10*x) - cos(9*x)
 + cos(7*x) + cos(6*x) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(18*x) + cos(
18*x)^2 - 2*(cos(15*x) + cos(14*x) - cos(12*x) + cos(10*x) + cos(9*x) - cos
(7*x) - cos(6*x) + cos(4*x) + cos(3*x) - cos(x) - 1)*cos(17*x) + cos(17*x)^
2 + 2*(cos(14*x) - cos(12*x) + cos(10*x) + cos(9*x) - cos(7*x) - cos(6*x) +
 cos(4*x) + cos(3*x) - cos(x) - 1)*cos(15*x) + cos(15*x)^2 - 2*(cos(12*x) -
 cos(10*x) - cos(9*x) + cos(7*x) + cos(6*x) - cos(4*x) - cos(3*x) + cos(x)
+ 1)*cos(14*x) + cos(14*x)^2 - 2*(cos(10*x) + cos(9*x) - cos(7*x) - cos(6*x
) + cos(4*x) + cos(3*x) - cos(x) - 1)*cos(12*x) + cos(12*x)^2 + 2*(cos(9*x)
 - cos(7*x) - cos(6*x) + cos(4*x) + cos(3*x) - cos(x) - 1)*cos(10*x) + cos(
10*x)^2 - 2*(cos(7*x) + cos(6*x) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(9*
x) + cos(9*x)^2 + 2*(cos(6*x) - cos(4*x) - cos(3*x) + cos(x) + 1)*cos(7*x)
+ cos(7*x)^2 - 2*(cos(4*x) + cos(3*x) - cos(x) - 1)*cos(6*x) + cos(6*x)^2 +
 2*(cos(3*x) - cos(x) - 1)*cos(4*x) + cos(4*x)^2 - 2*(cos(x) + 1)*cos(3*x)
+ cos(3*x)^2 + cos(x)^2 + 2*(sin(23*x) - sin(21*x) - sin(20*x) + sin(18*x)
+ sin(17*x) - sin(15*x) - sin(14*x) + sin(12*x) - sin(10*x) - sin(9*x) + si
n(7*x) + sin(6*x) - sin(4*x) - sin(3*x) + sin(x))*sin(24*x) + sin(24*x)^2 -
 2*(sin(21*x) + sin(20*x) - sin(18*x) - sin(17*x) + sin(15*x) + sin(14*x) -
 sin(12*x) + sin(10*x) + sin(9*x) - sin(7*x) - sin(6*x) + sin(4*x) + sin(3*
x) - sin(x))*sin(23*x) + sin(23*x)^2 + 2*(sin(20*x) - sin(18*x) - sin(17*x)
 + sin(15*x) + sin(14*x) - sin(12*x) + sin(10*x) + sin(9*x) - sin(7*x) - si
n(6*x) + sin(4*x) + sin(3*x) - sin(x))*sin(21*x) + sin(21*x)^2 - 2*(sin(18*
x) + sin(17*x) - sin(15*x) - sin(14*x) + sin(12*x) - sin(10*x) - sin(9*x) +
 sin(7*x) + sin(6*x) - sin(4*x) - sin(3*x) + sin(x))*sin(20*x) + sin(20*x)^
2 + 2*(sin(17*x) - sin(15*x) - sin(14*x) + sin(12*x) - sin(10*x) - sin(9*x)
```

```
 + sin(7*x) + sin(6*x) - sin(4*x) - sin(3*x) + sin(x))*sin(18*x) + sin(18*x
)^2 - 2*(sin(15*x) + sin(14*x) - sin(12*x) + sin(10*x) + sin(9*x) - sin(7*x
) - sin(6*x) + sin(4*x) + sin(3*x) - sin(x))*sin(17*x) + sin(17*x)^2 + 2*(s
in(14*x) - sin(12*x) + sin(10*x) + sin(9*x) - sin(7*x) - sin(6*x) + sin(4*x
) + sin(3*x) - sin(x))*sin(15*x) + sin(15*x)^2 - 2*(sin(12*x) - sin(10*x) -
 sin(9*x) + sin(7*x) + sin(6*x) - sin(4*x) - sin(3*x) + sin(x))*sin(14*x) +
 sin(14*x)^2 - 2*(sin(10*x) + sin(9*x) - sin(7*x) - sin(6*x) + sin(4*x) + s
in(3*x) - sin(x))*sin(12*x) + sin(12*x)^2 + 2*(sin(9*x) - sin(7*x) - sin(6*
x) + sin(4*x) + sin(3*x) - sin(x))*sin(10*x) + sin(10*x)^2 - 2*(sin(7*x) +
sin(6*x) - sin(4*x) - sin(3*x) + sin(x))*sin(9*x) + sin(9*x)^2 + 2*(sin(6*x
) - sin(4*x) - sin(3*x) + sin(x))*sin(7*x) + sin(7*x)^2 - 2*(sin(4*x) + sin
(3*x) - sin(x))*sin(6*x) + sin(6*x)^2 + 2*(sin(3*x) - sin(x))*sin(4*x) + si
n(4*x)^2 + sin(3*x)^2 - 2*sin(3*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/39
*log(-2*(cos(11*x) - cos(10*x) + cos(9*x) - cos(8*x) + cos(7*x) - cos(6*x)
+ cos(5*x) - cos(4*x) + cos(3*x) - cos(2*x) + cos(x) - 1)*cos(12*x) + cos(1
2*x)^2 - 2*(cos(10*x) - cos(9*x) + cos(8*x) - cos(7*x) + cos(6*x) - cos(5*x
) + cos(4*x) - cos(3*x) + cos(2*x) - cos(x) + 1)*cos(11*x) + cos(11*x)^2 -
2*(cos(9*x) - cos(8*x) + cos(7*x) - cos(6*x) + cos(5*x) - cos(4*x) + cos(3*
x) - cos(2*x) + cos(x) - 1)*cos(10*x) + cos(10*x)^2 - 2*(cos(8*x) - cos(7*x
) + cos(6*x) - cos(5*x) + cos(4*x) - cos(3*x) + cos(2*x) - cos(x) + 1)*cos(
9*x) + cos(9*x)^2 - 2*(cos(7*x) - cos(6*x) + cos(5*x) - cos(4*x) + cos(3*x)
 - cos(2*x) + cos(x) - 1)*cos(8*x) + cos(8*x)^2 - 2*(cos(6*x) - cos(5*x) +
cos(4*x) - cos(3*x) + cos(2*x) - cos(x) + 1)*cos(7*x) + cos(7*x)^2 - 2*(cos
(5*x) - cos(4*x) + cos(3*x) - cos(2*x) + cos(x) - 1)*cos(6*x) + cos(6*x)^2
- 2*(cos(4*x) - cos(3*x) + cos(2*x) - cos(x) + 1)*cos(5*x) + cos(5*x)^2 - 2
*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x) + cos(4*x)^2 - 2*(cos(2*x) - c
os(x) + 1)*cos(3*x) + cos(3*x)^2 - 2*(cos(x) - 1)*cos(2*x) + cos(2*x)^2 + c
os(x)^2 - 2*(sin(11*x) - sin(10*x) + sin(9*x) - sin(8*x) + sin(7*x) - sin(6
*x) + sin(5*x) - sin(4*x) + sin(3*x) - sin(2*x) + sin(x))*sin(12*x) + sin(1
2*x)^2 - 2*(sin(10*x) - sin(9*x) + sin(8*x) - sin(7*x) + sin(6*x) - sin(5*x
) + sin(4*x) - sin(3*x) + sin(2*x) - sin(x))*sin(11*x) + sin(11*x)^2 - 2*(s
in(9*x) - sin(8*x) + sin(7*x) - sin(6*x) + sin(...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(7) = 14$.

time = 0.87, size = 123, normalized size = 11.18

$$-\frac{1}{39}\log\left(137438953472\cos\left(x\right)^{39} - 1340029796352\cos\left(x\right)^{37} + 6030134083584\cos\left(x\right)^{35} - 16610786017280\cos\left(x\right)^{33} + 31323196489728\cos\left(x\right)^{31} - 42839077552128\cos\left(x\right)^{29} + 43920872439808\cos\left(x\right)^{27} - 34411219255296\cos\left(x\right)^{25} + 20813237452800\cos\left(x\right)^{23} - 9751387176960\cos\left(x\right)^{21} + 3530674667520\cos\left(x\right)^{19} - 980106117120\cos\left(x\right)^{17} + 205701283840\cos\left(x\right)^{15} - 31950643200\cos\left(x\right)^{13} + 3560214528\cos\left(x\right)^{11} - 271960832\cos\left(x\right)^{9} + 13302432\cos\left(x\right)^{7} - 373464\cos\left(x\right)^{5} + 4940\cos\left(x\right)^{3} - \frac{39}{2}\cos\left(x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(20*x)+sin(19*x))/(cos(20*x)+cos(19*x)),x, algorithm="fricas"
)
```

```
[Out] -1/39*log(137438953472*cos(x)^39 - 1340029796352*cos(x)^37 + 6030134083584*
cos(x)^35 - 16610786017280*cos(x)^33 + 31323196489728*cos(x)^31 - 428390775
```

```
52128*cos(x)^29 + 43920872439808*cos(x)^27 - 34411219255296*cos(x)^25 + 208
13237452800*cos(x)^23 - 9751387176960*cos(x)^21 + 3530674667520*cos(x)^19 -
 980106117120*cos(x)^17 + 205701283840*cos(x)^15 - 31950643200*cos(x)^13 +
3560214528*cos(x)^11 - 271960832*cos(x)^9 + 13302432*cos(x)^7 - 373464*cos(
x)^5 + 4940*cos(x)^3 - 39/2*cos(x) + 1/2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(20*x)+sin(19*x))/(cos(20*x)+cos(19*x)),x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(7) = 14$.
time = 0.46, size = 134, normalized size = 12.18

$$-\frac{1}{39}\log\left(\cos\left(x\right)+1\right)-\frac{2}{39}\log\left(\left|4096\cos\left(x\right)^{12}+2048\cos\left(x\right)^{11}-12288\cos\left(x\right)^{10}-6144\cos\left(x\right)^{9}+13568\cos\left(x\right)^{8}+6784\cos\left(x\right)^{7}-6592\cos\left(x\right)^{6}-3296\cos\left(x\right)^{5}+1264\cos\left(x\right)^{4}+632\cos\left(x\right)^{3}-48\cos\left(x\right)^{2}-24\cos\left(x\right)+1\right|\right)-\frac{2}{39}\log\left(\left|64\cos\left(x\right)^{6}-32\cos\left(x\right)^{5}-80\cos\left(x\right)^{4}+32\cos\left(x\right)^{3}+24\cos\left(x\right)^{2}-6\cos\left(x\right)-1\right|\right)-\frac{2}{39}\log\left(\left|2\cos\left(x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(20*x)+sin(19*x))/(cos(20*x)+cos(19*x)),x, algorithm="giac")
[Out] -1/39*log(cos(x) + 1) - 2/39*log(abs(4096*cos(x)^12 + 2048*cos(x)^11 - 1228
8*cos(x)^10 - 6144*cos(x)^9 + 13568*cos(x)^8 + 6784*cos(x)^7 - 6592*cos(x)^
6 - 3296*cos(x)^5 + 1264*cos(x)^4 + 632*cos(x)^3 - 48*cos(x)^2 - 24*cos(x)
+ 1)) - 2/39*log(abs(64*cos(x)^6 - 32*cos(x)^5 - 80*cos(x)^4 + 32*cos(x)^3
+ 24*cos(x)^2 - 6*cos(x) - 1)) - 2/39*log(abs(2*cos(x) - 1))
```

Mupad [B]
time = 1.43, size = 15, normalized size = 1.36

$$x\,1\mathrm{i}-\frac{2\,\ln\left(\mathrm{e}^{x\,39\mathrm{i}}+1\right)}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(19*x) + sin(20*x))/(cos(19*x) + cos(20*x)),x)
[Out] x*1i - (2*log(exp(x*39i) + 1))/39
```

Chatgpt [F] Failed to verify
time = 1.00, size = 7, normalized size = 0.64

$$-\frac{39\ln\left(\cos\left(\frac{39x}{2}\right)\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((sin(20*x)+sin(19*x))/(cos(20*x)+cos(19*x)),x)
[Out] -39/2*ln(cos(39/2*x))
```

3.208 $\int e^x(\cos^2(x)+\cos(x)\sin(x)-\sin^2(x))\,dx$

Optimal. Leaf size=8

$$e^x\cos(x)\sin(x)$$

```
[Out] exp(x)*cos(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = $\{2326\}$

$$e^x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*(Cos[x]^2 + Cos[x]*Sin[x] - Sin[x]^2),x]
```

```
[Out] E^x*Cos[x]*Sin[x]
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{Integral}=e^x\cos(x)\sin(x)$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.38

$$\frac{1}{2}e^x\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*(Cos[x]^2 + Cos[x]*Sin[x] - Sin[x]^2),x]
```

```
[Out] (E^x*Sin[2*x])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(7)=14$.
time = 0.15, size = 43, normalized size = 5.38

method	result	size

parallelrisch	$\frac{e^{x} \sin(2x)}{2}$	9
risch	$-\frac{i e^{(1+2 i) x}}{4}+\frac{i e^{(1-2 i) x}}{4}$	20
norman	$\frac{2 e^{x} \tan\left(\frac{x}{2}\right)-2 e^{x} \left(\tan^{3}\left(\frac{x}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	31
default	$\frac{(\cos(x)+2 \sin(x)) e^{x} \cos(x)}{5}+\frac{e^{x} (\sin(2x)-2 \cos(2x))}{10}-\frac{(\sin(x)-2 \cos(x)) e^{x} \sin(x)}{5}$	43
parts	$\frac{(\cos(x)+2 \sin(x)) e^{x} \cos(x)}{5}+\frac{e^{x} (\sin(2x)-2 \cos(2x))}{10}-\frac{(\sin(x)-2 \cos(x)) e^{x} \sin(x)}{5}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(cos(x)+2*sin(x))*exp(x)*cos(x)+1/10*exp(x)*(sin(2*x)-2*cos(2*x))-1/5*(
sin(x)-2*cos(x))*exp(x)*sin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(7) = 14$.
time = 0.32, size = 34, normalized size = 4.25

$$-\frac{1}{10}\left(2 \cos\left(2 x\right)-\sin\left(2 x\right)\right)e^{x}+\frac{1}{5} \cos\left(2 x\right) e^{x}+\frac{2}{5} e^{x} \sin\left(2 x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/10*(2*cos(2*x) - sin(2*x))*e^x + 1/5*cos(2*x)*e^x + 2/5*e^x*sin(2*x)
```

Fricas [A]
time = 0.59, size = 7, normalized size = 0.88

$$\cos\left(x\right) e^{x} \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x, algorithm="fricas")
```

```
[Out] cos(x)*e^x*sin(x)
```

Sympy [A]
time = 0.16, size = 8, normalized size = 1.00

$$e^{x} \sin\left(x\right) \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(cos(x)**2+cos(x)*sin(x)-sin(x)**2),x)
```

[Out] exp(x)*sin(x)*cos(x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(7) = 14$.
time = 0.46, size = 33, normalized size = 4.12

$$-\frac{1}{10}\left(2 \cos\left(2 x\right)-\sin\left(2 x\right)\right)e^{x}+\frac{1}{5}\left(\cos\left(2 x\right)+2 \sin\left(2 x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x, algorithm="giac")

[Out] -1/10*(2*cos(2*x) - sin(2*x))*e^x + 1/5*(cos(2*x) + 2*sin(2*x))*e^x

Mupad [B]
time = 0.12, size = 8, normalized size = 1.00

$$\frac{\sin\left(2 x\right) e^{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)*(cos(x)*sin(x) + cos(x)^2 - sin(x)^2),x)

[Out] (sin(2*x)*exp(x))/2

Chatgpt [F] Failed to verify
time = 1.00, size = 34, normalized size = 4.25

$$e^{x} \sin\left(x\right) \cos\left(x\right)-e^{x} \tan\left(x\right)+e^{x} \sin\left(x\right)+\frac{e^{x} (\tan^{2}\left(x\right))}{2}+\frac{e^{x} \ln\left(\cos\left(x\right)\right)}{2}$$

Warning: Unable to verify antiderivative.

[In] int(exp(x)*(cos(x)^2+cos(x)*sin(x)-sin(x)^2),x)

[Out] exp(x)*sin(x)*cos(x)-exp(x)*tan(x)+exp(x)*sin(x)+1/2*exp(x)*tan(x)^2+1/2*ex
p(x)*ln(cos(x))

3.209 $\int \csc\left(\frac{\pi}{4}+x\right)\sin(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{\sqrt{2}}-\frac{\log\left(\sin\left(\frac{\pi}{4}+x\right)\right)}{\sqrt{2}}$$

```
[Out] 1/2*2^(1/2)*x-1/2*ln(sin(x+1/4*Pi))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4678, 3556, 8}

$$\frac{x}{\sqrt{2}}-\frac{\log\left(\sin\left(x+\frac{\pi}{4}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[Pi/4 + x]*Sin[x],x]
```

```
[Out] x/Sqrt[2] - Log[Sin[Pi/4 + x]]/Sqrt[2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4678

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{\int 1\,dx}{\sqrt{2}}-\frac{\int \cot\left(\frac{\pi}{4}+x\right)\,dx}{\sqrt{2}}\\&=\frac{x}{\sqrt{2}}-\frac{\log\left(\sin\left(\frac{\pi}{4}+x\right)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 29, normalized size = 1.21

$$\left(-\frac{1}{4}-\frac{i}{4}\right)(-1)^{3/4}(2x-2\text{arctanh}(\cot(x))-\log(\cos(2x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[Pi/4 + x]*Sin[x],x]
[Out] (-1/4 - I/4)*(-1)^(3/4)*(2*x - 2*ArcTanh[Cot[x]] - Log[Cos[2*x]])
```

Maple [A]
time = 0.12, size = 27, normalized size = 1.12

method	result	size
default	$\sqrt{2}\left(-\frac{\ln(1+\tan(x))}{2}+\frac{\ln(1+\tan^2(x))}{4}+\frac{\arctan(\tan(x))}{2}\right)$	27
risch	$\frac{\sqrt{2}\,x}{2}+\frac{i\sqrt{2}\,x}{2}-\frac{\sqrt{2}\,\ln(e^{2ix}+i)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(x+1/4*Pi),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*(-1/2*ln(1+tan(x))+1/4*ln(1+tan(x)^2)+1/2*arctan(tan(x)))
```

Maxima [A]
time = 0.42, size = 33, normalized size = 1.38

$$\frac{1}{2}\sqrt{2}x-\frac{1}{4}\sqrt{2}\log\left(\cos(2x)^2+\sin(2x)^2+2\sin(2x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(x+1/4*pi),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*x - 1/4*sqrt(2)*log(cos(2*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1)
```

Fricas [A]
time = 0.64, size = 21, normalized size = 0.88

$$\frac{1}{2}\sqrt{2}x-\frac{1}{2}\sqrt{2}\log\left(\frac{1}{2}\sin\left(\frac{1}{4}\pi+x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(x+1/4*pi),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*x - 1/2*sqrt(2)*log(1/2*sin(1/4*pi + x))
```

Sympy [A]
time = 8.97, size = 22, normalized size = 0.92

$$\frac{\sqrt{2}x}{2} - \frac{\sqrt{2}\log\left(\sin\left(x\right) + \cos\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(x+1/4*pi),x)
```

```
[Out] sqrt(2)*x/2 - sqrt(2)*log(sin(x) + cos(x))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.
time = 0.48, size = 44, normalized size = 1.83

$$\frac{1}{8}\sqrt{2}(\pi + 4x) + \frac{1}{2}\sqrt{2}\log\left(\tan\left(\frac{1}{8}\pi + \frac{1}{2}x\right)^2 + 1\right) - \frac{1}{2}\sqrt{2}\log\left(\left|\tan\left(\frac{1}{8}\pi + \frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(x+1/4*pi),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(pi + 4*x) + 1/2*sqrt(2)*log(tan(1/8*pi + 1/2*x)^2 + 1) - 1/2*s
qrt(2)*log(abs(tan(1/8*pi + 1/2*x)))
```

Mupad [B]
time = 0.28, size = 46, normalized size = 1.92

$$x\,e^{\frac{\Pi\,1i}{4}} - \frac{e^{-\frac{\Pi\,1i}{2}}\,\ln\left(e^{\frac{\Pi\,1i}{2}}\,e^{x\,2i} - 1\right)\,\left(e^{\frac{\Pi\,1i}{4}}\,2i - e^{\frac{\Pi\,3i}{4}}\,2i\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(Pi/4 + x),x)
```

```
[Out] x*exp((Pi*1i)/4) - (exp(-(Pi*1i)/2)*log(exp((Pi*1i)/2)*exp(x*2i) - 1)*(exp(
(Pi*1i)/4)*2i - exp((Pi*3i)/4)*2i))/4
```

Chatgpt [F] Failed to verify
time = 1.00, size = 1, normalized size = 0.04

$$x$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)/sin(x+1/4*Pi),x)
```

```
[Out] x
```

3.210 $\int \frac{1}{\sqrt[3]{x}+x}\,dx$

Optimal. Leaf size=12

$$\frac{3}{2}\log\left(1+x^{2/3}\right)$$

```
[Out] 3/2*ln(1+x^(2/3))
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1607, 266}

$$\frac{3}{2}\log\left(x^{2/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(2/3)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{1}{\left(1+x^{2/3}\right)\sqrt[3]{x}}\,dx\\ &= \frac{3}{2}\log\left(1+x^{2/3}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{3}{2}\log\left(1+x^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(2/3)])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(8) = 16.
time = 0.08, size = 29, normalized size = 2.42

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{2}{3}}\right)}{2}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{2}{3}}\right)}{2}$	9
trager	$\frac{\ln\left(3x^{\frac{2}{3}}+3x^{\frac{4}{3}}+x^2+1\right)}{2}$	19
default	$-\frac{\ln\left(x^{\frac{4}{3}}-x^{\frac{2}{3}}+1\right)}{2}+\ln\left(1+x^{\frac{2}{3}}\right)+\frac{\ln(x^2+1)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(1/3)),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^(4/3)-x^(2/3)+1)+ln(1+x^(2/3))+1/2*ln(x^2+1)
```

Maxima [A]
time = 0.43, size = 8, normalized size = 0.67

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/3)),x, algorithm="maxima")
[Out] 3/2*log(x^(2/3) + 1)
```

Fricas [A]
time = 0.58, size = 8, normalized size = 0.67

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/3)),x, algorithm="fricas")
[Out] 3/2*log(x^(2/3) + 1)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{2}{3}}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(1/3)),x)
```

```
[Out] 3*log(x**(2/3) + 1)/2
```

Giac [A]
time = 0.47, size = 8, normalized size = 0.67

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*log(x^(2/3) + 1)
```

Mupad [B]
time = 0.19, size = 8, normalized size = 0.67

$$\frac{3\ln\left(x^{2/3}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^(1/3)),x)
```

```
[Out] (3*log(x^(2/3) + 1))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 8, normalized size = 0.67

$$3\ln\left(1+x^{\frac{1}{3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x+x^(1/3)),x)
```

```
[Out] 3*ln(1+x^(1/3))
```

3.211 $\int x^{1+x^2}(1+2\log(x))\,dx$

Optimal. Leaf size=5

$$x^{x^2}$$

[Out] x^(x^2)

Rubi [F]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^{1+x^2}(1+2\log(x))\,dx$$

Verification is not applicable to the result.

[In] Int[x^(1 + x^2)*(1 + 2*Log[x]),x]

[Out] Defer[Int][x^(1 + x^2), x] + 2*Log[x]*Defer[Int][x^(1 + x^2), x] - 2*Defer[Int][Defer[Int][x^(1 + x^2), x]/x, x]

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(x^{1+x^2}+2x^{1+x^2}\log(x)\right)\,dx\\ &= 2\int x^{1+x^2}\log(x)\,dx+\int x^{1+x^2}\,dx\\ &= -\left(2\int \frac{\int x^{1+x^2}\,dx}{x}\,dx\right)+(2\log(x))\int x^{1+x^2}\,dx+\int x^{1+x^2}\,dx\end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$x^{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1 + x^2)*(1 + 2*Log[x]),x]

[Out] x^x^2

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.02, size = 12, normalized size = 2.40

method	result	size
risch	$\frac{x^{x^2+1}}{x}$	12
parallelrisch	$\frac{x^{x^2+1}}{x}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(x^2+1)*(1+2*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x^(x^2+1)/x
```

Maxima [A]
time = 0.41, size = 5, normalized size = 1.00

$$x^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(x^2+1)*(1+2*log(x)),x, algorithm="maxima")
```

```
[Out] x^(x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.
time = 0.60, size = 11, normalized size = 2.20

$$\frac{x^{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(x^2+1)*(1+2*log(x)),x, algorithm="fricas")
```

```
[Out] x^(x^2 + 1)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.
time = 0.05, size = 7, normalized size = 1.40

$$\frac{x^{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(x**2+1)*(1+2*ln(x)),x)
```

```
[Out] x**(x**2 + 1)/x
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(x^2+1)*(1+2*log(x)),x, algorithm="giac")
```

```
[Out] integrate(x^(x^2 + 1)*(2*log(x) + 1), x)
```

Mupad [B]
time = 0.23, size = 5, normalized size = 1.00

$$x^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(x^2 + 1)*(2*log(x) + 1),x)
```

```
[Out] x^(x^2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 44, normalized size = 8.80

$$x^{x^2+2} + 2x^{x^2+1}\ln(x) - \frac{x^{x^2}}{x^2+1} - 2\ \mathrm{expIntegral}\left(\left(x^2+1\right)\ln(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(x^2+1)*(1+2*ln(x)),x)
```

```
[Out] x^(x^2+2)+2*x^(x^2+1)*ln(x)-1/(x^2+1)*x^(x^2)-2*Ei((x^2+1)*ln(x))
```

3.212 $\int \frac{-1+2x^3}{x(1+x^3)}\,dx$

Optimal. Leaf size=11

$$-\log(x)+\log\left(1+x^3\right)$$

[Out] -ln(x)+ln(x^3+1)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {457, 78}

$$\log\left(x^3+1\right)-\log(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x^3)/(x*(1 + x^3)),x]

[Out] -Log[x] + Log[1 + x^3]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{-1+2x}{x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{3}{1+x}\right)\,dx, x, x^3\right)\\
&= -\log(x)+\log\left(1+x^3\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\log(x) + \log\left(1 + x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^3)/(x*(1 + x^3)),x]
[Out] -Log[x] + Log[1 + x^3]
```

Maple [A]
time = 0.07, size = 19, normalized size = 1.73

method	result	size
meijerg	$-\ln(x) + \ln(x^3 + 1)$	12
risch	$-\ln(x) + \ln(x^3 + 1)$	12
default	$\ln(x+1) - \ln(x) + \ln(x^2 - x + 1)$	19
norman	$\ln(x+1) - \ln(x) + \ln(x^2 - x + 1)$	19
parallelrisch	$\ln(x+1) - \ln(x) + \ln(x^2 - x + 1)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-1)/x/(x^3+1),x,method=_RETURNVERBOSE)
[Out] ln(x+1)-ln(x)+ln(x^2-x+1)
```

Maxima [A]
time = 0.35, size = 13, normalized size = 1.18

$$\log\left(x^3 + 1\right) - \frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/x/(x^3+1),x, algorithm="maxima")
[Out] log(x^3 + 1) - 1/3*log(x^3)
```

Fricas [A]
time = 0.59, size = 11, normalized size = 1.00

$$\log\left(x^3 + 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/x/(x^3+1),x, algorithm="fricas")
[Out] log(x^3 + 1) - log(x)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$-\log(x) + \log(x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-1)/x/(x**3+1),x)
[Out] -log(x) + log(x**3 + 1)
```

Giac [A]
time = 0.46, size = 13, normalized size = 1.18

$$\log(|x^3 + 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-1)/x/(x^3+1),x, algorithm="giac")
[Out] log(abs(x^3 + 1)) - log(abs(x))
```

Mupad [B]
time = 0.12, size = 11, normalized size = 1.00

$$\ln(x^3 + 1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - 1)/(x*(x^3 + 1)),x)
[Out] log(x^3 + 1) - log(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 10, normalized size = 0.91

$$\ln\left(\frac{x^3 + 1}{x^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((2*x^3-1)/x/(x^3+1),x)
[Out] ln((x^3+1)/x^3)
```

3.213 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=2

$$\operatorname{arcsinh}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\operatorname{arcsinh}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\text{Integral} = \operatorname{arcsinh}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(2) = 4.
time = 0.01, size = 16, normalized size = 8.00

$$-\log\left(-x+\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] -Log[-x + Sqrt[1 + x^2]]
```

Maple [A]
time = 0.09, size = 3, normalized size = 1.50

method	result	size

default	$\text{arcsinh}\,(x)$	3
meijerg	$\text{arcsinh}\,(x)$	3
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{x^2+1}}{x}\right)$	13
trager	$-\ln\left(x-\sqrt{x^2+1}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)
```

Maxima [A]
time = 0.46, size = 2, normalized size = 1.00

$$\text{arsinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(2) = 4.
time = 0.57, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A]
time = 0.05, size = 2, normalized size = 1.00

$$\text{asinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(2) = 4.
time = 0.45, size = 25, normalized size = 12.50

$$\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [**B**]
time = 0.02, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
```

```
[Out] asinh(x)
```

Chatgpt [**A**]
time = 1.00, size = 10, normalized size = 5.00

$$\ln\left(x+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] int(1/(x^2+1)^(1/2),x)
```

```
[Out] ln(x+(x^2+1)^(1/2))
```

3.214 $\int \frac{\log(2x)}{x\log(x)}\,dx$

Optimal. Leaf size=9

$$\log(x) + \log(2)\log(\log(x))$$

[Out] ln(x)+ln(2)*ln(ln(x))

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 2.00, number of steps used = 2, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2339, 29, 2413, 2601}

$$-\log(\log(x))\log(x) + \log(x) + \log(2x)\log(\log(x))$$

Antiderivative was successfully verified.

[In] Int[Log[2*x]/(x*Log[x]),x]

[Out] Log[x] - Log[x]*Log[Log[x]] + Log[2*x]*Log[Log[x]]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2413

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.))*((g_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(g*x)^m*(a +
b*Log[c*x^n])^p, x]}, Dist[d + e*Log[f*x^r], u, x] - Dist[e*r, Int[Simplify
Integrand[u/x, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, r}, x] &
&  !(EqQ[p, 1] && EqQ[a, 0] && NeQ[d, 0])
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\text{Integral} = \log(2x)\log(\log(x)) - \int \frac{\log(\log(x))}{x}\,dx$$
$$= \log(x) - \log(x)\log(\log(x)) + \log(2x)\log(\log(x))$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\log(x) + \log(2)\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2*x]/(x*Log[x]),x]
[Out] Log[x] + Log[2]*Log[Log[x]]
```

Maple [A]
time = 0.03, size = 10, normalized size = 1.11

method	result	size
default	$\ln(x)+\ln(2)\ln(\ln(x))$	10
risch	$\ln(x)+\ln(2)\ln(\ln(x))$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x)/x/ln(x),x,method=_RETURNVERBOSE)
[Out] ln(x)+ln(2)*ln(ln(x))
```

Maxima [A]
time = 0.34, size = 18, normalized size = 2.00

$$\log(2x)\log(\log(x)) - \log(x)\log(\log(x)) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x)/x/log(x),x, algorithm="maxima")
[Out] log(2*x)*log(log(x)) - log(x)*log(log(x)) + log(x)
```

Fricas [A]
time = 0.59, size = 9, normalized size = 1.00

$$\log(2)\log(\log(x)) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x)/x/log(x),x, algorithm="fricas")
```

```
[Out] log(2)*log(log(x)) + log(x)
```

Sympy [**A**]
time = 0.04, size = 10, normalized size = 1.11

$$\log(x) + \log(2)\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x)/x/ln(x),x)
[Out] log(x) + log(2)*log(log(x))
```

Giac [**A**]
time = 0.46, size = 10, normalized size = 1.11

$$\log(2)\log(|\log(x)|) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x)/x/log(x),x, algorithm="giac")
[Out] log(2)*log(abs(log(x))) + log(x)
```

Mupad [**B**]
time = 0.14, size = 9, normalized size = 1.00

$$\ln(x) + \ln(\ln(x))\ \ln(2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(2*x)/(x*log(x)),x)
[Out] log(x) + log(log(x))*log(2)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 9, normalized size = 1.00

$$\text{hyperbolicCosineIntegral}(2x) - \text{hyperbolicCosineIntegral}(x)$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(2*x)/x/ln(x),x)
[Out] Li(2*x)-Li(x)
```

3.215 $\int \frac{1}{1+e^x} \, dx$

Optimal. Leaf size=10

$$x - \log(1 + e^x)$$

[Out] x-ln(1+exp(x))

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2320, 36, 29, 31}

$$x - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 + E^x)^(-1),x]

[Out] x - Log[1 + E^x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right)\\ &= x - \log\left(1+e^x\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-2\text{arctanh}(1+2e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)^(-1),x]
```

```
[Out] -2*ArcTanh[1 + 2*E^x]
```

Maple [A]
time = 0.02, size = 12, normalized size = 1.20

method	result	size
norman	$x-\ln\left(1+e^x\right)$	10
risch	$x-\ln\left(1+e^x\right)$	10
parallelrisch	$x-\ln\left(1+e^x\right)$	10
derivativedivides	$\ln\left(e^x\right)-\ln\left(1+e^x\right)$	12
default	$\ln\left(e^x\right)-\ln\left(1+e^x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(exp(x))-ln(1+exp(x))
```

Maxima [A]
time = 0.35, size = 9, normalized size = 0.90

$$x-\log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] x - log(e^x + 1)
```

Fricas [A]
time = 0.58, size = 9, normalized size = 0.90

$$x - \log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="fricas")
[Out] x - log(e^x + 1)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.70

$$x - \log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x)
[Out] x - log(exp(x) + 1)
```

Giac [A]
time = 0.45, size = 9, normalized size = 0.90

$$x - \log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="giac")
[Out] x - log(e^x + 1)
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.90

$$x - \ln\left(\mathrm{e}^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x) + 1),x)
[Out] x - log(exp(x) + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 0.90

$$\ln\left(\mathrm{e}^{x} + 1\right) - x$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(exp(x)+1),x)
[Out] ln(exp(x)+1)-x
```

3.216 $\int \frac{\log(x)\log(\log(x))}{x}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{4}\log^2(x)+\frac{1}{2}\log^2(x)\log(\log(x))$$

```
[Out] -1/4*ln(x)^2+1/2*ln(x)^2*ln(ln(x))
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2601, 2341}

$$\frac{1}{2}\log^2(x)\log(\log(x))-\frac{\log^2(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]*Log[Log[x]])/x,x]
```

```
[Out] -1/4*Log[x]^2 + (Log[x]^2*Log[Log[x]])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int x\log(x)\,dx, x, \log(x)\right)\\ &= -\frac{1}{4}\log^2(x)+\frac{1}{2}\log^2(x)\log(\log(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{4}\log^2(x)+\frac{1}{2}\log^2(x)\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]*Log[Log[x]])/x,x]
[Out] -1/4*Log[x]^2 + (Log[x]^2*Log[Log[x]])/2
```

Maple [A]
time = 0.03, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17
default	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17
norman	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17
risch	$-\frac{\ln(x)^2}{4}+\frac{\ln(x)^2\ln(\ln(x))}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*ln(ln(x))/x,x,method=_RETURNVERBOSE)
[Out] -1/4*ln(x)^2+1/2*ln(x)^2*ln(ln(x))
```

Maxima [A]
time = 0.34, size = 16, normalized size = 0.80

$$\frac{1}{2}\log\left(x\right)^2\log\left(\log\left(x\right)\right)-\frac{1}{4}\log\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(log(x))/x,x, algorithm="maxima")
[Out] 1/2*log(x)^2*log(log(x)) - 1/4*log(x)^2
```

Fricas [A]
time = 0.57, size = 16, normalized size = 0.80

$$\frac{1}{2}\log\left(x\right)^2\log\left(\log\left(x\right)\right)-\frac{1}{4}\log\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(log(x))/x,x, algorithm="fricas")
[Out] 1/2*log(x)^2*log(log(x)) - 1/4*log(x)^2
```

Sympy [A]
time = 0.09, size = 17, normalized size = 0.85

$$\frac{\log\left(x\right)^2\log\left(\log\left(x\right)\right)}{2}-\frac{\log\left(x\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*ln(ln(x))/x,x)
```

```
[Out] log(x)**2*log(log(x))/2 - log(x)**2/4
```

Giac [A]
time = 0.47, size = 16, normalized size = 0.80

$$\frac{1}{2}\log\left(x\right)^{2}\log\left(\log\left(x\right)\right)-\frac{1}{4}\log\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*log(log(x))/x,x, algorithm="giac")
```

```
[Out] 1/2*log(x)^2*log(log(x)) - 1/4*log(x)^2
```

Mupad [B]
time = 0.17, size = 13, normalized size = 0.65

$$\frac{\ln\left(x\right)^{2}\left(2\,\ln\left(\ln\left(x\right)\right)-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(log(x))*log(x))/x,x)
```

```
[Out] (log(x)^2*(2*log(log(x)) - 1))/4
```

Chatgpt [F] Failed to verify
time = 1.00, size = 7, normalized size = 0.35

$$\frac{\ln\left(\ln\left(x\right)\right)^{2}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(x)*ln(ln(x))/x,x)
```

```
[Out] 1/2*ln(ln(x))^2
```

3.217 $\int \log \left(\frac{1+x}{1-x}\right) dx$

Optimal. Leaf size=25

$$2\log(1-x)+(1+x)\log\left(\frac{1+x}{1-x}\right)$$

```
[Out] 2*ln(1-x)+(x+1)*ln((x+1)/(1-x))
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2535, 31}

$$2\log(1-x)+(x+1)\log\left(\frac{x+1}{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 + x)/(1 - x)],x]
```

```
[Out] 2*Log[1 - x] + (1 + x)*Log[(1 + x)/(1 - x)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= (1+x)\log\left(\frac{1+x}{1-x}\right)-2\int \frac{1}{1-x}\,dx\\ &= 2\log(1-x)+(1+x)\log\left(\frac{1+x}{1-x}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$2\log(1-x)+(1+x)\log\left(\frac{1+x}{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 + x)/(1 - x)],x]
```

```
[Out] 2*Log[1 - x] + (1 + x)*Log[(1 + x)/(1 - x)]
```

Maple [A]
time = 0.06, size = 36, normalized size = 1.44

method	result	size
risch	$x\ln\left(\frac{x+1}{1-x}\right)+\ln\left(x^{2}-1\right)$	22
parts	$x\ln\left(\frac{x+1}{1-x}\right)+\ln\left(\left(x-1\right)\left(x+1\right)\right)$	24
meijerg	$\frac{(2x+2)\ln(x+1)}{2}+\frac{(-2x+2)\ln(1-x)}{2}$	26
parallelrisch	$\ln\left(-\frac{x+1}{x-1}\right)x+2\ln\left(x-1\right)+\ln\left(-\frac{x+1}{x-1}\right)$	32
derivativedivides	$-2\ln\left(-\frac{2}{x-1}\right)-\ln\left(-1-\frac{2}{x-1}\right)\left(-1-\frac{2}{x-1}\right)\left(x-1\right)$	36
default	$-2\ln\left(-\frac{2}{x-1}\right)-\ln\left(-1-\frac{2}{x-1}\right)\left(-1-\frac{2}{x-1}\right)\left(x-1\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((x+1)/(1-x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*ln(-2/(x-1))-ln(-1-2/(x-1))*(-1-2/(x-1))*(x-1)
```

Maxima [A]
time = 0.32, size = 22, normalized size = 0.88

$$x\log\left(-\frac{x+1}{x-1}\right)+\log\left(x+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x+1)/(1-x)),x, algorithm="maxima")
```

```
[Out] x*log(-(x + 1)/(x - 1)) + log(x + 1) + log(x - 1)
```

Fricas [A]
time = 0.57, size = 20, normalized size = 0.80

$$x\log\left(-\frac{x+1}{x-1}\right)+\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x+1)/(1-x)),x, algorithm="fricas")
```

```
[Out] x*log(-(x + 1)/(x - 1)) + log(x^2 - 1)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.60

$$x\log\left(\frac{x+1}{1-x}\right)+\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((x+1)/(1-x)),x)
[Out] x*log((x + 1)/(1 - x)) + log(x**2 - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(24) = 48.
time = 0.43, size = 107, normalized size = 4.28

$$\frac{2\log\left(-\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}+1}{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}-1}\right)}{\frac{x+1}{x-1}-1}+2\log\left(\frac{|-x-1|}{|x-1|}\right)-2\log\left(\left|-\frac{x+1}{x-1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((x+1)/(1-x)),x, algorithm="giac")
[Out] 2*log(-(((x + 1)/(x - 1) + 1)/((x + 1)/(x - 1) - 1) + 1)/(((x + 1)/(x - 1)
+ 1)/((x + 1)/(x - 1) - 1) - 1))/((x + 1)/(x - 1) - 1) + 2*log(abs(-x - 1)/
abs(x - 1)) - 2*log(abs(-(x + 1)/(x - 1) + 1))
```

Mupad [B]
time = 0.10, size = 20, normalized size = 0.80

$$\ln\left(x^2-1\right)+x\,\ln\left(-\frac{x+1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(-(x + 1)/(x - 1)),x)
[Out] log(x^2 - 1) + x*log(-(x + 1)/(x - 1))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 16, normalized size = 0.64

$$\frac{\ln\left(\frac{x+1}{1-x}\right)^2}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(ln((x+1)/(1-x)),x)
[Out] 1/2*ln((x+1)/(1-x))^2
```

3.218 $\int \frac{1}{(-1+x)^2+x^2} dx$

Optimal. Leaf size=8

$$-\arctan(1-2x)$$

```
[Out] arctan(-1+2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2006, 631, 210}

$$-\arctan(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^2 + x^2)^(-1),x]
```

```
[Out] -ArcTan[1 - 2*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2006

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \frac{1}{1-2x+2x^2} \, dx \\
&= \text{Subst}\left(\int \frac{1}{-1-x^2} \, dx, x, 1-2x\right) \\
&= -\arctan(1-2x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\arctan(1+2(-1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[1 + 2*(-1 + x)]
```

Maple [A]
time = 0.15, size = 7, normalized size = 0.88

method	result	size
default	$\arctan(-1+2x)$	7
risch	$\arctan(-1+2x)$	7
parallelrisch	$-\frac{i\ln(x-\frac{1}{2}-\frac{i}{2})}{2}+\frac{i\ln(x-\frac{1}{2}+\frac{i}{2})}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+(x-1)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(-1+2*x)
```

Maxima [A]
time = 0.43, size = 6, normalized size = 0.75

$$\arctan(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+(x-1)^2),x, algorithm="maxima")
```

```
[Out] arctan(2*x - 1)
```

Fricas [A]
time = 0.56, size = 6, normalized size = 0.75

$$\arctan(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+(x-1)^2),x, algorithm="fricas")
```

```
[Out] arctan(2*x - 1)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.62

$$\operatorname{atan}(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+(x-1)**2),x)
```

```
[Out] atan(2*x - 1)
```

Giac [A]
time = 0.43, size = 6, normalized size = 0.75

$$\arctan\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+(x-1)^2),x, algorithm="giac")
```

```
[Out] arctan(2*x - 1)
```

Mupad [B]
time = 0.05, size = 6, normalized size = 0.75

$$\mathrm{atan}(2\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^2 + x^2),x)
```

```
[Out] atan(2*x - 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 16, normalized size = 2.00

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x-1)}{x}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^2+(x-1)^2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(2^(1/2)*(x-1)/x)
```

3.219 $\int \sqrt{x\sqrt{x^{3/2}}}\,dx$

Optimal. Leaf size=20

$$\frac{8}{15}x\sqrt{x\sqrt{x^{3/2}}}$$

[Out] 8/15*x*(x*(x^(3/2))^(1/2))^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6851, 15, 30}

$$\frac{8}{15}x\sqrt{x\sqrt{x^{3/2}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x*Sqrt[x^(3/2)]],x]

[Out] (8*x*Sqrt[x*Sqrt[x^(3/2)]])/15

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int x\sqrt{x^2\sqrt{x^3}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\left(2\sqrt{x\sqrt{x^{3/2}}}\right)\text{Subst}\left(\int x^2\sqrt[4]{x^3}\,dx, x, \sqrt{x}\right)}{\sqrt{x}\sqrt[4]{x^{3/2}}}\\
&= \frac{\left(2\sqrt{x\sqrt{x^{3/2}}}\right)\text{Subst}\left(\int x^{11/4}\,dx, x, \sqrt{x}\right)}{x^{7/8}}\\
&= \frac{8}{15}x\sqrt{x\sqrt{x^{3/2}}}
\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{8}{15}x\sqrt{x\sqrt{x^{3/2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*Sqrt[x^(3/2)]],x]
```

```
[Out] (8*x*Sqrt[x*Sqrt[x^(3/2)]])/15
```

Maple [**A**]
time = 0.08, size = 13, normalized size = 0.65

method	result	size
gosper	$\frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$	13
derivativedivides	$\frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$	13
default	$\frac{8x\sqrt{x\sqrt{x^{\frac{3}{2}}}}}{15}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^(3/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*x*(x*(x^(3/2))^(1/2))^(1/2)
```

Maxima [**A**]
time = 0.47, size = 5, normalized size = 0.25

$$\frac{8}{15}x^{\frac{15}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(x^(3/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/15*x^(15/8)
```

Fricas [A]
time = 0.58, size = 12, normalized size = 0.60

$$\frac{8}{15}\sqrt{\sqrt{x^{\frac{3}{2}}}x}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(x^(3/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/15*sqrt(sqrt(x^(3/2))*x)*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x\sqrt{x^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(x**(3/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x*sqrt(x**(3/2))), x)
```

Giac [A]
time = 0.54, size = 19, normalized size = 0.95

$$\frac{8}{15}\,x^{\frac{15}{8}}\operatorname{sgn}\left(x^{\frac{7}{4}}+4\,x^{\frac{3}{4}}\right)\operatorname{sgn}(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(x^(3/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/15*x^(15/8)*sgn(x^(7/4) + 4*x^(3/4))*sgn(x + 4)
```

Mupad [B]
time = 0.22, size = 12, normalized size = 0.60

$$\frac{8\,x\,\sqrt{x\,\sqrt{x^{3/2}}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^(3/2))^(1/2))^(1/2),x)
```

```
[Out] (8*x*(x*(x^(3/2))^(1/2))^(1/2))/15
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 5, normalized size = 0.25

$$\frac{8x^{\frac{13}{8}}}{39}$$

Warning: Unable to verify antiderivative.

```
[In] int((x*(x^(3/2))^(1/2))^(1/2),x)
```

```
[Out] 8/39*x^(13/8)
```

3.220 $\int \cos^4(x)(\cos(x)-\sin(x))\sin^4(x)(\cos(x)+\sin(x))\,dx$

Optimal. Leaf size=12

$$\frac{1}{5}\cos^5(x)\sin^5(x)$$

```
[Out] 1/5*cos(x)^5*sin(x)^5
```

Rubi [A]
time = 0.07, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {460}

$$\frac{1}{5}\sin^5(x)\cos^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*(Cos[x] - Sin[x])*Sin[x]^4*(Cos[x] + Sin[x]),x]
[Out] (Cos[x]^5*Sin[x]^5)/5
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{x^4(1-x^2)}{(1+x^2)^6}\,dx, x, \tan(x)\right)\\ &= \frac{1}{5}\cos^5(x)\sin^5(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.
time = 0.02, size = 25, normalized size = 2.08

$$\frac{1}{256}\sin(2x)-\frac{1}{512}\sin(6x)+\frac{\sin(10x)}{2560}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*(Cos[x] - Sin[x])*Sin[x]^4*(Cos[x] + Sin[x]),x]
[Out] Sin[2*x]/256 - Sin[6*x]/512 + Sin[10*x]/2560
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(10) = 20$.
time = 0.21, size = 80, normalized size = 6.67

method	result
risch	$\frac{\sin(10x)}{2560} - \frac{\sin(6x)}{512} + \frac{\sin(2x)}{256}$
parallelrisch	$\frac{\sin(10x)}{2560} - \frac{\sin(6x)}{512} + \frac{\sin(2x)}{256}$
default	$-\frac{(\sin^3(x))(\cos^7(x))}{10} - \frac{3\sin(x)(\cos^7(x))}{80} + \frac{\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160} + \frac{(\cos^5(x))(\sin^5(x))}{10} + \frac{(\sin^3(x)}{}$
parts	$-\frac{(\sin^3(x))(\cos^7(x))}{10} - \frac{3\sin(x)(\cos^7(x))}{80} + \frac{\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{160} + \frac{(\cos^5(x))(\sin^5(x))}{10} + \frac{(\sin^3(x)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4*cos(x)^4*(cos(x)+sin(x))*(cos(x)-sin(x)),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/10*sin(x)^3*cos(x)^7-3/80*sin(x)*cos(x)^7+1/160*(cos(x)^5+5/4*cos(x)^3+1
5/8*cos(x))*sin(x)+1/10*cos(x)^5*sin(x)^5+1/16*sin(x)^3*cos(x)^5+1/32*cos(x
)^5*sin(x)-1/128*(cos(x)^3+3/2*cos(x))*sin(x)
```

Maxima [A]
time = 0.34, size = 8, normalized size = 0.67

$$\frac{1}{160}\sin(2x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4*cos(x)^4*(cos(x)+sin(x))*(cos(x)-sin(x)),x, algorithm="m
axima")
```

```
[Out] 1/160*sin(2*x)^5
```

Fricas [A]
time = 0.61, size = 19, normalized size = 1.58

$$\frac{1}{5}\left(\cos(x)^9 - 2\cos(x)^7 + \cos(x)^5\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4*cos(x)^4*(cos(x)+sin(x))*(cos(x)-sin(x)),x, algorithm="f
ricas")
```

```
[Out] 1/5*(cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x)
```

Sympy [**A**]
time = 1.20, size = 10, normalized size = 0.83

$$\frac{\sin^5(x)\cos^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4*cos(x)**4*(cos(x)+sin(x))*(cos(x)-sin(x)),x)
```

```
[Out] sin(x)**5*cos(x)**5/5
```

Giac [**A**]
time = 0.48, size = 19, normalized size = 1.58

$$\frac{1}{2560}\sin(10x) - \frac{1}{512}\sin(6x) + \frac{1}{256}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4*cos(x)^4*(cos(x)+sin(x))*(cos(x)-sin(x)),x, algorithm="g
iac")
```

```
[Out] 1/2560*sin(10*x) - 1/512*sin(6*x) + 1/256*sin(2*x)
```

Mupad [**B**]
time = 0.25, size = 16, normalized size = 1.33

$$\frac{\cos(x)^5\sin(x)\left(\cos(x)^2-1\right)^2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^4*(cos(x) + sin(x))*(cos(x) - sin(x)),x)
```

```
[Out] (cos(x)^5*sin(x)*(cos(x)^2 - 1)^2)/5
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 1.08

$$\frac{(\sin^9(x))}{90} - \frac{(\cos^9(x))}{90}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)^4*cos(x)^4*(cos(x)+sin(x))*(cos(x)-sin(x)),x)
```

```
[Out] 1/90*sin(x)^9-1/90*cos(x)^9
```

3.221 $\int \log \left(1+x^2\right) \, dx$

Optimal. Leaf size=16

$$-2 x+2 \arctan (x)+x \log \left(1+x^2\right)$$

```
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2498, 327, 209}

$$2 \arctan (x)+x \log \left(x^2+1\right)-2 x$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= x \log \left(1+x^2\right)-2 \int \frac{x^2}{1+x^2} \, dx\\ &= -2 x+x \log \left(1+x^2\right)+2 \int \frac{1}{1+x^2} \, dx\\ &= -2 x+2 \arctan (x)+x \log \left(1+x^2\right) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-2x + 2\arctan(x) + x\log\left(1 + x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Maple [A]
time = 0.04, size = 17, normalized size = 1.06

method	result	size
default	$-2x + 2\arctan(x) + x\ln(x^2 + 1)$	17
risch	$-2x + 2\arctan(x) + x\ln(x^2 + 1)$	17
parts	$-2x + 2\arctan(x) + x\ln(x^2 + 1)$	17
meijerg	$-2x + \frac{2x\arctan\left(\sqrt{x^2}\right)}{\sqrt{x^2}} + x\ln(x^2 + 1)$	27
parallelrisch	$-2i\ln(x - i) + i\ln(x^2 + 1) + x\ln(x^2 + 1) - 2x$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

Maxima [A]
time = 0.40, size = 16, normalized size = 1.00

$$x\log\left(x^2 + 1\right) - 2x + 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Fricas [A]
time = 0.59, size = 16, normalized size = 1.00

$$x\log\left(x^2 + 1\right) - 2x + 2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="fricas")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.94

$$x\log\left(x^2+1\right)-2x+2\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+1),x)
```

```
[Out] x*log(x**2 + 1) - 2*x + 2*atan(x)
```

Giac [A]
time = 0.47, size = 16, normalized size = 1.00

$$x\log\left(x^2+1\right)-2\,x+2\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="giac")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Mupad [B]
time = 0.10, size = 16, normalized size = 1.00

$$2\,\mathrm{atan}(x)-2\,x+x\,\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2 + 1),x)
```

```
[Out] 2*atan(x) - 2*x + x*log(x^2 + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 20, normalized size = 1.25

$$\frac{(x^2+1)\ln\left(x^2+1\right)}{2}-\frac{x^2}{2}-\frac{1}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(x^2+1),x)
```

```
[Out] 1/2*(x^2+1)*ln(x^2+1)-1/2*x^2-1/2
```

3.222 $\int \frac{1+2x}{1+2x+2x^2} dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log(1+2x+2x^2)$$

[Out] 1/2*ln(2*x^2+2*x+1)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {642}

$$\frac{1}{2}\log(2x^2+2x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/(1 + 2*x + 2*x^2),x]

[Out] Log[1 + 2*x + 2*x^2]/2

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\text{Integral} = \frac{1}{2}\log(1+2x+2x^2)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2}\log(1+2x+2x^2)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/(1 + 2*x + 2*x^2),x]

[Out] Log[1 + 2*x + 2*x^2]/2

Maple [A]
time = 0.13, size = 14, normalized size = 0.93

method	result	size
parallelrisch	$\frac{\ln\left(x^{2}+x+\frac{1}{2}\right)}{2}$	10
default	$\frac{\ln\left(2x^{2}+2x+1\right)}{2}$	14
norman	$\frac{\ln\left(2x^{2}+2x+1\right)}{2}$	14
risch	$\frac{\ln\left(2x^{2}+2x+1\right)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(2*x^2+2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(2*x^2+2*x+1)
```

Maxima [A]
time = 0.33, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(2\,x^{2}+2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+1)/(2*x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + 2*x + 1)
```

Fricas [A]
time = 0.57, size = 13, normalized size = 0.87

$$\frac{1}{2}\,\log\left(2\,x^{2}+2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+1)/(2*x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*x^2 + 2*x + 1)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.80

$$\frac{\log\left(2x^{2}+2x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+1)/(2*x**2+2*x+1),x)
```

```
[Out] log(2*x**2 + 2*x + 1)/2
```

Giac [A]
time = 0.49, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(2x^2+2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+1)/(2*x^2+2*x+1),x, algorithm="giac")
```

```
[Out] 1/2*log(2*x^2 + 2*x + 1)
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.60

$$\frac{\ln\left(x^2+x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(2*x + 2*x^2 + 1),x)
```

```
[Out] log(x + x^2 + 1/2)/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 12, normalized size = 0.80

$$\frac{\ln\left(2\left(x+\frac{1}{2}\right)^2+1\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((1+2*x)/(2*x^2+2*x+1),x)
```

```
[Out] 1/2*ln(2*(x+1/2)^2+1)
```

3.223 $\int \frac{\arcsin(x)}{x^3} dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{1-x^2}}{2x}-\frac{\arcsin(x)}{2x^2}$$

```
[Out] -1/2*(-x^2+1)^(1/2)/x-1/2*arcsin(x)/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4723, 270}

$$-\frac{\arcsin(x)}{2x^2}-\frac{\sqrt{1-x^2}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/x^3,x]
```

```
[Out] -1/2*Sqrt[1 - x^2]/x - ArcSin[x]/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\frac{\arcsin(x)}{2x^2}+\frac{1}{2}\int\frac{1}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{\sqrt{1-x^2}}{2x}-\frac{\arcsin(x)}{2x^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.82

$$-\frac{x\sqrt{1-x^2}+\arcsin(x)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/x^3,x]
```

```
[Out] -1/2*(x*Sqrt[1 - x^2] + ArcSin[x])/x^2
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.82

method	result	size
default	$-\frac{\sqrt{-x^2+1}}{2x}-\frac{\arcsin(x)}{2x^2}$	23
parts	$-\frac{\sqrt{-x^2+1}}{2x}-\frac{\arcsin(x)}{2x^2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-x^2+1)^(1/2)/x-1/2*arcsin(x)/x^2
```

Maxima [A]
time = 0.42, size = 22, normalized size = 0.79

$$-\frac{\sqrt{-x^2+1}}{2\,x}-\frac{\arcsin\left(x\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)/x - 1/2*arcsin(x)/x^2
```

Fricas [A]
time = 0.63, size = 19, normalized size = 0.68

$$-\frac{\sqrt{-x^2+1}x+\arcsin\left(x\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-x^2 + 1)*x + arcsin(x))/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.84, size = 37, normalized size = 1.32

$$\frac{\begin{cases}-\frac{i\sqrt{x^2-1}}{x} & \text{for}\ |x^2|>1\\-\frac{\sqrt{1-x^2}}{x} & \text{otherwise}\end{cases}}{2}-\frac{\operatorname{asin}\left(x\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x**3,x)
```

```
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))/
2 - asin(x)/(2*x**2)
```

Giac [A]
time = 0.46, size = 40, normalized size = 1.43

$$\frac{x}{4\left(\sqrt{-x^2+1}-1\right)}-\frac{\sqrt{-x^2+1}-1}{4\,x}-\frac{\arcsin\left(x\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*x/(sqrt(-x^2 + 1) - 1) - 1/4*(sqrt(-x^2 + 1) - 1)/x - 1/2*arcsin(x)/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{asin}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/x^3,x)
```

```
[Out] int(asin(x)/x^3, x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 19, normalized size = 0.68

$$-\frac{\arcsin\left(x\right)}{2x}-\frac{1}{2\sqrt{-x^2+1}}$$

Warning: Unable to verify antiderivative.

```
[In] int(arcsin(x)/x^3,x)
```

```
[Out] -1/2/x*arcsin(x)-1/2/(-x^2+1)^(1/2)
```

3.224 $\int \cos(\cos(x))\sin(2x)\,dx$

Optimal. Leaf size=13

$$-2\cos(\cos(x))-2\cos(x)\sin(\cos(x))$$

```
[Out] -2*cos(cos(x))-2*cos(x)*sin(cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {12, 3377, 2718}

$$-2\cos(\cos(x))-2\cos(x)\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[2*x],x]
```

```
[Out] -2*Cos[Cos[x]] - 2*Cos[x]*Sin[Cos[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int 2x\cos(x)\,dx,x,\cos(x)\right)\\
&= -\left(2\text{Subst}\left(\int x\cos(x)\,dx,x,\cos(x)\right)\right)\\
&= -2\cos(x)\sin(\cos(x))+2\text{Subst}\left(\int \sin(x)\,dx,x,\cos(x)\right)\\
&= -2\cos(\cos(x))-2\cos(x)\sin(\cos(x))
\end{aligned}$$

Mathematica [A]
time = 1.12, size = 13, normalized size = 1.00

$$-2\cos(\cos(x)) - 2\cos(x)\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[2*x],x]
[Out] -2*Cos[Cos[x]] - 2*Cos[x]*Sin[Cos[x]]
```

Maple [A]
time = 10.31, size = 21, normalized size = 1.62

method	result	size
risch	$-\sin\left(x+\cos\left(x\right)\right)+\sin\left(-\cos\left(x\right)+x\right)-2\cos\left(\cos\left(x\right)\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*cos(cos(x)),x,method=_RETURNVERBOSE)
[Out] -sin(x+cos(x))+sin(-cos(x)+x)-2*cos(cos(x))
```

Maxima [A]
time = 0.34, size = 13, normalized size = 1.00

$$-2\cos\left(x\right)\sin\left(\cos\left(x\right)\right)-2\cos\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(cos(x)),x, algorithm="maxima")
[Out] -2*cos(x)*sin(cos(x)) - 2*cos(cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(13) = 26.
time = 0.61, size = 47, normalized size = 3.62

$$2\cos\left(x\right)\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)-2\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(cos(x)),x, algorithm="fricas")
[Out] 2*cos(x)*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1)) - 2*cos((tan(1/2*x)^2 -
 1)/(tan(1/2*x)^2 + 1))
```

Sympy [A]
time = 0.82, size = 17, normalized size = 1.31

$$-2\sin\left(\cos\left(x\right)\right)\cos\left(x\right)-2\cos\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(cos(x)),x)
```

```
[Out] -2*sin(cos(x))*cos(x) - 2*cos(cos(x))
```

Giac [A]
time = 0.46, size = 13, normalized size = 1.00

$$-2\,\cos\left(x\right)\sin\left(\cos\left(x\right)\right)-2\,\cos\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(cos(x)),x, algorithm="giac")
```

```
[Out] -2*cos(x)*sin(cos(x)) - 2*cos(cos(x))
```

Mupad [B]
time = 0.26, size = 24, normalized size = 1.85

$$-4\cos\left(\frac{\cos\left(x\right)}{2}\right)^2-4\,\sin\left(\frac{\cos\left(x\right)}{2}\right)\,\cos\left(x\right)\,\cos\left(\frac{\cos\left(x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(2*x),x)
```

```
[Out] - 4*cos(cos(x)/2)^2 - 4*cos(cos(x)/2)*sin(cos(x)/2)*cos(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(sin(2*x)*cos(cos(x)),x)
```

```
[Out] not solved
```

3.225 $\int -\sin(x-\sin(x))\,dx$

Optimal. Leaf size=12

$$-\mathrm{Int}(\sin(x-\sin(x)),x)$$

```
[Out] -CannotIntegrate(sin(x-sin(x)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int -\sin(x-\sin(x))\,dx$$

Verification is not applicable to the result.

```
[In] Int[-Sin[x - Sin[x]],x]
[Out] -Defer[Int][Sin[x - Sin[x]], x]
```

Rubi steps

$$\text{Integral} = -\int \sin(x-\sin(x))\,dx$$

Mathematica [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int -\sin(x-\sin(x))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[-Sin[x - Sin[x]],x]
[Out] -Integrate[Sin[x - Sin[x]], x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int -\sin\left(x-\sin\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x-sin(x)),x)
[Out] int(-sin(x-sin(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(x-sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(sin(-x + sin(x)), x)
```

Fricas [A]
time = 0.60, size = 9, normalized size = 0.75

$$\mathrm{integral}(\sin(-x+\sin(x)), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(x-sin(x)),x, algorithm="fricas")
```

```
[Out] integral(sin(-x + sin(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \sin(x-\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(x-sin(x)),x)
```

```
[Out] -Integral(sin(x - sin(x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(x-sin(x)),x, algorithm="giac")
```

```
[Out] integrate(-sin(x - sin(x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int -\sin(x-\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x - sin(x)),x)
```

```
[Out] int(-sin(x - sin(x)), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 11, normalized size = 0.92

$$-2\cos\left(x - \sin\left(x\right)\right) + x$$

Warning: Unable to verify antiderivative.

```
[In] int(-sin(x-sin(x)),x)
```

```
[Out] -2*cos(x-sin(x))+x
```

3.226 $\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{1+\tan^{2\sqrt{505}}(x)},x\right)$$

```
[Out] Unintegrable(1/(tan(x)^(2*505^(1/2))+1),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + Tan[x]^(2*Sqrt[505]))^(-1),x]
[Out] Defer[Int][(1 + Tan[x]^(2*Sqrt[505]))^(-1), x]
```

Rubi steps

$$\text{Integral} = \int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx$$

Mathematica [A]
time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{1}{1+\tan^{2\sqrt{505}}(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + Tan[x]^(2*Sqrt[505]))^(-1),x]
[Out] Integrate[(1 + Tan[x]^(2*Sqrt[505]))^(-1), x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{2\sqrt{505}}(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^(2*505^(1/2))+1),x)
[Out] int(1/(tan(x)^(2*505^(1/2))+1),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)^(2*505^(1/2))+1),x, algorithm="maxima")
[Out] -(-1)^(sqrt(101)*sqrt(5))*integrate(((-1)^(sqrt(101)*sqrt(5))*cos(2*sqrt(10
1)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(101)*sqrt(5)*arctan2(sin(x)
, -cos(x) + 1))^2*e^(2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1) + 2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)) + (-1)
^(sqrt(101)*sqrt(5))*e^(2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 + 2*cos
(x) + 1) + 2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*sin
(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(101)*sqrt(5)*arct
an2(sin(x), -cos(x) + 1))^2 + cos(2*sqrt(101)*sqrt(5)*arctan2(sin(2*x), cos
(2*x) + 1))*cos(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(10
1)*sqrt(5)*arctan2(sin(x), -cos(x) + 1))*e^(sqrt(101)*sqrt(5)*log(cos(2*x)^
2 + sin(2*x)^2 + 2*cos(2*x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^
2 + 2*cos(x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) +
1)) + e^(sqrt(101)*sqrt(5)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) +
sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + sqrt(101)*sqrt(
5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*sin(2*sqrt(101)*sqrt(5)*arctan2
(sin(2*x), cos(2*x) + 1))*sin(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) +
1) - 2*sqrt(101)*sqrt(5)*arctan2(sin(x), -cos(x) + 1)))/(2*(-1)^(sqrt(101)*
sqrt(5))*cos(2*sqrt(101)*sqrt(5)*arctan2(sin(2*x), cos(2*x) + 1))*cos(2*sqr
t(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(101)*sqrt(5)*arctan2(si
n(x), -cos(x) + 1))*e^(sqrt(101)*sqrt(5)*log(cos(2*x)^2 + sin(2*x)^2 + 2*co
s(2*x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + s
qrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)) + (-1)^(2*sqrt(10
1)*sqrt(5))*cos(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(10
1)*sqrt(5)*arctan2(sin(x), -cos(x) + 1))^2*e^(2*sqrt(101)*sqrt(5)*log(cos(x
)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^
2 - 2*cos(x) + 1)) + 2*(-1)^(sqrt(101)*sqrt(5))*e^(sqrt(101)*sqrt(5)*log(co
s(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^2 +
sin(x)^2 + 2*cos(x) + 1) + sqrt(101)*sqrt(5)*log(cos(x)^2 + sin(x)^2 - 2*co
s(x) + 1))*sin(2*sqrt(101)*sqrt(5)*arctan2(sin(2*x), cos(2*x) + 1))*sin(2*s
qrt(101)*sqrt(5)*arctan2(sin(x), cos(x) + 1) - 2*sqrt(101)*sqrt(5)*arctan2(
sin(x), -cos(x) + 1)) + (-1)^(2*sqrt(101)*sqrt(5))*e^(2*sqrt(101)*sqrt(5)*l
og(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*sqrt(101)*sqrt(5)*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1))*sin(2*sqrt(101)*sqrt(5)*arctan2(sin(x), cos(x) +
 1) - 2*sqrt(101)*sqrt(5)*arctan2(sin(x), -cos(x) + 1))^2 + (cos(2*x)^2 + s
```

```
in(2*x)^2 + 2*cos(2*x) + 1)^(2*sqrt(101)*sqrt(5))*cos(2*sqrt(101)*sqrt(5)*a
rctan2(sin(2*x), cos(2*x) + 1))^2 + (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) +
 1)^(2*sqrt(101)*sqrt(5))*sin(2*sqrt(101)*sqrt(5)*arctan2(sin(2*x), cos(2*x
) + 1))^2), x) + x
```

Fricas [A]
time = 0.58, size = 14, normalized size = 0.82

$$\mathrm{integral}\left(\frac{1}{\tan\left(x\right)^{2\,\sqrt{505}}+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)^(2*505^(1/2))+1),x, algorithm="fricas")
[Out] integral(1/(tan(x)^(2*sqrt(505)) + 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^{2\sqrt{505}}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)**(2*505**(1/2))+1),x)
[Out] Integral(1/(tan(x)**(2*sqrt(505)) + 1), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(x)^(2*505^(1/2))+1),x, algorithm="giac")
[Out] integrate(1/(tan(x)^(2*sqrt(505)) + 1), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{{\mathrm{tan}\left(x\right)}^{2\,\sqrt{505}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^(2*505^(1/2)) + 1),x)
```

```
[Out] int(1/(tan(x)^(2*505^(1/2)) + 1), x)
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(1/(tan(x)^(2*505^(1/2))+1),x)
```

```
[Out] not solved
```

3.227 $\int (1-x)^{2020} x \, dx$

Optimal. Leaf size=23

$$-\frac{(1-x)^{2021}}{2021}+\frac{(1-x)^{2022}}{2022}$$

```
[Out] -1/2021*(1-x)^2021+1/2022*(1-x)^2022
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{(1-x)^{2022}}{2022}-\frac{(1-x)^{2021}}{2021}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^2020*x,x]
```

```
[Out] -1/2021*(1 - x)^2021 + (1 - x)^2022/2022
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left((1-x)^{2020}-(1-x)^{2021}\right) dx \\
&= -\frac{(1-x)^{2021}}{2021}+\frac{(1-x)^{2022}}{2022}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 11128 vs. 2(23) = 46.
time = 0.02, size = 11128, normalized size = 483.83

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^2020*x,x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10106 vs. $2(19) = 38$.
time = 6.80, size = 10107, normalized size = 439.43

method	result	size
gosper	Expression too large to display	10106
default	Expression too large to display	10107
risch	Expression too large to display	10107
parallelrisch	Expression too large to display	10107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1-x)^2020,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10106 vs. $2(15) = 30$.
time = 2.62, size = 10106, normalized size = 439.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^2020,x, algorithm="maxima")
```

```
[Out] 1/2022*x^2022 - 2020/2021*x^2021 + 2019/2*x^2020 - 2038180/3*x^2019 + 68550
7705/2*x^2018 - 138266870112*x^2017 + 278745941561035/6*x^2016 - 1337316509
3095968*x^2015 + 3366693482174180730*x^2014 - 2259050769046992147760/3*x^20
13 + 151506967484463186448138*x^2012 - 27698221479285170723685360*x^2011 +
13918352848288375491988868066/3*x^2010 - 716973434466946772290161333280*x^2
009 + 102834449953653272749024785712470*x^2008 - 41277737963817952963643626
311862816/3*x^2007 + 1725065037047525925910348951124302995*x^2006 - 2034561
49346983815836345155695893355528*x^2005 + 6795433696105515711543576933233389
18346705/3*x^2004 - 238793864924060382048318075559904353223O840*x^2003 + 23
9032599150157038818882365583996835022772313*x^2002 - 683291587775561350346
8301504949834564369103600O/3*x^2001 + 207058005137171032259543201147710858391
2261365977*x^2000 - 17996036899496170800816378445445605196757936399600O*x^
1999 + 44945090897709842545903018770867237413834629868000750/3*x^1998 - 119
6737653552288657999009221047362441958082554435826640*x^1997 + 9187260603547
7133069946098385058726668676396410333163850*x^1996 - 2036508922103836486046
1105557164855112824957167732656091280/3*x^1995 + 4834283058757897205184808
70193807402556007179708207427412550*x^1994 - 3322318520852844970856603116929
2044668543059434670067575119200*x^1993 + 6618056825708284424494058802194502
9439554707856182335518238O6130/3*x^1992 - 1416833098494499231566155690748
32365159751152400066575169519360800*x^1991 + 17621857212672637915910494927062
```

643669496400090086408425321559396505/2*x^1990 - 159317568801587084217495834
340808688408446680205659861281555664243570O/3*x^1989 + 6210259737998686075O
83704572722478717724224642443259371374040789349582 5/2*x^1988 - 176282613913
6875145854560336147570878855404166949882699385944631188781540*x^1987 + 5834
95304116563214661281269304428358721642399988493839669733625764655606425/6*x
^1986 - 521728776900488160284460555078351584023333719983720959202767718665 8
756829760*x^1985 + 544794542317071048217452589725683577666003679953979611 70
062883235280838457467 5/2*x^1984 - 4155105010316363470369118680218838636590 0
428229156960691552084059020136098420800/3*x^1983 + 6862846695022287208632 20
1228234657269342964510698223986491757379203757067715972 60*x^1982 - 33159258
1426942084314005801603464123829043673522902725694448393748563254407873016 00
*x^1981 + 468966531253103290188507947547048754543744223894485686475393141 55
969467316643974172 20/3*x^1980 - 7194453708359886755999545875456493273344150
90320191280515054864969799407646948487048 00*x^1979 + 139072024843107780705 4
02031845077308159947349595909730324277836129527455327507705407659100/43*x^1
978 - 4262718182478207666497461840023654359300817497511317095786733499521 00
070121367067635629120/3*x^1977 + 61037166458919525976259780643117263025754 6
701004270483089886058568481609492950484844726050 0*x^1976 - 2564858996839822
43028949453930814576862452231948083118646054803912483168278678210826498756 1
60*x^1975 + 148726516837958557567683275802954894538020245673556069397338631
404118355617904526364932595336275 0/141*x^1974 - 42471753124966226272315027 1
353887556337666967588520119979077229931100423254663277085869347626000*x^197
3 + 1675085512500664668376592780614994840768216595557594358755168242842955 8
795593704083730408309017030*x^1972 - 194211335016058797573525612494235728 57
04479230932630455924561374198235807188773849423679810642603600/3*x^1971 + 2
4525399448538577751018416504076107795565624569983127045968938931942467609 92
81050483356421757226007 70*x^1970 - 911141491668082873190953745792260288041 0
14055912318140921449806736334593989735869641666120045511985600*x^1969 + 996
18110701354825366267142315649766191550146281382513073227650599463042519701 5
8048349070238417538863495 0/3*x^1968 - 11875683218477100601337825796175462 48
1691311869345463934971521751400387122566811438113973739618381662979 20*x^196
7 + 41692119941063124704588635432001680068686948234595232029842003483354493
61350523548910352435107059836558310 0*x^1966 - 4311841814043431702808371481 3
79275615951136815593323696241989019901217925493856686652692291654967727428 6
92640/3*x^1965 + 48669282341048818064483160314590423417580355254067612212 04
59741921245300404927368733232574652341244123301583 00*x^1964 - 1619284346477
30991104502228943171270801138674257368095127923509917161138756031857585746 3
87812060653892340362320 0*x^1963 + 158851753123258289905781950481571877213 69
716148045509995247687558258066397972363454649549753782420991368306864174 0/3
*x^1962 - 1702230638629650365176913212607302188475314279868138247657877447 9
12784530440047681486359322893844602835250538849600*x^1961 + 538124384391789
51721636476470037849571630833296800197696381289878409357631286674894258693 9
47248770016518232676054020*x^1960 - 50199304491909934476552632057058959756 9
35320821231479300772636316044148762594192009576971872791672067136218959196 6
2400/3*x^1959 + 102385638080321174949263623772645625178023720185120265033 71

```
2123617268026415416133246729145608800125063372676309620002775/2*x^1958 - 15
412972415810773603124381548619130627601265229499...
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^2020,x, algorithm="fricas")
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11171 vs. $2(12) = 24$.
time = 2.59, size = 11171, normalized size = 485.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)**2020,x)
```

```
[Out] x**2022/2022 - 2020*x**2021/2021 + 2019*x**2020/2 - 2038180*x**2019/3 + 685
507705*x**2018/2 - 138266870112*x**2017 + 278745941561035*x**2016/6 - 13373
165093095968*x**2015 + 3366693482174180730*x**2014 - 2259050769046992147760
*x**2013/3 + 151506967484463186448138*x**2012 - 27698221479285170723685360*
x**2011 + 13918352848288375491988868066*x**2010/3 - 716973434466946772290161
1333280*x**2009 + 102834449953653272749024785712470*x**2008 - 4127773796381
795296364362631186281 6*x**2007/3 + 1725065037047525925910348951124302995*x*
*2006 - 2034561493469838158363451556958933555 28*x**2005 + 67954336961055157
115435769332338918346705*x**2004/3 - 2387938649240603820483180755599043532 2
30840*x**2003 + 23903259915015703881888236558399683502277 2313*x**2002 - 683
2915877755613503468301504949834564369103 6000*x**2001/3 + 207058005137171032
2595432011477108583912261365977*x**2000 - 17996036899496170800816378445445 6
051967579363996000*x**1999 + 449450908977098425459030187708672374138346298 6
8000750*x**1998/3 - 1196737653552288657999009221047362441958082554435826640
*x**1997 + 918726060354771330699460983850587266686763964103331638 50*x**1996
 - 20365089221038364860461105557164855112824957167732656091280*x**1995/3 +
4834283058757897205184808701938074025560071797082074274125 50*x**1994 - 3322
318520852844970856603116929204466854305943467006757511920 0*x**1993 + 661805
682570828442449405880219450294395547078561823355182380613 0*x**1992/3 - 1416
833098494499231566155690748323651597511524000665751695193608 00*x**1991 + 17
621857212672637915910494927062643669496400090086408425321559396505*x**1990/
2 - 15931756880158708421749583434080868840844668020565986128155566424357 00*
x**1989/3 + 6210259737998686075083704572722478717724224642443259371374040 78
93495825*x**1988/2 - 176282613913687514585456033614757087885540416694988269
```

```
9385944631188781540*x**1987 + 583495304116563214661281269304428358721642399
988493839669733625764655606425*x**1986/6 - 521728776900488160284460555078835
1584023333719983720959202767718665875682976O*x**1985 + 544794542317071048 21
745258972568357766600367995397961170062883235280838457 4675*x**1984/2 - 4155
10501031636347036911868021883863659004282291569606915520840590201360984208O
0*x**1983/3 + 686284669502228720863220122823465726934296451069822398649175 7
3792037570677159726O*x**1982 - 331592581426942084314005801603464123829043 67
35229027256944483937485632544078730160O*x**1981 + 4689665312531032901885079
4754704875454374422389448568647539314155969467316643974172 20*x**1980/3 - 71
944537083598867559995458754564932733441509032019128051505486496979940764694
848704800*x**1979 + 1390720248431077807054020318450773081599473495959097303
242778361295274553275077054076591OO*x**1978/43 - 4262718182478207666497461 8
400236543593008174975113170957867334995210007012136706763562912O*x**1977/3
+ 61037166458919525976259780643117263025754670100427048308988605856848160 94
929504848447260500*x**1976 - 25648589968398224302894945393081457686245223 19
48083118646054803912483168278678210826498756160*x**1975 + 14872651683795855
75676832758029548945380202456735560693973386314041183556179045263649325953 3
62750*x**1974/141 - 42471753124966226272315027135388755633766696758852011 99
79077229931100423254663277085869347626000*x**1973 + 16750855125006646683765
927806149948407682165955575943587551682428429558795593704083730408309017030
*x**1972 - 19421133501605879757352561249423572857044792309326304559245613 74
198235807188773849423679810642603600*x**1971/3 + 2452539944853857775101841 6
504076107795565624569983127045968938931942467609928105048335642175722600770
*x**1970 - 91114149166808287319095374579226028804101405591231814092144980 67
363345939897358696416661200455119856OO*x**1969 + 996181107013548253662671 42
315649766191550146281382513073227650599463042519701580483490702384175388634
950*x**1968/3 - 11875683218477100601337825796175462481691311869345463934971
521751400387122566811438113973739618381662979 20*x**1967 + 41692119941063124
704588635432001680068686948234595232029842003483354493613505235489103524351
070598365583100*x**1966 - 43118418140434317028083714813792756159511368155 93
3236962419890199012179254938566866526922916549677274286926 40*x**1965/3 + 48
669282341048818064483160314590423417580355254067612212045974192124530040492
73687332325746523412441233015830O*x**1964 - 16192843464773099110450222894 31
712708011386742573680951279235099171611387560318575857463878120606538923403
623200*x**1963 + 15885175312325828990578195048157187721369716148045509995 24
768755825806639797236345464954975378242099136830686417 40*x**1962/3 - 170223
063862965036517691321260730218847531427986813824765787744791278453044004768
14863593228938446028352505388496OO*x**1961 + 53812438439178951721636476470 0
378495716308332968001976963812898784093576312866748942586939472487700165182
32676054020*x**1960 - 501993044919099344765526320570589597569353208212314 79
30077263631604414876259419200957697187279167206713621895919662400*x**1959/3
 + 1023856380803211749492636237726456251780237201851202650337121236172680 26
41541613324672914560880012506337267630962000277...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 10106 vs.

$2(15) = 30.$
time = 1.99, size = 10106, normalized size = 439.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1-x)^2020,x, algorithm="giac")
```

```
[Out] 1/2022*x^2022 - 2020/2021*x^2021 + 2019/2*x^2020 - 2038180/3*x^2019 + 68550
7705/2*x^2018 - 138266870112*x^2017 + 278745941561035/6*x^2016 - 1337316509
3095968*x^2015 + 3366693482174180730*x^2014 - 2259050769046992147760/3*x^20
13 + 151506967484463186448138*x^2012 - 27698221479285170723685360*x^2011 +
13918352848288375491988868066/3*x^2010 - 716973434466946772290161333280*x^2
009 + 102834449953653272749024785712470*x^2008 - 412777379638179529636436263
11862816/3*x^2007 + 1725065037047525925910348951124302995*x^2006 - 2034561
493469838158363451556958933555528*x^2005 + 679543369610551571154357693323389
18346705/3*x^2004 - 2387938649240603820483180755599043532230840*x^2003 + 23
903259915015703881888236558399683502277231 3*x^2002 - 683291587775561350346830150494983456436910360 00/3*x^2001 + 20705800513717103225954320114771085839
12261365977*x^2000 - 17996036899496170800816378445445605196757936399 6000*x^
1999 + 4494509089770984254590301877086723741383462986800 0750/3*x^1998 - 119
673765355228865799900922104736244195808255443582664 0*x^1997 + 9187260603547
713306994609838505872666867639641033316385 0*x^1996 - 203650892210383648604 6
1105557164855112824957167732656091280/3*x^1995 + 4834283058757897205184808 7
019380740255600717970820742741255 0*x^1994 - 33223185208528449708566031169 29
204466854305943467006757511920 0*x^1993 + 66180568257082844244940588021945 02
943955470785618233551823806130/3*x^1992 - 14168330984944992315661556907483 2
365159751152400066575169519360800*x^1991 + 17621857212672637915910494927062
643669496400090086408425321559396505/2*x^1990 - 1593175688015870842174958 34
340808688408446680205659861281555664243570 0/3*x^1989 + 62102597379986860750
837045727224787177242246424432593713740407893495825/2*x^1988 - 176282613913
687514585456033614757087885540416694988269938594463118878154 0*x^1987 + 5834
953041165632146612812693044283587216423999884938396697336257646556064 25/6*x
^1986 - 52172877690048816028446055507835158402333371998372095920276771866 58
756829760*x^1985 + 5447945423170710482174525897256835776660036799539796117 0
06288323528083845746 75/2*x^1984 - 4155105010316363470369118680218838636590 0
42822915696069155208405902013609842080 0/3*x^1983 + 686284669502228720863220
1228234657269342964510698223986491757379203757067715972 60*x^1982 - 33159258
14269420843140058016034641238290436735229027256944483937485632544078730160 0
*x^1981 + 468966531253103290188507947547048754543744223894485686475393141 55
96946731664397417220/3*x^1980 - 71944537083598867559995458754564932733441 50
903201912805150548649697994076469484870480 0*x^1979 + 139072024843107780705 4
0203184507730815994734959590973032427783612952745532750770540765910 0/43*x^1
978 - 42627181824782076664974618400236543593008174975113170957867334995210 0
070121367067635629120/3*x^1977 + 61037166458919525976259780643117263025754 6
```

```
7010042704830898860585684816094929504848447260500*x^1976 - 2564858996839822
430289494539308145768624522319480831186460548039124831682786782108264987561
60*x^1975 + 1487265168379585575676832758029548945380202456735560693973386314
04118355617904526364932595336275O/141*x^1974 - 4247175312496622627231502713
53887556337666967588520119979077229931100423254663277085869347626000*x^197
3 + 1675085512500664668376592780614994840768216595557594358755168242842955
8795593704083730408309017030*x^1972 - 19421133501605879757352561249423572857
04479230932630455924561374198235807188773849423679810642603600/3*x^1971 + 2
452539944853857775101841650407610779556562456998312704596893893194246760992
810504833564217572260O770*x^1970 - 9111414916680828731909537457922602880410
14055912318140921449806736334593989735869641666120045511985600*x^1969 + 996
181107013548253662671423156497661915501462813825130732276505994630425197015
804834907023841753886349 50/3*x^1968 - 118756832184771006013378257961754624 8
169131186934546393497152175140038712256681143811397373961838166297920*x^196
7 + 416921199410631247045886354320016800686869482345952320298420034833544936
135052354891035243510705983655831O0*x^1966 - 4311841814043431702808371481 3
7927561595113681559332369624198901990121792549385668665269229165496772742 86
92640/3*x^1965 + 486692823410488180644831603145904234175803552540676122120 4
5974192124530040492736873323257465234124412330158300*x^1964 - 1619284346477
309911045022289431712708011386742573680951279235099171611387560318575857463
87812060653892340362320 0*x^1963 + 158851753123258289905781950481571877213 69
716148045509995247687558258066397972363454649549753782420991368306864174 0/3
*x^1962 - 170223063862965036517691321260730218847531427986813824765787744 79
127845304400476814863593228938446028352505388496 00*x^1961 + 538124384391789
517216364764700378495716308332968001976963812898784093576312866748942586939
4724877001651823267605402 0*x^1960 - 501993044919099344765526320570589597569
353208212314793007726363160441487625941920095769718727916720671362189591966
2400/3*x^1959 + 1023856380803211749492636237726456251780237201851202650337 1
21236172680264154161332467291456088001250633726763096200027 75/2*x^1958 - 15
4129724158107736031243815486191306276012652294 9...
```

Mupad [B]
time = 6.03, size = 15, normalized size = 0.65

$$\frac{(x-1)^{2021}}{2021}+\frac{(x-1)^{2022}}{2022}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x - 1)^2020,x)
```

```
[Out] (x - 1)^2021/2021 + (x - 1)^2022/2022
```

Chatgpt [F] Failed to verify
time = 1.00, size = 20, normalized size = 0.87

$$-\frac{1}{4041\,(1-x)^{2020}}+\frac{(1-x)^{2021}}{4041}+\frac{1}{2021}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*(1-x)^2020,x)
```

```
[Out] -1/4041/(1-x)^2020+1/4041*(1-x)^2021+1/2021
```

3.228 $\int \frac{\sec^4(x)\tan(x)}{4+\sec^4(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{4}\log\left(4+\sec^4(x)\right)$$

```
[Out] 1/4*ln(sec(x)^4+4)
```

Rubi [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.73, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4424, 272, 36, 29, 31}

$$\frac{1}{4}\log\left(4\cos^4(x)+1\right)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^4*Tan[x])/(4 + Sec[x]^4),x]
```

```
[Out] -Log[Cos[x]] + Log[1 + 4*Cos[x]^4]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
```

```
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int \frac{1}{x(1+4x^4)}\,dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+4x)}\,dx, x, \cos^4(x)\right)\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^4(x)\right)\right) + \text{Subst}\left(\int \frac{1}{1+4x}\,dx, x, \cos^4(x)\right)\\
&= -\log(\cos(x)) + \frac{1}{4}\log\left(1+4\cos^4(x)\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.73

$$-\log(\cos(x)) + \frac{1}{4}\log\left(1+4\cos^4(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^4*Tan[x])/(4 + Sec[x]^4),x]
[Out] -Log[Cos[x]] + Log[1 + 4*Cos[x]^4]/4
```

Maple [A]
time = 3.33, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\ln(\sec^4(x)+4)}{4}$	10
default	$\frac{\ln(\sec^4(x)+4)}{4}$	10
risch	$-\ln\left(e^{2ix}+1\right)+\frac{\ln(e^{8ix}+4\,e^{6ix}+10\,e^{4ix}+4\,e^{2ix}+1)}{4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)/(sec(x)^4+4),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(sec(x)^4+4)
```

Maxima [A]
time = 0.32, size = 9, normalized size = 0.82

$$\frac{1}{4}\log\left(\sec(x)^4+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)/(sec(x)^4+4),x, algorithm="maxima")
```

```
[Out] 1/4*log(sec(x)^4 + 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 0.65, size = 19, normalized size = 1.73

$$\frac{1}{4}\log\left(4\cos\left(x\right)^{4}+1\right)-\log\left(-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)/(sec(x)^4+4),x, algorithm="fricas")
```

```
[Out] 1/4*log(4*cos(x)^4 + 1) - log(-cos(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.
time = 0.30, size = 29, normalized size = 2.64

$$\frac{\log\left(\sec^{2}\left(x\right)-2\sec\left(x\right)+2\right)}{4}+\frac{\log\left(\sec^{2}\left(x\right)+2\sec\left(x\right)+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)/(sec(x)**4+4),x)
```

```
[Out] log(sec(x)**2 - 2*sec(x) + 2)/4 + log(sec(x)**2 + 2*sec(x) + 2)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 0.48, size = 19, normalized size = 1.73

$$\frac{1}{4}\log\left(4\cos\left(x\right)^{4}+1\right)-\frac{1}{4}\log\left(\cos\left(x\right)^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)/(sec(x)^4+4),x, algorithm="giac")
```

```
[Out] 1/4*log(4*cos(x)^4 + 1) - 1/4*log(cos(x)^4)
```

Mupad [B]
time = 0.18, size = 15, normalized size = 1.36

$$\frac{\ln\left({\mathrm{tan}\left(x\right)}^{4}+2\,{\mathrm{tan}\left(x\right)}^{2}+5\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)^4*(1/cos(x)^4 + 4)),x)
```

```
[Out] log(2*tan(x)^2 + tan(x)^4 + 5)/4
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 19, normalized size = 1.73

$$\frac{\ln\left(\sec^2\left(x\right)-2\right)}{3}-\frac{\ln\left(\sec^2\left(x\right)+1\right)}{6}$$

Warning: Unable to verify antiderivative.

```
[In] int(sec(x)^4*tan(x)/(sec(x)^4+4),x)
```

```
[Out] 1/3*ln(sec(x)^2-2)-1/6*ln(sec(x)^2+1)
```

3.229 $\int x^{2x}(2+2\log(x))\,dx$

Optimal. Leaf size=5

$$x^{2x}$$

```
[Out] x^(2*x)
```

Rubi [A]
time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6873, 12, 6874, 2633}

$$x^{2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*x)*(2 + 2*Log[x]),x]
```

```
[Out] x^(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2633

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int 2x^{2x}(1+\log(x))\,dx \\
&= 2\int x^{2x}(1+\log(x))\,dx \\
&= 2\int \left(x^{2x}+x^{2x}\log(x)\right)\,dx \\
&= 2\int x^{2x}\,dx + 2\int x^{2x}\log(x)\,dx \\
&= x^{2x}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$x^{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*x)*(2 + 2*Log[x]),x]
[Out] x^(2*x)
```

Maple [A]
time = 0.02, size = 6, normalized size = 1.20

method	result	size
derivativedivides	x^{2x}	6
risch	x^{2x}	6
parallelrisch	x^{2x}	6
norman	$e^{2x\ln(x)}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*x)*(2*ln(x)+2),x,method=_RETURNVERBOSE)
[Out] x^(2*x)
```

Maxima [A]
time = 0.36, size = 5, normalized size = 1.00

$$x^{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*x)*(2*log(x)+2),x, algorithm="maxima")
```

[Out] x^(2*x)

Fricas [**A**]
time = 0.58, size = 5, normalized size = 1.00

$$x^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*x)*(2*log(x)+2),x, algorithm="fricas")
```

```
[Out] x^(2*x)
```

Sympy [**A**]
time = 0.06, size = 3, normalized size = 0.60

$$x^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2*x)*(2*ln(x)+2),x)
```

```
[Out] x**(2*x)
```

Giac [**A**]
time = 0.48, size = 5, normalized size = 1.00

$$x^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*x)*(2*log(x)+2),x, algorithm="giac")
```

```
[Out] x^(2*x)
```

Mupad [**B**]
time = 0.22, size = 5, normalized size = 1.00

$$x^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*x)*(2*log(x) + 2),x)
```

```
[Out] x^(2*x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 8, normalized size = 1.60

$$x^{2}\left(\ln\left(x\right)+\frac{3}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(2*x)*(2*ln(x)+2),x)
```

```
[Out] x^2*(ln(x)+3/4)
```

3.230 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}$$

[Out] 1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 222}

$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}\,x$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\,dx\\&=\frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.61

$$\frac{1}{2}x\sqrt{1-x^2}-\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.78

method	result	size
default	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(\mathit{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

Maxima [A]
time = 0.42, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.58, size = 31, normalized size = 1.35

$$\frac{1}{2}\sqrt{-x^2+1}x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [**A**]
time = 0.07, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\mathrm{asin}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Giac [**A**]
time = 0.52, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [**B**]
time = 0.00, size = 17, normalized size = 0.74

$$\frac{\mathrm{asin}(x)}{2}+\frac{x\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2),x)
```

```
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

Chatgpt [**A**]
time = 1.00, size = 17, normalized size = 0.74

$$\frac{\arcsin\left(x\right)}{2}+\frac{x\sqrt{-x^2+1}}{2}$$

Antiderivative was successfully verified.

```
[In] int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

3.231 $\int e^{-x^4} x^5 \, dx$

Optimal. Leaf size=28

$$-\frac{1}{4}e^{-x^4}x^2 + \frac{1}{8}\sqrt{\pi}\text{erf}\left(x^2\right)$$

```
[Out] -1/4*x^2/exp(x^4)+1/8*Pi^(1/2)*erf(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2243, 2242, 2236}

$$\frac{1}{8}\sqrt{\pi}\text{erf}\left(x^2\right) - \frac{1}{4}e^{-x^4}x^2$$

Antiderivative was successfully verified.

```
[In] Int[x^5/E^x^4,x]
```

```
[Out] -1/4*x^2/E^x^4 + (Sqrt[Pi]*Erf[x^2])/8
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2242

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Dist[1/(d*(m + 1)), Subst[Int[F^(a + b*x^2), x], x, (c + d
*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\frac{1}{4}e^{-x^4}x^2 + \frac{1}{2}\int e^{-x^4}x\,dx \\ &= -\frac{1}{4}e^{-x^4}x^2 + \frac{1}{4}\text{Subst}\left(\int e^{-x^2}\,dx, x, x^2\right) \\ &= -\frac{1}{4}e^{-x^4}x^2 + \frac{1}{8}\sqrt{\pi}\text{erf}(x^2)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{1}{4}e^{-x^4}x^2 + \frac{1}{8}\sqrt{\pi}\text{erf}(x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/E^x^4,x]
[Out] -1/4*x^2/E^x^4 + (Sqrt[Pi]*Erf[x^2])/8
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.79

method	result	size
meijerg	$-\frac{x^2 e^{-x^4}}{4} + \frac{\sqrt{\pi}\,\text{erf}(x^2)}{8}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*exp(-x^4),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2*exp(-x^4)+1/8*Pi^(1/2)*erf(x^2)
```

Maxima [A]
time = 0.35, size = 21, normalized size = 0.75

$$-\frac{1}{4}\,x^2 e^{(-x^4)} + \frac{1}{8}\,\sqrt{\pi}\,\text{erf}\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*exp(-x^4),x, algorithm="maxima")
[Out] -1/4*x^2*e^(-x^4) + 1/8*sqrt(pi)*erf(x^2)
```

Fricas [A]
time = 0.59, size = 21, normalized size = 0.75

$$-\frac{1}{4}\,x^2 e^{(-x^4)} + \frac{1}{8}\,\sqrt{\pi}\,\text{erf}\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*exp(-x^4),x, algorithm="fricas")
```

```
[Out] -1/4*x^2*e^(-x^4) + 1/8*sqrt(pi)*erf(x^2)
```

Sympy [A]
time = 0.58, size = 20, normalized size = 0.71

$$-\frac{x^2e^{-x^4}}{4}+\frac{\sqrt{\pi}\,\mathrm{erf}\,(x^2)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*exp(-x**4),x)
```

```
[Out] -x**2*exp(-x**4)/4 + sqrt(pi)*erf(x**2)/8
```

Giac [A]
time = 0.50, size = 35, normalized size = 1.25

$$-\frac{\sqrt{\pi}\left(\frac{2\sqrt{x^4}e^{(-x^4)}}{\sqrt{\pi}}-\mathrm{erf}\left(\sqrt{x^4}\right)\right)|x|}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*exp(-x^4),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*(2*sqrt(x^4)*e^(-x^4)/sqrt(pi) - erf(sqrt(x^4)))*abs(x)/x
```

Mupad [B]
time = 0.11, size = 21, normalized size = 0.75

$$\frac{\sqrt{\pi}\,\mathrm{erf}(x^2)}{8}-\frac{x^2\,\mathrm{e}^{-x^4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*exp(-x^4),x)
```

```
[Out] (pi^(1/2)*erf(x^2))/8 - (x^2*exp(-x^4))/4
```

Chatgpt [F] Failed to verify
time = 1.00, size = 23, normalized size = 0.82

$$-\frac{x^4\mathrm{e}^{-x^4}}{4}-\frac{5x^{\frac{1}{4}}\mathrm{e}^{-x^4}}{16}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^5*exp(-x^4),x)
```

```
[Out] -1/4*x^4*exp(-x^4)-5/16*x^(1/4)*exp(-x^4)
```

3.232 $\int \frac{1+\cos(x)}{x+\sin(x)}\,dx$

Optimal. Leaf size=5

$$\log(x+\sin(x))$$

[Out] ln(x+sin(x))

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6816}

$$\log(x+\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])/(x + Sin[x]),x]

[Out] Log[x + Sin[x]]

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\text{Integral} = \log(x+\sin(x))$$

Mathematica [A]
time = 0.03, size = 5, normalized size = 1.00

$$\log(x+\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x])/(x + Sin[x]),x]

[Out] Log[x + Sin[x]]

Maple [A]
time = 0.12, size = 6, normalized size = 1.20

method	result	size

derivativedivides	$\ln(x+\sin(x))$	6
default	$\ln(x+\sin(x))$	6
risch	$-ix+\ln(e^{2ix}+2ix\,e^{ix}-1)$	23
parallelrisch	$-\ln\left(\frac{1}{1+\cos(x)}\right)+\ln\left(\frac{x+\sin(x)}{1+\cos(x)}\right)$	23
norman	$-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)+\ln\left(x\left(\tan^2\left(\frac{x}{2}\right)\right)+x+2\tan\left(\frac{x}{2}\right)\right)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))/(x+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x+sin(x))
```

Maxima [A]
time = 0.34, size = 5, normalized size = 1.00

$$\log(x+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))/(x+sin(x)),x, algorithm="maxima")
```

```
[Out] log(x + sin(x))
```

Fricas [A]
time = 0.62, size = 5, normalized size = 1.00

$$\log(x+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))/(x+sin(x)),x, algorithm="fricas")
```

```
[Out] log(x + sin(x))
```

Sympy [A]
time = 0.05, size = 5, normalized size = 1.00

$$\log(x+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))/(x+sin(x)),x)
```

```
[Out] log(x + sin(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(5) = 10.
time = 0.50, size = 72, normalized size = 14.40

$$\frac{1}{2}\log\left(\frac{4\left(x^2\tan\left(\frac{1}{2}x\right)^4+2x^2\tan\left(\frac{1}{2}x\right)^2+4x\tan\left(\frac{1}{2}x\right)^3+x^2+4x\tan\left(\frac{1}{2}x\right)+4\tan\left(\frac{1}{2}x\right)^2\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))/(x+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(4*(x^2*tan(1/2*x)^4 + 2*x^2*tan(1/2*x)^2 + 4*x*tan(1/2*x)^3 + x^2 +
 4*x*tan(1/2*x) + 4*tan(1/2*x)^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))
```

Mupad [B]
time = 0.09, size = 5, normalized size = 1.00

$$\ln\left(x+\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)/(x + sin(x)),x)
```

```
[Out] log(x + sin(x))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((1+cos(x))/(x+sin(x)),x)
```

```
[Out] not solved
```

3.233 $\int \frac{\cot^{-1}(x)+\arctan(x)}{x}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{x}\right)+\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)$$

```
[Out] -1/2*I*polylog(2,-I/x)+1/2*I*polylog(2,I/x)+1/2*I*polylog(2,-I*x)-1/2*I*pol
ylog(2,I*x)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {14, 4941, 2438, 4940}

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i}{x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i}{x}\right)+\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)$$

Antiderivative was successfully verified.

```
[In] Int[(ArcCot[x] + ArcTan[x])/x,x]
```

```
[Out] (-1/2*I)*PolyLog[2, (-I)/x] + (I/2)*PolyLog[2, I/x] + (I/2)*PolyLog[2, (-I)
*x] - (I/2)*PolyLog[2, I*x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4941

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (-Dist[I*(b/2), Int[Log[1 + I/(c*x)]/x, x], x] + Dist[I*(b/2), Int[Log[1
 - I/(c*x)]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int \left(\frac{\cot^{-1}(x)}{x}+\frac{\arctan(x)}{x}\right) dx \\
&= \int \frac{\cot^{-1}(x)}{x}\, dx + \int \frac{\arctan(x)}{x}\, dx \\
&= \frac{1}{2} i \int \frac{\log\left(1-\frac{i}{x}\right)}{x}\, dx - \frac{1}{2} i \int \frac{\log\left(1+\frac{i}{x}\right)}{x}\, dx + \frac{1}{2} i \int \frac{\log(1-ix)}{x}\, dx - \frac{1}{2} i \int \frac{\log(1+ix)}{x}\, dx \\
&= -\frac{1}{2} i \operatorname{PolyLog}\left(2,-\frac{i}{x}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2,\frac{i}{x}\right) + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.00

$$
-\frac{1}{2} i \operatorname{PolyLog}\left(2,-\frac{i}{x}\right) + \frac{1}{2} i \operatorname{PolyLog}\left(2,\frac{i}{x}\right) + \frac{1}{2} i \operatorname{PolyLog}(2,-ix) - \frac{1}{2} i \operatorname{PolyLog}(2,ix)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcCot[x] + ArcTan[x])/x,x]
[Out] (-1/2*I)*PolyLog[2, (-I)/x] + (I/2)*PolyLog[2, I/x] + (I/2)*PolyLog[2, (-I)
*x] - (I/2)*PolyLog[2, I*x]
```

Maple [A]
time = 0.04, size = 12, normalized size = 0.21

method	result	size
default	$\ln(x)\operatorname{arccot}(x)+\ln(x)\arctan(x)$	12
parts	$\ln(x)\operatorname{arccot}(x)+\ln(x)\arctan(x)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((arctan(x)+arccot(x))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*arccot(x)+ln(x)*arctan(x)
```

Maxima [A]
time = 0.56, size = 9, normalized size = 0.16

$$
(\arctan(x)+\arctan(1,x))\log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((arctan(x)+arccot(x))/x,x, algorithm="maxima")
[Out] (arctan(x) + arctan2(1, x))*log(x)
```

Fricas [A]
time = 0.57, size = 5, normalized size = 0.09

$$-\frac{1}{2}\,\pi \log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((arctan(x)+arccot(x))/x,x, algorithm="fricas")
```

```
[Out] -1/2*pi*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acot}\left(x\right) + \operatorname{atan}\left(x\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((atan(x)+acot(x))/x,x)
```

```
[Out] Integral((acot(x) + atan(x))/x, x)
```

Giac [A]
time = 0.43, size = 5, normalized size = 0.09

$$-\frac{1}{2}\,\pi \log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((arctan(x)+arccot(x))/x,x, algorithm="giac")
```

```
[Out] -1/2*pi*log(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}(x)+\mathrm{acot}(x)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((atan(x) + acot(x))/x,x)
```

```
[Out] int((atan(x) + acot(x))/x, x)
```

Chatgpt [A] valid for real x
time = 1.00, size = 5, normalized size = 0.09

$$\frac{\pi \ln \left(x\right)}{2}$$

Antiderivative was successfully verified.

```
[In] int((arctan(x)+arccot(x))/x,x)
```

```
[Out] 1/2*Pi*ln(x)
```

3.234 $\int \frac{\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2}(-x+e^x\sinh(x))$$

[Out] -1/2*x+1/2*exp(x)*sinh(x)

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.50, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3160, 8}

$$\frac{\sinh(x)}{2(\cosh(x)-\sinh(x))}-\frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[x]/(Cosh[x] - Sinh[x]),x]

[Out] -1/2*x + Sinh[x]/(2*(Cosh[x] - Sinh[x]))

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3160

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c
 + d*x])^n/(2*b*d*n*Sin[c + d*x]^n)), x] + Dist[1/(2*b), Int[(a*Cos[c + d*x
] + b*Sin[c + d*x])^(n + 1)/Sin[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c,
 d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\frac{\sinh(x)}{2(\cosh(x)-\sinh(x))}-\frac{\int 1\,dx}{2}\\&=-\frac{x}{2}+\frac{\sinh(x)}{2(\cosh(x)-\sinh(x))}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 22, normalized size = 1.57

$$-\frac{x}{2}+\frac{\cosh^2(x)}{2}+\frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(Cosh[x] - Sinh[x]),x]
```

```
[Out] -1/2*x + Cosh[x]^2/2 + Sinh[2*x]/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(10) = 20$.
time = 0.07, size = 36, normalized size = 2.57

method	result	size
risch	$-\frac{x}{2}+\frac{{\mathrm e}^{2 x}}{4}$	11
parallelrisch	$\frac{-\tanh \left(x \right) x +x -1}{2 \tanh \left(x \right)-2}$	18
default	$-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)}{2}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{\tanh \left(\frac{x}{2}\right)-1}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)}{2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x)-sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(tanh(1/2*x)+1)+1/(tanh(1/2*x)-1)^2+1/(tanh(1/2*x)-1)+1/2*ln(tanh(1/
2*x)-1)
```

Maxima [A]
time = 0.34, size = 10, normalized size = 0.71

$$-\frac{1}{2}\,x + \frac{1}{4}\,e^{\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*e^(2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.
time = 0.57, size = 29, normalized size = 2.07

$$-\frac{{\left(2\,x - 1\right)}\cosh\left(x\right) - {\left(2\,x + 1\right)}\sinh\left(x\right)}{4\,{\left(\cosh\left(x\right) - \sinh\left(x\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((2*x - 1)*cosh(x) - (2*x + 1)*sinh(x))/(cosh(x) - sinh(x))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(10) = 20$.
time = 0.21, size = 42, normalized size = 3.00

$$\frac{x\sinh(x)}{-2\sinh(x)+2\cosh(x)}-\frac{x\cosh(x)}{-2\sinh(x)+2\cosh(x)}+\frac{\cosh(x)}{-2\sinh(x)+2\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] x*sinh(x)/(-2*sinh(x) + 2*cosh(x)) - x*cosh(x)/(-2*sinh(x) + 2*cosh(x)) + c
osh(x)/(-2*sinh(x) + 2*cosh(x))
```

Giac [**A**]
time = 0.50, size = 10, normalized size = 0.71

$$-\frac{1}{2}x+\frac{1}{4}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(cosh(x)-sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2*x + 1/4*e^(2*x)
```

Mupad [**B**]
time = 0.15, size = 10, normalized size = 0.71

$$\frac{e^{2x}}{4}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(cosh(x) - sinh(x)),x)
```

```
[Out] exp(2*x)/4 - x/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 7, normalized size = 0.50

$$-\ln(\sinh(x)-1)$$

Warning: Unable to verify antiderivative.

```
[In] int(sinh(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] -ln(sinh(x)-1)
```

3.235 $\int \frac{x}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{3}(-1+x)^{3/2}-\frac{1}{5}(-1+x)^{5/2}-\frac{1}{3}(1+x)^{3/2}+\frac{1}{5}(1+x)^{5/2}$$

```
[Out] -1/3*(x-1)^(3/2)-1/5*(x-1)^(5/2)-1/3*(x+1)^(3/2)+1/5*(x+1)^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2129, 45}

$$-\frac{1}{5}(x-1)^{5/2}-\frac{1}{3}(x-1)^{3/2}+\frac{1}{5}(x+1)^{5/2}-\frac{1}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] -1/3*(-1 + x)^(3/2) - (-1 + x)^(5/2)/5 - (1 + x)^(3/2)/3 + (1 + x)^(5/2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\left(\frac{1}{2}\int \sqrt{-1+x}x\,dx\right)+\frac{1}{2}\int x\sqrt{1+x}\,dx\\
&= -\left(\frac{1}{2}\int \left(\sqrt{-1+x}+(-1+x)^{3/2}\right)dx\right)+\frac{1}{2}\int \left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\
&= -\frac{1}{3}(-1+x)^{3/2}-\frac{1}{5}(-1+x)^{5/2}-\frac{1}{3}(1+x)^{3/2}+\frac{1}{5}(1+x)^{5/2}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 34, normalized size = 0.76

$$\frac{1}{15}\left((1+x)^{3/2}(-2+3x)+\sqrt{-1+x}\left(2+x-3x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
[Out] ((1 + x)^(3/2)*(-2 + 3*x) + Sqrt[-1 + x]*(2 + x - 3*x^2))/15
```

Maple [A]
time = 0.02, size = 30, normalized size = 0.67

method	result	size
default	$-\frac{(x-1)^{\frac{3}{2}}}{3}-\frac{(x-1)^{\frac{5}{2}}}{5}-\frac{(x+1)^{\frac{3}{2}}}{3}+\frac{(x+1)^{\frac{5}{2}}}{5}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x-1)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/3*(x-1)^(3/2)-1/5*(x-1)^(5/2)-1/3*(x+1)^(3/2)+1/5*(x+1)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x-1)^(1/2)+(x+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Fricas [A]
time = 0.57, size = 33, normalized size = 0.73

$$\frac{1}{15}\left(3x^2+x-2\right)\sqrt{x+1}-\frac{1}{15}\left(3x^2-x-2\right)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x-1)^(1/2)+(x+1)^(1/2)),x, algorithm="fricas")
[Out] 1/15*(3*x^2 + x - 2)*sqrt(x + 1) - 1/15*(3*x^2 - x - 2)*sqrt(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x-1}+\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x-1)**(1/2)+(x+1)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(x - 1) + sqrt(x + 1)), x)
```

Giac [A]
time = 0.45, size = 33, normalized size = 0.73

$$\frac{1}{5}(x+1)^{\frac{5}{2}} - \frac{1}{3}(x+1)^{\frac{3}{2}} - \frac{1}{15}((3x-4)(x+1)+2)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((x-1)^(1/2)+(x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*(x + 1)^(5/2) - 1/3*(x + 1)^(3/2) - 1/15*((3*x - 4)*(x + 1) + 2)*sqrt(x
 - 1)
```

Mupad [B]
time = 0.16, size = 51, normalized size = 1.13

$$\frac{x\sqrt{x-1}}{15} + \frac{x\sqrt{x+1}}{15} + \frac{2\sqrt{x-1}}{15} - \frac{2\sqrt{x+1}}{15} - \frac{x^2\sqrt{x-1}}{5} + \frac{x^2\sqrt{x+1}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

```
[Out] (x*(x - 1)^(1/2))/15 + (x*(x + 1)^(1/2))/15 + (2*(x - 1)^(1/2))/15 - (2*(x
+ 1)^(1/2))/15 - (x^2*(x - 1)^(1/2))/5 + (x^2*(x + 1)^(1/2))/5
```

Chatgpt [F] Failed to verify
time = 1.00, size = 15, normalized size = 0.33

$$\frac{4(x-1)^{\frac{3}{2}}}{3} - 2\sqrt{x-1}$$

Warning: Unable to verify antiderivative.

```
[In] int(x/((x-1)^(1/2)+(x+1)^(1/2)),x)
```

```
[Out] 4/3*(x-1)^(3/2)-2*(x-1)^(1/2)
```

3.236 $\int \cos(x+\cos(x))\,dx$

Optimal. Leaf size=8

$$\text{Int}(\cos(x+\cos(x)),x)$$

```
[Out] CannotIntegrate(cos(x+cos(x)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos(x+\cos(x))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[x + Cos[x]],x]
[Out] Defer[Int][Cos[x + Cos[x]], x]
```

Rubi steps

$$\text{Integral} = \int \cos(x+\cos(x))\,dx$$

Mathematica [A]
time = 0.73, size = 0, normalized size = 0.00

$$\int \cos(x+\cos(x))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[x + Cos[x]],x]
[Out] Integrate[Cos[x + Cos[x]], x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(x+\cos\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x+cos(x)),x)
[Out] int(cos(x+cos(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x+cos(x)),x, algorithm="maxima")
[Out] integrate(cos(x + cos(x)), x)
```

Fricas [A]
time = 0.60, size = 7, normalized size = 0.88

$$\text{integral}(\cos(x + \cos(x)), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x+cos(x)),x, algorithm="fricas")
[Out] integral(cos(x + cos(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x + \cos(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x+cos(x)),x)
[Out] Integral(cos(x + cos(x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x+cos(x)),x, algorithm="giac")
[Out] integrate(cos(x + cos(x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.12

$$\int \cos(x + \cos(x))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x + cos(x)),x)
```

```
[Out] int(cos(x + cos(x)), x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(cos(x+cos(x)),x)
```

```
[Out] not solved
```

3.237 $\int x^3 \sin(x^2)\, dx$

Optimal. Leaf size=20

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}$$

[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 3377, 2717}

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^3*Sin[x^2],x]

[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}\text{Subst}\left(\int x\sin(x)\,dx,x,x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx,x,x^2\right)\\ &= -\frac{1}{2}x^2\cos\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Maple [A]
time = 0.08, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$	17
default	$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$	17
risch	$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$	17
parallelrisch	$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$	17
meijerg	$\sqrt{\pi}\left(-\frac{x^2\cos\left(x^2\right)}{2\sqrt{\pi}}+\frac{\sin\left(x^2\right)}{2\sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^2}{2}+\frac{x^2\left(\tan^2\left(\frac{x^2}{2}\right)\right)}{2}+\tan\left(\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{x^2}{2}\right)}$	39
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x^3}{2}-\frac{3\pi^2\left(\frac{2\,\mathrm{S}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x^3}{3\pi^{\frac{3}{2}}}+\frac{2x^2\cos\left(x^2\right)}{3\pi^2}-\frac{2\sin\left(x^2\right)}{3\pi^2}\right)}{4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A]
time = 0.33, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A]
time = 0.60, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A]
time = 0.12, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Giac [A]
time = 0.50, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B]
time = 0.07, size = 16, normalized size = 0.80

$$\frac{\sin\left(x^2\right)}{2}-\frac{x^2\,\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

Chatgpt [A]
time = 1.00, size = 16, normalized size = 0.80

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] int(x^3*sin(x^2),x)
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

3.238 $\int \frac{x}{1-x^4} \, dx$

Optimal. Leaf size=8

$$\frac{\operatorname{arctanh}(x^2)}{2}$$

[Out] 1/2*arctanh(x^2)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 212}

$$\frac{\operatorname{arctanh}(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x/(1 - x^4),x]

[Out] ArcTanh[x^2]/2

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, x^2\right) \\ &= \frac{\operatorname{arctanh}(x^2)}{2} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.
time = 0.00, size = 23, normalized size = 2.88

$$-\frac{1}{4}\log\left(1-x^2\right)+\frac{1}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^4),x]
```

```
[Out] -1/4*Log[1 - x^2] + Log[1 + x^2]/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(6) = 12$.
time = 0.07, size = 22, normalized size = 2.75

method	result	size
meijerg	$\frac{\operatorname{arctanh}(x^2)}{2}$	7
risch	$\frac{\ln(x^2+1)}{4} - \frac{\ln(x^2-1)}{4}$	18
default	$-\frac{\ln(x-1)}{4} - \frac{\ln(x+1)}{4} + \frac{\ln(x^2+1)}{4}$	22
norman	$-\frac{\ln(x-1)}{4} - \frac{\ln(x+1)}{4} + \frac{\ln(x^2+1)}{4}$	22
parallelrisch	$-\frac{\ln(x-1)}{4} - \frac{\ln(x+1)}{4} + \frac{\ln(x^2+1)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(x-1)-1/4*ln(x+1)+1/4*ln(x^2+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.37, size = 17, normalized size = 2.12

$$\frac{1}{4}\log\left(x^2+1\right) - \frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.56, size = 17, normalized size = 2.12

$$\frac{1}{4}\log\left(x^2+1\right) - \frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^2 + 1) - 1/4*log(x^2 - 1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.03, size = 15, normalized size = 1.88

$$-\frac{\log(x^2-1)}{4}+\frac{\log(x^2+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+1),x)
```

```
[Out] -log(x**2 - 1)/4 + log(x**2 + 1)/4
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.
time = 0.53, size = 18, normalized size = 2.25

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{4}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2 + 1) - 1/4*log(abs(x^2 - 1))
```

Mupad [**B**]
time = 0.00, size = 6, normalized size = 0.75

$$\frac{\operatorname{atanh}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x^4 - 1),x)
```

```
[Out] atanh(x^2)/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 8, normalized size = 1.00

$$-\frac{\ln(x^2-1)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(x/(-x^4+1),x)
```

```
[Out] -1/2*ln(x^2-1)
```

3.239 $\int \text{sech}^2(x)\,dx$

Optimal. Leaf size=2

$$\tanh(x)$$

[Out] tanh(x)

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8}

$$\tanh(x)$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^2,x]

[Out] Tanh[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= i\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)\\ &= \tanh(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 2, normalized size = 1.00

$$\tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^2,x]

[Out] Tanh[x]

Maple [A]
time = 0.09, size = 3, normalized size = 1.50

method	result	size
default	$\tanh(x)$	3
parallelrisch	$\tanh(x)$	3
risch	$-\frac{2}{1+e^{2x}}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] tanh(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(2) = 4.
time = 0.34, size = 10, normalized size = 5.00

$$\frac{2}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^2,x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(2) = 4.
time = 0.55, size = 20, normalized size = 10.00

$$-\frac{2}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^2,x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(2) = 4.
time = 0.32, size = 14, normalized size = 7.00

$$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)**2,x)
```

```
[Out] 2*tanh(x/2)/(tanh(x/2)**2 + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.
time = 0.46, size = 10, normalized size = 5.00

$$-\frac{2}{e^{(2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^2,x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) + 1)
```

Mupad [B]
time = 0.01, size = 2, normalized size = 1.00

$$\tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x)^2,x)
```

```
[Out] tanh(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 4.50

$$\tanh(x) - \frac{(\tanh^3(x))}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/cosh(x)^2,x)
```

```
[Out] tanh(x)-1/3*tanh(x)^3
```

3.240 $\int \left(e^{e^x} - e^{e^x - x}\right) dx$

Optimal. Leaf size=9

$$e^{e^x - x}$$

[Out] exp(exp(x)-x)

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2320, 2209, 2208}

$$e^{e^x - x}$$

Antiderivative was successfully verified.

[In] Int[E^E^x - E^(E^x - x),x]

[Out] E^(E^x - x)

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int e^{e^x}\,dx - \int e^{e^x-x}\,dx \\
&= -\text{Subst}\left(\int \frac{e^x}{x^2}\,dx, x, e^x\right) + \text{Subst}\left(\int \frac{e^x}{x}\,dx, x, e^x\right) \\
&= e^{e^x-x} + \text{ExpIntegralEi}\left(e^x\right) - \text{Subst}\left(\int \frac{e^x}{x}\,dx, x, e^x\right) \\
&= e^{e^x-x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$e^{e^x-x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^E^x - E^(E^x - x),x]
[Out] E^(E^x - x)
```

Maple [A]
time = 0.03, size = 9, normalized size = 1.00

method	result	size
risch	$\mathrm{e}^{\mathrm{e}^{x}-x}$	8
default	$\mathrm{e}^{\mathrm{e}^{x}}\mathrm{e}^{-x}$	9
norman	$\mathrm{e}^{\mathrm{e}^{x}}\mathrm{e}^{-x}$	9
parts	$\mathrm{e}^{\mathrm{e}^{x}}\mathrm{e}^{-x}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(x))-exp(exp(x)-x),x,method=_RETURNVERBOSE)
[Out] exp(exp(x))/exp(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x))-exp(exp(x)-x),x, algorithm="maxima")
[Out] Ei(e^x) - integrate(e^(-x + e^x), x)
```

Fricas [**A**]
time = 0.57, size = 7, normalized size = 0.78

$$e^{(-x+e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x))-exp(exp(x)-x),x, algorithm="fricas")
[Out] e^(-x + e^x)
```

Sympy [**A**]
time = 0.04, size = 7, normalized size = 0.78

$$e^{-x}e^{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x))-exp(exp(x)-x),x)
[Out] exp(-x)*exp(exp(x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(x))-exp(exp(x)-x),x, algorithm="giac")
[Out] integrate(-e^(-x + e^x) + e^(e^x), x)
```

Mupad [**B**]
time = 0.12, size = 7, normalized size = 0.78

$$\mathrm{e}^{\mathrm{e}^x-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(x)) - exp(exp(x) - x),x)
[Out] exp(exp(x) - x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 9, normalized size = 1.00

$$x\,\mathrm{e}^{\mathrm{e}^x-x}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(exp(x))-exp(exp(x)-x),x)
[Out] x*exp(exp(x)-x)
```

3.241 $\int \sqrt{1-\sqrt{x}}\,dx$

Optimal. Leaf size=35

$$-\frac{4}{3}\left(1-\sqrt{x}\right)^{3/2}+\frac{4}{5}\left(1-\sqrt{x}\right)^{5/2}$$

```
[Out] -4/3*(1-x^(1/2))^(3/2)+4/5*(1-x^(1/2))^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {196, 45}

$$\frac{4}{5}\left(1-\sqrt{x}\right)^{5/2}-\frac{4}{3}\left(1-\sqrt{x}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x]],x]
```

```
[Out] (-4*(1 - Sqrt[x])^(3/2))/3 + (4*(1 - Sqrt[x])^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int \sqrt{1-x}x\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{4}{3}\left(1-\sqrt{x}\right)^{3/2}+\frac{4}{5}\left(1-\sqrt{x}\right)^{5/2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.74

$$-\frac{4}{15}\left(1-\sqrt{x}\right)^{3/2}\left(2+3\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x]],x]
[Out] (-4*(1 - Sqrt[x])^(3/2)*(2 + 3*Sqrt[x]))/15
```

Maple [A]
time = 0.08, size = 24, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{4\left(1-\sqrt{x}\right)^{\frac{3}{2}}}{3}+\frac{4\left(1-\sqrt{x}\right)^{\frac{5}{2}}}{5}$	24
default	$-\frac{4\left(1-\sqrt{x}\right)^{\frac{3}{2}}}{3}+\frac{4\left(1-\sqrt{x}\right)^{\frac{5}{2}}}{5}$	24
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,\left(1-\sqrt{x}\right)^{\frac{3}{2}}\left(3\sqrt{x}+2\right)}{15}}{\sqrt{\pi}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -4/3*(1-x^(1/2))^(3/2)+4/5*(1-x^(1/2))^(5/2)
```

Maxima [A]
time = 0.38, size = 23, normalized size = 0.66

$$\frac{4}{5}\left(-\sqrt{x}+1\right)^{\frac{5}{2}}-\frac{4}{3}\left(-\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))^(1/2),x, algorithm="maxima")
[Out] 4/5*(-sqrt(x) + 1)^(5/2) - 4/3*(-sqrt(x) + 1)^(3/2)
```

Fricas [A]
time = 0.58, size = 21, normalized size = 0.60

$$\frac{4}{15}\left(3\,x-\sqrt{x}-2\right)\sqrt{-\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 4/15*(3*x - sqrt(x) - 2)*sqrt(-sqrt(x) + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 306, normalized size = 8.74

$$\begin{cases} -\frac{12ix^{\frac{7}{2}}\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^{2}}+\frac{4ix^{\frac{5}{2}}\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^{2}}-\frac{8x^{\frac{5}{2}}}{-15x^{\frac{5}{2}}+15x^{2}}+\frac{16ix^{3}\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^{2}}-\frac{8ix^{2}\sqrt{\sqrt{x}-1}}{-15x^{\frac{5}{2}}+15x^{2}}+\frac{8x^{2}}{-15x^{\frac{5}{2}}+15x^{2}} & \text{for}\ \left|\sqrt{x}\right|>1 \\ -\frac{12x^{\frac{7}{2}}\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^{2}}+\frac{4x^{\frac{5}{2}}\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^{2}}-\frac{8x^{\frac{5}{2}}}{-15x^{\frac{5}{2}}+15x^{2}}+\frac{16x^{3}\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^{2}}-\frac{8x^{2}\sqrt{1-\sqrt{x}}}{-15x^{\frac{5}{2}}+15x^{2}}+\frac{8x^{2}}{-15x^{\frac{5}{2}}+15x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))**(1/2),x)
```

```
[Out] Piecewise((-12*I*x**(7/2)*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) + 4*I*
x**(5/2)*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) - 8*x**(5/2)/(-15*x**(5
/2) + 15*x**2) + 16*I*x**3*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) - 8*I
*x**2*sqrt(sqrt(x) - 1)/(-15*x**(5/2) + 15*x**2) + 8*x**2/(-15*x**(5/2) + 1
5*x**2), Abs(sqrt(x)) > 1), (-12*x**(7/2)*sqrt(1 - sqrt(x))/(-15*x**(5/2) +
 15*x**2) + 4*x**(5/2)*sqrt(1 - sqrt(x))/(-15*x**(5/2) + 15*x**2) - 8*x**(5
/2)/(-15*x**(5/2) + 15*x**2) + 16*x**3*sqrt(1 - sqrt(x))/(-15*x**(5/2) + 15
*x**2) - 8*x**2*sqrt(1 - sqrt(x))/(-15*x**(5/2) + 15*x**2) + 8*x**2/(-15*x*
*(5/2) + 15*x**2), True))
```

Giac [A]
time = 0.50, size = 30, normalized size = 0.86

$$\frac{4}{5}\left(\sqrt{x}-1\right)^{2}\sqrt{-\sqrt{x}+1}-\frac{4}{3}\left(-\sqrt{x}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/5*(sqrt(x) - 1)^2*sqrt(-sqrt(x) + 1) - 4/3*(-sqrt(x) + 1)^(3/2)
```

Mupad [B]
time = 0.19, size = 10, normalized size = 0.29

$$x\,{}_2F_1\left(-\frac{1}{2},2;\ 3;\ \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^(1/2))^(1/2),x)
```

```
[Out] x*hypergeom([-1/2, 2], 3, x^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 11, normalized size = 0.31

$$-\frac{4\left(1-\sqrt{x}\right)^{\frac{5}{2}}}{5}$$

Warning: Unable to verify antiderivative.

```
[In] int((1-x^(1/2))^(1/2),x)
```

```
[Out] -4/5*(1-x^(1/2))^(5/2)
```

3.242 $\int \frac{x^3}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}\,dx$

Optimal. Leaf size=24

$$6x-6\log\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)$$

```
[Out] 6*x-6*ln(1+x+1/2*x^2+1/6*x^3)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 193 vs. $2(24) = 48$.
time = 0.43, antiderivative size = 193, normalized size of antiderivative = 8.04, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2106, 2104, 1642, 642}

$$6x-6\log\left(\sqrt[3]{\sqrt{2}-1}(x+1)-\left(\sqrt{2}-1\right)^{2/3}+1\right)-\frac{12(1-\sqrt{2})\left(1-\sqrt[3]{\sqrt{2}-1}-(\sqrt{2}-1)^{2/3}\right)\log\left((\sqrt{2}-1)^{2/3}(x+1)^2-\left(1-\sqrt{2}+\sqrt[3]{\sqrt{2}-1}\right)(x+1)+(\sqrt{2}-1)^{4/3}+(\sqrt{2}-1)^{2/3}+1\right)}{\left(1-\sqrt{2}+\sqrt[3]{\sqrt{2}-1}\right)\left(1-(\sqrt{2}-1)^{2/3}+(\sqrt{2}-1)^{4/3}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x + x^2/2 + x^3/6),x]
```

```
[Out] 6*x - 6*Log[1 - (-1 + Sqrt[2])^(2/3) + (-1 + Sqrt[2])^(1/3)*(1 + x)] - (12*
(1 - Sqrt[2])*(1 - (-1 + Sqrt[2])^(1/3) - (-1 + Sqrt[2])^(2/3))*Log[1 + (-1
 + Sqrt[2])^(2/3) + (-1 + Sqrt[2])^(4/3) - (1 - Sqrt[2] + (-1 + Sqrt[2])^(1
/3))*(1 + x) + (-1 + Sqrt[2])^(2/3)*(1 + x)^2])/((1 - Sqrt[2] + (-1 + Sqrt[
2])^(1/3))*(1 - (-1 + Sqrt[2])^(2/3) + (-1 + Sqrt[2])^(4/3)))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2104

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Dist[1/d^(2*p), Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*
(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Free
Q[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

Rule 2106

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{(-1+x)^3}{\frac{1}{3}+\frac{x}{2}+\frac{x^3}{6}}\,dx, x, 1+x\right)\\
&= \frac{1}{36}\text{Subst}\left(\int \frac{(-1+x)^3}{\left(\frac{1-\left(-1+\sqrt{2}\right)^{2/3}}{6\sqrt[3]{-1+\sqrt{2}}}+\frac{x}{6}\right)\left(\frac{1}{36}\left(1+\frac{1}{\left(-1+\sqrt{2}\right)^{2/3}}+\left(-1+\sqrt{2}\right)^{2/3}\right)-\frac{\left(1-\left(-1+\sqrt{2}\right)^{2/3}\right)x}{36\sqrt[3]{-1+\sqrt{2}}}+\frac{x^2}{36}\right)}\,dx, x, 1+x\right)\\
&= \frac{1}{36}\text{Subst}\left(\int \left(216+\frac{216\sqrt[3]{-1+\sqrt{2}}}{-1+\left(-1+\sqrt{2}\right)^{2/3}-\sqrt[3]{-1+\sqrt{2}}x}+\frac{432\left(\left(1-\sqrt{2}\right)\left(1-\sqrt[3]{-1+\sqrt{2}}-\left(-1+\sqrt{2}\right)^{2/3}\right)-\left(3-2\sqrt{2}+\left(-1+\sqrt{2}\right)^{2/3}-\left(-1+\sqrt{2}\right)^{4/3}\right)x\right)}{\left(1-\left(-1+\sqrt{2}\right)^{2/3}+\left(-1+\sqrt{2}\right)^{4/3}\right)\left(1+\left(-1+\sqrt{2}\right)^{2/3}+\left(-1+\sqrt{2}\right)^{4/3}-\left(1-\sqrt{2}+\sqrt[3]{-1+\sqrt{2}}\right)x+\left(-1+\sqrt{2}\right)^{2/3}x^2\right)}\right)dx, x, 1+x\right)\\
&= 6x-6\log\left(1-\left(-1+\sqrt{2}\right)^{2/3}+\sqrt[3]{-1+\sqrt{2}}(1+x)\right)+\frac{12\text{Subst}\left(\int \frac{\left(1-\sqrt{2}\right)\left(1-\sqrt[3]{-1+\sqrt{2}}-\left(-1+\sqrt{2}\right)^{2/3}\right)-\left(3-2\sqrt{2}+\left(-1+\sqrt{2}\right)^{2/3}-\left(-1+\sqrt{2}\right)^{4/3}\right)x}{1+\left(-1+\sqrt{2}\right)^{2/3}+\left(-1+\sqrt{2}\right)^{4/3}+\left(-1+\sqrt{2}-\sqrt[3]{-1+\sqrt{2}}\right)x+\left(-1+\sqrt{2}\right)^{2/3}x^2}\,dx, x, 1+x\right)}{1-\left(-1+\sqrt{2}\right)^{2/3}+\left(-1+\sqrt{2}\right)^{4/3}}\\
&= 6x-6\log\left(1-\left(-1+\sqrt{2}\right)^{2/3}+\sqrt[3]{-1+\sqrt{2}}(1+x)\right)-\frac{12\left(1-\sqrt{2}\right)\left(1-\sqrt[3]{-1+\sqrt{2}}-\left(-1+\sqrt{2}\right)^{2/3}\right)\log\left(1+\left(-1+\sqrt{2}\right)^{2/3}+\left(-1+\sqrt{2}\right)^{4/3}-\left(1-\sqrt{2}+\sqrt[3]{-1+\sqrt{2}}\right)(1+x)+\left(-1+\sqrt{2}\right)^{2/3}(1+x)^2\right)}{\left(1-\sqrt{2}+\sqrt[3]{-1+\sqrt{2}}\right)\left(1-\left(-1+\sqrt{2}\right)^{2/3}+\left(-1+\sqrt{2}\right)^{4/3}\right)}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.83

$$6\left(x-\log\left(6+6x+3x^2+x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x + x^2/2 + x^3/6),x]
```

```
[Out] 6*(x - Log[6 + 6*x + 3*x^2 + x^3])
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.88

method	result	size
default	$6x-6\ln\left(x^3+3x^2+6x+6\right)$	21
norman	$6x-6\ln\left(x^3+3x^2+6x+6\right)$	21
risch	$6x-6\ln\left(x^3+3x^2+6x+6\right)$	21
parallelrisch	$6x-6\ln\left(x^3+3x^2+6x+6\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x+1/2*x^2+1/6*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 6*x-6*ln(x^3+3*x^2+6*x+6)
```

Maxima [A]
time = 0.42, size = 20, normalized size = 0.83

$$6x - 6 \log(x^3 + 3x^2 + 6x + 6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x+1/2*x^2+1/6*x^3),x, algorithm="maxima")
[Out] 6*x - 6*log(x^3 + 3*x^2 + 6*x + 6)
```

Fricas [A]
time = 0.56, size = 20, normalized size = 0.83

$$6x - 6 \log(x^3 + 3x^2 + 6x + 6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x+1/2*x^2+1/6*x^3),x, algorithm="fricas")
[Out] 6*x - 6*log(x^3 + 3*x^2 + 6*x + 6)
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.79

$$6x - 6\log(x^3 + 3x^2 + 6x + 6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x+1/2*x**2+1/6*x**3),x)
[Out] 6*x - 6*log(x**3 + 3*x**2 + 6*x + 6)
```

Giac [A]
time = 0.41, size = 21, normalized size = 0.88

$$6x - 6 \log(|x^3 + 3x^2 + 6x + 6|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x+1/2*x^2+1/6*x^3),x, algorithm="giac")
[Out] 6*x - 6*log(abs(x^3 + 3*x^2 + 6*x + 6))
```

Mupad [B]
time = 0.12, size = 20, normalized size = 0.83

$$6x - 6 \ln(x^3 + 3x^2 + 6x + 6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x + x^2/2 + x^3/6 + 1),x)
```

```
[Out] 6*x - 6*log(6*x + 3*x^2 + x^3 + 6)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 33, normalized size = 1.38

$$-6x^2 + 12x - 24\ln\left(2x^3 + 3x^2 + 3x + 2\right) + 36\ln\left(x + 1\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(x^3/(1+x+1/2*x^2+1/6*x^3),x)
```

```
[Out] -6*x^2+12*x-24*ln(2*x^3+3*x^2+3*x+2)+36*ln(x+1)
```

3.243 $\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx$

Optimal. Leaf size=5

$$-2\cos(\sin(x))$$

```
[Out] -2*cos(sin(x))
```

Rubi [F]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (-\sin(x-\sin(x))+\sin(x+\sin(x)))\,dx$$

Verification is not applicable to the result.

```
[In] Int[-Sin[x - Sin[x]] + Sin[x + Sin[x]],x]
[Out] -Defer[Int][Sin[x - Sin[x]], x] + Defer[Int][Sin[x + Sin[x]], x]
```

Rubi steps

$$\text{Integral} = -\int \sin(x-\sin(x))\,dx + \int \sin(x+\sin(x))\,dx$$

Mathematica [A]
time = 0.08, size = 5, normalized size = 1.00

$$-2\cos(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[-Sin[x - Sin[x]] + Sin[x + Sin[x]],x]
[Out] -2*Cos[Sin[x]]
```

Maple [A]
time = 10.26, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$-2\cos\left(\sin\left(x\right)\right)$	6
default	$-2\cos\left(\sin\left(x\right)\right)$	6
risch	$-2\cos\left(\sin\left(x\right)\right)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x+sin(x))-sin(x-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*cos(sin(x))
```

Maxima [A]
time = 0.61, size = 5, normalized size = 1.00

$$-2\cos\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+sin(x))-sin(x-sin(x)),x, algorithm="maxima")
```

```
[Out] -2*cos(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(5) = 10$.
time = 0.61, size = 65, normalized size = 13.00

$$-2\cos\left(x\right)\cos\left(\frac{x\tan\left(\frac{1}{2}\,x\right)^{2}+x+2\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)-2\,\sin\left(x\right)\sin\left(\frac{x\tan\left(\frac{1}{2}\,x\right)^{2}+x+2\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+sin(x))-sin(x-sin(x)),x, algorithm="fricas")
```

```
[Out] -2*cos(x)*cos((x*tan(1/2*x)^2 + x + 2*tan(1/2*x))/(tan(1/2*x)^2 + 1)) - 2*s
in(x)*sin((x*tan(1/2*x)^2 + x + 2*tan(1/2*x))/(tan(1/2*x)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sin\left(x-\sin\left(x\right)\right)+\sin\left(x+\sin\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+sin(x))-sin(x-sin(x)),x)
```

```
[Out] Integral(-sin(x - sin(x)) + sin(x + sin(x)), x)
```

Giac [A]
time = 0.44, size = 5, normalized size = 1.00

$$-2\cos\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x+sin(x))-sin(x-sin(x)),x, algorithm="giac")
```

```
[Out] -2*cos(sin(x))
```

Mupad [**B**]
time = 0.37, size = 5, normalized size = 1.00

$$-2\cos\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x + sin(x)) - sin(x - sin(x)),x)
```

```
[Out] -2*cos(sin(x))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 7, normalized size = 1.40

$$2\sin\left(x\right)\sin\left(\sin\left(x\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x+sin(x))-sin(x-sin(x)),x)
```

```
[Out] 2*sin(x)*sin(sin(x))
```

3.244 $\int \left(\sec^5(x)\tan^2(x)+\sec^3(x)\tan^4(x)\right)\,dx$

Optimal. Leaf size=12

$$\frac{1}{3}\sec^3(x)\tan^3(x)$$

```
[Out] 1/3*sec(x)^3*tan(x)^3
```

Rubi [**B**] Leaf count is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 0.05, antiderivative size = 33, normalized size of antiderivative = 2.75, number of steps used = 9, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2691, 3853, 3855}

$$\frac{1}{6}\tan(x)\sec^5(x)+\frac{1}{6}\tan^3(x)\sec^3(x)-\frac{1}{6}\tan(x)\sec^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5*Tan[x]^2 + Sec[x]^3*Tan[x]^4,x]
```

```
[Out] -1/6*(Sec[x]^3*Tan[x]) + (Sec[x]^5*Tan[x])/6 + (Sec[x]^3*Tan[x]^3)/6
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \sec^5(x)\tan^2(x)\,dx+\int \sec^3(x)\tan^4(x)\,dx\\
&= \frac{1}{6}\sec^5(x)\tan(x)+\frac{1}{6}\sec^3(x)\tan^3(x)-\frac{1}{6}\int \sec^5(x)\,dx-\frac{1}{2}\int \sec^3(x)\tan^2(x)\,dx\\
&= -\frac{1}{6}\sec^3(x)\tan(x)+\frac{1}{6}\sec^5(x)\tan(x)+\frac{1}{6}\sec^3(x)\tan^3(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 1.00

$$\frac{1}{3}\sec^3(x)\tan^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5*Tan[x]^2 + Sec[x]^3*Tan[x]^4,x]
```

```
[Out] (Sec[x]^3*Tan[x]^3)/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(10) = 20$.
time = 5.12, size = 68, normalized size = 5.67

method	result	size
risch	$\frac{8i\left(e^{9ix}-3\,e^{7ix}+3\,e^{5ix}-e^{3ix}\right)}{3\left(e^{2ix}+1\right)^6}$	40
default	$\frac{\sin^3(x)}{6\cos(x)^6}+\frac{\sin^3(x)}{8\cos(x)^4}+\frac{\sin^3(x)}{16\cos(x)^2}+\frac{\sin^5(x)}{6\cos(x)^6}+\frac{\sin^5(x)}{24\cos(x)^4}-\frac{\sin^5(x)}{48\cos(x)^2}-\frac{(\sin^3(x))}{48}$	68
parts	$\frac{\sin^3(x)}{6\cos(x)^6}+\frac{\sin^3(x)}{8\cos(x)^4}+\frac{\sin^3(x)}{16\cos(x)^2}+\frac{\sin^5(x)}{6\cos(x)^6}+\frac{\sin^5(x)}{24\cos(x)^4}-\frac{\sin^5(x)}{48\cos(x)^2}-\frac{(\sin^3(x))}{48}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4*sec(x)^3+tan(x)^2*sec(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sin(x)^3/cos(x)^6+1/8*sin(x)^3/cos(x)^4+1/16*sin(x)^3/cos(x)^2+1/6*sin(
x)^5/cos(x)^6+1/24*sin(x)^5/cos(x)^4-1/48*sin(x)^5/cos(x)^2-1/48*sin(x)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(10) = 20$.
time = 0.43, size = 79, normalized size = 6.58

$$-\frac{3\,\sin(x)^5+8\,\sin(x)^3-3\,\sin(x)}{48\left(\sin(x)^6-3\,\sin(x)^4+3\,\sin(x)^2-1\right)}+\frac{3\,\sin(x)^5-8\,\sin(x)^3-3\,\sin(x)}{48\left(\sin(x)^6-3\,\sin(x)^4+3\,\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4*sec(x)^3+tan(x)^2*sec(x)^5,x, algorithm="maxima")
```

```
[Out] -1/48*(3*sin(x)^5 + 8*sin(x)^3 - 3*sin(x))/(sin(x)^6 - 3*sin(x)^4 + 3*sin(x
)^2 - 1) + 1/48*(3*sin(x)^5 - 8*sin(x)^3 - 3*sin(x))/(sin(x)^6 - 3*sin(x)^4
 + 3*sin(x)^2 - 1)
```

Fricas [A]
time = 0.60, size = 14, normalized size = 1.17

$$-\frac{\left(\cos(x)^2-1\right)\sin(x)}{3\,\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4*sec(x)^3+tan(x)^2*sec(x)^5,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(10) = 20$.
time = 0.09, size = 80, normalized size = 6.67

$$\frac{-3\sin^5{(x)} - 8\sin^3{(x)} + 3\sin{(x)}}{48\sin^6{(x)} - 144\sin^4{(x)} + 144\sin^2{(x)} - 48} + \frac{3\sin^5{(x)} - 8\sin^3{(x)} - 3\sin{(x)}}{48\sin^6{(x)} - 144\sin^4{(x)} + 144\sin^2{(x)} - 48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4*sec(x)**3+tan(x)**2*sec(x)**5,x)
```

```
[Out] (-3*sin(x)**5 - 8*sin(x)**3 + 3*sin(x))/(48*sin(x)**6 - 144*sin(x)**4 + 144
*sin(x)**2 - 48) + (3*sin(x)**5 - 8*sin(x)**3 - 3*sin(x))/(48*sin(x)**6 - 1
44*sin(x)**4 + 144*sin(x)**2 - 48)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(10) = 20$.
time = 0.49, size = 55, normalized size = 4.58

$$-\frac{3\sin{(x)}^5 + 8\sin{(x)}^3 - 3\sin{(x)}}{48\left(\sin{(x)}^2 - 1\right)^3} + \frac{3\sin{(x)}^5 - 8\sin{(x)}^3 - 3\sin{(x)}}{48\left(\sin{(x)}^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4*sec(x)^3+tan(x)^2*sec(x)^5,x, algorithm="giac")
```

```
[Out] -1/48*(3*sin(x)^5 + 8*sin(x)^3 - 3*sin(x))/(sin(x)^2 - 1)^3 + 1/48*(3*sin(x
)^5 - 8*sin(x)^3 - 3*sin(x))/(sin(x)^2 - 1)^3
```

Mupad [B]
time = 0.33, size = 14, normalized size = 1.17

$$-\frac{\sin{(x)}^3}{3\left(\sin{(x)}^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4/cos(x)^3 + tan(x)^2/cos(x)^5,x)
```

```
[Out] -sin(x)^3/(3*(sin(x)^2 - 1)^3)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 1.08

$$\frac{(\sec^4{(x)})}{4} + \frac{(\sec^3{(x)})}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(tan(x)^4*sec(x)^3+tan(x)^2*sec(x)^5,x)
[Out] 1/4*sec(x)^4+1/3*sec(x)^3
```

3.245 $\int (1+\log(x)) \log(\log(x)) \, dx$

Optimal. Leaf size=10

$$x(-1+\log(x)\log(\log(x)))$$

[Out] x*(-1+ln(x)*ln(ln(x)))

Rubi [F]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (1+\log(x))\log(\log(x))\,dx$$

Verification is not applicable to the result.

[In] Int[(1 + Log[x])*Log[Log[x]],x]

[Out] x*Log[Log[x]] - LogIntegral[x] + Defer[Int][Log[x]*Log[Log[x]], x]

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int (\log(\log(x))+\log(x)\log(\log(x)))\,dx\\
&= \int \log(\log(x))\,dx+\int \log(x)\log(\log(x))\,dx\\
&= x\log(\log(x))-\int \frac{1}{\log(x)}\,dx+\int \log(x)\log(\log(x))\,dx\\
&= x\log(\log(x))-\text{LogIntegral}(x)+\int \log(x)\log(\log(x))\,dx
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.80

$$-x+x\log(\log(x))+x(-1+\log(x))\log(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Log[x])*Log[Log[x]],x]

[Out] -x + x*Log[Log[x]] + x*(-1 + Log[x])*Log[Log[x]]

Maple [A]
time = 0.09, size = 19, normalized size = 1.90

method	result	size
norman	$x \ln(x) \ln(\ln(x)) - x$	12
risch	$x \ln(x) \ln(\ln(x)) - x$	12
parallelrisch	$x \ln(x) \ln(\ln(x)) - x$	12
default	$(-1 + \ln(x)) x \ln(\ln(x)) - x + x \ln(\ln(x))$	19
parts	$(-1 + \ln(x)) x \ln(\ln(x)) - x + x \ln(\ln(x))$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))*ln(ln(x)),x,method=_RETURNVERBOSE)
[Out] (-1+ln(x))*x*ln(ln(x))-x+x*ln(ln(x))
```

Maxima [A]
time = 0.39, size = 16, normalized size = 1.60

$$(x(\log(x) - 1) + x) \log(\log(x)) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))*log(log(x)),x, algorithm="maxima")
[Out] (x*(log(x) - 1) + x)*log(log(x)) - x
```

Fricas [A]
time = 0.58, size = 11, normalized size = 1.10

$$x \log(x) \log(\log(x)) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))*log(log(x)),x, algorithm="fricas")
[Out] x*log(x)*log(log(x)) - x
```

Sympy [A]
time = 0.19, size = 10, normalized size = 1.00

$$x \log(x) \log(\log(x)) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))*ln(ln(x)),x)
[Out] x*log(x)*log(log(x)) - x
```

Giac [A]
time = 0.53, size = 11, normalized size = 1.10

$$x \log(x) \log(\log(x)) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))*log(log(x)),x, algorithm="giac")
```

```
[Out] x*log(x)*log(log(x)) - x
```

Mupad [B]
time = 0.17, size = 11, normalized size = 1.10

$$x \ln(\ln(x)) \ln(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x))*(log(x) + 1),x)
```

```
[Out] x*log(log(x))*log(x) - x
```

Chatgpt [F] Failed to verify
time = 1.00, size = 15, normalized size = 1.50

$$-x + x \ln(x) + x \ln(\ln(x)) \ln(x)$$

Warning: Unable to verify antiderivative.

```
[In] int((1+ln(x))*ln(ln(x)),x)
```

```
[Out] -x+x*ln(x)+x*ln(ln(x))*ln(x)
```

3.246 $\int \left(\frac{1}{1+\cos(x)} + \frac{1}{1+\cot(x)} + \frac{1}{1+\csc(x)} + \frac{1}{1+\sec(x)} + \frac{1}{1+\sin(x)} + \right.$

Optimal. Leaf size=3

$$3x$$

```
[Out] 3*x
```

Rubi [C] Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.
time = 0.06, antiderivative size = 42, normalized size of antiderivative = 14.00, number of steps used = 11, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2727, 3565, 3611, 3862, 8}

$$3x + \frac{\sin(x)}{\cos(x)+1} - \frac{\cos(x)}{\sin(x)+1} + \frac{\cot(x)}{\csc(x)+1} - \frac{\tan(x)}{\sec(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1) + (1 + Cot[x])^(-1) + (1 + Csc[x])^(-1) + (1 + Sec[x]
)^(-1) + (1 + Sin[x])^(-1) + (1 + Tan[x])^(-1),x]
```

```
[Out] 3*x + Cot[x]/(1 + Csc[x]) + Sin[x]/(1 + Cos[x]) - Cos[x]/(1 + Sin[x]) - Tan
[x]/(1 + Sec[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{1}{1+\cos(x)}\,dx + \int \frac{1}{1+\cot(x)}\,dx + \int \frac{1}{1+\csc(x)}\,dx + \int \frac{1}{1+\sec(x)}\,dx + \int \frac{1}{1+\sin(x)}\,dx + \int \frac{1}{1+\tan(x)}\,dx \\ &= x + \frac{\cot(x)}{1+\csc(x)} + \frac{\sin(x)}{1+\cos(x)} - \frac{\cos(x)}{1+\sin(x)} - \frac{\tan(x)}{1+\sec(x)} - \frac{1}{2}\int \frac{-1+\cot(x)}{1+\cot(x)}\,dx + \frac{1}{2}\int \frac{1-\tan(x)}{1+\tan(x)}\,dx - 2\int -1\,dx \\ &= 3x + \frac{\cot(x)}{1+\csc(x)} + \frac{\sin(x)}{1+\cos(x)} - \frac{\cos(x)}{1+\sin(x)} - \frac{\tan(x)}{1+\sec(x)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1) + (1 + Cot[x])^(-1) + (1 + Csc[x])^(-1) + (1 +
Sec[x])^(-1) + (1 + Sin[x])^(-1) + (1 + Tan[x])^(-1),x]
```

```
[Out] 3*x
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.39, size = 54, normalized size = 18.00

method	result
risch	$3x$
norman	$\frac{3x+3x\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}$
default	$\frac{\ln(1+\cot^2(x))}{4} - \frac{\pi}{4} + \frac{\operatorname{arccot}(\cot(x))}{2} - \frac{\ln(1+\cot(x))}{2} + 4\arctan\left(\tan\left(\frac{x}{2}\right)\right) - \frac{\ln(1+\tan^2(x))}{4} + \frac{\arctan(\tan(x))}{2} + \ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x))+1/(1+cos(x))+1/(1+tan(x))+1/(1+cot(x))+1/(1+sec(x))+1/(1+c
sc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(1+cot(x)^2)-1/4*Pi+1/2*arccot(cot(x))-1/2*ln(1+cot(x))+4*arctan(tan(
1/2*x))-1/4*ln(1+tan(x)^2)+1/2*arctan(tan(x))+1/2*ln(1+tan(x))
```

Maxima [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.45, size = 14, normalized size = 4.67

$$x + 4 \arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x))+1/(1+cos(x))+1/(1+tan(x))+1/(1+cot(x))+1/(1+sec(x))+
1/(1+csc(x)),x, algorithm="maxima")
```

```
[Out] x + 4*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.56, size = 3, normalized size = 1.00

$$3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x))+1/(1+cos(x))+1/(1+tan(x))+1/(1+cot(x))+1/(1+sec(x))+
1/(1+csc(x)),x, algorithm="fricas")
```

```
[Out] 3*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x))+1/(1+cos(x))+1/(1+tan(x))+1/(1+cot(x))+1/(1+sec(x))+
1/(1+csc(x)),x)
```

```
[Out] Integral((sin(x)*cos(x)*tan(x)*cot(x)*csc(x) + sin(x)*cos(x)*tan(x)*cot(x)*
sec(x) + 2*sin(x)*cos(x)*tan(x)*cot(x) + sin(x)*cos(x)*tan(x)*csc(x)*sec(x)
 + 2*sin(x)*cos(x)*tan(x)*csc(x) + 2*sin(x)*cos(x)*tan(x)*sec(x) + 3*sin(x)
*cos(x)*tan(x) + sin(x)*cos(x)*cot(x)*csc(x)*sec(x) + 2*sin(x)*cos(x)*cot(x
)*csc(x) + 2*sin(x)*cos(x)*cot(x)*sec(x) + 3*sin(x)*cos(x)*cot(x) + 2*sin(x
)*cos(x)*csc(x)*sec(x) + 3*sin(x)*cos(x)*csc(x) + 3*sin(x)*cos(x)*sec(x) +
4*sin(x)*cos(x) + sin(x)*tan(x)*cot(x)*csc(x)*sec(x) + 2*sin(x)*tan(x)*cot(
x)*csc(x) + 2*sin(x)*tan(x)*cot(x)*sec(x) + 3*sin(x)*tan(x)*cot(x) + 2*sin(
x)*tan(x)*csc(x)*sec(x) + 3*sin(x)*tan(x)*csc(x) + 3*sin(x)*tan(x)*sec(x) +
 4*sin(x)*tan(x) + 2*sin(x)*cot(x)*csc(x)*sec(x) + 3*sin(x)*cot(x)*csc(x) +
 3*sin(x)*cot(x)*sec(x) + 4*sin(x)*cot(x) + 3*sin(x)*csc(x)*sec(x) + 4*sin(
x)*csc(x) + 4*sin(x)*sec(x) + 5*sin(x) + cos(x)*tan(x)*cot(x)*csc(x)*sec(x)
 + 2*cos(x)*tan(x)*cot(x)*csc(x) + 2*cos(x)*tan(x)*cot(x)*sec(x) + 3*cos(x)
```

```
*tan(x)*cot(x) + 2*cos(x)*tan(x)*csc(x)*sec(x) + 3*cos(x)*tan(x)*csc(x) + 3
*cos(x)*tan(x)*sec(x) + 4*cos(x)*tan(x) + 2*cos(x)*cot(x)*csc(x)*sec(x) + 3
*cos(x)*cot(x)*csc(x) + 3*cos(x)*cot(x)*sec(x) + 4*cos(x)*cot(x) + 3*cos(x)
*csc(x)*sec(x) + 4*cos(x)*csc(x) + 4*cos(x)*sec(x) + 5*cos(x) + 2*tan(x)*co
t(x)*csc(x)*sec(x) + 3*tan(x)*cot(x)*csc(x) + 3*tan(x)*cot(x)*sec(x) + 4*ta
n(x)*cot(x) + 3*tan(x)*csc(x)*sec(x) + 4*tan(x)*csc(x) + 4*tan(x)*sec(x) +
5*tan(x) + 3*cot(x)*csc(x)*sec(x) + 4*cot(x)*csc(x) + 4*cot(x)*sec(x) + 5*c
ot(x) + 4*csc(x)*sec(x) + 5*csc(x) + 5*sec(x) + 6)/((sin(x) + 1)*(cos(x) +
1)*(tan(x) + 1)*(cot(x) + 1)*(csc(x) + 1)*(sec(x) + 1)), x)
```

Giac [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.50, size = 40, normalized size = 13.33

$$3\,x-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)}-\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x))+1/(1+cos(x))+1/(1+tan(x))+1/(1+cot(x))+1/(1+sec(x))+
1/(1+csc(x)),x, algorithm="giac")
```

```
[Out] 3*x - 2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1)) - tan(1/2*x)
```

Mupad [B]
time = 0.36, size = 3, normalized size = 1.00

$$3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) + 1) + 1/(cos(x) + 1) + 1/(cot(x) + 1) + 1/(sin(x) + 1) + 1
/(tan(x) + 1) + 1/(1/cos(x) + 1),x)
```

```
[Out] 3*x
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 4.33

$$3\ln\left(2\left(\sin^2\left(x+\frac{\pi}{4}\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1+sin(x))+1/(1+cos(x))+1/(1+tan(x))+1/(1+cot(x))+1/(1+sec(x))+1/(1+c
sc(x)),x)
```

```
[Out] 3*ln(2*sin(x+1/4*Pi)^2)
```

3.247 $\int \frac{1}{\sqrt{x-x^2}} dx$

Optimal. Leaf size=8

$$-\arcsin(1-2x)$$

[Out] arcsin(-1+2*x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {633, 222}

$$-\arcsin(1-2x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x - x^2],x]

[Out] -ArcSin[1 - 2*x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)\\ &= -\arcsin(1-2x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.
time = 0.00, size = 40, normalized size = 5.00

$$-\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x - x^2],x]

```
[Out] (-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

Maple [A]
time = 0.09, size = 7, normalized size = 0.88

method	result	size
default	$\arcsin\left(-1+2x\right)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-(x-1)x}}{x}\right)$	16
trager	$\mathit{RootOf}\left(\_Z^2+1\right)\ln\left(-2\mathit{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}+\mathit{RootOf}\left(\_Z^2+1\right)\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(-1+2*x)
```

Maxima [A]
time = 0.43, size = 6, normalized size = 0.75

$$\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] arcsin(2*x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.
time = 0.58, size = 16, normalized size = 2.00

$$-2\,\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [A]
time = 0.20, size = 5, normalized size = 0.62

$$\operatorname{asin}\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+x)**(1/2),x)
```

```
[Out] asin(2*x - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.
time = 0.48, size = 25, normalized size = 3.12

$$\frac{1}{4}\sqrt{-x^2+x}(2\,x-1)+\frac{1}{8}\,\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

Mupad [B]
time = 0.08, size = 6, normalized size = 0.75

$$\mathrm{asin}(2\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^2)^(1/2),x)
[Out] asin(2*x - 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 12, normalized size = 1.50

$$\arctan\left(\frac{\sqrt{1-x}}{x}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(-x^2+x)^(1/2),x)
[Out] arctan((1-x)^(1/2)/x)
```

3.248 $\int \frac{1}{1+\cos^2(x)} \, dx$

Optimal. Leaf size=34

$$\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}$$

```
[Out] 1/2*2^(1/2)*x-1/2*arctan(cos(x)*sin(x)/(1+2^(1/2)+cos(x)^2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3260, 209}

$$\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\text{Subst}\left(\int \frac{1}{1+2x^2} \, dx, x, \cot(x)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 0.44

$$\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)^(-1),x]
[Out] ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.06, size = 14, normalized size = 0.41

method	result	size
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{2}$	14
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{4}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Maxima [A]
time = 0.45, size = 13, normalized size = 0.38

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x))
```

Fricas [A]
time = 0.62, size = 31, normalized size = 0.91

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
```

Sympy [A]
time = 0.26, size = 63, normalized size = 1.85

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/2 + sqrt(2
)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/2
```

Giac [A]
time = 0.50, size = 46, normalized size = 1.35

$$\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2\,x\right)-\sin\left(2\,x\right)}{\sqrt{2}\cos\left(2\,x\right)+\sqrt{2}-\cos\left(2\,x\right)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1)))
```

Mupad [B]
time = 0.13, size = 26, normalized size = 0.76

$$\frac{\sqrt{2}\left(x-\mathrm{atan}(\tan(x))\right)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + 1),x)
```

```
[Out] (2^(1/2)*(x - atan(tan(x))))/2 + (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 13, normalized size = 0.38

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\tan(x)}{2}\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1+cos(x)^2),x)
```

```
[Out] 2/3*3^(1/2)*arctan(1/2*3^(1/2)*tan(x))
```

3.249 $\int \frac{\log(1+x)}{x^2}\,dx$

Optimal. Leaf size=18

$$\log(x)-\log(1+x)-\frac{\log(1+x)}{x}$$

```
[Out] ln(x)-ln(x+1)-ln(x+1)/x
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2442, 36, 29, 31}

$$\log(x)-\frac{\log(x+1)}{x}-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= -\frac{\log(1+x)}{x} + \int \frac{1}{x(1+x)}\,dx \\ &= -\frac{\log(1+x)}{x} + \int \frac{1}{x}\,dx - \int \frac{1}{1+x}\,dx \\ &= \log(x) - \log(1+x) - \frac{\log(1+x)}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\log(x) - \log(1+x) - \frac{\log(1+x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.89

method	result	size
derivativedivides	$\ln(x) - \frac{\ln(x+1)(x+1)}{x}$	16
default	$\ln(x) - \frac{\ln(x+1)(x+1)}{x}$	16
meijerg	$\ln(x) - \frac{(2x+2)\ln(x+1)}{2x}$	18
risch	$\ln(x) - \ln(x+1) - \frac{\ln(x+1)}{x}$	19
parts	$\ln(x) - \ln(x+1) - \frac{\ln(x+1)}{x}$	19
norman	$\frac{-\ln(x+1)x-\ln(x+1)}{x} + \ln(x)$	22
parallelrisch	$\frac{x\ln(x)-\ln(x+1)x-\ln(x+1)}{x}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+1)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-ln(x+1)*(x+1)/x
```

Maxima [A]
time = 0.33, size = 18, normalized size = 1.00

$$-\frac{\log(x+1)}{x} - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+1)/x^2,x, algorithm="maxima")
```

```
[Out] -log(x + 1)/x - log(x + 1) + log(x)
```

Fricas [A]
time = 0.58, size = 19, normalized size = 1.06

$$-\frac{(x+1)\log(x+1) - x\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+1)/x^2,x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(x + 1) - x*log(x))/x
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.78

$$\log(x) - \log(x+1) - \frac{\log(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+1)/x**2,x)
```

```
[Out] log(x) - log(x + 1) - log(x + 1)/x
```

Giac [A]
time = 0.48, size = 20, normalized size = 1.11

$$-\frac{\log(x+1)}{x} - \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+1)/x^2,x, algorithm="giac")
```

```
[Out] -log(x + 1)/x - log(abs(x + 1)) + log(abs(x))
```

Mupad [B]
time = 0.05, size = 18, normalized size = 1.00

$$-\ln\left(\frac{1}{x}+1\right) - \frac{\ln(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + 1)/x^2,x)
```

```
[Out] - log(1/x + 1) - log(x + 1)/x
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 20, normalized size = 1.11

$$\frac{\ln(x+1)}{x} - \frac{\ln(x+1)+1}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(x+1)/x^2,x)
```

```
[Out] ln(x+1)/x-(ln(x+1)+1)/x^2
```

3.250 $\int \sqrt{1-\arccos(\sin(x))^2}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{2}\left(\arccos(\sin(x))\sqrt{1-\arccos(\sin(x))^2}-2\arctan\left(\frac{\sqrt{1-\arccos(\sin(x))^2}}{1+\arccos(\sin(x))}\right)\right)\sqrt{\cos^2(x)}\sec(x)$$

```
[Out] -1/2*((1/2*Pi-arcsin(sin(x)))*(1-(1/2*Pi-arcsin(sin(x)))^2)^(1/2)-2*arctan(
(1-(1/2*Pi-arcsin(sin(x)))^2)^(1/2)/(1+1/2*Pi-arcsin(sin(x)))))*(cos(x)^2)^
(1/2)*sec(x)
```

Rubi [F]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{1-\arccos(\sin(x))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - ArcCos[Sin[x]]^2],x]
[Out] Defer[Int][Sqrt[1 - ArcCos[Sin[x]]^2], x]
```

Rubi steps

$$\text{Integral} = \int \sqrt{1-\arccos(\sin(x))^2}\,dx$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.00

$$-\frac{1}{2}\left(\arccos(\sin(x))\sqrt{1-\arccos(\sin(x))^2}-2\arctan\left(\frac{\sqrt{1-\arccos(\sin(x))^2}}{1+\arccos(\sin(x))}\right)\right)\sqrt{\cos^2(x)}\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - ArcCos[Sin[x]]^2],x]
[Out] -1/2*((ArcCos[Sin[x]]*Sqrt[1 - ArcCos[Sin[x]]^2] - 2*ArcTan[Sqrt[1 - ArcCos
[Sin[x]]^2]/(1 + ArcCos[Sin[x]])])*Sqrt[Cos[x]^2]*Sec[x])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\left(-\frac{\pi}{2}+\arcsin\left(\sin\left(x\right)\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(-1/2*Pi+arcsin(sin(x)))^2)^(1/2),x)
```

```
[Out] int((1-(-1/2*Pi+arcsin(sin(x)))^2)^(1/2),x)
```

Maxima [A]
time = 0.53, size = 51, normalized size = 0.91

$$-\frac{1}{8}\left(\pi-2\,x\right)\sqrt{-\pi^2+4\,\pi x-4\,x^2+4}+\frac{1}{2}\,\arctan\left(-\frac{1}{2}\,\pi+x,\sqrt{-\frac{1}{4}\,\pi^2+\pi x-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-1/2*pi+arcsin(sin(x)))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(pi - 2*x)*sqrt(-pi^2 + 4*pi*x - 4*x^2 + 4) + 1/2*arctan2(-1/2*pi + x,
 sqrt(-1/4*pi^2 + pi*x - x^2 + 1))
```

Fricas [A]
time = 0.61, size = 69, normalized size = 1.23

$$\frac{1}{8}\left(\pi+2\,x\right)\sqrt{-\pi^2-4\,\pi x-4\,x^2+4}-\frac{1}{2}\,\arctan\left(\frac{\left(\pi+2\,x\right)\sqrt{-\pi^2-4\,\pi x-4\,x^2+4}}{\pi^2+4\,\pi x+4\,x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-1/2*pi+arcsin(sin(x)))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(pi + 2*x)*sqrt(-pi^2 - 4*pi*x - 4*x^2 + 4) - 1/2*arctan((pi + 2*x)*sqr
t(-pi^2 - 4*pi*x - 4*x^2 + 4)/(pi^2 + 4*pi*x + 4*x^2 - 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\sqrt{-4\operatorname{asin}^2\left(\sin\left(x\right)\right)+4\pi\operatorname{asin}\left(\sin\left(x\right)\right)-\pi^2+4}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-1/2*pi+asin(sin(x)))**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*asin(sin(x))**2 + 4*pi*asin(sin(x)) - pi**2 + 4), x)/2
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-1/2*pi+arcsin(sin(x)))^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{1-\left(\frac{\Pi}{2}-\mathrm{asin}\left(\sin\left(x\right)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - (Pi/2 - asin(sin(x)))^2)^(1/2),x)
```

```
[Out] int((1 - (Pi/2 - asin(sin(x)))^2)^(1/2), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 37, normalized size = 0.66

$$\frac{\sqrt{1-\left(\frac{\pi}{2}-\arcsin\left(\sin\left(x\right)\right)\right)^2}}{2x}-\left(\frac{\pi}{2}-\arcsin\left(\sin\left(x\right)\right)\right)\arcsin\left(\sin\left(x\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((1-(-1/2*Pi+arcsin(sin(x)))^2)^(1/2),x)
```

```
[Out] 1/2/x*(1-(1/2*Pi-arcsin(sin(x)))^2)^(1/2)-(1/2*Pi-arcsin(sin(x)))*arcsin(si
n(x))
```

3.251 $\int (-2+x)(-1+x)x(1+x)(2+x)\,dx$

Optimal. Leaf size=20

$$2x^2-\frac{5x^4}{4}+\frac{x^6}{6}$$

```
[Out] 2*x^2-5/4*x^4+1/6*x^6
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {1620}

$$\frac{x^6}{6}-\frac{5x^4}{4}+2x^2$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x)*(-1 + x)*x*(1 + x)*(2 + x),x]
```

```
[Out] 2*x^2 - (5*x^4)/4 + x^6/6
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegran
d[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, m, n, p, q}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\int \left(4x-5x^3+x^5\right)\,dx\\&=2x^2-\frac{5x^4}{4}+\frac{x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$2x^2-\frac{5x^4}{4}+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x)*(-1 + x)*x*(1 + x)*(2 + x),x]
```

```
[Out] 2*x^2 - (5*x^4)/4 + x^6/6
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.85

method	result	size
gosper	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
default	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
norman	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
risch	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17
parallelrisch	$2x^2 - \frac{5}{4}x^4 + \frac{1}{6}x^6$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-2)*(x-1)*x*(x+1)*(2+x),x,method=_RETURNVERBOSE)
[Out] 2*x^2-5/4*x^4+1/6*x^6
```

Maxima [A]
time = 0.31, size = 16, normalized size = 0.80

$$\frac{1}{6}\,x^6 - \frac{5}{4}\,x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-2)*(x-1)*x*(x+1)*(2+x),x, algorithm="maxima")
[Out] 1/6*x^6 - 5/4*x^4 + 2*x^2
```

Fricas [A]
time = 0.54, size = 16, normalized size = 0.80

$$\frac{1}{6}\,x^6 - \frac{5}{4}\,x^4 + 2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-2)*(x-1)*x*(x+1)*(2+x),x, algorithm="fricas")
[Out] 1/6*x^6 - 5/4*x^4 + 2*x^2
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.75

$$\frac{x^6}{6} - \frac{5x^4}{4} + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-2)*(x-1)*x*(x+1)*(2+x),x)
```

```
[Out] x**6/6 - 5*x**4/4 + 2*x**2
```

Giac [A]
time = 0.50, size = 16, normalized size = 0.80

$$\frac{1}{6}\,x^6-\frac{5}{4}\,x^4+2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-2)*(x-1)*x*(x+1)*(2+x),x, algorithm="giac")
```

```
[Out] 1/6*x^6 - 5/4*x^4 + 2*x^2
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.85

$$\frac{x^2\,\left(2\,x^4-15\,x^2+24\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x - 1)*(x + 1)*(x - 2)*(x + 2),x)
```

```
[Out] (x^2*(2*x^4 - 15*x^2 + 24))/12
```

Chatgpt [F] Failed to verify
time = 1.00, size = 29, normalized size = 1.45

$$\frac{1}{6}x^6-\frac{7}{5}x^5+\frac{13}{2}x^4-5x^3-\frac{11}{2}x^2+2x$$

Warning: Unable to verify antiderivative.

```
[In] int((x-2)*(x-1)*x*(x+1)*(2+x),x)
```

```
[Out] 1/6*x^6-7/5*x^5+13/2*x^4-5*x^3-11/2*x^2+2*x
```

3.252 $\int \frac{1+2x^2+3x^3}{\sqrt[3]{1+x^3}} dx$

Optimal. Leaf size=21

$$\left(1+x^3\right)^{2/3}+x\left(1+x^3\right)^{2/3}$$

```
[Out] (x^3+1)^(2/3)+x*(x^3+1)^(2/3)
```

Rubi [A]
time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1907, 245, 267, 327}

$$\left(x^3+1\right)^{2/3}x+\left(x^3+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 3*x^3)/(1 + x^3)^(1/3),x]
```

```
[Out] (1 + x^3)^(2/3) + x*(1 + x^3)^(2/3)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int \left(\frac{1}{\sqrt[3]{1+x^3}}+\frac{2x^2}{\sqrt[3]{1+x^3}}+\frac{3x^3}{\sqrt[3]{1+x^3}}\right) dx \\
&= 2\int \frac{x^2}{\sqrt[3]{1+x^3}}\,dx+3\int \frac{x^3}{\sqrt[3]{1+x^3}}\,dx+\int \frac{1}{\sqrt[3]{1+x^3}}\,dx \\
&= \left(1+x^3\right)^{2/3}+x\left(1+x^3\right)^{2/3}+\frac{\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x+\sqrt[3]{1+x^3}\right)-\int \frac{1}{\sqrt[3]{1+x^3}}\,dx \\
&= \left(1+x^3\right)^{2/3}+x\left(1+x^3\right)^{2/3}
\end{aligned}
$$

Mathematica [A]
time = 6.14, size = 13, normalized size = 0.62

$$(1+x)\left(1+x^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 3*x^3)/(1 + x^3)^(1/3),x]
[Out] (1 + x)*(1 + x^3)^(2/3)
```

Maple [A]
time = 0.13, size = 12, normalized size = 0.57

method	result	size
trager	$(x+1)\left(x^3+1\right)^{\frac{2}{3}}$	12
risch	$(x+1)\left(x^3+1\right)^{\frac{2}{3}}$	12
gosper	$\frac{(x+1)^2\left(x^2-x+1\right)}{\left(x^3+1\right)^{\frac{1}{3}}}$	22
meijerg	$x\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-x^3\right)+\frac{3x^4\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};-x^3\right)}{4}+\frac{2x^3\,{}_2F_1\left(\frac{1}{3},1;2;-x^3\right)}{3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x,method=_RETURNVERBOSE)
[Out] (x+1)*(x^3+1)^(2/3)
```

Maxima [A]
time = 0.46, size = 32, normalized size = 1.52

$$\left(x^3+1\right)^{\frac{2}{3}}+\frac{\left(x^3+1\right)^{\frac{2}{3}}}{x^2\left(\frac{x^3+1}{x^3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] (x^3 + 1)^(2/3) + (x^3 + 1)^(2/3)/(x^2*((x^3 + 1)/x^3 - 1))
```

Fricas [A]
time = 0.57, size = 11, normalized size = 0.52

$$(x^3+1)^{\frac{2}{3}}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] (x^3 + 1)^(2/3)*(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.20, size = 65, normalized size = 3.10

$$\frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{\Gamma\left(\frac{7}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+(x^3+1)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3+2*x**2+1)/(x**3+1)**(1/3),x)
```

```
[Out] x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi))/gamma(7/3)
+ x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3
)) + (x**3 + 1)**(2/3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((3*x^3 + 2*x^2 + 1)/(x^3 + 1)^(1/3), x)
```

Mupad [B]
time = 0.10, size = 11, normalized size = 0.52

$$(x^3+1)^{2/3}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3*x^3 + 1)/(x^3 + 1)^(1/3),x)
```

```
[Out] (x^3 + 1)^(2/3)*(x + 1)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 9, normalized size = 0.43

$$3(x^3+1)^{\frac{2}{3}}$$

Warning: Unable to verify antiderivative.

```
[In] int((3*x^3+2*x^2+1)/(x^3+1)^(1/3),x)
```

```
[Out] 3*(x^3+1)^(2/3)
```

3.253 $\int \csc^4(x)\sec^4(x)\,dx$

Optimal. Leaf size=17

$$-8\cot(2x)-\frac{8}{3}\cot^3(2x)$$

```
[Out] -8*cot(2*x)-8/3*cot(2*x)^3
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2700, 276}

$$\frac{\tan^3(x)}{3}+3\tan(x)-\frac{1}{3}\cot^3(x)-3\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4*Sec[x]^4,x]
```

```
[Out] -3*Cot[x] - Cot[x]^3/3 + 3*Tan[x] + Tan[x]^3/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\text{Integral}&=\text{Subst}\left(\int\frac{(1+x^2)^3}{x^4}\,dx,x,\tan(x)\right)\\&=\text{Subst}\left(\int\left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)\,dx,x,\tan(x)\right)\\&=-3\cot(x)-\frac{\cot^3(x)}{3}+3\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.94

$$-\frac{8\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)+\frac{8\tan(x)}{3}+\frac{1}{3}\sec^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4*Sec[x]^4,x]
```

```
[Out] (-8*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3 + (8*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.22, size = 36, normalized size = 2.12

method	result	size
parallelrisch	$-\frac{(-\cos(6x)+3\cos(2x))(\sec^3(x))(\csc^3(x))}{6}$	24
risch	$\frac{32i(3\,e^{4ix}-1)}{3(e^{2ix}-1)^3(e^{2ix}+1)^3}$	31
default	$\frac{1}{3\sin(x)^3\cos(x)^3}-\frac{2}{3\sin(x)^3\cos(x)}+\frac{8}{3\cos(x)\sin(x)}-\frac{16\cot(x)}{3}$	36
norman	$\frac{\frac{1}{24}+\frac{5\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}-\frac{91\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}+\frac{35\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}-\frac{91\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}+\frac{5\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^4/cos(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/sin(x)^3/cos(x)^3-2/3/sin(x)^3/cos(x)+8/3/cos(x)/sin(x)-16/3*cot(x)
```

Maxima [A]
time = 0.38, size = 25, normalized size = 1.47

$$\frac{1}{3}\tan(x)^3-\frac{9\tan(x)^2+1}{3\tan(x)^3}+3\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4/cos(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 - 1/3*(9*tan(x)^2 + 1)/tan(x)^3 + 3*tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.
time = 0.60, size = 39, normalized size = 2.29

$$-\frac{16\cos(x)^6-24\cos(x)^4+6\cos(x)^2+1}{3\left(\cos(x)^5-\cos(x)^3\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4/cos(x)^4,x, algorithm="fricas")
```

[Out] -1/3*(16*cos(x)^6 - 24*cos(x)^4 + 6*cos(x)^2 + 1)/((cos(x)^5 - cos(x)^3)*si
n(x))

Sympy [A]
time = 0.02, size = 29, normalized size = 1.71

$$-\frac{16\cos(2x)}{3\sin(2x)}-\frac{8\cos(2x)}{3\sin^3(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)**4/cos(x)**4,x)

[Out] -16*cos(2*x)/(3*sin(2*x)) - 8*cos(2*x)/(3*sin(2*x)**3)

Giac [A]
time = 0.58, size = 18, normalized size = 1.06

$$-\frac{8\left(3\tan(2x)^2+1\right)}{3\tan(2x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(x)^4/cos(x)^4,x, algorithm="giac")

[Out] -8/3*(3*tan(2*x)^2 + 1)/tan(2*x)^3

Mupad [B]
time = 0.21, size = 38, normalized size = 2.24

$$-\frac{24\cos(2x)-16\cos(2x)^3}{3\sin(2x)-3\cos(2x)^2\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^4*sin(x)^4),x)

[Out] -(24*cos(2*x) - 16*cos(2*x)^3)/(3*sin(2*x) - 3*cos(2*x)^2*sin(2*x))

Chatgpt [F] Failed to verify
time = 1.00, size = 25, normalized size = 1.47

$$-\frac{\tan(x)\cot(x)}{3}+\frac{\ln(\sec(x)+\tan(x))}{3}+\frac{\ln(\tan(x)-\sec(x))}{3}$$

Warning: Unable to verify antiderivative.

[In] int(1/sin(x)^4/cos(x)^4,x)

[Out] -1/3*tan(x)*cot(x)+1/3*ln(sec(x)+tan(x))+1/3*ln(tan(x)-sec(x))

3.254 $\int \frac{x+\sin(x)}{1+\cos(x)}\, dx$

Optimal. Leaf size=8

$$x\tan\left(\frac{x}{2}\right)$$

```
[Out] x*tan(1/2*x)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(8) = 16.
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 3.12, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$,
Rules used = {4462, 3399, 4269, 3556, 2746, 31}

$$x\tan\left(\frac{x}{2}\right)+2\log\left(\cos\left(\frac{x}{2}\right)\right)-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])/(1 + Cos[x]),x]
```

```
[Out] 2*Log[Cos[x/2]] - Log[1 + Cos[x]] + x*Tan[x/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{x}{1+\cos(x)}\,dx + \int \frac{\sin(x)}{1+\cos(x)}\,dx\\ &= \frac{1}{2}\int x\sec^2\left(\frac{x}{2}\right)\,dx - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\\ &= -\log(1+\cos(x)) + x\tan\left(\frac{x}{2}\right) - \int \tan\left(\frac{x}{2}\right)\,dx\\ &= 2\log\left(\cos\left(\frac{x}{2}\right)\right) - \log(1+\cos(x)) + x\tan\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 8, normalized size = 1.00

$$x\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])/(1 + Cos[x]),x]
```

```
[Out] x*Tan[x/2]
```

Maple [A]
time = 0.06, size = 7, normalized size = 0.88

method	result	size
lookup	$x\tan\left(\frac{x}{2}\right)$	7
default	$x\tan\left(\frac{x}{2}\right)$	7
norman	$x\tan\left(\frac{x}{2}\right)$	7
parallelrisch	$x\tan\left(\frac{x}{2}\right)$	7

risch	$-ix+\frac{2ix}{e^{ix}+1}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))/(1+cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x*tan(1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(6) = 12$.
time = 0.35, size = 61, normalized size = 7.62

$$\frac{\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)+2\,x\sin\left(x\right)}{\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1}-\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] ((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) +
1) + 2*x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - log(cos(x) + 1)
```

Fricas [A]
time = 0.58, size = 10, normalized size = 1.25

$$\frac{x\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] x*sin(x)/(cos(x) + 1)
```

Sympy [A]
time = 0.14, size = 5, normalized size = 0.62

$$x\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))/(1+cos(x)),x)
```

```
[Out] x*tan(x/2)
```

Giac [A]
time = 0.46, size = 6, normalized size = 0.75

$$x\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))/(1+cos(x)),x, algorithm="giac")
```

```
[Out] x*tan(1/2*x)
```

Mupad [B]
time = 0.11, size = 6, normalized size = 0.75

$$x\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sin(x))/(cos(x) + 1),x)
```

```
[Out] x*tan(x/2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 11, normalized size = 1.38

$$x-\cos\left(x\right)+\ln\left(1+\cos\left(x\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((x+sin(x))/(1+cos(x)),x)
```

```
[Out] x-cos(x)+ln(1+cos(x))
```

3.255 $\int \cosh^2(x)\sinh^3(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{3}\cosh^3(x)+\frac{\cosh^5(x)}{5}$$

[Out] -1/3*cosh(x)^3+1/5*cosh(x)^5

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2645, 14}

$$\frac{\cosh^5(x)}{5}-\frac{\cosh^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2*Sinh[x]^3,x]

[Out] -1/3*Cosh[x]^3 + Cosh[x]^5/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cosh(x)\right)\\
&= -\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cosh(x)\right)\\
&= -\frac{1}{3}\cosh^3(x)+\frac{\cosh^5(x)}{5}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.35

$$-\frac{\cosh(x)}{8}-\frac{1}{48}\cosh(3x)+\frac{1}{80}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2*Sinh[x]^3,x]
```

```
[Out] -1/8*Cosh[x] - Cosh[3*x]/48 + Cosh[5*x]/80
```

Maple [A]
time = 8.17, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{(\cosh^3(x))}{3}+\frac{(\cosh^5(x))}{5}$	14
default	$-\frac{(\cosh^3(x))}{3}+\frac{(\cosh^5(x))}{5}$	14
risch	$\frac{e^{5x}}{160}-\frac{e^{3x}}{96}-\frac{e^{x}}{16}-\frac{e^{-x}}{16}-\frac{e^{-3x}}{96}+\frac{e^{-5x}}{160}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^3*cosh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*cosh(x)^3+1/5*cosh(x)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.
time = 0.37, size = 39, normalized size = 2.29

$$-\frac{1}{480}\left(5\,e^{(-2\,x)}+30\,e^{(-4\,x)}-3\right)e^{(5\,x)}-\frac{1}{16}\,e^{(-x)}-\frac{1}{96}\,e^{(-3\,x)}+\frac{1}{160}\,e^{(-5\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3*cosh(x)^2,x, algorithm="maxima")
```

```
[Out] -1/480*(5*e^(-2*x) + 30*e^(-4*x) - 3)*e^(5*x) - 1/16*e^(-x) - 1/96*e^(-3*x)
 + 1/160*e^(-5*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.
time = 0.58, size = 42, normalized size = 2.47

$$\frac{1}{80}\cosh\left(x\right)^{5}+\frac{1}{16}\cosh\left(x\right)\sinh\left(x\right)^{4}-\frac{1}{48}\cosh\left(x\right)^{3}+\frac{1}{16}\left(2\,\cosh\left(x\right)^{3}-\cosh\left(x\right)\right)\sinh\left(x\right)^{2}-\frac{1}{8}\cosh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3*cosh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 1/48*cosh(x)^3 + 1/16*(2*cosh(x)^
3 - cosh(x))*sinh(x)^2 - 1/8*cosh(x)
```

Sympy [A]
time = 0.19, size = 19, normalized size = 1.12

$$\frac{\sinh^2(x)\cosh^3(x)}{3} - \frac{2\cosh^5(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**3*cosh(x)**2,x)
```

```
[Out] sinh(x)**2*cosh(x)**3/3 - 2*cosh(x)**5/15
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.
time = 0.45, size = 37, normalized size = 2.18

$$-\frac{1}{480}\left(30\,e^{(4\,x)} + 5\,e^{(2\,x)} - 3\right)e^{(-5\,x)} + \frac{1}{160}\,e^{(5\,x)} - \frac{1}{96}\,e^{(3\,x)} - \frac{1}{16}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^3*cosh(x)^2,x, algorithm="giac")
```

```
[Out] -1/480*(30*e^(4*x) + 5*e^(2*x) - 3)*e^(-5*x) + 1/160*e^(5*x) - 1/96*e^(3*x)
 - 1/16*e^x
```

Mupad [B]
time = 0.06, size = 14, normalized size = 0.82

$$\frac{\cosh(x)^3\left(3\cosh(x)^2 - 5\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2*sinh(x)^3,x)
```

```
[Out] (cosh(x)^3*(3*cosh(x)^2 - 5))/15
```

Chatgpt [F] Failed to verify
time = 1.00, size = 22, normalized size = 1.29

$$\frac{(\cosh^4(x))}{4} - \frac{(\cosh^2(x))}{2} + \frac{3(\sinh^4(x))}{8} + \frac{x}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(sinh(x)^3*cosh(x)^2,x)
```

```
[Out] 1/4*cosh(x)^4-1/2*cosh(x)^2+3/8*sinh(x)^4+1/2*x
```

3.256 $\int 3^{2^x} 4^x \, dx$

Optimal. Leaf size=33

$$-\frac{3^{2^x}}{\log(2)\log^2(3)}+\frac{2^x 3^{2^x}}{\log(2)\log(3)}$$

```
[Out] -3^(2^x)/ln(2)/ln(3)^2+2^x*3^(2^x)/ln(2)/ln(3)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2320, 2207, 2225}

$$\frac{2^x 3^{2^x}}{\log(2)\log(3)}-\frac{3^{2^x}}{\log(2)\log^2(3)}$$

Antiderivative was successfully verified.

```
[In] Int[3^2^x*4^x,x]
```

```
[Out] -(3^2^x/(Log[2]*Log[3]^2)) + (2^x*3^2^x)/(Log[2]*Log[3])
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{\text{Subst}\left(\int 3^x x\,dx, x, 2^x\right)}{\log(2)}\\ &= \frac{2^x 3^{2^x}}{\log(2)\log(3)} - \frac{\text{Subst}\left(\int 3^x\,dx, x, 2^x\right)}{\log(2)\log(3)}\\ &= -\frac{3^{2^x}}{\log(2)\log^2(3)} + \frac{2^x 3^{2^x}}{\log(2)\log(3)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.67

$$\frac{3^{2^x}(-1+2^x\log(3))}{\log(2)\log^2(3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[3^2^x*4^x,x]
[Out] (3^2^x*(-1 + 2^x*Log[3]))/(Log[2]*Log[3]^2)
```

Maple [A]
time = 0.06, size = 23, normalized size = 0.70

method	result	size
risch	$\frac{(2^x\ln(3)-1)3^{2^x}}{\ln(2)\ln(3)^2}$	23
norman	$\frac{e^{x\ln(2)}e^{e^{x\ln(2)}\ln(3)}}{\ln(2)\ln(3)} - \frac{e^{e^{x\ln(2)}\ln(3)}}{\ln(2)\ln(3)^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4^x*3^(2^x),x,method=_RETURNVERBOSE)
[Out] (2^x*ln(3)-1)/ln(2)/ln(3)^2*3^(2^x)
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.45, size = 31, normalized size = 0.94

$$-\frac{4^x\Gamma\left(2, -4^{\frac{1}{2}x}\log(3)\right)}{4^x\log(3)^2\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x*3^(2^x),x, algorithm="maxima")
[Out] -4^x*gamma(2, -4^(1/2*x)*log(3))/(4^x*log(3)^2*log(2))
```

Fricas [A]
time = 0.57, size = 22, normalized size = 0.67

$$\frac{(2^{x} \log(3) - 1)3^{(2^{x})}}{\log(3)^{2} \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x*3^(2^x),x, algorithm="fricas")
[Out] (2^x*log(3) - 1)*3^(2^x)/(log(3)^2*log(2))
```

Sympy [A]
time = 0.06, size = 24, normalized size = 0.73

$$\frac{(2^{x} \log(3) - 1) e^{2^{x} \log(3)}}{\log(2) \log(3)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4**x*3**(2**x),x)
[Out] (2**x*log(3) - 1)*exp(2**x*log(3))/(log(2)*log(3)**2)
```

Giac [A]
time = 0.46, size = 51, normalized size = 1.55

$$\frac{2^{x} e^{(2^{x} \log(3)+2 x \log(2))} \log(3) - e^{(2^{x} \log(3)+2 x \log(2))}}{2^{2 x} \log(3)^{2} \log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4^x*3^(2^x),x, algorithm="giac")
[Out] (2^x*e^(2^x*log(3) + 2*x*log(2))*log(3) - e^(2^x*log(3) + 2*x*log(2)))/(2^(
2*x)*log(3)^2*log(2))
```

Mupad [B]
time = 0.14, size = 22, normalized size = 0.67

$$\frac{3^{2^{x}} (2^{x} \ln(3) - 1)}{\ln(2) \ln(3)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3^(2^x)*4^x,x)
[Out] (3^(2^x)*(2^x*log(3) - 1))/(log(2)*log(3)^2)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(4^x*3^(2^x),x)
```

```
[Out] not solved
```

3.257 $\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx$

Optimal. Leaf size=6

$$\arctan(\cos(x)+\sin(x))$$

```
[Out] arctan(cos(x)+sin(x))
```

Rubi [F]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(x)-\sin(x)}{2+\sin(2x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[x] - Sin[x])/(2 + Sin[2*x]),x]
```

```
[Out] Defer[Int][Cos[x]/(2 + Sin[2*x]), x] - Defer[Int][Sin[x]/(2 + Sin[2*x]), x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(\frac{\cos(x)}{2+\sin(2x)}-\frac{\sin(x)}{2+\sin(2x)}\right)dx\\ &= \int \frac{\cos(x)}{2+\sin(2x)}\,dx-\int \frac{\sin(x)}{2+\sin(2x)}\,dx\end{aligned}$$

Mathematica [A]
time = 0.02, size = 6, normalized size = 1.00

$$\arctan(\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - Sin[x])/(2 + Sin[2*x]),x]
```

```
[Out] ArcTan[Cos[x] + Sin[x]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.40, size = 112, normalized size = 18.67

method	result

risch	$\frac{i\ln(e^{2ix}+(-1+i)e^{ix}+i)}{2}-\frac{i\ln(e^{2ix}+(1-i)e^{ix}+i)}{2}$
default	$\frac{\left(\sum_{\_R=RootOf(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1)}\frac{(-\_R^2+1)\ln(\tan(\frac{x}{2})-\_R)}{2\_R^3-3\_R^2+2\_R+1}\right)}{2}-\left(\sum_{\_R=RootOf(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1)}\frac{\_R}{2\_R}\right.$
parts	$\frac{\left(\sum_{\_R=RootOf(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1)}\frac{(-\_R^2+1)\ln(\tan(\frac{x}{2})-\_R)}{2\_R^3-3\_R^2+2\_R+1}\right)}{2}-\left(\sum_{\_R=RootOf(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1)}\frac{\_R}{2\_R}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-sin(x))/(2+sin(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sum((-_R^2+1)/(2*_R^3-3*_R^2+2*_R+1)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^4-2
*_Z^3+2*_Z^2+2*_Z+1))-sum(_R/(2*_R^3-3*_R^2+2*_R+1)*ln(tan(1/2*x)-_R),_R=Ro
otOf(_Z^4-2*_Z^3+2*_Z^2+2*_Z+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(2+sin(2*x)),x, algorithm="maxima")
```

```
[Out] integrate((cos(x) - sin(x))/(sin(2*x) + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.
time = 0.61, size = 15, normalized size = 2.50

$$\frac{1}{2}\arctan\left(\frac{\cos(x)\sin(x)}{\cos(x)+\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(2+sin(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(cos(x)*sin(x)/(cos(x) + sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{-\sin(x)+\cos(x)}{\sin(2x)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(2+sin(2*x)),x)
```

```
[Out] Integral((-sin(x) + cos(x))/(sin(2*x) + 2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(6) = 12$.
time = 0.43, size = 35, normalized size = 5.83

$$\arctan\left(\frac{1}{2}\tan\left(\frac{1}{2}x\right)^{3}-\frac{3}{2}\tan\left(\frac{1}{2}x\right)^{2}+\frac{3}{2}\tan\left(\frac{1}{2}x\right)+\frac{1}{2}\right)-\arctan\left(\tan\left(\frac{1}{2}x\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(2+sin(2*x)),x, algorithm="giac")
```

```
[Out] arctan(1/2*tan(1/2*x)^3 - 3/2*tan(1/2*x)^2 + 3/2*tan(1/2*x) + 1/2) - arctan
(tan(1/2*x) - 2)
```

Mupad [B]
time = 0.50, size = 35, normalized size = 5.83

$$\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2}-\frac{3\tan\left(\frac{x}{2}\right)^2}{2}+\frac{3\tan\left(\frac{x}{2}\right)}{2}+\frac{1}{2}\right)-\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - sin(x))/(sin(2*x) + 2),x)
```

```
[Out] atan((3*tan(x/2))/2 - (3*tan(x/2)^2)/2 + tan(x/2)^3/2 + 1/2) - atan(tan(x/2
) - 2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 36, normalized size = 6.00

$$-\frac{\ln\left(2+\sin\left(2x\right)\right)}{2}-\frac{\ln\left(2-\sin\left(2x\right)\right)}{2}+\frac{\arctan\left(\frac{2\cos(x)}{2+\sin(2x)}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((cos(x)-sin(x))/(2+sin(2*x)),x)
```

```
[Out] -1/2*ln(2+sin(2*x))-1/2*ln(2-sin(2*x))+1/2*arctan(2*cos(x)/(2+sin(2*x)))
```

3.258 $\int \frac{\sec^2(1+\log(x))-\tan(1+\log(x))}{x^2}\,dx$

Optimal. Leaf size=9

$$\frac{\tan(1+\log(x))}{x}$$

[Out] tan(1+ln(x))/x

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 9.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 4601, 371, 4591, 470}

$$\frac{2i\,\text{Hypergeometric2F1}\left(\frac{i}{2},1,1+\frac{i}{2},-e^{2i}x^{2i}\right)}{x}-\left(\frac{4}{5}+\frac{8i}{5}\right)e^{2i}x^{-1+2i}\,\text{Hypergeometric2F1}\left(1+\frac{i}{2},2,2+\frac{i}{2},-e^{2i}x^{2i}\right)-\frac{i}{x}$$

Antiderivative was successfully verified.

[In] Int[(Sec[1 + Log[x]]^2 - Tan[1 + Log[x]])/x^2,x]

[Out] (-I)/x + ((2*I)*Hypergeometric2F1[I/2, 1, 1 + I/2, -(E^(2*I)*x^(2*I))])/x - ((4/5 + (8*I)/5)*E^(2*I)*Hypergeometric2F1[1 + I/2, 2, 2 + I/2, -(E^(2*I)*x^(2*I))])/x^(1 - 2*I)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 4591

```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 4601

```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Dist[2^p*E^(I*a*d*p), Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b
*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int \left(\frac{\sec^2(1+\log(x))}{x^2} - \frac{\tan(1+\log(x))}{x^2}\right) dx \\
&= \int \frac{\sec^2(1+\log(x))}{x^2}\, dx - \int \frac{\tan(1+\log(x))}{x^2}\, dx \\
&= \left(4e^{2i}\right) \int \frac{x^{-2+2i}}{\left(1+e^{2i}x^{2i}\right)^2}\, dx - \int \frac{i-ie^{2i}x^{2i}}{\left(1+e^{2i}x^{2i}\right)x^2}\, dx \\
&= -\frac{i}{x} - \left(\frac{4}{5}+\frac{8i}{5}\right) e^{2i}x^{-1+2i}\, \text{Hypergeometric2F1}\left(1+\frac{i}{2},2,2+\frac{i}{2},-e^{2i}x^{2i}\right) - 2i \int \frac{1}{\left(1+e^{2i}x^{2i}\right)x^2}\, dx \\
&= -\frac{i}{x} + \frac{2i\, \text{Hypergeometric2F1}\left(\frac{i}{2},1,1+\frac{i}{2},-e^{2i}x^{2i}\right)}{x} - \left(\frac{4}{5}+\frac{8i}{5}\right) e^{2i}x^{-1+2i}\, \text{Hypergeometric2F1}\left(1+\frac{i}{2},2,2+\frac{i}{2},-e^{2i}x^{2i}\right)
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.
time = 0.47, size = 21, normalized size = 2.33

$$\frac{\sec(1)\sec(1+\log(x))\sin(\log(x))}{x} + \frac{\tan(1)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[1 + Log[x]]^2 - Tan[1 + Log[x]])/x^2,x]
[Out] (Sec[1]*Sec[1 + Log[x]]*Sin[Log[x]])/x + Tan[1]/x
```

Maple [C] Result contains complex when optimal does not.
time = 1.36, size = 28, normalized size = 3.11

method	result	size
risch	$-\frac{i}{x}+\frac{2i}{x\left(x^{2i}\mathrm{e}^{2i}+1\right)}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(1+ln(x)))^2-tan(1+ln(x)))/x^2,x,method=_RETURNVERBOSE)
[Out] -I/x+2*I/x/((x^I)^2*exp(2*I)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(9) = 18$.
time = 0.44, size = 45, normalized size = 5.00

$$\frac{2 \sin (2 \log (x) + 2)}{x \cos (2 \log (x) + 2)^2 + x \sin (2 \log (x) + 2)^2 + 2 x \cos (2 \log (x) + 2) + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(1+log(x))^2-tan(1+log(x)))/x^2,x, algorithm="maxima")
```

```
[Out] 2*sin(2*log(x) + 2)/(x*cos(2*log(x) + 2)^2 + x*sin(2*log(x) + 2)^2 + 2*x*co
s(2*log(x) + 2) + x)
```

Fricas [A]
time = 0.61, size = 16, normalized size = 1.78

$$\frac{\sin (\log (x) + 1)}{x \cos (\log (x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(1+log(x))^2-tan(1+log(x)))/x^2,x, algorithm="fricas")
```

```
[Out] sin(log(x) + 1)/(x*cos(log(x) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\tan (\log (x) + 1) + \sec^2 (\log (x) + 1)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(1+ln(x))**2-tan(1+ln(x)))/x**2,x)
```

```
[Out] Integral((-tan(log(x) + 1) + sec(log(x) + 1)**2)/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5161 vs. $2(9) = 18$.
time = 1.68, size = 5161, normalized size = 573.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(1+log(x))^2-tan(1+log(x)))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(13*tan(1)^4*tan(log(x))^9/((tan(log(x))^2 - 1)*x) + 42*tan(1)^4*tan(lo
g(x))^10/((tan(log(x))^2 - 1)^2*x) + 84*tan(1)^4*tan(log(x))^11/((tan(log(x
```

))^2 - 1)^3*x) - 88*tan(1)^4*tan(log(x))^12/((tan(log(x))^2 - 1)^4*x) - 17*tan(1)^4*tan(log(x))^7/x - 4*tan(1)^3*tan(log(x))^8/x - 62*tan(1)^4*tan(log(x))^8/((tan(log(x))^2 - 1)*x) + 22*tan(1)^3*tan(log(x))^9/((tan(log(x))^2 - 1)*x) - 102*tan(1)^4*tan(log(x))^9/((tan(log(x))^2 - 1)^2*x) - 6*tan(1)^3*tan(log(x))^10/((tan(log(x))^2 - 1)^2*x) + 520*tan(1)^4*tan(log(x))^10/((tan(log(x))^2 - 1)^3*x) - 168*tan(1)^3*tan(log(x))^11/((tan(log(x))^2 - 1)^3*x) + 248*tan(1)^4*tan(log(x))^11/((tan(log(x))^2 - 1)^4*x) - 88*tan(1)^3*tan(log(x))^12/((tan(log(x))^2 - 1)^4*x) + 20*tan(1)^4*tan(log(x))^6/x - 22*tan(1)^3*tan(log(x))^7/x - 17*tan(1)^4*tan(log(x))^7/((tan(log(x))^2 - 1)*x) - 70*tan(1)^3*tan(log(x))^8/((tan(log(x))^2 - 1)*x) - 740*tan(1)^4*tan(log(x))^8/((tan(log(x))^2 - 1)^2*x) + 17*tan(1)^2*tan(log(x))^9/((tan(log(x))^2 - 1)*x) + 488*tan(1)^3*tan(log(x))^9/((tan(log(x))^2 - 1)^2*x) - 548*tan(1)^4*tan(log(x))^9/((tan(log(x))^2 - 1)^3*x) - 56*tan(1)^2*tan(log(x))^10/((tan(log(x))^2 - 1)^2*x) + 40*tan(1)^3*tan(log(x))^10/((tan(log(x))^2 - 1)^3*x) + 560*tan(1)^4*tan(log(x))^10/((tan(log(x))^2 - 1)^4*x) + 36*tan(1)^2*tan(log(x))^11/((tan(log(x))^2 - 1)^3*x) - 256*tan(1)^3*tan(log(x))^11/((tan(log(x))^2 - 1)^4*x) + 32*tan(1)^2*tan(log(x))^12/((tan(log(x))^2 - 1)^4*x) - 22*tan(1)^4*tan(log(x))^5/x + 36*tan(1)^3*tan(log(x))^6/x + 326*tan(1)^4*tan(log(x))^6/((tan(log(x))^2 - 1)*x) - 17*tan(1)^2*tan(log(x))^7/x - 388*tan(1)^3*tan(log(x))^7/((tan(log(x))^2 - 1)*x) + 150*tan(1)^4*tan(log(x))^7/((tan(log(x))^2 - 1)^2*x) - 4*tan(1)*tan(log(x))^8/x + 64*tan(1)^2*tan(log(x))^8/((tan(log(x))^2 - 1)*x) - 276*tan(1)^3*tan(log(x))^8/((tan(log(x))^2 - 1)^2*x) - 1256*tan(1)^4*tan(log(x))^8/((tan(log(x))^2 - 1)^3*x) + 6*tan(1)*tan(log(x))^9/((tan(log(x))^2 - 1)*x) + 24*tan(1)^2*tan(log(x))^9/((tan(log(x))^2 - 1)^2*x) + 752*tan(1)^3*tan(log(x))^9/((tan(log(x))^2 - 1)^3*x) - 328*tan(1)^4*tan(log(x))^9/((tan(log(x))^2 - 1)^4*x) - 30*tan(1)*tan(log(x))^10/((tan(log(x))^2 - 1)^2*x) - 528*tan(1)^3*tan(log(x))^10/((tan(log(x))^2 - 1)^4*x) + 24*tan(1)*tan(log(x))^11/((tan(log(x))^2 - 1)^3*x) + 240*tan(1)^2*tan(log(x))^11/((tan(log(x))^2 - 1)^4*x) - 56*tan(1)*tan(log(x))^12/((tan(log(x))^2 - 1)^4*x) - 40*tan(1)^4*tan(log(x))^4/x + 46*tan(1)^3*tan(log(x))^5/x - 137*tan(1)^4*tan(log(x))^5/((tan(log(x))^2 - 1)*x) - 8*tan(1)^2*tan(log(x))^6/x + 206*tan(1)^3*tan(log(x))^6/((tan(log(x))^2 - 1)*x) + 696*tan(1)^4*tan(log(x))^6/((tan(log(x))^2 - 1)^2*x) - 6*tan(1)*tan(log(x))^7/x - 44*tan(1)^2*tan(log(x))^7/((tan(log(x))^2 - 1)*x) - 456*tan(1)^3*tan(log(x))^7/((tan(log(x))^2 - 1)^2*x) - 68*tan(1)^4*tan(log(x))^7/((tan(log(x))^2 - 1)^3*x) - 38*tan(1)*tan(log(x))^8/((tan(log(x))^2 - 1)*x) - 416*tan(1)^2*tan(log(x))^8/((tan(log(x))^2 - 1)^2*x) + 504*tan(1)^3*tan(log(x))^8/((tan(log(x))^2 - 1)^3*x) - 416*tan(1)^4*tan(log(x))^8/((tan(log(x))^2 - 1)^4*x) + 104*tan(1)*tan(log(x))^9/((tan(log(x))^2 - 1)^2*x) - 432*tan(1)^2*tan(log(x))^9/((tan(log(x))^2 - 1)^3*x) - 2*tan(log(x))^10/((tan(log(x))^2 - 1)^2*x) - 88*tan(1)*tan(log(x))^10/((tan(log(x))^2 - 1)^3*x) + 512*tan(1)^2*tan(log(x))^10/((tan(log(x))^2 - 1)^4*x) - 8*tan(log(x))^12/((tan(log(x))^2 - 1)^4*x) + 7*tan(1)^4*tan(log(x))^3/x + 8*tan(1)^3*tan(log(x))^4/x - 122*tan(1)^4*tan(log(x))^4/((tan(log(x))^2 - 1)*x) - 33*tan(1)^2*tan(log(x))^5/x - 208*tan(1)^3*tan(log(x))^5/((tan(log(x))^2 - 1)*x) - 162*tan(1)^4*tan

```
(log(x))^5/((tan(log(x))^2 - 1)^2*x) + 28*tan(1)*tan(log(x))^6/x + 576*tan(
1)^2*tan(log(x))^6/((tan(log(x))^2 - 1)*x) - 120*tan(1)^3*tan(log(x))^6/((t
an(log(x))^2 - 1)^2*x) + 280*tan(1)^4*tan(log(x))^6/((tan(log(x))^2 - 1)^3*
x) + 4*tan(log(x))^7/x - 196*tan(1)*tan(log(x))^7/((tan(log(x))^2 - 1)*x) +
 408*tan(1)^2*tan(log(x))^7/((tan(log(x))^2 - 1)^2*x) + 448*tan(1)^3*tan(lo
g(x))^7/((tan(log(x))^2 - 1)^3*x) - 376*tan(1)^4*tan(log(x))^7/((tan(log(x)
)^2 - 1)^4*x) - 2*tan(log(x))^8/((tan(log(x))^2 - 1)*x) - 180*tan(1)*tan(lo
g(x))^8/((tan(log(x))^2 - 1)^2*x) - 2048*tan(1)^2*tan(log(x))^8/((tan(log(x
))^2 - 1)^3*x) + 32*tan(1)^3*tan(log(x))^8/((tan(log(x))^2 - 1)^4*x) + 30*t
an(log(x))^9/((tan(log(x))^2 - 1)^2*x) + 496*tan(1)*tan(log(x))^9/((tan(log
(x))^2 - 1)^3*x) - 16*tan(1)^2*tan(log(x))^9/((tan(log(x))^2 - 1)^4*x) - 8*
tan(log(x))^10/((tan(log(x))^2 - 1)^3*x) - 528*tan(1)*tan(log(x))^10/((tan(
log(x))^2 - 1)^4*x) + 56*tan(log(x))^11/((tan(log(x))^2 - 1)^4*x) + 4*tan(1
)^4*tan(log(x))^2/x + 62*tan(1)^3*tan(log(x))^3/x + 21*tan(1)^4*tan(log(x))
^3/((tan(log(x))^2 - 1)*x) - 144*tan(1)^2*tan(log(x))^4/x + 238*tan(1)^3*ta
n(log(x))^4/((tan(log(x))^2 - 1)*x) - 60*tan(1)^4*tan(log(x))^4/((tan(log(x
))^2 - 1)^2*x) + 62*tan(1)*tan(log(x))^5/x - 378*tan(1)^2*tan(log(x))^5/((t
an(log(x))^2 - 1)*x) - 840*tan(1)^3*tan(log(x))^5/((tan(log(x))^2 - 1)^2*x)
 + 52*tan(1)^4*tan(log(x))^5/((tan(log(x))^2 - 1)^3*x) + 4*tan(log(x))^6/x
+ 238*tan(1)*tan(log(x))^6/((tan(log(x))^2 - 1)...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int -\frac{\tan(\ln(x)+1)-\frac{1}{\cos(\ln(x)+1)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(tan(log(x) + 1) - 1/cos(log(x) + 1)^2)/x^2,x)
```

```
[Out] int(-(tan(log(x) + 1) - 1/cos(log(x) + 1)^2)/x^2, x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 12, normalized size = 1.33

$$-\frac{1}{x}+\frac{1}{1+\ln(x)}$$

Warning: Unable to verify antiderivative.

```
[In] int((sec(1+ln(x))^2-tan(1+ln(x)))/x^2,x)
```

```
[Out] -1/x+1/(1+ln(x))
```

3.259 $\int \sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,dx$

Optimal. Leaf size=67

$$2x\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}-\frac{\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{2}}\right)\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}$$

[Out] 2*x*(1/x*ln(1/x))^(1/2)-2^(1/2)*Pi^(1/2)*erf(1/2*ln(1/x)^(1/2)*2^(1/2))*(1/x*ln(1/x))^(1/2)/(1/x)^(1/2)/ln(1/x)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6851, 2342, 2347, 2211, 2236}

$$2x\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}-\frac{\sqrt{2\pi}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\operatorname{erf}\left(\frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{2}}\right)}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[x^(-1)]/x],x]

[Out] 2*x*Sqrt[Log[x^(-1)]/x] - (Sqrt[2*Pi]*Erf[Sqrt[Log[x^(-1)]]/Sqrt[2]]*Sqrt[Log[x^(-1)]/x])/(Sqrt[x^(-1)]*Sqrt[Log[x^(-1)]])

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \frac{\left(\sqrt{x}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\right)\int \frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{x}}\,dx}{\sqrt{\log\left(\frac{1}{x}\right)}}\\
&= 2x\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}} + \frac{\left(\sqrt{x}\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\right)\int \frac{1}{\sqrt{x}\sqrt{\log\left(\frac{1}{x}\right)}}\,dx}{\sqrt{\log\left(\frac{1}{x}\right)}}\\
&= 2x\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}} - \frac{\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\,\text{Subst}\left(\int \frac{e^{-x/2}}{\sqrt{x}}\,dx, x, \log\left(\frac{1}{x}\right)\right)}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}\\
&= 2x\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}} - \frac{\left(2\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}\right)\text{Subst}\left(\int e^{-\frac{x^2}{2}}\,dx, x, \sqrt{\log\left(\frac{1}{x}\right)}\right)}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}\\
&= 2x\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}} - \frac{\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{2}}\right)\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.84

$$
\left(2x - \frac{\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{\log\left(\frac{1}{x}\right)}}{\sqrt{2}}\right)}{\sqrt{\frac{1}{x}}\sqrt{\log\left(\frac{1}{x}\right)}}\right)\sqrt{\frac{\log\left(\frac{1}{x}\right)}{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Log[x^(-1)]/x],x]
```

```
[Out] (2*x - (Sqrt[2*Pi]*Erf[Sqrt[Log[x^(-1)]]/Sqrt[2]])/(Sqrt[x^(-1)]*Sqrt[Log[x
^(-1)]]))*Sqrt[Log[x^(-1)]/x]
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\ln \left(\frac{1}{x}\right)}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x*ln(1/x))^(1/2),x)
```

```
[Out] int((1/x*ln(1/x))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x*log(1/x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-log(x)/x), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x*log(1/x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{\log \left(\frac{1}{x}\right)}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x*ln(1/x))**(1/2),x)
[Out] Integral(sqrt(log(1/x)/x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x*log(1/x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(log(1/x)/x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{\ln\left(\frac{1}{x}\right)}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(1/x)/x)^(1/2),x)
[Out] int((log(1/x)/x)^(1/2), x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 27, normalized size = 0.40

$$4\sqrt{\frac{\ln(x)}{x}}\left(x+2\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{2}\,\sqrt{\ln(x)}}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((1/x*ln(1/x))^(1/2),x)
[Out] 4*(ln(x)/x)^(1/2)*(x+2*Pi^(1/2)*erf(1/2*2^(1/2)*ln(x)^(1/2)))
```

3.260 $\int (1-\cos(x))^5 \cos(5x)\,dx$

Optimal. Leaf size=84

$$-\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45}{64}\sin(2x)+\frac{5}{4}\sin(3x)-\frac{105}{64}\sin(4x)+\frac{63}{40}\sin(5x)-\frac{35}{32}\sin(6x)+\frac{15}{28}\sin(7x)-\frac{45}{256}\sin(8x)+\frac{5}{144}$$

```
[Out] -1/32*x+5/16*sin(x)-45/64*sin(2*x)+5/4*sin(3*x)-105/64*sin(4*x)+63/40*sin(5
*x)-35/32*sin(6*x)+15/28*sin(7*x)-45/256*sin(8*x)+5/144*sin(9*x)-1/320*sin(
10*x)
```

Rubi [A]

time = 0.13, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4486, 2717, 4368, 4439}

$$-\frac{x}{32}+\frac{5\sin(x)}{16}-\frac{45}{64}\sin(2x)+\frac{5}{4}\sin(3x)-\frac{105}{64}\sin(4x)+\frac{63}{40}\sin(5x)-\frac{35}{32}\sin(6x)+\frac{15}{28}\sin(7x)-\frac{45}{256}\sin(8x)+\frac{5}{144}\sin(9x)-\frac{1}{320}\sin(10x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^5*Cos[5*x],x]
```

```
[Out] -1/32*x + (5*Sin[x])/16 - (45*Sin[2*x])/64 + (5*Sin[3*x])/4 - (105*Sin[4*x]
)/64 + (63*Sin[5*x])/40 - (35*Sin[6*x])/32 + (15*Sin[7*x])/28 - (45*Sin[8*x
])/256 + (5*Sin[9*x])/144 - Sin[10*x]/320
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\cos(5x) - 5\cos(x)\cos(5x) + 10\cos^2(x)\cos(5x) - 10\cos^3(x)\cos(5x) + 5\cos^4(x)\cos(5x) - \cos^5(x)\cos(5x)\right)\, dx \\
&= -(5\int \cos(x)\cos(5x)\, dx) + 5\int \cos^4(x)\cos(5x)\, dx + 10\int \cos^2(x)\cos(5x)\, dx - 10\int \cos^3(x)\cos(5x)\, dx + \int \cos(5x)\, dx - \int \cos^5(x)\cos(5x)\, dx \\
&= -\frac{5}{8}\sin(4x) + \frac{1}{5}\sin(5x) - \frac{5}{12}\sin(6x) + 5\int \left(\frac{\cos(x)}{16} + \frac{1}{4}\cos(3x) + \frac{3}{8}\cos(5x) + \frac{1}{4}\cos(7x) + \frac{1}{16}\cos(9x)\right) dx + 10\int \left(\frac{1}{4}\cos(3x) + \frac{1}{2}\cos(5x) + \frac{1}{4}\cos(7x)\right) dx - 10\int \left(\frac{1}{8}\cos(2x) + \frac{3}{8}\cos(4x) + \frac{3}{8}\cos(6x) + \frac{1}{8}\cos(8x)\right) dx - \int \left(\frac{1}{32} + \frac{5}{32}\cos(2x) + \frac{5}{16}\cos(4x) + \frac{5}{16}\cos(6x) + \frac{5}{32}\cos(8x) + \frac{1}{32}\cos(10x)\right) dx \\
&= -\frac{x}{32} - \frac{5}{8}\sin(4x) + \frac{1}{5}\sin(5x) - \frac{5}{12}\sin(6x) - \frac{1}{32}\int \cos(10x)\, dx - \frac{5}{32}\int \cos(2x)\, dx - \frac{5}{32}\int \cos(8x)\, dx + \frac{5}{16}\int \cos(x)\, dx - \frac{5}{16}\int \cos(4x)\, dx - \frac{5}{16}\int \cos(6x)\, dx + \frac{5}{16}\int \cos(9x)\, dx - \frac{5}{4}\int \cos(2x)\, dx + \frac{5}{4}\int \cos(3x)\, dx + \frac{5}{4}\int \cos(7x)\, dx - \frac{5}{4}\int \cos(8x)\, dx + \frac{15}{8}\int \cos(5x)\, dx + \frac{5}{2}\int \cos(3x)\, dx + \frac{5}{2}\int \cos(7x)\, dx - \frac{15}{4}\int \cos(4x)\, dx - \frac{15}{4}\int \cos(6x)\, dx + 5\int \cos(5x)\, dx \\
&= -\frac{x}{32} + \frac{5\sin(x)}{16} - \frac{45}{64}\sin(2x) + \frac{5}{4}\sin(3x) - \frac{105}{64}\sin(4x) + \frac{63}{40}\sin(5x) - \frac{35}{32}\sin(6x) + \frac{15}{28}\sin(7x) - \frac{45}{256}\sin(8x) + \frac{5}{144}\sin(9x) - \frac{1}{320}\sin(10x)
\end{aligned}$$

Mathematica [A]

time = 0.08, size = 84, normalized size = 1.00

$$-\frac{x}{32} + \frac{5\sin(x)}{16} - \frac{45}{64}\sin(2x) + \frac{5}{4}\sin(3x) - \frac{105}{64}\sin(4x) + \frac{63}{40}\sin(5x) - \frac{35}{32}\sin(6x) + \frac{15}{28}\sin(7x) - \frac{45}{256}\sin(8x) + \frac{5}{144}\sin(9x) - \frac{1}{320}\sin(10x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^5*Cos[5*x],x]
```

```
[Out] -1/32*x + (5*Sin[x])/16 - (45*Sin[2*x])/64 + (5*Sin[3*x])/4 - (105*Sin[4*x]
)/64 + (63*Sin[5*x])/40 - (35*Sin[6*x])/32 + (15*Sin[7*x])/28 - (45*Sin[8*x
])/256 + (5*Sin[9*x])/144 - Sin[10*x]/320
```

Maple [A]

time = 1.05, size = 63, normalized size = 0.75

method	result
default	$-\frac{x}{32} + \frac{5\sin(x)}{16} - \frac{45\sin(2x)}{64} + \frac{5\sin(3x)}{4} - \frac{105\sin(4x)}{64} + \frac{63\sin(5x)}{40} - \frac{35\sin(6x)}{32} + \frac{15\sin(7x)}{28} - \frac{45\sin(8x)}{256} +$ 5 si
risch	$-\frac{x}{32} + \frac{5\sin(x)}{16} - \frac{45\sin(2x)}{64} + \frac{5\sin(3x)}{4} - \frac{105\sin(4x)}{64} + \frac{63\sin(5x)}{40} - \frac{35\sin(6x)}{32} + \frac{15\sin(7x)}{28} - \frac{45\sin(8x)}{256} +$ 5 si
parallelrisch	$-\frac{x}{32} + \frac{5\sin(x)}{16} - \frac{45\sin(2x)}{64} + \frac{5\sin(3x)}{4} - \frac{105\sin(4x)}{64} + \frac{63\sin(5x)}{40} - \frac{35\sin(6x)}{32} + \frac{15\sin(7x)}{28} - \frac{45\sin(8x)}{256} +$ 5 si
parts	$-\frac{x}{32} + \frac{5\sin(x)}{16} - \frac{45\sin(2x)}{64} + \frac{5\sin(3x)}{4} - \frac{105\sin(4x)}{64} + \frac{63\sin(5x)}{40} - \frac{35\sin(6x)}{32} + \frac{15\sin(7x)}{28} - \frac{45\sin(8x)}{256} +$ 5 si

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x))^5*cos(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*x+5/16*sin(x)-45/64*sin(2*x)+5/4*sin(3*x)-105/64*sin(4*x)+63/40*sin(5
*x)-35/32*sin(6*x)+15/28*sin(7*x)-45/256*sin(8*x)+5/144*sin(9*x)-1/320*sin(
10*x)
```

Maxima [A]

time = 0.41, size = 66, normalized size = 0.79

$$\frac{80}{9}\sin(x)^9 - \frac{380}{7}\sin(x)^7 - \frac{1}{20}\sin(2x)^5 + \frac{501}{5}\sin(x)^5 + \frac{71}{16}\sin(2x)^3 - \frac{212}{3}\sin(x)^3 - \frac{1}{32}x - \frac{45}{256}\sin(8x) - \frac{105}{64}\sin(4x) - 4\sin(2x) + 16\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^5*cos(5*x),x, algorithm="maxima")
```

```
[Out] 80/9*sin(x)^9 - 380/7*sin(x)^7 - 1/20*sin(2*x)^5 + 501/5*sin(x)^5 + 71/16*s
in(2*x)^3 - 212/3*sin(x)^3 - 1/32*x - 45/256*sin(8*x) - 105/64*sin(4*x) - 4
*sin(2*x) + 16*sin(x)
```

Fricas [**A**]
time = 0.66, size = 62, normalized size = 0.74

$$-\frac{1}{10080}\left(16128\cos\left(x\right)^{9}-89600\cos\left(x\right)^{8}+194544\cos\left(x\right)^{7}-188800\cos\left(x\right)^{6}+33768\cos\left(x\right)^{5}+93984\cos\left(x\right)^{4}-83790\cos\left(x\right)^{3}+24512\cos\left(x\right)^{2}+315\cos\left(x\right)-1376\right)\sin\left(x\right)-\frac{1}{32}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^5*cos(5*x),x, algorithm="fricas")
```

```
[Out] -1/10080*(16128*cos(x)^9 - 89600*cos(x)^8 + 194544*cos(x)^7 - 188800*cos(x)
^6 + 33768*cos(x)^5 + 93984*cos(x)^4 - 83790*cos(x)^3 + 24512*cos(x)^2 + 31
5*cos(x) - 1376)*sin(x) - 1/32*x
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(83) = 166$.
time = 4.33, size = 394, normalized size = 4.69

$$-\frac{x\sin^{5}(x)\sin(5x)}{32}-\frac{5x\sin^{4}(x)\cos(x)\cos(5x)}{32}+\frac{5x\sin^{3}(x)\sin(5x)\cos^{2}(x)}{16}+\frac{5x\sin^{2}(x)\cos^{3}(x)\cos(5x)}{16}-\frac{5x\sin(x)\sin(5x)\cos^{4}(x)}{32}-\frac{x\cos^{5}(x)\cos(5x)}{32}-\frac{\sin^{5}(x)\cos(5x)}{64}+\frac{3\sin^{4}(x)\sin(5x)\cos(x)}{64}+\frac{8\sin^{4}(x)\sin(5x)}{63}+\frac{40\sin^{3}(x)\cos(x)\cos(5x)}{63}-\frac{5\sin^{3}(x)\cos(5x)}{32}+\frac{\sin^{2}(x)\sin(5x)\cos^{3}(x)}{6}-\frac{4\sin^{2}(x)\sin(5x)\cos^{2}(x)}{3}+\frac{25\sin^{2}(x)\sin(5x)\cos(x)}{32}-\frac{4\sin^{2}(x)\sin(5x)}{21}+\frac{55\sin(x)\cos^{4}(x)\cos(5x)}{192}-\frac{100\sin(x)\cos^{3}(x)\cos(5x)}{63}+\frac{55\sin(x)\cos^{2}(x)\cos(5x)}{32}-\frac{20\sin(x)\cos(x)\cos(5x)}{21}+\frac{5\sin(x)\cos(5x)}{24}-\frac{241\sin(5x)\cos^{5}(x)}{960}+\frac{83\sin(5x)\cos^{4}(x)}{63}-\frac{75\sin(5x)\cos^{3}(x)}{32}+\frac{46\sin(5x)\cos^{2}(x)}{21}-\frac{25\sin(5x)\cos(x)}{24}+\frac{\sin(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))**5*cos(5*x),x)
```

```
[Out] -x*sin(x)**5*sin(5*x)/32 - 5*x*sin(x)**4*cos(x)*cos(5*x)/32 + 5*x*sin(x)**3
*sin(5*x)*cos(x)**2/16 + 5*x*sin(x)**2*cos(x)**3*cos(5*x)/16 - 5*x*sin(x)*s
in(5*x)*cos(x)**4/32 - x*cos(x)**5*cos(5*x)/32 - sin(x)**5*cos(5*x)/64 + 3*
sin(x)**4*sin(5*x)*cos(x)/64 + 8*sin(x)**4*sin(5*x)/63 + 40*sin(x)**3*cos(x
)*cos(5*x)/63 - 5*sin(x)**3*cos(5*x)/32 + sin(x)**2*sin(5*x)*cos(x)**3/6 -
4*sin(x)**2*sin(5*x)*cos(x)**2/3 + 25*sin(x)**2*sin(5*x)*cos(x)/32 - 4*sin(
x)**2*sin(5*x)/21 + 55*sin(x)*cos(x)**4*cos(5*x)/192 - 100*sin(x)*cos(x)**3
*cos(5*x)/63 + 55*sin(x)*cos(x)**2*cos(5*x)/32 - 20*sin(x)*cos(x)*cos(5*x)/
21 + 5*sin(x)*cos(5*x)/24 - 241*sin(5*x)*cos(x)**5/960 + 83*sin(5*x)*cos(x)
**4/63 - 75*sin(5*x)*cos(x)**3/32 + 46*sin(5*x)*cos(x)**2/21 - 25*sin(5*x)*
cos(x)/24 + sin(5*x)/5
```

Giac [**A**]
time = 0.71, size = 62, normalized size = 0.74

$$-\frac{1}{32}x-\frac{1}{320}\sin\left(10x\right)+\frac{5}{144}\sin\left(9x\right)-\frac{45}{256}\sin\left(8x\right)+\frac{15}{28}\sin\left(7x\right)-\frac{35}{32}\sin\left(6x\right)+\frac{63}{40}\sin\left(5x\right)-\frac{105}{64}\sin\left(4x\right)+\frac{5}{4}\sin\left(3x\right)-\frac{45}{64}\sin\left(2x\right)+\frac{5}{16}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^5*cos(5*x),x, algorithm="giac")
```

```
[Out] -1/32*x - 1/320*sin(10*x) + 5/144*sin(9*x) - 45/256*sin(8*x) + 15/28*sin(7*
x) - 35/32*sin(6*x) + 63/40*sin(5*x) - 105/64*sin(4*x) + 5/4*sin(3*x) - 45/
64*sin(2*x) + 5/16*sin(x)
```

Mupad [**B**]
time = 0.34, size = 78, normalized size = 0.93

$$-\frac{8\sin(x)\cos(x)^9}{5}+\frac{80\sin(x)\cos(x)^8}{9}-\frac{193\sin(x)\cos(x)^7}{10}+\frac{1180\sin(x)\cos(x)^6}{63}-\frac{67\sin(x)\cos(x)^5}{20}-\frac{979\sin(x)\cos(x)^4}{105}+\frac{133\sin(x)\cos(x)^3}{16}-\frac{766\sin(x)\cos(x)^2}{315}-\frac{\sin(x)\cos(x)}{32}-\frac{x}{32}+\frac{43\sin(x)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(5*x)*(cos(x) - 1)^5,x)
```

```
[Out] (43*sin(x))/315 - x/32 - (cos(x)*sin(x))/32 - (766*cos(x)^2*sin(x))/315 + (
133*cos(x)^3*sin(x))/16 - (979*cos(x)^4*sin(x))/105 - (67*cos(x)^5*sin(x))/
20 + (1180*cos(x)^6*sin(x))/63 - (193*cos(x)^7*sin(x))/10 + (80*cos(x)^8*si
n(x))/9 - (8*cos(x)^9*sin(x))/5
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 25, normalized size = 0.30

$$\frac{(\sin^{10}(x))}{10}-\frac{5(\sin^{8}(x))}{2}+\frac{25(\sin^{6}(x))}{3}-\frac{25(\sin^{4}(x))}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((1-cos(x))^5*cos(5*x),x)
```

```
[Out] 1/10*sin(x)^10-5/2*sin(x)^8+25/3*sin(x)^6-25/2*sin(x)^4
```

3.261 $\int \frac{3\cos(x)+4\sin(x)}{4\cos(x)+3\sin(x)}\,dx$

Optimal. Leaf size=20

$$\frac{24x}{25} - \frac{7}{25}\log(4\cos(x) + 3\sin(x))$$

```
[Out] 24/25*x-7/25*ln(3*sin(x)+4*cos(x))
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3212}

$$\frac{24x}{25} - \frac{7}{25}\log(3\sin(x) + 4\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(3*Cos[x] + 4*Sin[x])/(4*Cos[x] + 3*Sin[x]),x]
```

```
[Out] (24*x)/25 - (7*Log[4*Cos[x] + 3*Sin[x]])/25
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\text{Integral} = \frac{24x}{25} - \frac{7}{25}\log(4\cos(x) + 3\sin(x))$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$\frac{24x}{25} - \frac{7}{25}\log(4\cos(x) + 3\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*Cos[x] + 4*Sin[x])/(4*Cos[x] + 3*Sin[x]),x]
```

```
[Out] (24*x)/25 - (7*Log[4*Cos[x] + 3*Sin[x]])/25
```

Maple [A]
time = 0.22, size = 25, normalized size = 1.25

method	result	size
risch	$\frac{24x}{25}+\frac{7ix}{25}-\frac{7\ln\left(e^{2ix}+\frac{7}{25}+\frac{24i}{25}\right)}{25}$	21
default	$\frac{7\ln\left(1+\tan^2(x)\right)}{50}+\frac{24\arctan(\tan(x))}{25}-\frac{7\ln(3\tan(x)+4)}{25}$	25
parallelrisch	$\ln\left(\frac{2}{\left(-2048(\cot(x)-\csc(x)+2)^7\right)^{\frac{1}{25}}}\right)+\ln\left(\frac{1}{(-2\cot(x)+2\csc(x)+1)^{\frac{7}{25}}}\right)+\ln\left(\left(\frac{1}{1+\cos(x)}\right)^{\frac{7}{25}}\right)+\frac{24x}{25}$	44
norman	$\frac{\frac{24x}{25}+\frac{24x\left(\tan^2\left(\frac{x}{2}\right)\right)}{25}}{1+\tan^2\left(\frac{x}{2}\right)}-\frac{7\ln\left(\tan\left(\frac{x}{2}\right)-2\right)}{25}-\frac{7\ln\left(2\tan\left(\frac{x}{2}\right)+1\right)}{25}+\frac{7\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{25}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*sin(x)+3*cos(x))/(3*sin(x)+4*cos(x)),x,method=_RETURNVERBOSE)
[Out] 7/50*ln(1+tan(x)^2)+24/25*arctan(tan(x))-7/25*ln(3*tan(x)+4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.
time = 0.44, size = 58, normalized size = 2.90

$$\frac{48}{25}\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\frac{7}{25}\log\left(\frac{2\sin(x)}{\cos(x)+1}+1\right)-\frac{7}{25}\log\left(\frac{\sin(x)}{\cos(x)+1}-2\right)+\frac{7}{25}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*sin(x)+3*cos(x))/(3*sin(x)+4*cos(x)),x, algorithm="maxima")
[Out] 48/25*arctan(sin(x)/(cos(x) + 1)) - 7/25*log(2*sin(x)/(cos(x) + 1) + 1) - 7
/25*log(sin(x)/(cos(x) + 1) - 2) + 7/25*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 0.62, size = 16, normalized size = 0.80

$$\frac{24}{25}x-\frac{7}{25}\log\left(-2\cos(x)-\frac{3}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*sin(x)+3*cos(x))/(3*sin(x)+4*cos(x)),x, algorithm="fricas")
[Out] 24/25*x - 7/25*log(-2*cos(x) - 3/2*sin(x))
```

Sympy [A]
time = 0.06, size = 19, normalized size = 0.95

$$\frac{24x}{25}-\frac{7\log(3\sin(x)+4\cos(x))}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*sin(x)+3*cos(x))/(3*sin(x)+4*cos(x)),x)
```

```
[Out] 24*x/25 - 7*log(3*sin(x) + 4*cos(x))/25
```

Giac [A]
time = 0.41, size = 23, normalized size = 1.15

$$\frac{24}{25}\,x + \frac{7}{50}\,\log\left(\tan\left(x\right)^{2} + 1\right) - \frac{7}{25}\,\log\left(\left|3\,\tan\left(x\right) + 4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*sin(x)+3*cos(x))/(3*sin(x)+4*cos(x)),x, algorithm="giac")
```

```
[Out] 24/25*x + 7/50*log(tan(x)^2 + 1) - 7/25*log(abs(3*tan(x) + 4))
```

Mupad [B]
time = 0.58, size = 32, normalized size = 1.60

$$\frac{24\,x}{25} - \frac{7\,\ln\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2 - \frac{3\,\mathrm{tan}\left(\frac{x}{2}\right)}{2} - 1\right)}{25} + \frac{7\,\ln\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2 + 1\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*cos(x) + 4*sin(x))/(4*cos(x) + 3*sin(x)),x)
```

```
[Out] (24*x)/25 - (7*log(tan(x/2)^2 - (3*tan(x/2))/2 - 1))/25 + (7*log(tan(x/2)^2
 + 1))/25
```

Chatgpt [F] Failed to verify
time = 1.00, size = 11, normalized size = 0.55

$$-\frac{4\ln\left(\cos\left(x\right)\right)}{3} + \frac{3\ln\left(\sin\left(x\right)\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int((4*sin(x)+3*cos(x))/(3*sin(x)+4*cos(x)),x)
```

```
[Out] -4/3*ln(cos(x))+3/4*ln(sin(x))
```

3.262 $\int \left(-\sqrt{3}-\sqrt{4-x^2}+\sqrt{4-(1+x)^2}\right) dx$

Optimal. Leaf size=63

$$-\sqrt{3}x-\frac{1}{2}x\sqrt{4-x^2}+\frac{1}{2}(1+x)\sqrt{4-(1+x)^2}-2\arcsin\left(\frac{x}{2}\right)+2\arcsin\left(\frac{1+x}{2}\right)$$

```
[Out] -x*3^(1/2)-1/2*x*(-x^2+4)^(1/2)+1/2*(x+1)*(4-(x+1)^2)^(1/2)-2*arcsin(1/2*x)
+2*arcsin(1/2*x+1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {201, 222, 253}

$$-2\arcsin\left(\frac{x}{2}\right)+2\arcsin\left(\frac{x+1}{2}\right)-\frac{1}{2}\sqrt{4-x^2}x-\sqrt{3}x+\frac{1}{2}(x+1)\sqrt{4-(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[-Sqrt[3] - Sqrt[4 - x^2] + Sqrt[4 - (1 + x)^2],x]
```

```
[Out] -(Sqrt[3]*x) - (x*Sqrt[4 - x^2])/2 + ((1 + x)*Sqrt[4 - (1 + x)^2])/2 - 2*Ar
cSin[x/2] + 2*ArcSin[(1 + x)/2]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeqQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= -\sqrt{3}x - \int \sqrt{4-x^2}\,dx + \int \sqrt{4-(1+x)^2}\,dx \\
&= -\sqrt{3}x - \frac{1}{2}x\sqrt{4-x^2} - 2\int \frac{1}{\sqrt{4-x^2}}\,dx + \text{Subst}\left(\int \sqrt{4-x^2}\,dx, x, 1+x\right) \\
&= -\sqrt{3}x - \frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}(1+x)\sqrt{4-(1+x)^2} - 2\arcsin\left(\frac{x}{2}\right) + 2\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, 1+x\right) \\
&= -\sqrt{3}x - \frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}(1+x)\sqrt{4-(1+x)^2} - 2\arcsin\left(\frac{x}{2}\right) + 2\arcsin\left(\frac{1+x}{2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 89, normalized size = 1.41

$$
-\sqrt{3}x - \frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}(1+x)\sqrt{3-2x-x^2} + 4\arctan\left(\frac{\sqrt{4-x^2}}{2+x}\right) - 4\arctan\left(\frac{\sqrt{3-2x-x^2}}{3+x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[-Sqrt[3] - Sqrt[4 - x^2] + Sqrt[4 - (1 + x)^2],x]
```

```
[Out] -(Sqrt[3]*x) - (x*Sqrt[4 - x^2])/2 + ((1 + x)*Sqrt[3 - 2*x - x^2])/2 + 4*Ar
cTan[Sqrt[4 - x^2]/(2 + x)] - 4*ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)]
```

Maple [A]
time = 0.10, size = 53, normalized size = 0.84

method	result	size
default	$-\frac{(-2x-2)\sqrt{-x^2-2x+3}}{4} + 2\arcsin\left(\frac{x}{2}+\frac{1}{2}\right) - x\sqrt{3} - \frac{x\sqrt{-x^2+4}}{2} - 2\arcsin\left(\frac{x}{2}\right)$	53
parts	$-\frac{(-2x-2)\sqrt{-x^2-2x+3}}{4} + 2\arcsin\left(\frac{x}{2}+\frac{1}{2}\right) - x\sqrt{3} - \frac{x\sqrt{-x^2+4}}{2} - 2\arcsin\left(\frac{x}{2}\right)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4-(x+1)^2)^(1/2)-3^(1/2)-(-x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*x-2)*(-x^2-2*x+3)^(1/2)+2*arcsin(1/2*x+1/2)-x*3^(1/2)-1/2*x*(-x^2+
4)^(1/2)-2*arcsin(1/2*x)
```

Maxima [A]
time = 0.61, size = 62, normalized size = 0.98

$$
-\sqrt{3}x + \frac{1}{2}\sqrt{-x^2-2x+3}x - \frac{1}{2}\sqrt{-x^2+4}x + \frac{1}{2}\sqrt{-x^2-2x+3} - 2\arcsin\left(\frac{1}{2}x\right) - 2\arcsin\left(-\frac{1}{2}x - \frac{1}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-(x+1)^2)^(1/2)-3^(1/2)-(-x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*x + 1/2*sqrt(-x^2 - 2*x + 3)*x - 1/2*sqrt(-x^2 + 4)*x + 1/2*sqrt(-
x^2 - 2*x + 3) - 2*arcsin(1/2*x) - 2*arcsin(-1/2*x - 1/2)
```

Fricas [A]
time = 0.60, size = 83, normalized size = 1.32

$$\frac{1}{2}\sqrt{-x^2-2x+3}(x+1)-\sqrt{3}x-\frac{1}{2}\sqrt{-x^2+4}x-2\arctan\left(\frac{\sqrt{-x^2-2x+3}(x+1)}{x^2+2x-3}\right)+4\arctan\left(\frac{\sqrt{-x^2+4}-2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-(x+1)^2)^(1/2)-3^(1/2)-(-x^2+4)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 - 2*x + 3)*(x + 1) - sqrt(3)*x - 1/2*sqrt(-x^2 + 4)*x - 2*arc
tan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 4*arctan((sqrt(-x^2 + 4
) - 2)/x)
```

Sympy [A]
time = 1.01, size = 119, normalized size = 1.89

$$-\frac{x\sqrt{4-x^2}}{2}-\sqrt{3}x+\begin{cases}\frac{i(x+1)^3}{2\sqrt{(x+1)^2-4}}-\frac{2i(x+1)}{\sqrt{(x+1)^2-4}}-2i\operatorname{acosh}\left(\frac{x}{2}+\frac{1}{2}\right) & \text{for } \left|(x+1)^2\right|>4\\ 2\operatorname{asin}\left(\frac{x}{2}+\frac{1}{2}\right)-\frac{(x+1)^3}{2\sqrt{4-(x+1)^2}}+\frac{2(x+1)}{\sqrt{4-(x+1)^2}} & \text{otherwise}\end{cases}-2\operatorname{asin}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-(x+1)**2)**(1/2)-3**(1/2)-(-x**2+4)**(1/2),x)
[Out] -x*sqrt(4 - x**2)/2 - sqrt(3)*x + Piecewise((I*(x + 1)**3/(2*sqrt((x + 1)**
2 - 4)) - 2*I*(x + 1)/sqrt((x + 1)**2 - 4) - 2*I*acosh(x/2 + 1/2), Abs((x +
 1)**2) > 4), (2*asin(x/2 + 1/2) - (x + 1)**3/(2*sqrt(4 - (x + 1)**2)) + 2*
(x + 1)/sqrt(4 - (x + 1)**2), True)) - 2*asin(x/2)
```

Giac [A]
time = 0.46, size = 50, normalized size = 0.79

$$\frac{1}{2}\sqrt{-x^2-2x+3}(x+1)-\sqrt{3}x-\frac{1}{2}\sqrt{-x^2+4}x-2\arcsin\left(\frac{1}{2}x\right)+2\arcsin\left(\frac{1}{2}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-(x+1)^2)^(1/2)-3^(1/2)-(-x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 2*x + 3)*(x + 1) - sqrt(3)*x - 1/2*sqrt(-x^2 + 4)*x - 2*arc
sin(1/2*x) + 2*arcsin(1/2*x + 1/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{4-(x+1)^2}-\sqrt{3}-\sqrt{4-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - (x + 1)^2)^(1/2) - 3^(1/2) - (4 - x^2)^(1/2),x)
```

```
[Out] int((4 - (x + 1)^2)^(1/2) - 3^(1/2) - (4 - x^2)^(1/2), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 46, normalized size = 0.73

$$x\sqrt{4-(x+1)^{2}}-2\arcsin\left(\frac{x}{2}+\frac{1}{2}\right)-\sqrt{3}\,x-\frac{x\sqrt{-x^{2}+4}}{2}+\frac{\operatorname{arcsinh}\left(\frac{x}{2}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((4-(x+1)^2)^(1/2)-3^(1/2)-(-x^2+4)^(1/2),x)
```

```
[Out] x*(4-(x+1)^2)^(1/2)-2*arcsin(1/2*x+1/2)-3^(1/2)*x-1/2*x*(-x^2+4)^(1/2)+1/2*
arcsinh(1/2*x)
```

3.263 $\int x^2 \sin(\log(x))\, dx$

Optimal. Leaf size=21

$$-\frac{1}{10}x^3\cos(\log(x))+\frac{3}{10}x^3\sin(\log(x))$$

```
[Out] -1/10*x^3*cos(ln(x))+3/10*x^3*sin(ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4573\}$

$$\frac{3}{10}x^3\sin(\log(x))-\frac{1}{10}x^3\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[Log[x]],x]
[Out] -1/10*(x^3*Cos[Log[x]]) + (3*x^3*Sin[Log[x]])/10
```

Rule 4573

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Rubi steps

$$\text{Integral}=-\frac{1}{10}x^3\cos(\log(x))+\frac{3}{10}x^3\sin(\log(x))$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$-\frac{1}{10}x^3\cos(\log(x))+\frac{3}{10}x^3\sin(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[Log[x]],x]
[Out] -1/10*(x^3*Cos[Log[x]]) + (3*x^3*Sin[Log[x]])/10
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 26, normalized size = 1.24

method	result	size
risch	$\left(-\frac{1}{20}-\frac{3i}{20}\right)x^{3}x^{i}+\left(-\frac{1}{20}+\frac{3i}{20}\right)x^{3}x^{-i}$	26
norman	$\frac{-\frac{x^{3}}{10}+\frac{3x^{3}\tan\left(\frac{\ln(x)}{2}\right)}{5}+\frac{x^{3}\left(\tan^{2}\left(\frac{\ln(x)}{2}\right)\right)}{10}}{1+\tan^{2}\left(\frac{\ln(x)}{2}\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(ln(x)),x,method=_RETURNVERBOSE)
[Out] (-1/20-3/20*I)*x^3*x^I+(-1/20+3/20*I)*x^3/(x^I)
```

Maxima [A]
time = 0.34, size = 14, normalized size = 0.67

$$-\frac{1}{10}x^{3}(\cos(\log(x))-3\sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(log(x)),x, algorithm="maxima")
[Out] -1/10*x^3*(cos(log(x)) - 3*sin(log(x)))
```

Fricas [A]
time = 0.60, size = 17, normalized size = 0.81

$$-\frac{1}{10}x^{3}\cos(\log(x))+\frac{3}{10}x^{3}\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(log(x)),x, algorithm="fricas")
[Out] -1/10*x^3*cos(log(x)) + 3/10*x^3*sin(log(x))
```

Sympy [A]
time = 0.40, size = 20, normalized size = 0.95

$$\frac{3x^{3}\sin(\log(x))}{10}-\frac{x^{3}\cos(\log(x))}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(ln(x)),x)
[Out] 3*x**3*sin(log(x))/10 - x**3*cos(log(x))/10
```

Giac [**A**]
time = 0.49, size = 17, normalized size = 0.81

$$-\frac{1}{10}\,x^3\cos\left(\log\left(x\right)\right)+\frac{3}{10}\,x^3\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(log(x)),x, algorithm="giac")
```

```
[Out] -1/10*x^3*cos(log(x)) + 3/10*x^3*sin(log(x))
```

Mupad [**B**]
time = 0.14, size = 14, normalized size = 0.67

$$-\frac{\sqrt{10}\,x^3\,\cos\left(\mathrm{atan}(3)+\ln\left(x\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(log(x)),x)
```

```
[Out] -(10^(1/2)*x^3*cos(atan(3) + log(x)))/10
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 17, normalized size = 0.81

$$-\frac{x^2\cos\left(\ln\left(x\right)\right)}{2}-\frac{x^2\sin\left(\ln\left(x\right)\right)}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^2*sin(ln(x)),x)
```

```
[Out] -1/2*x^2*cos(ln(x))-1/4*x^2*sin(ln(x))
```

3.264 $\int e^{-x}(36x^5 - 12x^6 + x^7)\,dx$

Optimal. Leaf size=76

$$-720e^{-x} - 720e^{-x}x - 360e^{-x}x^2 - 120e^{-x}x^3 - 30e^{-x}x^4 - 6e^{-x}x^5 + 5e^{-x}x^6 - e^{-x}x^7$$

```
[Out] -720/exp(x)-720*x/exp(x)-360*x^2/exp(x)-120*x^3/exp(x)-30*x^4/exp(x)-6*x^5/
exp(x)+5*x^6/exp(x)-x^7/exp(x)
```

Rubi [A]
time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1608, 27, 2227, 2207, 2225}

$$-e^{-x}x^7 + 5e^{-x}x^6 - 6e^{-x}x^5 - 30e^{-x}x^4 - 120e^{-x}x^3 - 360e^{-x}x^2 - 720e^{-x}x - 720e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[(36*x^5 - 12*x^6 + x^7)/E^x,x]
```

```
[Out] -720/E^x - (720*x)/E^x - (360*x^2)/E^x - (120*x^3)/E^x - (30*x^4)/E^x - (6*
x^5)/E^x + (5*x^6)/E^x - x^7/E^x
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2227

```
Int[(F_)^((c_.)*(v_))*(u_), x_Symbol] :> Int[ExpandIntegrand[F^(c*ExpandToS
um[v, x]), u, x], x] /; FreeQ[{F, c}, x] && PolynomialQ[u, x] && LinearQ[v,
 x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int e^{-x}x^5\left(36-12x+x^2\right)\,dx\\
&= \int e^{-x}(-6+x)^2x^5\,dx\\
&= \int \left(36e^{-x}x^5-12e^{-x}x^6+e^{-x}x^7\right)\,dx\\
&= -\left(12\int e^{-x}x^6\,dx\right)+36\int e^{-x}x^5\,dx+\int e^{-x}x^7\,dx\\
&= -36e^{-x}x^5+12e^{-x}x^6-e^{-x}x^7+7\int e^{-x}x^6\,dx-72\int e^{-x}x^5\,dx+180\int e^{-x}x^4\,dx\\
&= -180e^{-x}x^4+36e^{-x}x^5+5e^{-x}x^6-e^{-x}x^7+42\int e^{-x}x^5\,dx-360\int e^{-x}x^4\,dx+720\int e^{-x}x^3\,dx\\
&= -720e^{-x}x^3+180e^{-x}x^4-6e^{-x}x^5+5e^{-x}x^6-e^{-x}x^7+210\int e^{-x}x^4\,dx-1440\int e^{-x}x^3\,dx+2160\int e^{-x}x^2\,dx\\
&= -2160e^{-x}x^2+720e^{-x}x^3-30e^{-x}x^4-6e^{-x}x^5+5e^{-x}x^6-e^{-x}x^7+840\int e^{-x}x^3\,dx+4320\int e^{-x}x\,dx-4320\int e^{-x}x^2\,dx\\
&= -4320e^{-x}x+2160e^{-x}x^2-120e^{-x}x^3-30e^{-x}x^4-6e^{-x}x^5+5e^{-x}x^6-e^{-x}x^7+2520\int e^{-x}x^2\,dx+4320\int e^{-x}\,dx-8640\int e^{-x}x\,dx\\
&= -4320e^{-x}+4320e^{-x}x-360e^{-x}x^2-120e^{-x}x^3-30e^{-x}x^4-6e^{-x}x^5+5e^{-x}x^6-e^{-x}x^7+5040\int e^{-x}x\,dx-8640\int e^{-x}\,dx\\
&= 4320e^{-x}-720e^{-x}x-360e^{-x}x^2-120e^{-x}x^3-30e^{-x}x^4-6e^{-x}x^5+5e^{-x}x^6-e^{-x}x^7+5040\int e^{-x}\,dx\\
&= -720e^{-x}-720e^{-x}x-360e^{-x}x^2-120e^{-x}x^3-30e^{-x}x^4-6e^{-x}x^5+5e^{-x}x^6-e^{-x}x^7
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 41, normalized size = 0.54

$$e^{-x}\left(-720-720x-360x^2-120x^3-30x^4-6x^5+5x^6-x^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(36*x^5 - 12*x^6 + x^7)/E^x,x]
[Out] (-720 - 720*x - 360*x^2 - 120*x^3 - 30*x^4 - 6*x^5 + 5*x^6 - x^7)/E^x
```

Maple [A]
time = 0.05, size = 69, normalized size = 0.91

method	result
gosper	$-\mathrm{e}^{-x}\left(x^7-5x^6+6x^5+30x^4+120x^3+360x^2+720x+720\right)$
risch	$\left(-x^7+5x^6-6x^5-30x^4-120x^3-360x^2-720x-720\right)\mathrm{e}^{-x}$
derivativedivides	$-6\,\mathrm{e}^{-x}x^5-30x^4\mathrm{e}^{-x}-120x^3\mathrm{e}^{-x}-360x^2\mathrm{e}^{-x}-720x\,\mathrm{e}^{-x}-720\,\mathrm{e}^{-x}+5\,\mathrm{e}^{-x}x^6-\mathrm{e}^{-x}x^7$
default	$-6\,\mathrm{e}^{-x}x^5-30x^4\mathrm{e}^{-x}-120x^3\mathrm{e}^{-x}-360x^2\mathrm{e}^{-x}-720x\,\mathrm{e}^{-x}-720\,\mathrm{e}^{-x}+5\,\mathrm{e}^{-x}x^6-\mathrm{e}^{-x}x^7$

norman	$-6\,\mathrm{e}^{-x}x^5-30x^4\mathrm{e}^{-x}-120x^3\mathrm{e}^{-x}-360x^2\mathrm{e}^{-x}-720x\,\mathrm{e}^{-x}-720\,\mathrm{e}^{-x}+5\,\mathrm{e}^{-x}x^6-\mathrm{e}^{-x}x^7$
parallelrisch	$-6\,\mathrm{e}^{-x}x^5-30x^4\mathrm{e}^{-x}-120x^3\mathrm{e}^{-x}-360x^2\mathrm{e}^{-x}-720x\,\mathrm{e}^{-x}-720\,\mathrm{e}^{-x}+5\,\mathrm{e}^{-x}x^6-\mathrm{e}^{-x}x^7$
parts	$-6\,\mathrm{e}^{-x}x^5-30x^4\mathrm{e}^{-x}-120x^3\mathrm{e}^{-x}-360x^2\mathrm{e}^{-x}-720x\,\mathrm{e}^{-x}-720\,\mathrm{e}^{-x}+5\,\mathrm{e}^{-x}x^6-\mathrm{e}^{-x}x^7$
meijerg	$720-\frac{(8x^7+56x^6+336x^5+1680x^4+6720x^3+20160x^2+40320x+40320)\mathrm{e}^{-x}}{8}+\frac{12(7x^6+42x^5+210x^4+840x^3+2520}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x)*(x^7-12*x^6+36*x^5),x,method=_RETURNVERBOSE)
```

```
[Out] -6*exp(-x)*x^5-30*x^4*exp(-x)-120*x^3*exp(-x)-360*x^2*exp(-x)-720*x*exp(-x)
-720*exp(-x)+5*exp(-x)*x^6-exp(-x)*x^7
```

Maxima [A]
time = 0.33, size = 103, normalized size = 1.36

$$-(x^7+7\,x^6+42\,x^5+210\,x^4+840\,x^3+2520\,x^2+5040\,x+5040)e^{(-x)}+12\,(x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720)e^{(-x)}-36\,(x^5+5\,x^4+20\,x^3+60\,x^2+120\,x+120)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x)*(x^7-12*x^6+36*x^5),x, algorithm="maxima")
```

```
[Out] -(x^7 + 7*x^6 + 42*x^5 + 210*x^4 + 840*x^3 + 2520*x^2 + 5040*x + 5040)*e^(-
x) + 12*(x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x + 720)*e^(-x) - 3
6*(x^5 + 5*x^4 + 20*x^3 + 60*x^2 + 120*x + 120)*e^(-x)
```

Fricas [A]
time = 0.56, size = 39, normalized size = 0.51

$$-\left(x^7-5\,x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x)*(x^7-12*x^6+36*x^5),x, algorithm="fricas")
```

```
[Out] -(x^7 - 5*x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x + 720)*e^(-x)
```

Sympy [A]
time = 0.03, size = 36, normalized size = 0.47

$$\left(-x^7+5x^6-6x^5-30x^4-120x^3-360x^2-720x-720\right)e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x)*(x**7-12*x**6+36*x**5),x)
```

```
[Out] (-x**7 + 5*x**6 - 6*x**5 - 30*x**4 - 120*x**3 - 360*x**2 - 720*x - 720)*exp
(-x)
```

Giac [A]
time = 0.43, size = 39, normalized size = 0.51

$$-\left(x^{7} - 5\,x^{6} + 6\,x^{5} + 30\,x^{4} + 120\,x^{3} + 360\,x^{2} + 720\,x + 720\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-x)*(x^7-12*x^6+36*x^5),x, algorithm="giac")
```

```
[Out] -(x^7 - 5*x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x + 720)*e^(-x)
```

Mupad [B]
time = 0.06, size = 39, normalized size = 0.51

$$-\mathrm{e}^{-x}\,\left(x^7-5\,x^6+6\,x^5+30\,x^4+120\,x^3+360\,x^2+720\,x+720\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x)*(36*x^5 - 12*x^6 + x^7),x)
```

```
[Out] -exp(-x)*(720*x + 360*x^2 + 120*x^3 + 30*x^4 + 6*x^5 - 5*x^6 + x^7 + 720)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 41, normalized size = 0.54

$$\frac{(-45360x^7 + 184756x^6 - 409512x^5 + 569395x^4 - 483840x^3 + 241920x^2 - 60480x + 5040)\,\mathrm{e}^{-x}}{720}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(-x)*(x^7-12*x^6+36*x^5),x)
```

```
[Out] 1/720*(-45360*x^7+184756*x^6-409512*x^5+569395*x^4-483840*x^3+241920*x^2-60
480*x+5040)*exp(-x)
```

3.265 $\int \arccos(x) \arcsin(x) \, dx$

Optimal. Leaf size=38

$$2x - \sqrt{1-x^2} \arcsin(x) + \arccos(x) \left(\sqrt{1-x^2} + x \arcsin(x)\right)$$

```
[Out] 2*x-(-x^2+1)^(1/2)*arcsin(x)+arccos(x)*((-x^2+1)^(1/2)+x*arcsin(x))
```

Rubi [F]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \arccos(x) \arcsin(x) \, dx$$

Verification is not applicable to the result.

```
[In] Int[ArcCos[x]*ArcSin[x],x]
[Out] Defer[Int][ArcCos[x]*ArcSin[x], x]
```

Rubi steps

$$\text{Integral} = \int \arccos(x) \arcsin(x) \, dx$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$2x - \sqrt{1-x^2} \arcsin(x) + \arccos(x) \left(\sqrt{1-x^2} + x \arcsin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x]*ArcSin[x],x]
[Out] 2*x - Sqrt[1 - x^2]*ArcSin[x] + ArcCos[x]*(Sqrt[1 - x^2] + x*ArcSin[x])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \arcsin(x) \arccos(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)*arccos(x),x)
```

```
[Out] int(arcsin(x)*arccos(x),x)
```

Maxima [A]
time = 0.42, size = 34, normalized size = 0.89

$$\left(x \arcsin\left(x\right) + \sqrt{-x^{2} + 1}\right) \arccos\left(x\right) - \sqrt{-x^{2} + 1} \arcsin\left(x\right) + 2 \, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*arccos(x),x, algorithm="maxima")
```

```
[Out] (x*arcsin(x) + sqrt(-x^2 + 1))*arccos(x) - sqrt(-x^2 + 1)*arcsin(x) + 2*x
```

Fricas [A]
time = 0.63, size = 34, normalized size = 0.89

$$\frac{1}{2} \, \pi x \arccos\left(x\right) - x \arccos\left(x\right)^{2} - \frac{1}{2} \, {\left(\pi - 4 \, \arccos\left(x\right)\right)} \sqrt{-x^{2} + 1} + 2 \, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*arccos(x),x, algorithm="fricas")
```

```
[Out] 1/2*pi*x*arccos(x) - x*arccos(x)^2 - 1/2*(pi - 4*arccos(x))*sqrt(-x^2 + 1)
+ 2*x
```

Sympy [A]
time = 0.10, size = 34, normalized size = 0.89

$$x \operatorname{acos}{\left(x \right)} \operatorname{asin}{\left(x \right)} + 2x + \sqrt{1 - x^{2}} \operatorname{acos}{\left(x \right)} - \sqrt{1 - x^{2}} \operatorname{asin}{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)*acos(x),x)
```

```
[Out] x*acos(x)*asin(x) + 2*x + sqrt(1 - x**2)*acos(x) - sqrt(1 - x**2)*asin(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(34) = 68.
time = 0.46, size = 163, normalized size = 4.29

$$-\pi \left(-1\right)^{\left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor} x \arccos\left(x\right) \left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor + \frac{1}{2} \, \pi \left(-1\right)^{\left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor} x \arccos\left(x\right) - \left(-1\right)^{\left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor} x \arccos\left(x\right)^{2} + \pi \sqrt{-x^{2} + 1} \left(-1\right)^{\left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor} \left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor - \frac{1}{2} \, \pi \sqrt{-x^{2} + 1} \left(-1\right)^{\left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor} + 2 \, \sqrt{-x^{2} + 1} \left(-1\right)^{\left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor} \arccos\left(x\right) + 2 \, \left(-1\right)^{\left \lfloor -\frac{\arccos\left(x\right)}{\pi} + 1 \right \rfloor} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*arccos(x),x, algorithm="giac")
```

```
[Out] -pi*(-1)^floor(-arccos(x)/pi + 1)*x*arccos(x)*floor(-arccos(x)/pi + 1) + 1/
2*pi*(-1)^floor(-arccos(x)/pi + 1)*x*arccos(x) - (-1)^floor(-arccos(x)/pi +
 1)*x*arccos(x)^2 + pi*sqrt(-x^2 + 1)*(-1)^floor(-arccos(x)/pi + 1)*floor(-
arccos(x)/pi + 1) - 1/2*pi*sqrt(-x^2 + 1)*(-1)^floor(-arccos(x)/pi + 1) + 2
```

```
*sqrt(-x^2 + 1)*(-1)^floor(-arccos(x)/pi + 1)*arccos(x) + 2*(-1)^floor(-arc
cos(x)/pi + 1)*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{acos}(x) \operatorname{asin}(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(x)*asin(x),x)
```

```
[Out] int(acos(x)*asin(x), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 11, normalized size = 0.29

$$x \arcsin(x) \arccos(x) + \frac{\arcsin(x)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(arcsin(x)*arccos(x),x)
```

```
[Out] x*arcsin(x)*arccos(x)+1/2*arcsin(x)
```

3.266 $\int (1+6x-7x^2+4x^3-x^4)^n \, dx$

Optimal. Leaf size=52

$$3^n(-1+x)\,\text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},-\frac{2(-1+x)^2}{1+\sqrt{13}},\frac{2(-1+x)^2}{-1+\sqrt{13}}\right)$$

```
[Out] 3^n*(x-1)*AppellF1(1/2,-n,-n,3/2,2*(x-1)^2/(-1+13^(1/2)),-2*(x-1)^2/(1+13^(
1/2)))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 115 vs. $2(52) = 104$.
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 2.21, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$,
Rules used = {1120, 1119, 440}

$$(x-1)\left(\frac{2(x-1)^2}{1-\sqrt{13}}+1\right)^{-n}\left(\frac{2(x-1)^2}{1+\sqrt{13}}+1\right)^{-n}\left(-(x-1)^4-(x-1)^2+3\right)^n \text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},-\frac{2(x-1)^2}{1+\sqrt{13}},-\frac{2(x-1)^2}{1-\sqrt{13}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 6*x - 7*x^2 + 4*x^3 - x^4)^n,x]
```

```
[Out] ((3 - (-1 + x)^2 - (-1 + x)^4)^n*(-1 + x)*AppellF1[1/2, -n, -n, 3/2, (-2*(-
1 + x)^2)/(1 + Sqrt[13]), (-2*(-1 + x)^2)/(1 - Sqrt[13])])/((1 + (2*(-1 + x
)^2)/(1 - Sqrt[13]))^n*(1 + (2*(-1 + x)^2)/(1 + Sqrt[13]))^n)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \left(3-x^2-x^4\right)^n\, dx, x, -1+x\right)\\
&= \left(\left(1-\frac{2(-1+x)^2}{-1-\sqrt{13}}\right)^{-n}\left(1-\frac{2(-1+x)^2}{-1+\sqrt{13}}\right)^{-n}\left(3-(-1+x)^2-(-1+x)^4\right)^n\right)\text{Subst}\left(\int \left(1-\frac{2x^2}{-1-\sqrt{13}}\right)^n\left(1-\frac{2x^2}{-1+\sqrt{13}}\right)^n dx, x, -1+x\right)\\
&= -\left(1+\frac{2(1-x)^2}{1-\sqrt{13}}\right)^{-n}\left(1+\frac{2(1-x)^2}{1+\sqrt{13}}\right)^{-n}\left(3-(1-x)^2-(1-x)^4\right)^n(1-x)\,\text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},-\frac{2(1-x)^2}{1+\sqrt{13}},-\frac{2(1-x)^2}{1-\sqrt{13}}\right)
\end{aligned}
$$

Mathematica [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(1+6x-7x^2+4x^3-x^4\right)^n\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + 6*x - 7*x^2 + 4*x^3 - x^4)^n,x]
[Out] Integrate[(1 + 6*x - 7*x^2 + 4*x^3 - x^4)^n, x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4+4x^3-7x^2+6x+1\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-7*x^2+6*x+1)^n,x)
[Out] int((-x^4+4*x^3-7*x^2+6*x+1)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-7*x^2+6*x+1)^n,x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 7*x^2 + 6*x + 1)^n, x)
```

Fricas [F]
time = 0.59, size = 24, normalized size = 0.46

$$\text{integral}\left(\left(-x^4+4\,x^3-7\,x^2+6\,x+1\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-7*x^2+6*x+1)^n,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 7*x^2 + 6*x + 1)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^{4}+4x^{3}-7x^{2}+6x+1\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-7*x**2+6*x+1)**n,x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 7*x**2 + 6*x + 1)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-7*x^2+6*x+1)^n,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 7*x^2 + 6*x + 1)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(-x^4+4\,x^3-7\,x^2+6\,x+1\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - 7*x^2 + 4*x^3 - x^4 + 1)^n,x)
```

```
[Out] int((6*x - 7*x^2 + 4*x^3 - x^4 + 1)^n, x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((-x^4+4*x^3-7*x^2+6*x+1)^n,x)
```

```
[Out] not solved
```

3.267 $\int \frac{x^4}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=64

$$-2\sqrt{1-x}+\frac{8}{3}(1-x)^{3/2}-\frac{12}{5}(1-x)^{5/2}+\frac{8}{7}(1-x)^{7/2}-\frac{2}{9}(1-x)^{9/2}$$

```
[Out] -2*(1-x)^(1/2)+8/3*(1-x)^(3/2)-12/5*(1-x)^(5/2)+8/7*(1-x)^(7/2)-2/9*(1-x)^(
9/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2}{9}(1-x)^{9/2}+\frac{8}{7}(1-x)^{7/2}-\frac{12}{5}(1-x)^{5/2}+\frac{8}{3}(1-x)^{3/2}-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x] + (8*(1 - x)^(3/2))/3 - (12*(1 - x)^(5/2))/5 + (8*(1 - x)^(7
/2))/7 - (2*(1 - x)^(9/2))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(\frac{1}{\sqrt{1-x}}-4\sqrt{1-x}+6(1-x)^{3/2}-4(1-x)^{5/2}+(1-x)^{7/2}\right)dx\\ &= -2\sqrt{1-x}+\frac{8}{3}(1-x)^{3/2}-\frac{12}{5}(1-x)^{5/2}+\frac{8}{7}(1-x)^{7/2}-\frac{2}{9}(1-x)^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.52

$$-\frac{2}{315}\sqrt{1-x}\left(128+64x+48x^2+40x^3+35x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[1 - x],x]
```

```
[Out] (-2*Sqrt[1 - x]*(128 + 64*x + 48*x^2 + 40*x^3 + 35*x^4))/315
```

Maple [A]
time = 0.10, size = 47, normalized size = 0.73

method	result	size
trager	$\left(-\frac{2}{9}x^4-\frac{16}{63}x^3-\frac{32}{105}x^2-\frac{128}{315}x-\frac{256}{315}\right)\sqrt{1-x}$	29
gosper	$-\frac{2\sqrt{1-x}\,(35x^4+40x^3+48x^2+64x+128)}{315}$	30
pseudoelliptic	$-\frac{2\sqrt{1-x}\,(35x^4+40x^3+48x^2+64x+128)}{315}$	30
risch	$\frac{2(x-1)(35x^4+40x^3+48x^2+64x+128)}{315\sqrt{1-x}}$	33
meijerg	$-\frac{-\frac{256\sqrt{\pi}}{315}+\frac{\sqrt{\pi}\left(70x^4+80x^3+96x^2+128x+256\right)\sqrt{1-x}}{315}}{\sqrt{\pi}}$	44
derivativedivides	$-2\sqrt{1-x}+\frac{8(1-x)^{\frac{3}{2}}}{3}-\frac{12(1-x)^{\frac{5}{2}}}{5}+\frac{8(1-x)^{\frac{7}{2}}}{7}-\frac{2(1-x)^{\frac{9}{2}}}{9}$	47
default	$-2\sqrt{1-x}+\frac{8(1-x)^{\frac{3}{2}}}{3}-\frac{12(1-x)^{\frac{5}{2}}}{5}+\frac{8(1-x)^{\frac{7}{2}}}{7}-\frac{2(1-x)^{\frac{9}{2}}}{9}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1-x)^(1/2)+8/3*(1-x)^(3/2)-12/5*(1-x)^(5/2)+8/7*(1-x)^(7/2)-2/9*(1-x)^(
9/2)
```

Maxima [A]
time = 0.44, size = 46, normalized size = 0.72

$$-\frac{2}{9}\,(-x+1)^{\frac{9}{2}}+\frac{8}{7}\,(-x+1)^{\frac{7}{2}}-\frac{12}{5}\,(-x+1)^{\frac{5}{2}}+\frac{8}{3}\,(-x+1)^{\frac{3}{2}}-2\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(-x + 1)^(9/2) + 8/7*(-x + 1)^(7/2) - 12/5*(-x + 1)^(5/2) + 8/3*(-x +
1)^(3/2) - 2*sqrt(-x + 1)
```

Fricas [A]
time = 0.57, size = 29, normalized size = 0.45

$$-\frac{2}{315}\,\left(35\,x^4+40\,x^3+48\,x^2+64\,x+128\right)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(35*x^4 + 40*x^3 + 48*x^2 + 64*x + 128)*sqrt(-x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.91, size = 138, normalized size = 2.16

$$\begin{cases} -\frac{2ix^4\sqrt{x-1}}{9} - \frac{16ix^3\sqrt{x-1}}{63} - \frac{32ix^2\sqrt{x-1}}{105} - \frac{128ix\sqrt{x-1}}{315} - \frac{256i\sqrt{x-1}}{315} & \text{for } |x| > 1 \\ -\frac{2x^4\sqrt{1-x}}{9} - \frac{16x^3\sqrt{1-x}}{63} - \frac{32x^2\sqrt{1-x}}{105} - \frac{128x\sqrt{1-x}}{315} - \frac{256\sqrt{1-x}}{315} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*x**4*sqrt(x - 1)/9 - 16*I*x**3*sqrt(x - 1)/63 - 32*I*x**2*s
qrt(x - 1)/105 - 128*I*x*sqrt(x - 1)/315 - 256*I*sqrt(x - 1)/315, Abs(x) >
1), (-2*x**4*sqrt(1 - x)/9 - 16*x**3*sqrt(1 - x)/63 - 32*x**2*sqrt(1 - x)/1
05 - 128*x*sqrt(1 - x)/315 - 256*sqrt(1 - x)/315, True))
```

Giac [A]
time = 0.43, size = 61, normalized size = 0.95

$$-\frac{2}{9}(x-1)^4\sqrt{-x+1} - \frac{8}{7}(x-1)^3\sqrt{-x+1} - \frac{12}{5}(x-1)^2\sqrt{-x+1} + \frac{8}{3}(-x+1)^{\frac{3}{2}} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(x - 1)^4*sqrt(-x + 1) - 8/7*(x - 1)^3*sqrt(-x + 1) - 12/5*(x - 1)^2*s
qrt(-x + 1) + 8/3*(-x + 1)^(3/2) - 2*sqrt(-x + 1)
```

Mupad [B]
time = 0.03, size = 46, normalized size = 0.72

$$\frac{8(1-x)^{3/2}}{3} - 2\sqrt{1-x} - \frac{12(1-x)^{5/2}}{5} + \frac{8(1-x)^{7/2}}{7} - \frac{2(1-x)^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x)^(1/2),x)
```

```
[Out] (8*(1 - x)^(3/2))/3 - 2*(1 - x)^(1/2) - (12*(1 - x)^(5/2))/5 + (8*(1 - x)^(
7/2))/7 - (2*(1 - x)^(9/2))/9
```

Chatgpt [F] Failed to verify
time = 1.00, size = 27, normalized size = 0.42

$$\frac{2(1-x)^{\frac{3}{2}}(x^2+4x+8)}{3} + \frac{16(1-x)^{\frac{5}{2}}}{15}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^4/(1-x)^(1/2),x)
```

```
[Out] 2/3*(1-x)^(3/2)*(x^2+4*x+8)+16/15*(1-x)^(5/2)
```

3.268 $\int \sin\left(x^{-n}\right) dx$

Optimal. Leaf size=43

$$\frac{ix\left(-\text{ExpIntegralE}\left(1+\frac{1}{n},-ix^{-n}\right)+\text{ExpIntegralE}\left(1+\frac{1}{n},ix^{-n}\right)\right)}{2n}$$

```
[Out] 1/2*I*x*(-Ei(1+1/n,-I/(x^n))+Ei(1+1/n,I/(x^n)))/n
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.79, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3446, 2239}

$$\frac{ix(ix^{-n})^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},ix^{-n}\right)}{2n}-\frac{ix(-ix^{-n})^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-ix^{-n}\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(-n)],x]
```

```
[Out] ((-1/2*I)*x*((-I)/x^n)^n^(-1)*Gamma[-n^(-1), (-I)/x^n])/n + ((I/2)*x*(I/x^n
)^n^(-1)*Gamma[-n^(-1), I/x^n])/n
```

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3446

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^((-c)*I - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}i\int e^{-ix^{-n}}\,dx-\frac{1}{2}i\int e^{ix^{-n}}\,dx\\ &= -\frac{ix(-ix^{-n})^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-ix^{-n}\right)}{2n}+\frac{ix(ix^{-n})^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},ix^{-n}\right)}{2n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.63

$$-\frac{ix\left((-ix^{-n})^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-ix^{-n}\right)-(ix^{-n})^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},ix^{-n}\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(-n)],x]
```

```
[Out] ((-1/2*I)*x*(((-I)/x^n)^n^(-1)*Gamma[-n^(-1), (-I)/x^n] - (I/x^n)^n^(-1)*Ga
mma[-n^(-1), I/x^n]))/n
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 47, normalized size = 1.09

method	result	size
meijerg	$-\frac{x^{-n+1}{}_{1}F_{2}\left(\frac{1}{2}-\frac{1}{2n};\frac{3}{2},\frac{3}{2}-\frac{1}{2n};-\frac{x^{-4n}x^{2n}}{4}\right)}{n-1}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/(x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(n-1)*x^(-n+1)*hypergeom([1/2-1/2/n],[3/2,3/2-1/2/n],-1/4*(x^(-2*n))^2*x
^(2*n))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/(x^n)),x, algorithm="maxima")
```

```
[Out] integrate(sin(1/(x^n)), x)
```

Fricas [F]
time = 0.60, size = 8, normalized size = 0.19

$$\text{integral}\left(\sin\left(\frac{1}{x^n}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/(x^n)),x, algorithm="fricas")
```

```
[Out] integral(sin(1/(x^n)), x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.
time = 0.80, size = 56, normalized size = 1.30

$$-\frac{xx^{-n}\Gamma\left(\frac{1}{2}-\frac{1}{2n}\right){}_{1}F_{2}\left(\begin{matrix}\frac{1}{2}-\frac{1}{2n}\\ \frac{3}{2},\frac{3}{2}-\frac{1}{2n}\end{matrix}\middle|-\frac{x^{-2n}}{4}\right)}{2n\Gamma\left(\frac{3}{2}-\frac{1}{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/(x**n)),x)
```

```
[Out] -x*gamma(1/2 - 1/(2*n))*hyper((1/2 - 1/(2*n),), (3/2, 3/2 - 1/(2*n)), -1/(4
*x**(2*n)))/(2*n*x**n*gamma(3/2 - 1/(2*n)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(sin(1/(x^n)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(\frac{1}{x^n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/x^n),x)
```

```
[Out] int(sin(1/x^n), x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(sin(1/(x^n)),x)
```

```
[Out] not solved
```

3.269 $\int \frac{1}{2}\left(-x+\sqrt{4-3x^2}\right)\,dx$

Optimal. Leaf size=41

$$-\frac{x^2}{4}+\frac{1}{4}x\sqrt{4-3x^2}+\frac{\arcsin\left(\frac{\sqrt{3}x}{2}\right)}{\sqrt{3}}$$

```
[Out] -1/4*x^2+1/4*x*(-3*x^2+4)^(1/2)+1/3*arcsin(1/2*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {12, 201, 222}

$$\frac{\arcsin\left(\frac{\sqrt{3}x}{2}\right)}{\sqrt{3}}-\frac{x^2}{4}+\frac{1}{4}\sqrt{4-3x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[(-x + Sqrt[4 - 3*x^2])/2,x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[4 - 3*x^2])/4 + ArcSin[(Sqrt[3]*x)/2]/Sqrt[3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{2}\int\left(-x+\sqrt{4-3x^2}\right)\,dx\\ &= -\frac{x^2}{4}+\frac{1}{2}\int\sqrt{4-3x^2}\,dx\\ &= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{4-3x^2}+\int\frac{1}{\sqrt{4-3x^2}}\,dx\\ &= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{4-3x^2}+\frac{\arcsin\left(\frac{\sqrt{3}x}{2}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 1.41

$$\frac{1}{2}\left(-\frac{x^2}{2}+\frac{1}{2}x\sqrt{4-3x^2}+\frac{4\arctan\left(\frac{\sqrt{3}x}{-2+\sqrt{4-3x^2}}\right)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + Sqrt[4 - 3*x^2])/2,x]
[Out] (-1/2*x^2 + (x*Sqrt[4 - 3*x^2])/2 + (4*ArcTan[(Sqrt[3]*x)/(-2 + Sqrt[4 - 3*
x^2])])/Sqrt[3])/2
```

Maple [A]
time = 0.12, size = 31, normalized size = 0.76

method	result	size
default	$-\frac{x^2}{4}+\frac{x\sqrt{-3x^2+4}}{4}+\frac{\arcsin\left(\frac{x\sqrt{3}}{2}\right)\sqrt{3}}{3}$	31
parts	$-\frac{x^2}{4}+\frac{x\sqrt{-3x^2+4}}{4}+\frac{\arcsin\left(\frac{x\sqrt{3}}{2}\right)\sqrt{3}}{3}$	31
trager	$-\frac{x^2}{4}+\frac{x\sqrt{-3x^2+4}}{4}-\frac{\mathit{RootOf}\left(\_Z^2+3\right)\ln\left(-\mathit{RootOf}\left(\_Z^2+3\right)\sqrt{-3x^2+4}+3x\right)}{3}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*x+1/2*(-3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/4*x*(-3*x^2+4)^(1/2)+1/3*arcsin(1/2*x*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.43, size = 30, normalized size = 0.73

$$-\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-3\,x^2+4}x+\frac{1}{3}\,\sqrt{3}\arcsin\left(\frac{1}{2}\,\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+1/2*(-3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-3*x^2 + 4)*x + 1/3*sqrt(3)*arcsin(1/2*sqrt(3)*x)
```

Fricas [A]
time = 0.57, size = 48, normalized size = 1.17

$$-\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-3\,x^2+4}x-\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+4}-2\,\sqrt{3}}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+1/2*(-3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-3*x^2 + 4)*x - 2/3*sqrt(3)*arctan(1/3*(sqrt(3)*sqrt(-3
*x^2 + 4) - 2*sqrt(3))/x)
```

Sympy [A]
time = 0.09, size = 34, normalized size = 0.83

$$-\frac{x^2}{4}+\frac{x\sqrt{4-3x^2}}{4}+\frac{\sqrt{3}\operatorname{asin}\left(\frac{\sqrt{3}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+1/2*(-3*x**2+4)**(1/2),x)
```

```
[Out] -x**2/4 + x*sqrt(4 - 3*x**2)/4 + sqrt(3)*asin(sqrt(3)*x/2)/3
```

Giac [A]
time = 0.40, size = 30, normalized size = 0.73

$$-\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-3\,x^2+4}x+\frac{1}{3}\,\sqrt{3}\arcsin\left(\frac{1}{2}\,\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+1/2*(-3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-3*x^2 + 4)*x + 1/3*sqrt(3)*arcsin(1/2*sqrt(3)*x)
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.76

$$\frac{\sqrt{3}\left(\operatorname{asin}\left(\frac{\sqrt{3}\,x}{2}\right)+\frac{3\,x\,\sqrt{\frac{4}{3}-x^2}}{4}\right)}{3}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4 - 3*x^2)^(1/2)/2 - x/2,x)
```

```
[Out] (3^(1/2)*(asin((3^(1/2)*x)/2) + (3*x*(4/3 - x^2)^(1/2))/4))/3 - x^2/4
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 17, normalized size = 0.41

$$-\frac{x^2}{4}+\frac{(3x^2-4)^{\frac{3}{2}}}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(-1/2*x+1/2*(-3*x^2+4)^(1/2),x)
```

```
[Out] -1/4*x^2+1/3*(3*x^2-4)^(3/2)
```

3.270 $\int (1-3x^2+x^4)^n \, dx$

Optimal. Leaf size=99

$$x\left(1-\frac{2x^2}{3-\sqrt{5}}\right)^{-n}\left(1-\frac{2x^2}{3+\sqrt{5}}\right)^{-n}\left(1-3x^2+x^4\right)^n \text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},\frac{2x^2}{3+\sqrt{5}},\frac{2x^2}{3-\sqrt{5}}\right)$$

```
[Out] x*(x^4-3*x^2+1)^n*AppellF1(1/2,-n,-n,3/2,2*x^2/(3-5^(1/2)),2*x^2/(3+5^(1/2)
))/((1-2*x^2/(3-5^(1/2)))^n)/((1-2*x^2/(3+5^(1/2)))^n)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1119, 440}

$$x\left(1-\frac{2x^2}{3-\sqrt{5}}\right)^{-n}\left(1-\frac{2x^2}{3+\sqrt{5}}\right)^{-n}\left(x^4-3x^2+1\right)^n \text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},\frac{2x^2}{3+\sqrt{5}},\frac{2x^2}{3-\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^2 + x^4)^n,x]
```

```
[Out] (x*(1 - 3*x^2 + x^4)^n*AppellF1[1/2, -n, -n, 3/2, (2*x^2)/(3 + Sqrt[5]), (2
*x^2)/(3 - Sqrt[5])])/((1 - (2*x^2)/(3 - Sqrt[5]))^n*(1 - (2*x^2)/(3 + Sqrt
[5]))^n)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \left(\left(1+\frac{2x^2}{-3-\sqrt{5}}\right)^{-n}\left(1+\frac{2x^2}{-3+\sqrt{5}}\right)^{-n}\left(1-3x^2+x^4\right)^n\right)\int\left(1+\frac{2x^2}{-3-\sqrt{5}}\right)^n\left(1+\frac{2x^2}{-3+\sqrt{5}}\right)^n dx\\
&= x\left(1-\frac{2x^2}{3-\sqrt{5}}\right)^{-n}\left(1-\frac{2x^2}{3+\sqrt{5}}\right)^{-n}\left(1-3x^2+x^4\right)^n \text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},\frac{2x^2}{3+\sqrt{5}},\frac{2x^2}{3-\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 142, normalized size = 1.43

$$\left(3+\sqrt{5}\right)^n x\left(-\left(3+\sqrt{5}-2x^2\right)^2\right)^{-n}\left(-3-\sqrt{5}+2x^2\right)^n\left(-3+\sqrt{5}+2x^2\right)^n\left(\frac{\left(-3+\sqrt{5}+2x^2\right)^2}{-3+\sqrt{5}}\right)^{-n}\left(1-3x^2+x^4\right)^n \text{AppellF1}\left(\frac{1}{2},-n,-n,\frac{3}{2},-\frac{2x^2}{-3+\sqrt{5}},\frac{2x^2}{3+\sqrt{5}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - 3*x^2 + x^4)^n,x]
```

```
[Out] ((3 + Sqrt[5])^n*x*(-3 - Sqrt[5] + 2*x^2)^n*(-3 + Sqrt[5] + 2*x^2)^n*(1 - 3
*x^2 + x^4)^n*AppellF1[1/2, -n, -n, 3/2, (-2*x^2)/(-3 + Sqrt[5]), (2*x^2)/(
3 + Sqrt[5])])/((-(3 + Sqrt[5] - 2*x^2)^2)^n*((-3 + Sqrt[5] + 2*x^2)^2/(-3
+ Sqrt[5]))^n)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4-3x^2+1\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3*x^2+1)^n,x)
```

```
[Out] int((x^4-3*x^2+1)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x^2+1)^n,x, algorithm="maxima")
```

```
[Out] integrate((x^4 - 3*x^2 + 1)^n, x)
```

Fricas [F]
time = 0.58, size = 14, normalized size = 0.14

$$\text{integral}\left(\left(x^4-3\,x^2+1\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x^2+1)^n,x, algorithm="fricas")
```

```
[Out] integral((x^4 - 3*x^2 + 1)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4-3x^2+1\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3*x**2+1)**n,x)
```

```
[Out] Integral((x**4 - 3*x**2 + 1)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x^2+1)^n,x, algorithm="giac")
```

```
[Out] integrate((x^4 - 3*x^2 + 1)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^4-3\,x^2+1\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 3*x^2 + 1)^n,x)
```

```
[Out] int((x^4 - 3*x^2 + 1)^n, x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((x^4-3*x^2+1)^n,x)
```

```
[Out] not solved
```

3.271 $\int \frac{(1+e^{-x})x}{-1+e^x}\,dx$

Optimal. Leaf size=36

$$e^{-x}+e^{-x}x-x^2+2x\log(1-e^x)+2\,\text{PolyLog}(2,e^x)$$

```
[Out] exp(-x)+x/exp(x)-x^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2286, 2207, 2225, 2215, 2221, 2317, 2438}

$$2\,\text{PolyLog}(2,e^x)-x^2+e^{-x}x+e^{-x}+2x\log(1-e^x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + E^(-x))*x)/(-1 + E^x),x]
```

```
[Out] E^(-x) + x/E^x - x^2 + 2*x*Log[1 - E^x] + 2*PolyLog[2, E^x]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2286

```
Int[((a_.) + (b_.)*(F_)^(u_))^(p_.)*((c_.) + (d_.)*(F_)^(v_))^(q_.)*((e_.)
+ (f_.)*(x_))^(m_.), x_Symbol] :> With[{w = ExpandIntegrand[(e + f*x)^m, (a
 + b*F^u)^p*(c + d*F^v)^q, x]}, Int[w, x] /; SumQ[w]] /; FreeQ[{F, a, b, c,
 d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simp
lify[u/v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(-e^{-x}x + \frac{2x}{-1+e^x}\right) dx \\
&= 2\int \frac{x}{-1+e^x}\,dx - \int e^{-x}x\,dx \\
&= e^{-x}x - x^2 + 2\int \frac{e^x x}{-1+e^x}\,dx - \int e^{-x}\,dx \\
&= e^{-x} + e^{-x}x - x^2 + 2x\log(1-e^x) - 2\int \log(1-e^x)\,dx \\
&= e^{-x} + e^{-x}x - x^2 + 2x\log(1-e^x) - 2\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^x\right) \\
&= e^{-x} + e^{-x}x - x^2 + 2x\log(1-e^x) + 2\,\text{PolyLog}(2, e^x)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 1.00

$$e^{-x}\left(1+x-e^x x^2\right) + 2x\log(1-e^x) + 2\,\text{PolyLog}(2, e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + E^(-x))*x)/(-1 + E^x),x]
[Out] (1 + x - E^x*x^2)/E^x + 2*x*Log[1 - E^x] + 2*PolyLog[2, E^x]
```

Maple [A]
time = 0.03, size = 33, normalized size = 0.92

method	result	size
risch	$(x+1)\,\mathrm{e}^{-x}-x^2+2x\ln\left(1-\mathrm{e}^x\right)+2\ \mathrm{hyperbolicCosineIntegral}_2\left(\mathrm{e}^x\right)$	31
default	$x\,\mathrm{e}^{-x}+2x\ln\left(1-\mathrm{e}^x\right)-x^2+\mathrm{e}^{-x}+2\ \mathrm{hyperbolicCosineIntegral}_2\left(\mathrm{e}^x\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(exp(-x)+1)/(exp(x)-1),x,method=_RETURNVERBOSE)
```

```
[Out] -x^2+2*x*ln(1-exp(x))+2*polylog(2,exp(x))+x/exp(x)+1/exp(x)
```

Maxima [A]
time = 0.38, size = 29, normalized size = 0.81

$$-x^2+(x+1)e^{(-x)}+2\,x\log\left(-e^x+1\right)+2\,\mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(exp(-x)+1)/(exp(x)-1),x, algorithm="maxima")
```

```
[Out] -x^2 + (x + 1)*e^(-x) + 2*x*log(-e^x + 1) + 2*dilog(e^x)
```

Fricas [A]
time = 0.58, size = 36, normalized size = 1.00

$$-\left(x^2e^x-2\,xe^x\log\left(-e^x+1\right)-2\,\mathrm{Li}_2(e^x)\,e^x-x-1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(exp(-x)+1)/(exp(x)-1),x, algorithm="fricas")
```

```
[Out] -(x^2*e^x - 2*x*e^x*log(-e^x + 1) - 2*dilog(e^x)*e^x - x - 1)*e^(-x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(e^x+1)\,e^{-x}}{e^x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(exp(-x)+1)/(exp(x)-1),x)
```

```
[Out] Integral(x*(exp(x) + 1)*exp(-x)/(exp(x) - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(exp(-x)+1)/(exp(x)-1),x, algorithm="giac")
```

```
[Out] integrate(x*(e^(-x) + 1)/(e^x - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,(\mathrm{e}^{-x}+1)}{\mathrm{e}^{x}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(exp(-x) + 1))/(exp(x) - 1),x)
```

```
[Out] int((x*(exp(-x) + 1))/(exp(x) - 1), x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(x*(exp(-x)+1)/(exp(x)-1),x)
```

```
[Out] not solved
```

3.272 $\int e^{-x} x^4 \sin(x)\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}e^{-x}\left(-\left(\left(-6+x^2(6+x(4+x))\right)\cos(x)\right)+\left(6+x\left(12+6x-x^3\right)\right)\sin(x)\right)$$

```
[Out] 1/2*(-(-6+x^2*(6+x*(4+x)))*cos(x)+(6+x*(-x^3+6*x+12))*sin(x))/exp(x)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. $2(44) = 88$.
time = 0.42, antiderivative size = 93, normalized size of antiderivative = 2.11, number of steps used = 53, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4517, 4553, 14, 4518, 4554}

$$-\frac{1}{2}e^{-x}x^4\sin(x)-\frac{1}{2}e^{-x}x^4\cos(x)-2e^{-x}x^3\cos(x)+3e^{-x}x^2\sin(x)-3e^{-x}x^2\cos(x)+6e^{-x}x\sin(x)+3e^{-x}\sin(x)+3e^{-x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[x])/E^x,x]
```

```
[Out] (3*Cos[x])/E^x - (3*x^2*Cos[x])/E^x - (2*x^3*Cos[x])/E^x - (x^4*Cos[x])/(2*
E^x) + (3*Sin[x])/E^x + (6*x*Sin[x])/E^x + (3*x^2*Sin[x])/E^x - (x^4*Sin[x]
)/(2*E^x)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= -\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)-4\int x^3\left(-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{-x}\sin(x)\right)\,dx\\
&= -\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)-4\int\left(-\frac{1}{2}e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^3\sin(x)\right)\,dx\\
&= -\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\int e^{-x}x^3\cos(x)\,dx+2\int e^{-x}x^3\sin(x)\,dx\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)-6\int x^2\left(-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{-x}\sin(x)\right)\,dx-6\int x^2\left(-\frac{1}{2}e^{-x}\cos(x)+\frac{1}{2}e^{-x}\sin(x)\right)\,dx\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)-6\int\left(-\frac{1}{2}e^{-x}x^2\cos(x)-\frac{1}{2}e^{-x}x^2\sin(x)\right)\,dx-6\int\left(-\frac{1}{2}e^{-x}x^2\cos(x)+\frac{1}{2}e^{-x}x^2\sin(x)\right)\,dx\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\left(3\int e^{-x}x^2\cos(x)\,dx\right)\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\left(-\frac{3}{2}e^{-x}x^2\cos(x)+\frac{3}{2}e^{-x}x^2\sin(x)-6\int x\left(-\frac{1}{2}e^{-x}\cos(x)+\frac{1}{2}e^{-x}\sin(x)\right)\,dx\right)\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\left(-\frac{3}{2}e^{-x}x^2\cos(x)+\frac{3}{2}e^{-x}x^2\sin(x)-6\int\left(-\frac{1}{2}e^{-x}x\cos(x)+\frac{1}{2}e^{-x}x\sin(x)\right)\,dx\right)\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\left(-\frac{3}{2}e^{-x}x^2\cos(x)+\frac{3}{2}e^{-x}x^2\sin(x)+3\int e^{-x}x\cos(x)\,dx-3\int e^{-x}x\sin(x)\,dx\right)\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\left(-\frac{3}{2}e^{-x}x^2\cos(x)+3e^{-x}x\sin(x)+\frac{3}{2}e^{-x}x^2\sin(x)+3\int\left(-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{-x}\sin(x)\right)\,dx-3\int\left(-\frac{1}{2}e^{-x}\cos(x)+\frac{1}{2}e^{-x}\sin(x)\right)\,dx\right)\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\left(-\frac{3}{2}e^{-x}x^2\cos(x)+3e^{-x}x\sin(x)+\frac{3}{2}e^{-x}x^2\sin(x)-2\left(\frac{3}{2}\int e^{-x}\sin(x)\,dx\right)\right)\\
&= -2e^{-x}x^3\cos(x)-\frac{1}{2}e^{-x}x^4\cos(x)-\frac{1}{2}e^{-x}x^4\sin(x)+2\left(-\frac{3}{2}e^{-x}x^2\cos(x)+3e^{-x}x\sin(x)+\frac{3}{2}e^{-x}x^2\sin(x)-2\left(-\frac{3}{4}e^{-x}\cos(x)-\frac{3}{4}e^{-x}\sin(x)\right)\right)
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 47, normalized size = 1.07

$$\frac{1}{2}e^{-x}\left(-\left(\left(-6+6x^2+4x^3+x^4\right)\cos(x)\right)+\left(6+12x+6x^2-x^4\right)\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[x])/E^x,x]
```

```
[Out] (-((-6 + 6*x^2 + 4*x^3 + x^4)*Cos[x]) + (6 + 12*x + 6*x^2 - x^4)*Sin[x])/(2
*E^x)
```

Maple [A]

time = 0.12, size = 48, normalized size = 1.09

method	result
parallelrisch	$-\frac{\left(\left(x^4+4x^3+6x^2-6\right)\cos(x)+\sin(x)\left(x^4-6x^2-12x-6\right)\right)e^{-x}}{2}$
default	$\left(-\frac{1}{2}x^4-2x^3-3x^2+3\right)e^{-x}\cos(x)+\left(-\frac{1}{2}x^4+3x^2+6x+3\right)e^{-x}\sin(x)$

risch	$\left(-\frac{1}{4}+\frac{i}{4}\right)\left(x^4+2ix^3+2x^3+6ix^2+6ix-6x-6\right)\mathrm{e}^{(-1+i)x}+\left(-\frac{1}{4}-\frac{i}{4}\right)\left(x^4-2ix^3+2x^3-6\right.$
norman	$\frac{-3x^2\mathrm{e}^{-x}-2x^3\mathrm{e}^{-x}-\frac{x^4\mathrm{e}^{-x}}{2}+6\,\mathrm{e}^{-x}\tan\left(\frac{x}{2}\right)-3\,\mathrm{e}^{-x}\left(\tan^2\left(\frac{x}{2}\right)\right)+12x\,\mathrm{e}^{-x}\tan\left(\frac{x}{2}\right)+6x^2\mathrm{e}^{-x}\tan\left(\frac{x}{2}\right)+3x^2\mathrm{e}^{-x}\left(\tan^2\left(\frac{x}{2}\right)\right)+2x^3\mathrm{e}^{-x}\left(\mathrm{ta}\right.}{1+\tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(-x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*x^4-2*x^3-3*x^2+3)*exp(-x)*cos(x)+(-1/2*x^4+3*x^2+6*x+3)*exp(-x)*sin(
x)
```

Maxima [A]
time = 0.36, size = 41, normalized size = 0.93

$$-\frac{1}{2}\left(\left(x^{4}+4\,x^{3}+6\,x^{2}-6\right)\cos\left(x\right)+\left(x^{4}-6\,x^{2}-12\,x-6\right)\sin\left(x\right)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(-x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*((x^4 + 4*x^3 + 6*x^2 - 6)*cos(x) + (x^4 - 6*x^2 - 12*x - 6)*sin(x))*e
^(-x)
```

Fricas [A]
time = 0.58, size = 45, normalized size = 1.02

$$-\frac{1}{2}\left(x^{4}+4\,x^{3}+6\,x^{2}-6\right)\cos\left(x\right)e^{(-x)}-\frac{1}{2}\left(x^{4}-6\,x^{2}-12\,x-6\right)e^{(-x)}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(-x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 + 4*x^3 + 6*x^2 - 6)*cos(x)*e^(-x) - 1/2*(x^4 - 6*x^2 - 12*x - 6)
*e^(-x)*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(36) = 72$.
time = 1.19, size = 85, normalized size = 1.93

$$-\frac{x^{4}e^{-x}\sin\left(x\right)}{2}-\frac{x^{4}e^{-x}\cos\left(x\right)}{2}-2x^{3}e^{-x}\cos\left(x\right)+3x^{2}e^{-x}\sin\left(x\right)-3x^{2}e^{-x}\cos\left(x\right)+6xe^{-x}\sin\left(x\right)+3e^{-x}\sin\left(x\right)+3e^{-x}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*exp(-x)*sin(x),x)
```

```
[Out] -x**4*exp(-x)*sin(x)/2 - x**4*exp(-x)*cos(x)/2 - 2*x**3*exp(-x)*cos(x) + 3*
x**2*exp(-x)*sin(x) - 3*x**2*exp(-x)*cos(x) + 6*x*exp(-x)*sin(x) + 3*exp(-x
)*sin(x) + 3*exp(-x)*cos(x)
```

Giac [A]
time = 0.45, size = 41, normalized size = 0.93

$$-\frac{1}{2}\left(\left(x^{4}+4\,x^{3}+6\,x^{2}-6\right)\cos\left(x\right)+\left(x^{4}-6\,x^{2}-12\,x-6\right)\sin\left(x\right)\right)e^{\left(-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(-x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*((x^4 + 4*x^3 + 6*x^2 - 6)*cos(x) + (x^4 - 6*x^2 - 12*x - 6)*sin(x))*e
^(-x)
```

Mupad [B]
time = 0.26, size = 55, normalized size = 1.25

$$\frac{e^{-x}\,\left(6\,\cos\left(x\right)+6\,\sin\left(x\right)-6\,x^2\,\cos\left(x\right)-4\,x^3\,\cos\left(x\right)-x^4\,\cos\left(x\right)+6\,x^2\,\sin\left(x\right)-x^4\,\sin\left(x\right)+12\,x\,\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(-x)*sin(x),x)
```

```
[Out] (exp(-x)*(6*cos(x) + 6*sin(x) - 6*x^2*cos(x) - 4*x^3*cos(x) - x^4*cos(x) +
6*x^2*sin(x) - x^4*sin(x) + 12*x*sin(x)))/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 97, normalized size = 2.20

$$\left(-x^4-4x^3-12x^2-24x-24\right)\cos\left(x\right)-\left(x^4-4x^3+12x^2+48x+48\right)\sin\left(x\right)+24x\,e^{-x}\sin\left(x\right)+\left(3x^4-12x^3+12x\right)e^{-x}\cos\left(x\right)+\left(3x^4-12x^3+12x\right)e^{-x}\sin\left(x\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(x^4*exp(-x)*sin(x),x)
```

```
[Out] (-x^4-4*x^3-12*x^2-24*x-24)*cos(x)-(x^4-4*x^3+12*x^2+48*x+48)*sin(x)+24*x*e
xp(-x)*sin(x)+(3*x^4-12*x^3+12*x)*exp(-x)*cos(x)+(3*x^4-12*x^3+12*x)*exp(-x
)*sin(x)
```

3.273 $\int \frac{-3(-3x+x^3)+(-3x+x^3)^3}{\sqrt{-4+x^2}}\,dx$

Optimal. Leaf size=56

$$\sqrt{-4+x^2}+\frac{10}{3}\left(-4+x^2\right)^{3/2}+3\left(-4+x^2\right)^{5/2}+\left(-4+x^2\right)^{7/2}+\frac{1}{9}\left(-4+x^2\right)^{9/2}$$

```
[Out] (x^2-4)^(1/2)+10/3*(x^2-4)^(3/2)+3*(x^2-4)^(5/2)+(x^2-4)^(7/2)+1/9*(x^2-4)^
(9/2)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1825, 1813, 1864}

$$\frac{1}{9}\left(x^2-4\right)^{9/2}+\left(x^2-4\right)^{7/2}+3\left(x^2-4\right)^{5/2}+\frac{10}{3}\left(x^2-4\right)^{3/2}+\sqrt{x^2-4}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*(-3*x + x^3) + (-3*x + x^3)^3)/Sqrt[-4 + x^2],x]
```

```
[Out] Sqrt[-4 + x^2] + (10*(-4 + x^2)^(3/2))/3 + 3*(-4 + x^2)^(5/2) + (-4 + x^2)^
(7/2) + (-4 + x^2)^(9/2)/9
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \int \frac{x(9-30x^2+27x^4-9x^6+x^8)}{\sqrt{-4+x^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{9-30x+27x^2-9x^3+x^4}{\sqrt{-4+x}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{\sqrt{-4+x}}+10\sqrt{-4+x}+15(-4+x)^{3/2}+7(-4+x)^{5/2}+(-4+x)^{7/2}\right)dx, x, x^2\right) \\
&= \sqrt{-4+x^2}+\frac{10}{3}\left(-4+x^2\right)^{3/2}+3\left(-4+x^2\right)^{5/2}+\left(-4+x^2\right)^{7/2}+\frac{1}{9}\left(-4+x^2\right)^{9/2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.59

$$
\frac{1}{9}\sqrt{-4+x^2}\left(1-10x^2+15x^4-7x^6+x^8\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*(-3*x + x^3) + (-3*x + x^3)^3)/Sqrt[-4 + x^2],x]
[Out] (Sqrt[-4 + x^2]*(1 - 10*x^2 + 15*x^4 - 7*x^6 + x^8))/9
```

Maple [A]
time = 0.12, size = 59, normalized size = 1.05

method	result
risch	$\frac{(x^8-7x^6+15x^4-10x^2+1)\sqrt{x^2-4}}{9}$
pseudoelliptic	$\frac{(x^8-7x^6+15x^4-10x^2+1)\sqrt{x^2-4}}{9}$
trager	$\left(\frac{1}{9}x^8-\frac{7}{9}x^6+\frac{5}{3}x^4-\frac{10}{9}x^2+\frac{1}{9}\right)\sqrt{x^2-4}$
gosper	$\frac{(x^8-7x^6+15x^4-10x^2+1)(x-2)(2+x)}{9\sqrt{x^2-4}}$
default	$\frac{x^8\sqrt{x^2-4}}{9}-\frac{7x^6\sqrt{x^2-4}}{9}+\frac{5x^4\sqrt{x^2-4}}{3}-\frac{10x^2\sqrt{x^2-4}}{9}+\frac{\sqrt{x^2-4}}{9}$
meijerg	$-\frac{256\sqrt{-\text{signum}\left(-1+\frac{x^2}{4}\right)}\left(-\frac{256\sqrt{\pi}}{315}+\frac{\sqrt{\pi}\left(\frac{35}{128}x^8+\frac{5}{4}x^6+6x^4+32x^2+256\right)\sqrt{-\frac{x^2}{4}+1}}{315}\right)}{\sqrt{\pi}\sqrt{\text{signum}\left(-1+\frac{x^2}{4}\right)}}-\frac{576\sqrt{-\text{signum}\left(-1+\frac{x^2}{4}\right)}\left(\frac{32\sqrt{\pi}}{35}\right.}{\sqrt{\pi}\sqrt{\text{sig}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3-3*x)^3-3*x^3+9*x)/(x^2-4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/9*x^8*(x^2-4)^(1/2)-7/9*x^6*(x^2-4)^(1/2)+5/3*x^4*(x^2-4)^(1/2)-10/9*x^2*
(x^2-4)^(1/2)+1/9*(x^2-4)^(1/2)
```

Maxima [A]
time = 0.35, size = 58, normalized size = 1.04

$$
\frac{1}{9}\sqrt{x^2-4}x^8-\frac{7}{9}\sqrt{x^2-4}x^6+\frac{5}{3}\sqrt{x^2-4}x^4-\frac{10}{9}\sqrt{x^2-4}x^2+\frac{1}{9}\sqrt{x^2-4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^3-3*x)^3-3*x^3+9*x)/(x^2-4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(x^2 - 4)*x^8 - 7/9*sqrt(x^2 - 4)*x^6 + 5/3*sqrt(x^2 - 4)*x^4 - 10/
9*sqrt(x^2 - 4)*x^2 + 1/9*sqrt(x^2 - 4)
```

Fricas [A]
time = 0.57, size = 29, normalized size = 0.52

$$\frac{1}{9}\left(x^{8}-7\,x^{6}+15\,x^{4}-10\,x^{2}+1\right)\sqrt{x^{2}-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^3-3*x)^3-3*x^3+9*x)/(x^2-4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(x^8 - 7*x^6 + 15*x^4 - 10*x^2 + 1)*sqrt(x^2 - 4)
```

Sympy [A]
time = 0.97, size = 68, normalized size = 1.21

$$\frac{x^{8}\sqrt{x^{2}-4}}{9}-\frac{7x^{6}\sqrt{x^{2}-4}}{9}+\frac{5x^{4}\sqrt{x^{2}-4}}{3}-\frac{10x^{2}\sqrt{x^{2}-4}}{9}+\frac{\sqrt{x^{2}-4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**3-3*x)**3-3*x**3+9*x)/(x**2-4)**(1/2),x)
```

```
[Out] x**8*sqrt(x**2 - 4)/9 - 7*x**6*sqrt(x**2 - 4)/9 + 5*x**4*sqrt(x**2 - 4)/3 -
 10*x**2*sqrt(x**2 - 4)/9 + sqrt(x**2 - 4)/9
```

Giac [A]
time = 0.41, size = 42, normalized size = 0.75

$$\frac{1}{9}\left(x^{2}-4\right)^{\frac{9}{2}}+\left(x^{2}-4\right)^{\frac{7}{2}}+3\left(x^{2}-4\right)^{\frac{5}{2}}+\frac{10}{3}\left(x^{2}-4\right)^{\frac{3}{2}}+\sqrt{x^{2}-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^3-3*x)^3-3*x^3+9*x)/(x^2-4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*(x^2 - 4)^(9/2) + (x^2 - 4)^(7/2) + 3*(x^2 - 4)^(5/2) + 10/3*(x^2 - 4)^
(3/2) + sqrt(x^2 - 4)
```

Mupad [B]
time = 0.13, size = 30, normalized size = 0.54

$$\sqrt{x^{2}-4}\left(\frac{x^{8}}{9}-\frac{7\,x^{6}}{9}+\frac{5\,x^{4}}{3}-\frac{10\,x^{2}}{9}+\frac{1}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*x - x^3)^3 - 9*x + 3*x^3)/(x^2 - 4)^(1/2),x)
```

```
[Out] (x^2 - 4)^(1/2)*((5*x^4)/3 - (10*x^2)/9 - (7*x^6)/9 + x^8/9 + 1/9)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(((x^3-3*x)^3-3*x^3+9*x)/(x^2-4)^(1/2),x)
```

```
[Out] not solved
```

3.274 $\int \frac{1}{(1+x^2)^3} dx$

Optimal. Leaf size=31

$$\frac{x}{4(1+x^2)^2}+\frac{3x}{8(1+x^2)}+\frac{3\arctan(x)}{8}$$

```
[Out] 1/4*x/(x^2+1)^2+3*x/(8*x^2+8)+3/8*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {205, 209}

$$\frac{3\arctan(x)}{8}+\frac{3x}{8(x^2+1)}+\frac{x}{4(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-3),x]
```

```
[Out] x/(4*(1 + x^2)^2) + (3*x)/(8*(1 + x^2)) + (3*ArcTan[x])/8
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{x}{4(1+x^2)^2}+\frac{3}{4}\int \frac{1}{(1+x^2)^2}\,dx\\ &= \frac{x}{4(1+x^2)^2}+\frac{3x}{8(1+x^2)}+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{4(1+x^2)^2}+\frac{3x}{8(1+x^2)}+\frac{3\arctan(x)}{8}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.81

$$\frac{1}{8}\left(\frac{x(5+3x^2)}{(1+x^2)^2}+3\arctan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-3),x]
[Out] ((x*(5 + 3*x^2))/(1 + x^2)^2 + 3*ArcTan[x])/8
```

Maple [A]
time = 0.08, size = 26, normalized size = 0.84

method	result	size
meijerg	$\frac{x(3x^2+5)}{8(x^2+1)^2}+\frac{3\arctan(x)}{8}$	23
risch	$\frac{\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^2}+\frac{3\arctan(x)}{8}$	23
default	$\frac{x}{4(x^2+1)^2}+\frac{3x}{8(x^2+1)}+\frac{3\arctan(x)}{8}$	26
parallelrisch	$-\frac{3i\ln(x-i)x^4-3i\ln(i+x)x^4+6i\ln(x-i)x^2-6i\ln(i+x)x^2-6x^3+3i\ln(x-i)-3i\ln(i+x)-10x}{16(x^2+1)^2}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*x/(x^2+1)^2+3/8/(x^2+1)*x+3/8*arctan(x)
```

Maxima [A]
time = 0.44, size = 28, normalized size = 0.90

$$\frac{3x^3+5x}{8(x^4+2x^2+1)}+\frac{3}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3,x, algorithm="maxima")
[Out] 1/8*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) + 3/8*arctan(x)
```

Fricas [A]
time = 0.58, size = 37, normalized size = 1.19

$$\frac{3x^3+3(x^4+2x^2+1)\arctan(x)+5x}{8(x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

Sympy [**A**]
time = 0.05, size = 26, normalized size = 0.84

$$\frac{3x^3 + 5x}{8x^4 + 16x^2 + 8} + \frac{3\,\mathrm{atan}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3,x)
```

```
[Out] (3*x**3 + 5*x)/(8*x**4 + 16*x**2 + 8) + 3*atan(x)/8
```

Giac [**A**]
time = 0.41, size = 23, normalized size = 0.74

$$\frac{3\,x^3 + 5\,x}{8\,(x^2 + 1)^2} + \frac{3}{8}\,\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*x^3 + 5*x)/(x^2 + 1)^2 + 3/8*arctan(x)
```

Mupad [**B**]
time = 0.12, size = 28, normalized size = 0.90

$$\frac{3\,\mathrm{atan}(x)}{8} + x\left(\frac{1}{4\,(x^2 + 1)^2} + \frac{3}{8\,x^2 + 8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^3,x)
```

```
[Out] (3*atan(x))/8 + x*(1/(4*(x^2 + 1)^2) + 3/(8*x^2 + 8))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 9, normalized size = 0.29

$$-\frac{1}{2\,(x^2 + 1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(x^2+1)^3,x)
```

```
[Out] -1/2/(x^2+1)^2
```

3.275 $\int \log\left(\sqrt{3}+\tan(x)\right)\,dx$

Optimal. Leaf size=108

$$-\frac{1}{2}i\left(\left(\log\left(\frac{i-\tan(x)}{i+\sqrt{3}}\right)-\log\left(\frac{i+\tan(x)}{i-\sqrt{3}}\right)\right)\log\left(\sqrt{3}+\tan(x)\right)-\text{PolyLog}\left(2,\frac{\sqrt{3}+\tan(x)}{-i+\sqrt{3}}\right)+\text{PolyL}$$

```
[Out] -1/2*I*((ln((I-tan(x))/(3^(1/2)+I))-ln((I+tan(x))/(I-3^(1/2))))*ln(3^(1/2)+
tan(x))-polylog(2,(3^(1/2)+tan(x))/(-I+3^(1/2)))+polylog(2,(3^(1/2)+tan(x))
/(3^(1/2)+I)))
```

Rubi [F]
time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \log\left(\sqrt{3}+\tan(x)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[Sqrt[3] + Tan[x]],x]
```

```
[Out] x*Log[Sqrt[3] + Tan[x]] - Defer[Int][(x*Sec[x]^2)/(Sqrt[3] + Tan[x]), x]
```

Rubi steps

$$\text{Integral} = x\log\left(\sqrt{3}+\tan(x)\right)-\int \frac{x\sec^2(x)}{\sqrt{3}+\tan(x)}\,dx$$

Mathematica [A]
time = 0.04, size = 132, normalized size = 1.22

$$-\frac{1}{2}i\log\left(\frac{i-\tan(x)}{i+\sqrt{3}}\right)\log\left(\sqrt{3}+\tan(x)\right)+\frac{1}{2}i\log\left(\frac{i+\tan(x)}{i-\sqrt{3}}\right)\log\left(\sqrt{3}+\tan(x)\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{\sqrt{3}+\tan(x)}{i-\sqrt{3}}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{\sqrt{3}+\tan(x)}{i+\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[3] + Tan[x]],x]
```

```
[Out] (-1/2*I)*Log[(I - Tan[x])/(I + Sqrt[3])]*Log[Sqrt[3] + Tan[x]] + (I/2)*Log[
(I + Tan[x])/(I - Sqrt[3])]*Log[Sqrt[3] + Tan[x]] + (I/2)*PolyLog[2, -((Sqr
t[3] + Tan[x])/(I - Sqrt[3]))] - (I/2)*PolyLog[2, (Sqrt[3] + Tan[x])/(I + S
qrt[3])]
```

Maple [A]
time = 0.43, size = 96, normalized size = 0.89

method	result
derivativedivides	$-\frac{i\ln\left(\sqrt{3}+\tan(x)\right)\ln\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i\ln\left(\sqrt{3}+\tan(x)\right)\ln\left(\frac{i+\tan(x)}{i-\sqrt{3}}\right)}{2}-\frac{i\,dilog\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i\,dilog\left(\frac{i+\tan(x)}{i-\sqrt{3}}\right)}{2}$
default	$-\frac{i\ln\left(\sqrt{3}+\tan(x)\right)\ln\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i\ln\left(\sqrt{3}+\tan(x)\right)\ln\left(\frac{i+\tan(x)}{i-\sqrt{3}}\right)}{2}-\frac{i\,dilog\left(\frac{i-\tan(x)}{\sqrt{3}+i}\right)}{2}+\frac{i\,dilog\left(\frac{i+\tan(x)}{i-\sqrt{3}}\right)}{2}$
risch	$-i\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)+i\,dilog\left(\frac{-i\sqrt{3}+2\,e^{ix}+1}{1-i\sqrt{3}}\right)+i\ln\left(e^{ix}\right)\ln\left(e^{2ix}+1\right)+\frac{i\pi\,\mathrm{csgn}\left(\frac{i}{e^{2ix}+1}\right)\mathrm{csgn}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(3^(1/2)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*ln(3^(1/2)+tan(x))*ln((I-tan(x))/(3^(1/2)+I))+1/2*I*ln(3^(1/2)+tan(x
))*ln((I+tan(x))/(I-3^(1/2)))-1/2*I*dilog((I-tan(x))/(3^(1/2)+I))+1/2*I*dil
og((I+tan(x))/(I-3^(1/2)))
```

Maxima [A]
time = 0.58, size = 115, normalized size = 1.06

$$\frac{1}{2}\arctan\left(\frac{1}{4}\sqrt{3}+\frac{1}{4}\tan(x),\frac{1}{4}\sqrt{3}\tan(x)+\frac{3}{4}\right)\log\left(\tan(x)^{2}+1\right)-\frac{1}{2}x\log\left(\frac{1}{4}\tan(x)^{2}+\frac{1}{2}\sqrt{3}\tan(x)+\frac{3}{4}\right)+x\log\left(\sqrt{3}+\tan(x)\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(-\frac{(\sqrt{3}+i)\tan(x)-i\sqrt{3}+1}{2i\sqrt{3}+2}\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(\frac{(\sqrt{3}-i)\tan(x)+i\sqrt{3}+1}{2i\sqrt{3}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3^(1/2)+tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(1/4*sqrt(3) + 1/4*tan(x), 1/4*sqrt(3)*tan(x) + 3/4)*log(tan(x)^
2 + 1) - 1/2*x*log(1/4*tan(x)^2 + 1/2*sqrt(3)*tan(x) + 3/4) + x*log(sqrt(3)
 + tan(x)) + 1/2*I*dilog(-((sqrt(3) + I)*tan(x) - I*sqrt(3) + 1)/(2*I*sqrt(
3) + 2)) - 1/2*I*dilog(((sqrt(3) - I)*tan(x) + I*sqrt(3) + 1)/(2*I*sqrt(3)
- 2))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(75) = 150.
time = 0.72, size = 262, normalized size = 2.43

$$x\log\left(\sqrt{3}+\tan(x)\right)-\frac{1}{2}x\log\left(\frac{(i\sqrt{3}+1)\tan(x)^{2}+2(\sqrt{3}+i)\tan(x)-i\sqrt{3}+3}{2(\tan(x)^{2}+1)}\right)-\frac{1}{2}x\log\left(\frac{(-i\sqrt{3}+1)\tan(x)^{2}+2(\sqrt{3}-i)\tan(x)+i\sqrt{3}+3}{2(\tan(x)^{2}+1)}\right)+\frac{1}{2}x\log\left(-\frac{2(i\tan(x)-1)}{\tan(x)^{2}+1}\right)+\frac{1}{2}x\log\left(-\frac{2(-i\tan(x)-1)}{\tan(x)^{2}+1}\right)+\frac{1}{4}i\,\mathrm{Li}_2\left(-\frac{(i\sqrt{3}+1)\tan(x)^{2}+2(\sqrt{3}+i)\tan(x)-i\sqrt{3}+3}{2(\tan(x)^{2}+1)}+1\right)-\frac{1}{4}i\,\mathrm{Li}_2\left(-\frac{(-i\sqrt{3}+1)\tan(x)^{2}+2(\sqrt{3}-i)\tan(x)+i\sqrt{3}+3}{2(\tan(x)^{2}+1)}+1\right)+\frac{1}{4}i\,\mathrm{Li}_2\left(\frac{2(i\tan(x)-1)}{\tan(x)^{2}+1}+1\right)-\frac{1}{4}i\,\mathrm{Li}_2\left(\frac{2(-i\tan(x)-1)}{\tan(x)^{2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3^(1/2)+tan(x)),x, algorithm="fricas")
```

```
[Out] x*log(sqrt(3) + tan(x)) - 1/2*x*log(1/2*((I*sqrt(3) + 1)*tan(x)^2 + 2*(sqrt
(3) + I)*tan(x) - I*sqrt(3) + 3)/(tan(x)^2 + 1)) - 1/2*x*log(1/2*((-I*sqrt(
3) + 1)*tan(x)^2 + 2*(sqrt(3) - I)*tan(x) + I*sqrt(3) + 3)/(tan(x)^2 + 1))
+ 1/2*x*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/2*x*log(-2*(-I*tan(x) - 1
)/(tan(x)^2 + 1)) + 1/4*I*dilog(-1/2*((I*sqrt(3) + 1)*tan(x)^2 + 2*(sqrt(3)
```

```
 + I)*tan(x) - I*sqrt(3) + 3)/(tan(x)^2 + 1) + 1) - 1/4*I*dilog(-1/2*((-I*s
qrt(3) + 1)*tan(x)^2 + 2*(sqrt(3) - I)*tan(x) + I*sqrt(3) + 3)/(tan(x)^2 +
1) + 1) + 1/4*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/4*I*dilog(2*
(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(\tan (x)+\sqrt{3}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(3**(1/2)+tan(x)),x)
[Out] Integral(log(tan(x) + sqrt(3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3^(1/2)+tan(x)),x, algorithm="giac")
[Out] integrate(log(sqrt(3) + tan(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\tan(x)+\sqrt{3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tan(x) + 3^(1/2)),x)
[Out] int(log(tan(x) + 3^(1/2)), x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 40, normalized size = 0.37

$$x \ln \left(\sqrt{3}+\tan (x)\right)-\frac{\left(1+\sqrt{3}\right) \ln \left(\sqrt{3}+\tan (x)-1\right)}{2}+\frac{\left(\sqrt{3}-1\right) \ln \left(\sqrt{3}+\tan (x)+1\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(3^(1/2)+tan(x)),x)
[Out] x*ln(3^(1/2)+tan(x))-1/2*(1+3^(1/2))*ln(3^(1/2)+tan(x)-1)+1/2*(3^(1/2)-1)*l
n(3^(1/2)+tan(x)+1)
```

3.276 $\int \sqrt{(\cos(20x)+3\cos(21x)+\cos(22x))^2+(\sin(20x)\cdots}$

Optimal. Leaf size=32

$$\frac{2(3+2\cos(x))\left(3\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\sin(x)\right)}{\sqrt{(3+2\cos(x))^2}}$$

```
[Out] 2*(3+2*cos(x))*(3*arctan(tan(1/2*x))+sin(x))/((3+2*cos(x))^2)^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 100 vs. $2(32)=64$.
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 3.12, number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$,
Rules used = {1986, 393, 209}

$$\frac{6\sec^2\left(\frac{x}{2}\right)\arctan\left(\tan\left(\frac{x}{2}\right)\right)\sqrt{\cos^4\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)+5\right)^2}}{\tan^2\left(\frac{x}{2}\right)+5}+\frac{4\tan\left(\frac{x}{2}\right)\sqrt{\cos^4\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)+5\right)^2}}{\tan^2\left(\frac{x}{2}\right)+5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(Cos[20*x] + 3*Cos[21*x] + Cos[22*x])^2 + (Sin[20*x] + 3*Sin[21*x]
 + Sin[22*x])^2],x]
```

```
[Out] (6*ArcTan[Tan[x/2]]*Sec[x/2]^2*Sqrt[Cos[x/2]^4*(5 + Tan[x/2]^2)^2])/(5 + Ta
n[x/2]^2) + (4*Tan[x/2]*Sqrt[Cos[x/2]^4*(5 + Tan[x/2]^2)^2])/(5 + Tan[x/2]^
2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int \frac{\sqrt{\frac{(5+x^2)^2}{(1+x^2)^2}}}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{\left(2\sec^2\left(\frac{x}{2}\right)\sqrt{\cos^4\left(\frac{x}{2}\right)\left(5+\tan^2\left(\frac{x}{2}\right)\right)^2}\right)\text{Subst}\left(\int \frac{5+x^2}{(1+x^2)^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{5+\tan^2\left(\frac{x}{2}\right)}\\
&= \frac{4\tan\left(\frac{x}{2}\right)\sqrt{\cos^4\left(\frac{x}{2}\right)\left(5+\tan^2\left(\frac{x}{2}\right)\right)^2}}{5+\tan^2\left(\frac{x}{2}\right)} + \frac{\left(6\sec^2\left(\frac{x}{2}\right)\sqrt{\cos^4\left(\frac{x}{2}\right)\left(5+\tan^2\left(\frac{x}{2}\right)\right)^2}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{5+\tan^2\left(\frac{x}{2}\right)}\\
&= \frac{3x\sec^2\left(\frac{x}{2}\right)\sqrt{\cos^4\left(\frac{x}{2}\right)\left(5+\tan^2\left(\frac{x}{2}\right)\right)^2}}{5+\tan^2\left(\frac{x}{2}\right)} + \frac{4\tan\left(\frac{x}{2}\right)\sqrt{\cos^4\left(\frac{x}{2}\right)\left(5+\tan^2\left(\frac{x}{2}\right)\right)^2}}{5+\tan^2\left(\frac{x}{2}\right)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 29, normalized size = 0.91

$$\frac{\sqrt{(3+2\cos(x))^2}(3x+2\sin(x))}{3+2\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(Cos[20*x] + 3*Cos[21*x] + Cos[22*x])^2 + (Sin[20*x] + 3*Sin
[21*x] + Sin[22*x])^2],x]
```

```
[Out] (Sqrt[(3 + 2*Cos[x])^2]*(3*x + 2*Sin[x]))/(3 + 2*Cos[x])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.60, size = 17, normalized size = 0.53

method	result	size
default	$\operatorname{csgn}(3+2\cos(x))\,(3x+2\sin(x))$	17
risch	$\frac{3\sqrt{(e^{2ix}+3\,e^{ix}+1)^2 e^{-2ix}}\,e^{ix}x}{e^{2ix}+3\,e^{ix}+1} - \frac{i\sqrt{(e^{2ix}+3\,e^{ix}+1)^2 e^{-2ix}}\,e^{2ix}}{e^{2ix}+3\,e^{ix}+1} + \frac{i\sqrt{(e^{2ix}+3\,e^{ix}+1)^2 e^{-2ix}}}{e^{2ix}+3\,e^{ix}+1}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((sin(20*x)+3*sin(21*x)+sin(22*x))^2+(cos(20*x)+3*cos(21*x)+cos(22*x))^
2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(3+2*cos(x))*(3*x+2*sin(x))
```

Maxima [**A**]
time = 0.39, size = 8, normalized size = 0.25

$$3\,x+2\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(20*x)+3*sin(21*x)+sin(22*x))^2+(cos(20*x)+3*cos(21*x)+cos(2
2*x))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3*x + 2*sin(x)
```

Fricas [A]
time = 0.64, size = 8, normalized size = 0.25

$$3x + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(20*x)+3*sin(21*x)+sin(22*x))^2+(cos(20*x)+3*cos(21*x)+cos(2
2*x))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3*x + 2*sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(\sin(20x) + 3\sin(21x) + \sin(22x))^2 + (\cos(20x) + 3\cos(21x) + \cos(22x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(20*x)+3*sin(21*x)+sin(22*x))**2+(cos(20*x)+3*cos(21*x)+cos(
22*x))**2)**(1/2),x)
```

```
[Out] Integral(sqrt((sin(20*x) + 3*sin(21*x) + sin(22*x))**2 + (cos(20*x) + 3*cos
(21*x) + cos(22*x))**2), x)
```

Giac [A]
time = 1.15, size = 32, normalized size = 1.00

$$-6\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor + 3x + \frac{4 \tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(20*x)+3*sin(21*x)+sin(22*x))^2+(cos(20*x)+3*cos(21*x)+cos(2
2*x))^2)^(1/2),x, algorithm="giac")
```

```
[Out] -6*pi*floor(1/2*x/pi + 1/2) + 3*x + 4*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 0.26, size = 8, normalized size = 0.25

$$3x + 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((cos(20*x) + 3*cos(21*x) + cos(22*x))^2 + (sin(20*x) + 3*sin(21*x) + s
in(22*x))^2)^(1/2),x)
```

```
[Out] 3*x + 2*sin(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(((sin(20*x)+3*sin(21*x)+sin(22*x))^2+(cos(20*x)+3*cos(21*x)+cos(22*x))^
2)^(1/2),x)
```

```
[Out] not solved
```

3.277 $\int \frac{e^{-2x}\sin(3x)}{x}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}i(\text{ExpIntegralEi}((-2-3i)x)-\text{ExpIntegralEi}((-2+3i)x))$$

```
[Out] 1/2*I*(Ei((-2-3*I)*x)-Ei((-2+3*I)*x))
```

Rubi [F]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{-2x}\sin(3x)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[3*x]/(E^(2*x)*x),x]
[Out] Defer[Int][Sin[3*x]/(E^(2*x)*x), x]
```

Rubi steps

$$\text{Integral} = \int \frac{e^{-2x}\sin(3x)}{x}\,dx$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 1.00

$$-\frac{1}{2}i(-\text{ExpIntegralEi}((-2-3i)x)+\text{ExpIntegralEi}((-2+3i)x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]/(E^(2*x)*x),x]
[Out] (-1/2*I)*(-ExpIntegralEi[(-2 - 3*I)*x] + ExpIntegralEi[(-2 + 3*I)*x])
```

Maple [A]
time = 0.08, size = 22, normalized size = 1.05

method	result	size
risch	$\frac{i\,\text{expIntegral}_1((2-3i)x)}{2}-\frac{i\,\text{expIntegral}_1((2+3i)x)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(-2*x)*sin(3*x)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*I*Ei(1,(2-3*I)*x)-1/2*I*Ei(1,(2+3*I)*x)

Maxima [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 27, normalized size = 1.29

$$-\frac{1}{4}i\,\mathrm{Ei}((3i-2)\;x)+\frac{1}{4}i\,\mathrm{Ei}(-(3i+2)\;x)+\frac{1}{4}i\,\overline{\mathrm{Ei}\left((3i-2)\;x\right)}-\frac{1}{4}i\,\overline{\mathrm{Ei}\left(-\left(3i+2\right)\;x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-2*x)*sin(3*x)/x,x, algorithm="maxima")

[Out] -1/4*I*Ei((3*I - 2)*x) + 1/4*I*Ei(-(3*I + 2)*x) + 1/4*I*conjugate(Ei((3*I - 2)*x)) - 1/4*I*conjugate(Ei(-(3*I + 2)*x))

Fricas [A]
time = 0.58, size = 13, normalized size = 0.62

$$-\frac{1}{2}i\,\mathrm{Ei}((3i-2)\;x)+\frac{1}{2}i\,\mathrm{Ei}(-(3i+2)\;x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-2*x)*sin(3*x)/x,x, algorithm="fricas")

[Out] -1/2*I*Ei((3*I - 2)*x) + 1/2*I*Ei(-(3*I + 2)*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{-2x}\sin\left(3x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-2*x)*sin(3*x)/x,x)

[Out] Integral(exp(-2*x)*sin(3*x)/x, x)

Giac [A]
time = 0.59, size = 13, normalized size = 0.62

$$-\frac{1}{2}i\,\mathrm{Ei}((3i-2)\;x)+\frac{1}{2}i\,\mathrm{Ei}(-(3i+2)\;x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-2*x)*sin(3*x)/x,x, algorithm="giac")
[Out] -1/2*I*Ei((3*I - 2)*x) + 1/2*I*Ei(-(3*I + 2)*x)
```

Mupad [**B**]
time = 0.03, size = 19, normalized size = 0.90

$$\frac{\mathrm{ei}(x\,(-2-3\mathrm{i}))\ 1\mathrm{i}}{2}-\frac{\mathrm{ei}(x\,(-2+3\mathrm{i}))\ 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*exp(-2*x))/x,x)
[Out] (ei(x*(- 2 - 3i))*1i)/2 - (ei(x*(- 2 + 3i))*1i)/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 32, normalized size = 1.52

$$\mathrm{expIntegral}\,(2-x)\sin\,(3x)+\frac{\mathrm{cosineIntegral}\,(2-3i-x)}{2}-\frac{\mathrm{cosineIntegral}\,(2+3i-x)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(-2*x)*sin(3*x)/x,x)
[Out] Ei(2-x)*sin(3*x)+1/2*Ci(2-3*I-x)-1/2*Ci(2+3*I-x)
```

3.278 $\int (1-x)^{2/3} \sqrt[3]{x}\, dx$

Optimal. Leaf size=102

$$\frac{1}{6}(1-x)^{2/3}\sqrt[3]{x}-\frac{1}{2}(1-x)^{5/3}\sqrt[3]{x}+\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{x}}\right)+\frac{\log(x)}{18}$$

```
[Out] 1/6*x^(1/3)*(1-x)^(2/3)-1/2*(1-x)^(5/3)*x^(1/3)-1/9*arctan(-1/3*3^(1/2)+2/3
*(1-x)^(1/3)*3^(1/2)/x^(1/3))*3^(1/2)+1/6*ln(1+(1-x)^(1/3)/x^(1/3))+1/18*ln
(x)
```

Rubi [A]
time = 0.01, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {52, 62}

$$\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x}}\right)}{3\sqrt{3}}-\frac{1}{2}\sqrt[3]{x}(1-x)^{5/3}+\frac{1}{6}\sqrt[3]{x}(1-x)^{2/3}+\frac{1}{6}\log\left(\frac{\sqrt[3]{1-x}}{\sqrt[3]{x}}+1\right)+\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(2/3)*x^(1/3),x]
```

```
[Out] ((1 - x)^(2/3)*x^(1/3))/6 - ((1 - x)^(5/3)*x^(1/3))/2 + ArcTan[1/Sqrt[3] -
(2*(1 - x)^(1/3))/(Sqrt[3]*x^(1/3))]/(3*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/x^
(1/3)]/6 + Log[x]/18
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 62

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\frac{1}{2}(1-x)^{5/3}\sqrt[3]{x}+\frac{1}{6}\int\frac{(1-x)^{2/3}}{x^{2/3}}\,dx\\
&=\frac{1}{6}(1-x)^{2/3}\sqrt[3]{x}-\frac{1}{2}(1-x)^{5/3}\sqrt[3]{x}+\frac{1}{9}\int\frac{1}{\sqrt[3]{1-x}x^{2/3}}\,dx\\
&=\frac{1}{6}(1-x)^{2/3}\sqrt[3]{x}-\frac{1}{2}(1-x)^{5/3}\sqrt[3]{x}+\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{x}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1+\frac{\sqrt[3]{1-x}}{\sqrt[3]{x}}\right)+\frac{\log(x)}{18}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 114, normalized size = 1.12

$$\frac{1}{18}\left(3(1-x)^{2/3}\sqrt[3]{x}(-2+3x)+2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{x}}{2\sqrt[3]{1-x}-\sqrt[3]{x}}\right)+2\log\left(\sqrt[3]{1-x}+\sqrt[3]{x}\right)-\log\left((1-x)^{2/3}+x^{2/3}-\sqrt[3]{-((-1+x)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(2/3)*x^(1/3),x]
```

```
[Out] (3*(1 - x)^(2/3)*x^(1/3)*(-2 + 3*x) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*x^(1/3))/(2
*(1 - x)^(1/3) - x^(1/3))] + 2*Log[(1 - x)^(1/3) + x^(1/3)] - Log[(1 - x)^(
2/3) + x^(2/3) - (-((-1 + x)*x))^(1/3)])/18
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.11, size = 13, normalized size = 0.13

method	result	size
meijerg	$\frac{3x^{\frac{4}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{4}{3};\frac{7}{3};x\right)}{4}$	13
risch	$-\frac{(-2+3x)(x-1)x^{\frac{1}{3}}\left(x^2(1-x)\right)^{\frac{1}{3}}}{6\left(-x^2(x-1)\right)^{\frac{1}{3}}(1-x)^{\frac{1}{3}}}+\frac{{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};x\right)\left(x^2(1-x)\right)^{\frac{1}{3}}}{3x^{\frac{1}{3}}(1-x)^{\frac{1}{3}}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(1-x)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^(4/3)*hypergeom([-2/3,4/3],[7/3],x)
```

Maxima [A]
time = 0.48, size = 117, normalized size = 1.15

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x+1)^{\frac{1}{3}}}{x^{\frac{1}{3}}}-1\right)\right)+\frac{\frac{(-x+1)^{\frac{2}{3}}}{x^{\frac{2}{3}}}-\frac{2(-x+1)^{\frac{5}{3}}}{x^{\frac{5}{3}}}}{6\left(\frac{(x-1)^2}{x^2}-\frac{2(x-1)}{x}+1\right)}+\frac{1}{9}\log\left(\frac{(-x+1)^{\frac{1}{3}}}{x^{\frac{1}{3}}}+1\right)-\frac{1}{18}\log\left(-\frac{(-x+1)^{\frac{1}{3}}}{x^{\frac{1}{3}}}+\frac{(-x+1)^{\frac{2}{3}}}{x^{\frac{2}{3}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(1-x)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x + 1)^(1/3)/x^(1/3) - 1)) + 1/6*((-x
+ 1)^(2/3)/x^(2/3) - 2*(-x + 1)^(5/3)/x^(5/3))/((x - 1)^2/x^2 - 2*(x - 1)/x
 + 1) + 1/9*log((-x + 1)^(1/3)/x^(1/3) + 1) - 1/18*log(-(-x + 1)^(1/3)/x^(1
/3) + (-x + 1)^(2/3)/x^(2/3) + 1)
```

Fricas [A]
time = 0.60, size = 114, normalized size = 1.12

$$\frac{1}{6}\,(3\,x-2)x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}-\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-1)+2\,\sqrt{3}x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}}{3\,(x-1)}\right)-\frac{1}{18}\,\log\left(\frac{x-x^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}+x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}-1}{x-1}\right)+\frac{1}{9}\,\log\left(-\frac{x-x^{\frac{1}{3}}(-x+1)^{\frac{2}{3}}-1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(1-x)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x - 2)*x^(1/3)*(-x + 1)^(2/3) - 1/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x -
 1) + 2*sqrt(3)*x^(1/3)*(-x + 1)^(2/3))/(x - 1)) - 1/18*log((x - x^(2/3)*(-
x + 1)^(1/3) + x^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 1/9*log(-(x - x^(1/3)
*(-x + 1)^(2/3) - 1)/(x - 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.92, size = 31, normalized size = 0.30

$$\frac{x^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\,xe^{2i\pi}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(1-x)**(2/3),x)
```

```
[Out] x**(4/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), x*exp_polar(2*I*pi))/gamma(7
/3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(1-x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^(1/3)*(-x + 1)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{1/3}\,{\left(1-x\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(1 - x)^(2/3),x)
```

```
[Out] int(x^(1/3)*(1 - x)^(2/3), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 14, normalized size = 0.14

$$\frac{x^{\frac{2}{3}}}{2} - \frac{x}{3} + \frac{x^{\frac{4}{3}}}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(1/3)*(1-x)^(2/3),x)
```

```
[Out] 1/2*x^(2/3)-1/3*x+1/4*x^(4/3)
```

3.279 $\int e\,dx$

Optimal. Leaf size=3

$$ex$$

```
[Out] exp(1)*x
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$ex$$

Antiderivative was successfully verified.

```
[In] Int[E,x]
```

```
[Out] E*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\text{Integral} = ex$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$ex$$

Antiderivative was successfully verified.

```
[In] Integrate[E,x]
```

```
[Out] E*x
```

Maple [A]
time = 0.01, size = 9, normalized size = 3.00

method	result	size
parallelrisch	$x^{\frac{1}{\ln(x)}}x$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/ln(x))*x
```

Maxima [A]
time = 0.35, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/log(x)),x, algorithm="maxima")
```

```
[Out] x*e
```

Fricas [A]
time = 0.55, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/log(x)),x, algorithm="fricas")
```

```
[Out] x*e
```

Sympy [A]
time = 0.00, size = 3, normalized size = 1.00

$$ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/ln(x)),x)
```

```
[Out] E*x
```

Giac [A]
time = 0.43, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/log(x)),x, algorithm="giac")
```

```
[Out] x*e
```

Mupad [**B**]
time = 0.00, size = 4, normalized size = 1.33

$$x\,\mathrm{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/log(x)),x)
```

```
[Out] x*exp(1)
```

Chatgpt [**A**] valid for real x
time = 1.00, size = 4, normalized size = 1.33

$$\mathrm{e}x$$

Antiderivative was successfully verified.

```
[In] int(x^(1/ln(x)),x)
```

```
[Out] exp(1)*x
```

3.280 $\int \mathbf{sech}(x)\,dx$

Optimal. Leaf size=3

$$\arctan(\sinh(x))$$

[Out] arctan(sinh(x))

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\arctan(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Sech[x],x]

[Out] ArcTan[Sinh[x]]

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{Integral} = \arctan(\sinh(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 9 vs. 2(3) = 6.
time = 0.01, size = 9, normalized size = 3.00

$$2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x],x]

[Out] 2*ArcTan[Tanh[x/2]]

Maple [A]
time = 0.05, size = 4, normalized size = 1.33

method	result	size

lookup	$\arctan(\sinh(x))$	4
default	$\arctan(\sinh(x))$	4
risch	$i\ln(e^x+i)-i\ln(e^x-i)$	20
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sinh(x))
```

Maxima [A]
time = 0.32, size = 3, normalized size = 1.00

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.
time = 0.56, size = 8, normalized size = 2.67

$$2\arctan(\cosh(x)+\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. $2(3) = 6$.
time = 0.18, size = 7, normalized size = 2.33

$$2\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x)
```

```
[Out] 2*atan(tanh(x/2))
```

Giac [A]
time = 0.41, size = 5, normalized size = 1.67

$$2\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

Mupad [**B**]
time = 0.00, size = 5, normalized size = 1.67

$$2\,\mathrm{atan}(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x),x)
```

```
[Out] 2*atan(exp(x))
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 5, normalized size = 1.67

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(sech(x),x)
```

```
[Out] ln(tanh(1/2*x))
```

3.281 $\int \frac{e^x}{(1+e^x)\log(1+e^x)}\,dx$

Optimal. Leaf size=7

$$\log(\log(1+e^x))$$

[Out] ln(ln(1+exp(x)))

Rubi [A]
time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2320, 2437, 2339, 29}

$$\log(\log(e^x+1))$$

Antiderivative was successfully verified.

[In] Int[E^x/((1 + E^x)*Log[1 + E^x]),x]

[Out] Log[Log[1 + E^x]]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \text{Subst}\left(\int \frac{1}{(1+x)\log(1+x)}\,dx, x, e^x\right)\\ &= \text{Subst}\left(\int \frac{1}{x\log(x)}\,dx, x, 1+e^x\right)\\ &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log\left(1+e^x\right)\right)\\ &= \log\left(\log\left(1+e^x\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$\log\left(\log\left(1+e^x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/((1 + E^x)*Log[1 + E^x]),x]
```

```
[Out] Log[Log[1 + E^x]]
```

Maple [A]
time = 0.03, size = 7, normalized size = 1.00

method	result	size
derivativedivides	$\ln\left(\ln\left(1+\mathrm{e}^x\right)\right)$	7
default	$\ln\left(\ln\left(1+\mathrm{e}^x\right)\right)$	7
norman	$\ln\left(\ln\left(1+\mathrm{e}^x\right)\right)$	7
risch	$\ln\left(\ln\left(1+\mathrm{e}^x\right)\right)$	7
parallelrisch	$\ln\left(\ln\left(1+\mathrm{e}^x\right)\right)$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(x))/ln(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(ln(1+exp(x)))
```

Maxima [A]
time = 0.33, size = 6, normalized size = 0.86

$$\log\left(\log\left(e^x+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x))/log(1+exp(x)),x, algorithm="maxima")
```

```
[Out] log(log(e^x + 1))
```

Fricas [**A**]
time = 0.56, size = 6, normalized size = 0.86

$$\log(\log(e^x + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x))/log(1+exp(x)),x, algorithm="fricas")
[Out] log(log(e^x + 1))
```

Sympy [**A**]
time = 0.05, size = 7, normalized size = 1.00

$$\log(\log(e^x + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x))/ln(1+exp(x)),x)
[Out] log(log(exp(x) + 1))
```

Giac [**A**]
time = 0.44, size = 6, normalized size = 0.86

$$\log(\log(e^x + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x))/log(1+exp(x)),x, algorithm="giac")
[Out] log(log(e^x + 1))
```

Mupad [**B**]
time = 0.14, size = 6, normalized size = 0.86

$$\ln(\ln(\mathrm{e}^x + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(log(exp(x) + 1)*(exp(x) + 1)),x)
[Out] log(log(exp(x) + 1))
```

Chatgpt [**A**]
time = 1.00, size = 6, normalized size = 0.86

$$\ln(\ln(\mathrm{e}^x + 1))$$

Antiderivative was successfully verified.

```
[In] int(exp(x)/(exp(x)+1)/ln(exp(x)+1),x)
[Out] ln(ln(exp(x)+1))
```

3.282 $\int (1-x+x^2-x^3+x^4) (1+x+x^2+x^3+x^4) \, dx$

Optimal. Leaf size=30

$$x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\frac{x^9}{9}$$

```
[Out] x+1/3*x^3+1/5*x^5+1/7*x^7+1/9*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {6820}

$$\frac{x^9}{9}+\frac{x^7}{7}+\frac{x^5}{5}+\frac{x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + x^2 - x^3 + x^4)*(1 + x + x^2 + x^3 + x^4),x]
[Out] x + x^3/3 + x^5/5 + x^7/7 + x^9/9
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \int \left(1+x^2+x^4+x^6+x^8\right) \, dx \\ &= x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\frac{x^9}{9} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\frac{x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + x^2 - x^3 + x^4)*(1 + x + x^2 + x^3 + x^4),x]
[Out] x + x^3/3 + x^5/5 + x^7/7 + x^9/9
```

Maple [A]
time = 0.05, size = 23, normalized size = 0.77

method	result	size
gosper	$x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7+\frac{1}{9}x^9$	23
default	$x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7+\frac{1}{9}x^9$	23
norman	$x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7+\frac{1}{9}x^9$	23
risch	$x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7+\frac{1}{9}x^9$	23
parallelrisch	$x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7+\frac{1}{9}x^9$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3+x^2+x+1)*(x^4-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/3*x^3+1/5*x^5+1/7*x^7+1/9*x^9
```

Maxima [**A**]
time = 0.34, size = 22, normalized size = 0.73

$$\frac{1}{9}\,x^{9}+\frac{1}{7}\,x^{7}+\frac{1}{5}\,x^{5}+\frac{1}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)*(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/9*x^9 + 1/7*x^7 + 1/5*x^5 + 1/3*x^3 + x
```

Fricas [**A**]
time = 0.55, size = 22, normalized size = 0.73

$$\frac{1}{9}\,x^{9}+\frac{1}{7}\,x^{7}+\frac{1}{5}\,x^{5}+\frac{1}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)*(x^4-x^3+x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/9*x^9 + 1/7*x^7 + 1/5*x^5 + 1/3*x^3 + x
```

Sympy [**A**]
time = 0.01, size = 20, normalized size = 0.67

$$\frac{x^{9}}{9}+\frac{x^{7}}{7}+\frac{x^{5}}{5}+\frac{x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3+x**2+x+1)*(x**4-x**3+x**2-x+1),x)
```

```
[Out] x**9/9 + x**7/7 + x**5/5 + x**3/3 + x
```

Giac [**A**]
time = 0.48, size = 22, normalized size = 0.73

$$\frac{1}{9}\,x^{9}+\frac{1}{7}\,x^{7}+\frac{1}{5}\,x^{5}+\frac{1}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3+x^2+x+1)*(x^4-x^3+x^2-x+1),x, algorithm="giac")
[Out] 1/9*x^9 + 1/7*x^7 + 1/5*x^5 + 1/3*x^3 + x
```

Mupad [**B**]
time = 0.02, size = 22, normalized size = 0.73

$$\frac{x^9}{9}+\frac{x^7}{7}+\frac{x^5}{5}+\frac{x^3}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x - x^3 + x^4 + 1)*(x + x^2 + x^3 + x^4 + 1),x)
[Out] x + x^3/3 + x^5/5 + x^7/7 + x^9/9
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 45, normalized size = 1.50

$$\frac{x^9}{9}+\frac{x^7}{9}+\frac{2x^6}{9}-\frac{x^5}{9}+\frac{2x^4}{9}+\frac{x^3}{9}+\frac{x}{9}+\frac{5\ln\left(x^2-x+1\right)}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int((x^4+x^3+x^2+x+1)*(x^4-x^3+x^2-x+1),x)
[Out] 1/9*x^9+1/9*x^7+2/9*x^6-1/9*x^5+2/9*x^4+1/9*x^3+1/9*x+5/3*ln(x^2-x+1)
```

3.283 $\int \frac{1}{120}(-4+x)(-3+x)(-2+x)(-1+x)x\,dx$

Optimal. Leaf size=36

$$\frac{x^2}{10}-\frac{5x^3}{36}+\frac{7x^4}{96}-\frac{x^5}{60}+\frac{x^6}{720}$$

```
[Out] 1/10*x^2-5/36*x^3+7/96*x^4-1/60*x^5+1/720*x^6
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {12, 1620}

$$\frac{x^6}{720}-\frac{x^5}{60}+\frac{7x^4}{96}-\frac{5x^3}{36}+\frac{x^2}{10}$$

Antiderivative was successfully verified.

```
[In] Int[((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)*x)/120,x]
```

```
[Out] x^2/10 - (5*x^3)/36 + (7*x^4)/96 - x^5/60 + x^6/720
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegran
d[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, m, n, p, q}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{1}{120}\int(-4+x)(-3+x)(-2+x)(-1+x)x\,dx\\ &= \frac{1}{120}\int\left(24x-50x^2+35x^3-10x^4+x^5\right)\,dx\\ &= \frac{x^2}{10}-\frac{5x^3}{36}+\frac{7x^4}{96}-\frac{x^5}{60}+\frac{x^6}{720}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 1.00

$$\frac{1}{120}\left(12x^2-\frac{50x^3}{3}+\frac{35x^4}{4}-2x^5+\frac{x^6}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)*x)/120,x]
[Out] (12*x^2 - (50*x^3)/3 + (35*x^4)/4 - 2*x^5 + x^6/6)/120
```

Maple [A]
time = 0.01, size = 27, normalized size = 0.75

method	result	size
gosper	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
default	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
norman	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
risch	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27
parallelrisch	$\frac{1}{10}x^2-\frac{5}{36}x^3+\frac{7}{96}x^4-\frac{1}{60}x^5+\frac{1}{720}x^6$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/120*(-4+x)*(-3+x)*(x-2)*(x-1)*x,x,method=_RETURNVERBOSE)
[Out] 1/10*x^2-5/36*x^3+7/96*x^4-1/60*x^5+1/720*x^6
```

Maxima [A]
time = 0.33, size = 26, normalized size = 0.72

$$\frac{1}{720}\,x^6-\frac{1}{60}\,x^5+\frac{7}{96}\,x^4-\frac{5}{36}\,x^3+\frac{1}{10}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/120*(-4+x)*(-3+x)*(x-2)*(x-1)*x,x, algorithm="maxima")
[Out] 1/720*x^6 - 1/60*x^5 + 7/96*x^4 - 5/36*x^3 + 1/10*x^2
```

Fricas [A]
time = 0.56, size = 26, normalized size = 0.72

$$\frac{1}{720}\,x^6-\frac{1}{60}\,x^5+\frac{7}{96}\,x^4-\frac{5}{36}\,x^3+\frac{1}{10}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/120*(-4+x)*(-3+x)*(x-2)*(x-1)*x,x, algorithm="fricas")
[Out] 1/720*x^6 - 1/60*x^5 + 7/96*x^4 - 5/36*x^3 + 1/10*x^2
```

Sympy [A]
time = 0.01, size = 27, normalized size = 0.75

$$\frac{x^6}{720}-\frac{x^5}{60}+\frac{7x^4}{96}-\frac{5x^3}{36}+\frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/120*(-4+x)*(-3+x)*(x-2)*(x-1)*x,x)
```

```
[Out] x**6/720 - x**5/60 + 7*x**4/96 - 5*x**3/36 + x**2/10
```

Giac [A]
time = 0.44, size = 23, normalized size = 0.64

$$\frac{1}{720}\left(x^{2}-4\,x\right)^{3}+\frac{1}{160}\left(x^{2}-4\,x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/120*(-4+x)*(-3+x)*(x-2)*(x-1)*x,x, algorithm="giac")
```

```
[Out] 1/720*(x^2 - 4*x)^3 + 1/160*(x^2 - 4*x)^2
```

Mupad [B]
time = 0.02, size = 26, normalized size = 0.72

$$\frac{x^{6}}{720}-\frac{x^{5}}{60}+\frac{7\,x^{4}}{96}-\frac{5\,x^{3}}{36}+\frac{x^{2}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x - 1)*(x - 2)*(x - 3)*(x - 4))/120,x)
```

```
[Out] x^2/10 - (5*x^3)/36 + (7*x^4)/96 - x^5/60 + x^6/720
```

Chatgpt [F] Failed to verify
time = 1.00, size = 26, normalized size = 0.72

$$-\frac{1}{720}x^{6}+\frac{2}{3}x^{5}-\frac{25}{24}x^{4}+\frac{17}{3}x^{3}-\frac{8}{3}x^{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/120*(-4+x)*(x-3)*(x-2)*(x-1)*x,x)
```

```
[Out] -1/720*x^6+2/3*x^5-25/24*x^4+17/3*x^3-8/3*x^2
```

3.284 $\int (x+x\cos(x)+\sin(x)+\cos(x)\sin(x))\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}(x+\sin(x))^2$$

```
[Out] 1/2*(x+sin(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 2.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3377, 2718, 2644, 30}

$$\frac{x^2}{2}+\frac{\sin^2(x)}{2}+x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[x + x*Cos[x] + Sin[x] + Cos[x]*Sin[x],x]
```

```
[Out] x^2/2 + x*Sin[x] + Sin[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \frac{x^2}{2} + \int x\cos(x)\,dx + \int \sin(x)\,dx + \int \cos(x)\sin(x)\,dx \\ &= \frac{x^2}{2} - \cos(x) + x\sin(x) - \int \sin(x)\,dx + \text{Subst}\left(\int x\,dx, x, \sin(x)\right) \\ &= \frac{x^2}{2} + x\sin(x) + \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 2.00

$$\frac{x^2}{2} - \frac{\cos^2(x)}{2} + x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x + x*Cos[x] + Sin[x] + Cos[x]*Sin[x],x]
[Out] x^2/2 - Cos[x]^2/2 + x*Sin[x]
```

Maple [A]
time = 0.11, size = 17, normalized size = 1.70

method	result	size
default	$\frac{x^2}{2} + x\sin(x) + \frac{(\sin^2(x))}{2}$	17
risch	$\frac{x^2}{2} + x\sin(x) - \frac{\cos(2x)}{4}$	17
parts	$\frac{x^2}{2} - \frac{(\cos^2(x))}{2} + x\sin(x)$	17
norman	$\frac{x^2(\tan^2(\frac{x}{2})) + 2(\tan^2(\frac{x}{2})) + \frac{x^2}{2} + 2x\tan(\frac{x}{2}) + 2x(\tan^3(\frac{x}{2})) + \frac{x^2(\tan^4(\frac{x}{2}))}{2}}{(1+\tan^2(\frac{x}{2}))^2}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+sin(x)+x*cos(x)+cos(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+x*sin(x)+1/2*sin(x)^2
```

Maxima [A]
time = 0.33, size = 16, normalized size = 1.60

$$\frac{1}{2}x^2 - \frac{1}{2}\cos(x)^2 + x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+sin(x)+x*cos(x)+cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*cos(x)^2 + x*sin(x)
```

Fricas [A]
time = 0.58, size = 16, normalized size = 1.60

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,\cos\left(x\right)^2 + x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+sin(x)+x*cos(x)+cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*cos(x)^2 + x*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.
time = 0.06, size = 15, normalized size = 1.50

$$\frac{x^2}{2} + x\sin\left(x\right) + \frac{\sin^2\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+sin(x)+x*cos(x)+cos(x)*sin(x),x)
```

```
[Out] x**2/2 + x*sin(x) + sin(x)**2/2
```

Giac [A]
time = 0.56, size = 16, normalized size = 1.60

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,\cos\left(x\right)^2 + x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+sin(x)+x*cos(x)+cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*cos(x)^2 + x*sin(x)
```

Mupad [B]
time = 0.06, size = 8, normalized size = 0.80

$$\frac{\left(x+\sin\left(x\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + sin(x) + cos(x)*sin(x) + x*cos(x),x)
```

```
[Out] (x + sin(x))^2/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 14, normalized size = 1.40

$$\frac{x^2}{2} - \cos(x) + x\sin(x)$$

Warning: Unable to verify antiderivative.

```
[In] int(x+sin(x)+x*cos(x)+cos(x)*sin(x),x)
```

```
[Out] 1/2*x^2-cos(x)+x*sin(x)
```

3.285 $\int (\cos^2(x)+\cot^2(x)+\csc^2(x)+\sec^2(x)+\sin^2(x)+\text{t}$

Optimal. Leaf size=12

$$-x-2\cot(x)+2\tan(x)$$

[Out] -x-2*cot(x)+2*tan(x)

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2715, 8, 3554, 3852}

$$-x+2\tan(x)-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2 + Cot[x]^2 + Csc[x]^2 + Sec[x]^2 + Sin[x]^2 + Tan[x]^2,x]
[Out] -x - 2*Cot[x] + 2*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \cos^2(x)\,dx + \int \cot^2(x)\,dx + \int \csc^2(x)\,dx + \int \sec^2(x)\,dx + \int \sin^2(x)\,dx + \int \tan^2(x)\,dx \\
&= -\cot(x)+\tan(x)+2\frac{\int 1\,dx}{2} - 2\int 1\,dx - \text{Subst}(\int 1\,dx, x, \cot(x)) - \text{Subst}(\int 1\,dx, x, -\tan(x)) \\
&= -x-2\cot(x)+2\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 12, normalized size = 1.00

$$-x-2\cot(x)+2\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2 + Cot[x]^2 + Csc[x]^2 + Sec[x]^2 + Sin[x]^2 + Tan[x]^2,x
]
```

```
[Out] -x - 2*Cot[x] + 2*Tan[x]
```

Maple [A]
time = 0.91, size = 24, normalized size = 2.00

method	result	size
parallelrisch	$-x-4\cot\left(x\right)+2\csc\left(x\right)\sec\left(x\right)$	15
default	$x-2\cot\left(x\right)+\frac{\pi}{2}-\operatorname{arccot}(\cot\left(x\right))+2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	24
parts	$x-2\cot\left(x\right)+\frac{\pi}{2}-\operatorname{arccot}(\cot\left(x\right))+2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	24
risch	$\frac{4i}{e^{2ix}+1}-x-\frac{4i}{e^{2ix}-1}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2+cos(x)^2+tan(x)^2+cot(x)^2+sec(x)^2+csc(x)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] x-2*cot(x)+1/2*Pi-arccot(cot(x))+2*tan(x)-arctan(tan(x))
```

Maxima [A]
time = 0.46, size = 14, normalized size = 1.17

$$-x-\frac{2}{\tan\left(x\right)}+2\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2+cos(x)^2+tan(x)^2+cot(x)^2+sec(x)^2+csc(x)^2,x, algorith
m="maxima")
```

```
[Out] -x - 2/tan(x) + 2*tan(x)
```

Fricas [A]
time = 0.60, size = 24, normalized size = 2.00

$$-\frac{x\cos\left(x\right)\sin\left(x\right)+4\,\cos\left(x\right)^{2}-2}{\cos\left(x\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2+cos(x)^2+tan(x)^2+cot(x)^2+sec(x)^2+csc(x)^2,x, algorith
m="fricas")
```

```
[Out] -(x*cos(x)*sin(x) + 4*cos(x)^2 - 2)/(cos(x)*sin(x))
```

Sympy [A]
time = 0.02, size = 17, normalized size = 1.42

$$-x+\frac{2\sin(x)}{\cos(x)}-\frac{2\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2+cos(x)**2+tan(x)**2+cot(x)**2+sec(x)**2+csc(x)**2,x)
```

```
[Out] -x + 2*sin(x)/cos(x) - 2*cos(x)/sin(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 0.42, size = 28, normalized size = 2.33

$$-x-\frac{1}{2\tan\left(\frac{1}{2}x\right)}-\frac{1}{\tan(x)}+\frac{1}{2}\tan\left(\frac{1}{2}x\right)+2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2+cos(x)^2+tan(x)^2+cot(x)^2+sec(x)^2+csc(x)^2,x, algorith
m="giac")
```

```
[Out] -x - 1/2/tan(1/2*x) - 1/tan(x) + 1/2*tan(1/2*x) + 2*tan(x)
```

Mupad [B]
time = 0.19, size = 14, normalized size = 1.17

$$2\tan(x)-x-\frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2 + cos(x)^2 + cot(x)^2 + 1/sin(x)^2 + sin(x)^2 + tan(x)^2,x)
```

```
[Out] 2*tan(x) - x - 2/tan(x)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 0.75

$$-\cot(x)-\tan(x)$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)^2+cos(x)^2+tan(x)^2+cot(x)^2+sec(x)^2+csc(x)^2,x)
```

```
[Out] -cot(x)-tan(x)
```

3.286 $\int e^{\log^2(x)}(1+2\log(x))\,dx$

Optimal. Leaf size=8

$$e^{\log^2(x)}x$$

```
[Out] x*exp(ln(x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2326}

$$xe^{\log^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^Log[x]^2*(1 + 2*Log[x]),x]
```

```
[Out] E^Log[x]^2*x
```

Rule 2326

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u,
 x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\text{Integral} = e^{\log^2(x)}x$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$e^{\log^2(x)}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Log[x]^2*(1 + 2*Log[x]),x]
```

```
[Out] E^Log[x]^2*x
```

Maple [A]
time = 0.03, size = 8, normalized size = 1.00

method	result	size

norman	$x\,\mathrm{e}^{\ln(x)^2}$	8
risch	$x\,\mathrm{e}^{\ln(x)^2}$	8
parallelrisch	$x\,\mathrm{e}^{\ln(x)^2}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*ln(x))*exp(ln(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*exp(ln(x)^2)
```

Maxima [A]
time = 0.39, size = 7, normalized size = 0.88

$$xe^{\left(\log(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*log(x))*exp(log(x)^2),x, algorithm="maxima")
```

```
[Out] x*e^(log(x)^2)
```

Fricas [A]
time = 0.56, size = 7, normalized size = 0.88

$$xe^{\left(\log(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*log(x))*exp(log(x)^2),x, algorithm="fricas")
```

```
[Out] x*e^(log(x)^2)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.88

$$xe^{\log(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*ln(x))*exp(ln(x)**2),x)
```

```
[Out] x*exp(log(x)**2)
```

Giac [A]
time = 0.40, size = 7, normalized size = 0.88

$$xe^{\left(\log(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*log(x))*exp(log(x)^2),x, algorithm="giac")
```

```
[Out] x*e^(log(x)^2)
```

Mupad [**B**]
time = 0.25, size = 7, normalized size = 0.88

$$x\,\mathrm{e}^{\ln(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(log(x)^2)*(2*log(x) + 1),x)
```

```
[Out] x*exp(log(x)^2)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 19, normalized size = 2.38

$$\frac{\sqrt{\pi}\,\operatorname{erf}\left(\ln\left(x\right)\right)}{2}+\frac{x^2\mathrm{e}^{\ln(x)^2}}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int((1+2*ln(x))*exp(ln(x)^2),x)
```

```
[Out] 1/2*Pi^(1/2)*erf(ln(x))+1/4*x^2*exp(ln(x)^2)
```

3.287 $\int \left((1-x)^3+(x-x^2)^3-3(1-x)(x-x^2)(-1+x\right.$

Optimal. Leaf size=1

$$0$$

[Out] 0

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 3 vs. $2(1) = 2$.
time = 0.03, antiderivative size = 3, normalized size of antiderivative = 3.00, number of steps used = 8, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {625, 1607, 786, 200}

$$-\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^3 + (x - x^2)^3 - 3*(1 - x)*(x - x^2)*(-1 + x^2) + (-1 + x^2)^3
,x]
```

```
[Out] -1/4
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= -\frac{1}{4}(1-x)^4 - 3\int (1-x)\left(x-x^2\right)\left(-1+x^2\right)\,dx + \int \left(x-x^2\right)^3\,dx + \int \left(-1+x^2\right)^3\,dx \\
&= -\frac{1}{4}(1-x)^4 - 3\int (1-x)^2 x\left(-1+x^2\right)\,dx + \int \left(x^3-3x^4+3x^5-x^6\right)\,dx + \int \left(-1+3x^2-3x^4+x^6\right)\,dx \\
&= -\frac{1}{4}(1-x)^4 - x + x^3 + \frac{x^4}{4} - \frac{6x^5}{5} + \frac{x^6}{2} - 3\int \left(-x+2x^2-2x^4+x^5\right)\,dx \\
&= -\frac{1}{4}(1-x)^4 - x + \frac{3x^2}{2} - x^3 + \frac{x^4}{4}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$0$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^3 + (x - x^2)^3 - 3*(1 - x)*(x - x^2)*(-1 + x^2) + (-1 +
x^2)^3,x]
```

```
[Out] 0
```

Maple [A]
time = 0.10, size = 2, normalized size = 2.00

method	result	size
default	0	2
norman	0	2
meijerg	0	2
risch	$-\frac{1}{4}$	2
parallelrisch	0	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^3+(-x^2+x)^3+(x^2-1)^3-3*(1-x)*(-x^2+x)*(x^2-1),x,method=_RETURNV
ERBOSE)
```

```
[Out] 0
```

Maxima [A]
time = 0.41, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^3+(-x^2+x)^3+(x^2-1)^3-3*(1-x)*(-x^2+x)*(x^2-1),x, algorith
m="maxima")
```

[Out] 0

Fricas [A]
time = 0.57, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^3+(-x^2+x)^3+(x^2-1)^3-3*(1-x)*(-x^2+x)*(x^2-1),x, algorith
m="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 0.01, size = 0, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**3+(-x**2+x)**3+(x**2-1)**3-3*(1-x)*(-x**2+x)*(x**2-1),x)
```

```
[Out] 0
```

Giac [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.47, size = 26, normalized size = 26.00

$$-\frac{1}{4}\,(x-1)^4+\frac{1}{4}\,x^4-x^3+\frac{3}{2}\,x^2-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^3+(-x^2+x)^3+(x^2-1)^3-3*(1-x)*(-x^2+x)*(x^2-1),x, algorith
m="giac")
```

```
[Out] -1/4*(x - 1)^4 + 1/4*x^4 - x^3 + 3/2*x^2 - x
```

Mupad [B]
time = 0.00, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)^3 - (x - 1)^3 + (x^2 - 1)^3 + 3*(x - x^2)*(x^2 - 1)*(x - 1),x
)
```

```
[Out] 0
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 41, normalized size = 41.00

$$\frac{(x-1)^4}{4}-\frac{x^4}{4}-\frac{x^3}{3}+\frac{(x-1)^2\,(-x^2+x)}{2}+\frac{(x^2-1)^2}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int((1-x)^3+(-x^2+x)^3+(x^2-1)^3-3*(1-x)*(-x^2+x)*(x^2-1),x)
```

```
[Out] 1/4*(x-1)^4-1/4*x^4-1/3*x^3+1/2*(x-1)^2*(-x^2+x)+1/2*(x^2-1)^2
```

3.288 $\int \left(\cos^6(x)+3\cos^2(x)\sin^2(x)+\sin^6(x)\right)\,dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [C] Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 50.00, number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2715, 8, 2648}

$$x+\frac{1}{6}\sin(x)\cos^5(x)-\frac{13}{24}\sin(x)\cos^3(x)-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)+\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6 + 3*Cos[x]^2*Sin[x]^2 + Sin[x]^6,x]
```

```
[Out] x + (3*Cos[x]*Sin[x])/8 - (13*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x])/6 - (
5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= 3\int \cos^2(x)\sin^2(x)\,dx + \int \cos^6(x)\,dx + \int \sin^6(x)\,dx \\
&= -\frac{3}{4}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{3}{4}\int \cos^2(x)\,dx + \frac{5}{6}\int \cos^4(x)\,dx + \frac{5}{6}\int \sin^4(x)\,dx \\
&= \frac{3}{8}\cos(x)\sin(x) - \frac{13}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{3\int 1\,dx}{8} + \frac{5}{8}\int \cos^2(x)\,dx + \frac{5}{8}\int \sin^2(x)\,dx \\
&= \frac{3x}{8} + \frac{3}{8}\cos(x)\sin(x) - \frac{13}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + 2\frac{5\int 1\,dx}{16} \\
&= x + \frac{3}{8}\cos(x)\sin(x) - \frac{13}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6 + 3*Cos[x]^2*Sin[x]^2 + Sin[x]^6,x]
[Out] x
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.16, size = 55, normalized size = 55.00

method	result
risch	x
default	$\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+x-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}-\frac{3\left(\cos^3(x)\right)\sin(x)}{4}+\frac{3\cos(x)\sin(x)}{8}$
parts	$\frac{\left(\cos^5(x)+\frac{5\left(\cos^3(x)\right)}{4}+\frac{15\cos(x)}{8}\right)\sin(x)}{6}+x-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}-\frac{3\left(\cos^3(x)\right)\sin(x)}{4}+\frac{3\cos(x)\sin(x)}{8}$
norman	$\frac{x+x\left(\tan^{12}\left(\frac{x}{2}\right)\right)+6x\left(\tan^2\left(\frac{x}{2}\right)\right)+15x\left(\tan^4\left(\frac{x}{2}\right)\right)+20x\left(\tan^6\left(\frac{x}{2}\right)\right)+15x\left(\tan^8\left(\frac{x}{2}\right)\right)+6\left(\tan^{10}\left(\frac{x}{2}\right)\right)x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6+cos(x)^6+3*sin(x)^2*cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+x-1/6*(sin(x)^5+5/4*sin(x)^3
+15/8*sin(x))*cos(x)-3/4*cos(x)^3*sin(x)+3/8*cos(x)*sin(x)
```

Maxima [A]
time = 0.40, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6+cos(x)^6+3*sin(x)^2*cos(x)^2,x, algorithm="maxima")
```

[Out] x

Fricas [A]
time = 0.55, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6+cos(x)^6+3*sin(x)^2*cos(x)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(0) = 0$.
time = 0.02, size = 58, normalized size = 58.00

$$x-\frac{\sin^5(x)\cos(x)}{6}-\frac{5\sin^3(x)\cos(x)}{24}+\frac{\sin(x)\cos^5(x)}{6}+\frac{5\sin(x)\cos^3(x)}{24}-\frac{3\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6+cos(x)**6+3*sin(x)**2*cos(x)**2,x)
```

```
[Out] x - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 + sin(x)*cos(x)**5/6 + 5*sin
(x)*cos(x)**3/24 - 3*sin(2*x)*cos(2*x)/16
```

Giac [A]
time = 0.45, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6+cos(x)^6+3*sin(x)^2*cos(x)^2,x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 0.15, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6 + sin(x)^6 + 3*cos(x)^2*sin(x)^2,x)
```

```
[Out] x
```

Chatgpt [F] Failed to verify
time = 1.00, size = 35, normalized size = 35.00

$$\frac{x}{8}-\frac{\cos(6x)}{64}-\frac{3\cos(2x)}{32}+\frac{3x\sin(2x)}{16}-\frac{\sin(4x)}{16}+\frac{\sin(6x)}{32}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)^6+cos(x)^6+3*sin(x)^2*cos(x)^2,x)
```

```
[Out] 1/8*x-1/64*cos(6*x)-3/32*cos(2*x)+3/16*x*sin(2*x)-1/16*sin(4*x)+1/32*sin(6*
x)
```

3.289 $\int e^x x^e (1+e+x)\,dx$

Optimal. Leaf size=9

$$e^x x^{1+e}$$

[Out] exp(x)*x^(1+exp(1))

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2228}

$$e^x x^{1+e}$$

Antiderivative was successfully verified.

[In] Int[E^x*x^E*(1 + E + x),x]

[Out] E^x*x^(1 + E)

Rule 2228

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[g*u^(m + 1)*(F^(c*v)/(b*c*e*Log
[F])), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\text{Integral} = e^x x^{1+e}$$

Mathematica [A]
time = 0.02, size = 9, normalized size = 1.00

$$e^x x^{1+e}$$

Antiderivative was successfully verified.

[In] Integrate[E^x*x^E*(1 + E + x),x]

[Out] E^x*x^(1 + E)

Maple [A]
time = 0.20, size = 9, normalized size = 1.00

method	result
risch	$x\,x^{\mathrm{e}}\mathrm{e}^{x}$
parallelrisch	$x\,x^{\mathrm{e}}\mathrm{e}^{x}$
gosper	$\mathrm{e}^{x}x^{1+\mathrm{e}}$
norman	$x\,\mathrm{e}^{x}\mathrm{e}^{\mathrm{e}\ln(x)}$
meijerg	$(-1)^{-\mathrm{e}}\left(x^{\mathrm{e}}(-1)^{\mathrm{e}}\,(1+\mathrm{e})\,\mathrm{e}\Gamma(\mathrm{e})\,(-x)^{-\mathrm{e}}-x^{\mathrm{e}}(-1)^{\mathrm{e}}\,(\mathrm{e}-x+1)\,\mathrm{e}^{x}-x^{\mathrm{e}}(-1)^{\mathrm{e}}\,(1+\mathrm{e})\,\mathrm{e}(-x)^{-\mathrm{e}}\,\Gamma\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+exp(1)+1)*x^exp(1)*exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*x^exp(1)*exp(x)
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 81, normalized size = 9.00

$$-(-x)^{-e-1}\,x^{e+1}e\Gamma(e+1,-x)-(-x)^{-e-2}\,x^{e+2}\Gamma(e+2,-x)-(-x)^{-e-1}\,x^{e+1}\Gamma(e+1,-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(1)+1)*x^exp(1)*exp(x),x, algorithm="maxima")
```

```
[Out] -(-x)^(-e - 1)*x^(e + 1)*e*gamma(e + 1, -x) - (-x)^(-e - 2)*x^(e + 2)*gamma
(e + 2, -x) - (-x)^(-e - 1)*x^(e + 1)*gamma(e + 1, -x)
```

Fricas [A]
time = 0.59, size = 8, normalized size = 0.89

$$xx^{e}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(1)+1)*x^exp(1)*exp(x),x, algorithm="fricas")
```

```
[Out] x*x^e*e^x
```

Sympy [A]
time = 0.17, size = 8, normalized size = 0.89

$$xx^{e}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(1)+1)*x**exp(1)*exp(x),x)
```

```
[Out] x*x**E*exp(x)
```

Giac [A]
time = 0.48, size = 8, normalized size = 0.89

$$xx^e e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(1)+1)*x^exp(1)*exp(x),x, algorithm="giac")
```

```
[Out] x*x^e*e^x
```

Mupad [B]
time = 0.19, size = 9, normalized size = 1.00

$$x^{e+1} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^exp(1)*exp(x)*(x + exp(1) + 1),x)
```

```
[Out] x^(exp(1) + 1)*exp(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((x+exp(1)+1)*x^exp(1)*exp(x),x)
```

```
[Out] not solved
```

3.290 $\int \left(\sqrt{2}\sqrt{\frac{x}{1+x}} + \frac{x^2}{2-x^2}\right) dx$

Optimal. Leaf size=49

$$-x + \sqrt{2}\sqrt{x}\sqrt{1+x} - \sqrt{2}\text{arcsinh}\left(\sqrt{x}\right) + \sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -x+2^(1/2)*x^(1/2)*(x+1)^(1/2)-2^(1/2)*arcsinh(x^(1/2))+2^(1/2)*arctanh(1/2
*2^(1/2)*x)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1978, 52, 56, 221, 327, 212}

$$-\sqrt{2}\text{arcsinh}\left(\sqrt{x}\right) + \sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right) - x + \sqrt{2}\sqrt{x+1}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2]*Sqrt[x/(1 + x)] + x^2/(2 - x^2),x]
```

```
[Out] -x + Sqrt[2]*Sqrt[x]*Sqrt[1 + x] - Sqrt[2]*ArcSinh[Sqrt[x]] + Sqrt[2]*ArcTa
nh[x/Sqrt[2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \sqrt{2}\int \sqrt{\frac{x}{1+x}}\,dx+\int \frac{x^2}{2-x^2}\,dx \\
&= -x+2\int \frac{1}{2-x^2}\,dx+\sqrt{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx \\
&= -x+\sqrt{2}\sqrt{x}\sqrt{1+x}+\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)-\frac{\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx}{\sqrt{2}} \\
&= -x+\sqrt{2}\sqrt{x}\sqrt{1+x}+\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)-\sqrt{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right) \\
&= -x+\sqrt{2}\sqrt{x}\sqrt{1+x}-\sqrt{2}\text{arcsinh}\left(\sqrt{x}\right)+\sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.
time = 0.07, size = 100, normalized size = 2.04

$$-x+\sqrt{2}\sqrt{\frac{x}{1+x}}(1+x)-\frac{\log\left(\sqrt{2}-x\right)}{\sqrt{2}}+\frac{\log\left(\sqrt{2}+x\right)}{\sqrt{2}}+\frac{\sqrt{2}\sqrt{\frac{x}{1+x}}\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2]*Sqrt[x/(1 + x)] + x^2/(2 - x^2),x]
```

```
[Out] -x + Sqrt[2]*Sqrt[x/(1 + x)]*(1 + x) - Log[Sqrt[2] - x]/Sqrt[2] + Log[Sqrt[
2] + x]/Sqrt[2] + (Sqrt[2]*Sqrt[x/(1 + x)]*Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[
1 + x]])/Sqrt[x]
```

Maple [A]
time = 0.23, size = 61, normalized size = 1.24

method	result
default	$-\frac{\sqrt{2}\,\sqrt{\frac{x}{x+1}}\,(x+1)\left(-2\sqrt{x^{2}+x}+\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)\right)}{2\sqrt{x(x+1)}}-x+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,x}{2}\right)$
trager	$1-x+\sqrt{2}\,(x+1)\,\sqrt{\frac{x}{x+1}}+\frac{\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{x}{x+1}}\,x^{2}-2\sqrt{2}\,x^{2}-2\sqrt{\frac{x}{x+1}}\,x^{2}-2\sqrt{2}\,\sqrt{\frac{x}{x+1}}+\sqrt{2}\,x+2x\sqrt{\frac{x}{x+1}}+2x^{2}+\sqrt{2}+4\sqrt{\frac{x}{x+1}}-3x-2}{\sqrt{2}\,x-\sqrt{2}+x-2}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+2)+2^(1/2)*(x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)*(x/(x+1))^(1/2)*(x+1)*(-2*(x^2+x)^(1/2)+ln(1/2+x+(x^2+x)^(1/2)
))/(x*(x+1))^(1/2)-x+2^(1/2)*arctanh(1/2*2^(1/2)*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.
time = 0.43, size = 79, normalized size = 1.61

$$-\frac{1}{2}\sqrt{2}\left(\frac{2\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}+\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)\right)-\frac{1}{2}\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+2)+2^(1/2)*(x/(x+1))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(x/(x + 1))/(x/(x + 1) - 1) + log(sqrt(x/(x + 1)) + 1)
- log(sqrt(x/(x + 1)) - 1)) - 1/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2)))
- x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(36) = 72.
time = 0.58, size = 114, normalized size = 2.33

$$\sqrt{2}(x+1)\sqrt{\frac{x}{x+1}}+\frac{1}{2}\sqrt{2}\log\left(-\frac{6\,x^{3}+19\,x^{2}+2\,\sqrt{2}(2\,x^{3}+7\,x^{2}+7\,x+2)-2\,(3\,x^{3}+11\,x^{2}+2\,\sqrt{2}(x^{3}+4\,x^{2}+5\,x+2)+14\,x+6)\sqrt{\frac{x}{x+1}}+20\,x+6}{x^{2}-2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+2)+2^(1/2)*(x/(x+1))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(x + 1)*sqrt(x/(x + 1)) + 1/2*sqrt(2)*log(-(6*x^3 + 19*x^2 + 2*sqrt
(2)*(2*x^3 + 7*x^2 + 7*x + 2) - 2*(3*x^3 + 11*x^2 + 2*sqrt(2)*(x^3 + 4*x^2
+ 5*x + 2) + 14*x + 6)*sqrt(x/(x + 1)) + 20*x + 6)/(x^2 - 2)) - x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2}x^{2}\sqrt{\frac{x}{x+1}}-x^{2}-2\sqrt{2}\sqrt{\frac{x}{x+1}}}{x^{2}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+2)+2**(1/2)*(x/(x+1))**(1/2),x)
```

```
[Out] Integral((sqrt(2)*x**2*sqrt(x/(x + 1)) - x**2 - 2*sqrt(2)*sqrt(x/(x + 1)))/
(x**2 - 2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.
time = 0.57, size = 73, normalized size = 1.49

$$\frac{1}{2}\sqrt{2}\Big(\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+2\sqrt{x^2+x}\operatorname{sgn}(x+1)\Big)-\frac{1}{2}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}|}{|2x+2\sqrt{2}|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+2)+2^(1/2)*(x/(x+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + 2*sqrt(x^2 +
 x)*sgn(x + 1)) - 1/2*sqrt(2)*log(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2))
) - x
```

Mupad [B]
time = 0.54, size = 238, normalized size = 4.86

$$-\sqrt{2}\,\mathrm{atanh}\left(\frac{96\sqrt{\frac{2x}{x+1}}}{64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}}-\frac{80\left(\frac{2x}{x+1}\right)^{3/2}}{64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}}+\frac{128}{64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}}-\frac{224x}{(x+1)\left(64\sqrt{2}\sqrt{\frac{2x}{x+1}}-56\sqrt{2}\left(\frac{2x}{x+1}\right)^{3/2}+96\sqrt{2}-\frac{160\sqrt{2}x}{x+1}\right)}\right)-\frac{\sqrt{\frac{2x}{x+1}}-1}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(1/2)*(x/(x + 1))^(1/2) - x^2/(x^2 - 2),x)
```

```
[Out] - 2^(1/2)*atanh((96*((2*x)/(x + 1))^(1/2))/(64*2^(1/2)*((2*x)/(x + 1))^(1/2
) - 56*2^(1/2)*((2*x)/(x + 1))^(3/2) + 96*2^(1/2) - (160*2^(1/2)*x)/(x + 1)
) - (80*((2*x)/(x + 1))^(3/2))/(64*2^(1/2)*((2*x)/(x + 1))^(1/2) - 56*2^(1/
2)*((2*x)/(x + 1))^(3/2) + 96*2^(1/2) - (160*2^(1/2)*x)/(x + 1)) + 128/(64*
2^(1/2)*((2*x)/(x + 1))^(1/2) - 56*2^(1/2)*((2*x)/(x + 1))^(3/2) + 96*2^(1/
2) - (160*2^(1/2)*x)/(x + 1)) - (224*x)/((x + 1)*(64*2^(1/2)*((2*x)/(x + 1)
)^(1/2) - 56*2^(1/2)*((2*x)/(x + 1))^(3/2) + 96*2^(1/2) - (160*2^(1/2)*x)/(
x + 1)))) - (((2*x)/(x + 1))^(1/2) - 1)/(x/(x + 1) - 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 28, normalized size = 0.57

$$-\frac{\ln\left(x^2-2\right)}{2}+\frac{\left(x^2-2\right)^{\frac{5}{2}}}{5}+\frac{4\sqrt{2}\,(x+1)^{\frac{3}{2}}}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^2/(-x^2+2)+2^(1/2)*(x/(x+1))^(1/2),x)
```

```
[Out] -1/2*ln(x^2-2)+1/5*(x^2-2)^(5/2)+4/3*2^(1/2)*(x+1)^(3/2)
```

3.291 $\int \frac{1+2x^{2022}}{x+x^{2023}}\,dx$

Optimal. Leaf size=13

$$\log(x)+\frac{\log\left(1+x^{2022}\right)}{2022}$$

[Out] ln(x)+1/2022*ln(x^2022+1)

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 457, 78}

$$\frac{\log\left(x^{2022}+1\right)}{2022}+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2022)/(x + x^2023),x]

[Out] Log[x] + Log[1 + x^2022]/2022

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \frac{1+2x^{2022}}{x\left(1+x^{2022}\right)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1+2x}{x(1+x)}\,dx, x, x^{2022}\right)}{2022}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{1}{1+x}\right)\,dx, x, x^{2022}\right)}{2022}\\ &= \log(x)+\frac{\log\left(1+x^{2022}\right)}{2022}\end{aligned}$$

Mathematica [**A**]
time = 1.00, size = 13, normalized size = 1.00

$$\log(x)+\frac{\log\left(1+x^{2022}\right)}{2022}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2022)/(x + x^2023),x]
```

```
[Out] Log[x] + Log[1 + x^2022]/2022
```

Maple [**F(-1)**]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2022+1)/(x^2023+x),x)
```

```
[Out] int((2*x^2022+1)/(x^2023+x),x)
```

Maxima [**A**]
time = 0.33, size = 11, normalized size = 0.85

$$\frac{1}{2022}\log\left(x^{2022}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2022+1)/(x^2023+x),x, algorithm="maxima")
```

```
[Out] 1/2022*log(x^2022 + 1) + log(x)
```

Fricas [**A**]
time = 188.88, size = 11, normalized size = 0.85

$$\frac{1}{2022}\log\left(x^{2022}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2022+1)/(x^2023+x),x, algorithm="fricas")
```

```
[Out] 1/2022*log(x^2022 + 1) + log(x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2022+1)/(x**2023+x),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.42, size = 15, normalized size = 1.15

$$\frac{1}{2022}\log\left(x^{2022}+1\right)+\frac{1}{2022}\log\left(x^{2022}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2022+1)/(x^2023+x),x, algorithm="giac")
```

```
[Out] 1/2022*log(x^2022 + 1) + 1/2022*log(x^2022)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{2\,x^{2022}+1}{x^{2023}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2022 + 1)/(x + x^2023),x)
```

```
[Out] int((2*x^2022 + 1)/(x + x^2023), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 1.00

$$3\ln\left(x\right)-\frac{\ln\left(x^{2022}+1\right)}{1011}$$

Warning: Unable to verify antiderivative.

```
[In] int((2*x^2022+1)/(x^2023+x),x)
```

```
[Out] 3*ln(x)-1/1011*ln(x^2022+1)
```

3.292 $\int (3\cos(23x)\sin(20x)+20\sin(43x))\,dx$

Optimal. Leaf size=9

$$\sin(20x)\sin(23x)$$

```
[Out] sin(20*x)*sin(23*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.89, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4369, 2718}

$$\frac{1}{2}\cos(3x)-\frac{1}{2}\cos(43x)$$

Antiderivative was successfully verified.

```
[In] Int[3*Cos[23*x]*Sin[20*x] + 20*Sin[43*x],x]
```

```
[Out] Cos[3*x]/2 - Cos[43*x]/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 3\int \cos(23x)\sin(20x)\,dx+20\int \sin(43x)\,dx\\ &= \frac{1}{2}\cos(3x)-\frac{1}{2}\cos(43x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.89

$$\frac{1}{2}\cos(3x)-\frac{1}{2}\cos(43x)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*Cos[23*x]*Sin[20*x] + 20*Sin[43*x],x]
```

[Out] Cos[3*x]/2 - Cos[43*x]/2

Maple [A]
time = 1.56, size = 14, normalized size = 1.56

method	result
default	$\frac{\cos(3x)}{2}-\frac{\cos(43x)}{2}$
risch	$\frac{\cos(3x)}{2}-\frac{\cos(43x)}{2}$
parts	$\frac{\cos(3x)}{2}-\frac{\cos(43x)}{2}$
parallelrisch	$\frac{-\frac{40}{43}+\frac{40\left(-\left(\tan^2(10x)\right)-\left(\tan^2\left(\frac{43x}{2}\right)\right)-2\right)\left(\tan^2\left(\frac{23x}{2}\right)\right)}{43}+\frac{92\tan(10x)\left(1+\tan^2\left(\frac{43x}{2}\right)\right)\tan\left(\frac{23x}{2}\right)}{43}-\frac{40\left(\tan^2\left(\frac{43x}{2}\right)\right)\left(\tan^2(10x)\right)}{43}-\frac{80(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{23x}{2}\right)\right)\left(1+\tan^2(10x)\right)\left(1+\tan^2\left(\frac{43x}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(3*sin(20*x)*cos(23*x)+20*sin(43*x),x,method=_RETURNVERBOSE)

[Out] 1/2*cos(3*x)-1/2*cos(43*x)

Maxima [A]
time = 0.33, size = 13, normalized size = 1.44

$$-\frac{1}{2}\cos\left(43\,x\right)+\frac{1}{2}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*sin(20*x)*cos(23*x)+20*sin(43*x),x, algorithm="maxima")

[Out] -1/2*cos(43*x) + 1/2*cos(3*x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(9) = 18.
time = 1.84, size = 131, normalized size = 14.56

$$-2199023255552\,\cos\left(x\right)^{43}+23639499997184\,\cos\left(x\right)^{41}-118197499985920\,\cos\left(x\right)^{39}+364934781206528\,\cos\left(x\right)^{37}-778995398344704\,\cos\left(x\right)^{35}+1219742794776576\,\cos\left(x\right)^{33}-1450504945139712\,\cos\left(x\right)^{31}+1338263491051520\,\cos\left(x\right)^{29}-970241031012352\,\cos\left(x\right)^{27}+556461767786496\,\cos\left(x\right)^{25}-252937167175680\,\cos\left(x\right)^{23}+90899294453760\,\cos\left(x\right)^{21}-25657058918400\,\cos\left(x\right)^{19}+5624816762880\,\cos\left(x\right)^{17}-942087536640\,\cos\left(x\right)^{15}+117760942080\,\cos\left(x\right)^{13}-10631196160\,\cos\left(x\right)^{11}+661443200\,\cos\left(x\right)^{9}-26457728\,\cos\left(x\right)^{7}+609224\,\cos\left(x\right)^{5}-6620\,\cos\left(x\right)^{3}+20\,\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*sin(20*x)*cos(23*x)+20*sin(43*x),x, algorithm="fricas")

[Out] -2199023255552*cos(x)^43 + 23639499997184*cos(x)^41 - 118197499985920*cos(x
)^39 + 364934781206528*cos(x)^37 - 778995398344704*cos(x)^35 + 121974279477
6576*cos(x)^33 - 1450504945139712*cos(x)^31 + 1338263491051520*cos(x)^29 -
970241031012352*cos(x)^27 + 556461767786496*cos(x)^25 - 252937167175680*cos
(x)^23 + 90899294453760*cos(x)^21 - 25657058918400*cos(x)^19 + 562481676288
0*cos(x)^17 - 942087536640*cos(x)^15 + 117760942080*cos(x)^13 - 10631196160
*cos(x)^11 + 661443200*cos(x)^9 - 26457728*cos(x)^7 + 609224*cos(x)^5 - 662
0*cos(x)^3 + 20*cos(x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(8) = 16$.
time = 0.13, size = 34, normalized size = 3.78

$$\frac{23\sin(20x)\sin(23x)}{43}+\frac{20\cos(20x)\cos(23x)}{43}-\frac{20\cos(43x)}{43}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*sin(20*x)*cos(23*x)+20*sin(43*x),x)
[Out] 23*sin(20*x)*sin(23*x)/43 + 20*cos(20*x)*cos(23*x)/43 - 20*cos(43*x)/43
```

Giac [A]
time = 0.47, size = 13, normalized size = 1.44

$$-\frac{1}{2}\cos(43x)+\frac{1}{2}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*sin(20*x)*cos(23*x)+20*sin(43*x),x, algorithm="giac")
[Out] -1/2*cos(43*x) + 1/2*cos(3*x)
```

Mupad [B]
time = 0.20, size = 13, normalized size = 1.44

$$\frac{\cos(3x)}{2}-\frac{\cos(43x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(20*sin(43*x) + 3*cos(23*x)*sin(20*x),x)
[Out] cos(3*x)/2 - cos(43*x)/2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 25, normalized size = 2.78

$$\frac{3\cos(20x)}{23}+\frac{3\cos(43x)}{43}+\frac{10\sin(20x)}{23}-\frac{20\sin(43x)}{43}$$

Warning: Unable to verify antiderivative.

```
[In] int(3*sin(20*x)*cos(23*x)+20*sin(43*x),x)
[Out] 3/23*cos(20*x)+3/43*cos(43*x)+10/23*sin(20*x)-20/43*sin(43*x)
```

3.293 $\int \frac{\sin(x)}{2e^x+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}(x-\log(2e^x+\cos(x)+\sin(x)))$$

```
[Out] 1/2*x-1/2*ln(2*exp(x)+cos(x)+sin(x))
```

Rubi [F]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sin(x)}{2e^x+\cos(x)+\sin(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[x]/(2*E^x + Cos[x] + Sin[x]),x]
[Out] Defer[Int][Sin[x]/(2*E^x + Cos[x] + Sin[x]), x]
```

Rubi steps

$$\text{Integral} = \int \frac{\sin(x)}{2e^x+\cos(x)+\sin(x)}\,dx$$

Mathematica [A]
time = 0.04, size = 21, normalized size = 1.11

$$\frac{x}{2}-\frac{1}{2}\log(2e^x+\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(2*E^x + Cos[x] + Sin[x]),x]
[Out] x/2 - Log[2*E^x + Cos[x] + Sin[x]]/2
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 30, normalized size = 1.58

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\frac{\ln\left(e^{2ix}+(2+2i)e^{(1+i)x}+i\right)}{2}$	30
parallelrisch	$\ln\left(\frac{\sqrt{2}}{\sqrt{\frac{2\,e^{x}+\cos(x)+\sin(x)}{1+\cos(x)}}}\right)+\ln\left(\sqrt{\frac{1}{1+\cos(x)}}\right)+\frac{x}{2}$	37

norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{\ln\left(2\,\mathrm{e}^x\left(\tan^2\left(\frac{x}{2}\right)\right)-\left(\tan^2\left(\frac{x}{2}\right)\right)+2\,\mathrm{e}^x+2\tan\left(\frac{x}{2}\right)+1\right)}{2}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(2*exp(x)+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/2*I*x-1/2*ln(exp(2*I*x)+(2+2*I)*exp((1+I)*x)+I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(16) = 32$.
time = 0.34, size = 91, normalized size = 4.79

$$\frac{1}{2}\,x-\frac{1}{4}\,\log\left(8\,\cos\left(x\right)^2e^{(2\,x)}+8\,e^{(2\,x)}\sin\left(x\right)^2+4\left(\cos\left(x\right)e^x-e^x\sin\left(x\right)\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^2+4\,\cos\left(x\right)e^x+2\left(2\,\cos\left(x\right)e^x+2\,e^x\sin\left(x\right)+1\right)\sin\left(2\,x\right)+\sin\left(2\,x\right)^2+4\,e^x\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*log(8*cos(x)^2*e^(2*x) + 8*e^(2*x)*sin(x)^2 + 4*(cos(x)*e^x - e
^x*sin(x))*cos(2*x) + cos(2*x)^2 + 4*cos(x)*e^x + 2*(2*cos(x)*e^x + 2*e^x*s
in(x) + 1)*sin(2*x) + sin(2*x)^2 + 4*e^x*sin(x) + 1)
```

Fricas [A]
time = 0.61, size = 16, normalized size = 0.84

$$\frac{1}{2}\,x-\frac{1}{2}\,\log\left(\cos\left(x\right)+2\,e^x+\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*log(cos(x) + 2*e^x + sin(x))
```

Sympy [A]
time = 0.07, size = 17, normalized size = 0.89

$$\frac{x}{2}-\frac{\log\left(2e^x+\sin\left(x\right)+\cos\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(2*exp(x) + sin(x) + cos(x))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(16) = 32$.
time = 0.54, size = 119, normalized size = 6.26

$$\frac{1}{2}\,x-\frac{1}{4}\,\log\left(\frac{2\left(4\,e^{(2\,x)}\tan\left(\frac{1}{2}\,x\right)^4-4\,e^x\tan\left(\frac{1}{2}\,x\right)^4+8\,e^x\tan\left(\frac{1}{2}\,x\right)^3+\tan\left(\frac{1}{2}\,x\right)^4+8\,e^{(2\,x)}\tan\left(\frac{1}{2}\,x\right)^2-4\,\tan\left(\frac{1}{2}\,x\right)^3+8\,e^x\tan\left(\frac{1}{2}\,x\right)+2\,\tan\left(\frac{1}{2}\,x\right)^2+4\,e^{(2\,x)}+4\,e^x+4\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(2*exp(x)+cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*log(2*(4*e^(2*x)*tan(1/2*x)^4 - 4*e^x*tan(1/2*x)^4 + 8*e^x*tan(
1/2*x)^3 + tan(1/2*x)^4 + 8*e^(2*x)*tan(1/2*x)^2 - 4*tan(1/2*x)^3 + 8*e^x*t
an(1/2*x) + 2*tan(1/2*x)^2 + 4*e^(2*x) + 4*e^x + 4*tan(1/2*x) + 1)/(tan(1/2
*x)^4 + 2*tan(1/2*x)^2 + 1))
```

Mupad [**B**]
time = 0.16, size = 22, normalized size = 1.16

$$\frac{x}{2}-\frac{\ln\left(2\,\mathrm{e}^{x}+\sqrt{2}\,\cos\left(x-\frac{\pi}{4}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + 2*exp(x) + sin(x)),x)
```

```
[Out] x/2 - log(2*exp(x) + 2^(1/2)*cos(x - pi/4))/2
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 21, normalized size = 1.11

$$2\arctan\left(\mathrm{e}^{x}+\sin\left(x\right)\right)-2\ln\left(\mathrm{e}^{x}-\cos\left(x\right)+\sin\left(x\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(x)/(2*exp(x)+cos(x)+sin(x)),x)
```

```
[Out] 2*arctan(exp(x)+sin(x))-2*ln(exp(x)-cos(x)+sin(x))
```

3.294 $\int \log^{-\log(e\pi)}(x) \log\left(\frac{x}{\pi}\right) \, dx$

Optimal. Leaf size=9

$$x \log^{-\log(\pi)}(x)$$

```
[Out] x/(ln(x)^ln(Pi))
```

Rubi [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 0.03, antiderivative size = 87, normalized size of antiderivative = 9.67, number of steps used = 4, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2336, 2212, 2408, 19, 6692}

$$(-\log(x))^{1+\log(\pi)} \log\left(\frac{x}{\pi}\right) \log^{-\log(e\pi)}(x)\Gamma(-\log(\pi), -\log(x)) + (-\log(x))^{\log(\pi)} \log^{-\log(\pi)}(x)\Gamma(1-\log(\pi), -\log(x)) + (-\log(x))^{\log(\pi)} \log^{1-\log(\pi)}(x)\Gamma(-\log(\pi), -\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[x/Pi]/Log[x]^Log[E*Pi],x]
```

```
[Out] Gamma[-Log[Pi], -Log[x]]*(-Log[x])^Log[Pi]*Log[x]^(1 - Log[Pi]) + (Gamma[1
- Log[Pi], -Log[x]]*(-Log[x])^Log[Pi])/Log[x]^Log[Pi] + (Gamma[-Log[Pi], -L
og[x]]*(-Log[x])^(1 + Log[Pi])*Log[x/Pi])/Log[x]^Log[E*Pi]
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] &&  !I
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2408

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(f_.)*(x_)^(r_
.)]*(e_.)), x_Symbol] :> With[{u = IntHide[(a + b*Log[c*x^n])^p, x]}, Dist[
d + e*Log[f*x^r], u, x] - Dist[e*r, Int[SimplifyIntegrand[u/x, x], x], x]]
```

```
/; FreeQ[{a, b, c, d, e, f, n, p, r}, x]
```

Rule 6692

```
Int[Gamma[n_, (a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Gamma[n, a
+ b*x]/b), x] - Simp[Gamma[n + 1, a + b*x]/b, x] /; FreeQ[{a, b, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \Gamma(-\log(\pi),-\log(x))(-\log(x))^{1+\log(\pi)}\log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)+\int\frac{\Gamma(-\log(\pi),-\log(x))(-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)}{x}\,dx\\
&= \Gamma(-\log(\pi),-\log(x))(-\log(x))^{1+\log(\pi)}\log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)+\left((-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)\right)\int\frac{\Gamma(-\log(\pi),-\log(x))}{x}\,dx\\
&= \Gamma(-\log(\pi),-\log(x))(-\log(x))^{1+\log(\pi)}\log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)+\left((-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)\right)\text{Subst}\left(\int\Gamma(-\log(\pi),-x)\,dx,x,\log(x)\right)\\
&= \Gamma(-\log(\pi),-\log(x))(-\log(x))^{\log(\pi)}\log^{1-\log(\pi)}(x)+\Gamma(1-\log(\pi),-\log(x))(-\log(x))^{\log(\pi)}\log^{-\log(\pi)}(x)+\Gamma(-\log(\pi),-\log(x))(-\log(x))^{1+\log(\pi)}\log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)
\end{aligned}$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \log^{-\log(e\pi)}(x)\log\left(\frac{x}{\pi}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[x/Pi]/Log[x]^Log[E*Pi],x]
[Out] Integrate[Log[x/Pi]/Log[x]^Log[E*Pi], x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \ln\left(\frac{x}{\pi}\right)\ln\left(x\right)^{-\ln(\pi\,\mathrm{e})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x/Pi)/(ln(x)^ln(Pi*exp(1))),x)
[Out] int(ln(x/Pi)/(ln(x)^ln(Pi*exp(1))),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x/pi)/(log(x)^log(pi*exp(1))),x, algorithm="maxima")
```

```
[Out] integrate(log(x)^(-log(pi*e))*log(x/pi), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.
time = 0.61, size = 30, normalized size = 3.33

$$\frac{x\log(\pi)+x\log\left(\frac{x}{\pi}\right)}{\left(\log(\pi)+\log\left(\frac{x}{\pi}\right)\right)^{\log(\pi)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x/pi)/(log(x)^log(pi*exp(1))),x, algorithm="fricas")
```

```
[Out] (x*log(pi) + x*log(x/pi))/(log(pi) + log(x/pi))^(log(pi) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(7) = 14.
time = 19.09, size = 56, normalized size = 6.22

$$-\frac{(-\log(x))^{1+\log(\pi)}\log(\pi)\Gamma(-\log(\pi),-\log(x))}{\log(x)^{1+\log(\pi)}}+\frac{(-\log(x))^{\log(\pi)}\Gamma(1-\log(\pi),-\log(x))}{\log(x)^{\log(\pi)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x/pi)/(ln(x)**ln(pi*exp(1))),x)
```

```
[Out] -(-log(x))**(1 + log(pi))*log(pi)*log(x)**(-log(pi) - 1)*uppergamma(-log(pi
), -log(x)) + (-log(x))**log(pi)*uppergamma(1 - log(pi), -log(x))/log(x)**l
og(pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x/pi)/(log(x)^log(pi*exp(1))),x, algorithm="giac")
```

```
[Out] integrate(log(x/pi)/log(x)^log(pi*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int\frac{\ln\left(\frac{x}{\Pi}\right)}{{\ln(x)}^{\ln(\Pi\,\mathrm{e})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x/Pi)/log(x)^log(Pi*exp(1)),x)
```

```
[Out] int(log(x/Pi)/log(x)^log(Pi*exp(1)), x)
```

<u>**Chatgpt**</u> [**F**] Failed to verify
time = 1.00, size = 27, normalized size = 3.00

$$\frac{\ln(x)^{\ln(\pi \mathrm{e})}}{(1-\ln(\pi \mathrm{e}))\ln(\pi \mathrm{e})}$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(x/Pi)/(ln(x)^ln(Pi*exp(1))),x)
```

```
[Out] 1/(1-ln(Pi*exp(1)))*ln(x)^ln(Pi*exp(1))/ln(Pi*exp(1))
```

3.295 $\int (x+8x^2+10x^3+5x^4+x^5)\,dx$

Optimal. Leaf size=32

$$\frac{x^2}{2}+\frac{8x^3}{3}+\frac{5x^4}{2}+x^5+\frac{x^6}{6}$$

```
[Out] 1/2*x^2+8/3*x^3+5/2*x^4+x^5+1/6*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\frac{x^6}{6}+x^5+\frac{5x^4}{2}+\frac{8x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x + 8*x^2 + 10*x^3 + 5*x^4 + x^5,x]
[Out] x^2/2 + (8*x^3)/3 + (5*x^4)/2 + x^5 + x^6/6
```

Rubi steps

$$\text{Integral}=\frac{x^2}{2}+\frac{8x^3}{3}+\frac{5x^4}{2}+x^5+\frac{x^6}{6}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$\frac{x^2}{2}+\frac{8x^3}{3}+\frac{5x^4}{2}+x^5+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x + 8*x^2 + 10*x^3 + 5*x^4 + x^5,x]
[Out] x^2/2 + (8*x^3)/3 + (5*x^4)/2 + x^5 + x^6/6
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.78

method	result	size
gosper	$\frac{x^2(x^4+6x^3+15x^2+16x+3)}{6}$	24
default	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
norman	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25

risch	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
parallelrisch	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25
parts	$\frac{1}{2}x^2+\frac{8}{3}x^3+\frac{5}{2}x^4+x^5+\frac{1}{6}x^6$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5+5*x^4+10*x^3+8*x^2+x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+8/3*x^3+5/2*x^4+x^5+1/6*x^6
```

Maxima [A]
time = 0.35, size = 24, normalized size = 0.75

$$\frac{1}{6}x^6+x^5+\frac{5}{2}x^4+\frac{8}{3}x^3+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5+5*x^4+10*x^3+8*x^2+x,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 + x^5 + 5/2*x^4 + 8/3*x^3 + 1/2*x^2
```

Fricas [A]
time = 0.56, size = 24, normalized size = 0.75

$$\frac{1}{6}x^6+x^5+\frac{5}{2}x^4+\frac{8}{3}x^3+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5+5*x^4+10*x^3+8*x^2+x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6 + x^5 + 5/2*x^4 + 8/3*x^3 + 1/2*x^2
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.81

$$\frac{x^6}{6}+x^5+\frac{5x^4}{2}+\frac{8x^3}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5+5*x**4+10*x**3+8*x**2+x,x)
```

```
[Out] x**6/6 + x**5 + 5*x**4/2 + 8*x**3/3 + x**2/2
```

Giac [A]
time = 0.62, size = 24, normalized size = 0.75

$$\frac{1}{6}x^6+x^5+\frac{5}{2}x^4+\frac{8}{3}x^3+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5+5*x^4+10*x^3+8*x^2+x,x, algorithm="giac")
```

```
[Out] 1/6*x^6 + x^5 + 5/2*x^4 + 8/3*x^3 + 1/2*x^2
```

Mupad [B]
time = 0.02, size = 24, normalized size = 0.75

$$\frac{x^6}{6}+x^5+\frac{5\,x^4}{2}+\frac{8\,x^3}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 8*x^2 + 10*x^3 + 5*x^4 + x^5,x)
```

```
[Out] x^2/2 + (8*x^3)/3 + (5*x^4)/2 + x^5 + x^6/6
```

Chatgpt [A]
time = 1.00, size = 24, normalized size = 0.75

$$\frac{1}{6}x^6+x^5+\frac{5}{2}x^4+\frac{8}{3}x^3+\frac{1}{2}x^2$$

Antiderivative was successfully verified.

```
[In] int(x^5+5*x^4+10*x^3+8*x^2+x,x)
```

```
[Out] 1/6*x^6+x^5+5/2*x^4+8/3*x^3+1/2*x^2
```

3.296 $\int \frac{(1+x)(4+x)}{(2+x)(3+x)}\,dx$

Optimal. Leaf size=14

$$x-2\log(2+x)+2\log(3+x)$$

```
[Out] x-2*ln(2+x)+2*ln(x+3)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {147}

$$x-2\log(x+2)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)*(4 + x))/((2 + x)*(3 + x)),x]
```

```
[Out] x - 2*Log[2 + x] + 2*Log[3 + x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{Integral} &= \int \left(1-\frac{2}{2+x}+\frac{2}{3+x}\right)dx\\ &= x-2\log(2+x)+2\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$x-2\log(2+x)+2\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)*(4 + x))/((2 + x)*(3 + x)),x]
```

```
[Out] x - 2*Log[2 + x] + 2*Log[3 + x]
```

Maple [A]
time = 0.07, size = 15, normalized size = 1.07

method	result	size
default	$x-2\ln(2+x)+2\ln(x+3)$	15
norman	$x-2\ln(2+x)+2\ln(x+3)$	15
risch	$x-2\ln(2+x)+2\ln(x+3)$	15
parallelrisch	$x-2\ln(2+x)+2\ln(x+3)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(2+x)/(x+3)*(4+x),x,method=_RETURNVERBOSE)
```

```
[Out] x-2*ln(2+x)+2*ln(x+3)
```

Maxima [A]
time = 0.35, size = 14, normalized size = 1.00

$$x+2\log(x+3)-2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(2+x)/(x+3)*(4+x),x, algorithm="maxima")
```

```
[Out] x + 2*log(x + 3) - 2*log(x + 2)
```

Fricas [A]
time = 0.58, size = 14, normalized size = 1.00

$$x+2\log(x+3)-2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(2+x)/(x+3)*(4+x),x, algorithm="fricas")
```

```
[Out] x + 2*log(x + 3) - 2*log(x + 2)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 1.00

$$x-2\log(x+2)+2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(2+x)/(x+3)*(4+x),x)
```

```
[Out] x - 2*log(x + 2) + 2*log(x + 3)
```

Giac [A]
time = 0.50, size = 16, normalized size = 1.14

$$x+2\log(|x+3|)-2\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+1)/(2+x)/(x+3)*(4+x),x, algorithm="giac")
```

```
[Out] x + 2*log(abs(x + 3)) - 2*log(abs(x + 2))
```

Mupad [**B**]
time = 0.13, size = 10, normalized size = 0.71

$$x + 4\,\mathrm{atanh}(2\,x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)*(x + 4))/((x + 2)*(x + 3)),x)
```

```
[Out] x + 4*atanh(2*x + 5)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 0.93

$$-5\ln\left(x+3\right)+6\ln\left(2+x\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((x+1)/(2+x)/(x+3)*(x+4),x)
```

```
[Out] -5*ln(x+3)+6*ln(2+x)
```

3.297 $\int x\cot(x)\,dx$

Optimal. Leaf size=39

$$-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)$$

```
[Out] -1/2*I*x^2+x*ln(1-exp(2*I*x))-1/2*I*polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3798, 2221, 2317, 2438}

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cot[x],x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 - E^((2*I)*x)] - (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= -\frac{ix^2}{2} - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) - \int \log\left(1-e^{2ix}\right)\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2, e^{2ix}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.87

$$x\log\left(1-e^{2ix}\right) - \frac{1}{2}i\left(x^2 + \text{PolyLog}\left(2, e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[x],x]
[Out] x*Log[1 - E^((2*I)*x)] - (I/2)*(x^2 + PolyLog[2, E^((2*I)*x)])
```

Maple [A]
time = 0.03, size = 28, normalized size = 0.72

method	result
parts	$x\ln\left(1-e^{2ix}\right) - \frac{i\left(x^2+\text{hyperbolicCosineIntegral}_2\left(e^{2ix}\right)\right)}{2}$
risch	$-\frac{ix^2}{2} + x\ln\left(e^{ix}+1\right) - i\,\text{hyperbolicCosineIntegral}_2\left(-e^{ix}\right) + x\ln\left(1-e^{ix}\right) - i\,\text{hyperbolicCosineI}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(x),x,method=_RETURNVERBOSE)
[Out] x*ln(1-exp(2*I*x))-1/2*I*(x^2+polylog(2,exp(2*I*x)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(24) = 48.
time = 0.36, size = 80, normalized size = 2.05

$$-\frac{1}{2}i\,x^2 + i\,x\arctan\left(\sin(x), \cos(x)+1\right) - i\,x\arctan\left(\sin(x), -\cos(x)+1\right) + \frac{1}{2}x\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) + \frac{1}{2}x\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right) - i\,\text{Li}_2\left(-e^{(ix)}\right) - i\,\text{Li}_2\left(e^{(ix)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(x), cos(x) + 1) - I*x*arctan2(sin(x), -cos(x)
+ 1) + 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*x*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1) - I*dilog(-e^(I*x)) - I*dilog(e^(I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(24) = 48$.
time = 0.62, size = 65, normalized size = 1.67

$$\frac{1}{2}x\log\left(-\cos\left(2x\right)+i\sin\left(2x\right)+1\right)+\frac{1}{2}x\log\left(-\cos\left(2x\right)-i\sin\left(2x\right)+1\right)-\frac{1}{4}i\operatorname{Li}_2(\cos\left(2x\right)+i\sin\left(2x\right))+\frac{1}{4}i\operatorname{Li}_2(\cos\left(2x\right)-i\sin\left(2x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(-cos(2*x) + I*sin(2*x) + 1) + 1/2*x*log(-cos(2*x) - I*sin(2*x) +
1) - 1/4*I*dilog(cos(2*x) + I*sin(2*x)) + 1/4*I*dilog(cos(2*x) - I*sin(2*x)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\cot\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x),x)
```

```
[Out] Integral(x*cot(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x),x, algorithm="giac")
```

```
[Out] integrate(x*cot(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(x),x)
```

```
[Out] int(x*cot(x), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 13, normalized size = 0.33

$$x\ln\left(\sin\left(x\right)\right)-\frac{\ln\left(\sin\left(x\right)\right)^{2}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*cot(x),x)
```

```
[Out] x*ln(sin(x))-1/2*ln(sin(x))^2
```

3.298 $\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx$

Optimal. Leaf size=24

$$e^{\frac{1}{x}+x}\left(4+\frac{1}{x^2}-\frac{2}{x}-2x+x^2\right)$$

```
[Out] exp(x+1/x)*(4+1/x^2-2/x-2*x+x^2)
```

Rubi [F]
time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{\frac{1}{x}+x}(-1-x^2+x^4+x^6)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(E^(x^(-1) + x)*(-1 - x^2 + x^4 + x^6))/x^4,x]
```

```
[Out] Defer[Int][E^(x^(-1) + x), x] - Defer[Int][E^(x^(-1) + x)/x^4, x] - Defer[I
nt][E^(x^(-1) + x)/x^2, x] + Defer[Int][E^(x^(-1) + x)*x^2, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \frac{e^{\frac{1}{x}+x}(-1+x^2)(1+x^2)^2}{x^4}\,dx \\
&= \int \left(e^{\frac{1}{x}+x}-\frac{e^{\frac{1}{x}+x}}{x^4}-\frac{e^{\frac{1}{x}+x}}{x^2}+e^{\frac{1}{x}+x}x^2\right)dx \\
&= \int e^{\frac{1}{x}+x}\,dx-\int \frac{e^{\frac{1}{x}+x}}{x^4}\,dx-\int \frac{e^{\frac{1}{x}+x}}{x^2}\,dx+\int e^{\frac{1}{x}+x}x^2\,dx
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$e^{\frac{1}{x}+x}\left(4+\frac{1}{x^2}-\frac{2}{x}-2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(x^(-1) + x)*(-1 - x^2 + x^4 + x^6))/x^4,x]
```

```
[Out] E^(x^(-1) + x)*(4 + x^(-2) - 2/x - 2*x + x^2)
```

Maple [A]
time = 0.10, size = 33, normalized size = 1.38

method	result	size
gosper	$\frac{\mathrm{e}^{\frac{x^2+1}{x}}\left(x^4-2x^3+4x^2-2x+1\right)}{x^2}$	33
risch	$\frac{\mathrm{e}^{\frac{x^2+1}{x}}\left(x^4-2x^3+4x^2-2x+1\right)}{x^2}$	33
norman	$\frac{x\,\mathrm{e}^{x+\frac{1}{x}}+x^5\mathrm{e}^{x+\frac{1}{x}}+4x^3\mathrm{e}^{x+\frac{1}{x}}-2x^4\mathrm{e}^{x+\frac{1}{x}}-2\,\mathrm{e}^{x+\frac{1}{x}}x^2}{x^3}$	57
parallelrisch	$\frac{\mathrm{e}^{\frac{x^2+1}{x}}x^4-2\,\mathrm{e}^{\frac{x^2+1}{x}}x^3+4\,\mathrm{e}^{\frac{x^2+1}{x}}x^2-2\,\mathrm{e}^{\frac{x^2+1}{x}}x+\mathrm{e}^{\frac{x^2+1}{x}}}{x^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x+1/x)*(x^6+x^4-x^2-1)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] exp((x^2+1)/x)*(x^4-2*x^3+4*x^2-2*x+1)/x^2
```

Maxima [A]
time = 0.39, size = 28, normalized size = 1.17

$$\frac{(x^4-2\,x^3+4\,x^2-2\,x+1)e^{\left(x+\frac{1}{x}\right)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+1/x)*(x^6+x^4-x^2-1)/x^4,x, algorithm="maxima")
```

```
[Out] (x^4 - 2*x^3 + 4*x^2 - 2*x + 1)*e^(x + 1/x)/x^2
```

Fricas [A]
time = 0.58, size = 32, normalized size = 1.33

$$\frac{(x^4-2\,x^3+4\,x^2-2\,x+1)e^{\left(\frac{x^2+1}{x}\right)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+1/x)*(x^6+x^4-x^2-1)/x^4,x, algorithm="fricas")
```

```
[Out] (x^4 - 2*x^3 + 4*x^2 - 2*x + 1)*e^((x^2 + 1)/x)/x^2
```

Sympy [A]
time = 0.05, size = 27, normalized size = 1.12

$$\frac{(x^4-2x^3+4x^2-2x+1)\,e^{x+\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+1/x)*(x**6+x**4-x**2-1)/x**4,x)
```

```
[Out] (x**4 - 2*x**3 + 4*x**2 - 2*x + 1)*exp(x + 1/x)/x**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(23) = 46$.
time = 0.48, size = 72, normalized size = 3.00

$$\frac{x^4 e^{\left(\frac{x^2+1}{x}\right)} - 2\,x^3 e^{\left(\frac{x^2+1}{x}\right)} + 4\,x^2 e^{\left(\frac{x^2+1}{x}\right)} - 2\,x e^{\left(\frac{x^2+1}{x}\right)} + e^{\left(\frac{x^2+1}{x}\right)}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+1/x)*(x^6+x^4-x^2-1)/x^4,x, algorithm="giac")
```

```
[Out] (x^4*e^((x^2 + 1)/x) - 2*x^3*e^((x^2 + 1)/x) + 4*x^2*e^((x^2 + 1)/x) - 2*x*
e^((x^2 + 1)/x) + e^((x^2 + 1)/x))/x^2
```

Mupad [B]
time = 0.17, size = 28, normalized size = 1.17

$$\frac{e^{x+\frac{1}{x}}\,\left(x^4 - 2\,x^3 + 4\,x^2 - 2\,x + 1\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(x + 1/x)*(x^2 - x^4 - x^6 + 1))/x^4,x)
```

```
[Out] (exp(x + 1/x)*(4*x^2 - 2*x - 2*x^3 + x^4 + 1))/x^2
```

Chatgpt [F] Failed to verify
time = 1.00, size = 35, normalized size = 1.46

$$\frac{e^{x+\frac{1}{x}}(x^2+1)}{2} + e^{x+\frac{1}{x}}\left(\frac{1}{5}x^5 - \frac{1}{3}x^3 - x\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(exp(x+1/x)*(x^6+x^4-x^2-1)/x^4,x)
```

```
[Out] 1/2*exp(x+1/x)*(x^2+1)+exp(x+1/x)*(1/5*x^5-1/3*x^3-x)
```

3.299 $\int \frac{1}{\sqrt{(-1+x)(1+x)^3}} \, dx$

Optimal. Leaf size=26

$$-\frac{(1-x)(1+x)}{\sqrt{-\left((1-x)(1+x)^3\right)}}$$

```
[Out] -(1-x)*(x+1)/(-(1-x)*(x+1)^3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{6851, 37\}$

$$-\frac{(1-x)(x+1)}{\sqrt{-\left((1-x)(x+1)^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-1 + x)*(1 + x)^3],x]
```

```
[Out] -(((1 - x)*(1 + x))/Sqrt[-((1 - x)*(1 + x)^3)])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned} \text{Integral} &= \frac{\left(\sqrt{-1+x}(1+x)^{3/2}\right) \int \frac{1}{\sqrt{-1+x}(1+x)^{3/2}} \, dx}{\sqrt{(-1+x)(1+x)^3}} \\ &= -\frac{(1-x)(1+x)}{\sqrt{-\left((1-x)(1+x)^3\right)}} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.77

$$\frac{(-1+x)(1+x)}{\sqrt{(-1+x)(1+x)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-1 + x)*(1 + x)^3],x]
[Out] ((-1 + x)*(1 + x))/Sqrt[(-1 + x)*(1 + x)^3]
```

Maple [A]
time = 0.19, size = 29, normalized size = 1.12

method	result	size
gosper	$\frac{(x-1)(x+1)}{\sqrt{(x+1)^3(x-1)}}$	19
risch	$\frac{(x-1)(x+1)}{\sqrt{(x+1)^3(x-1)}}$	19
trager	$\frac{\sqrt{x^4+2x^3-2x-1}}{(x+1)^2}$	22
default	$\frac{\sqrt{(x-1)(x+1)}\,\sqrt{x^2-1}}{\sqrt{(x+1)^3(x-1)}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x+1)^3*(x-1))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((x+1)^3*(x-1))^(1/2)*((x-1)*(x+1))^(1/2)*(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x+1)^3*(x-1))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((x + 1)^3*(x - 1)), x)
```

Fricas [A]
time = 0.57, size = 34, normalized size = 1.31

$$\frac{x^2+2\,x+\sqrt{x^4+2\,x^3-2\,x-1}+1}{x^2+2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x+1)^3*(x-1))^(1/2),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*x + sqrt(x^4 + 2*x^3 - 2*x - 1) + 1)/(x^2 + 2*x + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x+1)**3*(x-1))**(1/2),x)
```

```
[Out] Integral(1/sqrt((x - 1)*(x + 1)**3), x)
```

Giac [**A**]
time = 0.43, size = 22, normalized size = 0.85

$$\frac{2}{\left(x-\sqrt{x^2-1}+1\right)\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x+1)^3*(x-1))^(1/2),x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 - 1) + 1)*sgn(x + 1))
```

Mupad [**B**]
time = 0.05, size = 17, normalized size = 0.65

$$\frac{x^2-1}{\sqrt{(x-1)\,(x+1)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(x + 1)^3)^(1/2),x)
```

```
[Out] (x^2 - 1)/((x - 1)*(x + 1)^3)^(1/2)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 29, normalized size = 1.12

$$6\sqrt{\frac{x+1}{x-1}+2}+\frac{1}{\frac{x+1}{x-1}+2}$$

Warning: Unable to verify antiderivative.

```
[In] int(1/((x+1)^3*(x-1))^(1/2),x)
```

```
[Out] 6*((x+1)/(x-1)+2)^(1/2)+1/((x+1)/(x-1)+2)
```

3.300 $\int x \sin^4(x)\,dx$

Optimal. Leaf size=44

$$\frac{3x^2}{16} - \frac{3}{8}x\cos(x)\sin(x) + \frac{3\sin^2(x)}{16} - \frac{1}{4}x\cos(x)\sin^3(x) + \frac{\sin^4(x)}{16}$$

```
[Out] 3/16*x^2-3/8*x*cos(x)*sin(x)+3/16*sin(x)^2-1/4*x*cos(x)*sin(x)^3+1/16*sin(x
)^4
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3391, 30}

$$\frac{3x^2}{16} + \frac{\sin^4(x)}{16} + \frac{3\sin^2(x)}{16} - \frac{1}{4}x\sin^3(x)\cos(x) - \frac{3}{8}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^4,x]
```

```
[Out] (3*x^2)/16 - (3*x*Cos[x]*Sin[x])/8 + (3*Sin[x]^2)/16 - (x*Cos[x]*Sin[x]^3)/
4 + Sin[x]^4/16
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\frac{1}{4}x\cos(x)\sin^3(x) + \frac{\sin^4(x)}{16} + \frac{3}{4}\int x\sin^2(x)\,dx \\
&= -\frac{3}{8}x\cos(x)\sin(x) + \frac{3\sin^2(x)}{16} - \frac{1}{4}x\cos(x)\sin^3(x) + \frac{\sin^4(x)}{16} + \frac{3\int x\,dx}{8} \\
&= \frac{3x^2}{16} - \frac{3}{8}x\cos(x)\sin(x) + \frac{3\sin^2(x)}{16} - \frac{1}{4}x\cos(x)\sin^3(x) + \frac{\sin^4(x)}{16}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.95

$$\frac{3x^2}{16}-\frac{1}{8}\cos(2x)+\frac{1}{128}\cos(4x)-\frac{1}{4}x\sin(2x)+\frac{1}{32}x\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^4,x]
```

```
[Out] (3*x^2)/16 - Cos[2*x]/8 + Cos[4*x]/128 - (x*Sin[2*x])/4 + (x*Sin[4*x])/32
```

Maple [A]
time = 0.20, size = 38, normalized size = 0.86

method	result
risch	$\frac{3x^2}{16}+\frac{\cos(4x)}{128}+\frac{x\sin(4x)}{32}-\frac{\cos(2x)}{8}-\frac{x\sin(2x)}{4}$
default	$x\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-\frac{3x^2}{16}+\frac{\left(2\left(\cos^2(x)\right)-5\right)^2}{64}$
norman	$\frac{\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+\frac{5\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{3x^2}{16}-\frac{3x\tan\left(\frac{x}{2}\right)}{4}-\frac{11x\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{11x\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}+\frac{9x^2(\tan}{8}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-3/16*x^2+1/64*(2*cos(x)^2-5)^2
```

Maxima [A]
time = 0.33, size = 32, normalized size = 0.73

$$\frac{3}{16}x^2+\frac{1}{32}x\sin\left(4x\right)-\frac{1}{4}x\sin\left(2x\right)+\frac{1}{128}\cos\left(4x\right)-\frac{1}{8}\cos\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/16*x^2 + 1/32*x*sin(4*x) - 1/4*x*sin(2*x) + 1/128*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A]
time = 0.59, size = 35, normalized size = 0.80

$$\frac{1}{16}\cos\left(x\right)^4+\frac{3}{16}x^2-\frac{5}{16}\cos\left(x\right)^2+\frac{1}{8}\left(2x\cos\left(x\right)^3-5x\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/16*cos(x)^4 + 3/16*x^2 - 5/16*cos(x)^2 + 1/8*(2*x*cos(x)^3 - 5*x*cos(x))*
sin(x)
```

Sympy [A]
time = 0.18, size = 83, normalized size = 1.89

$$\frac{3x^2\sin^4(x)}{16}+\frac{3x^2\sin^2(x)\cos^2(x)}{8}+\frac{3x^2\cos^4(x)}{16}-\frac{5x\sin^3(x)\cos(x)}{8}-\frac{3x\sin(x)\cos^3(x)}{8}+\frac{5\sin^4(x)}{32}-\frac{3\cos^4(x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**4,x)
```

```
[Out] 3*x**2*sin(x)**4/16 + 3*x**2*sin(x)**2*cos(x)**2/8 + 3*x**2*cos(x)**4/16 -
5*x*sin(x)**3*cos(x)/8 - 3*x*sin(x)*cos(x)**3/8 + 5*sin(x)**4/32 - 3*cos(x)
**4/32
```

Giac [A]
time = 0.43, size = 32, normalized size = 0.73

$$\frac{3}{16}\,x^2+\frac{1}{32}\,x\sin(4\,x)-\frac{1}{4}\,x\sin(2\,x)+\frac{1}{128}\,\cos(4\,x)-\frac{1}{8}\,\cos(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/32*x*sin(4*x) - 1/4*x*sin(2*x) + 1/128*cos(4*x) - 1/8*cos(2*x)
```

Mupad [B]
time = 0.15, size = 38, normalized size = 0.86

$$\frac{\cos(2\,x)^2}{64}-\frac{x\,\sin(2\,x)}{4}-\frac{\cos(2\,x)}{8}+\frac{3\,x^2}{16}+\frac{x\,\cos(2\,x)\,\sin(2\,x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^4,x)
```

```
[Out] cos(2*x)^2/64 - (x*sin(2*x))/4 - cos(2*x)/8 + (3*x^2)/16 + (x*cos(2*x)*sin(
2*x))/16
```

Chatgpt [F] Failed to verify
time = 1.00, size = 34, normalized size = 0.77

$$\frac{3x^2}{8}-\frac{x\sin(2x)}{4}+\frac{\sin(4x)}{32}-\frac{3x}{8}+\frac{\cos(2x)}{8}-\frac{\cos(4x)}{128}$$

Warning: Unable to verify antiderivative.

```
[In] int(x*sin(x)^4,x)
```

```
[Out] 3/8*x^2-1/4*x*sin(2*x)+1/32*sin(4*x)-3/8*x+1/8*cos(2*x)-1/128*cos(4*x)
```

3.301 $\int \cos(3x)\csc^2(x)\sec^3(x)\sin(2x)\,dx$

Optimal. Leaf size=11

$$6\log(\cos(x)) + 2\log(\sin(x))$$

```
[Out] 6*ln(cos(x))+2*ln(sin(x))
```

Rubi [A]
time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4450, 457, 78}

$$2\log(\sin(x)) + 6\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Csc[x]^2*Sec[x]^3*Sin[2*x],x]
```

```
[Out] 6*Log[Cos[x]] + 2*Log[Sin[x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4450

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((-n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sec] || EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{2-8x^2}{x(1-x^2)}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{2-8x}{(1-x)x}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{6}{-1+x}+\frac{2}{x}\right)\,dx, x, \sin^2(x)\right)\\
&= 6\log(\cos(x)) + 2\log(\sin(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$6\log(\cos(x)) + 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Csc[x]^2*Sec[x]^3*Sin[2*x],x]
[Out] 6*Log[Cos[x]] + 2*Log[Sin[x]]
```

Maple [A]
time = 0.11, size = 12, normalized size = 1.09

$$-6\ln(\tan(x)) + 8\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*cos(3*x)/sin(x)^2/cos(x)^3,x)
[Out] -6*ln(tan(x))+8*ln(sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(11) = 22.
time = 0.43, size = 54, normalized size = 4.91

$$3\log\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)+\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x)/sin(x)^2/cos(x)^3,x, algorithm="maxima")
[Out] 3*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + log(cos(x)^2 + sin(x)^2 +
 2*cos(x) + 1) + log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A]
time = 0.64, size = 17, normalized size = 1.55

$$3\log\left(\cos(x)^2\right)+\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x)/sin(x)^2/cos(x)^3,x, algorithm="fricas")
```

```
[Out] 3*log(cos(x)^2) + log(-1/4*cos(x)^2 + 1/4)
```

Sympy [A]
time = 112.52, size = 15, normalized size = 1.36

$$3\log\left(\sin^2\left(x\right)-1\right)+2\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x)/sin(x)**2/cos(x)**3,x)
```

```
[Out] 3*log(sin(x)**2 - 1) + 2*log(sin(x))
```

Giac [A]
time = 0.49, size = 16, normalized size = 1.45

$$\log\left(-\cos\left(x\right)^2+1\right)+6\,\log\left(\left|\cos\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*cos(3*x)/sin(x)^2/cos(x)^3,x, algorithm="giac")
```

```
[Out] log(-cos(x)^2 + 1) + 6*log(abs(cos(x)))
```

Mupad [B]
time = 0.08, size = 11, normalized size = 1.00

$$6\,\ln\left(\cos\left(x\right)\right)+\ln\left(\sin\left(x\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(3*x)*sin(2*x))/(cos(x)^3*sin(x)^2),x)
```

```
[Out] 6*log(cos(x)) + log(sin(x)^2)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 4, normalized size = 0.36

$$\frac{1}{\cos\left(x\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(2*x)*cos(3*x)/sin(x)^2/cos(x)^3,x)
```

```
[Out] 1/cos(x)^2
```

3.302 $\int \left(\frac{1}{\sqrt{2}\sqrt{\log(x)}}+\sqrt{2}\sqrt{\log(x)}\right) dx$

Optimal. Leaf size=13

$$\sqrt{2}x\sqrt{\log(x)}$$

[Out] 2^(1/2)*x*ln(x)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2336, 2211, 2235, 2333}

$$\sqrt{2}x\sqrt{\log(x)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2]*Sqrt[Log[x]]) + Sqrt[2]*Sqrt[Log[x]],x]

[Out] Sqrt[2]*x*Sqrt[Log[x]]

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= \frac{\int \frac{1}{\sqrt{\log(x)}}\,dx}{\sqrt{2}} + \sqrt{2}\int \sqrt{\log(x)}\,dx \\
&= \sqrt{2}x\sqrt{\log(x)} - \frac{\int \frac{1}{\sqrt{\log(x)}}\,dx}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(x)\right)}{\sqrt{2}} \\
&= \sqrt{2}x\sqrt{\log(x)} - \frac{\text{Subst}\left(\int \frac{e^x}{\sqrt{x}}\,dx, x, \log(x)\right)}{\sqrt{2}} + \sqrt{2}\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(x)}\right) \\
&= \sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{\log(x)}\right) + \sqrt{2}x\sqrt{\log(x)} - \sqrt{2}\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\log(x)}\right) \\
&= \sqrt{2}x\sqrt{\log(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\sqrt{2}x\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2]*Sqrt[Log[x]]) + Sqrt[2]*Sqrt[Log[x]],x]
[Out] Sqrt[2]*x*Sqrt[Log[x]]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2}\,\sqrt{\ln\left(x\right)} + \frac{\sqrt{2}}{2\sqrt{\ln\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(1/2)*ln(x)^(1/2)+1/2*2^(1/2)/ln(x)^(1/2),x)
[Out] int(2^(1/2)*ln(x)^(1/2)+1/2*2^(1/2)/ln(x)^(1/2),x)
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.44, size = 41, normalized size = 3.15

$$-\frac{1}{2}i\,\sqrt{2}\sqrt{\pi}\,\text{erf}\left(i\,\sqrt{\log\left(x\right)}\right) - \frac{1}{2}\,\sqrt{2}\left(-i\,\sqrt{\pi}\,\text{erf}\left(i\,\sqrt{\log\left(x\right)}\right) - 2\,x\sqrt{\log\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(1/2)*log(x)^(1/2)+1/2*2^(1/2)/log(x)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*I*sqrt(2)*sqrt(pi)*erf(I*sqrt(log(x))) - 1/2*sqrt(2)*(-I*sqrt(pi)*erf(
I*sqrt(log(x))) - 2*x*sqrt(log(x)))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(1/2)*log(x)^(1/2)+1/2*2^(1/2)/log(x)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(12) = 24.
time = 0.25, size = 83, normalized size = 6.38

$$\frac{\sqrt{2}\sqrt{\pi}\sqrt{-\log{(x)}}\operatorname{erfc}{\left(\sqrt{-\log{(x)}}\right)}}{2\sqrt{\log{(x)}}} + \frac{\sqrt{2}\left(x\sqrt{-\log{(x)}} + \frac{\sqrt{\pi}\operatorname{erfc}{\left(\sqrt{-\log{(x)}}\right)}}{2}\right)\sqrt{\log{(x)}}}{\sqrt{-\log{(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**(1/2)*ln(x)**(1/2)+1/2*2**(1/2)/ln(x)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*sqrt(-log(x))*erfc(sqrt(-log(x)))/(2*sqrt(log(x))) + sqrt(
2)*(x*sqrt(-log(x)) + sqrt(pi)*erfc(sqrt(-log(x)))/2)*sqrt(log(x))/sqrt(-lo
g(x))
```

Giac [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.41, size = 41, normalized size = 3.15

$$\frac{1}{2}i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-i\sqrt{\log(x)}\right) - \frac{1}{2}\sqrt{2}\left(i\sqrt{\pi}\operatorname{erf}\left(-i\sqrt{\log(x)}\right) - 2x\sqrt{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(1/2)*log(x)^(1/2)+1/2*2^(1/2)/log(x)^(1/2),x, algorithm="giac"
)
```

```
[Out] 1/2*I*sqrt(2)*sqrt(pi)*erf(-I*sqrt(log(x))) - 1/2*sqrt(2)*(I*sqrt(pi)*erf(-
I*sqrt(log(x))) - 2*x*sqrt(log(x)))
```

Mupad [B]
time = 0.22, size = 9, normalized size = 0.69

$$\sqrt{2}\,x\,\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(1/2)/(2*log(x)^(1/2)) + 2^(1/2)*log(x)^(1/2),x)
```

```
[Out] 2^(1/2)*x*log(x)^(1/2)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 18, normalized size = 1.38

$$\frac{2\sqrt{2}\,\ln\left(x\right)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}}{\sqrt{\ln\left(x\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] int(2^(1/2)*ln(x)^(1/2)+1/2*2^(1/2)/ln(x)^(1/2),x)
```

```
[Out] 2/3*2^(1/2)*ln(x)^(3/2)+2^(1/2)/ln(x)^(1/2)
```

3.303 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal. Leaf size=12

$$-x+\tan(x)+\log(\cos(x))\tan(x)$$

```
[Out] -x+tan(x)+ln(cos(x))*tan(x)
```

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8, 2634, 3554}

$$-x+\tan(x)+\tan(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \log(\cos(x))\tan(x)+\int \tan^2(x)\,dx\\
&= \tan(x)+\log(\cos(x))\tan(x)-\int 1\,dx\\
&= -x+\tan(x)+\log(\cos(x))\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-x+\tan(x)+\log(\cos(x))\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 67, normalized size = 5.58

method	result
parallelrisch	$-x+\tan(x)+\ln(\cos(x))\tan(x)$
norman	$\frac{x-x\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}$
default	$-4i\left(\frac{\frac{e^{2ix}\ln\left(\left(e^{2ix}+1\right)e^{-ix}\right)}{2}-\frac{1}{2}}{e^{2ix}+1}-\frac{\ln\left(e^{2ix}+1\right)}{4}+\frac{\ln(2)}{2e^{2ix}+2}\right)$
risch	$-\frac{2i\ln\left(e^{ix}\right)}{e^{2ix}+1}+\frac{-i\ln\left(e^{2ix}+1\right)e^{2ix}-\pi\operatorname{csgn}\left(i\left(e^{2ix}+1\right)\right)\operatorname{csgn}\left(i\cos(x)\right)^2+\pi\operatorname{csgn}\left(i\left(e^{2ix}+1\right)\right)\operatorname{csgn}\left(i\cos(x)\right)\operatorname{csgn}\left(ie^{-ix}\right)+\pi\operatorname{csgn}(\cdots}{e^{2ix}+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))/cos(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*((1/2*exp(2*I*x)*ln((exp(I*x)^2+1)/exp(I*x))-1/2)/(exp(2*I*x)+1)-1/4*l
n(exp(2*I*x)+1)+1/2*ln(2)/(exp(I*x)^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.
time = 0.45, size = 94, normalized size = 7.83

$$-\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)}-\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))/cos(x)^2,x, algorithm="maxima")
```

```
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.61, size = 22, normalized size = 1.83

$$-\frac{x\cos\left(x\right)-\log\left(\cos\left(x\right)\right)\sin\left(x\right)-\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))/cos(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(12) = 24$.
time = 1.77, size = 83, normalized size = 6.92

$$-\frac{x\tan^{2}\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}+\frac{x}{\tan^{2}\left(\frac{x}{2}\right)-1}-\frac{2\log\left(-\frac{\tan^{2}\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}-\frac{2\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))/cos(x)**2,x)
```

```
[Out] -x*tan(x/2)**2/(tan(x/2)**2 - 1) + x/(tan(x/2)**2 - 1) - 2*log(-tan(x/2)**2
/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)/(tan(x/2)**2 - 1) - 2*ta
n(x/2)/(tan(x/2)**2 - 1)
```

Giac [A]
time = 0.48, size = 12, normalized size = 1.00

$$\log\left(\cos\left(x\right)\right)\tan\left(x\right)-x+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))/cos(x)^2,x, algorithm="giac")
```

```
[Out] log(cos(x))*tan(x) - x + tan(x)
```

Mupad [B]
time = 0.35, size = 35, normalized size = 2.92

$$\tan(x)-2\,x+\ln\left(\cos\left(x\right)\right)\,\tan(x)+\ln\left(\cos\left(x\right)\right)\,1\mathrm{i}-\ln\left(\cos\left(2\,x\right)+1+\sin\left(2\,x\right)\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))/cos(x)^2,x)
```

```
[Out] log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(co
s(x))*tan(x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 12, normalized size = 1.00

$$-\frac{\ln\left(\cos\left(x\right)\right)}{\cos\left(x\right)}+\sec\left(x\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(ln(cos(x))/cos(x)^2,x)
```

```
[Out] -ln(cos(x))/cos(x)+sec(x)
```

3.304 $\int \frac{\frac{5}{(-5+x)^6}+\frac{3}{(-3+x)^4}+\frac{1}{(-1+x)^2}}{\left(1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}\right)^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}}$$

```
[Out] 1/(1+1/(x-1)+1/(-3+x)^3+1/(x-5)^5)
```

Rubi [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {6818}

$$\frac{1}{\frac{1}{(x-3)^3}+\frac{1}{(x-5)^5}+\frac{1}{x-1}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(5/(-5 + x)^6 + 3/(-3 + x)^4 + (-1 + x)^(-2))/(1 + (-5 + x)^(-5) + (-3
+ x)^(-3) + (-1 + x)^(-1))^2,x]
```

```
[Out] (1 + (-5 + x)^(-5) + (-3 + x)^(-3) + (-1 + x)^(-1))^(-1)
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\text{Integral} = \frac{1}{1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$\frac{1}{1+\frac{1}{(-5+x)^5}+\frac{1}{(-3+x)^3}+\frac{1}{-1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5/(-5 + x)^6 + 3/(-3 + x)^4 + (-1 + x)^(-2))/(1 + (-5 + x)^(-5)
+ (-3 + x)^(-3) + (-1 + x)^(-1))^2,x]
```

```
[Out] (1 + (-5 + x)^(-5) + (-3 + x)^(-3) + (-1 + x)^(-1))^(-1)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(19) = 38$.
time = 0.12, size = 87, normalized size = 4.58

method	result
gosper	$-\frac{x^8-34x^7+503x^6-4228x^5+22076x^4-73260x^3+150661x^2-175054x+87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
default	$\frac{-x^8+34x^7-503x^6+4228x^5-22076x^4+73260x^3-150661x^2+175054x-87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
risch	$\frac{-x^8+34x^7-503x^6+4228x^5-22076x^4+73260x^3-150661x^2+175054x-87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
parallelrisch	$\frac{-x^8+34x^7-503x^6+4228x^5-22076x^4+73260x^3-150661x^2+175054x-87527}{x^9-34x^8+502x^7-4201x^6+21774x^5-71474x^4+144740x^3-164339x^2+78071x+3152}$
norman	$\frac{-x^{17}+69x^{16}-2229x^{15}+44761x^{14}-625602x^{13}+6455858x^{12}-50913715x^{11}+313280159x^{10}-1521750430x^9+5864557678x^8-1}{(x-1)(-3+x)^3(x-5)^5(x^9-34x^8+502x^7-4201x^6+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x-1)^2+3/(-3+x)^4+5/(x-5)^6)/(1+1/(x-1)+1/(-3+x)^3+1/(x-5)^5)^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (-x^8+34*x^7-503*x^6+4228*x^5-22076*x^4+73260*x^3-150661*x^2+175054*x-87527
)/(x^9-34*x^8+502*x^7-4201*x^6+21774*x^5-71474*x^4+144740*x^3-164339*x^2+78
071*x+3152)
```

Maxima [A]
time = 0.35, size = 19, normalized size = 1.00

$$\frac{1}{\frac{1}{x-1} + \frac{1}{(x-3)^3} + \frac{1}{(x-5)^5} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x-1)^2+3/(-3+x)^4+5/(x-5)^6)/(1+1/(x-1)+1/(-3+x)^3+1/(x-5)^5)
^2,x, algorithm="maxima")
```

```
[Out] 1/(1/(x - 1) + 1/(x - 3)^3 + 1/(x - 5)^5 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(19) = 38$.
time = 0.57, size = 85, normalized size = 4.47

$$-\frac{x^8 - 34\,x^7 + 503\,x^6 - 4228\,x^5 + 22076\,x^4 - 73260\,x^3 + 150661\,x^2 - 175054\,x + 87527}{x^9 - 34\,x^8 + 502\,x^7 - 4201\,x^6 + 21774\,x^5 - 71474\,x^4 + 144740\,x^3 - 164339\,x^2 + 78071\,x + 3152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x-1)^2+3/(-3+x)^4+5/(x-5)^6)/(1+1/(x-1)+1/(-3+x)^3+1/(x-5)^5)
^2,x, algorithm="fricas")
```

```
[Out] -(x^8 - 34*x^7 + 503*x^6 - 4228*x^5 + 22076*x^4 - 73260*x^3 + 150661*x^2 -
175054*x + 87527)/(x^9 - 34*x^8 + 502*x^7 - 4201*x^6 + 21774*x^5 - 71474*x^
4 + 144740*x^3 - 164339*x^2 + 78071*x + 3152)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(20) = 40$.
time = 0.50, size = 82, normalized size = 4.32

$$\frac{-x^8 + 34x^7 - 503x^6 + 4228x^5 - 22076x^4 + 73260x^3 - 150661x^2 + 175054x - 87527}{x^9 - 34x^8 + 502x^7 - 4201x^6 + 21774x^5 - 71474x^4 + 144740x^3 - 164339x^2 + 78071x + 3152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x-1)**2+3/(-3+x)**4+5/(x-5)**6)/(1+1/(x-1)+1/(-3+x)**3+1/(x-5
)**5)**2,x)
```

```
[Out] (-x**8 + 34*x**7 - 503*x**6 + 4228*x**5 - 22076*x**4 + 73260*x**3 - 150661*
x**2 + 175054*x - 87527)/(x**9 - 34*x**8 + 502*x**7 - 4201*x**6 + 21774*x**
5 - 71474*x**4 + 144740*x**3 - 164339*x**2 + 78071*x + 3152)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(19) = 38$.
time = 0.48, size = 85, normalized size = 4.47

$$-\frac{x^8 - 34\,x^7 + 503\,x^6 - 4228\,x^5 + 22076\,x^4 - 73260\,x^3 + 150661\,x^2 - 175054\,x + 87527}{x^9 - 34\,x^8 + 502\,x^7 - 4201\,x^6 + 21774\,x^5 - 71474\,x^4 + 144740\,x^3 - 164339\,x^2 + 78071\,x + 3152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x-1)^2+3/(-3+x)^4+5/(x-5)^6)/(1+1/(x-1)+1/(-3+x)^3+1/(x-5)^5)
^2,x, algorithm="giac")
```

```
[Out] -(x^8 - 34*x^7 + 503*x^6 - 4228*x^5 + 22076*x^4 - 73260*x^3 + 150661*x^2 -
175054*x + 87527)/(x^9 - 34*x^8 + 502*x^7 - 4201*x^6 + 21774*x^5 - 71474*x^
4 + 144740*x^3 - 164339*x^2 + 78071*x + 3152)
```

Mupad [B]
time = 0.31, size = 85, normalized size = 4.47

$$-\frac{x^8 - 34\,x^7 + 503\,x^6 - 4228\,x^5 + 22076\,x^4 - 73260\,x^3 + 150661\,x^2 - 175054\,x + 87527}{x^9 - 34\,x^8 + 502\,x^7 - 4201\,x^6 + 21774\,x^5 - 71474\,x^4 + 144740\,x^3 - 164339\,x^2 + 78071\,x + 3152}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x - 1)^2 + 3/(x - 3)^4 + 5/(x - 5)^6)/(1/(x - 1) + 1/(x - 3)^3 + 1/
(x - 5)^5 + 1)^2,x)
```

```
[Out] -(150661*x^2 - 175054*x - 73260*x^3 + 22076*x^4 - 4228*x^5 + 503*x^6 - 34*x
^7 + x^8 + 87527)/(78071*x - 164339*x^2 + 144740*x^3 - 71474*x^4 + 21774*x^
5 - 4201*x^6 + 502*x^7 - 34*x^8 + x^9 + 3152)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((1/(x-1)^2+3/(x-3)^4+5/(x-5)^6)/(1+1/(x-1)+1/(x-3)^3+1/(x-5)^5)^2,x)
[Out] not solved
```

3.305 $\int \csc(x)\sin(23x)\,dx$

Optimal. Leaf size=86

$$x+\sin(2x)+\frac{1}{2}\sin(4x)+\frac{1}{3}\sin(6x)+\frac{1}{4}\sin(8x)+\frac{1}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{1}{7}\sin(14x)+\frac{1}{8}\sin(16x)+\frac{1}{9}\sin(18x)+\frac{1}{1}$$

```
[Out] x+sin(2*x)+1/2*sin(4*x)+1/3*sin(6*x)+1/4*sin(8*x)+1/5*sin(10*x)+1/6*sin(12*
x)+1/7*sin(14*x)+1/8*sin(16*x)+1/9*sin(18*x)+1/10*sin(20*x)+1/11*sin(22*x)
```

Rubi [A]
time = 0.23, antiderivative size = 108, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1828, 1171, 393, 209}

$$x+\frac{2097152}{11}\sin(x)\cos^{21}(x)-\frac{49545216}{55}\sin(x)\cos^{19}(x)+\frac{899022848}{495}\sin(x)\cos^{17}(x)-\frac{1009455104}{495}\sin(x)\cos^{15}(x)+\frac{322500608}{231}\sin(x)\cos^{13}(x)-\frac{415607296}{693}\sin(x)\cos^{11}(x)+\frac{10100480}{63}\sin(x)\cos^{9}(x)-\frac{178880}{7}\sin(x)\cos^{7}(x)+\frac{6656}{3}\sin(x)\cos^{5}(x)-\frac{260}{3}\sin(x)\cos^{3}(x)+2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sin[23*x],x]
```

```
[Out] x + 2*Cos[x]*Sin[x] - (260*Cos[x]^3*Sin[x])/3 + (6656*Cos[x]^5*Sin[x])/3 -
(178880*Cos[x]^7*Sin[x])/7 + (10100480*Cos[x]^9*Sin[x])/63 - (415607296*Cos
[x]^11*Sin[x])/693 + (322500608*Cos[x]^13*Sin[x])/231 - (1009455104*Cos[x]^
15*Sin[x])/495 + (899022848*Cos[x]^17*Sin[x])/495 - (49545216*Cos[x]^19*Sin
[x])/55 + (2097152*Cos[x]^21*Sin[x])/11
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
```

```
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{23-1771x^2+33649x^4-245157x^6+817190x^8-1352078x^{10}+1144066x^{12}-490314x^{14}+100947x^{16}-8855x^{18}+253x^{20}-x^{22}}{(1+x^2)^{12}}\,dx,x,\tan(x)\right)\\
&= \frac{2097152}{11}\cos^{21}(x)\sin(x)-\frac{1}{22}\text{Subst}\left(\int \frac{4193798-92235220x^2+91494942x^4-86101488x^6+68123308x^8-38377592x^{10}+13208140x^{12}-2421232x^{14}+200398x^{16}-5588x^{18}+22x^{20}}{(1+x^2)^{11}}\,dx,x,\tan(x)\right)\\
&= -\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)+\frac{1}{440}\text{Subst}\left(\int \frac{312485768-5998654200x^2+4168755360x^4-2446725600x^6+1084259440x^8-316707600x^{10}+52544800x^{12}-4120160x^{14}+112200x^{16}-440x^{18}}{(1+x^2)^{10}}\,dx,x,\tan(x)\right)\\
&= \frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)-\frac{\text{Subst}\left(\int \frac{8759621744-145318060800x^2+70280464320x^4-26239403520x^6+6722733600x^8-1021996800x^{10}+76190400x^{12}-2027520x^{14}+7920x^{16}}{(1+x^2)^9}\,dx,x,\tan(x)\right)}{7920}\\
&= -\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)+\frac{\text{Subst}\left(\int \frac{118266558720-1669485185280x^2+544997756160x^4-125167299840x^6+17603562240x^8-1251613440x^{10}+32567040x^{12}-126720x^{14}}{(1+x^2)^8}\,dx,x,\tan(x)\right)}{126720}\\
&= \frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)-\frac{\text{Subst}\left(\int \frac{821072847360-9646740956160x^2+2016772369920x^4-264430172160x^6+17980300800x^8-457712640x^{10}+1774080x^{12}}{(1+x^2)^7}\,dx,x,\tan(x)\right)}{1774080}\\
&= -\frac{415607296}{693}\cos^{11}(x)\sin(x)+\frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)+\frac{\text{Subst}\left(\int \frac{2914581964800-27595707955200x^2+3394439516160x^4-221277450240x^6+5513840640x^8-21288960x^{10}}{(1+x^2)^6}\,dx,x,\tan(x)\right)}{21288960}\\
&= \frac{10100480}{63}\cos^{9}(x)\sin(x)-\frac{415607296}{693}\cos^{11}(x)\sin(x)+\frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)-\frac{\text{Subst}\left(\int \frac{4985722368000-36212520960000x^2+2268125798400x^4-55351296000x^6+212889600x^8}{(1+x^2)^5}\,dx,x,\tan(x)\right)}{212889600}\\
&= -\frac{178880}{7}\cos^{7}(x)\sin(x)+\frac{10100480}{63}\cos^{9}(x)\sin(x)-\frac{415607296}{693}\cos^{11}(x)\sin(x)+\frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)+\frac{\text{Subst}\left(\int \frac{3636154368000-18589519872000x^2+444513484800x^4-1703116800x^6}{(1+x^2)^4}\,dx,x,\tan(x)\right)}{1703116800}\\
&= \frac{6656}{3}\cos^{5}(x)\sin(x)-\frac{178880}{7}\cos^{7}(x)\sin(x)+\frac{10100480}{63}\cos^{9}(x)\sin(x)-\frac{415607296}{693}\cos^{11}(x)\sin(x)+\frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)-\frac{\text{Subst}\left(\int \frac{854964633600-2677299609600x^2+10218700800x^4}{(1+x^2)^3}\,dx,x,\tan(x)\right)}{10218700800}\\
&= -\frac{260}{3}\cos^{3}(x)\sin(x)+\frac{6656}{3}\cos^{5}(x)\sin(x)-\frac{178880}{7}\cos^{7}(x)\sin(x)+\frac{10100480}{63}\cos^{9}(x)\sin(x)-\frac{415607296}{693}\cos^{11}(x)\sin(x)+\frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)+\frac{\text{Subst}\left(\int \frac{122624409600-40874803200x^2}{(1+x^2)^2}\,dx,x,\tan(x)\right)}{40874803200}\\
&= 2\cos(x)\sin(x)-\frac{260}{3}\cos^{3}(x)\sin(x)+\frac{6656}{3}\cos^{5}(x)\sin(x)-\frac{178880}{7}\cos^{7}(x)\sin(x)+\frac{10100480}{63}\cos^{9}(x)\sin(x)-\frac{415607296}{693}\cos^{11}(x)\sin(x)+\frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= x+2\cos(x)\sin(x)-\frac{260}{3}\cos^{3}(x)\sin(x)+\frac{6656}{3}\cos^{5}(x)\sin(x)-\frac{178880}{7}\cos^{7}(x)\sin(x)+\frac{10100480}{63}\cos^{9}(x)\sin(x)-\frac{415607296}{693}\cos^{11}(x)\sin(x)+\frac{322500608}{231}\cos^{13}(x)\sin(x)-\frac{1009455104}{495}\cos^{15}(x)\sin(x)+\frac{899022848}{495}\cos^{17}(x)\sin(x)-\frac{49545216}{55}\cos^{19}(x)\sin(x)+\frac{2097152}{11}\cos^{21}(x)\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 86, normalized size = 1.00

$$x+\sin(2x)+\frac{1}{2}\sin(4x)+\frac{1}{3}\sin(6x)+\frac{1}{4}\sin(8x)+\frac{1}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{1}{7}\sin(14x)+\frac{1}{8}\sin(16x)+\frac{1}{9}\sin(18x)+\frac{1}{10}\sin(20x)+\frac{1}{11}\sin(22x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sin[23*x],x]
```

```
[Out] x + Sin[2*x] + Sin[4*x]/2 + Sin[6*x]/3 + Sin[8*x]/4 + Sin[10*x]/5 + Sin[12*
x]/6 + Sin[14*x]/7 + Sin[16*x]/8 + Sin[18*x]/9 + Sin[20*x]/10 + Sin[22*x]/1
1
```

Maple [A]
time = 0.21, size = 67, normalized size = 0.78

method	result
risch	$x+\sin(2x)+\frac{\sin(4x)}{2}+\frac{\sin(6x)}{3}+\frac{\sin(8x)}{4}+\frac{\sin(10x)}{5}+\frac{\sin(12x)}{6}+\frac{\sin(14x)}{7}+\frac{\sin(16x)}{8}+\frac{\sin(18x)}{9}+\frac{\sin(20x)}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(23*x)/sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] x+sin(2*x)+1/2*sin(4*x)+1/3*sin(6*x)+1/4*sin(8*x)+1/5*sin(10*x)+1/6*sin(12*
x)+1/7*sin(14*x)+1/8*sin(16*x)+1/9*sin(18*x)+1/10*sin(20*x)+1/11*sin(22*x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(23*x)/sin(x),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 0.85, size = 71, normalized size = 0.83

$$\frac{2}{3465}\left(330301440\cos(x)^{21}-1560674304\cos(x)^{19}+3146579968\cos(x)^{17}-3533092864\cos(x)^{15}+2418754560\cos(x)^{13}-1039018240\cos(x)^{11}+277763200\cos(x)^{9}-44272800\cos(x)^{7}+3843840\cos(x)^{5}-150150\cos(x)^{3}+3465\cos(x)\right)\sin(x)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(23*x)/sin(x),x, algorithm="fricas")
```

```
[Out] 2/3465*(330301440*cos(x)^21 - 1560674304*cos(x)^19 + 3146579968*cos(x)^17 -
 3533092864*cos(x)^15 + 2418754560*cos(x)^13 - 1039018240*cos(x)^11 + 27776
3200*cos(x)^9 - 44272800*cos(x)^7 + 3843840*cos(x)^5 - 150150*cos(x)^3 + 34
65*cos(x))*sin(x) + x
```

Sympy [A]
time = 71.46, size = 100, normalized size = 1.16

$$x-\frac{1024\sin^{11}(2x)}{11}+\frac{2560\sin^{9}(2x)}{9}-\frac{2304\sin^{7}(2x)}{7}+\frac{896\sin^{5}(2x)}{5}-\frac{140\sin^{3}(2x)}{3}+6\sin(2x)+\frac{8\sin^{5}(4x)}{5}-\frac{8\sin^{3}(4x)}{3}+\frac{3\sin(4x)}{2}+\frac{\sin(8x)}{4}+\frac{\sin(16x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(23*x)/sin(x),x)
```

```
[Out] x - 1024*sin(2*x)**11/11 + 2560*sin(2*x)**9/9 - 2304*sin(2*x)**7/7 + 896*si
n(2*x)**5/5 - 140*sin(2*x)**3/3 + 6*sin(2*x) + 8*sin(4*x)**5/5 - 8*sin(4*x)
**3/3 + 3*sin(4*x)/2 + sin(8*x)/4 + sin(16*x)/8
```

Giac [A]
time = 0.44, size = 66, normalized size = 0.77

$$x+\frac{1}{11}\sin(22x)+\frac{1}{10}\sin(20x)+\frac{1}{9}\sin(18x)+\frac{1}{8}\sin(16x)+\frac{1}{7}\sin(14x)+\frac{1}{6}\sin(12x)+\frac{1}{5}\sin(10x)+\frac{1}{4}\sin(8x)+\frac{1}{3}\sin(6x)+\frac{1}{2}\sin(4x)+\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(23*x)/sin(x),x, algorithm="giac")
```

```
[Out] x + 1/11*sin(22*x) + 1/10*sin(20*x) + 1/9*sin(18*x) + 1/8*sin(16*x) + 1/7*s
in(14*x) + 1/6*sin(12*x) + 1/5*sin(10*x) + 1/4*sin(8*x) + 1/3*sin(6*x) + 1/
2*sin(4*x) + sin(2*x)
```

Mupad [B]
time = 0.24, size = 66, normalized size = 0.77

$$x+\sin\left(2\,x\right)+\frac{\sin\left(4\,x\right)}{2}+\frac{\sin\left(6\,x\right)}{3}+\frac{\sin\left(8\,x\right)}{4}+\frac{\sin\left(10\,x\right)}{5}+\frac{\sin\left(12\,x\right)}{6}+\frac{\sin\left(14\,x\right)}{7}+\frac{\sin\left(16\,x\right)}{8}+\frac{\sin\left(18\,x\right)}{9}+\frac{\sin\left(20\,x\right)}{10}+\frac{\sin\left(22\,x\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(23*x)/sin(x),x)
```

```
[Out] x + sin(2*x) + sin(4*x)/2 + sin(6*x)/3 + sin(8*x)/4 + sin(10*x)/5 + sin(12*
x)/6 + sin(14*x)/7 + sin(16*x)/8 + sin(18*x)/9 + sin(20*x)/10 + sin(22*x)/1
1
```

Chatgpt [F] Failed to verify
time = 1.00, size = 15, normalized size = 0.17

$$\frac{22\sin\left(23x\right)\ln\left(x\right)}{23}-\text{cosineIntegral}\left(23x\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(23*x)/sin(x),x)
```

```
[Out] 22/23*sin(23*x)*ln(x)-Ci(23*x)
```

3.306 $\int \frac{(1-x)^2x^4}{1+x^2} dx$

Optimal. Leaf size=26

$$x^2 - \frac{x^4}{2} + \frac{x^5}{5} - \log\left(1 + x^2\right)$$

```
[Out] x^2-1/2*x^4+1/5*x^5-ln(x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1643, 266}

$$\frac{x^5}{5} - \frac{x^4}{2} + x^2 - \log\left(x^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)^2*x^4)/(1 + x^2),x]
```

```
[Out] x^2 - x^4/2 + x^5/5 - Log[1 + x^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(2x - 2x^3 + x^4 - \frac{2x}{1 + x^2}\right) dx \\
&= x^2 - \frac{x^4}{2} + \frac{x^5}{5} - 2\int \frac{x}{1 + x^2}\, dx \\
&= x^2 - \frac{x^4}{2} + \frac{x^5}{5} - \log\left(1 + x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 1.00

$$x^2 - \frac{x^4}{2} + \frac{x^5}{5} - \log\left(1 + x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)^2*x^4)/(1 + x^2),x]
[Out] x^2 - x^4/2 + x^5/5 - Log[1 + x^2]
```

Maple [A]
time = 0.10, size = 23, normalized size = 0.88

method	result	size
default	$x^2-\frac{x^4}{2}+\frac{x^5}{5}-\ln\left(x^2+1\right)$	23
norman	$x^2-\frac{x^4}{2}+\frac{x^5}{5}-\ln\left(x^2+1\right)$	23
risch	$x^2-\frac{x^4}{2}+\frac{x^5}{5}-\ln\left(x^2+1\right)$	23
parallelrisch	$x^2-\frac{x^4}{2}+\frac{x^5}{5}-\ln\left(x^2+1\right)$	23
meijerg	$-\frac{x(-5x^2+15)}{15}+\frac{x^2(-3x^2+6)}{6}-\ln\left(x^2+1\right)+\frac{x(21x^4-35x^2+105)}{105}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(1-x)^2/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x^2-1/2*x^4+1/5*x^5-ln(x^2+1)
```

Maxima [A]
time = 0.46, size = 22, normalized size = 0.85

$$\frac{1}{5}\,x^5-\frac{1}{2}\,x^4+x^2-\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(1-x)^2/(x^2+1),x, algorithm="maxima")
[Out] 1/5*x^5 - 1/2*x^4 + x^2 - log(x^2 + 1)
```

Fricas [A]
time = 0.56, size = 22, normalized size = 0.85

$$\frac{1}{5}\,x^5-\frac{1}{2}\,x^4+x^2-\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(1-x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/5*x^5 - 1/2*x^4 + x^2 - log(x^2 + 1)
```

Sympy [A]
time = 0.02, size = 19, normalized size = 0.73

$$\frac{x^5}{5}-\frac{x^4}{2}+x^2-\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(1-x)**2/(x**2+1),x)
```

```
[Out] x**5/5 - x**4/2 + x**2 - log(x**2 + 1)
```

Giac [A]
time = 0.71, size = 22, normalized size = 0.85

$$\frac{1}{5}x^5 - \frac{1}{2}x^4 + x^2 - \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(1-x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 1/2*x^4 + x^2 - log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 22, normalized size = 0.85

$$x^2 - \ln\left(x^2 + 1\right) - \frac{x^4}{2} + \frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x - 1)^2)/(x^2 + 1),x)
```

```
[Out] x^2 - log(x^2 + 1) - x^4/2 + x^5/5
```

Chatgpt [F] Failed to verify
time = 1.00, size = 36, normalized size = 1.38

$$\frac{(x^2 - 2)\arctan(x)}{2} - \frac{x(x^2 - 1)}{2} + \frac{x^5}{5x^2 + 5} + \frac{x^3}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^4*(1-x)^2/(x^2+1),x)
```

```
[Out] 1/2*(x^2-2)*arctan(x)-1/2*x*(x^2-1)+x^5/(5*x^2+5)+1/3*x^3
```

3.307 $\int x^{-\log(x)}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\sqrt[4]{e}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}-\log(x)\right)$$

```
[Out] 1/2*exp(1/4)*Pi^(1/2)*erf(-1/2+ln(x))
```

Rubi [F]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^{-\log(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^(-Log[x]),x]
[Out] Defer[Int][x^(-Log[x]), x]
```

Rubi steps

$$\text{Integral} = \int x^{-\log(x)}\,dx$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.09

$$\frac{1}{2}\sqrt[4]{e}\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}(-1+2\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-Log[x]),x]
[Out] (E^(1/4)*Sqrt[Pi]*Erf[(-1 + 2*Log[x])/2])/2
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-ln(x)),x)
```

```
[Out] int(x^(-ln(x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-log(x)),x, algorithm="maxima")
[Out] integrate(1/(x^log(x)), x)
```

Fricas [A]
time = 0.59, size = 12, normalized size = 0.52

$$\frac{1}{2}\sqrt{\pi}\operatorname{erf}\left(\log\left(x\right)-\frac{1}{2}\right)e^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-log(x)),x, algorithm="fricas")
[Out] 1/2*sqrt(pi)*erf(log(x) - 1/2)*e^(1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-\log\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-ln(x)),x)
[Out] Integral(x**(-log(x)), x)
```

Giac [A]
time = 0.44, size = 12, normalized size = 0.52

$$\frac{1}{2}\sqrt{\pi}\operatorname{erf}\left(\log\left(x\right)-\frac{1}{2}\right)e^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-log(x)),x, algorithm="giac")
[Out] 1/2*sqrt(pi)*erf(log(x) - 1/2)*e^(1/4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \mathrm{e}^{-{\ln\left(x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^log(x),x)
```

```
[Out] int(exp(-log(x)^2), x)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 9, normalized size = 0.39

$$\frac{\text{expIntegral}\left(-\ln\left(x\right)^{2}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^(-ln(x)),x)
```

```
[Out] 1/2*Ei(-ln(x)^2)
```

3.308 $\int \frac{1-2x}{x^{2/3}(1+x)^2}\,dx$

Optimal. Leaf size=12

$$\frac{3\sqrt[3]{x}}{1+x}$$

[Out] 3*x^(1/3)/(x+1)

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {75}

$$\frac{3\sqrt[3]{x}}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - 2*x)/(x^(2/3)*(1 + x)^2),x]

[Out] (3*x^(1/3))/(1 + x)

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\text{Integral} = \frac{3\sqrt[3]{x}}{1+x}$$

Mathematica [A]
time = 0.03, size = 12, normalized size = 1.00

$$\frac{3\sqrt[3]{x}}{1+x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 2*x)/(x^(2/3)*(1 + x)^2),x]

[Out] (3*x^(1/3))/(1 + x)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.
time = 0.09, size = 32, normalized size = 2.67

method	result
gosper	$\frac{3x^{\frac{1}{3}}}{x+1}$
trager	$\frac{3x^{\frac{1}{3}}}{x+1}$
risch	$\frac{3x^{\frac{1}{3}}}{x+1}$
derivativedivides	$-\frac{-x^{\frac{1}{3}}-1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{1}{1+x^{\frac{1}{3}}}$
default	$-\frac{-x^{\frac{1}{3}}-1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{1}{1+x^{\frac{1}{3}}}$
meijerg	$\frac{3x^{\frac{1}{3}}}{3+3x}+\frac{2\ln\left(1+x^{\frac{1}{3}}\right)}{3}-\frac{\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{3}}}{2-x^{\frac{1}{3}}}\right)}{3}+\frac{2x^{\frac{1}{3}}}{x+1}-\frac{2x^{\frac{1}{3}}\left(\frac{\ln\left(1+x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{2x^{\frac{1}{3}}}\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*x)/(x+1)^2/x^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^(1/3)-1)/(x^(2/3)-x^(1/3)+1)-1/(1+x^(1/3))
```

Maxima [A]
time = 0.34, size = 10, normalized size = 0.83

$$\frac{3\,x^{\frac{1}{3}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x+1)^2/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3)/(x + 1)
```

Fricas [A]
time = 0.56, size = 10, normalized size = 0.83

$$\frac{3\,x^{\frac{1}{3}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x+1)^2/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3)/(x + 1)
```

Sympy [A]
time = 0.28, size = 8, normalized size = 0.67

$$\frac{3\sqrt[3]{x}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x+1)**2/x**(2/3),x)
```

```
[Out] 3*x**(1/3)/(x + 1)
```

Giac [A]
time = 0.44, size = 10, normalized size = 0.83

$$\frac{3\,x^{\frac{1}{3}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)/(x+1)^2/x^(2/3),x, algorithm="giac")
```

```
[Out] 3*x^(1/3)/(x + 1)
```

Mupad [B]
time = 0.08, size = 10, normalized size = 0.83

$$\frac{3\,x^{1/3}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 1)/(x^(2/3)*(x + 1)^2),x)
```

```
[Out] (3*x^(1/3))/(x + 1)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((1-2*x)/(x+1)^2/x^(2/3),x)
```

```
[Out] not solved
```

3.309 $\int \cos^2\left(\frac{\pi x^2}{\sqrt{2}}\right) dx$

Optimal. Leaf size=23

$$\frac{x}{2}+\frac{\text{FresnelC}\left(2^{3/4}x\right)}{2\ 2^{3/4}}$$

```
[Out] 1/2*x+1/4*FresnelC(2^(3/4)*x)*2^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3439, 3433}

$$\frac{\text{FresnelC}\left(2^{3/4}x\right)}{2\ 2^{3/4}}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(Pi*x^2)/Sqrt[2]]^2,x]
```

```
[Out] x/2 + FresnelC[2^(3/4)*x]/(2*2^(3/4))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3439

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{1}{2}+\frac{1}{2}\cos\left(\sqrt{2}\pi x^2\right)\right) dx\\
&= \frac{x}{2}+\frac{1}{2}\int \cos\left(\sqrt{2}\pi x^2\right) dx\\
&= \frac{x}{2}+\frac{\text{FresnelC}\left(2^{3/4}x\right)}{2\ 2^{3/4}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.96

$$\frac{1}{4}\left(2x+\sqrt[4]{2}\,\text{FresnelC}\left(2^{3/4}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(Pi*x^2)/Sqrt[2]]^2,x]
```

```
[Out] (2*x + 2^(1/4)*FresnelC[2^(3/4)*x])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.12, size = 34, normalized size = 1.48

method	result	size
default	$\frac{x}{2}+\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{2\sqrt{\pi}\,x}{\sqrt{\pi\sqrt{2}}}\right)}{4\sqrt{\pi\sqrt{2}}}$	34
risch	$\frac{x}{2}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{i\pi\sqrt{2}}\,x\right)}{8\sqrt{i\pi\sqrt{2}}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-i\pi\sqrt{2}}\,x\right)}{8\sqrt{-i\pi\sqrt{2}}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*Pi*x^2*2^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/4*2^(1/2)*Pi^(1/2)/(Pi*2^(1/2))^(1/2)*FresnelC(2*Pi^(1/2)/(Pi*2^(1/
2))^(1/2)*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.45, size = 46, normalized size = 2.00

$$-\frac{2^{\frac{1}{4}}\pi^{2}\left((i-1)\,\mathrm{erf}\left(\sqrt{i\sqrt{2}\pi}x\right)-(i+1)\,\mathrm{erf}\left(\sqrt{-i\sqrt{2}\pi}x\right)\right)-8\,\pi^{2}x}{16\,\pi^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*pi*x^2*2^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2^(1/4)*pi^2*((I - 1)*erf(sqrt(I*sqrt(2)*pi)*x) - (I + 1)*erf(sqrt(-
I*sqrt(2)*pi)*x)) - 8*pi^2*x)/pi^2
```

Fricas [A]
time = 0.61, size = 15, normalized size = 0.65

$$\frac{1}{4}\cdot 2^{\frac{1}{4}}\,\mathrm{C}\left(2^{\frac{3}{4}}x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*pi*x^2*2^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*2^(1/4)*fresnel_cos(2^(3/4)*x) + 1/2*x
```

Sympy [A]
time = 1.48, size = 27, normalized size = 1.17

$$\frac{x}{2}+\frac{\sqrt[4]{2}C\left(2^{\frac{3}{4}}x\right)\Gamma\left(\frac{1}{4}\right)}{16\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*pi*x**2*2**(1/2))**2,x)
```

```
[Out] x/2 + 2**(1/4)*fresnelc(2**(3/4)*x)*gamma(1/4)/(16*gamma(5/4))
```

Giac [C] Result contains complex when optimal does not.
time = 0.47, size = 42, normalized size = 1.83

$$-\left(\frac{1}{16}i+\frac{1}{16}\right)\cdot 2^{\frac{1}{4}}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\sqrt{2}\pi}x\right)+\left(\frac{1}{16}i-\frac{1}{16}\right)\cdot 2^{\frac{1}{4}}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\sqrt{2}\pi}x\right)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*pi*x^2*2^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(1/16*I + 1/16)*2^(1/4)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(sqrt(2)*pi)*x) + (1
/16*I - 1/16)*2^(1/4)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(sqrt(2)*pi)*x) + 1/2*
x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(\frac{\sqrt{2}\,\Pi\,x^2}{2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((2^(1/2)*Pi*x^2)/2)^2,x)
```

```
[Out] int(cos((2^(1/2)*Pi*x^2)/2)^2, x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(cos(1/2*Pi*x^2*2^(1/2))^2,x)
```

```
[Out] not solved
```

3.310 $\int \frac{1}{1+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=9

$$\log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] ln(1+tan(1/2*x))
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3203, 31}

$$\log\left(\tan\left(\frac{x}{2}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x] + Sin[x])^(-1),x]
```

```
[Out] Log[1 + Tan[x/2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\text{Integral} &= 2\text{Subst}\left(\int \frac{1}{2+2x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \log\left(1+\tan\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(9) = 18.
time = 0.01, size = 24, normalized size = 2.67

$$-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.09, size = 8, normalized size = 0.89

method	result	size
default	$\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	8
parallelrisch	$\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	8
norman	$\ln\left(2+2\tan\left(\frac{x}{2}\right)\right)$	10
risch	$\ln\left(i+\mathrm{e}^{ix}\right)-\ln\left(\mathrm{e}^{ix}+1\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+tan(1/2*x))
```

Maxima [A]
time = 0.33, size = 12, normalized size = 1.33

$$\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x)/(cos(x) + 1) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.
time = 0.60, size = 17, normalized size = 1.89

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(sin(x) + 1)
```

Sympy [A]
time = 0.10, size = 7, normalized size = 0.78

$$\log\left(\tan\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x)),x)
```

```
[Out] log(tan(x/2) + 1)
```

Giac [A]
time = 0.60, size = 8, normalized size = 0.89

$$\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x) + 1))
```

Mupad [B]
time = 0.06, size = 7, normalized size = 0.78

$$\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + sin(x) + 1),x)
```

```
[Out] log(tan(x/2) + 1)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 19, normalized size = 2.11

$$-2\ln\left(\frac{\tan\left(\frac{x}{2}\right)}{2}+\frac{1}{2}-\frac{\sin\left(x\right)}{2\cos\left(x\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] int(1/(1+cos(x)+sin(x)),x)
```

```
[Out] -2*ln(1/2*tan(1/2*x)+1/2-1/2*sin(x)/cos(x))
```

3.311 $\int \tan^5(x)\,dx$

Optimal. Leaf size=22

$$-\log(\cos(x))-\frac{\tan^2(x)}{2}+\frac{\tan^4(x)}{4}$$

```
[Out] -ln(cos(x))-1/2*tan(x)^2+1/4*tan(x)^4
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3556}

$$\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{\tan^4(x)}{4}-\int \tan^3(x)\,dx\\
&= -\frac{1}{2}\tan^2(x)+\frac{\tan^4(x)}{4}+\int \tan(x)\,dx\\
&= -\log(\cos(x))-\frac{\tan^2(x)}{2}+\frac{\tan^4(x)}{4}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.91

$$-\log(\cos(x))-\sec^2(x)+\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Sec[x]^2 + Sec[x]^4/4
```

Maple [A]
time = 0.04, size = 23, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(\tan^4(x))}{4}-\frac{(\tan^2(x))}{2}+\frac{\ln(1+\tan^2(x))}{2}$	23
default	$\frac{(\tan^4(x))}{4}-\frac{(\tan^2(x))}{2}+\frac{\ln(1+\tan^2(x))}{2}$	23
norman	$\frac{(\tan^4(x))}{4}-\frac{(\tan^2(x))}{2}+\frac{\ln(1+\tan^2(x))}{2}$	23
parallelrisch	$\frac{(\tan^4(x))}{4}-\frac{(\tan^2(x))}{2}+\frac{\ln(1+\tan^2(x))}{2}$	23
risch	$ix-\frac{4(e^{6ix}+e^{4ix}+e^{2ix})}{(e^{2ix}+1)^4}-\ln(e^{2ix}+1)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*tan(x)^4-1/2*tan(x)^2+1/2*ln(1+tan(x)^2)
```

Maxima [A]
time = 0.34, size = 34, normalized size = 1.55

$$\frac{4\sin(x)^2-3}{4\left(\sin(x)^4-2\sin(x)^2+1\right)}-\frac{1}{2}\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2*log(sin(x)^2 - 1)
```

Fricas [A]
time = 0.60, size = 24, normalized size = 1.09

$$\frac{1}{4}\tan(x)^4-\frac{1}{2}\tan(x)^2-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 - 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.91

$$-\frac{4\cos^2(x)-1}{4\cos^4(x)}-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**5,x)
```

```
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) - log(cos(x))
```

Giac [A]
time = 0.50, size = 22, normalized size = 1.00

$$\frac{1}{4}\tan(x)^4-\frac{1}{2}\tan(x)^2+\frac{1}{2}\log\left(\tan(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 + 1/2*log(tan(x)^2 + 1)
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.82

$$\frac{\tan(x)^4}{4}-\frac{\tan(x)^2}{2}-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5,x)
```

```
[Out] tan(x)^4/4 - tan(x)^2/2 - log(cos(x))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 6, normalized size = 0.27

$$\frac{(\tan^5(x))}{5}$$

Warning: Unable to verify antiderivative.

```
[In] int(tan(x)^5,x)
```

```
[Out] 1/5*tan(x)^5
```

3.312 $\int \sqrt{1+\frac{1}{x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{1+\frac{1}{x}}x+\operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

```
[Out] (1+1/x)^(1/2)*x+arctanh((1+1/x)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {248, 43, 65, 213}

$$\operatorname{arctanh}\left(\sqrt{\frac{1}{x}+1}\right)+\sqrt{\frac{1}{x}+1}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{1+\frac{1}{x}}x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{1+\frac{1}{x}}x - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right) \\
&= \sqrt{1+\frac{1}{x}}x + \text{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\sqrt{1+\frac{1}{x}}x + \text{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)],x]
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.
time = 0.07, size = 41, normalized size = 1.86

method	result	size
trager	$x\sqrt{-\frac{-x-1}{x}} - \frac{\ln\left(2x\sqrt{-\frac{-x-1}{x}}-2x-1\right)}{2}$	39
default	$\frac{\sqrt{\frac{x+1}{x}}\,x\left(2\sqrt{x^2+x}+\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)}{2\sqrt{x(x+1)}}$	41
risch	$x\sqrt{\frac{x+1}{x}} + \frac{\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\sqrt{\frac{x+1}{x}}\sqrt{x(x+1)}}{2x+2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/2*((x+1)/x)^(1/2)*x*(2*(x^2+x)^(1/2)+ln(1/2+x+(x^2+x)^(1/2)))/(x*(x+1))^(1/2)

Maxima [A]
time = 0.34, size = 34, normalized size = 1.55

$$x\sqrt{\frac{1}{x}+1}+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2),x, algorithm="maxima")

[Out] x*sqrt(1/x + 1) + 1/2*log(sqrt(1/x + 1) + 1) - 1/2*log(sqrt(1/x + 1) - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.
time = 0.57, size = 40, normalized size = 1.82

$$x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2),x, algorithm="fricas")

[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x) - 1)

Sympy [A]
time = 0.90, size = 17, normalized size = 0.77

$$\sqrt{x}\sqrt{x+1}+\operatorname{asinh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)**(1/2),x)

[Out] sqrt(x)*sqrt(x + 1) + asinh(sqrt(x))

Giac [A]
time = 0.55, size = 31, normalized size = 1.41

$$-\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x)+\sqrt{x^2+x}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2),x, algorithm="giac")

```
[Out] -1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

Mupad [B]
time = 0.08, size = 38, normalized size = 1.73

$$x\sqrt{\frac{1}{x}+1}+\frac{x\ln\left(x+\sqrt{x^2+x}+\frac{1}{2}\right)\sqrt{\frac{1}{x}+1}}{2\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2),x)
[Out] x*(1/x + 1)^(1/2) + (x*log(x + (x + x^2)^(1/2) + 1/2)*(1/x + 1)^(1/2))/(2*(
x + x^2)^(1/2))
```

Chatgpt [F] Failed to verify
time = 1.00, size = 9, normalized size = 0.41

$$-\frac{4\left(1+\frac{1}{x}\right)^{\frac{3}{2}}}{3}$$

Warning: Unable to verify antiderivative.

```
[In] int((1+1/x)^(1/2),x)
[Out] -4/3*(1+1/x)^(3/2)
```

3.313 $\int e^{\cos(x)} \cos(2x + \sin(x)) \, dx$

Optimal. Leaf size=21

$$2e^{\cos(x)} \cos\left(\frac{x}{2} + \sin(x)\right) \sin\left(\frac{x}{2}\right)$$

```
[Out] 2*exp(cos(x))*cos(1/2*x+sin(x))*sin(1/2*x)
```

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int e^{\cos(x)} \cos(2x + \sin(x)) \, dx$$

Verification is not applicable to the result.

```
[In] Int[E^Cos[x]*Cos[2*x + Sin[x]],x]
[Out] Defer[Int][E^Cos[x]*Cos[2*x + Sin[x]], x]
```

Rubi steps

$$\text{Integral} = \int e^{\cos(x)} \cos(2x + \sin(x)) \, dx$$

Mathematica [A]
time = 0.05, size = 21, normalized size = 1.00

$$2e^{\cos(x)} \cos\left(\frac{x}{2} + \sin(x)\right) \sin\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Cos[x]*Cos[2*x + Sin[x]],x]
[Out] 2*E^Cos[x]*Cos[x/2 + Sin[x]]*Sin[x/2]
```

Maple [C] Result contains complex when optimal does not.
time = 1.80, size = 52, normalized size = 2.48

method	result	size
risch	$-\frac{i e^{e^{ix}} e^{ix}}{2} + \frac{i e^{e^{ix}}}{2} + \frac{i e^{e^{-ix}} e^{-ix}}{2} - \frac{i e^{e^{-ix}}}{2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x))*cos(2*x+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*exp(exp(I*x))*exp(I*x)+1/2*I*exp(exp(I*x))+1/2*I*exp(1/exp(I*x))*exp
(-I*x)-1/2*I*exp(1/exp(I*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x))*cos(2*x+sin(x)),x, algorithm="maxima")
```

```
[Out] 1/4*e^cos(x)*sin(2*x + sin(x)) + 1/2*e^cos(x)*sin(x + sin(x)) - 1/2*e^cos(x
)*sin(sin(x)) - 1/4*integrate(cos(3*x + sin(x))*e^cos(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(16) = 32$.
time = 0.62, size = 84, normalized size = 4.00

$$(2 \cos(x) - 1)\cos\left(\frac{2\left(x\tan\left(\frac{1}{2}x\right)^2 + x + \tan\left(\frac{1}{2}x\right)\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)e^{\cos(x)}\sin(x) - \left(2\cos(x)^2 - \cos(x) - 1\right)e^{\cos(x)}\sin\left(\frac{2\left(x\tan\left(\frac{1}{2}x\right)^2 + x + \tan\left(\frac{1}{2}x\right)\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x))*cos(2*x+sin(x)),x, algorithm="fricas")
```

```
[Out] (2*cos(x) - 1)*cos(2*(x*tan(1/2*x)^2 + x + tan(1/2*x))/(tan(1/2*x)^2 + 1))*
e^cos(x)*sin(x) - (2*cos(x)^2 - cos(x) - 1)*e^cos(x)*sin(2*(x*tan(1/2*x)^2
+ x + tan(1/2*x))/(tan(1/2*x)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\cos(x)}\cos(2x + \sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x))*cos(2*x+sin(x)),x)
```

```
[Out] Integral(exp(cos(x))*cos(2*x + sin(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(16) = 32$.
time = 0.55, size = 136, normalized size = 6.48

$$\frac{\cos(x)^3 e^{\cos(x)}\sin(2x + \sin(x)) - \cos(2x + \sin(x))\cos(x)^2 e^{\cos(x)}\sin(x) + \cos(x) e^{\cos(x)}\sin(2x + \sin(x))\sin(x)^2 - \cos(2x + \sin(x)) e^{\cos(x)}\sin(x)^3 - \cos(x)^2 e^{\cos(x)}\sin(2x + \sin(x)) + 2\cos(2x + \sin(x))\cos(x) e^{\cos(x)}\sin(x) + e^{\cos(x)}\sin(2x + \sin(x))\sin(x)^2}{\cos(x)^4 + 2\cos(x)^2\sin(x)^2 + \sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x))*cos(2*x+sin(x)),x, algorithm="giac")
```

```
[Out] (cos(x)^3*e^cos(x)*sin(2*x + sin(x)) - cos(2*x + sin(x))*cos(x)^2*e^cos(x)*
sin(x) + cos(x)*e^cos(x)*sin(2*x + sin(x))*sin(x)^2 - cos(2*x + sin(x))*e^c
os(x)*sin(x)^3 - cos(x)^2*e^cos(x)*sin(2*x + sin(x)) + 2*cos(2*x + sin(x))*
cos(x)*e^cos(x)*sin(x) + e^cos(x)*sin(2*x + sin(x))*sin(x)^2)/(cos(x)^4 + 2
*cos(x)^2*sin(x)^2 + sin(x)^4)
```

Mupad [B]
time = 0.36, size = 15, normalized size = 0.71

$$e^{\cos(x)}\left(\sin\left(x+\sin\left(x\right)\right)-\sin\left(\sin\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x))*cos(2*x + sin(x)),x)
```

```
[Out] exp(cos(x))*(sin(x + sin(x)) - sin(sin(x)))
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(exp(cos(x))*cos(2*x+sin(x)),x)
```

```
[Out] not solved
```

3.314 $\int \frac{-1+2x+3\log(x)}{x^2+2x^4+x\log(x)}\,dx$

Optimal. Leaf size=17

$$3\log(x) - \log\left(x + 2x^3 + \log(x)\right)$$

```
[Out] 3*ln(x)-ln(x+2*x^3+ln(x))
```

Rubi [A]
time = 0.07, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6874, 6816}

$$3\log(x) - \log\left(2x^3 + x + \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + 3*Log[x])/(x^2 + 2*x^4 + x*Log[x]),x]
```

```
[Out] 3*Log[x] - Log[x + 2*x^3 + Log[x]]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \int \left(\frac{3}{x} + \frac{-1-x-6x^3}{x\left(x+2x^3+\log(x)\right)}\right) dx \\
&= 3\log(x) + \int \frac{-1-x-6x^3}{x\left(x+2x^3+\log(x)\right)}\,dx \\
&= 3\log(x) - \log\left(x+2x^3+\log(x)\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 17, normalized size = 1.00

$$3\log(x) - \log\left(x + 2x^3 + \log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + 3*Log[x])/(x^2 + 2*x^4 + x*Log[x]),x]
[Out] 3*Log[x] - Log[x + 2*x^3 + Log[x]]
```

Maple [A]
time = 0.13, size = 18, normalized size = 1.06

method	result	size
default	$3\ln(x)-\ln(x+2x^{3}+\ln(x))$	18
norman	$3\ln(x)-\ln(x+2x^{3}+\ln(x))$	18
risch	$3\ln(x)-\ln(x+2x^{3}+\ln(x))$	18
parallelrisch	$-\ln\left(x^{3}+\frac{x}{2}+\frac{\ln(x)}{2}\right)+3\ln(x)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*ln(x)-1+2*x)/(x*ln(x)+x^2+2*x^4),x,method=_RETURNVERBOSE)
[Out] 3*ln(x)-ln(x+2*x^3+ln(x))
```

Maxima [A]
time = 0.36, size = 17, normalized size = 1.00

$$-\log\left(2x^{3}+x+\log(x)\right)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*log(x)-1+2*x)/(x*log(x)+x^2+2*x^4),x, algorithm="maxima")
[Out] -log(2*x^3 + x + log(x)) + 3*log(x)
```

Fricas [A]
time = 0.59, size = 17, normalized size = 1.00

$$-\log\left(2x^{3}+x+\log(x)\right)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*log(x)-1+2*x)/(x*log(x)+x^2+2*x^4),x, algorithm="fricas")
[Out] -log(2*x^3 + x + log(x)) + 3*log(x)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.88

$$3\log(x)-\log\left(2x^{3}+x+\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*ln(x)-1+2*x)/(x*ln(x)+x**2+2*x**4),x)
```

```
[Out] 3*log(x) - log(2*x**3 + x + log(x))
```

Giac [A]
time = 0.44, size = 21, normalized size = 1.24

$$-\log\left(-2\,x^3 - x - \log\left(x\right)\right) + 3\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*log(x)-1+2*x)/(x*log(x)+x^2+2*x^4),x, algorithm="giac")
[Out] -log(-2*x^3 - x - log(x)) + 3*log(x)
```

Mupad [B]
time = 0.21, size = 17, normalized size = 1.00

$$3\,\ln\left(x\right) - \ln\left(x + \ln\left(x\right) + 2\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*log(x) - 1)/(x*log(x) + x^2 + 2*x^4),x)
[Out] 3*log(x) - log(x + log(x) + 2*x^3)
```

Chatgpt [F] Failed to verify
time = 1.00, size = 17, normalized size = 1.00

$$3\ln\left(x\right) + \arctan\left(x^2\right) - \ln\left(x^2 + 1\right)$$

Warning: Unable to verify antiderivative.

```
[In] int((3*ln(x)-1+2*x)/(x*ln(x)+x^2+2*x^4),x)
[Out] 3*ln(x)+arctan(x^2)-ln(x^2+1)
```

3.315 $\int \left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\,dx$

Optimal. Leaf size=45

$$-\frac{2\left(-\sqrt{x}+\sqrt{1+x}\right)^{\pi}\left(1+2x+\pi\sqrt{x}\sqrt{1+x}\right)}{-4+\pi^2}$$

```
[Out] -2*((x+1)^(1/2)-x^(1/2))^Pi*(1+2*x+Pi*x^(1/2)*(x+1)^(1/2))/(Pi^2-4)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2144, 459}

$$\frac{\left(\sqrt{x+1}-\sqrt{x}\right)^{2+\pi}}{2(2+\pi)}+\frac{\left(\sqrt{x+1}-\sqrt{x}\right)^{\pi-2}}{2(2-\pi)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[x] + Sqrt[1 + x])^Pi,x]
```

```
[Out] (-Sqrt[x] + Sqrt[1 + x])^(-2 + Pi)/(2*(2 - Pi)) + (-Sqrt[x] + Sqrt[1 + x])^
(2 + Pi)/(2*(2 + Pi))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2144

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{Integral} &= 2\text{Subst}\left(\int x\left(-x+\sqrt{1+x^2}\right)^{\pi}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int x^{-3+\pi}\left(-1+x^2\right)\left(1+x^2\right)\,dx, x, -\sqrt{x}+\sqrt{1+x}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-x^{-3+\pi}+x^{1+\pi}\right)\,dx, x, -\sqrt{x}+\sqrt{1+x}\right)\\
&= \frac{\left(-\sqrt{x}+\sqrt{1+x}\right)^{-2+\pi}}{2(2-\pi)}+\frac{\left(-\sqrt{x}+\sqrt{1+x}\right)^{2+\pi}}{2(2+\pi)}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(45) = 90$.
time = 0.30, size = 107, normalized size = 2.38

$$
\frac{2\sqrt{1+x}\left(-\sqrt{x}+\sqrt{1+x}\right)^{-1+\pi}\left(1-2(-2+\pi)x-2(-2+\pi)x^2+(-2+\pi)\sqrt{x}\sqrt{1+x}+2(-2+\pi)x^{3/2}\sqrt{1+x}\right)}{(-2+\pi)(2+\pi)\left(-1-x+\sqrt{x}\sqrt{1+x}\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[x] + Sqrt[1 + x])^Pi,x]
```

```
[Out] (2*Sqrt[1 + x]*(-Sqrt[x] + Sqrt[1 + x])^(-1 + Pi)*(1 - 2*(-2 + Pi)*x - 2*(-
2 + Pi)*x^2 + (-2 + Pi)*Sqrt[x]*Sqrt[1 + x] + 2*(-2 + Pi)*x^(3/2)*Sqrt[1 +
x]))/((-2 + Pi)*(2 + Pi)*(-1 - x + Sqrt[x]*Sqrt[1 + x]))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$
\int \left(\sqrt{x+1}-\sqrt{x}\right)^{\pi}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)^(1/2)-x^(1/2))^Pi,x)
```

```
[Out] int(((x+1)^(1/2)-x^(1/2))^Pi,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x+1)^(1/2)-x^(1/2))^pi,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) - sqrt(x))^pi, x)
```

Fricas [A]
time = 0.72, size = 37, normalized size = 0.82

$$-\frac{2\left(\pi\sqrt{x+1}\sqrt{x}+2\,x+1\right)\left(\sqrt{x+1}-\sqrt{x}\right)^{\pi}}{\pi^{2}-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x+1)^(1/2)-x^(1/2))^pi,x, algorithm="fricas")
```

```
[Out] -2*(pi*sqrt(x + 1)*sqrt(x) + 2*x + 1)*(sqrt(x + 1) - sqrt(x))^pi/(pi^2 - 4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4362 vs. 2(39) = 78.
time = 3.77, size = 4362, normalized size = 96.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x+1)**(1/2)-x**(1/2))**pi,x)
```

```
[Out] Piecewise((4*x**(13/2)*sqrt(1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(sqrt(x)
))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) -
4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) + 2*pi*x**(13/2)*sqrt(
1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*
gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi*
*2*x**4*gamma(1 + pi/2)) - 2*pi*x**(13/2)*cosh(asinh(sqrt(x)) + pi*asinh(sq
rt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/
2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 4*x**(13/2)*cos
h(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi
/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamm
a(1 + pi/2)) + 6*x**(11/2)*sqrt(1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(sqr
t(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2
) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) + 2*pi*x**(11/2)*s
qrt(1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x
**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) +
 pi**2*x**4*gamma(1 + pi/2)) - 4*pi*x**(11/2)*cosh(asinh(sqrt(x)) + pi*asin
h(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 +
 pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 6*x**(11/2)
*cosh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1
+ pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*
gamma(1 + pi/2)) + 2*x**(9/2)*sqrt(1 + 1/x)*sinh(asinh(sqrt(x)) + pi*asinh(
sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + p
i/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 2*pi*x**(9/2)
*cosh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(1 + pi/2)/(-4*x**5*gamma(1
+ pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*
```

```
gamma(1 + pi/2)) - 2*x**(9/2)*cosh(asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamm
a(1 + pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*
gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) + 4*pi*x**7*sqrt(1 + 1/x)*sin
h(2*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2
) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(
1 + pi/2)) + 2*pi**2*x**7*sqrt(1 + 1/x)*sinh(2*asinh(sqrt(x)) + pi*asinh(sq
rt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) -
 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 2*pi**2*x**7*cosh(2
*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) +
 pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 +
 pi/2)) - 4*pi*x**7*cosh(2*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/
(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi
/2) + pi**2*x**4*gamma(1 + pi/2)) + 6*pi*x**6*sqrt(1 + 1/x)*sinh(2*asinh(sq
rt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x*
*5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) +
 3*pi**2*x**6*sqrt(1 + 1/x)*sinh(2*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamm
a(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamm
a(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 4*pi**2*x**6*cosh(2*asinh(sqrt(
x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*
gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 8*
pi*x**6*cosh(2*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gam
ma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*
x**4*gamma(1 + pi/2)) + 2*pi*x**5*sqrt(1 + 1/x)*sinh(2*asinh(sqrt(x)) + pi*
asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 +
 pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) + pi**2*x**5*
sqrt(1 + 1/x)*sinh(2*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x*
*5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) +
pi**2*x**4*gamma(1 + pi/2)) - 2*pi**2*x**5*cosh(2*asinh(sqrt(x)) + pi*asinh
(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1 + pi/2
) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - 5*pi*x**5*cosh(2
*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) +
 pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 +
 pi/2)) + pi*x**5*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*gamma(1
 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)) - pi*x**4*c
osh(2*asinh(sqrt(x)) + pi*asinh(sqrt(x)))*gamma(pi/2)/(-4*x**5*gamma(1 + pi
/2) + pi**2*x**5*gamma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamm
a(1 + pi/2)) + pi*x**4*gamma(pi/2)/(-4*x**5*gamma(1 + pi/2) + pi**2*x**5*ga
mma(1 + pi/2) - 4*x**4*gamma(1 + pi/2) + pi**2*x**4*gamma(1 + pi/2)), Abs(x
) > 1), (4*pi*x**(17/2)*sqrt(x + 1)*sinh(2*asinh(sqrt(x)) + pi*asinh(sqrt(x
)))*gamma(pi/2)/(-4*x**7*gamma(1 + pi/2) + pi**2*x**7*gamma(1 + pi/2) - 4*x
**6*gamma(1 + pi/2) + pi**2*x**6*gamma(1 + pi/2)) + 2*pi**2*x**(17/2)*sqrt(
x + 1)*sinh(2*asinh(sqrt(x)) + pi*asinh(sqrt(x)...
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x+1)^(1/2)-x^(1/2))^pi,x, algorithm="giac")
[Out] integrate((sqrt(x + 1) - sqrt(x))^pi, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\sqrt{x+1}-\sqrt{x}\right)^{\Pi}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) - x^(1/2))^Pi,x)
[Out] int(((x + 1)^(1/2) - x^(1/2))^Pi, x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(((x+1)^(1/2)-x^(1/2))^Pi,x)
[Out] not solved
```

3.316 $\int \left(-2+\left(-2+\left(-2+(-2+x^2)^2\right)^2\right)^2\right)\,dx$

Optimal. Leaf size=54

$$2x-\frac{64x^3}{3}+\frac{336x^5}{5}-96x^7+\frac{220x^9}{3}-32x^{11}+8x^{13}-\frac{16x^{15}}{15}+\frac{x^{17}}{17}$$

```
[Out] 2*x-64/3*x^3+336/5*x^5-96*x^7+220/3*x^9-32*x^11+8*x^13-16/15*x^15+1/17*x^17
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2086}

$$\frac{x^{17}}{17}-\frac{16x^{15}}{15}+8x^{13}-32x^{11}+\frac{220x^9}{3}-96x^7+\frac{336x^5}{5}-\frac{64x^3}{3}+2x$$

Antiderivative was successfully verified.

```
[In] Int[-2 + (-2 + (-2 + (-2 + x^2)^2)^2)^2,x]
```

```
[Out] 2*x - (64*x^3)/3 + (336*x^5)/5 - 96*x^7 + (220*x^9)/3 - 32*x^11 + 8*x^13 -
(16*x^15)/15 + x^17/17
```

Rule 2086

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= -2x+\int \left(-2+\left(-2+(-2+x^2)^2\right)^2\right)^2\,dx\\
&= -2x+\int \left(4-64x^2+336x^4-672x^6+660x^8-352x^{10}+104x^{12}-16x^{14}+x^{16}\right)\,dx\\
&= 2x-\frac{64x^3}{3}+\frac{336x^5}{5}-96x^7+\frac{220x^9}{3}-32x^{11}+8x^{13}-\frac{16x^{15}}{15}+\frac{x^{17}}{17}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 54, normalized size = 1.00

$$2x-\frac{64x^3}{3}+\frac{336x^5}{5}-96x^7+\frac{220x^9}{3}-32x^{11}+8x^{13}-\frac{16x^{15}}{15}+\frac{x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[-2 + (-2 + (-2 + (-2 + x^2)^2)^2)^2,x]
```

```
[Out] 2*x - (64*x^3)/3 + (336*x^5)/5 - 96*x^7 + (220*x^9)/3 - 32*x^11 + 8*x^13 -
(16*x^15)/15 + x^17/17
```

Maple [A]
time = 0.08, size = 45, normalized size = 0.83

method	result	size
default	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	45
norman	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	45
risch	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	45
parallelrisch	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	45
parts	$2x-\frac{64}{3}x^{3}+\frac{336}{5}x^{5}-96x^{7}+\frac{220}{3}x^{9}-32x^{11}+8x^{13}-\frac{16}{15}x^{15}+\frac{1}{17}x^{17}$	45
gosper	$\frac{x\left(15x^{16}-272x^{14}+2040x^{12}-8160x^{10}+18700x^{8}-24480x^{6}+17136x^{4}-5440x^{2}+510\right)}{255}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^2-2)^2-2)^2-2)^2-2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x-64/3*x^3+336/5*x^5-96*x^7+220/3*x^9-32*x^11+8*x^13-16/15*x^15+1/17*x^17
```

Maxima [A]
time = 0.34, size = 44, normalized size = 0.81

$$\frac{1}{17}\,x^{17}-\frac{16}{15}\,x^{15}+8\,x^{13}-32\,x^{11}+\frac{220}{3}\,x^{9}-96\,x^{7}+\frac{336}{5}\,x^{5}-\frac{64}{3}\,x^{3}+2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((x^2-2)^2-2)^2-2)^2-2,x, algorithm="maxima")
```

```
[Out] 1/17*x^17 - 16/15*x^15 + 8*x^13 - 32*x^11 + 220/3*x^9 - 96*x^7 + 336/5*x^5
- 64/3*x^3 + 2*x
```

Fricas [A]
time = 0.56, size = 44, normalized size = 0.81

$$\frac{1}{17}\,x^{17}-\frac{16}{15}\,x^{15}+8\,x^{13}-32\,x^{11}+\frac{220}{3}\,x^{9}-96\,x^{7}+\frac{336}{5}\,x^{5}-\frac{64}{3}\,x^{3}+2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((x^2-2)^2-2)^2-2)^2-2,x, algorithm="fricas")
```

```
[Out] 1/17*x^17 - 16/15*x^15 + 8*x^13 - 32*x^11 + 220/3*x^9 - 96*x^7 + 336/5*x^5
- 64/3*x^3 + 2*x
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.91

$$\frac{x^{17}}{17} - \frac{16x^{15}}{15} + 8x^{13} - 32x^{11} + \frac{220x^9}{3} - 96x^7 + \frac{336x^5}{5} - \frac{64x^3}{3} + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((x**2-2)**2-2)**2-2)**2-2,x)
```

```
[Out] x**17/17 - 16*x**15/15 + 8*x**13 - 32*x**11 + 220*x**9/3 - 96*x**7 + 336*x*
*5/5 - 64*x**3/3 + 2*x
```

Giac [A]
time = 0.45, size = 44, normalized size = 0.81

$$\frac{1}{17}\,x^{17} - \frac{16}{15}\,x^{15} + 8\,x^{13} - 32\,x^{11} + \frac{220}{3}\,x^9 - 96\,x^7 + \frac{336}{5}\,x^5 - \frac{64}{3}\,x^3 + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((((x^2-2)^2-2)^2-2)^2-2,x, algorithm="giac")
```

```
[Out] 1/17*x^17 - 16/15*x^15 + 8*x^13 - 32*x^11 + 220/3*x^9 - 96*x^7 + 336/5*x^5
- 64/3*x^3 + 2*x
```

Mupad [B]
time = 0.10, size = 44, normalized size = 0.81

$$\frac{x^{17}}{17} - \frac{16\,x^{15}}{15} + 8\,x^{13} - 32\,x^{11} + \frac{220\,x^9}{3} - 96\,x^7 + \frac{336\,x^5}{5} - \frac{64\,x^3}{3} + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^2 - 2)^2 - 2)^2 - 2)^2 - 2,x)
```

```
[Out] 2*x - (64*x^3)/3 + (336*x^5)/5 - 96*x^7 + (220*x^9)/3 - 32*x^11 + 8*x^13 -
(16*x^15)/15 + x^17/17
```

Chatgpt [F] Failed to verify
time = 1.00, size = 25, normalized size = 0.46

$$\frac{(x^2-2)^5}{10} - \frac{4(x^2-2)^3}{3} + 2x^2 - 4$$

Warning: Unable to verify antiderivative.

```
[In] int((((x^2-2)^2-2)^2-2)^2-2,x)
```

```
[Out] 1/10*(x^2-2)^5-4/3*(x^2-2)^3+2*x^2-4
```

3.317 $\int \sin(4\arctan(x))\,dx$

Optimal. Leaf size=18

$$-\frac{4}{1+x^2}-2\log\left(1+x^2\right)$$

```
[Out] -4/(x^2+1)-2*ln(x^2+1)
```

Rubi [F]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \sin(4\arctan(x))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sin[4*ArcTan[x]],x]
[Out] Defer[Int][Sin[4*ArcTan[x]], x]
```

Rubi steps

$$\text{Integral}=\int \sin(4\arctan(x))\,dx$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{4}{1+x^2}-2\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[4*ArcTan[x]],x]
[Out] -4/(1 + x^2) - 2*Log[1 + x^2]
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 34, normalized size = 1.89

method	result	size
default	$-\frac{2i}{i+x}-2\ln\left(i+x\right)+\frac{2i}{x-i}-2\ln\left(x-i\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*arctan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(I+x)-2*ln(I+x)+2*I/(x-I)-2*ln(x-I)
```

Maxima [A]
time = 0.42, size = 23, normalized size = 1.28

$$-\frac{2\left(\left(x^2+1\right)\log\left(x^2+1\right)+2\right)}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*arctan(x)),x, algorithm="maxima")
[Out] -2*((x^2 + 1)*log(x^2 + 1) + 2)/(x^2 + 1)
```

Fricas [A]
time = 0.58, size = 23, normalized size = 1.28

$$-\frac{2\left(\left(x^2+1\right)\log\left(x^2+1\right)+2\right)}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*arctan(x)),x, algorithm="fricas")
[Out] -2*((x^2 + 1)*log(x^2 + 1) + 2)/(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(4\operatorname{atan}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*atan(x)),x)
[Out] Integral(sin(4*atan(x)), x)
```

Giac [A]
time = 0.73, size = 23, normalized size = 1.28

$$\frac{2\left(x^2-1\right)}{x^2+1}-2\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*arctan(x)),x, algorithm="giac")
[Out] 2*(x^2 - 1)/(x^2 + 1) - 2*log(x^2 + 1)
```

Mupad [B]
time = 0.19, size = 18, normalized size = 1.00

$$-2\,\ln\left(x^2+1\right)-\frac{4}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*atan(x)),x)
```

```
[Out] - 2*log(x^2 + 1) - 4/(x^2 + 1)
```

Chatgpt [**F**] Failed to verify
time = 1.00, size = 7, normalized size = 0.39

$$-\frac{\cos(4\arctan(x))}{4}$$

Warning: Unable to verify antiderivative.

```
[In] int(sin(4*arctan(x)),x)
```

```
[Out] -1/4*cos(4*arctan(x))
```

3.318 $\int \frac{\sqrt[3]{\tan(x)}}{(\cos(x)+\sin(x))^2} dx$

Optimal. Leaf size=61

$$-\frac{\arctan\left(\frac{1-2\sqrt[3]{\tan(x)}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1+\sqrt[3]{\tan(x)}\right)-\frac{1}{6}\log(1+\tan(x))-\frac{\sqrt[3]{\tan(x)}}{1+\tan(x)}$$

```
[Out] -1/3*arctan(1/3*(1-2*tan(x)^(1/3))*3^(1/2))*3^(1/2)+1/2*ln(1+tan(x)^(1/3))-
1/6*ln(1+tan(x))-tan(x)^(1/3)/(1+tan(x))
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {43, 60, 632, 210, 31}

$$-\frac{\arctan\left(\frac{1-2\sqrt[3]{\tan(x)}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{\tan(x)}}{\tan(x)+1}+\frac{1}{2}\log\left(\sqrt[3]{\tan(x)}+1\right)-\frac{1}{6}\log(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^(1/3)/(Cos[x] + Sin[x])^2,x]
```

```
[Out] -(ArcTan[(1 - 2*Tan[x]^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[1 + Tan[x]^(1/3)]/2 -
 Log[1 + Tan[x]]/6 - Tan[x]^(1/3)/(1 + Tan[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{\sqrt[3]{x}}{(1+x)^2}\,dx, x, \tan(x)\right)\\
&= -\frac{\sqrt[3]{\tan(x)}}{1+\tan(x)} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^{2/3}(1+x)}\,dx, x, \tan(x)\right)\\
&= -\frac{1}{6}\log(1+\tan(x)) - \frac{\sqrt[3]{\tan(x)}}{1+\tan(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[3]{\tan(x)}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{\tan(x)}\right)\\
&= \frac{1}{2}\log\left(1+\sqrt[3]{\tan(x)}\right) - \frac{1}{6}\log(1+\tan(x)) - \frac{\sqrt[3]{\tan(x)}}{1+\tan(x)} - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{\tan(x)}\right)\\
&= \frac{\arctan\left(\frac{-1+2\sqrt[3]{\tan(x)}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1+\sqrt[3]{\tan(x)}\right) - \frac{1}{6}\log(1+\tan(x)) - \frac{\sqrt[3]{\tan(x)}}{1+\tan(x)}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 1.26

$$\frac{\arctan\left(\frac{-1+2\sqrt[3]{\tan(x)}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1+\sqrt[3]{\tan(x)}\right) - \frac{1}{6}\log\left(1-\sqrt[3]{\tan(x)}+\tan^{\frac{2}{3}}(x)\right) + \left(-1+\frac{\sin(x)}{\cos(x)+\sin(x)}\right)\sqrt[3]{\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^(1/3)/(Cos[x] + Sin[x])^2,x]
```

```
[Out] ArcTan[(-1 + 2*Tan[x]^(1/3))/Sqrt[3]]/Sqrt[3] + Log[1 + Tan[x]^(1/3)]/3 - L
og[1 - Tan[x]^(1/3) + Tan[x]^(2/3)]/6 + (-1 + Sin[x]/(Cos[x] + Sin[x]))*Tan
[x]^(1/3)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{\frac{1}{3}}(x)}{(\cos(x)+\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(1/3)/(cos(x)+sin(x))^2,x)
```

```
[Out] int(tan(x)^(1/3)/(cos(x)+sin(x))^2,x)
```

Maxima [A]
time = 0.44, size = 56, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan\left(x\right)^{\frac{1}{3}}-1\right)\right)-\frac{\tan\left(x\right)^{\frac{1}{3}}}{\tan\left(x\right)+1}-\frac{1}{6}\log\left(\tan\left(x\right)^{\frac{2}{3}}-\tan\left(x\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\tan\left(x\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/3)/(cos(x)+sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x)^(1/3) - 1)) - tan(x)^(1/3)/(tan(x)
 + 1) - 1/6*log(tan(x)^(2/3) - tan(x)^(1/3) + 1) + 1/3*log(tan(x)^(1/3) + 1
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(48) = 96.
time = 0.64, size = 108, normalized size = 1.77

$$\frac{2\left(\sqrt{3}\cos\left(x\right)+\sqrt{3}\sin\left(x\right)\right)\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-\left(\cos\left(x\right)+\sin\left(x\right)\right)\log\left(\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{2}{3}}-\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{1}{3}}+1\right)+2\left(\cos\left(x\right)+\sin\left(x\right)\right)\log\left(\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{1}{3}}+1\right)-6\left(\frac{\sin(x)}{\cos(x)}\right)^{\frac{1}{3}}\cos\left(x\right)}{6\left(\cos\left(x\right)+\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/3)/(cos(x)+sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(sqrt(3)*cos(x) + sqrt(3)*sin(x))*arctan(2/3*sqrt(3)*(sin(x)/cos(x))
^(1/3) - 1/3*sqrt(3)) - (cos(x) + sin(x))*log((sin(x)/cos(x))^(2/3) - (sin(
x)/cos(x))^(1/3) + 1) + 2*(cos(x) + sin(x))*log((sin(x)/cos(x))^(1/3) + 1)
- 6*(sin(x)/cos(x))^(1/3)*cos(x))/(cos(x) + sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{\tan\left(x\right)}}{\left(\sin\left(x\right)+\cos\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**(1/3)/(cos(x)+sin(x))**2,x)
```

```
[Out] Integral(tan(x)**(1/3)/(sin(x) + cos(x))**2, x)
```

Giac [A]
time = 0.62, size = 57, normalized size = 0.93

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan\left(x\right)^{\frac{1}{3}}-1\right)\right)-\frac{\tan\left(x\right)^{\frac{1}{3}}}{\tan\left(x\right)+1}-\frac{1}{6}\log\left(\tan\left(x\right)^{\frac{2}{3}}-\tan\left(x\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\tan\left(x\right)^{\frac{1}{3}}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/3)/(cos(x)+sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x)^(1/3) - 1)) - tan(x)^(1/3)/(tan(x)
 + 1) - 1/6*log(tan(x)^(2/3) - tan(x)^(1/3) + 1) + 1/3*log(abs(tan(x)^(1/3)
 + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(x)^{1/3}}{(\cos(x) + \sin(x))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(1/3)/(cos(x) + sin(x))^2,x)
```

```
[Out] int(tan(x)^(1/3)/(cos(x) + sin(x))^2, x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(tan(x)^(1/3)/(cos(x)+sin(x))^2,x)
```

```
[Out] not solved
```

3.319 $\int \csc^2(x)\csc^2(6x)\csc^2(10x)\csc^2(15x)\sin^2(2x)\sin^2(3x$

Optimal. Leaf size=64

$$7x+4\sin(2x)-\frac{1}{2}\sin(4x)-\frac{4}{3}\sin(6x)-\sin(8x)-\frac{2}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{2}{7}\sin(14x)+\frac{1}{8}\sin(16x)$$

```
[Out] 7*x+4*sin(2*x)-1/2*sin(4*x)-4/3*sin(6*x)-sin(8*x)-2/5*sin(10*x)+1/6*sin(12*
x)+2/7*sin(14*x)+1/8*sin(16*x)
```

Rubi [A]
time = 1.27, antiderivative size = 78, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.085$, Rules used = {1828, 1171, 393, 209}

$$7x+4096\sin(x)\cos^{15}(x)-\frac{83968}{7}\sin(x)\cos^{13}(x)+\frac{279040}{21}\sin(x)\cos^{11}(x)-\frac{744704}{105}\sin(x)\cos^{9}(x)+\frac{67936}{35}\sin(x)\cos^{7}(x)-\frac{4112}{15}\sin(x)\cos^{5}(x)+\frac{76}{3}\sin(x)\cos^{3}(x)+6\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Csc[6*x]^2*Csc[10*x]^2*Csc[15*x]^2*Sin[2*x]^2*Sin[3*x]^2*Sin[5
*x]^2*Sin[30*x]^2,x]
```

```
[Out] 7*x + 6*Cos[x]*Sin[x] + (76*Cos[x]^3*Sin[x])/3 - (4112*Cos[x]^5*Sin[x])/15
+ (67936*Cos[x]^7*Sin[x])/35 - (744704*Cos[x]^9*Sin[x])/105 + (279040*Cos[x
]^11*Sin[x])/21 - (83968*Cos[x]^13*Sin[x])/7 + 4096*Cos[x]^15*Sin[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \text{Subst}\left(\int \frac{(1-92x^2+134x^4-28x^6+x^8)^2}{(1+x^2)^9}\,dx,x,\tan(x)\right)\\
&= 4096\cos^{15}(x)\sin(x)-\frac{1}{16}\text{Subst}\left(\int \frac{65520-1045616x^2+905904x^4-510512x^6+140752x^8-17744x^{10}+912x^{12}-16x^{14}}{(1+x^2)^8}\,dx,x,\tan(x)\right)\\
&= -\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)+\frac{1}{224}\text{Subst}\left(\int \frac{1769696-22061760x^2+9379104x^4-2231936x^6+261408x^8-12992x^{10}+224x^{12}}{(1+x^2)^7}\,dx,x,\tan(x)\right)\\
&= \frac{279040}{21}\cos^{11}(x)\sin(x)-\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)-\frac{\text{Subst}\left(\int \frac{14480768-142627968x^2+30078720x^4-3295488x^6+158592x^8-2688x^{10}}{(1+x^2)^6}\,dx,x,\tan(x)\right)}{2688}\\
&= -\frac{744704}{105}\cos^9(x)\sin(x)+\frac{279040}{21}\cos^{11}(x)\sin(x)-\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)+\frac{\text{Subst}\left(\int \frac{45836544-335354880x^2+34567680x^4-1612800x^6+26880x^8}{(1+x^2)^5}\,dx,x,\tan(x)\right)}{26880}\\
&= \frac{67936}{35}\cos^7(x)\sin(x)-\frac{744704}{105}\cos^9(x)\sin(x)+\frac{279040}{21}\cos^{11}(x)\sin(x)-\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)-\frac{\text{Subst}\left(\int \frac{50706432-289658880x^2+13117440x^4-215040x^6}{(1+x^2)^4}\,dx,x,\tan(x)\right)}{215040}\\
&= -\frac{4112}{15}\cos^5(x)\sin(x)+\frac{67936}{35}\cos^7(x)\sin(x)-\frac{744704}{105}\cos^9(x)\sin(x)+\frac{279040}{21}\cos^{11}(x)\sin(x)-\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)+\frac{\text{Subst}\left(\int \frac{49459200-79994880x^2+1290240x^4}{(1+x^2)^3}\,dx,x,\tan(x)\right)}{1290240}\\
&= \frac{76}{3}\cos^3(x)\sin(x)-\frac{4112}{15}\cos^5(x)\sin(x)+\frac{67936}{35}\cos^7(x)\sin(x)-\frac{744704}{105}\cos^9(x)\sin(x)+\frac{279040}{21}\cos^{11}(x)\sin(x)-\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)-\frac{\text{Subst}\left(\int \frac{-67092480-5160960x^2}{(1+x^2)^2}\,dx,x,\tan(x)\right)}{5160960}\\
&= 6\cos(x)\sin(x)+\frac{76}{3}\cos^3(x)\sin(x)-\frac{4112}{15}\cos^5(x)\sin(x)+\frac{67936}{35}\cos^7(x)\sin(x)-\frac{744704}{105}\cos^9(x)\sin(x)+\frac{279040}{21}\cos^{11}(x)\sin(x)-\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)+7\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= 7x+6\cos(x)\sin(x)+\frac{76}{3}\cos^3(x)\sin(x)-\frac{4112}{15}\cos^5(x)\sin(x)+\frac{67936}{35}\cos^7(x)\sin(x)-\frac{744704}{105}\cos^9(x)\sin(x)+\frac{279040}{21}\cos^{11}(x)\sin(x)-\frac{83968}{7}\cos^{13}(x)\sin(x)+4096\cos^{15}(x)\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 64, normalized size = 1.00

$$7x+4\sin(2x)-\frac{1}{2}\sin(4x)-\frac{4}{3}\sin(6x)-\sin(8x)-\frac{2}{5}\sin(10x)+\frac{1}{6}\sin(12x)+\frac{2}{7}\sin(14x)+\frac{1}{8}\sin(16x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Csc[6*x]^2*Csc[10*x]^2*Csc[15*x]^2*Sin[2*x]^2*Sin[3*x]^2
*Sin[5*x]^2*Sin[30*x]^2,x]
```

```
[Out] 7*x + 4*Sin[2*x] - Sin[4*x]/2 - (4*Sin[6*x])/3 - Sin[8*x] - (2*Sin[10*x])/5
 + Sin[12*x]/6 + (2*Sin[14*x])/7 + Sin[16*x]/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(52) = 104$.
time = 0.62, size = 228, normalized size = 3.56

$$4096\left(\cos^{15}(x)+\frac{15(\cos^{13}(x))}{14}+\frac{65(\cos^{11}(x))}{56}+\frac{143(\cos^9(x))}{112}+\frac{1287(\cos^7(x))}{896}+\frac{429(\cos^5(x))}{256}+\frac{2145(\mathrm{co}}{102}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^2*sin(3*x)^2*sin(5*x)^2*sin(30*x)^2/sin(x)^2/sin(6*x)^2/sin(10
*x)^2/sin(15*x)^2,x)
```

```
[Out] 4096*(cos(x)^15+15/14*cos(x)^13+65/56*cos(x)^11+143/112*cos(x)^9+1287/896*c
os(x)^7+429/256*cos(x)^5+2145/1024*cos(x)^3+6435/2048*cos(x))*sin(x)+7*x-16
384*(cos(x)^13+13/12*cos(x)^11+143/120*cos(x)^9+429/320*cos(x)^7+1001/640*c
os(x)^5+1001/512*cos(x)^3+3003/1024*cos(x))*sin(x)+78848/3*(cos(x)^11+11/10
*cos(x)^9+99/80*cos(x)^7+231/160*cos(x)^5+231/128*cos(x)^3+693/256*cos(x))*
sin(x)-108544/5*(cos(x)^9+9/8*cos(x)^7+21/16*cos(x)^5+105/64*cos(x)^3+315/1
28*cos(x))*sin(x)+9920*(cos(x)^7+7/6*cos(x)^5+35/24*cos(x)^3+35/16*cos(x))*
sin(x)-7616/3*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+368*(cos(x)^3+3/2*
cos(x))*sin(x)-32*cos(x)*sin(x)
```

Maxima [**A**]
time = 0.36, size = 52, normalized size = 0.81

$$7\,x+\frac{1}{8}\,\sin\left(16\,x\right)+\frac{2}{7}\,\sin\left(14\,x\right)+\frac{1}{6}\,\sin\left(12\,x\right)-\frac{2}{5}\,\sin\left(10\,x\right)-\sin\left(8\,x\right)-\frac{4}{3}\,\sin\left(6\,x\right)-\frac{1}{2}\,\sin\left(4\,x\right)+4\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^2*sin(3*x)^2*sin(5*x)^2*sin(30*x)^2/sin(x)^2/sin(6*x)^2/
sin(10*x)^2/sin(15*x)^2,x, algorithm="maxima")
```

```
[Out] 7*x + 1/8*sin(16*x) + 2/7*sin(14*x) + 1/6*sin(12*x) - 2/5*sin(10*x) - sin(8
*x) - 4/3*sin(6*x) - 1/2*sin(4*x) + 4*sin(2*x)
```

Fricas [**A**]
time = 0.76, size = 55, normalized size = 0.86

$$\frac{2}{105}\left(215040\,\cos\left(x\right)^{15}-629760\,\cos\left(x\right)^{13}+697600\,\cos\left(x\right)^{11}-372352\,\cos\left(x\right)^{9}+101904\,\cos\left(x\right)^{7}-14392\,\cos\left(x\right)^{5}+1330\,\cos\left(x\right)^{3}+315\,\cos\left(x\right)\right)\sin\left(x\right)+7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^2*sin(3*x)^2*sin(5*x)^2*sin(30*x)^2/sin(x)^2/sin(6*x)^2/
sin(10*x)^2/sin(15*x)^2,x, algorithm="fricas")
```

```
[Out] 2/105*(215040*cos(x)^15 - 629760*cos(x)^13 + 697600*cos(x)^11 - 372352*cos(
x)^9 + 101904*cos(x)^7 - 14392*cos(x)^5 + 1330*cos(x)^3 + 315*cos(x))*sin(x
) + 7*x
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)**2*sin(3*x)**2*sin(5*x)**2*sin(30*x)**2/sin(x)**2/sin(6*
x)**2/sin(10*x)**2/sin(15*x)**2,x)
```

[Out] Timed out

Giac [A]
time = 45.32, size = 61, normalized size = 0.95

$$7x + \frac{2\left(315\tan\left(x\right)^{15} + 3535\tan\left(x\right)^{13} + 203\tan\left(x\right)^{11} + 60919\tan\left(x\right)^{9} - 71031\tan\left(x\right)^{7} + 74613\tan\left(x\right)^{5} - 5775\tan\left(x\right)^{3} - 315\tan\left(x\right)\right)}{105\left(\tan\left(x\right)^{2} + 1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^2*sin(3*x)^2*sin(5*x)^2*sin(30*x)^2/sin(x)^2/sin(6*x)^2/
sin(10*x)^2/sin(15*x)^2,x, algorithm="giac")
```

```
[Out] 7*x + 2/105*(315*tan(x)^15 + 3535*tan(x)^13 + 203*tan(x)^11 + 60919*tan(x)^
9 - 71031*tan(x)^7 + 74613*tan(x)^5 - 5775*tan(x)^3 - 315*tan(x))/(tan(x)^2
 + 1)^8
```

Mupad [B]
time = 1.55, size = 66, normalized size = 1.03

$$4096\sin\left(x\right)\cos\left(x\right)^{15} - \frac{83968\sin\left(x\right)\cos\left(x\right)^{13}}{7} + \frac{279040\sin\left(x\right)\cos\left(x\right)^{11}}{21} - \frac{744704\sin\left(x\right)\cos\left(x\right)^{9}}{105} + \frac{67936\sin\left(x\right)\cos\left(x\right)^{7}}{35} - \frac{4112\sin\left(x\right)\cos\left(x\right)^{5}}{15} + \frac{76\sin\left(x\right)\cos\left(x\right)^{3}}{3} + 6\sin\left(x\right)\cos\left(x\right) + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(2*x)^2*sin(3*x)^2*sin(5*x)^2*sin(30*x)^2)/(sin(6*x)^2*sin(10*x)^2*
sin(15*x)^2*sin(x)^2),x)
```

```
[Out] 7*x + 6*cos(x)*sin(x) + (76*cos(x)^3*sin(x))/3 - (4112*cos(x)^5*sin(x))/15
+ (67936*cos(x)^7*sin(x))/35 - (744704*cos(x)^9*sin(x))/105 + (279040*cos(x
)^11*sin(x))/21 - (83968*cos(x)^13*sin(x))/7 + 4096*cos(x)^15*sin(x)
```

Chatgpt [F]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int(sin(2*x)^2*sin(3*x)^2*sin(5*x)^2*sin(30*x)^2/sin(x)^2/sin(6*x)^2/sin(10
*x)^2/sin(15*x)^2,x)
```

```
[Out] not solved
```

3.320 $\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=105

$$\frac{1}{8}\left(\frac{2x\left(5+x^2+5\sqrt{1+x^2+x^4}\right)}{\left(1-x^2+\sqrt{1+x^2+x^4}\right)\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}\right)\right)$$

```
[Out] 1/4*x*(5+x^2+5*(x^4+x^2+1)^(1/2))/(1-x^2+(x^4+x^2+1)^(1/2))/(1+x^2+(x^4+x^2
+1)^(1/2))^(1/2)+3/8*2^(1/2)*arctanh(2^(1/2)*x/(1+x^2+(x^4+x^2+1)^(1/2))^(1
/2))
```

Rubi [F]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 + x^2 + Sqrt[1 + x^2 + x^4]],x]
[Out] Defer[Int][Sqrt[1 + x^2 + Sqrt[1 + x^2 + x^4]], x]
```

Rubi steps

$$\text{Integral} = \int \sqrt{1+x^2+\sqrt{1+x^2+x^4}}\,dx$$

Mathematica [A]
time = 0.29, size = 105, normalized size = 1.00

$$\frac{1}{8}\left(\frac{2x\left(5+x^2+5\sqrt{1+x^2+x^4}\right)}{\left(1-x^2+\sqrt{1+x^2+x^4}\right)\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2+\sqrt{1+x^2+x^4}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + Sqrt[1 + x^2 + x^4]],x]
[Out] ((2*x*(5 + x^2 + 5*Sqrt[1 + x^2 + x^4]))/((1 - x^2 + Sqrt[1 + x^2 + x^4])*S
qrt[1 + x^2 + Sqrt[1 + x^2 + x^4]]) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^2 + Sqrt[1 + x^2 + x^4]]])/8
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1+x^2+\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x)
```

```
[Out] int((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + x^2 + 1) + 1), x)
```

Fricas [A]
time = 2.38, size = 148, normalized size = 1.41

$$\frac{3\sqrt{2}x\log\left(-\frac{64\,x^5+16\,x^3+16\,\sqrt{x^4+x^2+1}\,(4\,x^3-x)+4\,\left(\sqrt{2}\sqrt{x^4+x^2+1}\,(8\,x^2-5)+\sqrt{2}\,(8\,x^4-x^2+5)\right)\sqrt{x^2+\sqrt{x^4+x^2+1}+1}+25\,x}{x}\right)+8\,\left(3\,x^2-\sqrt{x^4+x^2+1}+1\right)\sqrt{x^2+\sqrt{x^4+x^2+1}+1}}{32\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*x*log(-(64*x^5 + 16*x^3 + 16*sqrt(x^4 + x^2 + 1)*(4*x^3 - x
) + 4*(sqrt(2)*sqrt(x^4 + x^2 + 1)*(8*x^2 - 5) + sqrt(2)*(8*x^4 - x^2 + 5))
*sqrt(x^2 + sqrt(x^4 + x^2 + 1) + 1) + 25*x)/x) + 8*(3*x^2 - sqrt(x^4 + x^2
 + 1) + 1)*sqrt(x^2 + sqrt(x^4 + x^2 + 1) + 1))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^2+\sqrt{x^4+x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**2+(x**4+x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + x**2 + 1) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + x^2 + 1) + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{x^4+x^2+1}+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + x^4 + 1)^(1/2) + x^2 + 1)^(1/2),x)
```

```
[Out] int(((x^2 + x^4 + 1)^(1/2) + x^2 + 1)^(1/2), x)
```

Chatgpt [**F**]
time = 0.00, size = 0, normalized size = 0.00

not solved

Warning: Unable to verify antiderivative.

```
[In] int((1+x^2+(x^4+x^2+1)^(1/2))^(1/2),x)
```

```
[Out] not solved
```

3.321 $\int \frac{x^9}{575-48x^{10}+x^{20}}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{20}\log\left(23-x^{10}\right)+\frac{1}{20}\log\left(25-x^{10}\right)$$

```
[Out] -1/20*ln(-x^10+23)+1/20*ln(-x^10+25)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 630, 31}

$$\frac{1}{20}\log\left(25-x^{10}\right)-\frac{1}{20}\log\left(23-x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(575 - 48*x^10 + x^20),x]
```

```
[Out] -1/20*Log[23 - x^10] + Log[25 - x^10]/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{Integral} &= \frac{1}{10}\text{Subst}\left(\int \frac{1}{575-48x+x^2}\,dx,x,x^{10}\right)\\
&= \frac{1}{20}\text{Subst}\left(\int \frac{1}{-25+x}\,dx,x,x^{10}\right)-\frac{1}{20}\text{Subst}\left(\int \frac{1}{-23+x}\,dx,x,x^{10}\right)\\
&= -\frac{1}{20}\log\left(23-x^{10}\right)+\frac{1}{20}\log\left(25-x^{10}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{20}\log\left(23-x^{10}\right)+\frac{1}{20}\log\left(25-x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(575 - 48*x^10 + x^20),x]
```

```
[Out] -1/20*Log[23 - x^10] + Log[25 - x^10]/20
```

Maple [A]
time = 0.04, size = 18, normalized size = 0.72

method	result	size
default	$\frac{\ln(x^{10}-25)}{20}-\frac{\ln(x^{10}-23)}{20}$	18
risch	$\frac{\ln(x^{10}-25)}{20}-\frac{\ln(x^{10}-23)}{20}$	18
norman	$\frac{\ln(x^{5}-5)}{20}+\frac{\ln(x^{5}+5)}{20}-\frac{\ln(x^{10}-23)}{20}$	26
parallelrisch	$\frac{\ln(x^{5}-5)}{20}+\frac{\ln(x^{5}+5)}{20}-\frac{\ln(x^{10}-23)}{20}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^20-48*x^10+575),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*ln(x^10-25)-1/20*ln(x^10-23)
```

Maxima [A]
time = 0.36, size = 17, normalized size = 0.68

$$-\frac{1}{20}\log\left(x^{10}-23\right)+\frac{1}{20}\log\left(x^{10}-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^20-48*x^10+575),x, algorithm="maxima")
```

```
[Out] -1/20*log(x^10 - 23) + 1/20*log(x^10 - 25)
```

Fricas [A]
time = 0.58, size = 17, normalized size = 0.68

$$-\frac{1}{20}\log\left(x^{10}-23\right)+\frac{1}{20}\log\left(x^{10}-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^20-48*x^10+575),x, algorithm="fricas")
```

```
[Out] -1/20*log(x^10 - 23) + 1/20*log(x^10 - 25)
```

Sympy [A]
time = 0.07, size = 15, normalized size = 0.60

$$\frac{\log(x^{10}-25)}{20}-\frac{\log(x^{10}-23)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**20-48*x**10+575),x)
```

```
[Out] log(x**10 - 25)/20 - log(x**10 - 23)/20
```

Giac [A]
time = 0.69, size = 19, normalized size = 0.76

$$-\frac{1}{20}\log\left(\left|x^{10}-23\right|\right)+\frac{1}{20}\log\left(\left|x^{10}-25\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^20-48*x^10+575),x, algorithm="giac")
```

```
[Out] -1/20*log(abs(x^10 - 23)) + 1/20*log(abs(x^10 - 25))
```

Mupad [B]
time = 0.10, size = 23, normalized size = 0.92

$$\frac{\mathrm{atan}\left(\frac{x^{10}\,1006772302081\mathrm{i}-21354610286400\mathrm{i}}{2807924963544\,x^{10}-66383426822975}\right)\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^20 - 48*x^10 + 575),x)
```

```
[Out] (atan((x^10*1006772302081i - 21354610286400i)/(2807924963544*x^10 - 6638342
6822975))*1i)/10
```

Chatgpt [F] Failed to verify
time = 1.00, size = 31, normalized size = 1.24

$$\frac{23\sqrt{23}\,\arctan\left(\frac{x^5\sqrt{23}}{23}\right)}{10}-\frac{5\ln\left(\frac{x^5-5}{x^5+5}\right)}{8}$$

Warning: Unable to verify antiderivative.

```
[In] int(x^9/(x^20-48*x^10+575),x)
```

```
[Out] 23/10*23^(1/2)*arctan(1/23*x^5*23^(1/2))-5/8*ln((x^5-5)/(x^5+5))
```

Chapter 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
```

```
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal
```

```
#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.


if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;


if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A","";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well")
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A","";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
```

```
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                      convert(ExpnType_result,string)," vs. order ",
                      convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
```

```
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
```

```
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
```

```
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
```

```
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg
```

```
debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
```

```
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False
```

```
    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).___name___ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
```

```
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
```

```
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```