Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/8-Moses-Problems

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 3:58am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [113]. This is test number [8].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Maple	100.00 (113)	0.00 (0)
Fricas	99.12 (112)	0.88 (1)
Maxima	98.23 (111)	1.77 (2)
Giac	96.46 (109)	3.54 (4)
Mupad	93.81 (106)	6.19 (7)
Sympy	92.92 (105)	7.08 (8)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

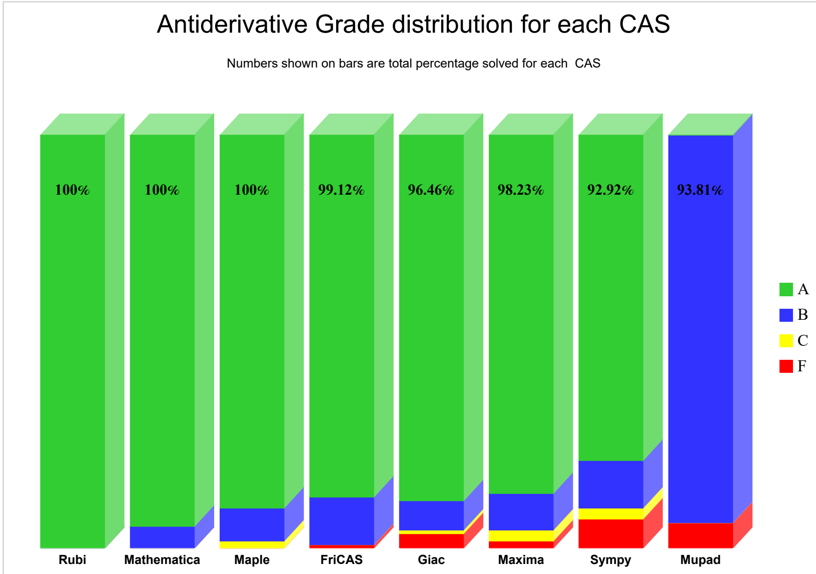
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

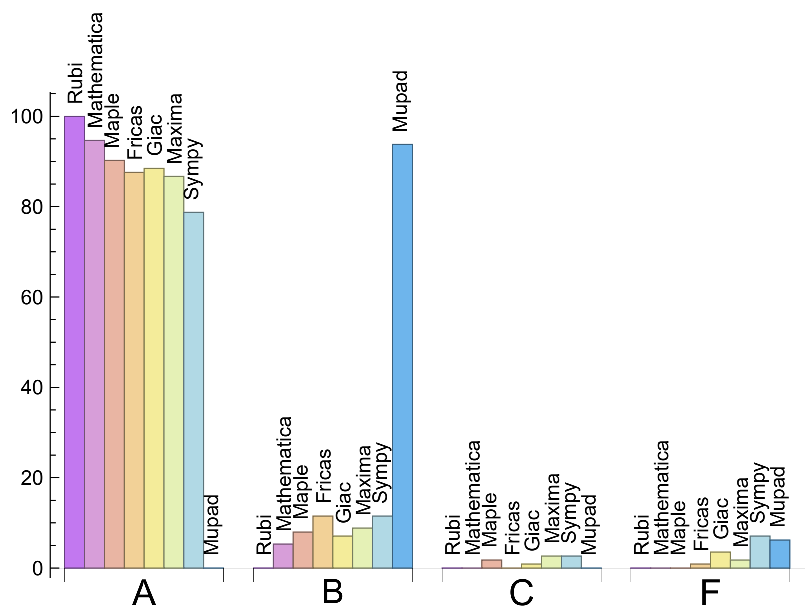
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	94.69	5.31	0.00	0.00
Maple	90.27	7.96	1.77	0.00
Giac	88.50	7.08	0.88	3.54
Fricas	87.61	11.50	0.00	0.88
Maxima	86.73	8.85	2.65	1.77
Sympy	78.76	11.50	2.65	7.08
Mupad	N/A	93.81	0.00	6.19

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	1	100.00 %	0.00 %	0.00 %
Giac	4	50.00 %	0.00 %	50.00 %
Maxima	2	100.00 %	0.00 %	0.00 %
Sympy	8	100.00 %	0.00 %	0.00 %
Mupad	7	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

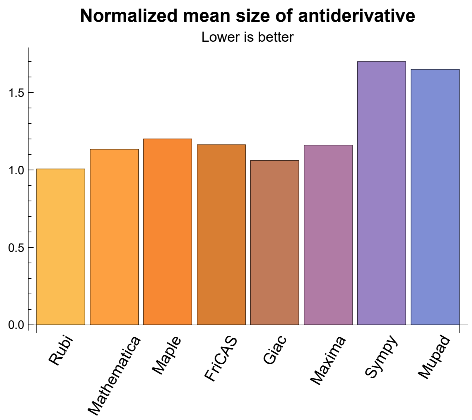
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

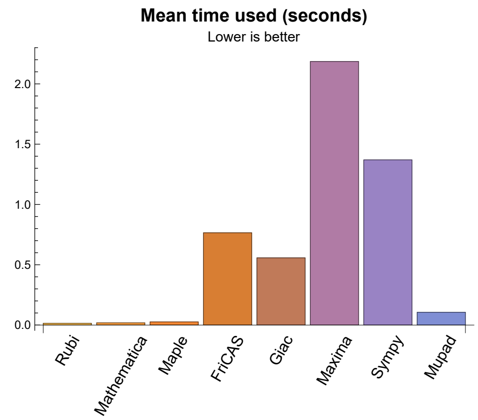
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.01	20.14	1.01	16.00	1.00
Mathematica	0.02	24.71	1.13	17.00	1.00
Maple	0.03	24.57	1.20	14.00	0.92
Maxima	2.19	23.60	1.16	14.00	0.88
Fricas	0.77	24.82	1.16	14.00	0.93
Sympy	1.37	31.10	1.70	15.00	0.83
Giac	0.56	18.54	1.06	13.00	0.83
Mupad	0.11	28.25	1.65	12.00	0.83

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

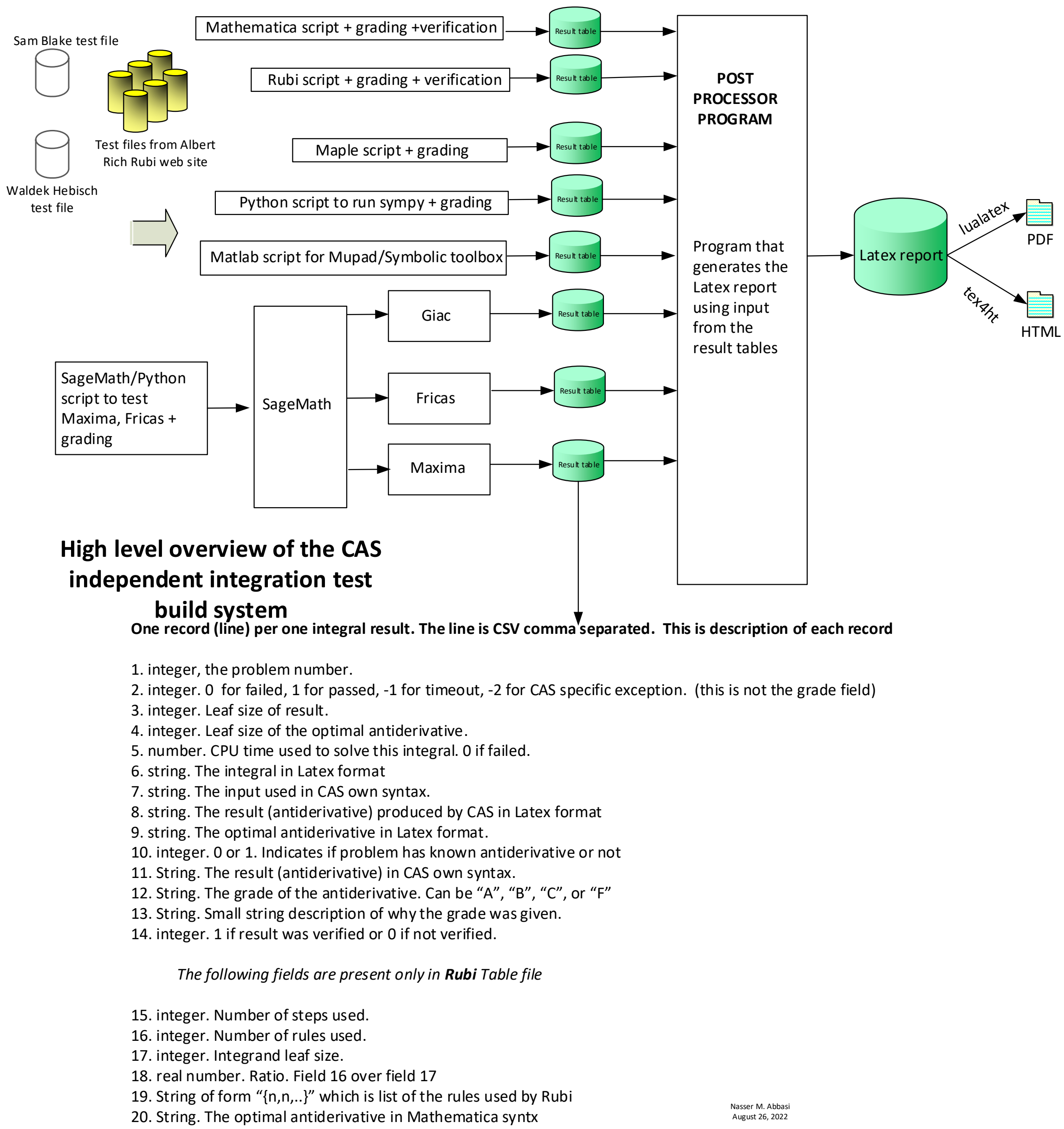
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 31, 42, 70, 71, 72, 84 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 37, 38, 39, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 35, 36, 40, 48, 57, 69, 70, 71, 72 }

C grade: { 32, 42 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 39, 41, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 36, 38, 40, 42, 68, 69, 70, 71, 72, 87 }

C grade: { 10, 11, 47 }

F grade: { 32, 67 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 9, 37, 39, 40, 42, 69, 70, 73, 74, 83, 84, 100 }

C grade: { }

F grade: { 32 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 70, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 88, 89, 91, 92, 93, 94, 95, 96, 97, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 7, 18, 37, 39, 50, 51, 56, 66, 73, 82, 83, 84, 100 }

C grade: { 38, 71, 72 }

F grade: { 35, 36, 40, 42, 69, 87, 90, 98 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 44, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 25, 36, 43, 45, 70, 71, 72 }

C grade: { 47 }

F grade: { 32, 40, 42, 69 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

C grade: { }

F grade: { 10, 11, 32, 38, 40, 42, 69 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	16	16	48	19	34	10
N.S.	1	1.00	1.50	1.33	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.008	0.003	0.000	2.962	0.544	0.016	0.457	0.004

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	19	14	15	12
N.S.	1	1.00	1.00	0.92	1.15	1.46	1.08	1.15	0.92
time (sec)	N/A	0.004	0.003	0.034	3.908	0.586	0.036	0.438	0.158

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	12	11	14	15	11	10
N.S.	1	1.00	0.74	0.63	0.58	0.74	0.79	0.58	0.53
time (sec)	N/A	0.002	0.008	0.033	2.592	0.513	0.065	0.469	0.030

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.000	1.371	0.503	0.023	0.450	0.028

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.004	0.001	0.007	2.285	0.497	0.022	0.457	0.017

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.008	0.004	0.025	2.620	0.679	0.011	0.445	0.022

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.001	0.001	0.000	2.730	0.646	0.059	0.488	0.029

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	11	13	15	11	11
N.S.	1	1.00	0.74	0.74	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.005	0.011	0.013	1.617	0.785	0.083	0.473	0.019

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.75	1.00	0.75	0.75
time (sec)	N/A	0.010	0.004	0.033	2.658	0.568	0.011	0.442	0.053

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	15	3	3	3	-1
N.S.	1	1.00	1.00	1.00	3.75	0.75	0.75	0.75	-0.25
time (sec)	N/A	0.009	0.005	0.025	1.232	0.649	0.318	0.477	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	-1
N.S.	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	-0.50
time (sec)	N/A	0.008	0.012	0.020	1.968	0.686	0.311	0.429	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.62	0.88	0.88
time (sec)	N/A	0.004	0.006	0.016	5.908	1.079	0.019	0.443	0.042

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.013	0.025	0.014	4.372	1.388	0.023	0.446	0.153

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	22	19	19	20	19	21
N.S.	1	1.00	0.93	0.79	0.68	0.68	0.71	0.68	0.75
time (sec)	N/A	0.015	0.026	0.016	1.493	1.086	0.028	0.427	0.054

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.73
time (sec)	N/A	0.003	0.009	0.010	2.860	1.150	0.026	0.446	0.047

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.005	0.001	0.000	1.858	1.482	0.024	0.438	0.017

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.005	0.001	0.005	1.478	1.369	0.021	0.442	0.002

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.001	0.000	0.000	3.300	1.065	0.059	0.421	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.011	0.010	0.009	1.088	1.006	0.019	0.481	0.029

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.001	0.000	0.012	1.981	1.185	0.023	0.490	0.044

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.002	0.006	0.021	1.906	0.972	0.036	0.466	0.072

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.00	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.002	0.001	0.008	3.319	1.264	0.026	0.457	0.016

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	8	7	7	8	7	7
N.S.	1	1.00	1.90	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.019	0.019	0.029	1.533	1.000	0.325	0.448	0.187

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	16	15	15	34	15	12
N.S.	1	1.00	0.78	0.70	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.003	0.010	0.033	1.460	0.905	0.458	0.446	0.031

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.002	0.003	0.000	2.459	0.714	0.048	0.450	0.158

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	13	13	15	13	13
N.S.	1	1.00	1.00	0.78	0.72	0.72	0.83	0.72	0.72
time (sec)	N/A	0.014	0.024	0.017	1.668	0.590	0.039	0.436	0.107

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	20	19	76	22	19	19
N.S.	1	1.00	1.00	0.65	0.61	2.45	0.71	0.61	0.61
time (sec)	N/A	0.025	0.024	0.027	2.016	0.785	0.065	0.463	0.231

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	11	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.38	1.00	1.00	1.00
time (sec)	N/A	0.020	0.010	0.013	2.054	0.637	0.029	0.465	0.184

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	15	14	14	10	13	12
N.S.	1	1.00	1.00	1.25	1.17	1.17	0.83	1.08	1.00
time (sec)	N/A	0.004	0.002	0.020	2.008	0.696	0.026	0.492	0.072

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.012	0.003	0.015	1.232	0.582	0.128	0.446	0.208

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	40	40	46	40	52
N.S.	1	1.00	5.31	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.026	0.077	0.024	1.790	0.600	0.065	0.475	0.138

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	142	41	0	0	54	0	-1
N.S.	1	1.00	1.23	0.36	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.038	0.206	0.077	0.000	0.000	1.410	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.008	0.014	0.022	1.698	0.619	0.085	0.439	0.212

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	16	15	15	34	15	12
N.S.	1	1.00	0.78	0.70	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.003	0.002	0.034	1.216	0.517	0.455	0.446	0.002

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	92	24	24	0	24	24
N.S.	1	1.00	1.03	2.88	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.010	0.005	0.031	2.265	0.473	0.000	0.498	0.002

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	75	75	80	55	0	61	57
N.S.	1	1.00	1.70	1.70	1.82	1.25	0.00	1.39	1.30
time (sec)	N/A	0.008	0.135	0.083	1.551	0.519	0.000	0.469	0.206

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.005	0.073	0.083	1.970	0.598	0.447	0.512	0.151

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	47	70	113	44	190	90	-1
N.S.	1	1.00	0.63	0.93	1.51	0.59	2.53	1.20	-0.01
time (sec)	N/A	0.008	0.066	0.043	1.256	0.641	19.857	0.475	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.002	0.000	1.517	0.620	0.444	0.459	0.002

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	100	262	114	129	0	0	-1
N.S.	1	1.00	1.96	5.14	2.24	2.53	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.130	0.075	3.236	0.754	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.004	0.003	0.016	2.106	0.920	0.009	0.459	0.033

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	99	149	116	244	0	0	-1
N.S.	1	1.16	2.02	3.04	2.37	4.98	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.080	0.168	2.129	1.396	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.007	0.004	0.011	3.344	1.180	0.076	0.459	0.182

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	10	6	6	5	6	6
N.S.	1	1.00	0.64	0.91	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.004	0.002	0.000	2.916	1.131	0.020	0.462	0.017

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	5	30	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.56	3.33	0.89
time (sec)	N/A	0.016	0.039	0.033	1.651	0.762	0.026	0.449	0.086

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.020	0.005	0.010	1.406	0.854	0.024	0.440	0.145

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	9	7
N.S.	1	1.00	1.00	0.73	0.82	0.64	0.73	0.82	0.64
time (sec)	N/A	0.001	0.002	0.006	2.499	0.883	0.067	0.444	0.019

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
N.S.	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.006	0.005	0.011	1.813	0.822	0.305	0.454	0.008

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	34	34	41	35	46
N.S.	1	1.00	0.98	0.85	0.83	0.83	1.00	0.85	1.12
time (sec)	N/A	0.015	0.007	0.030	1.353	0.670	0.047	0.449	0.106

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	65	65	83	67	88
N.S.	1	1.55	1.60	1.40	1.38	1.38	1.77	1.43	1.87
time (sec)	N/A	0.074	0.010	0.034	2.227	0.431	0.117	0.435	0.090

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	34	39	27	70	41	21
N.S.	1	1.00	1.00	1.62	1.86	1.29	3.33	1.95	1.00
time (sec)	N/A	0.006	0.004	0.056	2.960	0.469	0.136	0.459	0.240

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.001	0.000	1.021	0.392	0.024	0.453	0.035

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	33	24	32	38	24
N.S.	1	1.00	0.72	0.85	0.82	0.60	0.80	0.95	0.60
time (sec)	N/A	0.016	0.009	0.003	4.150	0.481	0.091	0.430	0.002

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.001	0.001	0.043	4.775	0.418	0.018	0.474	0.021

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.030	0.008	0.039	1.160	0.426	0.025	0.447	0.264

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.013	0.010	0.012	1.805	0.434	0.052	0.458	0.307

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.001	0.002	0.003	2.828	0.452	0.230	0.433	0.011

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.016	0.006	0.034	1.947	0.537	0.054	0.451	0.059

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	13	16	11	14	8	11	15
N.S.	1	1.00	0.76	0.94	0.65	0.82	0.47	0.65	0.88
time (sec)	N/A	0.022	0.009	0.009	3.538	0.665	0.024	0.448	0.058

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	29	24	24	26	24	28
N.S.	1	1.00	0.76	0.76	0.63	0.63	0.68	0.63	0.74
time (sec)	N/A	0.023	0.038	0.011	1.978	0.561	0.028	0.452	0.157

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.007	0.003	0.000	2.716	0.676	0.053	0.434	0.018

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.008	0.012	0.007	1.323	0.493	0.085	0.461	0.002

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.007	0.003	0.000	1.164	0.809	0.052	0.465	0.002

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.008	0.002	0.000	2.913	0.800	0.034	0.441	0.032

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	1.36	0.73	0.82	0.82
time (sec)	N/A	0.010	0.003	0.042	1.304	1.021	0.130	0.433	0.040

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.013	0.009	0.000	1.429	0.996	0.052	0.479	0.002

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	3	3	3	43
N.S.	1	1.00	1.00	1.33	0.00	1.00	1.00	1.00	14.33
time (sec)	N/A	0.032	0.019	0.034	0.000	0.749	0.126	0.437	3.099

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	23	53	15	12	21	13
N.S.	1	1.00	1.00	1.44	3.31	0.94	0.75	1.31	0.81
time (sec)	N/A	0.028	0.017	0.049	1.297	0.921	0.047	0.456	0.029

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	98	262	115	128	0	0	-1
N.S.	1	1.00	1.85	4.94	2.17	2.42	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.090	0.046	2.161	1.347	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	74	69	67	202	83	360
N.S.	1	1.00	2.19	4.62	4.31	4.19	12.62	5.19	22.50
time (sec)	N/A	0.057	0.070	0.115	2.292	1.846	112.058	0.487	0.499

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	71	68	26	422	82	352
N.S.	1	1.00	2.19	4.44	4.25	1.62	26.38	5.12	22.00
time (sec)	N/A	0.090	0.014	0.058	1.251	1.102	1.058	0.440	0.236

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	68	65	26	422	79	352
N.S.	1	1.00	2.19	4.25	4.06	1.62	26.38	4.94	22.00
time (sec)	N/A	0.013	0.011	0.064	1.541	1.307	1.039	0.429	0.058

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.010	0.037	0.987	1.218	0.457	0.463	0.002

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	12	26	19	12	12
N.S.	1	1.00	1.29	0.93	0.86	1.86	1.36	0.86	0.86
time (sec)	N/A	0.006	0.006	0.032	1.813	0.966	0.011	0.455	0.072

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.004	0.004	0.036	1.272	0.931	0.024	0.435	0.028

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.00	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.020	0.005	0.008	1.428	0.684	0.025	0.453	0.002

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	24	23	23	20	30	24
N.S.	1	1.00	0.79	1.00	0.96	0.96	0.83	1.25	1.00
time (sec)	N/A	0.236	0.224	0.043	3.305	0.790	0.038	0.440	0.223

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	0.89
time (sec)	N/A	0.002	0.009	0.010	1.479	0.536	0.018	0.432	0.032

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	24	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.96	0.76	0.76	0.76	0.76
time (sec)	N/A	0.023	0.004	0.008	1.500	1.196	0.030	0.443	0.227

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.002	0.003	0.034	2.409	1.335	0.027	0.429	0.056

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.004	0.004	0.034	2.203	1.161	0.028	0.460	0.022

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	19	16	20	36	16	10
N.S.	1	1.00	1.00	1.36	1.14	1.43	2.57	1.14	0.71
time (sec)	N/A	0.009	0.003	0.019	3.777	0.760	0.174	0.469	0.178

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.006	0.001	0.000	3.242	0.669	0.452	0.478	0.002

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	35	16	15	45	34	21	37
N.S.	1	1.00	2.06	0.94	0.88	2.65	2.00	1.24	2.18
time (sec)	N/A	0.003	0.053	0.068	3.601	0.609	0.185	0.526	0.163

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	11	13	15	11	11
N.S.	1	1.00	0.74	0.74	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.005	0.007	0.010	1.998	0.551	0.083	0.853	0.002

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.003	2.455	0.555	0.022	0.719	0.006

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	31	256	71	0	33	32
N.S.	1	1.00	1.62	0.69	5.69	1.58	0.00	0.73	0.71
time (sec)	N/A	0.083	0.143	0.203	2.513	0.541	0.000	0.783	0.715

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	16	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	1.14	0.71
time (sec)	N/A	0.005	0.003	0.015	0.977	0.467	0.011	0.756	0.002

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	12	11	10	14	11	10
N.S.	1	1.00	0.82	0.71	0.65	0.59	0.82	0.65	0.59
time (sec)	N/A	0.002	0.008	0.013	1.477	0.414	0.066	0.731	0.024

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	20	19	27	0	27	27
N.S.	1	1.00	1.35	0.87	0.83	1.17	0.00	1.17	1.17
time (sec)	N/A	0.007	0.045	0.130	1.765	0.427	0.000	0.750	0.078

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.25	0.62	0.75	0.75
time (sec)	N/A	0.008	0.001	0.017	1.550	0.413	0.027	0.717	0.027

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	0.83
time (sec)	N/A	0.011	0.002	0.005	1.305	0.434	0.020	0.577	0.002

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.015	0.002	0.007	1.927	0.401	0.027	0.691	0.053

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.007	0.009	0.012	1.665	0.374	0.157	0.654	0.262

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.005	0.014	1.239	0.394	0.012	0.735	0.002

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.002	0.000	2.035	0.369	0.024	0.802	0.002

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.005	0.001	0.000	2.415	0.380	0.024	0.829	0.002

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	43	21	22	32	0	20	20
N.S.	1	1.00	1.79	0.88	0.92	1.33	0.00	0.83	0.83
time (sec)	N/A	0.007	0.045	0.071	1.710	0.367	0.000	1.265	0.268

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	13	19	15	13	13
N.S.	1	1.00	1.00	0.70	0.65	0.95	0.75	0.65	0.65
time (sec)	N/A	0.016	0.004	0.013	2.015	0.388	0.039	1.243	0.091

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	44	30	44	63	105	29	91
N.S.	1	1.00	1.26	0.86	1.26	1.80	3.00	0.83	2.60
time (sec)	N/A	0.007	0.008	0.037	2.712	0.410	0.456	1.177	0.002

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	14	14	24	14	14
N.S.	1	1.00	0.90	0.75	0.70	0.70	1.20	0.70	0.70
time (sec)	N/A	0.015	0.022	0.017	2.023	0.411	0.044	1.110	0.067

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	32	31	26	26
N.S.	1	1.00	1.00	0.82	0.79	0.97	0.94	0.79	0.79
time (sec)	N/A	0.007	0.011	0.003	4.335	0.396	0.047	0.850	0.002

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	31	15	19	19
N.S.	1	1.00	1.00	0.95	0.90	1.48	0.71	0.90	0.90
time (sec)	N/A	0.011	0.000	0.018	3.433	0.578	0.005	0.918	0.002

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	23	40	19	23	24
N.S.	1	1.00	1.00	0.96	0.88	1.54	0.73	0.88	0.92
time (sec)	N/A	0.013	0.000	0.017	2.421	0.678	0.007	1.087	0.003

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	43	22	25	25
N.S.	1	1.00	1.00	0.96	0.93	1.59	0.81	0.93	0.93
time (sec)	N/A	0.015	0.000	0.020	1.926	0.542	0.006	1.185	0.002

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	29	51	26	29	30
N.S.	1	1.00	1.00	0.97	0.91	1.59	0.81	0.91	0.94
time (sec)	N/A	0.017	0.000	0.020	2.316	0.687	0.007	1.062	0.002

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	24	23	41	20	23	23
N.S.	1	1.00	1.04	1.00	0.96	1.71	0.83	0.96	0.96
time (sec)	N/A	0.017	0.000	0.017	2.043	0.677	0.006	0.707	0.002

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	29	27	50	24	27	28
N.S.	1	1.00	1.03	1.00	0.93	1.72	0.83	0.93	0.97
time (sec)	N/A	0.022	0.000	0.017	1.296	0.863	0.007	0.835	0.002

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	18	17	18	18
N.S.	1	1.00	1.00	1.00	0.95	0.95	0.89	0.95	0.95
time (sec)	N/A	0.006	0.000	0.015	2.128	0.777	0.004	0.606	0.002

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	22	22	20	22	23
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.83	0.92	0.96
time (sec)	N/A	0.010	0.000	0.014	2.572	1.283	0.007	0.598	0.004

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	24	44	24	24	24
N.S.	1	1.00	1.00	1.00	0.96	1.76	0.96	0.96	0.96
time (sec)	N/A	0.009	0.000	0.016	2.580	1.076	0.006	0.592	0.002

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	30	28	50	27	28	29
N.S.	1	1.00	1.00	1.00	0.93	1.67	0.90	0.93	0.97
time (sec)	N/A	0.012	0.000	0.015	2.100	0.667	0.007	0.757	0.003

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	27	25	25	24	25	26
N.S.	1	1.00	1.04	1.00	0.93	0.93	0.89	0.93	0.96
time (sec)	N/A	0.004	0.000	0.011	3.241	2.425	0.007	0.772	0.003

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [71] had the largest ratio of [48]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	4	0.500
2	A	3	2	1.00	11	0.182
3	A	2	1	1.00	11	0.091
4	A	1	1	1.00	2	0.500
5	A	1	1	1.00	7	0.143
6	A	2	2	1.00	7	0.286
7	A	1	1	1.00	11	0.091
8	A	1	1	1.00	6	0.167
9	A	2	2	1.00	7	0.286
10	A	2	2	1.00	4	0.500
11	A	1	1	1.00	6	0.167
12	A	3	2	1.00	6	0.333
13	A	4	2	1.00	16	0.125
14	A	5	3	1.00	7	0.429
15	A	3	1	1.00	14	0.071
16	A	2	2	1.00	5	0.400
17	A	1	1	1.00	7	0.143
18	A	1	1	1.00	11	0.091
19	A	2	2	1.00	11	0.182
20	A	1	1	1.00	5	0.200
21	A	1	1	1.00	6	0.167
22	A	2	2	1.00	9	0.222
23	A	3	3	1.00	14	0.214
24	A	2	1	1.00	9	0.111
25	A	3	3	1.00	7	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	15	0.133
27	A	2	2	1.00	17	0.118
28	A	3	3	1.00	13	0.231
29	A	1	1	1.00	5	0.200
30	A	3	3	1.00	8	0.375
31	A	7	7	1.00	11	0.636
32	A	3	2	1.00	12	0.167
33	A	3	3	1.00	6	0.500
34	A	2	1	1.00	9	0.111
35	A	4	3	1.00	13	0.231
36	A	4	4	1.00	15	0.267
37	A	3	2	1.00	15	0.133
38	A	6	3	1.00	13	0.231
39	A	3	2	1.00	15	0.133
40	A	5	5	1.00	29	0.172
41	A	2	2	1.00	4	0.500
42	A	6	6	1.16	19	0.316
43	A	1	1	1.00	6	0.167
44	A	2	2	1.00	5	0.400
45	A	1	1	1.00	10	0.100
46	A	5	3	1.00	13	0.231
47	A	1	1	1.00	5	0.200
48	A	1	1	1.00	7	0.143
49	A	6	6	1.00	9	0.667
50	A	10	6	1.55	7	0.857
51	A	1	1	1.00	27	0.037
52	A	1	1	1.00	4	0.250
53	A	4	3	1.00	6	0.500
54	A	2	2	1.00	10	0.200
55	A	7	4	1.00	9	0.444
56	A	2	1	1.00	12	0.083
57	A	1	1	1.00	4	0.250
58	A	6	4	1.00	7	0.571
59	A	4	3	1.00	11	0.273
60	A	6	3	1.00	9	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	4	0.500
62	A	3	3	1.00	6	0.500
63	A	2	2	1.00	4	0.500
64	A	2	2	1.00	6	0.333
65	A	2	1	1.00	9	0.111
66	A	2	1	1.00	12	0.083
67	A	2	2	1.00	20	0.100
68	A	2	2	1.00	10	0.200
69	A	6	6	1.00	30	0.200
70	A	5	5	1.00	39	0.128
71	A	6	5	1.00	48	0.104
72	A	4	4	1.00	31	0.129
73	A	3	2	1.00	15	0.133
74	A	3	2	1.00	4	0.500
75	A	3	2	1.00	11	0.182
76	A	5	3	1.00	13	0.231
77	A	10	6	1.00	33	0.182
78	A	1	1	1.00	7	0.143
79	A	5	5	1.00	8	0.625
80	A	2	2	1.00	9	0.222
81	A	3	3	1.00	11	0.273
82	A	3	3	1.00	8	0.375
83	A	3	2	1.00	15	0.133
84	A	2	2	1.00	16	0.125
85	A	1	1	1.00	6	0.167
86	A	1	1	1.00	3	0.333
87	A	4	2	1.00	17	0.118
88	A	2	2	1.00	4	0.500
89	A	2	1	1.00	11	0.091
90	A	3	3	1.00	14	0.214
91	A	2	2	1.00	7	0.286
92	A	2	2	1.00	11	0.182
93	A	3	2	1.00	13	0.154
94	A	1	1	1.00	8	0.125
95	A	2	2	1.00	7	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	1	1	1.00	4	0.250
97	A	2	2	1.00	5	0.400
98	A	3	3	1.00	17	0.176
99	A	3	3	1.00	16	0.188
100	A	3	2	1.00	15	0.133
101	A	3	3	1.00	15	0.200
102	A	3	3	1.00	8	0.375
103	A	1	1	1.00	20	0.050
104	A	1	1	1.00	25	0.040
105	A	1	1	1.00	26	0.038
106	A	1	1	1.00	31	0.032
107	A	1	1	1.00	24	0.042
108	A	1	1	1.00	29	0.034
109	A	1	1	1.00	18	0.056
110	A	1	1	1.00	23	0.043
111	A	1	1	1.00	24	0.042
112	A	1	1	1.00	29	0.034
113	A	1	1	1.00	27	0.037

Chapter 3

Listing of integrals

Local contents

3.1 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x+\cot(x)-\frac{\cot^3(x)}{3}$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\ &= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\ &= x+\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.50

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A]
time = 0.00, size = 16, normalized size = 1.33

method	result	size
derivativedivides	$-\frac{(\cot^3(x))}{3} + \cot(x) - \frac{\pi}{2} + \operatorname{arccot}(\cot(x))$	16
default	$-\frac{(\cot^3(x))}{3} + \cot(x) - \frac{\pi}{2} + \operatorname{arccot}(\cot(x))$	16
norman	$\frac{-\frac{1}{3}+\tan^2(x)+x(\tan^3(x))}{\tan(x)^3}$	18
risch	$x + \frac{4i(3\,e^{4ix}-3\,e^{2ix}+2)}{3(e^{2ix}-1)^3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+arccot(cot(x))
```

Maxima [A]
time = 2.96, size = 16, normalized size = 1.33

$$x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.
time = 0.54, size = 48, normalized size = 4.00

$$\frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A]
time = 0.02, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(10) = 20.
time = 0.46, size = 34, normalized size = 2.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

Mupad [B]
time = 0.00, size = 10, normalized size = 0.83

$$-\frac{\cot(x)^3}{3} + \cot(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] x + cot(x) - cot(x)^3/3
```

3.2 $\int \frac{1}{x^4(1+x^2)}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

```
[Out] -1/3/x^3+1/x+arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {331, 209}

$$\text{ArcTan}(x)-\frac{1}{3x^3}+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^2)),x]
```

```
[Out] -1/3*1/x^3 + x^(-1) + ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(1+x^2\right)}\,dx &= -\frac{1}{3x^3}-\int \frac{1}{x^2\left(1+x^2\right)}\,dx\\
&= -\frac{1}{3x^3}+\frac{1}{x}+\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^2)),x]
```

```
[Out] -1/3*1/x^3 + x^(-1) + ArcTan[x]
```

Maple [A]
time = 0.03, size = 12, normalized size = 0.92

method	result	size
default	$-\frac{1}{3x^3}+\frac{1}{x}+\arctan(x)$	12
meijerg	$-\frac{1}{3x^3}+\frac{1}{x}+\arctan(x)$	12
risch	$\frac{x^2-\frac{1}{3}}{x^3}+\arctan(x)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3+1/x+arctan(x)
```

Maxima [A]
time = 3.91, size = 15, normalized size = 1.15

$$\frac{3\,x^2-1}{3\,x^3}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

Fricas [A]
time = 0.59, size = 19, normalized size = 1.46

$$\frac{3\,x^3\arctan(x)+3\,x^2-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*arctan(x) + 3*x^2 - 1)/x^3
```

Sympy [A]
time = 0.04, size = 14, normalized size = 1.08

$$\operatorname{atan}(x)+\frac{3x^2-1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**2+1),x)
```

```
[Out] atan(x) + (3*x**2 - 1)/(3*x**3)
```

Giac [A]
time = 0.44, size = 15, normalized size = 1.15

$$\frac{3\,x^2-1}{3\,x^3}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

Mupad [B]
time = 0.16, size = 12, normalized size = 0.92

$$\operatorname{atan}(x)+\frac{x^2-\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^2 + 1)),x)
```

```
[Out] atan(x) + (x^2 - 1/3)/x^3
```

3.3 $\int \frac{x+x^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=19

$$\frac{2x^{3/2}}{3} + \frac{2x^{5/2}}{5}$$

```
[Out] 2/3*x^(3/2)+2/5*x^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{x+x^2}{\sqrt{x}} \, dx &= \int \left(\sqrt{x} + x^{3/2}\right) \, dx \\ &= \frac{2x^{3/2}}{3} + \frac{2x^{5/2}}{5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.74

$$\frac{2}{15} x^{3/2} (5+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2)*(5 + 3*x))/15
```

Maple [A]
time = 0.03, size = 12, normalized size = 0.63

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
trager	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
risch	$\frac{2x^{\frac{3}{2}}(5+3x)}{15}$	11
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{5}{2}}}{5}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)+2/5*x^(5/2)
```

Maxima [A]
time = 2.59, size = 11, normalized size = 0.58

$$\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

Fricas [A]
time = 0.51, size = 14, normalized size = 0.74

$$\frac{2}{15}\,\left(3\,x^{2}+5\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + 5*x)*sqrt(x)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/3
```

Giac [A]
time = 0.47, size = 11, normalized size = 0.58

$$\frac{2}{5}\,x^{\frac{5}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.53

$$\frac{2\,x^{3/2}\,\left(3\,x+5\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)/x^(1/2),x)
```

```
[Out] (2*x^(3/2)*(3*x + 5))/15
```

3.4 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2717}

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\,dx = \sin(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

Maple [A]
time = 0.00, size = 3, normalized size = 1.50

method	result	size

lookup	$\sin(x)$	3
default	$\sin(x)$	3
meijerg	$\sin(x)$	3
risch	$\sin(x)$	3
norman	$\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x,method=_RETURNVERBOSE)
[Out] sin(x)
```

Maxima [A]
time = 1.37, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
[Out] sin(x)
```

Fricas [A]
time = 0.50, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
[Out] sin(x)
```

Sympy [A]
time = 0.02, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
[Out] sin(x)
```

Giac [A]
time = 0.45, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

Mupad [B]
time = 0.03, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

3.5 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] 1/2*exp(x^2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2240}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x,x]

[Out] E^x^2/2

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*x,x]

[Out] E^x^2/2

Maple [A]
time = 0.01, size = 7, normalized size = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x^2)
```

Maxima [A]
time = 2.28, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
[Out] 1/2*e^(x^2)
```

Fricas [A]
time = 0.50, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
[Out] 1/2*e^(x^2)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Giac [A]
time = 0.46, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.67

$$\frac{\mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
```

```
[Out] exp(x^2)/2
```

3.6 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] 1/2*sec(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right)\\ &= \frac{\sec^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Maple [A]
time = 0.02, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\sec^2(x))}{2}$	7
default	$\frac{(\sec^2(x))}{2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sec(x)^2
```

Maxima [A]
time = 2.62, size = 6, normalized size = 0.75

$$\frac{1}{2}\tan\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2
```

Fricas [A]
time = 0.68, size = 6, normalized size = 0.75

$$\frac{1}{2\,\cos\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

Giac [A]
time = 0.45, size = 6, normalized size = 0.75

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.75

$$\frac{\tan(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^2,x)
```

```
[Out] tan(x)^2/2
```

3.7 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}(1+x^2)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{267\}$

$$\frac{1}{3}(x^2+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}(1+x^2)^{3/2}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{3}(1+x^2)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Maple [A]
time = 0.00, size = 10, normalized size = 0.77

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
default	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2x^2+2)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A]
time = 2.73, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A]
time = 0.65, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.
time = 0.06, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Giac [A]
time = 0.49, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.69

$$\frac{\left(x^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/2),x)
```

```
[Out] (x^2 + 1)^(3/2)/3
```

3.8 $\int e^x \sin(x)\, dx$

Optimal. Leaf size=19

$$-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4517}

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(x)\,dx=-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(-\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.74

method	result	size
default	$-\frac{e^{x} \cos(x)}{2}+\frac{e^{x} \sin(x)}{2}$	14
norman	$\frac{e^{x} \tan\left(\frac{x}{2}\right)+\frac{e^{x} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i)x}}{4}-\frac{i e^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{i e^{(1-i)x}}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A]
time = 1.62, size = 11, normalized size = 0.58

$$-\frac{1}{2} (\cos(x) - \sin(x))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A]
time = 0.78, size = 13, normalized size = 0.68

$$-\frac{1}{2} \cos(x) e^{x} + \frac{1}{2} e^{x} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.79

$$\frac{e^{x} \sin(x)}{2} - \frac{e^{x} \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Giac [A]
time = 0.47, size = 11, normalized size = 0.58

$$-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.58

$$-\frac{e^{x}\left(\cos\left(x\right)-\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

3.9 $\int \cot(x)\csc^3(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{3}\csc^3(x)$$

```
[Out] -1/3*csc(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 30}

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x]^3,x]
```

```
[Out] -1/3*Csc[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc^3(x)\,dx &= -\text{Subst}\left(\int x^2\,dx, x, \csc(x)\right)\\ &= -\frac{1}{3}\csc^3(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x]^3,x]
[Out] -1/3*Csc[x]^3
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{1}{3\sin(x)^3}$	7
default	$-\frac{1}{3\sin(x)^3}$	7
risch	$\frac{8i e^{3ix}}{3(e^{2ix}-1)^3}$	18
norman	$\frac{-\frac{1}{24}-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/3/sin(x)^3
```

Maxima [A]
time = 2.66, size = 6, normalized size = 0.75

$$-\frac{1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="maxima")
[Out] -1/3/sin(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.57, size = 14, normalized size = 1.75

$$\frac{1}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="fricas")
[Out] 1/3/((cos(x)^2 - 1)*sin(x))
```

Sympy [A]
time = 0.01, size = 8, normalized size = 1.00

$$-\frac{1}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)**2/sin(x)**2,x)
```

```
[Out] -1/(3*sin(x)**3)
```

Giac [A]
time = 0.44, size = 6, normalized size = 0.75

$$-\frac{1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/3/sin(x)^3
```

Mupad [B]
time = 0.05, size = 6, normalized size = 0.75

$$-\frac{1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)^4,x)
```

```
[Out] -1/(3*sin(x)^3)
```

3.10 $\int \sin(e^x)\,dx$

Optimal. Leaf size=4

$$\text{Si}(e^x)$$

[Out] Si(exp(x))

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2320, 3380}

$$\text{Si}(e^x)$$

Antiderivative was successfully verified.

[In] Int[Sin[E^x],x]

[Out] SinIntegral[E^x]

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(e^x)\,dx &= \text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, e^x\right)\\ &= \text{Si}(e^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\text{Si}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[E^x],x]
```

```
[Out] SinIntegral[E^x]
```

Maple [A]
time = 0.02, size = 4, normalized size = 1.00

method	result	size
derivativedivides	$\mathrm{sinIntegral}\left(\mathrm{e}^{x}\right)$	4
default	$\mathrm{sinIntegral}\left(\mathrm{e}^{x}\right)$	4
risch	$-\frac{\pi\,\mathrm{csgn}(\mathrm{e}^{x})}{2}+\mathrm{sinIntegral}\left(\mathrm{e}^{x}\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] Si(exp(x))
```

Maxima [C] Result contains complex when optimal does not.
time = 1.23, size = 15, normalized size = 3.75

$$-\frac{1}{2}i\,\mathrm{Ei}(i\,e^{x})+\frac{1}{2}i\,\mathrm{Ei}(-i\,e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*e^x) + 1/2*I*Ei(-I*e^x)
```

Fricas [A]
time = 0.65, size = 3, normalized size = 0.75

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="fricas")
```

```
[Out] sin_integral(e^x)
```

Sympy [A]
time = 0.32, size = 3, normalized size = 0.75

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x)
```

```
[Out] Si(exp(x))
```

Giac [A]
time = 0.48, size = 3, normalized size = 0.75

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="giac")
```

```
[Out] sin_integral(e^x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.25

$$\mathrm{sinint}(\mathrm{e}^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(exp(x)),x)
```

```
[Out] sinint(exp(x))
```

3.11 $\int \frac{\sin(y)}{y} \, dy$

Optimal. Leaf size=2

$$\mathrm{Si}(y)$$

[Out] Si(y)

Rubi [A]
time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3380}

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

[In] Int[Sin[y]/y,y]

[Out] SinIntegral[y]

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(y)}{y} \, dy = \mathrm{Si}(y)$$

Mathematica [A]
time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[y]/y,y]

[Out] SinIntegral[y]

Maple [A]
time = 0.02, size = 3, normalized size = 1.50

method	result	size

default	$\mathrm{sinIntegral}(y)$	3
meijerg	$\mathrm{sinIntegral}(y)$	3
risch	$-\frac{\pi\,\mathrm{csgn}(y)}{2}+\mathrm{sinIntegral}(y)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)/y,y,method=_RETURNVERBOSE)
```

```
[Out] Si(y)
```

Maxima [C] Result contains complex when optimal does not.
time = 1.97, size = 13, normalized size = 6.50

$$-\frac{1}{2}i\,\mathrm{Ei}(i\,y)+\frac{1}{2}i\,\mathrm{Ei}(-i\,y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*y) + 1/2*I*Ei(-I*y)
```

Fricas [A]
time = 0.69, size = 2, normalized size = 1.00

$$\mathrm{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y, algorithm="fricas")
```

```
[Out] sin_integral(y)
```

Sympy [A]
time = 0.31, size = 2, normalized size = 1.00

$$\mathrm{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y)
```

```
[Out] Si(y)
```

Giac [A]
time = 0.43, size = 2, normalized size = 1.00

$$\mathrm{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y, algorithm="giac")
```

```
[Out] sin_integral(y)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.50

$$\mathrm{sinint}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)/y,y)
```

```
[Out] sinint(y)
```

3.12 $\int (e^x+\sin(x)) \, dx$

Optimal. Leaf size=8

$$e^x-\cos(x)$$

```
[Out] exp(x)-cos(x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225, 2718}

$$e^x-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x + Sin[x],x]
```

```
[Out] E^x - Cos[x]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (e^x+\sin(x)) \, dx &= \int e^x \, dx+\int \sin(x) \, dx \\ &= e^x-\cos(x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$e^x-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x + Sin[x],x]
```

```
[Out] E^x - Cos[x]
```

Maple [A]
time = 0.02, size = 8, normalized size = 1.00

method	result	size
default	$e^{x}-\cos \left(x \right)$	8
risch	$e^{x}-\cos \left(x \right)$	8
meijerg	$-1+e^{x}+\sqrt{\pi}\, \left(\frac{1}{\sqrt{\pi}}-\frac{\cos \left(x \right)}{\sqrt{\pi}}\right)$	20
norman	$\frac{e^{x} \left(\tan^{2}\left(\frac{x}{2}\right)\right)-2+e^{x}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)+sin(x),x,method=_RETURNVERBOSE)
[Out] exp(x)-cos(x)
```

Maxima [A]
time = 5.91, size = 7, normalized size = 0.88

$$-\cos \left(x\right) + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="maxima")
[Out] -cos(x) + e^x
```

Fricas [A]
time = 1.08, size = 7, normalized size = 0.88

$$-\cos \left(x\right) + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="fricas")
[Out] -cos(x) + e^x
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.62

$$e^{x} - \cos{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x)
```

```
[Out] exp(x) - cos(x)
```

Giac [A]
time = 0.44, size = 7, normalized size = 0.88

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="giac")
```

```
[Out] -cos(x) + e^x
```

Mupad [B]
time = 0.04, size = 7, normalized size = 0.88

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x) + sin(x),x)
```

```
[Out] exp(x) - cos(x)
```

3.13 $\int \left(e^{x^2}+2e^{x^2}x^2\right) dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

```
[Out] exp(x^2)*x
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2235, 2243}

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int \left(e^{x^2}+2e^{x^2}x^2\right) dx &= 2\int e^{x^2}x^2\, dx+\int e^{x^2}\, dx\\ &= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)-\int e^{x^2}\, dx\\ &= e^{x^2}x\end{aligned}$$

Mathematica [A]
time = 0.02, size = 7, normalized size = 1.00

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Maple [A]
time = 0.01, size = 7, normalized size = 1.00

method	result	size
gosper	$e^{x^2}x$	7
default	$e^{x^2}x$	7
norman	$e^{x^2}x$	7
risch	$e^{x^2}x$	7
meijerg	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}+i\left(-ix\,e^{x^2}+\frac{i\operatorname{erfi}(x)\sqrt{\pi}}{2}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)+2*exp(x^2)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(x^2)*x
```

Maxima [A]
time = 4.37, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A]
time = 1.39, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)+2*exp(x**2)*x**2,x)
```

```
[Out] x*exp(x**2)
```

Giac [A]
time = 0.45, size = 6, normalized size = 0.86

$$xe^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

Mupad [B]
time = 0.15, size = 6, normalized size = 0.86

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2) + 2*x^2*exp(x^2),x)
```

```
[Out] x*exp(x^2)
```

3.14 $\int (e^x + x)^2 \, dx$

Optimal. Leaf size=28

$$-2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3}$$

```
[Out] -2*exp(x)+1/2*exp(2*x)+2*exp(x)*x+1/3*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6874, 2225, 2207}

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + x)^2,x]
```

```
[Out] -2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (e^x+x)^2\,dx &= \int \left(e^{2x}+2e^x x+x^2\right)\,dx\\ &= \frac{x^3}{3}+2\int e^x x\,dx+\int e^{2x}\,dx\\ &= \frac{e^{2x}}{2}+2e^x x+\frac{x^3}{3}-2\int e^x\,dx\\ &= -2e^x+\frac{e^{2x}}{2}+2e^x x+\frac{x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.93

$$\frac{e^{2x}}{2}+\frac{x^3}{3}+e^x(-2+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + x)^2,x]
```

```
[Out] E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.79

method	result	size
risch	$\frac{x^3}{3}+(2x-2)\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}$	21
default	$-2\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^x x+\frac{x^3}{3}$	22
norman	$-2\,\mathrm{e}^x+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^x x+\frac{x^3}{3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+1/2*exp(x)^2+2*exp(x)*x-2*exp(x)
```

Maxima [A]
time = 1.49, size = 19, normalized size = 0.68

$$\frac{1}{3}\,x^3+2\,(x-1)e^x+\frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Fricas [A]
time = 1.09, size = 19, normalized size = 0.68

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.71

$$\frac{x^3}{3} + \frac{(4x-4)\,e^x}{2} + \frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))**2,x)
```

```
[Out] x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2
```

Giac [A]
time = 0.43, size = 19, normalized size = 0.68

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Mupad [B]
time = 0.05, size = 21, normalized size = 0.75

$$\frac{\mathrm{e}^{2\,x}}{2} - 2\,\mathrm{e}^x + 2\,x\,\mathrm{e}^x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + exp(x))^2,x)
```

```
[Out] exp(2*x)/2 - 2*exp(x) + 2*x*exp(x) + x^3/3
```

3.15 $\int (2e^x + e^{2x} + x^2) \, dx$

Optimal. Leaf size=22

$$2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}$$

```
[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2225}

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[2*E^x + E^(2*x) + x^2,x]
```

```
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(2e^x + e^{2x} + x^2\right) dx &= \frac{x^3}{3} + 2\int e^x \, dx + \int e^{2x} \, dx \\ &= 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*E^x + E^(2*x) + x^2,x]
```

```
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
default	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
norman	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17
risch	$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+\frac{x^{3}}{3}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(x)+exp(2*x)+x^2,x,method=_RETURNVERBOSE)
[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

Maxima [A]
time = 2.86, size = 16, normalized size = 0.73

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,e^{(2\,x)}+2\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Fricas [A]
time = 1.15, size = 16, normalized size = 0.73

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,e^{(2\,x)}+2\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.68

$$\frac{x^{3}}{3}+\frac{e^{2x}}{2}+2e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x**2,x)
[Out] x**3/3 + exp(2*x)/2 + 2*exp(x)
```

Giac [A]
time = 0.45, size = 16, normalized size = 0.73

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,e^{(2\,x)} + 2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Mupad [B]
time = 0.05, size = 16, normalized size = 0.73

$$\frac{\mathrm{e}^{2\,x}}{2} + 2\,\mathrm{e}^x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x) + 2*exp(x) + x^2,x)
```

```
[Out] exp(2*x)/2 + 2*exp(x) + x^3/3
```

3.16 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

[Out] 1/2*sin(x)^2

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2644, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]
```

```
[Out] -1/2*Cos[x]^2
```

Maple [A]
time = 0.00, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\left(\sin^2(x)\right)}{2}$	7
default	$\frac{\left(\sin^2(x)\right)}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
norman	$\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)^2
```

Maxima [A]
time = 1.86, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A]
time = 1.48, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

Giac [A]
time = 0.44, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.75

$$\frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
```

```
[Out] sin(x)^2/2
```

3.17 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] 1/2*exp(x^2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2240}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x,x]

[Out] E^x^2/2

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*x,x]

[Out] E^x^2/2

Maple [A]
time = 0.00, size = 7, normalized size = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x^2)
```

Maxima [A]
time = 1.48, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
[Out] 1/2*e^(x^2)
```

Fricas [A]
time = 1.37, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
[Out] 1/2*e^(x^2)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Giac [A]
time = 0.44, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

Mupad [B]
time = 0.00, size = 6, normalized size = 0.67

$$\frac{\mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
```

```
[Out] exp(x^2)/2
```

3.18 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx = \frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Maple [A]
time = 0.00, size = 10, normalized size = 0.77

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
default	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2x^2+2)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A]
time = 3.30, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A]
time = 1.07, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.
time = 0.06, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Giac [A]
time = 0.42, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Mupad [B]
time = 0.00, size = 9, normalized size = 0.69

$$\frac{\left(x^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/2),x)
```

```
[Out] (x^2 + 1)^(3/2)/3
```

3.19 $\int \frac{e^x}{1+e^x} \, dx$

Optimal. Leaf size=6

$$\log (1+e^x)$$

```
[Out] ln(1+exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2278, 31}

$$\log (e^x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1+e^x} \, dx &= \text{Subst}\left(\int \frac{1}{1+x} \, dx, x, e^x\right) \\ &= \log (1+e^x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$\log (1+e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^x),x]
[Out] Log[1 + E^x]
```

Maple [A]
time = 0.01, size = 6, normalized size = 1.00

method	result	size
derivativedivides	$\ln(1+e^x)$	6
default	$\ln(1+e^x)$	6
norman	$\ln(1+e^x)$	6
risch	$\ln(1+e^x)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] ln(1+exp(x))
```

Maxima [A]
time = 1.09, size = 5, normalized size = 0.83

$$\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
[Out] log(e^x + 1)
```

Fricas [A]
time = 1.01, size = 5, normalized size = 0.83

$$\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
[Out] log(e^x + 1)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.83

$$\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x)
```

```
[Out] log(exp(x) + 1)
```

Giac [A]
time = 0.48, size = 5, normalized size = 0.83

$$\log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] log(e^x + 1)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.83

$$\ln\left(\mathrm{e}^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(x) + 1),x)
```

```
[Out] log(exp(x) + 1)
```

3.20 $\int x^{3/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

[Out] 2/5*x^(5/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2),x]

[Out] (2*x^(5/2))/5

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2),x]

[Out] (2*x^(5/2))/5

Maple [A]
time = 0.01, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}}{5}$	6
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}$	6
default	$\frac{2x^{\frac{5}{2}}}{5}$	6
trager	$\frac{2x^{\frac{5}{2}}}{5}$	6
risch	$\frac{2x^{\frac{5}{2}}}{5}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)
```

Maxima [A]
time = 1.98, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)
```

Fricas [A]
time = 1.19, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5
```

Giac [A]
time = 0.49, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)
```

Mupad [B]
time = 0.04, size = 5, normalized size = 0.56

$$\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
```

```
[Out] (2*x^(5/2))/5
```

3.21 $\int \cos(3+2x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin(3+2x)$$

```
[Out] 1/2*sin(3+2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2717}

$$\frac{1}{2}\sin(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(3+2x)$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{2}\sin(3+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\frac{\sin(3+2x)}{2}$	9
default	$\frac{\sin(3+2x)}{2}$	9
risch	$\frac{\sin(3+2x)}{2}$	9
norman	$\frac{\tan\left(\frac{3}{2}+x\right)}{1+\tan^2\left(\frac{3}{2}+x\right)}$	16
meijerg	$\frac{\cos(3)\sin(2x)}{2}-\frac{\sin(3)\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(3+2*x)
```

Maxima [A]
time = 1.91, size = 8, normalized size = 0.80

$$\frac{1}{2}\,\sin\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*x + 3)
```

Fricas [A]
time = 0.97, size = 8, normalized size = 0.80

$$\frac{1}{2}\,\sin\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sin(2*x + 3)
```

Sympy [A]
time = 0.04, size = 7, normalized size = 0.70

$$\frac{\sin\left(2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x)
```

```
[Out] sin(2*x + 3)/2
```

Giac [A]
time = 0.47, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="giac")
```

```
[Out] 1/2*sin(2*x + 3)
```

Mupad [B]
time = 0.07, size = 8, normalized size = 0.80

$$\frac{\sin\left(2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x + 3),x)
```

```
[Out] sin(2*x + 3)/2
```

3.22 $\int 2e^{2x}yz\,dx$

Optimal. Leaf size=8

$$e^{2x}yz$$

[Out] exp(2*x)*y*z

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 2225}

$$e^{2x}yz$$

Antiderivative was successfully verified.

[In] Int[2*E^(2*x)*y*z,x]

[Out] E^(2*x)*y*z

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int 2e^{2x}yz\,dx &= (2yz)\int e^{2x}\,dx \\ &= e^{2x}yz \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$e^{2x}yz$$

Antiderivative was successfully verified.

[In] Integrate[2*E^(2*x)*y*z,x]

```
[Out] E^(2*x)*y*z
```

Maple [A]
time = 0.01, size = 8, normalized size = 1.00

method	result	size
gosper	$e^{2x}yz$	8
derivativedivides	$e^{2x}yz$	8
default	$e^{2x}yz$	8
norman	$e^{2x}yz$	8
risch	$e^{2x}yz$	8
meijerg	$-yz(1-e^{2x})$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(2*x)*y*z,x,method=_RETURNVERBOSE)
```

```
[Out] exp(2*x)*y*z
```

Maxima [A]
time = 3.32, size = 7, normalized size = 0.88

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="maxima")
```

```
[Out] y*z*e^(2*x)
```

Fricas [A]
time = 1.26, size = 7, normalized size = 0.88

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="fricas")
```

```
[Out] y*z*e^(2*x)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.88

$$yze^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x)
```

```
[Out] y*z*exp(2*x)
```

Giac [**A**]
time = 0.46, size = 7, normalized size = 0.88

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="giac")
```

```
[Out] y*z*e^(2*x)
```

Mupad [**B**]
time = 0.02, size = 7, normalized size = 0.88

$$y\,z\,\mathrm{e}^{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*y*z*exp(2*x),x)
```

```
[Out] y*z*exp(2*x)
```

3.23 $\int e^x \cos^2(e^x) \sin(e^x)\, dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\cos^3(e^x)$$

```
[Out] -1/3*cos(exp(x))^3
```

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 2645, 30}

$$-\frac{1}{3}\cos^3(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[E^x]^2*Sin[E^x],x]
```

```
[Out] -1/3*Cos[E^x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int e^x \cos^2(e^x) \sin(e^x) \, dx = \text{Subst}\left(\int \cos^2(x) \sin(x) \, dx, x, e^x\right)$$
$$= -\text{Subst}\left(\int x^2 \, dx, x, \cos(e^x)\right)$$
$$= -\frac{1}{3} \cos^3(e^x)$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.90

$$-\frac{1}{4} \cos(e^x) - \frac{1}{12} \cos(3e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[E^x]^2*Sin[E^x],x]
[Out] -1/4*Cos[E^x] - Cos[3*E^x]/12
```

Maple [A]
time = 0.03, size = 8, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{(\cos^3(e^x))}{3}$	8
default	$-\frac{(\cos^3(e^x))}{3}$	8
risch	$-\frac{\cos(e^x)}{4} - \frac{\cos(3\,e^x)}{12}$	14
norman	$\frac{2\left(\tan^2\left(\frac{e^x}{2}\right)\right)+\frac{2\left(\tan^6\left(\frac{e^x}{2}\right)\right)}{3}}{\left(1+\tan^2\left(\frac{e^x}{2}\right)\right)^3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(exp(x))^2*sin(exp(x)),x,method=_RETURNVERBOSE)
[Out] -1/3*cos(exp(x))^3
```

Maxima [A]
time = 1.53, size = 7, normalized size = 0.70

$$-\frac{1}{3} \cos(e^x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/3*cos(e^x)^3
```

Fricas [A]
time = 1.00, size = 7, normalized size = 0.70

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="fricas")
```

```
[Out] -1/3*cos(e^x)^3
```

Sympy [A]
time = 0.32, size = 8, normalized size = 0.80

$$-\frac{\cos^{3}\left(e^{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)
```

```
[Out] -cos(exp(x))**3/3
```

Giac [A]
time = 0.45, size = 7, normalized size = 0.70

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="giac")
```

```
[Out] -1/3*cos(e^x)^3
```

Mupad [B]
time = 0.19, size = 7, normalized size = 0.70

$$-\frac{\cos\left(\mathrm{e}^{x}\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(exp(x))^2*sin(exp(x))*exp(x),x)
```

```
[Out] -cos(exp(x))^3/3
```

3.24 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x],x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x}\,dx &= \int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$\frac{2}{15}(1+x)^{3/2}(-5+3(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(-5 + 3*(1 + x)))/15
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.70

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(-2+3x)}{15}$	13
derivativedivides	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
default	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
risch	$\frac{2(3x^2+x-2)\sqrt{1+x}}{15}$	16
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{4}{15}\right)\sqrt{1+x}$	17
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Maxima [A]
time = 1.46, size = 15, normalized size = 0.65

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Fricas [A]
time = 0.90, size = 15, normalized size = 0.65

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A]
time = 0.46, size = 34, normalized size = 1.48

$$\frac{2x^2\sqrt{x+1}}{5}+\frac{2x\sqrt{x+1}}{15}-\frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

Giac [A]
time = 0.45, size = 15, normalized size = 0.65

$$\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.52

$$\frac{2\,(3\,x-2)\,\left(x+1\right)^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(1/2),x)
```

```
[Out] (2*(3*x - 2)*(x + 1)^(3/2))/15
```

3.25 $\int \frac{1}{-1+x^4} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {218, 212, 209}

$$-\frac{\text{ArcTan}(x)}{2}-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x] - ArcTanh[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.92

$$-\frac{1}{2}\tan^{-1}(x)+\frac{1}{4}\log(1-x)-\frac{1}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(-1),x]
[Out] -1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A]
time = 0.00, size = 10, normalized size = 0.77

method	result	size
default	$-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$\frac{\ln(-1+x)}{4}-\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{4}$	18
meijerg	$\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1),x,method=_RETURNVERBOSE)
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Maxima [A]
time = 2.46, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A]
time = 0.71, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [**A**]
time = 0.05, size = 17, normalized size = 1.31

$$\frac{\log{(x-1)}}{4}-\frac{\log{(x+1)}}{4}-\frac{\operatorname{atan}{(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1),x)
```

```
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 0.45, size = 19, normalized size = 1.46

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(|x+1|)+\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [**B**]
time = 0.16, size = 9, normalized size = 0.69

$$-\frac{\mathrm{atan}(x)}{2}-\frac{\mathrm{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 1),x)
```

```
[Out] - atan(x)/2 - atanh(x)/2
```

3.26 $\int \frac{e^x}{2+3e^{2x}}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,e^x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2281, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(2 + 3*E^(2*x)),x]
```

```
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right) = \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,e^x\right)}{\sqrt{6}}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(2 + 3*E^(2*x)),x]
```

```
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.78

method	result	size
default	$\frac{\arctan\left(\frac{e^x\sqrt{6}}{2}\right)\sqrt{6}}{6}$	14
risch	$\frac{i\sqrt{6}\,\ln\left(e^x+\frac{i\sqrt{6}}{3}\right)}{12}-\frac{i\sqrt{6}\,\ln\left(e^x-\frac{i\sqrt{6}}{3}\right)}{12}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(2+3*exp(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Maxima [A]
time = 1.67, size = 13, normalized size = 0.72

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Fricas [A]
time = 0.59, size = 13, normalized size = 0.72

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.83

$$\operatorname{RootSum}\left(24z^{2}+1,\left(i\mapsto i\log\left(4i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] RootSum(24*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))
```

Giac [A]
time = 0.44, size = 13, normalized size = 0.72

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Mupad [B]
time = 0.11, size = 13, normalized size = 0.72

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,\mathrm{e}^{x}}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(3*exp(2*x) + 2),x)
```

```
[Out] (6^(1/2)*atan((6^(1/2)*exp(x))/2))/6
```

3.27 $\int \frac{e^{2x}}{A+Be^{4x}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}\,e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\,\sqrt{B}}$$

```
[Out] 1/2*arctan(exp(2*x)*B^(1/2)/A^(1/2))/A^(1/2)/B^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2281, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{B}\,e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\,\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(A + B*E^(4*x)),x]
```

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2x}}{A+Be^{4x}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{A+Bx^2}\,dx, x, e^{2x}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{B}\,e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\,\sqrt{B}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(A + B*E^(4*x)),x]
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.65

method	result	size
default	$\frac{\arctan\left(\frac{B\,e^{2x}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$	20
risch	$-\frac{\ln\left(e^{2x}-\frac{A}{\sqrt{-AB}}\right)}{4\sqrt{-AB}}+\frac{\ln\left(e^{2x}+\frac{A}{\sqrt{-AB}}\right)}{4\sqrt{-AB}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(A+B*exp(4*x)),x,method=_RETURNVERBOSE)
[Out] 1/2/(A*B)^(1/2)*arctan(B*exp(x)^2/(A*B)^(1/2))
```

Maxima [A]
time = 2.02, size = 19, normalized size = 0.61

$$\frac{\arctan\left(\frac{Be^{(2\,x)}}{\sqrt{AB}}\right)}{2\,\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="maxima")
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

Fricas [A]
time = 0.79, size = 76, normalized size = 2.45

$$\left[-\frac{\sqrt{-AB}\log\left(\frac{Be^{(4\,x)}-2\,\sqrt{-AB}\,e^{(2\,x)}-A}{Be^{(4\,x)}+A}\right)}{4\,AB}, -\frac{\sqrt{AB}\arctan\left(\frac{\sqrt{AB}\,e^{(-2\,x)}}{B}\right)}{2\,AB}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-A*B)*log((B*e^(4*x) - 2*sqrt(-A*B)*e^(2*x) - A)/(B*e^(4*x) + A)
)/(A*B), -1/2*sqrt(A*B)*arctan(sqrt(A*B)*e^(-2*x)/B)/(A*B)]
```

Sympy [A]
time = 0.06, size = 22, normalized size = 0.71

$$\operatorname{RootSum}\left(16z^2AB + 1, \left(i \mapsto i\log\left(4iA + e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x)
```

```
[Out] RootSum(16*_z**2*A*B + 1, Lambda(_i, _i*log(4*_i*A + exp(2*x))))
```

Giac [A]
time = 0.46, size = 19, normalized size = 0.61

$$\frac{\arctan\left(\frac{Be^{(2x)}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

Mupad [B]
time = 0.23, size = 19, normalized size = 0.61

$$\frac{\operatorname{atan}\left(\frac{B\,\mathrm{e}^{2\,x}}{\sqrt{A\,B}}\right)}{2\,\sqrt{A\,B}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(A + B*exp(4*x)),x)
```

```
[Out] atan((B*exp(2*x))/(A*B)^(1/2))/(2*(A*B)^(1/2))
```

3.28 $\int \frac{e^{1+x}}{1+e^x} \, dx$

Optimal. Leaf size=8

$$e \log (1 + e^x)$$

```
[Out] exp(1)*ln(1+exp(x))
```

Rubi [A]

time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2279, 2278, 31}

$$e \log (e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(1 + x)/(1 + E^x),x]
```

```
[Out] E*Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rule 2279

```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Dist[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n, Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p, x
], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Log
[F], g*h*m*Log[G]]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{1+x}}{1+e^x} \, dx &= e \int \frac{e^x}{1+e^x} \, dx \\ &= e\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, e^x\right) \\ &= e \log (1 + e^x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$e \log (1 + e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(1 + x)/(1 + E^x),x]
[Out] E*Log[1 + E^x]
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size
default	$\mathrm{e}\ln(1 + \mathrm{e}^x)$	9
norman	$\mathrm{e}\ln(1 + \mathrm{e}^x)$	9
risch	$\mathrm{e}\ln(1 + \mathrm{e}^x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1+x)/(1+exp(x)),x,method=_RETURNVERBOSE)
[Out] exp(1)*ln(1+exp(x))
```

Maxima [A]
time = 2.05, size = 8, normalized size = 1.00

$$e \log (e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="maxima")
[Out] e*log(e^x + 1)
```

Fricas [A]
time = 0.64, size = 11, normalized size = 1.38

$$e \log \left(e + e^{(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="fricas")
[Out] e*log(e + e^(x + 1))
```

Sympy [A]
time = 0.03, size = 8, normalized size = 1.00

$$e \log (e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x)
```

```
[Out] E*log(exp(x) + 1)
```

Giac [A]
time = 0.46, size = 8, normalized size = 1.00

$$e \log (e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e*log(e^x + 1)
```

Mupad [B]
time = 0.18, size = 8, normalized size = 1.00

$$\mathrm{e} \ln (\mathrm{e}^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x + 1)/(exp(x) + 1),x)
```

```
[Out] exp(1)*log(exp(x) + 1)
```

3.29 $\int (10e)^x \, dx$

Optimal. Leaf size=12

$$\frac{(10e)^x}{1+\log(10)}$$

```
[Out] (10*exp(1))^x/(1+ln(10))
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2225}

$$\frac{(10e)^x}{1+\log(10)}$$

Antiderivative was successfully verified.

```
[In] Int[(10*E)^x,x]
```

```
[Out] (10*E)^x/(1 + Log[10])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int (10e)^x \, dx = \frac{(10e)^x}{1+\log(10)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{(10e)^x}{\log(10e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(10*E)^x,x]
```

```
[Out] (10*E)^x/Log[10*E]
```

Maple [A]
time = 0.02, size = 15, normalized size = 1.25

method	result	size
gosper	$\frac{(10\,\mathrm{e})^{x}}{\ln(10\,\mathrm{e})}$	15
derivativedivides	$\frac{(10\,\mathrm{e})^{x}}{\ln(10\,\mathrm{e})}$	15
default	$\frac{(10\,\mathrm{e})^{x}}{\ln(10\,\mathrm{e})}$	15
norman	$\frac{\mathrm{e}^{x\ln(10\,\mathrm{e})}}{1+\ln(10)}$	16
risch	$\frac{5^{x}2^{x}\mathrm{e}^{x}}{1+\ln(2)+\ln(5)}$	18
meijerg	$-\frac{1-\mathrm{e}^{x(1+\ln(10))}}{1+\ln(10)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*exp(1))^x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/ln(10*exp(1))*(10*exp(1))^x
```

Maxima [A]
time = 2.01, size = 14, normalized size = 1.17

$$\frac{(10\,e)^{x}}{\log(10\,e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*exp(1))^x,x, algorithm="maxima")
```

```
[Out] (10*e)^x/log(10*e)
```

Fricas [A]
time = 0.70, size = 14, normalized size = 1.17

$$\frac{e^{(x\log(10)+x)}}{\log(10)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*exp(1))^x,x, algorithm="fricas")
```

```
[Out] e^(x*log(10) + x)/(log(10) + 1)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.83

$$\frac{(10e)^{x}}{1+\log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*exp(1))**x,x)
[Out] (10*E)**x/(1 + log(10))
```

Giac [A]
time = 0.49, size = 13, normalized size = 1.08

$$\frac{(10\,e)^x}{\log(10)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*exp(1))^x,x, algorithm="giac")
[Out] (10*e)^x/(log(10) + 1)
```

Mupad [B]
time = 0.07, size = 12, normalized size = 1.00

$$\frac{10^x\,\mathrm{e}^x}{\ln(10)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*exp(1))^x,x)
[Out] (10^x*exp(x))/(log(10) + 1)
```

3.30 $\int x^3 \sin(x^2) \, dx$

Optimal. Leaf size=20

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}$$

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 3377, 2717}

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \sin\left(x^2\right) dx = \frac{1}{2}\text{Subst}\left(\int x\sin(x)\,dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2}x^2\cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
```

```
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{x^2\cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
default	$-\frac{x^2\cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
risch	$-\frac{x^2\cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
meijerg	$\sqrt{\pi}\left(-\frac{x^2\cos(x^2)}{2\sqrt{\pi}} + \frac{\sin(x^2)}{2\sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^2}{2}+\frac{x^2\left(\tan^2\left(\frac{x^2}{2}\right)\right)}{2}+\tan\left(\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{x^2}{2}\right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A]
time = 1.23, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A]
time = 0.58, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A]
time = 0.13, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Giac [A]
time = 0.45, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B]
time = 0.21, size = 16, normalized size = 0.80

$$\frac{\sin\left(x^2\right)}{2}-\frac{x^2\,\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.31 $\int \frac{x^7}{1+x^{12}} dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)$$

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {281, 298, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^12),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+x^{12}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\right)+\frac{1}{12}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 260 vs. 2(49) = 98.
time = 0.08, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt{3}+2\sqrt{2}x}{-1+\sqrt{3}}\right)-2\log\left(1-\sqrt{2}x+x^2\right)-2\log\left(1+\sqrt{2}x+x^2\right)+\log\left(2+\sqrt{2}x-\sqrt{6}x+2x^2\right)+\log\left(2+\sqrt{2}\left(-1+\sqrt{3}\right)x+2x^2\right)+\log\left(2-\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)+\log\left(2+\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*Ar
cTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sq
rt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqr
t[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x
+ x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqr
t[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2
] + Sqrt[6])*x + 2*x^2])/24
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.84

method	result	size
risch	$-\frac{\ln(x^4+1)}{12}+\frac{\ln(x^8-x^4+1)}{24}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	39
default	$-\frac{\ln(x^4+1)}{12}+\frac{\ln(x^8-x^4+1)}{24}+\frac{\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	41
meijerg	$-\frac{x^8\ln\left(1+(x^{12})^{\frac{1}{3}}\right)}{12(x^{12})^{\frac{2}{3}}}+\frac{x^8\ln\left(1-(x^{12})^{\frac{1}{3}}+(x^{12})^{\frac{2}{3}}\right)}{24(x^{12})^{\frac{2}{3}}}+\frac{x^8\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^{12})^{\frac{1}{3}}}{2-(x^{12})^{\frac{1}{3}}}\right)}{12(x^{12})^{\frac{2}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*arctan(1/3*(2*x^4-1)*3^(1/2))*3^(1/
2)
```

Maxima [A]
time = 1.79, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4-1)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Fricas [A]
time = 0.60, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4-1)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Sympy [A]
time = 0.06, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3
 - sqrt(3)/3)/12
```

Giac [A]
time = 0.47, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Mupad [B]
time = 0.14, size = 52, normalized size = 1.06

$$-\frac{\ln\left(x^4+1\right)}{12}-\ln\left(x^4-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)+\ln\left(x^4+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12 + 1),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^4 - 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x^4 - (3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x^4 + 1)/12
```

3.32 $\int x^{3a}\sin\left(x^{2a}\right)\,dx$

Optimal. Leaf size=115

$$\frac{ix^{1+3a}\left(-ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),-ix^{2a}\right)}{4a}-\frac{ix^{1+3a}\left(ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),ix^{2a}\right)}{4a}$$

```
[Out] 1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,-I*x^(2*a))/a/((-I*x^(2*a))^(1/2*(1+3*a)/a)
)-1/4*I*x^(1+3*a)*GAMMA(3/2+1/2/a,I*x^(2*a))/a/((I*x^(2*a))^(1/2*(1+3*a)/a)
)
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3504, 2250}

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),ix^{2a}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*x^(2*a))^(
(1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, I*x^(2*a)])/(a
*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3504

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^((-c)*I - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int x^{3a}\sin\left(x^{2a}\right)\,dx &= \frac{1}{2}i\int e^{-ix^{2a}}x^{3a}\,dx-\frac{1}{2}i\int e^{ix^{2a}}x^{3a}\,dx\\ &= \frac{ix^{1+3a}\left(-ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),-ix^{2a}\right)}{4a}-\frac{ix^{1+3a}\left(ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),ix^{2a}\right)}{4a}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 142, normalized size = 1.23

$$-\frac{x^{1+a}(x^{4a})^{-\frac{1+a}{2a}}\left(4a(x^{4a})^{\frac{1+a}{2a}}\cos\left(x^{2a}\right)+(1+a)\left(ix^{2a}\right)^{\frac{1+a}{2a}}\Gamma\left(\frac{1+a}{2a},-ix^{2a}\right)+(1+a)\left(-ix^{2a}\right)^{\frac{1+a}{2a}}\Gamma\left(\frac{1+a}{2a},ix^{2a}\right)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] -1/8*(x^(1 + a)*(4*a*(x^(4*a))^((1 + a)/(2*a))*Cos[x^(2*a)] + (1 + a)*(I*x^
(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), (-I)*x^(2*a)] + (1 + a)*((-I)*x
^(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), I*x^(2*a)]))/(a^2*(x^(4*a))^((
1 + a)/(2*a)))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.08, size = 41, normalized size = 0.36

method	result	size
meijerg	$\frac{x^{5a+1}\,\mathrm{hypergeom}\left(\left[\frac{5}{4}+\frac{1}{4a}\right],\left[\frac{3}{2},\frac{9}{4}+\frac{1}{4a}\right],-\frac{x^{4a}}{4}\right)}{5a+1}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*a)*sin(x^(2*a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(5*a+1)*x^(5*a+1)*hypergeom([5/4+1/4/a],[3/2,9/4+1/4/a],-1/4*x^(4*a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="maxima")
```

```
[Out] -1/2*(x*x^a*cos(x^(2*a)) - (a + 1)*integrate(x^a*cos(x^(2*a)), x))/a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="fricas")
```

```
[Out] integral(x^(3*a)*sin(x^(2*a)), x)
```

Sympy [A]
time = 1.41, size = 54, normalized size = 0.47

$$\frac{xx^{5a}\Gamma\left(\frac{5}{4}+\frac{1}{4a}\right){}_1F_2\left(\begin{matrix}\frac{5}{4}+\frac{1}{4a}\\ \frac{3}{2},\frac{9}{4}+\frac{1}{4a}\end{matrix}\middle| -\frac{x^{4a}}{4}\right)}{4a\Gamma\left(\frac{9}{4}+\frac{1}{4a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3*a)*sin(x**(2*a)),x)
```

```
[Out] x*x**(5*a)*gamma(5/4 + 1/(4*a))*hyper((5/4 + 1/(4*a),), (3/2, 9/4 + 1/(4*a)
), -x**(4*a)/4)/(4*a*gamma(9/4 + 1/(4*a)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*a)*sin(x^(2*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3\,a}\,\sin\left(x^{2\,a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*a)*sin(x^(2*a)),x)
```

```
[Out] int(x^(3*a)*sin(x^(2*a)), x)
```

3.33 $\int \cos\left(\sqrt{x}\right) \, dx$

Optimal. Leaf size=22

$$2\cos\left(\sqrt{x}\right)+2\sqrt{x}\sin\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3443, 3377, 2718}

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\sqrt{x}\right)\,dx &= 2\text{Subst}\left(\int x\cos(x)\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{x}\sin\left(\sqrt{x}\right)-2\text{Subst}\left(\int \sin(x)\,dx,x,\sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right)+2\sqrt{x}\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$2\cos\left(\sqrt{x}\right)+2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sqrt{x}\,\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A]
time = 1.70, size = 16, normalized size = 0.73

$$2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A]
time = 0.62, size = 16, normalized size = 0.73

$$2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [**A**]
time = 0.09, size = 20, normalized size = 0.91

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [**A**]
time = 0.44, size = 16, normalized size = 0.73

$$2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Mupad [**B**]
time = 0.21, size = 16, normalized size = 0.73

$$2\,\cos\left(\sqrt{x}\right)+2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.34 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x],x]
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x}\,dx &= \int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 0.78

$$\frac{2}{15}(1+x)^{3/2}(-5+3(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2)*(-5 + 3*(1 + x)))/15
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.70

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(-2+3x)}{15}$	13
derivativedivides	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
default	$-\frac{2(1+x)^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{5}{2}}}{5}$	16
risch	$\frac{2(3x^2+x-2)\sqrt{1+x}}{15}$	16
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{4}{15}\right)\sqrt{1+x}$	17
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)
```

Maxima [A]
time = 1.22, size = 15, normalized size = 0.65

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Fricas [A]
time = 0.52, size = 15, normalized size = 0.65

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A]
time = 0.46, size = 34, normalized size = 1.48

$$\frac{2x^2\sqrt{x+1}}{5}+\frac{2x\sqrt{x+1}}{15}-\frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

Giac [A]
time = 0.45, size = 15, normalized size = 0.65

$$\frac{2}{5}\left(x+1\right)^{\frac{5}{2}}-\frac{2}{3}\left(x+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Mupad [B]
time = 0.00, size = 12, normalized size = 0.52

$$\frac{2\,\left(3\,x-2\right)\,{\left(x+1\right)}^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + 1)^(1/2),x)
```

```
[Out] (2*(3*x - 2)*(x + 1)^(3/2))/15
```

3.35 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 45}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx$$
$$= 6\mathrm{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\mathrm{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.03

$$\left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] (6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(24) = 48.
time = 0.03, size = 92, normalized size = 2.88

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$2\ln\left(-1+x^{\frac{1}{6}}\right)-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+2\sqrt{x}+\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(-1+x^(1/6))-ln(x^(1/3)+x^(1/6)+1)-2*ln(1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1
)+2*x^(1/2)+ln(x^(1/2)-1)-ln(1+x^(1/2))+6*x^(1/6)-ln(-1+x)-2*ln(-1+x^(1/3))
+ln(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

Maxima [A]
time = 2.27, size = 24, normalized size = 0.75

$$2\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A]
time = 0.47, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Giac [A]
time = 0.50, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Mupad [B]
time = 0.00, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6\ln\left(x^{1/6} + 1\right) - 3x^{1/3} + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
```

```
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.36 $\int \sqrt{\frac{1+x}{3+2x}}\, dx$

Optimal. Leaf size=44

$$\frac{1}{2}\sqrt{1+x}\,\sqrt{3+2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,\sqrt{1+x}\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arcsinh(2^(1/2)*(1+x)^(1/2))*2^(1/2)+1/2*(1+x)^(1/2)*(3+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1978, 52, 56, 221}

$$\frac{1}{2}\sqrt{x+1}\,\sqrt{2x+3}-\frac{\sinh^{-1}\left(\sqrt{2}\,\sqrt{x+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
```

```
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1+x}{3+2x}}\,dx &= \int \frac{\sqrt{1+x}}{\sqrt{3+2x}}\,dx \\
&= \frac{1}{2}\sqrt{1+x}\,\sqrt{3+2x} - \frac{1}{4}\int \frac{1}{\sqrt{1+x}\,\sqrt{3+2x}}\,dx \\
&= \frac{1}{2}\sqrt{1+x}\,\sqrt{3+2x} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{1}{2}\sqrt{1+x}\,\sqrt{3+2x} - \frac{\sinh^{-1}\left(\sqrt{2}\,\sqrt{1+x}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 75, normalized size = 1.70

$$
\frac{\sqrt{\frac{1+x}{3+2x}}\left(\sqrt{1+x}\,(3+2x) - \sqrt{6+4x}\,\tanh^{-1}\left(\frac{\sqrt{2+2x}}{-1+\sqrt{3+2x}}\right)\right)}{2\sqrt{1+x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[(1 + x)/(3 + 2*x)]*(Sqrt[1 + x]*(3 + 2*x) - Sqrt[6 + 4*x]*ArcTanh[Sqr
t[2 + 2*x]/(-1 + Sqrt[3 + 2*x])]))/(2*Sqrt[1 + x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(30) = 60$.
time = 0.08, size = 75, normalized size = 1.70

method	result
default	$-\frac{\sqrt{\frac{1+x}{3+2x}}\,(3+2x)\left(\ln\left(\frac{5\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2x^2+5x+3}\right)\sqrt{2}-4\sqrt{2x^2+5x+3}\right)}{8\sqrt{(3+2x)(1+x)}}$
risch	$\frac{(3+2x)\sqrt{\frac{1+x}{3+2x}}}{2} - \frac{\ln\left(\frac{\left(\frac{5}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^2+5x+3}\right)\sqrt{2}\,\sqrt{\frac{1+x}{3+2x}}\,\sqrt{(3+2x)(1+x)}}{8(1+x)}$
trager	$3\left(\frac{1}{2}+\frac{x}{3}\right)\sqrt{-\frac{-1-x}{3+2x}} - \frac{\text{RootOf}\left(\_Z^2-2\right)\ln\left(8\sqrt{-\frac{-1-x}{3+2x}}\,x+4\,\text{RootOf}\left(\_Z^2-2\right)x+12\sqrt{-\frac{-1-x}{3+2x}}+5\,\text{RootOf}\left(\_Z^2-\right.\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(3+2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((1+x)/(3+2*x))^(1/2)*(3+2*x)*(ln(5/4*2^(1/2)+x*2^(1/2)+(2*x^2+5*x+3)^
(1/2))*2^(1/2)-4*(2*x^2+5*x+3)^(1/2))/((3+2*x)*(1+x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(30) = 60$.
time = 1.55, size = 80, normalized size = 1.82

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+1}{2x+3}}}{\sqrt{2}+2\sqrt{\frac{x+1}{2x+3}}}\right)-\frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2(x+1)}{2x+3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 1)/(2*x + 3)))/(sqrt(2) + 2*sqrt((x
 + 1)/(2*x + 3)))) - 1/2*sqrt((x + 1)/(2*x + 3))/(2*(x + 1)/(2*x + 3) - 1)
```

Fricas [A]
time = 0.52, size = 55, normalized size = 1.25

$$\frac{1}{2}(2x+3)\sqrt{\frac{x+1}{2x+3}}+\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}(2x+3)\sqrt{\frac{x+1}{2x+3}}-4x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) + 1/8*sqrt(2)*log(2*sqrt(2)*(2*x + 3)
*sqrt((x + 1)/(2*x + 3)) - 4*x - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/(2*x + 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.
time = 0.47, size = 61, normalized size = 1.39

$$\frac{1}{8}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,\left(\sqrt{2}\,x-\sqrt{2\,x^2+5\,x+3}\right)-5\right|\right)\,\mathrm{sgn}(2\,x+3)+\frac{1}{2}\,\sqrt{2\,x^2+5\,x+3}\,\mathrm{sgn}(2\,x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + 5*x + 3)) - 5))*sg
n(2*x + 3) + 1/2*sqrt(2*x^2 + 5*x + 3)*sgn(2*x + 3)
```

Mupad [B]
time = 0.21, size = 57, normalized size = 1.30

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sqrt{\frac{x+1}{2\,x+3}}\right)}{4}-\frac{\sqrt{\frac{x+1}{2\,x+3}}}{2\,\left(\frac{2\,x+2}{2\,x+3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/(2*x + 3))^(1/2),x)
```

```
[Out] - (2^(1/2)*atanh(2^(1/2)*((x + 1)/(2*x + 3))^(1/2)))/4 - ((x + 1)/(2*x + 3)
)^(1/2)/(2*((2*x + 2)/(2*x + 3) - 1))
```

3.37 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{x^3}{3\,(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {294, 222}

$$\text{ArcSin}(x)-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\,(1-x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(1-x^2)^{5/2}}\,dx &= \frac{x^3}{3\,(1-x^2)^{3/2}}-\int \frac{x^2}{(1-x^2)^{3/2}}\,dx\\ &= \frac{x^3}{3\,(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^3}{3\,(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 1.26

$$\frac{x(-3+4x^2)}{3\,(1-x^2)^{3/2}}+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A]
time = 0.08, size = 30, normalized size = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x(-20x^2+15)}{10(-x^2+1)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\mathrm{RootOf}(\_Z^2+1)\ln\left(\mathrm{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A]
time = 1.97, size = 44, normalized size = 1.26

$$\frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

time = 0.60, size = 63, normalized size = 1.80

$$-\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.

time = 0.45, size = 105, normalized size = 3.00

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A]

time = 0.51, size = 29, normalized size = 0.83

$$\frac{(4\,x^2-3)\sqrt{-x^2+1}\,x}{3\,(x^2-1)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B]

time = 0.15, size = 91, normalized size = 2.60

$$\operatorname{asin}(x)+\frac{3\sqrt{1-x^2}}{4\,(x-1)}+\frac{3\sqrt{1-x^2}}{4\,(x+1)}-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^2}\right)-\sqrt{1-x^2}\,\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.38 $\int \sqrt{x}\,(1+x)^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{5}{64}\sqrt{x}\,\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 5/24*x^(3/2)*(1+x)^(3/2)+1/4*x^(3/2)*(1+x)^(5/2)-5/64*arcsinh(x^(1/2))+5/32
*x^(3/2)*(1+x)^(1/2)+5/64*x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 56, 221}

$$\frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\,\sqrt{x+1}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3/2)*(1 + x
)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sqrt[x]])/64
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(1+x)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \sqrt{x}\,(1+x)^{3/2}\,dx\\ &= \frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{16}\int \sqrt{x}\,\sqrt{1+x}\,dx\\ &= \frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{64}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\ &= \frac{5}{64}\sqrt{x}\,\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{128}\int \frac{}{\sqrt{}}\\ &= \frac{5}{64}\sqrt{x}\,\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\mathrm{Subst}(\\ &= \frac{5}{64}\sqrt{x}\,\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\sinh^{-1}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 47, normalized size = 0.63

$$\frac{1}{192}\left(\sqrt{x}\,\sqrt{1+x}\,\left(15+118x+136x^2+48x^3\right)-15\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(1 + x)^(5/2),x]
[Out] (Sqrt[x]*Sqrt[1 + x]*(15 + 118*x + 136*x^2 + 48*x^3) - 15*ArcTanh[Sqrt[x/(1
 + x)]])/192
```

Maple [A]
time = 0.04, size = 70, normalized size = 0.93

method	result	si
meijerg	$-\frac{15\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\left(48x^3+136x^2+118x+15\right)\sqrt{1+x}}{360}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{24}\right)}{8\sqrt{\pi}}$	44
risch	$\frac{\left(48x^3+136x^2+118x+15\right)\sqrt{x}\,\sqrt{1+x}}{192}-\frac{5\sqrt{x\,(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{128\sqrt{1+x}\,\sqrt{x}}$	55
default	$\frac{\sqrt{x}\,(1+x)^{\frac{7}{2}}}{4}-\frac{\sqrt{x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{x}\,(1+x)^{\frac{3}{2}}}{96}-\frac{5\sqrt{x}\,\sqrt{1+x}}{64}-\frac{5\sqrt{x\,(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{128\sqrt{1+x}\,\sqrt{x}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(1/2)*(1+x)^(7/2)-1/24*x^(1/2)*(1+x)^(5/2)-5/96*x^(1/2)*(1+x)^(3/2)-5
/64*x^(1/2)*(1+x)^(1/2)-5/128*(x*(1+x))^(1/2)/(1+x)^(1/2)/x^(1/2)*ln(x+1/2+
(x^2+x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(47) = 94$.
time = 1.26, size = 113, normalized size = 1.51

$$\frac{\frac{15\,(x+1)^{\frac{7}{2}}}{x^{\frac{7}{2}}} + \frac{73\,(x+1)^{\frac{5}{2}}}{x^{\frac{5}{2}}} - \frac{55\,(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}} + \frac{15\,\sqrt{x+1}}{\sqrt{x}}}{192\left(\frac{(x+1)^4}{x^4} - \frac{4\,(x+1)^3}{x^3} + \frac{6\,(x+1)^2}{x^2} - \frac{4\,(x+1)}{x} + 1\right)} - \frac{5}{128}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{5}{128}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/192*(15*(x + 1)^(7/2)/x^(7/2) + 73*(x + 1)^(5/2)/x^(5/2) - 55*(x + 1)^(3/
2)/x^(3/2) + 15*sqrt(x + 1)/sqrt(x))/((x + 1)^4/x^4 - 4*(x + 1)^3/x^3 + 6*(
x + 1)^2/x^2 - 4*(x + 1)/x + 1) - 5/128*log(sqrt(x + 1)/sqrt(x) + 1) + 5/12
8*log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A]
time = 0.64, size = 44, normalized size = 0.59

$$\frac{1}{192}\left(48\,x^3 + 136\,x^2 + 118\,x + 15\right)\sqrt{x+1}\,\sqrt{x} + \frac{5}{128}\log\left(2\,\sqrt{x+1}\,\sqrt{x} - 2\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(48*x^3 + 136*x^2 + 118*x + 15)*sqrt(x + 1)*sqrt(x) + 5/128*log(2*sqr
t(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 19.86, size = 190, normalized size = 2.53

$$\begin{cases} -\frac{5\operatorname{acosh}\left(\sqrt{x+1}\right)}{64} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{x}} - \frac{7(x+1)^{\frac{7}{2}}}{24\sqrt{x}} - \frac{(x+1)^{\frac{5}{2}}}{96\sqrt{x}} - \frac{5(x+1)^{\frac{3}{2}}}{192\sqrt{x}} + \frac{5\sqrt{x+1}}{64\sqrt{x}} & \text{for } |x+1| > 1 \\ \frac{5i\operatorname{asin}\left(\sqrt{x+1}\right)}{64} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{-x}} + \frac{7i(x+1)^{\frac{7}{2}}}{24\sqrt{-x}} + \frac{i(x+1)^{\frac{5}{2}}}{96\sqrt{-x}} + \frac{5i(x+1)^{\frac{3}{2}}}{192\sqrt{-x}} - \frac{5i\sqrt{x+1}}{64\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*acosh(sqrt(x + 1))/64 + (x + 1)**(9/2)/(4*sqrt(x)) - 7*(x + 1
)**(7/2)/(24*sqrt(x)) - (x + 1)**(5/2)/(96*sqrt(x)) - 5*(x + 1)**(3/2)/(192
```

```
*sqrt(x)) + 5*sqrt(x + 1)/(64*sqrt(x)), Abs(x + 1) > 1), (5*I*asin(sqrt(x +
 1))/64 - I*(x + 1)**(9/2)/(4*sqrt(-x)) + 7*I*(x + 1)**(7/2)/(24*sqrt(-x))
+ I*(x + 1)**(5/2)/(96*sqrt(-x)) + 5*I*(x + 1)**(3/2)/(192*sqrt(-x)) - 5*I*
sqrt(x + 1)/(64*sqrt(-x)), True))
```

Giac [A]
time = 0.48, size = 90, normalized size = 1.20

$$\frac{1}{192}\left(2\left(4\left(6x-19\right)\left(x+1\right)+163\right)\left(x+1\right)-279\right)\sqrt{x+1}\,\sqrt{x}+\frac{1}{8}\left(2\left(4x-9\right)\left(x+1\right)+33\right)\sqrt{x+1}\,\sqrt{x}+\frac{3}{4}\left(2x-3\right)\sqrt{x+1}\,\sqrt{x}+\sqrt{x+1}\,\sqrt{x}+\frac{5}{64}\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x - 19)*(x + 1) + 163)*(x + 1) - 279)*sqrt(x + 1)*sqrt(x) +
1/8*(2*(4*x - 9)*(x + 1) + 33)*sqrt(x + 1)*sqrt(x) + 3/4*(2*x - 3)*sqrt(x +
 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 5/64*log(sqrt(x + 1) - sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(x+1\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x + 1)^(5/2),x)
```

```
[Out] int(x^(1/2)*(x + 1)^(5/2), x)
```

3.39 $\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {294, 222}

$$\text{ArcSin}(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(1-x^2)^{5/2}} \, dx &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} \, dx \\
&= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} \, dx \\
&= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.26

$$\frac{x(-3+4x^2)}{3(1-x^2)^{3/2}}+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A]
time = 0.00, size = 30, normalized size = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x(-20x^2+15)}{10(-x^2+1)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\mathrm{RootOf}(\_Z^2+1)\ln\left(\mathrm{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A]
time = 1.52, size = 44, normalized size = 1.26

$$\frac{1}{3}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

time = 0.62, size = 63, normalized size = 1.80

$$-\frac{6\left(x^{4}-2\,x^{2}+1\right)\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\left(4\,x^{3}-3\,x\right)\sqrt{-x^{2}+1}}{3\left(x^{4}-2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.
time = 0.44, size = 105, normalized size = 3.00

$$\frac{3x^{4}\operatorname{asin}(x)}{3x^{4}-6x^{2}+3}+\frac{4x^{3}\sqrt{1-x^{2}}}{3x^{4}-6x^{2}+3}-\frac{6x^{2}\operatorname{asin}(x)}{3x^{4}-6x^{2}+3}-\frac{3x\sqrt{1-x^{2}}}{3x^{4}-6x^{2}+3}+\frac{3\operatorname{asin}(x)}{3x^{4}-6x^{2}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A]
time = 0.46, size = 29, normalized size = 0.83

$$\frac{\left(4\,x^{2}-3\right)\sqrt{-x^{2}+1}\,x}{3\left(x^{2}-1\right)^{2}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B]
time = 0.00, size = 91, normalized size = 2.60

$$\operatorname{asin}(x)+\frac{3\,\sqrt{1-x^2}}{4\,\left(x-1\right)}+\frac{3\,\sqrt{1-x^2}}{4\,\left(x+1\right)}-\sqrt{1-x^2}\,\left(\frac{1}{12\,\left(x-1\right)}-\frac{1}{12\,{\left(x-1\right)}^2}\right)-\sqrt{1-x^2}\,\left(\frac{1}{12\,\left(x+1\right)}+\frac{1}{12\,{\left(x+1\right)}^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.40 $\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy$

Optimal. Leaf size=51

$$B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

```
[Out] B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))+A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(1/
2))
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {399, 223, 209, 385, 212}

$$B\text{ArcTan}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2
- B^2*y^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy &= A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}\,dy + B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy \\ &= A^2\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right) + B^2\text{Subst}\left(\int \frac{1}{1+B^2y^2}\,dy\right. \\ &= B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) + A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.13, size = 100, normalized size = 1.96

$$\frac{\sqrt{-B^2}\left(-A\tan^{-1}\left(\frac{B^2(-1+y^2)+\sqrt{-B^2}\,y\sqrt{A^2+B^2-B^2y^2}}{AB}\right)+B\log\left(-\sqrt{-B^2}\,y+\sqrt{A^2+B^2-B^2y^2}\right)\right)}{B}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] (Sqrt[-B^2]*(-(A*ArcTan[(B^2*(-1 + y^2) + Sqrt[-B^2]*y*Sqrt[A^2 + B^2 - B^2
*y^2])/(A*B)]) + B*Log[-(Sqrt[-B^2]*y) + Sqrt[A^2 + B^2 - B^2*y^2]]))/B
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(47) = 94$.
time = 0.08, size = 262, normalized size = 5.14

method	result
default	$-\frac{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}{2}+\frac{B^2\arctan\left(\frac{\sqrt{B^2}\,y}{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}\right)}{2\sqrt{B^2}}+\frac{A^2\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^
(1/2)*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2
-2*B^2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))/(y-1))+1/2
*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2
)*y/(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2+2*B
^2*(1+y)+2*(A^2)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.
time = 3.24, size = 114, normalized size = 2.24

$$B \arcsin\left(\frac{B^2 y}{\sqrt{A^2 B^2 + B^4}}\right) - \frac{1}{2} A \log\left(B^2 + \frac{A^2}{y+1} + \frac{\sqrt{-B^2 y^2 + A^2 + B^2} A}{y+1}\right) + \frac{1}{2} A \log\left(-B^2 + \frac{2 A^2}{|2 y - 2|} + \frac{2 \sqrt{-B^2 y^2 + A^2 + B^2} A}{|2 y - 2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="maxima")
```

```
[Out] B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 + A^2/(y + 1) + sqrt(-B
^2*y^2 + A^2 + B^2)*A/(y + 1)) + 1/2*A*log(-B^2 + 2*A^2/abs(2*y - 2) + 2*sq
rt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(47) = 94$.
time = 0.75, size = 129, normalized size = 2.53

$$-B \arctan\left(\frac{\sqrt{-B^2 y^2 + A^2 + B^2}}{B y}\right) + \frac{1}{4} A \log\left(-\frac{(A^2 - B^2) y^2 + 2 \sqrt{-B^2 y^2 + A^2 + B^2} A y + A^2 + B^2}{y^2}\right) - \frac{1}{4} A \log\left(-\frac{(A^2 - B^2) y^2 - 2 \sqrt{-B^2 y^2 + A^2 + B^2} A y + A^2 + B^2}{y^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] -B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) + 1/4*A*log(-((A^2 - B^2)*y^2 +
 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) - 1/4*A*log(-((A^2 - B^
2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{A^2 - B^2 y^2 + B^2}}{y^2 - 1} \, dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)
```

```
[Out] -Integral(sqrt(A**2 - B**2*y**2 + B**2)/(y**2 - 1), y)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[2,0,0]%%%}+%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,
0]%%%},0,%%%{1,[0,4,4]%%%}] at parameters values [88,76,-66]Warning, choosi
ng root of [1,0,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} -\int \frac{\sqrt{-B^2\,y^2}}{y^2-1}\,dy & \text{if}\ \ A^2+B^2=0 \\ -\ln\left(2\,y\,\sqrt{-B^2}+2\,\sqrt{A^2-B^2\,y^2+B^2}\right)\,\sqrt{-B^2}-\mathrm{atan}\left(\frac{y\,\sqrt{A^2}\,1\mathrm{i}}{\sqrt{A^2-B^2\,y^2+B^2}}\right)\,\sqrt{A^2}\,1\mathrm{i} & \text{if}\ \ A^2+B^2\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A^2 + B^2 - B^2*y^2)^(1/2)/(y^2 - 1),y)
```

```
[Out] piecewise(A^2 + B^2 == 0, -int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~=
 0, - atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i -
 log(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

3.41 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
[Out] x/2 - Sin[2*x]/4
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3(\frac{x}{2})+x(\tan^2(\frac{x}{2}))+\frac{x}{2}+\frac{x(\tan^4(\frac{x}{2}))}{2}-\tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A]
time = 2.11, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x-\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A]
time = 0.92, size = 10, normalized size = 0.71

$$-\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

Giac [A]
time = 0.46, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] x/2 - sin(2*x)/4
```

3.42 $\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx$

Optimal. Leaf size=49

$$-B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)-A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)$$

```
[Out] -B*arctan(B*cos(x)/(A^2+B^2*sin(x)^2)^(1/2))-A*arctanh(A*cos(x)/(A^2+B^2*si
n(x)^2)^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3265, 399, 223, 209, 385, 212}

$$-B\text{ArcTan}\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)-A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]) - A*ArcTanh[(A*Cos[x
])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{A^2+B^2-B^2x^2}}{1-x^2}\,dx, x, \cos(x)\right)\\ &= -\left(A^2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{A^2+B^2-B^2x^2}}\,dx, x, \cos(x)\right)\right) - B^2\text{Subs}\\ &= -\left(A^2\text{Subst}\left(\int \frac{1}{1-A^2x^2}\,dx, x, \frac{\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)\right) - B^2\text{Subs}\\ &= -B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right) - A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2-B^2\text{c}}}\right.\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.
time = 0.08, size = 99, normalized size = 2.02

$$-\sqrt{A^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{A^2}\cos(x)}{\sqrt{2A^2+B^2-B^2\cos(2x)}}\right)+\sqrt{-B^2}\log\left(\sqrt{2}\sqrt{-B^2}\cos(x)+\sqrt{2A^2+B^2-B^2\cos(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(Sqrt[A^2]*ArcTanh[(Sqrt[2]*Sqrt[A^2]*Cos[x])/Sqrt[2*A^2 + B^2 - B^2*Cos[2
*x]]]) + Sqrt[-B^2]*Log[Sqrt[2]*Sqrt[-B^2]*Cos[x] + Sqrt[2*A^2 + B^2 - B^2*
Cos[2*x]]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 149, normalized size = 3.04

method	result
default	$-\frac{\sqrt{(A^2+B^2(\sin^2(x)))(\cos^2(x))}\left(A\,\mathrm{csgn}(A)\ln\left(-\frac{A^2(\sin^2(x))-B^2(\sin^2(x))-2\,\mathrm{csgn}(A)A\sqrt{(A^2+B^2(\sin^2(x))}}{\sin(x)^2}\right.\right.}{2\cos(x)\sqrt{A^2+B^2(\sin^2(x)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)*(A*csgn(A)*ln(-(A^2*sin(x)^2-B^2*s
in(x)^2-2*csgn(A)*A*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)-2*A^2)/sin(x)^2)-B*
csgn(B)*arctan(1/2*csgn(B)/B*(2*B^2*sin(x)^2+A^2-B^2)/((A^2+B^2*sin(x)^2)*c
os(x)^2)^(1/2)))/cos(x)/(A^2+B^2*sin(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(45) = 90.
time = 2.13, size = 116, normalized size = 2.37

$$-B\arcsin\left(\frac{B^2\cos(x)}{\sqrt{A^2B^2+B^4}}\right)-\frac{1}{2}A\log\left(B^2-\frac{A^2}{\cos(x)-1}-\frac{\sqrt{-B^2\cos(x)^2+A^2+B^2}\,A}{\cos(x)-1}\right)+\frac{1}{2}A\log\left(-B^2+\frac{A^2}{\cos(x)+1}+\frac{\sqrt{-B^2\cos(x)^2+A^2+B^2}\,A}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="maxima")
```

```
[Out] -B*arcsin(B^2*cos(x)/sqrt(A^2*B^2 + B^4)) - 1/2*A*log(B^2 - A^2/(cos(x) - 1
) - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) - 1)) + 1/2*A*log(-B^2 + A^2/
(cos(x) + 1) + sqrt(-B^2*cos(x)^2 + A^2 + B^2)*A/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(45) = 90.
time = 1.40, size = 244, normalized size = 4.98

$$\frac{1}{2}B\arctan\left(-\frac{(A^4+2A^2B^2+B^4)\cos(x)\sin(x)-2\left(2B^3\cos(x)^3-(A^2B+B^3)\cos(x)\right)\sqrt{-B^2\cos(x)^2+A^2+B^2}}{4B^4\cos(x)^4+A^4+2A^2B^2+B^4-(A^4+6A^2B^2+5B^4)\cos(x)^2}\right)-\frac{1}{2}B\arctan\left(\frac{\sin(x)}{\cos(x)}\right)-\frac{1}{2}A\log\left(-B^2\cos(x)^2+AB\cos(x)\sin(x)+A^2+B^2+\sqrt{-B^2\cos(x)^2+A^2+B^2}\,(A\cos(x)+B\sin(x))\right)+\frac{1}{2}A\log\left(-B^2\cos(x)^2-AB\cos(x)\sin(x)+A^2+B^2-\sqrt{-B^2\cos(x)^2+A^2+B^2}\,(A\cos(x)-B\sin(x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="fricas")
```

```
[Out] 1/2*B*arctan(-((A^4 + 2*A^2*B^2 + B^4)*cos(x)*sin(x) - 2*(2*B^3*cos(x)^3 -
(A^2*B + B^3)*cos(x))*sqrt(-B^2*cos(x)^2 + A^2 + B^2))/(4*B^4*cos(x)^4 + A^
4 + 2*A^2*B^2 + B^4 - (A^4 + 6*A^2*B^2 + 5*B^4)*cos(x)^2)) - 1/2*B*arctan(s
in(x)/cos(x)) - 1/2*A*log(-B^2*cos(x)^2 + A*B*cos(x)*sin(x) + A^2 + B^2 + s
qrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) + B*sin(x))) + 1/2*A*log(-B^2*cos(
```

```
x)^2 - A*B*cos(x)*sin(x) + A^2 + B^2 - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*c
os(x) - B*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{A^2 + B^2 \sin^2{(x)}}}{\sin{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)
[Out] Integral(sqrt(A**2 + B**2*sin(x)**2)/sin(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="giac")
[Out] integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{A^2 + B^2 \sin{(x)}^2}}{\sin{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x),x)
[Out] int((B^2*sin(x)^2 + A^2)^(1/2)/sin(x), x)
```

3.43 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{1+\cos(x)}$$

[Out] sin(x)/(cos(x)+1)

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2727}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])^(-1),x]

[Out] Sin[x]/(1 + Cos[x])

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x])^(-1),x]

[Out] Tan[x/2]

Maple [A]
time = 0.01, size = 5, normalized size = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{1+e^{ix}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] tan(1/2*x)
```

Maxima [A]
time = 3.34, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A]
time = 1.18, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A]
time = 0.08, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(9) = 18.
time = 0.46, size = 30, normalized size = 3.33

$$-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

Mupad [B]
time = 0.18, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 1),x)
```

```
[Out] tan(x/2)
```

3.44 $\int e^x x\,dx$

Optimal. Leaf size=11

$$-e^x + e^x x$$

[Out] -exp(x)+exp(x)*x

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2207, 2225}

$$e^x x - e^x$$

Antiderivative was successfully verified.

[In] Int[E^x*x,x]

[Out] -E^x + E^x*x

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^x x\,dx &= e^x x - \int e^x\,dx\\ &= -e^x + e^x x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 0.64

$$e^x(-1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x,x]
```

```
[Out] E^x*(-1 + x)
```

Maple [A]
time = 0.00, size = 10, normalized size = 0.91

method	result	size
gosper	$(-1+x)\,\mathrm{e}^{x}$	7
risch	$(-1+x)\,\mathrm{e}^{x}$	7
default	$-\mathrm{e}^{x}+\mathrm{e}^{x} x$	10
norman	$-\mathrm{e}^{x}+\mathrm{e}^{x} x$	10
meijerg	$1-\frac{(-2x+2)\mathrm{e}^{x}}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(x)+exp(x)*x
```

Maxima [A]
time = 2.92, size = 6, normalized size = 0.55

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x
```

Fricas [A]
time = 1.13, size = 6, normalized size = 0.55

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.45

$$(x-1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x)
```

```
[Out] (x - 1)*exp(x)
```

Giac [**A**]
time = 0.46, size = 6, normalized size = 0.55

$$(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="giac")
```

```
[Out] (x - 1)*e^x
```

Mupad [**B**]
time = 0.02, size = 6, normalized size = 0.55

$$\mathrm{e}^x\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x),x)
```

```
[Out] exp(x)*(x - 1)
```

3.45 $\int \frac{e^x x}{(1+x)^2} \, dx$

Optimal. Leaf size=9

$$\frac{e^x}{1+x}$$

```
[Out] exp(x)/(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2228}

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*x)/(1 + x)^2,x]
```

```
[Out] E^x/(1 + x)
```

Rule 2228

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[g*u^(m + 1)*(F^(c*v)/(b*c*e*Log
[F])), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{1+x}$$

Mathematica [A]
time = 0.04, size = 9, normalized size = 1.00

$$\frac{e^x}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*x)/(1 + x)^2,x]
```

```
[Out] E^x/(1 + x)
```

Maple [A]
time = 0.03, size = 9, normalized size = 1.00

method	result	size
gosper	$\frac{e^x}{1+x}$	9
default	$\frac{e^x}{1+x}$	9
norman	$\frac{e^x}{1+x}$	9
risch	$\frac{e^x}{1+x}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] exp(x)/(1+x)
```

Maxima [A]
time = 1.65, size = 8, normalized size = 0.89

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="maxima")
[Out] e^x/(x + 1)
```

Fricas [A]
time = 0.76, size = 8, normalized size = 0.89

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="fricas")
[Out] e^x/(x + 1)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.56

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)**2,x)
```

```
[Out] exp(x)/(x + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(8) = 16$.
time = 0.45, size = 30, normalized size = 3.33

$$-\frac{e^{\left(-(x+1)\left(\frac{1}{x+1}-1\right)\right)}}{(x+1)\left(\frac{1}{x+1}-1\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="giac")
[Out] -e^(-(x + 1)*(1/(x + 1) - 1))/((x + 1)*(1/(x + 1) - 1) - 1)
```

Mupad [B]
time = 0.09, size = 8, normalized size = 0.89

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(x))/(x + 1)^2,x)
[Out] exp(x)/(x + 1)
```

3.46 $\int e^{x^2}(1+2x^2)\,dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

[Out] exp(x^2)*x

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2258, 2235, 2243}

$$e^{x^2}x$$

Antiderivative was successfully verified.

[In] Int[E^x^2*(1 + 2*x^2),x]

[Out] E^x^2*x

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2258

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^{x^2}(1+2x^2)\,dx &= \int \left(e^{x^2}+2e^{x^2}x^2\right)dx \\ &= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx \\ &= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)-\int e^{x^2}\,dx \\ &= e^{x^2}x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Maple [A]
time = 0.01, size = 7, normalized size = 1.00

method	result	size
gosper	$\mathrm{e}^{x^2}x$	7
default	$\mathrm{e}^{x^2}x$	7
norman	$\mathrm{e}^{x^2}x$	7
risch	$\mathrm{e}^{x^2}x$	7
meijerg	$\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}+i\left(-ix\,\mathrm{e}^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x^2)*x
```

Maxima [A]
time = 1.41, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [**A**]
time = 0.85, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [**A**]
time = 0.02, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

Giac [**A**]
time = 0.44, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

Mupad [B]
time = 0.15, size = 6, normalized size = 0.86

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2 + 1),x)
```

```
[Out] x*exp(x^2)
```

3.47 $\int e^{x^2} dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)$$

[Out] 1/2*erfi(x)*Pi^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2235}

$$\frac{1}{2}\sqrt{\pi}\,\text{Erfi}(x)$$

Antiderivative was successfully verified.

[In] Int[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2}\,dx = \frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\sqrt{\pi}\,\text{erfi}(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

Maple [A]
time = 0.01, size = 8, normalized size = 0.73

method	result	size
default	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8
meijerg	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8
risch	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 2.50, size = 9, normalized size = 0.82

$$-\frac{1}{2}i\sqrt{\pi}\,\operatorname{erf}(i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

Fricas [A]
time = 0.88, size = 7, normalized size = 0.64

$$\frac{1}{2}\sqrt{\pi}\,\operatorname{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erfi(x)
```

Sympy [A]
time = 0.07, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\operatorname{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2),x)
```

```
[Out] sqrt(pi)*erfi(x)/2
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 9, normalized size = 0.82

$$\frac{1}{2}i\sqrt{\pi}\,\mathrm{erf}\left(-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

Mupad [B]
time = 0.02, size = 7, normalized size = 0.64

$$\frac{\sqrt{\pi}\,\mathrm{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
```

```
[Out] (pi^(1/2)*erfi(x))/2
```

3.48 $\int \frac{e^x}{x} dx$

Optimal. Leaf size=2

$$\text{Ei}(x)$$

[Out] Ei(x)

Rubi [A]
time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2209}

$$\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

[In] Int[E^x/x,x]

[Out] ExpIntegralEi[x]

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\int \frac{e^x}{x} dx = \text{Ei}(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\text{Ei}(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/x,x]

[Out] ExpIntegralEi[x]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.
time = 0.01, size = 8, normalized size = 4.00

method	result	size
default	$-\text{expIntegral}(1, -x)$	8
risch	$-\text{expIntegral}(1, -x)$	8
meijerg	$\ln(x) + i\pi - \ln(-x) - \text{expIntegral}(1, -x)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-x)
```

Maxima [A]
time = 1.81, size = 2, normalized size = 1.00

$$\text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="maxima")
```

```
[Out] Ei(x)
```

Fricas [A]
time = 0.82, size = 2, normalized size = 1.00

$$\text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="fricas")
```

```
[Out] Ei(x)
```

Sympy [A]
time = 0.31, size = 2, normalized size = 1.00

$$\text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x)
```

```
[Out] Ei(x)
```

Giac [A]
time = 0.45, size = 2, normalized size = 1.00

$$\text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="giac")
```

```
[Out] Ei(x)
```

Mupad [B]
time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/x,x)
```

```
[Out] ei(x)
```

3.49 $\int \frac{x}{1+x^3} \, dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {298, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right)+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A]
time = 0.03, size = 35, normalized size = 0.85

method	result	size
default	$-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	35
risch	$\frac{\ln(4x^2-4x+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$	37
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 1.35, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Fricas [A]
time = 0.67, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.00

$$-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

Giac [A]
time = 0.45, size = 35, normalized size = 0.85

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\,\log\left(x^2-x+1\right)-\frac{1}{3}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

Mupad [B]
time = 0.11, size = 46, normalized size = 1.12

$$-\frac{\ln\left(x+1\right)}{3}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,\,1\mathrm{i}}{6}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

3.50 $\int \frac{1}{-1+x^6}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,x}{1-x^2}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{1+x^2}\right)$$

```
[Out] -1/3*arctanh(x)-1/6*arctanh(x/(x^2+1))-1/6*arctan(x*3^(1/2)/(-x^2+1))*3^(1/
2)
```

Rubi [A]
time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {216, 648, 632, 210, 642, 212}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^6}\,dx &= -\left(\frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx\right)-\frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx-\frac{1}{3}\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\right.\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.60

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

Maple [A]
time = 0.03, size = 66, normalized size = 1.40

method	result
risch	$-\frac{\ln(1+x)}{6}+\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,(x+\frac{1}{2})}{3}\right)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2(x-\frac{1}{2})\sqrt{3}}{3}\right)}{6}$
default	$\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\right.\right.}{6(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*l
n(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 2.23, size = 65, normalized size = 1.38

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Fricas [A]
time = 0.43, size = 65, normalized size = 1.38

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(36) = 72$.
time = 0.12, size = 83, normalized size = 1.77

$$\frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\log(x^2-x+1)}{12} - \frac{\log(x^2+x+1)}{12} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 -
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/6
```

Giac [A]
time = 0.43, size = 67, normalized size = 1.43

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log(x^2+x+1) + \frac{1}{12}\log(x^2-x+1) - \frac{1}{6}\log(|x+1|) + \frac{1}{6}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1)) + 1/6*log(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 88, normalized size = 1.87

$$-\frac{\operatorname{atanh}(x)}{3} - \operatorname{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}} + \frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6} + \frac{1}{6}\mathrm{i}\right) - \operatorname{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}} - \frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6} - \frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - 1),x)
```

```
[Out] - atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))
*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1
i - 1))*(3^(1/2)/6 - 1i/6)
```

3.51 $\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)x^2}\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

```
[Out] arctanh(x/A)/A/(A^2-B^2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {214}

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

```
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{A^4-A^2B^2+\left(-A^2+B^2\right)x^2}\,dx=\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\,(A^2-B^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

```
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Maple [A]
time = 0.06, size = 34, normalized size = 1.62

method	result	size
default	$\frac{-\frac{\ln(A-x)}{2A}+\frac{\ln(A+x)}{2A}}{A^2-B^2}$	34
norman	$-\frac{\ln(A-x)}{2A(A^2-B^2)}+\frac{\ln(A+x)}{2A(A^2-B^2)}$	44
risch	$-\frac{\ln(A-x)}{2A(A^2-B^2)}+\frac{\ln(A+x)}{2A(A^2-B^2)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(A^2-B^2)*(-1/2/A*ln(A-x)+1/2/A*ln(A+x))
```

Maxima [A]
time = 2.96, size = 39, normalized size = 1.86

$$\frac{\log\left(A+x\right)}{2\left(A^3-AB^2\right)}-\frac{\log\left(-A+x\right)}{2\left(A^3-AB^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(A + x)/(A^3 - A*B^2) - 1/2*log(-A + x)/(A^3 - A*B^2)
```

Fricas [A]
time = 0.47, size = 27, normalized size = 1.29

$$\frac{\log\left(A+x\right)-\log\left(-A+x\right)}{2\left(A^3-AB^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(A + x) - log(-A + x))/(A^3 - A*B^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(12) = 24$.
time = 0.14, size = 70, normalized size = 3.33

$$-\frac{\log\left(-\frac{A^3}{(A-B)(A+B)}+\frac{AB^2}{(A-B)(A+B)}+x\right)}{2A\left(A-B\right)\left(A+B\right)}+\frac{\log\left(\frac{A^3}{(A-B)(A+B)}-\frac{AB^2}{(A-B)(A+B)}+x\right)}{2A\left(A-B\right)\left(A+B\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)
```

```
[Out] -log(-A**3/((A - B)*(A + B)) + A*B**2/((A - B)*(A + B)) + x)/(2*A*(A - B)*(
A + B)) + log(A**3/((A - B)*(A + B)) - A*B**2/((A - B)*(A + B)) + x)/(2*A*(
A - B)*(A + B))
```

Giac [**A**]
time = 0.46, size = 41, normalized size = 1.95

$$\frac{\log\left(|A+x|\right)}{2\left(A^3-AB^2\right)}-\frac{\log\left(|-A+x|\right)}{2\left(A^3-AB^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(A + x))/(A^3 - A*B^2) - 1/2*log(abs(-A + x))/(A^3 - A*B^2)
```

Mupad [**B**]
time = 0.24, size = 21, normalized size = 1.00

$$-\frac{\operatorname{atanh}\left(\frac{x}{A}\right)}{A\,B^2-A^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^2*(A^2 - B^2) - A^4 + A^2*B^2),x)
```

```
[Out] -atanh(x/A)/(A*B^2 - A^3)
```

3.52 $\int x \log(x) \, dx$

Optimal. Leaf size=17

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2341}

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A]
time = 0.00, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A]
time = 1.02, size = 13, normalized size = 0.76

$$\frac{1}{2}x^2\log(x)-\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A]
time = 0.39, size = 13, normalized size = 0.76

$$\frac{1}{2}x^2\log(x)-\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.71

$$\frac{x^2\log(x)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A]
time = 0.45, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.53

$$\frac{x^2\,\left(\ln\left(x\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
```

```
[Out] (x^2*(log(x) - 1/2))/2
```

3.53 $\int x^2 \sin^{-1}(x)\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-x^2}}{3}-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\sin^{-1}(x)$$

```
[Out] -1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4723, 272, 45}

$$\frac{1}{3}x^3\text{ArcSin}(x)-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \sin^{-1}(x)\,dx &= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{6}\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^2\right)\\ &= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right)dx, x, x^2\right)\\ &= \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(\sqrt{1-x^2}\left(2+x^2\right) + 3x^3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x],x]
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

Maple [A]
time = 0.00, size = 34, normalized size = 0.85

method	result	size
default	$\frac{x^3\arcsin(x)}{3} + \frac{x^2\sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

Maxima [A]
time = 4.15, size = 33, normalized size = 0.82

$$\frac{1}{3}\,x^3\arcsin\left(x\right) + \frac{1}{9}\,\sqrt{-x^2+1}\,x^2 + \frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="maxima")
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

Fricas [A]
time = 0.48, size = 24, normalized size = 0.60

$$\frac{1}{3}\,x^3 \arcsin\left(x\right) + \frac{1}{9}\,\left(x^2 + 2\right)\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A]
time = 0.09, size = 32, normalized size = 0.80

$$\frac{x^3 \operatorname{asin}\left(x\right)}{3} + \frac{x^2\sqrt{1 - x^2}}{9} + \frac{2\sqrt{1 - x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

Giac [A]
time = 0.43, size = 38, normalized size = 0.95

$$\frac{1}{3}\,\left(x^2 - 1\right)x \arcsin\left(x\right) + \frac{1}{3}\,x \arcsin\left(x\right) - \frac{1}{9}\,\left(-x^2 + 1\right)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sq
rt(-x^2 + 1)
```

Mupad [B]
time = 0.00, size = 24, normalized size = 0.60

$$\frac{x^3 \operatorname{asin}(x)}{3} + \frac{\sqrt{1 - x^2}\,\left(x^2 + 2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asin(x),x)
```

```
[Out] (x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.54 $\int \frac{1}{1+2x+x^2} dx$

Optimal. Leaf size=7

$$-\frac{1}{1+x}$$

[Out] -1/(1+x)

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {27, 32}

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + x^2)^(-1),x]

[Out] -(1 + x)^(-1)

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+2x+x^2} dx &= \int \frac{1}{(1+x)^2} dx\\ &= -\frac{1}{1+x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)^(-1),x]
```

```
[Out] -(1 + x)^(-1)
```

Maple [A]
time = 0.04, size = 8, normalized size = 1.14

method	result	size
gosper	$-\frac{1}{1+x}$	8
default	$-\frac{1}{1+x}$	8
norman	$-\frac{1}{1+x}$	8
meijerg	$\frac{x}{1+x}$	8
risch	$-\frac{1}{1+x}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1+x)
```

Maxima [A]
time = 4.78, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] -1/(x + 1)
```

Fricas [A]
time = 0.42, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] -1/(x + 1)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.71

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+1),x)
```

```
[Out] -1/(x + 1)
```

Giac [A]
time = 0.47, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -1/(x + 1)
```

Mupad [B]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + x^2 + 1),x)
```

```
[Out] -1/(x + 1)
```

3.55 $\int \frac{\log(x)}{(1+\log(x))^2} \, dx$

Optimal. Leaf size=8

$$\frac{x}{1+\log(x)}$$

```
[Out] x/(1+ln(x))
```

Rubi [A]
time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2407, 2334, 2336, 2209}

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(1 + Log[x])^2,x]
```

```
[Out] x/(1 + Log[x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2407

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{(1+\log(x))^2}\,dx &= \int \left(-\frac{1}{(1+\log(x))^2}+\frac{1}{1+\log(x)}\right)dx\\ &= -\int \frac{1}{(1+\log(x))^2}\,dx+\int \frac{1}{1+\log(x)}\,dx\\ &= \frac{x}{1+\log(x)}-\int \frac{1}{1+\log(x)}\,dx+\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,\log(x)\right)\\ &= \frac{\text{Ei}(1+\log(x))}{e}+\frac{x}{1+\log(x)}-\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,\log(x)\right)\\ &= \frac{x}{1+\log(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{x}{1+\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(1 + Log[x])^2,x]
```

```
[Out] x/(1 + Log[x])
```

Maple [A]
time = 0.04, size = 9, normalized size = 1.12

method	result	size
default	$\frac{x}{1+\ln(x)}$	9
norman	$\frac{x}{1+\ln(x)}$	9
risch	$\frac{x}{1+\ln(x)}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(1+ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(1+ln(x))
```

Maxima [A]
time = 1.16, size = 8, normalized size = 1.00

$$\frac{x}{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="maxima")
```

```
[Out] x/(log(x) + 1)
```

Fricas [A]
time = 0.43, size = 8, normalized size = 1.00

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="fricas")
```

```
[Out] x/(log(x) + 1)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.62

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(1+ln(x))**2,x)
```

```
[Out] x/(log(x) + 1)
```

Giac [A]
time = 0.45, size = 8, normalized size = 1.00

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="giac")
```

```
[Out] x/(log(x) + 1)
```

Mupad [B]
time = 0.26, size = 8, normalized size = 1.00

$$\frac{x}{\ln(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(log(x) + 1)^2,x)
```

```
[Out] x/(log(x) + 1)
```

3.56 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {209}

$$\text{ArcTan}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right)\\ &= \tan^{-1}(\log(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i\ln(\ln(x)+i)}{2} - \frac{i\ln(\ln(x)-i)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(ln(x))
```

Maxima [A]
time = 1.80, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A]
time = 0.43, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.
time = 0.05, size = 15, normalized size = 5.00

$$\operatorname{RootSum}\left(4z^2 + 1, \left(i \mapsto i\log(2i + \log(x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A]
time = 0.46, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

Mupad [B]
time = 0.31, size = 3, normalized size = 1.00

$$\operatorname{atan}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
```

```
[Out] atan(log(x))
```

3.57 $\int \frac{1}{\log(x)} \, dx$

Optimal. Leaf size=2

$$\text{li}(x)$$

```
[Out] Li(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2335}

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(x)} \, dx = \text{li}(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\text{li}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.
time = 0.00, size = 9, normalized size = 4.50

method	result	size

default	$-\text{expIntegral}(1, -\ln(x))$	9
risch	$-\text{expIntegral}(1, -\ln(x))$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x,method=_RETURNVERBOSE)
[Out] -Ei(1,-ln(x))
```

Maxima [A]
time = 2.83, size = 3, normalized size = 1.50

$$\text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="maxima")
[Out] Ei(log(x))
```

Fricas [A]
time = 0.45, size = 2, normalized size = 1.00

$$\text{log\_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="fricas")
[Out] log_integral(x)
```

Sympy [A]
time = 0.23, size = 2, normalized size = 1.00

$$\text{li}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x),x)
[Out] li(x)
```

Giac [A]
time = 0.43, size = 3, normalized size = 1.50

$$\text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="giac")
```

```
[Out] Ei(log(x))
```

Mupad [**B**]
time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{logint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(x),x)
```

```
[Out] logint(x)
```

3.58 $\int x(\cos(x)+\sin(x))\,dx$

Optimal. Leaf size=14

$$\cos(x)-x\cos(x)+\sin(x)+x\sin(x)$$

[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {14, 3377, 2718, 2717}

$$x\sin(x)+\sin(x)-x\cos(x)+\cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*(Cos[x] + Sin[x]),x]

[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int x(\cos(x)+\sin(x))\,dx &= \int (x\cos(x)+x\sin(x))\,dx \\
&= \int x\cos(x)\,dx + \int x\sin(x)\,dx \\
&= -x\cos(x)+x\sin(x)+\int \cos(x)\,dx - \int \sin(x)\,dx \\
&= \cos(x)-x\cos(x)+\sin(x)+x\sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\cos(x)-x\cos(x)+\sin(x)+x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Cos[x] + Sin[x]),x]
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Maple [A]
time = 0.03, size = 15, normalized size = 1.07

method	result	size
default	$\cos(x)-x\cos(x)+\sin(x)+x\sin(x)$	15
risch	$(1-x)\cos(x)+(1+x)\sin(x)$	16
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-x+2x\tan\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+2}{1+\tan^2\left(\frac{x}{2}\right)}$	38
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)+2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

Maxima [A]
time = 1.95, size = 14, normalized size = 1.00

$$-x\cos(x)+x\sin(x)+\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

Fricas [**A**]
time = 0.54, size = 14, normalized size = 1.00

$$-(x-1)\cos(x) + (x+1)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="fricas")
[Out] -(x - 1)*cos(x) + (x + 1)*sin(x)
```

Sympy [**A**]
time = 0.05, size = 15, normalized size = 1.07

$$x\sin(x) - x\cos(x) + \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x)
[Out] x*sin(x) - x*cos(x) + sin(x) + cos(x)
```

Giac [**A**]
time = 0.45, size = 14, normalized size = 1.00

$$-x\cos(x) + x\sin(x) + \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="giac")
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

Mupad [B]
time = 0.06, size = 14, normalized size = 1.00

$$\cos(x) + \sin(x) - x\,\cos(x) + x\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x) + sin(x)),x)
[Out] cos(x) + sin(x) - x*cos(x) + x*sin(x)
```

3.59 $\int e^{-x}(e^x + x)\, dx$

Optimal. Leaf size=17

$$-e^{-x} + x - e^{-x}x$$

[Out] -1/exp(x)+x-x/exp(x)

Rubi [A]

time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6874, 2207, 2225}

$$-e^{-x}x + x - e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + x)/E^x,x]
```

```
[Out] -E^(-x) + x - x/E^x
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int e^{-x}(e^x + x)\, dx &= \int \left(1 + e^{-x}x\right)\, dx \\
&= x + \int e^{-x}x\, dx \\
&= x - e^{-x}x + \int e^{-x}\, dx \\
&= -e^{-x} + x - e^{-x}x
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 0.76

$$e^{-x}(-1-x)+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + x)/E^x,x]
```

```
[Out] (-1 - x)/E^x + x
```

Maple [A]
time = 0.01, size = 16, normalized size = 0.94

method	result	size
risch	$x+(-1-x)\,\mathrm{e}^{-x}$	13
norman	$(-1+\mathrm{e}^{x}x-x)\,\mathrm{e}^{-x}$	15
default	$-\mathrm{e}^{-x}+x-x\,\mathrm{e}^{-x}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+x)/exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/exp(x)+x-x/exp(x)
```

Maxima [A]
time = 3.54, size = 11, normalized size = 0.65

$$-(x+1)e^{(-x)}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x) + x
```

Fricas [A]
time = 0.67, size = 14, normalized size = 0.82

$$(xe^{x}-x-1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] (x*e^x - x - 1)*e^(-x)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.47

$$x+(-x-1)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x)
```

```
[Out] x + (-x - 1)*exp(-x)
```

Giac [A]
time = 0.45, size = 11, normalized size = 0.65

$$-(x+1)e^{(-x)} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x) + x
```

Mupad [B]
time = 0.06, size = 15, normalized size = 0.88

$$x - \mathrm{e}^{-x} - x\,\mathrm{e}^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-x)*(x + exp(x)),x)
```

```
[Out] x - exp(-x) - x*exp(-x)
```

3.60 $\int (1+e^x)^2 x \, dx$

Optimal. Leaf size=38

$$-2e^x - \frac{e^{2x}}{4} + 2e^x x + \frac{1}{2}e^{2x}x + \frac{x^2}{2}$$

```
[Out] -2*exp(x)-1/4*exp(2*x)+2*exp(x)*x+1/2*exp(2*x)*x+1/2*x^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2214, 2207, 2225}

$$\frac{x^2}{2} + 2e^x x + \frac{1}{2}e^{2x}x - 2e^x - \frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + E^x)^2*x,x]
```

```
[Out] -2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2214

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int (1+e^x)^2\, x\, dx &= \int \left(x+2e^x x+e^{2x} x\right)\, dx \\
&= \frac{x^2}{2}+2\int e^x x\, dx+\int e^{2x} x\, dx \\
&= 2e^x x+\frac{1}{2}e^{2x} x+\frac{x^2}{2}-\frac{1}{2}\int e^{2x}\, dx-2\int e^x\, dx \\
&= -2e^x-\frac{e^{2x}}{4}+2e^x x+\frac{1}{2}e^{2x} x+\frac{x^2}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.76

$$\frac{1}{4}\left(8e^x(-1+x)+2x^2+e^{2x}(-1+2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)^2*x,x]
```

```
[Out] (8*E^x*(-1 + x) + 2*x^2 + E^(2*x)*(-1 + 2*x))/4
```

Maple [A]
time = 0.01, size = 29, normalized size = 0.76

method	result	size
risch	$\frac{x^2}{2}+\left(-\frac{1}{4}+\frac{x}{2}\right) e^{2x}+(2x-2)\, e^x$	25
default	$-2\, e^x-\frac{e^{2x}}{4}+2\, e^x x+\frac{e^{2x} x}{2}+\frac{x^2}{2}$	29
norman	$-2\, e^x-\frac{e^{2x}}{4}+2\, e^x x+\frac{e^{2x} x}{2}+\frac{x^2}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+exp(x))^2*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/2*x*exp(x)^2-1/4*exp(x)^2+2*exp(x)*x-2*exp(x)
```

Maxima [A]
time = 1.98, size = 24, normalized size = 0.63

$$\frac{1}{2}\,x^2+\frac{1}{4}\,(2\,x-1)e^{(2\,x)}+2\,(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Fricas [A]
time = 0.56, size = 24, normalized size = 0.63

$$\frac{1}{2}\,x^{2}+\frac{1}{4}\,(2\,x-1)e^{(2\,x)}+2\,(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Sympy [A]
time = 0.03, size = 26, normalized size = 0.68

$$\frac{x^{2}}{2}+\frac{(2x-1)\,e^{2x}}{4}+\frac{(8x-8)\,e^{x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))**2*x,x)
```

```
[Out] x**2/2 + (2*x - 1)*exp(2*x)/4 + (8*x - 8)*exp(x)/4
```

Giac [A]
time = 0.45, size = 24, normalized size = 0.63

$$\frac{1}{2}\,x^{2}+\frac{1}{4}\,(2\,x-1)e^{(2\,x)}+2\,(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Mupad [B]
time = 0.16, size = 28, normalized size = 0.74

$$\frac{x\,\mathrm{e}^{2\,x}}{2}-2\,\mathrm{e}^{x}-\frac{\mathrm{e}^{2\,x}}{4}+2\,x\,\mathrm{e}^{x}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(exp(x) + 1)^2,x)
```

```
[Out] (x*exp(2*x))/2 - 2*exp(x) - exp(2*x)/4 + 2*x*exp(x) + x^2/2
```

3.61 $\int x\cos(x)\,dx$

Optimal. Leaf size=7

$$\cos(x)+x\sin(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3377, 2718}

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x)-\int \sin(x)\,dx\\ &= \cos(x)+x\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\cos(x)+x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A]
time = 0.00, size = 8, normalized size = 1.14

method	result	size
default	$\cos(x) + x\sin(x)$	8
risch	$\cos(x) + x\sin(x)$	8
norman	$\frac{2x\tan(\frac{x}{2})+2}{1+\tan^2(\frac{x}{2})}$	21
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+x*sin(x)
```

Maxima [A]
time = 2.72, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
[Out] x*sin(x) + cos(x)
```

Fricas [A]
time = 0.68, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
[Out] x*sin(x) + cos(x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [A]
time = 0.43, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

Mupad [B]
time = 0.02, size = 7, normalized size = 1.00

$$\cos(x) + x\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x) + x*sin(x)
```

3.62 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\cos\left(\sqrt{x}\right)+2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3443, 3377, 2718}

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3443

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\sqrt{x}\right)\,dx &= 2\text{Subst}\left(\int x\cos(x)\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{x}\,\sin\left(\sqrt{x}\right)-2\text{Subst}\left(\int \sin(x)\,dx,x,\sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right)+2\sqrt{x}\,\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sqrt{x}\,\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A]
time = 1.32, size = 16, normalized size = 0.73

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A]
time = 0.49, size = 16, normalized size = 0.73

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A]
time = 0.09, size = 20, normalized size = 0.91

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A]
time = 0.46, size = 16, normalized size = 0.73

$$2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Mupad [B]
time = 0.00, size = 16, normalized size = 0.73

$$2\,\cos\left(\sqrt{x}\right)+2\,\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

3.63 $\int x\cos(x)\,dx$

Optimal. Leaf size=7

$$\cos(x) + x\sin(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x) - \int \sin(x)\,dx\\ &= \cos(x) + x\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\cos(x) + x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A]
time = 0.00, size = 8, normalized size = 1.14

method	result	size
default	$\cos(x)+x\sin(x)$	8
risch	$\cos(x)+x\sin(x)$	8
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2}{1+\tan^2\left(\frac{x}{2}\right)}$	21
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+x*sin(x)
```

Maxima [A]
time = 1.16, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
[Out] x*sin(x) + cos(x)
```

Fricas [A]
time = 0.81, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
[Out] x*sin(x) + cos(x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 1.00

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [**A**]
time = 0.47, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
[Out] x*sin(x) + cos(x)
```

Mupad [**B**]
time = 0.00, size = 7, normalized size = 1.00

$$\cos(x) + x\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
[Out] cos(x) + x*sin(x)
```

3.64 $\int x \log^2(x) \, dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2342, 2341}

$$\frac{x^2}{4} + \frac{1}{2}x^2\log^2(x) - \frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\log^2(x)\,dx &= \frac{1}{2}x^2\log^2(x) - \int x\log(x)\,dx\\ &= \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A]
time = 0.00, size = 23, normalized size = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A]
time = 2.91, size = 17, normalized size = 0.61

$$\frac{1}{4}\left(2\,\log\left(x\right)^{2}-2\,\log\left(x\right)+1\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A]
time = 0.80, size = 22, normalized size = 0.79

$$\frac{1}{2}\,x^{2}\log\left(x\right)^{2}-\frac{1}{2}\,x^{2}\log\left(x\right)+\frac{1}{4}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.79

$$\frac{x^{2}\log\left(x\right)^{2}}{2}-\frac{x^{2}\log\left(x\right)}{2}+\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Giac [A]
time = 0.44, size = 22, normalized size = 0.79

$$\frac{1}{2}\,x^2\log\left(x\right)^2-\frac{1}{2}\,x^2\log\left(x\right)+\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.61

$$\frac{x^2\,\left(2\ln\left(x\right)^2-2\,\ln\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x)^2,x)
```

```
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.65 $\int \cos(x)\left(1+\sin^3(x)\right)\,dx$

Optimal. Leaf size=11

$$\sin(x)+\frac{\sin^4(x)}{4}$$

```
[Out] sin(x)+1/4*sin(x)^4
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3302}

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(1 + Sin[x]^3),x]
```

```
[Out] Sin[x] + Sin[x]^4/4
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\left(1+\sin^3(x)\right)\,dx &= \text{Subst}\left(\int \left(1+x^3\right)\,dx, x, \sin(x)\right)\\ &= \sin(x)+\frac{\sin^4(x)}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\sin(x)+\frac{\sin^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(1 + Sin[x]^3),x]
```

```
[Out] Sin[x] + Sin[x]^4/4
```

Maple [A]
time = 0.04, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$\sin(x)+\frac{(\sin^4(x))}{4}$	10
default	$\sin(x)+\frac{(\sin^4(x))}{4}$	10
risch	$\sin(x)+\frac{\cos(4x)}{32}-\frac{\cos(2x)}{8}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+sin(x)^3),x,method=_RETURNVERBOSE)
[Out] sin(x)+1/4*sin(x)^4
```

Maxima [A]
time = 1.30, size = 9, normalized size = 0.82

$$\frac{1}{4}\sin(x)^4+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="maxima")
[Out] 1/4*sin(x)^4 + sin(x)
```

Fricas [A]
time = 1.02, size = 15, normalized size = 1.36

$$\frac{1}{4}\cos(x)^4-\frac{1}{2}\cos(x)^2+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="fricas")
[Out] 1/4*cos(x)^4 - 1/2*cos(x)^2 + sin(x)
```

Sympy [A]
time = 0.13, size = 8, normalized size = 0.73

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)**3),x)
```

```
[Out] sin(x)**4/4 + sin(x)
```

Giac [A]
time = 0.43, size = 9, normalized size = 0.82

$$\frac{1}{4}\sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/4*sin(x)^4 + sin(x)
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.82

$$\frac{\sin(x)^4}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sin(x)^3 + 1),x)
```

```
[Out] sin(x) + sin(x)^4/4
```

3.66 $\int \frac{1}{x(1+\log^2(x))} dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {209}

$$\text{ArcTan}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\left(1+\log^2(x)\right)} dx &= \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \log(x)\right) \\ &= \tan^{-1}(\log(x)) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A]
time = 0.00, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(ln(x))
```

Maxima [A]
time = 1.43, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
[Out] arctan(log(x))
```

Fricas [A]
time = 1.00, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
[Out] arctan(log(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.
time = 0.05, size = 15, normalized size = 5.00

$$\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log(2i+\log(x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A]
time = 0.48, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

Mupad [B]
time = 0.00, size = 3, normalized size = 1.00

$$\operatorname{atan}(\ln{(x)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
```

```
[Out] atan(log(x))
```

3.67 $\int \frac{1}{\sqrt{1-x^2}\,(1+\sin^{-1}(x)^2)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

```
[Out] arctan(arcsin(x))
```

Rubi [A]
time = 0.03, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6828, 209}

$$\text{ArcTan}(\text{ArcSin}(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

```
[Out] ArcTan[ArcSin[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6828

```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fals
eQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-x^2}\,(1+\sin^{-1}(x)^2)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin^{-1}(x)\right) \\ &= \tan^{-1}\left(\sin^{-1}(x)\right) \end{aligned}$$

Mathematica [A]
time = 0.02, size = 3, normalized size = 1.00

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

```
[Out] ArcTan[ArcSin[x]]
```

Maple [A]
time = 0.03, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\arctan(\arcsin(x))$	4
default	$\arctan(\arcsin(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(arcsin(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)), x)
```

Fricas [A]
time = 0.75, size = 3, normalized size = 1.00

$$\arctan(\arcsin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(arcsin(x))
```

Sympy [A]
time = 0.13, size = 3, normalized size = 1.00

$$\operatorname{atan}(\operatorname{asin}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)
```

```
[Out] atan(asin(x))
```

Giac [A]
time = 0.44, size = 3, normalized size = 1.00

$$\arctan\left(\arcsin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(arcsin(x))
```

Mupad [B]
time = 3.10, size = 43, normalized size = 14.33

$$\frac{\ln\left(\frac{-1+\mathrm{asin}(x)\,1\mathrm{i}}{\sqrt{1-x^2}}\right)\,1\mathrm{i}}{2}-\frac{\ln\left(\frac{1+\mathrm{asin}(x)\,1\mathrm{i}}{\sqrt{1-x^2}}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/2)*(asin(x)^2 + 1)),x)
```

```
[Out] (log((asin(x)*1i - 1)/(1 - x^2)^(1/2))*1i)/2 - (log((asin(x)*1i + 1)/(1 - x
^2)^(1/2))*1i)/2
```

3.68 $\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2}\log(\cos(x)+\sin(x))$$

```
[Out] 1/2*x-1/2*ln(cos(x)+sin(x))
```

Rubi [A]
time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3176, 3212}

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx &= \frac{x}{2}-\frac{1}{2}\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx\\ &= \frac{x}{2}-\frac{1}{2}\log(\cos(x)+\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{2}\log(\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Maple [A]
time = 0.05, size = 23, normalized size = 1.44

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\frac{\ln(e^{2ix}+i)}{2}$	20
default	$-\frac{\ln(\tan(x)+1)}{2}+\frac{\ln(1+\tan^2(x))}{4}+\frac{\arctan(\tan(x))}{2}$	23
norman	$\frac{\frac{x}{2}+\frac{x(\tan^2(\frac{x}{2}))}{2}}{1+\tan^2(\frac{x}{2})}+\frac{\ln(1+\tan^2(\frac{x}{2}))}{2}-\frac{\ln(\tan^2(\frac{x}{2})-2\tan(\frac{x}{2})-1)}{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(tan(x)+1)+1/4*ln(1+tan(x)^2)+1/2*arctan(tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(12) = 24.
time = 1.30, size = 53, normalized size = 3.31

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\frac{1}{2}\log\left(-\frac{2\sin(x)}{\cos(x)+1}+\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)+\frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/2*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(co
s(x) + 1)^2 - 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 0.92, size = 15, normalized size = 0.94

$$\frac{1}{2}x-\frac{1}{4}\log(2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*x - 1/4*log(2*cos(x)*sin(x) + 1)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.75

$$\frac{x}{2} - \frac{\log\left(\sin\left(x\right) + \cos\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x)
[Out] x/2 - log(sin(x) + cos(x))/2
```

Giac [A]
time = 0.46, size = 21, normalized size = 1.31

$$\frac{1}{2}\,x + \frac{1}{4}\,\log\left(\tan\left(x\right)^{2} + 1\right) - \frac{1}{2}\,\log\left(\left|\tan\left(x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="giac")
[Out] 1/2*x + 1/4*log(tan(x)^2 + 1) - 1/2*log(abs(tan(x) + 1))
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.81

$$\frac{x}{2} - \frac{\ln\left(\cos\left(x - \frac{\pi}{4}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + sin(x)),x)
[Out] x/2 - log(cos(x - pi/4))/2
```

3.69 $\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$

Optimal. Leaf size=53

$$-B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)$$

```
[Out] -B*arctan(B*y/(-B^2*y^2+A^2+B^2)^(1/2))-A*arctanh(A*y/(-B^2*y^2+A^2+B^2)^(1
/2))
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {1999, 399, 223, 209, 385, 212}

$$-B\text{ArcTan}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] -(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sqrt[A^2 + B
^2 - B^2*y^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1999

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] &&  !BinomialMatchQ[{u, v}, x]
```

Rubi steps

$$
\begin{aligned}
\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy &= -\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy \\
&= -\left(A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}\,dy\right) - B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy \\
&= -\left(A^2\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy, y, \frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right)\right) - B^2\text{Subst}\Bigg(\int \frac{}{1-} \\
&= -B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right) - A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.09, size = 98, normalized size = 1.85

$$
\frac{B\left(-A\tan^{-1}\left(\frac{B^2(-1+y^2)+\sqrt{-B^2}\,y\sqrt{A^2+B^2-B^2y^2}}{AB}\right)+B\log\left(-\sqrt{-B^2}\,y+\sqrt{A^2+B^2-B^2y^2}\right)\right)}{\sqrt{-B^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] (B*(-(A*ArcTan[(B^2*(-1 + y^2) + Sqrt[-B^2]*y*Sqrt[A^2 + B^2 - B^2*y^2])/(A
*B)]) + B*Log[-(Sqrt[-B^2]*y) + Sqrt[A^2 + B^2 - B^2*y^2]]))/Sqrt[-B^2]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(49) = 98$.
time = 0.05, size = 262, normalized size = 4.94

method	result
default	$\frac{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}{2}-\frac{B^2\arctan\left(\frac{\sqrt{B^2}\,y}{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}\right)}{2\sqrt{B^2}}-\frac{A^2\ln\left(\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)-1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(
1/2)*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2-
2*B^2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))/(y-1))-1/2*
(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2)-1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)
*y/(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2+2*B^
2*(1+y)+2*(A^2)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(49) = 98$.
time = 2.16, size = 115, normalized size = 2.17

$$-B\arcsin\left(\frac{B^2y}{\sqrt{A^2B^2+B^4}}\right)+\frac{1}{2}A\log\left(B^2+\frac{A^2}{y+1}+\frac{\sqrt{-B^2y^2+A^2+B^2}\,A}{y+1}\right)-\frac{1}{2}A\log\left(-B^2+\frac{2A^2}{|2y-2|}+\frac{2\sqrt{-B^2y^2+A^2+B^2}\,A}{|2y-2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="maxima")
```

```
[Out] -B*arcsin(B^2*y/sqrt(A^2*B^2 + B^4)) + 1/2*A*log(B^2 + A^2/(y + 1) + sqrt(-
B^2*y^2 + A^2 + B^2)*A/(y + 1)) - 1/2*A*log(-B^2 + 2*A^2/abs(2*y - 2) + 2*s
qrt(-B^2*y^2 + A^2 + B^2)*A/abs(2*y - 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(49) = 98$.
time = 1.35, size = 128, normalized size = 2.42

$$B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)-\frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2+2\sqrt{-B^2y^2+A^2+B^2}\,Ay+A^2+B^2}{y^2}\right)+\frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2-2\sqrt{-B^2y^2+A^2+B^2}\,Ay+A^2+B^2}{y^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) - 1/4*A*log(-((A^2 - B^2)*y^2 +
2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) + 1/4*A*log(-((A^2 - B^2
)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{A^2 - B^2y^2 + B^2}}{(y-1)(y+1)}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)
```

```
[Out] Integral(sqrt(A**2 - B**2*y**2 + B**2)/((y - 1)*(y + 1)), y)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[2,0,0]%%%}+%%%{2,[0,2,2]%%%}+%%%{-4,[0,2,
0]%%%},0,%%%{1,[0,4,4]%%%}] at parameters values [88,76,-66]Warning, choosi
ng root of [1,0,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \int \frac{\sqrt{-B^2\,y^2}}{y^2-1}\,dy & \text{if}\ \ A^2+B^2=0 \\ \ln\left(2\,y\,\sqrt{-B^2}+2\,\sqrt{A^2-B^2\,y^2+B^2}\right)\,\sqrt{-B^2}+\mathrm{atan}\left(\frac{y\,\sqrt{A^2}\,1\mathrm{i}}{\sqrt{A^2-B^2\,y^2+B^2}}\right)\,\sqrt{A^2}\,1\mathrm{i} & \text{if}\ \ A^2+B^2\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A^2 - B^2*(y^2 - 1))^(1/2)/(y^2 - 1),y)
```

```
[Out] piecewise(A^2 + B^2 == 0, int((-B^2*y^2)^(1/2)/(y^2 - 1), y), A^2 + B^2 ~=
0, atan((y*(A^2)^(1/2)*1i)/(A^2 + B^2 - B^2*y^2)^(1/2))*(A^2)^(1/2)*1i + lo
g(2*y*(-B^2)^(1/2) + 2*(A^2 + B^2 - B^2*y^2)^(1/2))*(-B^2)^(1/2))
```

3.70 $\int \frac{\left(-A^2-B^2\right) \cos^2(z)}{B\left(1-\frac{\left(A^2+B^2\right) \sin^2(z)}{B^2}\right)} \, dz$

Optimal. Leaf size=16

$$-Bz - A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)$$

```
[Out] -B*z-A*arctanh(A*tan(z)/B)
```

Rubi [A]
time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {12, 3270, 400, 209, 212}

$$-A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right) - Bz$$

Antiderivative was successfully verified.

```
[In] Int[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
[Out] -(B*z) - A*ArcTanh[(A*Tan[z])/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3270

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz &= -\frac{(A^2+B^2)\int \frac{\cos^2(z)}{1-\frac{(A^2+B^2)\sin^2(z)}{B^2}}\,dz}{B} \\
&= -\frac{(A^2+B^2)\,\mathrm{Subst}\left(\int \frac{1}{(1+z^2)\left(1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2\right)}\,dz, z, \tan(z)\right)}{B} \\
&= -\frac{A^2\mathrm{Subst}\left(\int \frac{1}{1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2}\,dz, z, \tan(z)\right)}{B} - B\mathrm{Subst}\left(\int \frac{1}{1+z^2}\,dz, z, \tan(
\end{aligned}
$$

$$
= -Bz - A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.
time = 0.07, size = 35, normalized size = 2.19

$$
-\frac{B(A^2+B^2)\left(Bz+A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)\right)}{A^2B+B^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
[Out] -((B*(A^2 + B^2)*(B*z + A*ArcTanh[(A*Tan[z])/B]))/(A^2*B + B^3))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(16) = 32.
time = 0.12, size = 74, normalized size = 4.62

method	result
default	$(-A^2-B^2)\,B\left(\frac{A\ln(A\tan(z)+B)}{2B(A^2+B^2)}-\frac{A\ln(A\tan(z)-B)}{2B(A^2+B^2)}+\frac{\arctan(\tan(z))}{A^2+B^2}\right)$

norman	$\frac{-Bz-2Bz\left(\tan^2\left(\frac{z}{2}\right)\right)-Bz\left(\tan^4\left(\frac{z}{2}\right)\right)}{\left(1+\tan^2\left(\frac{z}{2}\right)\right)^2}-\frac{A\ln\left(-B\left(\tan^2\left(\frac{z}{2}\right)\right)+2A\tan\left(\frac{z}{2}\right)+B\right)}{2}+\frac{A\ln\left(B\left(\tan^2\left(\frac{z}{2}\right)\right)+2A\tan\left(\frac{z}{2}\right)-B\right)}{2}$
risch	$-\frac{Bz\,A^2}{A^2+B^2}-\frac{B^3z}{A^2+B^2}+\frac{A^3\ln\left(\mathrm{e}^{2iz}-\frac{iB+A}{-iB+A}\right)}{2A^2+2B^2}+\frac{A\ln\left(\mathrm{e}^{2iz}-\frac{iB+A}{-iB+A}\right)B^2}{2A^2+2B^2}-\frac{A^3\ln\left(\mathrm{e}^{2iz}-\frac{-iB+A}{iB+A}\right)}{2(A^2+B^2)}-\frac{A\ln\left(\mathrm{e}^{2iz}-\frac{-iB+A}{iB+A}\right)B^2}{2(A^2+B^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z,method=_RETURNVERBOS
E)
```

```
[Out] (-A^2-B^2)*B*(1/2*A/B/(A^2+B^2)*ln(A*tan(z)+B)-1/2*A/B/(A^2+B^2)*ln(A*tan(z
)-B)+1/(A^2+B^2)*arctan(tan(z)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(16) = 32$.
time = 2.29, size = 69, normalized size = 4.31

$$-\frac{(A^2+B^2)\left(\frac{2\,B^2z}{A^2+B^2}+\frac{AB\log(A\tan(z)+B)}{A^2+B^2}-\frac{AB\log(A\tan(z)-B)}{A^2+B^2}\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="ma
xima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*z/(A^2 + B^2) + A*B*log(A*tan(z) + B)/(A^2 + B^2) -
 A*B*log(A*tan(z) - B)/(A^2 + B^2))/B
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(16) = 32$.
time = 1.85, size = 67, normalized size = 4.19

$$-Bz-\frac{1}{4}\,A\log\left(2\,AB\cos(z)\sin(z)-(A^2-B^2)\cos(z)^2+A^2\right)+\frac{1}{4}\,A\log\left(-2\,AB\cos(z)\sin(z)-(A^2-B^2)\cos(z)^2+A^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="fr
icas")
```

```
[Out] -B*z - 1/4*A*log(2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2) + 1/4*A*
log(-2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2)
```

Sympy [A]
time = 112.06, size = 202, normalized size = 12.62

$$\frac{(-A^2-B^2)\left(\begin{cases} z & \text{for}\ A=0\wedge B=0 \\ \frac{z\sin^2(z)}{2}+\frac{z\cos^2(z)}{2}+\frac{\sin(z)\cos(z)}{2} & \text{for}\ A=-iB\vee A=iB \\ \frac{AB\log\left(-\frac{A}{B}+\tan\left(\frac{z}{2}\right)-\frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2}+\frac{AB\log\left(-\frac{A}{B}+\tan\left(\frac{z}{2}\right)+\frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2}-\frac{AB\log\left(\frac{A}{B}+\tan\left(\frac{z}{2}\right)-\frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2}-\frac{AB\log\left(\frac{A}{B}+\tan\left(\frac{z}{2}\right)+\frac{\sqrt{A^2+B^2}}{B}\right)}{2A^2+2B^2}+\frac{2B^2z}{2A^2+2B^2} & \text{otherwise} \end{cases}\right)}{B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)
```

```
[Out] (-A**2 - B**2)*Piecewise((z, Eq(A, 0) & Eq(B, 0)), (z*sin(z)**2/2 + z*cos(z
)**2/2 + sin(z)*cos(z)/2, Eq(A, I*B) | Eq(A, -I*B)), (A*B*log(-A/B + tan(z/
2) - sqrt(A**2 + B**2)/B)/(2*A**2 + 2*B**2) + A*B*log(-A/B + tan(z/2) + sqr
t(A**2 + B**2)/B)/(2*A**2 + 2*B**2) - A*B*log(A/B + tan(z/2) - sqrt(A**2 +
B**2)/B)/(2*A**2 + 2*B**2) - A*B*log(A/B + tan(z/2) + sqrt(A**2 + B**2)/B)/
(2*A**2 + 2*B**2) + 2*B**2*z/(2*A**2 + 2*B**2), True))/B
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(16) = 32$.
time = 0.49, size = 83, normalized size = 5.19

$$-\frac{\left(\frac{A^3 B \log(|A \tan(z)+B|)}{A^4+A^2B^2} - \frac{A^3 B \log(|A \tan(z)-B|)}{A^4+A^2B^2} + \frac{2\,B^2 z}{A^2+B^2}\right)(A^2+B^2)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="gi
ac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*tan(z) + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*tan(z)
 - B))/(A^4 + A^2*B^2) + 2*B^2*z/(A^2 + B^2))*(A^2 + B^2)/B
```

Mupad [B]
time = 0.50, size = 360, normalized size = 22.50

$$-A\,\mathrm{atanh}\left(\frac{2\,A^{13}\tan(z)}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{2\,A^7\,B^6\tan(z)}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{6\,A^9\,B^4\tan(z)}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{6\,A^{11}\,B^2\tan(z)}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}\right)-B\,\mathrm{atan}\left(\frac{2\,A^4\,B^9\tan(z)}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{6\,A^6\,B^7\tan(z)}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{6\,A^8\,B^5\tan(z)}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{2\,A^{10}\,B^3\tan(z)}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(z)^2*(A^2 + B^2))/(B*((sin(z)^2*(A^2 + B^2))/B^2 - 1)),z)
```

```
[Out] - A*atanh((2*A^13*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) +
 (2*A^7*B^6*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^
9*B^4*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2
*tan(z))/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B
^9*tan(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*ta
n(z))/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*tan(z))
/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (2*A^10*B^3*tan(z))/(2*
A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3))
```

3.71 $\int \frac{-A^2-B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw$

Optimal. Leaf size=16

$$-B\tan^{-1}(w) - A\tanh^{-1}\left(\frac{Aw}{B}\right)$$

```
[Out] -B*arctan(w)-A*arctanh(A*w/B)
```

Rubi [A]
time = 0.09, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$, Rules used = {12, 6820, 400, 209, 214}

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right) - B\text{ArcTan}(w)$$

Antiderivative was successfully verified.

```
[In] Int[(-A^2 - B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2)))),w]
```

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int -\frac{A^2+B^2}{B\left(1+w^2\right)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw = -\frac{\left(A^2+B^2\right)\int \frac{1}{(1+w^2)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw}{B}$$

$$= -\frac{\left(A^2+B^2\right)\int \frac{B^2}{(1+w^2)(B^2-A^2w^2)}\,dw}{B}$$

$$= -\left(\left(B\left(A^2+B^2\right)\right)\int \frac{1}{(1+w^2)\left(B^2-A^2w^2\right)}\,dw\right)$$

$$= -\left(B\int \frac{1}{1+w^2}\,dw\right) - \left(A^2B\right)\int \frac{1}{B^2-A^2w^2}\,dw$$

$$= -B\tan^{-1}(w) - A\tanh^{-1}\left(\frac{Aw}{B}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.
time = 0.01, size = 35, normalized size = 2.19

$$-\frac{B(A^2+B^2)\left(B\tan^{-1}(w)+A\tanh^{-1}\left(\frac{Aw}{B}\right)\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-A^2 - B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2)
))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(16) = 32.
time = 0.06, size = 71, normalized size = 4.44

method	result
default	$(-A^2-B^2)\,B\left(\frac{\arctan(w)}{A^2+B^2}-\frac{A\ln(Aw-B)}{2B(A^2+B^2)}+\frac{A\ln(Aw+B)}{2B(A^2+B^2)}\right)$

risch	$-\frac{A^3\ln(-Aw-B)}{2(A^2+B^2)} - \frac{A\ln(-Aw-B)B^2}{2(A^2+B^2)} + \frac{A^3\ln(-Aw+B)}{2A^2+2B^2} + \frac{A\ln(-Aw+B)B^2}{2A^2+2B^2} - \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(A^4+2A^2B^2+B^4\right)\_Z^2+B^4\right)} -\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w,method=_RETURNVE
RBOSE)
```

```
[Out] (-A^2-B^2)*B*(1/(A^2+B^2)*arctan(w)-1/2*A/B/(A^2+B^2)*ln(A*w-B)+1/2*A/B/(A^
2+B^2)*ln(A*w+B))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(16) = 32$.
time = 1.25, size = 68, normalized size = 4.25

$$-\frac{(A^2+B^2)\left(\frac{2B^2\arctan(w)}{A^2+B^2} + \frac{AB\log(Aw+B)}{A^2+B^2} - \frac{AB\log(Aw-B)}{A^2+B^2}\right)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*arctan(w)/(A^2 + B^2) + A*B*log(A*w + B)/(A^2 + B^2
) - A*B*log(A*w - B)/(A^2 + B^2))/B
```

Fricas [A]
time = 1.10, size = 26, normalized size = 1.62

$$-B\arctan(w) - \frac{1}{2}A\log(Aw+B) + \frac{1}{2}A\log(Aw-B)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 422, normalized size = 26.38

$$(A^2B+B^3)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B(A^2+B^2)^3}-\frac{A^7B}{(A^2+B^2)^3}+\frac{A^5B^3}{(A^2+B^2)^3}+\frac{A^5}{B(A^2+B^2)}+\frac{A^3B^5}{(A^2+B^2)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)} + \frac{A\log\left(w+\frac{\frac{A^9}{B(A^2+B^2)^3}+\frac{A^7B}{(A^2+B^2)^3}-\frac{A^5B^3}{(A^2+B^2)^3}-\frac{A^5}{B(A^2+B^2)}-\frac{A^3B^5}{(A^2+B^2)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)} + \frac{i\log\left(w+\frac{-\frac{iA^6B^2}{(A^2+B^2)^3}-\frac{iA^4B^4}{(A^2+B^2)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{(A^2+B^2)^3}+\frac{iB^8}{(A^2+B^2)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)} - \frac{i\log\left(w+\frac{\frac{iA^6B^2}{(A^2+B^2)^3}+\frac{iA^4B^4}{(A^2+B^2)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{(A^2+B^2)^3}-\frac{iB^8}{(A^2+B^2)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(16) = 32$.
time = 0.44, size = 82, normalized size = 5.12

$$-\frac{\left(\frac{A^3B\log(|Aw+B|)}{A^4+A^2B^2}-\frac{A^3B\log(|Aw-B|)}{A^4+A^2B^2}+\frac{2B^2\arctan(w)}{A^2+B^2}\right)(A^2+B^2)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="giac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*w + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*w - B))/(A^
4 + A^2*B^2) + 2*B^2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)/B
```

Mupad [B]
time = 0.24, size = 352, normalized size = 22.00

$$-A\,\mathrm{atanh}\left(\frac{2A^{13}w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{2A^7B^6w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^9B^4w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}+\frac{6A^{11}B^2w}{2A^{12}B+6A^{10}B^3+6A^8B^5+2A^6B^7}\right)-B\,\mathrm{atan}\left(\frac{2A^4B^9w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^6B^7w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{6A^8B^5w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}+\frac{2A^{10}B^3w}{2A^{10}B^3+6A^8B^5+6A^6B^7+2A^4B^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A^2 + B^2)/(B*(w^2 + 1)^2*((w^2*(A^2 + B^2))/(B^2*(w^2 + 1)) - 1)),w)
```

```
[Out] - A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*A
^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)/(
2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*B +
 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9 + 6
*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*B^7 +
 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5
 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10
*B^3))
```

3.72 $\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$

Optimal. Leaf size=16

$$-B\tan^{-1}(w)-A\tanh^{-1}\left(\frac{Aw}{B}\right)$$

```
[Out] -B*arctan(w)-A*arctanh(A*w/B)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {12, 400, 209, 214}

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\text{ArcTan}(w)$$

Antiderivative was successfully verified.

```
[In] Int[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int -\frac{B(A^2+B^2)}{(1+w^2)\,(B^2-A^2w^2)}\,dw = -\left((B(A^2+B^2))\int \frac{1}{(1+w^2)\,(B^2-A^2w^2)}\,dw\right)$$
$$= -\left(B\int \frac{1}{1+w^2}\,dw\right) - (A^2B)\int \frac{1}{B^2-A^2w^2}\,dw$$
$$= -B\tan^{-1}(w) - A\tanh^{-1}\left(\frac{Aw}{B}\right)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.
time = 0.01, size = 35, normalized size = 2.19

$$-\frac{B(A^2+B^2)\,\left(B\tan^{-1}(w)+A\tanh^{-1}\left(\frac{Aw}{B}\right)\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(16) = 32.
time = 0.06, size = 68, normalized size = 4.25

method	result
default	$-(A^2+B^2)\,B\left(\frac{\arctan(w)}{A^2+B^2}-\frac{A\ln(Aw-B)}{2B(A^2+B^2)}+\frac{A\ln(Aw+B)}{2B(A^2+B^2)}\right)$
risch	$-\frac{A\ln(-Aw-B)}{2}-\frac{A^2B\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(A^4+2A^2B^2+B^4\right)\_Z^2\right)}\_R\ln\left(\left(\left(-A^6-B^2A^4+A^2B^4+B^6\right)\_R^2-2A^2\right)w+\left(-A^4-2A\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w,method=_RETURNVERBOSE)
```

```
[Out] -(A^2+B^2)*B*(1/(A^2+B^2)*arctan(w)-1/2*A/B/(A^2+B^2)*ln(A*w-B)+1/2*A/B/(A^
2+B^2)*ln(A*w+B))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(16) = 32.
time = 1.54, size = 65, normalized size = 4.06

$$-\frac{1}{2}\,(A^2+B^2)B\left(\frac{A\log\left(Aw+B\right)}{A^2B+B^3}-\frac{A\log\left(Aw-B\right)}{A^2B+B^3}+\frac{2\,\arctan\left(w\right)}{A^2+B^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*B*(A*log(A*w + B)/(A^2*B + B^3) - A*log(A*w - B)/(A^2*B +
B^3) + 2*arctan(w)/(A^2 + B^2))
```

Fricas [**A**]
time = 1.31, size = 26, normalized size = 1.62

$$-B\arctan(w)-\frac{1}{2}A\log(Aw+B)+\frac{1}{2}A\log(Aw-B)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 1.04, size = 422, normalized size = 26.38

$$(A^2B+B^3)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B(A^2+B^2)^3}-\frac{A^7B}{(A^2+B^2)^3}+\frac{A^5B^3}{(A^2+B^2)^3}+\frac{A^5}{B(A^2+B^2)}+\frac{A^3B^5}{(A^2+B^2)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}+\frac{A\log\left(w+\frac{\frac{A^9}{B(A^2+B^2)^3}+\frac{A^7B}{(A^2+B^2)^3}-\frac{A^5B^3}{(A^2+B^2)^3}-\frac{A^5}{B(A^2+B^2)}-\frac{A^3B^5}{(A^2+B^2)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B(A^2+B^2)}+\frac{i\log\left(w+\frac{-\frac{iA^6B^2}{(A^2+B^2)^3}-\frac{iA^4B^4}{(A^2+B^2)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{(A^2+B^2)^3}+\frac{iB^8}{(A^2+B^2)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}-\frac{i\log\left(w+\frac{\frac{iA^6B^2}{(A^2+B^2)^3}+\frac{iA^4B^4}{(A^2+B^2)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{(A^2+B^2)^3}-\frac{iB^8}{(A^2+B^2)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2(A^2+B^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(16) = 32$.
time = 0.43, size = 79, normalized size = 4.94

$$-\frac{1}{2}\left(\frac{A^3\log(|Aw+B|)}{A^4B+A^2B^3}-\frac{A^3\log(|Aw-B|)}{A^4B+A^2B^3}+\frac{2\arctan(w)}{A^2+B^2}\right)(A^2+B^2)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="giac")
```

```
[Out] -1/2*(A^3*log(abs(A*w + B))/(A^4*B + A^2*B^3) - A^3*log(abs(A*w - B))/(A^4*
B + A^2*B^3) + 2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)*B
```

Mupad [B]
time = 0.06, size = 352, normalized size = 22.00

$$-A\,\mathrm{atanh}\left(\frac{2\,A^{13}\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{2\,A^7\,B^6\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{6\,A^9\,B^4\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}+\frac{6\,A^{11}\,B^2\,w}{2\,A^{12}\,B+6\,A^{10}\,B^3+6\,A^8\,B^5+2\,A^6\,B^7}\right)-B\,\mathrm{atan}\left(\frac{2\,A^4\,B^9\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{6\,A^6\,B^7\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{6\,A^8\,B^5\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}+\frac{2\,A^{10}\,B^3\,w}{2\,A^{10}\,B^3+6\,A^8\,B^5+6\,A^6\,B^7+2\,A^4\,B^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(A^2 + B^2))/((w^2 + 1)*(B^2 - A^2*w^2)),w)
```

```
[Out] - A*atanh((2*A^13*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (2*A
^7*B^6*w)/(2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^9*B^4*w)/(
2*A^12*B + 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3) + (6*A^11*B^2*w)/(2*A^12*B +
 2*A^6*B^7 + 6*A^8*B^5 + 6*A^10*B^3)) - B*atan((2*A^4*B^9*w)/(2*A^4*B^9 + 6
*A^6*B^7 + 6*A^8*B^5 + 2*A^10*B^3) + (6*A^6*B^7*w)/(2*A^4*B^9 + 6*A^6*B^7 +
 6*A^8*B^5 + 2*A^10*B^3) + (6*A^8*B^5*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5
 + 2*A^10*B^3) + (2*A^10*B^3*w)/(2*A^4*B^9 + 6*A^6*B^7 + 6*A^8*B^5 + 2*A^10
*B^3))
```

3.73 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{x^3}{3\,(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {294, 222}

$$\text{ArcSin}(x)-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\,(1-x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(1-x^2)^{5/2}}\,dx &= \frac{x^3}{3\,(1-x^2)^{3/2}}-\int \frac{x^2}{(1-x^2)^{3/2}}\,dx\\ &= \frac{x^3}{3\,(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^3}{3\,(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.26

$$\frac{x(-3+4x^2)}{3(1-x^2)^{3/2}}+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x(-20x^2+15)}{10(-x^2+1)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\mathrm{RootOf}(\_Z^2+1)\ln\left(\mathrm{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.99, size = 44, normalized size = 1.26

$$\frac{1}{3}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

time = 1.22, size = 63, normalized size = 1.80

$$-\frac{6\,(x^4-2\,x^2+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-(4\,x^3-3\,x)\sqrt{-x^2+1}}{3\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.
time = 0.46, size = 105, normalized size = 3.00

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A]
time = 0.46, size = 29, normalized size = 0.83

$$\frac{(4\,x^2-3)\sqrt{-x^2+1}\,x}{3\,(x^2-1)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B]
time = 0.00, size = 91, normalized size = 2.60

$$\operatorname{asin}(x)+\frac{3\sqrt{1-x^2}}{4\,(x-1)}+\frac{3\sqrt{1-x^2}}{4\,(x+1)}-\sqrt{1-x^2}\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^2}\right)-\sqrt{1-x^2}\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.74 $\int \tan^4(y)\,dy$

Optimal. Leaf size=14

$$y-\tan(y)+\frac{\tan^3(y)}{3}$$

```
[Out] y-tan(y)+1/3*tan(y)^3
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[y]^4,y]
```

```
[Out] y - Tan[y] + Tan[y]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(y)\,dy &= \frac{\tan^3(y)}{3}-\int \tan^2(y)\,dy\\ &= -\tan(y)+\frac{\tan^3(y)}{3}+\int 1\,dy\\ &= y-\tan(y)+\frac{\tan^3(y)}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.29

$$y-\frac{4\tan(y)}{3}+\frac{1}{3}\sec^2(y)\tan(y)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[y]^4,y]
```

```
[Out] y - (4*Tan[y])/3 + (Sec[y]^2*Tan[y])/3
```

Maple [A]
time = 0.03, size = 13, normalized size = 0.93

method	result	size
default	$y-\tan\left(y\right)+\frac{\left(\tan^{3}\left(y\right)\right)}{3}$	13
risch	$y-\frac{4 i \left(3\,{\mathrm e}^{4 i y}+3\,{\mathrm e}^{2 i y}+2\right)}{3 \left({\mathrm e}^{2 i y}+1\right)^{3}}$	31
norman	$\frac{y \left(\tan^{6}\left(\frac{y}{2}\right)\right)-y-\frac{20 \left(\tan^{3}\left(\frac{y}{2}\right)\right)}{3}+2 \left(\tan^{5}\left(\frac{y}{2}\right)\right)+3 y \left(\tan^{2}\left(\frac{y}{2}\right)\right)-3 y \left(\tan^{4}\left(\frac{y}{2}\right)\right)+2 \tan\left(\frac{y}{2}\right)}{\left(\tan^{2}\left(\frac{y}{2}\right)-1\right)^{3}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)^4/cos(y)^4,y,method=_RETURNVERBOSE)
```

```
[Out] y-tan(y)+1/3*tan(y)^3
```

Maxima [A]
time = 1.81, size = 12, normalized size = 0.86

$$\frac{1}{3}\,\tan\left(y\right)^{3}+y-\tan\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="maxima")
```

```
[Out] 1/3*tan(y)^3 + y - tan(y)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.97, size = 26, normalized size = 1.86

$$\frac{3\,y\cos\left(y\right)^{3}-\left(4\,\cos\left(y\right)^{2}-1\right)\sin\left(y\right)}{3\,\cos\left(y\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="fricas")
```

```
[Out] 1/3*(3*y*cos(y)^3 - (4*cos(y)^2 - 1)*sin(y))/cos(y)^3
```

Sympy [A]
time = 0.01, size = 19, normalized size = 1.36

$$y+\frac{\sin^{3}\left(y\right)}{3\cos^{3}\left(y\right)}-\frac{\sin\left(y\right)}{\cos\left(y\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)**4/cos(y)**4,y)
```

```
[Out] y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)
```

Giac [A]
time = 0.46, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan\left(y\right)^{3}+y-\tan\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="giac")
```

```
[Out] 1/3*tan(y)^3 + y - tan(y)
```

Mupad [B]
time = 0.07, size = 12, normalized size = 0.86

$$\frac{\tan(y)^3}{3}-\tan(y)+y$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)^4/cos(y)^4,y)
```

```
[Out] y - tan(y) + tan(y)^3/3
```

3.75 $\int \frac{z^4}{1+z^2} \, dz$

Optimal. Leaf size=13

$$-z+\frac{z^3}{3}+\tan^{-1}(z)$$

[Out] -z+1/3*z^3+arctan(z)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {308, 209}

$$\text{ArcTan}(z)+\frac{z^3}{3}-z$$

Antiderivative was successfully verified.

[In] Int[z^4/(1 + z^2),z]

[Out] -z + z^3/3 + ArcTan[z]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{z^4}{1+z^2} \, dz &= \int \left(-1+z^2+\frac{1}{1+z^2}\right) \, dz\\ &= -z+\frac{z^3}{3}+\int \frac{1}{1+z^2} \, dz\\ &= -z+\frac{z^3}{3}+\tan^{-1}(z)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-z+\frac{z^3}{3}+\tan^{-1}(z)$$

Antiderivative was successfully verified.

```
[In] Integrate[z^4/(1 + z^2),z]
```

```
[Out] -z + z^3/3 + ArcTan[z]
```

Maple [A]
time = 0.04, size = 12, normalized size = 0.92

method	result	size
default	$-z+\frac{z^{3}}{3}+\arctan\left(z\right)$	12
risch	$-z+\frac{z^{3}}{3}+\arctan\left(z\right)$	12
meijerg	$-\frac{z\left(-5z^{2}+15\right)}{15}+\arctan\left(z\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^4/(z^2+1),z,method=_RETURNVERBOSE)
```

```
[Out] -z+1/3*z^3+arctan(z)
```

Maxima [A]
time = 1.27, size = 11, normalized size = 0.85

$$\frac{1}{3}\,z^{3}-z+\arctan\left(z\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="maxima")
```

```
[Out] 1/3*z^3 - z + arctan(z)
```

Fricas [A]
time = 0.93, size = 11, normalized size = 0.85

$$\frac{1}{3}\,z^{3}-z+\arctan\left(z\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="fricas")
```

```
[Out] 1/3*z^3 - z + arctan(z)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.62

$$\frac{z^{3}}{3}-z+\operatorname{atan}\left(z\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z**4/(z**2+1),z)
```

```
[Out] z**3/3 - z + atan(z)
```

Giac [A]
time = 0.44, size = 11, normalized size = 0.85

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="giac")
```

```
[Out] 1/3*z^3 - z + arctan(z)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.85

$$\mathrm{atan}(z) - z + \frac{z^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^4/(z^2 + 1),z)
```

```
[Out] atan(z) - z + z^3/3
```

3.76 $\int e^{x^2}(1+2x^2)\,dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

[Out] exp(x^2)*x

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2258, 2235, 2243}

$$e^{x^2}x$$

Antiderivative was successfully verified.

[In] Int[E^x^2*(1 + 2*x^2),x]

[Out] E^x^2*x

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2258

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\left(1+2x^2\right)\,dx &= \int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx\\
&= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx\\
&= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)-\int e^{x^2}\,dx\\
&= e^{x^2}x
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Maple [A]
time = 0.01, size = 7, normalized size = 1.00

method	result	size
gosper	$e^{x^2}x$	7
default	$e^{x^2}x$	7
norman	$e^{x^2}x$	7
risch	$e^{x^2}x$	7
meijerg	$\frac{\mathrm{erfi}(x)\sqrt{\pi}}{2}+i\left(-ix\,e^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x^2)*x
```

Maxima [A]
time = 1.43, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A]
time = 0.68, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

Giac [A]
time = 0.45, size = 6, normalized size = 0.86

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

Mupad [B]
time = 0.00, size = 6, normalized size = 0.86

$$x\,\mathrm{e}^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2 + 1),x)
```

```
[Out] x*exp(x^2)
```

3.77 $\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx$

Optimal. Leaf size=24

$$e^{x^2}x+\frac{e^{x^2}}{2(1+x^2)}$$

```
[Out] exp(x^2)*x+1/2*exp(x^2)/(x^2+1)
```

Rubi [A]
time = 0.24, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6874, 2235, 2243, 6847, 2208, 2209}

$$e^{x^2}x+\frac{e^{x^2}}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
```

```
[Out] E^x^2*x + E^x^2/(2*(1 + x^2))
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
```

```
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx &= \int \left(e^{x^2}+2e^{x^2}x^2-\frac{e^{x^2}x}{(1+x^2)^2}+\frac{e^{x^2}x}{1+x^2}\right)dx\\ &= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx-\int \frac{e^{x^2}x}{(1+x^2)^2}\,dx+\int \frac{e^{x^2}x}{1+x^2}\,dx\\ &= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)-\frac{1}{2}\mathrm{Subst}\left(\int \frac{e^x}{(1+x)^2}\,dx,x,x^2\right)+\frac{1}{2}\mathrm{Subst}\left(\int\right.\\ &= e^{x^2}x+\frac{e^{x^2}}{2(1+x^2)}+\frac{\mathrm{Ei}(1+x^2)}{2e}-\frac{1}{2}\mathrm{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,x^2\right)\\ &= e^{x^2}x+\frac{e^{x^2}}{2(1+x^2)}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 19, normalized size = 0.79

$$e^{x^2}\left(x+\frac{1}{2(1+x^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
[Out] E^x^2*(x + 1/(2*(1 + x^2)))
```

Maple [A]
time = 0.04, size = 24, normalized size = 1.00

method	result	size
gosper	$\frac{(2x^3+2x+1)e^{x^2}}{2x^2+2}$	24
risch	$\frac{(2x^3+2x+1)e^{x^2}}{2x^2+2}$	24
norman	$\frac{x^3e^{x^2}+e^{x^2}x+\frac{e^{x^2}}{2}}{x^2+1}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*x^3+2*x+1)*exp(x^2)/(x^2+1)
```

Maxima [A]
time = 3.31, size = 23, normalized size = 0.96

$$\frac{(2\,x^3+2\,x+1)e^{(x^2)}}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Fricas [A]
time = 0.79, size = 23, normalized size = 0.96

$$\frac{(2\,x^3+2\,x+1)e^{(x^2)}}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.83

$$\frac{(2x^3+2x+1)\,e^{x^2}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)
```

```
[Out] (2*x**3 + 2*x + 1)*exp(x**2)/(2*x**2 + 2)
```

Giac [A]
time = 0.44, size = 30, normalized size = 1.25

$$\frac{2\,x^{3}e^{\left(x^{2}\right)}+2\,xe^{\left(x^{2}\right)}+e^{\left(x^{2}\right)}}{2\,\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(2*x^3*e^(x^2) + 2*x*e^(x^2) + e^(x^2))/(x^2 + 1)
```

Mupad [B]
time = 0.22, size = 24, normalized size = 1.00

$$\frac{\mathrm{e}^{x^{2}}\,\left(2\,x^{3}+2\,x+1\right)}{2\,\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x^2)*(4*x^2 + x^3 + 5*x^4 + 2*x^6 + 1))/(x^2 + 1)^2,x)
[Out] (exp(x^2)*(2*x + 2*x^3 + 1))/(2*(x^2 + 1))
```

3.78 $\int e^{-1-x} dx$

Optimal. Leaf size=9

$$-e^{-1-x}$$

[Out] -exp(-1-x)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2225}

$$-e^{-x-1}$$

Antiderivative was successfully verified.

[In] Int[E^(-1 - x),x]

[Out] -E^(-1 - x)

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{-1-x} dx = -e^{-1-x}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$-e^{-1-x}$$

Antiderivative was successfully verified.

[In] Integrate[E^(-1 - x),x]

[Out] -E^(-1 - x)

Maple [A]
time = 0.01, size = 9, normalized size = 1.00

method	result	size

gosper	$-e^{-1-x}$	9
derivativedivides	$-e^{-1-x}$	9
default	$-e^{-1-x}$	9
norman	$-e^{-1-x}$	9
risch	$-e^{-1-x}$	9
meijerg	$e^{-1}(1-e^{-x})$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-1-x),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(-1-x)
```

Maxima [**A**]
time = 1.48, size = 8, normalized size = 0.89

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="maxima")
```

```
[Out] -e^(-x - 1)
```

Fricas [**A**]
time = 0.54, size = 8, normalized size = 0.89

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="fricas")
```

```
[Out] -e^(-x - 1)
```

Sympy [**A**]
time = 0.02, size = 7, normalized size = 0.78

$$-e^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x)
```

```
[Out] -exp(-x - 1)
```

Giac [**A**]
time = 0.43, size = 8, normalized size = 0.89

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="giac")
```

```
[Out] -e^(-x - 1)
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.89

$$-e^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(- x - 1),x)
```

```
[Out] -exp(- x - 1)
```

3.79 $\int \left(\frac{1}{x}+x\right) \log(x)\,dx$

Optimal. Leaf size=25

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1607, 14, 2393, 2338, 2341}

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1) + x)*Log[x],x]

[Out] -1/4*x^2 + (x^2*Log[x])/2 + Log[x]^2/2

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
```

```
(f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{x}+x\right)\log(x)\,dx &= \int \frac{(1+x^2)\log(x)}{x}\,dx\\ &= \int \left(\frac{\log(x)}{x}+x\log(x)\right)\,dx\\ &= \int \frac{\log(x)}{x}\,dx+\int x\log(x)\,dx\\ &= -\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + x)*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2 + Log[x]^2/2
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{\ln(x)^2}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+x)*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

Maxima [A]
time = 1.50, size = 24, normalized size = 0.96

$$-\frac{1}{4}x^2 + \frac{1}{2}\left(x^2 + 2\log(x)\right)\log(x) - \frac{1}{2}\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="maxima")
[Out] -1/4*x^2 + 1/2*(x^2 + 2*log(x))*log(x) - 1/2*log(x)^2
```

Fricas [A]
time = 1.20, size = 19, normalized size = 0.76

$$\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 + \frac{1}{2}\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^2\log(x)}{2} - \frac{x^2}{4} + \frac{\log(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4 + log(x)**2/2
```

Giac [A]
time = 0.44, size = 19, normalized size = 0.76

$$\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 + \frac{1}{2}\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="giac")
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

Mupad [B]
time = 0.23, size = 19, normalized size = 0.76

$$\frac{x^2\ln(x)}{2} - \frac{x^2}{4} + \frac{\ln(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(x + 1/x),x)
[Out] (x^2*log(x))/2 + log(x)^2/2 - x^2/4
```

3.80 $\int \frac{x}{1+x^4} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

```
[Out] 1/2*arctan(x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {281, 209}

$$\frac{\text{ArcTan}(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, x^2\right)\\ &= \frac{1}{2}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.88

method	result	size
default	$\frac{\arctan(x^2)}{2}$	7
meijerg	$\frac{\arctan(x^2)}{2}$	7
risch	$\frac{\arctan(x^2)}{2}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x^2)
```

Maxima [A]
time = 2.41, size = 6, normalized size = 0.75

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2)
```

Fricas [A]
time = 1.33, size = 6, normalized size = 0.75

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x^2)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+1),x)
```

```
[Out] atan(x**2)/2
```

Giac [A]
time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x^2)
```

Mupad [B]
time = 0.06, size = 6, normalized size = 0.75

$$\frac{\operatorname{atan}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 1),x)
```

```
[Out] atan(x^2)/2
```

3.81 $\int \frac{x^5}{1+x^4} \, dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right)$$

```
[Out] 1/2*x^2-1/2*arctan(x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {281, 327, 209}

$$\frac{x^2}{2} - \frac{\text{ArcTan}(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{1+x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2} \, dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^4),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Maple [A]
time = 0.03, size = 13, normalized size = 0.81

method	result	size
default	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
meijerg	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13
risch	$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/2*arctan(x^2)
```

Maxima [A]
time = 2.20, size = 12, normalized size = 0.75

$$\frac{1}{2}\,x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Fricas [A]
time = 1.16, size = 12, normalized size = 0.75

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.62

$$\frac{x^2}{2} - \frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1),x)
```

```
[Out] x**2/2 - atan(x**2)/2
```

Giac [A]
time = 0.46, size = 12, normalized size = 0.75

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{x^2}{2} - \frac{\operatorname{atan}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + 1),x)
```

```
[Out] x^2/2 - atan(x^2)/2
```

3.82 $\int \frac{1}{1+\tan^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3738, 2715, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\tan^2(x)}\,dx &= \int \cos^2(x)\,dx\\ &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.36

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
derivativedivides	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
default	$\frac{\tan(x)}{2+2(\tan^2(x))}+\frac{\arctan(\tan(x))}{2}$	19
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2(x)\right)}{2}+\frac{\tan(x)}{2}}{1+\tan^2(x)}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(1+tan(x)^2)*tan(x)+1/2*arctan(tan(x))
```

Maxima [A]
time = 3.78, size = 16, normalized size = 1.14

$$\frac{1}{2}\,x+\frac{\tan\left(x\right)}{2\,\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Fricas [A]
time = 0.76, size = 20, normalized size = 1.43

$$\frac{x\tan\left(x\right)^2+x+\tan\left(x\right)}{2\,\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.
time = 0.17, size = 36, normalized size = 2.57

$$\frac{x\tan^2(x)}{2\tan^2(x)+2}+\frac{x}{2\tan^2(x)+2}+\frac{\tan(x)}{2\tan^2(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**2),x)
```

```
[Out] x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2 +
 2)
```

Giac [A]
time = 0.47, size = 16, normalized size = 1.14

$$\frac{1}{2}\,x+\frac{\tan(x)}{2\,\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Mupad [B]
time = 0.18, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(2\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^2 + 1),x)
```

```
[Out] x/2 + sin(2*x)/4
```

3.83 $\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {294, 222}

$$\text{ArcSin}(x) - \frac{x}{\sqrt{1-x^2}} + \frac{x^3}{3(1-x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(1-x^2)^{5/2}} \, dx &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} \, dx \\
&= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} \, dx \\
&= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.26

$$\frac{x(-3+4x^2)}{3(1-x^2)^{3/2}}+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A]
time = 0.00, size = 30, normalized size = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin(x)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x(4x^2-3)}{3(x^2-1)\sqrt{-x^2+1}}+\arcsin(x)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x(-20x^2+15)}{10(-x^2+1)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
trager	$\frac{(4x^2-3)x\sqrt{-x^2+1}}{3(x^2-1)^2}+\mathrm{RootOf}(\_Z^2+1)\ln\left(\mathrm{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A]
time = 3.24, size = 44, normalized size = 1.26

$$\frac{1}{3}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

time = 0.67, size = 63, normalized size = 1.80

$$-\frac{6\left(x^{4}-2\,x^{2}+1\right)\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\left(4\,x^{3}-3\,x\right)\sqrt{-x^{2}+1}}{3\left(x^{4}-2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.
time = 0.45, size = 105, normalized size = 3.00

$$\frac{3x^{4}\operatorname{asin}\left(x\right)}{3x^{4}-6x^{2}+3}+\frac{4x^{3}\sqrt{1-x^{2}}}{3x^{4}-6x^{2}+3}-\frac{6x^{2}\operatorname{asin}\left(x\right)}{3x^{4}-6x^{2}+3}-\frac{3x\sqrt{1-x^{2}}}{3x^{4}-6x^{2}+3}+\frac{3\operatorname{asin}\left(x\right)}{3x^{4}-6x^{2}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A]
time = 0.48, size = 29, normalized size = 0.83

$$\frac{\left(4\,x^{2}-3\right)\sqrt{-x^{2}+1}\,x}{3\left(x^{2}-1\right)^{2}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B]
time = 0.00, size = 91, normalized size = 2.60

$$\operatorname{asin}(x)+\frac{3\sqrt{1-x^{2}}}{4\,(x-1)}+\frac{3\sqrt{1-x^{2}}}{4\,(x+1)}-\sqrt{1-x^{2}}\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^{2}}\right)-\sqrt{1-x^{2}}\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.84 $\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=17

$$-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

[Out] arcsin(x)-x/(-x^2+1)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {294, 222}

$$\text{ArcSin}(x)-\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In] Int[-(x^2/(1 - x^2)^(3/2)),x]

[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx &= -\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

time = 0.05, size = 35, normalized size = 2.06

$$-\frac{x}{\sqrt{1-x^2}}+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^2/(1 - x^2)^(3/2)),x]
[Out] -(x/Sqrt[1 - x^2]) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A]
time = 0.07, size = 16, normalized size = 0.94

method	result	size
default	$\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	16
risch	$\arcsin\left(x\right)-\frac{x}{\sqrt{-x^{2}+1}}$	16
meijerg	$-\frac{i\left(-\frac{i\sqrt{\pi}\, x}{\sqrt{-x^{2}+1}}+i\sqrt{\pi}\, \arcsin\left(x\right)\right)}{\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^{2}+1}}{x^{2}-1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A]
time = 3.60, size = 15, normalized size = 0.88

$$-\frac{x}{\sqrt{-x^2+1}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="maxima")
[Out] -x/sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.
time = 0.61, size = 45, normalized size = 2.65

$$-\frac{2\left(x^2-1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}\,x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*(x^2 - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*x)/(x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.
time = 0.19, size = 34, normalized size = 2.00

$$\frac{x^2 \operatorname{asin}(x)}{x^2-1}+\frac{x\sqrt{1-x^2}}{x^2-1}-\frac{\operatorname{asin}(x)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2/(-x**2+1)**(3/2),x)
```

```
[Out] x**2*asin(x)/(x**2 - 1) + x*sqrt(1 - x**2)/(x**2 - 1) - asin(x)/(x**2 - 1)
```

Giac [A]
time = 0.53, size = 21, normalized size = 1.24

$$\frac{\sqrt{-x^2+1}\,x}{x^2-1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)*x/(x^2 - 1) + arcsin(x)
```

Mupad [B]
time = 0.16, size = 37, normalized size = 2.18

$$\operatorname{asin}(x)+\frac{\sqrt{1-x^2}}{2\,(x-1)}+\frac{\sqrt{1-x^2}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(1 - x^2)^(3/2),x)
```

```
[Out] asin(x) + (1 - x^2)^(1/2)/(2*(x - 1)) + (1 - x^2)^(1/2)/(2*(x + 1))
```

3.85 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4517}

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(x)\,dx=-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(-\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.74

method	result	size
default	$-\frac{e^{x} \cos \left(x \right)}{2}+\frac{e^{x} \sin \left(x \right)}{2}$	14
norman	$\frac{e^{x} \tan \left(\frac{x}{2}\right)+\frac{e^{x} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i) x}}{4}-\frac{i e^{(1+i) x}}{4}-\frac{e^{(1-i) x}}{4}+\frac{i e^{(1-i) x}}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A]
time = 2.00, size = 11, normalized size = 0.58

$$-\frac{1}{2} \, {\left(\cos\left(x\right) - \sin\left(x\right)\right)} e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A]
time = 0.55, size = 13, normalized size = 0.68

$$-\frac{1}{2} \, \cos\left(x\right) e^{x} + \frac{1}{2} \, e^{x} \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.79

$$\frac{e^{x} \sin{\left(x \right)}}{2} - \frac{e^{x} \cos{\left(x \right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Giac [A]
time = 0.85, size = 11, normalized size = 0.58

$$-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Mupad [B]
time = 0.00, size = 11, normalized size = 0.58

$$-\frac{e^{x}\left(\cos\left(x\right)-\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

3.86 $\int \frac{1}{x}\,dx$

Optimal. Leaf size=2

$$\log(x)$$

[Out] ln(x)

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{29\}$

$$\log(x)$$

Antiderivative was successfully verified.

[In] Int[x^(-1),x]

[Out] Log[x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x}\,dx = \log(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1),x]

[Out] Log[x]

Maple [A]
time = 0.00, size = 3, normalized size = 1.50

method	result	size

default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)
```

Maxima [A]
time = 2.46, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A]
time = 0.56, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A]
time = 0.02, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A]
time = 0.72, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

Mupad [**B**]
time = 0.01, size = 2, normalized size = 1.00

$$\ln (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

3.87 $\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$

Optimal. Leaf size=45

$$-\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\cos(t)+\sin(t))+\frac{1}{3}\log(2\cos(t)+\sin(t))-\frac{1}{2(1+\tan(t))}$$

```
[Out] -1/12*ln(cos(t)-sin(t))-1/4*ln(cos(t)+sin(t))+1/3*ln(2*cos(t)+sin(t))-1/2/(
1+tan(t))
```

Rubi [A]
time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {723, 814}

$$-\frac{1}{2(\tan(t)+1)}-\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\sin(t)+\cos(t))+\frac{1}{3}\log(\sin(t)+2\cos(t))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -1/12*Log[Cos[t] - Sin[t]] - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t] + Sin[t]
]/3 - 1/(2*(1 + Tan[t]))
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt &= \text{Subst}\left(\int \frac{1}{(1+t)^2\,(2-t-t^2)}\,dt,t,\tan(t)\right)\\ &= -\frac{1}{2(1+\tan(t))}+\frac{1}{2}\text{Subst}\left(\int \frac{t}{(1+t)\,(2-t-t^2)}\,dt,t,\tan(t)\right)\\ &= -\frac{1}{2(1+\tan(t))}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{6(-1+t)}-\frac{1}{2(1+t)}+\frac{2}{3(2+t)}\right)\,dt,t,\tan(t)\right)\\ &= -\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\cos(t)+\sin(t))+\frac{1}{3}\log(2\cos(t)+\sin(t))-\frac{1}{2}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 73, normalized size = 1.62

$$-\frac{\cos(t)(\log(\cos(t)-\sin(t))+3\log(\cos(t)+\sin(t))-4\log(2\cos(t)+\sin(t)))+(-6+\log(\cos(t)-\sin(t))+3\log(\cos(t)+\sin(t))-4\log(2\cos(t)+\sin(t)))\sin(t)}{12(\cos(t)+\sin(t))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -1/12*(Cos[t]*(Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Cos[
t] + Sin[t]]) + (-6 + Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log
[2*Cos[t] + Sin[t]])*Sin[t])/(Cos[t] + Sin[t])
```

Maple [A]
time = 0.20, size = 31, normalized size = 0.69

method	result	size
default	$-\frac{1}{2(1+\tan(t))}-\frac{\ln(1+\tan(t))}{4}+\frac{\ln(\tan(t)+2)}{3}-\frac{\ln(\tan(t)-1)}{12}$	31
risch	$-\frac{1}{2(\mathrm{e}^{2it}+i)}+\frac{\ln\left(\mathrm{e}^{2it}+\frac{3}{5}+\frac{4i}{5}\right)}{3}-\frac{\ln(\mathrm{e}^{2it}+i)}{4}-\frac{\ln(\mathrm{e}^{2it}-i)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*t)/(1+sec(t)^2+3*tan(t)),t,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(1+tan(t))-1/4*ln(1+tan(t))+1/3*ln(tan(t)+2)-1/12*ln(tan(t)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(37) = 74.
time = 2.51, size = 256, normalized size = 5.69

$$\frac{3\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)\log\left(953674316406250\left(3\cos(2t)+\sin(2t)+4\right)\cos(4t)+2384185791015625\cos(4t)^2+953674316406250\cos(2t)^2-953674316406250\left(\cos(2t)-3\sin(2t)+3\right)\sin(4t)+2384185791015625\sin(4t)^2+953674316406250\sin(2t)^2+2861022949218750\cos(2t)-953674316406250\sin(2t)+2384185791015625\right)-6\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)\log\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)+5\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)\log\left(\frac{5\cos(2t)^2+5\sin(2t)^2+6\cos(2t)+8\sin(2t)+5}{5\left(\cos(2t)^2+\sin(2t)^2-2\sin(2t)+1\right)}\right)-24\cos(2t)}{48\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="maxima")
```

```
[Out] 1/48*(3*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(953674316406250*(3*c
os(2*t) + sin(2*t) + 4)*cos(4*t) + 2384185791015625*cos(4*t)^2 + 9536743164
06250*cos(2*t)^2 - 953674316406250*(cos(2*t) - 3*sin(2*t) + 3)*sin(4*t) + 2
384185791015625*sin(4*t)^2 + 953674316406250*sin(2*t)^2 + 2861022949218750*
cos(2*t) - 953674316406250*sin(2*t) + 2384185791015625) - 6*(cos(2*t)^2 + s
in(2*t)^2 + 2*sin(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1) +
 5*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(1/5*(5*cos(2*t)^2 + 5*sin
(2*t)^2 + 6*cos(2*t) + 8*sin(2*t) + 5)/(cos(2*t)^2 + sin(2*t)^2 - 2*sin(2*t
) + 1)) - 24*cos(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)
```

Fricas [A]
time = 0.54, size = 71, normalized size = 1.58

$$\frac{4\left(\cos\left(t\right)+\sin\left(t\right)\right)\log\left(\frac{3}{4}\cos\left(t\right)^{2}+\cos\left(t\right)\sin\left(t\right)+\frac{1}{4}\right)-3\left(\cos\left(t\right)+\sin\left(t\right)\right)\log\left(2\cos\left(t\right)\sin\left(t\right)+1\right)-\left(\cos\left(t\right)+\sin\left(t\right)\right)\log\left(-2\cos\left(t\right)\sin\left(t\right)+1\right)-6\cos\left(t\right)+6\sin\left(t\right)}{24\left(\cos\left(t\right)+\sin\left(t\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="fricas")
```

```
[Out] 1/24*(4*(cos(t) + sin(t))*log(3/4*cos(t)^2 + cos(t)*sin(t) + 1/4) - 3*(cos(
t) + sin(t))*log(2*cos(t)*sin(t) + 1) - (cos(t) + sin(t))*log(-2*cos(t)*sin
(t) + 1) - 6*cos(t) + 6*sin(t))/(cos(t) + sin(t))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(2t\right)}{3\tan\left(t\right)+\sec^{2}\left(t\right)+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
```

```
[Out] Integral(sec(2*t)/(3*tan(t) + sec(t)**2 + 1), t)
```

Giac [A]
time = 0.78, size = 33, normalized size = 0.73

$$-\frac{1}{2\left(\tan\left(t\right)+1\right)}+\frac{1}{3}\log\left(\left|\tan\left(t\right)+2\right|\right)-\frac{1}{4}\log\left(\left|\tan\left(t\right)+1\right|\right)-\frac{1}{12}\log\left(\left|\tan\left(t\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="giac")
```

```
[Out] -1/2/(tan(t) + 1) + 1/3*log(abs(tan(t) + 2)) - 1/4*log(abs(tan(t) + 1)) - 1
/12*log(abs(tan(t) - 1))
```

Mupad [B]
time = 0.72, size = 32, normalized size = 0.71

$$\frac{\ln\left(\tan(t)+2\right)}{3}-\frac{\ln\left(\tan(t)+1\right)}{4}-\frac{\ln\left(\tan(t)-1\right)}{12}-\frac{1}{2\,\left(\tan\left(t\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*t)*(3*tan(t) + 1/cos(t)^2 + 1)),t)
[Out] log(tan(t) + 2)/3 - log(tan(t) + 1)/4 - log(tan(t) - 1)/12 - 1/(2*(tan(t) +
 1))
```

3.88 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A]
time = 0.98, size = 10, normalized size = 0.71

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A]
time = 0.47, size = 10, normalized size = 0.71

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A]
time = 0.76, size = 16, normalized size = 1.14

$$\frac{1}{2}\,x+\frac{\tan\left(x\right)}{2\,\left(\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Mupad [B]
time = 0.00, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] x/2 + sin(2*x)/4
```

3.89 $\int \frac{1+x^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=17

$$2\sqrt{x} + \frac{2x^{5/2}}{5}$$

```
[Out] 2/5*x^(5/2)+2*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{2x^{5/2}}{5} + 2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x] + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{\sqrt{x}} \, dx &= \int \left(\frac{1}{\sqrt{x}} + x^{3/2}\right) \, dx \\ &= 2\sqrt{x} + \frac{2x^{5/2}}{5} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.82

$$\frac{2}{5}\sqrt{x}\left(5+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(5 + x^2))/5
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.71

method	result	size
gosper	$\frac{2\sqrt{x}\,(x^2+5)}{5}$	11
risch	$\frac{2\sqrt{x}\,(x^2+5)}{5}$	11
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$	12
default	$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$	12
trager	$\left(\frac{2x^2}{5}+2\right)\sqrt{x}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*x^(5/2)+2*x^(1/2)
```

Maxima [A]
time = 1.48, size = 11, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}}+2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

Fricas [A]
time = 0.41, size = 10, normalized size = 0.59

$$\frac{2}{5}\,(x^2+5)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="fricas")
[Out] 2/5*(x^2 + 5)*sqrt(x)
```

Sympy [A]
time = 0.07, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*sqrt(x)
```

Giac [A]
time = 0.73, size = 11, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}} + 2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.59

$$\frac{2\,\sqrt{x}\,\left(x^2+5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(x^2 + 5))/5
```

3.90 $\int \frac{x}{\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=23

$$\sqrt{5+2x+x^2}-\sinh^{-1}\left(\frac{1+x}{2}\right)$$

```
[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {654, 633, 221}

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[5 + 2*x + x^2],x]
```

```
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{5+2x+x^2}}\,dx = \sqrt{5+2x+x^2} - \int \frac{1}{\sqrt{5+2x+x^2}}\,dx$$
$$= \sqrt{5+2x+x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}}\,dx, x, 2+2x\right)$$
$$= \sqrt{5+2x+x^2} - \sinh^{-1}\left(\frac{1+x}{2}\right)$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 1.35

$$\sqrt{5+2x+x^2} + \log\left(-1-x+\sqrt{5+2x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[5 + 2*x + x^2],x]
[Out] Sqrt[5 + 2*x + x^2] + Log[-1 - x + Sqrt[5 + 2*x + x^2]]
```

Maple [A]
time = 0.13, size = 20, normalized size = 0.87

method	result	size
default	$-\operatorname{arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^2+2x+5}$	20
risch	$-\operatorname{arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^2+2x+5}$	20
trager	$\sqrt{x^2+2x+5}-\ln\left(x+1+\sqrt{x^2+2x+5}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

Maxima [A]
time = 1.76, size = 19, normalized size = 0.83

$$\sqrt{x^2+2\,x+5} - \operatorname{arsinh}\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 2*x + 5) - arcsinh(1/2*x + 1/2)
```

Fricas [**A**]
time = 0.43, size = 27, normalized size = 1.17

$$\sqrt{x^2+2\,x+5}+\log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2 + 2*x + 5), x)
```

Giac [**A**]
time = 0.75, size = 27, normalized size = 1.17

$$\sqrt{x^2+2\,x+5}+\log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

Mupad [**B**]
time = 0.08, size = 27, normalized size = 1.17

$$\sqrt{x^2+2\,x+5}-\ln\left(x+\sqrt{x^2+2\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x + x^2 + 5)^(1/2),x)
```

```
[Out] (2*x + x^2 + 5)^(1/2) - log(x + (2*x + x^2 + 5)^(1/2) + 1)
```

3.91 $\int \cos(x)\sin^2(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^3(x)}{3}$$

```
[Out] 1/3*sin(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2644, 30}

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]^2,x]
```

```
[Out] Sin[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned} \int \cos(x)\sin^2(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sin(x)\right) \\ &= \frac{\sin^3(x)}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]^2,x]
```

```
[Out] Sin[x]^3/3
```

Maple [A]
time = 0.02, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\sin^3(x))}{3}$	7
default	$\frac{(\sin^3(x))}{3}$	7
risch	$\frac{\sin(x)}{4} - \frac{\sin(3x)}{12}$	12
norman	$\frac{8\left(\tan^3\left(\frac{x}{2}\right)\right)}{3\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sin(x)^3
```

Maxima [A]
time = 1.55, size = 6, normalized size = 0.75

$$\frac{1}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sin(x)^3
```

Fricas [A]
time = 0.41, size = 10, normalized size = 1.25

$$-\frac{1}{3}\left(\cos(x)^2 - 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.62

$$\frac{\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2,x)
```

```
[Out] sin(x)**3/3
```

Giac [A]
time = 0.72, size = 6, normalized size = 0.75

$$\frac{1}{3}\sin\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3
```

Mupad [B]
time = 0.03, size = 6, normalized size = 0.75

$$\frac{\sin\left(x\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2,x)
```

```
[Out] sin(x)^3/3
```

3.92 $\int \frac{e^x}{1+e^x} \, dx$

Optimal. Leaf size=6

$$\log(1+e^x)$$

```
[Out] ln(1+exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2278, 31}

$$\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2278

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{1+e^x} \, dx &= \text{Subst}\left(\int \frac{1}{1+x} \, dx, x, e^x\right) \\
&= \log(1+e^x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\log(1+e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Maple [A]
time = 0.00, size = 6, normalized size = 1.00

method	result	size
derivativedivides	$\ln(1+e^x)$	6
default	$\ln(1+e^x)$	6
norman	$\ln(1+e^x)$	6
risch	$\ln(1+e^x)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+exp(x))
```

Maxima [A]
time = 1.30, size = 5, normalized size = 0.83

$$\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] log(e^x + 1)
```

Fricas [A]
time = 0.43, size = 5, normalized size = 0.83

$$\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] log(e^x + 1)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.83

$$\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x)
```

```
[Out] log(exp(x) + 1)
```

Giac [**A**]
time = 0.58, size = 5, normalized size = 0.83

$$\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] log(e^x + 1)
```

Mupad [**B**]
time = 0.00, size = 5, normalized size = 0.83

$$\ln\left(\mathrm{e}^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(x) + 1),x)
```

```
[Out] log(exp(x) + 1)
```

3.93 $\int \frac{e^{2x}}{1+e^x}\,dx$

Optimal. Leaf size=12

$$e^x - \log(1 + e^x)$$

[Out] exp(x)-ln(1+exp(x))

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2280, 45}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2x}}{1+e^x}\,dx &= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1 + \frac{1}{-1-x}\right)\,dx, x, e^x\right)\\
&= e^x - \log(1 + e^x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$e^x - \log(1 + e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
default	$e^x - \ln(1 + e^x)$	11
norman	$e^x - \ln(1 + e^x)$	11
risch	$e^x - \ln(1 + e^x)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)-ln(1+exp(x))
```

Maxima [A]
time = 1.93, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

Fricas [A]
time = 0.40, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

Giac [A]
time = 0.69, size = 10, normalized size = 0.83

$$e^x - \log\left(e^x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

Mupad [B]
time = 0.05, size = 10, normalized size = 0.83

$$\mathrm{e}^x - \ln\left(\mathrm{e}^x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x) + 1),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

3.94 $\int \frac{1}{1-\cos(x)} \, dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)} \, dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
[Out] -Cot[x/2]
```

Maple [A]
time = 0.01, size = 9, normalized size = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A]
time = 1.66, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A]
time = 0.37, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A]
time = 0.16, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

Giac [A]
time = 0.65, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

Mupad [B]
time = 0.26, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1),x)
```

```
[Out] -cot(x/2)
```

3.95 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

[Out] 1/2*sec(x)^2

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*Tan[x],x]

[Out] Sec[x]^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right)\\ &= \frac{\sec^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Maple [A]
time = 0.01, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\sec^2(x))}{2}$	7
default	$\frac{(\sec^2(x))}{2}$	7
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sec(x)^2
```

Maxima [A]
time = 1.24, size = 6, normalized size = 0.75

$$\frac{1}{2}\tan\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2
```

Fricas [A]
time = 0.39, size = 6, normalized size = 0.75

$$\frac{1}{2\cos\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

Giac [A]
time = 0.74, size = 6, normalized size = 0.75

$$\frac{1}{2\cos{(x)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

Mupad [B]
time = 0.00, size = 6, normalized size = 0.75

$$\frac{\tan(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/cos(x)^2,x)
```

```
[Out] tan(x)^2/2
```

3.96 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2341}

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\, dx = -\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A]
time = 0.00, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A]
time = 2.03, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.71

$$\frac{x^2\log\left(x\right)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A]
time = 0.80, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Mupad [B]
time = 0.00, size = 9, normalized size = 0.53

$$\frac{x^2\,\left(\ln\left(x\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
```

```
[Out] (x^2*(log(x) - 1/2))/2
```

3.97 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

[Out] 1/2*sin(x)^2

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2644, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]
```

```
[Out] -1/2*Cos[x]^2
```

Maple [A]
time = 0.00, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\sin^2(x))}{2}$	7
default	$\frac{(\sin^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
norman	$\frac{2(\tan^2(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)^2
```

Maxima [A]
time = 2.41, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A]
time = 0.38, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

Giac [A]
time = 0.83, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

Mupad [B]
time = 0.00, size = 6, normalized size = 0.75

$$\frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
```

```
[Out] sin(x)^2/2
```

3.98 $\int \frac{1+x}{\sqrt{2x-x^2}} \, dx$

Optimal. Leaf size=24

$$-\sqrt{2x-x^2}-2\sin^{-1}(1-x)$$

[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {654, 633, 222}

$$-2\text{ArcSin}(1-x)-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/Sqrt[2*x - x^2],x]

[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{2x-x^2} + 2\int \frac{1}{\sqrt{2x-x^2}}\,dx$$
$$= -\sqrt{2x-x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, 2-2x\right)$$
$$= -\sqrt{2x-x^2} - 2\sin^{-1}(1-x)$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 1.79

$$\frac{(-2+x)x + 4\sqrt{-2+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-2+x}{x}}}\right)}{\sqrt{-((-2+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[2*x - x^2],x]
[Out] ((-2 + x)*x + 4*Sqrt[-2 + x]*Sqrt[x]*ArcTanh[1/Sqrt[(-2 + x)/x]])/Sqrt[-((-
2 + x)*x)]
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.88

method	result	size
default	$2\arcsin(-1+x) - \sqrt{-x^2+2x}$	21
risch	$\frac{x(-2+x)}{\sqrt{-x(-2+x)}} + 2\arcsin(-1+x)$	21
trager	$-\sqrt{-x^2+2x} + 2\,\text{RootOf}(\_Z^2+1)\ln\left(\text{RootOf}(\_Z^2+1)\sqrt{-x^2+2x} + x - 1\right)$	45
meijerg	$2\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right) + \frac{2i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{1-\frac{x}{2}}}{2} - i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

Maxima [A]
time = 1.71, size = 22, normalized size = 0.92

$$-\sqrt{-x^2+2\,x}-2\,\arcsin\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 2*x) - 2*arcsin(-x + 1)
```

Fricas [A]
time = 0.37, size = 32, normalized size = 1.33

$$-\sqrt{-x^2+2\,x}-4\,\arctan\left(\frac{\sqrt{-x^2+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 2*x) - 4*arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{-x\,(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+2*x)**(1/2),x)
[Out] Integral((x + 1)/sqrt(-x*(x - 2)), x)
```

Giac [A]
time = 1.26, size = 20, normalized size = 0.83

$$-\sqrt{-x^2+2\,x}+2\,\arcsin\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 2*x) + 2*arcsin(x - 1)
```

Mupad [B]
time = 0.27, size = 20, normalized size = 0.83

$$2\,\mathrm{asin}(x-1)-\sqrt{2\,x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(2*x - x^2)^(1/2),x)
[Out] 2*asin(x - 1) - (2*x - x^2)^(1/2)
```

3.99 $\int \frac{2e^x}{2+3e^{2x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,e^x\right)$$

```
[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {12, 2281, 209}

$$\sqrt{\frac{2}{3}}\,\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*E^x)/(2 + 3*E^(2*x)),x]
```

```
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\int \frac{2e^x}{2+3e^{2x}}\,dx = 2\int \frac{e^x}{2+3e^{2x}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right)$$
$$= \sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*E^x)/(2 + 3*E^(2*x)),x]
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.70

method	result	size
default	$\frac{\arctan\left(\frac{e^x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	14
risch	$\frac{i\sqrt{6}\,\ln\left(e^x+\frac{i\sqrt{6}}{3}\right)}{6}-\frac{i\sqrt{6}\,\ln\left(e^x-\frac{i\sqrt{6}}{3}\right)}{6}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(x)/(2+3*exp(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Maxima [A]
time = 2.02, size = 13, normalized size = 0.65

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Fricas [A]
time = 0.39, size = 19, normalized size = 0.95

$$\frac{1}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*e^x)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.75

$$\operatorname{RootSum}\left(6z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x)
[Out] RootSum(6*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Giac [A]
time = 1.24, size = 13, normalized size = 0.65

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}\,e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Mupad [B]
time = 0.09, size = 13, normalized size = 0.65

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,\mathrm{e}^{x}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*exp(x))/(3*exp(2*x) + 2),x)
[Out] (6^(1/2)*atan((6^(1/2)*exp(x))/2))/3
```

3.100 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=35

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {294, 222}

$$\text{ArcSin}(x)-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\int \frac{x^2}{\left(1-x^2\right)^{3/2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.26

$$\frac{x(-3+4x^2)}{3\left(1-x^2\right)^{3/2}}+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.86

method	result	size
default	$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}+\arcsin\left(x\right)-\frac{x}{\sqrt{-x^2+1}}$	30
risch	$-\frac{x\left(4x^2-3\right)}{3\left(x^2-1\right)\sqrt{-x^2+1}}+\arcsin\left(x\right)$	30
meijerg	$-\frac{2i\left(-\frac{i\sqrt{\pi}\,x\left(-20x^2+15\right)}{10\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{3i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{3\sqrt{\pi}}$	39
trager	$\frac{\left(4x^2-3\right)x\sqrt{-x^2+1}}{3\left(x^2-1\right)^2}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A]
time = 2.71, size = 44, normalized size = 1.26

$$\frac{1}{3}\,x\left(\frac{3\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

time = 0.41, size = 63, normalized size = 1.80

$$-\frac{6\left(x^{4}-2\,x^{2}+1\right)\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)-\left(4\,x^{3}-3\,x\right)\sqrt{-x^{2}+1}}{3\left(x^{4}-2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(26) = 52$.
time = 0.46, size = 105, normalized size = 3.00

$$\frac{3x^{4}\operatorname{asin}(x)}{3x^{4}-6x^{2}+3}+\frac{4x^{3}\sqrt{1-x^{2}}}{3x^{4}-6x^{2}+3}-\frac{6x^{2}\operatorname{asin}(x)}{3x^{4}-6x^{2}+3}-\frac{3x\sqrt{1-x^{2}}}{3x^{4}-6x^{2}+3}+\frac{3\operatorname{asin}(x)}{3x^{4}-6x^{2}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A]
time = 1.18, size = 29, normalized size = 0.83

$$\frac{\left(4\,x^{2}-3\right)\sqrt{-x^{2}+1}\,x}{3\left(x^{2}-1\right)^{2}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

Mupad [B]
time = 0.00, size = 91, normalized size = 2.60

$$\operatorname{asin}(x)+\frac{3\,\sqrt{1-x^{2}}}{4\,(x-1)}+\frac{3\,\sqrt{1-x^{2}}}{4\,(x+1)}-\sqrt{1-x^{2}}\,\left(\frac{1}{12\,(x-1)}-\frac{1}{12\,(x-1)^{2}}\right)-\sqrt{1-x^{2}}\,\left(\frac{1}{12\,(x+1)}+\frac{1}{12\,(x+1)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(1 - x^2)^(5/2),x)
```

```
[Out] asin(x) + (3*(1 - x^2)^(1/2))/(4*(x - 1)) + (3*(1 - x^2)^(1/2))/(4*(x + 1))
 - (1 - x^2)^(1/2)*(1/(12*(x - 1)) - 1/(12*(x - 1)^2)) - (1 - x^2)^(1/2)*(1
/(12*(x + 1)) + 1/(12*(x + 1)^2))
```

3.101 $\int \frac{e^{6x}}{1+e^{4x}}\,dx$

Optimal. Leaf size=20

$$\frac{e^{2x}}{2} - \frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

```
[Out] 1/2*exp(2*x)-1/2*arctan(exp(2*x))
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2280, 327, 209}

$$\frac{e^{2x}}{2} - \frac{1}{2}\text{ArcTan}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(6*x)/(1 + E^(4*x)),x]
```

```
[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^{6x}}{1+e^{4x}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{e^{2x}}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{e^{2x}}{2} - \frac{1}{2}\tan^{-1}\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.90

$$\frac{1}{2}\left(e^{2x} - \tan^{-1}\left(e^{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(6*x)/(1 + E^(4*x)),x]
[Out] (E^(2*x) - ArcTan[E^(2*x)])/2
```

Maple [A]
time = 0.02, size = 15, normalized size = 0.75

method	result	size
default	$\frac{e^{2x}}{2} - \frac{\arctan(e^{2x})}{2}$	15
risch	$\frac{e^{2x}}{2} + \frac{i\ln(e^{2x}-i)}{4} - \frac{i\ln(e^{2x}+i)}{4}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(6*x)/(exp(4*x)+1),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)^2-1/2*arctan(exp(x)^2)
```

Maxima [A]
time = 2.02, size = 14, normalized size = 0.70

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="maxima")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Fricas [A]
time = 0.41, size = 14, normalized size = 0.70

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right)+\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="fricas")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Sympy [A]
time = 0.04, size = 24, normalized size = 1.20

$$\frac{e^{2x}}{2}+\operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(-4i+e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x)
[Out] exp(2*x)/2 + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-4*_i + exp(2*x))))
```

Giac [A]
time = 1.11, size = 14, normalized size = 0.70

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right)+\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="giac")
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Mupad [B]
time = 0.07, size = 14, normalized size = 0.70

$$\frac{\mathrm{e}^{2x}}{2}-\frac{\mathrm{atan}\left(\mathrm{e}^{2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(6*x)/(exp(4*x) + 1),x)
[Out] exp(2*x)/2 - atan(exp(2*x))/2
```

3.102 $\int \log(2+3x^2)\,dx$

Optimal. Leaf size=33

$$-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)+x\log(2+3x^2)$$

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2498, 327, 209}

$$2\sqrt{\frac{2}{3}}\,\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)+x\log(3x^2+2)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log(2+3x^2)\,dx &= x\log(2+3x^2) - 6\int \frac{x^2}{2+3x^2}\,dx\\ &= -2x + x\log(2+3x^2) + 4\int \frac{1}{2+3x^2}\,dx\\ &= -2x + 2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) + x\log(2+3x^2)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$-2x + 2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) + x\log(2+3x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + 3*x^2],x]
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Maple [A]
time = 0.00, size = 27, normalized size = 0.82

method	result	size
default	$-2x + x\ln(3x^2+2) + \frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
risch	$-2x + x\ln(3x^2+2) + \frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(3*x^2+2),x,method=_RETURNVERBOSE)
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A]
time = 4.33, size = 26, normalized size = 0.79

$$x\log(3\,x^2+2) + \frac{2}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}\,x\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="maxima")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Fricas [A]
time = 0.40, size = 32, normalized size = 0.97

$$\frac{2}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right)+x\log\left(3x^2+2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

Sympy [A]
time = 0.05, size = 31, normalized size = 0.94

$$x\log\left(3x^2+2\right)-2x+\frac{2\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(3*x**2+2),x)
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

Giac [A]
time = 0.85, size = 26, normalized size = 0.79

$$x\log\left(3x^2+2\right)+\frac{2}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="giac")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Mupad [B]
time = 0.00, size = 26, normalized size = 0.79

$$\frac{2\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}-2x+x\ln\left(3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(3*x^2 + 2),x)
[Out] (2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)
```

3.103 $\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx$

Optimal. Leaf size=21

$$\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

```
[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {8}

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2}}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.95

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^2-a^2}}$	20
norman	$\frac{x}{r\sqrt{2H\,r^2-a^2}}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

Maxima [A]
time = 3.43, size = 19, normalized size = 0.90

$$\frac{x}{\sqrt{2\,Hr^2-a^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

Fricas [A]
time = 0.58, size = 31, normalized size = 1.48

$$\frac{\sqrt{2\,Hr^2-a^2}\,x}{2\,Hr^3-a^2r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*H*r^2 - a^2)*x/(2*H*r^3 - a^2*r)
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.71

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - a**2))
```

Giac [A]
time = 0.92, size = 19, normalized size = 0.90

$$\frac{x}{\sqrt{2\,Hr^2-a^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

Mupad [B]
time = 0.00, size = 19, normalized size = 0.90

$$\frac{x}{r\,\sqrt{2\,H\,r^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - a^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - a^2)^(1/2))
```

3.104 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

```
[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{8\}$

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.96

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^{2}-a^{2}-e^{2}}}$	25
norman	$\frac{x}{r\sqrt{2H\,r^{2}-a^{2}-e^{2}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

Maxima [A]
time = 2.42, size = 23, normalized size = 0.88

$$\frac{x}{\sqrt{2\,Hr^{2}-a^{2}-e^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

Fricas [A]
time = 0.68, size = 40, normalized size = 1.54

$$\frac{\sqrt{2\,Hr^{2}-a^{2}-e^{2}}\,x}{2\,Hr^{3}-a^{2}r-re^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*H*r^2 - a^2 - e^2)*x/(2*H*r^3 - a^2*r - r*e^2)
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.73

$$\frac{x}{r\sqrt{2Hr^{2}-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - a**2 - e**2))
```

Giac [A]
time = 1.09, size = 23, normalized size = 0.88

$$\frac{x}{\sqrt{2\,Hr^{2}-a^{2}-e^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

Mupad [B]
time = 0.00, size = 24, normalized size = 0.92

$$\frac{x}{r\,\sqrt{-a^2-e^2+2\,H\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - a^2 - e^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - a^2 - e^2)^(1/2))
```

3.105 $\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
```

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]
```

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.96

method	result	size
default	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2}}$	26
norman	$\frac{x}{r\sqrt{-2K\,r^4+2H\,r^2-a^2}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

Maxima [A]
time = 1.93, size = 25, normalized size = 0.93

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

Fricas [A]
time = 0.54, size = 43, normalized size = 1.59

$$-\frac{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}\,x}{2\,Kr^5-2\,Hr^3+a^2r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*x/(2*K*r^5 - 2*H*r^3 + a^2*r)
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2))
```

Giac [A]
time = 1.18, size = 25, normalized size = 0.93

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

Mupad [B]
time = 0.00, size = 25, normalized size = 0.93

$$\frac{x}{r\,\sqrt{-a^2-2\,K\,r^4+2\,H\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - 2*K*r^4 - a^2)^(1/2))
```

3.106 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=32

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.97

method	result	size
default	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}-e^{2}}}$	31
norman	$\frac{x}{r\sqrt{-2K\,r^{4}+2H\,r^{2}-a^{2}-e^{2}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

Maxima [A]
time = 2.32, size = 29, normalized size = 0.91

$$\frac{x}{\sqrt{-2\,Kr^{4}+2\,Hr^{2}-a^{2}-e^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

Fricas [A]
time = 0.69, size = 51, normalized size = 1.59

$$-\frac{\sqrt{-2\,Kr^{4}+2\,Hr^{2}-a^{2}-e^{2}}\,x}{2\,Kr^{5}-2\,Hr^{3}+a^{2}r+re^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*x/(2*K*r^5 - 2*H*r^3 + a^2*r + r*e^2)
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^{2}-2Kr^{4}-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2 - e**2))
```

Giac [A]
time = 1.06, size = 29, normalized size = 0.91

$$\frac{x}{\sqrt{-2\,Kr^{4}+2\,Hr^{2}-a^{2}-e^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

Mupad [B]
time = 0.00, size = 30, normalized size = 0.94

$$\frac{x}{r\sqrt{-a^2-e^2-2\,K\,r^4+2\,H\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - 2*K*r^4 - a^2 - e^2)^(1/2))
```

3.107 $\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

```
[Out] x/r/(-a^2-2*r*(-H*r+K))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{8\}$

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.04

$$\frac{x}{r\sqrt{-a^2-2Kr+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2])
```

Maple [A]
time = 0.02, size = 24, normalized size = 1.00

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^{2}-2Kr-a^{2}}}$	24
norman	$\frac{x}{r\sqrt{2H\,r^{2}-2Kr-a^{2}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x/r/(2*H*r^2-2*K*r-a^2)^(1/2)
```

Maxima [A]
time = 2.04, size = 23, normalized size = 0.96

$$\frac{x}{\sqrt{2\,Hr^{2}-a^{2}-2\,Kr}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

Fricas [A]
time = 0.68, size = 41, normalized size = 1.71

$$\frac{\sqrt{2\,Hr^{2}-a^{2}-2\,Kr}\,x}{2\,Hr^{3}-a^{2}r-2\,Kr^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*H*r^2 - a^2 - 2*K*r)*x/(2*H*r^3 - a^2*r - 2*K*r^2)
```

Sympy [A]
time = 0.01, size = 20, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^{2}-2Kr-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2))
```

Giac [A]
time = 0.71, size = 23, normalized size = 0.96

$$\frac{x}{\sqrt{2\,Hr^{2}-a^{2}-2\,Kr}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

Mupad [B]
time = 0.00, size = 23, normalized size = 0.96

$$\frac{x}{r\sqrt{-a^2+2\,H\,r^2-2\,K\,r}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - 2*K*r - a^2)^(1/2))
```

3.108 $\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=29

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{8\}$

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.03

$$\frac{x}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2])
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.00

method	result	size
default	$\frac{x}{r\sqrt{2H\,r^{2}-2Kr-a^{2}-e^{2}}}$	29
norman	$\frac{x}{r\sqrt{2H\,r^{2}-2Kr-a^{2}-e^{2}}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Maxima [A]
time = 1.30, size = 27, normalized size = 0.93

$$\frac{x}{\sqrt{2\,Hr^{2}-a^{2}-2\,Kr-e^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)*r)
```

Fricas [A]
time = 0.86, size = 50, normalized size = 1.72

$$\frac{\sqrt{2\,Hr^{2}-a^{2}-2\,Kr-e^{2}}\,x}{2\,Hr^{3}-a^{2}r-2\,Kr^{2}-re^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)*x/(2*H*r^3 - a^2*r - 2*K*r^2 - r*e^2)
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^{2}-2Kr-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2 - e**2))
```

Giac [A]
time = 0.83, size = 27, normalized size = 0.93

$$\frac{x}{\sqrt{2\,Hr^{2}-a^{2}-2\,Kr-e^{2}}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)*r)
```

Mupad [B]
time = 0.00, size = 28, normalized size = 0.97

$$\frac{x}{r\sqrt{-a^2-e^2+2\,H\,r^2-2\,K\,r}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)),x)
```

```
[Out] x/(r*(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2))
```

3.109 $\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx$

Optimal. Leaf size=19

$$\frac{rx}{\sqrt{-a^2+2er^2}}$$

```
[Out] r*x/(-a^2+2*exp(1)*r^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {8}

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2+2er^2}}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{rx}{\sqrt{-a^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2}}$	19
norman	$\frac{rx}{\sqrt{-a^2+2\,\mathrm{e}r^2}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-a^2+2*exp(1)*r^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] r*x/(-a^2+2*exp(1)*r^2)^(1/2)
```

Maxima [A]
time = 2.13, size = 18, normalized size = 0.95

$$\frac{rx}{\sqrt{2\,r^2e-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(2*r^2*e - a^2)
```

Fricas [A]
time = 0.78, size = 18, normalized size = 0.95

$$\frac{rx}{\sqrt{2\,r^2e-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*r^2*e - a^2)
```

Sympy [A]
time = 0.00, size = 17, normalized size = 0.89

$$\frac{rx}{\sqrt{-a^2+2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a**2+2*exp(1)*r**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-a**2 + 2*E*r**2)
```

Giac [A]
time = 0.61, size = 18, normalized size = 0.95

$$\frac{rx}{\sqrt{2\,r^2e-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2+2*exp(1)*r^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*r^2*e - a^2)
```

Mupad [B]
time = 0.00, size = 18, normalized size = 0.95

$$\frac{r\,x}{\sqrt{2\,r^2\,\mathrm{e}-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - a^2)^(1/2),x)
```

```
[Out] (r*x)/(2*r^2*exp(1) - a^2)^(1/2)
```

3.110 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx$

Optimal. Leaf size=24

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{8\}$

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Maple [A]
time = 0.01, size = 24, normalized size = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^{2}-e^{2}+2\,\mathrm{e}r^{2}}}$	24
norman	$\frac{rx}{\sqrt{-a^{2}-e^{2}+2\,\mathrm{e}r^{2}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2)^(1/2)
```

Maxima [A]
time = 2.57, size = 22, normalized size = 0.92

$$\frac{rx}{\sqrt{2\,r^{2}e-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(2*r^2*e - a^2 - e^2)
```

Fricas [A]
time = 1.28, size = 22, normalized size = 0.92

$$\frac{rx}{\sqrt{2\,r^{2}e-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*r^2*e - a^2 - e^2)
```

Sympy [A]
time = 0.01, size = 20, normalized size = 0.83

$$\frac{rx}{\sqrt{-a^{2}-e^{2}+2er^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a**2-e**2+2*exp(1)*r**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-a**2 - e**2 + 2*E*r**2)
```

Giac [A]
time = 0.60, size = 22, normalized size = 0.92

$$\frac{rx}{\sqrt{2\,r^{2}e-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*r^2*e - a^2 - e^2)
```

Mupad [B]
time = 0.00, size = 23, normalized size = 0.96

$$\frac{r\,x}{\sqrt{-a^2-e^2+2\,\mathrm{e}\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - a^2 - e^2)^(1/2),x)
```

```
[Out] (r*x)/(2*r^2*exp(1) - a^2 - e^2)^(1/2)
```

3.111 $\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx$

Optimal. Leaf size=25

$$\frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

```
[Out] r*x/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A]
time = 0.02, size = 25, normalized size = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}r^{2}-2K\,r^{4}}}$	25
norman	$\frac{rx}{\sqrt{-a^{2}+2\,\mathrm{e}r^{2}-2K\,r^{4}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] r*x/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Maxima [A]
time = 2.58, size = 24, normalized size = 0.96

$$\frac{rx}{\sqrt{-2\,Kr^{4}+2\,r^{2}e-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2)
```

Fricas [A]
time = 1.08, size = 44, normalized size = 1.76

$$-\frac{\sqrt{-2\,Kr^{4}+2\,r^{2}e-a^{2}}\,rx}{2\,Kr^{4}-2\,r^{2}e+a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*r^2*e - a^2)*r*x/(2*K*r^4 - 2*r^2*e + a^2)
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.96

$$\frac{rx}{\sqrt{-2Kr^{4}-a^{2}+2er^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a**2+2*exp(1)*r**2-2*K*r**4)**(1/2),x)
```

```
[Out] r*x/sqrt(-2*K*r**4 - a**2 + 2*E*r**2)
```

Giac [A]
time = 0.59, size = 24, normalized size = 0.96

$$\frac{rx}{\sqrt{-2\,Kr^{4}+2\,r^{2}e-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2)
```

Mupad [B]
time = 0.00, size = 24, normalized size = 0.96

$$\frac{r\,x}{\sqrt{-a^2-2\,K\,r^4+2\,\mathrm{e}\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2),x)
```

```
[Out] (r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2)^(1/2)
```

3.112 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx$

Optimal. Leaf size=30

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

```
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{8\}$

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A]
time = 0.02, size = 30, normalized size = 1.00

method	result	size
default	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2-2K\,r^4}}$	30
norman	$\frac{rx}{\sqrt{-a^2-e^2+2\,\mathrm{e}r^2-2K\,r^4}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] r*x/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2)
```

Maxima [A]
time = 2.10, size = 28, normalized size = 0.93

$$\frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="maxima")
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)
```

Fricas [A]
time = 0.67, size = 50, normalized size = 1.67

$$-\frac{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}\,rx}{2\,Kr^4-2\,r^2e+a^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)*r*x/(2*K*r^4 - 2*r^2*e + a^2 + e^2)
```

Sympy [A]
time = 0.01, size = 27, normalized size = 0.90

$$\frac{rx}{\sqrt{-2Kr^4-a^2-e^2+2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a**2-e**2+2*exp(1)*r**2-2*K*r**4)**(1/2),x)
[Out] r*x/sqrt(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)
```

Giac [A]
time = 0.76, size = 28, normalized size = 0.93

$$\frac{rx}{\sqrt{-2\,Kr^4+2\,r^2e-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-a^2-e^2+2*exp(1)*r^2-2*K*r^4)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*r^2*e - a^2 - e^2)
```

Mupad [B]
time = 0.00, size = 29, normalized size = 0.97

$$\frac{r\,x}{\sqrt{-a^2-e^2-2\,K\,r^4+2\,\mathrm{e}\,r^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2),x)
```

```
[Out] (r*x)/(2*r^2*exp(1) - 2*K*r^4 - a^2 - e^2)^(1/2)
```

3.113 $\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=27

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{8\}$

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx = \frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.04

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]
```

Maple [A]
time = 0.01, size = 27, normalized size = 1.00

method	result	size
default	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27
norman	$\frac{rx}{\sqrt{2H\,r^2-2Kr-a^2-e^2}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Maxima [A]
time = 3.24, size = 25, normalized size = 0.93

$$\frac{rx}{\sqrt{2\,Hr^2-a^2-2\,Kr-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)
```

Fricas [A]
time = 2.43, size = 25, normalized size = 0.93

$$\frac{rx}{\sqrt{2\,Hr^2-a^2-2\,Kr-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.89

$$\frac{rx}{\sqrt{2Hr^2-2Kr-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
```

```
[Out] r*x/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

Giac [A]
time = 0.77, size = 25, normalized size = 0.93

$$\frac{rx}{\sqrt{2\,Hr^2-a^2-2\,Kr-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)
```

Mupad [B]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{r\,x}{\sqrt{-a^2-e^2+2\,H\,r^2-2\,K\,r}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2),x)
```

```
[Out] (r*x)/(2*H*r^2 - 2*K*r - a^2 - e^2)^(1/2)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```python
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```