Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/5-Hearn-Problems

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [284]. This is test number [5].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (284)	0.00 (0)
Maple	99.30 (282)	0.70 (2)
Fricas	98.94 (281)	1.06 (3)
Rubi	98.24 (279)	1.76 (5)
Mupad	95.07 (270)	4.93 (14)
Giac	94.72 (269)	5.28 (15)
Sympy	89.08 (253)	10.92 (31)
Maxima	88.73 (252)	11.27 (32)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

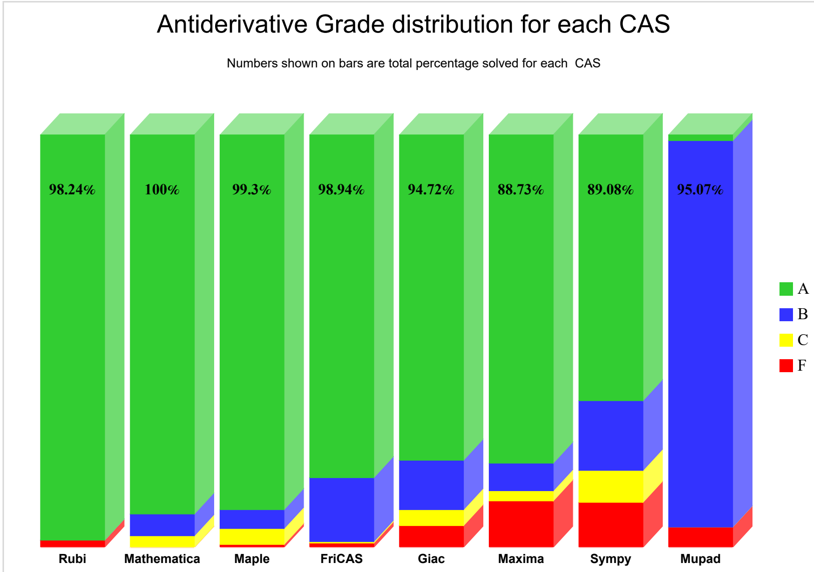
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

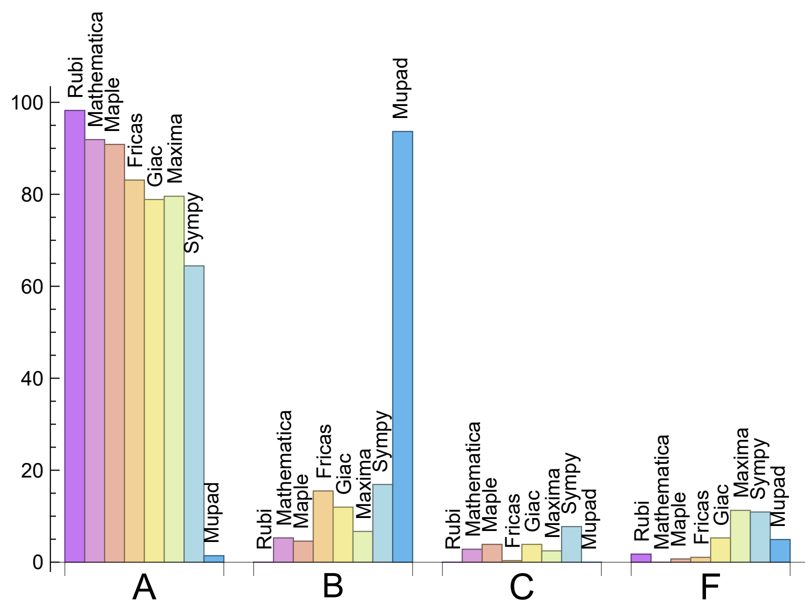
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.24	0.00	0.00	1.76
Mathematica	91.90	5.28	2.82	0.00
Maple	90.85	4.58	3.87	0.70
Fricas	83.10	15.49	0.35	1.06
Maxima	79.58	6.69	2.46	11.27
Giac	78.87	11.97	3.87	5.28
Sympy	64.44	16.90	7.75	10.92
Mupad	N/A	93.66	0.00	4.93

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	5	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	3	100.00 %	0.00 %	0.00 %
Giac	15	100.00 %	0.00 %	0.00 %
Maxima	32	68.75 %	0.00 %	31.25 %
Sympy	31	87.10 %	9.68 %	3.23 %
Mupad	14	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

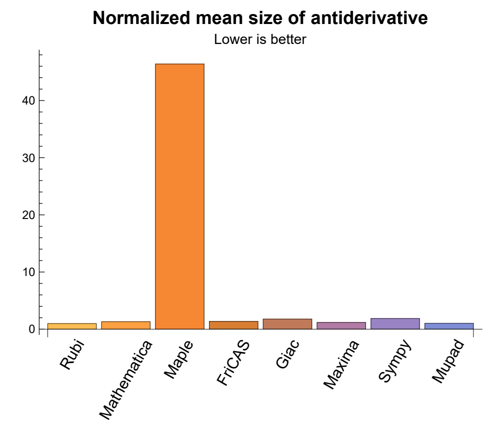
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

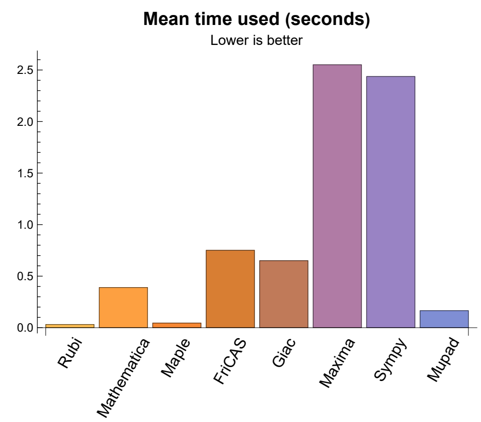
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.03	39.11	0.99	22.00	1.00
Mathematica	0.39	65.82	1.31	23.00	1.00
Maple	0.04	4305.99	46.39	22.00	0.92
Maxima	2.55	35.17	1.18	19.00	0.88
Fricas	0.75	49.68	1.36	22.00	1.00
Sympy	2.44	86.70	1.87	20.00	1.00
Giac	0.65	104.87	1.77	23.00	0.96
Mupad	0.17	35.62	1.03	18.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
10
20
30
40
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{75, 145, 170, 273\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {145}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{279, 281\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

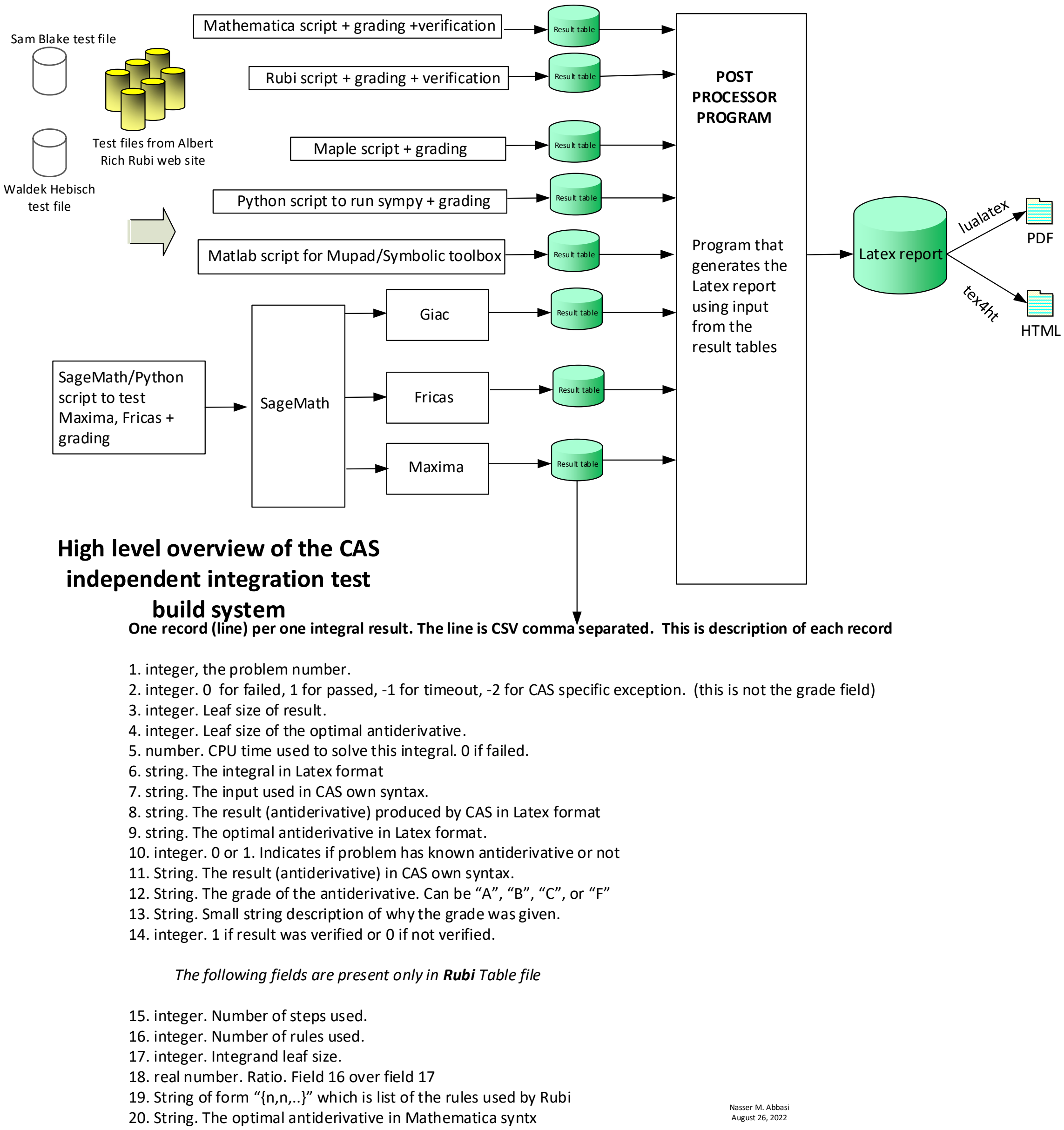
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 282, 283 }

B grade: { }

C grade: { }

F grade: { 169, 278, 279, 281, 284 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 195, 196, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 282, 283, 284 }

B grade: { 52, 81, 82, 108, 111, 120, 121, 190, 194, 199, 211, 235, 236, 256, 260 }

C grade: { 20, 44, 45, 51, 197, 278, 279, 281 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 281, 283, 284 }

B grade: { 32, 60, 61, 110, 124, 126, 204, 237, 242, 256, 257, 278, 280 }

C grade: { 49, 51, 136, 137, 138, 139, 140, 141, 203, 279, 282 }

F grade: { 86, 251 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 164, 165, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 212, 215, 216, 217, 218, 219, 220, 221, 223, 224, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 283 }

B grade: { 32, 81, 82, 111, 112, 128, 129, 130, 146, 191, 194, 195, 204, 222, 225, 226, 241, 256, 280 }

C grade: { 102, 103, 104, 105, 166, 168, 197 }

F grade: { 8, 39, 40, 42, 43, 44, 45, 49, 51, 86, 122, 123, 147, 160, 161, 163, 176, 196, 203, 211, 213, 214, 235, 236, 239, 251, 257, 278, 279, 281, 282, 284 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 43, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 196, 198, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 279, 282, 283 }

B grade: { 36, 37, 39, 40, 41, 42, 44, 48, 49, 78, 79, 80, 81, 82, 90, 103, 110, 111, 112, 127, 128, 133, 194, 195, 199, 204, 220, 222, 227, 228, 235, 236, 244, 245, 246, 247, 254, 256, 260, 268, 269, 278, 280, 284 }

C grade: { 197 }

F grade: { 86, 257, 281 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 40, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 111, 114, 116, 118, 119, 120, 121, 124, 125, 126, 127, 130, 132, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 177, 181, 182, 185, 186, 187, 190, 192, 193, 195, 199, 200, 201, 202, 206, 209, 210, 215, 216, 217, 218, 219, 221, 223, 225, 226, 227, 230, 231, 232, 240, 241, 243, 244, 245, 248, 250, 252, 253, 255, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 277, 282, 283, 284 }

B grade: { 7, 8, 13, 22, 23, 32, 39, 41, 44, 56, 80, 81, 82, 90, 103, 104, 110, 112, 113, 115, 117, 122, 123, 131, 133, 142, 143, 161, 178, 179, 180, 183, 184, 188, 189, 204, 222, 228, 229, 233, 234, 237, 239, 246, 247, 254, 256, 272 }

C grade: { 9, 14, 31, 50, 70, 72, 73, 134, 135, 136, 137, 138, 139, 140, 141, 175, 191, 194, 203, 205, 220, 224 }

F grade: { 12, 86, 128, 129, 146, 147, 160, 162, 163, 176, 196, 197, 198, 207, 208, 211, 212, 213, 214, 235, 236, 238, 242, 249, 251, 257, 265, 278, 279, 280, 281 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 196, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 282, 283, 284 }

B grade: { 11, 23, 32, 67, 79, 81, 82, 110, 111, 112, 118, 127, 164, 176, 178, 179, 190, 195, 198, 199, 202, 220, 222, 228, 235, 236, 245, 246, 247, 256, 260, 268, 269, 280 }

C grade: { 134, 135, 136, 137, 138, 139, 140, 141, 160, 161, 166 }

F grade: { 56, 63, 86, 128, 129, 162, 163, 197, 200, 201, 203, 257, 278, 279, 281 }

2.1.8 Mupad

A grade: { 75, 145, 170, 273 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 280, 282, 283, 284 }

C grade: { }

F grade: { 102, 103, 104, 105, 128, 129, 163, 169, 203, 249, 257, 278, 279, 281 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81
time (sec)	N/A	0.001	0.000	0.004	5.663	0.796	0.005	0.593	0.021

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	15
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.68
time (sec)	N/A	0.009	0.001	0.018	2.560	0.677	0.006	0.571	0.039

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	15
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.68
time (sec)	N/A	0.003	0.001	0.022	3.248	0.651	0.006	0.527	0.029

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.003	2.379	0.582	0.022	0.527	0.009

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	32	46	29	23	22
N.S.	1	1.00	0.67	0.75	0.89	1.28	0.81	0.64	0.61
time (sec)	N/A	0.013	0.009	0.041	2.585	0.697	0.038	0.482	0.118

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	33	24	34	26
N.S.	1	1.00	0.88	0.78	0.75	1.03	0.75	1.06	0.81
time (sec)	N/A	0.012	0.009	0.035	3.197	0.671	0.053	0.506	0.034

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	43	40	34	144	42	40
N.S.	1	1.00	0.85	1.08	1.00	0.85	3.60	1.05	1.00
time (sec)	N/A	0.028	0.012	0.043	2.361	0.924	0.516	0.452	0.250

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35
time (sec)	N/A	0.022	0.008	0.115	0.000	0.632	0.093	0.470	0.198

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	26	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	1.62	0.88	0.88
time (sec)	N/A	0.004	0.005	0.024	4.547	0.844	0.065	0.477	0.038

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	17	14	14	22	14	14
N.S.	1	1.00	0.84	0.89	0.74	0.74	1.16	0.74	0.74
time (sec)	N/A	0.008	0.005	0.212	2.790	0.648	0.032	0.538	0.031

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	36	39	71	41	80	49
N.S.	1	1.00	0.71	0.73	0.80	1.45	0.84	1.63	1.00
time (sec)	N/A	0.021	0.019	0.043	2.898	0.700	0.069	0.477	0.126

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	69	78	81	0	81	87
N.S.	1	1.00	0.91	1.01	1.15	1.19	0.00	1.19	1.28
time (sec)	N/A	0.038	0.021	0.077	2.899	0.703	0.000	0.452	0.568

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	44	43	32	121	43	256
N.S.	1	1.00	0.72	0.94	0.91	0.68	2.57	0.91	5.45
time (sec)	N/A	0.016	0.008	0.048	2.166	0.829	0.351	0.441	0.304

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	41	40	30	393	40	191
N.S.	1	1.00	0.75	1.02	1.00	0.75	9.82	1.00	4.78
time (sec)	N/A	0.014	0.011	0.061	2.407	0.730	0.671	0.448	0.175

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	19	19	19	20	25
N.S.	1	1.00	1.00	0.74	0.70	0.70	0.70	0.74	0.93
time (sec)	N/A	0.014	0.004	0.030	2.519	0.903	0.047	0.502	0.046

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	34	34	41	35	46
N.S.	1	1.00	0.98	0.85	0.83	0.83	1.00	0.85	1.12
time (sec)	N/A	0.016	0.008	0.028	2.896	0.651	0.052	0.497	0.206

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	31	40	31	36	33
N.S.	1	1.00	0.79	0.74	0.72	0.93	0.72	0.84	0.77
time (sec)	N/A	0.089	0.017	0.038	2.663	0.518	0.053	0.490	0.043

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	52	72	100	73	72	33
N.S.	1	1.00	0.75	0.61	0.85	1.18	0.86	0.85	0.39
time (sec)	N/A	0.033	0.015	0.000	2.157	0.624	0.055	0.462	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	52	72	100	73	72	33
N.S.	1	1.00	0.75	0.61	0.85	1.18	0.86	0.85	0.39
time (sec)	N/A	0.031	0.008	0.023	2.522	0.497	0.054	0.449	0.205

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	53	53	70	53	47
N.S.	1	1.00	1.09	0.81	0.79	0.79	1.04	0.79	0.70
time (sec)	N/A	0.029	0.043	0.018	2.881	0.601	0.086	0.482	0.179

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	18	20	20	18	18
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	1.00	1.00
time (sec)	N/A	0.003	0.017	0.023	1.495	0.514	0.007	0.451	0.220

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	42	53	201	76	94
N.S.	1	1.00	0.85	0.92	1.08	1.36	5.15	1.95	2.41
time (sec)	N/A	0.010	0.028	0.020	1.164	0.666	0.227	0.438	0.392

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	68	96	597	140	192
N.S.	1	1.00	0.95	1.22	1.13	1.60	9.95	2.33	3.20
time (sec)	N/A	0.020	0.038	0.026	2.155	0.579	0.374	0.496	0.601

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.001	0.001	0.021	1.532	0.574	0.008	0.469	0.026

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00
time (sec)	N/A	0.001	0.002	0.022	0.962	0.820	0.046	0.466	0.118

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.007	0.002	0.023	2.662	0.612	0.031	0.435	0.035

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.011	0.003	0.023	1.995	0.745	0.035	0.459	0.127

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	16	10	20	15
N.S.	1	1.00	1.00	1.06	1.00	0.89	0.56	1.11	0.83
time (sec)	N/A	0.002	0.003	0.023	1.775	0.747	0.053	0.464	0.134

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.010	0.003	0.025	2.360	1.054	0.071	0.439	0.054

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	52	45
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.07
time (sec)	N/A	0.017	0.027	0.026	2.644	0.885	0.114	0.461	0.188

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.032	1.842	0.696	0.038	0.452	0.037

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	22	21	18	15	23	10
N.S.	1	1.00	1.00	2.20	2.10	1.80	1.50	2.30	1.00
time (sec)	N/A	0.002	0.002	0.024	6.074	0.690	0.043	0.459	0.159

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	58	66	63	78	57	72
N.S.	1	1.00	0.85	0.74	0.85	0.81	1.00	0.73	0.92
time (sec)	N/A	0.033	0.015	0.026	4.649	0.729	0.138	0.456	0.277

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	65	54	56	68	71	57	72
N.S.	1	1.00	0.88	0.73	0.76	0.92	0.96	0.77	0.97
time (sec)	N/A	0.026	0.013	0.029	3.986	0.689	0.136	0.442	0.356

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	92	97	300	20	104	101
N.S.	1	1.00	0.77	0.80	0.84	2.61	0.17	0.90	0.88
time (sec)	N/A	0.047	0.018	0.024	1.736	0.871	0.061	0.459	0.315

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	32	34	56	46	39	20
N.S.	1	1.00	1.23	0.91	0.97	1.60	1.31	1.11	0.57
time (sec)	N/A	0.008	0.012	0.026	5.054	0.709	0.145	0.445	0.158

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	33	41	60	48	39	18
N.S.	1	1.00	1.23	0.94	1.17	1.71	1.37	1.11	0.51
time (sec)	N/A	0.013	0.011	0.030	4.546	0.947	0.148	0.447	0.175

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	120	93	151	161	151	95	45
N.S.	1	1.00	0.70	0.54	0.88	0.94	0.88	0.56	0.26
time (sec)	N/A	0.079	0.034	0.024	2.671	0.746	0.195	0.486	0.111

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	132	146	74	93
N.S.	1	1.00	0.93	0.77	0.00	1.81	2.00	1.01	1.27
time (sec)	N/A	0.056	0.037	0.033	0.000	0.685	0.228	0.462	0.224

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	132	24	74	93
N.S.	1	1.00	0.93	0.77	0.00	1.81	0.33	1.01	1.27
time (sec)	N/A	0.014	0.019	0.029	0.000	0.587	0.180	0.447	0.135

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	54	75	91	158	81	67
N.S.	1	1.00	1.15	0.75	1.04	1.26	2.19	1.12	0.93
time (sec)	N/A	0.051	0.023	0.023	5.958	0.688	0.191	0.471	0.098

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	123	24	101	98
N.S.	1	1.00	1.00	0.90	0.00	1.84	0.36	1.51	1.46
time (sec)	N/A	0.043	0.022	0.034	0.000	0.565	0.184	0.477	0.231

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	87	92	51	117
N.S.	1	1.00	1.00	0.90	0.00	1.30	1.37	0.76	1.75
time (sec)	N/A	0.009	0.015	0.029	0.000	0.611	0.096	0.492	0.197

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	251	0	298	994	252	61
N.S.	1	1.00	0.46	1.28	0.00	1.52	5.07	1.29	0.31
time (sec)	N/A	0.111	0.046	0.061	0.000	0.521	0.647	0.866	0.225

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	91	253	0	268	24	256	132
N.S.	1	1.00	0.46	1.29	0.00	1.37	0.12	1.31	0.67
time (sec)	N/A	0.101	0.055	0.061	0.000	0.826	0.309	0.908	0.110

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	66	65	65	83	67	88
N.S.	1	1.00	1.03	0.90	0.89	0.89	1.14	0.92	1.21
time (sec)	N/A	0.074	0.011	0.040	4.621	0.791	0.122	0.979	0.091

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	122	111	112	176	14	114	140
N.S.	1	1.00	0.88	0.80	0.81	1.28	0.10	0.83	1.01
time (sec)	N/A	0.141	0.028	0.083	2.744	0.911	0.289	0.867	0.210

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	115	95	107	279	14	107	135
N.S.	1	1.00	0.83	0.69	0.78	2.02	0.10	0.78	0.98
time (sec)	N/A	0.201	0.019	0.098	3.239	1.224	0.128	0.967	0.129

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	1043	14	239	288
N.S.	1	1.00	0.62	0.06	0.00	3.08	0.04	0.71	0.85
time (sec)	N/A	0.165	0.005	0.027	0.000	1.264	1.173	0.733	0.304

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	61	88	116	44	90	45
N.S.	1	1.00	1.01	0.63	0.91	1.20	0.45	0.93	0.46
time (sec)	N/A	0.036	0.015	0.038	2.885	1.057	82.254	0.474	0.165

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	220	165	205	53
N.S.	1	1.00	0.15	0.11	0.00	0.80	0.60	0.75	0.19
time (sec)	N/A	0.191	0.009	0.016	0.000	1.036	0.093	0.949	0.100

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	40	40	46	40	52
N.S.	1	1.00	5.31	0.84	0.82	0.82	0.94	0.82	1.06
time (sec)	N/A	0.027	0.075	0.027	1.797	1.038	0.066	0.843	0.219

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	5	8	6
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.62	1.00	0.75
time (sec)	N/A	0.001	0.001	0.003	1.265	0.878	0.021	0.905	0.023

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.003	0.001	0.000	1.431	0.791	0.024	0.913	0.034

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.004	0.002	0.003	2.030	0.705	0.026	1.069	0.032

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	19	26	25	56	0	32
N.S.	1	1.00	0.73	0.73	1.00	0.96	2.15	0.00	1.23
time (sec)	N/A	0.007	0.006	0.014	2.518	0.782	0.203	0.000	0.239

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	15	12
N.S.	1	1.00	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.003	0.002	0.000	3.549	0.847	0.031	0.825	0.030

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	104	71	103	133	103	71
N.S.	1	1.00	1.00	0.82	0.56	0.81	1.05	0.81	0.56
time (sec)	N/A	0.097	0.003	0.007	1.802	1.084	0.148	0.619	0.177

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.008	0.002	0.007	2.280	0.971	0.024	0.587	0.060

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.001	0.002	0.004	1.182	0.825	0.243	0.771	0.004

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	11	5	4	3	5	4
N.S.	1	1.00	1.00	2.75	1.25	1.00	0.75	1.25	1.00
time (sec)	N/A	0.003	0.005	0.010	1.415	1.131	0.233	0.859	0.016

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.008	0.005	0.000	1.770	1.143	0.028	0.662	0.120

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	6	19	14	0	17
N.S.	1	1.00	1.00	0.88	0.35	1.12	0.82	0.00	1.00
time (sec)	N/A	0.022	0.020	0.007	2.495	1.200	0.327	0.000	0.030

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	15	12	22
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.25	1.00	1.83
time (sec)	N/A	0.011	0.004	0.013	1.331	0.909	0.443	0.899	0.146

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	25	25	22	24	21
N.S.	1	1.00	1.00	0.96	0.89	0.89	0.79	0.86	0.75
time (sec)	N/A	0.007	0.004	0.006	1.475	0.828	0.041	1.061	0.141

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	48	47	47	44	47	47
N.S.	1	1.00	0.98	0.89	0.87	0.87	0.81	0.87	0.87
time (sec)	N/A	0.019	0.009	0.020	2.424	0.955	0.052	1.830	0.170

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	38	34	24	138	35
N.S.	1	1.00	0.93	1.03	1.31	1.17	0.83	4.76	1.21
time (sec)	N/A	0.009	0.011	0.025	1.781	1.031	0.102	1.643	0.232

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	53	44	39	42	58	66
N.S.	1	1.00	1.00	1.15	0.96	0.85	0.91	1.26	1.43
time (sec)	N/A	0.016	0.012	0.015	2.273	0.763	0.074	1.063	0.189

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	82	57	49	54	94	75
N.S.	1	1.00	1.00	1.39	0.97	0.83	0.92	1.59	1.27
time (sec)	N/A	0.022	0.015	0.013	5.978	0.742	0.084	1.161	0.182

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	36	23	23
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.57	1.00	1.00
time (sec)	N/A	0.007	0.002	0.017	5.411	0.661	0.064	1.199	0.066

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	29	28	23	31	28	51
N.S.	1	1.00	0.96	1.07	1.04	0.85	1.15	1.04	1.89
time (sec)	N/A	0.022	0.004	0.013	4.055	0.603	0.062	1.174	0.041

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	36	36	53	36	65
N.S.	1	1.00	1.00	0.84	0.82	0.82	1.20	0.82	1.48
time (sec)	N/A	0.018	0.003	0.018	3.374	0.564	0.080	1.103	0.036

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	63	44	73
N.S.	1	1.00	1.00	0.83	0.81	0.81	1.17	0.81	1.35
time (sec)	N/A	0.018	0.003	0.021	7.413	0.636	0.085	0.651	0.128

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	31	31	29	33	25
N.S.	1	1.00	1.00	1.28	1.24	1.24	1.16	1.32	1.00
time (sec)	N/A	0.007	0.003	0.014	3.950	0.510	0.066	0.643	0.074

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.12
time (sec)	N/A	0.004	0.028	0.025	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.002	0.002	0.013	1.906	0.619	0.025	0.917	0.021

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.002	0.013	2.540	0.515	0.024	1.204	0.027

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	3	11	5	6	5
N.S.	1	1.00	1.00	1.20	0.60	2.20	1.00	1.20	1.00
time (sec)	N/A	0.002	0.002	0.011	2.962	0.554	0.027	0.886	0.029

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	16	3	17	13
N.S.	1	1.00	1.00	1.33	1.00	5.33	1.00	5.67	4.33
time (sec)	N/A	0.002	0.002	0.016	3.047	0.805	0.030	0.683	0.197

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	26	25	37	75	26	27
N.S.	1	1.00	1.29	1.24	1.19	1.76	3.57	1.24	1.29
time (sec)	N/A	0.019	0.033	0.029	2.971	1.006	0.159	0.638	0.246

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	33	7	15	17	15	17	11
N.S.	1	1.00	11.00	2.33	5.00	5.67	5.00	5.67	3.67
time (sec)	N/A	0.002	0.003	0.017	1.923	1.198	0.051	0.569	0.129

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	15	19	15	17	5
N.S.	1	1.00	3.40	1.80	3.00	3.80	3.00	3.40	1.00
time (sec)	N/A	0.002	0.003	0.020	2.384	1.097	0.053	0.584	0.045

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.004	0.003	0.000	3.881	1.054	0.009	0.567	0.032

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.013	0.003	0.022	2.407	1.051	0.133	0.623	0.158

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.005	0.002	0.000	2.353	0.973	0.011	0.636	0.034

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0	35
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.008	0.031	0.017	0.000	0.000	0.000	0.000	0.315

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	18	17	15	15
N.S.	1	1.00	1.00	0.84	0.79	0.95	0.89	0.79	0.79
time (sec)	N/A	0.016	0.005	0.027	1.474	0.837	0.211	0.682	0.045

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.006	0.002	0.000	4.403	0.733	0.009	0.689	0.026

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	9	10	8	9	9
N.S.	1	1.00	1.36	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.005	0.003	0.013	2.539	0.776	0.011	0.896	0.031

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	7	5	2	2
N.S.	1	1.00	1.00	1.50	1.00	3.50	2.50	1.00	1.00
time (sec)	N/A	0.005	0.002	0.039	1.862	0.662	0.026	0.727	0.023

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	10	20	6	6
N.S.	1	1.00	1.00	0.47	0.73	0.67	1.33	0.40	0.40
time (sec)	N/A	0.005	0.007	0.028	1.605	0.926	0.128	0.735	0.028

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.006	0.003	0.011	2.259	0.758	0.053	0.793	0.021

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	15	15	17	15	15
N.S.	1	1.00	0.88	1.06	0.88	0.88	1.00	0.88	0.88
time (sec)	N/A	0.018	0.012	0.000	2.435	0.760	0.085	0.635	0.028

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	19	19
N.S.	1	1.00	1.00	1.00	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.010	0.010	0.000	1.087	0.625	0.089	0.738	0.055

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	26	28
N.S.	1	1.00	0.71	0.90	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.020	0.029	0.017	1.187	0.554	0.141	0.689	0.155

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	23	39	23	25
N.S.	1	1.00	0.94	0.70	0.70	0.70	1.18	0.70	0.76
time (sec)	N/A	0.017	0.008	0.023	1.785	0.509	0.139	0.859	0.066

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	7	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.006	0.003	0.019	2.055	0.477	0.055	0.888	0.019

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	14	14	17	14	14
N.S.	1	1.00	0.88	1.06	0.88	0.88	1.06	0.88	0.88
time (sec)	N/A	0.017	0.011	0.020	2.584	0.549	0.087	0.982	0.026

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	19	19
N.S.	1	1.00	1.00	1.00	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.010	0.010	0.022	1.497	0.539	0.091	0.735	0.151

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	26	28
N.S.	1	1.00	0.71	0.90	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.025	0.027	0.034	2.847	0.521	0.137	0.627	0.065

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	25	39	23	25
N.S.	1	1.00	0.94	0.70	0.70	0.76	1.18	0.70	0.76
time (sec)	N/A	0.016	0.008	0.032	1.379	0.468	0.135	0.627	0.169

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	2	2	2	-1
N.S.	1	1.00	1.00	1.50	6.50	1.00	1.00	1.00	-0.50
time (sec)	N/A	0.008	0.010	0.012	1.282	0.569	0.331	0.708	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	13	11	12	2	-1
N.S.	1	1.00	1.00	1.50	6.50	5.50	6.00	1.00	-0.50
time (sec)	N/A	0.010	0.003	0.033	1.534	0.537	0.492	0.886	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	15	20	17	13	-1
N.S.	1	1.00	1.00	1.10	1.50	2.00	1.70	1.30	-0.10
time (sec)	N/A	0.018	0.003	0.036	1.980	0.597	0.737	0.800	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	17	17	10	11	-1
N.S.	1	1.00	1.00	0.80	1.13	1.13	0.67	0.73	-0.07
time (sec)	N/A	0.031	0.011	0.031	1.071	0.510	0.582	0.726	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	20	18	12	16	16
N.S.	1	1.00	1.00	1.42	1.67	1.50	1.00	1.33	1.33
time (sec)	N/A	0.005	0.003	0.015	1.004	0.575	0.029	0.789	0.021

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	11	11	14	11	11
N.S.	1	1.00	2.00	1.09	1.00	1.00	1.27	1.00	1.00
time (sec)	N/A	0.003	0.008	0.019	0.929	0.807	0.047	1.152	0.021

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	10	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	1.00	1.00
time (sec)	N/A	0.003	0.007	0.021	1.682	0.781	0.045	1.149	0.022

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	13	16
N.S.	1	1.00	1.00	1.42	0.92	1.50	1.58	1.08	1.33
time (sec)	N/A	0.003	0.008	0.012	1.571	0.872	0.047	1.366	0.234

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	19	26	11	27	29	56	28
N.S.	1	1.00	1.73	2.36	1.00	2.45	2.64	5.09	2.55
time (sec)	N/A	0.003	0.008	0.014	1.541	0.754	0.156	1.188	0.190

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	21	26	30	17	51	12
N.S.	1	1.00	3.17	1.75	2.17	2.50	1.42	4.25	1.00
time (sec)	N/A	0.004	0.013	0.033	1.824	0.804	0.270	0.788	0.117

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	26	28	34	28	11
N.S.	1	1.00	1.00	1.73	2.36	2.55	3.09	2.55	1.00
time (sec)	N/A	0.004	0.002	0.027	6.203	1.069	0.307	0.862	0.024

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	24	23	46	18	18
N.S.	1	1.00	0.92	1.08	0.96	0.92	1.84	0.72	0.72
time (sec)	N/A	0.006	0.021	0.027	3.320	1.081	0.078	0.681	0.209

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	22	22	37	25	24
N.S.	1	1.00	1.07	0.81	0.81	0.81	1.37	0.93	0.89
time (sec)	N/A	0.008	0.010	0.038	2.362	0.788	0.108	0.716	0.147

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	22	22	46	18	18
N.S.	1	1.00	0.92	1.08	0.88	0.88	1.84	0.72	0.72
time (sec)	N/A	0.007	0.014	0.031	2.160	0.604	0.077	0.713	0.177

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	22	21	36	22	24
N.S.	1	1.00	1.00	0.85	0.85	0.81	1.38	0.85	0.92
time (sec)	N/A	0.008	0.006	0.036	3.531	1.192	0.109	1.037	0.025

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	18	58	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.80	5.80	1.00	1.00
time (sec)	N/A	0.008	0.003	0.027	2.730	1.084	0.532	0.914	0.145

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	3	30	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.33	3.33	0.44
time (sec)	N/A	0.007	0.004	0.016	2.614	0.833	0.080	0.858	0.171

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	7	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.58	0.67	0.50
time (sec)	N/A	0.009	0.009	0.018	2.323	0.973	0.161	0.873	0.166

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	8	10	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.80	1.00	1.00
time (sec)	N/A	0.005	0.008	0.020	1.538	1.071	0.170	1.079	0.024

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	8	10	10
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.73	0.91	0.91
time (sec)	N/A	0.007	0.009	0.032	2.269	1.037	0.179	0.843	0.034

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	148	141	48	45
N.S.	1	1.00	1.00	0.98	0.00	3.70	3.52	1.20	1.12
time (sec)	N/A	0.031	0.028	0.048	0.000	0.906	4.028	0.967	0.383

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	43	0	287	1872	60	58
N.S.	1	1.00	0.94	0.91	0.00	6.11	39.83	1.28	1.23
time (sec)	N/A	0.046	0.044	0.094	0.000	0.990	210.904	1.098	0.260

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	117	54	105	45	52
N.S.	1	1.00	0.64	2.16	1.60	0.74	1.44	0.62	0.71
time (sec)	N/A	0.032	0.091	0.038	1.646	0.971	0.185	1.243	0.137

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	9
N.S.	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	0.60
time (sec)	N/A	0.006	0.007	0.042	7.030	0.624	0.121	1.161	0.025

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	113	54	105	45	52
N.S.	1	1.00	0.64	2.16	1.55	0.74	1.44	0.62	0.71
time (sec)	N/A	0.032	0.085	0.042	2.976	0.733	0.184	1.109	0.103

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	14	31	14	29	21
N.S.	1	1.00	1.00	1.57	1.00	2.21	1.00	2.07	1.50
time (sec)	N/A	0.006	0.003	0.017	2.571	1.023	0.028	1.026	0.168

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	138	491	182	0	0	-1
N.S.	1	1.00	0.97	1.33	4.72	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.124	0.054	2.539	0.915	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	133	237	777	212	0	0	-1
N.S.	1	1.00	0.87	1.55	5.08	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.247	0.045	4.453	1.156	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	107	21	19	23	13
N.S.	1	1.00	1.00	1.33	7.13	1.40	1.27	1.53	0.87
time (sec)	N/A	0.011	0.012	0.013	2.782	0.793	0.057	0.598	0.022

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	13	26	11	13
N.S.	1	1.00	1.00	0.80	0.73	0.87	1.73	0.73	0.87
time (sec)	N/A	0.006	0.008	0.057	2.087	0.473	0.128	0.585	0.044

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	10	19	14	10	18
N.S.	1	1.00	0.58	0.79	0.42	0.79	0.58	0.42	0.75
time (sec)	N/A	0.020	0.006	0.046	1.186	0.521	0.012	0.636	0.044

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	9	18	12	9	6
N.S.	1	1.00	0.86	2.14	1.29	2.57	1.71	1.29	0.86
time (sec)	N/A	0.016	0.008	0.028	2.455	0.531	0.015	0.763	0.064

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	33	25	22	104	328	22
N.S.	1	1.00	0.69	1.03	0.78	0.69	3.25	10.25	0.69
time (sec)	N/A	0.008	0.017	0.033	2.204	0.533	0.267	0.985	0.024

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	32	24	20	104	329	20
N.S.	1	1.00	0.65	1.03	0.77	0.65	3.35	10.61	0.65
time (sec)	N/A	0.006	0.015	0.031	1.352	0.542	0.269	1.472	0.017

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	50	58	60	60	308	1156	57
N.S.	1	1.00	0.60	0.69	0.71	0.71	3.67	13.76	0.68
time (sec)	N/A	0.036	0.042	0.042	2.475	0.558	0.487	1.062	0.285

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	56	58	58	308	1155	55
N.S.	1	1.00	0.59	0.67	0.70	0.70	3.71	13.92	0.66
time (sec)	N/A	0.034	0.038	0.039	1.467	0.588	0.484	1.386	0.213

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	94	102	107	115	665	2631	133
N.S.	1	1.00	0.58	0.63	0.66	0.71	4.10	16.24	0.82
time (sec)	N/A	0.128	0.065	0.057	0.948	0.481	1.062	0.924	0.392

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	93	100	105	111	665	2631	132
N.S.	1	1.00	0.58	0.62	0.65	0.69	4.13	16.34	0.82
time (sec)	N/A	0.126	0.057	0.057	2.081	0.571	1.014	1.095	0.348

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	169	166	186	203	1355	5069	231
N.S.	1	1.00	0.65	0.64	0.71	0.78	5.19	19.42	0.89
time (sec)	N/A	0.312	0.111	0.074	1.212	0.546	2.103	1.248	0.627

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	168	164	184	202	1355	5065	232
N.S.	1	1.00	0.65	0.63	0.71	0.78	5.21	19.48	0.89
time (sec)	N/A	0.301	0.090	0.073	1.590	0.518	2.143	0.900	0.640

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	114	17	14
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.56	0.68	0.56
time (sec)	N/A	0.022	0.014	0.063	2.235	0.662	1.238	0.831	0.166

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	116	22	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.87	0.73	0.73
time (sec)	N/A	0.023	0.014	0.061	1.299	0.545	1.229	0.665	0.276

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	55	64	55	59	100	60	67
N.S.	1	1.00	0.65	0.75	0.65	0.69	1.18	0.71	0.79
time (sec)	N/A	0.054	0.054	0.067	1.159	0.713	0.257	1.117	0.230

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	240	0	0	0	-1
N.S.	1	0.00	0.00	0.00	17.14	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.319	0.628	0.262	1.903	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	41	16	0	18	10
N.S.	1	1.00	1.00	0.92	3.42	1.33	0.00	1.50	0.83
time (sec)	N/A	0.023	0.012	0.069	3.419	0.498	0.000	1.318	0.175

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	107	0	312	0	98	133
N.S.	1	1.00	0.99	1.39	0.00	4.05	0.00	1.27	1.73
time (sec)	N/A	0.070	0.192	0.153	0.000	0.519	0.000	0.748	0.485

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	12	13	15	13	13
N.S.	1	1.00	1.00	0.82	0.71	0.76	0.88	0.76	0.76
time (sec)	N/A	0.002	0.003	0.023	1.274	0.410	0.107	0.778	0.133

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	10	13	15	13	13
N.S.	1	1.00	1.00	0.82	0.59	0.76	0.88	0.76	0.76
time (sec)	N/A	0.002	0.003	0.018	0.864	0.447	0.106	0.854	0.148

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67
time (sec)	N/A	0.001	0.000	0.007	1.104	0.399	0.017	0.709	0.007

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.001	0.012	1.553	0.670	0.023	0.650	0.155

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	0.89
time (sec)	N/A	0.001	0.005	0.008	0.645	1.139	0.023	0.695	0.027

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4
N.S.	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.008	0.011	0.013	3.623	1.118	0.336	0.646	0.014

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	38	29	23	22	15	26	22
N.S.	1	1.00	1.58	1.21	0.96	0.92	0.62	1.08	0.92
time (sec)	N/A	0.011	0.021	0.014	2.639	0.616	0.042	0.918	0.093

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.014	0.014	0.012	3.152	0.860	0.025	0.792	0.045

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	14	12	14	14	14
N.S.	1	1.00	1.00	1.15	1.08	0.92	1.08	1.08	1.08
time (sec)	N/A	0.006	0.007	0.010	3.255	0.847	0.024	0.770	0.052

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	23	85	24	21	21
N.S.	1	1.00	1.00	0.71	0.74	2.74	0.77	0.68	0.68
time (sec)	N/A	0.020	0.027	0.020	6.529	1.011	0.071	0.827	0.228

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	14	19	13	13	19	13	13
N.S.	1	1.00	0.67	0.90	0.62	0.62	0.90	0.62	0.62
time (sec)	N/A	0.006	0.010	0.010	5.632	1.037	0.029	1.098	0.027

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	102	143	101	101	102	101	101
N.S.	1	1.00	0.63	0.88	0.62	0.62	0.63	0.62	0.62
time (sec)	N/A	0.170	0.070	0.035	3.590	0.708	0.040	0.856	0.362

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	18	0	216	18
N.S.	1	1.00	1.00	1.06	0.00	1.00	0.00	12.00	1.00
time (sec)	N/A	0.015	0.016	0.024	0.000	0.783	0.000	0.880	0.204

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	14	31	237	14
N.S.	1	1.00	1.00	1.07	0.00	1.00	2.21	16.93	1.00
time (sec)	N/A	0.011	0.013	0.022	0.000	1.167	0.247	0.866	0.167

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	10	19	0	0	19
N.S.	1	1.00	1.00	1.24	0.59	1.12	0.00	0.00	1.12
time (sec)	N/A	0.013	0.014	0.029	3.057	0.886	0.000	0.000	0.131

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	43	79	0	54	0	0	-1
N.S.	1	1.00	0.67	1.23	0.00	0.84	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.141	0.040	0.000	0.778	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	16	16	12	45	15
N.S.	1	1.00	1.00	1.00	1.00	1.00	0.75	2.81	0.94
time (sec)	N/A	0.021	0.148	0.027	1.776	0.712	0.042	0.646	0.196

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	15	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	1.15	0.85	0.85
time (sec)	N/A	0.005	0.011	0.015	1.838	0.590	0.029	0.747	0.026

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	7	8	9	7
N.S.	1	1.00	1.00	0.73	0.82	0.64	0.73	0.82	0.64
time (sec)	N/A	0.002	0.012	0.016	1.700	0.510	0.067	0.935	0.017

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.005	0.009	0.007	4.147	0.502	0.021	0.790	0.018

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	16	25	17	17	14	24	17
N.S.	1	1.00	0.59	0.93	0.63	0.63	0.52	0.89	0.63
time (sec)	N/A	0.076	0.011	0.022	1.786	0.455	0.028	0.811	0.158

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	25	23	22	36	31	36	-1
N.S.	1	0.00	1.00	0.92	0.88	1.44	1.24	1.44	-0.04
time (sec)	N/A	0.714	0.081	0.046	2.663	0.523	0.098	0.786	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.032	0.015	0.010	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	10	10	8	10	10
N.S.	1	1.00	1.00	1.09	0.91	0.91	0.73	0.91	0.91
time (sec)	N/A	0.011	0.006	0.015	1.800	0.503	0.981	0.720	0.026

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	17	21	15	15	17	15	28
N.S.	1	1.00	0.77	0.95	0.68	0.68	0.77	0.68	1.27
time (sec)	N/A	0.036	0.010	0.014	5.680	0.462	1.669	0.753	0.219

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	17	28	11	11	10	11	11
N.S.	1	1.00	0.74	1.22	0.48	0.48	0.43	0.48	0.48
time (sec)	N/A	0.009	0.005	0.010	1.939	0.467	0.030	0.610	0.042

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	12	12	12	12	12
N.S.	1	1.00	1.00	1.19	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.002	0.000	0.017	2.937	0.452	0.006	0.528	0.030

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	43	58	89	1166	48	49
N.S.	1	1.00	0.75	0.75	1.02	1.56	20.46	0.84	0.86
time (sec)	N/A	0.022	0.017	0.036	1.798	0.432	1.945	0.524	0.094

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	95	140	0	300	0	232	88
N.S.	1	1.00	0.82	1.21	0.00	2.59	0.00	2.00	0.76
time (sec)	N/A	0.051	0.242	0.039	0.000	0.452	0.000	0.503	0.169

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.002	0.008	0.026	1.220	0.512	0.006	0.465	0.025

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	26	30	202	66	25
N.S.	1	1.00	1.00	0.76	0.76	0.88	5.94	1.94	0.74
time (sec)	N/A	0.006	0.016	0.035	2.014	0.495	0.541	0.453	0.030

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	93	37
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	1.75	0.70
time (sec)	N/A	0.010	0.021	0.041	2.948	0.463	0.825	0.458	0.148

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	42	73	68	32	27
N.S.	1	1.00	1.00	0.80	1.20	2.09	1.94	0.91	0.77
time (sec)	N/A	0.008	0.022	0.031	2.167	0.506	0.668	0.468	0.146

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	47	93	44	41	31
N.S.	1	1.00	1.00	0.95	1.21	2.38	1.13	1.05	0.79
time (sec)	N/A	0.008	0.055	0.032	2.240	0.494	0.883	0.471	0.054

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.001	0.007	0.026	2.806	0.582	0.006	0.475	0.018

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	26	26	19	162	23	25
N.S.	1	1.00	0.72	0.81	0.81	0.59	5.06	0.72	0.78
time (sec)	N/A	0.006	0.013	0.026	2.660	0.654	0.526	0.527	0.029

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	41	31	600	37	37
N.S.	1	1.00	0.69	0.73	0.80	0.61	11.76	0.73	0.73
time (sec)	N/A	0.009	0.020	0.027	2.039	0.610	0.814	0.464	0.038

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.005	0.017	0.026	2.084	0.627	0.453	0.473	0.144

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	60	93	44	47	33
N.S.	1	1.00	1.00	0.98	1.46	2.27	1.07	1.15	0.80
time (sec)	N/A	0.009	0.050	0.031	3.316	0.616	1.132	0.478	0.159

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	25	21	25	24	21	21
N.S.	1	1.00	1.04	1.09	0.91	1.09	1.04	0.91	0.91
time (sec)	N/A	0.003	0.021	0.029	1.616	0.813	0.008	0.449	0.251

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	45	58	216	86	94
N.S.	1	1.00	0.79	0.90	0.94	1.21	4.50	1.79	1.96
time (sec)	N/A	0.011	0.028	0.031	1.903	0.728	0.230	0.519	0.448

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	37	52	37	230	52	43
N.S.	1	1.00	0.87	0.67	0.95	0.67	4.18	0.95	0.78
time (sec)	N/A	0.021	0.025	0.050	3.147	0.783	0.391	0.504	0.162

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	38	11	14	14	2	26	10
N.S.	1	1.00	3.17	0.92	1.17	1.17	0.17	2.17	0.83
time (sec)	N/A	0.002	0.003	0.053	3.207	0.745	0.044	0.503	0.197

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	50	71	34	119	39	30
N.S.	1	1.00	0.86	1.16	1.65	0.79	2.77	0.91	0.70
time (sec)	N/A	0.005	0.038	0.034	2.336	0.742	1.570	0.497	0.217

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.008	0.012	0.013	2.010	0.996	0.088	0.459	0.245

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	18	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.38	0.62	0.85	0.85
time (sec)	N/A	0.002	0.002	0.049	1.476	0.762	0.284	0.493	0.348

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	20	7	16	18	19	6	16
N.S.	1	1.00	2.50	0.88	2.00	2.25	2.38	0.75	2.00
time (sec)	N/A	0.002	0.064	0.132	3.826	0.625	0.408	0.488	0.312

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	8	25	10
N.S.	1	1.00	1.00	0.79	1.79	1.79	0.57	1.79	0.71
time (sec)	N/A	0.005	0.013	0.095	2.174	0.770	0.402	0.460	0.186

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	26	14	0	22	0	22	18
N.S.	1	1.00	1.44	0.78	0.00	1.22	0.00	1.22	1.00
time (sec)	N/A	0.013	0.049	0.075	0.000	0.771	0.000	0.481	0.359

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	27	23	17	55	0	0	32
N.S.	1	1.00	0.90	0.77	0.57	1.83	0.00	0.00	1.07
time (sec)	N/A	0.015	0.049	0.139	4.221	0.607	0.000	0.000	0.538

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	14	17	0	35	14
N.S.	1	1.00	1.00	1.12	0.82	1.00	0.00	2.06	0.82
time (sec)	N/A	0.007	0.125	0.038	2.678	0.712	0.000	0.735	0.151

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	12	3	2	14	2	25	2
N.S.	1	1.00	6.00	1.50	1.00	7.00	1.00	12.50	1.00
time (sec)	N/A	0.001	0.012	0.041	3.563	0.597	0.043	0.780	0.028

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	30	15	14	14	17	0	14
N.S.	1	1.00	1.50	0.75	0.70	0.70	0.85	0.00	0.70
time (sec)	N/A	0.634	0.017	0.051	2.024	0.615	0.230	0.000	0.345

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	30	20	19	0	20
N.S.	1	1.00	1.00	0.88	1.25	0.83	0.79	0.00	0.83
time (sec)	N/A	0.223	23.841	0.070	2.519	0.649	0.285	0.000	0.310

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	52	24	31	29	8	51	23
N.S.	1	1.00	1.93	0.89	1.15	1.07	0.30	1.89	0.85
time (sec)	N/A	0.006	0.018	0.057	1.932	0.577	0.062	0.688	0.687

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	25	0	216	51	0	-1
N.S.	1	1.00	1.00	0.30	0.00	2.63	0.62	0.00	-0.01
time (sec)	N/A	0.066	0.073	0.026	0.000	0.954	1.712	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	39	40	29	48	10	169
N.S.	1	1.00	1.00	3.25	3.33	2.42	4.00	0.83	14.08
time (sec)	N/A	0.036	0.014	0.113	1.725	0.675	47.964	0.586	0.061

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	36	85	66	55	32
N.S.	1	1.00	1.00	0.82	0.90	2.12	1.65	1.38	0.80
time (sec)	N/A	0.008	0.007	0.036	1.312	0.918	0.492	0.571	0.479

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	66	59	41	42	40	40
N.S.	1	1.00	1.00	1.43	1.28	0.89	0.91	0.87	0.87
time (sec)	N/A	0.024	0.028	0.041	2.053	0.720	0.539	0.587	0.665

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	52	46	52	0	40	51
N.S.	1	1.00	1.22	1.41	1.24	1.41	0.00	1.08	1.38
time (sec)	N/A	0.012	0.083	0.116	2.135	0.747	0.000	0.559	0.105

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	66	74	84	97	0	57	72
N.S.	1	1.00	1.08	1.21	1.38	1.59	0.00	0.93	1.18
time (sec)	N/A	0.017	0.106	0.129	2.240	0.594	0.000	0.529	0.246

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	29	19	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	1.26	0.83	0.83
time (sec)	N/A	0.003	0.003	0.040	2.355	0.733	0.210	0.572	0.289

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	36	24	24
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.29	0.86	0.86
time (sec)	N/A	0.003	0.004	0.043	1.798	0.662	0.284	0.548	0.272

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	150	47	0	155	0	60	50
N.S.	1	1.00	2.68	0.84	0.00	2.77	0.00	1.07	0.89
time (sec)	N/A	0.030	0.177	0.026	0.000	0.692	0.000	0.454	0.979

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	86	70	68	94	0	72	67
N.S.	1	1.00	1.06	0.86	0.84	1.16	0.00	0.89	0.83
time (sec)	N/A	0.022	0.197	0.117	1.110	0.557	0.000	0.498	0.237

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	56	0	58	0	45	54
N.S.	1	1.00	1.20	1.27	0.00	1.32	0.00	1.02	1.23
time (sec)	N/A	0.028	0.081	0.028	0.000	0.584	0.000	0.467	0.400

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	72	78	0	106	0	62	72
N.S.	1	1.00	1.06	1.15	0.00	1.56	0.00	0.91	1.06
time (sec)	N/A	0.035	0.110	0.028	0.000	0.574	0.000	0.454	0.442

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	22	10	11	11
N.S.	1	1.00	1.00	0.92	0.85	1.69	0.77	0.85	0.85
time (sec)	N/A	0.013	0.005	0.039	3.117	0.618	0.082	0.448	0.089

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	11	17	15	11
N.S.	1	1.00	1.00	0.84	0.79	0.58	0.89	0.79	0.58
time (sec)	N/A	0.005	0.002	0.008	1.560	0.554	0.031	0.418	0.165

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	21	20	12	10	12	12
N.S.	1	1.00	1.00	1.75	1.67	1.00	0.83	1.00	1.00
time (sec)	N/A	0.008	0.021	0.025	2.204	0.888	0.054	0.486	0.083

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	10	10	7	10	10
N.S.	1	1.00	1.00	1.00	0.91	0.91	0.64	0.91	0.91
time (sec)	N/A	0.017	0.007	0.013	3.580	0.803	0.050	0.448	0.216

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
N.S.	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.012	0.004	0.026	5.543	0.850	0.048	0.438	0.154

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	32	20	50	167	66	15
N.S.	1	1.00	0.64	0.97	0.61	1.52	5.06	2.00	0.45
time (sec)	N/A	0.002	0.051	0.033	1.704	0.786	2.880	0.497	0.299

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	11	8
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.22	0.89
time (sec)	N/A	0.001	0.002	0.031	1.721	0.813	0.025	0.454	0.197

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	28	28	39	32	13
N.S.	1	1.00	1.00	0.74	1.47	1.47	2.05	1.68	0.68
time (sec)	N/A	0.004	0.023	0.072	1.597	0.617	0.099	0.480	0.075

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	12	11	14	15	11	12
N.S.	1	1.00	0.84	0.63	0.58	0.74	0.79	0.58	0.63
time (sec)	N/A	0.002	0.009	0.012	3.376	0.707	0.506	0.453	0.024

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	112	89	86	104	153	103	119
N.S.	1	1.00	1.27	1.01	0.98	1.18	1.74	1.17	1.35
time (sec)	N/A	0.028	0.063	0.039	3.132	0.702	0.945	0.676	0.181

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9
N.S.	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00
time (sec)	N/A	0.008	0.003	0.016	2.108	0.620	0.060	0.472	0.021

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	34	9	7	17	9
N.S.	1	1.00	1.00	1.11	3.78	1.00	0.78	1.89	1.00
time (sec)	N/A	0.008	0.004	0.015	1.310	0.632	0.058	0.544	0.026

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	26	7	13	7
N.S.	1	1.00	1.00	0.89	0.78	2.89	0.78	1.44	0.78
time (sec)	N/A	0.003	0.005	0.014	5.812	0.568	0.039	0.515	0.186

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	13	26	22	23	13
N.S.	1	1.00	1.00	1.33	1.08	2.17	1.83	1.92	1.08
time (sec)	N/A	0.023	0.042	0.032	1.702	0.829	0.146	0.492	0.321

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	32	76	22	22
N.S.	1	1.00	1.00	0.82	0.79	1.14	2.71	0.79	0.79
time (sec)	N/A	0.016	0.016	0.051	1.252	1.095	0.334	0.489	0.332

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	15	14
N.S.	1	1.00	1.00	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.006	0.004	0.000	2.303	1.007	0.077	0.482	0.019

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	19	15
N.S.	1	1.00	1.00	0.76	0.71	0.62	0.71	0.90	0.71
time (sec)	N/A	0.006	0.003	0.034	1.679	1.026	0.082	0.474	0.048

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	23	22	19	26	22	41
N.S.	1	1.00	0.96	1.00	0.96	0.83	1.13	0.96	1.78
time (sec)	N/A	0.014	0.004	0.012	1.228	0.558	0.061	0.515	0.144

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.56	1.78	0.78	0.78
time (sec)	N/A	0.012	0.015	0.066	1.557	0.938	0.127	0.464	0.026

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	28	32	14	14
N.S.	1	1.00	1.00	0.83	0.78	1.56	1.78	0.78	0.78
time (sec)	N/A	0.009	0.011	0.047	2.222	0.991	0.126	0.463	0.024

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	0	24	0	22	16
N.S.	1	1.00	3.33	1.42	0.00	2.00	0.00	1.83	1.33
time (sec)	N/A	0.005	0.009	0.062	0.000	1.114	0.000	0.452	0.139

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	23	0	26	0	35	18
N.S.	1	1.00	3.00	1.64	0.00	1.86	0.00	2.50	1.29
time (sec)	N/A	0.007	0.009	0.087	0.000	1.271	0.000	0.455	0.136

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	16	22	9	10	36	14	10
N.S.	1	1.00	1.33	1.83	0.75	0.83	3.00	1.17	0.83
time (sec)	N/A	0.005	0.007	0.043	2.631	0.824	0.358	0.452	0.145

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	20	18	0	23	12
N.S.	1	1.00	1.29	1.57	1.43	1.29	0.00	1.64	0.86
time (sec)	N/A	0.007	0.008	0.052	1.929	0.821	0.000	0.480	0.033

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	0	13	63	13	21
N.S.	1	1.00	0.81	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.006	0.068	0.025	0.000	0.769	0.168	0.405	0.193

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	31	18	18	20	19	18
N.S.	1	1.00	0.92	1.29	0.75	0.75	0.83	0.79	0.75
time (sec)	N/A	0.007	0.011	0.039	2.463	0.589	0.044	0.431	0.106

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	9	33	16	7	16	8
N.S.	1	1.00	1.50	0.75	2.75	1.33	0.58	1.33	0.67
time (sec)	N/A	0.003	0.070	0.079	1.769	0.701	0.398	0.421	0.037

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	92	24	24	0	24	24
N.S.	1	1.00	1.03	2.88	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.009	0.023	0.033	2.443	0.610	0.000	0.444	0.030

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	32	31	26	26
N.S.	1	1.00	1.00	0.82	0.79	0.97	0.94	0.79	0.79
time (sec)	N/A	0.008	0.010	0.017	2.170	0.665	0.046	0.441	0.060

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.002	0.003	0.004	1.376	0.561	0.026	0.450	0.025

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	16	16	48	19	34	10
N.S.	1	1.00	1.50	1.33	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.009	0.004	0.001	1.661	0.613	0.015	0.437	0.028

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	7	11	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	2.33	3.67	1.00
time (sec)	N/A	0.003	0.002	0.009	1.328	0.629	0.038	0.444	0.011

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	18	12	12	3
N.S.	1	1.00	1.00	1.33	1.00	6.00	4.00	4.00	1.00
time (sec)	N/A	0.003	0.004	0.008	2.455	0.619	0.134	0.451	0.137

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	8	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.002	0.000	0.012	3.451	1.001	0.024	0.470	0.165

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	32	10	10	0	11	-1
N.S.	1	1.00	0.59	0.65	0.20	0.20	0.00	0.22	-0.02
time (sec)	N/A	0.011	0.007	0.013	3.629	0.747	0.000	0.476	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	12	12	12	13	10
N.S.	1	1.00	1.06	0.81	0.75	0.75	0.75	0.81	0.62
time (sec)	N/A	0.005	0.003	0.028	2.949	0.777	0.017	0.439	0.145

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	36	0	0	30	0	40	32
N.S.	1	1.00	0.90	0.00	0.00	0.75	0.00	1.00	0.80
time (sec)	N/A	0.014	0.008	0.003	0.000	0.728	0.000	0.469	0.042

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	67	63	58	55	109	57	69
N.S.	1	1.00	0.52	0.49	0.45	0.43	0.85	0.45	0.54
time (sec)	N/A	0.133	0.107	0.041	2.082	0.881	0.699	0.459	0.278

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	17	17	27	15	17
N.S.	1	1.00	0.60	0.67	0.57	0.57	0.90	0.50	0.57
time (sec)	N/A	0.027	0.019	0.026	2.459	1.118	0.165	0.457	0.171

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	7	17	17	7	7
N.S.	1	1.00	0.82	0.73	0.64	1.55	1.55	0.64	0.64
time (sec)	N/A	0.001	0.002	0.038	5.060	1.421	0.031	0.453	0.066

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	32	32	41	33	46
N.S.	1	1.00	1.00	0.82	0.80	0.80	1.02	0.82	1.15
time (sec)	N/A	0.014	0.007	0.036	2.406	1.060	0.046	0.482	0.115

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	23	22	17	17	15	18	6
N.S.	1	1.00	2.88	2.75	2.12	2.12	1.88	2.25	0.75
time (sec)	N/A	0.002	0.003	0.034	4.490	0.936	0.029	0.459	0.067

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	194	394	0	0	0	0	-1
N.S.	1	1.00	0.85	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.193	0.043	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	26	25	24	37	25	51
N.S.	1	1.00	1.25	1.08	1.04	1.00	1.54	1.04	2.12
time (sec)	N/A	0.007	0.006	0.011	2.423	0.774	0.687	0.457	0.443

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	71	54	57	69	73	58	70
N.S.	1	1.00	0.91	0.69	0.73	0.88	0.94	0.74	0.90
time (sec)	N/A	0.032	0.018	0.027	3.608	0.807	0.131	0.454	0.272

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	12	3	2	14	2	25	2
N.S.	1	1.00	6.00	1.50	1.00	7.00	1.00	12.50	1.00
time (sec)	N/A	0.001	0.001	0.028	4.330	0.663	0.043	0.451	0.002

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	21	20	25	24	25	20
N.S.	1	1.00	1.15	0.78	0.74	0.93	0.89	0.93	0.74
time (sec)	N/A	0.002	0.021	0.049	3.816	0.615	0.065	0.454	0.048

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.60	0.80	1.10	1.00
time (sec)	N/A	0.003	0.003	0.028	1.824	0.584	0.020	0.454	0.031

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	0.88	0.88
time (sec)	N/A	0.003	0.002	0.003	2.429	0.917	0.042	0.446	0.220

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	33	24	32	38	24
N.S.	1	1.00	0.72	0.85	0.82	0.60	0.80	0.95	0.60
time (sec)	N/A	0.016	0.010	0.004	4.266	0.930	0.092	0.503	0.029

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	13	29	0	15	9
N.S.	1	1.00	1.38	0.67	0.62	1.38	0.00	0.71	0.43
time (sec)	N/A	0.083	0.026	0.105	3.120	0.858	0.000	0.443	0.810

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	16	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	1.14	0.71
time (sec)	N/A	0.005	0.003	0.018	1.495	0.984	0.011	0.450	0.002

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	18	14	18	16
N.S.	1	1.00	1.00	0.94	0.89	1.00	0.78	1.00	0.89
time (sec)	N/A	0.010	0.004	0.043	2.539	1.461	0.039	0.454	0.152

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	7	6	16	7	29	6
N.S.	1	1.00	2.00	0.70	0.60	1.60	0.70	2.90	0.60
time (sec)	N/A	0.001	0.013	0.047	2.988	1.113	0.051	0.464	0.038

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	12	5	4	14	3	25	4
N.S.	1	1.00	2.00	0.83	0.67	2.33	0.50	4.17	0.67
time (sec)	N/A	0.001	0.012	0.047	3.493	1.027	0.049	0.441	0.029

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	16	16	22	16	16
N.S.	1	1.00	1.00	0.81	0.76	0.76	1.05	0.76	0.76
time (sec)	N/A	0.012	0.007	0.159	2.652	0.927	0.036	0.444	0.142

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	42	47	73	46	46	45
N.S.	1	1.00	0.96	0.79	0.89	1.38	0.87	0.87	0.85
time (sec)	N/A	0.021	0.015	0.020	2.895	1.140	0.068	0.463	0.061

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	41	41	762	44	46
N.S.	1	1.00	1.00	0.94	0.84	0.84	15.55	0.90	0.94
time (sec)	N/A	0.053	0.019	0.044	2.248	1.277	56.795	0.466	0.210

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.003	2.591	0.055	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	27	44	26	27	27
N.S.	1	1.00	1.00	1.00	0.96	1.57	0.93	0.96	0.96
time (sec)	N/A	0.148	40.005	0.190	3.925	0.846	0.159	0.435	0.338

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	136	128	160	145	1277	114	140
N.S.	1	1.00	0.68	0.64	0.80	0.73	6.42	0.57	0.70
time (sec)	N/A	0.074	0.155	0.177	5.234	0.759	147.269	0.451	1.111

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	15	20	15	15	15
N.S.	1	1.00	1.00	0.89	0.83	1.11	0.83	0.83	0.83
time (sec)	N/A	0.004	0.006	0.013	3.164	0.954	0.133	0.444	0.155

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	21	37	36	37	21
N.S.	1	1.00	1.00	0.92	0.88	1.54	1.50	1.54	0.88
time (sec)	N/A	0.006	0.021	0.016	2.985	0.821	0.219	0.491	0.066

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	B	F	B	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	5137	1197351	0	179	0	0	-1
N.S.	1	0.00	54.65	12737.78	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	1.282	16.125	1.343	0.000	1.473	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	A	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	0	630	354	0	223	0	0	-1
N.S.	1	0.00	4.44	2.49	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	2.180	3.627	0.339	0.000	1.358	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	31	1088	171	162	0	76	172
N.S.	1	1.00	1.48	51.81	8.14	7.71	0.00	3.62	8.19
time (sec)	N/A	0.193	0.619	0.199	2.533	0.835	0.000	0.444	2.373

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	4030	0	3168	4640	0	0	0	0	-1
N.S.	1	0.00	0.79	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.019	16.687	0.868	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	181	1356	0	269	330	472	265
N.S.	1	1.00	0.55	4.11	0.00	0.82	1.00	1.43	0.80
time (sec)	N/A	0.634	0.103	0.135	0.000	0.648	9.754	0.606	0.767

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	5	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	1.25	1.00	1.00
time (sec)	N/A	0.008	0.002	0.031	2.657	0.639	0.382	0.495	0.020

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	B	A	A	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	0	71	61	0	137	76	94	103
N.S.	1	0.00	1.00	0.86	0.00	1.93	1.07	1.32	1.45
time (sec)	N/A	0.510	0.026	0.037	0.000	0.592	0.094	0.459	0.280

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [282] had the largest ratio of [107]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	0	1.00	6	0.000
2	A	3	2	1.00	13	0.154
3	A	2	1	1.00	10	0.100
4	A	1	1	1.00	3	0.333
5	A	2	1	1.00	11	0.091
6	A	2	1	1.00	14	0.071
7	A	2	1	1.00	20	0.050
8	A	2	2	1.00	12	0.167
9	A	3	3	1.00	13	0.231
10	A	2	2	1.00	10	0.200
11	A	6	5	1.00	13	0.385
12	A	2	1	1.00	23	0.043
13	A	4	3	1.00	20	0.150
14	A	3	2	1.00	22	0.091
15	A	5	4	1.00	14	0.286
16	A	6	6	1.00	9	0.667
17	A	3	2	1.00	16	0.125
18	A	9	6	1.00	7	0.857
19	A	9	6	1.00	11	0.546
20	A	9	5	1.00	10	0.500
21	A	1	1	1.00	7	0.143
22	A	2	1	1.00	9	0.111
23	A	2	1	1.00	11	0.091
24	A	1	1	1.00	7	0.143
25	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	1	1.00	9	0.111
27	A	2	1	1.00	11	0.091
28	A	3	3	1.00	11	0.273
29	A	2	1	1.00	11	0.091
30	A	2	1	1.00	11	0.091
31	A	1	1	1.00	9	0.111
32	A	1	1	1.00	11	0.091
33	A	6	6	1.00	9	0.667
34	A	6	6	1.00	7	0.857
35	A	6	6	1.00	11	0.546
36	A	3	3	1.00	7	0.429
37	A	3	3	1.00	9	0.333
38	A	9	6	1.00	9	0.667
39	A	3	3	1.00	12	0.250
40	A	3	3	1.00	12	0.250
41	A	3	2	1.00	12	0.167
42	A	3	2	1.00	12	0.167
43	A	3	2	1.00	12	0.167
44	A	9	5	1.00	10	0.500
45	A	9	5	1.00	12	0.417
46	A	10	6	1.00	7	0.857
47	A	10	6	1.00	7	0.857
48	A	10	6	1.00	7	0.857
49	A	19	6	1.00	7	0.857
50	A	13	10	1.00	7	1.429
51	A	19	6	1.00	12	0.500
52	A	7	7	1.00	11	0.636
53	A	1	1	1.00	2	0.500
54	A	1	1	1.00	4	0.250
55	A	1	1	1.00	6	0.167
56	A	1	1	1.00	6	0.167
57	A	2	2	1.00	4	0.500
58	A	11	2	1.00	8	0.250
59	A	2	2	1.00	8	0.250
60	A	1	1	1.00	4	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	6	0.333
62	A	2	2	1.00	8	0.250
63	A	3	3	1.00	8	0.375
64	A	2	2	1.00	8	0.250
65	A	2	1	1.00	8	0.125
66	A	4	4	1.00	10	0.400
67	A	2	2	1.00	10	0.200
68	A	3	2	1.00	8	0.250
69	A	3	2	1.00	10	0.200
70	A	3	3	1.00	8	0.375
71	A	3	3	1.00	10	0.300
72	A	4	3	1.00	12	0.250
73	A	4	3	1.00	12	0.250
74	A	3	3	1.00	10	0.300
75	A	0	0	0.00	0	0.000
76	A	1	1	1.00	2	0.500
77	A	1	1	1.00	2	0.500
78	A	1	1	1.00	2	0.500
79	A	1	1	1.00	2	0.500
80	A	2	2	1.00	6	0.333
81	A	1	1	1.00	2	0.500
82	A	1	1	1.00	2	0.500
83	A	2	2	1.00	4	0.500
84	A	3	3	1.00	8	0.375
85	A	2	1	1.00	4	0.250
86	A	1	1	1.00	4	0.250
87	A	3	2	1.00	11	0.182
88	A	2	2	1.00	4	0.500
89	A	2	1	1.00	4	0.250
90	A	2	2	1.00	4	0.500
91	A	1	1	1.00	7	0.143
92	A	2	2	1.00	4	0.500
93	A	3	2	1.00	6	0.333
94	A	2	2	1.00	6	0.333
95	A	4	4	1.00	8	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	6	0.500
97	A	2	2	1.00	4	0.500
98	A	3	2	1.00	6	0.333
99	A	2	2	1.00	6	0.333
100	A	4	4	1.00	8	0.500
101	A	3	3	1.00	6	0.500
102	A	1	1	1.00	6	0.167
103	A	1	1	1.00	6	0.167
104	A	2	2	1.00	6	0.333
105	A	3	2	1.00	8	0.250
106	A	2	2	1.00	4	0.500
107	A	1	1	1.00	6	0.167
108	A	1	1	1.00	6	0.167
109	A	1	1	1.00	6	0.167
110	A	1	1	1.00	6	0.167
111	A	1	1	1.00	6	0.167
112	A	1	1	1.00	6	0.167
113	A	2	2	1.00	8	0.250
114	A	2	1	1.00	8	0.125
115	A	2	2	1.00	8	0.250
116	A	2	1	1.00	8	0.125
117	A	2	2	1.00	8	0.250
118	A	1	1	1.00	6	0.167
119	A	1	1	1.00	8	0.125
120	A	1	1	1.00	6	0.167
121	A	1	1	1.00	8	0.125
122	A	3	3	1.00	8	0.375
123	A	3	3	1.00	10	0.300
124	A	4	4	1.00	12	0.333
125	A	1	1	1.00	7	0.143
126	A	4	4	1.00	12	0.333
127	A	2	2	1.00	4	0.500
128	A	17	8	1.00	8	1.000
129	A	34	9	1.00	8	1.125
130	A	3	3	1.00	6	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	1	1	1.00	9	0.111
132	A	3	3	1.00	9	0.333
133	A	3	2	1.00	9	0.222
134	A	1	1	1.00	6	0.167
135	A	1	1	1.00	6	0.167
136	A	4	3	1.00	7	0.429
137	A	4	3	1.00	7	0.429
138	A	11	5	1.00	9	0.556
139	A	11	5	1.00	9	0.556
140	A	25	5	1.00	9	0.556
141	A	25	5	1.00	9	0.556
142	A	5	2	1.00	11	0.182
143	A	5	2	1.00	11	0.182
144	A	6	4	1.00	10	0.400
145	A	0	0	0.00	0	0.000
146	A	4	3	1.00	9	0.333
147	A	5	5	1.00	15	0.333
148	A	1	1	1.00	3	0.333
149	A	1	1	1.00	3	0.333
150	A	1	1	1.00	3	0.333
151	A	1	1	1.00	3	0.333
152	A	1	1	1.00	5	0.200
153	A	1	1	1.00	9	0.111
154	A	4	4	1.00	11	0.364
155	A	3	2	1.00	13	0.154
156	A	2	2	1.00	9	0.222
157	A	2	2	1.00	18	0.111
158	A	2	2	1.00	7	0.286
159	A	21	2	1.00	7	0.286
160	A	2	2	1.00	9	0.222
161	A	2	2	1.00	7	0.286
162	A	2	2	1.00	7	0.286
163	A	5	3	1.00	12	0.250
164	A	1	1	1.00	14	0.071
165	A	1	1	1.00	7	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	1	1	1.00	5	0.200
167	A	1	1	1.00	7	0.143
168	A	7	3	1.00	12	0.250
169	F	0	0	N/A	0.	N/A
170	A	0	0	0.00	0	0.000
171	A	2	3	1.00	6	0.500
172	A	5	5	1.00	7	0.714
173	A	3	3	1.00	10	0.300
174	A	1	0	1.00	13	0.000
175	A	4	4	1.00	12	0.333
176	A	5	4	1.00	19	0.210
177	A	1	1	1.00	9	0.111
178	A	2	1	1.00	11	0.091
179	A	2	1	1.00	13	0.077
180	A	3	3	1.00	13	0.231
181	A	3	3	1.00	13	0.231
182	A	1	1	1.00	9	0.111
183	A	2	1	1.00	11	0.091
184	A	2	1	1.00	13	0.077
185	A	2	2	1.00	13	0.154
186	A	3	3	1.00	13	0.231
187	A	1	1	1.00	11	0.091
188	A	2	1	1.00	13	0.077
189	A	6	6	1.00	18	0.333
190	A	2	2	1.00	9	0.222
191	A	4	3	1.00	13	0.231
192	A	3	3	1.00	6	0.500
193	A	1	1	1.00	13	0.077
194	A	2	2	1.00	13	0.154
195	A	3	3	1.00	13	0.231
196	A	3	3	1.00	14	0.214
197	A	3	3	1.00	18	0.167
198	A	1	1	1.00	20	0.050
199	A	1	1	1.00	9	0.111
200	A	3	2	1.00	65	0.031

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	4	1.00	68	0.059
202	A	5	2	1.00	21	0.095
203	A	6	6	1.00	21	0.286
204	A	4	3	1.00	23	0.130
205	A	2	2	1.00	16	0.125
206	A	3	3	1.00	25	0.120
207	A	2	2	1.00	24	0.083
208	A	2	2	1.00	29	0.069
209	A	1	1	1.00	18	0.056
210	A	1	1	1.00	23	0.043
211	A	3	3	1.00	24	0.125
212	A	3	3	1.00	22	0.136
213	A	3	3	1.00	26	0.115
214	A	3	3	1.00	31	0.097
215	A	3	3	1.00	16	0.188
216	A	1	1	1.00	8	0.125
217	A	2	2	1.00	6	0.333
218	A	1	1	1.00	20	0.050
219	A	3	2	1.00	13	0.154
220	A	2	2	1.00	13	0.154
221	A	3	3	1.00	9	0.333
222	A	2	2	1.00	13	0.154
223	A	2	1	1.00	11	0.091
224	A	5	5	1.00	13	0.385
225	A	2	2	1.00	4	0.500
226	A	2	2	1.00	4	0.500
227	A	1	1	1.00	4	0.250
228	A	1	1	1.00	28	0.036
229	A	4	2	1.00	11	0.182
230	A	3	3	1.00	4	0.750
231	A	3	3	1.00	4	0.750
232	A	3	3	1.00	8	0.375
233	A	3	2	1.00	7	0.286
234	A	3	2	1.00	7	0.286
235	A	1	1	1.00	8	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	1	1	1.00	10	0.100
237	A	1	1	1.00	8	0.125
238	A	1	1	1.00	10	0.100
239	A	3	3	1.00	17	0.176
240	A	4	3	1.00	13	0.231
241	A	2	2	1.00	11	0.182
242	A	4	3	1.00	13	0.231
243	A	3	3	1.00	8	0.375
244	A	1	1	1.00	2	0.500
245	A	3	2	1.00	4	0.500
246	A	1	1	1.00	2	0.500
247	A	1	1	1.00	2	0.500
248	A	1	1	1.00	3	0.333
249	A	4	3	1.00	12	0.250
250	A	2	1	1.00	13	0.077
251	A	3	3	1.00	14	0.214
252	A	14	8	1.00	12	0.667
253	A	4	3	1.00	7	0.429
254	A	1	1	1.00	5	0.200
255	A	6	6	1.00	9	0.667
256	A	2	2	1.00	9	0.222
257	A	10	3	1.00	15	0.200
258	A	6	2	1.00	11	0.182
259	A	6	6	1.00	7	0.857
260	A	1	1	1.00	9	0.111
261	A	2	2	1.00	9	0.222
262	A	2	1	1.00	7	0.143
263	A	2	2	1.00	2	1.000
264	A	4	3	1.00	6	0.500
265	A	4	2	1.00	17	0.118
266	A	2	2	1.00	4	0.500
267	A	2	1	1.00	19	0.053
268	A	1	1	1.00	11	0.091
269	A	1	1	1.00	9	0.111
270	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	2	1.00	24	0.083
272	A	2	1	1.00	29	0.034
273	A	0	0	0.00	0	0.000
274	A	9	7	1.00	54	0.130
275	A	4	3	1.00	21	0.143
276	A	1	1	1.00	2	0.500
277	A	1	1	1.00	4	0.250
278	F	0	0	N/A	0.	N/A
279	F	0	0	N/A	0.	N/A
280	A	1	1	1.00	85	0.012
281	F	0	0	N/A	0.	N/A
282	A	20	8	1.00	107	0.075
283	A	2	2	1.00	7	0.286
284	F	0	0	N/A	0.	N/A

Chapter 3

Listing of integrals

Local contents

3.1 $\int (1+x+x^2)\, dx$

Optimal. Leaf size=16

$$x+\frac{x^2}{2}+\frac{x^3}{3}$$

```
[Out] x+1/2*x^2+1/3*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 + x + x^2,x]
[Out] x + x^2/2 + x^3/3
```

Rubi steps

$$\int (1+x+x^2)\, dx = x+\frac{x^2}{2}+\frac{x^3}{3}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$x+\frac{x^2}{2}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x + x^2,x]
[Out] x + x^2/2 + x^3/3
```

Maple [A]
time = 0.00, size = 13, normalized size = 0.81

method	result	size
gosper	$x+\frac{1}{2}x^2+\frac{1}{3}x^3$	13
default	$x+\frac{1}{2}x^2+\frac{1}{3}x^3$	13

norman	$x+\frac{1}{2}x^2+\frac{1}{3}x^3$	13
risch	$x+\frac{1}{2}x^2+\frac{1}{3}x^3$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2+x+1,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+1/3*x^3
```

Maxima [A]
time = 5.66, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Fricas [A]
time = 0.80, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.62

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2+x+1,x)
```

```
[Out] x**3/3 + x**2/2 + x
```

Giac [A]
time = 0.59, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.81

$$\frac{x\,\left(2\,x^2+3\,x+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + x^2 + 1,x)
```

```
[Out] (x*(3*x + 2*x^2 + 6))/6
```

3.2 $\int x^2(x+2x^2)^2\,dx$

Optimal. Leaf size=22

$$\frac{x^5}{5}+\frac{2x^6}{3}+\frac{4x^7}{7}$$

```
[Out] 1/5*x^5+2/3*x^6+4/7*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {661, 45}

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(x + 2*x^2)^2,x]
```

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 661

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(x+2x^2\right)^2\,dx&=\int x^4(1+2x)^2\,dx\\&=\int\left(x^4+4x^5+4x^6\right)\,dx\\&=\frac{x^5}{5}+\frac{2x^6}{3}+\frac{4x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^5}{5} + \frac{2x^6}{3} + \frac{4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(x + 2*x^2)^2,x]
```

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
gosper	$\frac{x^5(60x^2+70x+21)}{105}$	16
default	$\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$	17
norman	$\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$	17
risch	$\frac{1}{5}x^5 + \frac{2}{3}x^6 + \frac{4}{7}x^7$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^2+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5+2/3*x^6+4/7*x^7
```

Maxima [A]
time = 2.56, size = 16, normalized size = 0.73

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="maxima")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

Fricas [A]
time = 0.68, size = 16, normalized size = 0.73

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="fricas")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

Sympy [A]
time = 0.01, size = 17, normalized size = 0.77

$$\frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**2+x)**2,x)
```

```
[Out] 4*x**7/7 + 2*x**6/3 + x**5/5
```

Giac [A]
time = 0.57, size = 16, normalized size = 0.73

$$\frac{4}{7}\,x^7 + \frac{2}{3}\,x^6 + \frac{1}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="giac")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

Mupad [B]
time = 0.04, size = 15, normalized size = 0.68

$$\frac{x^5\,\left(60\,x^2+70\,x+21\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 2*x^2)^2,x)
```

```
[Out] (x^5*(70*x + 60*x^2 + 21))/105
```

3.3 $\int x(1+2x+x^2)\,dx$

Optimal. Leaf size=22

$$\frac{x^2}{2}+\frac{2x^3}{3}+\frac{x^4}{4}$$

```
[Out] 1/2*x^2+2/3*x^3+1/4*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {14}

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + 2*x + x^2),x]
```

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(1+2x+x^2\right)\,dx &= \int \left(x+2x^2+x^3\right)\,dx\\ &= \frac{x^2}{2}+\frac{2x^3}{3}+\frac{x^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^2}{2}+\frac{2x^3}{3}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 2*x + x^2),x]
```

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.77

method	result	size
gosper	$\frac{x^2(3x^2+8x+6)}{12}$	16
default	$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4$	17
norman	$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4$	17
risch	$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+2/3*x^3+1/4*x^4
```

Maxima [A]
time = 3.25, size = 16, normalized size = 0.73

$$\frac{1}{4}\,x^4+\frac{2}{3}\,x^3+\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

Fricas [A]
time = 0.65, size = 16, normalized size = 0.73

$$\frac{1}{4}\,x^4+\frac{2}{3}\,x^3+\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.68

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2*x+1),x)
```

```
[Out] x**4/4 + 2*x**3/3 + x**2/2
```

Giac [A]
time = 0.53, size = 16, normalized size = 0.73

$$\frac{1}{4}\,x^4+\frac{2}{3}\,x^3+\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.68

$$\frac{x^2\,\left(3\,x^2+8\,x+6\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x + x^2 + 1),x)
```

```
[Out] (x^2*(8*x + 3*x^2 + 6))/12
```

3.4 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{29\}$

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Maple [A]
time = 0.00, size = 3, normalized size = 1.50

method	result	size

default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)
```

Maxima [A]
time = 2.38, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A]
time = 0.58, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A]
time = 0.02, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A]
time = 0.53, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

Mupad [**B**]
time = 0.01, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

3.5 $\int \frac{(1+x)^3}{(-1+x)^4} \, dx$

Optimal. Leaf size=36

$$\frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x)$$

```
[Out] 8/3/(1-x)^3-6/(1-x)^2+6/(1-x)+ln(1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{6}{1-x} - \frac{6}{(1-x)^2} + \frac{8}{3(1-x)^3} + \log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^3/(-1 + x)^4,x]
```

```
[Out] 8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^3}{(-1+x)^4} \, dx &= \int \left(\frac{8}{(-1+x)^4} + \frac{12}{(-1+x)^3} + \frac{6}{(-1+x)^2} + \frac{1}{-1+x}\right) dx \\
&= \frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.67

$$-\frac{2(4-9x+9x^2)}{3(-1+x)^3} + \log(-1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3/(-1 + x)^4,x]
[Out] (-2*(4 - 9*x + 9*x^2))/(3*(-1 + x)^3) + Log[-1 + x]
```

Maple [A]
time = 0.04, size = 27, normalized size = 0.75

method	result	size
norman	$\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3}+\ln(-1+x)$	22
risch	$\frac{-6x^2+6x-\frac{8}{3}}{(-1+x)^3}+\ln(-1+x)$	22
default	$\ln(-1+x)-\frac{6}{-1+x}-\frac{6}{(-1+x)^2}-\frac{8}{3(-1+x)^3}$	27
meijerg	$\frac{x(x^2-3x+3)}{3(1-x)^3}+\frac{x^2(3-x)}{2(1-x)^3}+\frac{x^3}{(1-x)^3}+\frac{x(22x^2-30x+12)}{12(1-x)^3}+\ln(1-x)$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3/(-1+x)^4,x,method=_RETURNVERBOSE)
[Out] ln(-1+x)-6/(-1+x)-6/(-1+x)^2-8/3/(-1+x)^3
```

Maxima [A]
time = 2.59, size = 32, normalized size = 0.89

$$-\frac{2\,(9\,x^2-9\,x+4)}{3\,(x^3-3\,x^2+3\,x-1)}+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="maxima")
[Out] -2/3*(9*x^2 - 9*x + 4)/(x^3 - 3*x^2 + 3*x - 1) + log(x - 1)
```

Fricas [A]
time = 0.70, size = 46, normalized size = 1.28

$$-\frac{18\,x^2-3\,(x^3-3\,x^2+3\,x-1)\log(x-1)-18\,x+8}{3\,(x^3-3\,x^2+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="fricas")
[Out] -1/3*(18*x^2 - 3*(x^3 - 3*x^2 + 3*x - 1)*log(x - 1) - 18*x + 8)/(x^3 - 3*x^
2 + 3*x - 1)
```

Sympy [A]
time = 0.04, size = 29, normalized size = 0.81

$$\frac{-18x^2+18x-8}{3x^3-9x^2+9x-3}+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/(-1+x)**4,x)
[Out] (-18*x**2 + 18*x - 8)/(3*x**3 - 9*x**2 + 9*x - 3) + log(x - 1)
```

Giac [A]
time = 0.48, size = 23, normalized size = 0.64

$$-\frac{2\,(9\,x^2-9\,x+4)}{3\,(x-1)^3}+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="giac")
[Out] -2/3*(9*x^2 - 9*x + 4)/(x - 1)^3 + log(abs(x - 1))
```

Mupad [B]
time = 0.12, size = 22, normalized size = 0.61

$$\ln\left(x-1\right)-\frac{6\,x^2-6\,x+\frac{8}{3}}{(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^3/(x - 1)^4,x)
[Out] log(x - 1) - (6*x^2 - 6*x + 8/3)/(x - 1)^3
```

3.6 $\int \frac{1}{(-1+x)x(1+x)^2}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(1+x)$$

```
[Out] -1/2/(1+x)+1/4*ln(1-x)-ln(x)+3/4*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {84}

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*x*(1 + x)^2),x]
```

```
[Out] -1/2*1/(1 + x) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)x(1+x)^2}\,dx &= \int \left(\frac{1}{4(-1+x)}-\frac{1}{x}+\frac{1}{2(1+x)^2}+\frac{3}{4(1+x)}\right)dx\\ &= -\frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-\frac{2}{1+x}+\log(1-x)-4\log(x)+3\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*x*(1 + x)^2),x]
```

```
[Out] (-2/(1 + x) + Log[1 - x] - 4*Log[x] + 3*Log[1 + x])/4
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.78

method	result	size
default	$-\ln(x)+\frac{\ln(-1+x)}{4}-\frac{1}{2(1+x)}+\frac{3\ln(1+x)}{4}$	25
norman	$-\ln(x)+\frac{\ln(-1+x)}{4}-\frac{1}{2(1+x)}+\frac{3\ln(1+x)}{4}$	25
risch	$-\ln(x)+\frac{\ln(-1+x)}{4}-\frac{1}{2(1+x)}+\frac{3\ln(1+x)}{4}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/x/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] -ln(x)+1/4*ln(-1+x)-1/2/(1+x)+3/4*ln(1+x)
```

Maxima [A]
time = 3.20, size = 24, normalized size = 0.75

$$-\frac{1}{2(x+1)}+\frac{3}{4}\log(x+1)+\frac{1}{4}\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="maxima")
[Out] -1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1) - log(x)
```

Fricas [A]
time = 0.67, size = 33, normalized size = 1.03

$$\frac{3(x+1)\log(x+1)+(x+1)\log(x-1)-4(x+1)\log(x)-2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="fricas")
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) - 4*(x + 1)*log(x) - 2)/(x +
 1)
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.75

$$-\log(x)+\frac{\log(x-1)}{4}+\frac{3\log(x+1)}{4}-\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)**2,x)
[Out] -log(x) + log(x - 1)/4 + 3*log(x + 1)/4 - 1/(2*x + 2)
```

Giac [A]
time = 0.51, size = 34, normalized size = 1.06

$$-\frac{1}{2(x+1)} - \log\left(\left|-\frac{1}{x+1}+1\right|\right) + \frac{1}{4}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="giac")
[Out] -1/2/(x + 1) - log(abs(-1/(x + 1) + 1)) + 1/4*log(abs(-2/(x + 1) + 1))
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.81

$$\frac{\ln(x-1)}{4} + \frac{3\ln(x+1)}{4} - \ln(x) - \frac{1}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x - 1)*(x + 1)^2),x)
[Out] log(x - 1)/4 + (3*log(x + 1))/4 - log(x) - 1/(2*(x + 1))
```

3.7 $\int \frac{b+ax}{(-p+x)(-q+x)}\,dx$

Optimal. Leaf size=40

$$\frac{(b+ap)\log(p-x)}{p-q}-\frac{(b+aq)\log(q-x)}{p-q}$$

```
[Out] (a*p+b)*ln(p-x)/(p-q)-(a*q+b)*ln(q-x)/(p-q)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = {78}

$$\frac{(ap+b)\log(p-x)}{p-q}-\frac{(aq+b)\log(q-x)}{p-q}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/((-p + x)*(-q + x)),x]
```

```
[Out] ((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax}{(-p+x)(-q+x)}\,dx &= \int \left(\frac{-b-ap}{(p-q)(p-x)}+\frac{b+aq}{(p-q)(q-x)}\right)dx\\ &= \frac{(b+ap)\log(p-x)}{p-q}-\frac{(b+aq)\log(q-x)}{p-q}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.85

$$\frac{(b+ap)\log(-p+x)-(b+aq)\log(-q+x)}{p-q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/((-p + x)*(-q + x)),x]
[Out] ((b + a*p)*Log[-p + x] - (b + a*q)*Log[-q + x])/(p - q)
```

Maple [A]
time = 0.04, size = 43, normalized size = 1.08

method	result	size
norman	$\frac{(ap+b)\ln(p-x)}{p-q}-\frac{(aq+b)\ln(q-x)}{p-q}$	41
default	$\frac{(-aq-b)\ln(q-x)}{p-q}+\frac{(ap+b)\ln(p-x)}{p-q}$	43
risch	$-\frac{\ln(-q+x)aq}{p-q}-\frac{\ln(-q+x)b}{p-q}+\frac{\ln(p-x)ap}{p-q}+\frac{\ln(p-x)b}{p-q}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(-p+x)/(-q+x),x,method=_RETURNVERBOSE)
[Out] (-a*q-b)/(p-q)*ln(q-x)+(a*p+b)*ln(p-x)/(p-q)
```

Maxima [A]
time = 2.36, size = 40, normalized size = 1.00

$$\frac{(ap+b)\log(-p+x)}{p-q}-\frac{(aq+b)\log(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="maxima")
[Out] (a*p + b)*log(-p + x)/(p - q) - (a*q + b)*log(-q + x)/(p - q)
```

Fricas [A]
time = 0.92, size = 34, normalized size = 0.85

$$\frac{(ap+b)\log(-p+x)-(aq+b)\log(-q+x)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="fricas")
[Out] ((a*p + b)*log(-p + x) - (a*q + b)*log(-q + x))/(p - q)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(26) = 52.
time = 0.52, size = 144, normalized size = 3.60

$$\frac{(ap+b)\log\left(x+\frac{-2apq-bp-bq-\frac{p^2(ap+b)}{p-q}+\frac{2pq(ap+b)}{p-q}-\frac{q^2(ap+b)}{p-q}}{ap+aq+2b}\right)}{p-q}-\frac{(aq+b)\log\left(x+\frac{-2apq-bp-bq+\frac{p^2(aq+b)}{p-q}-\frac{2pq(aq+b)}{p-q}+\frac{q^2(aq+b)}{p-q}}{ap+aq+2b}\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x)
```

```
[Out] (a*p + b)*log(x + (-2*a*p*q - b*p - b*q - p**2*(a*p + b)/(p - q) + 2*p*q*(a
*p + b)/(p - q) - q**2*(a*p + b)/(p - q))/(a*p + a*q + 2*b))/(p - q) - (a*q
 + b)*log(x + (-2*a*p*q - b*p - b*q + p**2*(a*q + b)/(p - q) - 2*p*q*(a*q +
 b)/(p - q) + q**2*(a*q + b)/(p - q))/(a*p + a*q + 2*b))/(p - q)
```

Giac [A]
time = 0.45, size = 42, normalized size = 1.05

$$\frac{(ap+b)\log\left(|-p+x|\right)}{p-q}-\frac{(aq+b)\log\left(|-q+x|\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="giac")
```

```
[Out] (a*p + b)*log(abs(-p + x))/(p - q) - (a*q + b)*log(abs(-q + x))/(p - q)
```

Mupad [B]
time = 0.25, size = 40, normalized size = 1.00

$$\frac{\ln\left(x-p\right)\,\left(b+a\,p\right)}{p-q}-\frac{\ln\left(x-q\right)\,\left(b+a\,q\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x)/((p - x)*(q - x)),x)
```

```
[Out] (log(x - p)*(b + a*p))/(p - q) - (log(x - q)*(b + a*q))/(p - q)
```

3.8 $\int \frac{1}{c+bx+ax^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -2*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + b*x + a*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{c+bx+ax^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2ax\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + b*x + a*x^2)^(-1),x]
[Out] (2*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A]
time = 0.12, size = 35, normalized size = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2ax+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2ax+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x+c),x,method=_RETURNVERBOSE)
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.63, size = 120, normalized size = 3.53

$$\left[\frac{\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)}{\sqrt{b^2-4\,ac}},\,-\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="fricas")
```

```
[Out] [log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a
*x^2 + b*x + c))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(34) = 68$.
time = 0.09, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x+c),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*a)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a))
```

Giac [A]
time = 0.47, size = 34, normalized size = 1.00

$$\frac{2\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="giac")
```

```
[Out] 2*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B]
time = 0.20, size = 46, normalized size = 1.35

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,a\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + b*x + a*x^2),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*a*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

3.9 $\int \frac{b+ax}{1+x^2}\,dx$

Optimal. Leaf size=16

$$b\tan^{-1}(x)+\frac{1}{2}a\log\left(1+x^2\right)$$

```
[Out] b*arctan(x)+1/2*a*ln(x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {649, 209, 266}

$$\frac{1}{2}a\log\left(x^2+1\right)+b\text{ArcTan}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/(1 + x^2),x]
```

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\int\frac{b+ax}{1+x^2}\,dx&=a\int\frac{x}{1+x^2}\,dx+b\int\frac{1}{1+x^2}\,dx\\&=b\tan^{-1}(x)+\frac{1}{2}a\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$b\tan^{-1}(x)+\frac{1}{2}a\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/(1 + x^2),x]
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Maple [A]
time = 0.02, size = 15, normalized size = 0.94

method	result	size
default	$b\arctan\left(x\right)+\frac{a\ln\left(x^2+1\right)}{2}$	15
meijerg	$b\arctan\left(x\right)+\frac{a\ln\left(x^2+1\right)}{2}$	15
risch	$b\arctan\left(x\right)+\frac{a\ln\left(x^2+1\right)}{2}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] b*arctan(x)+1/2*a*ln(x^2+1)
```

Maxima [A]
time = 4.55, size = 14, normalized size = 0.88

$$b\arctan\left(x\right)+\frac{1}{2}\,a\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="maxima")
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

Fricas [A]
time = 0.84, size = 14, normalized size = 0.88

$$b\arctan\left(x\right)+\frac{1}{2}\,a\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="fricas")
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 26, normalized size = 1.62

$$\left(\frac{a}{2}-\frac{ib}{2}\right)\log\left(x-i\right)+\left(\frac{a}{2}+\frac{ib}{2}\right)\log\left(x+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x**2+1),x)
```

```
[Out] (a/2 - I*b/2)*log(x - I) + (a/2 + I*b/2)*log(x + I)
```

Giac [A]
time = 0.48, size = 14, normalized size = 0.88

$$b\arctan\left(x\right)+\frac{1}{2}\,a\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="giac")
```

```
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.88

$$\frac{a\,\ln\left(x^2+1\right)}{2}+b\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + a*x)/(x^2 + 1),x)
```

```
[Out] (a*log(x^2 + 1))/2 + b*atan(x)
```

3.10 $\int \frac{1}{3-2x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(1/2*(1-x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {632, 210}

$$-\frac{\text{ArcTan}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x + x^2)^(-1),x]
```

```
[Out] -(ArcTan[(1 - x)/Sqrt[2]]/Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3-2x+x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+2x\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{-1+x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x + x^2)^(-1),x]
```

```
[Out] ArcTan[(-1 + x)/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.21, size = 17, normalized size = 0.89

method	result	size
risch	$\frac{\sqrt{2}\,\arctan\left(\frac{(-1+x)\sqrt{2}}{2}\right)}{2}$	15
default	$\frac{\sqrt{2}\,\arctan\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(2*x-2)*2^(1/2))
```

Maxima [A]
time = 2.79, size = 14, normalized size = 0.74

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

Fricas [A]
time = 0.65, size = 14, normalized size = 0.74

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

Sympy [A]
time = 0.03, size = 22, normalized size = 1.16

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}-\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3),x)
[Out] sqrt(2)*atan(sqrt(2)*x/2 - sqrt(2)/2)/2
```

Giac [A]
time = 0.54, size = 14, normalized size = 0.74

$$\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.74

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,(x-1)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 2*x + 3),x)
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1))/2))/2
```

3.11 $\int \frac{1}{(-1+x)^2(1+x^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}+\frac{1}{4}\tan^{-1}(x)-\frac{1}{2}\log(1-x)+\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/4/(1-x)-1/4/(x^2+1)+1/4*arctan(x)-1/2*ln(1-x)+1/4*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {755, 815, 649, 209, 266}

$$\frac{\text{ArcTan}(x)}{4}-\frac{1}{4\left(x^2+1\right)}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*(1 + x^2)^2),x]
```

```
[Out] 1/(4*(1 - x)) - 1/(4*(1 + x^2)) + ArcTan[x]/4 - Log[1 - x]/2 + Log[1 + x^2]
/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
```

```
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+x)^2\left(1+x^2\right)^2}\,dx &= -\frac{1}{4\left(1+x^2\right)}-\frac{1}{4}\int \frac{-4+2x}{(-1+x)^2\left(1+x^2\right)}\,dx \\
&= -\frac{1}{4\left(1+x^2\right)}-\frac{1}{4}\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}+\frac{-1-2x}{1+x^2}\right)dx \\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}-\frac{1}{2}\log(1-x)-\frac{1}{4}\int \frac{-1-2x}{1+x^2}\,dx \\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}+\frac{1}{4}\tan^{-1}(x)-\frac{1}{2}\log(1-x)+\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.71

$$\frac{1}{4}\left(\frac{1}{1-x}-\frac{1}{1+x^2}+\tan^{-1}(x)-2\log(-1+x)+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*(1 + x^2)^2),x]
[Out] ((1 - x)^(-1) - (1 + x^2)^(-1) + ArcTan[x] - 2*Log[-1 + x] + Log[1 + x^2])/
4
```

Maple [A]
time = 0.04, size = 36, normalized size = 0.73

method	result	size
default	$-\frac{1}{4(-1+x)}-\frac{\ln(-1+x)}{2}-\frac{1}{4(x^2+1)}+\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{4}$	36
risch	$\frac{-\frac{1}{4}x^2-\frac{1}{4}x}{(x^2+1)(-1+x)}-\frac{\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-1+x)-1/2*ln(-1+x)-1/4/(x^2+1)+1/4*ln(x^2+1)+1/4*arctan(x)
```

Maxima [A]
time = 2.90, size = 39, normalized size = 0.80

$$-\frac{x^2+x}{4(x^3-x^2+x-1)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\log(x^2+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^2 + x)/(x^3 - x^2 + x - 1) + 1/4*arctan(x) + 1/4*log(x^2 + 1) - 1/2
*log(x - 1)
```

Fricas [A]
time = 0.70, size = 71, normalized size = 1.45

$$-\frac{x^2-(x^3-x^2+x-1)\arctan(x)-(x^3-x^2+x-1)\log(x^2+1)+2(x^3-x^2+x-1)\log(x-1)+x}{4(x^3-x^2+x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x^2 - (x^3 - x^2 + x - 1)*arctan(x) - (x^3 - x^2 + x - 1)*log(x^2 + 1
) + 2*(x^3 - x^2 + x - 1)*log(x - 1) + x)/(x^3 - x^2 + x - 1)
```

Sympy [A]
time = 0.07, size = 41, normalized size = 0.84

$$\frac{-x^2-x}{4x^3-4x^2+4x-4}-\frac{\log(x-1)}{2}+\frac{\log(x^2+1)}{4}+\frac{\operatorname{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/(x**2+1)**2,x)
```

```
[Out] (-x**2 - x)/(4*x**3 - 4*x**2 + 4*x - 4) - log(x - 1)/2 + log(x**2 + 1)/4 +
atan(x)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(37) = 74.
time = 0.48, size = 80, normalized size = 1.63

$$\frac{1}{16}\pi-\frac{1}{4}\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{\frac{2}{x-1}+1}{8\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)}-\frac{1}{4(x-1)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*pi - 1/4*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/8*(2/(x - 1) +
1)/(2/(x - 1) + 2/(x - 1)^2 + 1) - 1/4/(x - 1) + 1/4*arctan(x) + 1/4*log(2/
(x - 1) + 2/(x - 1)^2 + 1)
```

Mupad [B]
time = 0.13, size = 49, normalized size = 1.00

$$-\frac{\ln(x-1)}{2}-\frac{\frac{x^2}{4}+\frac{x}{4}}{x^3-x^2+x-1}+\ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{8}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^2*(x - 1)^2),x)
```

```
[Out] log(x - 1i)*(1/4 - 1i/8) - log(x - 1)/2 + log(x + 1i)*(1/4 + 1i/8) - (x/4 +
 x^2/4)/(x - x^2 + x^3 - 1)
```

3.12 $\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx$

Optimal. Leaf size=68

$$\frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}$$

```
[Out] a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {153}

$$\frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c)) + (c*Lo
g[c - x])/((a - c)*(b - c))
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(-a+x)(-b+x)(-c+x)}\,dx &= \int \left(-\frac{a}{(a-b)(a-c)(a-x)}+\frac{b}{(a-b)(b-c)(b-x)}+\frac{c}{(a-c)(-b+c)\ldots}\right)dx\\ &= \frac{a\log(a-x)}{(a-b)(a-c)}-\frac{b\log(b-x)}{(a-b)(b-c)}+\frac{c\log(c-x)}{(a-c)(b-c)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.91

$$\frac{a(b-c)\log(-a+x)+b(-a+c)\log(-b+x)+(a-b)c\log(-c+x)}{(a-b)(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*(b - c)*Log[-a + x] + b*(-a + c)*Log[-b + x] + (a - b)*c*Log[-c + x])/((
a - b)*(a - c)*(b - c))
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.01

method	result	size
default	$\frac{a\ln(a-x)}{(a-b)(a-c)}-\frac{b\ln(b-x)}{(a-b)(b-c)}+\frac{c\ln(c-x)}{(a-c)(b-c)}$	69
norman	$\frac{c\ln(c-x)}{ab-ac-bc+c^2}+\frac{a\ln(a-x)}{(a-b)(a-c)}-\frac{b\ln(b-x)}{(a-b)(b-c)}$	72
risch	$\frac{c\ln(c-x)}{ab-ac-bc+c^2}-\frac{b\ln(-b+x)}{ab-ac-b^2+bc}+\frac{a\ln(-a+x)}{a^2-ab-ac+bc}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a+x)/(-b+x)/(-c+x),x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(a-x)/(a-b)/(a-c)-b*ln(b-x)/(a-b)/(b-c)+c*ln(c-x)/(a-c)/(b-c)
```

Maxima [A]
time = 2.90, size = 78, normalized size = 1.15

$$\frac{a\log(-a+x)}{a^2-ab-(a-b)c}-\frac{b\log(-b+x)}{ab-b^2-(a-b)c}+\frac{c\log(-c+x)}{ab-(a+b)c+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="maxima")
```

```
[Out] a*log(-a + x)/(a^2 - a*b - (a - b)*c) - b*log(-b + x)/(a*b - b^2 - (a - b)*
c) + c*log(-c + x)/(a*b - (a + b)*c + c^2)
```

Fricas [A]
time = 0.70, size = 81, normalized size = 1.19

$$\frac{(a-b)c\log(-c+x)+(ab-ac)\log(-a+x)-(ab-bc)\log(-b+x)}{a^2b-ab^2+(a-b)c^2-(a^2-b^2)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="fricas")
```

```
[Out] ((a - b)*c*log(-c + x) + (a*b - a*c)*log(-a + x) - (a*b - b*c)*log(-b + x))
/(a^2*b - a*b^2 + (a - b)*c^2 - (a^2 - b^2)*c)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 81, normalized size = 1.19

$$\frac{a\log\left(\left|-a+x\right|\right)}{a^2-ab-ac+bc}-\frac{b\log\left(\left|-b+x\right|\right)}{ab-b^2-ac+bc}+\frac{c\log\left(\left|-c+x\right|\right)}{ab-ac-bc+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="giac")
```

```
[Out] a*log(abs(-a + x))/(a^2 - a*b - a*c + b*c) - b*log(abs(-b + x))/(a*b - b^2
- a*c + b*c) + c*log(abs(-c + x))/(a*b - a*c - b*c + c^2)
```

Mupad [B]
time = 0.57, size = 87, normalized size = 1.28

$$\ln\left(x-a\right)\,\left(\frac{b}{\left(a-b\right)\,\left(b-c\right)}-\frac{c}{\left(a-c\right)\,\left(b-c\right)}\right)-\frac{b\,\ln\left(x-b\right)}{\left(a-b\right)\,\left(b-c\right)}+\frac{c\,\ln\left(x-c\right)}{\left(a-c\right)\,\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - x)*(b - x)*(c - x)),x)
```

```
[Out] log(x - a)*(b/((a - b)*(b - c)) - c/((a - c)*(b - c))) - (b*log(x - b))/((a
 - b)*(b - c)) + (c*log(x - c))/((a - c)*(b - c))
```

3.13 $\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx$

Optimal. Leaf size=47

$$-\frac{\log(a^2+x^2)}{2(a^2-b^2)}+\frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

```
[Out] -1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {455, 36, 31}

$$\frac{\log(b^2+x^2)}{2(a^2-b^2)}-\frac{\log(a^2+x^2)}{2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] -1/2*Log[a^2 + x^2]/(a^2 - b^2) + Log[b^2 + x^2]/(2*(a^2 - b^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a^2+x)(b^2+x)}\,dx, x, x^2\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{a^2+x}\,dx, x, x^2\right)}{2(a^2-b^2)} + \frac{\text{Subst}\left(\int \frac{1}{b^2+x}\,dx, x, x^2\right)}{2(a^2-b^2)}$$
$$= -\frac{\log(a^2+x^2)}{2(a^2-b^2)} + \frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.72

$$\frac{-\log(a^2+x^2)+\log(b^2+x^2)}{2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a^2 + x^2)*(b^2 + x^2)),x]
[Out] (-Log[a^2 + x^2] + Log[b^2 + x^2])/(2*(a^2 - b^2))
```

Maple [A]
time = 0.05, size = 44, normalized size = 0.94

method	result	size
default	$-\frac{\ln(a^2+x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$	44
norman	$-\frac{\ln(a^2+x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$	44
risch	$-\frac{\ln(-a^2-x^2)}{2(a^2-b^2)} + \frac{\ln(b^2+x^2)}{2a^2-2b^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)
```

Maxima [A]
time = 2.17, size = 43, normalized size = 0.91

$$-\frac{\log(a^2+x^2)}{2(a^2-b^2)} + \frac{\log(b^2+x^2)}{2(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

Fricas [A]
time = 0.83, size = 32, normalized size = 0.68

$$-\frac{\log\left(a^2+x^2\right)-\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
[Out] -1/2*(log(a^2 + x^2) - log(b^2 + x^2))/(a^2 - b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(36) = 72$.
time = 0.35, size = 121, normalized size = 2.57

$$\frac{\log\left(-\frac{a^4}{2(a-b)(a+b)}+\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}-\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2\left(a-b\right)\left(a+b\right)}-\frac{\log\left(\frac{a^4}{2(a-b)(a+b)}-\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}+\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2\left(a-b\right)\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+x**2)/(b**2+x**2),x)
[Out] log(-a**4/(2*(a - b)*(a + b)) + a**2*b**2/((a - b)*(a + b)) + a**2/2 - b**4
/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b)) - log(a**4/(2*(a
- b)*(a + b)) - a**2*b**2/((a - b)*(a + b)) + a**2/2 + b**4/(2*(a - b)*(a +
 b)) + b**2/2 + x**2)/(2*(a - b)*(a + b))
```

Giac [A]
time = 0.44, size = 43, normalized size = 0.91

$$-\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}+\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
[Out] -1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

Mupad [B]
time = 0.30, size = 256, normalized size = 5.45

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x^2\,\left(8\,a^2+8\,b^2\right)+16\,a^2\,b^2}{2\,\left(a^2-b^2\right)}-4\,x^2\right)\,1{}\mathrm{i}}{2\,\left(a^2-b^2\right)}-\frac{\left(\frac{x^2\,\left(8\,a^2+8\,b^2\right)+16\,a^2\,b^2}{2\,\left(a^2-b^2\right)}+4\,x^2\right)\,1{}\mathrm{i}}{2\,\left(a^2-b^2\right)}}{\frac{\frac{x^2\,\left(8\,a^2+8\,b^2\right)+16\,a^2\,b^2}{2\,\left(a^2-b^2\right)}-4\,x^2}{2\,\left(a^2-b^2\right)}+\frac{\frac{x^2\,\left(8\,a^2+8\,b^2\right)+16\,a^2\,b^2}{2\,\left(a^2-b^2\right)}+4\,x^2}{2\,\left(a^2-b^2\right)}}\right)\,1{}\mathrm{i}}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a^2 + x^2)*(b^2 + x^2)),x)
```

```
[Out] (atan(((((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) - 4*x^2)*1i)/(2
*(a^2 - b^2)) - (((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)) + 4*x^
2)*1i)/(2*(a^2 - b^2)))/(((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 - b^2)
) - 4*x^2)/(2*(a^2 - b^2)) + ((x^2*(8*a^2 + 8*b^2) + 16*a^2*b^2)/(2*(a^2 -
b^2)) + 4*x^2)/(2*(a^2 - b^2))))*1i)/(a^2 - b^2)
```

3.14 $\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx$

Optimal. Leaf size=40

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

```
[Out] a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {492, 209}

$$\frac{a\text{ArcTan}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\text{ArcTan}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx &= \frac{a^2\int \frac{1}{a^2+x^2}\,dx}{a^2-b^2}-\frac{b^2\int \frac{1}{b^2+x^2}\,dx}{a^2-b^2}\\ &= \frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.75

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)-b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/a] - b*ArcTan[x/b])/(a^2 - b^2)
```

Maple [A]
time = 0.06, size = 41, normalized size = 1.02

method	result	size
default	$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$	41
risch	$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2+x^2)/(b^2+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

Maxima [A]
time = 2.41, size = 40, normalized size = 1.00

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

```
[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

Fricas [A]
time = 0.73, size = 30, normalized size = 0.75

$$\frac{a\arctan\left(\frac{x}{a}\right)-b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
```

```
[Out] (a*arctan(x/a) - b*arctan(x/b))/(a^2 - b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 393, normalized size = 9.82

$$-\frac{ia\log\left(-\frac{2ia^7}{(a-b)^3(a+b)^3}+\frac{4ia^5b^2}{(a-b)^3(a+b)^3}-\frac{2ia^3b^4}{(a-b)^3(a+b)^3}+\frac{ia^3}{(a-b)(a+b)}+\frac{iab^2}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}+\frac{ia\log\left(\frac{2ia^7}{(a-b)^3(a+b)^3}-\frac{4ia^5b^2}{(a-b)^3(a+b)^3}+\frac{2ia^3b^4}{(a-b)^3(a+b)^3}-\frac{ia^3}{(a-b)(a+b)}-\frac{iab^2}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}-\frac{ib\log\left(-\frac{2ia^4b^3}{(a-b)^3(a+b)^3}+\frac{4ia^2b^5}{(a-b)^3(a+b)^3}+\frac{ia^2b}{(a-b)(a+b)}-\frac{2ib^7}{(a-b)^3(a+b)^3}+\frac{ib^3}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}+\frac{ib\log\left(\frac{2ia^4b^3}{(a-b)^3(a+b)^3}-\frac{4ia^2b^5}{(a-b)^3(a+b)^3}-\frac{ia^2b}{(a-b)(a+b)}+\frac{2ib^7}{(a-b)^3(a+b)^3}-\frac{ib^3}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] -I*a*log(-2*I*a**7/((a - b)**3*(a + b)**3) + 4*I*a**5*b**2/((a - b)**3*(a +
 b)**3) - 2*I*a**3*b**4/((a - b)**3*(a + b)**3) + I*a**3/((a - b)*(a + b))
+ I*a*b**2/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b)) + I*a*log(2*I*a**7/((
a - b)**3*(a + b)**3) - 4*I*a**5*b**2/((a - b)**3*(a + b)**3) + 2*I*a**3*b*
*4/((a - b)**3*(a + b)**3) - I*a**3/((a - b)*(a + b)) - I*a*b**2/((a - b)*(
a + b)) + x)/(2*(a - b)*(a + b)) - I*b*log(-2*I*a**4*b**3/((a - b)**3*(a +
b)**3) + 4*I*a**2*b**5/((a - b)**3*(a + b)**3) + I*a**2*b/((a - b)*(a + b))
 - 2*I*b**7/((a - b)**3*(a + b)**3) + I*b**3/((a - b)*(a + b)) + x)/(2*(a -
 b)*(a + b)) + I*b*log(2*I*a**4*b**3/((a - b)**3*(a + b)**3) - 4*I*a**2*b**
5/((a - b)**3*(a + b)**3) - I*a**2*b/((a - b)*(a + b)) + 2*I*b**7/((a - b)*
*3*(a + b)**3) - I*b**3/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b))
```

Giac [A]
time = 0.45, size = 40, normalized size = 1.00

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
```

```
[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

Mupad [B]
time = 0.18, size = 191, normalized size = 4.78

$$-\frac{a\,\mathrm{atan}\left(\frac{x\,(2\,a^4+2\,b^4)-\frac{a^2\,x\,\left(8\,a^6-8\,a^4\,b^2-8\,a^2\,b^4+8\,b^6\right)}{\left(2\,a^2-2\,b^2\right)^2}}{a\,b^2\,(2\,a^2-2\,b^2)}\right)}{a^2-b^2}-\frac{b\,\mathrm{atan}\left(\frac{x\,(2\,a^4+2\,b^4)-\frac{b^2\,x\,\left(8\,a^6-8\,a^4\,b^2-8\,a^2\,b^4+8\,b^6\right)}{\left(2\,a^2-2\,b^2\right)^2}}{a^2\,b\,(2\,a^2-2\,b^2)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a^2 + x^2)*(b^2 + x^2)),x)
```

```
[Out] - (a*atan((x*(2*a^4 + 2*b^4) - (a^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b^
2))/(2*a^2 - 2*b^2)^2)/(a*b^2*(2*a^2 - 2*b^2))))/(a^2 - b^2) - (b*atan((x*(
2*a^4 + 2*b^4) - (b^2*x*(8*a^6 + 8*b^6 - 8*a^2*b^4 - 8*a^4*b^2))/(2*a^2 - 2
*b^2)^2)/(a^2*b*(2*a^2 - 2*b^2))))/(a^2 - b^2)
```

3.15 $\int \frac{x}{(-1+x)(1+x^2)}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1-x)-\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2*arctan(x)+1/2*ln(1-x)-1/4*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {815, 649, 209, 266}

$$\frac{\text{ArcTan}(x)}{2}-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{(-1+x)\left(1+x^2\right)} \, dx &= \int \left(\frac{1}{2(-1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right) dx \\
&= \frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1-x}{1+x^2} \, dx \\
&= \frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{1+x^2} \, dx-\frac{1}{2}\int \frac{x}{1+x^2} \, dx \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1-x)-\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$
\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1-x)-\frac{1}{4}\log\left(1+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.74

method	result	size
default	$\frac{\ln(-1+x)}{2}-\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{2}$	20
risch	$\frac{\ln(-1+x)}{2}-\frac{\ln(x^2+1)}{4}+\frac{\arctan(x)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+x)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(-1+x)-1/4*ln(x^2+1)+1/2*arctan(x)
```

Maxima [A]
time = 2.52, size = 19, normalized size = 0.70

$$
\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Fricas [A]
time = 0.90, size = 19, normalized size = 0.70

$$\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^{2} + 1\right) + \frac{1}{2}\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.70

$$\frac{\log\left(x - 1\right)}{2} - \frac{\log\left(x^{2} + 1\right)}{4} + \frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x**2+1),x)
[Out] log(x - 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Giac [A]
time = 0.50, size = 20, normalized size = 0.74

$$\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x^{2} + 1\right) + \frac{1}{2}\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.93

$$\frac{\ln\left(x - 1\right)}{2} + \ln\left(x - \mathrm{i}\right)\left(-\frac{1}{4} - \frac{1}{4}\mathrm{i}\right) + \ln\left(x + 1\mathrm{i}\right)\left(-\frac{1}{4} + \frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x - 1)),x)
[Out] log(x - 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

3.16 $\int \frac{x}{1+x^3}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {298, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right)+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx\\ &= -\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\ &= -\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3),x]
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A]
time = 0.03, size = 35, normalized size = 0.85

method	result	size
default	$-\frac{\ln(1+x)}{3}+\frac{\ln(x^2-x+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	35
risch	$\frac{\ln(4x^2-4x+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\ln(1+x)}{3}$	37
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 2.90, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\,\log\left(x^2-x+1\right)-\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Fricas [A]
time = 0.65, size = 34, normalized size = 0.83

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\,\log\left(x^2-x+1\right)-\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.00

$$-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

Giac [A]
time = 0.50, size = 35, normalized size = 0.85

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

Mupad [B]
time = 0.21, size = 46, normalized size = 1.12

$$-\frac{\ln\left(x+1\right)}{3}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/6) - log(x - (3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x + 1)/3
```

3.17 $\int \frac{x^3}{(-1+x)^2(1+x^3)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)$$

```
[Out] 1/2/(1-x)+3/4*ln(1-x)-1/12*ln(1+x)-1/3*ln(x^2-x+1)
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6857, 642}

$$-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-1 + x)^2*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^2]/3
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(-1+x)^2\left(1+x^3\right)}\,dx &= \int \left(\frac{1}{2(-1+x)^2}+\frac{3}{4(-1+x)}-\frac{1}{12(1+x)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx\\ &= \frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(1+x)+\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx\\ &= \frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.79

$$\frac{1}{12}\left(-\frac{6}{-1+x}+9\log(-1+x)-\log(1+x)-4\log\left((-1+x)^2+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-1 + x)^2*(1 + x^3)),x]
[Out] (-6/(-1 + x) + 9*Log[-1 + x] - Log[1 + x] - 4*Log[(-1 + x)^2 + x])/12
```

Maple [A]
time = 0.04, size = 32, normalized size = 0.74

method	result	size
default	$-\frac{1}{2(-1+x)}+\frac{3\ln(-1+x)}{4}-\frac{\ln(1+x)}{12}-\frac{\ln(x^2-x+1)}{3}$	32
norman	$-\frac{1}{2(-1+x)}+\frac{3\ln(-1+x)}{4}-\frac{\ln(1+x)}{12}-\frac{\ln(x^2-x+1)}{3}$	32
risch	$-\frac{1}{2(-1+x)}+\frac{3\ln(-1+x)}{4}-\frac{\ln(1+x)}{12}-\frac{\ln(x^2-x+1)}{3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-1+x)^2/(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/2/(-1+x)+3/4*ln(-1+x)-1/12*ln(1+x)-1/3*ln(x^2-x+1)
```

Maxima [A]
time = 2.66, size = 31, normalized size = 0.72

$$-\frac{1}{2\,(x-1)}-\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{1}{12}\,\log\left(x+1\right)+\frac{3}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="maxima")
[Out] -1/2/(x - 1) - 1/3*log(x^2 - x + 1) - 1/12*log(x + 1) + 3/4*log(x - 1)
```

Fricas [A]
time = 0.52, size = 40, normalized size = 0.93

$$-\frac{4\,(x-1)\log\left(x^2-x+1\right)+(x-1)\log\left(x+1\right)-9\,(x-1)\log\left(x-1\right)+6}{12\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/12*(4*(x - 1)*log(x^2 - x + 1) + (x - 1)*log(x + 1) - 9*(x - 1)*log(x -
1) + 6)/(x - 1)
```

Sympy [A]
time = 0.05, size = 31, normalized size = 0.72

$$\frac{3\log\left(x-1\right)}{4}-\frac{\log\left(x+1\right)}{12}-\frac{\log\left(x^{2}-x+1\right)}{3}-\frac{1}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-1+x)**2/(x**3+1),x)
```

```
[Out] 3*log(x - 1)/4 - log(x + 1)/12 - log(x**2 - x + 1)/3 - 1/(2*x - 2)
```

Giac [A]
time = 0.49, size = 36, normalized size = 0.84

$$-\frac{1}{2\left(x-1\right)}-\frac{1}{3}\log\left(\frac{1}{x-1}+\frac{1}{\left(x-1\right)^{2}}+1\right)-\frac{1}{12}\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/2/(x - 1) - 1/3*log(1/(x - 1) + 1/(x - 1)^2 + 1) - 1/12*log(abs(-2/(x -
1) - 1))
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.77

$$\frac{3\ \ln\left(x-1\right)}{4}-\frac{\ln\left(x+1\right)}{12}-\frac{\ln\left(x^{2}-x+1\right)}{3}-\frac{1}{2\ \left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((x^3 + 1)*(x - 1)^2),x)
```

```
[Out] (3*log(x - 1))/4 - log(x + 1)/12 - log(x^2 - x + 1)/3 - 1/(2*(x - 1))
```

3.18 $\int \frac{1}{1+x^4}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+x^4}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}}\\
&= -\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{\text{Subst}}{}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 64, normalized size = 0.75

$$
\frac{-2\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\tan^{-1}\left(1+\sqrt{2}\,x\right)-\log\left(1-\sqrt{2}\,x+x^2\right)+\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A]
time = 0.00, size = 52, normalized size = 0.61

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(-\_R+x\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}$
meijerg	$-\frac{x\sqrt{2}\ln\left(1-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^4\right)^{\frac{1}{4}}}\right)}{4\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln\left(1+\sqrt{2}\left(x^4\right)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8\left(x^4\right)^{\frac{1}{4}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+2*arctan(1+x*2^(1/2))+
2*arctan(-1+x*2^(1/2)))
```

Maxima [A]
time = 2.16, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A]
time = 0.62, size = 100, normalized size = 1.18

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(4x^2+4\sqrt{2}x+4\right)-\frac{1}{8}\sqrt{2}\log\left(4x^2-4\sqrt{2}x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
```

```
*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) - 1/8*sqrt(2)*log(4*x^2 - 4*sqrt(2)*x
 + 4)
```

Sympy [A]
time = 0.06, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Giac [A]
time = 0.46, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B]
time = 0.00, size = 33, normalized size = 0.39

$$\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.19 $\int \frac{x^2}{1+x^4}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)-1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^4),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx+\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}}\\
&= \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{\mathrm{Subst}\left(\int\right.}{}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 64, normalized size = 0.75

$$\frac{-2\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\tan^{-1}\left(1+\sqrt{2}\,x\right)+\log\left(1-\sqrt{2}\,x+x^2\right)-\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] + Log[1 - Sqrt[2]*x + x
^2] - Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A]
time = 0.02, size = 52, normalized size = 0.61

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(-\_R+x\right)}{\_R}\right)}{4}$
default	$\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}$
meijerg	$\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(ln((1+x^2-x*2^(1/2))/(1+x^2+x*2^(1/2)))+2*arctan(1+x*2^(1/2))+
2*arctan(-1+x*2^(1/2)))
```

Maxima [A]
time = 2.52, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{1}{8}\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A]
time = 0.50, size = 100, normalized size = 1.18

$$-\frac{1}{2}\sqrt{2}\,\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}-1\right)-\frac{1}{2}\sqrt{2}\,\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2-\sqrt{2}\,x+1}+1\right)-\frac{1}{8}\sqrt{2}\,\log\left(4x^2+4\sqrt{2}\,x+4\right)+\frac{1}{8}\sqrt{2}\,\log\left(4x^2-4\sqrt{2}\,x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/8
```

```
*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) + 1/8*sqrt(2)*log(4*x^2 - 4*sqrt(2)*x
 + 4)
```

Sympy [A]
time = 0.05, size = 73, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{8}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 +
 sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Giac [A]
time = 0.45, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B]
time = 0.21, size = 33, normalized size = 0.39

$$\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 - 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 + 1i/4)
```

3.20 $\int \frac{1}{1+x^2+x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(1-x+x^2\right)+\frac{1}{4}\log\left(1+x+x^2\right)$$

```
[Out] -1/4*ln(x^2-x+1)+1/4*ln(x^2+x+1)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/
6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1-x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1+x}{1+x+x^2}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{1}{4}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\frac{1}{4}\log\left(1-x+x^2\right)+\frac{1}{4}\log\left(1+x+x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\text{S}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(1-x+x^2\right)+\frac{1}{4}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 73, normalized size = 1.09

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)-\sqrt{1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*
ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A]
time = 0.02, size = 54, normalized size = 0.81

method	result	size

default	$\frac{\ln(x^2+x+1)}{4}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(x^2-x+1)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	54
risch	$-\frac{\ln(4x^2-4x+4)}{4}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(4x^2+4x+4)}{4}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(x^2+x+1)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/4*ln(x^2-x+1)+1/6
*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 2.88, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Fricas [A]
time = 0.60, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Sympy [A]
time = 0.09, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Giac [A]
time = 0.48, size = 53, normalized size = 0.79

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Mupad [B]
time = 0.18, size = 47, normalized size = 0.70

$$\mathrm{atanh}\left(\frac{2x}{-1+\sqrt{3}\ 1\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)+\mathrm{atanh}\left(\frac{2x}{1+\sqrt{3}\ 1\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^4 + 1),x)
```

```
[Out] atanh((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/6 - 1/2) + atanh((2*x)/(3^(1/2)
*1i + 1))*((3^(1/2)*1i)/6 + 1/2)
```

3.21 $\int (a+bx)^p \, dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{1+p}}{b(1+p)}$$

```
[Out] (b*x+a)^(1+p)/b/(1+p)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^p,x]
```

```
[Out] (a + b*x)^(1 + p)/(b*(1 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^p \, dx = \frac{(a+bx)^{1+p}}{b(1+p)}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{1+p}}{b+bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^p,x]
```

```
[Out] (a + b*x)^(1 + p)/(b + b*p)
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.06

method	result	size
gosper	$\frac{(bx+a)^{1+p}}{b(1+p)}$	19
default	$\frac{(bx+a)^{1+p}}{b(1+p)}$	19
risch	$\frac{(bx+a)(bx+a)^{p}}{b(1+p)}$	22
norman	$\frac{x\,e^{p\ln(bx+a)}}{1+p}+\frac{a\,e^{p\ln(bx+a)}}{b(1+p)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+p)/b/(1+p)
```

Maxima [A]
time = 1.50, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(p + 1)/(b*(p + 1))
```

Fricas [A]
time = 0.51, size = 20, normalized size = 1.11

$$\frac{(bx+a)(bx+a)^{p}}{bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^p/(b*p + b)
```

Sympy [A]
time = 0.01, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**p,x)
[Out] Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x), True))/b
```

Giac [A]
time = 0.45, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="giac")
[Out] (b*x + a)^(p + 1)/(b*(p + 1))
```

Mupad [B]
time = 0.22, size = 18, normalized size = 1.00

$$\frac{(a+b\,x)^{p+1}}{b\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^p,x)
[Out] (a + b*x)^(p + 1)/(b*(p + 1))
```

3.22 $\int x(a+bx)^p\,dx$

Optimal. Leaf size=39

$$-\frac{a(a+bx)^{1+p}}{b^2(1+p)}+\frac{(a+bx)^{2+p}}{b^2(2+p)}$$

```
[Out] -a*(b*x+a)^(1+p)/b^2/(1+p)+(b*x+a)^(2+p)/b^2/(2+p)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^p,x]
```

```
[Out] -((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^p\,dx&=\int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx\\&=-\frac{a(a+bx)^{1+p}}{b^2(1+p)}+\frac{(a+bx)^{2+p}}{b^2(2+p)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{1+p}(-a+b(1+p)x)}{b^2(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.92

method	result	size
gosper	$-\frac{(bx+a)^{1+p}(-xpb-bx+a)}{b^2(p^2+3p+2)}$	36
risch	$-\frac{(-x^2b^2p-xapb-x^2b^2+a^2)(bx+a)^p}{b^2(2+p)(1+p)}$	50
norman	$\frac{x^2 e^{p\ln(bx+a)}}{2+p}+\frac{pax\, e^{p\ln(bx+a)}}{b(p^2+3p+2)}-\frac{a^2 e^{p\ln(bx+a)}}{b^2(p^2+3p+2)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^p,x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+p)*(-b*p*x-b*x+a)/b^2/(p^2+3*p+2)
```

Maxima [A]
time = 1.16, size = 42, normalized size = 1.08

$$\frac{(b^2(p+1)x^2+abpx-a^2)(bx+a)^p}{(p^2+3\,p+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="maxima")
[Out] (b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2)
```

Fricas [A]
time = 0.67, size = 53, normalized size = 1.36

$$\frac{(abpx+(b^2p+b^2)x^2-a^2)(bx+a)^p}{b^2p^2+3\,b^2p+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="fricas")
[Out] (a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(31)=62$.
time = 0.23, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^p x^2}{2} & \text{for}\ b=0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b^3x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\ p=-2 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{x}{b} & \text{for}\ p=-1 \\ -\frac{a^2(a+bx)^p}{b^2p^2+3b^2p+2b^2}+\frac{abpx(a+bx)^p}{b^2p^2+3b^2p+2b^2}+\frac{b^2px^2(a+bx)^p}{b^2p^2+3b^2p+2b^2}+\frac{b^2x^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p +
 2*b**2) + a*b*p*x*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*p*x*
*2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)**p/(b
**2*p**2 + 3*b**2*p + 2*b**2), True))
```

Giac [A]
time = 0.44, size = 76, normalized size = 1.95

$$\frac{(bx+a)^p b^2 p x^2 + (bx+a)^p abpx + (bx+a)^p b^2 x^2 - (bx+a)^p a^2}{b^2 p^2 + 3\,b^2 p + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((b*x + a)^p*b^2*p*x^2 + (b*x + a)^p*a*b*p*x + (b*x + a)^p*b^2*x^2 - (b*x +
 a)^p*a^2)/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

Mupad [B]
time = 0.39, size = 94, normalized size = 2.41

$$\begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if } p=-1\\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if } p=-2\\ \frac{2\left(\frac{(a+b\,x)^{p+2}}{2\,p+4}-\frac{a\,(a+b\,x)^{p+1}}{2\,p+2}\right)}{b^2} & \text{if } p\neq -1\wedge p\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^p,x)
```

```
[Out] piecewise(p == -1, -(a*log(a + b*x) - b*x)/b^2, p == -2, (log(a + b*x) + a/
(a + b*x))/b^2, p ~= -1 & p ~= -2, (2*((a + b*x)^(p + 2)/(2*p + 4) - (a*(a
+ b*x)^(p + 1))/(2*p + 2)))/b^2)
```

3.23 $\int x^2(a+bx)^p\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{1+p}}{b^3(1+p)}-\frac{2a(a+bx)^{2+p}}{b^3(2+p)}+\frac{(a+bx)^{3+p}}{b^3(3+p)}$$

```
[Out] a^2*(b*x+a)^(1+p)/b^3/(1+p)-2*a*(b*x+a)^(2+p)/b^3/(2+p)+(b*x+a)^(3+p)/b^3/(
3+p)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^p,x]
```

```
[Out] (a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(2 + p
)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^p\,dx&=\int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{1+p}}{b^3(1+p)}-\frac{2a(a+bx)^{2+p}}{b^3(2+p)}+\frac{(a+bx)^{3+p}}{b^3(3+p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{1+p}\left(2a^2-2ab(1+p)x+b^2\left(2+3p+p^2\right)x^2\right)}{b^3(1+p)(2+p)(3+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/(b^
3*(1 + p)*(2 + p)*(3 + p))
```

Maple [A]
time = 0.03, size = 73, normalized size = 1.22

method	result	size
gosper	$\frac{(bx+a)^{1+p}(b^2p^2x^2+3x^2b^2p-2xapb+2x^2b^2-2axb+2a^2)}{b^3(p^3+6p^2+11p+6)}$	73
risch	$\frac{(b^3p^2x^3+a\,b^2p^2x^2+3b^3p\,x^3+x^2ap\,b^2+2x^3b^3-2a^2pxb+2a^3)(bx+a)^p}{(2+p)(3+p)(1+p)b^3}$	88
norman	$\frac{x^3\mathrm{e}^{p\ln(bx+a)}}{3+p}+\frac{ap\,x^2\mathrm{e}^{p\ln(bx+a)}}{b(p^2+5p+6)}+\frac{2a^3\mathrm{e}^{p\ln(bx+a)}}{b^3(p^3+6p^2+11p+6)}-\frac{2p\,a^2x\,\mathrm{e}^{p\ln(bx+a)}}{b^2(p^3+6p^2+11p+6)}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+p)*(b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(p^3+6*p^2+11*p+6)
```

Maxima [A]
time = 2.16, size = 68, normalized size = 1.13

$$\frac{((p^2+3\,p+2)b^3x^3+(p^2+p)ab^2x^2-2\,a^2bpx+2\,a^3)(bx+a)^p}{(p^3+6\,p^2+11\,p+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 + 3*p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*a^3)*(b*x
+ a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Fricas [A]
time = 0.58, size = 96, normalized size = 1.60

$$-\frac{(2\,a^2bpx-(b^3p^2+3\,b^3p+2\,b^3)x^3-2\,a^3-(ab^2p^2+ab^2p)x^2)(bx+a)^p}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b^
2*p)*x^2)*(b*x + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(51) = 102$.
time = 0.37, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^p x^3}{3} & \text{for } b = 0 \\ \frac{2a^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} & \text{for } p = -3 \\ -\frac{2a^2 \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} & \text{for } p = -2 \\ \frac{a^2 \log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ \frac{2a^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} - \frac{2a^2bpx(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2p^2x^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{ab^2px^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{b^3p^2x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{3b^3px^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} + \frac{2b^3x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b**3 +
b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x)
 + b**2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3 - a*x/b
**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2
 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p/(b**3*p**3 + 6*b**3*p**2
 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3*
p**2 + 11*b**3*p + 6*b**3) + a*b**2*p*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**3
*p**2 + 11*b**3*p + 6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3*p**3 + 6*b*
*3*p**2 + 11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b**3*p**3 + 6*b
**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b**3*p**3 + 6*b*
*3*p**2 + 11*b**3*p + 6*b**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(60) = 120$.
time = 0.50, size = 140, normalized size = 2.33

$$\frac{(bx+a)^p b^3 p^2 x^3 + (bx+a)^p a b^2 p^2 x^2 + 3\,(bx+a)^p b^3 p x^3 + (bx+a)^p a b^2 p x^2 + 2\,(bx+a)^p b^3 x^3 - 2\,(bx+a)^p a^2 b p x + 2\,(bx+a)^p a^3}{b^3 p^3 + 6\,b^3 p^2 + 11\,b^3 p + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((b*x + a)^p*b^3*p^2*x^3 + (b*x + a)^p*a*b^2*p^2*x^2 + 3*(b*x + a)^p*b^3*p*
x^3 + (b*x + a)^p*a*b^2*p*x^2 + 2*(b*x + a)^p*b^3*x^3 - 2*(b*x + a)^p*a^2*b
*p*x + 2*(b*x + a)^p*a^3)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Mupad [B]

time = 0.60, size = 192, normalized size = 3.20

$$\begin{cases} \frac{2 a^2 \ln(a+b x)+b^2 x^2-2 a b x}{2 b^3} & \text{if} \;\; p=-1 \\ \frac{x}{b^2}-\frac{a^2}{b^3 (a+b x)}-\frac{2 a \ln(a+b x)}{b^3} & \text{if} \;\; p=-2 \\ \frac{\ln(a+b x)+\frac{2 a}{a+b x}-\frac{a^2}{2 (a+b x)^2}}{b^3} & \text{if} \;\; p=-3 \\ \frac{2 (a+b x)^{p+1} (8 a^2-8 a b p x-8 a b x+4 b^2 p^2 x^2+12 b^2 p x^2+8 b^2 x^2)}{b^3 (8 p^3+48 p^2+88 p+48)} & \text{if} \;\; p \neq -1 \land p \neq -2 \land p \neq -3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^p,x)
```

```
[Out] piecewise(p == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), p == -
2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, p == -3, (log(a +
b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, p ~= -1 & p ~= -2 & p ~=
 -3, (2*(a + b*x)^(p + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*p*x^2 - 8*a*b*x + 4*b
^2*p^2*x^2 - 8*a*b*p*x))/(b^3*(88*p + 48*p^2 + 8*p^3 + 48)))
```

3.24 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

[Out] ln(b*x+a)/b

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Maple [A]
time = 0.02, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A]
time = 1.53, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A]
time = 0.57, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A]
time = 0.47, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

Mupad [B]
time = 0.03, size = 10, normalized size = 1.00

$$\frac{\ln\left(a + b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

3.25 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Maple [A]
time = 0.02, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A]
time = 0.96, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A]
time = 0.82, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A]
time = 0.47, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

Mupad [B]
time = 0.12, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
```

```
[Out] -1/(b*(a + b*x))
```

3.26 $\int \frac{x}{a+bx}\,dx$

Optimal. Leaf size=18

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx}\,dx &= \int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx\\ &= \frac{x}{b}-\frac{a\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] x/b-a*ln(b*x+a)/b^2
```

Maxima [A]
time = 2.66, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A]
time = 0.61, size = 17, normalized size = 0.94

$$\frac{bx-a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a+bx\right)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

<u>Giac</u> [**A**]
time = 0.43, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a\log\left(|bx + a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
[Out] x/b - a*log(abs(b*x + a))/b^2
```

<u>Mupad</u> [**B**]
time = 0.03, size = 18, normalized size = 1.00

$$-\frac{a\,\ln\left(a+b\,x\right)-b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x),x)
[Out] -(a*log(a + b*x) - b*x)/b^2
```

3.27 $\int \frac{x^2}{a+bx}\,dx$

Optimal. Leaf size=31

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx}\,dx &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A]
time = 0.02, size = 30, normalized size = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}x^2b+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
norman	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*x^2*b+a*x)+a^2*ln(b*x+a)/b^3
```

Maxima [A]
time = 1.99, size = 29, normalized size = 0.94

$$\frac{a^2\log(bx+a)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A]
time = 0.75, size = 29, normalized size = 0.94

$$\frac{b^2x^2-2\,abx+2\,a^2\log(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A]
time = 0.04, size = 26, normalized size = 0.84

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Giac [**A**]
time = 0.46, size = 30, normalized size = 0.97

$$\frac{a^2 \log(|bx + a|)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [**B**]
time = 0.13, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln(a + b\,x) + b^2\,x^2 - 2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.28 $\int \frac{1}{x(a+bx)} \, dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)} \, dx &= \frac{\int \frac{1}{x} \, dx}{a} - \frac{b \int \frac{1}{a+bx} \, dx}{a}\\ &= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.06

method	result	size
default	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$	19
risch	$\frac{\ln(-x)}{a} - \frac{\ln(bx+a)}{a}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Maxima [A]
time = 1.77, size = 18, normalized size = 1.00

$$-\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A]
time = 0.75, size = 16, normalized size = 0.89

$$-\frac{\log(bx+a) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.56

$$\frac{\log\left(x\right) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [A]
time = 0.46, size = 20, normalized size = 1.11

$$-\frac{\log\left(|bx + a|\right)}{a} + \frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B]
time = 0.13, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)),x)
```

```
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

3.29 $\int \frac{1}{x^2(a+bx)} dx$

Optimal. Leaf size=28

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A]
time = 2.36, size = 28, normalized size = 1.00

$$\frac{b\log\left(bx+a\right)}{a^2}-\frac{b\log\left(x\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A]
time = 1.05, size = 26, normalized size = 0.93

$$\frac{bx\log\left(bx+a\right)-bx\log\left(x\right)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A]
time = 0.07, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A]
time = 0.44, size = 30, normalized size = 1.07

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)),x)
```

```
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.30 $\int \frac{1}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A]
time = 0.03, size = 43, normalized size = 1.02

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b}{a^{2} \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	43
risch	$\frac{-\frac{2 b x}{a^{2}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(-b x -a \right)}{a^{3}}$	49
norman	$\frac{\frac{2 b^{2} x^{2}}{a^{3}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Maxima [A]
time = 2.64, size = 45, normalized size = 1.07

$$-\frac{2 \, b x + a}{a^{2} b x^{2} + a^{3} x} + \frac{2 \, b \log\left(b x + a\right)}{a^{3}} - \frac{2 \, b \log\left(x\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A]
time = 0.89, size = 63, normalized size = 1.50

$$-\frac{2 \, a b x + a^{2} - 2 \, {\left(b^{2} x^{2} + a b x\right)} \log\left(b x + a\right) + 2 \, {\left(b^{2} x^{2} + a b x\right)} \log\left(x\right)}{a^{3} b x^{2} + a^{4} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A]
time = 0.11, size = 37, normalized size = 0.88

$$\frac{-a - 2bx}{a^{3} x + a^{2} b x^{2}} + \frac{2b\left(-\log{\left(x \right)} + \log{\left(\frac{a}{b} + x \right)}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Giac [A]
time = 0.46, size = 52, normalized size = 1.24

$$-\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3}-\frac{b}{(bx+a)a^2}+\frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

Mupad [B]
time = 0.19, size = 45, normalized size = 1.07

$$\frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3}-\frac{1}{a\,x\,\left(a+b\,x\right)}-\frac{2\,b}{a^2\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^2),x)
```

```
[Out] (2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

3.31 $\int \frac{1}{c^2+x^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

[Out] arctan(x/c)/c

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {209}

$$\frac{\text{ArcTan}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(c^2 + x^2)^(-1),x]

[Out] ArcTan[x/c]/c

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^2+x^2} \, dx = \frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(c^2 + x^2)^(-1),x]

[Out] ArcTan[x/c]/c

Maple [A]
time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{c}\right)}{c}$	11
risch	$\frac{\arctan\left(\frac{x}{c}\right)}{c}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x/c)/c
```

Maxima [A]
time = 1.84, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/c)/c
```

Fricas [A]
time = 0.70, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/c)/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.04, size = 20, normalized size = 2.00

$$\frac{-\frac{i\log\left(-ic+x\right)}{2}+\frac{i\log\left(ic+x\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2+x**2),x)
```

```
[Out] (-I*log(-I*c + x)/2 + I*log(I*c + x)/2)/c
```

Giac [A]
time = 0.45, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/c)/c
```

Mupad [B]
time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2 + x^2),x)
```

```
[Out] atan(x/c)/c
```

3.32 $\int \frac{1}{c^2-x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

```
[Out] arctanh(x/c)/c
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {212}

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x/c]/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^2-x^2}\,dx = \frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x/c]/c
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.02, size = 22, normalized size = 2.20

method	result	size
default	$\frac{\ln(c+x)}{2c} - \frac{\ln(c-x)}{2c}$	22
norman	$\frac{\ln(c+x)}{2c} - \frac{\ln(c-x)}{2c}$	22
risch	$-\frac{\ln(-c+x)}{2c} + \frac{\ln(c+x)}{2c}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2-x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*ln(c+x)-1/2/c*ln(c-x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 6.07, size = 21, normalized size = 2.10

$$\frac{\log\left(c+x\right)}{2\,c} - \frac{\log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c + x)/c - 1/2*log(-c + x)/c
```

Fricas [A]
time = 0.69, size = 18, normalized size = 1.80

$$\frac{\log\left(c+x\right) - \log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(c + x) - log(-c + x))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.04, size = 15, normalized size = 1.50

$$-\frac{\frac{\log\left(-c+x\right)}{2} - \frac{\log\left(c+x\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2-x**2),x)
```

```
[Out] -(log(-c + x)/2 - log(c + x)/2)/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.46, size = 23, normalized size = 2.30

$$\frac{\log\left(|c+x|\right)}{2\,c}-\frac{\log\left(|-c+x|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c + x))/c - 1/2*log(abs(-c + x))/c
```

Mupad [B]
time = 0.16, size = 10, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2 - x^2),x)
```

```
[Out] atanh(x/c)/c
```

3.33 $\int \frac{1}{-1+2x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{2}\,x}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}-\frac{\log\left(1+\sqrt[3]{2}\,x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

```
[Out] 1/6*ln(1-2^(1/3)*x)*2^(2/3)-1/12*ln(1+2^(1/3)*x+2^(2/3)*x^2)*2^(2/3)-1/6*ar
ctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{2}\,x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}\,x+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) + Log[1 - 2^(1/3)*x]
/(3*2^(1/3)) - Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(6*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-1+2x^3}\,dx &= \frac{1}{3}\int \frac{1}{-1+\sqrt[3]{2}\,x}\,dx + \frac{1}{3}\int \frac{-2-\sqrt[3]{2}\,x}{1+\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx \\
&= \frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}} - \frac{1}{2}\int \frac{1}{1+\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx - \frac{\int \frac{\sqrt[3]{2}+2\ 2^{2/3}x}{1+\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx}{6\sqrt[3]{2}} \\
&= \frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\sqrt[3]{2}\,x+2^{2/3}x^2\right)}{6\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{2}\,x\right)}{\sqrt[3]{2}} \\
&= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{2}\,x}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} + \frac{\log\left(1-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}} - \frac{\log\left(1+\sqrt[3]{2}\,x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 66, normalized size = 0.85

$$-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{2}\,x}{\sqrt{3}}\right) - 2\log\left(1-\sqrt[3]{2}\,x\right) + \log\left(1+\sqrt[3]{2}\,x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^3)^(-1),x]
```

[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]] - 2*Log[1 - 2^(1/3)*x] + Log[1 + 2^(1/3)*x + 2^(2/3)*x^2])/2^(1/3)

Maple [A]
time = 0.03, size = 58, normalized size = 0.74

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^{3}-1\right)}\frac{\ln\left(-\_R+x\right)}{\_R^{2}}\right)}{6}$	24
default	$\frac{2^{\frac{2}{3}}\ln\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\ln\left(x^{2}+\frac{2^{\frac{2}{3}}x}{2}+\frac{2^{\frac{1}{3}}}{2}\right)}{12}-\frac{\arctan\left(\frac{\left(1+2\,2^{\frac{1}{3}}x\right)\sqrt{3}}{3}\right)2^{\frac{2}{3}}\sqrt{3}}{6}$	58
meijerg	$\frac{2^{\frac{2}{3}}x\left(\ln\left(1-2^{\frac{1}{3}}\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+2^{\frac{1}{3}}\left(x^{3}\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{2+2^{\frac{1}{3}}\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{6\left(x^{3}\right)^{\frac{1}{3}}}$	82

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^3-1),x,method=_RETURNVERBOSE)

[Out] 1/6*2^(2/3)*ln(x-1/2*2^(2/3))-1/12*2^(2/3)*ln(x^2+1/2*2^(2/3)*x+1/2*2^(1/3))-1/6*arctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/2)

Maxima [A]
time = 4.65, size = 66, normalized size = 0.85

$$-\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}x^{2}+2^{\frac{1}{3}}x+1\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{1}{2}\cdot 2^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^3-1),x, algorithm="maxima")

[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*2^(2/3)*x + 2^(1/3))) - 1/12*2^(2/3)*log(2^(2/3)*x^2 + 2^(1/3)*x + 1) + 1/6*2^(2/3)*log(1/2*2^(2/3)*(2^(1/3)*x - 1))

Fricas [A]
time = 0.73, size = 63, normalized size = 0.81

$$-\frac{1}{6}\,\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\,\sqrt{6}\,2^{\frac{1}{6}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2\,x^{2}+2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(2\,x-2^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*x^3-1),x, algorithm="fricas")

```
[Out] -1/6*sqrt(6)*2^(1/6)*arctan(1/6*sqrt(6)*2^(1/6)*(2*2^(2/3)*x + 2^(1/3))) -
1/12*2^(2/3)*log(2*x^2 + 2^(2/3)*x + 2^(1/3)) + 1/6*2^(2/3)*log(2*x - 2^(2/
3))
```

Sympy [A]
time = 0.14, size = 78, normalized size = 1.00

$$\frac{2^{\frac{2}{3}}\log\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x}{2}+\frac{\sqrt[3]{2}}{2}\right)}{12}-\frac{2^{\frac{2}{3}}\sqrt{3}\,\operatorname{atan}\left(\frac{2\cdot\sqrt[3]{2}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**3-1),x)
```

```
[Out] 2**(2/3)*log(x - 2**(2/3)/2)/6 - 2**(2/3)*log(x**2 + 2**(2/3)*x/2 + 2**(1/3
)/2)/12 - 2**(2/3)*sqrt(3)*atan(2*2**(1/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Giac [A]
time = 0.46, size = 57, normalized size = 0.73

$$-\frac{1}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{1}{2}\right)^{\frac{2}{3}}\left(2x+\left(\frac{1}{2}\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 4^{\frac{1}{3}}\log\left(x^2+\left(\frac{1}{2}\right)^{\frac{1}{3}}x+\left(\frac{1}{2}\right)^{\frac{2}{3}}\right)+\frac{1}{3}\left(\frac{1}{2}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{1}{2}\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^3-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(1/2)^(1/3)*arctan(2/3*sqrt(3)*(1/2)^(2/3)*(2*x + (1/2)^(1/3))
) - 1/12*4^(1/3)*log(x^2 + (1/2)^(1/3)*x + (1/2)^(2/3)) + 1/3*(1/2)^(1/3)*l
og(abs(x - (1/2)^(1/3)))
```

Mupad [B]
time = 0.28, size = 72, normalized size = 0.92

$$\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}}{2}\right)}{6}+\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^3 - 1),x)
```

```
[Out] (2^(2/3)*log(x - 2^(2/3)/2))/6 + (2^(2/3)*log(x - (2^(2/3)*(3^(1/2)*1i - 1)
)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 1))/4)*
(3^(1/2)*1i + 1))/12
```

3.34 $\int \frac{1}{-2+x^3} \, dx$

Optimal. Leaf size=74

$$-\frac{\tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\log\left(2^{2/3}+\sqrt[3]{2}\,x+x^2\right)}{6\ 2^{2/3}}$$

```
[Out] 1/6*ln(2^(1/3)-x)*2^(1/3)-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arctan
(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(x^2+\sqrt[3]{2}\,x+2^{2/3}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[2^(1/3) - x]/(3*
2^(2/3)) - Log[2^(2/3) + 2^(1/3)*x + x^2]/(6*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-2+x^3}\,dx &= \frac{\int \frac{1}{-\sqrt[3]{2}+x}\,dx}{3\ 2^{2/3}}+\frac{\int \frac{-2\sqrt[3]{2}-x}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx}{3\ 2^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\int \frac{\sqrt[3]{2}+2x}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx}{6\ 2^{2/3}}-\frac{\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx}{2\sqrt[3]{2}}\\
&= \frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\log\left(2^{2/3}+\sqrt[3]{2}\,x+x^2\right)}{6\ 2^{2/3}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}x\right)}{2^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\log\left(2^{2/3}+\sqrt[3]{2}\,x+x^2\right)}{6\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 0.88

$$
-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)-2\log\left(2-2^{2/3}x\right)+\log\left(2+2^{2/3}x+\sqrt[3]{2}\,x^2\right)}{6\ 2^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] - 2*Log[2 - 2^(2/3)*x] + Lo
g[2 + 2^(2/3)*x + 2^(1/3)*x^2])/2^(2/3)
```

Maple [A]
time = 0.03, size = 54, normalized size = 0.73

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}-2\right)}\frac{\ln\left(-\_R+x\right)}{\_R^{2}}\right)}{3}$	22
default	$\frac{2^{\frac{1}{3}}\ln\left(x-2^{\frac{1}{3}}\right)}{6}-\frac{\ln\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}x+x^{2}\right)2^{\frac{1}{3}}}{12}-\frac{\arctan\left(\frac{\left(1+2^{\frac{2}{3}}x\right)\sqrt{3}}{3}\right)2^{\frac{1}{3}}\sqrt{3}}{6}$	54
meijerg	$\frac{2^{\frac{1}{3}}x\left(\ln\left(1-\frac{2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{2}\right)-\frac{\ln\left(1+\frac{2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{2}+\frac{2^{\frac{1}{3}}\left(x^{3}\right)^{\frac{2}{3}}}{2}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{4+2^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{6\left(x^{3}\right)^{\frac{1}{3}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/3)*ln(x-2^(1/3))-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arctan
(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)
```

Maxima [A]
time = 3.99, size = 56, normalized size = 0.76

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2\,x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(x^{2}+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(x-2^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^(
1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(x - 2^(1/3))
```

Fricas [A]
time = 0.69, size = 68, normalized size = 0.92

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,x+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(2\,x^{2}+4^{\frac{2}{3}}x+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(2\,x-4^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*x + 4^(1/3)*sqrt(3
))) - 1/24*4^(2/3)*log(2*x^2 + 4^(2/3)*x + 2*4^(1/3)) + 1/12*4^(2/3)*log(2*
x - 4^(2/3))
```

Sympy [A]
time = 0.14, size = 71, normalized size = 0.96

$$\frac{\sqrt[3]{2}\,\log\left(x-\sqrt[3]{2}\right)}{6}-\frac{\sqrt[3]{2}\,\log\left(x^2+\sqrt[3]{2}\,x+2^{\frac{2}{3}}\right)}{12}-\frac{\sqrt[3]{2}\,\sqrt{3}\,\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-2),x)
```

```
[Out] 2**(1/3)*log(x - 2**(1/3))/6 - 2**(1/3)*log(x**2 + 2**(1/3)*x + 2**(2/3))/1
2 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Giac [A]
time = 0.44, size = 57, normalized size = 0.77

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2\,x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(x^2+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|x-2^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^(
1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(abs(x - 2^(1/3)))
```

Mupad [B]
time = 0.36, size = 72, normalized size = 0.97

$$\frac{2^{1/3}\,\ln\left(x-2^{1/3}\right)}{6}+\frac{2^{1/3}\,\ln\left(x-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{1/3}\,\ln\left(x+\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 - 2),x)
```

```
[Out] (2^(1/3)*log(x - 2^(1/3)))/6 + (2^(1/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 1))/
2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(x + (2^(1/3)*(3^(1/2)*1i + 1))/2)*(3
^(1/2)*1i + 1))/12
```

3.35 $\int \frac{1}{-b+ax^3} dx$

Optimal. Leaf size=115

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}+2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] 1/3*ln(b^(1/3)-a^(1/3)*x)/a^(1/3)/b^(2/3)-1/6*ln(b^(2/3)+a^(1/3)*b^(1/3)*x+
a^(2/3)*x^2)/a^(1/3)/b^(2/3)-1/3*arctan(1/3*(b^(1/3)+2*a^(1/3)*x)/b^(1/3)*3
^(1/2))/a^(1/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\right)}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{a}\,x+\sqrt[3]{b}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) + Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) + a^(1/3)*
b^(1/3)*x + a^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-b+ax^3}\,dx &= \frac{\int \frac{1}{-\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx}{3b^{2/3}}+\frac{\int \frac{-2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{3b^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\int \frac{\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{2/3}}-\frac{\int \frac{1}{b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&= \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a}}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}\,b^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}+2\sqrt[3]{a}\,x}{\sqrt{3}\,\sqrt[3]{b}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 89, normalized size = 0.77

$$
-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a}\,x}{\sqrt[3]{b}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}-\sqrt[3]{a}\,x\right)+\log\left(b^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3)
 - a^(1/3)*x] + Log[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(a^(1/3)*b^
(2/3))
```

Maple [A]
time = 0.02, size = 92, normalized size = 0.80

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^3-b\right)}\frac{\ln\left(-\_R+x\right)}{\_R^2}}{3a}$	29
default	$\frac{\ln\left(x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2+\left(\frac{b}{a}\right)^{\frac{1}{3}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3-b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/(b/a)^(2/3)*ln(x-(b/a)^(1/3))-1/6/a/(b/a)^(2/3)*ln(x^2+(b/a)^(1/3)*x+
(b/a)^(2/3))-1/3/a/(b/a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*x+
1))
```

Maxima [A]
time = 1.74, size = 97, normalized size = 0.84

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2+x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\log\left(x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*(b/a)^(
2/3)) - 1/6*log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*(b/a)^(2/3)) + 1/3*lo
g(x - (b/a)^(1/3))/(a*(b/a)^(2/3))
```

Fricas [A]
time = 0.87, size = 300, normalized size = 2.61

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2abx^3-3\left(ab^2\right)^{\frac{1}{3}}bx+b^2-3\sqrt{\frac{1}{3}}\left(2abx^2-\left(ab^2\right)^{\frac{2}{3}}x-\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{ax^3-b}\right)-\left(ab^2\right)^{\frac{2}{3}}\log\left(abx^2+\left(ab^2\right)^{\frac{2}{3}}x+\left(ab^2\right)^{\frac{1}{3}}b\right)+2\left(ab^2\right)^{\frac{2}{3}}\log\left(abx-\left(ab^2\right)^{\frac{2}{3}}\right)}{6ab^2},\ -\frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(ab^2\right)^{\frac{2}{3}}x+\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{b^2}\right)+\left(ab^2\right)^{\frac{2}{3}}\log\left(abx^2+\left(ab^2\right)^{\frac{2}{3}}x+\left(ab^2\right)^{\frac{1}{3}}b\right)-2\left(ab^2\right)^{\frac{2}{3}}\log\left(abx-\left(ab^2\right)^{\frac{2}{3}}\right)}{6ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*x^3 - 3*(a*b^2)^(1/
3)*b*x + b^2 - 3*sqrt(1/3)*(2*a*b*x^2 - (a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*
sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 - b)) - (a*b^2)^(2/3)*log(a*b*x^2 + (a*b^2)^
(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x - (a*b^2)^(2/3)))/(a
*b^2), -1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(a*b
^2)^(2/3)*x + (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) + (a*b^2)^(2/3)*l
og(a*b*x^2 + (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) - 2*(a*b^2)^(2/3)*log(a*b*x
 - (a*b^2)^(2/3)))/(a*b^2)]
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**3-b),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-3*_t*b + x)))
```

Giac [A]
time = 0.46, size = 104, normalized size = 0.90

$$\frac{\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b}-\frac{\sqrt{3}\left(a^2b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab}-\frac{\left(a^2b\right)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="giac")
```

```
[Out] 1/3*(b/a)^(1/3)*log(abs(x - (b/a)^(1/3)))/b - 1/3*sqrt(3)*(a^2*b)^(1/3)*arc
tan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*b) - 1/6*(a^2*b)^(1/3)*
log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*b)
```

Mupad [B]
time = 0.31, size = 101, normalized size = 0.88

$$\frac{\ln\left(a^{1/3}\,x-b^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(3\,a^2\,x-\frac{3\,a^{5/3}\,b^{1/3}\,\left(-1+\sqrt{3}\,\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}-\frac{\ln\left(3\,a^2\,x+\frac{3\,a^{5/3}\,b^{1/3}\,\left(1+\sqrt{3}\,\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b - a*x^3),x)
```

```
[Out] log(a^(1/3)*x - b^(1/3))/(3*a^(1/3)*b^(2/3)) + (log(3*a^2*x - (3*a^(5/3)*b^
(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(1/3)*b^(2/3)) - (log(3*a
^2*x + (3*a^(5/3)*b^(1/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(1/3)
*b^(2/3))
```

3.36 $\int \frac{1}{-2+x^4}\,dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

```
[Out] -1/4*arctan(1/2*x*2^(3/4))*2^(1/4)-1/4*arctanh(1/2*x*2^(3/4))*2^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {218, 212, 209}

$$-\frac{\text{ArcTan}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x/2^(1/4)]/2^(3/4) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\int \frac{1}{-2+x^4}\,dx = -\frac{\int \frac{1}{\sqrt{2}-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{1}{\sqrt{2}+x^2}\,dx}{2\sqrt{2}}$$
$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.23

$$-\frac{2\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right) - \log\left(2-2^{3/4}x\right) + \log\left(2+2^{3/4}x\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^4)^(-1),x]
```

```
[Out] -1/4*(2*ArcTan[x/2^(1/4)] - Log[2 - 2^(3/4)*x] + Log[2 + 2^(3/4)*x])/2^(3/4
)
```

Maple [A]
time = 0.03, size = 32, normalized size = 0.91

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-2\right)} \frac{\ln\left(-\_R+x\right)}{\_R^3}\right)}{4}$	22
default	$-\frac{2^{\frac{1}{4}}\left(\ln\left(\frac{x+2^{\frac{1}{4}}}{x-2^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x 2^{\frac{3}{4}}}{2}\right)\right)}{8}$	32
meijerg	$\frac{2^{\frac{1}{4}} x\left(\ln\left(1-\frac{2^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{2^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{2^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{2}\right)\right)}{8\left(x^4\right)^{\frac{1}{4}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*2^(1/4)*(ln((x+2^(1/4))/(x-2^(1/4)))+2*arctan(1/2*x*2^(3/4)))
```

Maxima [A]
time = 5.05, size = 34, normalized size = 0.97

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right) + \frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{x-2^{\frac{1}{4}}}{x+2^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="maxima")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) + 1/8*2^(1/4)*log((x - 2^(1/4))/(x + 2^(
1/4)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.
time = 0.71, size = 56, normalized size = 1.60

$$\frac{1}{8}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 8^{\frac{1}{4}}x+\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{x^2+\sqrt{2}}\right)-\frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4x+8^{\frac{3}{4}}\right)+\frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4x-8^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="fricas")
```

```
[Out] 1/8*8^(3/4)*arctan(-1/2*8^(1/4)*x + 1/2*8^(1/4)*sqrt(x^2 + sqrt(2))) - 1/32
*8^(3/4)*log(4*x + 8^(3/4)) + 1/32*8^(3/4)*log(4*x - 8^(3/4))
```

Sympy [A]
time = 0.15, size = 46, normalized size = 1.31

$$\frac{\sqrt[4]{2}\log\left(x-\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\log\left(x+\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-2),x)
```

```
[Out] 2**(1/4)*log(x - 2**(1/4))/8 - 2**(1/4)*log(x + 2**(1/4))/8 - 2**(1/4)*atan
(2**(3/4)*x/2)/4
```

Giac [A]
time = 0.45, size = 39, normalized size = 1.11

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x+2^{\frac{1}{4}}\right|\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x-2^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="giac")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) - 1/8*2^(1/4)*log(abs(x + 2^(1/4))) + 1/
8*2^(1/4)*log(abs(x - 2^(1/4)))
```

Mupad [B]
time = 0.16, size = 20, normalized size = 0.57

$$-\frac{2^{1/4}\left(\operatorname{atan}\left(\frac{2^{3/4}x}{2}\right)+\operatorname{atanh}\left(\frac{2^{3/4}x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 2),x)
[Out] -(2^(1/4)*(atan((2^(3/4)*x)/2) + atanh((2^(3/4)*x)/2)))/4
```

3.37 $\int \frac{1}{-1+5x^4} dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}\,x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}\,x\right)}{2\sqrt[4]{5}}$$

```
[Out] -1/10*arctan(5^(1/4)*x)*5^(3/4)-1/10*arctanh(5^(1/4)*x)*5^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {218, 212, 209}

$$-\frac{\text{ArcTan}\left(\sqrt[4]{5}\,x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}\,x\right)}{2\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[5^(1/4)*x]/5^(1/4) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\int \frac{1}{-1+5x^4}\,dx = -\left(\frac{1}{2}\int \frac{1}{1-\sqrt{5}\,x^2}\,dx\right) - \frac{1}{2}\int \frac{1}{1+\sqrt{5}\,x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\sqrt[4]{5}\,x\right)}{2\sqrt[4]{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{5}\,x\right)}{2\sqrt[4]{5}}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.23

$$-\frac{2\tan^{-1}\left(\sqrt[4]{5}\,x\right) - \log\left(1-\sqrt[4]{5}\,x\right) + \log\left(1+\sqrt[4]{5}\,x\right)}{4\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x^4)^(-1),x]
[Out] -1/4*(2*ArcTan[5^(1/4)*x] - Log[1 - 5^(1/4)*x] + Log[1 + 5^(1/4)*x])/5^(1/4
)
```

Maple [A]
time = 0.03, size = 33, normalized size = 0.94

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(5\_Z^4-1\right)}\frac{\ln\left(-\_R+x\right)}{\_R^3}\right)}{20}$	24
default	$-\frac{5^{\frac{3}{4}}\left(\ln\left(\frac{x+\frac{5^{\frac{3}{4}}}{5}}{x-\frac{5^{\frac{3}{4}}}{5}}\right)+2\arctan\left(5^{\frac{1}{4}}x\right)\right)}{20}$	33
meijerg	$\frac{5^{\frac{3}{4}}x\left(\ln\left(1-5^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+5^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right)-2\arctan\left(5^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}\right)\right)}{20\left(x^4\right)^{\frac{1}{4}}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4-1),x,method=_RETURNVERBOSE)
[Out] -1/20*5^(3/4)*(ln((x+1/5*5^(3/4))/(x-1/5*5^(3/4)))+2*arctan(5^(1/4)*x))
```

Maxima [A]
time = 4.55, size = 41, normalized size = 1.17

$$-\frac{1}{10}\cdot 5^{\frac{3}{4}}\arctan\left(5^{\frac{1}{4}}x\right) + \frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\frac{\sqrt{5}\,x - 5^{\frac{1}{4}}}{\sqrt{5}\,x + 5^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="maxima")
```

```
[Out] -1/10*5^(3/4)*arctan(5^(1/4)*x) + 1/20*5^(3/4)*log((sqrt(5)*x - 5^(1/4))/(s
qrt(5)*x + 5^(1/4)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(23) = 46$.

time = 0.95, size = 60, normalized size = 1.71

$$\frac{1}{5} \cdot 5^{\frac{3}{4}} \arctan\left(-5^{\frac{1}{4}} x + \frac{1}{5} \cdot 5^{\frac{1}{4}} \sqrt{25 x^2 + 5\sqrt{5}}\right) - \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(5x + 5^{\frac{3}{4}}\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(5x - 5^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="fricas")
```

```
[Out] 1/5*5^(3/4)*arctan(-5^(1/4)*x + 1/5*5^(1/4)*sqrt(25*x^2 + 5*sqrt(5))) - 1/2
0*5^(3/4)*log(5*x + 5^(3/4)) + 1/20*5^(3/4)*log(5*x - 5^(3/4))
```

Sympy [A]

time = 0.15, size = 48, normalized size = 1.37

$$\frac{5^{\frac{3}{4}} \log\left(x - \frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}} \log\left(x + \frac{5^{\frac{3}{4}}}{5}\right)}{20} - \frac{5^{\frac{3}{4}} \operatorname{atan}\left(\sqrt[4]{5}\, x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**4-1),x)
```

```
[Out] 5**(3/4)*log(x - 5**(3/4)/5)/20 - 5**(3/4)*log(x + 5**(3/4)/5)/20 - 5**(3/4
)*atan(5**(1/4)*x)/10
```

Giac [A]

time = 0.45, size = 39, normalized size = 1.11

$$-\frac{1}{10} \cdot 5^{\frac{3}{4}} \arctan\left(5\left(\frac{1}{5}\right)^{\frac{3}{4}} x\right) - \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\left|x + \left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right) + \frac{1}{20} \cdot 5^{\frac{3}{4}} \log\left(\left|x - \left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="giac")
```

```
[Out] -1/10*5^(3/4)*arctan(5*(1/5)^(3/4)*x) - 1/20*5^(3/4)*log(abs(x + (1/5)^(1/4
))) + 1/20*5^(3/4)*log(abs(x - (1/5)^(1/4)))
```

Mupad [B]

time = 0.18, size = 18, normalized size = 0.51

$$-\frac{5^{3/4}\left(\operatorname{atan}\left(5^{1/4}\, x\right) + \operatorname{atanh}\left(5^{1/4}\, x\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4 - 1),x)
[Out] -(5^(3/4)*(atan(5^(1/4)*x) + atanh(5^(1/4)*x)))/10
```

3.38 $\int \frac{1}{7+3x^4}\,dx$

Optimal. Leaf size=171

$$-\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}}+\frac{\tan^{-1}\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}}-\frac{\log\left(\sqrt{21}-\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}}+\frac{\log\left(\sqrt{21}+\sqrt{\ldots}\right.}{4\sqrt{2}\ldots}$$

```
[Out] 1/84*arctan(-1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/84*
arctan(1+1/7*3^(1/4)*7^(3/4)*x*2^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)-1/168*ln(3*
x^2-3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)+1/168*ln(3*
x^2+3^(3/4)*7^(1/4)*x*2^(1/2)+21^(1/2))*3^(3/4)*7^(1/4)*2^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}}+\frac{\text{ArcTan}\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x+1\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}}-\frac{\log\left(3x^2-\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}}+\frac{\log\left(3x^2+\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 3*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(Sqrt[2]*3^(1/4)*7^(3/4)) + ArcTan[1
 + (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) - Log[Sqrt[21] - Sqrt
[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4)) + Log[Sqrt[21] +
 Sqrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{7+3x^4}\,dx = \frac{\int \frac{\sqrt{7}-\sqrt{3}\,x^2}{7+3x^4}\,dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7}+\sqrt{3}\,x^2}{7+3x^4}\,dx}{2\sqrt{7}}$$

$$= -\frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}+2x}{-\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}\,x-x^2}\,dx}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}-2x}{-\sqrt{\frac{7}{3}}+\sqrt{2}\sqrt[4]{\frac{7}{3}}\,x-x^2}\,dx}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\int \frac{1}{\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}\,x+x^2}\,dx}{4\sqrt{21}} + \frac{\int \cdots}{\cdots}$$

$$= -\frac{\log\left(\sqrt{21}-\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\log\left(\sqrt{21}+\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-\cdots}\cdots\right)}{2\cdots}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}} + \frac{\tan^{-1}\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x\right)}{2\sqrt{2}\sqrt[4]{3}\,7^{3/4}} - \frac{\log\left(\sqrt{21}-\sqrt{2}\,3^{3/4}\sqrt[4]{7}\,x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}}\cdots$$

Mathematica [A]
time = 0.03, size = 120, normalized size = 0.70

$$\frac{-2\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x\right)+2\tan^{-1}\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}\,x\right)-\log\left(7-\sqrt{2}\sqrt[4]{3}\,7^{3/4}x+\sqrt{21}\,x^2\right)+\log\left(7+\sqrt{2}\sqrt[4]{3}\,7^{3/4}x+\sqrt{21}\,x^2\right)}{4\sqrt{2}\sqrt[4]{3}\,7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 3*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x] + 2*ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x]
 - Log[7 - Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2] + Log[7 + Sqrt[2]*3^(1
/4)*7^(3/4)*x + Sqrt[21]*x^2])/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

Maple [A]
time = 0.02, size = 93, normalized size = 0.54

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+7\right)}\frac{\ln\left(-\_R+x\right)}{\_R^3}\right)}{12}$
default	$\frac{\sqrt{3}\,21^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,21^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{21}}{3}}{x^2-\frac{\sqrt{3}\,21^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{21}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,21^{\frac{3}{4}}x}{21}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,21^{\frac{3}{4}}x}{21}-1\right)\right)}{168}$
meijerg	$\frac{1029^{\frac{3}{4}}\left(-\frac{x\sqrt{2}\,\ln\left(1-\frac{\sqrt{2}\,3^{\frac{1}{4}}7^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{7}+\frac{\sqrt{3}\sqrt{7}\sqrt{x^4}}{7}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,3^{\frac{1}{4}}7^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{14-\sqrt{2}\,3^{\frac{1}{4}}7^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\frac{\sqrt{2}\,3^{\frac{1}{4}}\cdots}{\cdots}\right.}{\cdots}\right.}{4116}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/168*3^(1/2)*21^(1/4)*2^(1/2)*(ln((x^2+1/3*3^(1/2)*21^(1/4)*x*2^(1/2)+1/3*
21^(1/2))/(x^2-1/3*3^(1/2)*21^(1/4)*x*2^(1/2)+1/3*21^(1/2)))+2*arctan(1/21*
2^(1/2)*3^(1/2)*21^(3/4)*x+1)+2*arctan(1/21*2^(1/2)*3^(1/2)*21^(3/4)*x-1))
```

Maxima [A]
time = 2.67, size = 151, normalized size = 0.88

$$\frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}\,x+7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{84}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\,\arctan\left(\frac{1}{42}\cdot 7^{\frac{3}{4}}3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}\,x-7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{168}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\,\log\left(\sqrt{3}\,x^2+7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{7}\right)-\frac{1}{168}\cdot 7^{\frac{1}{4}}3^{\frac{3}{4}}\sqrt{2}\,\log\left(\sqrt{3}\,x^2-7^{\frac{1}{4}}3^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="maxima")
```

```
[Out] 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)
*x + 7^(1/4)*3^(1/4)*sqrt(2))) + 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7
^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)*x - 7^(1/4)*3^(1/4)*sqrt(2))) + 1/168*7^(
1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 + 7^(1/4)*3^(1/4)*sqrt(2)*x + sqrt(7))
 - 1/168*7^(1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 - 7^(1/4)*3^(1/4)*sqrt(2)*
x + sqrt(7))
```

Fricas [A]
time = 0.75, size = 161, normalized size = 0.94

$$-\frac{1}{2058}\cdot 1029^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{147}\cdot 1029^{\frac{1}{4}}\sqrt{3}\sqrt{2}\sqrt{1029^{\frac{3}{4}}\sqrt{2}x+147x^2+49\sqrt{21}}-\frac{1}{7}\cdot 1029^{\frac{1}{4}}\sqrt{2}x-1\right)-\frac{1}{2058}\cdot 1029^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot 1029^{\frac{1}{4}}\sqrt{2}x+\frac{1}{2058}\cdot 1029^{\frac{1}{4}}\sqrt{2}\sqrt{-588\cdot 1029^{\frac{3}{4}}\sqrt{2}x+86436x^2+28812\sqrt{21}}+1\right)+\frac{1}{8232}\cdot 1029^{\frac{3}{4}}\sqrt{2}\log\left(588\cdot 1029^{\frac{3}{4}}\sqrt{2}x+86436x^2+28812\sqrt{21}\right)-\frac{1}{8232}\cdot 1029^{\frac{3}{4}}\sqrt{2}\log\left(-588\cdot 1029^{\frac{3}{4}}\sqrt{2}x+86436x^2+28812\sqrt{21}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="fricas")
```

```
[Out] -1/2058*1029^(3/4)*sqrt(2)*arctan(1/147*1029^(1/4)*sqrt(3)*sqrt(2)*sqrt(102
9^(3/4)*sqrt(2)*x + 147*x^2 + 49*sqrt(21)) - 1/7*1029^(1/4)*sqrt(2)*x - 1)
- 1/2058*1029^(3/4)*sqrt(2)*arctan(-1/7*1029^(1/4)*sqrt(2)*x + 1/2058*1029^
(1/4)*sqrt(2)*sqrt(-588*1029^(3/4)*sqrt(2)*x + 86436*x^2 + 28812*sqrt(21))
+ 1) + 1/8232*1029^(3/4)*sqrt(2)*log(588*1029^(3/4)*sqrt(2)*x + 86436*x^2 +
 28812*sqrt(21)) - 1/8232*1029^(3/4)*sqrt(2)*log(-588*1029^(3/4)*sqrt(2)*x
+ 86436*x^2 + 28812*sqrt(21))
```

Sympy [A]
time = 0.20, size = 151, normalized size = 0.88

$$-\frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2-\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{168}+\frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2+\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{168}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{84}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\cdot\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}+1\right)}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+7),x)
```

```
[Out] -189**(1/4)*sqrt(2)*log(x**2 - 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + 1
89**(1/4)*sqrt(2)*log(x**2 + 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + sqr
t(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 - 1)/84 + sqrt(2)
*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 + 1)/84
```

Giac [A]
time = 0.49, size = 95, normalized size = 0.56

$$\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{168}\cdot 756^{\frac{1}{4}}\log\left(x^2+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{7}{3}}\right)-\frac{1}{168}\cdot 756^{\frac{1}{4}}\log\left(x^2-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{7}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="giac")
```

```
[Out] 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x + (7/3)^(1/4)*sqrt(2)))
 + 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x - (7/3)^(1/4)*sqrt(2
```

```
))) + 1/168*756^(1/4)*log(x^2 + (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3)) - 1/168*
756^(1/4)*log(x^2 - (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3))
```

Mupad [B]
time = 0.11, size = 45, normalized size = 0.26

$$\sqrt{2}\,189^{1/4}\,\mathrm{atan}\left(\sqrt{2}\,189^{3/4}\,x\left(\frac{1}{126}-\frac{1}{126}\mathrm{i}\right)\right)\left(\frac{1}{84}+\frac{1}{84}\mathrm{i}\right)+\sqrt{2}\,189^{1/4}\,\mathrm{atan}\left(\sqrt{2}\,189^{3/4}\,x\left(\frac{1}{126}+\frac{1}{126}\mathrm{i}\right)\right)\left(\frac{1}{84}-\frac{1}{84}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + 7),x)
```

```
[Out] 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 - 1i/126))*(1/84 + 1i/84)
 + 2^(1/2)*189^(1/4)*atan(2^(1/2)*189^(3/4)*x*(1/126 + 1i/126))*(1/84 - 1i/
84)
```

3.39 $\int \frac{1}{-1+3x^2+x^4} \, dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}\,x\right)$$

```
[Out] -1/13*arctan(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/26
*arctanh(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1107, 213, 209}

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\,\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) - Sqrt[(3 +
 Sqrt[13])/26]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{-1+3x^2+x^4}\,dx = \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}}$$
$$= -\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right) - \sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{}{-3}}\right.$$

Mathematica [**A**]
time = 0.04, size = 68, normalized size = 0.93

$$-\frac{\sqrt{-3+\sqrt{13}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)+\sqrt{3+\sqrt{13}}\tanh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}\,x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[-3 + Sqrt[13]]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x] + Sqrt[3 + Sqrt[13]
]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x])/Sqrt[26])
```

Maple [**A**]
time = 0.03, size = 56, normalized size = 0.77

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+3\_Z^2-1\right)}\frac{\ln\left(-\_R+x\right)}{2\_R^3+3\_R}\right)}{2}$	35
default	$-\frac{2\sqrt{13}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}-\frac{2\sqrt{13}\,\arctan\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/13*13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctanh(2*x/(-6+2*13^(1/2))^(1/2))-2/1
3*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctan(2*x/(6+2*13^(1/2))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 3*x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(50) = 100$.
time = 0.68, size = 132, normalized size = 1.81

$$\frac{1}{13}\sqrt{26}\sqrt{\sqrt{13}-3}\arctan\left(\frac{1}{52}\sqrt{26}\sqrt{13}\sqrt{2}\sqrt{2x^2+\sqrt{13}+3}\sqrt{\sqrt{13}-3}-\frac{1}{26}\sqrt{26}\sqrt{13}x\sqrt{\sqrt{13}-3}\right)+\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}+3}\log\left(\sqrt{26}\left(3\sqrt{13}-13\right)\sqrt{\sqrt{13}+3}+52x\right)-\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}+3}\log\left(-\sqrt{26}\left(3\sqrt{13}-13\right)\sqrt{\sqrt{13}+3}+52x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/13*sqrt(26)*sqrt(sqrt(13) - 3)*arctan(1/52*sqrt(26)*sqrt(13)*sqrt(2)*sqrt
(2*x^2 + sqrt(13) + 3)*sqrt(sqrt(13) - 3) - 1/26*sqrt(26)*sqrt(13)*x*sqrt(s
qrt(13) - 3)) + 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(sqrt(26)*(3*sqrt(13) -
 13)*sqrt(sqrt(13) + 3) + 52*x) - 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(-sqr
t(26)*(3*sqrt(13) - 13)*sqrt(sqrt(13) + 3) + 52*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.
time = 0.23, size = 146, normalized size = 2.00

$$\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\log\left(x-22\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}+312\left(\frac{3}{104}+\frac{\sqrt{13}}{104}\right)^{\frac{3}{2}}\right)-\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\log\left(x-312\left(\frac{3}{104}+\frac{\sqrt{13}}{104}\right)^{\frac{3}{2}}+22\sqrt{\frac{3}{104}+\frac{\sqrt{13}}{104}}\right)-2\sqrt{-\frac{3}{104}+\frac{\sqrt{13}}{104}}\operatorname{atan}\left(\frac{2\sqrt{2}x}{3\sqrt{-3+\sqrt{13}}+\sqrt{13}\sqrt{-3+\sqrt{13}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2-1),x)
```

```
[Out] sqrt(3/104 + sqrt(13)/104)*log(x - 22*sqrt(3/104 + sqrt(13)/104) + 312*(3/1
04 + sqrt(13)/104)**(3/2)) - sqrt(3/104 + sqrt(13)/104)*log(x - 312*(3/104
+ sqrt(13)/104)**(3/2) + 22*sqrt(3/104 + sqrt(13)/104)) - 2*sqrt(-3/104 + s
qrt(13)/104)*atan(2*sqrt(2)*x/(3*sqrt(-3 + sqrt(13)) + sqrt(13)*sqrt(-3 + s
qrt(13))))
```

Giac [A]
time = 0.46, size = 74, normalized size = 1.01

$$-\frac{1}{26}\sqrt{26\sqrt{13}-78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}+78}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right)+\frac{1}{52}\sqrt{26\sqrt{13}+78}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) - 78)*arctan(x/sqrt(1/2*sqrt(13) + 3/2)) - 1/52*sqrt
(26*sqrt(13) + 78)*log(abs(x + sqrt(1/2*sqrt(13) - 3/2))) + 1/52*sqrt(26*sq
rt(13) + 78)*log(abs(x - sqrt(1/2*sqrt(13) - 3/2)))
```

Mupad [B]
time = 0.22, size = 93, normalized size = 1.27

$$-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{\sqrt{13}+3}}+\frac{3\sqrt{13}\,\sqrt{26}\,x}{26\sqrt{\sqrt{13}+3}}\right)\sqrt{\sqrt{13}+3}}{26}-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{3-\sqrt{13}}}-\frac{3\sqrt{13}\,\sqrt{26}\,x}{26\sqrt{3-\sqrt{13}}}\right)\sqrt{3-\sqrt{13}}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 - 1),x)
```

```
[Out] - (26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) + 3)^(1/2)) + (3*13^(1/2)*26^(1
/2)*x)/(26*(13^(1/2) + 3)^(1/2)))*(13^(1/2) + 3)^(1/2))/26 - (26^(1/2)*atan
h((26^(1/2)*x)/(2*(3 - 13^(1/2))^(1/2)) - (3*13^(1/2)*26^(1/2)*x)/(26*(3 -
13^(1/2))^(1/2)))*(3 - 13^(1/2))^(1/2))/26
```

3.40 $\int \frac{1}{-1-3x^2+x^4}\,dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}\,x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\,\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)$$

```
[Out] -1/13*arctanh(x*2^(1/2)/(3+13^(1/2))^(1/2))*26^(1/2)/(3+13^(1/2))^(1/2)-1/2
6*arctan(x*2^(1/2)/(-3+13^(1/2))^(1/2))*(78+26*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1107, 213, 209}

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{13}-3}}\,x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\,\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[13])/26]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sqrt[2/(13*(
3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{-1-3x^2+x^4}\,dx = \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}}$$
$$= -\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}\,x\right) - \sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{}{3}}\right.$$

Mathematica [**A**]
time = 0.02, size = 68, normalized size = 0.93

$$-\frac{\sqrt{3+\sqrt{13}}\tan^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}\,x\right)+\sqrt{-3+\sqrt{13}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}\,x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[3 + Sqrt[13]]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x] + Sqrt[-3 + Sqrt[13
]]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x])/Sqrt[26])
```

Maple [**A**]
time = 0.03, size = 56, normalized size = 0.77

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-3\_Z^2-1\right)}\frac{\ln\left(-\_R+x\right)}{2\_R^3-3\_R}\right)}{2}$	35
default	$-\frac{2\sqrt{13}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{6+2\sqrt{13}}}\right)}{13\sqrt{6+2\sqrt{13}}}-\frac{2\sqrt{13}\,\arctan\left(\frac{2x}{\sqrt{-6+2\sqrt{13}}}\right)}{13\sqrt{-6+2\sqrt{13}}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/13*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctanh(2*x/(6+2*13^(1/2))^(1/2))-2/13*
13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctan(2*x/(-6+2*13^(1/2))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 3*x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(50) = 100$.
time = 0.59, size = 132, normalized size = 1.81

$$\frac{1}{13}\sqrt{26}\sqrt{\sqrt{13}+3}\arctan\left(\frac{1}{52}\sqrt{26}\sqrt{13}\sqrt{2}\sqrt{2x^2+\sqrt{13}-3}\sqrt{\sqrt{13}+3}-\frac{1}{26}\sqrt{26}\sqrt{13}x\sqrt{\sqrt{13}+3}\right)-\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}-3}\log\left(\sqrt{26}\left(3\sqrt{13}+13\right)\sqrt{\sqrt{13}-3}+52x\right)+\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}-3}\log\left(-\sqrt{26}\left(3\sqrt{13}+13\right)\sqrt{\sqrt{13}-3}+52x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/13*sqrt(26)*sqrt(sqrt(13) + 3)*arctan(1/52*sqrt(26)*sqrt(13)*sqrt(2)*sqrt
(2*x^2 + sqrt(13) - 3)*sqrt(sqrt(13) + 3) - 1/26*sqrt(26)*sqrt(13)*x*sqrt(s
qrt(13) + 3)) - 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(sqrt(26)*(3*sqrt(13) +
 13)*sqrt(sqrt(13) - 3) + 52*x) + 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(-sqr
t(26)*(3*sqrt(13) + 13)*sqrt(sqrt(13) - 3) + 52*x)
```

Sympy [A]
time = 0.18, size = 24, normalized size = 0.33

$$\operatorname{RootSum}\left(2704t^4+156t^2-1,\left(t\mapsto t\log\left(-312t^3-22t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**2-1),x)
```

```
[Out] RootSum(2704*_t**4 + 156*_t**2 - 1, Lambda(_t, _t*log(-312*_t**3 - 22*_t +
x)))
```

Giac [A]
time = 0.45, size = 74, normalized size = 1.01

$$-\frac{1}{26}\sqrt{26\sqrt{13}+78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}-78}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}\right|\right)+\frac{1}{52}\sqrt{26\sqrt{13}-78}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) + 78)*arctan(x/sqrt(1/2*sqrt(13) - 3/2)) - 1/52*sqrt
(26*sqrt(13) - 78)*log(abs(x + sqrt(1/2*sqrt(13) + 3/2))) + 1/52*sqrt(26*sq
rt(13) - 78)*log(abs(x - sqrt(1/2*sqrt(13) + 3/2)))
```

Mupad [B]
time = 0.13, size = 93, normalized size = 1.27

$$-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{\sqrt{13}-3}}-\frac{3\sqrt{13}\,\sqrt{26}\,x}{26\sqrt{\sqrt{13}-3}}\right)\sqrt{\sqrt{13}-3}}{26}-\frac{\sqrt{26}\,\mathrm{atanh}\left(\frac{\sqrt{26}\,x}{2\sqrt{-\sqrt{13}-3}}+\frac{3\sqrt{13}\,\sqrt{26}\,x}{26\sqrt{-\sqrt{13}-3}}\right)\sqrt{-\sqrt{13}-3}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(3*x^2 - x^4 + 1),x)
```

```
[Out] - (26^(1/2)*atanh((26^(1/2)*x)/(2*(13^(1/2) - 3)^(1/2)) - (3*13^(1/2)*26^(1
/2)*x)/(26*(13^(1/2) - 3)^(1/2)))*(13^(1/2) - 3)^(1/2))/26 - (26^(1/2)*atan
h((26^(1/2)*x)/(2*(- 13^(1/2) - 3)^(1/2)) + (3*13^(1/2)*26^(1/2)*x)/(26*(-
13^(1/2) - 3)^(1/2)))*(- 13^(1/2) - 3)^(1/2))/26
```

3.41 $\int \frac{1}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=72

$$-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)+\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)$$

```
[Out] -1/5*arctanh(x*2^(1/2)/(3+5^(1/2))^(1/2))*10^(1/2)/(3+5^(1/2))^(1/2)+arctan
h(x*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1107, 213}

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-3x^2+x^4}\,dx &= \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{\sqrt{5}}-\frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{\sqrt{5}}\\ &= -\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)+\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 1.15

$$\frac{1}{20}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x\right)\right)-\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x\right)+\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x\right)+\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5]
- 2*x] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x] + (-5 + Sqrt[5])*Log[1 + Sqr
t[5] + 2*x])/20
```

Maple [A]
time = 0.02, size = 54, normalized size = 0.75

method	result
default	$-\frac{\ln(x^2+x-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)}{10}+\frac{\ln(x^2-x-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{10}$
risch	$\frac{\ln\left(2x-1+\sqrt{5}\right)}{4}+\frac{\ln\left(2x-1+\sqrt{5}\right)\sqrt{5}}{20}+\frac{\ln\left(2x-1-\sqrt{5}\right)}{4}-\frac{\ln\left(2x-1-\sqrt{5}\right)\sqrt{5}}{20}+\frac{\ln\left(2x+\sqrt{5}+1\right)\sqrt{5}}{20}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(x^2+x-1)+1/10*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))+1/4*ln(x^2-x-1)+
1/10*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))
```

Maxima [A]
time = 5.96, size = 75, normalized size = 1.04

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)-\frac{1}{20}\sqrt{5}\log\left(\frac{2x-\sqrt{5}-1}{2x+\sqrt{5}-1}\right)-\frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/20*sqrt(5)*l
og((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1)) - 1/4*log(x^2 + x - 1) + 1/4*lo
g(x^2 - x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(42) = 84.
time = 0.69, size = 91, normalized size = 1.26

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)+\frac{1}{20}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x-1)-2x+3}{x^2-x-1}\right)-\frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/2
0*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x - 1) - 2*x + 3)/(x^2 - x - 1)) - 1/4*lo
g(x^2 + x - 1) + 1/4*log(x^2 - x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(58) = 116$.
time = 0.19, size = 158, normalized size = 2.19

$$\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)\log\left(x-\frac{7}{2}-\frac{7\sqrt{5}}{10}+120\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)^{3}\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x-\frac{7}{2}+120\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)^{3}+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right)\log\left(x-\frac{7\sqrt{5}}{10}+120\left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right)^{3}+\frac{7}{2}\right)+\left(-\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x+120\left(-\frac{1}{4}-\frac{\sqrt{5}}{20}\right)^{3}+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**2+1),x)
```

```
[Out] (sqrt(5)/20 + 1/4)*log(x - 7/2 - 7*sqrt(5)/10 + 120*(sqrt(5)/20 + 1/4)**3)
+ (1/4 - sqrt(5)/20)*log(x - 7/2 + 120*(1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/10
) + (-1/4 + sqrt(5)/20)*log(x - 7*sqrt(5)/10 + 120*(-1/4 + sqrt(5)/20)**3 +
 7/2) + (-1/4 - sqrt(5)/20)*log(x + 120*(-1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/
10 + 7/2)
```

Giac [A]
time = 0.47, size = 81, normalized size = 1.12

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}-1\right|}{\left|2x+\sqrt{5}-1\right|}\right)-\frac{1}{4}\log\left(\left|x^{2}+x-1\right|\right)+\frac{1}{4}\log\left(\left|x^{2}-x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/20*sqr
t(5)*log(abs(2*x - sqrt(5) - 1)/abs(2*x + sqrt(5) - 1)) - 1/4*log(abs(x^2 +
 x - 1)) + 1/4*log(abs(x^2 - x - 1))
```

Mupad [B]
time = 0.10, size = 67, normalized size = 0.93

$$\operatorname{atanh}\left(\frac{4x}{\sqrt{5}-3}-\frac{2\sqrt{5}\,x}{\sqrt{5}-3}\right)\left(\frac{\sqrt{5}}{10}-\frac{1}{2}\right)+\operatorname{atanh}\left(\frac{4x}{\sqrt{5}+3}+\frac{2\sqrt{5}\,x}{\sqrt{5}+3}\right)\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 3*x^2 + 1),x)
```

```
[Out] atanh((4*x)/(5^(1/2) - 3) - (2*5^(1/2)*x)/(5^(1/2) - 3))*(5^(1/2)/10 - 1/2)
 + atanh((4*x)/(5^(1/2) + 3) + (2*5^(1/2)*x)/(5^(1/2) + 3))*(5^(1/2)/10 + 1
/2)
```

3.42 $\int \frac{1}{1-4x^2+x^4} \, dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] 1/2*arctanh(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arct
anh(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1107, 213}

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-4x^2+x^4}\,dx = \frac{\int \frac{1}{-2-\sqrt{3}+x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{-2+\sqrt{3}+x^2}\,dx}{2\sqrt{3}}$$
$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A]
time = 0.03, size = 60, normalized size = 0.90

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^2+1\right)}\frac{\ln\left(-\_R+x\right)}{\_R^3-2\_R}\right)}{4}$	33
default	$\frac{\sqrt{3}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}} - \frac{\sqrt{3}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))-1/3*3^(1/2)/(6
^(1/2)+2^(1/2))*arctanh(2*x/(6^(1/2)+2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 4*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(55) = 110$.
time = 0.57, size = 123, normalized size = 1.84

$$-\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right)+\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right)-\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+x\right)+\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}+2}\log\left(-\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x) +
1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(-sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x) -
1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + x) +
 1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log(-(sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + x)
```

Sympy [A]
time = 0.18, size = 24, normalized size = 0.36

$$\operatorname{RootSum}\left(2304t^4-192t^2+1,\left(t\mapsto t\log\left(384t^3-28t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-4*x**2+1),x)
```

```
[Out] RootSum(2304*_t**4 - 192*_t**2 + 1, Lambda(_t, _t*log(384*_t**3 - 28*_t + x
)))
```

Giac [A]
time = 0.48, size = 101, normalized size = 1.51

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(\left|x-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) + 1/2*sqrt(2))) + 1/24*(
sqrt(6) + 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) - 1/2*sqrt(2))) - 1/24*(sqrt(6
```

```
) + 3*sqrt(2))*log(abs(x - 1/2*sqrt(6) + 1/2*sqrt(2))) - 1/24*(sqrt(6) - 3*
sqrt(2))*log(abs(x - 1/2*sqrt(6) - 1/2*sqrt(2)))
```

Mupad [B]
time = 0.23, size = 98, normalized size = 1.46

$$\operatorname{atanh}\left(\frac{5\sqrt{2}\,x}{\sqrt{2}\,\sqrt{6}+4}+\frac{3\sqrt{6}\,x}{\sqrt{2}\,\sqrt{6}+4}\right)\left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{12}\right)-\operatorname{atanh}\left(\frac{5\sqrt{2}\,x}{\sqrt{2}\,\sqrt{6}-4}-\frac{3\sqrt{6}\,x}{\sqrt{2}\,\sqrt{6}-4}\right)\left(\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 4*x^2 + 1),x)
```

```
[Out] atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) + 4) + (3*6^(1/2)*x)/(2^(1/2)*6^(1/2)
+ 4))*(2^(1/2)/4 + 6^(1/2)/12) - atanh((5*2^(1/2)*x)/(2^(1/2)*6^(1/2) - 4)
- (3*6^(1/2)*x)/(2^(1/2)*6^(1/2) - 4))*(2^(1/2)/4 - 6^(1/2)/12)
```

3.43 $\int \frac{1}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] 1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/2*arcta
n(x/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1107, 209}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+4x^2+x^4}\,dx = \frac{\int \frac{1}{2-\sqrt{3}+x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{2+\sqrt{3}+x^2}\,dx}{2\sqrt{3}}$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x^2 + x^4)^(-1),x]
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A]
time = 0.03, size = 60, normalized size = 0.90

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+4\_Z^2+1\right)} \frac{\ln\left(-\_R+x\right)}{\_R^3+2\_R}\right)}{4}$	33
default	$-\frac{\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}+\frac{\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/3*3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))+1/3*3^(1/2)/(6
^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 4*x^2 + 1), x)
```

Fricas [A]
time = 0.61, size = 87, normalized size = 1.30

$$-\frac{1}{3}\sqrt{3}\sqrt{\sqrt{3}+2}\arctan\left(-\left(x-\sqrt{x^2-\sqrt{3}+2}\right)\sqrt{\sqrt{3}+2}\right)+\frac{1}{3}\sqrt{3}\sqrt{-\sqrt{3}+2}\arctan\left(-x\sqrt{-\sqrt{3}+2}+\sqrt{x^2+\sqrt{3}+2}\sqrt{-\sqrt{3}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*sqrt(sqrt(3) + 2)*arctan(-(x - sqrt(x^2 - sqrt(3) + 2))*sqrt(s
qrt(3) + 2)) + 1/3*sqrt(3)*sqrt(-sqrt(3) + 2)*arctan(-x*sqrt(-sqrt(3) + 2)
+ sqrt(x^2 + sqrt(3) + 2)*sqrt(-sqrt(3) + 2))
```

Sympy [A]
time = 0.10, size = 92, normalized size = 1.37

$$-2\sqrt{\frac{1}{24}-\frac{\sqrt{3}}{48}}\operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{2-\sqrt{3}}+2\sqrt{2-\sqrt{3}}}\right)-2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3}+2}+\sqrt{3}\sqrt{\sqrt{3}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**2+1),x)
```

```
[Out] -2*sqrt(1/24 - sqrt(3)/48)*atan(x/(sqrt(3)*sqrt(2 - sqrt(3)) + 2*sqrt(2 - s
qrt(3)))) - 2*sqrt(sqrt(3)/48 + 1/24)*atan(x/(-2*sqrt(sqrt(3) + 2) + sqrt(3
)*sqrt(sqrt(3) + 2)))
```

Giac [A]
time = 0.49, size = 51, normalized size = 0.76

$$\frac{1}{12}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(6) - 3*sqrt(2))*arctan(2*x/(sqrt(6) + sqrt(2))) + 1/12*(sqrt(6)
+ 3*sqrt(2))*arctan(2*x/(sqrt(6) - sqrt(2)))
```

Mupad [B]
time = 0.20, size = 117, normalized size = 1.75

$$2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}\,-4}-\frac{16\,\sqrt{3}\,x\,\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}\,-4}\right)\,\sqrt{\frac{\sqrt{3}}{48}-\frac{1}{24}}-2\,\mathrm{atanh}\left(\frac{24\,x\,\sqrt{-\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}\,+4}+\frac{16\,\sqrt{3}\,x\,\sqrt{-\frac{\sqrt{3}}{48}-\frac{1}{24}}}{2\,\sqrt{3}\,+4}\right)\,\sqrt{-\frac{\sqrt{3}}{48}-\frac{1}{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((24*x*(3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4) - (16*3^(1/2)*x*(3
^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) - 4))*(3^(1/2)/48 - 1/24)^(1/2) - 2*ata
nh((24*x*(- 3^(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4) + (16*3^(1/2)*x*(- 3^
(1/2)/48 - 1/24)^(1/2))/(2*3^(1/2) + 4))*(- 3^(1/2)/48 - 1/24)^(1/2)
```

3.44 $\int \frac{1}{2+x^2+x^4}\,dx$

Optimal. Leaf size=196

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2}{\sqrt{1+2\sqrt{2}}}\right.$$

```
[Out] -1/28*arctan((-2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-14+28*2^(1/
2))^(1/2)+1/28*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-14+
28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))/(-2+4*2^(1/2))
^(1/2)+1/4*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))/(-2+4*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\,\text{ArcTan}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*(Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1
+ 2*Sqrt[2]]]) + (Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] +
2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/
(4*Sqrt[2*(-1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/
(4*Sqrt[2*(-1 + 2*Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2+x^2+x^4}\,dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}}-x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\int \frac{\sqrt{-1+2\sqrt{2}}+x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$=\frac{\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{-\sqrt{-1+2\sqrt{2}}+\cdots}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\cdots}}{4\sqrt{2\left(-1+2\sqrt{2}\right.}\cdots}$$

$$=-\frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}-\frac{\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}-\sqrt{-1+2\cdots}\right.}{4\sqrt{2\left(-1+2\cdots\right.}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.05, size = 91, normalized size = 0.46

$$-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1-i\sqrt{7}\right)}}+\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[(7*(1 - I*Sqrt[7]))/2] + (I*A
rcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[(7*(1 + I*Sqrt[7]))/2]
```

Maple [A]
time = 0.06, size = 251, normalized size = 1.28

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2+2\right)}\frac{\ln\left(-\_R+x\right)}{2\_R^3+\_R}\right)}{2}$
default	$\frac{\left(\sqrt{-1+2\sqrt{2}}\,\sqrt{2}+4\sqrt{-1+2\sqrt{2}}\right)\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)}{56}+\frac{\left(7\sqrt{2}-\frac{\left(\sqrt{-1+2\sqrt{2}}\,\sqrt{\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*((-1+2*2^(1/2))^(1/2)*2^(1/2)+4*(-1+2*2^(1/2))^(1/2))*ln(x^2+2^(1/2)+x
*(-1+2*2^(1/2))^(1/2))+1/14*(7*2^(1/2)-1/2*((-1+2*2^(1/2))^(1/2)*2^(1/2)+4*
(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2)*arctan((2*x
+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))-1/56*((-1+2*2^(1/2))^(1/2)*2^(1
/2)+4*(-1+2*2^(1/2))^(1/2))*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))-1/14*(-7
*2^(1/2)+1/2*((-1+2*2^(1/2))^(1/2)*2^(1/2)+4*(-1+2*2^(1/2))^(1/2))*(-1+2*2^
(1/2))^(1/2))/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^
(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + x^2 + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(135) = 270$.
time = 0.52, size = 298, normalized size = 1.52

$$-\frac{1}{56}\cdot 8^{\frac{1}{4}}\sqrt{7}\sqrt{2}\sqrt{-4\sqrt{2}+16}\arctan\left(-\frac{1}{56}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}x\sqrt{-4\sqrt{2}+16}+\frac{1}{112}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}\sqrt{4x^{2}+8^{\frac{1}{4}}x\sqrt{-4\sqrt{2}+16}+4\sqrt{2}}\sqrt{-4\sqrt{2}+16}-\frac{1}{7}\sqrt{7}\left(2\sqrt{2}-1\right)\right)-\frac{1}{56}\cdot 8^{\frac{1}{4}}\sqrt{7}\sqrt{2}\sqrt{-4\sqrt{2}+16}\arctan\left(-\frac{1}{56}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}x\sqrt{-4\sqrt{2}+16}+\frac{1}{112}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}\sqrt{4x^{2}-8^{\frac{1}{4}}x\sqrt{-4\sqrt{2}+16}+4\sqrt{2}}\sqrt{-4\sqrt{2}+16}+\frac{1}{7}\sqrt{7}\left(2\sqrt{2}-1\right)\right)+\frac{1}{224}\cdot 8^{\frac{1}{4}}\left(\sqrt{2}+4\right)\sqrt{-4\sqrt{2}+16}\log\left(16x^{2}+4\cdot 8^{\frac{1}{4}}x\sqrt{-4\sqrt{2}+16}+16\sqrt{2}\right)-\frac{1}{224}\cdot 8^{\frac{1}{4}}\left(\sqrt{2}+4\right)\sqrt{-4\sqrt{2}+16}\log\left(16x^{2}-4\cdot 8^{\frac{1}{4}}x\sqrt{-4\sqrt{2}+16}+16\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] -1/56*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(-4*sqrt(2) + 16)*arctan(-1/56*8^(3/4)*sq
rt(7)*sqrt(2)*x*sqrt(-4*sqrt(2) + 16) + 1/112*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(
4*x^2 + 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4*sqrt(2))*sqrt(-4*sqrt(2) + 16)
- 1/7*sqrt(7)*(2*sqrt(2) - 1)) - 1/56*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(-4*sqrt(
2) + 16)*arctan(-1/56*8^(3/4)*sqrt(7)*sqrt(2)*x*sqrt(-4*sqrt(2) + 16) + 1/1
12*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(4*x^2 - 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4
*sqrt(2))*sqrt(-4*sqrt(2) + 16) + 1/7*sqrt(7)*(2*sqrt(2) - 1)) + 1/224*8^(1
/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 16)*log(16*x^2 + 4*8^(1/4)*x*sqrt(-4*sq
rt(2) + 16) + 16*sqrt(2)) - 1/224*8^(1/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 1
6)*log(16*x^2 - 4*8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 16*sqrt(2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(158) = 316$.
time = 0.65, size = 994, normalized size = 5.07

$$\sqrt{\frac{1}{224}+\frac{\sqrt{2}}{112}}\log\left(x^{2}+x\left(-\frac{4\sqrt{7}\sqrt{1+2\sqrt{2}}}{7}+\frac{5\sqrt{14}\sqrt{1+2\sqrt{2}}}{28}+\frac{3\sqrt{14}\sqrt{1+2\sqrt{2}}\sqrt{4\sqrt{2}+9}}{28}\right)-\frac{33\sqrt{4\sqrt{2}+9}}{28}-\frac{11}{28}+\frac{11\sqrt{2}\sqrt{4\sqrt{2}+9}}{28}+\frac{83\sqrt{2}}{28}\right)-\sqrt{\frac{1}{224}+\frac{\sqrt{2}}{112}}\log\left(x^{2}+x\left(-\frac{3\sqrt{14}\sqrt{1+2\sqrt{2}}\sqrt{4\sqrt{2}+9}}{28}-\frac{5\sqrt{14}\sqrt{1+2\sqrt{2}}}{28}+\frac{4\sqrt{7}\sqrt{1+2\sqrt{2}}}{7}\right)-\frac{33\sqrt{4\sqrt{2}+9}}{28}-\frac{11}{28}+\frac{11\sqrt{2}\sqrt{4\sqrt{2}+9}}{28}+\frac{83\sqrt{2}}{28}\right)+2\sqrt{-\frac{\sqrt{4\sqrt{2}+9}}{112}+\frac{1}{224}+\frac{3\sqrt{2}}{112}}\operatorname{atan}\left(\frac{4\sqrt{14}x}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}-\frac{8\sqrt{2}\sqrt{1+2\sqrt{2}}}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}+\frac{5\sqrt{1+2\sqrt{2}}}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}+\frac{3\sqrt{1+2\sqrt{2}}\sqrt{4\sqrt{2}+9}}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}\right)+2\sqrt{-\frac{\sqrt{4\sqrt{2}+9}}{112}+\frac{1}{224}+\frac{3\sqrt{2}}{112}}\operatorname{atan}\left(\frac{4\sqrt{14}x}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}-\frac{3\sqrt{1+2\sqrt{2}}\sqrt{4\sqrt{2}+9}}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}-\frac{5\sqrt{1+2\sqrt{2}}}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}+\frac{8\sqrt{2}\sqrt{1+2\sqrt{2}}}{\sqrt{4\sqrt{2}+9}\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}+7\sqrt{-2\sqrt{4\sqrt{2}+9}+1+6\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+2),x)
```

```
[Out] sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-4*sqrt(7)*sqrt(1 + 2*sqrt(2))/7 +
5*sqrt(14)*sqrt(1 + 2*sqrt(2))/28 + 3*sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4*s
qrt(2) + 9)/28) - 33*sqrt(4*sqrt(2) + 9)/28 - 11/28 + 11*sqrt(2)*sqrt(4*sqr
t(2) + 9)/28 + 83*sqrt(2)/28) - sqrt(1/224 + sqrt(2)/112)*log(x**2 + x*(-3*
sqrt(14)*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/28 - 5*sqrt(14)*sqrt(1 + 2
*sqrt(2))/28 + 4*sqrt(7)*sqrt(1 + 2*sqrt(2))/7) - 33*sqrt(4*sqrt(2) + 9)/28
 - 11/28 + 11*sqrt(2)*sqrt(4*sqrt(2) + 9)/28 + 83*sqrt(2)/28) + 2*sqrt(-sqr
t(4*sqrt(2) + 9)/112 + 1/224 + 3*sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqr
t(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*s
qrt(2) + 9) + 1 + 6*sqrt(2))) - 8*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(
2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqr
t(2) + 9) + 1 + 6*sqrt(2))) + 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sq
rt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9)
```

```
+ 1 + 6*sqrt(2))) + 3*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(
2) + 9)*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqr
t(2) + 9) + 1 + 6*sqrt(2)))) + 2*sqrt(-sqrt(4*sqrt(2) + 9)/112 + 1/224 + 3*
sqrt(2)/112)*atan(4*sqrt(14)*x/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2)
+ 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) - 3
*sqrt(1 + 2*sqrt(2))*sqrt(4*sqrt(2) + 9)/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(
4*sqrt(2) + 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqr
t(2))) - 5*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2)
+ 9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))) + 8
*sqrt(2)*sqrt(1 + 2*sqrt(2))/(sqrt(4*sqrt(2) + 9)*sqrt(-2*sqrt(4*sqrt(2) +
9) + 1 + 6*sqrt(2)) + 7*sqrt(-2*sqrt(4*sqrt(2) + 9) + 1 + 6*sqrt(2))))
```

Giac [A]
time = 0.87, size = 252, normalized size = 1.29

$$\frac{1}{112}\sqrt{7}\left(\sqrt{7}\,2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}-2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}\right)\arctan\left(\frac{2\cdot 2^{\frac{3}{4}}\sqrt{\frac{1}{2}}\left(x+2^{\frac{1}{4}}\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}\right)}{\sqrt{\sqrt{2}+4}}\right)+\frac{1}{112}\sqrt{7}\left(\sqrt{7}\,2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}-2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}\right)\arctan\left(\frac{2\cdot 2^{\frac{3}{4}}\sqrt{\frac{1}{2}}\left(x-2^{\frac{1}{4}}\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}\right)}{\sqrt{\sqrt{2}+4}}\right)+\frac{1}{224}\sqrt{7}\left(\sqrt{7}\,2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}+2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}\right)\log\left(x^{2}+2\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}+\sqrt{2}\right)-\frac{1}{224}\sqrt{7}\left(\sqrt{7}\,2^{\frac{1}{4}}\sqrt{-2\sqrt{2}+8}+2^{\frac{1}{4}}\sqrt{2\sqrt{2}+8}\right)\log\left(x^{2}-2\cdot 2^{\frac{1}{4}}x\sqrt{-\frac{1}{8}\sqrt{2}+\frac{1}{2}}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="giac")
```

```
[Out] 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(2*sqrt(2) + 8) - 2^(1/4)*sqrt(-2*sqrt(2
) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/s
qrt(sqrt(2) + 4)) + 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(2*sqrt(2) + 8) - 2^
(1/4)*sqrt(-2*sqrt(2) + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1
/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)) + 1/224*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt
(-2*sqrt(2) + 8) + 2^(1/4)*sqrt(2*sqrt(2) + 8))*log(x^2 + 2*2^(1/4)*x*sqrt(
-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/224*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(-2*sqr
t(2) + 8) + 2^(1/4)*sqrt(2*sqrt(2) + 8))*log(x^2 - 2*2^(1/4)*x*sqrt(-1/8*sq
rt(2) + 1/2) + sqrt(2))
```

Mupad [B]
time = 0.22, size = 61, normalized size = 0.31

$$\frac{\mathrm{atan}\left(\frac{\sqrt{7}\,x\,\sqrt{7-\sqrt{7}\,7\mathrm{i}}}{14}\right)\,\sqrt{7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{14}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{x\,\sqrt{1+\sqrt{7}\,1\mathrm{i}}}{2}\right)\,\sqrt{1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^4 + 2),x)
```

```
[Out] (atan((7^(1/2)*x*(7 - 7^(1/2)*7i)^(1/2))/14)*(7 - 7^(1/2)*7i)^(1/2)*1i)/14
- (7^(1/2)*atan((x*(7^(1/2)*1i + 1)^(1/2))/2)*(7^(1/2)*1i + 1)^(1/2)*1i)/14
```

3.45 $\int \frac{1}{2-x^2+x^4}\,dx$

Optimal. Leaf size=196

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right)-\frac{\log}{}$$

```
[Out] -1/28*arctan((-2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+28*2^(1/2
))^(1/2)+1/28*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(14+28
*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^(1/
2)+1/4*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^(1/2))/(2+4*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1108, 648, 632, 210, 642}

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\text{ArcTan}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{2\sqrt{2}-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{1+2\sqrt{2}}}{\sqrt{2\sqrt{2}-1}}\right)-\frac{\log\left(x^2-\sqrt{1+2\sqrt{2}}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{1+2\sqrt{2}}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] -1/2*(Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] - 2*x)/Sqrt[-1 +
 2*Sqrt[2]]]) + (Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] + 2*x
)/Sqrt[-1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*
Sqrt[2*(1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sq
rt[2*(1 + 2*Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2-x^2+x^4}\,dx = \frac{\int \frac{\sqrt{1+2\sqrt{2}}-x}{\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{1+2\sqrt{2}}+x}{\sqrt{2}+\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{1+2\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2}}{4\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$= -\frac{\log\left(\sqrt{2}-\sqrt{1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}+\sqrt{1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}} - \frac{\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2\left(-1+2\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}-\sqrt{1+2\sqrt{2}}\right.}{4\sqrt{2\left(1+2\sqrt{2}\right)}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.05, size = 91, normalized size = 0.46

$$-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1-i\sqrt{7}\right)}}+\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(-1 - I*Sqrt[7])/2]])/Sqrt[(7*(-1 - I*Sqrt[7]))/2] + (I
*ArcTan[x/Sqrt[(-1 + I*Sqrt[7])/2]])/Sqrt[(7*(-1 + I*Sqrt[7]))/2]
```

Maple [A]
time = 0.06, size = 253, normalized size = 1.29

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2+2\right)}\frac{\ln\left(-\_R+x\right)}{2\_R^3-\_R}\right)}{2}$
default	$\frac{\left(-\sqrt{1+2\sqrt{2}}\,\sqrt{2}+4\sqrt{1+2\sqrt{2}}\right)\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)}{56}+\frac{\left(7\sqrt{2}-\frac{\left(-\sqrt{1+2\sqrt{2}}\,\sqrt{2}+4\sqrt{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*(-(1+2*2^(1/2))^(1/2)*2^(1/2)+4*(1+2*2^(1/2))^(1/2))*ln(x^2+2^(1/2)+x*
(1+2*2^(1/2))^(1/2))+1/14*(7*2^(1/2)-1/2*(-(1+2*2^(1/2))^(1/2)*2^(1/2)+4*(1
+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2)*arctan((2*x+(1
+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))+1/56*((1+2*2^(1/2))^(1/2)*2^(1/2)-
4*(1+2*2^(1/2))^(1/2))*ln(x^2+2^(1/2)-x*(1+2*2^(1/2))^(1/2))+1/14*(7*2^(1/2
)+1/2*((1+2*2^(1/2))^(1/2)*2^(1/2)-4*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/
2))/(-1+2*2^(1/2))^(1/2)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 2), x)
```

Fricas [A]
time = 0.83, size = 268, normalized size = 1.37

$$-\frac{1}{28} \cdot 8^{\frac{1}{4}} \sqrt{7} \sqrt{2} \sqrt{\sqrt{2}+4} \arctan\left(-\frac{1}{28} \cdot 8^{\frac{3}{4}} \sqrt{7} \sqrt{2} x \sqrt{\sqrt{2}+4}+\frac{1}{28} \cdot 8^{\frac{3}{4}} \sqrt{7} \sqrt{2 x^{2}+8^{\frac{1}{4}} x \sqrt{\sqrt{2}+4}+2 \sqrt{2}} \sqrt{\sqrt{2}+4}-\frac{1}{7} \sqrt{7}\left(2 \sqrt{2}+1\right)\right)-\frac{1}{28} \cdot 8^{\frac{1}{4}} \sqrt{7} \sqrt{2} \sqrt{\sqrt{2}+4} \arctan\left(-\frac{1}{28} \cdot 8^{\frac{3}{4}} \sqrt{7} \sqrt{2} x \sqrt{\sqrt{2}+4}+\frac{1}{28} \cdot 8^{\frac{3}{4}} \sqrt{7} \sqrt{2 x^{2}-8^{\frac{1}{4}} x \sqrt{\sqrt{2}+4}+2 \sqrt{2}} \sqrt{\sqrt{2}+4}+\frac{1}{7} \sqrt{7}\left(2 \sqrt{2}+1\right)\right)-\frac{1}{112} \cdot 8^{\frac{1}{4}} \sqrt{\sqrt{2}+4}\left(\sqrt{2}-4\right) \log\left(16 x^{2}+8 \cdot 8^{\frac{1}{4}} x \sqrt{\sqrt{2}+4}+16 \sqrt{2}\right)+\frac{1}{112} \cdot 8^{\frac{1}{4}} \sqrt{\sqrt{2}+4}\left(\sqrt{2}-4\right) \log\left(16 x^{2}-8 \cdot 8^{\frac{1}{4}} x \sqrt{\sqrt{2}+4}+16 \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="fricas")
```

```
[Out] -1/28*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(2) + 4)*arctan(-1/28*8^(3/4)*sqrt(7
)*sqrt(2)*x*sqrt(sqrt(2) + 4) + 1/28*8^(3/4)*sqrt(7)*sqrt(2*x^2 + 8^(1/4)*x
*sqrt(sqrt(2) + 4) + 2*sqrt(2))*sqrt(sqrt(2) + 4) - 1/7*sqrt(7)*(2*sqrt(2)
+ 1)) - 1/28*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(2) + 4)*arctan(-1/28*8^(3/4)
*sqrt(7)*sqrt(2)*x*sqrt(sqrt(2) + 4) + 1/28*8^(3/4)*sqrt(7)*sqrt(2*x^2 - 8^
(1/4)*x*sqrt(sqrt(2) + 4) + 2*sqrt(2))*sqrt(sqrt(2) + 4) + 1/7*sqrt(7)*(2*s
qrt(2) + 1)) - 1/112*8^(1/4)*sqrt(sqrt(2) + 4)*(sqrt(2) - 4)*log(16*x^2 + 8
*8^(1/4)*x*sqrt(sqrt(2) + 4) + 16*sqrt(2)) + 1/112*8^(1/4)*sqrt(sqrt(2) + 4
)*(sqrt(2) - 4)*log(16*x^2 - 8*8^(1/4)*x*sqrt(sqrt(2) + 4) + 16*sqrt(2))
```

Sympy [A]
time = 0.31, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(1568t^{4}+28t^{2}+1,\left(t\mapsto t\log\left(-112t^{3}+6t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + 28*_t**2 + 1, Lambda(_t, _t*log(-112*_t**3 + 6*_t + x)
))
```

Giac [A]
time = 0.91, size = 256, normalized size = 1.31

$$\frac{1}{112} \sqrt{7}\left(\sqrt{7} 2^{\frac{1}{4}} \sqrt{-2 \sqrt{2}+8}+2^{\frac{1}{4}} \sqrt{2 \sqrt{2}+8}\right) \arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}} \sqrt{\frac{1}{2}} \sqrt{\sqrt{2}+4}+2 x\right)}{4 \sqrt{-\frac{1}{8} \sqrt{2}+\frac{1}{2}}}\right)+\frac{1}{112} \sqrt{7}\left(\sqrt{7} 2^{\frac{1}{4}} \sqrt{-2 \sqrt{2}+8}+2^{\frac{1}{4}} \sqrt{2 \sqrt{2}+8}\right) \arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}} \sqrt{\frac{1}{2}} \sqrt{\sqrt{2}+4}-2 x\right)}{4 \sqrt{-\frac{1}{8} \sqrt{2}+\frac{1}{2}}}\right)+\frac{1}{224} \sqrt{7}\left(\sqrt{7} 2^{\frac{1}{4}} \sqrt{2 \sqrt{2}+8}-2^{\frac{1}{4}} \sqrt{-2 \sqrt{2}+8}\right) \log\left(2^{\frac{1}{4}} \sqrt{\frac{1}{2}} x \sqrt{\sqrt{2}+4}+x^{2}+\sqrt{2}\right)-\frac{1}{224} \sqrt{7}\left(\sqrt{7} 2^{\frac{1}{4}} \sqrt{2 \sqrt{2}+8}-2^{\frac{1}{4}} \sqrt{-2 \sqrt{2}+8}\right) \log\left(-2^{\frac{1}{4}} \sqrt{\frac{1}{2}} x \sqrt{\sqrt{2}+4}+x^{2}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="giac")
```

```
[Out] 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(-2*sqrt(2) + 8) + 2^(1/4)*sqrt(2*sqrt(2
) + 8))*arctan(1/4*2^(3/4)*(2^(1/4)*sqrt(1/2)*sqrt(sqrt(2) + 4) + 2*x)/sqrt
(-1/8*sqrt(2) + 1/2)) + 1/112*sqrt(7)*(sqrt(7)*2^(1/4)*sqrt(-2*sqrt(2) + 8)
 + 2^(1/4)*sqrt(2*sqrt(2) + 8))*arctan(-1/4*2^(3/4)*(2^(1/4)*sqrt(1/2)*sqrt
(sqrt(2) + 4) - 2*x)/sqrt(-1/8*sqrt(2) + 1/2)) + 1/224*sqrt(7)*(sqrt(7)*2^(
```

```
1/4)*sqrt(2*sqrt(2) + 8) - 2^(1/4)*sqrt(-2*sqrt(2) + 8))*log(2^(1/4)*sqrt(1
/2)*x*sqrt(sqrt(2) + 4) + x^2 + sqrt(2)) - 1/224*sqrt(7)*(sqrt(7)*2^(1/4)*s
qrt(2*sqrt(2) + 8) - 2^(1/4)*sqrt(-2*sqrt(2) + 8))*log(-2^(1/4)*sqrt(1/2)*x
*sqrt(sqrt(2) + 4) + x^2 + sqrt(2))
```

Mupad [B]
time = 0.11, size = 132, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{-7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{4\left(\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}+\frac{\sqrt{7}\,x\sqrt{-7-\sqrt{7}\,7\mathrm{i}}}{28\left(\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}\right)\sqrt{-7-\sqrt{7}\,7\mathrm{i}}\,1\mathrm{i}}{14}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{x\sqrt{-1+\sqrt{7}\,1\mathrm{i}}}{4\left(-\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}-\frac{\sqrt{7}\,x\sqrt{-1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{4\left(-\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)}\right)\sqrt{-1+\sqrt{7}\,1\mathrm{i}}\,1\mathrm{i}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - x^2 + 2),x)
```

```
[Out] (atan((x*(- 7^(1/2)*7i - 7)^(1/2)*1i)/(4*((7^(1/2)*1i)/2 + 1/2)) + (7^(1/2)
*x*(- 7^(1/2)*7i - 7)^(1/2))/(28*((7^(1/2)*1i)/2 + 1/2)))*(- 7^(1/2)*7i - 7
)^(1/2)*1i)/14 + (7^(1/2)*atan((x*(7^(1/2)*1i - 1)^(1/2))/(4*((7^(1/2)*1i)/
2 - 1/2)) - (7^(1/2)*x*(7^(1/2)*1i - 1)^(1/2)*1i)/(4*((7^(1/2)*1i)/2 - 1/2)
))*(7^(1/2)*1i - 1)^(1/2)*1i)/14
```

3.46 $\int \frac{1}{-1+x^6}\,dx$

Optimal. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)$$

```
[Out] -1/3*arctanh(x)+1/12*ln(x^2-x+1)-1/12*ln(x^2+x+1)+1/6*arctan(1/3*(1-2*x)*3^
(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {216, 648, 632, 210, 642, 212}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^6}\,dx &= -\left(\frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx\right)-\frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx-\frac{1}{3}\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-\right.\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right.\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 75, normalized size = 1.03

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+2\log(1-x)-2\log(1+x)+\log\left(1-x+x^2\right)-\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

Maple [A]
time = 0.04, size = 66, normalized size = 0.90

method	result
risch	$-\frac{\ln(1+x)}{6}+\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,(x+\frac{1}{2})}{3}\right)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2(x-\frac{1}{2})\sqrt{3}}{3}\right)}{6}$
default	$\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$
meijerg	$\frac{x\left(\ln\left(1-(x^6)^{\frac{1}{6}}\right)-\ln\left(1+(x^6)^{\frac{1}{6}}\right)+\frac{\ln\left(1-(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{2-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+(x^6)^{\frac{1}{6}}+(x^6)^{\frac{1}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\right.\right.}{6(x^6)^{\frac{1}{6}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*l
n(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 4.62, size = 65, normalized size = 0.89

$$-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Fricas [A]
time = 0.79, size = 65, normalized size = 0.89

$$-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)+\frac{1}{6}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Sympy [A]
time = 0.12, size = 83, normalized size = 1.14

$$\frac{\log(x-1)}{6}-\frac{\log(x+1)}{6}+\frac{\log(x^2-x+1)}{12}-\frac{\log(x^2+x+1)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 -
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/6
```

Giac [A]
time = 0.98, size = 67, normalized size = 0.92

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log(x^2+x+1)+\frac{1}{12}\log(x^2-x+1)-\frac{1}{6}\log(|x+1|)+\frac{1}{6}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1)) + 1/6*log(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 88, normalized size = 1.21

$$-\frac{\mathrm{atanh}(x)}{3}-\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}+\frac{\sqrt{3}\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}-\frac{\sqrt{3}\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - 1),x)
```

```
[Out] - atanh(x)/3 - atan((x*1i)/(3^(1/2)*1i + 1) + (3^(1/2)*x)/(3^(1/2)*1i + 1))
*(3^(1/2)/6 + 1i/6) - atan((x*1i)/(3^(1/2)*1i - 1) - (3^(1/2)*x)/(3^(1/2)*1
i - 1))*(3^(1/2)/6 - 1i/6)
```

3.47 $\int \frac{1}{-2+x^6} \, dx$

Optimal. Leaf size=138

$$\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\,x+x^2\right)}{12\ 2^{5/6}}-\frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\right.}{12\ 2^{5/6}}$$

```
[Out] -1/6*arctanh(1/2*x*2^(5/6))*2^(1/6)+1/24*ln(2^(1/3)-2^(1/6)*x+x^2)*2^(1/6)-
1/24*ln(2^(1/3)+2^(1/6)*x+x^2)*2^(1/6)-1/12*arctan(-1/3*3^(1/2)+1/3*2^(5/6)
*x*3^(1/2))*2^(1/6)*3^(1/2)-1/12*arctan(1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*
2^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {216, 648, 632, 210, 642, 212}

$$\frac{\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2^{5/6}x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2-\sqrt[6]{2}\,x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\log\left(x^2+\sqrt[6]{2}\,x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)^(-1),x]
```

```
[Out] ArcTan[1/Sqrt[3] - (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTan[1/Sqrt
[3] + (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTanh[x/2^(1/6)]/(3*2^(5
/6)) + Log[2^(1/3) - 2^(1/6)*x + x^2]/(12*2^(5/6)) - Log[2^(1/3) + 2^(1/6)*
x + x^2]/(12*2^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
```

```
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-2+x^6}\,dx &= -\frac{\int \frac{\sqrt[6]{2}-\frac{x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}\,x+x^2}\,dx}{3\ 2^{5/6}}-\frac{\int \frac{\sqrt[6]{2}+\frac{x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}\,x+x^2}\,dx}{3\ 2^{5/6}}-\frac{\int \frac{1}{\sqrt[3]{2}-x^2}\,dx}{3\ 2^{2/3}}\\
&= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}+\frac{\int \frac{-\sqrt[6]{2}+2x}{\sqrt[3]{2}-\sqrt[6]{2}\,x+x^2}\,dx}{12\ 2^{5/6}}-\frac{\int \frac{\sqrt[6]{2}+2x}{\sqrt[3]{2}+\sqrt[6]{2}\,x+x^2}\,dx}{12\ 2^{5/6}}-\frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}\,x+x^2}\,dx}{4\ 2^{2/3}}-\cdots\\
&= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\,x+x^2\right)}{12\ 2^{5/6}}-\frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\,x+x^2\right)}{12\ 2^{5/6}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\cdots\right.}{2\ 2\cdots}\\
&= \frac{\tan^{-1}\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\,x+x^2\right)}{12\ 2^{5/6}}-\frac{\log\left(\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 122, normalized size = 0.88

$$
-\frac{2\sqrt{3}\tan^{-1}\left(\frac{-1+2^{5/6}x}{\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right)-2\log\left(2-2^{5/6}x\right)+2\log\left(2+2^{5/6}x\right)-\log\left(2-2^{5/6}x+2^{2/3}x^2\right)+\log\left(2+2^{5/6}x+2^{2/3}x^2\right)}{12\ 2^{5/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)^(-1),x]
```

```
[Out] -1/12*(2*Sqrt[3]*ArcTan[(-1 + 2^(5/6)*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2
^(5/6)*x)/Sqrt[3]] - 2*Log[2 - 2^(5/6)*x] + 2*Log[2 + 2^(5/6)*x] - Log[2 -
2^(5/6)*x + 2^(2/3)*x^2] + Log[2 + 2^(5/6)*x + 2^(2/3)*x^2])/2^(5/6)
```

Maple [A]
time = 0.08, size = 111, normalized size = 0.80

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^6-2)}\frac{\ln(-\_R+x)}{\_R^5}\right)}{6}$
default	$-\frac{\ln\left(2^{\frac{1}{3}}+2^{\frac{1}{6}}x+x^2\right)2^{\frac{1}{6}}}{24}-\frac{\arctan\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)2^{\frac{1}{6}}\sqrt{3}}{12}-\frac{2^{\frac{1}{6}}\ln\left(x+2^{\frac{1}{6}}\right)}{12}+\frac{\ln\left(2^{\frac{1}{3}}-2^{\frac{1}{6}}x+x^2\right)2^{\frac{1}{6}}}{24}-\frac{\arctan\left(-\frac{\sqrt{3}}{3}\cdots\right.}{\cdots}$
meijerg	$\frac{2^{\frac{1}{6}}x\left(\ln\left(1-\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right)-\ln\left(1+\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}\right)+\frac{\ln\left(1-\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}(x^6)^{\frac{1}{3}}}{2}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{4-2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2^{\frac{5}{6}}(x^6)^{\frac{1}{6}}}{2}+\cdots\right.}{2}\cdots\right.}{12(x^6)^{\frac{1}{6}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*ln(2^(1/3)+2^(1/6)*x+x^2)*2^(1/6)-1/12*arctan(1/3*3^(1/2)+1/3*2^(5/6)
*x*3^(1/2))*2^(1/6)*3^(1/2)-1/12*2^(1/6)*ln(x+2^(1/6))+1/24*ln(2^(1/3)-2^(1
/6)*x+x^2)*2^(1/6)-1/12*arctan(-1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*
3^(1/2)+1/12*2^(1/6)*ln(x-2^(1/6))
```

Maxima [A]
time = 2.74, size = 112, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2x+2^{\frac{1}{6}}\right)\right)-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2x-2^{\frac{1}{6}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2-2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(x+2^{\frac{1}{6}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(x-2^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*sq
rt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*lo
g(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3))
- 1/12*2^(1/6)*log(x + 2^(1/6)) + 1/12*2^(1/6)*log(x - 2^(1/6))
```

Fricas [A]
time = 0.91, size = 176, normalized size = 1.28

$$\frac{1}{96}\cdot 32^{\frac{5}{6}}\sqrt{3}\arctan\left(-\frac{1}{3}\cdot 32^{\frac{1}{6}}\sqrt{3}\,x+\frac{1}{12}\cdot 32^{\frac{1}{6}}\sqrt{3}\sqrt{16\,x^2+32^{\frac{5}{6}}x+8\cdot 4^{\frac{2}{3}}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{96}\cdot 32^{\frac{5}{6}}\sqrt{3}\arctan\left(-\frac{1}{3}\cdot 32^{\frac{1}{6}}\sqrt{3}\,x+\frac{1}{12}\cdot 32^{\frac{1}{6}}\sqrt{3}\sqrt{16\,x^2-32^{\frac{5}{6}}x+8\cdot 4^{\frac{2}{3}}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{384}\cdot 32^{\frac{5}{6}}\log\left(1024\,x^2+64\cdot 32^{\frac{5}{6}}x+512\cdot 4^{\frac{2}{3}}\right)+\frac{1}{384}\cdot 32^{\frac{5}{6}}\log\left(1024\,x^2-64\cdot 32^{\frac{5}{6}}x+512\cdot 4^{\frac{2}{3}}\right)-\frac{1}{192}\cdot 32^{\frac{5}{6}}\log\left(16\,x+32^{\frac{5}{6}}\right)+\frac{1}{192}\cdot 32^{\frac{5}{6}}\log\left(16\,x-32^{\frac{5}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="fricas")
```

```
[Out] 1/96*32^(5/6)*sqrt(3)*arctan(-1/3*32^(1/6)*sqrt(3)*x + 1/12*32^(1/6)*sqrt(3
)*sqrt(16*x^2 + 32^(5/6)*x + 8*4^(2/3)) - 1/3*sqrt(3)) + 1/96*32^(5/6)*sqrt
(3)*arctan(-1/3*32^(1/6)*sqrt(3)*x + 1/12*32^(1/6)*sqrt(3)*sqrt(16*x^2 - 32
^(5/6)*x + 8*4^(2/3)) + 1/3*sqrt(3)) - 1/384*32^(5/6)*log(1024*x^2 + 64*32^
(5/6)*x + 512*4^(2/3)) + 1/384*32^(5/6)*log(1024*x^2 - 64*32^(5/6)*x + 512*
4^(2/3)) - 1/192*32^(5/6)*log(16*x + 32^(5/6)) + 1/192*32^(5/6)*log(16*x -
32^(5/6))
```

Sympy [A]
time = 0.29, size = 14, normalized size = 0.10

$$\operatorname{RootSum}\left(1492992t^6-1,\left(t\mapsto t\log\left(-12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-2),x)
```

```
[Out] RootSum(1492992*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x)))
```

Giac [A]
time = 0.87, size = 114, normalized size = 0.83

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2\,x+2^{\frac{1}{6}}\right)\right)-\frac{1}{12}\sqrt{3}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{5}{6}}\left(2\,x-2^{\frac{1}{6}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2-2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(\left|x+2^{\frac{1}{6}}\right|\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\log\left(\left|x-2^{\frac{1}{6}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*sq
rt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*lo
g(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3))
- 1/12*2^(1/6)*log(abs(x + 2^(1/6))) + 1/12*2^(1/6)*log(abs(x - 2^(1/6)))
```

Mupad [B]
time = 0.21, size = 140, normalized size = 1.01

$$-\frac{2^{1/6}\,\mathrm{atanh}\left(\frac{2^{5/6}\,x}{2}\right)}{6}+\frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x\,1{}\mathrm{i}}{2\,\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,1{}\mathrm{i}}{2}\right)}-\frac{2^{1/6}\,\sqrt{3}\,x}{2\,\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,1{}\mathrm{i}}{2}\right)}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{12}+\frac{2^{1/6}\,\mathrm{atan}\left(\frac{2^{1/6}\,x\,1{}\mathrm{i}}{2\,\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,1{}\mathrm{i}}{2}\right)}+\frac{2^{1/6}\,\sqrt{3}\,x}{2\,\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\,\sqrt{3}\,1{}\mathrm{i}}{2}\right)}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - 2),x)
```

```
[Out] (2^(1/6)*atan((2^(1/6)*x*1i)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)) - (2^
(1/6)*3^(1/2)*x)/(2*((2^(1/3)*3^(1/2)*1i)/2 - 2^(1/3)/2)))*(3^(1/2)*1i + 1)
*1i)/12 - (2^(1/6)*atanh((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x*1i)/(
2*((2^(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x)/(2*((2^(1/3)*
3^(1/2)*1i)/2 + 2^(1/3)/2)))*(3^(1/2)*1i - 1)*1i)/12
```

3.48 $\int \frac{1}{2+x^6} \, dx$

Optimal. Leaf size=138

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\tan^{-1}\left(\sqrt{3}+2^{5/6}x\right)}{6\ 2^{5/6}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}\,x+x^2\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\cdots\right.}{4\ 2^{5/\cdots}}$$

```
[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/
12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))
*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {215, 648, 632, 210, 642, 209}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\text{ArcTan}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\text{ArcTan}\left(2^{5/6}x+\sqrt{3}\right)}{6\ 2^{5/6}}-\frac{\log\left(x^2-\sqrt[6]{2}\sqrt{3}\,x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2+\sqrt[6]{2}\sqrt{3}\,x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^6)^(-1),x]
```

```
[Out] ArcTan[x/2^(1/6)]/(3*2^(5/6)) - ArcTan[Sqrt[3] - 2^(5/6)*x]/(6*2^(5/6)) + A
rcTan[Sqrt[3] + 2^(5/6)*x]/(6*2^(5/6)) - Log[2^(1/3) - 2^(1/6)*Sqrt[3]*x +
x^2]/(4*2^(5/6)*Sqrt[3]) + Log[2^(1/3) + 2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6
)*Sqrt[3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
```

```
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{2+x^6}\,dx = \frac{\int \frac{\sqrt[6]{2}-\frac{\sqrt{3}x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{3\ 2^{5/6}} + \frac{\int \frac{\sqrt[6]{2}+\frac{\sqrt{3}x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{3\ 2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}+x^2}\,dx}{3\ 2^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{12\ 2^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{12\ 2^{2/3}} - \frac{\int \frac{-\sqrt[6]{2}\sqrt{3}+2x}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,d}{4\ 2^{5/6}\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\ 2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\ 2^{5/6}\sqrt{3}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}} + \frac{\tan^{-1}\left(\sqrt{3}+2^{5/6}x\right)}{6\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+\right.}{4\ 2^{5/6}\sqrt{3}}$$

Mathematica [A]

time = 0.02, size = 115, normalized size = 0.83

$$\frac{4\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)-2\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)+2\tan^{-1}\left(\sqrt{3}+2^{5/6}x\right)-\sqrt{3}\log\left(2-2^{5/6}\sqrt{3}x+2^{2/3}x^2\right)+\sqrt{3}\log\left(2+2^{5/6}\sqrt{3}x+2^{2/3}x^2\right)}{12\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^6)^(-1),x]
```

```
[Out] (4*ArcTan[x/2^(1/6)] - 2*ArcTan[Sqrt[3] - 2^(5/6)*x] + 2*ArcTan[Sqrt[3] + 2
^(5/6)*x] - Sqrt[3]*Log[2 - 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2] + Sqrt[3]*Log[
2 + 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2])/(12*2^(5/6))
```

Maple [A]
time = 0.10, size = 95, normalized size = 0.69

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+2\right)}\frac{\ln\left(-\_R+x\right)}{\_R^{5}}\right)}{6}$
default	$\frac{\arctan\left(\frac{x2^{\frac{5}{6}}}{2}\right)2^{\frac{1}{6}}}{6}+\frac{\arctan\left(x2^{\frac{5}{6}}-\sqrt{3}\right)2^{\frac{1}{6}}}{12}+\frac{\arctan\left(x2^{\frac{5}{6}}+\sqrt{3}\right)2^{\frac{1}{6}}}{12}-\frac{\ln\left(2^{\frac{1}{3}}+x^{2}-2^{\frac{1}{6}}x\sqrt{3}\right)2^{\frac{1}{6}}\sqrt{3}}{24}+\frac{\ln\left(2^{\frac{1}{3}}+x^{2}+2\right.}{}$
meijerg	$\frac{2^{\frac{1}{6}}\left(-\frac{x\sqrt{3}\,\ln\left(1-\frac{\sqrt{3}\,2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}+\frac{2^{\frac{2}{3}}\left(x^{6}\right)^{\frac{1}{3}}}{2}\right)}{2\left(x^{6}\right)^{\frac{1}{6}}}+\frac{x\arctan\left(\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{4-\sqrt{3}\,2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}\right)}{\left(x^{6}\right)^{\frac{1}{6}}}+\frac{2x\arctan\left(\frac{2^{\frac{5}{6}}\left(x^{6}\right)^{\frac{1}{6}}}{2}\right)}{\left(x^{6}\right)^{\frac{1}{6}}}+\frac{x\sqrt{3}\,\ln\left(1+\frac{\sqrt{3}\,2^{\frac{5}{6}}\left(x^{6}\right.}{2}\right.}{2\left(x^{6}\right)^{\frac{1}{6}}}\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/
12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))
*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

Maxima [A]
time = 3.24, size = 107, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^{2}+\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^{2}-\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2\,x+\sqrt{3}\,2^{\frac{1}{6}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2\,x-\sqrt{3}\,2^{\frac{1}{6}}\right)\right)+\frac{1}{6}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)*
2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*2^
(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x - sqr
t(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(94) = 188$.
time = 1.22, size = 279, normalized size = 2.02

$$\frac{1}{768}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(64\cdot 32^{\frac{5}{6}}\sqrt{3}\,x+1024\,x^{2}+512\cdot 4^{\frac{2}{3}}\right)+\frac{1}{768}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(16\cdot 32^{\frac{5}{6}}\sqrt{3}\,x+256\,x^{2}+128\cdot 4^{\frac{2}{3}}\right)-\frac{1}{768}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(-16\cdot 32^{\frac{5}{6}}\sqrt{3}\,x+256\,x^{2}+128\cdot 4^{\frac{2}{3}}\right)-\frac{1}{768}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(-64\cdot 32^{\frac{5}{6}}\sqrt{3}\,x+1024\,x^{2}+512\cdot 4^{\frac{2}{3}}\right)-\frac{1}{48}\cdot 32^{\frac{5}{6}}\arctan\left(\frac{1}{4}\cdot 32^{\frac{1}{6}}\sqrt{2}\sqrt{2\,x^{2}+4^{\frac{2}{3}}}-\frac{1}{2}\cdot 32^{\frac{1}{6}}x\right)-\frac{1}{96}\cdot 32^{\frac{5}{6}}\arctan\left(-32^{\frac{1}{6}}x+\frac{1}{4}\cdot 32^{\frac{1}{6}}\sqrt{32^{\frac{5}{6}}\sqrt{3}\,x+16\,x^{2}+8\cdot 4^{\frac{2}{3}}}-\sqrt{3}\right)-\frac{1}{192}\cdot 32^{\frac{5}{6}}\arctan\left(-32^{\frac{1}{6}}x+\frac{1}{16}\cdot 32^{\frac{1}{6}}\sqrt{-16\cdot 32^{\frac{5}{6}}\sqrt{3}\,x+256\,x^{2}+128\cdot 4^{\frac{2}{3}}}+\sqrt{3}\right)-\frac{1}{192}\cdot 32^{\frac{5}{6}}\arctan\left(-32^{\frac{1}{6}}x+\frac{1}{32}\cdot 32^{\frac{1}{6}}\sqrt{-64\cdot 32^{\frac{5}{6}}\sqrt{3}\,x+1024\,x^{2}+512\cdot 4^{\frac{2}{3}}}+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="fricas")
```

```
[Out] 1/768*32^(5/6)*sqrt(3)*log(64*32^(5/6)*sqrt(3)*x + 1024*x^2 + 512*4^(2/3))
+ 1/768*32^(5/6)*sqrt(3)*log(16*32^(5/6)*sqrt(3)*x + 256*x^2 + 128*4^(2/3))
 - 1/768*32^(5/6)*sqrt(3)*log(-16*32^(5/6)*sqrt(3)*x + 256*x^2 + 128*4^(2/3
)) - 1/768*32^(5/6)*sqrt(3)*log(-64*32^(5/6)*sqrt(3)*x + 1024*x^2 + 512*4^(
2/3)) - 1/48*32^(5/6)*arctan(1/4*32^(1/6)*sqrt(2)*sqrt(2*x^2 + 4^(2/3)) - 1
/2*32^(1/6)*x) - 1/96*32^(5/6)*arctan(-32^(1/6)*x + 1/4*32^(1/6)*sqrt(32^(5
/6)*sqrt(3)*x + 16*x^2 + 8*4^(2/3)) - sqrt(3)) - 1/192*32^(5/6)*arctan(-32^
(1/6)*x + 1/16*32^(1/6)*sqrt(-16*32^(5/6)*sqrt(3)*x + 256*x^2 + 128*4^(2/3)
) + sqrt(3)) - 1/192*32^(5/6)*arctan(-32^(1/6)*x + 1/32*32^(1/6)*sqrt(-64*3
2^(5/6)*sqrt(3)*x + 1024*x^2 + 512*4^(2/3)) + sqrt(3))
```

Sympy [A]
time = 0.13, size = 14, normalized size = 0.10

$$\operatorname{RootSum}\left(1492992t^6 + 1, \left(t \mapsto t\log\left(12t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+2),x)
```

```
[Out] RootSum(1492992*_t**6 + 1, Lambda(_t, _t*log(12*_t + x)))
```

Giac [A]
time = 0.97, size = 107, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^2+\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}\,2^{\frac{1}{6}}\log\left(x^2-\sqrt{3}\,2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2x+\sqrt{3}\,2^{\frac{1}{6}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2x-\sqrt{3}\,2^{\frac{1}{6}}\right)\right)+\frac{1}{6}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)*
2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*2^
(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x - sqr
t(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)
```

Mupad [B]
time = 0.13, size = 135, normalized size = 0.98

$$\frac{2^{1/6}\operatorname{atan}\left(\frac{2^{5/6}x}{2}\right)}{6}+\frac{2^{1/6}\operatorname{atan}\left(\frac{2^{1/6}x}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}+\frac{2^{1/6}\sqrt{3}\,x\,1\mathrm{i}}{2\left(-\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(\sqrt{3}-\mathrm{i}\right)1\mathrm{i}}{12}+\frac{2^{1/6}\operatorname{atan}\left(\frac{2^{1/6}x}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}-\frac{2^{1/6}\sqrt{3}\,x\,1\mathrm{i}}{2\left(\frac{2^{1/3}}{2}+\frac{2^{1/3}\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)\left(\sqrt{3}+1\mathrm{i}\right)1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 + 2),x)
```

```
[Out] (2^(1/6)*atan((2^(5/6)*x)/2))/6 + (2^(1/6)*atan((2^(1/6)*x)/(2*((2^(1/3)*3^
(1/2)*1i)/2 - 2^(1/3)/2)) + (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)*3^(1/2)*1i)
/2 - 2^(1/3)/2)))*(3^(1/2) - 1i)*1i)/12 + (2^(1/6)*atan((2^(1/6)*x)/(2*((2^
(1/3)*3^(1/2)*1i)/2 + 2^(1/3)/2)) - (2^(1/6)*3^(1/2)*x*1i)/(2*((2^(1/3)*3^(
1/2)*1i)/2 + 2^(1/3)/2)))*(3^(1/2) + 1i)*1i)/12
```

3.49 $\int \frac{1}{1+x^8} \, dx$

Optimal. Leaf size=339

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] -1/16*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/16*ln(1+x^2+x*(2-2^
(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/4*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/4*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/16*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(2+2^(1/2
))^(1/2)+1/16*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/4*arctan((-
2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)+1/4*arctan((2
*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {219, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2+\sqrt{2+\sqrt{2}}x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^8)^(-1),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2*(2 - Sqrt[2
])] - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sq
rt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt
[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])
/16 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 - (Sqrt[2 +
 Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[2]]*Log[1
 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1+x^8}\,dx = \frac{\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}}{1-\sqrt{2+}}}{4\sqrt{2\left(2-\right.}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x}$$

$$= -\frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right) + \frac{1}{16}\sqrt{2-\sqrt{2}}\,\log\left(1+\sqrt{2-\sqrt{2}}\,x+x^2\right.$$

$$= -\frac{1}{8}\sqrt{2+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) +$$

Mathematica [A]
time = 0.01, size = 209, normalized size = 0.62

$$\frac{1}{4}\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x-\sin\left(\frac{\pi}{8}\right)\right)\right)\cos\left(\frac{\pi}{8}\right)+\frac{1}{4}\tan^{-1}\left(\sec\left(\frac{\pi}{8}\right)\left(x+\sin\left(\frac{\pi}{8}\right)\right)\right)\cos\left(\frac{\pi}{8}\right)-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(1+x^2-2x\cos\left(\frac{\pi}{8}\right)\right)+\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(1+x^2+2x\cos\left(\frac{\pi}{8}\right)\right)+\frac{1}{4}\tan^{-1}\left(\left(x-\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)+\frac{1}{4}\tan^{-1}\left(\left(x+\cos\left(\frac{\pi}{8}\right)\right)\csc\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)-\frac{1}{8}\log\left(1+x^2-2x\sin\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)+\frac{1}{8}\log\left(1+x^2+2x\sin\left(\frac{\pi}{8}\right)\right)\sin\left(\frac{\pi}{8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^8)^(-1),x]
```

```
[Out] (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Si
n[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 + (Cos[
Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*
Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 - (Log[1 + x
^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi/8])
/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 22, normalized size = 0.06

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+1\right)}\frac{\ln\left(-\_R+x\right)}{\_R^7}\right)}{8}$

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^8+1)} \frac{\ln(-\_R+x)}{\_R^7}\right)}{8}$
meijerg	$-\frac{x\cos(\frac{\pi}{8})\ln\left(1-2\cos(\frac{\pi}{8})(x^8)^{\frac{1}{8}}+(x^8)^{\frac{1}{4}}\right)}{8(x^8)^{\frac{1}{8}}}+\frac{x\sin(\frac{\pi}{8})\arctan\left(\frac{\sin(\frac{\pi}{8})(x^8)^{\frac{1}{8}}}{1-\cos(\frac{\pi}{8})(x^8)^{\frac{1}{8}}}\right)}{4(x^8)^{\frac{1}{8}}}-\frac{x\cos(\frac{3\pi}{8})\ln\left(1-2\cos(\frac{3\pi}{8})(x^8)^{\frac{1}{8}}+(x^8)^{\frac{1}{4}}\right)}{8(x^8)^{\frac{1}{8}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sum(1/_R^7*ln(-_R+x),_R=RootOf(_Z^8+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1043 vs. 2(245) = 490.
time = 1.26, size = 1043, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*s
qrt(2)*x - sqrt(2)*sqrt(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 2*sqrt(2)*x
*sqrt(-sqrt(2) + 2) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sq
rt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(
2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - sqrt(2)*sqrt(4*x^2 - 2*sqrt(2
)*x*sqrt(sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - sqrt(sqrt(2)
+ 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/16
*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)
*x - sqrt(2)*sqrt(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(
-sqrt(2) + 2) + 4) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2)
+ 2) - sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqr
t(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - sqrt(2)*sqrt(4*x^2 - 2*sqrt(2)*x*sqr
t(sqrt(2) + 2) - 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - sqrt(sqrt(2) + 2) -
```

```
sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/64*(sqrt(
2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 + 2*sqrt(2)*x*
sqrt(sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) + 1/64*(sqrt(2)*sqr
t(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 + 2*sqrt(2)*x*sqrt(s
qrt(2) + 2) - 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - 1/64*(sqrt(2)*sqrt(sqrt
(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 - 2*sqrt(2)*x*sqrt(sqrt(2)
 + 2) + 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - 1/64*(sqrt(2)*sqrt(sqrt(2) +
2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 - 2*sqrt(2)*x*sqrt(sqrt(2) + 2)
- 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - 1/8*sqrt(sqrt(2) + 2)*arctan(-(2*x
- 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2)
 + 2)) - 1/8*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-sqrt(2)
+ 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)
*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2))/
sqrt(-sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*
sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/32*sqrt
(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/32*sqrt(sqrt(2) + 2)*l
og(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/32*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqr
t(-sqrt(2) + 2) + 1) - 1/32*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(-sqrt(2) +
2) + 1)
```

Sympy [A]
time = 1.17, size = 14, normalized size = 0.04

$$\operatorname{RootSum}\left(16777216t^{8}+1,\left(t\mapsto t\log\left(8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))
```

Giac [A]
time = 0.73, size = 239, normalized size = 0.71

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{16}\sqrt{\sqrt{2}+2}\log\left(x^{2}+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{\sqrt{2}+2}\log\left(x^{2}-x\sqrt{\sqrt{2}+2}+1\right)+\frac{1}{16}\sqrt{-\sqrt{2}+2}\log\left(x^{2}+x\sqrt{-\sqrt{2}+2}+1\right)-\frac{1}{16}\sqrt{-\sqrt{2}+2}\log\left(x^{2}-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
+ 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) +
 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*
sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) +
 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2 -
 x*sqrt(-sqrt(2) + 2) + 1)
```

Mupad [B]
time = 0.30, size = 288, normalized size = 0.85

$$\mathrm{atan}\left(\frac{x\,\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}-\frac{x\,\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{\sqrt{2-\sqrt{2}}\,\sqrt{-\sqrt{2}-2}+\sqrt{2}}\right)\left(\frac{\sqrt{-\sqrt{2}-2}\,1\mathrm{i}}{8}-\frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)-\mathrm{atan}\left(\frac{x\,\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{\sqrt{2}+\sqrt{\sqrt{2}-2}\,\sqrt{\sqrt{2}+2}}+\frac{x\,\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{\sqrt{2}+\sqrt{\sqrt{2}-2}\,\sqrt{\sqrt{2}+2}}\right)\left(\frac{\sqrt{\sqrt{2}-2}\,1\mathrm{i}}{8}+\frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right)+\mathrm{atan}\left(-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}+2}}{2}+x\,\sqrt{\sqrt{2}+2}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{16}-\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,2\mathrm{i}-\mathrm{atan}\left(x\,\sqrt{\sqrt{2}+2}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\frac{\sqrt{2}\,x\,\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{2}}{16}-\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)\sqrt{\sqrt{2}+2}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 + 1),x)
```

```
[Out] atan((x*(- 2^(1/2) - 2)^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) - 2)^(1/2
) + 2^(1/2)) - (x*(2 - 2^(1/2))^(1/2)*1i)/((2 - 2^(1/2))^(1/2)*(- 2^(1/2) -
 2)^(1/2) + 2^(1/2)))*(((- 2^(1/2) - 2)^(1/2)*1i)/8 - ((2 - 2^(1/2))^(1/2)*
1i)/8) - atan((x*(2^(1/2) - 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/2)*(2^
(1/2) + 2)^(1/2)) + (x*(2^(1/2) + 2)^(1/2)*1i)/(2^(1/2) + (2^(1/2) - 2)^(1/
2)*(2^(1/2) + 2)^(1/2)))*(((2^(1/2) - 2)^(1/2)*1i)/8 + ((2^(1/2) + 2)^(1/2)
*1i)/8) + atan(x*(2^(1/2) + 2)^(1/2)*(1/2 + 1i/2) - (2^(1/2)*x*(2^(1/2) + 2
)^(1/2))/2)*((2^(1/2)*1i)/16 - (1/16 + 1i/16))*(2^(1/2) + 2)^(1/2)*2i - ata
n(x*(2^(1/2) + 2)^(1/2)*(1/2 - 1i/2) + (2^(1/2)*x*(2^(1/2) + 2)^(1/2)*1i)/2
)*(2^(1/2)/16 - (1/16 - 1i/16))*(2^(1/2) + 2)^(1/2)*2i
```

3.50 $\int \frac{1}{-1+x^8} dx$

Optimal. Leaf size=97

$$-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)+\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\right.}{8\sqrt{2}}$$

```
[Out] -1/4*arctan(x)-1/4*arctanh(x)-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(1
+x*2^(1/2))*2^(1/2)+1/16*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/16*ln(1+x^2+x*2^(1/2
))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$,
Rules used = {220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}(x)}{4}+\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)^(-1),x]
```

```
[Out] -1/4*ArcTan[x] + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/
(4*Sqrt[2]) - ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) - Log[1 +
 Sqrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{-1+x^8}\,dx = -\left(\frac{1}{2}\int \frac{1}{1-x^4}\,dx\right) - \frac{1}{2}\int \frac{1}{1+x^4}\,dx$$
$$= -\left(\frac{1}{4}\int \frac{1}{1-x^2}\,dx\right) - \frac{1}{4}\int \frac{1}{1+x^2}\,dx - \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{1}{4}\tan^{-1}(x) - \frac{1}{4}\tanh^{-1}(x) - \frac{1}{8}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{\int \frac{\sqrt{}}{-1-}}{8}$$
$$= -\frac{1}{4}\tan^{-1}(x) - \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{1}{4}\tan^{-1}(x) + \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{4\sqrt{2}} - \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}\,x\right.}{8\sqrt{2}}$$

Mathematica [A]

time = 0.01, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(-4\tan^{-1}(x)+2\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\,x\right)-2\sqrt{2}\,\tan^{-1}\left(1+\sqrt{2}\,x\right)+2\log(1-x)-2\log(1+x)+\sqrt{2}\,\log\left(1-\sqrt{2}\,x+x^2\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)^(-1),x]
[Out] (-4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt
[2]*x] + 2*Log[1 - x] - 2*Log[1 + x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - S
qrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

Maple [A]

time = 0.04, size = 61, normalized size = 0.63

method	result
risch	$-\frac{\ln(1+x)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln(-\_R+x)\right)}{8}+\frac{\ln(-1+x)}{8}-\frac{\arctan(x)}{4}$
default	$-\frac{\arctan(x)}{4}-\frac{\mathrm{arctanh}(x)}{4}-\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{16}$
meijerg	$\frac{x\left(\ln\left(1-(x^8)^{\frac{1}{8}}\right)-\ln\left(1+(x^8)^{\frac{1}{8}}\right)+\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^8)^{\frac{1}{8}}+(x^8)^{\frac{1}{4}}\right)}{2}-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^8)^{\frac{1}{8}}}{2-\sqrt{2}\,(x^8)^{\frac{1}{8}}}\right)-2\arctan\left((x^8)^{\frac{1}{8}}\right)-\right.}{8(x^8)^{\frac{1}{8}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arctan(x)-1/4*arctanh(x)-1/16*2^(1/2)*(ln((1+x^2+x*2^(1/2))/(1+x^2-x*2
^(1/2)))+2*arctan(1+x*2^(1/2))+2*arctan(-1+x*2^(1/2)))
```

Maxima [A]
time = 2.89, size = 88, normalized size = 0.91

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{1}{4}\arctan\left(x\right)-\frac{1}{8}\log\left(x+1\right)+\frac{1}{8}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x -
 1)
```

Fricas [A]
time = 1.06, size = 116, normalized size = 1.20

$$\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\frac{1}{16}\sqrt{2}\log\left(4x^2+4\sqrt{2}x+4\right)+\frac{1}{16}\sqrt{2}\log\left(4x^2-4\sqrt{2}x+4\right)-\frac{1}{4}\arctan\left(x\right)-\frac{1}{8}\log\left(x+1\right)+\frac{1}{8}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) + 1/
4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/16
*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) + 1/16*sqrt(2)*log(4*x^2 - 4*sqrt(2)*
x + 4) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 82.25, size = 44, normalized size = 0.45

$$\frac{\log\left(x-1\right)}{8}-\frac{\log\left(x+1\right)}{8}+\frac{i\log\left(x-i\right)}{8}-\frac{i\log\left(x+i\right)}{8}+\operatorname{RootSum}\left(4096t^4+1,\left(t\mapsto t\log\left(-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-1),x)
```

```
[Out] log(x - 1)/8 - log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 + RootSum(409
6*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x)))
```

Giac [A]
time = 0.47, size = 90, normalized size = 0.93

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^{2}+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x^{2}-\sqrt{2}x+1\right)-\frac{1}{4}\arctan\left(x\right)-\frac{1}{8}\log\left(|x+1|\right)+\frac{1}{8}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(abs(x + 1)) + 1/8*lo
g(abs(x - 1))
```

Mupad [B]
time = 0.16, size = 45, normalized size = 0.46

$$\frac{\mathrm{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}-\frac{\mathrm{atan}(x)}{4}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 1),x)
```

```
[Out] (atan(x*1i)*1i)/4 - atan(x)/4 - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/8 +
 1i/8) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(1/8 - 1i/8)
```

3.51 $\int \frac{1}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1360, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[6] - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1360

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{\right.}}}{1-\sqrt{2}}}{4\sqrt{3}}$$

$$= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{}}{1+\sqrt{}}}{}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\right.}{4\sqrt{6}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 30, normalized size = 0.11

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 1.04, size = 220, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4+36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4-36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(36*x^4 + 36*sqrt(3)*sqrt(2)*(x^3 + x) + 108
*x^2 + 36) - 1/24*sqrt(3)*sqrt(2)*log(36*x^4 - 36*sqrt(3)*sqrt(2)*(x^3 + x)
 + 108*x^2 + 36)
```

Sympy [A]
time = 0.09, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3-4x^2+2\sqrt{6}\,x-3\right)\right)}{24}+\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3+4x^2+2\sqrt{6}\,x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}\,x^3+3x^2-\sqrt{6}\,x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}\,x^3+3x^2+\sqrt{6}\,x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

Giac [A]
time = 0.95, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B]
time = 0.10, size = 53, normalized size = 0.19

$$\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)
```

3.52 $\int \frac{x^7}{1+x^{12}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)$$

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {281, 298, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^12),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+x^{12}}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\right)+\frac{1}{12}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 260 vs. 2(49) = 98.
time = 0.07, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt{3}-2\sqrt{2}x}{1-\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{-1+\sqrt{3}+2\sqrt{2}x}{1+\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt{3}+2\sqrt{2}x}{-1+\sqrt{3}}\right)-2\log\left(1-\sqrt{2}x+x^2\right)-2\log\left(1+\sqrt{2}x+x^2\right)+\log\left(2+\sqrt{2}x-\sqrt{6}x+2x^2\right)+\log\left(2+\sqrt{2}\left(-1+\sqrt{3}\right)x+2x^2\right)+\log\left(2-\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)+\log\left(2+\left(\sqrt{2}+\sqrt{6}\right)x+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*Ar
cTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sq
rt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqr
t[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x
+ x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqr
t[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2
] + Sqrt[6])*x + 2*x^2])/24
```

Maple [A]
time = 0.03, size = 41, normalized size = 0.84

method	result	size
risch	$-\frac{\ln\left(x^4+1\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	39
default	$-\frac{\ln\left(x^4+1\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{24}+\frac{\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	41
meijerg	$-\frac{x^8\ln\left(1+\left(x^{12}\right)^{\frac{1}{3}}\right)}{12\left(x^{12}\right)^{\frac{2}{3}}}+\frac{x^8\ln\left(1-\left(x^{12}\right)^{\frac{1}{3}}+\left(x^{12}\right)^{\frac{2}{3}}\right)}{24\left(x^{12}\right)^{\frac{2}{3}}}+\frac{x^8\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{12}\right)^{\frac{1}{3}}}{2-\left(x^{12}\right)^{\frac{1}{3}}}\right)}{12\left(x^{12}\right)^{\frac{2}{3}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*arctan(1/3*(2*x^4-1)*3^(1/2))*3^(1/
2)
```

Maxima [A]
time = 1.80, size = 40, normalized size = 0.82

$$\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4-1\right)\right)+\frac{1}{24}\,\log\left(x^8-x^4+1\right)-\frac{1}{12}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Fricas [A]
time = 1.04, size = 40, normalized size = 0.82

$$\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4-1\right)\right)+\frac{1}{24}\,\log\left(x^8-x^4+1\right)-\frac{1}{12}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Sympy [A]
time = 0.07, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3
 - sqrt(3)/3)/12
```

Giac [A]
time = 0.84, size = 40, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Mupad [B]
time = 0.22, size = 52, normalized size = 1.06

$$-\frac{\ln\left(x^4+1\right)}{12}-\ln\left(x^4-\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)+\ln\left(x^4+\frac{\sqrt{3}\,1\mathrm{i}}{2}-\frac{1}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12 + 1),x)
```

```
[Out] log((3^(1/2)*1i)/2 + x^4 - 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x^4 - (3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x^4 + 1)/12
```

3.53 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$-x+x\log(x)$$

[Out] -x+x*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2332\}$

$$x\log(x)-x$$

Antiderivative was successfully verified.

[In] Int[Log[x],x]

[Out] -x + x*Log[x]

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log(x)\,dx=-x+x\log(x)$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-x+x\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x],x]

[Out] -x + x*Log[x]

Maple [A]
time = 0.00, size = 9, normalized size = 1.12

method	result	size

lookup	$-x + x\ln(x)$	9
default	$-x + x\ln(x)$	9
norman	$-x + x\ln(x)$	9
risch	$-x + x\ln(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -x+x*ln(x)
```

Maxima [A]
time = 1.27, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A]
time = 0.88, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.62

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

Giac [A]
time = 0.90, size = 8, normalized size = 1.00

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="giac")
[Out] x*log(x) - x
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.75

$$x(\ln(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x),x)
[Out] x*(log(x) - 1)
```

3.54 $\int x \log(x) \, dx$

Optimal. Leaf size=17

$$-\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2341}

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A]
time = 0.00, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$	14
norman	$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$	14
risch	$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A]
time = 1.43, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A]
time = 0.79, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.71

$$\frac{x^2\log\left(x\right)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

<u>**Giac**</u> [**A**]
time = 0.91, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

<u>**Mupad**</u> [**B**]
time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^2\left(\ln\left(x\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
```

```
[Out] (x^2*(log(x) - 1/2))/2
```

3.55 $\int x^2 \log(x) \, dx$

Optimal. Leaf size=17

$$-\frac{x^3}{9}+\frac{1}{3} x^3 \log(x)$$

[Out] -1/9*x^3+1/3*x^3*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2341}

$$\frac{1}{3} x^3 \log(x)-\frac{x^3}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2*Log[x],x]

[Out] -1/9*x^3 + (x^3*Log[x])/3

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \log(x) \, dx=-\frac{x^3}{9}+\frac{1}{3} x^3 \log(x)$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{x^3}{9}+\frac{1}{3} x^3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Log[x],x]

[Out] -1/9*x^3 + (x^3*Log[x])/3

Maple [A]
time = 0.00, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14
norman	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14
risch	$-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x),x,method=_RETURNVERBOSE)
[Out] -1/9*x^3+1/3*x^3*ln(x)
```

Maxima [A]
time = 2.03, size = 13, normalized size = 0.76

$$\frac{1}{3}\,x^3\log\left(x\right)-\frac{1}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="maxima")
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

Fricas [A]
time = 0.71, size = 13, normalized size = 0.76

$$\frac{1}{3}\,x^3\log\left(x\right)-\frac{1}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="fricas")
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{x^3\log\left(x\right)}{3}-\frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x),x)
[Out] x**3*log(x)/3 - x**3/9
```

Giac [A]
time = 1.07, size = 13, normalized size = 0.76

$$\frac{1}{3}\,x^{3}\log\left(x\right)-\frac{1}{9}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^{3}\left(\ln\left(x\right)-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(x),x)
```

```
[Out] (x^3*(log(x) - 1/3))/3
```

3.56 $\int x^p \log(x)\,dx$

Optimal. Leaf size=26

$$-\frac{x^{1+p}}{(1+p)^2}+\frac{x^{1+p}\log(x)}{1+p}$$

```
[Out] -x^(1+p)/(1+p)^2+x^(1+p)*ln(x)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2341}

$$\frac{x^{p+1}\log(x)}{p+1}-\frac{x^{p+1}}{(p+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*Log[x],x]
[Out] -(x^(1 + p)/(1 + p)^2) + (x^(1 + p)*Log[x])/(1 + p)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^p \log(x)\,dx=-\frac{x^{1+p}}{(1+p)^2}+\frac{x^{1+p}\log(x)}{1+p}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.73

$$\frac{x^{1+p}(-1+(1+p)\log(x))}{(1+p)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*Log[x],x]
[Out] (x^(1 + p)*(-1 + (1 + p)*Log[x]))/(1 + p)^2
```

Maple [A]
time = 0.01, size = 19, normalized size = 0.73

method	result	size
risch	$\frac{x(\ln(x)p+\ln(x)-1)x^{p}}{(1+p)^{2}}$	19
norman	$\frac{x\ln(x)e^{\ln(x)p}}{1+p}-\frac{x\,e^{\ln(x)p}}{p^{2}+2p+1}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*(ln(x)*p+ln(x)-1)/(1+p)^2*x^p
```

Maxima [A]
time = 2.52, size = 26, normalized size = 1.00

$$\frac{x^{p+1}\log\left(x\right)}{p+1}-\frac{x^{p+1}}{\left(p+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="maxima")
```

```
[Out] x^(p + 1)*log(x)/(p + 1) - x^(p + 1)/(p + 1)^2
```

Fricas [A]
time = 0.78, size = 25, normalized size = 0.96

$$\frac{\left((p+1)x\log\left(x\right)-x\right)x^{p}}{p^{2}+2\,p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="fricas")
```

```
[Out] ((p + 1)*x*log(x) - x)*x^p/(p^2 + 2*p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(20)=40$.
time = 0.20, size = 56, normalized size = 2.15

$$\begin{cases}\frac{pxx^{p}\log\left(x\right)}{p^{2}+2p+1}+\frac{xx^{p}\log\left(x\right)}{p^{2}+2p+1}-\frac{xx^{p}}{p^{2}+2p+1} & \text{for}\ p\neq-1\\ \frac{\log\left(x\right)^{2}}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*ln(x),x)
```

```
[Out] Piecewise((p*x*x**p*log(x)/(p**2 + 2*p + 1) + x*x**p*log(x)/(p**2 + 2*p + 1
) - x*x**p/(p**2 + 2*p + 1), Ne(p, -1)), (log(x)**2/2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^p*log(x), x)
```

Mupad [B]
time = 0.24, size = 32, normalized size = 1.23

$$\begin{cases} \frac{\ln(x)^2}{2} & \text{if } p = -1 \\ \frac{x^{p+1}\,(\ln(x)\,(p+1)-1)}{(p+1)^2} & \text{if } p \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*log(x),x)
```

```
[Out] piecewise(p == -1, log(x)^2/2, p ~= -1, (x^(p + 1)*(log(x)*(p + 1) - 1))/(p
 + 1)^2)
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x - 2x\log(x) + x\log^2(x)$$

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2333, 2332}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$2x - 2x\log(x) + x\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A]
time = 0.00, size = 16, normalized size = 1.07

method	result	size
default	$2x - 2x\ln(x) + x\ln(x)^2$	16
norman	$2x - 2x\ln(x) + x\ln(x)^2$	16
risch	$2x - 2x\ln(x) + x\ln(x)^2$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A]
time = 3.55, size = 12, normalized size = 0.80

$$\left(\log(x)^2 - 2\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A]
time = 0.85, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="fricas")
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A]
time = 0.03, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Giac [**A**]
time = 0.83, size = 15, normalized size = 1.00

$$x\log\left(x\right)^2 - 2\,x\log\left(x\right) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Mupad [**B**]
time = 0.03, size = 12, normalized size = 0.80

$$x\,\left(\ln\left(x\right)^2 - 2\,\ln\left(x\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2,x)
```

```
[Out] x*(log(x)^2 - 2*log(x) + 2)
```

3.58 $\int x^9 \log^{11}(x)\,dx$

Optimal. Leaf size=127

$$-\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\log(x)}{15625000}-\frac{6237x^{10}\log^2(x)}{3125000}+\frac{2079x^{10}\log^3(x)}{312500}-\frac{2079x^{10}\log^4(x)}{125000}+\frac{2079x^{10}\log^5(x)}{62500}-\frac{693x^{1}}{12}$$

```
[Out] -6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+207
9/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-693/
12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10*ln
(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2342, 2341}

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^8(x)}{1000}+\frac{99x^{10}\log^7(x)}{1250}-\frac{693x^{10}\log^6(x)}{12500}+\frac{2079x^{10}\log^5(x)}{62500}-\frac{2079x^{10}\log^4(x)}{125000}+\frac{2079x^{10}\log^3(x)}{312500}-\frac{6237x^{10}\log^2(x)}{3125000}+\frac{6237x^{10}\log(x)}{15625000}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Log[x]^11,x]
```

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2)
/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (207
9*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/125
0 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/
100 + (x^10*Log[x]^11)/10
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^9 \log^{11}(x)\,dx &= \frac{1}{10}x^{10}\log^{11}(x) - \frac{11}{10}\int x^9 \log^{10}(x)\,dx \\
&= -\frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) + \frac{11}{10}\int x^9\log^9(x)\,dx \\
&= \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) - \frac{99}{100}\int x^9\log^8(x)\,dx \\
&= -\frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) + \frac{99}{125}\int x^9\log^7(x \\
&= \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) - \\
&= -\frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \\
&= \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \\
&= -\frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} \\
&= \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} \\
&= -\frac{6237x^{10}\log^2(x)}{3125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log}{12500} \\
&= -\frac{6237x^{10}}{156250000} + \frac{6237x^{10}\log(x)}{15625000} - \frac{6237x^{10}\log^2(x)}{3125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 127, normalized size = 1.00

$$
-\frac{6237x^{10}}{156250000} + \frac{6237x^{10}\log(x)}{15625000} - \frac{6237x^{10}\log^2(x)}{3125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Log[x]^11,x]
```

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2)
/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (207
9*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/125
0 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/
100 + (x^10*Log[x]^11)/10
```

Maple [A]
time = 0.01, size = 104, normalized size = 0.82

method	result
default	$-\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\ln(x)}{15625000}-\frac{6237x^{10}\ln(x)^{2}}{3125000}+\frac{2079x^{10}\ln(x)^{3}}{312500}-\frac{2079x^{10}\ln(x)^{4}}{125000}+\frac{2079x^{10}\ln(x)^{5}}{62500}-\frac{693x^{10}\ln(x)^{6}}{12500}+9$
risch	$-\frac{6237x^{10}}{156250000}+\frac{6237x^{10}\ln(x)}{15625000}-\frac{6237x^{10}\ln(x)^{2}}{3125000}+\frac{2079x^{10}\ln(x)^{3}}{312500}-\frac{2079x^{10}\ln(x)^{4}}{125000}+\frac{2079x^{10}\ln(x)^{5}}{62500}-\frac{693x^{10}\ln(x)^{6}}{12500}+9$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*ln(x)^11,x,method=_RETURNVERBOSE)
```

```
[Out] -6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+207
9/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-693/
12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10*ln
(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

Maxima [A]
time = 1.80, size = 71, normalized size = 0.56

$$\frac{1}{156250000}\left(15625000\log(x)^{11}-17187500\log(x)^{10}+17187500\log(x)^{9}-15468750\log(x)^{8}+12375000\log(x)^{7}-8662500\log(x)^{6}+5197500\log(x)^{5}-2598750\log(x)^{4}+1039500\log(x)^{3}-311850\log(x)^{2}+62370\log(x)-6237\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="maxima")
```

```
[Out] 1/156250000*(15625000*log(x)^11 - 17187500*log(x)^10 + 17187500*log(x)^9 -
15468750*log(x)^8 + 12375000*log(x)^7 - 8662500*log(x)^6 + 5197500*log(x)^5
 - 2598750*log(x)^4 + 1039500*log(x)^3 - 311850*log(x)^2 + 62370*log(x) - 6
237)*x^10
```

Fricas [A]
time = 1.08, size = 103, normalized size = 0.81

$$\frac{1}{10}x^{10}\log(x)^{11}-\frac{11}{100}x^{10}\log(x)^{10}+\frac{11}{100}x^{10}\log(x)^{9}-\frac{99}{1000}x^{10}\log(x)^{8}+\frac{99}{1250}x^{10}\log(x)^{7}-\frac{693}{12500}x^{10}\log(x)^{6}+\frac{2079}{62500}x^{10}\log(x)^{5}-\frac{2079}{125000}x^{10}\log(x)^{4}+\frac{2079}{312500}x^{10}\log(x)^{3}-\frac{6237}{3125000}x^{10}\log(x)^{2}+\frac{6237}{15625000}x^{10}\log(x)-\frac{6237}{156250000}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/100
0*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62
500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 -
 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^
10
```

Sympy [A]
time = 0.15, size = 133, normalized size = 1.05

$$\frac{x^{10}\log(x)^{11}}{10}-\frac{11x^{10}\log(x)^{10}}{100}+\frac{11x^{10}\log(x)^{9}}{100}-\frac{99x^{10}\log(x)^{8}}{1000}+\frac{99x^{10}\log(x)^{7}}{1250}-\frac{693x^{10}\log(x)^{6}}{12500}+\frac{2079x^{10}\log(x)^{5}}{62500}-\frac{2079x^{10}\log(x)^{4}}{125000}+\frac{2079x^{10}\log(x)^{3}}{312500}-\frac{6237x^{10}\log(x)^{2}}{3125000}+\frac{6237x^{10}\log(x)}{15625000}-\frac{6237x^{10}}{156250000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*ln(x)**11,x)
```

```
[Out] x**10*log(x)**11/10 - 11*x**10*log(x)**10/100 + 11*x**10*log(x)**9/100 - 99
*x**10*log(x)**8/1000 + 99*x**10*log(x)**7/1250 - 693*x**10*log(x)**6/12500
 + 2079*x**10*log(x)**5/62500 - 2079*x**10*log(x)**4/125000 + 2079*x**10*lo
g(x)**3/312500 - 6237*x**10*log(x)**2/3125000 + 6237*x**10*log(x)/15625000
- 6237*x**10/156250000
```

Giac [A]
time = 0.62, size = 103, normalized size = 0.81

$$\frac{1}{10}\,x^{10}\log\left(x\right)^{11}-\frac{11}{100}\,x^{10}\log\left(x\right)^{10}+\frac{11}{100}\,x^{10}\log\left(x\right)^{9}-\frac{99}{1000}\,x^{10}\log\left(x\right)^{8}+\frac{99}{1250}\,x^{10}\log\left(x\right)^{7}-\frac{693}{12500}\,x^{10}\log\left(x\right)^{6}+\frac{2079}{62500}\,x^{10}\log\left(x\right)^{5}-\frac{2079}{125000}\,x^{10}\log\left(x\right)^{4}+\frac{2079}{312500}\,x^{10}\log\left(x\right)^{3}-\frac{6237}{3125000}\,x^{10}\log\left(x\right)^{2}+\frac{6237}{15625000}\,x^{10}\log\left(x\right)-\frac{6237}{156250000}\,x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="giac")
```

```
[Out] 1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/100
0*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62
500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 -
 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^
10
```

Mupad [B]
time = 0.18, size = 71, normalized size = 0.56

$$\frac{6237\,x^{10}\,\left(\frac{15625000\,{\ln\left(x\right)}^{11}}{6237}-\frac{1562500\,{\ln\left(x\right)}^{10}}{567}+\frac{1562500\,{\ln\left(x\right)}^{9}}{567}-\frac{156250\,{\ln\left(x\right)}^{8}}{63}+\frac{125000\,{\ln\left(x\right)}^{7}}{63}-\frac{12500\,{\ln\left(x\right)}^{6}}{9}+\frac{2500\,{\ln\left(x\right)}^{5}}{3}-\frac{1250\,{\ln\left(x\right)}^{4}}{3}+\frac{500\,{\ln\left(x\right)}^{3}}{3}-50\,{\ln\left(x\right)}^{2}+10\,\ln\left(x\right)-1\right)}{156250000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*log(x)^11,x)
```

```
[Out] (6237*x^10*(10*log(x) - 50*log(x)^2 + (500*log(x)^3)/3 - (1250*log(x)^4)/3
+ (2500*log(x)^5)/3 - (12500*log(x)^6)/9 + (125000*log(x)^7)/63 - (156250*l
og(x)^8)/63 + (1562500*log(x)^9)/567 - (1562500*log(x)^10)/567 + (15625000*
log(x)^11)/6237 - 1))/156250000
```

3.59 $\int \frac{\log^2(x)}{x} \, dx$

Optimal. Leaf size=8

$$\frac{\log^3(x)}{3}$$

```
[Out] 1/3*ln(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 30}

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2/x,x]
```

```
[Out] Log[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2(x)}{x} \, dx &= \text{Subst}\left(\int x^2 \, dx, x, \log(x)\right)\\ &= \frac{\log^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2/x,x]
```

```
[Out] Log[x]^3/3
```

Maple [A]
time = 0.01, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\ln(x)^3}{3}$	7
default	$\frac{\ln(x)^3}{3}$	7
norman	$\frac{\ln(x)^3}{3}$	7
risch	$\frac{\ln(x)^3}{3}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(x)^3
```

Maxima [A]
time = 2.28, size = 6, normalized size = 0.75

$$\frac{1}{3}\log\left(x\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*log(x)^3
```

Fricas [A]
time = 0.97, size = 6, normalized size = 0.75

$$\frac{1}{3}\log\left(x\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*log(x)^3
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.62

$$\frac{\log\left(x\right)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2/x,x)
```

```
[Out] log(x)**3/3
```

Giac [A]
time = 0.59, size = 6, normalized size = 0.75

$$\frac{1}{3}\,\log\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="giac")
```

```
[Out] 1/3*log(x)^3
```

Mupad [B]
time = 0.06, size = 6, normalized size = 0.75

$$\frac{\ln\left(x\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2/x,x)
```

```
[Out] log(x)^3/3
```

3.60 $\int \frac{1}{\log(x)} \, dx$

Optimal. Leaf size=2

$$\mathrm{li}(x)$$

```
[Out] Li(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2335}

$$\mathrm{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(x)} \, dx = \mathrm{li}(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\mathrm{li}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.
time = 0.00, size = 9, normalized size = 4.50

method	result	size

default	$-\text{expIntegral}(1, -\ln(x))$	9
risch	$-\text{expIntegral}(1, -\ln(x))$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-ln(x))
```

Maxima [A]
time = 1.18, size = 3, normalized size = 1.50

$$\text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="maxima")
```

```
[Out] Ei(log(x))
```

Fricas [A]
time = 0.82, size = 2, normalized size = 1.00

$$\text{log\_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

Sympy [A]
time = 0.24, size = 2, normalized size = 1.00

$$\text{li}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x),x)
```

```
[Out] li(x)
```

Giac [A]
time = 0.77, size = 3, normalized size = 1.50

$$\text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="giac")
```

```
[Out] Ei(log(x))
```

Mupad [B]
time = 0.00, size = 2, normalized size = 1.00

$$\mathrm{logint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(x),x)
```

```
[Out] logint(x)
```

3.61 $\int \frac{1}{\log(1+x)}\,dx$

Optimal. Leaf size=4

$$\text{li}(1+x)$$

[Out] Li(1+x)

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2436, 2335}

$$\text{LogIntegral}(x+1)$$

Antiderivative was successfully verified.

[In] Int[Log[1 + x]^(-1),x]

[Out] LogIntegral[1 + x]

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\log(1+x)}\,dx &= \text{Subst}\left(\int \frac{1}{\log(x)}\,dx, x, 1+x\right)\\ &= \text{li}(1+x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\text{li}(1+x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[1 + x]^(-1),x]

```
[Out] LogIntegral[1 + x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. 2(4) = 8.
time = 0.01, size = 11, normalized size = 2.75

method	result	size
derivativedivides	$-\operatorname{expIntegral}(1, -\ln(1+x))$	11
default	$-\operatorname{expIntegral}(1, -\ln(1+x))$	11
risch	$-\operatorname{expIntegral}(1, -\ln(1+x))$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-ln(1+x))
```

Maxima [A]
time = 1.42, size = 5, normalized size = 1.25

$$\operatorname{Ei}(\log(x+1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="maxima")
```

```
[Out] Ei(log(x + 1))
```

Fricas [A]
time = 1.13, size = 4, normalized size = 1.00

$$\operatorname{log\_integral}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="fricas")
```

```
[Out] log_integral(x + 1)
```

Sympy [A]
time = 0.23, size = 3, normalized size = 0.75

$$\operatorname{li}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(1+x),x)
```

```
[Out] li(x + 1)
```

Giac [A]
time = 0.86, size = 5, normalized size = 1.25

$$\operatorname{Ei}(\log(x+1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="giac")
```

```
[Out] Ei(log(x + 1))
```

Mupad [B]
time = 0.02, size = 4, normalized size = 1.00

$$\operatorname{logint}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(x + 1),x)
```

```
[Out] logint(x + 1)
```

3.62 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] ln(ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Maple [A]
time = 0.00, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x,method=_RETURNVERBOSE)
[Out] ln(ln(x))
```

Maxima [A]
time = 1.77, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
[Out] log(log(x))
```

Fricas [A]
time = 1.14, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
[Out] log(log(x))
```

Sympy [A]
time = 0.03, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
[Out] log(log(x))
```

Giac [**A**]
time = 0.66, size = 4, normalized size = 1.33

$$\log(|\log(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
```

```
[Out] log(abs(log(x)))
```

Mupad [**B**]
time = 0.12, size = 3, normalized size = 1.00

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)),x)
```

```
[Out] log(log(x))
```

3.63 $\int \frac{1}{x^2 \log^2(x)} \, dx$

Optimal. Leaf size=17

$$-\text{Ei}(-\log(x)) - \frac{1}{x\log(x)}$$

```
[Out] -Ei(-ln(x))-1/x/ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2343, 2346, 2209}

$$-\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Log[x]^2),x]
```

```
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \log^2(x)}\,dx &= -\frac{1}{x\log(x)} - \int \frac{1}{x^2\log(x)}\,dx\\ &= -\frac{1}{x\log(x)} - \text{Subst}\left(\int \frac{e^{-x}}{x}\,dx, x, \log(x)\right)\\ &= -\text{Ei}(-\log(x)) - \frac{1}{x\log(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$-\text{Ei}(-\log(x)) - \frac{1}{x\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Log[x]^2),x]
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Maple [A]
time = 0.01, size = 15, normalized size = 0.88

method	result	size
default	$-\frac{1}{x\ln(x)} + \text{expIntegral}\left(1, \ln\left(x\right)\right)$	15
risch	$-\frac{1}{x\ln(x)} + \text{expIntegral}\left(1, \ln\left(x\right)\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/x/ln(x)+Ei(1,ln(x))
```

Maxima [A]
time = 2.50, size = 6, normalized size = 0.35

$$-\Gamma\left(-1, \log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="maxima")
[Out] -gamma(-1, log(x))
```

Fricas [A]
time = 1.20, size = 19, normalized size = 1.12

$$-\frac{x\log(x)\,\mathrm{log\_integral}\left(\frac{1}{x}\right)+1}{x\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="fricas")
[Out] -(x*log(x)*log_integral(1/x) + 1)/(x*log(x))
```

Sympy [A]
time = 0.33, size = 14, normalized size = 0.82

$$-\operatorname{Ei}(-\log(x))-\frac{1}{x\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(x)**2,x)
[Out] -Ei(-log(x)) - 1/(x*log(x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="giac")
[Out] integrate(1/(x^2*log(x)^2), x)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 1.00

$$-\mathrm{ei}(-\ln(x))-\frac{1}{x\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*log(x)^2),x)
[Out] - ei(-log(x)) - 1/(x*log(x))
```

3.64 $\int \frac{\log^p(x)}{x} \, dx$

Optimal. Leaf size=12

$$\frac{\log^{1+p}(x)}{1+p}$$

```
[Out] ln(x)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 30}

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^p/x,x]
[Out] Log[x]^(1 + p)/(1 + p)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log^p(x)}{x} \, dx &= \text{Subst}\left(\int x^p \, dx, x, \log(x)\right) \\ &= \frac{\log^{1+p}(x)}{1+p} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{\log^{1+p}(x)}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^p/x,x]
[Out] Log[x]^(1 + p)/(1 + p)
```

Maple [A]
time = 0.01, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\ln(x)^{1+p}}{1+p}$	13
default	$\frac{\ln(x)^{1+p}}{1+p}$	13
risch	$\frac{\ln(x)\ln(x)^{p}}{1+p}$	13
norman	$\frac{\ln(x)e^{p\ln(\ln(x))}}{1+p}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^p/x,x,method=_RETURNVERBOSE)
[Out] ln(x)^(1+p)/(1+p)
```

Maxima [A]
time = 1.33, size = 12, normalized size = 1.00

$$\frac{\log(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="maxima")
[Out] log(x)^(p + 1)/(p + 1)
```

Fricas [A]
time = 0.91, size = 12, normalized size = 1.00

$$\frac{\log(x)^{p}\log(x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="fricas")
[Out] log(x)^p*log(x)/(p + 1)
```

Sympy [A]
time = 0.44, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(x)^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**p/x,x)
```

```
[Out] Piecewise((log(x)**(p + 1)/(p + 1), Ne(p, -1)), (log(log(x)), True))
```

Giac [A]
time = 0.90, size = 12, normalized size = 1.00

$$\frac{\log(x)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="giac")
```

```
[Out] log(x)^(p + 1)/(p + 1)
```

Mupad [B]
time = 0.15, size = 22, normalized size = 1.83

$$\begin{cases} \ln(\ln(x)) & \text{if } p = -1 \\ \frac{\ln(x)^{p+1}}{p+1} & \text{if } p \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^p/x,x)
```

```
[Out] piecewise(p == -1, log(log(x)), p ~= -1, log(x)^(p + 1)/(p + 1))
```

3.65 $\int (b+ax)\log(x)\,dx$

Optimal. Leaf size=28

$$-bx-\frac{ax^2}{4}+bx\log(x)+\frac{1}{2}ax^2\log(x)$$

```
[Out] -b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2350}

$$-\frac{ax^2}{4}+\frac{1}{2}ax^2\log(x)-bx+bx\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)*Log[x],x]
```

```
[Out] -(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (b+ax)\log(x)\,dx &= \frac{1}{2}\left(2bx+ax^2\right)\log(x)-\int \left(b+\frac{ax}{2}\right)\,dx\\ &= -bx-\frac{ax^2}{4}+\frac{1}{2}\left(2bx+ax^2\right)\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-bx-\frac{ax^2}{4}+bx\log(x)+\frac{1}{2}ax^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)*Log[x],x]
```

```
[Out] -(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2
```

Maple [A]
time = 0.01, size = 27, normalized size = 0.96

method	result	size
norman	$-bx-\frac{a\,x^{2}}{4}+bx\ln\left(x\right)+\frac{a\,x^{2}\ln\left(x\right)}{2}$	25
risch	$\left(\frac{1}{2}a\,x^{2}+bx\right)\ln\left(x\right)-\frac{a\,x^{2}}{4}-bx$	25
default	$a\left(-\frac{x^{2}}{4}+\frac{x^{2}\ln\left(x\right)}{2}\right)+b(-x+x\ln\left(x\right))$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)*ln(x),x,method=_RETURNVERBOSE)
[Out] a*(-1/4*x^2+1/2*x^2*ln(x))+b*(-x+x*ln(x))
```

Maxima [A]
time = 1.47, size = 25, normalized size = 0.89

$$-\frac{1}{4}\,ax^{2}-bx+\frac{1}{2}\,\left(ax^{2}+2\,bx\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="maxima")
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

Fricas [A]
time = 0.83, size = 25, normalized size = 0.89

$$-\frac{1}{4}\,ax^{2}-bx+\frac{1}{2}\,\left(ax^{2}+2\,bx\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="fricas")
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.79

$$-\frac{ax^{2}}{4}-bx+\left(\frac{ax^{2}}{2}+bx\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*ln(x),x)
```

```
[Out] -a*x**2/4 - b*x + (a*x**2/2 + b*x)*log(x)
```

Giac [**A**]
time = 1.06, size = 24, normalized size = 0.86

$$\frac{1}{2}\,ax^2\log\left(x\right)-\frac{1}{4}\,ax^2+bx\log\left(x\right)-bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2*log(x) - 1/4*a*x^2 + b*x*log(x) - b*x
```

Mupad [**B**]
time = 0.14, size = 21, normalized size = 0.75

$$-\frac{x\left(4\,b+a\,x-4\,b\,\ln\left(x\right)-2\,a\,x\,\ln\left(x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(b + a*x),x)
```

```
[Out] -(x*(4*b + a*x - 4*b*log(x) - 2*a*x*log(x)))/4
```

3.66 $\int (b+ax)^2 \log(x)\,dx$

Optimal. Leaf size=54

$$-b^2x-\frac{1}{2}abx^2-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}+\frac{(b+ax)^3\log(x)}{3a}$$

```
[Out] -b^2*x-1/2*a*b*x^2-1/9*a^2*x^3-1/3*b^3*ln(x)/a+1/3*(a*x+b)^3*ln(x)/a
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {32, 2350, 12, 45}

$$-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}-\frac{1}{2}abx^2+\frac{\log(x)(ax+b)^3}{3a}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)^2*Log[x],x]
```

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 - (b^3*Log[x])/(3*a) + ((b + a*x)^3*Lo
g[x])/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2350

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Dist[a + b*Log[c*x^n], u
, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\int (b+ax)^2\log(x)\,dx &= \frac{(b+ax)^3\log(x)}{3a} - \int \frac{(b+ax)^3}{3ax}\,dx \\
&= \frac{(b+ax)^3\log(x)}{3a} - \frac{\int \frac{(b+ax)^3}{x}\,dx}{3a} \\
&= \frac{(b+ax)^3\log(x)}{3a} - \frac{\int \left(3ab^2+\frac{b^3}{x}+3a^2bx+a^3x^2\right)dx}{3a} \\
&= -b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} - \frac{b^3\log(x)}{3a} + \frac{(b+ax)^3\log(x)}{3a}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.98

$$-b^2x - \frac{1}{2}abx^2 - \frac{a^2x^3}{9} + b^2x\log(x) + abx^2\log(x) + \frac{1}{3}a^2x^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)^2*Log[x],x]
```

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 + b^2*x*Log[x] + a*b*x^2*Log[x] + (a^2
*x^3*Log[x])/3
```

Maple [A]
time = 0.02, size = 48, normalized size = 0.89

method	result	size
risch	$-b^2x - \frac{ab\,x^2}{2} - \frac{a^2x^3}{9} - \frac{b^3\ln(x)}{3a} + \frac{(ax+b)^3\ln(x)}{3a}$	47
default	$a^2\left(-\frac{x^3}{9}+\frac{x^3\ln(x)}{3}\right)+2ab\left(-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}\right)+b^2(-x+x\ln(x))$	48
norman	$b^2x\ln(x) + ab\,x^2\ln(x) - \frac{a^2x^3}{9} - b^2x - \frac{ab\,x^2}{2} + \frac{a^2x^3\ln(x)}{3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)^2*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/9*x^3+1/3*x^3*ln(x))+2*a*b*(-1/4*x^2+1/2*x^2*ln(x))+b^2*(-x+x*ln(x)
)
```

Maxima [A]
time = 2.42, size = 47, normalized size = 0.87

$$-\frac{1}{9}\,a^2x^3 - \frac{1}{2}\,abx^2 - b^2x + \frac{1}{3}\left(a^2x^3 + 3\,abx^2 + 3\,b^2x\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="maxima")
```

```
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*lo
g(x)
```

Fricas [A]
time = 0.95, size = 47, normalized size = 0.87

$$-\frac{1}{9}\,a^2x^3-\frac{1}{2}\,abx^2-b^2x+\frac{1}{3}\left(a^2x^3+3\,abx^2+3\,b^2x\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="fricas")
```

```
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*lo
g(x)
```

Sympy [A]
time = 0.05, size = 44, normalized size = 0.81

$$-\frac{a^2x^3}{9}-\frac{abx^2}{2}-b^2x+\left(\frac{a^2x^3}{3}+abx^2+b^2x\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)**2*ln(x),x)
```

```
[Out] -a**2*x**3/9 - a*b*x**2/2 - b**2*x + (a**2*x**3/3 + a*b*x**2 + b**2*x)*log(
x)
```

Giac [A]
time = 1.83, size = 47, normalized size = 0.87

$$\frac{1}{3}\,a^2x^3\log\left(x\right)-\frac{1}{9}\,a^2x^3+abx^2\log\left(x\right)-\frac{1}{2}\,abx^2+b^2x\log\left(x\right)-b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/3*a^2*x^3*log(x) - 1/9*a^2*x^3 + a*b*x^2*log(x) - 1/2*a*b*x^2 + b^2*x*log
(x) - b^2*x
```

Mupad [B]
time = 0.17, size = 47, normalized size = 0.87

$$b^2\,x\,\ln\left(x\right)-\frac{a^2\,x^3}{9}-b^2\,x+\frac{a^2\,x^3\,\ln\left(x\right)}{3}-\frac{a\,b\,x^2}{2}+a\,b\,x^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)*(b + a*x)^2,x)
```

```
[Out] b^2*x*log(x) - (a^2*x^3)/9 - b^2*x + (a^2*x^3*log(x))/3 - (a*b*x^2)/2 + a*b
*x^2*log(x)
```

3.67 $\int \frac{\log(x)}{(b+ax)^2}\,dx$

Optimal. Leaf size=29

$$\frac{x\log(x)}{b(b+ax)}-\frac{\log(b+ax)}{ab}$$

```
[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2351, 31}

$$\frac{x\log(x)}{b(ax+b)}-\frac{\log(ax+b)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(b + a*x)^2,x]
```

```
[Out] (x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{(b+ax)^2}\,dx &= \frac{x\log(x)}{b(b+ax)}-\frac{\int \frac{1}{b+ax}\,dx}{b}\\ &= \frac{x\log(x)}{b(b+ax)}-\frac{\log(b+ax)}{ab}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.93

$$\frac{\frac{x\log(x)}{b+ax}-\frac{\log(b+ax)}{a}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(b + a*x)^2,x]
```

```
[Out] ((x*Log[x])/(b + a*x) - Log[b + a*x]/a)/b
```

Maple [A]
time = 0.02, size = 30, normalized size = 1.03

method	result	size
default	$\frac{x \ln(x)}{b(ax+b)} - \frac{\ln(ax+b)}{ab}$	30
norman	$\frac{x \ln(x)}{b(ax+b)} - \frac{\ln(ax+b)}{ab}$	30
risch	$-\frac{\ln(x)}{a(ax+b)} - \frac{\ln(ax+b)}{ab} + \frac{\ln(-x)}{ba}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a*x+b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

Maxima [A]
time = 1.78, size = 38, normalized size = 1.31

$$-\frac{\frac{\log(ax+b)}{b} - \frac{\log(x)}{b}}{a} - \frac{\log(x)}{(ax+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="maxima")
```

```
[Out] -(log(a*x + b)/b - log(x)/b)/a - log(x)/((a*x + b)*a)
```

Fricas [A]
time = 1.03, size = 34, normalized size = 1.17

$$\frac{ax\log(x) - (ax+b)\log(ax+b)}{a^2bx + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - (a*x + b)*log(a*x + b))/(a^2*b*x + a*b^2)
```

Sympy [A]
time = 0.10, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{a^2x + ab} + \frac{\log(x) - \log\left(x + \frac{b}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a*x+b)**2,x)
```

```
[Out] -log(x)/(a**2*x + a*b) + (log(x) - log(x + b/a))/(a*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(29) = 58.
time = 1.64, size = 138, normalized size = 4.76

$$a^2\left(\frac{\log\left(\frac{(ax+b)^2|a|\left|\frac{b}{ax+b}-1\right|}{a^2|ax+b|}\right)}{a^3b}+\frac{\log\left(-\frac{b+\frac{(ax+b)a\left(\frac{b}{ax+b}-1\right)-ab}{a}}{a}\right)}{\left((ax+b)\left(\frac{b}{ax+b}-1\right)-b\right)a^3}-\frac{\log\left(\left|-(ax+b)\left(\frac{b}{ax+b}-1\right)+b\right|\right)}{a^3b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="giac")
```

```
[Out] a^2*(log((a*x + b)^2*abs(a)*abs(b/(a*x + b) - 1)/(a^2*abs(a*x + b)))/(a^3*b
) + log(-(b + ((a*x + b)*a*(b/(a*x + b) - 1) - a*b)/a)/a)/(((a*x + b)*(b/(a
*x + b) - 1) - b)*a^3) - log(abs(-(a*x + b)*(b/(a*x + b) - 1) + b))/(a^3*b)
)
```

Mupad [B]
time = 0.23, size = 35, normalized size = 1.21

$$\frac{x^2\ln(x)}{b\left(a\,x^2+b\,x\right)}-\frac{\ln(b+a\,x)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(b + a*x)^2,x)
```

```
[Out] (x^2*log(x))/(b*(b*x + a*x^2)) - log(b + a*x)/(a*b)
```

3.68 $\int x \log(b+ax)\,dx$

Optimal. Leaf size=46

$$\frac{bx}{2a}-\frac{x^2}{4}-\frac{b^2\log(b+ax)}{2a^2}+\frac{1}{2}x^2\log(b+ax)$$

```
[Out] 1/2*b*x/a-1/4*x^2-1/2*b^2*ln(a*x+b)/a^2+1/2*x^2*ln(a*x+b)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2442, 45}

$$-\frac{b^2\log(ax+b)}{2a^2}+\frac{1}{2}x^2\log(ax+b)+\frac{bx}{2a}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\log(b+ax)\,dx &= \frac{1}{2}x^2\log(b+ax)-\frac{1}{2}a\int\frac{x^2}{b+ax}\,dx\\ &= \frac{1}{2}x^2\log(b+ax)-\frac{1}{2}a\int\left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right)dx\\ &= \frac{bx}{2a}-\frac{x^2}{4}-\frac{b^2\log(b+ax)}{2a^2}+\frac{1}{2}x^2\log(b+ax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$\frac{bx}{2a}-\frac{x^2}{4}-\frac{b^2\log(b+ax)}{2a^2}+\frac{1}{2}x^2\log(b+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

Maple [A]
time = 0.02, size = 53, normalized size = 1.15

method	result	size
norman	$\frac{bx}{2a}-\frac{x^2}{4}-\frac{b^2\ln(ax+b)}{2a^2}+\frac{x^2\ln(ax+b)}{2}$	39
risch	$\frac{bx}{2a}-\frac{x^2}{4}-\frac{b^2\ln(ax+b)}{2a^2}+\frac{x^2\ln(ax+b)}{2}$	39
derivativedivides	$\frac{-b(\ln(ax+b)(ax+b)-ax-b)+\frac{(ax+b)^2\ln(ax+b)}{2}-\frac{(ax+b)^2}{4}}{a^2}$	53
default	$\frac{-b(\ln(ax+b)(ax+b)-ax-b)+\frac{(ax+b)^2\ln(ax+b)}{2}-\frac{(ax+b)^2}{4}}{a^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a*x+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-b*(ln(a*x+b)*(a*x+b)-a*x-b)+1/2*(a*x+b)^2*ln(a*x+b)-1/4*(a*x+b)^2)
```

Maxima [A]
time = 2.27, size = 44, normalized size = 0.96

$$\frac{1}{2}\,x^2\log\left(ax+b\right)-\frac{1}{4}\,a\left(\frac{2\,b^2\log\left(ax+b\right)}{a^3}+\frac{ax^2-2\,bx}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + b) - 1/4*a*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b*x)/a^2)
```

Fricas [A]
time = 0.76, size = 39, normalized size = 0.85

$$-\frac{a^2x^2-2\,abx-2\,(a^2x^2-b^2)\log\left(ax+b\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*a*b*x - 2*(a^2*x^2 - b^2)*log(a*x + b))/a^2
```

Sympy [A]
time = 0.07, size = 42, normalized size = 0.91

$$-a\left(\frac{x^2}{4a}-\frac{bx}{2a^2}+\frac{b^2\log{(ax+b)}}{2a^3}\right)+\frac{x^2\log{(ax+b)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a*x+b),x)
```

```
[Out] -a*(x**2/(4*a) - b*x/(2*a**2) + b**2*log(a*x + b)/(2*a**3)) + x**2*log(a*x
+ b)/2
```

Giac [A]
time = 1.06, size = 58, normalized size = 1.26

$$\frac{(ax+b)^2\log{(ax+b)}}{2\,a^2}-\frac{(ax+b)b\log{(ax+b)}}{a^2}-\frac{(ax+b)^2}{4\,a^2}+\frac{(ax+b)b}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="giac")
```

```
[Out] 1/2*(a*x + b)^2*log(a*x + b)/a^2 - (a*x + b)*b*log(a*x + b)/a^2 - 1/4*(a*x
+ b)^2/a^2 + (a*x + b)*b/a^2
```

Mupad [B]
time = 0.19, size = 66, normalized size = 1.43

$$\begin{cases}\frac{x^2\,\left(\ln(a\,x)-\frac{1}{2}\right)}{2} & \text{if}\ \ b=0\\ \frac{\ln(b+a\,x)\,\left(x^2-\frac{b^2}{a^2}\right)}{2}-\frac{b^2\,\left(\frac{a^2\,x^2}{2\,b^2}-\frac{a\,x}{b}\right)}{2\,a^2} & \text{if}\ \ b\neq 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(b + a*x),x)
```

```
[Out] piecewise(b == 0, (x^2*(log(a*x) - 1/2))/2, b ~= 0, (log(b + a*x)*(x^2 - b^
2/a^2))/2 - (b^2*((a^2*x^2)/(2*b^2) - (a*x)/b))/(2*a^2))
```

3.69 $\int x^2 \log(b+ax)\,dx$

Optimal. Leaf size=59

$$-\frac{b^2x}{3a^2}+\frac{bx^2}{6a}-\frac{x^3}{9}+\frac{b^3\log(b+ax)}{3a^3}+\frac{1}{3}x^3\log(b+ax)$$

```
[Out] -1/3*b^2*x/a^2+1/6*b*x^2/a-1/9*x^3+1/3*b^3*ln(a*x+b)/a^3+1/3*x^3*ln(a*x+b)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2442, 45}

$$\frac{b^3\log(ax+b)}{3a^3}-\frac{b^2x}{3a^2}+\frac{1}{3}x^3\log(ax+b)+\frac{bx^2}{6a}-\frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[b + a*x],x]
```

```
[Out] -1/3*(b^2*x)/a^2 + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^
3*Log[b + a*x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log(b+ax)\,dx&=\frac{1}{3}x^3\log(b+ax)-\frac{1}{3}a\int\frac{x^3}{b+ax}\,dx\\&=\frac{1}{3}x^3\log(b+ax)-\frac{1}{3}a\int\left(\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}-\frac{b^3}{a^3(b+ax)}\right)dx\\&=-\frac{b^2x}{3a^2}+\frac{bx^2}{6a}-\frac{x^3}{9}+\frac{b^3\log(b+ax)}{3a^3}+\frac{1}{3}x^3\log(b+ax)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.00

$$-\frac{b^2x}{3a^2}+\frac{bx^2}{6a}-\frac{x^3}{9}+\frac{b^3\log(b+ax)}{3a^3}+\frac{1}{3}x^3\log(b+ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[b + a*x],x]
```

```
[Out] -1/3*(b^2*x)/a^2 + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^
3*Log[b + a*x])/3
```

Maple [A]
time = 0.01, size = 82, normalized size = 1.39

method	result	size
norman	$-\frac{b^2x}{3a^2}+\frac{b\,x^2}{6a}-\frac{x^3}{9}+\frac{b^3\ln(ax+b)}{3a^3}+\frac{x^3\ln(ax+b)}{3}$	50
risch	$-\frac{b^2x}{3a^2}+\frac{b\,x^2}{6a}-\frac{x^3}{9}+\frac{b^3\ln(ax+b)}{3a^3}+\frac{x^3\ln(ax+b)}{3}$	50
derivativedivides	$\frac{b^2(\ln(ax+b)(ax+b)-ax-b)-2b\left(\frac{(ax+b)^2\ln(ax+b)}{2}-\frac{(ax+b)^2}{4}\right)+\frac{(ax+b)^3\ln(ax+b)}{3}-\frac{(ax+b)^3}{9}}{a^3}$	82
default	$\frac{b^2(\ln(ax+b)(ax+b)-ax-b)-2b\left(\frac{(ax+b)^2\ln(ax+b)}{2}-\frac{(ax+b)^2}{4}\right)+\frac{(ax+b)^3\ln(ax+b)}{3}-\frac{(ax+b)^3}{9}}{a^3}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a*x+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(b^2*(ln(a*x+b)*(a*x+b)-a*x-b)-2*b*(1/2*(a*x+b)^2*ln(a*x+b)-1/4*(a*x+
b)^2)+1/3*(a*x+b)^3*ln(a*x+b)-1/9*(a*x+b)^3)
```

Maxima [A]
time = 5.98, size = 57, normalized size = 0.97

$$\frac{1}{3}\,x^3\log\left(ax+b\right)+\frac{1}{18}\,a\left(\frac{6\,b^3\log\left(ax+b\right)}{a^4}-\frac{2\,a^2x^3-3\,abx^2+6\,b^2x}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x + b) + 1/18*a*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3 - 3*a*b*
x^2 + 6*b^2*x)/a^3)
```

Fricas [A]
time = 0.74, size = 49, normalized size = 0.83

$$-\frac{2\,a^3x^3-3\,a^2bx^2+6\,ab^2x-6\,(a^3x^3+b^3)\log\left(ax+b\right)}{18\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="fricas")
```

```
[Out] -1/18*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*(a^3*x^3 + b^3)*log(a*x + b)
)/a^3
```

Sympy [A]
time = 0.08, size = 54, normalized size = 0.92

$$-a\left(\frac{x^3}{9a}-\frac{bx^2}{6a^2}+\frac{b^2x}{3a^3}-\frac{b^3\log\left(ax+b\right)}{3a^4}\right)+\frac{x^3\log\left(ax+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a*x+b),x)
```

```
[Out] -a*(x**3/(9*a) - b*x**2/(6*a**2) + b**2*x/(3*a**3) - b**3*log(a*x + b)/(3*a
**4)) + x**3*log(a*x + b)/3
```

Giac [A]
time = 1.16, size = 94, normalized size = 1.59

$$\frac{(ax+b)^3\log\left(ax+b\right)}{3\,a^3}-\frac{(ax+b)^2b\log\left(ax+b\right)}{a^3}+\frac{(ax+b)b^2\log\left(ax+b\right)}{a^3}-\frac{(ax+b)^3}{9\,a^3}+\frac{(ax+b)^2b}{2\,a^3}-\frac{(ax+b)b^2}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + b)^3*log(a*x + b)/a^3 - (a*x + b)^2*b*log(a*x + b)/a^3 + (a*x +
b)*b^2*log(a*x + b)/a^3 - 1/9*(a*x + b)^3/a^3 + 1/2*(a*x + b)^2*b/a^3 - (a*
x + b)*b^2/a^3
```

Mupad [B]
time = 0.18, size = 75, normalized size = 1.27

$$\begin{cases} \frac{x^3\,\left(\ln(a\,x)-\frac{1}{3}\right)}{3} & \text{if}\ \ b=0\\ \frac{\ln(b+a\,x)\,\left(x^3+\frac{b^3}{a^3}\right)}{3}-\frac{b^3\,\left(\frac{a^3\,x^3}{3\,b^3}-\frac{a^2\,x^2}{2\,b^2}+\frac{a\,x}{b}\right)}{3\,a^3} & \text{if}\ \ b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(b + a*x),x)
```

```
[Out] piecewise(b == 0, (x^3*(log(a*x) - 1/3))/3, b ~= 0, (log(b + a*x)*(x^3 + b^
3/a^3))/3 - (b^3*(- (a^2*x^2)/(2*b^2) + (a^3*x^3)/(3*b^3) + (a*x)/b))/(3*a^
3))
```

3.70 $\int \log(a^2 + x^2)\, dx$

Optimal. Leaf size=23

$$-2x + 2a\tan^{-1}\left(\frac{x}{a}\right) + x\log\left(a^2 + x^2\right)$$

```
[Out] -2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2498, 327, 209}

$$x\log\left(a^2 + x^2\right) + 2a\text{ArcTan}\left(\frac{x}{a}\right) - 2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\int \log \left(a^2+x^2\right) dx = x \log \left(a^2+x^2\right) - 2 \int \frac{x^2}{a^2+x^2} dx$$
$$= -2x + x \log \left(a^2+x^2\right) + \left(2a^2\right) \int \frac{1}{a^2+x^2} dx$$
$$= -2x + 2a \tan^{-1}\left(\frac{x}{a}\right) + x \log \left(a^2+x^2\right)$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-2x + 2a \tan^{-1}\left(\frac{x}{a}\right) + x \log \left(a^2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a^2 + x^2],x]
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Maple [A]
time = 0.02, size = 24, normalized size = 1.04

method	result	size
default	$-2x+2a\arctan\left(\frac{x}{a}\right)+x\ln\left(a^2+x^2\right)$	24
risch	$-2x+2a\arctan\left(\frac{x}{a}\right)+x\ln\left(a^2+x^2\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a^2+x^2),x,method=_RETURNVERBOSE)
[Out] -2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)
```

Maxima [A]
time = 5.41, size = 23, normalized size = 1.00

$$2\,a \arctan\left(\frac{x}{a}\right) + x \log \left(a^2+x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="maxima")
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

Fricas [A]
time = 0.66, size = 23, normalized size = 1.00

$$2\,a \arctan\left(\frac{x}{a}\right) + x \log \left(a^2+x^2\right) - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.06, size = 36, normalized size = 1.57

$$-2a\left(\frac{i\log\left(-ia+x\right)}{2}-\frac{i\log\left(ia+x\right)}{2}\right)+x\log\left(a^2+x^2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a**2+x**2),x)
```

```
[Out] -2*a*(I*log(-I*a + x)/2 - I*log(I*a + x)/2) + x*log(a**2 + x**2) - 2*x
```

Giac [A]
time = 1.20, size = 23, normalized size = 1.00

$$2\,a\arctan\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

Mupad [B]
time = 0.07, size = 23, normalized size = 1.00

$$x\,\ln\left(a^2+x^2\right)-2\,x+2\,a\,\mathrm{atan}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a^2 + x^2),x)
```

```
[Out] x*log(a^2 + x^2) - 2*x + 2*a*atan(x/a)
```

3.71 $\int x \log (a^2 + x^2) \, dx$

Optimal. Leaf size=27

$$-\frac{x^2}{2} + \frac{1}{2}(a^2 + x^2) \log (a^2 + x^2)$$

```
[Out] -1/2*x^2+1/2*(a^2+x^2)*ln(a^2+x^2)
```

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2504, 2436, 2332}

$$\frac{1}{2}(a^2 + x^2) \log (a^2 + x^2) - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a^2 + x^2],x]
```

```
[Out] -1/2*x^2 + ((a^2 + x^2)*Log[a^2 + x^2])/2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x \log \left(a^2+x^2\right) \, dx &= \frac{1}{2}\text{Subst}\left(\int \log \left(a^2+x\right) \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \log(x) \, dx, x, a^2+x^2\right) \\
&= -\frac{x^2}{2}+\frac{1}{2}\left(a^2+x^2\right) \log \left(a^2+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{1}{2}\left(-x^2+\left(a^2+x^2\right) \log \left(a^2+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a^2 + x^2],x]
```

```
[Out] (-x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2
```

Maple [A]
time = 0.01, size = 29, normalized size = 1.07

method	result	size
derivativedivides	$\frac{(a^2+x^2)\ln(a^2+x^2)}{2}-\frac{a^2}{2}-\frac{x^2}{2}$	29
default	$\frac{(a^2+x^2)\ln(a^2+x^2)}{2}-\frac{a^2}{2}-\frac{x^2}{2}$	29
norman	$-\frac{x^2}{2}+\frac{x^2\ln(a^2+x^2)}{2}+\frac{\ln(a^2+x^2)a^2}{2}$	33
risch	$-\frac{x^2}{2}+\frac{x^2\ln(a^2+x^2)}{2}+\frac{\ln(a^2+x^2)a^2}{2}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a^2+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a^2+x^2)*ln(a^2+x^2)-1/2*a^2-1/2*x^2
```

Maxima [A]
time = 4.06, size = 28, normalized size = 1.04

$$-\frac{1}{2}\,a^2-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(a^2+x^2\right)\log\left(a^2+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

Fricas [A]
time = 0.60, size = 23, normalized size = 0.85

$$-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(a^2+x^2\right)\log\left(a^2+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

Sympy [A]
time = 0.06, size = 31, normalized size = 1.15

$$\frac{a^2\log\left(a^2+x^2\right)}{2}+\frac{x^2\log\left(a^2+x^2\right)}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a**2+x**2),x)
```

```
[Out] a**2*log(a**2 + x**2)/2 + x**2*log(a**2 + x**2)/2 - x**2/2
```

Giac [A]
time = 1.17, size = 28, normalized size = 1.04

$$-\frac{1}{2}\,a^2-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(a^2+x^2\right)\log\left(a^2+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="giac")
```

```
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

Mupad [B]
time = 0.04, size = 51, normalized size = 1.89

$$\frac{a^2\,\ln\left(x-\sqrt{-a^2}\right)}{2}+\frac{x^2\,\ln\left(a^2+x^2\right)}{2}-\frac{x^2}{2}+\frac{a^2\,\ln\left(x+\sqrt{-a^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a^2 + x^2),x)
```

```
[Out] (a^2*log(x - (-a^2)^(1/2)))/2 + (x^2*log(a^2 + x^2))/2 - x^2/2 + (a^2*log(x
 + (-a^2)^(1/2)))/2
```

3.72 $\int x^2 \log(a^2+x^2)\, dx$

Optimal. Leaf size=44

$$\frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3\log\left(a^2+x^2\right)$$

```
[Out] 2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2505, 308, 209}

$$-\frac{2}{3}a^3\text{ArcTan}\left(\frac{x}{a}\right)+\frac{1}{3}x^3\log\left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[a^2 + x^2],x]
```

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \log\left(a^2+x^2\right)\,dx &= \frac{1}{3}x^3\log\left(a^2+x^2\right)-\frac{2}{3}\int \frac{x^4}{a^2+x^2}\,dx\\ &= \frac{1}{3}x^3\log\left(a^2+x^2\right)-\frac{2}{3}\int\left(-a^2+x^2+\frac{a^4}{a^2+x^2}\right)dx\\ &= \frac{2a^2x}{3}-\frac{2x^3}{9}+\frac{1}{3}x^3\log\left(a^2+x^2\right)-\frac{1}{3}\left(2a^4\right)\int\frac{1}{a^2+x^2}\,dx\\ &= \frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3\log\left(a^2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$\frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{3}x^3\log\left(a^2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[a^2 + x^2],x]
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.84

method	result	size
default	$\frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2a^3\arctan\left(\frac{x}{a}\right)}{3}+\frac{x^3\ln\left(a^2+x^2\right)}{3}$	37
risch	$\frac{2a^2x}{3}-\frac{2x^3}{9}-\frac{2a^3\arctan\left(\frac{x}{a}\right)}{3}+\frac{x^3\ln\left(a^2+x^2\right)}{3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a^2+x^2),x,method=_RETURNVERBOSE)
[Out] 2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)
```

Maxima [A]
time = 3.37, size = 36, normalized size = 0.82

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

Fricas [A]
time = 0.56, size = 36, normalized size = 0.82

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.08, size = 53, normalized size = 1.20

$$-2a^3\left(-\frac{i\log\left(-ia+x\right)}{6}+\frac{i\log\left(ia+x\right)}{6}\right)+\frac{2a^2x}{3}+\frac{x^3\log\left(a^2+x^2\right)}{3}-\frac{2x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a**2+x**2),x)
```

```
[Out] -2*a**3*(-I*log(-I*a + x)/6 + I*log(I*a + x)/6) + 2*a**2*x/3 + x**3*log(a**
2 + x**2)/3 - 2*x**3/9
```

Giac [A]
time = 1.10, size = 36, normalized size = 0.82

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="giac")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

Mupad [B]
time = 0.04, size = 65, normalized size = 1.48

$$\frac{2\,a^2\,x}{3}-\frac{\ln\left(x-\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{3/2}}{3}+\frac{x^3\,\ln\left(a^2+x^2\right)}{3}+\frac{\ln\left(x+\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{3/2}}{3}-\frac{2\,x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(a^2 + x^2),x)
```

```
[Out] (2*a^2*x)/3 - (log(x - (-a^2)^(1/2))*(-a^2)^(3/2))/3 + (x^3*log(a^2 + x^2))
/3 + (log(x + (-a^2)^(1/2))*(-a^2)^(3/2))/3 - (2*x^3)/9
```

3.73 $\int x^4 \log\left(a^2+x^2\right)\,dx$

Optimal. Leaf size=54

$$-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{5}x^5\log\left(a^2+x^2\right)$$

```
[Out] -2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 308, 209}

$$\frac{2}{5}a^5\text{ArcTan}\left(\frac{x}{a}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2x^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[a^2 + x^2],x]
```

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*L
og[a^2 + x^2])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4\log\left(a^2+x^2\right)\,dx = \frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2}{5}\int\frac{x^6}{a^2+x^2}\,dx$$
$$= \frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2}{5}\int\left(a^4-a^2x^2+x^4-\frac{a^6}{a^2+x^2}\right)dx$$
$$= -\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{1}{5}x^5\log\left(a^2+x^2\right)+\frac{1}{5}\left(2a^6\right)\int\frac{1}{a^2+x^2}\,dx$$
$$= -\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{5}x^5\log\left(a^2+x^2\right)$$

Mathematica [A]
time = 0.00, size = 54, normalized size = 1.00

$$-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{5}x^5\log\left(a^2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[a^2 + x^2],x]
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*L
og[a^2 + x^2])/5
```

Maple [A]
time = 0.02, size = 45, normalized size = 0.83

method	result	size
default	$-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2a^5\arctan\left(\frac{x}{a}\right)}{5}+\frac{x^5\ln\left(a^2+x^2\right)}{5}$	45
risch	$-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2a^5\arctan\left(\frac{x}{a}\right)}{5}+\frac{x^5\ln\left(a^2+x^2\right)}{5}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(a^2+x^2),x,method=_RETURNVERBOSE)
[Out] -2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)
```

Maxima [A]
time = 7.41, size = 44, normalized size = 0.81

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

Fricas [A]
time = 0.64, size = 44, normalized size = 0.81

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.09, size = 63, normalized size = 1.17

$$-2a^5\left(\frac{i\log\left(-ia+x\right)}{10}-\frac{i\log\left(ia+x\right)}{10}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{x^5\log\left(a^2+x^2\right)}{5}-\frac{2x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(a**2+x**2),x)
```

```
[Out] -2*a**5*(I*log(-I*a + x)/10 - I*log(I*a + x)/10) - 2*a**4*x/5 + 2*a**2*x**3
/15 + x**5*log(a**2 + x**2)/5 - 2*x**5/25
```

Giac [A]
time = 0.65, size = 44, normalized size = 0.81

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="giac")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

Mupad [B]
time = 0.13, size = 73, normalized size = 1.35

$$\frac{x^5\,\ln\left(a^2+x^2\right)}{5}-\frac{2\,a^4\,x}{5}-\frac{\ln\left(x-\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{5/2}}{5}+\frac{\ln\left(x+\sqrt{-a^2}\right)\,{\left(-a^2\right)}^{5/2}}{5}-\frac{2\,x^5}{25}+\frac{2\,a^2\,x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(a^2 + x^2),x)
```

```
[Out] (x^5*log(a^2 + x^2))/5 - (2*a^4*x)/5 - (log(x - (-a^2)^(1/2))*(-a^2)^(5/2))
/5 + (log(x + (-a^2)^(1/2))*(-a^2)^(5/2))/5 - (2*x^5)/25 + (2*a^2*x^3)/15
```

3.74 $\int \log(-a^2+x^2)\,dx$

Optimal. Leaf size=25

$$-2x+2a\tanh^{-1}\left(\frac{x}{a}\right)+x\log\left(-a^2+x^2\right)$$

```
[Out] -2*x+2*a*arctanh(x/a)+x*ln(-a^2+x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2498, 327, 213}

$$x\log\left(x^2-a^2\right)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(-a^2+x^2\right)\,dx &= x\log\left(-a^2+x^2\right)-2\int \frac{x^2}{-a^2+x^2}\,dx\\
&=-2x+x\log\left(-a^2+x^2\right)-\left(2a^2\right)\int \frac{1}{-a^2+x^2}\,dx\\
&=-2x+2a\tanh^{-1}\left(\frac{x}{a}\right)+x\log\left(-a^2+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$-2x+2a\tanh^{-1}\left(\frac{x}{a}\right)+x\log\left(-a^2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Maple [A]
time = 0.01, size = 32, normalized size = 1.28

method	result	size
default	$x\ln\left(-a^2+x^2\right)-2x+a\ln\left(a+x\right)-a\ln\left(a-x\right)$	32
risch	$x\ln\left(-a^2+x^2\right)-2x+a\ln\left(a+x\right)-a\ln\left(-a+x\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-a^2+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(-a^2+x^2)-2*x+a*ln(a+x)-a*ln(a-x)
```

Maxima [A]
time = 3.95, size = 31, normalized size = 1.24

$$x\log\left(-a^2+x^2\right)+a\log\left(a+x\right)-a\log\left(-a+x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="maxima")
```

```
[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

Fricas [A]
time = 0.51, size = 31, normalized size = 1.24

$$x\log\left(-a^2+x^2\right)+a\log\left(a+x\right)-a\log\left(-a+x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="fricas")
```

```
[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

Sympy [A]
time = 0.07, size = 29, normalized size = 1.16

$$-2a\left(\frac{\log\left(-a+x\right)}{2}-\frac{\log\left(a+x\right)}{2}\right)+x\log\left(-a^{2}+x^{2}\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-a**2+x**2),x)
```

```
[Out] -2*a*(log(-a + x)/2 - log(a + x)/2) + x*log(-a**2 + x**2) - 2*x
```

Giac [A]
time = 0.64, size = 33, normalized size = 1.32

$$x\log\left(-a^{2}+x^{2}\right)+a\log\left(|a+x|\right)-a\log\left(|-a+x|\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="giac")
```

```
[Out] x*log(-a^2 + x^2) + a*log(abs(a + x)) - a*log(abs(-a + x)) - 2*x
```

Mupad [B]
time = 0.07, size = 25, normalized size = 1.00

$$x\,\ln\left(x^{2}-a^{2}\right)-2\,x+2\,a\,\mathrm{atanh}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2 - a^2),x)
```

```
[Out] x*log(x^2 - a^2) - 2*x + 2*a*atanh(x/a)
```

3.75 $\int \log(\log(\log(\log(x))))\,dx$

Optimal. Leaf size=8

$$\text{Int}(\log(\log(\log(\log(x)))), x)$$

```
[Out] CannotIntegrate(ln(ln(ln(ln(x)))),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \log(\log(\log(\log(x))))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[Log[Log[Log[x]]]],x]
```

```
[Out] Defer[Int][Log[Log[Log[Log[x]]]], x]
```

Rubi steps

$$\int \log(\log(\log(\log(x))))\,dx = \int \log(\log(\log(\log(x))))\,dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \log(\log(\log(\log(x))))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[Log[Log[Log[x]]]],x]
```

```
[Out] Integrate[Log[Log[Log[Log[x]]]], x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \ln\left(\ln\left(\ln\left(\ln\left(x\right)\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(ln(ln(x)))),x)
```

```
[Out] int(ln(ln(ln(ln(x)))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="maxima")
[Out] x*log(log(log(log(x)))) - integrate(1/(log(x)*log(log(x))*log(log(log(x))))
, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="fricas")
[Out] integral(log(log(log(log(x)))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$x \log \left(\log \left(\log \left(\log \left(x\right)\right)\right)\right) - \int \frac{1}{\log \left(x\right) \log \left(\log \left(x\right)\right) \log \left(\log \left(\log \left(x\right)\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(ln(ln(x)))),x)
[Out] x*log(log(log(log(x)))) - Integral(1/(log(x)*log(log(x))*log(log(log(x)))),
 x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="giac")
[Out] integrate(log(log(log(log(x)))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.12

$$\int \ln \left(\ln \left(\ln \left(\ln \left(x\right)\right)\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(log(log(x)))),x)
[Out] int(log(log(log(log(x)))), x)
```

3.76 $\int \sin(x) \, dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -cos(x)
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{2718\}$

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x],x]
```

```
[Out] -Cos[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(x) \, dx = -\cos(x)$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x],x]
```

```
[Out] -Cos[x]
```

Maple [A]
time = 0.01, size = 5, normalized size = 1.25

method	result	size

lookup	$-\cos(x)$	5
default	$-\cos(x)$	5
risch	$-\cos(x)$	5
norman	$-\frac{2}{1+\tan^2\left(\frac{x}{2}\right)}$	13
meijerg	$\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x)}{\sqrt{\pi}}\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)
```

Maxima [A]
time = 1.91, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="maxima")
```

```
[Out] -cos(x)
```

Fricas [A]
time = 0.62, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)
```

Sympy [A]
time = 0.03, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x)
```

```
[Out] -cos(x)
```

Giac [A]
time = 0.92, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="giac")
```

```
[Out] -cos(x)
```

Mupad [B]
time = 0.02, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
```

```
[Out] -cos(x)
```

3.77 $\int \cos(x) \, dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2717}

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x) \, dx = \sin(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

Maple [A]
time = 0.01, size = 3, normalized size = 1.50

method	result	size

lookup	$\sin(x)$	3
default	$\sin(x)$	3
meijerg	$\sin(x)$	3
risch	$\sin(x)$	3
norman	$\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)
```

Maxima [A]
time = 2.54, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A]
time = 0.52, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A]
time = 0.02, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
```

```
[Out] sin(x)
```

Giac [A]
time = 1.20, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

Mupad [B]
time = 0.03, size = 2, normalized size = 1.00

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

3.78 $\int \tan(x)\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -ln(cos(x))
```

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(x)\,dx = -\log(\cos(x))$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Maple [A]
time = 0.01, size = 6, normalized size = 1.20

method	result	size

lookup	$-\ln(\cos(x))$	6
default	$-\ln(\cos(x))$	6
derivativedivides	$\frac{\ln(1+\tan^2(x))}{2}$	10
norman	$\frac{\ln(1+\tan^2(x))}{2}$	10
risch	$ix - \ln(e^{2ix} + 1)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x,method=_RETURNVERBOSE)
[Out] -ln(cos(x))
```

Maxima [A]
time = 2.96, size = 3, normalized size = 0.60

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="maxima")
[Out] log(sec(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.
time = 0.55, size = 11, normalized size = 2.20

$$-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="fricas")
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A]
time = 0.03, size = 5, normalized size = 1.00

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x)
[Out] -log(cos(x))
```

Giac [A]
time = 0.89, size = 6, normalized size = 1.20

$$-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="giac")
[Out] -log(abs(cos(x)))
```

Mupad [B]
time = 0.03, size = 5, normalized size = 1.00

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
[Out] -log(cos(x))
```

3.79 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Maple [A]
time = 0.02, size = 4, normalized size = 1.33

method	result	size

lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
risch	$-ix+\ln(e^{2ix}-1)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sin(x))
```

Maxima [A]
time = 3.05, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.
time = 0.81, size = 16, normalized size = 5.33

$$\frac{1}{2}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(x)^2/(tan(x)^2 + 1))
```

Sympy [A]
time = 0.03, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x)
```

```
[Out] log(sin(x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.
time = 0.68, size = 17, normalized size = 5.67

$$-\frac{1}{2}\log\left(\tan\left(x\right)^{2}+1\right)+\frac{1}{2}\log\left(\tan\left(x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="giac")
```

```
[Out] -1/2*log(tan(x)^2 + 1) + 1/2*log(tan(x)^2)
```

Mupad [**B**]
time = 0.20, size = 13, normalized size = 4.33

$$\ln\left(\tan(x)\right)-\frac{\ln\left(\tan(x)^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x),x)
```

```
[Out] log(tan(x)) - log(tan(x)^2 + 1)/2
```

3.80 $\int \frac{1}{(1+\tan(x))^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(\cos(x)+\sin(x))-\frac{1}{2(1+\tan(x))}$$

```
[Out] 1/2*ln(cos(x)+sin(x))-1/2/(1+tan(x))
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3611}

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x])^(-2),x]
```

```
[Out] Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\tan(x))^2}\,dx &= -\frac{1}{2(1+\tan(x))}+\frac{1}{2}\int \frac{1-\tan(x)}{1+\tan(x)}\,dx\\ &= \frac{1}{2}\log(\cos(x)+\sin(x))-\frac{1}{2(1+\tan(x))}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 1.29

$$\frac{\log(\cos(x)+\sin(x))+\tan(x)+\log(\cos(x)+\sin(x))\tan(x)}{2+2\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x])^(-2),x]
```

```
[Out] (Log[Cos[x] + Sin[x]] + Tan[x] + Log[Cos[x] + Sin[x]]*Tan[x])/(2 + 2*Tan[x]
)
```

Maple [A]
time = 0.03, size = 26, normalized size = 1.24

method	result	size
derivativedivides	$-\frac{1}{2(\tan(x)+1)}+\frac{\ln(\tan(x)+1)}{2}-\frac{\ln(1+\tan^2(x))}{4}$	26
default	$-\frac{1}{2(\tan(x)+1)}+\frac{\ln(\tan(x)+1)}{2}-\frac{\ln(1+\tan^2(x))}{4}$	26
norman	$-\frac{1}{2(\tan(x)+1)}+\frac{\ln(\tan(x)+1)}{2}-\frac{\ln(1+\tan^2(x))}{4}$	26
risch	$-\frac{ix}{2}-\frac{1}{2(e^{2ix}+i)}+\frac{\ln(e^{2ix}+i)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(tan(x)+1)+1/2*ln(tan(x)+1)-1/4*ln(1+tan(x)^2)
```

Maxima [A]
time = 2.97, size = 25, normalized size = 1.19

$$-\frac{1}{2(\tan(x)+1)}-\frac{1}{4}\log\left(\tan(x)^2+1\right)+\frac{1}{2}\log\left(\tan(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.
time = 1.01, size = 37, normalized size = 1.76

$$\frac{(\tan(x)+1)\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)+\tan(x)-1}{4(\tan(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((tan(x) + 1)*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) + tan(x) -
1)/(tan(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(17) = 34$.
time = 0.16, size = 75, normalized size = 3.57

$$\frac{2\log(\tan(x)+1)\tan(x)}{4\tan(x)+4} + \frac{2\log(\tan(x)+1)}{4\tan(x)+4} - \frac{\log(\tan^2(x)+1)\tan(x)}{4\tan(x)+4} - \frac{\log(\tan^2(x)+1)}{4\tan(x)+4} - \frac{2}{4\tan(x)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))**2,x)
```

```
[Out] 2*log(tan(x) + 1)*tan(x)/(4*tan(x) + 4) + 2*log(tan(x) + 1)/(4*tan(x) + 4)
- log(tan(x)**2 + 1)*tan(x)/(4*tan(x) + 4) - log(tan(x)**2 + 1)/(4*tan(x) +
 4) - 2/(4*tan(x) + 4)
```

Giac [A]
time = 0.64, size = 26, normalized size = 1.24

$$-\frac{1}{2(\tan(x)+1)} - \frac{1}{4}\log\left(\tan(x)^2+1\right) + \frac{1}{2}\log(|\tan(x)+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="giac")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(abs(tan(x) + 1))
```

Mupad [B]
time = 0.25, size = 27, normalized size = 1.29

$$\frac{\ln(\tan(x)+1)}{2} - \frac{\ln\left(\tan(x)^2+1\right)}{4} - \frac{1}{2\,(\tan(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1)^2,x)
```

```
[Out] log(tan(x) + 1)/2 - log(tan(x)^2 + 1)/4 - 1/(2*(tan(x) + 1))
```

3.81 $\int \sec(x)\,dx$

Optimal. Leaf size=3

$$\tanh^{-1}(\sin(x))$$

```
[Out] arctanh(sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(x)\,dx = \tanh^{-1}(\sin(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(3) = 6.
time = 0.00, size = 33, normalized size = 11.00

$$-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.02, size = 7, normalized size = 2.33

method	result	size

default	$\ln(\sec(x)+\tan(x))$	7
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$	18
risch	$\ln(e^{ix}+i)-\ln(e^{ix}-i)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sec(x)+tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3)=6$.
time = 1.92, size = 15, normalized size = 5.00

$$\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3)=6$.
time = 1.20, size = 17, normalized size = 5.67

$$\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3)=6$.
time = 0.05, size = 15, normalized size = 5.00

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.
time = 0.57, size = 17, normalized size = 5.67

$$\frac{1}{2}\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Mupad [**B**]
time = 0.13, size = 11, normalized size = 3.67

$$\ln\left(\frac{1}{\cos\left(x\right)}\right)+\ln\left(\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x)
```

```
[Out] log(1/cos(x)) + log(sin(x) + 1)
```

3.82 $\int \csc(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

```
[Out] -arctanh(cos(x))
```

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3855}

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(x)\,dx = -\tanh^{-1}(\cos(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.
time = 0.00, size = 17, normalized size = 3.40

$$-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A]
time = 0.02, size = 9, normalized size = 1.80

method	result	size

norman	$\ln\left(\tan\left(\frac{x}{2}\right)\right)$	6
default	$\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)$	9
risch	$-\ln\left(1+e^{ix}\right)+\ln\left(e^{ix}-1\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(csc(x)-cot(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 2.38, size = 15, normalized size = 3.00

$$-\frac{1}{2}\log\left(\cos\left(x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.
time = 1.10, size = 19, normalized size = 3.80

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 0.05, size = 15, normalized size = 3.00

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.
time = 0.58, size = 17, normalized size = 3.40

$$-\frac{1}{2}\log\left(\cos\left(x\right)+1\right)+\frac{1}{2}\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

Mupad [B]
time = 0.04, size = 5, normalized size = 1.00

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x),x)
```

```
[Out] log(tan(x/2))
```

3.83 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
```

```
[Out] x/2 - Sin[2*x]/4
```

Maple [A]
time = 0.00, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^3\left(\frac{x}{2}\right)+x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A]
time = 3.88, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x-\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A]
time = 1.05, size = 10, normalized size = 0.71

$$-\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

Giac [A]
time = 0.57, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] x/2 - sin(2*x)/4
```

3.84 $\int x^3 \sin(x^2)\, dx$

Optimal. Leaf size=20

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}$$

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 3377, 2717}

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \sin(x^2)\,dx = \frac{1}{2}\text{Subst}\left(\int x\sin(x)\,dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos(x^2) + \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
```

```
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
default	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
risch	$-\frac{x^2\cos(x^2)}{2}+\frac{\sin(x^2)}{2}$	17
meijerg	$\sqrt{\pi}\left(-\frac{x^2\cos(x^2)}{2\sqrt{\pi}}+\frac{\sin(x^2)}{2\sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^2}{2}+\frac{x^2\left(\tan^2\left(\frac{x^2}{2}\right)\right)}{2}+\tan\left(\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{x^2}{2}\right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A]
time = 2.41, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos(x^2) + \frac{1}{2}\,\sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A]
time = 1.05, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A]
time = 0.13, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Giac [A]
time = 0.62, size = 16, normalized size = 0.80

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B]
time = 0.16, size = 16, normalized size = 0.80

$$\frac{\sin\left(x^2\right)}{2}-\frac{x^2\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.85 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$-\cos(x)+\frac{\cos^3(x)}{3}$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.15

$$-\frac{3\cos(x)}{4}+\frac{1}{12}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A]
time = 0.00, size = 11, normalized size = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
norman	$\frac{-4(\tan^2(\frac{x}{2}))-\frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A]
time = 2.35, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A]
time = 0.97, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

Giac [A]
time = 0.64, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos\left(x\right)^{3}-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.77

$$\frac{\cos\left(x\right)\,\left(\cos\left(x\right)^2-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

3.86 $\int \sin^p(x)\,dx$

Optimal. Leaf size=44

$$\frac{\cos(x)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2(x)\right)\sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

```
[Out] cos(x)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],sin(x)^2)*sin(x)^(1+p)/(1+p)/
(cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{2722\}$

$$\frac{\cos(x)\sin^{p+1}(x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^p,x]
```

```
[Out] (Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sin[x]^(1 +
p))/((1 + p)*Sqrt[Cos[x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^p(x)\,dx = \frac{\cos(x)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2(x)\right)\sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.00

$$-\cos(x)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3}{2};\cos^2(x)\right)\sin^{1+p}(x)\sin^2(x)^{\frac{1}{2}(-1-p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^p,x]
```

```
[Out] -(Cos[x]*Hypergeometric2F1[1/2, (1 - p)/2, 3/2, Cos[x]^2]*Sin[x]^(1 + p)*(S
in[x]^2)^((-1 - p)/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sin^p(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^p,x)
```

```
[Out] int(sin(x)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="fricas")
```

```
[Out] integral(sin(x)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^p(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**p,x)
```

```
[Out] Integral(sin(x)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="giac")
```

```
[Out] integrate(sin(x)^p, x)
```

Mupad [B]
time = 0.32, size = 35, normalized size = 0.80

$$-\frac{\cos(x)\sin(x)^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-\frac{p}{2};\ \frac{3}{2};\ \cos(x)^2\right)}{\left(\sin(x)^2\right)^{\frac{p}{2}+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^p,x)
```

```
[Out] -(cos(x)*sin(x)^(p + 1)*hypergeom([1/2, 1/2 - p/2], 3/2, cos(x)^2))/(sin(x)
^2)^(p/2 + 1/2)
```

3.87 $\int \cos(x) \left(1+\sin^2(x)\right)^2 \, dx$

Optimal. Leaf size=19

$$\sin(x)+\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}$$

```
[Out] sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3269, 200}

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(1 + Sin[x]^2)^2,x]
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\left(1+\sin^2(x)\right)^2 \, dx &= \text{Subst}\left(\int \left(1+x^2\right)^2 \, dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \left(1+2x^2+x^4\right) \, dx, x, \sin(x)\right) \\
&= \sin(x)+\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\sin(x)+\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(1 + Sin[x]^2)^2,x]
```

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.84

method	result	size
derivativedivides	$\sin(x)+\frac{2(\sin^3(x))}{3}+\frac{(\sin^5(x))}{5}$	16
default	$\sin(x)+\frac{2(\sin^3(x))}{3}+\frac{(\sin^5(x))}{5}$	16
risch	$\frac{13\sin(x)}{8}+\frac{\sin(5x)}{80}-\frac{11\sin(3x)}{48}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

Maxima [A]
time = 1.47, size = 15, normalized size = 0.79

$$\frac{1}{5}\sin(x)^5+\frac{2}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

Fricas [A]
time = 0.84, size = 18, normalized size = 0.95

$$\frac{1}{15}\left(3\cos(x)^4-16\cos(x)^2+28\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 - 16*cos(x)^2 + 28)*sin(x)
```

Sympy [A]
time = 0.21, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)**2)**2,x)
```

```
[Out] sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)
```

Giac [A]
time = 0.68, size = 15, normalized size = 0.79

$$\frac{1}{5}\sin\left(x\right)^{5}+\frac{2}{3}\sin\left(x\right)^{3}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

Mupad [B]
time = 0.04, size = 15, normalized size = 0.79

$$\frac{\sin\left(x\right)^{5}}{5}+\frac{2\sin\left(x\right)^{3}}{3}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sin(x)^2 + 1)^2,x)
```

```
[Out] sin(x) + (2*sin(x)^3)/3 + sin(x)^5/5
```

3.88 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A]
time = 0.00, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A]
time = 4.40, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A]
time = 0.73, size = 10, normalized size = 0.71

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A]
time = 0.69, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] x/2 + sin(2*x)/4
```

3.89 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x)-\frac{\sin^3(x)}{3}$$

[Out] sin(x)-1/3*sin(x)^3

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\sin(x)-\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3,x]

[Out] Sin[x] - Sin[x]^3/3

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x)-\frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^3,x]

[Out] (3*Sin[x])/4 + Sin[3*x]/12

Maple [A]
time = 0.01, size = 11, normalized size = 1.00

method	result	size
default	$\frac{(2+\cos^2(x))\sin(x)}{3}$	11
risch	$\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A]
time = 2.54, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A]
time = 0.78, size = 10, normalized size = 0.91

$$\frac{1}{3}\left(\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
[Out] -sin(x)**3/3 + sin(x)
```

Giac [A]
time = 0.90, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.82

$$\sin(x) - \frac{\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] sin(x) - sin(x)^3/3
```

3.90 $\int \sec^2(x)\,dx$

Optimal. Leaf size=2

$$\tan(x)$$

[Out] tan(x)

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3852, 8\}$

$$\tan(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2,x]

[Out] Tan[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\,dx &= -\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= \tan(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2,x]

```
[Out] Tan[x]
```

Maple [A]
time = 0.04, size = 3, normalized size = 1.50

method	result	size
default	$\tan\left(x\right)$	3
risch	$\frac{2i}{e^{2ix}+1}$	13
norman	$-\frac{2\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] tan(x)
```

Maxima [A]
time = 1.86, size = 2, normalized size = 1.00

$$\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.
time = 0.66, size = 7, normalized size = 3.50

$$\frac{\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5 vs. 2(2) = 4.
time = 0.03, size = 5, normalized size = 2.50

$$\frac{\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

Giac [A]
time = 0.73, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="giac")
```

```
[Out] tan(x)
```

Mupad [B]
time = 0.02, size = 2, normalized size = 1.00

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2,x)
```

```
[Out] tan(x)
```

3.91 $\int \sin(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

```
[Out] 1/2*sin(x)-1/6*sin(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4367}

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(2x)\,dx=\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.47

method	result	size
derivativedivides	$\frac{2(\sin^3(x))}{3}$	7
default	$\frac{2(\sin^3(x))}{3}$	7
risch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{2\tan(x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{4\left(\tan^2(x)\right)\tan\left(\frac{x}{2}\right)}{3}+\frac{2\tan(x)}{3}-\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(x)\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*sin(x)^3
```

Maxima [A]
time = 1.60, size = 11, normalized size = 0.73

$$-\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A]
time = 0.93, size = 10, normalized size = 0.67

$$-\frac{2}{3}\left(\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A]
time = 0.13, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3}+\frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x)
```

```
[Out] -2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

Giac [A]
time = 0.73, size = 6, normalized size = 0.40

$$\frac{2}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] 2/3*sin(x)^3
```

Mupad [B]
time = 0.03, size = 6, normalized size = 0.40

$$\frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(x),x)
```

```
[Out] (2*sin(x)^3)/3
```

3.92 $\int x \sin(x)\, dx$

Optimal. Leaf size=8

$$-x\cos(x)+\sin(x)$$

```
[Out] -x*cos(x)+sin(x)
```

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2717}

$$\sin(x)-x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin(x)\,dx &= -x\cos(x)+\int \cos(x)\,dx\\ &= -x\cos(x)+\sin(x)\end{aligned}$$

Mathematica [A]

time = 0.00, size = 8, normalized size = 1.00

$$-x\cos(x)+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size
default	$-x\cos\left(x\right)+\sin\left(x\right)$	9
risch	$-x\cos\left(x\right)+\sin\left(x\right)$	9
meijerg	$2\sqrt{\pi}\,\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	22
norman	$\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)-x+2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x,method=_RETURNVERBOSE)
[Out] -x*cos(x)+sin(x)
```

Maxima [A]
time = 2.26, size = 8, normalized size = 1.00

$$-x\cos\left(x\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
[Out] -x*cos(x) + sin(x)
```

Fricas [A]
time = 0.76, size = 8, normalized size = 1.00

$$-x\cos\left(x\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
[Out] -x*cos(x) + sin(x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.88

$$-x\cos\left(x\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

Giac [A]
time = 0.79, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

Mupad [B]
time = 0.02, size = 8, normalized size = 1.00

$$\sin(x) - x\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] sin(x) - x*cos(x)
```

3.93 $\int x^2 \sin(x) \, dx$

Optimal. Leaf size=17

$$2\cos(x) - x^2\cos(x) + 2x\sin(x)$$

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2718}

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x],x]
```

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sin(x) \, dx &= -x^2\cos(x) + 2\int x\cos(x) \, dx \\
&= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x) \, dx \\
&= 2\cos(x) - x^2\cos(x) + 2x\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.88

$$-\left(\left(-2+x^2\right)\cos(x)\right) + 2x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x],x]
```

```
[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

Maple [A]
time = 0.00, size = 18, normalized size = 1.06

method	result	size
risch	$\left(-x^{2}+2\right) \cos \left(x \right)+2 x \sin \left(x \right)$	17
default	$2 \cos \left(x \right)-x^{2} \cos \left(x \right)+2 x \sin \left(x \right)$	18
meijerg	$4 \sqrt{\pi}\, \left(-\frac{1}{2 \sqrt{\pi}}+\frac{\left(-\frac{x^{2}}{2}+1\right) \cos \left(x \right)}{2 \sqrt{\pi}}+\frac{x \sin \left(x \right)}{2 \sqrt{\pi}}\right)$	34
norman	$\frac{x^{2} \left(\tan^{2}\left(\frac{x}{2}\right)\right)-x^{2}+4 x \tan \left(\frac{x}{2}\right)+4}{1+\tan^{2}\left(\frac{x}{2}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Maxima [A]
time = 2.44, size = 15, normalized size = 0.88

$$-{\left(x^{2} - 2\right)} \cos\left(x\right) + 2 \, x \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Fricas [A]
time = 0.76, size = 15, normalized size = 0.88

$$-{\left(x^{2} - 2\right)} \cos\left(x\right) + 2 \, x \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Sympy [A]
time = 0.08, size = 17, normalized size = 1.00

$$- x^{2} \cos{\left(x \right)} + 2 x \sin{\left(x \right)} + 2 \cos{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x),x)
```

```
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

Giac [A]
time = 0.64, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="giac")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$2\,x\,\sin\left(x\right)-\cos\left(x\right)\,\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
```

```
[Out] 2*x*sin(x) - cos(x)*(x^2 - 2)
```

3.94 $\int x \sin^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}$$

```
[Out] 1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3391, 30}

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\sin^2(x)\,dx&=-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}\\&=\frac{x^2}{4}-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4}-\frac{1}{8}\cos(2x)-\frac{1}{4}x\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

Maple [A]
time = 0.00, size = 25, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{4}-\frac{\cos(2x)}{8}-\frac{x\sin(2x)}{4}$	20
default	$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin^2(x))}{4}$	25
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x^2+1}{2\sqrt{\pi}}-\frac{\cos(2x)}{2\sqrt{\pi}}-\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{4}$	38
norman	$\frac{x(\tan^3(\frac{x}{2}))+\tan^2(\frac{x}{2})+\frac{x^2}{4}-x\tan(\frac{x}{2})+\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{x^2\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(1/2*x-1/2*cos(x)*sin(x))-1/4*x^2+1/4*sin(x)^2
```

Maxima [A]
time = 1.09, size = 19, normalized size = 0.76

$$\frac{1}{4}x^2-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Fricas [A]
time = 0.63, size = 19, normalized size = 0.76

$$-\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}x^2-\frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

Sympy [A]
time = 0.09, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2(x)}{4}+\frac{x^2\cos^2(x)}{4}-\frac{x\sin(x)\cos(x)}{2}-\frac{\cos^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 - cos(x)**2/4
```

Giac [A]
time = 0.74, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^2-\frac{1}{4}\,x\sin(2\,x)-\frac{1}{8}\,\cos(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Mupad [B]
time = 0.06, size = 19, normalized size = 0.76

$$\frac{\sin(x)^2}{4}-\frac{x\,\sin(2\,x)}{4}+\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
```

```
[Out] sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4
```

3.95 $\int x^2 \sin^2(x)\,dx$

Optimal. Leaf size=41

$$-\frac{x}{4}+\frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)$$

```
[Out] -1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3392, 30, 2715, 8}

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^2,x]
```

```
[Out] -1/4*x + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sin^2(x)\,dx &= -\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)+\frac{\int x^2\,dx}{2}-\frac{1}{2}\int \sin^2(x)\,dx\\
&= \frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)-\frac{\int 1\,dx}{4}\\
&= -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3-6x\cos(2x)+\left(3-6x^2\right)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^2,x]
```

```
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.90

method	result	size
meijerg	$\frac{x^5\,\mathrm{hypergeom}\left(\left[1,\frac{5}{2}\right],\left[\frac{3}{2},2,\frac{7}{2}\right],-x^2\right)}{5}$	19
risch	$\frac{x^3}{6}-\frac{x\cos(2x)}{4}-\frac{(2x^2-1)\sin(2x)}{8}$	27
default	$x^2\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x\left(\cos^2(x)\right)}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}-\frac{x^3}{3}$	37
norman	$\frac{x^2\left(\tan^3\left(\frac{x}{2}\right)\right)-\frac{x}{4}+\frac{x^3}{6}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}-x^2\tan\left(\frac{x}{2}\right)+\frac{x^3\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{x^3\left(\tan^4\left(\frac{x}{2}\right)\right)}{6}+\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(1/2*x-1/2*cos(x)*sin(x))-1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x-1/3*x^
3
```

Maxima [A]
time = 1.19, size = 26, normalized size = 0.63

$$\frac{1}{6}\,x^3-\frac{1}{4}\,x\cos\left(2\,x\right)-\frac{1}{8}\,\left(2\,x^2-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A]
time = 0.55, size = 29, normalized size = 0.71

$$\frac{1}{6}x^3 - \frac{1}{2}x\cos(x)^2 - \frac{1}{4}\left(2x^2 - 1\right)\cos(x)\sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

Sympy [A]
time = 0.14, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6} + \frac{x^3\cos^2(x)}{6} - \frac{x^2\sin(x)\cos(x)}{2} + \frac{x\sin^2(x)}{4} - \frac{x\cos^2(x)}{4} + \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
- x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Giac [A]
time = 0.69, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Mupad [B]
time = 0.16, size = 28, normalized size = 0.68

$$\frac{\sin(2x)}{8} - \frac{x\cos(2x)}{4} - \frac{x^2\sin(2x)}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6
```

3.96 $\int x\sin^3(x)\,dx$

Optimal. Leaf size=33

$$-\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}$$

```
[Out] -2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3391, 3377, 2717}

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\sin^3(x)\,dx &= -\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int x\sin(x)\,dx\\ &= -\frac{2}{3}x\cos(x)-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int\cos(x)\,dx\\ &= -\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.94

$$-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)+\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^3,x]
```

```
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

Maple [A]
time = 0.02, size = 23, normalized size = 0.70

method	result	size
default	$-\frac{x(2+\sin^2(x))\cos(x)}{3}+\frac{(\sin^3(x))}{9}+\frac{2\sin(x)}{3}$	23
risch	$-\frac{3x\cos(x)}{4}+\frac{3\sin(x)}{4}+\frac{x\cos(3x)}{12}-\frac{\sin(3x)}{36}$	24
norman	$\frac{-\frac{2x}{3}+\frac{32\left(\tan^3\left(\frac{x}{2}\right)\right)}{9}+\frac{4\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)+2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

Maxima [A]
time = 1.79, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos\left(3\,x\right)-\frac{3}{4}\,x\cos\left(x\right)-\frac{1}{36}\,\sin\left(3\,x\right)+\frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Fricas [A]
time = 0.51, size = 23, normalized size = 0.70

$$\frac{1}{3}\,x\cos\left(x\right)^{3}-x\cos\left(x\right)-\frac{1}{9}\left(\cos\left(x\right)^{2}-7\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="fricas")
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

Sympy [A]
time = 0.14, size = 39, normalized size = 1.18

$$-x\sin^{2}\left(x\right)\cos\left(x\right)-\frac{2x\cos^{3}\left(x\right)}{3}+\frac{7\sin^{3}\left(x\right)}{9}+\frac{2\sin\left(x\right)\cos^{2}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3,x)
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/
3
```

Giac [A]
time = 0.86, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos\left(3\,x\right)-\frac{3}{4}\,x\cos\left(x\right)-\frac{1}{36}\,\sin\left(3\,x\right)+\frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="giac")
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Mupad [B]
time = 0.07, size = 25, normalized size = 0.76

$$\frac{x\cos\left(x\right)^{3}}{3}-\frac{\sin\left(x\right)\,\cos\left(x\right)^{2}}{9}-x\,\cos\left(x\right)+\frac{7\,\sin\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
[Out] (7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)
```

3.97 $\int x\cos(x)\,dx$

Optimal. Leaf size=7

$$\cos(x)+x\sin(x)$$

```
[Out] cos(x)+x*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x)-\int \sin(x)\,dx\\ &= \cos(x)+x\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\cos(x)+x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A]
time = 0.02, size = 8, normalized size = 1.14

method	result	size
default	$\cos(x) + x\sin(x)$	8
risch	$\cos(x) + x\sin(x)$	8
norman	$\frac{2x\tan(\frac{x}{2})+2}{1+\tan^2(\frac{x}{2})}$	21
meijerg	$2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x,method=_RETURNVERBOSE)
[Out] cos(x)+x*sin(x)
```

Maxima [A]
time = 2.06, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
[Out] x*sin(x) + cos(x)
```

Fricas [A]
time = 0.48, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
[Out] x*sin(x) + cos(x)
```

Sympy [A]
time = 0.06, size = 7, normalized size = 1.00

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [**A**]
time = 0.89, size = 7, normalized size = 1.00

$$x\sin\left(x\right)+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

Mupad [**B**]
time = 0.02, size = 7, normalized size = 1.00

$$\cos\left(x\right)+x\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x) + x*sin(x)
```

3.98 $\int x^2 \cos(x)\,dx$

Optimal. Leaf size=16

$$2x\cos(x) - 2\sin(x) + x^2\sin(x)$$

```
[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2717}

$$x^2\sin(x) - 2\sin(x) + 2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x],x]
```

```
[Out] 2*x*Cos[x] - 2*Sin[x] + x^2*Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\cos(x)\,dx &= x^2\sin(x) - 2\int x\sin(x)\,dx \\
&= 2x\cos(x) + x^2\sin(x) - 2\int \cos(x)\,dx \\
&= 2x\cos(x) - 2\sin(x) + x^2\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.88

$$2x\cos(x) + \left(-2 + x^2\right)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x],x]
```

```
[Out] 2*x*Cos[x] + (-2 + x^2)*Sin[x]
```

Maple [A]
time = 0.02, size = 17, normalized size = 1.06

method	result	size
risch	$2x\cos\left(x\right)+\left(x^{2}-2\right)\sin\left(x\right)$	15
default	$2x\cos\left(x\right)-2\sin\left(x\right)+x^{2}\sin\left(x\right)$	17
meijerg	$4\sqrt{\pi}\,\left(\frac{x\cos\left(x\right)}{2\sqrt{\pi}}-\frac{\left(-\frac{3x^{2}}{2}+3\right)\sin\left(x\right)}{6\sqrt{\pi}}\right)$	29
norman	$\frac{2x-2x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2x^{2}\tan\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)
```

Maxima [A]
time = 2.58, size = 14, normalized size = 0.88

$$2\,x\cos\left(x\right)+\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="maxima")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

Fricas [A]
time = 0.55, size = 14, normalized size = 0.88

$$2\,x\cos\left(x\right)+\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="fricas")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

Sympy [A]
time = 0.09, size = 17, normalized size = 1.06

$$x^{2}\sin\left(x\right)+2x\cos\left(x\right)-2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x),x)
[Out] x**2*sin(x) + 2*x*cos(x) - 2*sin(x)
```

Giac [A]
time = 0.98, size = 14, normalized size = 0.88

$$2\,x\cos\left(x\right) + \left(x^{2} - 2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="giac")
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.88

$$\sin\left(x\right)\,\left(x^2-2\right)+2\,x\,\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x),x)
[Out] sin(x)*(x^2 - 2) + 2*x*cos(x)
```

3.99 $\int x \cos^2(x) \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\cos(x)\sin(x)$$

```
[Out] 1/4*x^2+1/4*cos(x)^2+1/2*x*cos(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3391, 30}

$$\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2,x]
```

```
[Out] x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\cos^2(x)\,dx&=\frac{\cos^2(x)}{4}+\frac{1}{2}x\cos(x)\sin(x)+\frac{\int x\,dx}{2}\\&=\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4}+\frac{1}{8}\cos(2x)+\frac{1}{4}x\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2,x]
[Out] x^2/4 + Cos[2*x]/8 + (x*Sin[2*x])/4
```

Maple [A]
time = 0.02, size = 25, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{4}+\frac{\cos(2x)}{8}+\frac{x\sin(2x)}{4}$	20
default	$x\left(\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}-\frac{\left(\sin^2(x)\right)}{4}$	25
norman	$\frac{x\tan\left(\frac{x}{2}\right)-\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x^2}{4}-x\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{x^2\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2,x,method=_RETURNVERBOSE)
[Out] x*(1/2*x+1/2*cos(x)*sin(x))-1/4*x^2-1/4*sin(x)^2
```

Maxima [A]
time = 1.50, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^2+\frac{1}{4}\,x\sin\left(2\,x\right)+\frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="maxima")
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

Fricas [A]
time = 0.54, size = 19, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x^2+\frac{1}{4}\,\cos\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="fricas")
[Out] 1/2*x*cos(x)*sin(x) + 1/4*x^2 + 1/4*cos(x)^2
```

Sympy [A]
time = 0.09, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2\left(x\right)}{4}+\frac{x^2\cos^2\left(x\right)}{4}+\frac{x\sin\left(x\right)\cos\left(x\right)}{2}+\frac{\cos^2\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 + x*sin(x)*cos(x)/2 + cos(x)**2/4
```

Giac [A]
time = 0.74, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^2+\frac{1}{4}\,x\sin\left(2\,x\right)+\frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

Mupad [B]
time = 0.15, size = 19, normalized size = 0.76

$$\frac{x\,\sin\left(2\,x\right)}{4}-\frac{{\sin\left(x\right)}^2}{4}+\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2,x)
```

```
[Out] (x*sin(2*x))/4 - sin(x)^2/4 + x^2/4
```

3.100 $\int x^2 \cos^2(x) \, dx$

Optimal. Leaf size=41

$$-\frac{x}{4}+\frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)$$

```
[Out] -1/4*x+1/6*x^3+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)+1/2*x^2*cos(x)*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 30, 2715, 8}

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2,x]
```

```
[Out] -1/4*x + x^3/6 + (x*Cos[x]^2)/2 - (Cos[x]*Sin[x])/4 + (x^2*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cos^2(x)\,dx &= \frac{1}{2}x\cos^2(x)+\frac{1}{2}x^2\cos(x)\sin(x)+\frac{\int x^2\,dx}{2}-\frac{1}{2}\int \cos^2(x)\,dx\\
&= \frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)-\frac{\int 1\,dx}{4}\\
&= -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+6x\cos(2x)+\left(-3+6x^2\right)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2,x]
```

```
[Out] (4*x^3 + 6*x*Cos[2*x] + (-3 + 6*x^2)*Sin[2*x])/24
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.90

method	result	size
risch	$\frac{x^3}{6}+\frac{x\cos(2x)}{4}+\frac{(2x^2-1)\sin(2x)}{8}$	27
default	$x^2\left(\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}\right)+\frac{x(\cos^2(x))}{2}-\frac{\cos(x)\sin(x)}{4}-\frac{x}{4}-\frac{x^3}{3}$	37
norman	$\frac{x^2\tan(\frac{x}{2})+\frac{x}{4}+\frac{x^3}{6}+\frac{(\tan^3(\frac{x}{2}))}{2}-\frac{3x(\tan^2(\frac{x}{2}))}{2}+\frac{x(\tan^4(\frac{x}{2}))}{4}-x^2(\tan^3(\frac{x}{2}))+\frac{x^3(\tan^2(\frac{x}{2}))}{3}+\frac{x^3(\tan^4(\frac{x}{2}))}{6}-\frac{\tan(\frac{x}{2})}{2}}{(1+\tan^2(\frac{x}{2}))^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(1/2*x+1/2*cos(x)*sin(x))+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^
3
```

Maxima [A]
time = 2.85, size = 26, normalized size = 0.63

$$\frac{1}{6}\,x^3+\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\left(2\,x^2-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A]
time = 0.52, size = 29, normalized size = 0.71

$$\frac{1}{6}\,x^3+\frac{1}{2}\,x\cos\left(x\right)^2+\frac{1}{4}\,\left(2\,x^2-1\right)\cos\left(x\right)\sin\left(x\right)-\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x*cos(x)^2 + 1/4*(2*x^2 - 1)*cos(x)*sin(x) - 1/4*x
```

Sympy [A]
time = 0.14, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2\left(x\right)}{6}+\frac{x^3\cos^2\left(x\right)}{6}+\frac{x^2\sin\left(x\right)\cos\left(x\right)}{2}-\frac{x\sin^2\left(x\right)}{4}+\frac{x\cos^2\left(x\right)}{4}-\frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 + x**2*sin(x)*cos(x)/2 - x*sin(x)**2/4
+ x*cos(x)**2/4 - sin(x)*cos(x)/4
```

Giac [A]
time = 0.63, size = 26, normalized size = 0.63

$$\frac{1}{6}\,x^3+\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\left(2\,x^2-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

Mupad [B]
time = 0.06, size = 28, normalized size = 0.68

$$\frac{x\,\cos\left(2\,x\right)}{4}-\frac{\sin\left(2\,x\right)}{8}+\frac{x^2\,\sin\left(2\,x\right)}{4}+\frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2,x)
```

```
[Out] (x*cos(2*x))/4 - sin(2*x)/8 + (x^2*sin(2*x))/4 + x^3/6
```

3.101 $\int x\cos^3(x)\,dx$

Optimal. Leaf size=33

$$\frac{2\cos(x)}{3}+\frac{\cos^3(x)}{9}+\frac{2}{3}x\sin(x)+\frac{1}{3}x\cos^2(x)\sin(x)$$

```
[Out] 2/3*cos(x)+1/9*cos(x)^3+2/3*x*sin(x)+1/3*x*cos(x)^2*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3391, 3377, 2718}

$$\frac{2}{3}x\sin(x)+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}+\frac{1}{3}x\sin(x)\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^3,x]
```

```
[Out] (2*Cos[x])/3 + Cos[x]^3/9 + (2*x*Sin[x])/3 + (x*Cos[x]^2*Sin[x])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$
\begin{aligned}
\int x\cos^3(x)\,dx &= \frac{\cos^3(x)}{9}+\frac{1}{3}x\cos^2(x)\sin(x)+\frac{2}{3}\int x\cos(x)\,dx \\
&= \frac{\cos^3(x)}{9}+\frac{2}{3}x\sin(x)+\frac{1}{3}x\cos^2(x)\sin(x)-\frac{2}{3}\int \sin(x)\,dx \\
&= \frac{2\cos(x)}{3}+\frac{\cos^3(x)}{9}+\frac{2}{3}x\sin(x)+\frac{1}{3}x\cos^2(x)\sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.94

$$\frac{3\cos(x)}{4}+\frac{1}{36}\cos(3x)+\frac{3}{4}x\sin(x)+\frac{1}{12}x\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^3,x]
[Out] (3*Cos[x])/4 + Cos[3*x]/36 + (3*x*Sin[x])/4 + (x*Sin[3*x])/12
```

Maple [A]
time = 0.03, size = 23, normalized size = 0.70

method	result	size
default	$\frac{x\left(2+\cos^2(x)\right)\sin(x)}{3}+\frac{\left(\cos^3(x)\right)}{9}+\frac{2\cos(x)}{3}$	23
risch	$\frac{3\cos(x)}{4}+\frac{3x\sin(x)}{4}+\frac{\cos(3x)}{36}+\frac{x\sin(3x)}{12}$	24
norman	$\frac{-\frac{2\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}-\frac{8\left(\tan^6\left(\frac{x}{2}\right)\right)}{9}+2x\tan\left(\frac{x}{2}\right)+\frac{4x\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+2x\left(\tan^5\left(\frac{x}{2}\right)\right)+\frac{2}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*x*(2+cos(x)^2)*sin(x)+1/9*cos(x)^3+2/3*cos(x)
```

Maxima [A]
time = 1.38, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\sin\left(3\,x\right)+\frac{3}{4}\,x\sin\left(x\right)+\frac{1}{36}\,\cos\left(3\,x\right)+\frac{3}{4}\,\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

Fricas [A]
time = 0.47, size = 25, normalized size = 0.76

$$\frac{1}{9}\cos(x)^3 + \frac{1}{3}\left(x\cos(x)^2 + 2x\right)\sin(x) + \frac{2}{3}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="fricas")
[Out] 1/9*cos(x)^3 + 1/3*(x*cos(x)^2 + 2*x)*sin(x) + 2/3*cos(x)
```

Sympy [A]
time = 0.13, size = 39, normalized size = 1.18

$$\frac{2x\sin^3(x)}{3} + x\sin(x)\cos^2(x) + \frac{2\sin^2(x)\cos(x)}{3} + \frac{7\cos^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**3,x)
[Out] 2*x*sin(x)**3/3 + x*sin(x)*cos(x)**2 + 2*sin(x)**2*cos(x)/3 + 7*cos(x)**3/9
```

Giac [A]
time = 0.63, size = 23, normalized size = 0.70

$$\frac{1}{12}x\sin(3x) + \frac{3}{4}x\sin(x) + \frac{1}{36}\cos(3x) + \frac{3}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="giac")
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

Mupad [B]
time = 0.17, size = 25, normalized size = 0.76

$$\frac{\cos(x)^3}{9} + \frac{x\,\sin(x)\,\cos(x)^2}{3} + \frac{2\,\cos(x)}{3} + \frac{2\,x\,\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^3,x)
[Out] (2*cos(x))/3 + cos(x)^3/9 + (2*x*sin(x))/3 + (x*cos(x)^2*sin(x))/3
```

3.102 $\int \frac{\sin(x)}{x} \, dx$

Optimal. Leaf size=2

$$\mathrm{Si}(x)$$

[Out] Si(x)

Rubi [A]
time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3380}

$$\mathrm{Si}(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/x,x]

[Out] SinIntegral[x]

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{x} \, dx = \mathrm{Si}(x)$$

Mathematica [A]
time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{Si}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/x,x]

[Out] SinIntegral[x]

Maple [A]
time = 0.01, size = 3, normalized size = 1.50

method	result	size

default	$\text{sinIntegral}\left(x\right)$	3
meijerg	$\text{sinIntegral}\left(x\right)$	3
risch	$-\frac{\pi\,\text{csgn}(x)}{2}+\text{sinIntegral}\left(x\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] Si(x)
```

Maxima [C] Result contains complex when optimal does not.
time = 1.28, size = 13, normalized size = 6.50

$$-\frac{1}{2}i\,\text{Ei}(i\,x)+\frac{1}{2}i\,\text{Ei}(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*x) + 1/2*I*Ei(-I*x)
```

Fricas [A]
time = 0.57, size = 2, normalized size = 1.00

$$\text{Si}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="fricas")
```

```
[Out] sin_integral(x)
```

Sympy [A]
time = 0.33, size = 2, normalized size = 1.00

$$\text{Si}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x)
```

```
[Out] Si(x)
```

Giac [A]
time = 0.71, size = 2, normalized size = 1.00

$$\text{Si}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="giac")
```

```
[Out] sin_integral(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.50

$$\operatorname{sinint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x,x)
```

```
[Out] sinint(x)
```

3.103 $\int \frac{\cos(x)}{x}\,dx$

Optimal. Leaf size=2

$$\mathrm{Ci}(x)$$

```
[Out] Ci(x)
```

Rubi [A]
time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3383}

$$\mathrm{CosIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/x,x]
```

```
[Out] CosIntegral[x]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(x)}{x}\,dx = \mathrm{Ci}(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\mathrm{Ci}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/x,x]
```

```
[Out] CosIntegral[x]
```

Maple [A]
time = 0.03, size = 3, normalized size = 1.50

method	result	size
default	$\text{cosineIntegral}(x)$	3
risch	$\text{cosineIntegral}(x)-\frac{i\pi\,\text{csgn}(ix)\text{csgn}(x)}{2}+\frac{i\pi\,\text{csgn}(ix)}{2}$	24
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma+2\ln(x)}{\sqrt{\pi}}-\frac{2\gamma}{\sqrt{\pi}}-\frac{2\ln(2)}{\sqrt{\pi}}-\frac{2\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}}+\frac{2\,\text{cosineIntegral}(x)}{\sqrt{\pi}}\right)}{2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] Ci(x)
```

Maxima [C] Result contains complex when optimal does not.
time = 1.53, size = 13, normalized size = 6.50

$$\frac{1}{2}\,\text{Ei}(i\,x)+\frac{1}{2}\,\text{Ei}(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="maxima")
```

```
[Out] 1/2*Ei(I*x) + 1/2*Ei(-I*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(2)=4$.
time = 0.54, size = 11, normalized size = 5.50

$$\frac{1}{2}\,\text{Ci}\left(-x\right)+\frac{1}{2}\,\text{Ci}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="fricas")
```

```
[Out] 1/2*cos_integral(-x) + 1/2*cos_integral(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2)=4$.
time = 0.49, size = 12, normalized size = 6.00

$$-\log\left(x\right)+\frac{\log\left(x^{2}\right)}{2}+\text{Ci}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(x)
```

Giac [A]
time = 0.89, size = 2, normalized size = 1.00

$$\mathrm{Ci}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="giac")
```

```
[Out] cos_integral(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.50

$$\mathrm{cosint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x,x)
```

```
[Out] cosint(x)
```

3.104 $\int \frac{\sin(x)}{x^2} dx$

Optimal. Leaf size=10

$$\text{Ci}(x) - \frac{\sin(x)}{x}$$

[Out] Ci(x)-sin(x)/x

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3378, 3383}

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/x^2,x]

[Out] CosIntegral[x] - Sin[x]/x

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{x^2} dx &= -\frac{\sin(x)}{x} + \int \frac{\cos(x)}{x} dx \\ &= \text{Ci}(x) - \frac{\sin(x)}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\text{Ci}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/x^2,x]
```

```
[Out] CosIntegral[x] - Sin[x]/x
```

Maple [A]
time = 0.04, size = 11, normalized size = 1.10

method	result	size
default	$\operatorname{cosineIntegral}(x)-\frac{\sin(x)}{x}$	11
risch	$\operatorname{cosineIntegral}(x)-\frac{i\pi\,\operatorname{csgn}(ix)\operatorname{csgn}(x)}{2}+\frac{i\pi\,\operatorname{csgn}(ix)}{2}-\frac{\sin(x)}{x}$	31
meijerg	$\frac{\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(x)}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln\left(\frac{x}{2}\right)}{\sqrt{\pi}}-\frac{4\sin(x)}{\sqrt{\pi}\,x}+\frac{4\operatorname{cosineIntegral}(x)}{\sqrt{\pi}}\right)}{4}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] Ci(x)-sin(x)/x
```

Maxima [C] Result contains complex when optimal does not.
time = 1.98, size = 15, normalized size = 1.50

$$\frac{1}{2}\,\Gamma(-1,i\,x)+\frac{1}{2}\,\Gamma(-1,-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, I*x) + 1/2*gamma(-1, -I*x)
```

Fricas [A]
time = 0.60, size = 20, normalized size = 2.00

$$\frac{x\,\mathrm{Ci}\,(-x)+x\,\mathrm{Ci}\,(x)-2\,\sin\,(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos_integral(-x) + x*cos_integral(x) - 2*sin(x))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7)=14$.
time = 0.74, size = 17, normalized size = 1.70

$$-\log\,(x)+\frac{\log\,(x^2)}{2}+\mathrm{Ci}\,(x)-\frac{\sin\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x**2,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(x) - sin(x)/x
```

Giac [A]
time = 0.80, size = 13, normalized size = 1.30

$$\frac{x\,\mathrm{Ci}\,(x) - \sin\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="giac")
```

```
[Out] (x*cos_integral(x) - sin(x))/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\mathrm{cosint}(x) - \frac{\sin\,(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x^2,x)
```

```
[Out] cosint(x) - sin(x)/x
```

3.105 $\int \frac{\sin^2(x)}{x} \, dx$

Optimal. Leaf size=15

$$-\frac{\text{Ci}(2x)}{2}+\frac{\log(x)}{2}$$

```
[Out] -1/2*Ci(2*x)+1/2*ln(x)
```

Rubi [A]

time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3393, 3383}

$$\frac{\log(x)}{2}-\frac{1}{2}\text{CosIntegral}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/x,x]
```

```
[Out] -1/2*CosIntegral[2*x] + Log[x]/2
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{x} \, dx &= \int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right) \, dx \\
&= \frac{\log(x)}{2}-\frac{1}{2}\int \frac{\cos(2x)}{x} \, dx \\
&= -\frac{\text{Ci}(2x)}{2}+\frac{\log(x)}{2}
\end{aligned}$$

Mathematica [A]

time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\text{Ci}(2x)}{2}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/x,x]
```

```
[Out] -1/2*CosIntegral[2*x] + Log[x]/2
```

Maple [A]
time = 0.03, size = 12, normalized size = 0.80

method	result	size
default	$-\frac{\mathrm{cosineIntegral}(2x)}{2}+\frac{\ln(x)}{2}$	12
risch	$-\frac{\mathrm{cosineIntegral}(2x)}{2}+\frac{i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(x)}{4}-\frac{i\pi\,\mathrm{csgn}(ix)}{4}+\frac{\ln(x)}{2}$	32
meijerg	$\frac{\sqrt{\pi}\left(\frac{2\gamma}{\sqrt{\pi}}+\frac{2\ln(2)}{\sqrt{\pi}}+\frac{2\ln(x)}{\sqrt{\pi}}-\frac{2\,\mathrm{cosineIntegral}(2x)}{\sqrt{\pi}}\right)}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*Ci(2*x)+1/2*ln(x)
```

Maxima [C] Result contains complex when optimal does not.
time = 1.07, size = 17, normalized size = 1.13

$$-\frac{1}{4}\,\mathrm{Ei}(2i\,x)-\frac{1}{4}\,\mathrm{Ei}(-2i\,x)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="maxima")
```

```
[Out] -1/4*Ei(2*I*x) - 1/4*Ei(-2*I*x) + 1/2*log(x)
```

Fricas [A]
time = 0.51, size = 17, normalized size = 1.13

$$-\frac{1}{4}\,\mathrm{Ci}\left(2\,x\right)-\frac{1}{4}\,\mathrm{Ci}\left(-2\,x\right)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*cos_integral(2*x) - 1/4*cos_integral(-2*x) + 1/2*log(x)
```

Sympy [A]
time = 0.58, size = 10, normalized size = 0.67

$$\frac{\log\left(x\right)}{2}-\frac{\mathrm{Ci}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/x,x)
```

```
[Out] log(x)/2 - Ci(2*x)/2
```

Giac [**A**]
time = 0.73, size = 11, normalized size = 0.73

$$-\frac{1}{2}\operatorname{Ci}(2x) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="giac")
```

```
[Out] -1/2*cos_integral(2*x) + 1/2*log(x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\frac{\ln(x)}{2} - \frac{\operatorname{cosint}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/x,x)
```

```
[Out] log(x)/2 - cosint(2*x)/2
```

3.106 $\int \tan^3(x)\, dx$

Optimal. Leaf size=12

$$\log(\cos(x))+\frac{\tan^2(x)}{2}$$

[Out] ln(cos(x))+1/2*tan(x)^2

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3556}

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Tan[x]^3,x]

[Out] Log[Cos[x]] + Tan[x]^2/2

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(x)\, dx &= \frac{\tan^2(x)}{2}-\int \tan(x)\, dx\\ &= \log(\cos(x))+\frac{\tan^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\log(\cos(x))+\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Sec[x]^2/2
```

Maple [A]
time = 0.02, size = 17, normalized size = 1.42

method	result	size
derivativedivides	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
default	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
norman	$\frac{(\tan^2(x))}{2} - \frac{\ln(1+\tan^2(x))}{2}$	17
risch	$-ix + \frac{2\,e^{2ix}}{(e^{2ix}+1)^2} + \ln(e^{2ix}+1)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x)^2-1/2*ln(1+tan(x)^2)
```

Maxima [A]
time = 1.00, size = 20, normalized size = 1.67

$$-\frac{1}{2\left(\sin(x)^2-1\right)} + \frac{1}{2}\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

Fricas [A]
time = 0.57, size = 18, normalized size = 1.50

$$\frac{1}{2}\tan(x)^2 + \frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A]
time = 0.03, size = 12, normalized size = 1.00

$$\log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3,x)
```

```
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

Giac [A]
time = 0.79, size = 16, normalized size = 1.33

$$\frac{1}{2}\tan(x)^2 - \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)
```

Mupad [B]
time = 0.02, size = 16, normalized size = 1.33

$$\ln(\cos(x)) - \frac{\cos(x)^2 - 1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x)
```

```
[Out] log(cos(x)) - (cos(x)^2 - 1)/(2*cos(x)^2)
```

3.107 $\int \sin(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2718\}$

$$-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x],x]
```

```
[Out] -(Cos[a + b*x]/b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 2.00

$$-\frac{\cos(a)\cos(bx)}{b} + \frac{\sin(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x],x]
```

```
[Out] -((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

Maple [A]
time = 0.02, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$-\frac{\cos(bx+a)}{b}$	12
default	$-\frac{\cos(bx+a)}{b}$	12
risch	$-\frac{\cos(bx+a)}{b}$	12
norman	$\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	32
meijerg	$\frac{\sin(a)\sin(bx)}{b}+\frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(b*x+a)/b
```

Maxima [A]
time = 0.93, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -cos(b*x + a)/b
```

Fricas [A]
time = 0.81, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -cos(b*x + a)/b
```

Sympy [A]
time = 0.05, size = 14, normalized size = 1.27

$$\begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x\sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

Giac [A]
time = 1.15, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="giac")
```

```
[Out] -cos(b*x + a)/b
```

Mupad [B]
time = 0.02, size = 11, normalized size = 1.00

$$-\frac{\cos(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x),x)
```

```
[Out] -cos(a + b*x)/b
```

3.108 $\int \cos(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sin(a+bx)}{b}$$

```
[Out] sin(b*x+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2717}

$$\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x],x]
```

```
[Out] Sin[a + b*x]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(a+bx)\,dx = \frac{\sin(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.01, size = 21, normalized size = 2.10

$$\frac{\cos(bx)\sin(a)}{b} + \frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b
```

Maple [A]
time = 0.02, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\sin(bx+a)}{b}$	11
default	$\frac{\sin(bx+a)}{b}$	11
risch	$\frac{\sin(bx+a)}{b}$	11
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	30
meijerg	$\frac{\cos(a)\sin(bx)}{b}-\frac{\sin(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] sin(b*x+a)/b
```

Maxima [A]
time = 1.68, size = 10, normalized size = 1.00

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="maxima")
```

```
[Out] sin(b*x + a)/b
```

Fricas [A]
time = 0.78, size = 10, normalized size = 1.00

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/b
```

Sympy [A]
time = 0.05, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\sin(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

Giac [A]
time = 1.15, size = 10, normalized size = 1.00

$$\frac{\sin(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="giac")
```

```
[Out] sin(b*x + a)/b
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\sin(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x),x)
```

```
[Out] sin(a + b*x)/b
```

3.109 $\int \tan(a + bx)\, dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(a + bx))}{b}$$

```
[Out] -ln(cos(b*x+a))/b
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(a + bx)\, dx = -\frac{\log(\cos(a + bx))}{b}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(\cos(a + bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Maple [A]
time = 0.01, size = 17, normalized size = 1.42

method	result	size
derivativedivides	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
default	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
norman	$\frac{\ln(1+\tan^2(bx+a))}{2b}$	17
risch	$ix+\frac{2ia}{b}-\frac{\ln(e^{2i(bx+a)}+1)}{b}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*ln(1+tan(b*x+a)^2)
```

Maxima [A]
time = 1.57, size = 11, normalized size = 0.92

$$\frac{\log(\sec(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sec(b*x + a))/b
```

Fricas [A]
time = 0.87, size = 18, normalized size = 1.50

$$-\frac{\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(b*x + a)^2 + 1))/b
```

Sympy [A]
time = 0.05, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\log(\tan^2(a+bx)+1)}{2b} & \text{for}\ b\neq 0 \\ x\tan(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a), True))
```

Giac [A]
time = 1.37, size = 13, normalized size = 1.08

$$-\frac{\log\left(|\cos\left(bx+a\right)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(cos(b*x + a)))/b
```

Mupad [B]
time = 0.23, size = 16, normalized size = 1.33

$$\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x),x)
```

```
[Out] log(tan(a + b*x)^2 + 1)/(2*b)
```

3.110 $\int \cot(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a+bx))}{b}$$

```
[Out] ln(sin(b*x+a))/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.73

$$\frac{\log(\cos(a+bx)) + \log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.01, size = 26, normalized size = 2.36

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(1+\tan^2(bx+a)\right)}{2}+\ln(\tan(bx+a))}{b}$	26
default	$\frac{-\frac{\ln\left(1+\tan^2(bx+a)\right)}{2}+\ln(\tan(bx+a))}{b}$	26
norman	$\frac{\ln(\tan(bx+a))}{b}-\frac{\ln(1+\tan^2(bx+a))}{2b}$	29
risch	$-ix-\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*ln(1+tan(b*x+a)^2)+ln(tan(b*x+a)))
```

Maxima [A]
time = 1.54, size = 11, normalized size = 1.00

$$\frac{\log\left(\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sin(b*x + a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.
time = 0.75, size = 27, normalized size = 2.45

$$\frac{\log\left(\frac{\tan(bx+a)^2}{\tan(bx+a)^2+1}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(b*x + a)^2/(tan(b*x + a)^2 + 1))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.
time = 0.16, size = 29, normalized size = 2.64

$$\begin{cases} -\frac{\log\left(\tan^2\left(a+bx\right)+1\right)}{2b}+\frac{\log\left(\tan\left(a+bx\right)\right)}{b} & \text{for } b\neq 0\\ \frac{x}{\tan\left(a\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0))
, (x/tan(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(11) = 22$.
time = 1.19, size = 56, normalized size = 5.09

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

Mupad [B]
time = 0.19, size = 28, normalized size = 2.55

$$\frac{\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(a + b*x),x)
```

```
[Out] log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b)
```

3.111 $\int \csc(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3855}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x],x]
[Out] -(ArcTanh[Cos[a + b*x]]/b)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(a+bx)\,dx = -\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.
time = 0.01, size = 38, normalized size = 3.17

$$-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x],x]
[Out] -(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b
```

Maple [A]
time = 0.03, size = 21, normalized size = 1.75

method	result	size
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	15
derivativedivides	$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$	21
default	$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$	21
risch	$-\frac{\ln(e^{i(bx+a)}+1)}{b}+\frac{\ln(e^{i(bx+a)}-1)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 1.82, size = 26, normalized size = 2.17

$$-\frac{\log\left(\cos\left(bx+a\right)+1\right)-\log\left(\cos\left(bx+a\right)-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.
time = 0.80, size = 30, normalized size = 2.50

$$-\frac{\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [A]
time = 0.27, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for}\ b\neq 0 \\ \frac{x}{\sin\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a/2 + b*x/2))/b, Ne(b, 0)), (x/sin(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(12) = 24$.
time = 0.79, size = 51, normalized size = 4.25

$$\frac{\log\left(\left|-\frac{\cos(bx+a)}{b}+\frac{1}{|b|}\right|\right)}{2\,|b|}-\frac{\log\left(\left|-\frac{\cos(bx+a)}{b}-\frac{1}{|b|}\right|\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-cos(b*x + a)/b + 1/abs(b)))/abs(b) - 1/2*log(abs(-cos(b*x + a)
/b - 1/abs(b)))/abs(b)
```

Mupad [B]
time = 0.12, size = 12, normalized size = 1.00

$$-\frac{\mathrm{atanh}(\cos\left(a+b\,x\right))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*x),x)
```

```
[Out] -atanh(cos(a + b*x))/b
```

3.112 $\int \sec(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

[Out] arctanh(sin(b*x+a))/b

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3855}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/b

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(a+bx)\,dx = \frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/b

Maple [A]
time = 0.03, size = 19, normalized size = 1.73

method	result	size
derivativedivides	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
default	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	35
risch	$\frac{\ln(e^{i(bx+a)}+i)}{b}-\frac{\ln(e^{i(bx+a)}-i)}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.
time = 6.20, size = 26, normalized size = 2.36

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 1.07, size = 28, normalized size = 2.55

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(8) = 16$.
time = 0.31, size = 34, normalized size = 3.09

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b} & \text{for}\ b\neq 0 \\ \frac{x}{\cos\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2) - 1)/b + log(tan(a/2 + b*x/2) + 1)/b, Ne(b
, 0)), (x/cos(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 0.86, size = 28, normalized size = 2.55

$$\frac{\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B]
time = 0.02, size = 11, normalized size = 1.00

$$\frac{\mathrm{atanh}(\sin\left(a+b\,x\right))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x),x)
```

```
[Out] atanh(sin(a + b*x))/b
```

3.113 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2715, 8}

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\,dx&=-\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\&=\frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.92

$$-\frac{-2(a+bx)+\sin(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2,x]
```

```
[Out] -1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Maple [A]
time = 0.03, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{x}{2}-\frac{\sin(2bx+2a)}{4b}$	19
derivativedivides	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A]
time = 3.32, size = 24, normalized size = 0.96

$$\frac{2\,bx+2\,a-\sin\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 1.08, size = 23, normalized size = 0.92

$$\frac{bx-\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.08, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Giac [A]
time = 0.68, size = 18, normalized size = 0.72

$$\frac{1}{2}x - \frac{\sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B]
time = 0.21, size = 18, normalized size = 0.72

$$\frac{x}{2} - \frac{\sin(2a + 2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2,x)
```

```
[Out] x/2 - sin(2*a + 2*b*x)/(4*b)
```

3.114 $\int \sin^3(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2713}

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.07

$$-\frac{3\cos(a+bx)}{4b}+\frac{\cos(3(a+bx))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

Maple [A]
time = 0.04, size = 22, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{(2+\sin^2(bx+a))\cos(bx+a)}{3b}$	22
default	$-\frac{(2+\sin^2(bx+a))\cos(bx+a)}{3b}$	22
risch	$-\frac{3\cos(bx+a)}{4b}+\frac{\cos(3bx+3a)}{12b}$	27
norman	$\frac{-\frac{4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4}{3b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

Maxima [A]
time = 2.36, size = 22, normalized size = 0.81

$$\frac{\cos(bx+a)^3-3\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Fricas [A]
time = 0.79, size = 22, normalized size = 0.81

$$\frac{\cos(bx+a)^3-3\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Sympy [A]
time = 0.11, size = 37, normalized size = 1.37

$$\begin{cases} -\frac{\sin^2(a+bx)\cos(a+bx)}{b}-\frac{2\cos^3(a+bx)}{3b} & \text{for } b\neq 0 \\ x\sin^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b,
 0)), (x*sin(a)**3, True))
```

Giac [A]
time = 0.72, size = 25, normalized size = 0.93

$$\frac{\cos(bx+a)^3}{3b} - \frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

Mupad [B]
time = 0.15, size = 24, normalized size = 0.89

$$-\frac{3\cos(a+b\,x)-\cos(a+b\,x)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3,x)
```

```
[Out] -(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)
```

3.115 $\int \cos^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2,x]
```

```
[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\,dx &= \frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A]
time = 0.03, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2bx+2a)}{4b}$	19
derivativedivides	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A]
time = 2.16, size = 22, normalized size = 0.88

$$\frac{2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 0.60, size = 22, normalized size = 0.88

$$\frac{bx+\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.08, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} + \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

Giac [A]
time = 0.71, size = 18, normalized size = 0.72

$$\frac{1}{2}x + \frac{\sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B]
time = 0.18, size = 18, normalized size = 0.72

$$\frac{x}{2} + \frac{\sin(2a + 2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2,x)
```

```
[Out] x/2 + sin(2*a + 2*b*x)/(4*b)
```

3.116 $\int \cos^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[Out] sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3,x]
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3,x]
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Maple [A]
time = 0.04, size = 22, normalized size = 0.85

method	result	size
derivativedivides	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
default	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
risch	$\frac{3\sin(bx+a)}{4b}+\frac{\sin(3bx+3a)}{12b}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A]
time = 3.53, size = 22, normalized size = 0.85

$$-\frac{\sin\left(bx+a\right)^3-3\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Fricas [A]
time = 1.19, size = 21, normalized size = 0.81

$$\frac{\left(\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [A]
time = 0.11, size = 36, normalized size = 1.38

$$\begin{cases} \frac{2\sin^3\left(a+bx\right)}{3b}+\frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{b} & \text{for } b\neq 0 \\ x\cos^3\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b,
0)), (x*cos(a)**3, True))
```

Giac [A]
time = 1.04, size = 22, normalized size = 0.85

$$-\frac{\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.92

$$\frac{3\,\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3,x)
```

```
[Out] (3*sin(a + b*x) - sin(a + b*x)^3)/(3*b)
```

3.117 $\int \sec^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tan(a+bx)}{b}$$

[Out] tan(b*x+a)/b

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852, 8}

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]^2,x]

[Out] Tan[a + b*x]/b

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\,dx &= -\frac{\text{Subst}(\int 1\,dx, x, -\tan(a+bx))}{b}\\ &= \frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

Maple [A]
time = 0.03, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\tan(bx+a)}{b}$	11
default	$\frac{\tan(bx+a)}{b}$	11
risch	$\frac{2i}{b\left(e^{2i(bx+a)}+1\right)}$	20
norman	$-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] tan(b*x+a)/b
```

Maxima [A]
time = 2.73, size = 10, normalized size = 1.00

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] tan(b*x + a)/b
```

Fricas [A]
time = 1.08, size = 18, normalized size = 1.80

$$\frac{\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(7) = 14$.

time = 0.53, size = 58, normalized size = 5.80

$$\begin{cases} \tilde{\infty}x & \text{for } \left(a = -\frac{\pi}{2} \vee a = -bx - \frac{\pi}{2}\right) \wedge \left(a = -bx - \frac{\pi}{2} \vee b = 0\right) \\ \frac{x}{\cos^2(a)} & \text{for } b = 0 \\ -\frac{2\tan\left(\frac{a}{2} + \frac{bx}{2}\right)}{b\tan^2\left(\frac{a}{2} + \frac{bx}{2}\right) - b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x, (Eq(b, 0) | Eq(a, -b*x - pi/2)) & (Eq(a, -pi/2) | Eq(a, -
b*x - pi/2))), (x/cos(a)**2, Eq(b, 0)), (-2*tan(a/2 + b*x/2)/(b*tan(a/2 + b
*x/2)**2 - b), True))
```

Giac [A]
time = 0.91, size = 10, normalized size = 1.00

$$\frac{\tan(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] tan(b*x + a)/b
```

Mupad [B]
time = 0.14, size = 10, normalized size = 1.00

$$\frac{\tan(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^2,x)
```

```
[Out] tan(a + b*x)/b
```

3.118 $\int \frac{1}{1+\cos(x)} \, dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{1+\cos(x)}$$

```
[Out] sin(x)/(cos(x)+1)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2727}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)} \, dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

Maple [A]
time = 0.02, size = 5, normalized size = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
norman	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{1+e^{ix}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] tan(1/2*x)
```

Maxima [A]
time = 2.61, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A]
time = 0.83, size = 9, normalized size = 1.00

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A]
time = 0.08, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(9) = 18$.
time = 0.86, size = 30, normalized size = 3.33

$$-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(x^{2}+1\right)\left(\frac{x^{2}-1}{x^{2}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

Mupad [B]
time = 0.17, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + 1),x)
```

```
[Out] tan(x/2)
```

3.119 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -sin(x)/(1-cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
norman	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A]
time = 2.32, size = 10, normalized size = 0.83

$$-\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A]
time = 0.97, size = 10, normalized size = 0.83

$$-\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A]
time = 0.16, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

<u>**Giac**</u> [**A**]
time = 0.87, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

<u>**Mupad**</u> [**B**]
time = 0.17, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1),x)
```

```
[Out] -cot(x/2)
```

3.120 $\int \frac{1}{1+\sin(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{1+\sin(x)}$$

```
[Out] -cos(x)/(1+sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{2727\}$

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\sin(x)}\,dx = -\frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.
time = 0.01, size = 23, normalized size = 2.30

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.02, size = 11, normalized size = 1.10

method	result	size
default	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)}$	11
risch	$-\frac{2}{e^{ix}+i}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(1+tan(1/2*x))
```

Maxima [A]
time = 1.54, size = 15, normalized size = 1.50

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

Fricas [A]
time = 1.07, size = 18, normalized size = 1.80

$$-\frac{\cos\left(x\right)-\sin\left(x\right)+1}{\cos\left(x\right)+\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A]
time = 0.17, size = 8, normalized size = 0.80

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

Giac [A]
time = 1.08, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) + 1)
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + 1),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

3.121 $\int \frac{1}{1-\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] cos(x)/(1-sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2727}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x])^(-1),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.01, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A]
time = 0.03, size = 11, normalized size = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
norman	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(tan(1/2*x)-1)
```

Maxima [A]
time = 2.27, size = 15, normalized size = 1.36

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A]
time = 1.04, size = 17, normalized size = 1.55

$$\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A]
time = 0.18, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

Giac [A]
time = 0.84, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{1}{2}x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

3.122 $\int \frac{1}{a+b\sin(x)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] 2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\sin(x)}\,dx = 2\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\left(4\text{Subst}\left(\int \frac{1}{-4\,(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)\right)$$
$$= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-1),x]
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Maple [A]
time = 0.05, size = 39, normalized size = 0.98

method	result	size
default	$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	39
risch	$-\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)),x,method=_RETURNVERBOSE)
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.91, size = 148, normalized size = 3.70

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)}{2(a^2-b^2)}, -\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x)))/sqrt(a^2 - b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(31) = 62$.
time = 4.03, size = 141, normalized size = 3.52

$$\begin{cases} \tilde{\infty}\log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for}\ a=0\wedge b=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for}\ a=0 \\ \frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)-b\sqrt{b^2}} & \text{for}\ a=-\sqrt{b^2} \\ -\frac{2\sqrt{b^2}}{b^2\tan\left(\frac{x}{2}\right)+b\sqrt{b^2}} & \text{for}\ a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a,
 0)), (2*sqrt(b**2)/(b**2*tan(x/2) - b*sqrt(b**2)), Eq(a, -sqrt(b**2))), (-
2*sqrt(b**2)/(b**2*tan(x/2) + b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(tan(x
/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x/2) + b/a +
 sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))
```

Giac [A]
time = 0.97, size = 48, normalized size = 1.20

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))/sqrt(a^2 - b^2)
```

Mupad [B]
time = 0.38, size = 45, normalized size = 1.12

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)),x)
```

```
[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*tan(x/2))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^
(1/2)
```

3.123 $\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{1-a^2+b^2}}\right)}{\sqrt{1-a^2+b^2}}$$

```
[Out] -2*arctanh((b-(1-a)*tan(1/2*x))/(-a^2+b^2+1)^(1/2))/(-a^2+b^2+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3203, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - (1 - a)*Tan[x/2])/Sqrt[1 - a^2 + b^2]])/Sqrt[1 - a^2 + b^2
]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx = 2\text{Subst}\left(\int \frac{1}{1+a+2bx+(-1+a)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\left(4\text{Subst}\left(\int \frac{1}{4\left(1-a^2+b^2\right)-x^2}\,dx, x, 2b+2(-1+a)\tan\left(\frac{x}{2}\right)\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{1-a^2+b^2}}\right)}{\sqrt{1-a^2+b^2}}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.94

$$\frac{2\tan^{-1}\left(\frac{b+(-1+a)\tan\left(\frac{x}{2}\right)}{\sqrt{-1+a^2-b^2}}\right)}{\sqrt{-1+a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + (-1 + a)*Tan[x/2])/Sqrt[-1 + a^2 - b^2]])/Sqrt[-1 + a^2 - b^
2]
```

Maple [A]
time = 0.09, size = 43, normalized size = 0.91

method	result
default	$\frac{2\arctan\left(\frac{2(a-1)\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$
risch	$-\frac{\ln\left(e^{ix}+\frac{iab\sqrt{-a^2+b^2+1}+ia^2-ib^2-a^2b+b^3+a\sqrt{-a^2+b^2+1}-i+b}{\left(b^2+1\right)\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}+\frac{\ln\left(e^{ix}+\frac{iab\sqrt{-a^2+b^2+1}-ia^2+ib^2\cdots}{\left(b^2+1\right)\sqrt{-\cdots}}\right)}{\sqrt{-a^2+\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2-b^2-1)^(1/2)*arctan(1/2*(2*(a-1)*tan(1/2*x)+2*b)/(a^2-b^2-1)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-a^2+1>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.99, size = 287, normalized size = 6.11

$$\left[-\frac{\sqrt{-a^2+b^2+1}\log\left(-\frac{b^4+(a^2+3)b^2-(2a^2b^2-b^4-2a^2+1)\cos(x)^2-a^2+2(ab^2+a)\cos(x)+2(ab^3+ab-(b^3-(2a^2-1)b)\cos(x))\sin(x)-2(2ab\cos(x)^2-ab+(b^3+b)\cos(x)-(b^2-(ab^2-a)\cos(x)+1)\sin(x))\sqrt{-a^2+b^2+1}+2}{(b^2-1)\cos(x)^2-a^2-b^2-2a\cos(x)-2(ab+b\cos(x))\sin(x)}\right)}{2(a^2-b^2-1)}, \frac{\arctan\left(-\frac{(ab\sin(x)+b^2+a\cos(x)+1)\sqrt{a^2-b^2-1}}{(b^3-(a^2-1)b)\cos(x)+(a^2-b^2-1)\sin(x)}\right)}{\sqrt{a^2-b^2-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2 + 1)*log(-(b^4 + (a^2 + 3)*b^2 - (2*a^2*b^2 - b^4 - 2
*a^2 + 1)*cos(x)^2 - a^2 + 2*(a*b^2 + a)*cos(x) + 2*(a*b^3 + a*b - (b^3 - (
2*a^2 - 1)*b)*cos(x))*sin(x) - 2*(2*a*b*cos(x)^2 - a*b + (b^3 + b)*cos(x) -
 (b^2 - (a*b^2 - a)*cos(x) + 1)*sin(x))*sqrt(-a^2 + b^2 + 1) + 2)/((b^2 - 1
)*cos(x)^2 - a^2 - b^2 - 2*a*cos(x) - 2*(a*b + b*cos(x))*sin(x)))/(a^2 - b^
2 - 1), arctan(-(a*b*sin(x) + b^2 + a*cos(x) + 1)*sqrt(a^2 - b^2 - 1)/((b^3
 - (a^2 - 1)*b)*cos(x) + (a^2 - b^2 - 1)*sin(x)))/sqrt(a^2 - b^2 - 1)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1872 vs. 2(37) = 74.
time = 210.90, size = 1872, normalized size = 39.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] Piecewise((2*b**4*sqrt(b**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b
**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2
 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b
*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 10*b**
4/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan
(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(
b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqr
t(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 24*b**2*sqrt(b**2 + 1)/(b**6*tan(x/2)
 - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*
tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/
2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(
```

```
x/2) + 32*tan(x/2)) + 40*b**2/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5
 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 +
1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sq
rt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32*sqrt(b
**2 + 1)/(b**6*tan(x/2) - b**5*sqrt(b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 +
 1)*tan(x/2) + 18*b**4*tan(x/2) - 12*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**
2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b +
 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32/(b**6*tan(x/2) - b**5*sqrt(
b**2 + 1) - 5*b**5 + 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) - 12
*b**3*sqrt(b**2 + 1) - 20*b**3 + 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*
tan(x/2) - 16*b*sqrt(b**2 + 1) - 16*b + 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan
(x/2)), Eq(a, -sqrt(b**2 + 1))), (-2*b**4*sqrt(b**2 + 1)/(b**6*tan(x/2) + b
**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(
x/2) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) +
 48*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2)
 + 32*tan(x/2)) + 10*b**4/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6
*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2 + 1) -
 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b*sqrt(b
**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) - 24*b**2*sqrt(
b**2 + 1)/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2
+ 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 32*b*
*2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 16*b
- 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 40*b**2/(b**6*tan(x/2) + b**5
*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2
) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48
*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2) +
32*tan(x/2)) - 32*sqrt(b**2 + 1)/(b**6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b
**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2) + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2
 + 1) - 20*b**3 - 32*b**2*sqrt(b**2 + 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b
*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**2 + 1)*tan(x/2) + 32*tan(x/2)) + 32/(b*
*6*tan(x/2) + b**5*sqrt(b**2 + 1) - 5*b**5 - 6*b**4*sqrt(b**2 + 1)*tan(x/2)
 + 18*b**4*tan(x/2) + 12*b**3*sqrt(b**2 + 1) - 20*b**3 - 32*b**2*sqrt(b**2
+ 1)*tan(x/2) + 48*b**2*tan(x/2) + 16*b*sqrt(b**2 + 1) - 16*b - 32*sqrt(b**
2 + 1)*tan(x/2) + 32*tan(x/2)), Eq(a, sqrt(b**2 + 1))), (log(tan(x/2) + 1/b
)/b, Eq(a, 1)), (log(b/(a - 1) + tan(x/2) - sqrt(-a**2 + b**2 + 1)/(a - 1))
/sqrt(-a**2 + b**2 + 1) - log(b/(a - 1) + tan(x/2) + sqrt(-a**2 + b**2 + 1)
/(a - 1))/sqrt(-a**2 + b**2 + 1), True))
```

Giac [A]
time = 1.10, size = 60, normalized size = 1.28

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b-\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2-1}}\right)\right)}{\sqrt{a^2-b^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) + b - tan(1
/2*x))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1)
```

Mupad [B]
time = 0.26, size = 58, normalized size = 1.23

$$\begin{cases} \frac{\ln\left(b\tan\left(\frac{x}{2}\right)+1\right)}{b} & \text{if } a=1 \\ \frac{2\,\mathrm{atan}\left(\frac{b+\tan\left(\frac{x}{2}\right)\,(a-1)}{\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}} & \text{if } a\neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + cos(x) + b*sin(x)),x)
```

```
[Out] piecewise(a == 1, log(b*tan(x/2) + 1)/b, a ~= 1, (2*atan((b + tan(x/2)*(a -
 1))/(a^2 - b^2 - 1)^(1/2)))/(a^2 - b^2 - 1)^(1/2))
```

3.124 $\int x^2 \sin^2(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{x}{4b^2}+\frac{x^3}{6}+\frac{\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}$$

```
[Out] -1/4*x/b^2+1/6*x^3+1/4*cos(b*x+a)*sin(b*x+a)/b^3-1/2*x^2*cos(b*x+a)*sin(b*x
+a)/b+1/2*x*sin(b*x+a)^2/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3392, 30, 2715, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*x]^2,x]
```

```
[Out] -1/4*x/b^2 + x^3/6 + (Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - (x^2*Cos[a + b*x
]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^2 \sin^2(a+bx)\,dx = -\frac{x^2\cos(a+bx)\sin(a+bx)}{2b} + \frac{x\sin^2(a+bx)}{2b^2} + \frac{\int x^2\,dx}{2} - \frac{\int \sin^2(a+bx)\,dx}{2b^2}$$
$$= \frac{x^3}{6} + \frac{\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{x^2\cos(a+bx)\sin(a+bx)}{2b} + \frac{x\sin^2(a+bx)}{2b^2} - \frac{\int 1\,d}{4b^2}$$
$$= -\frac{x}{4b^2} + \frac{x^3}{6} + \frac{\cos(a+bx)\sin(a+bx)}{4b^3} - \frac{x^2\cos(a+bx)\sin(a+bx)}{2b} + \frac{x\sin^2(a+bx)}{2b^2}$$

Mathematica [A]
time = 0.09, size = 47, normalized size = 0.64

$$\frac{4b^3x^3 - 6bx\cos(2(a+bx)) + (3-6b^2x^2)\sin(2(a+bx))}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)])/(24
*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(63) = 126$.
time = 0.04, size = 158, normalized size = 2.16

method	result
risch	$\frac{x^3}{6} - \frac{x\cos(2bx+2a)}{4b^2} - \frac{(2x^2b^2-1)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{a^2(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2})-2a((bx+a)(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2})-\frac{(bx+a)^2}{4}+\frac{(\sin^2(bx+a))}{4})+(bx+a)^2(-}{b^3}$
default	$\frac{a^2(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2})-2a((bx+a)(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2})-\frac{(bx+a)^2}{4}+\frac{(\sin^2(bx+a))}{4})+(bx+a)^2(-}{b^3}$
norman	$\frac{\frac{x^2(\tan^3(\frac{bx}{2}+\frac{a}{2}))}{b}+\frac{x^3}{6}+\frac{\tan(\frac{bx}{2}+\frac{a}{2})}{2b^3}-\frac{\tan^3(\frac{bx}{2}+\frac{a}{2})}{2b^3}-\frac{x}{4b^2}+\frac{x^3(\tan^2(\frac{bx}{2}+\frac{a}{2}))}{3}+\frac{x^3(\tan^4(\frac{bx}{2}+\frac{a}{2}))}{6}+\frac{3x(\tan^2(\frac{bx}{2}+\frac{a}{2}))}{2b^2}-\frac{x}{}}{(1+\tan^2(\frac{bx}{2}+\frac{a}{2}))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(a^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2*a*((b*x+a)*(-1/2*co
s(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2)+(b*x+a)^
2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)*cos(b*x+a)^2+1/4*c
os(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3)
```

Maxima [A]
time = 1.65, size = 117, normalized size = 1.60

$$\frac{4\,(bx+a)^3+6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2-6\,\left(2\,(bx+a)^2-2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)\right)a-6\,(bx+a)\cos(2\,bx+2\,a)-3\,\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a - 6*(b*x + a)*cos
(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

Fricas [A]
time = 0.97, size = 54, normalized size = 0.74

$$\frac{2\,b^3x^3-6\,bx\cos(bx+a)^2-3\,(2\,b^2x^2-1)\cos(bx+a)\sin(bx+a)+3\,bx}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin
(b*x + a) + 3*b*x)/b^3
```

Sympy [A]
time = 0.19, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2(a+bx)}{6}+\frac{x^3\cos^2(a+bx)}{6}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{4b^2}-\frac{x\cos^2(a+bx)}{4b^2}+\frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for } b\neq 0\\ \frac{x^3\sin^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 - x**2*sin(a + b
*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2/(4*b**2) - x*cos(a + b*x)**2/(4*
b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*sin(a)**2/3, T
rue))
```

Giac [A]
time = 1.24, size = 45, normalized size = 0.62

$$\frac{1}{6}\,x^3-\frac{x\cos(2\,bx+2\,a)}{4\,b^2}-\frac{(2\,b^2x^2-1)\sin(2\,bx+2\,a)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*b*x + 2*a)/b^2 - 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

Mupad [B]
time = 0.14, size = 52, normalized size = 0.71

$$\frac{x^3}{6}+\frac{\sin\left(2\,a+2\,b\,x\right)}{8\,b^3}-\frac{x\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{x^2\,\sin\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(a + b*x)^2,x)
```

```
[Out] x^3/6 + sin(2*a + 2*b*x)/(8*b^3) - (x*cos(2*a + 2*b*x))/(4*b^2) - (x^2*sin(
2*a + 2*b*x))/(4*b)
```

3.125 $\int \cos(x)\cos(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

[Out] 1/2*sin(x)+1/6*sin(3*x)

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4368}

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(2x)\,dx=\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Maple [A]
time = 0.04, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{4\tan(x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^2(x)\right)\tan\left(\frac{x}{2}\right)}{3}+\frac{4\tan(x)}{3}-\frac{2\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(x)\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

Maxima [A]
time = 7.03, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A]
time = 0.62, size = 12, normalized size = 0.80

$$\frac{1}{3}\left(2\cos(x)^2+1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A]
time = 0.12, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3}+\frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x)
```

```
[Out] -sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

Giac [A]
time = 1.16, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.60

$$\sin(x) - \frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(x),x)
```

```
[Out] sin(x) - (2*sin(x)^3)/3
```

3.126 $\int x^2 \cos^2(a+bx)\,dx$

Optimal. Leaf size=73

$$-\frac{x}{4b^2}+\frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] -1/4*x/b^2+1/6*x^3+1/2*x*cos(b*x+a)^2/b^2-1/4*cos(b*x+a)*sin(b*x+a)/b^3+1/2
*x^2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3392, 30, 2715, 8}

$$-\frac{\sin(a+bx)\cos(a+bx)}{4b^3}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x]^2,x]
```

```
[Out] -1/4*x/b^2 + x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) - (Cos[a + b*x]*Sin[a + b*x
])/(4*b^3) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int x^2\cos^2(a+bx)\,dx &= \frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int x^2\,dx}{2}-\frac{\int\cos^2(a+bx)\,dx}{2b^2}\\ &= \frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}-\frac{\int 1\,dx}{4b^2}\\ &= -\frac{x}{4b^2}+\frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 47, normalized size = 0.64

$$\frac{4b^3x^3+6bx\cos(2(a+bx))+(-3+6b^2x^2)\sin(2(a+bx))}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 + 6*b*x*Cos[2*(a + b*x)] + (-3 + 6*b^2*x^2)*Sin[2*(a + b*x)])/(2
4*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(63) = 126$.
time = 0.04, size = 158, normalized size = 2.16

method	result
risch	$\frac{x^3}{6}+\frac{x\cos(2bx+2a)}{4b^2}+\frac{(2x^2b^2-1)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{a^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-2a\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\left(\sin^2(bx+a)\right)}{4}\right)+(bx+a)^2\left(\frac{\cos(bx+\cdots}{}\right.}{b^3}$
default	$\frac{a^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-2a\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\left(\sin^2(bx+a)\right)}{4}\right)+(bx+a)^2\left(\frac{\cos(bx+\cdots}{}\right.}{b^3}$
norman	$\frac{\frac{x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x^3}{6}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^3}+\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b^3}+\frac{x}{4b^2}+\frac{x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6}-\frac{3x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^2}+\frac{x\left(\tan^4\cdots\right.}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2*a*((b*x+a)*(1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+(b*x+a)^2*
(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(
b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)
```

Maxima [A]
time = 2.98, size = 113, normalized size = 1.55

$$\frac{4\,(bx+a)^3+6\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2-6\,\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)a+6\,(bx+a)\cos(2\,bx+2\,a)+3\,\left(2\,(bx+a)^2-1\right)\sin(2\,bx+2\,a)}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a + 6*(b*x + a)*cos
(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

Fricas [A]
time = 0.73, size = 54, normalized size = 0.74

$$\frac{2\,b^3x^3+6\,bx\cos(bx+a)^2+3\,(2\,b^2x^2-1)\cos(bx+a)\sin(bx+a)-3\,bx}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 6*b*x*cos(b*x + a)^2 + 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin
(b*x + a) - 3*b*x)/b^3
```

Sympy [A]
time = 0.18, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2(a+bx)}{6}+\frac{x^3\cos^2(a+bx)}{6}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x\sin^2(a+bx)}{4b^2}+\frac{x\cos^2(a+bx)}{4b^2}-\frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for } b\neq 0 \\ \frac{x^3\cos^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 + x**2*sin(a + b
*x)*cos(a + b*x)/(2*b) - x*sin(a + b*x)**2/(4*b**2) + x*cos(a + b*x)**2/(4*
b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*cos(a)**2/3, T
rue))
```

Giac [A]
time = 1.11, size = 45, normalized size = 0.62

$$\frac{1}{6}\,x^3+\frac{x\cos(2\,bx+2\,a)}{4\,b^2}+\frac{(2\,b^2x^2-1)\sin(2\,bx+2\,a)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

Mupad [B]
time = 0.10, size = 52, normalized size = 0.71

$$\frac{x^3}{6}-\frac{\sin\left(2\,a+2\,b\,x\right)}{8\,b^3}+\frac{x\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^2}+\frac{x^2\,\sin\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x)^2,x)
```

```
[Out] x^3/6 - sin(2*a + 2*b*x)/(8*b^3) + (x*cos(2*a + 2*b*x))/(4*b^2) + (x^2*sin(
2*a + 2*b*x))/(4*b)
```

3.127 $\int \cot^3(x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

```
[Out] -1/2*cot(x)^2-ln(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3554, 3556}

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3,x]
```

```
[Out] -1/2*Cot[x]^2 - Log[Sin[x]]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\,dx &= -\frac{1}{2}\cot^2(x)-\int \cot(x)\,dx\\ &= -\frac{1}{2}\cot^2(x)-\log(\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2}\csc^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3,x]
[Out] -1/2*Csc[x]^2 - Log[Sin[x]]
```

Maple [A]
time = 0.02, size = 22, normalized size = 1.57

method	result	size
derivativedivides	$\frac{\ln(1+\tan^2(x))}{2}-\frac{1}{2\tan(x)^2}-\ln(\tan(x))$	22
default	$\frac{\ln(1+\tan^2(x))}{2}-\frac{1}{2\tan(x)^2}-\ln(\tan(x))$	22
norman	$\frac{\ln(1+\tan^2(x))}{2}-\frac{1}{2\tan(x)^2}-\ln(\tan(x))$	22
risch	$ix+\frac{2\,e^{2ix}}{(e^{2ix}-1)^2}-\ln(e^{2ix}-1)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1+tan(x)^2)-1/2/tan(x)^2-ln(tan(x))
```

Maxima [A]
time = 2.57, size = 14, normalized size = 1.00

$$-\frac{1}{2\,\sin(x)^2}-\frac{1}{2}\,\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^3,x, algorithm="maxima")
[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 1.02, size = 31, normalized size = 2.21

$$-\frac{\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^2+\tan(x)^2+1}{2\,\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^3,x, algorithm="fricas")
[Out] -1/2*(log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^2 + tan(x)^2 + 1)/tan(x)^2
```

Sympy [A]
time = 0.03, size = 14, normalized size = 1.00

$$-\log(\sin(x))-\frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)**3,x)
```

```
[Out] -log(sin(x)) - 1/(2*sin(x)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 1.03, size = 29, normalized size = 2.07

$$\frac{\tan\left(x\right)^{2} - 1}{2\,\tan\left(x\right)^{2}} + \frac{1}{2}\,\log\left(\tan\left(x\right)^{2} + 1\right) - \frac{1}{2}\,\log\left(\tan\left(x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*(tan(x)^2 - 1)/tan(x)^2 + 1/2*log(tan(x)^2 + 1) - 1/2*log(tan(x)^2)
```

Mupad [B]
time = 0.17, size = 21, normalized size = 1.50

$$\frac{\ln\left(\tan(x)^2 + 1\right)}{2} - \ln\left(\tan(x)\right) - \frac{1}{2\tan\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x)^3,x)
```

```
[Out] log(tan(x)^2 + 1)/2 - log(tan(x)) - 1/(2*tan(x)^2)
```

3.128 $\int x^3 \tan^4(x)\,dx$

Optimal. Leaf size=104

$$-\frac{x^2}{2}+\frac{4ix^3}{3}+\frac{x^4}{4}-4x^2\log\left(1+e^{2ix}\right)+\log(\cos(x))+4ix\text{Li}_2\left(-e^{2ix}\right)-2\text{Li}_3\left(-e^{2ix}\right)+x\tan(x)-x^3\tan(x)-\frac{1}{2}x^2\tan$$

```
[Out] -1/2*x^2+4/3*I*x^3+1/4*x^4-4*x^2*ln(1+exp(2*I*x))+ln(cos(x))+4*I*x*polylog(
2,-exp(2*I*x))-2*polylog(3,-exp(2*I*x))+x*tan(x)-x^3*tan(x)-1/2*x^2*tan(x)^
2+1/3*x^3*tan(x)^3
```

Rubi [A]
time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3801, 3556, 30, 3800, 2221, 2611, 2320, 6724}

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right)-2\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}+\frac{1}{3}x^3\tan^3(x)-x^3\tan(x)-\frac{x^2}{2}-4x^2\log\left(1+e^{2ix}\right)-\frac{1}{2}x^2\tan^2(x)+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[x]^4,x]
```

```
[Out] -1/2*x^2 + ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]]
 + (4*I)*x*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] + x*Tan[x]
 - x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^3)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \tan^4(x)\,dx &= \frac{1}{3}x^3\tan^3(x) - \int x^3\tan^2(x)\,dx - \int x^2\tan^3(x)\,dx \\
&= -x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x^3\tan^3(x) + 3\int x^2\tan(x)\,dx + \int x^3\,dx + \int x^2\tan(x)\,dx \\
&= \frac{4ix^3}{3} + \frac{x^4}{4} + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x^3\tan^3(x) - 2i\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx - 6i\int \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\tan^2(x \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\mathrm{Li}_2\left(-e^{2ix}\right) + x\tan(x) - x^3\tan \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\mathrm{Li}_2\left(-e^{2ix}\right) + x\tan(x) - x^3\tan \\
&= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\mathrm{Li}_2\left(-e^{2ix}\right) - 2\mathrm{Li}_3\left(-e^{2ix}\right) + x\,\mathrm{t}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 101, normalized size = 0.97

$$
\frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\mathrm{Li}_2\left(-e^{2ix}\right) - 2\mathrm{Li}_3\left(-e^{2ix}\right) - \frac{1}{2}x^2\sec^2(x) + x\tan(x) - \frac{4}{3}x^3\tan(x) + \frac{1}{3}x^3\sec^2(x)\tan(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[x]^4,x]
```

```
[Out] ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] + (4*I)*x*
PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] - (x^2*Sec[x]^2)/2 +
x*Tan[x] - (4*x^3*Tan[x])/3 + (x^3*Sec[x]^2*Tan[x])/3
```

Maple [A]
time = 0.05, size = 138, normalized size = 1.33

method	result
risch	$\frac{x^4}{4} - \frac{2ix\left(6x^2e^{4ix}+6x^2e^{2ix}-3\,e^{4ix}-3ix\,e^{4ix}+4x^2-6\,e^{2ix}-3ix\,e^{2ix}-3\right)}{3\left(e^{2ix}+1\right)^3} - 2\ln\left(e^{ix}\right) + \ln\left(e^{2ix}+1\right) + \frac{8ix^3}{3} - 4x^2\ln\left(\mathrm{e}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4-2/3*I*x*(6*x^2*exp(4*I*x)+6*x^2*exp(2*I*x)-3*exp(4*I*x)-3*I*x*exp(4
*I*x)+4*x^2-6*exp(2*I*x)-3*I*x*exp(2*I*x)-3)/(exp(2*I*x)+1)^3-2*ln(exp(I*x)
)+ln(exp(2*I*x)+1)+8/3*I*x^3-4*x^2*ln(exp(2*I*x)+1)+4*I*x*polylog(2,-exp(2*
I*x))-2*polylog(3,-exp(2*I*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(80) = 160$.
time = 2.54, size = 491, normalized size = 4.72

$$-\frac{3 i x^{4} + 12 \left(4 x^{2} + \left(4 x^{2} - 1\right) \cos\left(6 x\right) + 3 \left(4 x^{2} - 1\right) \cos\left(4 x\right) + 3 \left(4 x^{2} - 1\right) \cos\left(2 x\right) - \left(-4 i x^{2} + i\right) \sin\left(6 x\right) - 3 \left(-4 i x^{2} + i\right) \sin\left(4 x\right) - 3 \left(-4 i x^{2} + i\right) \sin\left(2 x\right) - 1\right) \arctan\left(\sin\left(2 x\right), \cos\left(2 x\right) + 1\right) + \left(3 i x^{4} - 32 x^{3} + 24 x\right) \cos\left(6 x\right) - 3 \left(-3 i x^{4} + 16 x^{3} + 8 i x^{2} - 16 x\right) \cos\left(4 x\right) - 3 \left(-3 i x^{4} + 16 x^{3} + 8 i x^{2} - 8 x\right) \cos\left(2 x\right) - 48 \left(x \cos\left(6 x\right) + 3 x \cos\left(4 x\right) + 3 x \cos\left(2 x\right) + i x \sin\left(6 x\right) + 3 i x \sin\left(4 x\right) + 3 i x \sin\left(2 x\right) + x\right) \mathrm{Li}_2\left(-e^{\left(2 i x\right)}\right) - 6 \left(4 i x^{2} + \left(4 i x^{2} - i\right) \cos\left(6 x\right) + 3 \left(4 i x^{2} - i\right) \cos\left(4 x\right) + 3 \left(4 i x^{2} - i\right) \cos\left(2 x\right) - \left(4 x^{2} - 1\right) \sin\left(6 x\right) - 3 \left(4 x^{2} - 1\right) \sin\left(4 x\right) - 3 \left(4 x^{2} - 1\right) \sin\left(2 x\right) - i\right) \log\left(\cos\left(2 x\right)^{2} + \sin\left(2 x\right)^{2} + 2 \cos\left(2 x\right) + 1\right) - 24 \left(i \cos\left(6 x\right) + 3 i \cos\left(4 x\right) + 3 i \cos\left(2 x\right) - \sin\left(6 x\right) - 3 \sin\left(4 x\right) - 3 \sin\left(2 x\right) + i\right) \mathrm{Li}_3\left(-e^{\left(2 i x\right)}\right) - \left(3 x^{4} + 32 i x^{3} - 24 i x\right) \sin\left(6 x\right) - 3 \left(3 x^{4} + 16 i x^{3} - 8 x^{2} - 16 i x\right) \sin\left(4 x\right) - 3 \left(3 x^{4} + 16 i x^{3} - 8 x^{2} - 8 i x\right) \sin\left(2 x\right)}{-12 i \cos\left(6 x\right) - 36 i \cos\left(4 x\right) - 36 i \cos\left(2 x\right) + 12 \sin\left(6 x\right) + 36 \sin\left(4 x\right) + 36 \sin\left(2 x\right) - 12 i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="maxima")
```

```
[Out] -(3*I*x^4 + 12*(4*x^2 + (4*x^2 - 1)*cos(6*x) + 3*(4*x^2 - 1)*cos(4*x) + 3*(
4*x^2 - 1)*cos(2*x) - (-4*I*x^2 + I)*sin(6*x) - 3*(-4*I*x^2 + I)*sin(4*x) -
 3*(-4*I*x^2 + I)*sin(2*x) - 1)*arctan2(sin(2*x), cos(2*x) + 1) + (3*I*x^4
- 32*x^3 + 24*x)*cos(6*x) - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 16*x)*cos(4*x)
 - 3*(-3*I*x^4 + 16*x^3 + 8*I*x^2 - 8*x)*cos(2*x) - 48*(x*cos(6*x) + 3*x*co
s(4*x) + 3*x*cos(2*x) + I*x*sin(6*x) + 3*I*x*sin(4*x) + 3*I*x*sin(2*x) + x)
*dilog(-e^(2*I*x)) - 6*(4*I*x^2 + (4*I*x^2 - I)*cos(6*x) + 3*(4*I*x^2 - I)*
cos(4*x) + 3*(4*I*x^2 - I)*cos(2*x) - (4*x^2 - 1)*sin(6*x) - 3*(4*x^2 - 1)*
sin(4*x) - 3*(4*x^2 - 1)*sin(2*x) - I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(
2*x) + 1) - 24*(I*cos(6*x) + 3*I*cos(4*x) + 3*I*cos(2*x) - sin(6*x) - 3*sin
(4*x) - 3*sin(2*x) + I)*polylog(3, -e^(2*I*x)) - (3*x^4 + 32*I*x^3 - 24*I*x
)*sin(6*x) - 3*(3*x^4 + 16*I*x^3 - 8*x^2 - 16*I*x)*sin(4*x) - 3*(3*x^4 + 16
*I*x^3 - 8*x^2 - 8*I*x)*sin(2*x))/(-12*I*cos(6*x) - 36*I*cos(4*x) - 36*I*co
s(2*x) + 12*sin(6*x) + 36*sin(4*x) + 36*sin(2*x) - 12*I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(80) = 160$.
time = 0.92, size = 182, normalized size = 1.75

$$\frac{1}{3} x^{3} \tan\left(x\right)^{3} + \frac{1}{4} x^{4} - \frac{1}{2} x^{2} \tan\left(x\right)^{2} - \frac{1}{2} x^{2} - 2 i x \mathrm{Li}_2\left(\frac{2 \left(i \tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1} + 1\right) + 2 i x \mathrm{Li}_2\left(\frac{2 \left(-i \tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1} + 1\right) - \frac{1}{2} \left(4 x^{2} - 1\right) \log\left(-\frac{2 \left(i \tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1}\right) - \frac{1}{2} \left(4 x^{2} - 1\right) \log\left(-\frac{2 \left(-i \tan\left(x\right) - 1\right)}{\tan\left(x\right)^{2} + 1}\right) - \left(x^{3} - x\right) \tan\left(x\right) - \mathrm{polylog}\left(3, \frac{\tan\left(x\right)^{2} + 2 i \tan\left(x\right) - 1}{\tan\left(x\right)^{2} + 1}\right) - \mathrm{polylog}\left(3, \frac{\tan\left(x\right)^{2} - 2 i \tan\left(x\right) - 1}{\tan\left(x\right)^{2} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*tan(x)^3 + 1/4*x^4 - 1/2*x^2*tan(x)^2 - 1/2*x^2 - 2*I*x*dilog(2*(I*
tan(x) - 1)/(tan(x)^2 + 1) + 1) + 2*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 +
 1) + 1) - 1/2*(4*x^2 - 1)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*(4*x
^2 - 1)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - (x^3 - x)*tan(x) - polylog
(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) - polylog(3, (tan(x)^2 - 2*
I*tan(x) - 1)/(tan(x)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \tan^{4}\left(x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(x)**4,x)
```

```
[Out] Integral(x**3*tan(x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \tan(x)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^4,x)
```

```
[Out] int(x^3*tan(x)^4, x)
```

3.129 $\int x^3 \tan^6(x)\,dx$

Optimal. Leaf size=153

$$\frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{23}{5}ix\mathrm{Li}_2\left(-e^{2ix}\right)+\frac{23}{10}\mathrm{Li}_3\left(-e^{2ix}\right)-\frac{19}{10}x\tan(x)+x^3\tan($$

```
[Out] 19/20*x^2-23/15*I*x^3-1/4*x^4+23/5*x^2*ln(1+exp(2*I*x))-2*ln(cos(x))-23/5*I
*x*polylog(2,-exp(2*I*x))+23/10*polylog(3,-exp(2*I*x))-19/10*x*tan(x)+x^3*t
an(x)-1/20*tan(x)^2+4/5*x^2*tan(x)^2+1/10*x*tan(x)^3-1/3*x^3*tan(x)^3-3/20*
x^2*tan(x)^4+1/5*x^3*tan(x)^5
```

Rubi [A]
time = 0.30, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {3801, 3554, 3556, 30, 3800, 2221, 2611, 2320, 6724}

$$-\frac{23}{5}ix\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{23}{10}\text{PolyLog}\left(3,-e^{2ix}\right)-\frac{x^4}{4}-\frac{23ix^3}{15}+\frac{1}{5}x^3\tan^5(x)-\frac{1}{3}x^3\tan^3(x)+x^3\tan(x)+\frac{19x^2}{20}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-\frac{3}{20}x^2\tan^4(x)+\frac{4}{5}x^2\tan^2(x)+\frac{1}{10}x\tan^3(x)-\frac{\tan^2(x)}{20}-\frac{19}{10}x\tan(x)-2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[x]^6,x]
```

```
[Out] (19*x^2)/20 - ((23*I)/15)*x^3 - x^4/4 + (23*x^2*Log[1 + E^((2*I)*x)])/5 - 2
*Log[Cos[x]] - ((23*I)/5)*x*PolyLog[2, -E^((2*I)*x)] + (23*PolyLog[3, -E^((
2*I)*x)])/10 - (19*x*Tan[x])/10 + x^3*Tan[x] - Tan[x]^2/20 + (4*x^2*Tan[x]^
2)/5 + (x*Tan[x]^3)/10 - (x^3*Tan[x]^3)/3 - (3*x^2*Tan[x]^4)/20 + (x^3*Tan[
x]^5)/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x^3\tan^6(x)\,dx &= \frac{1}{5}x^3\tan^5(x)-\frac{3}{5}\int x^2\tan^5(x)\,dx-\int x^3\tan^4(x)\,dx\\ &= -\frac{1}{3}x^3\tan^3(x)-\frac{3}{20}x^2\tan^4(x)+\frac{1}{5}x^3\tan^5(x)+\frac{3}{10}\int x\tan^4(x)\,dx+\frac{3}{5}\int x^2\tan^3(x)\,dx - \\ &= x^3\tan(x)+\frac{4}{5}x^2\tan^2(x)+\frac{1}{10}x\tan^3(x)-\frac{1}{3}x^3\tan^3(x)-\frac{3}{20}x^2\tan^4(x)+\frac{1}{5}x^3\tan^5(x) - \\ &= -\frac{23ix^3}{15}-\frac{x^4}{4}-\frac{19}{10}x\tan(x)+x^3\tan(x)-\frac{\tan^2(x)}{20}+\frac{4}{5}x^2\tan^2(x)+\frac{1}{10}x\tan^3(x)-\frac{1}{3}x^3 \\ &= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{19}{10}x\tan(x)+x^3\tan(x)-\frac{\text{ta}}{} \\ &= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{23}{5}ix\text{Li}_2\left(-e^{2ix}\right)-\frac{19}{10}x\tan(\\ &= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{23}{5}ix\text{Li}_2\left(-e^{2ix}\right)-\frac{19}{10}x\tan(\\ &= \frac{19x^2}{20}-\frac{23ix^3}{15}-\frac{x^4}{4}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)-2\log(\cos(x))-\frac{23}{5}ix\text{Li}_2\left(-e^{2ix}\right)+\frac{23}{10}\text{Li}_3(-e\end{aligned}$$

Mathematica [A]
time = 0.25, size = 133, normalized size = 0.87

$$\frac{1}{60}\left(-92ix^3-15x^4+276x^2\log\left(1+e^{2ix}\right)-120\log(\cos(x))-276ix\text{Li}_2\left(-e^{2ix}\right)+138\text{Li}_3\left(-e^{2ix}\right)-3\sec^2(x)+66x^2\sec^2(x)-9x^2\sec^4(x)-120x\tan(x)+92x^3\tan(x)+6x\sec^2(x)\tan(x)-44x^3\sec^2(x)\tan(x)+12x^3\sec^4(x)\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[x]^6,x]
```

```
[Out] ((-92*I)*x^3 - 15*x^4 + 276*x^2*Log[1 + E^((2*I)*x)] - 120*Log[Cos[x]] - (2
76*I)*x*PolyLog[2, -E^((2*I)*x)] + 138*PolyLog[3, -E^((2*I)*x)] - 3*Sec[x]^
2 + 66*x^2*Sec[x]^2 - 9*x^2*Sec[x]^4 - 120*x*Tan[x] + 92*x^3*Tan[x] + 6*x*S
ec[x]^2*Tan[x] - 44*x^3*Sec[x]^2*Tan[x] + 12*x^3*Sec[x]^4*Tan[x])/60
```

Maple [A]
time = 0.04, size = 237, normalized size = 1.55

method	result
risch	$-\frac{x^4}{4}+\frac{i\left(9ie^{6ix}+90x^3e^{8ix}-162ix^2e^{4ix}-66ix^2e^{8ix}+180x^3e^{6ix}-66x\,e^{8ix}+3ie^{2ix}+3ie^{8ix}+280x^3e^{4ix}-246x\,e^{6ix}-66ix^2e^{2ix}+9ie^{4i}\right.}{15\left(e^{2ix}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4+1/15*I*(9*I*exp(6*I*x)+90*x^3*exp(8*I*x)-162*I*x^2*exp(4*I*x)-66*I
*x^2*exp(8*I*x)+180*x^3*exp(6*I*x)-66*x*exp(8*I*x)+3*I*exp(2*I*x)+3*I*exp(8
```

```
*I*x)+280*x^3*exp(4*I*x)-246*x*exp(6*I*x)-66*I*x^2*exp(2*I*x)+9*I*exp(4*I*x
)+140*x^3*exp(2*I*x)-354*x*exp(4*I*x)-162*I*x^2*exp(6*I*x)+46*x^3-234*x*exp
(2*I*x)-60*x)/(exp(2*I*x)+1)^5+4*ln(exp(I*x))-2*ln(exp(2*I*x)+1)-46/15*I*x^
3+23/5*x^2*ln(exp(2*I*x)+1)-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*polylog(3
,-exp(2*I*x))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(113) = 226.
time = 4.45, size = 777, normalized size = 5.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="maxima")
```

```
[Out] (15*I*x^4 + 12*(23*x^2 + (23*x^2 - 10)*cos(10*x) + 5*(23*x^2 - 10)*cos(8*x)
 + 10*(23*x^2 - 10)*cos(6*x) + 10*(23*x^2 - 10)*cos(4*x) + 5*(23*x^2 - 10)*
cos(2*x) - (-23*I*x^2 + 10*I)*sin(10*x) - 5*(-23*I*x^2 + 10*I)*sin(8*x) - 1
0*(-23*I*x^2 + 10*I)*sin(6*x) - 10*(-23*I*x^2 + 10*I)*sin(4*x) - 5*(-23*I*x
^2 + 10*I)*sin(2*x) - 10)*arctan2(sin(2*x), cos(2*x) + 1) + (15*I*x^4 - 184
*x^3 + 240*x)*cos(10*x) + (75*I*x^4 - 560*x^3 - 264*I*x^2 + 936*x + 12*I)*c
os(8*x) - 2*(-75*I*x^4 + 560*x^3 + 324*I*x^2 - 708*x - 18*I)*cos(6*x) - 6*(
-25*I*x^4 + 120*x^3 + 108*I*x^2 - 164*x - 6*I)*cos(4*x) - 3*(-25*I*x^4 + 12
0*x^3 + 88*I*x^2 - 88*x - 4*I)*cos(2*x) - 276*(x*cos(10*x) + 5*x*cos(8*x) +
 10*x*cos(6*x) + 10*x*cos(4*x) + 5*x*cos(2*x) + I*x*sin(10*x) + 5*I*x*sin(8
*x) + 10*I*x*sin(6*x) + 10*I*x*sin(4*x) + 5*I*x*sin(2*x) + x)*dilog(-e^(2*I
*x)) - 6*(23*I*x^2 + (23*I*x^2 - 10*I)*cos(10*x) + 5*(23*I*x^2 - 10*I)*cos(
8*x) + 10*(23*I*x^2 - 10*I)*cos(6*x) + 10*(23*I*x^2 - 10*I)*cos(4*x) + 5*(2
3*I*x^2 - 10*I)*cos(2*x) - (23*x^2 - 10)*sin(10*x) - 5*(23*x^2 - 10)*sin(8*
x) - 10*(23*x^2 - 10)*sin(6*x) - 10*(23*x^2 - 10)*sin(4*x) - 5*(23*x^2 - 10
)*sin(2*x) - 10*I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 138*(I*c
os(10*x) + 5*I*cos(8*x) + 10*I*cos(6*x) + 10*I*cos(4*x) + 5*I*cos(2*x) - si
n(10*x) - 5*sin(8*x) - 10*sin(6*x) - 10*sin(4*x) - 5*sin(2*x) + I)*polylog(
3, -e^(2*I*x)) - (15*x^4 + 184*I*x^3 - 240*I*x)*sin(10*x) - (75*x^4 + 560*I
*x^3 - 264*x^2 - 936*I*x + 12)*sin(8*x) - 2*(75*x^4 + 560*I*x^3 - 324*x^2 -
 708*I*x + 18)*sin(6*x) - 6*(25*x^4 + 120*I*x^3 - 108*x^2 - 164*I*x + 6)*si
n(4*x) - 3*(25*x^4 + 120*I*x^3 - 88*x^2 - 88*I*x + 4)*sin(2*x))/(-60*I*cos(
10*x) - 300*I*cos(8*x) - 600*I*cos(6*x) - 600*I*cos(4*x) - 300*I*cos(2*x) +
 60*sin(10*x) + 300*sin(8*x) + 600*sin(6*x) + 600*sin(4*x) + 300*sin(2*x) -
 60*I)
```

Fricas [A]
time = 1.16, size = 212, normalized size = 1.39

$$\frac{1}{5}x^{3}\tan\left(x\right)^{5}-\frac{3}{20}x^{2}\tan\left(x\right)^{4}-\frac{1}{4}x^{4}-\frac{1}{30}\left(10x^{3}-3x\right)\tan\left(x\right)^{3}+\frac{1}{20}\left(16x^{2}-1\right)\tan\left(x\right)^{2}+\frac{19}{20}x^{2}+\frac{23}{10}ix\mathrm{Li}_2\left(\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}+1\right)-\frac{23}{10}ix\mathrm{Li}_2\left(\frac{2\left(-i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}+1\right)+\frac{1}{10}\left(23x^{2}-10\right)\log\left(-\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{10}\left(23x^{2}-10\right)\log\left(-\frac{2\left(-i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{10}\left(10x^{3}-19x\right)\tan\left(x\right)+\frac{23}{20}\mathrm{polylog}\left(3,\frac{\tan\left(x\right)^{2}+2i\tan\left(x\right)-1}{\tan\left(x\right)^{2}+1}\right)+\frac{23}{20}\mathrm{polylog}\left(3,\frac{\tan\left(x\right)^{2}-2i\tan\left(x\right)-1}{\tan\left(x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*x^3*tan(x)^5 - 3/20*x^2*tan(x)^4 - 1/4*x^4 - 1/30*(10*x^3 - 3*x)*tan(x)
^3 + 1/20*(16*x^2 - 1)*tan(x)^2 + 19/20*x^2 + 23/10*I*x*dilog(2*(I*tan(x) -
 1)/(tan(x)^2 + 1) + 1) - 23/10*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1)
+ 1) + 1/10*(23*x^2 - 10)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(23*
x^2 - 10)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(10*x^3 - 19*x)*tan
(x) + 23/20*polylog(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) + 23/20*
polylog(3, (tan(x)^2 - 2*I*tan(x) - 1)/(tan(x)^2 + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \tan^6(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(x)**6,x)
```

```
[Out] Integral(x**3*tan(x)**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\tan\left(x\right)}^6 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^6,x)
```

```
[Out] int(x^3*tan(x)^6, x)
```

3.130 $\int x \tan^2(x)\,dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2}+\log(\cos(x))+x\tan(x)$$

```
[Out] -1/2*x^2+ln(cos(x))+x*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3801, 3556, 30}

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[x]^2,x]
```

```
[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\tan^2(x)\,dx &= x\tan(x)-\int x\,dx-\int \tan(x)\,dx\\ &= -\frac{x^2}{2}+\log(\cos(x))+x\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{x^2}{2} + \log(\cos(x)) + x\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[x]^2,x]
```

```
[Out] -1/2*x^2 + Log[Cos[x]] + x*Tan[x]
```

Maple [A]
time = 0.01, size = 20, normalized size = 1.33

method	result	size
norman	$x\tan\left(x\right)-\frac{x^{2}}{2}-\frac{\ln\left(1+\tan^{2}\left(x\right)\right)}{2}$	20
risch	$-\frac{x^{2}}{2}-2ix+\frac{2ix}{\mathrm{e}^{2ix}+1}+\ln\left(\mathrm{e}^{2ix}+1\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*tan(x)-1/2*x^2-1/2*ln(1+tan(x)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(13) = 26$.
time = 2.78, size = 107, normalized size = 7.13

$$-\frac{x^{2}\cos\left(2\,x\right)^{2}+x^{2}\sin\left(2\,x\right)^{2}+2\,x^{2}\cos\left(2\,x\right)+x^{2}-\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)\log\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)-4\,x\sin\left(2\,x\right)}{2\,\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A]
time = 0.79, size = 21, normalized size = 1.40

$$-\frac{1}{2}\,x^{2}+x\tan\left(x\right)+\frac{1}{2}\,\log\left(\frac{1}{\tan\left(x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A]
time = 0.06, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x\tan(x) - \frac{\log(\tan^2(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)**2,x)
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

Giac [A]
time = 0.60, size = 23, normalized size = 1.53

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\log\left(\frac{4}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="giac")
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.87

$$\ln(\cos(x)) + x\tan(x) - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x)
[Out] log(cos(x)) + x*tan(x) - x^2/2
```

3.131 $\int \cos(3x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

```
[Out] 1/2*cos(x)-1/10*cos(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {4369}

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\sin(2x)\,dx=\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
risch	$\frac{\cos(x)}{2}-\frac{\cos(5x)}{10}$	12
norman	$\frac{-\frac{4\left(\tan^2(x)\right)}{5}-\frac{4\left(\tan^2\left(\frac{3x}{2}\right)\right)}{5}+\frac{12\tan(x)\tan\left(\frac{3x}{2}\right)}{5}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2(x)\right)}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3*x)*sin(2*x),x,method=_RETURNVERBOSE)

[Out] 1/2*cos(x)-1/10*cos(5*x)

Maxima [A]
time = 2.09, size = 11, normalized size = 0.73

$$-\frac{1}{10}\cos\left(5\,x\right)+\frac{1}{2}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")

[Out] -1/10*cos(5*x) + 1/2*cos(x)

Fricas [A]
time = 0.47, size = 13, normalized size = 0.87

$$-\frac{8}{5}\cos\left(x\right)^5+2\cos\left(x\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")

[Out] -8/5*cos(x)^5 + 2*cos(x)^3

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.13, size = 26, normalized size = 1.73

$$\frac{3\sin\left(2x\right)\sin\left(3x\right)}{5}+\frac{2\cos\left(2x\right)\cos\left(3x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(3*x)*sin(2*x),x)

[Out] 3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5

Giac [A]
time = 0.58, size = 11, normalized size = 0.73

$$-\frac{1}{10}\cos\left(5\,x\right)+\frac{1}{2}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

Mupad [B]
time = 0.04, size = 13, normalized size = 0.87

$$2\cos\left(x\right)^3-\frac{8\cos\left(x\right)^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(2*x),x)
```

```
[Out] 2*cos(x)^3 - (8*cos(x)^5)/5
```

3.132 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)$$

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2648, 2715, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \cos^2(x)\sin^2(x)\,dx = -\frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\int \cos^2(x)\,dx$$
$$= \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x) + \frac{\int 1\,dx}{8}$$
$$= \frac{x}{8} + \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
[Out] x/8 - Sin[4*x]/32
```

Maple [A]
time = 0.05, size = 19, normalized size = 0.79

method	result	size
risch	$\frac{x}{8}-\frac{\sin(4x)}{32}$	11
default	$\frac{x}{8}+\frac{\cos(x)\sin(x)}{8}-\frac{(\cos^3(x))\sin(x)}{4}$	19
norman	$\frac{\frac{x}{8}+\frac{7\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{7\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A]
time = 1.19, size = 10, normalized size = 0.42

$$\frac{1}{8}\,x - \frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A]
time = 0.52, size = 19, normalized size = 0.79

$$-\frac{1}{8}\left(2\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A]
time = 0.01, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Giac [A]
time = 0.64, size = 10, normalized size = 0.42

$$\frac{1}{8}\,x-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*x)
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.75

$$\frac{\cos\left(x\right)\,\sin\left(x\right)^{3}}{4}-\frac{\cos\left(x\right)\,\sin\left(x\right)}{8}+\frac{x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
[Out] x/8 - (cos(x)*sin(x))/8 + (cos(x)*sin(x)^3)/4
```

3.133 $\int \csc^2(x)\sec^2(x)\,dx$

Optimal. Leaf size=7

$$-\cot(x)+\tan(x)$$

```
[Out] -cot(x)+tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2700, 14}

$$\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -Cot[x] + Tan[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(x)\sec^2(x)\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(x)\right)\\ &= -\cot(x)+\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -2*Cot[2*x]
```

Maple [A]
time = 0.03, size = 15, normalized size = 2.14

method	result	size
default	$\frac{1}{\cos(x)\sin(x)}-2\cot(x)$	15
risch	$-\frac{4i}{(e^{2ix}+1)(e^{2ix}-1)}$	22
norman	$\frac{\frac{1}{2}-3\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)\tan\left(\frac{x}{2}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2/sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/cos(x)/sin(x)-2*cot(x)
```

Maxima [A]
time = 2.46, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/tan(x) + tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.
time = 0.53, size = 18, normalized size = 2.57

$$-\frac{2\cos(x)^2-1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
```

```
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.
time = 0.01, size = 12, normalized size = 1.71

$$-\frac{2\cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2/sin(x)**2,x)
```

```
[Out] -2*cos(2*x)/sin(2*x)
```

Giac [A]
time = 0.76, size = 9, normalized size = 1.29

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(x) + tan(x)
```

Mupad [B]
time = 0.06, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*sin(x)^2),x)
```

```
[Out] -2*cot(2*x)
```

3.134 $\int d^x \sin(x)\,dx$

Optimal. Leaf size=32

$$-\frac{d^x\cos(x)}{1+\log^2(d)}+\frac{d^x\log(d)\sin(x)}{1+\log^2(d)}$$

```
[Out] -d^x*cos(x)/(1+ln(d)^2)+d^x*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4517}

$$\frac{d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{d^x\cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*Sin[x],x]
[Out] -((d^x*Cos[x])/(1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x\sin(x)\,dx=-\frac{d^x\cos(x)}{1+\log^2(d)}+\frac{d^x\log(d)\sin(x)}{1+\log^2(d)}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.69

$$\frac{d^x(-\cos(x)+\log(d)\sin(x))}{1+\log^2(d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*Sin[x],x]
[Out] (d^x*(-Cos[x] + Log[d]*Sin[x]))/(1 + Log[d]^2)
```

Maple [A]
time = 0.03, size = 33, normalized size = 1.03

method	result	size
risch	$-\frac{d^x\cos(x)}{1+\ln(d)^2}+\frac{d^x\ln(d)\sin(x)}{1+\ln(d)^2}$	33
norman	$\frac{\frac{e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}-\frac{e^{x\ln(d)}}{1+\ln(d)^2}+\frac{2\ln(d)e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -d^x*cos(x)/(1+ln(d)^2)+d^x*ln(d)*sin(x)/(1+ln(d)^2)
```

Maxima [A]
time = 2.20, size = 25, normalized size = 0.78

$$\frac{d^x\log(d)\sin(x)-d^x\cos(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="maxima")
```

```
[Out] (d^x*log(d)*sin(x) - d^x*cos(x))/(log(d)^2 + 1)
```

Fricas [A]
time = 0.53, size = 22, normalized size = 0.69

$$\frac{(\log(d)\sin(x)-\cos(x))d^x}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="fricas")
```

```
[Out] (log(d)*sin(x) - cos(x))*d^x/(log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.27, size = 104, normalized size = 3.25

$$\begin{cases} \frac{xe^{-ix}\sin(x)}{2}-\frac{ixe^{-ix}\cos(x)}{2}-\frac{e^{-ix}\cos(x)}{2} & \text{for}\ d=e^{-i} \\ \frac{xe^{ix}\sin(x)}{2}+\frac{ixe^{ix}\cos(x)}{2}-\frac{e^{ix}\cos(x)}{2} & \text{for}\ d=e^{i} \\ \frac{d^x\log(d)\sin(x)}{\log(d)^2+1}-\frac{d^x\cos(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*sin(x),x)
```

```
[Out] Piecewise((x*exp(-I*x)*sin(x)/2 - I*x*exp(-I*x)*cos(x)/2 - exp(-I*x)*cos(x)
/2, Eq(d, exp(-I))), (x*exp(I*x)*sin(x)/2 + I*x*exp(I*x)*cos(x)/2 - exp(I*x
)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*cos(
x)/(log(d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.98, size = 328, normalized size = 10.25

$$|d|^x \left(\frac{(\pi - \pi \text{sgn}(d) - 2)\cos\left(\frac{1}{2}\pi x \text{sgn}(d) - \frac{1}{2}\pi x + x\right)}{(\pi - \pi \text{sgn}(d) - 2)^2 + 4\log(|d|)^2} + \frac{2\log(|d|)\sin\left(\frac{1}{2}\pi x \text{sgn}(d) - \frac{1}{2}\pi x + x\right)}{(\pi - \pi \text{sgn}(d) - 2)^2 + 4\log(|d|)^2}\right) - |d|^x \left(\frac{(\pi - \pi \text{sgn}(d) + 2)\cos\left(\frac{1}{2}\pi x \text{sgn}(d) - \frac{1}{2}\pi x - x\right)}{(\pi - \pi \text{sgn}(d) + 2)^2 + 4\log(|d|)^2} + \frac{2\log(|d|)\sin\left(\frac{1}{2}\pi x \text{sgn}(d) - \frac{1}{2}\pi x - x\right)}{(\pi - \pi \text{sgn}(d) + 2)^2 + 4\log(|d|)^2}\right) - |d|^x \left(-\frac{i e^{\left(\frac{1}{2} i \pi x \text{sgn}(d) - \frac{1}{2} i \pi x + i x\right)}}{-2i\pi + 2i\pi \text{sgn}(d) + 4\log(|d|) + 4i} - \frac{i e^{\left(-\frac{1}{2} i \pi x \text{sgn}(d) + \frac{1}{2} i \pi x - i x\right)}}{2i\pi - 2i\pi \text{sgn}(d) + 4\log(|d|) - 4i}\right) - |d|^x \left(\frac{i e^{\left(\frac{1}{2} i \pi x \text{sgn}(d) - \frac{1}{2} i \pi x - i x\right)}}{-2i\pi + 2i\pi \text{sgn}(d) + 4\log(|d|) - 4i} + \frac{i e^{\left(-\frac{1}{2} i \pi x \text{sgn}(d) + \frac{1}{2} i \pi x + i x\right)}}{2i\pi - 2i\pi \text{sgn}(d) + 4\log(|d|) + 4i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="giac")
```

```
[Out] abs(d)^x*((pi - pi*sgn(d) - 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)/((pi - p
i*sgn(d) - 2)^2 + 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/
2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*((pi - p
i*sgn(d) + 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi - pi*sgn(d) + 2)^2 +
 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi
- pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*(-I*e^(1/2*I*pi*x*sgn(d)
- 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I) - I*e^(
-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(
d)) - 4*I)) - abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(-2*I*pi
 + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) + I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*p
i*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))
```

Mupad [B]
time = 0.02, size = 22, normalized size = 0.69

$$-\frac{d^x\left(\cos(x) - \ln(d)\sin(x)\right)}{\ln(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*sin(x),x)
```

```
[Out] -(d^x*(cos(x) - log(d)*sin(x)))/(log(d)^2 + 1)
```

3.135 $\int d^x \cos(x)\, dx$

Optimal. Leaf size=31

$$\frac{d^x \cos(x) \log(d)}{1+\log^2(d)}+\frac{d^x \sin(x)}{1+\log^2(d)}$$

```
[Out] d^x*cos(x)*ln(d)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(d)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4518}

$$\frac{d^x \sin(x)}{\log^2(d)+1}+\frac{d^x \log(d) \cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*Cos[x],x]
[Out] (d^x*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x \cos(x)\, dx=\frac{d^x \cos(x) \log(d)}{1+\log^2(d)}+\frac{d^x \sin(x)}{1+\log^2(d)}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.65

$$\frac{d^x (\cos(x) \log(d)+\sin(x))}{1+\log^2(d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*Cos[x],x]
[Out] (d^x*(Cos[x]*Log[d] + Sin[x]))/(1 + Log[d]^2)
```

Maple [A]
time = 0.03, size = 32, normalized size = 1.03

method	result	size
risch	$\frac{d^{x}\cos(x)\ln(d)}{1+\ln(d)^{2}}+\frac{d^{x}\sin(x)}{1+\ln(d)^{2}}$	32
norman	$\frac{\frac{\ln(d)e^{x\ln(d)}}{1+\ln(d)^{2}}+\frac{2\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^{2}}-\frac{\ln(d)e^{x\ln(d)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\ln(d)^{2}}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] d^x*cos(x)*ln(d)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(d)^2)
```

Maxima [A]
time = 1.35, size = 24, normalized size = 0.77

$$\frac{d^{x}\cos\left(x\right)\log\left(d\right)+d^{x}\sin\left(x\right)}{\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="maxima")
```

```
[Out] (d^x*cos(x)*log(d) + d^x*sin(x))/(log(d)^2 + 1)
```

Fricas [A]
time = 0.54, size = 20, normalized size = 0.65

$$\frac{\left(\cos\left(x\right)\log\left(d\right)+\sin\left(x\right)\right)d^{x}}{\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="fricas")
```

```
[Out] (cos(x)*log(d) + sin(x))*d^x/(log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.27, size = 104, normalized size = 3.35

$$\begin{cases} \frac{ixe^{-ix}\sin\left(x\right)}{2}+\frac{xe^{-ix}\cos\left(x\right)}{2}+\frac{e^{-ix}\sin\left(x\right)}{2} & \text{for}\ d=e^{-i} \\ -\frac{ixe^{ix}\sin\left(x\right)}{2}+\frac{xe^{ix}\cos\left(x\right)}{2}+\frac{e^{ix}\sin\left(x\right)}{2} & \text{for}\ d=e^{i} \\ \frac{d^{x}\log\left(d\right)\cos\left(x\right)}{\log\left(d\right)^{2}+1}+\frac{d^{x}\sin\left(x\right)}{\log\left(d\right)^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*cos(x),x)
```

```
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + exp(-I*x)*sin(x)
/2, Eq(d, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 + exp(I*
x)*sin(x)/2, Eq(d, exp(I))), (d**x*log(d)*cos(x)/(log(d)**2 + 1) + d**x*sin
(x)/(log(d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.47, size = 329, normalized size = 10.61

$$|d|^{x}\left(\frac{2\cos\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x+x\right)\log\left(|d|\right)}{\left(\pi-\pi\mathrm{sgn}\left(d\right)-2\right)^{2}+4\log\left(|d|\right)^{2}}-\frac{\left(\pi-\pi\mathrm{sgn}(d)-2\right)\sin\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x+x\right)}{\left(\pi-\pi\mathrm{sgn}\left(d\right)-2\right)^{2}+4\log\left(|d|\right)^{2}}\right)+|d|^{x}\left(\frac{2\cos\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x-x\right)\log\left(|d|\right)}{\left(\pi-\pi\mathrm{sgn}\left(d\right)+2\right)^{2}+4\log\left(|d|\right)^{2}}-\frac{\left(\pi-\pi\mathrm{sgn}(d)+2\right)\sin\left(\frac{1}{2}\pi x\mathrm{sgn}(d)-\frac{1}{2}\pi x-x\right)}{\left(\pi-\pi\mathrm{sgn}\left(d\right)+2\right)^{2}+4\log\left(|d|\right)^{2}}\right)+i|d|^{x}\left(\frac{ie^{\left(\frac{1}{2}i\pi x\mathrm{sgn}(d)-\frac{1}{2}i\pi x+ix\right)}}{-2i\pi+2i\pi\mathrm{sgn}\left(d\right)+4\log\left(|d|\right)+4i}-\frac{ie^{\left(-\frac{1}{2}i\pi x\mathrm{sgn}(d)+\frac{1}{2}i\pi x-ix\right)}}{2i\pi-2i\pi\mathrm{sgn}\left(d\right)+4\log\left(|d|\right)-4i}\right)+i|d|^{x}\left(\frac{ie^{\left(\frac{1}{2}i\pi x\mathrm{sgn}(d)-\frac{1}{2}i\pi x-ix\right)}}{-2i\pi+2i\pi\mathrm{sgn}\left(d\right)+4\log\left(|d|\right)-4i}-\frac{ie^{\left(-\frac{1}{2}i\pi x\mathrm{sgn}(d)+\frac{1}{2}i\pi x+ix\right)}}{2i\pi-2i\pi\mathrm{sgn}\left(d\right)+4\log\left(|d|\right)+4i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="giac")
```

```
[Out] abs(d)^x*(2*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)*log(abs(d))/((pi - pi*sgn(d
) - 2)^2 + 4*log(abs(d))^2) - (pi - pi*sgn(d) - 2)*sin(1/2*pi*x*sgn(d) - 1/
2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) + abs(d)^x*(2*cos(1
/2*pi*x*sgn(d) - 1/2*pi*x - x)*log(abs(d))/((pi - pi*sgn(d) + 2)^2 + 4*log(
abs(d))^2) - (pi - pi*sgn(d) + 2)*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi
- pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) + I*abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d)
 - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I) - I*e^
(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs
(d)) - 4*I)) + I*abs(d)^x*(I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(-2*I
*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) - I*e^(-1/2*I*pi*x*sgn(d) + 1/2*
I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))
```

Mupad [B]
time = 0.02, size = 20, normalized size = 0.65

$$\frac{d^x\,\left(\sin\left(x\right)+\ln\left(d\right)\,\cos\left(x\right)\right)}{{\ln\left(d\right)}^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*cos(x),x)
```

```
[Out] (d^x*(sin(x) + log(d)*cos(x)))/(log(d)^2 + 1)
```

3.136 $\int d^x x \sin(x) \, dx$

Optimal. Leaf size=84

$$\frac{2d^x \cos(x) \log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x \cos(x)}{1+\log^2(d)}+\frac{d^x \sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{d^x \log^2(d) \sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x \log(d) \sin(x)}{1+\log^2(d)}$$

```
[Out] 2*d^x*cos(x)*ln(d)/(1+ln(d)^2)^2-d^x*x*cos(x)/(1+ln(d)^2)+d^x*sin(x)/(1+ln(
d)^2)^2-d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4517, 4553, 4518}

$$\frac{x d^x \log(d) \sin(x)}{\log^2(d)+1}-\frac{d^x \log^2(d) \sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{x d^x \cos(x)}{\log^2(d)+1}+\frac{2d^x \log(d) \cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x*Cos[x])/(1 + Log[d]^2) + (d
^x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2 + (d^x
*x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int d^x x\sin(x)\,dx &= -\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}-\int\left(-\frac{d^x\cos(x)}{1+\log^2(d)}+\frac{d^x\log(d)\sin(x)}{1+\log^2(d)}\right)dx\\ &= -\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}+\frac{\int d^x\cos(x)\,dx}{1+\log^2(d)}-\frac{\log(d)\int d^x\sin(x)\,dx}{1+\log^2(d)}\\ &= \frac{2d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{d^x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.60

$$\frac{d^x\left(-\cos(x)\left(x-2\log(d)+x\log^2(d)\right)+\left(1+x\log(d)-\log^2(d)+x\log^3(d)\right)\sin(x)\right)}{\left(1+\log^2(d)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x - 2*Log[d] + x*Log[d]^2)) + (1 + x*Log[d] - Log[d]^2 + x*
Log[d]^3)*Sin[x]))/(1 + Log[d]^2)^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 58, normalized size = 0.69

method	result	size
risch	$-\frac{i(-1+x\ln(d)+ix)d^x e^{ix}}{2(\ln(d)+i)^2}+\frac{i(-1+x\ln(d)-ix)d^x e^{-ix}}{2(\ln(d)-i)^2}$	58
norman	$\frac{\frac{x\,e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}+\frac{2\ln(d)e^{x\ln(d)}}{\left(1+\ln(d)^2\right)^2}-\frac{x\,e^{x\ln(d)}}{1+\ln(d)^2}-\frac{2\ln(d)e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^2\right)^2}-\frac{2\left(\ln(d)^2-1\right)e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\left(1+\ln(d)^2\right)^2}+\frac{2\ln(d)x\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(-1+x*ln(d)+I*x)*d^x/(ln(d)+I)^2*exp(I*x)+1/2*I*(-1+x*ln(d)-I*x)*d^x
/(ln(d)-I)^2*exp(-I*x)
```

Maxima [A]
time = 2.48, size = 60, normalized size = 0.71

$$-\frac{\left(\left(\log(d)^2+1\right)x-2\log(d)\right)d^x\cos(x)-\left(\left(\log(d)^3+\log(d)\right)x-\log(d)^2+1\right)d^x\sin(x)}{\log(d)^4+2\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^2 + 1)*x - 2*log(d))*d^x*cos(x) - ((log(d)^3 + log(d))*x - log(d
)^2 + 1)*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

Fricas [A]
time = 0.56, size = 60, normalized size = 0.71

$$-\frac{\left(x\cos\left(x\right)\log\left(d\right)^{2}+x\cos\left(x\right)-2\,\cos\left(x\right)\log\left(d\right)-\left(x\log\left(d\right)^{3}+x\log\left(d\right)-\log\left(d\right)^{2}+1\right)\sin\left(x\right)\right)d^{x}}{\log\left(d\right)^{4}+2\,\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="fricas")
```

```
[Out] -(x*cos(x)*log(d)^2 + x*cos(x) - 2*cos(x)*log(d) - (x*log(d)^3 + x*log(d) -
 log(d)^2 + 1)*sin(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 308, normalized size = 3.67

$$\begin{cases} \frac{x^{2}e^{-ix}\sin\left(x\right)}{4}-\frac{ix^{2}e^{-ix}\cos\left(x\right)}{4}+\frac{ixe^{-ix}\sin\left(x\right)}{4}-\frac{xe^{-ix}\cos\left(x\right)}{4}+\frac{ie^{-ix}\cos\left(x\right)}{4} & \text{for}\ d=e^{-i} \\ \frac{x^{2}e^{ix}\sin\left(x\right)}{4}+\frac{ix^{2}e^{ix}\cos\left(x\right)}{4}-\frac{ixe^{ix}\sin\left(x\right)}{4}-\frac{xe^{ix}\cos\left(x\right)}{4}-\frac{ie^{ix}\cos\left(x\right)}{4} & \text{for}\ d=e^{i} \\ \frac{d^{x}x\log\left(d\right)^{3}\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}-\frac{d^{x}x\log\left(d\right)^{2}\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}x\log\left(d\right)\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}-\frac{d^{x}x\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}-\frac{d^{x}\log\left(d\right)^{2}\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{2d^{x}\log\left(d\right)\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x*sin(x),x)
```

```
[Out] Piecewise((x**2*exp(-I*x)*sin(x)/4 - I*x**2*exp(-I*x)*cos(x)/4 + I*x*exp(-I
*x)*sin(x)/4 - x*exp(-I*x)*cos(x)/4 + I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I)))
, (x**2*exp(I*x)*sin(x)/4 + I*x**2*exp(I*x)*cos(x)/4 - I*x*exp(I*x)*sin(x)/
4 - x*exp(I*x)*cos(x)/4 - I*exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(
d)**3*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*log(d)**2*cos(x)/(log(d
)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1
) - d**x*x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*sin(x)/(lo
g(d)**4 + 2*log(d)**2 + 1) + 2*d**x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2
+ 1) + d**x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.06, size = 1156, normalized size = 13.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="giac")
```

```
[Out] 1/2*(((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(pi*x
*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*
sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^
2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))*(x*log(abs(
d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2
 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos(1/2*p
i*x*sgn(d) - 1/2*pi*x + x) + 2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*
sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*lo
g(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)
) + 2*log(abs(d)))^2) + (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi
*sgn(d) - 2)*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d)
)^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*lo
g(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(((2*pi
- pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(pi*x*sgn(d) - pi
*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^
2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2) + 4*(pi*l
og(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))*(x*log(abs(d)) - 1)/((2
*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log
(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) -
 1/2*pi*x - x) - 2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*
log(abs(d)) - 2*log(abs(d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2
- 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(ab
s(d)))^2) - (2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)
*(x*log(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*s
gn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2
))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*abs(d)^x - 1/4*abs(d)^x*((pi*x*sgn(
d) - pi*x - 2*I*x*log(abs(d)) + 2*x + 2*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*
x + I*x)/(2*pi + pi^2*sgn(d) + 2*I*pi*log(abs(d))*sgn(d) - pi^2 - 2*I*pi*lo
g(abs(d)) + 2*log(abs(d))^2 - 2*pi*sgn(d) + 4*I*log(abs(d)) - 2) - (pi*x*sg
n(d) - pi*x + 2*I*x*log(abs(d)) + 2*x - 2*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*
pi*x - I*x)/(2*pi + pi^2*sgn(d) - 2*I*pi*log(abs(d))*sgn(d) - pi^2 + 2*I*pi
*log(abs(d)) + 2*log(abs(d))^2 - 2*pi*sgn(d) - 4*I*log(abs(d)) - 2)) - 1/4*
abs(d)^x*((pi*x*sgn(d) - pi*x - 2*I*x*log(abs(d)) - 2*x + 2*I)*e^(1/2*I*pi*
x*sgn(d) - 1/2*I*pi*x - I*x)/(2*pi - pi^2*sgn(d) - 2*I*pi*log(abs(d))*sgn(d
) + pi^2 + 2*I*pi*log(abs(d)) - 2*log(abs(d))^2 - 2*pi*sgn(d) + 4*I*log(abs
(d)) + 2) - (pi*x*sgn(d) - pi*x + 2*I*x*log(abs(d)) - 2*x - 2*I)*e^(-1/2*I*
pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(2*pi - pi^2*sgn(d) + 2*I*pi*log(abs(d))*sg
n(d) + pi^2 - 2*I*pi*log(abs(d)) - 2*log(abs(d))^2 - 2*pi*sgn(d) - 4*I*log(
abs(d)) + 2))
```

Mupad [B]
time = 0.28, size = 57, normalized size = 0.68

$$\frac{d^x\,\left(\sin\left(x\right)+2\,\ln\left(d\right)\,\cos\left(x\right)-{\ln\left(d\right)}^2\,\sin\left(x\right)-x\,\cos\left(x\right)+x\,\ln\left(d\right)\,\sin\left(x\right)-x\,{\ln\left(d\right)}^2\,\cos\left(x\right)+x\,{\ln\left(d\right)}^3\,\sin\left(x\right)\right)}{{\left({\ln\left(d\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*sin(x),x)
```

```
[Out] (d^x*(sin(x) + 2*log(d)*cos(x) - log(d)^2*sin(x) - x*cos(x) + x*log(d)*sin(
x) - x*log(d)^2*cos(x) + x*log(d)^3*sin(x)))/(log(d)^2 + 1)^2
```

3.137 $\int d^x x \cos(x)\,dx$

Optimal. Leaf size=83

$$\frac{d^x\cos(x)}{\left(1+\log^2(d)\right)^2}-\frac{d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}-\frac{2d^x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\sin(x)}{1+\log^2(d)}$$

```
[Out] d^x*cos(x)/(1+ln(d)^2)^2-d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x*cos(x)*ln(d
)/(1+ln(d)^2)-2*d^x*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x*sin(x)/(1+ln(d)^2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4518, 4554, 4517}

$$\frac{xd^x\sin(x)}{\log^2(d)+1}-\frac{2d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{xd^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x*Cos[x],x]
```

```
[Out] (d^x*Cos[x])/(1 + Log[d]^2)^2 - (d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d
^x*x*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^2
 + (d^x*x*Sin[x])/(1 + Log[d]^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int d^x x\cos(x)\,dx &= \frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x x\sin(x)}{1+\log^2(d)}-\int\left(\frac{d^x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x\sin(x)}{1+\log^2(d)}\right)dx\\ &= \frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x x\sin(x)}{1+\log^2(d)}-\frac{\int d^x\sin(x)\,dx}{1+\log^2(d)}-\frac{\log(d)\int d^x\cos(x)\,dx}{1+\log^2(d)}\\ &= \frac{d^x\cos(x)}{\left(1+\log^2(d)\right)^2}-\frac{d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}-\frac{2d^x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\sin(x)}{1+\log^2(d)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.59

$$\frac{d^x\left(\cos(x)\left(1+x\log(d)-\log^2(d)+x\log^3(d)\right)+\left(x-2\log(d)+x\log^2(d)\right)\sin(x)\right)}{\left(1+\log^2(d)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(1 + x*Log[d] - Log[d]^2 + x*Log[d]^3) + (x - 2*Log[d] + x*Log
[d]^2)*Sin[x]))/(1 + Log[d]^2)^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 56, normalized size = 0.67

method	result
risch	$\frac{(-1+x\ln(d)+ix)d^x e^{ix}}{2(\ln(d)+i)^2}+\frac{(-1+x\ln(d)-ix)d^x e^{-ix}}{2(\ln(d)-i)^2}$
norman	$\frac{\frac{\ln(d)x\,e^{x\ln(d)}}{1+\ln(d)^2}+\frac{\left(\ln(d)^2-1\right)e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^2\right)^2}-\frac{\left(\ln(d)^2-1\right)e^{x\ln(d)}}{\left(1+\ln(d)^2\right)^2}-\frac{4\ln(d)e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\left(1+\ln(d)^2\right)^2}+\frac{2x\,e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}-\frac{\ln(d)x\,e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1+x*ln(d)+I*x)*d^x/(ln(d)+I)^2*exp(I*x)+1/2*(-1+x*ln(d)-I*x)*d^x/(ln(
d)-I)^2*exp(-I*x)
```

Maxima [A]
time = 1.47, size = 58, normalized size = 0.70

$$\frac{\left(\left(\log\left(d\right)^3+\log\left(d\right)\right)x-\log\left(d\right)^2+1\right)d^x\cos\left(x\right)+\left(\left(\log\left(d\right)^2+1\right)x-2\,\log\left(d\right)\right)d^x\sin\left(x\right)}{\log\left(d\right)^4+2\,\log\left(d\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^3 + log(d))*x - log(d)^2 + 1)*d^x*cos(x) + ((log(d)^2 + 1)*x - 2*
log(d))*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

Fricas [A]
time = 0.59, size = 58, normalized size = 0.70

$$\frac{\left(x\cos\left(x\right)\log\left(d\right)^{3}+x\cos\left(x\right)\log\left(d\right)-\cos\left(x\right)\log\left(d\right)^{2}+\left(x\log\left(d\right)^{2}+x-2\,\log\left(d\right)\right)\sin\left(x\right)+\cos\left(x\right)\right)d^{x}}{\log\left(d\right)^{4}+2\,\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="fricas")
```

```
[Out] (x*cos(x)*log(d)^3 + x*cos(x)*log(d) - cos(x)*log(d)^2 + (x*log(d)^2 + x -
2*log(d))*sin(x) + cos(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 308, normalized size = 3.71

$$\begin{cases} \frac{ix^{2}e^{-ix}\sin\left(x\right)}{4}+\frac{x^{2}e^{-ix}\cos\left(x\right)}{4}+\frac{xe^{-ix}\sin\left(x\right)}{4}+\frac{ixe^{-ix}\cos\left(x\right)}{4}-\frac{ie^{-ix}\sin\left(x\right)}{4} & \text{for}\: d=e^{-i} \\ -\frac{ix^{2}e^{ix}\sin\left(x\right)}{4}+\frac{x^{2}e^{ix}\cos\left(x\right)}{4}+\frac{xe^{ix}\sin\left(x\right)}{4}-\frac{ixe^{ix}\cos\left(x\right)}{4}+\frac{ie^{ix}\sin\left(x\right)}{4} & \text{for}\: d=e^{i} \\ \frac{d^{x}x\log\left(d\right)^{3}\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}x\log\left(d\right)^{2}\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}x\log\left(d\right)\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}x\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}-\frac{d^{x}\log\left(d\right)^{2}\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}-\frac{2d^{x}\log\left(d\right)\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x*cos(x),x)
```

```
[Out] Piecewise((I*x**2*exp(-I*x)*sin(x)/4 + x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x
)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 - I*exp(-I*x)*sin(x)/4, Eq(d, exp(-I)))
, (-I*x**2*exp(I*x)*sin(x)/4 + x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/4
 - I*x*exp(I*x)*cos(x)/4 + I*exp(I*x)*sin(x)/4, Eq(d, exp(I))), (d**x*x*log
(d)**3*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)**2*sin(x)/(log(
d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 +
1) + d**x*x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*cos(x)/(l
og(d)**4 + 2*log(d)**2 + 1) - 2*d**x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2
 + 1) + d**x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.39, size = 1155, normalized size = 13.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="giac")
```

[Out] 1/2*(2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2) + (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x) - ((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(pi*x*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2) - (2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(x*log(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(pi*x*sgn(d) - pi*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2) + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))*(x*log(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*abs(d)^x - 1/4*I*abs(d)^x*((pi*x*sgn(d) - pi*x - 2*I*x*log(abs(d)) + 2*x + 2*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/(2*pi + pi^2*sgn(d) + 2*I*pi*log(abs(d))*sgn(d) - pi^2 - 2*I*pi*log(abs(d)) + 2*log(abs(d))^2 - 2*pi*sgn(d) + 4*I*log(abs(d)) - 2) + (pi*x*sgn(d) - pi*x + 2*I*x*log(abs(d)) + 2*x - 2*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*pi + pi^2*sgn(d) - 2*I*pi*log(abs(d))*sgn(d) - pi^2 + 2*I*pi*log(abs(d)) + 2*log(abs(d))^2 - 2*pi*sgn(d) - 4*I*log(abs(d)) - 2)) + 1/4*I*abs(d)^x*((pi*x*sgn(d) - pi*x - 2*I*x*log(abs(d)) - 2*x + 2*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(2*pi - pi^2*sgn(d) - 2*I*pi*log(abs(d))*sgn(d) + pi^2 + 2*I*pi*log(abs(d)) - 2*log(abs(d))^2 - 2*pi*sgn(d) + 4*I*log(abs(d)) + 2) + (pi*x*sgn(d) - pi*x + 2*I*x*log(abs(d)) - 2*x - 2*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(2*pi - pi^2*sgn(d) + 2*I*pi*log(abs(d))*sgn(d) + pi^2 - 2*I*pi*log(abs(d)) - 2*log(abs(d))^2 - 2*pi*sgn(d) - 4*I*log(abs(d)) + 2))

Mupad [B]
time = 0.21, size = 55, normalized size = 0.66

$$\frac{d^x\,\left(\cos\left(x\right)-2\,\ln\left(d\right)\,\sin\left(x\right)-{\ln\left(d\right)}^2\,\cos\left(x\right)+x\,\sin\left(x\right)+x\,\ln\left(d\right)\,\cos\left(x\right)+x\,{\ln\left(d\right)}^3\,\cos\left(x\right)+x\,{\ln\left(d\right)}^2\,\sin\left(x\right)\right)}{{\left({\ln\left(d\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*cos(x),x)
```

```
[Out] (d^x*(cos(x) - 2*log(d)*sin(x) - log(d)^2*cos(x) + x*sin(x) + x*log(d)*cos(
x) + x*log(d)^3*cos(x) + x*log(d)^2*sin(x)))/(log(d)^2 + 1)^2
```

3.138 $\int d^x x^2 \sin(x)\, dx$

Optimal. Leaf size=162

$$\frac{2d^x\cos(x)}{\left(1+\log^2(d)\right)^3}-\frac{6d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3}+\frac{4d^xx\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^xx^2\cos(x)}{1+\log^2(d)}-\frac{6d^x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^3}+\frac{2d^x\log^3(d)\sin(x)}{\left(1+\log^2(d)\right)^3}+$$

```
[Out] 2*d^x*cos(x)/(1+ln(d)^2)^3-6*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+4*d^x*x*cos(x
)*ln(d)/(1+ln(d)^2)^2-d^x*x^2*cos(x)/(1+ln(d)^2)-6*d^x*ln(d)*sin(x)/(1+ln(d
)^2)^3+2*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^3+2*d^x*x*sin(x)/(1+ln(d)^2)^2-2*d^
x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x^2*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4517, 4553, 14, 4518, 4554}

$$\frac{x^2d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2d^x\cos(x)}{\log^2(d)+1}-\frac{2xd^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x\log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{4xd^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x\cos(x)}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^2*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^3
+ (4*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^2*Cos[x])/(1 + Log[d]^2
) - (6*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (2*d^x*Log[d]^3*Sin[x])/(1 + L
og[d]^2)^3 + (2*d^x*x*Sin[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Log[d]^2*Sin[x])/
(1 + Log[d]^2)^2 + (d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int d^x x^2 \sin(x)\,dx &= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-2\int x\left(-\frac{d^x\cos(x)}{1+\log^2(d)}+\frac{d^x\log(d)\sin(x)}{1+\log^2(d)}\right)dx\\
&= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-2\int\left(-\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}\right)dx\\
&= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}+\frac{2\int d^x x\cos(x)\,dx}{1+\log^2(d)}-\frac{(2\log(d))\int d^x x\sin(x)\,dx}{1+\log^2(d)}\\
&= \frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{2d^x x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^x x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\log(d)}{1+\log^2}\\
&= \frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{2d^x x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^x x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\log(d)}{1+\log^2}\\
&= \frac{2d^x\cos(x)}{\left(1+\log^2(d)\right)^3}-\frac{2d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3}+\frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}-\frac{2d^x\log(d)\sin}{(1+\log^2(d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.58

$$\frac{d^x\left(-\cos(x)\left(-2+x^2-4x\log(d)+2(3+x^2)\log^2(d)-4x\log^3(d)+x^2\log^4(d)\right)+\left(2x+(-6+x^2)\log(d)+2(1+x^2)\log^3(d)-2x\log^4(d)+x^2\log^5(d)\right)\sin(x)\right)}{\left(1+\log^2(d)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^2*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[d]^3
 + x^2*Log[d]^4)) + (2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*L
og[d]^4 + x^2*Log[d]^5)*Sin[x]))/(1 + Log[d]^2)^3
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 102, normalized size = 0.63

method	result
risch	$-\frac{i\left(2+\ln(d)^{2}x^{2}+2i\ln(d)x^{2}-x^{2}-2x\ln(d)-2ix\right)d^{x}\mathrm{e}^{ix}}{2(\ln(d)+i)^{3}}+\frac{i\left(2-2x\ln(d)+2ix+\ln(d)^{2}x^{2}-2i\ln(d)x^{2}-x^{2}\right)d^{x}\mathrm{e}^{-ix}}{2(\ln(d)-i)^{3}}$
norman	$\frac{\frac{x^{2}\mathrm{e}^{x\ln(d)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\ln(d)^{2}}-\frac{x^{2}\mathrm{e}^{x\ln(d)}}{1+\ln(d)^{2}}-\frac{2\left(3\ln(d)^{2}-1\right)\mathrm{e}^{x\ln(d)}}{\left(1+\ln(d)^{2}\right)^{3}}+\frac{4\ln(d)x\,\mathrm{e}^{x\ln(d)}}{\left(1+\ln(d)^{2}\right)^{2}}+\frac{2\left(3\ln(d)^{2}-1\right)\mathrm{e}^{x\ln(d)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^{2}\right)^{3}}-\frac{4\ln(d)x\,\mathrm{e}^{x\ln(d)}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(1+\ln(d)^{2}\right)^{2}}+}{1+\tan^{2}\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(2+ln(d)^2*x^2+2*I*ln(d)*x^2-x^2-2*x*ln(d)-2*I*x)*d^x/(ln(d)+I)^3*ex
p(I*x)+1/2*I*(2-2*x*ln(d)+2*I*x+ln(d)^2*x^2-2*I*ln(d)*x^2-x^2)*d^x/(ln(d)-I
)^3*exp(-I*x)
```

Maxima [A]
time = 0.95, size = 107, normalized size = 0.66

$$-\frac{\left(\left(\log\left(d\right)^{4}+2\,\log\left(d\right)^{2}+1\right)x^{2}-4\,\left(\log\left(d\right)^{3}+\log\left(d\right)\right)x+6\,\log\left(d\right)^{2}-2\right)d^{x}\cos\left(x\right)-\left(\left(\log\left(d\right)^{5}+2\,\log\left(d\right)^{3}+\log\left(d\right)\right)x^{2}+2\,\log\left(d\right)^{3}-2\,\left(\log\left(d\right)^{4}-1\right)x-6\,\log\left(d\right)\right)d^{x}\sin\left(x\right)}{\log\left(d\right)^{6}+3\,\log\left(d\right)^{4}+3\,\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log(d))*x + 6*log(d)^2 -
 2)*d^x*cos(x) - ((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(lo
g(d)^4 - 1)*x - 6*log(d))*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 +
 1)
```

Fricas [A]
time = 0.48, size = 115, normalized size = 0.71

$$-\frac{\left(x^{2}\cos\left(x\right)\log\left(d\right)^{4}-4\,x\cos\left(x\right)\log\left(d\right)^{3}+2\,\left(x^{2}+3\right)\cos\left(x\right)\log\left(d\right)^{2}-4\,x\cos\left(x\right)\log\left(d\right)+\left(x^{2}-2\right)\cos\left(x\right)-\left(x^{2}\log\left(d\right)^{5}-2\,x\log\left(d\right)^{4}+2\,\left(x^{2}+1\right)\log\left(d\right)^{3}+\left(x^{2}-6\right)\log\left(d\right)+2\,x\right)\sin\left(x\right)\right)d^{x}}{\log\left(d\right)^{6}+3\,\log\left(d\right)^{4}+3\,\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2*cos(x)*log(d)^4 - 4*x*cos(x)*log(d)^3 + 2*(x^2 + 3)*cos(x)*log(d)^2 -
 4*x*cos(x)*log(d) + (x^2 - 2)*cos(x) - (x^2*log(d)^5 - 2*x*log(d)^4 + 2*(x
^2 + 1)*log(d)^3 + (x^2 - 6)*log(d) + 2*x)*sin(x))*d^x/(log(d)^6 + 3*log(d)
^4 + 3*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 665, normalized size = 4.10

$$\begin{cases} \frac{x^{3}e^{-ix}\sin(x)}{6} - \frac{ix^{3}e^{-ix}\cos(x)}{6} + \frac{ix^{2}e^{-ix}\sin(x)}{4} - \frac{x^{2}e^{-ix}\cos(x)}{4} + \frac{xe^{-ix}\sin(x)}{4} + \frac{ixe^{-ix}\cos(x)}{4} + \frac{e^{-ix}\cos(x)}{4} & \text{for}\: d = e^{-i} \\ \frac{x^{3}e^{ix}\sin(x)}{6} + \frac{ix^{3}e^{ix}\cos(x)}{6} - \frac{ix^{2}e^{ix}\sin(x)}{4} - \frac{x^{2}e^{ix}\cos(x)}{4} + \frac{xe^{ix}\sin(x)}{4} - \frac{ixe^{ix}\cos(x)}{4} + \frac{e^{ix}\cos(x)}{4} & \text{for}\: d = e^{i} \\ \frac{d^{x}x^{2}\log(d)^{5}\sin(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} - \frac{d^{x}x^{2}\log(d)^{4}\cos(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} + \frac{2d^{x}x^{2}\log(d)^{3}\sin(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} - \frac{2d^{x}x^{2}\log(d)^{2}\cos(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} + \frac{d^{x}x^{2}\log(d)\sin(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} - \frac{d^{x}x^{2}\cos(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} - \frac{2d^{x}x\log(d)^{4}\sin(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} + \frac{4d^{x}x\log(d)^{3}\cos(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} + \frac{4d^{x}x\log(d)\cos(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} + \frac{2d^{x}x\sin(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} + \frac{2d^{x}\log(d)^{3}\sin(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} - \frac{6d^{x}\log(d)^{2}\cos(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} - \frac{6d^{x}\log(d)\sin(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} + \frac{2d^{x}\cos(x)}{\log(d)^{6}+3\log(d)^{4}+3\log(d)^{2}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**2*sin(x),x)
```

```
[Out] Piecewise((x**3*exp(-I*x)*sin(x)/6 - I*x**3*exp(-I*x)*cos(x)/6 + I*x**2*exp
(-I*x)*sin(x)/4 - x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*exp(
-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))), (x**3*exp(I*x)*sin(x)
/6 + I*x**3*exp(I*x)*cos(x)/6 - I*x**2*exp(I*x)*sin(x)/4 - x**2*exp(I*x)*co
s(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, E
q(d, exp(I))), (d**x*x**2*log(d)**5*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log
(d)**2 + 1) - d**x*x**2*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1) + 2*d**x*x**2*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1) + d**x*x**2*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2
+ 1) - d**x*x**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d**
x*x*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x
*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*x*lo
g(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*sin(x)/(
log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*sin(x)/(log(d
)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)**2*cos(x)/(log(d)**6
+ 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*sin(x)/(log(d)**6 + 3*log(
d)**4 + 3*log(d)**2 + 1) + 2*d**x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d
)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.92, size = 2631, normalized size = 16.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi^2
*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 -
 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn
(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2
- 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*
log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))
^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d))
 - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)
```

```
) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(ab
s(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 +
(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*lo
g(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d
) - 1/2*pi*x + x) - (2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi
^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*s
gn(d) - 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(
d)) - pi*x*sgn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*s
gn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^
2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) +
2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)
))^2) + (pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
 + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(
d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^
3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sg
n(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d
))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(
1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(((3*pi - pi^3*sgn(d) + 3*p
i*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2
 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*l
og(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((
3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2
- 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*l
og(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))
*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(
d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)*(3*
pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(a
bs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^
2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(
d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) - (2*(3*pi - p
i^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2
*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi*x^2*log(abs(d))*s
gn(d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)/
((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^
2 - 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2
*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d
))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x^2*sgn(d) - pi^2*
x^2 + 2*x^2*log(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(ab
s(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^
3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi
^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*
```

```
sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d)
)*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) -
 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))
*abs(d)^x - 2*abs(d)^x*((-I*pi^2*x^2*sgn(d) + 2*pi*x^2*log(abs(d))*sgn(d) +
 I*pi^2*x^2 - 2*pi*x^2*log(abs(d)) - 2*I*x^2*log(abs(d))^2 + 2*I*pi*x^2*sgn
(d) - 2*I*pi*x^2 + 4*x^2*log(abs(d)) - 2*pi*x*sgn(d) + 2*pi*x + 2*I*x^2 + 4
*I*x*log(abs(d)) - 4*x - 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/(24*
I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(abs(d))^2
*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 + 16*log(a
bs(d))^3 + 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2 + 48*pi*
log(abs(d)) + 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d)) - 16*I)
+ (-I*pi^2*x^2*sgn(d) - 2*pi*x^2*log(abs(d))*sgn(d) + I*pi^2*x^2 + 2*pi*x^2
*log(abs(d)) - 2*I*x^2*log(abs(d))^2 + 2*I*pi*x...
```

Mupad [B]

time = 0.39, size = 133, normalized size = 0.82

$$\frac{d^x\,\left(2\,\cos\left(x\right)-x^2\,\cos\left(x\right)+2\,x\,\sin\left(x\right)\right)+d^x\,{\ln\left(d\right)}^3\,\left(2\,\sin\left(x\right)+2\,x^2\,\sin\left(x\right)+4\,x\,\cos\left(x\right)\right)-d^x\,{\ln\left(d\right)}^2\,\left(6\,\cos\left(x\right)+2\,x^2\,\cos\left(x\right)\right)+d^x\,\ln\left(d\right)\,\left(x^2\,\sin\left(x\right)-6\,\sin\left(x\right)+4\,x\,\cos\left(x\right)\right)-d^x\,{\ln\left(d\right)}^4\,\left(x^2\,\cos\left(x\right)+2\,x\,\sin\left(x\right)\right)+d^x\,x^2\,{\ln\left(d\right)}^5\,\sin\left(x\right)}{{\left({\ln\left(d\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*sin(x),x)
```

```
[Out] (d^x*(2*cos(x) - x^2*cos(x) + 2*x*sin(x)) + d^x*log(d)^3*(2*sin(x) + 2*x^2*
sin(x) + 4*x*cos(x)) - d^x*log(d)^2*(6*cos(x) + 2*x^2*cos(x)) + d^x*log(d)*
(x^2*sin(x) - 6*sin(x) + 4*x*cos(x)) - d^x*log(d)^4*(x^2*cos(x) + 2*x*sin(x
)) + d^x*x^2*log(d)^5*sin(x))/(log(d)^2 + 1)^3
```

3.139 $\int d^x x^2 \cos(x)\, dx$

Optimal. Leaf size=161

$$-\frac{6d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^3}+\frac{2d^x\cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3}+\frac{2d^xx\cos(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^xx\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2}+\frac{d^xx^2\cos(x)\log(d)}{1+\log^2(d)}-\frac{2d^x\sin(}{(1+\log^2(}$$

```
[Out] -6*d^x*cos(x)*ln(d)/(1+ln(d)^2)^3+2*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^3+2*d^x*
x*cos(x)/(1+ln(d)^2)^2-2*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^2+d^x*x^2*cos(x)*
ln(d)/(1+ln(d)^2)-2*d^x*sin(x)/(1+ln(d)^2)^3+6*d^x*ln(d)^2*sin(x)/(1+ln(d)^
2)^3-4*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^2+d^x*x^2*sin(x)/(1+ln(d)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4518, 4554, 14, 4517, 4553}

$$\frac{x^2d^x\sin(x)}{\log^2(d)+1}+\frac{x^2d^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{4xd^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6d^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2d^x\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2xd^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{6d^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{2d^x\log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^2*Cos[x],x]
```

```
[Out] (-6*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (2*d^x*Cos[x]*Log[d]^3)/(1 + Log[
d]^2)^3 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Cos[x]*Log[d]^2)/(1
+ Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Sin[x])/(1
+ Log[d]^2)^3 + (6*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (4*d^x*x*Log[d]*
Sin[x])/(1 + Log[d]^2)^2 + (d^x*x^2*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x^2 \cos(x)\,dx &= \frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)} - 2\int x\left(\frac{d^x \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x \sin(x)}{1+\log^2(d)}\right)dx \\
&= \frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)} - 2\int \left(\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x\sin(x)}{1+\log^2(d)}\right)dx \\
&= \frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)} - \frac{2\int d^x x\sin(x)\,dx}{1+\log^2(d)} - \frac{(2\log(d))\int d^x x\cos(x)\,dx}{1+\log^2(d)} \\
&= \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} - \frac{4d^x x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2}{1+}\\
&= \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} - \frac{4d^x x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2}{1+}\\
&= -\frac{2d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{2d^x\cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x}{}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.58

$$\frac{d^x\left(\cos(x)\left(2x+(-6+x^2)\log(d)+2(1+x^2)\log^3(d)-2x\log^4(d)+x^2\log^5(d)\right)+\left(-2+x^2-4x\log(d)+2(3+x^2)\log^2(d)-4x\log^3(d)+x^2\log^4(d)\right)\sin(x)\right)}{\left(1+\log^2(d)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^2*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*Log[d]^4
 + x^2*Log[d]^5) + (-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[
d]^3 + x^2*Log[d]^4)*Sin[x]))/(1 + Log[d]^2)^3
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 100, normalized size = 0.62

method	result
risch	$\frac{\left(2+\ln(d)^2x^2+2i\ln(d)x^2-x^2-2x\ln(d)-2ix\right)d^x \mathrm{e}^{ix}}{2(\ln(d)+i)^3}+\frac{\left(2-2x\ln(d)+2ix+\ln(d)^2x^2-2i\ln(d)x^2-x^2\right)d^x \mathrm{e}^{-ix}}{2(\ln(d)-i)^3}$
norman	$\frac{\frac{\ln(d)x^2\mathrm{e}^{x\ln(d)}}{1+\ln(d)^2}+\frac{2x^2\mathrm{e}^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}-\frac{2\left(\ln(d)^2-1\right)x\,\mathrm{e}^{x\ln(d)}}{\left(1+\ln(d)^2\right)^2}+\frac{4\left(3\ln(d)^2-1\right)\mathrm{e}^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\left(1+\ln(d)^2\right)^3}+\frac{2\ln(d)\left(\ln(d)^2-3\right)\mathrm{e}^{x\ln(d)}}{\left(1+\ln(d)^2\right)^3}-\frac{8\ln(d)x\,\mathrm{e}^{x\ln(d)}\tan\left(\cdots\right.}{\left(1+\ln(d)^2\right)^2}}{1+\tan^2\left(\frac{x}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2+ln(d)^2*x^2+2*I*ln(d)*x^2-x^2-2*x*ln(d)-2*I*x)*d^x/(ln(d)+I)^3*exp(I
*x)+1/2*(2-2*x*ln(d)+2*I*x+ln(d)^2*x^2-2*I*ln(d)*x^2-x^2)*d^x/(ln(d)-I)^3*e
xp(-I*x)
```

Maxima [A]
time = 2.08, size = 105, normalized size = 0.65

$$\frac{\left(\left(\log(d)^5+2\log(d)^3+\log(d)\right)x^2+2\log(d)^3-2\left(\log(d)^4-1\right)x-6\log(d)\right)d^x\cos(x)+\left(\left(\log(d)^4+2\log(d)^2+1\right)x^2-4\left(\log(d)^3+\log(d)\right)x+6\log(d)^2-2\right)d^x\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4 - 1)*x -
6*log(d))*d^x*cos(x) + ((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log
(d))*x + 6*log(d)^2 - 2)*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 +
1)
```

Fricas [A]
time = 0.57, size = 111, normalized size = 0.69

$$\frac{\left(x^2\cos(x)\log(d)^5-2x\cos(x)\log(d)^4+2\left(x^2+1\right)\cos(x)\log(d)^3+\left(x^2-6\right)\cos(x)\log(d)+2x\cos(x)+\left(x^2\log(d)^4-4x\log(d)^3+2\left(x^2+3\right)\log(d)^2+x^2-4x\log(d)-2\right)\sin(x)\right)d^x}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="fricas")
```

```
[Out] (x^2*cos(x)*log(d)^5 - 2*x*cos(x)*log(d)^4 + 2*(x^2 + 1)*cos(x)*log(d)^3 +
(x^2 - 6)*cos(x)*log(d) + 2*x*cos(x) + (x^2*log(d)^4 - 4*x*log(d)^3 + 2*(x^
2 + 3)*log(d)^2 + x^2 - 4*x*log(d) - 2)*sin(x))*d^x/(log(d)^6 + 3*log(d)^4
+ 3*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.01, size = 665, normalized size = 4.13

$$\begin{cases} \frac{ix^3e^{-ix}\sin(x)}{6}+\frac{x^3e^{-ix}\cos(x)}{6}+\frac{x^2e^{-ix}\sin(x)}{4}+\frac{ix^2e^{-ix}\cos(x)}{4}-\frac{ixe^{-ix}\sin(x)}{4}+\frac{xe^{-ix}\cos(x)}{4}-\frac{e^{-ix}\sin(x)}{4} & \text{for } d=e^{-i} \\ -\frac{ix^3e^{ix}\sin(x)}{6}+\frac{x^3e^{ix}\cos(x)}{6}+\frac{x^2e^{ix}\sin(x)}{4}-\frac{ix^2e^{ix}\cos(x)}{4}+\frac{ixe^{ix}\sin(x)}{4}+\frac{xe^{ix}\cos(x)}{4}-\frac{e^{ix}\sin(x)}{4} & \text{for } d=e^{i} \\ \frac{d^xx^2\log(d)^5\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{d^xx^2\log(d)^4\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{2d^xx^2\log(d)^3\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{2d^xx^2\log(d)^2\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{d^xx^2\log(d)\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{d^xx^2\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}-\frac{2d^xx\log(d)^4\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}-\frac{4d^xx\log(d)^3\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}-\frac{4d^xx\log(d)\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{2d^xx\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{2d^x\log(d)^3\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}+\frac{6d^x\log(d)^2\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}-\frac{6d^x\log(d)\cos(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}-\frac{2d^x\sin(x)}{\log(d)^6+3\log(d)^4+3\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**2*cos(x),x)
```

```
[Out] Piecewise((I*x**3*exp(-I*x)*sin(x)/6 + x**3*exp(-I*x)*cos(x)/6 + x**2*exp(-
I*x)*sin(x)/4 + I*x**2*exp(-I*x)*cos(x)/4 - I*x*exp(-I*x)*sin(x)/4 + x*exp(
-I*x)*cos(x)/4 - exp(-I*x)*sin(x)/4, Eq(d, exp(-I))), (-I*x**3*exp(I*x)*sin
(x)/6 + x**3*exp(I*x)*cos(x)/6 + x**2*exp(I*x)*sin(x)/4 - I*x**2*exp(I*x)*c
os(x)/4 + I*x*exp(I*x)*sin(x)/4 + x*exp(I*x)*cos(x)/4 - exp(I*x)*sin(x)/4,
Eq(d, exp(I))), (d**x*x**2*log(d)**5*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*lo
g(d)**2 + 1) + d**x*x**2*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1) + 2*d**x*x**2*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1) + 2*d**x*x**2*log(d)**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1) + d**x*x**2*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2
 + 1) + d**x*x**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d*
*x*x*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x*
x*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x*x*l
og(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*cos(x)/
(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*cos(x)/(log(
d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 6*d**x*log(d)**2*sin(x)/(log(d)**6
 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*cos(x)/(log(d)**6 + 3*log
(d)**4 + 3*log(d)**2 + 1) - 2*d**x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.09, size = 2631, normalized size = 16.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*((2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi*
x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d)) - pi*x*sgn(
d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d)
 - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3
 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x
^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 - 2
*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d))
```

+ 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs
(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (
3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log
(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d)
 - 1/2*pi*x + x) + ((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
- 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(
d) - 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
 + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(
abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*l
og(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log
(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6
log(abs(d)))^2) - 2(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^
2*log(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d
)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn
(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d)
)^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1
/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*((2*(3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^
2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*
log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)/((3*pi - pi^3*s
gn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(
d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sg
n(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*p
i*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log
(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*p
i^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(ab
s(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*p
i*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2
 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d
)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((3*pi - pi^3
*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sg
n(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi^2*x^2*sgn(d) - pi^2*
x^2 + 2*x^2*log(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(ab
s(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*l
og(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^
2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*p
i*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) - 2*(pi*x^2*log
(abs(d))*sgn(d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi
*x + 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3
 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^
3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*s

```
gn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))
*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) -
6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*
abs(d)^x + 2*I*abs(d)^x*((I*pi^2*x^2*sgn(d) - 2*pi*x^2*log(abs(d))*sgn(d) -
 I*pi^2*x^2 + 2*pi*x^2*log(abs(d)) + 2*I*x^2*log(abs(d))^2 - 2*I*pi*x^2*sgn
(d) + 2*I*pi*x^2 - 4*x^2*log(abs(d)) + 2*pi*x*sgn(d) - 2*pi*x - 2*I*x^2 - 4
*I*x*log(abs(d)) + 4*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/(24*
I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(abs(d))^2
*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 + 16*log(a
bs(d))^3 + 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2 + 48*pi*
log(abs(d)) + 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d)) - 16*I)
- (I*pi^2*x^2*sgn(d) + 2*pi*x^2*log(abs(d))*sgn(d) - I*pi^2*x^2 - 2*pi*x^2*
log(abs(d)) + 2*I*x^2*log(abs(d))^2 - 2*I*pi*x^...
```

Mupad [B]
time = 0.35, size = 132, normalized size = 0.82

$$\frac{d^x\,\left(x^2\,\sin\left(x\right)-2\,\sin\left(x\right)+2\,x\,\cos\left(x\right)\right)+d^x\,{\ln\left(d\right)}^3\,\left(2\,\cos\left(x\right)+2\,x^2\,\cos\left(x\right)-4\,x\,\sin\left(x\right)\right)+d^x\,{\ln\left(d\right)}^2\,\left(6\,\sin\left(x\right)+2\,x^2\,\sin\left(x\right)\right)-d^x\,\ln\left(d\right)\,\left(6\,\cos\left(x\right)-x^2\,\cos\left(x\right)+4\,x\,\sin\left(x\right)\right)+d^x\,{\ln\left(d\right)}^4\,\left(x^2\,\sin\left(x\right)-2\,x\,\cos\left(x\right)\right)+d^x\,x^2\,{\ln\left(d\right)}^5\,\cos\left(x\right)}{{\left({\ln\left(d\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*cos(x),x)
```

```
[Out] (d^x*(x^2*sin(x) - 2*sin(x) + 2*x*cos(x)) + d^x*log(d)^3*(2*cos(x) + 2*x^2*
cos(x) - 4*x*sin(x)) + d^x*log(d)^2*(6*sin(x) + 2*x^2*sin(x)) - d^x*log(d)*
(6*cos(x) - x^2*cos(x) + 4*x*sin(x)) + d^x*log(d)^4*(x^2*sin(x) - 2*x*cos(x
)) + d^x*x^2*log(d)^5*cos(x))/(log(d)^2 + 1)^3
```

3.140 $\int d^x x^3 \sin(x)\,dx$

Optimal. Leaf size=261

$$-\frac{24d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^4}+\frac{24d^x\cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^4}+\frac{6d^x x\cos(x)}{\left(1+\log^2(d)\right)^3}-\frac{18d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3}+\frac{6d^x x^2\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^3}{1+\text{l}}$$

```
[Out] -24*d^x*cos(x)*ln(d)/(1+ln(d)^2)^4+24*d^x*cos(x)*ln(d)^3/(1+ln(d)^2)^4+6*d^
x*x*cos(x)/(1+ln(d)^2)^3-18*d^x*x*cos(x)*ln(d)^2/(1+ln(d)^2)^3+6*d^x*x^2*co
s(x)*ln(d)/(1+ln(d)^2)^2-d^x*x^3*cos(x)/(1+ln(d)^2)-6*d^x*sin(x)/(1+ln(d)^2
)^4+36*d^x*ln(d)^2*sin(x)/(1+ln(d)^2)^4-6*d^x*ln(d)^4*sin(x)/(1+ln(d)^2)^4-
18*d^x*x*ln(d)*sin(x)/(1+ln(d)^2)^3+6*d^x*x*ln(d)^3*sin(x)/(1+ln(d)^2)^3+3*
d^x*x^2*sin(x)/(1+ln(d)^2)^2-3*d^x*x^2*ln(d)^2*sin(x)/(1+ln(d)^2)^2+d^x*x^3
*ln(d)*sin(x)/(1+ln(d)^2)
```

Rubi [A]

time = 0.31, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4517, 4553, 14, 4518, 4554}

$$\frac{x^3d^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{x^3d^x\cos(x)}{\log^2(d)+1}+\frac{3x^2d^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2d^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6x^2d^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}-\frac{18xd^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{36d^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x\log^4(d)\sin(x)}{\left(\log^2(d)+1\right)^4}+\frac{6xd^x\log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{18xd^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{6xd^x\cos(x)}{\left(\log^2(d)+1\right)^3}-\frac{24d^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^4}+\frac{24d^x\log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^3*Sin[x],x]
```

```
[Out] (-24*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (24*d^x*Cos[x]*Log[d]^3)/(1 + Lo
g[d]^2)^4 + (6*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (18*d^x*x*Cos[x]*Log[d]^2)/
(1 + Log[d]^2)^3 + (6*d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^3*Co
s[x])/(1 + Log[d]^2) - (6*d^x*Sin[x])/(1 + Log[d]^2)^4 + (36*d^x*Log[d]^2*S
in[x])/(1 + Log[d]^2)^4 - (6*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (18*d^
x*x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (6*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]
^2)^3 + (3*d^x*x^2*Sin[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Log[d]^2*Sin[x])/(
1 + Log[d]^2)^2 + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int d^x x^3 \sin(x)\,dx = -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - 3\int x^2\left(-\frac{d^x \cos(x)}{1+\log^2(d)} + \frac{d^x \log(d)\sin(x)}{1+\log^2(d)}\right)dx$$

$$= -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - 3\int\left(-\frac{d^x x^2 \cos(x)}{1+\log^2(d)} + \frac{d^x x^2 \log(d)\sin(x)}{1+\log^2(d)}\right)dx$$

$$= -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} + \frac{3\int d^x x^2 \cos(x)\,dx}{1+\log^2(d)} - \frac{(3\log(d))\int d^x x^2 \sin(x)\,dx}{1+\log^2(d)}$$

$$= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \cdots$$

$$= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \cdots$$

$$= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \cdots$$

$$= \frac{6d^x x \cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} - \cdots$$

$$= \frac{6d^x x \cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} - \cdots$$

$$= -\frac{12d^x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^4} + \frac{12d^x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^4} + \frac{6d^x x \cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \cdots$$

Mathematica [A]
time = 0.11, size = 169, normalized size = 0.65

$$\frac{d^x\left(-\cos(x)\left(x(-6+x^2)-6(-4+x^2)\log(d)+3x(4+x^2)\log^2(d)-12(2+x^2)\log^3(d)+3x(6+x^2)\log^4(d)-6x^2\log^5(d)+x^3\log^6(d)\right)+\left(3(-2+x^2)+x(-18+x^2)\log(d)+3(12+x^2)\log^2(d)+3x(-4+x^2)\log^3(d)-3(2+x^2)\log^4(d)+3x(2+x^2)\log^5(d)-3x^2\log^6(d)+x^3\log^7(d)\right)\sin(x)\right)}{\left(1+\log^2(d)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^3*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^2
 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*Lo
g[d]^6)) + (3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 + 3
*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 - 3*
x^2*Log[d]^6 + x^3*Log[d]^7)*Sin[x]))/(1 + Log[d]^2)^4
```

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 166, normalized size = 0.64

method	result
risch	$-\frac{i\left(-6+\ln(d)^3x^3+3i\ln(d)^2x^3-3\ln(d)x^3-ix^3+6x\ln(d)+6ix-3\ln(d)^2x^2-6i\ln(d)x^2+3x^2\right)d^x e^{ix}}{2(\ln(d)+i)^4}+\frac{i\left(-6+6x\ln(d)-6ix-3\ln(d)^2\right.}{}$
norman	$\frac{\frac{x^3 e^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}-\frac{x^3 e^{x\ln(d)}}{1+\ln(d)^2}+\frac{6\ln(d)x^2e^{x\ln(d)}}{\ln(d)^4+2\ln(d)^2+1}+\frac{2\ln(d)x^3e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}-\frac{6\left(\ln(d)^2-1\right)x^2e^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{\ln(d)^4+2\ln(d)^2+1}-\frac{6\left(3\ln(d)^2-1\right)x\, e^{x\ln(}}{\left(1+\ln(d)^2\right)\left(\ln(d)^4+2\ln(\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(-6+ln(d)^3*x^3+3*I*ln(d)^2*x^3-3*ln(d)*x^3-I*x^3+6*x*ln(d)+6*I*x-3*
ln(d)^2*x^2-6*I*ln(d)*x^2+3*x^2)*d^x/(ln(d)+I)^4*exp(I*x)+1/2*I*(-6+6*x*ln(
d)-6*I*x-3*ln(d)^2*x^2+6*I*ln(d)*x^2+3*x^2+ln(d)^3*x^3-3*I*ln(d)^2*x^3-3*ln
(d)*x^3+I*x^3)*d^x/(ln(d)-I)^4*exp(-I*x)
```

Maxima [A]
time = 1.21, size = 186, normalized size = 0.71

$$-\frac{\left(\left(\log(d)^6+3\log(d)^4+3\log(d)^2+1\right)x^3-6\left(\log(d)^5+2\log(d)^3+\log(d)\right)x^2-24\log(d)^3+6\left(3\log(d)^4+2\log(d)^2-1\right)x+24\log(d)\right)d^x\cos(x)-\left(\left(\log(d)^7+3\log(d)^5+3\log(d)^3+\log(d)\right)x^3-6\log(d)^4-3\left(\log(d)^6+\log(d)^4-\log(d)^2-1\right)x^2+6\left(\log(d)^5-2\log(d)^3-3\log(d)\right)x+36\log(d)^2-6\right)d^x\sin(x)}{\log(d)^8+4\log(d)^6+6\log(d)^4+4\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2*log(d)^3
 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4 + 2*log(d)^2 - 1)*x + 24*log(d)
)*d^x*cos(x) - ((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d
)^4 - 3*(log(d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3
 - 3*log(d))*x + 36*log(d)^2 - 6)*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*lo
g(d)^4 + 4*log(d)^2 + 1)
```

Fricas [A]
time = 0.55, size = 203, normalized size = 0.78

$$-\frac{\left(x^3\cos(x)\log(d)^6-6x^2\cos(x)\log(d)^5+3\left(x^3+6x\right)\cos(x)\log(d)^4-12\left(x^2+2\right)\cos(x)\log(d)^3+3\left(x^3+4x\right)\cos(x)\log(d)^2-6\left(x^2-4\right)\cos(x)\log(d)+\left(x^3-6x\right)\cos(x)-\left(x^3\log(d)^7-3x^2\log(d)^6+3\left(x^3+2x\right)\log(d)^5-3\left(x^2+2\right)\log(d)^4+3\left(x^3-4x\right)\log(d)^3+3\left(x^2+12\right)\log(d)^2+3x^2+\left(x^3-18x\right)\log(d)-6\right)\sin(x)\right)d^x}{\log(d)^8+4\log(d)^6+6\log(d)^4+4\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3*cos(x)*log(d)^6 - 6*x^2*cos(x)*log(d)^5 + 3*(x^3 + 6*x)*cos(x)*log(d)
^4 - 12*(x^2 + 2)*cos(x)*log(d)^3 + 3*(x^3 + 4*x)*cos(x)*log(d)^2 - 6*(x^2
- 4)*cos(x)*log(d) + (x^3 - 6*x)*cos(x) - (x^3*log(d)^7 - 3*x^2*log(d)^6 +
3*(x^3 + 2*x)*log(d)^5 - 3*(x^2 + 2)*log(d)^4 + 3*(x^3 - 4*x)*log(d)^3 + 3*
(x^2 + 12)*log(d)^2 + 3*x^2 + (x^3 - 18*x)*log(d) - 6)*sin(x))*d^x/(log(d)^
8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.10, size = 1355, normalized size = 5.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**3*sin(x),x)
```

```
[Out] Piecewise((x**4*exp(-I*x)*sin(x)/8 - I*x**4*exp(-I*x)*cos(x)/8 + I*x**3*exp
(-I*x)*sin(x)/4 - x**3*exp(-I*x)*cos(x)/4 + 3*x**2*exp(-I*x)*sin(x)/8 + 3*I
*x**2*exp(-I*x)*cos(x)/8 - 3*I*x*exp(-I*x)*sin(x)/8 + 3*x*exp(-I*x)*cos(x)/
8 - 3*I*exp(-I*x)*cos(x)/8, Eq(d, exp(-I))), (x**4*exp(I*x)*sin(x)/8 + I*x*
*4*exp(I*x)*cos(x)/8 - I*x**3*exp(I*x)*sin(x)/4 - x**3*exp(I*x)*cos(x)/4 +
3*x**2*exp(I*x)*sin(x)/8 - 3*I*x**2*exp(I*x)*cos(x)/8 + 3*I*x*exp(I*x)*sin(
x)/8 + 3*x*exp(I*x)*cos(x)/8 + 3*I*exp(I*x)*cos(x)/8, Eq(d, exp(I))), (d**x
*x**3*log(d)**7*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
 + 1) - d**x*x**3*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 +
 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**4*cos(x)/(log(d)**8 +
 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*sin(x
)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*l
og(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) +
 d**x*x**3*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)*
*2 + 1) - d**x*x**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d
)**2 + 1) - 3*d**x*x**2*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d
)**4 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)**5*cos(x)/(log(d)**8 + 4*log(d
)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*sin(x)/(log(d
)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x**2*log(d)**
3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x
*x**2*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
 + 1) + 6*d**x*x**2*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
4*log(d)**2 + 1) + 3*d**x*x**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**
4 + 4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 +
 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)**4*cos(x)/(log(d)**8 + 4
*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*sin(x)/(l
og(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)*
*2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d*
*x*x*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1
) + 6*d**x*x*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
1) - 6*d**x*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log
(d)**2 + 1) + 24*d**x*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)*
*4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 +
 6*log(d)**4 + 4*log(d)**2 + 1) - 24*d**x*log(d)*cos(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*sin(x)/(log(d)**8 + 4*log(d
)**6 + 6*log(d)**4 + 4*log(d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 1.25, size = 5069, normalized size = 19.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="giac")
```

```
[Out] 1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log
(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) +
 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 -
 4*pi*sgn(d) - 2)*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x
^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*log(ab
s(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x
^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x
)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(ab
s(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*
pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*
pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) -
pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*lo
g(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) +
2*log(abs(d)))^2) + 4*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d
)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs(d
)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))
^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)*(pi^3*log(
abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d)
)^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*
pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))/((4*pi + pi^4*sgn
(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs
(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(ab
s(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 +
16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) +
pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(a
bs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2))*
cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x) - ((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs
(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(
d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn
(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*
sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d)
)*sgn(d) + 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*lo
g(abs(d)) - 6*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x
*log(abs(d)) - 12)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^
4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(
d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*l
og(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(ab
```

```
s(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*s
gn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*
pi*log(abs(d)) + 2*log(abs(d)))^2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(
d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi
^2*x^3 - 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d
) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*s
gn(d) + 6*pi*x - 12*x)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) -
 pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*l
og(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) +
 2*log(abs(d)))/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 +
 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))
^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(
abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d
))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(
d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*
log(abs(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d
)^x + 1/2*(((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi
^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sg
n(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d
))^2 - 4*pi*sgn(d) + 2)*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) -
pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 + 3*pi^2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*
log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 -
6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x
+ 12*x)/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*
log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d
) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^
2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn
(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(
d)) - 2*log(abs(d)))^2) - 4*(3*pi^2*x^3*log(abs...
```

Mupad [**B**]
time = 0.63, size = 231, normalized size = 0.89

$$-\frac{d^x\,\left(6\,\sin\left(x\right)+x^3\,\cos\left(x\right)-3\,x^2\,\sin\left(x\right)-6\,x\,\cos\left(x\right)\right)-d^x\,\ln\left(d\right)^5\,\left(6\,x^2\,\cos\left(x\right)+3\,x^3\,\sin\left(x\right)+6\,x\,\sin\left(x\right)\right)+d^x\,\ln\left(d\right)^4\,\left(6\,\sin\left(x\right)+3\,x^3\,\cos\left(x\right)+3\,x^2\,\sin\left(x\right)+18\,x\,\cos\left(x\right)\right)-d^x\,\ln\left(d\right)^3\,\left(24\,\cos\left(x\right)+12\,x^2\,\cos\left(x\right)+3\,x^3\,\sin\left(x\right)-12\,x\,\sin\left(x\right)\right)-d^x\,\ln\left(d\right)^2\,\left(36\,\sin\left(x\right)-3\,x^3\,\cos\left(x\right)+3\,x^2\,\sin\left(x\right)-12\,x\,\cos\left(x\right)\right)+d^x\,\ln\left(d\right)^6\,\left(x^3\,\cos\left(x\right)+3\,x^2\,\sin\left(x\right)\right)+d^x\,\ln\left(d\right)\,\left(24\,\cos\left(x\right)-6\,x^2\,\cos\left(x\right)-x^3\,\sin\left(x\right)+18\,x\,\sin\left(x\right)\right)-d^x\,x^3\,\ln\left(d\right)^7\,\sin\left(x\right)}{{\left({\ln\left(d\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*sin(x),x)
```

```
[Out] -(d^x*(6*sin(x) + x^3*cos(x) - 3*x^2*sin(x) - 6*x*cos(x)) - d^x*log(d)^5*(6
*x^2*cos(x) + 3*x^3*sin(x) + 6*x*sin(x)) + d^x*log(d)^4*(6*sin(x) + 3*x^3*c
os(x) + 3*x^2*sin(x) + 18*x*cos(x)) - d^x*log(d)^3*(24*cos(x) + 12*x^2*cos(
x) + 3*x^3*sin(x) - 12*x*sin(x)) - d^x*log(d)^2*(36*sin(x) - 3*x^3*cos(x) +
 3*x^2*sin(x) - 12*x*cos(x)) + d^x*log(d)^6*(x^3*cos(x) + 3*x^2*sin(x)) + d
^x*log(d)*(24*cos(x) - 6*x^2*cos(x) - x^3*sin(x) + 18*x*sin(x)) - d^x*x^3*l
og(d)^7*sin(x))/(log(d)^2 + 1)^4
```

3.141 $\int d^x x^3 \cos(x) \, dx$

Optimal. Leaf size=260

$$-\frac{6d^x\cos(x)}{\left(1+\log^2(d)\right)^4}+\frac{36d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^4}-\frac{6d^x\cos(x)\log^4(d)}{\left(1+\log^2(d)\right)^4}-\frac{18d^xx\cos(x)\log(d)}{\left(1+\log^2(d)\right)^3}+\frac{6d^xx\cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3}+\frac{3d^x}{(1+}$$

```
[Out] -6*d^x*cos(x)/(1+ln(d)^2)^4+36*d^x*cos(x)*ln(d)^2/(1+ln(d)^2)^4-6*d^x*cos(x
)*ln(d)^4/(1+ln(d)^2)^4-18*d^x*x*cos(x)*ln(d)/(1+ln(d)^2)^3+6*d^x*x*cos(x)*
ln(d)^3/(1+ln(d)^2)^3+3*d^x*x^2*cos(x)/(1+ln(d)^2)^2-3*d^x*x^2*cos(x)*ln(d)
^2/(1+ln(d)^2)^2+d^x*x^3*cos(x)*ln(d)/(1+ln(d)^2)+24*d^x*ln(d)*sin(x)/(1+ln
(d)^2)^4-24*d^x*ln(d)^3*sin(x)/(1+ln(d)^2)^4-6*d^x*x*sin(x)/(1+ln(d)^2)^3+1
8*d^x*x*ln(d)^2*sin(x)/(1+ln(d)^2)^3-6*d^x*x^2*ln(d)*sin(x)/(1+ln(d)^2)^2+d
^x*x^3*sin(x)/(1+ln(d)^2)
```

Rubi [A]
time = 0.30, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {4518, 4554, 14, 4517, 4553}

$$\frac{x^3d^x\sin(x)}{\log^2(d)+1}+\frac{x^3d^x\log(d)\cos(x)}{\log^2(d)+1}-\frac{6x^2d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{3x^2d^x\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{18xd^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{6xd^x\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{24d^x\log(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{24d^x\log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^4}-\frac{18xd^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^3}+\frac{36d^x\log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x\cos(x)}{\left(\log^2(d)+1\right)^4}-\frac{6d^x\log^4(d)\cos(x)}{\left(\log^2(d)+1\right)^4}+\frac{6xd^x\log^3(d)\cos(x)}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^3*Cos[x],x]
```

```
[Out] (-6*d^x*Cos[x])/(1 + Log[d]^2)^4 + (36*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^
4 - (6*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 - (18*d^x*x*Cos[x]*Log[d])/(1
+ Log[d]^2)^3 + (6*d^x*x*Cos[x]*Log[d]^3)/(1 + Log[d]^2)^3 + (3*d^x*x^2*Cos
[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x
*x^3*Cos[x]*Log[d])/(1 + Log[d]^2) + (24*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^
4 - (24*d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Sin[x])/(1 + Log[d
]^2)^3 + (18*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x^2*Log[d]*Si
n[x])/(1 + Log[d]^2)^2 + (d^x*x^3*Sin[x])/(1 + Log[d]^2)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int d^x x^3 \cos(x)\,dx &= \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - 3\int x^2\left(\frac{d^x \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x \sin(x)}{1+\log^2(d)}\right)dx \\
&= \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - 3\int\left(\frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)}\right)dx \\
&= \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - \frac{3\int d^x x^2 \sin(x)\,dx}{1+\log^2(d)} - \frac{(3\log(d))\int d^x x^2 \cos(x)\,dx}{1+\log^2(d)} \\
&= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x}{1} \\
&= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x}{1} \\
&= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x}{1} \\
&= -\frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \\
&= -\frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \\
&= -\frac{6d^x \cos(x)}{\left(1+\log^2(d)\right)^4} + \frac{12d^x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^4} - \frac{6d^x \cos(x)\log^4(d)}{\left(1+\log^2(d)\right)^4} - \frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + 6d
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 168, normalized size = 0.65

$$\frac{d^x\left(\cos(x)\left(3(-2+x^2)+x(-18+x^2)\log(d)+3(12+x^2)\log^2(d)+3x(-4+x^2)\log^3(d)-3(2+x^2)\log^4(d)+3x(2+x^2)\log^5(d)-3x^2\log^6(d)+x^3\log^7(d)\right)+\left(x(-6+x^2)-6(-4+x^2)\log(d)+3x(4+x^2)\log^2(d)-12(2+x^2)\log^3(d)+3x(6+x^2)\log^4(d)-6x^2\log^5(d)+x^3\log^6(d)\right)\sin(x)\right)}{\left(1+\log^2(d)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^3*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 +
 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 -
3*x^2*Log[d]^6 + x^3*Log[d]^7) + (x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*
(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2
*Log[d]^5 + x^3*Log[d]^6)*Sin[x]))/(1 + Log[d]^2)^4
```

Maple [C] Result contains complex when optimal does not.
time = 0.07, size = 164, normalized size = 0.63

method	result
risch	$\frac{\left(-6+\ln(d)^3x^3+3i\ln(d)^2x^3-3\ln(d)x^3-ix^3+6x\ln(d)+6ix-3\ln(d)^2x^2-6i\ln(d)x^2+3x^2\right)d^x\mathrm{e}^{ix}}{2(\ln(d)+i)^4}+\frac{\left(-6+6x\ln(d)-6ix-3\ln(d)^2x^2+6\right.}{}$
norman	$\frac{\frac{\ln(d)x^3\mathrm{e}^{x\ln(d)}}{1+\ln(d)^2}+\frac{2x^3\mathrm{e}^{x\ln(d)}\tan\left(\frac{x}{2}\right)}{1+\ln(d)^2}-\frac{3\left(\ln(d)^2-1\right)x^2\mathrm{e}^{x\ln(d)}}{\ln(d)^4+2\ln(d)^2+1}-\frac{6\left(\ln(d)^4-6\ln(d)^2+1\right)\mathrm{e}^{x\ln(d)}}{\left(\ln(d)^6+3\ln(d)^4+3\ln(d)^2+1\right)\left(1+\ln(d)^2\right)}-\frac{\ln(d)x^3\mathrm{e}^{x\ln(d)}\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\ln(d)^2}+\frac{3\left(\ln(d)^2\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-6+ln(d)^3*x^3+3*I*ln(d)^2*x^3-3*ln(d)*x^3-I*x^3+6*x*ln(d)+6*I*x-3*ln(
d)^2*x^2-6*I*ln(d)*x^2+3*x^2)*d^x/(ln(d)+I)^4*exp(I*x)+1/2*(-6+6*x*ln(d)-6*
I*x-3*ln(d)^2*x^2+6*I*ln(d)*x^2+3*x^2+ln(d)^3*x^3-3*I*ln(d)^2*x^3-3*ln(d)*x
^3+I*x^3)*d^x/(ln(d)-I)^4*exp(-I*x)
```

Maxima [A]
time = 1.59, size = 184, normalized size = 0.71

$$\frac{\left(\left(\log(d)^7+3\log(d)^5+3\log(d)^3+\log(d)\right)x^3-6\log(d)^4-3\left(\log(d)^6+\log(d)^4-\log(d)^2-1\right)x^2+6\left(\log(d)^5-2\log(d)^3-3\log(d)\right)x+36\log(d)^2-6\right)d^x\cos(x)+\left(\left(\log(d)^6+3\log(d)^4+3\log(d)^2+1\right)x^3-6\left(\log(d)^5+2\log(d)^3+\log(d)\right)x^2-24\log(d)^3+6\left(3\log(d)^4+2\log(d)^2-1\right)x+24\log(d)\right)d^x\sin(x)}{\log(d)^8+4\log(d)^6+6\log(d)^4+4\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d)^4 - 3*(log(d
)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3 - 3*log(d))*x
 + 36*log(d)^2 - 6)*d^x*cos(x) + ((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*
x^3 - 6*(log(d)^5 + 2*log(d)^3 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4
+ 2*log(d)^2 - 1)*x + 24*log(d))*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*log
(d)^4 + 4*log(d)^2 + 1)
```

Fricas [A]
time = 0.52, size = 202, normalized size = 0.78

$$\frac{\left(x^3\cos(x)\log(d)^7-3x^2\cos(x)\log(d)^6+3\left(x^3+2x\right)\cos(x)\log(d)^5-3\left(x^2+2\right)\cos(x)\log(d)^4+3\left(x^3-4x\right)\cos(x)\log(d)^3+3\left(x^2+12\right)\cos(x)\log(d)^2+\left(x^3-18x\right)\cos(x)\log(d)+3\left(x^2-2\right)\cos(x)+\left(x^3\log(d)^6-6x^2\log(d)^5+3\left(x^3+6x\right)\log(d)^4-12\left(x^2+2\right)\log(d)^3+x^3+3\left(x^3+4x\right)\log(d)^2-6\left(x^2-4\right)\log(d)-6x\right)\sin(x)\right)d^x}{\log(d)^8+4\log(d)^6+6\log(d)^4+4\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="fricas")
```

```
[Out] (x^3*cos(x)*log(d)^7 - 3*x^2*cos(x)*log(d)^6 + 3*(x^3 + 2*x)*cos(x)*log(d)^
5 - 3*(x^2 + 2)*cos(x)*log(d)^4 + 3*(x^3 - 4*x)*cos(x)*log(d)^3 + 3*(x^2 +
12)*cos(x)*log(d)^2 + (x^3 - 18*x)*cos(x)*log(d) + 3*(x^2 - 2)*cos(x) + (x^
3*log(d)^6 - 6*x^2*log(d)^5 + 3*(x^3 + 6*x)*log(d)^4 - 12*(x^2 + 2)*log(d)^
3 + x^3 + 3*(x^3 + 4*x)*log(d)^2 - 6*(x^2 - 4)*log(d) - 6*x)*sin(x))*d^x/(l
og(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.14, size = 1355, normalized size = 5.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**3*cos(x),x)
```

```
[Out] Piecewise((I*x**4*exp(-I*x)*sin(x)/8 + x**4*exp(-I*x)*cos(x)/8 + x**3*exp(-
I*x)*sin(x)/4 + I*x**3*exp(-I*x)*cos(x)/4 - 3*I*x**2*exp(-I*x)*sin(x)/8 + 3
*x**2*exp(-I*x)*cos(x)/8 - 3*x*exp(-I*x)*sin(x)/8 - 3*I*x*exp(-I*x)*cos(x)/
8 + 3*I*exp(-I*x)*sin(x)/8, Eq(d, exp(-I))), (-I*x**4*exp(I*x)*sin(x)/8 + x
**4*exp(I*x)*cos(x)/8 + x**3*exp(I*x)*sin(x)/4 - I*x**3*exp(I*x)*cos(x)/4 +
 3*I*x**2*exp(I*x)*sin(x)/8 + 3*x**2*exp(I*x)*cos(x)/8 - 3*x*exp(I*x)*sin(x
)/8 + 3*I*x*exp(I*x)*cos(x)/8 - 3*I*exp(I*x)*sin(x)/8, Eq(d, exp(I))), (d**
x*x**3*log(d)**7*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**
2 + 1) + d**x*x**3*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
+ 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6
+ 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**4*sin(x)/(log(d)**8
+ 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*cos(
x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*
log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1)
+ d**x*x**3*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)
**2 + 1) + d**x*x**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(
d)**2 + 1) - 3*d**x*x**2*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(
d)**4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)**5*sin(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*cos(x)/(log(
d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x**2*log(d)*
*3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**
x*x**2*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**
2 + 1) - 6*d**x*x**2*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
 4*log(d)**2 + 1) + 3*d**x*x**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)*
*4 + 4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6
+ 6*log(d)**4 + 4*log(d)**2 + 1) + 18*d**x*x*log(d)**4*sin(x)/(log(d)**8 +
4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*cos(x)/(
log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x*log(d)
**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d
**x*x*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
1) - 6*d**x*x*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
 1) - 6*d**x*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*lo
g(d)**2 + 1) - 24*d**x*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)
**4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6
+ 6*log(d)**4 + 4*log(d)**2 + 1) + 24*d**x*log(d)*sin(x)/(log(d)**8 + 4*log
(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*cos(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.90, size = 5065, normalized size = 19.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="giac")
```

```
[Out] -1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*lo
g(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d)
+ 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2
- 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d))
+ 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs(d))
- 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))^2
+ 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi + pi^4
*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log
(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*lo
g(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^
2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)
) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*l
og(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^
2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x
^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*log(abs(d))^2 + 3
*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(
d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x)*(pi^3*log
(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d
))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3
*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))/((4*pi + pi^4*sg
n(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(ab
s(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(a
bs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 +
 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) +
 pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(
abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2))
*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x) + ((4*pi + pi^4*sgn(d) - 6*pi^2*log(ab
s(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn
(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sg
n(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)*(pi^3*x^3*sgn(d) - 3*pi
*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*
sgn(d) + 3*pi^2*x^3 - 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(
abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d
)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d
))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d)
 + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d
) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*s
```

```
gn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*p
i^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(ab
s(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2) + 4*(3*pi^2*x^3*log(abs
(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(a
bs(d))*sgn(d) + 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x
^3*log(abs(d)) - 6*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 +
 12*x*log(abs(d)) - 12)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d)
- pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*
log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d))
+ 2*log(abs(d)))/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4
+ 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d)
)^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log
(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(
d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn
(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi
*log(abs(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(
d)^x + 1/2*(((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*p
i^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*s
gn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(
d))^2 - 4*pi*sgn(d) + 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(ab
s(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(abs(d))*sgn(d) - 6*pi*x^3*log(ab
s(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(
d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi
- pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 +
 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12
*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d)
 + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(
abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d))
 + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs
(d)))^2) + 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(ab...
```

Mupad [B]
time = 0.64, size = 232, normalized size = 0.89

$$-\frac{d^x\,\left(6\,\cos\left(x\right)-3\,x^2\,\cos\left(x\right)-x^3\,\sin\left(x\right)+6\,x\,\sin\left(x\right)\right)-d^x\,{\ln\left(d\right)}^5\,\left(3\,x^3\,\cos\left(x\right)-6\,x^2\,\sin\left(x\right)+6\,x\,\cos\left(x\right)\right)+d^x\,{\ln\left(d\right)}^4\,\left(6\,\cos\left(x\right)+3\,x^2\,\cos\left(x\right)-3\,x^3\,\sin\left(x\right)-18\,x\,\sin\left(x\right)\right)+d^x\,{\ln\left(d\right)}^3\,\left(24\,\sin\left(x\right)-3\,x^3\,\cos\left(x\right)+12\,x^2\,\sin\left(x\right)+12\,x\,\cos\left(x\right)\right)-d^x\,{\ln\left(d\right)}^2\,\left(36\,\cos\left(x\right)+3\,x^2\,\cos\left(x\right)+3\,x^3\,\sin\left(x\right)+12\,x\,\sin\left(x\right)\right)+d^x\,{\ln\left(d\right)}^6\,\left(3\,x^2\,\cos\left(x\right)-x^3\,\sin\left(x\right)\right)-d^x\,\ln\left(d\right)\,\left(24\,\sin\left(x\right)+x^3\,\cos\left(x\right)-6\,x^2\,\sin\left(x\right)-18\,x\,\cos\left(x\right)\right)-d^x\,x^3\,{\ln\left(d\right)}^7\,\cos\left(x\right)}{{\left({\ln\left(d\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*cos(x),x)
```

```
[Out] -(d^x*(6*cos(x) - 3*x^2*cos(x) - x^3*sin(x) + 6*x*sin(x)) - d^x*log(d)^5*(3
*x^3*cos(x) - 6*x^2*sin(x) + 6*x*cos(x)) + d^x*log(d)^4*(6*cos(x) + 3*x^2*c
os(x) - 3*x^3*sin(x) - 18*x*sin(x)) + d^x*log(d)^3*(24*sin(x) - 3*x^3*cos(x
) + 12*x^2*sin(x) + 12*x*cos(x)) - d^x*log(d)^2*(36*cos(x) + 3*x^2*cos(x) +
 3*x^3*sin(x) + 12*x*sin(x)) + d^x*log(d)^6*(3*x^2*cos(x) - x^3*sin(x)) - d
^x*log(d)*(24*sin(x) + x^3*cos(x) - 6*x^2*sin(x) - 18*x*cos(x)) - d^x*x^3*l
og(d)^7*cos(x))/(log(d)^2 + 1)^4
```

3.142 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {4440, 2718}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)\,dx\\
&= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx\\
&= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A]
time = 0.06, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A]
time = 2.23, size = 19, normalized size = 0.76

$$\frac{1}{24}\cos\left(6\,x\right)-\frac{1}{16}\cos\left(4\,x\right)-\frac{1}{8}\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A]
time = 0.66, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos\left(x\right)^{6}-\frac{5}{2}\cos\left(x\right)^{4}+\cos\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(19) = 38$.
time = 1.24, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{5\sin(x)\sin(2x)\cos(3x)}{24} - \frac{\sin(2x)\sin(3x)\cos(x)}{8} - \frac{\cos(x)\cos(2x)\cos(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6
```

Giac [A]
time = 0.83, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Mupad [B]
time = 0.17, size = 14, normalized size = 0.56

$$-\frac{\sin(x)^4\left(8\sin(x)^2 - 9\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*sin(x),x)
```

```
[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6
```

3.143 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2717}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A]
time = 0.06, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A]
time = 1.30, size = 22, normalized size = 0.73

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A]
time = 0.54, size = 25, normalized size = 0.83

$$\frac{1}{12}\left(16\cos(x)^{5}-10\cos(x)^{3}+3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(22) = 44$.
time = 1.23, size = 116, normalized size = 3.87

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4}+\frac{x\sin(x)\sin(3x)\cos(2x)}{4}+\frac{x\sin(2x)\sin(3x)\cos(x)}{4}+\frac{x\cos(x)\cos(2x)\cos(3x)}{4}+\frac{3\sin(x)\sin(2x)\sin(3x)}{8}+\frac{\sin(x)\cos(2x)\cos(3x)}{3}+\frac{5\sin(2x)\cos(x)\cos(3x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + 3*sin(x)*sin(2*x)*sin(3*x
)/8 + sin(x)*cos(2*x)*cos(3*x)/3 + 5*sin(2*x)*cos(x)*cos(3*x)/24
```

Giac [A]
time = 0.67, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x+\frac{1}{24}\,\sin(6\,x)+\frac{1}{16}\,\sin(4\,x)+\frac{1}{8}\,\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Mupad [B]
time = 0.28, size = 22, normalized size = 0.73

$$\frac{x}{4}+\frac{\sin(2\,x)}{8}+\frac{\sin(4\,x)}{16}+\frac{\sin(6\,x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

3.144 $\int x^2 \sin^3(kx)\,dx$

Optimal. Leaf size=85

$$\frac{14\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{2\cos^3(kx)}{27k^3}+\frac{4x\sin(kx)}{3k^2}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}$$

```
[Out] 14/9*cos(k*x)/k^3-2/3*x^2*cos(k*x)/k-2/27*cos(k*x)^3/k^3+4/3*x*sin(k*x)/k^2
-1/3*x^2*cos(k*x)*sin(k*x)^2/k+2/9*x*sin(k*x)^3/k^2
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3392, 3377, 2718, 2713}

$$-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}+\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2x^2\cos(kx)}{3k}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[k*x]^3,x]
```

```
[Out] (14*Cos[k*x])/(9*k^3) - (2*x^2*Cos[k*x])/(3*k) - (2*Cos[k*x]^3)/(27*k^3) +
(4*x*Sin[k*x])/(3*k^2) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k) + (2*x*Sin[k*x]^3)
/(9*k^2)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\sin^3(kx)\,dx = -\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}+\frac{2}{3}\int x^2\sin(kx)\,dx-\frac{2\int\sin^3(kx)\,dx}{9k^2}$$
$$= -\frac{2x^2\cos(kx)}{3k}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}+\frac{2\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos(kx)\right)}{9k^3}$$
$$= \frac{2\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{2\cos^3(kx)}{27k^3}+\frac{4x\sin(kx)}{3k^2}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}$$
$$= \frac{14\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{2\cos^3(kx)}{27k^3}+\frac{4x\sin(kx)}{3k^2}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.65

$$\frac{-81(-2+k^2x^2)\cos(kx)+(-2+9k^2x^2)\cos(3kx)-6kx(-27\sin(kx)+\sin(3kx))}{108k^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[k*x]^3,x]
[Out] (-81*(-2 + k^2*x^2)*Cos[k*x] + (-2 + 9*k^2*x^2)*Cos[3*k*x] - 6*k*x*(-27*Sin
[k*x] + Sin[3*k*x]))/(108*k^3)
```

Maple [A]
time = 0.07, size = 64, normalized size = 0.75

method	result
risch	$-\frac{3(x^2k^2-2)\cos(kx)}{4k^3}+\frac{3x\sin(kx)}{2k^2}+\frac{(9x^2k^2-2)\cos(3kx)}{108k^3}-\frac{x\sin(3kx)}{18k^2}$
derivativedivides	$\frac{-\frac{k^2x^2\left(2+\sin^2(kx)\right)\cos(kx)}{3}+\frac{4\cos(kx)}{3}+\frac{4kx\sin(kx)}{3}+\frac{2kx\left(\sin^3(kx)\right)}{9}+\frac{2\left(2+\sin^2(kx)\right)\cos(kx)}{27}}{k^3}$
default	$\frac{-\frac{k^2x^2\left(2+\sin^2(kx)\right)\cos(kx)}{3}+\frac{4\cos(kx)}{3}+\frac{4kx\sin(kx)}{3}+\frac{2kx\left(\sin^3(kx)\right)}{9}+\frac{2\left(2+\sin^2(kx)\right)\cos(kx)}{27}}{k^3}$
norman	$\frac{-\frac{2x^2}{3k}+\frac{80}{27k^3}+\frac{8x\tan\left(\frac{kx}{2}\right)}{3k^2}+\frac{64x\left(\tan^3\left(\frac{kx}{2}\right)\right)}{9k^2}+\frac{8x\left(\tan^5\left(\frac{kx}{2}\right)\right)}{3k^2}-\frac{2x^2\left(\tan^2\left(\frac{kx}{2}\right)\right)}{k}+\frac{2x^2\left(\tan^4\left(\frac{kx}{2}\right)\right)}{k}+\frac{2x^2\left(\tan^6\left(\frac{kx}{2}\right)\right)}{3k}+\frac{8(\tan}{}}{\left(1+\tan^2\left(\frac{kx}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(k*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/k^3*(-1/3*k^2*x^2*(2+sin(k*x)^2)*cos(k*x)+4/3*cos(k*x)+4/3*k*x*sin(k*x)+2
/9*k*x*sin(k*x)^3+2/27*(2+sin(k*x)^2)*cos(k*x))
```

Maxima [A]
time = 1.16, size = 55, normalized size = 0.65

$$-\frac{6\,kx\sin\left(3\,kx\right)-162\,kx\sin\left(kx\right)-\left(9\,k^2x^2-2\right)\cos\left(3\,kx\right)+81\,\left(k^2x^2-2\right)\cos\left(kx\right)}{108\,k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(6*k*x*sin(3*k*x) - 162*k*x*sin(k*x) - (9*k^2*x^2 - 2)*cos(3*k*x) +
81*(k^2*x^2 - 2)*cos(k*x))/k^3
```

Fricas [A]
time = 0.71, size = 59, normalized size = 0.69

$$\frac{\left(9\,k^2x^2-2\right)\cos\left(kx\right)^3-3\left(9\,k^2x^2-14\right)\cos\left(kx\right)-6\left(kx\cos\left(kx\right)^2-7\,kx\right)\sin\left(kx\right)}{27\,k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*k^2*x^2 - 2)*cos(k*x)^3 - 3*(9*k^2*x^2 - 14)*cos(k*x) - 6*(k*x*cos
(k*x)^2 - 7*k*x)*sin(k*x))/k^3
```

Sympy [A]
time = 0.26, size = 100, normalized size = 1.18

$$\begin{cases} -\frac{x^2\sin^2\left(kx\right)\cos\left(kx\right)}{k}-\frac{2x^2\cos^3\left(kx\right)}{3k}+\frac{14x\sin^3\left(kx\right)}{9k^2}+\frac{4x\sin\left(kx\right)\cos^2\left(kx\right)}{3k^2}+\frac{14\sin^2\left(kx\right)\cos\left(kx\right)}{9k^3}+\frac{40\cos^3\left(kx\right)}{27k^3} & \text{for}\ k\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(k*x)**3,x)
```

```
[Out] Piecewise((-x**2*sin(k*x)**2*cos(k*x)/k - 2*x**2*cos(k*x)**3/(3*k) + 14*x*s
in(k*x)**3/(9*k**2) + 4*x*sin(k*x)*cos(k*x)**2/(3*k**2) + 14*sin(k*x)**2*co
s(k*x)/(9*k**3) + 40*cos(k*x)**3/(27*k**3), Ne(k, 0)), (0, True))
```

Giac [A]
time = 1.12, size = 60, normalized size = 0.71

$$-\frac{x\sin\left(3\,kx\right)}{18\,k^2}+\frac{3\,x\sin\left(kx\right)}{2\,k^2}+\frac{\left(9\,k^2x^2-2\right)\cos\left(3\,kx\right)}{108\,k^3}-\frac{3\,\left(k^2x^2-2\right)\cos\left(kx\right)}{4\,k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="giac")
```

```
[Out] -1/18*x*sin(3*k*x)/k^2 + 3/2*x*sin(k*x)/k^2 + 1/108*(9*k^2*x^2 - 2)*cos(3*k
*x)/k^3 - 3/4*(k^2*x^2 - 2)*cos(k*x)/k^3
```

Mupad [B]
time = 0.23, size = 67, normalized size = 0.79

$$\frac{\frac{14\,\cos(k\,x)}{9}-\frac{2\cos(k\,x)^3}{27}+k\left(\frac{14\,x\,\sin(k\,x)}{9}-\frac{2\,x\cos(k\,x)^2\,\sin(k\,x)}{9}\right)+k^2\left(\frac{x^2\cos(k\,x)^3}{3}-x^2\,\cos\left(k\,x\right)\right)}{k^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(k*x)^3,x)
```

```
[Out] ((14*cos(k*x))/9 - (2*cos(k*x)^3)/27 + k*((14*x*sin(k*x))/9 - (2*x*cos(k*x)
^2*sin(k*x))/9) + k^2*((x^2*cos(k*x)^3)/3 - x^2*cos(k*x)))/k^3
```

3.145 $\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$

Optimal. Leaf size=14

$$\text{Int}(x \cos(k \csc(x)) \cot(x) \csc(x), x)$$

```
[Out] CannotIntegrate(x*cos(k*csc(x))*cot(x)*csc(x),x)
```

Rubi [A]
time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$$

Verification is not applicable to the result.

```
[In] Int[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
[Out] Defer[Int][x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Rubi steps

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx = \int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
[Out] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Maple [A]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x \cos(x) \cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

```
[Out] int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

Maxima [A] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(13) = 26$.
time = 1.90, size = 240, normalized size = 17.14

$$-\frac{\left(xe^{\left(\frac{4\,k\cos\left(2\,x\right)\cos\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1}+\frac{4\,k\sin\left(2\,x\right)\sin\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1}\right)}+xe^{\left(\frac{4\,k\cos\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1}\right)}\right)e^{\left(-\frac{2\,k\cos\left(2\,x\right)\cos\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1}-\frac{2\,k\sin\left(2\,x\right)\sin\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1}-\frac{2\,k\cos\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1}\right)}\sin\left(\frac{2\,\left(k\cos\left(x\right)\sin\left(2\,x\right)-k\cos\left(2\,x\right)\sin\left(x\right)+k\sin\left(x\right)\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}-2\,\cos\left(2\,x\right)+1}\right)}{2\,k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x*e^(4*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) +
 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + x*e^(4*k
*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)))*e^(-2*k*cos(2*x)*cos(x
)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos(2*x
)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 -
2*cos(2*x) + 1))*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin(x))/(
cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x*cos(x)*cos(k/sin(x))/(cos(x)^2 - 1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(x\right)\cos\left(\frac{k}{\sin\left(x\right)}\right)}{\sin^{2}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)
```

```
[Out] Integral(x*cos(x)*cos(k/sin(x))/sin(x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x\cos\left(\frac{k}{\sin(x)}\right)\cos\left(x\right)}{\sin\left(x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(k/sin(x))*cos(x))/sin(x)^2,x)
```

```
[Out] int((x*cos(k/sin(x))*cos(x))/sin(x)^2, x)
```

3.146 $\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx$

Optimal. Leaf size=12

$$-x-\cot\left(\frac{x}{2}\right)$$

```
[Out] -x-cot(1/2*x)
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 464, 209}

$$-x-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x/2]*Cot[x],x]
```

```
[Out] -x - Cot[x/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx &= 2\text{Subst}\left(\int \frac{1-x^2}{2x^2(1+x^2)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \text{Subst}\left(\int \frac{1-x^2}{x^2(1+x^2)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\cot\left(\frac{x}{2}\right) - 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -x-\cot\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-x-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x/2]*Cot[x],x]
[Out] -x - Cot[x/2]
```

Maple [A]
time = 0.07, size = 11, normalized size = 0.92

method	result	size
default	$-x-\cot\left(\frac{x}{2}\right)$	11
norman	$\frac{-1-x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}$	17
risch	$-x-\frac{2i}{e^{ix}-1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)/tan(1/2*x),x,method=_RETURNVERBOSE)
[Out] -x-cot(1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(10) = 20.
time = 3.42, size = 41, normalized size = 3.42

$$-\frac{x\cos(x)^2+x\sin(x)^2-2x\cos(x)+x+2\sin(x)}{\cos(x)^2+\sin(x)^2-2\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="maxima")
```

```
[Out] -(x*cos(x)^2 + x*sin(x)^2 - 2*x*cos(x) + x + 2*sin(x))/(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1)
```

Fricas [A]
time = 0.50, size = 16, normalized size = 1.33

$$-\frac{x\tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="fricas")
```

```
[Out] -(x*tan(1/2*x) + 1)/tan(1/2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(x\right)}{\sin\left(x\right)\tan\left(\frac{x}{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x)
```

```
[Out] Integral(cos(x)/(sin(x)*tan(x/2)), x)
```

Giac [A]
time = 1.32, size = 18, normalized size = 1.50

$$-x-\frac{1}{2\,\tan\left(\frac{1}{4}\,x\right)}+\frac{1}{2}\,\tan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/4*x) + 1/2*tan(1/4*x)
```

Mupad [B]
time = 0.18, size = 10, normalized size = 0.83

$$-x-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(tan(x/2)*sin(x)),x)
```

```
[Out] - x - cot(x/2)
```

3.147 $\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2c\tan^{-1}\left(\frac{c+b\tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(b+c\sin(ax)\right)}$$

```
[Out] -2*c*arctan((c+b*tan(1/2*a*x))/(b^2-c^2)^(1/2))/a/(b^2-c^2)^(3/2)-b*cos(a*x
)/a/(b^2-c^2)/(b+c*sin(a*x))
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2833, 12, 2739, 632, 210}

$$-\frac{2c\text{ArcTan}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)\left(c\sin(ax)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] (-2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(a*(b^2 - c^2)^(3/2)) -
 (b*Cos[a*x])/(a*(b^2 - c^2)*(b + c*Sin[a*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx &= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}+\frac{\int \frac{c}{b+c\sin(ax)}\,dx}{-b^2+c^2}\\
&= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}-\frac{c\int \frac{1}{b+c\sin(ax)}\,dx}{b^2-c^2}\\
&= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}-\frac{(2c)\text{Subst}\left(\int \frac{1}{b+2cx+bx^2}\,dx,x,\tan\left(\frac{ax}{2}\right)\right)}{a\left(b^2-c^2\right)}\\
&= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}+\frac{(4c)\text{Subst}\left(\int \frac{1}{-4(b^2-c^2)-x^2}\,dx,x,2c+2b\tan\left(\frac{ax}{2}\right)\right)}{a\left(b^2-c^2\right)}\\
&= -\frac{2c\tan^{-1}\left(\frac{c+b\tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 76, normalized size = 0.99

$$
-\frac{\frac{2c\tan^{-1}\left(\frac{c+b\tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{b\cos(ax)}{(b-c)(b+c)(b+c\sin(ax))}}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] -(((2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (
b*Cos[a*x])/((b - c)*(b + c)*(b + c*Sin[a*x])))/a)
```

Maple [A]
time = 0.15, size = 107, normalized size = 1.39

method	result
derivativedivides	$\frac{\frac{-8c\tan\left(\frac{ax}{2}\right)-8b}{\left(4b^2-4c^2\right)\left(b\left(\tan^2\left(\frac{ax}{2}\right)\right)+2c\tan\left(\frac{ax}{2}\right)+b\right)}-\frac{8c\arctan\left(\frac{2b\tan\left(\frac{ax}{2}\right)+2c}{2\sqrt{b^2-c^2}}\right)}{\left(4b^2-4c^2\right)\sqrt{b^2-c^2}}}{a}$
default	$\frac{\frac{-8c\tan\left(\frac{ax}{2}\right)-8b}{\left(4b^2-4c^2\right)\left(b\left(\tan^2\left(\frac{ax}{2}\right)\right)+2c\tan\left(\frac{ax}{2}\right)+b\right)}-\frac{8c\arctan\left(\frac{2b\tan\left(\frac{ax}{2}\right)+2c}{2\sqrt{b^2-c^2}}\right)}{\left(4b^2-4c^2\right)\sqrt{b^2-c^2}}}{a}$
risch	$\frac{2ib(c-ib\,\mathrm{e}^{iax})}{c(-b^2+c^2)a(c\,\mathrm{e}^{2iax}-c+2ib\,\mathrm{e}^{iax})}-\frac{ic\ln\left(\mathrm{e}^{iax}+\frac{i\left(b\sqrt{b^2-c^2}+b^2-c^2\right)}{\sqrt{b^2-c^2}\,c}\right)}{\sqrt{b^2-c^2}\,(b+c)(b-c)a}+\frac{ic\ln\left(\mathrm{e}^{iax}+\frac{i\left(b\sqrt{b^2-c^2}-b^2+c^2\right)}{\sqrt{b^2-c^2}\,c}\right)}{\sqrt{b^2-c^2}\,(b+c)(b-c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)/(b+c*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(4*(-2*c*tan(1/2*a*x)-2*b)/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*
a*x)+b)-8*c/(4*b^2-4*c^2)/(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*a*x)+2*c)
/(b^2-c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.52, size = 312, normalized size = 4.05

$$\left[\frac{\left(c^2\sin(ax)+bc\right)\sqrt{-b^2+c^2}\,\log\left(\frac{\left(2b^2-c^2\right)\cos(ax)^2-2bc\sin(ax)-b^2-c^2+2\left(b\cos(ax)\sin(ax)+c\cos(ax)\right)\sqrt{-b^2+c^2}}{c^2\cos(ax)^2-2bc\sin(ax)-b^2-c^2}\right)-2\left(b^3-bc^2\right)\cos(ax)}{2\left(ab^5-2ab^3c^2+abc^4+\left(ab^4c-2ab^2c^3+ac^5\right)\sin(ax)\right)},\ \frac{\left(c^2\sin(ax)+bc\right)\sqrt{b^2-c^2}\,\arctan\left(-\frac{b\sin(ax)+c}{\sqrt{b^2-c^2}\cos(ax)}\right)-\left(b^3-bc^2\right)\cos(ax)}{ab^5-2ab^3c^2+abc^4+\left(ab^4c-2ab^2c^3+ac^5\right)\sin(ax)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*sin(a*x) + b*c)*sqrt(-b^2 + c^2)*log(((2*b^2 - c^2)*cos(a*x)^2 -
 2*b*c*sin(a*x) - b^2 - c^2 + 2*(b*cos(a*x)*sin(a*x) + c*cos(a*x))*sqrt(-b^
```

```
2 + c^2))/(c^2*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)) - 2*(b^3 - b*c^2)*
cos(a*x))/(a*b^5 - 2*a*b^3*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*
sin(a*x)), ((c^2*sin(a*x) + b*c)*sqrt(b^2 - c^2)*arctan(-(b*sin(a*x) + c)/(
sqrt(b^2 - c^2)*cos(a*x))) - (b^3 - b*c^2)*cos(a*x))/(a*b^5 - 2*a*b^3*c^2 +
 a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*sin(a*x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.75, size = 98, normalized size = 1.27

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ax}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ax\right)+c}{\sqrt{b^2-c^2}}\right)\right)c}{\left(b^2-c^2\right)^{\frac{3}{2}}}+\frac{c\tan\left(\frac{1}{2}ax\right)+b}{\left(b\tan\left(\frac{1}{2}ax\right)^2+2c\tan\left(\frac{1}{2}ax\right)+b\right)\left(b^2-c^2\right)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*a*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*a*x) + c)/sqrt(b
^2 - c^2)))*c/(b^2 - c^2)^(3/2) + (c*tan(1/2*a*x) + b)/((b*tan(1/2*a*x)^2 +
 2*c*tan(1/2*a*x) + b)*(b^2 - c^2)))/a
```

Mupad [**B**]
time = 0.49, size = 133, normalized size = 1.73

$$-\frac{\frac{2b}{b^2-c^2}+\frac{2c\tan\left(\frac{ax}{2}\right)}{b^2-c^2}}{a\left(b\tan\left(\frac{ax}{2}\right)^2+2c\tan\left(\frac{ax}{2}\right)+b\right)}-\frac{2c\,\mathrm{atan}\left(\frac{\left(\frac{2c^2}{(b+c)^{3/2}(b-c)^{3/2}}+\frac{2bc\tan\left(\frac{ax}{2}\right)}{(b+c)^{3/2}(b-c)^{3/2}}\right)\left(b^2-c^2\right)}{2c}\right)}{a\left(b+c\right)^{3/2}\left(b-c\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)/(b + c*sin(a*x))^2,x)
```

```
[Out] - ((2*b)/(b^2 - c^2) + (2*c*tan((a*x)/2))/(b^2 - c^2))/(a*(b + 2*c*tan((a*x
)/2) + b*tan((a*x)/2)^2)) - (2*c*atan((((2*c^2)/((b + c)^(3/2)*(b - c)^(3/2
)) + (2*b*c*tan((a*x)/2))/((b + c)^(3/2)*(b - c)^(3/2)))*(b^2 - c^2))/(2*c)
))/(a*(b + c)^(3/2)*(b - c)^(3/2))
```

3.148 $\int \sin(\log(x))\,dx$

Optimal. Leaf size=17

$$-\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {4563}

$$\frac{1}{2}x\sin(\log(x))-\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]],x]
```

```
[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

Rule 4563

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \sin(\log(x))\,dx=-\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]],x]
```

```
[Out] -1/2*(x*Cos[Log[x]]) + (x*Sin[Log[x]])/2
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.82

method	result	size
lookup	$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(-\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(-\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22
norman	$\frac{x\tan\left(\frac{\ln(x)}{2}\right)-\frac{x}{2}+\frac{x\left(\tan^{2}\left(\frac{\ln(x)}{2}\right)\right)}{2}}{1+\tan^{2}\left(\frac{\ln(x)}{2}\right)}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A]
time = 1.27, size = 12, normalized size = 0.71

$$-\frac{1}{2}\,x(\cos\left(\log\left(x\right)\right)-\sin\left(\log\left(x\right)\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

Fricas [A]
time = 0.41, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A]
time = 0.11, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}-\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

Giac [**A**]
time = 0.78, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Mupad [**B**]
time = 0.13, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2}\,x\,\cos\left(\frac{\pi}{4}+\ln\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(log(x)),x)
```

```
[Out] -(2^(1/2)*x*cos(pi/4 + log(x)))/2
```

3.149 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4564}

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4564

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(\log(x))\,dx=\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.82

method	result	size
lookup	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
default	$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$	14
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)x\,x^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)x\,x^{-i}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A]
time = 0.86, size = 10, normalized size = 0.59

$$\frac{1}{2}\,x(\cos\left(\log\left(x\right)\right)+\sin\left(\log\left(x\right)\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A]
time = 0.45, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A]
time = 0.11, size = 15, normalized size = 0.88

$$\frac{x\sin\left(\log\left(x\right)\right)}{2}+\frac{x\cos\left(\log\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Giac [A]
time = 0.85, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Mupad [B]
time = 0.15, size = 13, normalized size = 0.76

$$\frac{\sqrt{2}\,x\,\sin\left(\frac{\pi}{4}+\ln\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(log(x)),x)
```

```
[Out] (2^(1/2)*x*sin(pi/4 + log(x)))/2
```

3.150 $\int e^x \, dx$

Optimal. Leaf size=3

$$e^x$$

```
[Out] exp(x)
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x,x]
```

```
[Out] E^x
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x \, dx = e^x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x,x]
```

```
[Out] E^x
```

Maple [A]
time = 0.01, size = 3, normalized size = 1.00

method	result	size

gosper	e^x	3
lookup	e^x	3
derivativedivides	e^x	3
default	e^x	3
norman	e^x	3
risch	e^x	3
meijerg	$-1+e^x$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)
```

Maxima [A]
time = 1.10, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="maxima")
```

```
[Out] e^x
```

Fricas [A]
time = 0.40, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="fricas")
```

```
[Out] e^x
```

Sympy [A]
time = 0.02, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
```

```
[Out] exp(x)
```

Giac [**A**]
time = 0.71, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="giac")
```

```
[Out] e^x
```

Mupad [**B**]
time = 0.01, size = 2, normalized size = 0.67

$$\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

3.151 $\int a^x\,dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

```
[Out] a^x/ln(a)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2225}

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x,x]
```

```
[Out] a^x/Log[a]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x,x]
```

```
[Out] a^x/Log[a]
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size
gosper	$\frac{a^{x}}{\ln(a)}$	9
derivativedivides	$\frac{a^{x}}{\ln(a)}$	9
default	$\frac{a^{x}}{\ln(a)}$	9
risch	$\frac{a^{x}}{\ln(a)}$	9
norman	$\frac{e^{x\ln(a)}}{\ln(a)}$	11
meijerg	$-\frac{1-e^{x\ln(a)}}{\ln(a)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x,method=_RETURNVERBOSE)
```

```
[Out] a^x/ln(a)
```

Maxima [A]
time = 1.55, size = 8, normalized size = 1.00

$$\frac{a^{x}}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="maxima")
```

```
[Out] a^x/log(a)
```

Fricas [A]
time = 0.67, size = 8, normalized size = 1.00

$$\frac{a^{x}}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="fricas")
```

```
[Out] a^x/log(a)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 1.00

$$\begin{cases} \frac{a^{x}}{\log(a)} & \text{for}\ \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x,x)
```

```
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

Giac [**A**]
time = 0.65, size = 8, normalized size = 1.00

$$\frac{a^x}{\log{(a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="giac")
```

```
[Out] a^x/log(a)
```

Mupad [**B**]
time = 0.16, size = 8, normalized size = 1.00

$$\frac{a^x}{\ln{(a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
```

```
[Out] a^x/log(a)
```

3.152 $\int e^{ax}\,dx$

Optimal. Leaf size=9

$$\frac{e^{ax}}{a}$$

```
[Out] exp(a*x)/a
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{2225\}$

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x),x]
```

```
[Out] E^(a*x)/a
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x),x]
```

```
[Out] E^(a*x)/a
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.00

method	result	size
gosper	$\frac{e^{ax}}{a}$	9
derivativedivides	$\frac{e^{ax}}{a}$	9
default	$\frac{e^{ax}}{a}$	9
norman	$\frac{e^{ax}}{a}$	9
risch	$\frac{e^{ax}}{a}$	9
meijerg	$-\frac{1-e^{ax}}{a}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(a*x)/a
```

Maxima [A]
time = 0.64, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x, algorithm="maxima")
```

```
[Out] e^(a*x)/a
```

Fricas [A]
time = 1.14, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x, algorithm="fricas")
```

```
[Out] e^(a*x)/a
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\begin{cases} \frac{e^{ax}}{a} & \text{for}\ a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x)
```

```
[Out] Piecewise((exp(a*x)/a, Ne(a, 0)), (x, True))
```

Giac [A]
time = 0.69, size = 8, normalized size = 0.89

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x, algorithm="giac")
```

```
[Out] e^(a*x)/a
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.89

$$\frac{\mathrm{e}^{a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x),x)
```

```
[Out] exp(a*x)/a
```

3.153 $\int \frac{e^{ax}}{x} dx$

Optimal. Leaf size=4

$$\operatorname{Ei}(ax)$$

[Out] Ei(a*x)

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2209}

$$\operatorname{ExpIntegralEi}(ax)$$

Antiderivative was successfully verified.

[In] Int[E^(a*x)/x,x]

[Out] ExpIntegralEi[a*x]

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\int \frac{e^{ax}}{x} dx = \operatorname{Ei}(ax)$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\operatorname{Ei}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[E^(a*x)/x,x]

[Out] ExpIntegralEi[a*x]

Maple [A]
time = 0.01, size = 9, normalized size = 2.25

method	result	size
derivativedivides	$-\operatorname{expIntegral}(1,-ax)$	9
default	$-\operatorname{expIntegral}(1,-ax)$	9
risch	$-\operatorname{expIntegral}(1,-ax)$	9
meijerg	$\ln(x)+\ln(-a)-\ln(-ax)-\operatorname{expIntegral}(1,-ax)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)/x,x,method=_RETURNVERBOSE)
[Out] -Ei(1,-a*x)
```

Maxima [A]
time = 3.62, size = 4, normalized size = 1.00

$$\mathrm{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="maxima")
[Out] Ei(a*x)
```

Fricas [A]
time = 1.12, size = 4, normalized size = 1.00

$$\mathrm{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="fricas")
[Out] Ei(a*x)
```

Sympy [A]
time = 0.34, size = 3, normalized size = 0.75

$$\mathrm{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x)
[Out] Ei(a*x)
```

Giac [A]
time = 0.65, size = 4, normalized size = 1.00

$$\mathrm{Ei}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="giac")
```

```
[Out] Ei(a*x)
```

Mupad [B]
time = 0.01, size = 4, normalized size = 1.00

$$\mathrm{ei}(a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)/x,x)
```

```
[Out] ei(a*x)
```

3.154 $\int \frac{1}{a+be^{mx}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

```
[Out] x/a-ln(a+b*exp(m*x))/a/m
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {2320, 36, 29, 31}

$$\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(m*x))^(-1),x]
```

```
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\int \frac{1}{a+be^{mx}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, e^{mx}\right)}{m}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{mx}\right)}{am} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{mx}\right)}{am}$$
$$= \frac{x}{a} - \frac{\log(a+be^{mx})}{am}$$

Mathematica [**A**]
time = 0.02, size = 38, normalized size = 1.58

$$\frac{\log(e^{mx})}{am} - \frac{\log(a^2m + abe^{mx}m)}{am}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(m*x))^(-1),x]
```

```
[Out] Log[E^(m*x)]/(a*m) - Log[a^2*m + a*b*E^(m*x)*m]/(a*m)
```

Maple [**A**]
time = 0.01, size = 29, normalized size = 1.21

method	result	size
norman	$\frac{x}{a} - \frac{\ln(a+b\,e^{mx})}{am}$	24
risch	$\frac{x}{a} - \frac{\ln(e^{mx}+\frac{a}{b})}{am}$	26
derivativedivides	$\frac{-\frac{\ln(a+b\,e^{mx})}{a}+\frac{\ln(e^{mx})}{a}}{m}$	29
default	$\frac{-\frac{\ln(a+b\,e^{mx})}{a}+\frac{\ln(e^{mx})}{a}}{m}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(m*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/m*(-1/a*ln(a+b*exp(m*x))+1/a*ln(exp(m*x)))
```

Maxima [**A**]
time = 2.64, size = 23, normalized size = 0.96

$$\frac{x}{a} - \frac{\log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="maxima")
```

```
[Out] x/a - log(b*e^(m*x) + a)/(a*m)
```

Fricas [A]
time = 0.62, size = 22, normalized size = 0.92

$$\frac{mx - \log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="fricas")
```

```
[Out] (m*x - log(b*e^(m*x) + a))/(a*m)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.62

$$\frac{x}{a} - \frac{\log\left(\frac{a}{b} + e^{mx}\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x)
```

```
[Out] x/a - log(a/b + exp(m*x))/(a*m)
```

Giac [A]
time = 0.92, size = 26, normalized size = 1.08

$$\frac{\frac{mx}{a} - \frac{\log\left(\left|be^{(mx)} + a\right|\right)}{a}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="giac")
```

```
[Out] (m*x/a - log(abs(b*e^(m*x) + a))/a)/m
```

Mupad [B]
time = 0.09, size = 22, normalized size = 0.92

$$-\frac{\ln\left(a + b\,\mathrm{e}^{m\,x}\right) - m\,x}{a\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*exp(m*x)),x)
```

```
[Out] -(log(a + b*exp(m*x)) - m*x)/(a*m)
```

3.155 $\int \frac{e^{2x}}{1+e^x} \, dx$

Optimal. Leaf size=12

$$e^x - \log(1 + e^x)$$

```
[Out] exp(x)-ln(1+exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2280, 45}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{1+e^x} \, dx &= \text{Subst}\left(\int \frac{x}{1+x} \, dx, x, e^x\right) \\ &= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) \, dx, x, e^x\right) \\ &= e^x - \log(1+e^x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$e^x - \log(1 + e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
default	$e^x - \ln(1 + e^x)$	11
norman	$e^x - \ln(1 + e^x)$	11
risch	$e^x - \ln(1 + e^x)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)-ln(1+exp(x))
```

Maxima [A]
time = 3.15, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

Fricas [A]
time = 0.86, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

Giac [A]
time = 0.79, size = 10, normalized size = 0.83

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

Mupad [B]
time = 0.04, size = 10, normalized size = 0.83

$$\mathrm{e}^x - \ln(\mathrm{e}^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(exp(x) + 1),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

3.156 $\int e^{2x+ax}\,dx$

Optimal. Leaf size=13

$$\frac{e^{(2+a)x}}{2+a}$$

```
[Out] exp((2+a)*x)/(2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2259, 2225}

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x + a*x),x]
```

```
[Out] E^((2 + a)*x)/(2 + a)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2259

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x+ax}\,dx &= \int e^{(2+a)x}\,dx\\ &= \frac{e^{(2+a)x}}{2+a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{e^{(2+a)x}}{2+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x + a*x),x]
```

```
[Out] E^((2 + a)*x)/(2 + a)
```

Maple [A]
time = 0.01, size = 15, normalized size = 1.15

method	result	size
risch	$\frac{e^{(2+a)x}}{2+a}$	13
gosper	$\frac{e^{ax+2x}}{2+a}$	15
derivativedivides	$\frac{e^{ax+2x}}{2+a}$	15
default	$\frac{e^{ax+2x}}{2+a}$	15
norman	$\frac{e^{ax+2x}}{2+a}$	15
meijerg	$\frac{1-e^{-x(-a-2)}}{-a-2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2+a)*exp(a*x+2*x)
```

Maxima [A]
time = 3.26, size = 14, normalized size = 1.08

$$\frac{e^{(ax+2\,x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="maxima")
```

```
[Out] e^(a*x + 2*x)/(a + 2)
```

Fricas [A]
time = 0.85, size = 12, normalized size = 0.92

$$\frac{e^{((a+2)x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="fricas")
```

```
[Out] e^((a + 2)*x)/(a + 2)
```

Sympy [A]
time = 0.02, size = 14, normalized size = 1.08

$$\begin{cases} \frac{e^{ax+2x}}{a+2} & \text{for } a \neq -2 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x)
[Out] Piecewise((exp(a*x + 2*x)/(a + 2), Ne(a, -2)), (x, True))
```

Giac [A]
time = 0.77, size = 14, normalized size = 1.08

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="giac")
[Out] e^(a*x + 2*x)/(a + 2)
```

Mupad [B]
time = 0.05, size = 14, normalized size = 1.08

$$\frac{\mathrm{e}^{2x+ax}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x + a*x),x)
[Out] exp(2*x + a*x)/(a + 2)
```

3.157 $\int \frac{1}{be^{-mx}+ae^{mx}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}$$

```
[Out] arctan(exp(m*x)*a^(1/2)/b^(1/2))/m/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2320, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{be^{-mx}+ae^{mx}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, e^{mx}\right)}{m}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\,e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\,\sqrt{b}\,m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(m*x) + a*E^(m*x))^(-1),x]
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{a\,e^{mx}}{\sqrt{ab}}\right)}{m\sqrt{ab}}$	22
default	$\frac{\arctan\left(\frac{a\,e^{mx}}{\sqrt{ab}}\right)}{m\sqrt{ab}}$	22
risch	$-\frac{\ln\left(e^{mx}-\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,m}+\frac{\ln\left(e^{mx}+\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,m}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(m*x)+a*exp(m*x)),x,method=_RETURNVERBOSE)
[Out] 1/m/(a*b)^(1/2)*arctan(a*exp(m*x)/(a*b)^(1/2))
```

Maxima [A]
time = 6.53, size = 23, normalized size = 0.74

$$-\frac{\arctan\left(\frac{be^{(-mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="maxima")
[Out] -arctan(b*e^(-m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

Fricas [A]
time = 1.01, size = 85, normalized size = 2.74

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ae^{(2\,mx)}-2\,\sqrt{-ab}\,e^{(mx)}-b}{ae^{(2\,mx)}+b}\right)}{2\,abm}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,e^{(-mx)}}{a}\right)}{abm}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((a*e^(2*m*x) - 2*sqrt(-a*b)*e^(m*x) - b)/(a*e^(2*m*x)
+ b))/(a*b*m), -sqrt(a*b)*arctan(sqrt(a*b)*e^(-m*x)/a)/(a*b*m)]
```

Sympy [A]
time = 0.07, size = 24, normalized size = 0.77

$$\frac{\operatorname{RootSum}\left(4z^2ab+1,\left(i\mapsto i\log\left(2ib+e^{mx}\right)\right)\right)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x)
```

```
[Out] RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(2*_i*b + exp(m*x))))/m
```

Giac [A]
time = 0.83, size = 21, normalized size = 0.68

$$\frac{\arctan\left(\frac{ae^{(mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="giac")
```

```
[Out] arctan(a*e^(m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

Mupad [B]
time = 0.23, size = 21, normalized size = 0.68

$$\frac{\mathrm{atan}\left(\frac{a\,{\mathrm{e}}^{m\,x}}{\sqrt{a\,b}}\right)}{m\,\sqrt{a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*exp(m*x) + b*exp(-m*x)),x)
```

```
[Out] atan((a*exp(m*x))/(a*b)^(1/2))/(m*(a*b)^(1/2))
```

3.158 $\int e^{ax} x \, dx$

Optimal. Leaf size=21

$$-\frac{e^{ax}}{a^2}+\frac{e^{ax}x}{a}$$

```
[Out] -exp(a*x)/a^2+exp(a*x)*x/a
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2207, 2225}

$$\frac{xe^{ax}}{a}-\frac{e^{ax}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*x,x]
```

```
[Out] -(E^(a*x)/a^2) + (E^(a*x)*x)/a
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{ax}x\,dx &= \frac{e^{ax}x}{a}-\frac{\int e^{ax}\,dx}{a}\\ &= -\frac{e^{ax}}{a^2}+\frac{e^{ax}x}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.67

$$\frac{e^{ax}(-1+ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*x,x]
```

```
[Out] (E^(a*x)*(-1 + a*x))/a^2
```

Maple [A]
time = 0.01, size = 19, normalized size = 0.90

method	result	size
gosper	$\frac{(ax-1)\mathrm{e}^{ax}}{a^2}$	14
risch	$\frac{(ax-1)\mathrm{e}^{ax}}{a^2}$	14
derivativedivides	$\frac{a\,\mathrm{e}^{ax}x-\mathrm{e}^{ax}}{a^2}$	19
default	$\frac{a\,\mathrm{e}^{ax}x-\mathrm{e}^{ax}}{a^2}$	19
meijerg	$\frac{1-\frac{(-2ax+2)\mathrm{e}^{ax}}{2}}{a^2}$	19
norman	$-\frac{\mathrm{e}^{ax}}{a^2}+\frac{\mathrm{e}^{ax}x}{a}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(a*exp(a*x)*x-exp(a*x))
```

Maxima [A]
time = 5.63, size = 13, normalized size = 0.62

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="maxima")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

Fricas [A]
time = 1.04, size = 13, normalized size = 0.62

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="fricas")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.90

$$\begin{cases} \frac{(ax-1)e^{ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x)
[Out] Piecewise(((a*x - 1)*exp(a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))
```

Giac [A]
time = 1.10, size = 13, normalized size = 0.62

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="giac")
[Out] (a*x - 1)*e^(a*x)/a^2
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.62

$$\frac{e^{a\,x}\,(a\,x-1)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(a*x),x)
[Out] (exp(a*x)*(a*x - 1))/a^2
```

3.159 $\int e^x x^{20}\,dx$

Optimal. Leaf size=163

$$2432902008176640000e^x - 2432902008176640000e^x x + 1216451004088320000e^x x^2 - 405483668029440000e^x x$$

```
[Out] 2432902008176640000*exp(x)-2432902008176640000*exp(x)*x+1216451004088320000
*exp(x)*x^2-405483668029440000*exp(x)*x^3+101370917007360000*exp(x)*x^4-202
74183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*exp(x
)*x^7+60339831552000*exp(x)*x^8-6704425728000*exp(x)*x^9+670442572800*exp(x
)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)*x^13
+27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp(x)*x^
17+380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20
```

Rubi [A]
time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2207, 2225}

$$e^x x^{20} - 20e^x x^{19} + 380e^x x^{18} - 6840e^x x^{17} + 116280e^x x^{16} - 1860480e^x x^{15} + 27907200e^x x^{14} - 390700800e^x x^{13} + 5079110400e^x x^{12} - 60949324800e^x x^{11} + 670442572800e^x x^{10} - 6704425728000e^x x^9 + 60339831552000e^x x^8 - 482718652416000e^x x^7 + 3379030566912000e^x x^6 - 20274183401472000e^x x^5 + 101370917007360000e^x x^4 - 405483668029440000e^x x^3 + 1216451004088320000e^x x^2 - 2432902008176640000e^x x + 2432902008176640000e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^20,x]
```

```
[Out] 2432902008176640000*E^x - 2432902008176640000*E^x*x + 1216451004088320000*E
^x*x^2 - 405483668029440000*E^x*x^3 + 101370917007360000*E^x*x^4 - 20274183
401472000*E^x*x^5 + 3379030566912000*E^x*x^6 - 482718652416000*E^x*x^7 + 60
339831552000*E^x*x^8 - 6704425728000*E^x*x^9 + 670442572800*E^x*x^10 - 6094
9324800*E^x*x^11 + 5079110400*E^x*x^12 - 390700800*E^x*x^13 + 27907200*E^x*
x^14 - 1860480*E^x*x^15 + 116280*E^x*x^16 - 6840*E^x*x^17 + 380*E^x*x^18 -
20*E^x*x^19 + E^x*x^20
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x x^{20}\,dx = e^x x^{20} - 20\int e^x x^{19}\,dx$$
$$= -20e^x x^{19} + e^x x^{20} + 380\int e^x x^{18}\,dx$$
$$= 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 6840\int e^x x^{17}\,dx$$
$$= -6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} + 116280\int e^x x^{16}\,dx$$
$$= 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 1860480\int e^x x^{15}\,dx$$
$$= -1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} + 27907200\int e^x x^{14}$$
$$= 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 3$$
$$= -390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} -$$
$$= 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x$$
$$= -60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} +$$
$$= 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x$$
$$= -6704425728000e^x x^{9} + 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x^{12} - 39070$$
$$= 60339831552000e^x x^{8} - 6704425728000e^x x^{9} + 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079$$
$$= -482718652416000e^x x^{7} + 60339831552000e^x x^{8} - 6704425728000e^x x^{9} + 670442572800e^x x^{10} -$$
$$= 3379030566912000e^x x^{6} - 482718652416000e^x x^{7} + 60339831552000e^x x^{8} - 6704425728000e^x x^{9}$$
$$= -20274183401472000e^x x^{5} + 3379030566912000e^x x^{6} - 482718652416000e^x x^{7} + 6033983155200$$
$$= 101370917007360000e^x x^{4} - 20274183401472000e^x x^{5} + 3379030566912000e^x x^{6} - 48271865241$$
$$= -405483668029440000e^x x^{3} + 101370917007360000e^x x^{4} - 20274183401472000e^x x^{5} + 3379030$$
$$= 1216451004088320000e^x x^{2} - 405483668029440000e^x x^{3} + 101370917007360000e^x x^{4} - 2027418$$
$$= -2432902008176640000e^x x + 1216451004088320000e^x x^{2} - 405483668029440000e^x x^{3} + 10137$$
$$= 2432902008176640000e^x - 2432902008176640000e^x x + 1216451004088320000e^x x^{2} - 4054836$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 0.63

$$e^x(2432902008176640000 - 2432902008176640000x + 1216451004088320000x^2 - 405483668029440000x^3 + 101370917007360000x^4 - 20274183401472000x^5 + 3379030566912000x^6 - 482718652416000x^7 + 60339831552000x^8 - 6704425728000x^9 + 670442572800x^{10} - 60949324800x^{11} + 5079110400x^{12} - 390700800x^{13} + 27907200x^{14} - 1860480x^{15} + 116280x^{16} - 6840x^{17} + 380x^{18} - 20x^{19} + x^{20})$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^20,x]
```

```
[Out] E^x*(2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2
- 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 +
 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 67044257
28000*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700
800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^1
8 - 20*x^19 + x^20)
```

Maple [A]
time = 0.04, size = 143, normalized size = 0.88

method	result
gosper	$(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} +$
risch	$(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} +$
meijerg	$-2432902008176640000 + \frac{(21x^{20}-420x^{19}+7980x^{18}-143640x^{17}+2441880x^{16}-39070080x^{15}+586051200x^{14}-820471680}{}$
default	$2432902008176640000\,\mathrm{e}^x - 2432902008176640000\,\mathrm{e}^x x + 1216451004088320000\,\mathrm{e}^x x^2 - 4054836$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^20,x,method=_RETURNVERBOSE)
```

```
[Out] 2432902008176640000*exp(x)-2432902008176640000*exp(x)*x+1216451004088320000
*exp(x)*x^2-405483668029440000*exp(x)*x^3+101370917007360000*exp(x)*x^4-202
74183401472000*exp(x)*x^5+3379030566912000*exp(x)*x^6-482718652416000*exp(x
)*x^7+60339831552000*exp(x)*x^8-6704425728000*exp(x)*x^9+670442572800*exp(x
)*x^10-60949324800*exp(x)*x^11+5079110400*exp(x)*x^12-390700800*exp(x)*x^13
+27907200*exp(x)*x^14-1860480*exp(x)*x^15+116280*exp(x)*x^16-6840*exp(x)*x^
17+380*exp(x)*x^18-20*exp(x)*x^19+exp(x)*x^20
```

Maxima [A]
time = 3.59, size = 101, normalized size = 0.62

$$(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 60949324800\,x^{11} + 670442572800\,x^{10} - 6704425728000\,x^9 + 60339831552000\,x^8 - 482718652416000\,x^7 + 3379030566912000\,x^6 - 20274183401472000\,x^5 + 101370917007360000\,x^4 - 405483668029440000\,x^3 + 1216451004088320000\,x^2 - 2432902008176640000\,x + 2432902008176640000)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="maxima")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

Fricas [A]
time = 0.71, size = 101, normalized size = 0.62

$$(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 60949324800\,x^{11} + 670442572800\,x^{10} - 6704425728000\,x^{9} + 60339831552000\,x^{8} - 482718652416000\,x^{7} + 3379030566912000\,x^{6} - 20274183401472000\,x^{5} + 101370917007360000\,x^{4} - 405483668029440000\,x^{3} + 1216451004088320000\,x^{2} - 2432902008176640000\,x + 2432902008176640000)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="fricas")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

Sympy [A]
time = 0.04, size = 102, normalized size = 0.63

$$\left(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60949324800x^{11} + 670442572800x^{10} - 6704425728000x^{9} + 60339831552000x^{8} - 482718652416000x^{7} + 3379030566912000x^{6} - 20274183401472000x^{5} + 101370917007360000x^{4} - 405483668029440000x^{3} + 1216451004088320000x^{2} - 2432902008176640000x + 2432902008176640000\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**20,x)
```

```
[Out] (x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 1860480*x**15 +
 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 - 60949324800*x**11 +
670442572800*x**10 - 6704425728000*x**9 + 60339831552000*x**8 - 48271865241
6000*x**7 + 3379030566912000*x**6 - 20274183401472000*x**5 + 10137091700736
0000*x**4 - 405483668029440000*x**3 + 1216451004088320000*x**2 - 2432902008
176640000*x + 2432902008176640000)*exp(x)
```

Giac [A]
time = 0.86, size = 101, normalized size = 0.62

$$(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 60949324800\,x^{11} + 670442572800\,x^{10} - 6704425728000\,x^{9} + 60339831552000\,x^{8} - 482718652416000\,x^{7} + 3379030566912000\,x^{6} - 20274183401472000\,x^{5} + 101370917007360000\,x^{4} - 405483668029440000\,x^{3} + 1216451004088320000\,x^{2} - 2432902008176640000\,x + 2432902008176640000)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="giac")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
```

```
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

Mupad [B]
time = 0.36, size = 101, normalized size = 0.62

$$e^x\,\left(x^{20}-20\,x^{19}+380\,x^{18}-6840\,x^{17}+116280\,x^{16}-1860480\,x^{15}+27907200\,x^{14}-390700800\,x^{13}+5079110400\,x^{12}-60949324800\,x^{11}+670442572800\,x^{10}-6704425728000\,x^9+60339831552000\,x^8-482718652416000\,x^7+3379030566912000\,x^6-20274183401472000\,x^5+101370917007360000\,x^4-405483668029440000\,x^3+1216451004088320000\,x^2-2432902008176640000\,x+2432902008176640000\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^20*exp(x),x)
```

```
[Out] exp(x)*(1216451004088320000*x^2 - 2432902008176640000*x - 40548366802944000
0*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 + 3379030566912000*x
^6 - 482718652416000*x^7 + 60339831552000*x^8 - 6704425728000*x^9 + 6704425
72800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700800*x^13 + 27907200
*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^18 - 20*x^19 + x^20
+ 2432902008176640000)
```

3.160 $\int a^x b^{-x}\,dx$

Optimal. Leaf size=18

$$\frac{a^x b^{-x}}{\log(a)-\log(b)}$$

```
[Out] a^x/(b^x)/(ln(a)-ln(b))
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2325, 2225}

$$\frac{a^x b^{-x}}{\log(a)-\log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x/b^x,x]
```

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}\int a^x b^{-x}\,dx &= \int e^{x(\log(a)-\log(b))}\,dx\\ &= \frac{a^x b^{-x}}{\log(a)-\log(b)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{a^x b^{-x}}{\log(a)-\log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/b^x,x]
```

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.06

method	result	size
gosper	$\frac{a^{x} b^{-x}}{\ln(a)-\ln(b)}$	19
risch	$\frac{a^{x} b^{-x}}{\ln(a)-\ln(b)}$	19
norman	$\frac{e^{x\ln(a)} e^{-x\ln(b)}}{\ln(a)-\ln(b)}$	23
meijerg	$-\frac{1-e^{x\ln(a)\left(1-\frac{\ln(b)}{\ln(a)}\right)}}{\ln(a)\left(1-\frac{\ln(b)}{\ln(a)}\right)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/(b^x),x,method=_RETURNVERBOSE)
```

```
[Out] a^x/(b^x)/(ln(a)-ln(b))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?` f
or more
```

Fricas [A]
time = 0.78, size = 18, normalized size = 1.00

$$\frac{a^x}{b^x(\log(a) - \log(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="fricas")
```

```
[Out] a^x/(b^x*(log(a) - log(b)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x/(b**x),x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Giac [C] Result contains complex when optimal does not.
time = 0.88, size = 216, normalized size = 12.00

$$2\left(\frac{2\left(\log\left(|a|\right)-\log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi x\mathrm{sgn}(a)+\frac{1}{2}\pi x\mathrm{sgn}(b)\right)}{\left(\pi\mathrm{sgn}\left(a\right)-\pi\mathrm{sgn}\left(b\right)\right)^{2}+4\left(\log\left(|a|\right)-\log\left(|b|\right)\right)^{2}}-\frac{\left(\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)\sin\left(-\frac{1}{2}\pi x\mathrm{sgn}(a)+\frac{1}{2}\pi x\mathrm{sgn}(b)\right)}{\left(\pi\mathrm{sgn}\left(a\right)-\pi\mathrm{sgn}\left(b\right)\right)^{2}+4\left(\log\left(|a|\right)-\log\left(|b|\right)\right)^{2}}\right)e^{\left(x\left(\log(|a|)-\log(|b|)\right)\right)}+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi x\mathrm{sgn}(a)-\frac{1}{2}i\,\pi x\mathrm{sgn}(b)\right)}}{i\,\pi\mathrm{sgn}\left(a\right)-i\,\pi\mathrm{sgn}\left(b\right)+2\,\log\left(|a|\right)-2\,\log\left(|b|\right)}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi x\mathrm{sgn}(a)+\frac{1}{2}i\,\pi x\mathrm{sgn}(b)\right)}}{-i\,\pi\mathrm{sgn}\left(a\right)+i\,\pi\mathrm{sgn}\left(b\right)+2\,\log\left(|a|\right)-2\,\log\left(|b|\right)}\right)e^{\left(x\left(\log(|a|)-\log(|b|)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((
pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) -
pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))
^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) +
I*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn(b) +
 2*log(abs(a)) - 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(
b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*e^(x*(log
(abs(a)) - log(abs(b))))
```

Mupad [B]
time = 0.20, size = 18, normalized size = 1.00

$$\frac{a^x}{b^x\,\left(\ln\left(a\right)-\ln\left(b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/b^x,x)
```

```
[Out] a^x/(b^x*(log(a) - log(b)))
```

3.161 $\int a^x b^x \, dx$

Optimal. Leaf size=14

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

```
[Out] a^x*b^x/(ln(a)+ln(b))
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2325, 2225}

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x*b^x,x]
```

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rubi steps

$$\begin{aligned}\int a^x b^x \, dx &= \int e^{x(\log(a)+\log(b))} \, dx\\ &= \frac{a^x b^x}{\log(a) + \log(b)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x*b^x,x]
```

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Maple [A]
time = 0.02, size = 15, normalized size = 1.07

method	result	size
gosper	$\frac{a^{x} b^{x}}{\ln(a)+\ln(b)}$	15
risch	$\frac{a^{x} b^{x}}{\ln(a)+\ln(b)}$	15
norman	$\frac{e^{x \ln(a)} e^{x \ln(b)}}{\ln(a)+\ln(b)}$	19
meijerg	$-\frac{1-e^{x \ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)}}{\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*b^x,x,method=_RETURNVERBOSE)
```

```
[Out] a^x*b^x/(ln(a)+ln(b))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(log(b)/log(a)>0)', see `assume?` fo
r more
```

Fricas [A]
time = 1.17, size = 14, normalized size = 1.00

$$\frac{a^{x} b^{x}}{\log\left(a\right) + \log\left(b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="fricas")
```

```
[Out] a^x*b^x/(log(a) + log(b))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.25, size = 31, normalized size = 2.21

$$\begin{cases} \frac{a^x b^x}{\log(a)+\log(b)} & \text{for } a \neq \frac{1}{b} \\ \frac{b^x \left(\frac{1}{b}\right)^x}{\log\left(\frac{1}{b}\right)+\log(b)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*b**x,x)
```

```
[Out] Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (b**x*(1/b)**x/(log(1/
b) + log(b)), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.87, size = 237, normalized size = 16.93

$$2\left(\frac{2\left(\log(|a|)+\log(|b|)\right)\cos\left(-\frac{1}{2}\pi x\operatorname{sgn}(a)-\frac{1}{2}\pi x\operatorname{sgn}(b)+\pi x\right)}{\left(2\pi-\pi\operatorname{sgn}(a)-\pi\operatorname{sgn}(b)\right)^2+4\left(\log(|a|)+\log(|b|)\right)^2}+\frac{\left(2\pi-\pi\operatorname{sgn}(a)-\pi\operatorname{sgn}(b)\right)\sin\left(-\frac{1}{2}\pi x\operatorname{sgn}(a)-\frac{1}{2}\pi x\operatorname{sgn}(b)+\pi x\right)}{\left(2\pi-\pi\operatorname{sgn}(a)-\pi\operatorname{sgn}(b)\right)^2+4\left(\log(|a|)+\log(|b|)\right)^2}\right)e^{\left(x\left(\log(|a|)+\log(|b|)\right)\right)}+i\left(\frac{i e^{\left(\frac{1}{2}i\pi x\operatorname{sgn}(a)+\frac{1}{2}i\pi x\operatorname{sgn}(b)-i\pi x\right)}}{-2i\pi+i\pi\operatorname{sgn}(a)+i\pi\operatorname{sgn}(b)+2\log(|a|)+2\log(|b|)}-\frac{i e^{\left(-\frac{1}{2}i\pi x\operatorname{sgn}(a)-\frac{1}{2}i\pi x\operatorname{sgn}(b)+i\pi x\right)}}{2i\pi-i\pi\operatorname{sgn}(a)-i\pi\operatorname{sgn}(b)+2\log(|a|)+2\log(|b|)}\right)e^{\left(x\left(\log(|a|)+\log(|b|)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) + log(abs(b)))*cos(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + p
i*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2) +
 (2*pi - pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + pi
*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2))*e
^(x*(log(abs(a)) + log(abs(b)))) + I*(I*e^(1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*s
gn(b) - I*pi*x)/(-2*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) + 2*lo
g(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b) + I*pi*x)/(2*I*pi
- I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))))*e^(x*(log(abs
(a)) + log(abs(b))))
```

Mupad [B]
time = 0.17, size = 14, normalized size = 1.00

$$\frac{a^x\,b^x}{\ln(a)+\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*b^x,x)
```

```
[Out] (a^x*b^x)/(log(a) + log(b))
```

3.162 $\int \frac{a^x}{x^2} dx$

Optimal. Leaf size=17

$$-\frac{a^x}{x} + \text{Ei}(x\log(a))\log(a)$$

[Out] -a^x/x+Ei(x*ln(a))*ln(a)

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2208, 2209}

$$\log(a)\text{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

[In] Int[a^x/x^2,x]

[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned} \int \frac{a^x}{x^2} dx &= -\frac{a^x}{x} + \log(a)\int \frac{a^x}{x} dx \\ &= -\frac{a^x}{x} + \text{Ei}(x\log(a))\log(a) \end{aligned}$$

Mathematica [A]

time = 0.01, size = 17, normalized size = 1.00

$$-\frac{a^x}{x} + \text{Ei}(x\log(a))\log(a)$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/x^2,x]
```

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Maple [A]
time = 0.03, size = 21, normalized size = 1.24

method	result
risch	$-\frac{a^{x}}{x}-\ln \left(a \right) \operatorname{expIntegral}\left(1, -x \ln \left(a \right)\right)$
meijerg	$-\ln \left(a \right) \left(\frac{1}{x \ln \left(a \right)}+1-\ln \left(x \right)-i \pi -\ln \left(\ln \left(a \right)\right)-\frac{2+2 x \ln \left(a \right)}{2 x \ln \left(a \right)}+\frac{{\mathrm e}^{x \ln \left(a \right)}}{x \ln \left(a \right)}+\ln \left(-x \ln \left(a \right)\right)+\operatorname{expIntegral}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^x/x-ln(a)*Ei(1,-x*ln(a))
```

Maxima [A]
time = 3.06, size = 10, normalized size = 0.59

$$\Gamma\left(-1, -x \log \left(a\right)\right) \log \left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="maxima")
```

```
[Out] gamma(-1, -x*log(a))*log(a)
```

Fricas [A]
time = 0.89, size = 19, normalized size = 1.12

$$\frac{x \mathrm{Ei}\left(x \log \left(a\right)\right) \log \left(a\right) - a^{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="fricas")
```

```
[Out] (x*Ei(x*log(a))*log(a) - a^x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^{x}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x/x**2,x)
```

```
[Out] Integral(a**x/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="giac")
```

```
[Out] integrate(a^x/x^2, x)
```

Mupad [**B**]
time = 0.13, size = 19, normalized size = 1.12

$$-\ln(a)\ \mathrm{expint}(-x\ \ln(a)) - \frac{a^x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/x^2,x)
```

```
[Out] - log(a)*expint(-x*log(a)) - a^x/x
```

3.163 $\int \frac{a^x x}{(1+bx)^2}\,dx$

Optimal. Leaf size=64

$$\frac{a^x}{b^2(1+bx)}+\frac{a^{-1/b}\text{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2}-\frac{a^{-1/b}\text{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)\log(a)}{b^3}$$

```
[Out] a^x/b^2/(b*x+1)+Ei((b*x+1)*ln(a)/b)/(a^(1/b))/b^2-Ei((b*x+1)*ln(a)/b)*ln(a)
/(a^(1/b))/b^3
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {2230, 2208, 2209}

$$-\frac{a^{-1/b}\log(a)\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^3}+\frac{a^{-1/b}\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^2}+\frac{a^x}{b^2(bx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-1)*b^2) -
(ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/(a^b^(-1)*b^3)
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a^x x}{(1+bx)^2}\,dx &= \int \left(-\frac{a^x}{b(1+bx)^2}+\frac{a^x}{b(1+bx)}\right)dx \\
&= -\frac{\int \frac{a^x}{(1+bx)^2}\,dx}{b}+\frac{\int \frac{a^x}{1+bx}\,dx}{b} \\
&= \frac{a^x}{b^2(1+bx)}+\frac{a^{-1/b}\mathrm{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2}-\frac{\log(a)\int \frac{a^x}{1+bx}\,dx}{b^2} \\
&= \frac{a^x}{b^2(1+bx)}+\frac{a^{-1/b}\mathrm{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2}-\frac{a^{-1/b}\mathrm{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)\log(a)}{b^3}
\end{aligned}
$$

Mathematica [**A**]
time = 0.14, size = 43, normalized size = 0.67

$$
\frac{\frac{a^x b}{1+bx}+a^{-1/b}\mathrm{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)(b-\log(a))}{b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] ((a^x*b)/(1 + b*x) + (ExpIntegralEi[((1 + b*x)*Log[a])/b]*(b - Log[a]))/a^b
^(-1))/b^3
```

Maple [**A**]
time = 0.04, size = 79, normalized size = 1.23

method	result	size
risch	$-\frac{a^{-\frac{1}{b}}\,\mathrm{expIntegral}\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)}{b^2}+\frac{\ln(a)a^x}{b^3\left(x\ln(a)+\frac{\ln(a)}{b}\right)}+\frac{\ln(a)a^{-\frac{1}{b}}\,\mathrm{expIntegral}\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)}{b^3}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*x/(b*x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)+ln(a)/b^3*a^x/(x*ln(a)+ln(a)/b)+ln(a
)/b^3*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="maxima")
```

```
[Out] a^x*x/(b^2*x^2*log(a) + 2*b*x*log(a) + log(a)) + integrate((b*x - 1)*a^x/(b
^3*x^3*log(a) + 3*b^2*x^2*log(a) + 3*b*x*log(a) + log(a)), x)
```

Fricas [**A**]
time = 0.78, size = 54, normalized size = 0.84

$$\frac{a^x b + \frac{(b^2 x - (bx+1)\log(a)+b)\operatorname{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{a^{\left(\frac{1}{b}\right)}}}{b^4 x + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="fricas")
```

```
[Out] (a^x*b + (b^2*x - (b*x + 1)*log(a) + b)*Ei((b*x + 1)*log(a)/b)/a^(1/b))/(b^
4*x + b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^x x}{(bx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*x/(b*x+1)**2,x)
```

```
[Out] Integral(a**x*x/(b*x + 1)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(a^x*x/(b*x + 1)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a^x\,x}{{\left(b\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^x*x)/(b*x + 1)^2,x)
```

```
[Out] int((a^x*x)/(b*x + 1)^2, x)
```

3.164 $\int \frac{e^{ax}x}{(1+ax)^2}\,dx$

Optimal. Leaf size=16

$$\frac{e^{ax}}{a^2(1+ax)}$$

```
[Out] exp(a*x)/a^2/(a*x+1)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2228}

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(a*x)*x)/(1 + a*x)^2,x]
```

```
[Out] E^(a*x)/(a^2*(1 + a*x))
```

Rule 2228

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[g*u^(m + 1)*(F^(c*v)/(b*c*e*Log
[F])), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\int \frac{e^{ax}x}{(1+ax)^2}\,dx = \frac{e^{ax}}{a^2(1+ax)}$$

Mathematica [A]
time = 0.15, size = 16, normalized size = 1.00

$$\frac{e^{ax}}{a^2(1+ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(a*x)*x)/(1 + a*x)^2,x]
```

```
[Out] E^(a*x)/(a^2*(1 + a*x))
```

Maple [A]
time = 0.03, size = 16, normalized size = 1.00

method	result	size
gosper	$\frac{e^{ax}}{a^2(ax+1)}$	16
derivativedivides	$\frac{e^{ax}}{a^2(ax+1)}$	16
default	$\frac{e^{ax}}{a^2(ax+1)}$	16
norman	$\frac{e^{ax}}{a^2(ax+1)}$	16
risch	$\frac{e^{ax}}{a^2(ax+1)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*x/(a*x+1)^2,x,method=_RETURNVERBOSE)
[Out] exp(a*x)/a^2/(a*x+1)
```

Maxima [A]
time = 1.78, size = 16, normalized size = 1.00

$$\frac{e^{(ax)}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="maxima")
[Out] e^(a*x)/(a^3*x + a^2)
```

Fricas [A]
time = 0.71, size = 16, normalized size = 1.00

$$\frac{e^{(ax)}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="fricas")
[Out] e^(a*x)/(a^3*x + a^2)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.75

$$\frac{e^{ax}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)**2,x)
[Out] exp(a*x)/(a**3*x + a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(15) = 30$.
time = 0.65, size = 45, normalized size = 2.81

$$-\frac{e^{\left(-(ax+1)\left(\frac{1}{ax+1}-1\right)\right)}}{(ax+1)a^2\left(\frac{1}{ax+1}-1\right)-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="giac")
[Out] -e^(-(a*x + 1)*(1/(a*x + 1) - 1))/((a*x + 1)*a^2*(1/(a*x + 1) - 1) - a^2)
```

Mupad [B]
time = 0.20, size = 15, normalized size = 0.94

$$\frac{e^{a\,x}}{a^2\,\left(a\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(a*x))/(a*x + 1)^2,x)
[Out] exp(a*x)/(a^2*(a*x + 1))
```

3.165 $\int k^{x^2} x \, dx$

Optimal. Leaf size=13

$$\frac{k^{x^2}}{2\log(k)}$$

```
[Out] 1/2*k^(x^2)/ln(k)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2240}

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

```
[In] Int[k^x^2*x,x]
```

```
[Out] k^x^2/(2*Log[k])
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int k^{x^2} x \, dx = \frac{k^{x^2}}{2\log(k)}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

```
[In] Integrate[k^x^2*x,x]
```

```
[Out] k^x^2/(2*Log[k])
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.92

method	result	size
gosper	$\frac{k^{x^2}}{2\ln(k)}$	12
derivativedivides	$\frac{k^{x^2}}{2\ln(k)}$	12
default	$\frac{k^{x^2}}{2\ln(k)}$	12
risch	$\frac{k^{x^2}}{2\ln(k)}$	12
norman	$\frac{e^{\ln(k)x^2}}{2\ln(k)}$	14
meijerg	$-\frac{1-e^{\ln(k)x^2}}{2\ln(k)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(k^(x^2)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*k^(x^2)/ln(k)
```

Maxima [A]
time = 1.84, size = 11, normalized size = 0.85

$$\frac{k^{(x^2)}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="maxima")
[Out] 1/2*k^(x^2)/log(k)
```

Fricas [A]
time = 0.59, size = 11, normalized size = 0.85

$$\frac{k^{(x^2)}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="fricas")
[Out] 1/2*k^(x^2)/log(k)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 1.15

$$\begin{cases} \frac{k^{x^2}}{2\log(k)} & \text{for}\ \log(k) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k**(x**2)*x,x)
```

```
[Out] Piecewise((k**(x**2)/(2*log(k)), Ne(log(k), 0)), (x**2/2, True))
```

Giac [A]
time = 0.75, size = 11, normalized size = 0.85

$$\frac{k^{\left(x^2\right)}}{2 \log\left(k\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*k^(x^2)/log(k)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.85

$$\frac{k^{x^2}}{2 \ln\left(k\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(k^(x^2)*x,x)
```

```
[Out] k^(x^2)/(2*log(k))
```

3.166 $\int e^{x^2}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)$$

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2235}

$$\frac{1}{2}\sqrt{\pi}\,\mathrm{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2}\,dx = \frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Maple [A]
time = 0.02, size = 8, normalized size = 0.73

method	result	size
default	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8
meijerg	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8
risch	$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 1.70, size = 9, normalized size = 0.82

$$-\frac{1}{2}i\sqrt{\pi}\,\operatorname{erf}(ix)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="maxima")
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

Fricas [A]
time = 0.51, size = 7, normalized size = 0.64

$$\frac{1}{2}\sqrt{\pi}\,\operatorname{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="fricas")
[Out] 1/2*sqrt(pi)*erfi(x)
```

Sympy [A]
time = 0.07, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\operatorname{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2),x)
[Out] sqrt(pi)*erfi(x)/2
```

Giac [C] Result contains complex when optimal does not.
time = 0.94, size = 9, normalized size = 0.82

$$\frac{1}{2}i\sqrt{\pi}\,\operatorname{erf}\left(-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

Mupad [B]
time = 0.02, size = 7, normalized size = 0.64

$$\frac{\sqrt{\pi}\,\operatorname{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
```

```
[Out] (pi^(1/2)*erfi(x))/2
```

3.167 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] 1/2*exp(x^2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2240}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x,x]

[Out] E^x^2/2

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*x,x]

[Out] E^x^2/2

Maple [A]
time = 0.01, size = 7, normalized size = 0.78

method	result	size
gosper	$\frac{e^{x^2}}{2}$	7
derivativedivides	$\frac{e^{x^2}}{2}$	7
default	$\frac{e^{x^2}}{2}$	7
norman	$\frac{e^{x^2}}{2}$	7
risch	$\frac{e^{x^2}}{2}$	7
meijerg	$-\frac{1}{2}+\frac{e^{x^2}}{2}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x^2)
```

Maxima [A]
time = 4.15, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
[Out] 1/2*e^(x^2)
```

Fricas [A]
time = 0.50, size = 6, normalized size = 0.67

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
[Out] 1/2*e^(x^2)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Giac [A]
time = 0.79, size = 6, normalized size = 0.67

$$\frac{1}{2}e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.67

$$\frac{\mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x^2),x)
```

```
[Out] exp(x^2)/2
```

3.168 $\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx$

Optimal. Leaf size=27

$$-e^{\frac{1}{x}}-\frac{e^{\frac{1}{x}}}{x^2}+\frac{e^{\frac{1}{x}}}{x}$$

```
[Out] -exp(1/x)-exp(1/x)/x^2+exp(1/x)/x
```

Rubi [A]
time = 0.08, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6874, 2243, 2240}

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x^(-1)*(1 + x))/x^4,x]
```

```
[Out] -E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx &= \int \left(\frac{e^{\frac{1}{x}}}{x^4}+\frac{e^{\frac{1}{x}}}{x^3}\right)dx \\
&= \int \frac{e^{\frac{1}{x}}}{x^4}\,dx + \int \frac{e^{\frac{1}{x}}}{x^3}\,dx \\
&= -\frac{e^{\frac{1}{x}}}{x^2} - \frac{e^{\frac{1}{x}}}{x} - 2\int \frac{e^{\frac{1}{x}}}{x^3}\,dx - \int \frac{e^{\frac{1}{x}}}{x^2}\,dx \\
&= e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x} + 2\int \frac{e^{\frac{1}{x}}}{x^2}\,dx \\
&= -e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x^2} + \frac{e^{\frac{1}{x}}}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.59

$$e^{\frac{1}{x}}\left(-1-\frac{1}{x^2}+\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x^(-1)*(1 + x))/x^4,x]
[Out] E^x^(-1)*(-1 - x^(-2) + x^(-1))
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.93

method	result	size
gosper	$-\frac{(x^2-x+1)\,\mathrm{e}^{\frac{1}{x}}}{x^2}$	18
risch	$-\frac{(x^2-x+1)\,\mathrm{e}^{\frac{1}{x}}}{x^2}$	18
derivativedivides	$-\mathrm{e}^{\frac{1}{x}}-\frac{\mathrm{e}^{\frac{1}{x}}}{x^2}+\frac{\mathrm{e}^{\frac{1}{x}}}{x}$	25
default	$-\mathrm{e}^{\frac{1}{x}}-\frac{\mathrm{e}^{\frac{1}{x}}}{x^2}+\frac{\mathrm{e}^{\frac{1}{x}}}{x}$	25
norman	$\frac{x^2\mathrm{e}^{\frac{1}{x}}-x\,\mathrm{e}^{\frac{1}{x}}-x^3\mathrm{e}^{\frac{1}{x}}}{x^3}$	30
meijerg	$1-\frac{\left(\frac{3}{x^2}-\frac{6}{x}+6\right)\mathrm{e}^{\frac{1}{x}}}{3}+\frac{\left(2-\frac{2}{x}\right)\mathrm{e}^{\frac{1}{x}}}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/x)*(1+x)/x^4,x,method=_RETURNVERBOSE)
```

[Out] -exp(1/x)-exp(1/x)/x^2+exp(1/x)/x

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.79, size = 17, normalized size = 0.63

$$-\Gamma\left(3, -\frac{1}{x}\right) + \Gamma\left(2, -\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="maxima")

[Out] -gamma(3, -1/x) + gamma(2, -1/x)

Fricas [A]
time = 0.45, size = 17, normalized size = 0.63

$$-\frac{(x^2 - x + 1)e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="fricas")

[Out] -(x^2 - x + 1)*e^(1/x)/x^2

Sympy [A]
time = 0.03, size = 14, normalized size = 0.52

$$\frac{(-x^2 + x - 1)\,e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/x)*(1+x)/x**4,x)

[Out] (-x**2 + x - 1)*exp(1/x)/x**2

Giac [A]
time = 0.81, size = 24, normalized size = 0.89

$$\frac{e^{\frac{1}{x}}}{x} - \frac{e^{\frac{1}{x}}}{x^2} - e^{\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="giac")

[Out] e^(1/x)/x - e^(1/x)/x^2 - e^(1/x)

Mupad [B]
time = 0.16, size = 17, normalized size = 0.63

$$-\frac{e^{1/x}\left(x^2-x+1\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(1/x)*(x + 1))/x^4,x)
```

```
[Out] -(exp(1/x)*(x^2 - x + 1))/x^2
```

3.169 $\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

```
[Out] -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

Rubi [F]
time = 0.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

```
[Out] Defer[Int][(E^(1 - E^x^2*x + 2*x^2)*x)/(-1 + E^x^2*x)^2, x] + 2*Defer[Int][
(E^(1 - E^x^2*x + 2*x^2)*x^3)/(-1 + E^x^2*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{(1-e^{x^2}x)^2}\,dx &= \int \frac{e^{1-e^{x^2}x+2x^2}x(1+2x^2)}{(1-e^{x^2}x)^2}\,dx\\
&= \int \left(\frac{e^{1-e^{x^2}x+2x^2}x}{(-1+e^{x^2}x)^2}+\frac{2e^{1-e^{x^2}x+2x^2}x^3}{(-1+e^{x^2}x)^2}\right)dx\\
&= 2\int \frac{e^{1-e^{x^2}x+2x^2}x^3}{(-1+e^{x^2}x)^2}\,dx+\int \frac{e^{1-e^{x^2}x+2x^2}x}{(-1+e^{x^2}x)^2}\,dx
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 25, normalized size = 1.00

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

```
[Out] -(E^(1 - E^x^2*x)/(-1 + E^x^2*x))
```

Maple [A]
time = 0.05, size = 23, normalized size = 0.92

method	result	size
risch	$-\frac{e^{1-e^{x^2} x}}{-1+e^{x^2} x}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x,method=_RETURNVERB
OSE)
```

```
[Out] -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

Maxima [A]
time = 2.66, size = 22, normalized size = 0.88

$$-\frac{e^{\left(-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(x^2\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="
maxima")
```

```
[Out] -e^(-x*e^(x^2) + 1)/(x*e^(x^2) - 1)
```

Fricas [A]
time = 0.52, size = 36, normalized size = 1.44

$$-\frac{e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3\,x^2\right)}-e^{\left(2\,x^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="
fricas")
```

```
[Out] -e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))
```

Sympy [A]
time = 0.10, size = 31, normalized size = 1.24

$$-\frac{e^{2x^2-xe^{x^2}+1}}{xe^{3x^2}-e^{2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)
```

```
[Out] -exp(2*x**2 - x*exp(x**2) + 1)/(x*exp(3*x**2) - exp(2*x**2))
```

Giac [A]
time = 0.79, size = 36, normalized size = 1.44

$$-\frac{e^{\left(2x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3x^2\right)}-e^{\left(2x^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="
giac")
```

```
[Out] -e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\mathrm{e}^{2x^2-x\,\mathrm{e}^{x^2}+1}\,\left(2x^3+x\right)}{\left(x\,\mathrm{e}^{x^2}-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2,x)
```

```
[Out] int((exp(2*x^2 - x*exp(x^2) + 1)*(x + 2*x^3))/(x*exp(x^2) - 1)^2, x)
```

3.170 $\int e^{e^{e^{e^x}}} \, dx$

Optimal. Leaf size=12

$$\text{Int}\left(e^{e^{e^{e^x}}},x\right)$$

```
[Out] CannotIntegrate(exp(exp(exp(exp(x)))),x)
```

Rubi [A]

time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int e^{e^{e^{e^x}}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[E^E^E^E^x,x]
```

```
[Out] Defer[Subst][Defer[Int][E^E^E^x/x, x], x, E^x]
```

Rubi steps

$$\int e^{e^{e^{e^x}}} \, dx = \text{Subst}\left(\int \frac{e^{e^{e^x}}}{x} \, dx, x, e^x\right)$$

Mathematica [A]

time = 0.02, size = 0, normalized size = 0.00

$$\int e^{e^{e^{e^x}}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[E^E^E^E^x,x]
```

```
[Out] Integrate[E^E^E^E^x, x]
```

Maple [A]

time = 0.01, size = 0, normalized size = 0.00

$$\int e^{e^{e^{e^x}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(exp(exp(x)))),x)
[Out] int(exp(exp(exp(exp(x)))),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="maxima")
[Out] integrate(e^(e^(e^(e^x))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="fricas")
[Out] integral(e^(e^(e^(e^x))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{e^{e^{e^{x}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x)
[Out] Integral(exp(exp(exp(exp(x)))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="giac")
[Out] integrate(e^(e^(e^(e^x))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \mathrm{e}^{\mathrm{e}^{\mathrm{e}^{\mathrm{e}^{x}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(exp(exp(x)))),x)
```

```
[Out] int(exp(exp(exp(exp(x)))), x)
```

3.171 $\int e^x \log(x)\,dx$

Optimal. Leaf size=11

$$-\text{Ei}(x)+e^x\log(x)$$

[Out] -Ei(x)+exp(x)*ln(x)

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2225, 2634, 2209}

$$e^x\log(x)-\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

[In] Int[E^x*Log[x],x]

[Out] -ExpIntegralEi[x] + E^x*Log[x]

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^x\log(x)\,dx&=e^x\log(x)-\int\frac{e^x}{x}\,dx\\&=-\text{Ei}(x)+e^x\log(x)\end{aligned}$$

Mathematica [A]

time = 0.01, size = 11, normalized size = 1.00

$$-\text{Ei}(x)+e^x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[x],x]
```

```
[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Maple [A]
time = 0.02, size = 12, normalized size = 1.09

method	result	size
risch	$e^{x} \ln \left(x\right)+\text{expIntegral}\left(1,-x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)*ln(x)+Ei(1,-x)
```

Maxima [A]
time = 1.80, size = 10, normalized size = 0.91

$$e^{x} \log \left(x\right) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="maxima")
```

```
[Out] e^x*log(x) - Ei(x)
```

Fricas [A]
time = 0.50, size = 10, normalized size = 0.91

$$e^{x} \log \left(x\right) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="fricas")
```

```
[Out] e^x*log(x) - Ei(x)
```

Sympy [A]
time = 0.98, size = 8, normalized size = 0.73

$$e^{x} \log \left(x\right) - \mathrm{Ei}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(x),x)
```

```
[Out] exp(x)*log(x) - Ei(x)
```

Giac [**A**]
time = 0.72, size = 10, normalized size = 0.91

$$e^x \log(x) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="giac")
```

```
[Out] e^x*log(x) - Ei(x)
```

Mupad [**B**]
time = 0.03, size = 10, normalized size = 0.91

$$\mathrm{e}^x \ln(x) - \mathrm{ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*log(x),x)
```

```
[Out] exp(x)*log(x) - ei(x)
```

3.172 $\int e^x x \log(x) \, dx$

Optimal. Leaf size=22

$$-e^x + \text{Ei}(x) - e^x \log(x) + e^x x \log(x)$$

```
[Out] -exp(x)+Ei(x)-exp(x)*ln(x)+exp(x)*x*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {2207, 2225, 2634, 2230, 2209}

$$\text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Log[x],x]
```

```
[Out] -E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] &&  !TrueQ[$UseGamma]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^x x \log(x)\,dx &= -e^x\log(x)+e^x x\log(x)-\int \frac{e^x(-1+x)}{x}\,dx\\ &= -e^x\log(x)+e^x x\log(x)-\int\left(e^x-\frac{e^x}{x}\right)dx\\ &= -e^x\log(x)+e^x x\log(x)-\int e^x\,dx+\int\frac{e^x}{x}\,dx\\ &= -e^x+\text{Ei}(x)-e^x\log(x)+e^x x\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.77

$$-e^x+\text{Ei}(x)+e^x(-1+x)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Log[x],x]
```

```
[Out] -E^x + ExpIntegralEi[x] + E^x*(-1 + x)*Log[x]
```

Maple [A]
time = 0.01, size = 21, normalized size = 0.95

method	result	size
risch	$(-1+x)\,\mathrm{e}^x\ln(x)-\text{expIntegral}(1,-x)-\mathrm{e}^x$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+x)*exp(x)*ln(x)-Ei(1,-x)-exp(x)
```

Maxima [A]
time = 5.68, size = 15, normalized size = 0.68

$$(x-1)e^x\log(x)+\text{Ei}(x)-e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="maxima")
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

Fricas [**A**]
time = 0.46, size = 15, normalized size = 0.68

$$(x-1)e^{x}\log\left(x\right)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="fricas")
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

Sympy [**A**]
time = 1.67, size = 17, normalized size = 0.77

$$\left(xe^{x}-e^{x}\right)\log\left(x\right)-e^{x}+\mathrm{Ei}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*ln(x),x)
[Out] (x*exp(x) - exp(x))*log(x) - exp(x) + Ei(x)
```

Giac [**A**]
time = 0.75, size = 15, normalized size = 0.68

$$(x-1)e^{x}\log\left(x\right)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="giac")
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

Mupad [**B**]
time = 0.22, size = 28, normalized size = 1.27

$$\mathrm{ei}(x)-\frac{x\,\mathrm{e}^{x}+x\,\mathrm{e}^{x}\,\ln\left(x\right)-x^{2}\,\mathrm{e}^{x}\,\ln\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*log(x),x)
[Out] ei(x) - (x*exp(x) + x*exp(x)*log(x) - x^2*exp(x)*log(x))/x
```

3.173 $\int e^{2x}\log(e^x)\,dx$

Optimal. Leaf size=23

$$-\frac{e^{2x}}{4}+\frac{1}{2}e^{2x}\log(e^x)$$

```
[Out] -1/4*exp(2*x)+1/2*exp(2*x)*ln(exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {2225, 2634, 12}

$$\frac{1}{2}e^{2x}\log(e^x)-\frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Log[E^x],x]
```

```
[Out] -1/4*E^(2*x) + (E^(2*x)*Log[E^x])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x}\log(e^x)\,dx&=\frac{1}{2}e^{2x}\log(e^x)-\int\frac{e^{2x}}{2}\,dx\\&=\frac{1}{2}e^{2x}\log(e^x)-\frac{1}{2}\int e^{2x}\,dx\\&=-\frac{e^{2x}}{4}+\frac{1}{2}e^{2x}\log(e^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.74

$$\frac{1}{4}e^{2x}(-1+2\log(e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Log[E^x],x]
```

```
[Out] (E^(2*x)*(-1 + 2*Log[E^x]))/4
```

Maple [A]
time = 0.01, size = 28, normalized size = 1.22

method	result	size
norman	$-\frac{e^{2x}}{4}+\frac{e^{2x}\ln(e^x)}{2}$	17
risch	$-\frac{e^{2x}}{4}+\frac{e^{2x}\ln(e^x)}{2}$	17
default	$\frac{e^{2x}x}{2}-\frac{e^{2x}}{4}+\frac{e^{2x}(\ln(e^x)-x)}{2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*ln(exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(2*x)*x-1/4*exp(2*x)+1/2*exp(2*x)*(ln(exp(x))-x)
```

Maxima [A]
time = 1.94, size = 11, normalized size = 0.48

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Fricas [A]
time = 0.47, size = 11, normalized size = 0.48

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.43

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*ln(exp(x)),x)
[Out] (2*x - 1)*exp(2*x)/4
```

Giac [A]
time = 0.61, size = 11, normalized size = 0.48

$$\frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="giac")
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Mupad [B]
time = 0.04, size = 11, normalized size = 0.48

$$\frac{\mathrm{e}^{2\,x}\,(2\,x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x))*exp(2*x),x)
[Out] (exp(2*x)*(2*x - 1))/4
```

3.174 $\int \left(2x+\sqrt{2}\, x^2\right) \, dx$

Optimal. Leaf size=16

$$x^2+\frac{\sqrt{2}\, x^3}{3}$$

[Out] x^2+1/3*x^3*2^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{\sqrt{2}\, x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Int[2*x + Sqrt[2]*x^2,x]
```

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Rubi steps

$$\int \left(2x+\sqrt{2}\, x^2\right) \, dx = x^2+\frac{\sqrt{2}\, x^3}{3}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$x^2+\frac{\sqrt{2}\, x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x + Sqrt[2]*x^2,x]
```

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.19

method	result	size

norman	$x^2+\frac{x^3\sqrt{2}}{3}$	13
risch	$x^2+\frac{x^3\sqrt{2}}{3}$	13
default	$\sqrt{2}\left(\frac{x^3}{3}+\frac{x^2\sqrt{2}}{2}\right)$	19
gosper	$\frac{x^2\left(2x+3\sqrt{2}\right)\left(x\sqrt{2}+2\right)}{6x+6\sqrt{2}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x+x^2*2^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*(1/3*x^3+1/2*x^2*2^(1/2))
```

Maxima [A]
time = 2.94, size = 12, normalized size = 0.75

$$\frac{1}{3}\sqrt{2}\,x^3+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

Fricas [A]
time = 0.45, size = 12, normalized size = 0.75

$$\frac{1}{3}\sqrt{2}\,x^3+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{\sqrt{2}\,x^3}{3}+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x**2*2**(1/2),x)
```

```
[Out] sqrt(2)*x**3/3 + x**2
```

Giac [A]
time = 0.53, size = 12, normalized size = 0.75

$$\frac{1}{3}\sqrt{2}\,x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{x^2\left(\sqrt{2}\,x + 3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 2^(1/2)*x^2,x)
```

```
[Out] (x^2*(2^(1/2)*x + 3))/3
```

3.175 $\int \frac{\log(x)}{\sqrt{b+ax}}\,dx$

Optimal. Leaf size=57

$$-\frac{4\sqrt{b+ax}}{a}+\frac{4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a}+\frac{2\sqrt{b+ax}\,\log(x)}{a}$$

```
[Out] 4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln(x)*(a*x+b
)^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2356, 52, 65, 214}

$$-\frac{4\sqrt{ax+b}}{a}+\frac{2\log(x)\sqrt{ax+b}}{a}+\frac{4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[b + a*x],x]
```

```
[Out] (-4*Sqrt[b + a*x])/a + (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/a + (2*Sq
rt[b + a*x]*Log[x])/a
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{\sqrt{b+ax}}\,dx &= \frac{2\sqrt{b+ax}\,\log(x)}{a}-\frac{2\int \frac{\sqrt{b+ax}}{x}\,dx}{a}\\
&= -\frac{4\sqrt{b+ax}}{a}+\frac{2\sqrt{b+ax}\,\log(x)}{a}-\frac{(2b)\int \frac{1}{x\sqrt{b+ax}}\,dx}{a}\\
&= -\frac{4\sqrt{b+ax}}{a}+\frac{2\sqrt{b+ax}\,\log(x)}{a}-\frac{(4b)\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx,x,\sqrt{b+ax}\right)}{a^2}\\
&= -\frac{4\sqrt{b+ax}}{a}+\frac{4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a}+\frac{2\sqrt{b+ax}\,\log(x)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.75

$$\frac{4\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)+2\sqrt{b+ax}\,(-2+\log(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[b + a*x],x]
[Out] (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]] + 2*Sqrt[b + a*x]*(-2 + Log[x]))/
a
```

Maple [A]
time = 0.04, size = 43, normalized size = 0.75

method	result	size
derivativedivides	$\frac{2\sqrt{ax+b}\,\ln(x)-4\sqrt{ax+b}+4\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$	43
default	$\frac{2\sqrt{ax+b}\,\ln(x)-4\sqrt{ax+b}+4\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a*x+b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a*((a*x+b)^(1/2)*ln(x)-2*(a*x+b)^(1/2)+2*b^(1/2)*arctanh((a*x+b)^(1/2)/b^
(1/2)))
```

Maxima [A]
time = 1.80, size = 58, normalized size = 1.02

$$\frac{2\left(\sqrt{ax+b}\,\log(x)-\sqrt{b}\,\log\left(\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right)-2\sqrt{ax+b}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(a*x + b)*log(x) - sqrt(b)*log((sqrt(a*x + b) - sqrt(b))/(sqrt(a*x +
 b) + sqrt(b))) - 2*sqrt(a*x + b))/a
```

Fricas [A]
time = 0.43, size = 89, normalized size = 1.56

$$\left[\frac{2\left(\sqrt{ax+b}\,(\log(x)-2)+\sqrt{b}\,\log\left(\frac{ax+2\sqrt{ax+b}\,\sqrt{b}+2b}{x}\right)\right)}{a}, \frac{2\left(\sqrt{ax+b}\,(\log(x)-2)-2\sqrt{-b}\,\arctan\left(\frac{\sqrt{ax+b}\,\sqrt{-b}}{b}\right)\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(a*x + b)*(log(x) - 2) + sqrt(b)*log((a*x + 2*sqrt(a*x + b)*sqrt(b)
 + 2*b)/x))/a, 2*(sqrt(a*x + b)*(log(x) - 2) - 2*sqrt(-b)*arctan(sqrt(a*x +
 b)*sqrt(-b)/b))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.94, size = 1166, normalized size = 20.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a*x+b)**(1/2),x)
```

```
[Out] Piecewise((4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x +
b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)
))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/a)
< 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(
x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/
a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) - 2*I*pi*sqrt
(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (1/Abs(x + b/a) < 1)), (4*sqrt(b)*a
coth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a)
- 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b
/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sq
rt(a), (Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh(s
qrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sq
rt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x
+ b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), Abs(x + b/a) < 1), (4*sqrt(b)*a
coth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a)
- 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b
/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2*I*pi*sqrt(x + b/a)/sq
rt(a), (1/Abs(x + b/a) < 1) & (Abs(b/(a*(x + b/a))) > 1)), (4*sqrt(b)*atanh
(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*
sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(
x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), 1/Abs(x + b/a) < 1), (4*sqrt(
b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x +
 b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqr
t(x + b/a)/sqrt(a) + meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)*log(b
/a)/sqrt(a) + I*pi*meijerg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)/sqrt(a)
 + meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)*log(b/a)/sqrt(a) + I*pi
*meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)/sqrt(a), Abs(b/(a*(x + b/
a))) > 1), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x +
 b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x + b/a)
))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) - 2*I*pi*sqrt(x + b/a)/sqrt(a) + meije
rg(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg
(((1,), (3/2,)), ((1/2,), (0,)), x + b/a)/sqrt(a) + meijerg(((3/2, 1), ()),
 ((), (1/2, 0)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg(((3/2, 1), ()), (
(), (1/2, 0)), x + b/a)/sqrt(a), True))
```

Giac [A]
time = 0.52, size = 48, normalized size = 0.84

$$-\frac{2\left(\frac{2\,b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\sqrt{ax+b}\,\log\left(x\right)+2\,\sqrt{ax+b}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*log(x) + 2*
sqrt(a*x + b))/a
```

Mupad [B]
time = 0.09, size = 49, normalized size = 0.86

$$\frac{2\sqrt{b}\,\ln\left(\frac{2b+a\,x+2\sqrt{b}\,\sqrt{b+a\,x}}{x}\right)}{a}+\frac{2\left(\ln\left(x\right)-2\right)\,\sqrt{b+a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(b + a*x)^(1/2),x)
```

```
[Out] (2*b^(1/2)*log((2*b + a*x + 2*b^(1/2)*(b + a*x)^(1/2))/x))/a + (2*(log(x) -
 2)*(b + a*x)^(1/2))/a
```

3.176 $\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$\frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

```
[Out] -1/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3
/2)/d^(3/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b+1/4*(-a*d+b*c)*(b*x+a)^(1/2)*
(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 223, 212}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4b}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{c}\cdots}}{8bd}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt{c-\cdots}}\cdots\right.}{\cdots}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\cdots\right.}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\cdots}{\sqrt{b}\,\cdots}\right.}{4b^{3/2}d^{3/2}}$$

Mathematica [A]
time = 0.24, size = 95, normalized size = 0.82

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(ad+b(c+2dx))}{4bd} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(a*d + b*(c + 2*d*x)))/(4*b*d) - ((b*c - a*d)^
2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(3/2)*d^(3
/2))
```

Maple [A]
time = 0.04, size = 140, normalized size = 1.21

method	result
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}-\frac{(-ad+bc)\left(\frac{\sqrt{dx+c}\,\sqrt{bx+a}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x}\right.}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*(1/b*(d*x+c)^(1/2)*(b*x+
a)^(1/2)-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2
)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(
b*d)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.45, size = 300, normalized size = 2.59

$$\left[\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x+b^2cd+abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,b^2d^2},\ \frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(2\,b^2d^2x+b^2cd+abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(90) = 180.
time = 0.50, size = 232, normalized size = 2.00

$$-\frac{\frac{4\left(\frac{\left(b^{2}c-abd\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^{2}c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)a|b|}{b^{2}}-\frac{\left(\sqrt{b^{2}c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^{2}}{d^{2}}\right)\sqrt{bx+a}+\frac{\left(b^{3}c^{2}+2ab^{2}cd-3a^{2}bd^{2}\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)|b|}{b^{2}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*abs(b)/b^2)/b
```

Mupad [B]
time = 0.17, size = 88, normalized size = 0.76

$$\left(\frac{x}{2}+\frac{a\,d+b\,c}{4\,b\,d}\right)\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}-\frac{\ln\left(a\,d+b\,c+2\,b\,d\,x+2\,\sqrt{b}\,\sqrt{d}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{8\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

```
[Out] (x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b*
c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b*c
)^2)/(8*b^(3/2)*d^(3/2))
```

3.177 $\int \sqrt{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,dx = \frac{2(a+bx)^{3/2}}{3b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Maple [A]
time = 0.03, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
trager	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
risch	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Maxima [A]
time = 1.22, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A]
time = 0.51, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

Giac [A]
time = 0.46, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*(b*x + a)^(3/2)/b
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2),x)
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

3.178 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2+2/5*(b*x+a)^(5/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx &= \int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 1.00

$$\frac{2\sqrt{a+bx}\,(-2a^2+abx+3b^2x^2)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2)
```

Maple [A]
time = 0.04, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
trager	$-\frac{2(-3x^2b^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32
risch	$-\frac{2(-3x^2b^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))
```

Maxima [A]
time = 2.01, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

Fricas [A]
time = 0.49, size = 30, normalized size = 0.88

$$\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.
time = 0.54, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 0.45, size = 66, normalized size = 1.94

$$\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a}{b}+\frac{3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 10
*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{10\,a\,(a+b\,x)^{3/2}-6\,(a+b\,x)^{5/2}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/2),x)
```

```
[Out] -(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

3.179 $\int x^2\sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3-4/5*a*(b*x+a)^(5/2)/b^3+2/7*(b*x+a)^(7/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx&=\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

Maple [A]
time = 0.04, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}\left(15x^{2}b^{2}-12abx+8a^{2}\right)}{105b^{3}}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	38
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	38
trager	$\frac{2\left(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3}\right)\sqrt{bx+a}}{105b^{3}}$	43
risch	$\frac{2\left(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3}\right)\sqrt{bx+a}}{105b^{3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))
```

Maxima [A]
time = 2.95, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{3}}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^{3}}+\frac{2\,(bx+a)^{\frac{3}{2}}a^{2}}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

Fricas [A]
time = 0.46, size = 42, normalized size = 0.79

$$\frac{2\,(15\,b^{3}x^{3}+3\,ab^{2}x^{2}-4\,a^{2}bx+8\,a^{3})\sqrt{bx+a}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.
time = 0.82, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} - \frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3} + \frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(41) = 82$.
time = 0.46, size = 93, normalized size = 1.75

$$\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

Mupad [B]
time = 0.15, size = 37, normalized size = 0.70

$$\frac{30\,{\left(a+b\,x\right)}^{7/2}-84\,a\,{\left(a+b\,x\right)}^{5/2}+70\,a^2\,{\left(a+b\,x\right)}^{3/2}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/2),x)
```

```
[Out] (30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*b
^3)
```

3.180 $\int \frac{\sqrt{a+bx}}{x} \, dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}}{x}\,dx &= 2\sqrt{a+bx}+a\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 2\sqrt{a+bx}+\frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b} \\
&= 2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.00

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.03, size = 28, normalized size = 0.80

method	result	size
derivativedivides	$-2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28
default	$-2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Maxima [A]
time = 2.17, size = 42, normalized size = 1.20

$$\sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b
*x + a)
```

Fricas [A]
time = 0.51, size = 73, normalized size = 2.09

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a},2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.
time = 0.67, size = 68, normalized size = 1.94

$$-2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A]
time = 0.47, size = 32, normalized size = 0.91

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

Mupad [B]
time = 0.15, size = 27, normalized size = 0.77

$$2\sqrt{a+b\,x}-2\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x,x)
[Out] 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

3.181 $\int \frac{\sqrt{a+bx}}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^2}\,dx &= -\frac{\sqrt{a+bx}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= -\frac{\sqrt{a+bx}}{x}+\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= -\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 39, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.95

method	result	size
risch	$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$	32
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37
default	$2b\left(-\frac{\sqrt{bx+a}}{2bx}-\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 2.24, size = 47, normalized size = 1.21

$$\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - sq
rt(b*x + a)/x
```

Fricas [A]
time = 0.49, size = 93, normalized size = 2.38

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2ax},\ \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [A]
time = 0.88, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

Giac [A]
time = 0.47, size = 41, normalized size = 1.05

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx+a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.79

$$-\frac{\sqrt{a+b\,x}}{x}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.182 $\int \frac{1}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

[Out] 2*(b*x+a)^(1/2)/b

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*x],x]

[Out] (2*Sqrt[a + b*x])/b

Maple [A]
time = 0.03, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{2\sqrt{bx+a}}{b}$	13
derivativedivides	$\frac{2\sqrt{bx+a}}{b}$	13
default	$\frac{2\sqrt{bx+a}}{b}$	13
trager	$\frac{2\sqrt{bx+a}}{b}$	13
risch	$\frac{2\sqrt{bx+a}}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

Maxima [A]
time = 2.81, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A]
time = 0.58, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

Giac [A]
time = 0.47, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{a+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2))/b
```

3.183 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=32

$$-\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b^2-2*a*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx\\ &= -\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.72

$$\frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.81

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
trager	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
risch	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^{2}}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^{2}}$	26
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^{2}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/3*(b*x+a)^(3/2)-a*(b*x+a)^(1/2))
```

Maxima [A]
time = 2.66, size = 26, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^{2}}-\frac{2\,\sqrt{bx+a}\,a}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Fricas [A]
time = 0.65, size = 19, normalized size = 0.59

$$\frac{2\,\sqrt{bx+a}\,(bx-2\,a)}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.
time = 0.53, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x}-\frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x}+\frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

Giac [A]
time = 0.53, size = 23, normalized size = 0.72

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.78

$$-\frac{6\,a\,\sqrt{a+b\,x}-2\,{\left(a+b\,x\right)}^{3/2}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/2),x)
```

```
[Out] -(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

3.184 $\int \frac{x^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}$$

```
[Out] -4/3*a*(b*x+a)^(3/2)/b^3+2/5*(b*x+a)^(5/2)/b^3+2*a^2*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\,(8a^2-4abx+3b^2x^2)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.73

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4abx+8a^2)}{15b^3}$	32
trager	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4abx+8a^2)}{15b^3}$	32
risch	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4abx+8a^2)}{15b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*(b*x+a)^(5/2)-2/3*a*(b*x+a)^(3/2)+a^2*(b*x+a)^(1/2))
```

Maxima [A]
time = 2.04, size = 41, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3}+\frac{2\,\sqrt{bx+a}\,a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

Fricas [A]
time = 0.61, size = 31, normalized size = 0.61

$$\frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(48) = 96$.
time = 0.81, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

Giac [A]
time = 0.46, size = 37, normalized size = 0.73

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}\,a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,{\left(a+b\,x\right)}^{5/2}-20\,a\,{\left(a+b\,x\right)}^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/2),x)
```

```
[Out] (6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3
)
```

3.185 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.03, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 2.08, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.63, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A]
time = 0.45, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A]
time = 0.47, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 0.14, size = 17, normalized size = 0.74

$$-\frac{2\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.186 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{a+bx}}{ax}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out] b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {44, 65, 214}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + b*x]),x]

[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} - \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}$$
$$= -\frac{\sqrt{a+bx}}{ax} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a}$$
$$= -\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.03, size = 40, normalized size = 0.98

method	result	size
risch	$\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{bx+a}}{ax}$	34
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40
default	$2b\left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Maxima [A]
time = 3.32, size = 60, normalized size = 1.46

$$-\frac{\sqrt{bx+a}\,b}{(bx+a)a-a^2}-\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

Fricas [A]
time = 0.62, size = 93, normalized size = 2.27

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\,a}{2\,a^2x},-\frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [A]
time = 1.13, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

Giac [A]
time = 0.48, size = 47, normalized size = 1.15

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx+a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

Mupad [B]
time = 0.16, size = 33, normalized size = 0.80

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\,x}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

3.187 $\int (a+bx)^{p/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(a+bx)^{\frac{2+p}{2}}}{b(2+p)}$$

```
[Out] 2*(b*x+a)^(1+1/2*p)/b/(2+p)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {32}

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^((2 + p)/2))/(b*(2 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{p/2}\,dx = \frac{2(a+bx)^{\frac{2+p}{2}}}{b(2+p)}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.04

$$\frac{2(a+bx)^{1+\frac{p}{2}}}{2b+bp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2))/(2*b + b*p)
```

Maple [A]
time = 0.03, size = 25, normalized size = 1.09

method	result	size
gosper	$\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$	25
risch	$\frac{2(bx+a)(bx+a)^{\frac{p}{2}}}{b(2+p)}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*x+a)*((b*x+a)^(1/2))^p/b/(2+p)
```

Maxima [A]
time = 1.62, size = 21, normalized size = 0.91

$$\frac{2\,(bx+a)^{\frac{1}{2}\,p+1}}{b(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

Fricas [A]
time = 0.81, size = 25, normalized size = 1.09

$$\frac{2\,(bx+a)\sqrt{bx+a}^{\,p}}{bp+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(b*x + a)*sqrt(b*x + a)^p/(b*p + 2*b)
```

Sympy [A]
time = 0.01, size = 24, normalized size = 1.04

$$\frac{\begin{cases} \frac{(a+bx)^{\frac{p}{2}+1}}{\frac{p}{2}+1} & \text{for}\ p \neq -2 \\ \log\left(a+bx\right) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise(((a + b*x)**(p/2 + 1)/(p/2 + 1), Ne(p, -2)), (log(a + b*x), True)
)/b
```

Giac [A]
time = 0.45, size = 21, normalized size = 0.91

$$\frac{2\,(bx+a)^{\frac{1}{2}\,p+1}}{b(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

Mupad [B]
time = 0.25, size = 21, normalized size = 0.91

$$\frac{2\,{\left(a+b\,x\right)}^{\frac{p}{2}+1}}{b\,\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(p/2),x)
```

```
[Out] (2*(a + b*x)^(p/2 + 1))/(b*(p + 2))
```

3.188 $\int x(a+bx)^{p/2}\,dx$

Optimal. Leaf size=48

$$-\frac{2a(a+bx)^{\frac{2+p}{2}}}{b^2(2+p)}+\frac{2(a+bx)^{\frac{4+p}{2}}}{b^2(4+p)}$$

```
[Out] -2*a*(b*x+a)^(1+1/2*p)/b^2/(2+p)+2*(b*x+a)^(2+1/2*p)/b^2/(4+p)
```

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)}-\frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(p/2),x]
```

```
[Out] (-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p)/2))/(b^2
*(4 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{p/2}\,dx &= \int\left(\frac{(a+bx)^{1+\frac{p}{2}}}{b}-\frac{a(a+bx)^{p/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{\frac{2+p}{2}}}{b^2(2+p)}+\frac{2(a+bx)^{\frac{4+p}{2}}}{b^2(4+p)}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 38, normalized size = 0.79

$$\frac{2(a+bx)^{1+\frac{p}{2}}(-2a+b(2+p)x)}{b^2(2+p)(4+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2)*(-2*a + b*(2 + p)*x))/(b^2*(2 + p)*(4 + p))
```

Maple [A]
time = 0.03, size = 43, normalized size = 0.90

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{p}{2}}(-xpb-2bx+2a)(bx+a)}{b^2(p^2+6p+8)}$	43
risch	$-\frac{2(-x^2b^2p-xapb-2x^2b^2+2a^2)(bx+a)^{\frac{p}{2}}}{b^2(4+p)(2+p)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^(1/2))^p,x,method=_RETURNVERBOSE)
```

```
[Out] -2*((b*x+a)^(1/2))^p*(-b*p*x-2*b*x+2*a)*(b*x+a)/b^2/(p^2+6*p+8)
```

Maxima [A]
time = 1.90, size = 45, normalized size = 0.94

$$\frac{2\,(b^2(p+2)x^2+abpx-2\,a^2)(bx+a)^{\frac{1}{2}\,p}}{(p^2+6\,p+8)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(p + 2)*x^2 + a*b*p*x - 2*a^2)*(b*x + a)^(1/2*p)/((p^2 + 6*p + 8)*b^
2)
```

Fricas [A]
time = 0.73, size = 58, normalized size = 1.21

$$\frac{2\,(abpx+(b^2p+2\,b^2)x^2-2\,a^2)\sqrt{bx+a}^{\,p}}{b^2p^2+6\,b^2p+8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(a*b*p*x + (b^2*p + 2*b^2)*x^2 - 2*a^2)*sqrt(b*x + a)^p/(b^2*p^2 + 6*b^2*
p + 8*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(37) = 74$.

time = 0.23, size = 216, normalized size = 4.50

$$\begin{cases} \frac{a^{\frac{p}{2}}x^2}{2} & \text{for } b = 0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p = -4 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } p = -2 \\ -\frac{4a^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2abpx(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2b^2px^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{4b^2x^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise((a**(p/2)*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) +
a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -4)), (-a*l
og(a/b + x)/b**2 + x/b, Eq(p, -2)), (-4*a**2*(a + b*x)**(p/2)/(b**2*p**2 +
6*b**2*p + 8*b**2) + 2*a*b*p*x*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b
**2) + 2*b**2*p*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 4*b
**2*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2), True))
```

Giac [A]
time = 0.52, size = 86, normalized size = 1.79

$$\frac{2\left((bx+a)^{\frac{1}{2}p}b^2px^2 + (bx+a)^{\frac{1}{2}p}abpx + 2(bx+a)^{\frac{1}{2}p}b^2x^2 - 2(bx+a)^{\frac{1}{2}p}a^2\right)}{b^2p^2 + 6b^2p + 8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((b*x + a)^(1/2*p)*b^2*p*x^2 + (b*x + a)^(1/2*p)*a*b*p*x + 2*(b*x + a)^(1
/2*p)*b^2*x^2 - 2*(b*x + a)^(1/2*p)*a^2)/(b^2*p^2 + 6*b^2*p + 8*b^2)
```

Mupad [B]
time = 0.45, size = 94, normalized size = 1.96

$$\begin{cases} -\frac{a\ln(a+bx)-bx}{b^2} & \text{if } p=-2 \\ \frac{\ln(a+bx)+\frac{a}{a+bx}}{b^2} & \text{if } p=-4 \\ \frac{2\left(\frac{(a+bx)^{\frac{p}{2}+2}}{p+4} - \frac{a(a+bx)^{\frac{p}{2}+1}}{p+2}\right)}{b^2} & \text{if } p\neq -2 \land p\neq -4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(p/2),x)
```

```
[Out] piecewise(p == -2, -(a*log(a + b*x) - b*x)/b^2, p == -4, (log(a + b*x) + a/
(a + b*x))/b^2, p ~= -2 & p ~= -4, (2*((a + b*x)^(p/2 + 2)/(p + 4) - (a*(a
+ b*x)^(p/2 + 1))/(p + 2)))/b^2)
```

3.189 $\int \tan^{-1}\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$

Optimal. Leaf size=55

$$\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}\,x\right)-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{2\sqrt{2}}$$

```
[Out] x*arctan(-1+x*2^(1/2))-1/2*arctan(-1+x*2^(1/2))*2^(1/2)-1/4*ln(1+x^2-x*2^(1
/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5311, 12, 648, 631, 210, 642}

$$-x\text{ArcTan}\left(1-\sqrt{2}\,x\right)+\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - x*ArcTan[1 - Sqrt[2]*x] - Log[1 - Sqrt[2]*x
 + x^2]/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx &= -x\tan^{-1}\left(1-\sqrt{2}\,x\right) - \int \frac{x}{\sqrt{2}\left(1-\sqrt{2}\,x+x^2\right)}\,dx \\
&= -x\tan^{-1}\left(1-\sqrt{2}\,x\right) - \frac{\int \frac{x}{1-\sqrt{2}\,x+x^2}\,dx}{\sqrt{2}} \\
&= -x\tan^{-1}\left(1-\sqrt{2}\,x\right) - \frac{1}{2}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{\int \frac{-\sqrt{2}+2x}{1-\sqrt{2}\,x+x^2}\,dx}{2\sqrt{2}} \\
&= -x\tan^{-1}\left(1-\sqrt{2}\,x\right) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{}\right.}{\sqrt{2}} \\
&= \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}} - x\tan^{-1}\left(1-\sqrt{2}\,x\right) - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.87

$$\frac{1}{4}\left(2\left(\sqrt{2}-2x\right)\tan^{-1}\left(1-\sqrt{2}\,x\right)-\sqrt{2}\,\log\left(1-\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] (2*(Sqrt[2] - 2*x)*ArcTan[1 - Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
)/4
```

Maple [A]
time = 0.05, size = 37, normalized size = 0.67

method	result
derivativedivides	$\frac{\sqrt{2}\left(\left(-1+x\sqrt{2}\right)\arctan\left(-1+x\sqrt{2}\right)-\frac{\ln\left(\left(-1+x\sqrt{2}\right)^{2}+1\right)}{2}\right)}{2}$
default	$\frac{\sqrt{2}\left(\left(-1+x\sqrt{2}\right)\arctan\left(-1+x\sqrt{2}\right)-\frac{\ln\left(\left(-1+x\sqrt{2}\right)^{2}+1\right)}{2}\right)}{2}$
risch	$\frac{ix\ln\left(1+\frac{i\left(-2x+\sqrt{2}\right)\sqrt{2}}{2}\right)}{2}-\frac{ix\ln\left(1-\frac{i\left(-2x+\sqrt{2}\right)\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(4-4x\sqrt{2}+4x^{2}\right)}{4}-\frac{\sqrt{2}\,\text{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*((-1+x*2^(1/2))*arctan(-1+x*2^(1/2))-1/2*ln((-1+x*2^(1/2))^2+1)
)
```

Maxima [A]
time = 3.15, size = 52, normalized size = 0.95

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^{2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
log(1/2*(2*x - sqrt(2))^2 + 1))
```

Fricas [A]
time = 0.78, size = 37, normalized size = 0.67

$$\frac{1}{2}\left(2x-\sqrt{2}\right)\arctan\left(\sqrt{2}x-1\right)-\frac{1}{4}\sqrt{2}\log\left(x^{2}-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x - sqrt(2))*arctan(sqrt(2)*x - 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x
 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(48) = 96$.
time = 0.39, size = 230, normalized size = 4.18

$$\frac{4x^3 \operatorname{atan}\left(\sqrt{2}\,x-1\right)}{4x^2-4\sqrt{2}\,x+4}-\frac{\sqrt{2}\,x^2 \log\left(x^2-\sqrt{2}\,x+1\right)}{4x^2-4\sqrt{2}\,x+4}-\frac{6\sqrt{2}\,x^2 \operatorname{atan}\left(\sqrt{2}\,x-1\right)}{4x^2-4\sqrt{2}\,x+4}+\frac{2x \log\left(x^2-\sqrt{2}\,x+1\right)}{4x^2-4\sqrt{2}\,x+4}+\frac{8x \operatorname{atan}\left(\sqrt{2}\,x-1\right)}{4x^2-4\sqrt{2}\,x+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{4x^2-4\sqrt{2}\,x+4}-\frac{2\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{4x^2-4\sqrt{2}\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)
```

```
[Out] 4*x**3*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*x**2*log(x*
*2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 6*sqrt(2)*x**2*atan(sqrt(2
)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 2*x*log(x**2 - sqrt(2)*x + 1)/(4*x**2
 - 4*sqrt(2)*x + 4) + 8*x*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) -
sqrt(2)*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 2*sqrt(2)*at
an(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4)
```

Giac [A]
time = 0.50, size = 52, normalized size = 0.95

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
log(1/2*(2*x - sqrt(2))^2 + 1))
```

Mupad [B]
time = 0.16, size = 43, normalized size = 0.78

$$\operatorname{atan}\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\right)}{2}\right)\left(x-\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{2}\ln\left(\left(2x-\sqrt{2}\right)^2+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((2^(1/2)*(2*x - 2^(1/2)))/2),x)
```

```
[Out] atan((2^(1/2)*(2*x - 2^(1/2)))/2)*(x - 2^(1/2)/2) - (2^(1/2)*log((2*x - 2^(
1/2))^2 + 2))/4
```

3.190 $\int \frac{1}{\sqrt{-1+x^2}} \, dx$

Optimal. Leaf size=12

$$\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

```
[Out] arctanh(x/(x^2-1)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {223, 212}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}} \, dx &= \text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.
time = 0.00, size = 38, normalized size = 3.17

$$-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\frac{1}{2}\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + x^2],x]
```

```
[Out] -1/2*Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[-1 + x^2]]/2
```

Maple [A]
time = 0.05, size = 11, normalized size = 0.92

method	result	size
default	$\ln\left(x+\sqrt{x^2-1}\right)$	11
trager	$\ln\left(x+\sqrt{x^2-1}\right)$	11
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(x^2-1\right)}\ \arcsin(x)}{\sqrt{\mathrm{signum}\left(x^2-1\right)}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x+(x^2-1)^(1/2))
```

Maxima [A]
time = 3.21, size = 14, normalized size = 1.17

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A]
time = 0.75, size = 14, normalized size = 1.17

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [A]
time = 0.04, size = 2, normalized size = 0.17

$$\operatorname{acosh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2),x)
```

```
[Out] acosh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.50, size = 26, normalized size = 2.17

$$\frac{1}{2}\sqrt{x^2-1}\,x + \frac{1}{2}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 1)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [B]
time = 0.20, size = 10, normalized size = 0.83

$$\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^(1/2),x)
```

```
[Out] log(x + (x^2 - 1)^(1/2))
```

3.191 $\int \sqrt{x}\,\sqrt{1+x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{4}\sqrt{x}\,\sqrt{1+x}\,+\frac{1}{2}x^{3/2}\sqrt{1+x}\,-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\,\right)$$

```
[Out] -1/4*arcsinh(x^(1/2))+1/2*x^(3/2)*(1+x)^(1/2)+1/4*x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 56, 221}

$$\frac{1}{2}\sqrt{x+1}\,x^{3/2}+\frac{1}{4}\sqrt{x+1}\,\sqrt{x}\,-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\,\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x])/4 + (x^(3/2)*Sqrt[1 + x])/2 - ArcSinh[Sqrt[x]]/4
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{1+x}\,dx &= \frac{1}{2}x^{3/2}\sqrt{1+x}+\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{8}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 0.86

$$\frac{1}{4}\left(\sqrt{x}\sqrt{1+x}\,(1+2x)-\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(1 + 2*x) - ArcTanh[Sqrt[x/(1 + x)]])/4
```

Maple [A]
time = 0.03, size = 50, normalized size = 1.16

method	result	size
meijerg	$-\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,(3+6x)\sqrt{1+x}}{6}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{2}}{2\sqrt{\pi}}$	34
risch	$\frac{(1+2x)\sqrt{x}\,\sqrt{1+x}}{4}-\frac{\sqrt{x\,(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8\sqrt{1+x}\,\sqrt{x}}$	45
default	$\frac{\sqrt{x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{x}\,\sqrt{1+x}}{4}-\frac{\sqrt{x\,(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8\sqrt{1+x}\,\sqrt{x}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)*(1+x)^(3/2)-1/4*x^(1/2)*(1+x)^(1/2)-1/8*(x*(1+x))^(1/2)/(1+x)^(
1/2)/x^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(27) = 54$.

time = 2.34, size = 71, normalized size = 1.65

$$\frac{\frac{(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}+\frac{\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^2}{x^2}-\frac{2\,(x+1)}{x}+1\right)}-\frac{1}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{1}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1
)/x + 1) - 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(sqrt(x + 1)/sqrt(x) -
 1)
```

Fricas [A]
time = 0.74, size = 34, normalized size = 0.79

$$\frac{1}{4}\,(2\,x+1)\sqrt{x+1}\,\sqrt{x}+\frac{1}{8}\,\log\left(2\,\sqrt{x+1}\,\sqrt{x}-2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/8*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1
)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.57, size = 119, normalized size = 2.77

$$\begin{cases} -\frac{\operatorname{acosh}\left(\sqrt{x+1}\right)}{4}+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{x}}-\frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x}}+\frac{\sqrt{x+1}}{4\sqrt{x}} & \text{for } |x+1|>1 \\ \frac{i\operatorname{asin}\left(\sqrt{x+1}\right)}{4}-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}}+\frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}}-\frac{i\sqrt{x+1}}{4\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1))/4 + (x + 1)**(5/2)/(2*sqrt(x)) - 3*(x + 1)**
(3/2)/(4*sqrt(x)) + sqrt(x + 1)/(4*sqrt(x)), Abs(x + 1) > 1), (I*asin(sqrt(
x + 1))/4 - I*(x + 1)**(5/2)/(2*sqrt(-x)) + 3*I*(x + 1)**(3/2)/(4*sqrt(-x))
 - I*sqrt(x + 1)/(4*sqrt(-x)), True))
```

Giac [A]
time = 0.50, size = 39, normalized size = 0.91

$$\frac{1}{4}\,(2\,x-3)\sqrt{x+1}\,\sqrt{x}+\sqrt{x+1}\,\sqrt{x}+\frac{1}{4}\,\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 3)*sqrt(x + 1)*sqrt(x) + sqrt(x + 1)*sqrt(x) + 1/4*log(sqrt(x +
1) - sqrt(x))
```

Mupad [B]
time = 0.22, size = 30, normalized size = 0.70

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{1}{4}\right)\,\sqrt{x+1}-\frac{\ln\left(x+\sqrt{x}\,\sqrt{x+1}+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x + 1)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + 1/4)*(x + 1)^(1/2) - log(x + x^(1/2)*(x + 1)^(1/2) + 1/2)/8
```

3.192 $\int \sin\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3442, 3377, 2717}

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\,\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \sin\left(\sqrt{x}\right)\,dx&=2\text{Subst}\left(\int x\sin(x)\,dx,x,\sqrt{x}\right)\\&=-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\text{Subst}\left(\int \cos(x)\,dx,x,\sqrt{x}\right)\\&=-2\sqrt{x}\,\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$2\sin\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
default	$2\sin\left(\sqrt{x}\right)-2\cos\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$4\sqrt{\pi}\left(-\frac{\sqrt{x}\,\cos\left(\sqrt{x}\right)}{2\sqrt{\pi}}+\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{\pi}}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Maxima [A]
time = 2.01, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A]
time = 1.00, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A]
time = 0.09, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A]
time = 0.46, size = 16, normalized size = 0.73

$$-2\sqrt{x}\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Mupad [B]
time = 0.25, size = 16, normalized size = 0.73

$$2\sin\left(\sqrt{x}\right)-2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
[Out] 2*sin(x^(1/2)) - 2*x^(1/2)*cos(x^(1/2))
```

3.193 $\int \frac{x}{(1-x^2)^{9/8}} \, dx$

Optimal. Leaf size=13

$$\frac{4}{\sqrt[8]{1-x^2}}$$

```
[Out] 4/(-x^2+1)^(1/8)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2)^(9/8),x]
```

```
[Out] 4/(1 - x^2)^(1/8)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(1-x^2)^{9/8}} \, dx = \frac{4}{\sqrt[8]{1-x^2}}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2)^(9/8),x]
```

```
[Out] 4/(1 - x^2)^(1/8)
```

Maple [A]
time = 0.05, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$	12
default	$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$	12
risch	$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$	12
meijerg	$\frac{x^2 \operatorname{hypergeom}\left(\left[1, \frac{9}{8}\right], [2], x^2\right)}{2}$	15
gosper	$-\frac{4(-1+x)(1+x)}{(-x^2+1)^{\frac{9}{8}}}$	18
trager	$-\frac{4(-x^2+1)^{\frac{7}{8}}}{x^2-1}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(9/8),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(-x^2+1)^(1/8)
```

Maxima [A]
time = 1.48, size = 11, normalized size = 0.85

$$\frac{4}{(-x^2+1)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="maxima")
```

```
[Out] 4/(-x^2 + 1)^(1/8)
```

Fricas [A]
time = 0.76, size = 18, normalized size = 1.38

$$-\frac{4\,(-x^2+1)^{\frac{7}{8}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="fricas")
```

```
[Out] -4*(-x^2 + 1)^(7/8)/(x^2 - 1)
```

Sympy [A]
time = 0.28, size = 8, normalized size = 0.62

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(9/8),x)
```

```
[Out] 4/(1 - x**2)**(1/8)
```

Giac [A]
time = 0.49, size = 11, normalized size = 0.85

$$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="giac")
```

```
[Out] 4/(-x^2 + 1)^(1/8)
```

Mupad [B]
time = 0.35, size = 11, normalized size = 0.85

$$\frac{4}{\left(1-x^{2}\right)^{1/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^2)^(9/8),x)
```

```
[Out] 4/(1 - x^2)^(1/8)
```

3.194 $\int \frac{x}{\sqrt{1-x^4}} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

[Out] 1/2*arcsin(x^2)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {281, 222}

$$\frac{\text{ArcSin}(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[1 - x^4],x]

[Out] ArcSin[x^2]/2

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1-x^4}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, x^2\right)\\ &= \frac{1}{2}\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.
time = 0.06, size = 20, normalized size = 2.50

$$-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{1-x^4}}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^4],x]
```

```
[Out] -1/2*ArcTan[Sqrt[1 - x^4]/x^2]
```

Maple [A]
time = 0.13, size = 7, normalized size = 0.88

method	result	size
default	$\frac{\arcsin\left(x^{2}\right)}{2}$	7
meijerg	$\frac{\arcsin\left(x^{2}\right)}{2}$	7
elliptic	$\frac{\arcsin\left(x^{2}\right)}{2}$	7
trager	$\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right) \ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right) \sqrt{-x^{4}+1}+x^{2}\right)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(x^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 3.83, size = 16, normalized size = 2.00

$$-\frac{1}{2} \arctan\left(\frac{\sqrt{-x^{4}+1}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.
time = 0.63, size = 18, normalized size = 2.25

$$-\arctan\left(\frac{\sqrt{-x^{4}+1}-1}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 19, normalized size = 2.38

$$\begin{cases} -\frac{i\operatorname{acosh}\left(x^{2}\right)}{2} & \text{for}\ \left|x^{4}\right| > 1 \\ \frac{\operatorname{asin}\left(x^{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+1)**(1/2),x)
[Out] Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

Giac [A]
time = 0.49, size = 6, normalized size = 0.75

$$\frac{1}{2}\arcsin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] 1/2*arcsin(x^2)
```

Mupad [B]
time = 0.31, size = 16, normalized size = 2.00

$$\frac{\operatorname{atan}\left(\frac{x^{2}}{\sqrt{1-x^{4}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - x^4)^(1/2),x)
[Out] atan(x^2/(1 - x^4)^(1/2))/2
```

3.195 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)$$

```
[Out] -1/2*arctanh((x^4+1)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {272, 65, 213}

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*ArcTanh[Sqrt[1 + x^4]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)$$
$$= -\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^4]),x]
[Out] -1/2*ArcTanh[Sqrt[1 + x^4]]
```

Maple [A]
time = 0.10, size = 11, normalized size = 0.79

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	11
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	11
trager	$\frac{\ln\left(\frac{-1+\sqrt{x^4+1}}{x^2}\right)}{2}$	17
meijerg	$\frac{(4\ln(x)-2\ln(2))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.
time = 2.17, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 0.77, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Sympy [A]
time = 0.40, size = 8, normalized size = 0.57

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+1)**(1/2),x)
```

```
[Out] -asinh(x**(-2))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 0.46, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Mupad [B]
time = 0.19, size = 10, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + 1)^(1/2)),x)
```

```
[Out] -atanh((x^4 + 1)^(1/2))/2
```

3.196 $\int \frac{x}{\sqrt{1+x^2+x^4}} \, dx$

Optimal. Leaf size=18

$$\frac{1}{2}\sinh^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)$$

```
[Out] 1/2*arcsinh(1/3*(2*x^2+1)*3^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1121, 633, 221}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, x^2\right)$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x^2\right)}{2\sqrt{3}}$$

$$= \frac{1}{2}\sinh^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)$$

Mathematica [A]
time = 0.05, size = 26, normalized size = 1.44

$$-\frac{1}{2}\log\left(-1-2x^2+2\sqrt{1+x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^2 + x^4],x]
[Out] -1/2*Log[-1 - 2*x^2 + 2*Sqrt[1 + x^2 + x^4]]
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.78

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}$	14
elliptic	$\frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x^2+\frac{1}{2}\right)}{3}\right)}{2}$	14
trager	$-\frac{\ln\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(2/3*3^(1/2)*(x^2+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^4 + x^2 + 1), x)
```

Fricas [A]
time = 0.77, size = 22, normalized size = 1.22

$$-\frac{1}{2}\log\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1)**(1/2),x)
[Out] Integral(x/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [A]
time = 0.48, size = 22, normalized size = 1.22

$$-\frac{1}{2}\log\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

Mupad [B]
time = 0.36, size = 18, normalized size = 1.00

$$\frac{\ln\left(\sqrt{x^4+x^2+1}+x^2+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 + x^4 + 1)^(1/2),x)
[Out] log((x^2 + x^4 + 1)^(1/2) + x^2 + 1/2)/2
```

3.197 $\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2}\tan^{-1}\left(\frac{2-x^2}{2\sqrt{-1+x^2-x^4}}\right)$$

```
[Out] -1/2*arctan(1/2*(-x^2+2)/(-x^4+x^2-1)^(1/2))
```

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 738, 210}

$$-\frac{1}{2}\text{ArcTan}\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

```
[Out] -1/2*ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x-x^2}}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-2+x^2}{\sqrt{-1+x^2-x^4}}\right)$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{-2+x^2}{2\sqrt{-1+x^2-x^4}}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.05, size = 27, normalized size = 0.90

$$-i\tanh^{-1}\left(x^2+i\sqrt{-1+x^2-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

```
[Out] (-I)*ArcTanh[x^2 + I*Sqrt[-1 + x^2 - x^4]]
```

Maple [A]
time = 0.14, size = 23, normalized size = 0.77

method	result	size
default	$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$	23
elliptic	$\frac{\arctan\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)}{2}$	23
trager	$\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\text{RootOf}\left(\_Z^2+1\right)x^2+2\sqrt{-x^4+x^2-1}+2\,\text{RootOf}\left(\_Z^2+1\right)}{x^2}\right)}{2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^4+x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(1/2*(x^2-2)/(-x^4+x^2-1)^(1/2))
```

Maxima [C] Result contains complex when optimal does not.
time = 4.22, size = 17, normalized size = 0.57

$$-\frac{1}{2}i\,\text{arsinh}\left(-\frac{1}{3}\sqrt{3}+\frac{2\sqrt{3}}{3\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*arcsinh(-1/3*sqrt(3) + 2/3*sqrt(3)/x^2)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.61, size = 55, normalized size = 1.83

$$\frac{1}{4}i\log\left(\frac{x^2+2i\sqrt{-x^4+x^2-1}-2}{2x^2}\right)-\frac{1}{4}i\log\left(\frac{x^2-2i\sqrt{-x^4+x^2-1}-2}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*I*log(1/2*(x^2 + 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2) - 1/4*I*log(1/2*(x^
2 - 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{-x^4+x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**4+x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x**4 + x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 - 1)*x), x)
```

Mupad [B]
time = 0.54, size = 32, normalized size = 1.07

$$\frac{\ln\left(\frac{1}{x^2}\right)\,1\mathrm{i}}{2}+\frac{\ln\left(x^2-2+\sqrt{-x^4+x^2-1}\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - x^4 - 1)^(1/2)),x)
```

```
[Out] (log(1/x^2)*1i)/2 + (log((x^2 - x^4 - 1)^(1/2)*2i + x^2 - 2)*1i)/2
```

3.198 $\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{1+x^2}}{1-x}$$

[Out] (x^2+1)^(1/2)/(1-x)

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {817}

$$\frac{\sqrt{x^2+1}}{1-x}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]

[Out] Sqrt[1 + x^2]/(1 - x)

Rule 817

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*
g, 0]
```

Rubi steps

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = \frac{\sqrt{1+x^2}}{1-x}$$

Mathematica [A]
time = 0.12, size = 17, normalized size = 1.00

$$\frac{\sqrt{1+x^2}}{1-x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]

```
[Out] Sqrt[1 + x^2]/(1 - x)
```

Maple [A]
time = 0.04, size = 19, normalized size = 1.12

method	result	size
gosper	$-\frac{\sqrt{x^2+1}}{-1+x}$	15
trager	$-\frac{\sqrt{x^2+1}}{-1+x}$	15
risch	$-\frac{\sqrt{x^2+1}}{-1+x}$	15
default	$-\frac{\sqrt{(-1+x)^2+2x}}{-1+x}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(1-x)^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(-1+x)*((-1+x)^2+2*x)^(1/2)
```

Maxima [A]
time = 2.68, size = 14, normalized size = 0.82

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.71, size = 17, normalized size = 1.00

$$-\frac{x+\sqrt{x^2+1}-1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(x + sqrt(x^2 + 1) - 1)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x-1)^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x - 1)**2*sqrt(x**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.
time = 0.73, size = 35, normalized size = 2.06

$$-\frac{\sqrt{\frac{2}{x-1}+\frac{2}{(x-1)^2}+1}}{\operatorname{sgn}\left(\frac{1}{x-1}\right)}+\operatorname{sgn}\left(\frac{1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2/(x - 1) + 2/(x - 1)^2 + 1)/sgn(1/(x - 1)) + sgn(1/(x - 1))
```

Mupad [B]
time = 0.15, size = 14, normalized size = 0.82

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^2 + 1)^(1/2)*(x - 1)^2),x)
```

```
[Out] -(x^2 + 1)^(1/2)/(x - 1)
```

3.199 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \sinh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4.
time = 0.01, size = 12, normalized size = 6.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[1 + x^2]]
```

Maple [A]
time = 0.04, size = 3, normalized size = 1.50

method	result	size
default	$\operatorname{arcsinh}(x)$	3
meijerg	$\operatorname{arcsinh}(x)$	3
trager	$\ln\left(x+\sqrt{x^2+1}\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)
```

Maxima [A]
time = 3.56, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.
time = 0.60, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A]
time = 0.04, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(2) = 4$.
time = 0.78, size = 25, normalized size = 12.50

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B]
time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
```

```
[Out] asinh(x)
```

3.200 $\int \frac{\sqrt{x}\,\sqrt{1+x}+\sqrt{x}\,\sqrt{2+x}+\sqrt{1+x}\,\sqrt{2+x}}{2\sqrt{x}\,\sqrt{1+x}\,\sqrt{2+x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{x}+\sqrt{1+x}+\sqrt{2+x}$$

```
[Out] x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}=0.031$, Rules used = {12, 6820}

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(
2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

```
[Out] Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}\,\sqrt{1+x}+\sqrt{x}\,\sqrt{2+x}+\sqrt{1+x}\,\sqrt{2+x}}{2\sqrt{x}\,\sqrt{1+x}\,\sqrt{2+x}}\,dx &= \frac{1}{2}\int \frac{\sqrt{x}\,\sqrt{1+x}+\sqrt{x}\,\sqrt{2+x}+\sqrt{1+x}\,\sqrt{2+}}{\sqrt{x}\,\sqrt{1+x}\,\sqrt{2+x}}\\ &= \frac{1}{2}\int \left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{1+x}}+\frac{1}{\sqrt{2+x}}\right)dx\\ &= \sqrt{x}+\sqrt{1+x}+\sqrt{2+x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 1.50

$$\frac{1}{2}\left(2\sqrt{x}+2\sqrt{1+x}+2\sqrt{2+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 +
 x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

```
[Out] (2*Sqrt[x] + 2*Sqrt[1 + x] + 2*Sqrt[2 + x])/2
```

Maple [A]
time = 0.05, size = 15, normalized size = 0.75

method	result	size
default	$\sqrt{x}+\sqrt{1+x}+\sqrt{2+x}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x
^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

Maxima [A]
time = 2.02, size = 14, normalized size = 0.70

$$\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

Fricas [A]
time = 0.61, size = 14, normalized size = 0.70

$$\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

Sympy [A]
time = 0.23, size = 17, normalized size = 0.85

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(2+
x)**(1/2))/x**(1/2)/(1+x)**(1/2)/(2+x)**(1/2),x)
```

```
[Out] sqrt(x) + sqrt(x + 1) + sqrt(x + 2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)*
sqrt(x))/(sqrt(x + 2)*sqrt(x + 1)*sqrt(x)), x)
```

Mupad [**B**]
time = 0.34, size = 14, normalized size = 0.70

$$\sqrt{x+1} + \sqrt{x+2} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2)*(x + 1)^(1/2))/2 + (x^(1/2)*(x + 2)^(1/2))/2 + ((x + 1)^(1/2)
*(x + 2)^(1/2))/2)/(x^(1/2)*(x + 1)^(1/2)*(x + 2)^(1/2)),x)
```

```
[Out] (x + 1)^(1/2) + (x + 2)^(1/2) + x^(1/2)
```

3.201 $\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx$

Optimal. Leaf size=24

$$-\sqrt{1+x^3}+\sqrt{1-2x+x^5}$$

```
[Out] -(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 68, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {12, 6820, 267, 2124}

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2
*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2124

```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Dist
[Simplify[Pm - Coeff[Pm, x, m]*(D[Qn, x]/(n*Coeff[Qn, x, n]))], Int[Qn^p, x
], x] /; EqQ[m, n - 1] && EqQ[D[Simplify[Pm - (Coeff[Pm, x, m]/(n*Coeff[Qn,
 x, n]))*D[Qn, x]], x], 0]] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x]
&& NeQ[p, -1]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx = \frac{1}{2}\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+}}{\sqrt{1+x^3}\sqrt{1-2x+x^5}}$$

$$= \frac{1}{2}\int \left(-\frac{3x^2}{\sqrt{1+x^3}}-\frac{2}{\sqrt{1-2x+x^5}}+\frac{5x^4}{\sqrt{1-2x-}}\right.$$

$$= -\left(\frac{3}{2}\int \frac{x^2}{\sqrt{1+x^3}}\,dx\right)+\frac{5}{2}\int \frac{x^4}{\sqrt{1-2x+x^5}}\,dx$$

$$= -\sqrt{1+x^3}+\sqrt{1-2x+x^5}$$

Mathematica [**A**]
time = 23.84, size = 24, normalized size = 1.00

$$-\sqrt{1+x^3}+\sqrt{1-2x+x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^
5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Maple [**A**]
time = 0.07, size = 21, normalized size = 0.88

method	result	size
default	$-\sqrt{x^3+1}+\sqrt{x^5-2x+1}$	21
elliptic	$-\sqrt{x^3+1}+\frac{(-1+x)(x^4+x^3+x^2+x-1)}{\sqrt{x^5-2x+1}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3
+1)^(1/2)/(x^5-2*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

Maxima [**A**]
time = 2.52, size = 30, normalized size = 1.25

$$\sqrt{x^4+x^3+x^2+x-1}\,\sqrt{x-1}-\sqrt{x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + x^3 + x^2 + x - 1)*sqrt(x - 1) - sqrt(x^3 + 1)
```

Fricas [A]
time = 0.65, size = 20, normalized size = 0.83

$$\sqrt{x^5 - 2\,x + 1} - \sqrt{x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^5 - 2*x + 1) - sqrt(x^3 + 1)
```

Sympy [A]
time = 0.28, size = 19, normalized size = 0.79

$$-\sqrt{x^3 + 1} + \sqrt{x^5 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-2*x+1
)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)
```

```
[Out] -sqrt(x**3 + 1) + sqrt(x**5 - 2*x + 1)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^3
 + 1))/(sqrt(x^5 - 2*x + 1)*sqrt(x^3 + 1)), x)
```

Mupad [B]
time = 0.31, size = 20, normalized size = 0.83

$$\sqrt{x^5 - 2\,x + 1} - \sqrt{x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3 + 1)^(1/2) + (3*x^2*(x^5 - 2*x + 1)^(1/2))/2 - (5*x^4*(x^3 + 1)^
(1/2))/2)/((x^3 + 1)^(1/2)*(x^5 - 2*x + 1)^(1/2)),x)
```

```
[Out] (x^5 - 2*x + 1)^(1/2) - (x^3 + 1)^(1/2)
```

3.202 $\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) \, dx$

Optimal. Leaf size=27

$$10 \tanh ^{-1}\left(\frac{x}{\sqrt{-4+x^2}}\right)+\tanh ^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

```
[Out] 10*arctanh(x/(x^2-4)^(1/2))+arctanh(x/(x^2-1)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {223, 212}

$$10 \tanh ^{-1}\left(\frac{x}{\sqrt{x^2-4}}\right)+\tanh ^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

```
[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) \, dx &= 10 \int \frac{1}{\sqrt{-4+x^2}} \, dx+\int \frac{1}{\sqrt{-1+x^2}} \, dx\\
&= 10 \text{Subst}\left(\int \frac{1}{1-x^2} \, dx,x,\frac{x}{\sqrt{-4+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2} \, dx,x,\frac{x}{\sqrt{-
\end{aligned}$$

$$= 10 \tanh ^{-1}\left(\frac{x}{\sqrt{-4+x^2}}\right)+\tanh ^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 1.93

$$10 \tanh^{-1}\left(\frac{x}{\sqrt{-4+x^2}}\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\frac{1}{2}\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

```
[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] - Log[1 - x/Sqrt[-1 + x^2]]/2 + Log[1 + x/Sqrt
[-1 + x^2]]/2
```

Maple [A]
time = 0.06, size = 24, normalized size = 0.89

method	result	size
default	$\ln\left(x+\sqrt{x^2-1}\right)+10\ln\left(x+\sqrt{x^2-4}\right)$	24
meijerg	$\frac{10\sqrt{-\mathrm{signum}\left(-1+\frac{x^2}{4}\right)}\ \arcsin\left(\frac{x}{2}\right)}{\sqrt{\mathrm{signum}\left(-1+\frac{x^2}{4}\right)}}+\frac{\sqrt{-\mathrm{signum}\left(x^2-1\right)}\ \arcsin(x)}{\sqrt{\mathrm{signum}\left(x^2-1\right)}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x+(x^2-1)^(1/2))+10*ln(x+(x^2-4)^(1/2))
```

Maxima [A]
time = 1.93, size = 31, normalized size = 1.15

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)+10\,\log\left(2\,x+2\,\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1)) + 10*log(2*x + 2*sqrt(x^2 - 4))
```

Fricas [A]
time = 0.58, size = 29, normalized size = 1.07

$$-\log\left(-x+\sqrt{x^2-1}\right)-10\,\log\left(-x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1)) - 10*log(-x + sqrt(x^2 - 4))
```

Sympy [A]
time = 0.06, size = 8, normalized size = 0.30

$$10\,\mathrm{acosh}\left(\frac{x}{2}\right)+\mathrm{acosh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)
```

```
[Out] 10*acosh(x/2) + acosh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.
time = 0.69, size = 51, normalized size = 1.89

$$\frac{1}{2}\,\sqrt{x^2-1}\,x+5\,\sqrt{x^2-4}\,x+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)+20\,\log\left(\left|-x+\sqrt{x^2-4}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 1)*x + 5*sqrt(x^2 - 4)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
+ 20*log(abs(-x + sqrt(x^2 - 4)))
```

Mupad [B]
time = 0.69, size = 23, normalized size = 0.85

$$\ln\left(x+\sqrt{x^2-1}\right)+10\,\ln\left(x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^(1/2) + 10/(x^2 - 4)^(1/2),x)
```

```
[Out] log(x + (x^2 - 1)^(1/2)) + 10*log(x + (x^2 - 4)^(1/2))
```

3.203 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx$

Optimal. Leaf size=82

$$2\sqrt{x+\sqrt{a^2+x^2}}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2144, 470, 335, 218, 212, 209}

$$-2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2144

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx &= \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}} + (2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}} + (4a^2)\,\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\ &= 2\sqrt{x+\sqrt{a^2+x^2}} - (2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right) - (2a)\text{Subs}\\ &= 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x+}}{}\right.\end{aligned}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 1.00

$$2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.03, size = 25, normalized size = 0.30

method	result	size
meijerg	$2\sqrt{2}\,\sqrt{x}\,\,\text{hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4}\right],-\frac{a^2}{x^2}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Fricas [A]
time = 0.95, size = 216, normalized size = 2.63

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\,\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a^2+x^2}},2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,\sqrt{x+\sqrt{a^2+x^2}}}{a}\right)+\sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}\,a+\left(\sqrt{-a}\,(a+x)-\sqrt{a^2+x^2}\,\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.71, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\,\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

3.204 $\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx$

Optimal. Leaf size=12

$$\log\left(1+\sqrt{1+x^3}\right)$$

[Out] ln(1+(x^3+1)^(1/2))

Rubi [A]

time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {12, 2186, 31}

$$\log\left(\sqrt{x^3+1}+1\right)$$

Antiderivative was successfully verified.

[In] Int[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]

[Out] Log[1 + Sqrt[1 + x^3]]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \frac{3}{2}\int \frac{x^2}{1+x^3+\sqrt{1+x^3}}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+\sqrt{1+x}}\,dx, x, x^3\right)$$
$$= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{1+x^3}\right)$$
$$= \log\left(1+\sqrt{1+x^3}\right)$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\log\left(1+\sqrt{1+x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
```

```
[Out] Log[1 + Sqrt[1 + x^3]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(10) = 20$.
time = 0.11, size = 39, normalized size = 3.25

method	result	size
trager	$\frac{\ln\left(-x^3-2\sqrt{x^3+1}-2\right)}{2}$	20
default	$-\frac{\ln(1+x)}{2}+\frac{3\ln(x)}{2}-\frac{\ln(x^2-x+1)}{2}+\frac{\ln(x^3+1)}{2}+\operatorname{arctanh}\left(\sqrt{x^3+1}\right)$	39
elliptic	$\frac{\left(1+\sqrt{x^3+1}\right)\sqrt{x^3+1}\left(\frac{3\ln(x)}{2}+\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\right)}{1+x^3+\sqrt{x^3+1}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(1+x)+3/2*ln(x)-1/2*ln(x^2-x+1)+1/2*ln(x^3+1)+arctanh((x^3+1)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(10) = 20$.
time = 1.73, size = 40, normalized size = 3.33

$$-\frac{1}{2}\log\left(x^2-x+1\right)+\log\left(\frac{x^3+\sqrt{x^2-x+1}\sqrt{x+1}+1}{\sqrt{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 - x + 1) + log((x^3 + sqrt(x^2 - x + 1)*sqrt(x + 1) + 1)/sqrt(
x + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.
time = 0.68, size = 29, normalized size = 2.42

$$\frac{3}{2}\log\left(x\right)+\frac{1}{2}\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{2}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 3/2*log(x) + 1/2*log(sqrt(x^3 + 1) + 1) - 1/2*log(sqrt(x^3 + 1) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.
time = 47.96, size = 48, normalized size = 4.00

$$-\frac{\log\left(2\sqrt{x^3+1}\right)}{2}+\frac{\log\left(2\sqrt{x^3+1}+2\right)}{2}+\frac{\log\left(3x^3+3\sqrt{x^3+1}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)
```

```
[Out] -log(2*sqrt(x**3 + 1))/2 + log(2*sqrt(x**3 + 1) + 2)/2 + log(3*x**3 + 3*sqr
t(x**3 + 1) + 3)/2
```

Giac [A]
time = 0.59, size = 10, normalized size = 0.83

$$\log\left(\sqrt{x^3+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="giac")
```

```
[Out] log(sqrt(x^3 + 1) + 1)
```

Mupad [B]
time = 0.06, size = 169, normalized size = 14.08

$$\frac{3\,\ln\left(x\right)}{2}+\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\;\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2)/(2*((x^3 + 1)^(1/2) + x^3 + 1)),x)
```

```
[Out] (3*log(x))/2 + (3*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1
i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3
/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/(
(3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.205 $\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{h}\,r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\,\sqrt{h}}$$

```
[Out] 1/2*arctanh(r*2^(1/2)*h^(1/2)/(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{h}\,r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\,\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-alpha^2 + 2*h*r^2],r]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr &= \text{Subst}\left(\int \frac{1}{1-2hr^2}\,dr, r, \frac{r}{\sqrt{-\alpha^2+2hr^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{h}\,r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\,\sqrt{h}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}\,r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-alpha^2 + 2*h*r^2],r]
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

Maple [A]
time = 0.04, size = 33, normalized size = 0.82

method	result	size
default	$\frac{\ln\left(\sqrt{h}\,r\sqrt{2}+\sqrt{2h\,r^2-\alpha^2}\right)\sqrt{2}}{2\sqrt{h}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
[Out] 1/2*ln(h^(1/2)*r*2^(1/2)+(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

Maxima [A]
time = 1.31, size = 36, normalized size = 0.90

$$\frac{\sqrt{2}\,\log\left(4\,hr+2\,\sqrt{2}\,\sqrt{2\,hr^2-\alpha^2}\,\sqrt{h}\right)}{2\,\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
[Out] 1/2*sqrt(2)*log(4*h*r + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h))/sqrt(h)
```

Fricas [A]
time = 0.92, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{2}\,\log\left(4\,hr^2+2\,\sqrt{2}\,\sqrt{2\,hr^2-\alpha^2}\,\sqrt{h}\,r-\alpha^2\right)}{4\,\sqrt{h}}, -\frac{1}{2}\,\sqrt{2}\,\sqrt{-\frac{1}{h}}\,\arctan\left(\frac{\sqrt{2}\,hr\sqrt{-\frac{1}{h}}}{\sqrt{2\,hr^2-\alpha^2}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(4*h*r^2 + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h)*r - al
pha^2)/sqrt(h), -1/2*sqrt(2)*sqrt(-1/h)*arctan(sqrt(2)*h*r*sqrt(-1/h)/sqrt(
2*h*r^2 - alpha^2))]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 66, normalized size = 1.65

$$\begin{cases} \frac{\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{h}\,r}{\alpha}\right)}{2\sqrt{h}} & \text{for}\ \left|\frac{hr^2}{\alpha^2}\right| > \frac{1}{2} \\ -\frac{\sqrt{2}\,i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{h}\,r}{\alpha}\right)}{2\sqrt{h}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), Abs(h*r**2/a
lpha**2) > 1/2), (-sqrt(2)*I*asin(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), Tru
e))
```

Giac [A]
time = 0.57, size = 55, normalized size = 1.38

$$\frac{\sqrt{2}\,\alpha^2\log\left(\left|-\sqrt{2}\,\sqrt{h}\,r+\sqrt{2\,hr^2-\alpha^2}\right|\right)}{4\,\sqrt{h}}+\frac{1}{2}\,\sqrt{2\,hr^2-\alpha^2}\,r$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*alpha^2*log(abs(-sqrt(2)*sqrt(h)*r + sqrt(2*h*r^2 - alpha^2)))/
sqrt(h) + 1/2*sqrt(2*h*r^2 - alpha^2)*r
```

Mupad [B]
time = 0.48, size = 32, normalized size = 0.80

$$\frac{\sqrt{2}\,\ln\left(\sqrt{2\,h\,r^2-\alpha^2}+\sqrt{2}\,\sqrt{h}\,r\right)}{2\,\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*h*r^2 - alpha^2)^(1/2),r)
```

```
[Out] (2^(1/2)*log((2*h*r^2 - alpha^2)^(1/2) + 2^(1/2)*h^(1/2)*r))/(2*h^(1/2))
```

3.206 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] arctan((2*h*r^2-alpha^2-epsilon^2)^(1/2)/(alpha^2+epsilon^2)^(1/2))/(alpha^
2+epsilon^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {272, 65, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt
[alpha^2 + epsilon^2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr = \frac{1}{2}\text{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr}}\,dr, r, r^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{-\alpha^2-\epsilon^2}{2h}+\frac{r^2}{2h}}\,dr, r, \sqrt{-\alpha^2-\epsilon^2+2hr^2}\right)}{2h}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt
[alpha^2 + epsilon^2]
```

Maple [A]
time = 0.04, size = 66, normalized size = 1.43

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{2h\,r^2-\alpha^2-\epsilon^2}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*(-alpha^2-epsilo
n^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2)^(1/2))/r)
```

Maxima [A]
time = 2.05, size = 59, normalized size = 1.28

$$-\frac{\arcsin\left(\frac{\sqrt{2}\,\alpha^2}{2\sqrt{(\alpha^2+\epsilon^2)h}\,|r|}+\frac{\sqrt{2}\,\epsilon^2}{2\sqrt{(\alpha^2+\epsilon^2)h}\,|r|}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

```
[Out] -arcsin(1/2*sqrt(2)*alpha^2/(sqrt((alpha^2 + epsilon^2)*h)*abs(r)) + 1/2*sq
rt(2)*epsilon^2/(sqrt((alpha^2 + epsilon^2)*h)*abs(r)))/sqrt(alpha^2 + epsi
lon^2)
```

Fricas [A]
time = 0.72, size = 41, normalized size = 0.89

$$-\frac{\arctan\left(\frac{\sqrt{\alpha^2+\epsilon^2}}{\sqrt{2\,hr^2-\alpha^2-\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -arctan(sqrt(alpha^2 + epsilon^2)/sqrt(2*h*r^2 - alpha^2 - epsilon^2))/sqrt
(alpha^2 + epsilon^2)
```

Sympy [A]
time = 0.54, size = 42, normalized size = 0.91

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{\operatorname{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}{2\sqrt{h}\,r}\right)}{\sqrt{\operatorname{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] -asinh(sqrt(2)*sqrt(polar_lift(-alpha**2 - epsilon**2))/(2*sqrt(h)*r))/sqrt
(polar_lift(-alpha**2 - epsilon**2))
```

Giac [A]
time = 0.59, size = 40, normalized size = 0.87

$$\frac{\arctan\left(\frac{\sqrt{2\,hr^2-\alpha^2-\epsilon^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

```
[Out] arctan(sqrt(2*h*r^2 - alpha^2 - epsilon^2)/sqrt(alpha^2 + epsilon^2))/sqrt(
alpha^2 + epsilon^2)
```

Mupad [B]
time = 0.66, size = 40, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2\,h\,r^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - alpha^2 - epsilon^2)^(1/2)),r)
```

```
[Out] atan((2*h*r^2 - alpha^2 - epsilon^2)^(1/2)/(alpha^2 + epsilon^2)^(1/2))/(al
pha^2 + epsilon^2)^(1/2)
```

3.207 $\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=37

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2-2kr+2hr^2}}\right)}{\alpha}$$

```
[Out] -arctan((alpha^2+k*r)/alpha/(2*h*r^2-alpha^2-2*k*r)^(1/2))/alpha
```

Rubi [A]

time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {738, 210}

$$-\frac{\text{ArcTan}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])]/alpha)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr &= -\left(2\text{Subst}\left(\int \frac{1}{-4\alpha^2-r^2}\,dr, r, \frac{-2\alpha^2-2kr}{\sqrt{-\alpha^2-2kr+2hr^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2-2kr+2hr^2}}\right)}{\alpha}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 1.22

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{h}\,r-\sqrt{-\alpha^2-2kr+2hr^2}}{\alpha}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
[Out] (-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])/alpha])/a
lpha
```

Maple [A]
time = 0.12, size = 52, normalized size = 1.41

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2-2kr+2\sqrt{-\alpha^2}\,\sqrt{2h\,r^2-\alpha^2-2kr}}{r}\right)}{\sqrt{-\alpha^2}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)
[Out] -1/(-alpha^2)^(1/2)*ln((-2*alpha^2-2*k*r+2*(-alpha^2)^(1/2)*(2*h*r^2-alpha^
2-2*k*r)^(1/2))/r)
```

Maxima [A]
time = 2.14, size = 46, normalized size = 1.24

$$-\frac{\arcsin\left(\frac{\alpha^2}{\sqrt{2\,\alpha^2h+k^2}\,|r|}+\frac{kr}{\sqrt{2\,\alpha^2h+k^2}\,|r|}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
[Out] -arcsin(alpha^2/(sqrt(2*alpha^2*h + k^2)*abs(r)) + k*r/(sqrt(2*alpha^2*h +
k^2)*abs(r)))/alpha
```

Fricas [A]
time = 0.75, size = 52, normalized size = 1.41

$$-\frac{\arctan\left(\frac{\sqrt{2\,hr^2-\alpha^2-2\,kr}\,(\alpha^2+kr)}{2\,\alpha hr^2-\alpha^3-2\,\alpha kr}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

```
[Out] -arctan(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*(alpha^2 + k*r)/(2*alpha*h*r^2 - al
pha^3 - 2*alpha*k*r))/alpha
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2 + 2hr^2 - 2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r)), r)
```

Giac [A]
time = 0.56, size = 40, normalized size = 1.08

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{h}\,r-\sqrt{2\,hr^2-\alpha^2-2\,kr}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - 2*k*r))/alpha)/alph
a
```

Mupad [B]
time = 0.10, size = 51, normalized size = 1.38

$$-\frac{\ln\left(\frac{\sqrt{-\alpha^2}\,\sqrt{-\alpha^2+2\,h\,r^2-2\,k\,r}}{r}-\frac{\alpha^2}{r}-k\right)}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r - alpha^2)^(1/2)),r)
```

```
[Out] -log(((-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2)^(1/2))/r - alpha^2/r - k
)/(-alpha^2)^(1/2)
```

3.208 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -arctan((alpha^2+epsilon^2+k*r)/(alpha^2+epsilon^2)^(1/2)/(2*h*r^2-alpha^2-
epsilon^2-2*k*r)^(1/2))/(alpha^2+epsilon^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {738, 210}

$$-\frac{\text{ArcTan}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha
^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsilon^2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr &= -\left(2\text{Subst}\left(\int \frac{1}{4(-\alpha^2-\epsilon^2)-r^2}\,dr, r, \frac{2(-\alpha^2-\epsilon^2)-2kr}{\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 1.08

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{h}\,r-\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] (-2*ArcTan[(Sqrt[2]*Sqrt[h]*r - Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2
])/Sqrt[alpha^2 + epsilon^2]])/Sqrt[alpha^2 + epsilon^2]
```

Maple [A]
time = 0.13, size = 74, normalized size = 1.21

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2-2kr+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{2h\,r^2-\alpha^2-\epsilon^2-2kr}}{r}\right)}{\sqrt{-\alpha^2-\epsilon^2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2-2*k*r+2*(-alpha^2-
epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2))/r)
```

Maxima [A]
time = 2.24, size = 84, normalized size = 1.38

$$-\frac{\arcsin\left(\frac{\alpha^2}{\sqrt{2(\alpha^2+\epsilon^2)h+k^2}\,|r|}+\frac{\epsilon^2}{\sqrt{2(\alpha^2+\epsilon^2)h+k^2}\,|r|}+\frac{kr}{\sqrt{2(\alpha^2+\epsilon^2)h+k^2}\,|r|}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="maxima"
)
```

```
[Out] -arcsin(alpha^2/(sqrt(2*(alpha^2 + epsilon^2)*h + k^2)*abs(r)) + epsilon^2/
(sqrt(2*(alpha^2 + epsilon^2)*h + k^2)*abs(r)) + k*r/(sqrt(2*(alpha^2 + eps
ilon^2)*h + k^2)*abs(r)))/sqrt(alpha^2 + epsilon^2)
```

Fricas [A]
time = 0.59, size = 97, normalized size = 1.59

$$-\frac{\arctan\left(-\frac{\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}\,(\alpha^2+\epsilon^2+kr)\sqrt{\alpha^2+\epsilon^2}}{\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\,(\alpha^2+\epsilon^2)hr^2+2\,(\alpha^2+\epsilon^2)kr}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="fricas"
)
```

```
[Out] -arctan(-sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*(alpha^2 + epsilon^2 +
 k*r)*sqrt(alpha^2 + epsilon^2)/(alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4
- 2*(alpha^2 + epsilon^2)*h*r^2 + 2*(alpha^2 + epsilon^2)*k*r))/sqrt(alpha^
2 + epsilon^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2 - \epsilon^2 + 2hr^2 - 2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)), r)
```

Giac [A]
time = 0.53, size = 57, normalized size = 0.93

$$\frac{2\arctan\left(-\frac{\sqrt{2}\,\sqrt{h}\,r - \sqrt{2\,hr^2 - \alpha^2 - \epsilon^2 - 2\,kr}}{\sqrt{\alpha^2 + \epsilon^2}}\right)}{\sqrt{\alpha^2 + \epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r))
/sqrt(alpha^2 + epsilon^2))/sqrt(alpha^2 + epsilon^2)
```

Mupad [B]
time = 0.25, size = 72, normalized size = 1.18

$$-\frac{\ln\left(\frac{\sqrt{-\alpha^2 - \epsilon^2}\,\sqrt{-\alpha^2 - \epsilon^2 + 2\,h\,r^2 - 2\,k\,r}}{r} - \frac{\alpha^2 + \epsilon^2}{r} - k\right)}{\sqrt{-\alpha^2 - \epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2)^(1/2)),r)
```

```
[Out] -log(((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^2 - 2*k*r - alpha^2 - epsilon^2)
^(1/2))/r - (alpha^2 + epsilon^2)/r - k)/(- alpha^2 - epsilon^2)^(1/2)
```

3.209 $\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} \, dr$

Optimal. Leaf size=23

$$\frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

```
[Out] 1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{267\}$

$$\frac{\sqrt{2er^2 - \alpha^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2}} \, dr = \frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{\sqrt{-\alpha^2 + 2er^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Maple [A]
time = 0.04, size = 20, normalized size = 0.87

method	result	size
gosper	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
derivativedivides	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
default	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
trager	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}}}{2e}$	20
risch	$-\frac{-2e\,r^{2}+\alpha^{2}}{2e\sqrt{2e\,r^{2}-\alpha^{2}}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

Maxima [A]
time = 2.35, size = 19, normalized size = 0.83

$$\frac{1}{2}\,\sqrt{2\,r^{2}e-\alpha^{2}}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)
```

Fricas [A]
time = 0.73, size = 19, normalized size = 0.83

$$\frac{1}{2}\,\sqrt{2\,r^{2}e-\alpha^{2}}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)
```

Sympy [A]
time = 0.21, size = 29, normalized size = 1.26

$$\begin{cases} \frac{\sqrt{-\alpha^{2}+2er^{2}}}{2e} & \text{for}\ e\neq 0 \\ \frac{r^{2}}{2\sqrt{-\alpha^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alph
a**2)), True))
```

Giac [A]
time = 0.57, size = 19, normalized size = 0.83

$$\frac{1}{2}\sqrt{2\,r^2e-\alpha^2}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)
```

Mupad [B]
time = 0.29, size = 19, normalized size = 0.83

$$\frac{\sqrt{2\,e\,r^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - alpha^2)^(1/2),r)
```

```
[Out] (2*e*r^2 - alpha^2)^(1/2)/(2*e)
```

3.210 $\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr$

Optimal. Leaf size=28

$$\frac{\sqrt{-\alpha^2-\epsilon^2+2er^2}}{2e}$$

```
[Out] 1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{267\}$

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr = \frac{\sqrt{-\alpha^2-\epsilon^2+2er^2}}{2e}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{\sqrt{-\alpha^2-\epsilon^2+2er^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.89

method	result	size
gosper	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
derivativedivides	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
default	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
trager	$\frac{\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}{2e}$	25
risch	$-\frac{-2e\,r^{2}+\alpha^{2}+\epsilon^{2}}{2e\sqrt{2e\,r^{2}-\alpha^{2}-\epsilon^{2}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

Maxima [A]
time = 1.80, size = 24, normalized size = 0.86

$$\frac{1}{2}\,\sqrt{2\,r^{2}e-\alpha^{2}-\epsilon^{2}}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2 - epsilon^2)*e^(-1)
```

Fricas [A]
time = 0.66, size = 24, normalized size = 0.86

$$\frac{1}{2}\,\sqrt{2\,r^{2}e-\alpha^{2}-\epsilon^{2}}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2 - epsilon^2)*e^(-1)
```

Sympy [A]
time = 0.28, size = 36, normalized size = 1.29

$$\begin{cases} \frac{\sqrt{-\alpha^{2}+2er^{2}-\epsilon^{2}}}{2e} & \text{for}\ e\neq 0 \\ \frac{r^{2}}{2\sqrt{-\alpha^{2}-\epsilon^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e), Ne(e, 0)), (r**2/
(2*sqrt(-alpha**2 - epsilon**2)), True))
```

Giac [A]
time = 0.55, size = 24, normalized size = 0.86

$$\frac{1}{2}\,\sqrt{2\,r^2e-\alpha^2-\epsilon^2}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2 - epsilon^2)*e^(-1)
```

Mupad [B]
time = 0.27, size = 24, normalized size = 0.86

$$\frac{\sqrt{-\alpha^2-\epsilon^2+2\,e\,r^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - alpha^2 - epsilon^2)^(1/2),r)
```

```
[Out] (2*e*r^2 - alpha^2 - epsilon^2)^(1/2)/(2*e)
```

3.211 $\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr^4}} \, dr$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

```
[Out] -1/4*arctan(1/2*(-2*k*r^2+e)*2^(1/2)/k^(1/2)/(-2*k*r^4+2*e*r^2-alpha^2)^(1/
2))*2^(1/2)/k^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1121, 635, 210}

$$-\frac{\text{ArcTan}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

```
[Out] -1/2*ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^
4])]/(Sqrt[2]*Sqrt[k])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-\alpha^2+2er-2kr^2}}\,dr, r, r^2\right)$$
$$= \text{Subst}\left(\int \frac{1}{-8k-r^2}\,dr, r, \frac{2(e-2kr^2)}{\sqrt{-\alpha^2+2er^2-2kr^4}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(56) = 112.
time = 0.18, size = 150, normalized size = 2.68

$$-\frac{2\sqrt{-k}\tan^{-1}\left(\frac{\sqrt{k}\left(2\sqrt{-k}\,r^2-\sqrt{2}\sqrt{-\alpha^2+2er^2-2kr^4}\right)}{e}\right)+\sqrt{k}\log\left(e^2+4ekr^2-2k\left(\alpha^2+4kr^4+2\sqrt{2}\sqrt{-k}\,r^2\sqrt{-\alpha^2+2er^2-2kr^4}\right)\right)}{4\sqrt{2}\sqrt{-k^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

```
[Out] -1/4*(2*Sqrt[-k]*ArcTan[(Sqrt[k]*(2*Sqrt[-k]*r^2 - Sqrt[2]*Sqrt[-alpha^2 +
2*e*r^2 - 2*k*r^4]))/e] + Sqrt[k]*Log[e^2 + 4*e*k*r^2 - 2*k*(alpha^2 + 4*k*
r^4 + 2*Sqrt[2]*Sqrt[-k]*r^2*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4])])/(Sqrt[2]
*Sqrt[-k^2])
```

Maple [A]
time = 0.03, size = 47, normalized size = 0.84

method	result	size
default	$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{k}\left(r^2-\frac{e}{2k}\right)}{\sqrt{-2k\,r^4+2e\,r^2-\alpha^2}}\right)}{4\sqrt{k}}$	47
elliptic	$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{k}\left(r^2-\frac{e}{2k}\right)}{\sqrt{-2k\,r^4+2e\,r^2-\alpha^2}}\right)}{4\sqrt{k}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)/k^(1/2)*arctan(2^(1/2)*k^(1/2)*(r^2-1/2*e/k)/(-2*k*r^4+2*e*r^2-
alpha^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*alpha^2*k-%e^2>0)', see `assume?`
 for mo
```

Fricas [A]
time = 0.69, size = 155, normalized size = 2.77

$$\left[-\frac{\sqrt{2}\,\sqrt{-k}\,\log\left(8\,k^2r^4-8\,kr^2e+2\,\alpha^2k-2\,\sqrt{2}\,\sqrt{-2\,kr^4+2\,r^2e-\alpha^2}\,(2\,kr^2-e)\sqrt{-k}\,+e^2\right)}{8\,k}, -\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-2\,kr^4+2\,r^2e-\alpha^2}\,(2\,kr^2-e)\sqrt{k}}{2\,(2\,k^2r^4-2\,kr^2e+\alpha^2k)}\right)}{4\,\sqrt{k}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(2)*sqrt(-k)*log(8*k^2*r^4 - 8*k*r^2*e + 2*alpha^2*k - 2*sqrt(2)*
sqrt(-2*k*r^4 + 2*r^2*e - alpha^2)*(2*k*r^2 - e)*sqrt(-k) + e^2)/k, -1/4*sq
rt(2)*arctan(1/2*sqrt(2)*sqrt(-2*k*r^4 + 2*r^2*e - alpha^2)*(2*k*r^2 - e)*s
qrt(k)/(2*k^2*r^4 - 2*k*r^2*e + alpha^2*k))/sqrt(k)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r**4), r)
```

Giac [A]
time = 0.45, size = 60, normalized size = 1.07

$$-\frac{\sqrt{2}\,\log\left(\left|\sqrt{2}\,\left(\sqrt{2}\,\sqrt{-k}\,r^2-\sqrt{-2\,kr^4+2\,r^2e-\alpha^2}\right)\sqrt{-k}\,+e\right|\right)}{4\,\sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(sqrt(2)*(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*r^2*
e - alpha^2))*sqrt(-k) + e))/sqrt(-k)
```

Mupad [B]
time = 0.98, size = 50, normalized size = 0.89

$$\frac{\sqrt{2}\,\ln\left(\sqrt{-\alpha^2-2\,k\,r^4+2\,e\,r^2}+\frac{\sqrt{2}\,\left(e-2\,k\,r^2\right)}{2\,\sqrt{-k}}\right)}{4\,\sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2),r)
```

```
[Out] (2^(1/2)*log((2*e*r^2 - 2*k*r^4 - alpha^2)^(1/2) + (2^(1/2)*(e - 2*k*r^2))/
(2*(-k)^(1/2))))/(4*(-k)^(1/2))
```

3.212 $\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr$

Optimal. Leaf size=81

$$\frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\,\sqrt{e}\,\sqrt{-\alpha^2-2kr+2er^2}}\right)}{2\sqrt{2}\,e^{3/2}}$$

```
[Out] -1/4*k*arctanh(1/2*(-2*e*r+k)*2^(1/2)/e^(1/2)/(2*e*r^2-alpha^2-2*k*r)^(1/2)
)/e^(3/2)*2^(1/2)+1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {654, 635, 212}

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\,\sqrt{e}\,\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}\,e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(Sqrt[2]*Sq
rt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr=\frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}+\frac{k\int\frac{1}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr}{2e}$$
$$=\frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}+\frac{k\text{Subst}\left(\int\frac{1}{8e-r^2}\,dr,r,\frac{-2k+4er}{\sqrt{-\alpha^2-2kr+2er^2}}\right)}{e}$$
$$=\frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2-2kr+2er^2}}\right)}{2\sqrt{2}\,e^{3/2}}$$

Mathematica [A]
time = 0.20, size = 86, normalized size = 1.06

$$\frac{2\sqrt{e}\sqrt{-\alpha^2+2r(-k+er)}-\sqrt{2}\,k\log\left(-e\left(k-2er+\sqrt{2}\sqrt{e}\sqrt{-\alpha^2-2kr+2er^2}\right)\right)}{4e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] (2*Sqrt[e]*Sqrt[-alpha^2 + 2*r*(-k + e*r)] - Sqrt[2]*k*Log[-(e*(k - 2*e*r +
 Sqrt[2]*Sqrt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]))])/(4*e^(3/2))
```

Maple [A]
time = 0.12, size = 70, normalized size = 0.86

method	result	size
default	$\frac{\sqrt{2e\,r^2-\alpha^2-2kr}}{2e}+\frac{k\ln\left(\frac{(2er-k)\sqrt{2}}{2\sqrt{e}}+\sqrt{2e\,r^2-\alpha^2-2kr}\right)\sqrt{2}}{4e^{\frac{3}{2}}}$	70
risch	$-\frac{-2e\,r^2+\alpha^2+2kr}{2e\sqrt{2e\,r^2-\alpha^2-2kr}}+\frac{k\ln\left(\frac{(2er-k)\sqrt{2}}{2\sqrt{e}}+\sqrt{2e\,r^2-\alpha^2-2kr}\right)\sqrt{2}}{4e^{\frac{3}{2}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e+1/4*k/e^(3/2)*ln(1/2*(2*e*r-k)*2^(1/2)/
e^(1/2)+(2*e*r^2-alpha^2-2*k*r)^(1/2))*2^(1/2)
```

Maxima [A]
time = 1.11, size = 68, normalized size = 0.84

$$\frac{1}{4}\sqrt{2}\,ke^{\left(-\frac{3}{2}\right)}\log\left(4\,re+2\sqrt{2}\sqrt{2\,r^2e-\alpha^2-2\,kr}\,e^{\frac{1}{2}}-2\,k\right)+\frac{1}{2}\sqrt{2\,r^2e-\alpha^2-2\,kr}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*k*e^(-3/2)*log(4*r*e + 2*sqrt(2)*sqrt(2*r^2*e - alpha^2 - 2*k*r
)*e^(1/2) - 2*k) + 1/2*sqrt(2*r^2*e - alpha^2 - 2*k*r)*e^(-1)
```

Fricas [**A**]
time = 0.56, size = 94, normalized size = 1.16

$$\frac{1}{8}\left(\sqrt{2}\,ke^{\frac{1}{2}}\log\left(8\,r^2e^2+2\,\sqrt{2}\,\sqrt{2\,r^2e-\alpha^2-2\,kr}\,(2\,re-k)e^{\frac{1}{2}}+k^2-2\,(\alpha^2+4\,kr)e\right)+4\,\sqrt{2\,r^2e-\alpha^2-2\,kr}\,e\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*k*e^(1/2)*log(8*r^2*e^2 + 2*sqrt(2)*sqrt(2*r^2*e - alpha^2 - 2
*k*r)*(2*r*e - k)*e^(1/2) + k^2 - 2*(alpha^2 + 4*k*r)*e) + 4*sqrt(2*r^2*e -
 alpha^2 - 2*k*r)*e)*e^(-2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{r}{\sqrt{-\alpha^2+2er^2-2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r), r)
```

Giac [**A**]
time = 0.50, size = 72, normalized size = 0.89

$$-\frac{1}{4}\sqrt{2}\,ke^{\left(-\frac{3}{2}\right)}\log\left(\left|-\sqrt{2}\left(\sqrt{2}\,re^{\frac{1}{2}}-\sqrt{2\,r^2e-\alpha^2-2\,kr}\right)e^{\frac{1}{2}}+k\right|\right)+\frac{1}{2}\sqrt{2\,r^2e-\alpha^2-2\,kr}\,e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*k*e^(-3/2)*log(abs(-sqrt(2)*(sqrt(2)*r*e^(1/2) - sqrt(2*r^2*e
- alpha^2 - 2*k*r))*e^(1/2) + k)) + 1/2*sqrt(2*r^2*e - alpha^2 - 2*k*r)*e^(
-1)
```

Mupad [**B**]
time = 0.24, size = 67, normalized size = 0.83

$$\frac{\sqrt{-\alpha^2+2\,e\,r^2-2\,k\,r}}{2\,e}+\frac{\sqrt{2}\;k\;\ln\left(\sqrt{-\alpha^2+2\,e\,r^2-2\,k\,r}-\frac{\sqrt{2}\,(k-2\,e\,r)}{2\,\sqrt{e}}\right)}{4\,e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2 - 2*k*r - alpha^2)^(1/2),r)
```

```
[Out] (2*e*r^2 - 2*k*r - alpha^2)^(1/2)/(2*e) + (2^(1/2)*k*log((2*e*r^2 - 2*k*r -
 alpha^2)^(1/2) - (2^(1/2)*(k - 2*e*r))/(2*e^(1/2))))/(4*e^(3/2))
```

3.213 $\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

```
[Out] -1/2*arctan((-h*r^2+alpha^2)/alpha/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2))/alpha
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1128, 738, 210}

$$-\frac{\text{ArcTan}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -1/2*ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]/a
lpha
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr = \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2+2hr-2kr^2}}\,dr, r, r^2\right)$$
$$= -\mathrm{Subst}\left(\int \frac{1}{-4\alpha^2-r^2}\,dr, r, \frac{2(-\alpha^2+hr^2)}{\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{-\alpha^2+hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Mathematica [A]
time = 0.08, size = 53, normalized size = 1.20

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-k}\,r^2}{\alpha}-\frac{\sqrt{-\alpha^2+2hr^2-2kr^4}}{\alpha}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2)/alpha - Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]/
alpha]/alpha)
```

Maple [A]
time = 0.03, size = 56, normalized size = 1.27

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2+2h\,r^2+2\sqrt{-\alpha^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$	56
elliptic	$-\frac{\ln\left(\frac{-2\alpha^2+2h\,r^2+2\sqrt{-\alpha^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2}}{r^2}\right)}{2\sqrt{-\alpha^2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-alpha^2)^(1/2)*ln((-2*alpha^2+2*h*r^2+2*(-alpha^2)^(1/2)*(-2*k*r^4+2
*h*r^2-alpha^2)^(1/2))/r^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*alpha^2*k-h^2>0)', see `assume?`
for mor
```

Fricas [**A**]
time = 0.58, size = 58, normalized size = 1.32

$$-\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}\,(hr^2-\alpha^2)}{2\,\alpha kr^4-2\,\alpha hr^2+\alpha^3}\right)}{2\,\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*(h*r^2 - alpha^2)/(2*alpha*k
*r^4 - 2*alpha*h*r^2 + alpha^3))/alpha
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r**4)), r)
```

Giac [**A**]
time = 0.47, size = 45, normalized size = 1.02

$$\frac{\arctan\left(-\frac{\sqrt{2}\,\sqrt{-k}\,r^2-\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2))/alpha)/
alpha
```

Mupad [B]
time = 0.40, size = 54, normalized size = 1.23

$$-\frac{\ln\left(\frac{1}{r^2}\right)+\ln\left(h\,r^2-\alpha^2+\sqrt{-\alpha^2}\,\sqrt{-\alpha^2-2\,k\,r^4+2\,h\,r^2}\right)}{2\,\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2)^(1/2)),r)
```

```
[Out] -(log(1/r^2) + log(h*r^2 - alpha^2 + (-alpha^2)^(1/2)*(2*h*r^2 - 2*k*r^4 -
alpha^2)^(1/2)))/(2*(-alpha^2)^(1/2))
```

3.214 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=68

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -1/2*arctan((-h*r^2+alpha^2+epsilon^2)/(alpha^2+epsilon^2)^(1/2)/(-2*k*r^4+
2*h*r^2-alpha^2-epsilon^2)^(1/2))/(alpha^2+epsilon^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1128, 738, 210}

$$-\frac{\text{ArcTan}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -1/2*ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-
alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/Sqrt[alpha^2 + epsilon^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = \frac{1}{2}\text{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr-2kr^2}}\,dr, r, r^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{4\left(-\alpha^2-\epsilon^2\right)-r^2}\,dr, r, \frac{2(-\alpha^2-\epsilon^2+hr^2)}{\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{-\alpha^2-\epsilon^2+hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Mathematica [**A**]
time = 0.11, size = 72, normalized size = 1.06

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-k}\,r^2-\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -(ArcTan[(Sqrt[2]*Sqrt[-k]*r^2 - Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*
r^4])/Sqrt[alpha^2 + epsilon^2]]/Sqrt[alpha^2 + epsilon^2])
```

Maple [**A**]
time = 0.03, size = 78, normalized size = 1.15

method	result	size
default	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2h\,r^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2-\epsilon^2}}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$	78
elliptic	$-\frac{\ln\left(\frac{-2\alpha^2-2\epsilon^2+2h\,r^2+2\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-2k\,r^4+2h\,r^2-\alpha^2-\epsilon^2}}{r^2}\right)}{2\sqrt{-\alpha^2-\epsilon^2}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*h*r^2+2*(-alph
a^2-epsilon^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2))/r^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*epsilon^2*k+2*alpha^2*k>0)', see
`assume
```

Fricas [A]
time = 0.57, size = 106, normalized size = 1.56

$$-\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}\,(hr^2-\alpha^2-\epsilon^2)\sqrt{\alpha^2+\epsilon^2}}{2\,(\alpha^2+\epsilon^2)kr^4+\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\,(\alpha^2+\epsilon^2)hr^2}\right)}{2\,\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fric
as")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*(h*r^2 - alpha^2
 - epsilon^2)*sqrt(alpha^2 + epsilon^2)/(2*(alpha^2 + epsilon^2)*k*r^4 + al
pha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2))/s
qrt(alpha^2 + epsilon^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)), r)
```

Giac [A]
time = 0.45, size = 62, normalized size = 0.91

$$\frac{\arctan\left(-\frac{\sqrt{2}\,\sqrt{-k}\,r^2-\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac
")
```

```
[Out] arctan(-(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon
^2))/sqrt(alpha^2 + epsilon^2))/sqrt(alpha^2 + epsilon^2)
```

Mupad [B]
time = 0.44, size = 72, normalized size = 1.06

$$-\frac{\ln\left(h-\frac{\alpha^2+\epsilon^2}{r^2}+\frac{\sqrt{-\alpha^2-\epsilon^2}\,\sqrt{-\alpha^2-\epsilon^2-2\,k\,r^4+2\,h\,r^2}}{r^2}\right)}{2\,\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(r*(2*h*r^2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2)),r)
```

```
[Out] -log(h - (alpha^2 + epsilon^2)/r^2 + ((- alpha^2 - epsilon^2)^(1/2)*(2*h*r^
2 - 2*k*r^4 - alpha^2 - epsilon^2)^(1/2))/r^2)/(2*(- alpha^2 - epsilon^2)^(
1/2))
```

3.215 $\int a\cos(5+3x)\sin^2(5+3x)\,dx$

Optimal. Leaf size=13

$$\frac{1}{9}a\sin^3(5+3x)$$

```
[Out] 1/9*a*sin(5+3*x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {12, 2644, 30}

$$\frac{1}{9}a\sin^3(3x+5)$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
```

```
[Out] (a*Sin[5 + 3*x]^3)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int a\cos(5+3x)\sin^2(5+3x)\,dx &= a\int\cos(5+3x)\sin^2(5+3x)\,dx\\
&= \frac{1}{3}a\text{Subst}\left(\int x^2\,dx, x, \sin(5+3x)\right)\\
&= \frac{1}{9}a\sin^3(5+3x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{9}a\sin^3(5+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
[Out] (a*Sin[5 + 3*x]^3)/9
```

Maple [A]
time = 0.04, size = 12, normalized size = 0.92

method	result	size
derivativedivides	$\frac{a(\sin^3(5+3x))}{9}$	12
default	$\frac{a(\sin^3(5+3x))}{9}$	12
risch	$\frac{a\sin(5+3x)}{12}-\frac{a\sin(15+9x)}{36}$	20
norman	$\frac{8a\left(\tan^3\left(\frac{5}{2}+\frac{3x}{2}\right)\right)}{9\left(1+\tan^2\left(\frac{5}{2}+\frac{3x}{2}\right)\right)^3}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(5+3*x)*sin(5+3*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/9*a*sin(5+3*x)^3
```

Maxima [A]
time = 3.12, size = 11, normalized size = 0.85

$$\frac{1}{9}\,a\sin\left(3\,x+5\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="maxima")
[Out] 1/9*a*sin(3*x + 5)^3
```

Fricas [A]
time = 0.62, size = 22, normalized size = 1.69

$$-\frac{1}{9}\left(a\cos\left(3\,x+5\right)^2-a\right)\sin\left(3\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(a*cos(3*x + 5)^2 - a)*sin(3*x + 5)
```

Sympy [A]
time = 0.08, size = 10, normalized size = 0.77

$$\frac{a\sin^3(3x+5)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)
```

```
[Out] a*sin(3*x + 5)**3/9
```

Giac [A]
time = 0.45, size = 11, normalized size = 0.85

$$\frac{1}{9}\,a\sin\left(3\,x+5\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*a*sin(3*x + 5)^3
```

Mupad [B]
time = 0.09, size = 11, normalized size = 0.85

$$\frac{a\sin\left(3\,x+5\right)^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(3*x + 5)*sin(3*x + 5)^2,x)
```

```
[Out] (a*sin(3*x + 5)^3)/9
```

3.216 $\int \frac{\log(x^2)}{x^3} \, dx$

Optimal. Leaf size=19

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

```
[Out] -1/2/x^2-1/2*ln(x^2)/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^2]/x^3,x]
```

```
[Out] -1/2*1/x^2 - Log[x^2]/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x^2)}{x^3} \, dx = -\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$-\frac{1}{2x^2} - \frac{\log(x^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^2]/x^3,x]
```

```
[Out] -1/2*1/x^2 - Log[x^2]/(2*x^2)
```

Maple [A]
time = 0.01, size = 16, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{2}-\frac{\ln\left(x^{2}\right)}{2}}{x^{2}}$	13
default	$-\frac{1}{2x^{2}}-\frac{\ln\left(x^{2}\right)}{2x^{2}}$	16
risch	$-\frac{1}{2x^{2}}-\frac{\ln\left(x^{2}\right)}{2x^{2}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/2*ln(x^2)/x^2
```

Maxima [A]
time = 1.56, size = 15, normalized size = 0.79

$$-\frac{\log\left(x^{2}\right)}{2\,x^{2}}-\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2)/x^2 - 1/2/x^2
```

Fricas [A]
time = 0.55, size = 11, normalized size = 0.58

$$-\frac{\log\left(x^{2}\right)+1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(log(x^2) + 1)/x^2
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.89

$$-\frac{\log\left(x^{2}\right)}{2x^{2}}-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2)/x**3,x)
```

```
[Out] -log(x**2)/(2*x**2) - 1/(2*x**2)
```

Giac [A]
time = 0.42, size = 15, normalized size = 0.79

$$-\frac{\log\left(x^2\right)}{2\,x^2}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="giac")
[Out] -1/2*log(x^2)/x^2 - 1/2/x^2
```

Mupad [B]
time = 0.16, size = 11, normalized size = 0.58

$$-\frac{\ln\left(x^2\right)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2)/x^3,x)
[Out] -(log(x^2) + 1)/(2*x^2)
```

3.217 $\int x\sin(a+x)\,dx$

Optimal. Leaf size=12

$$-x\cos(a+x)+\sin(a+x)$$

```
[Out] -x*cos(a+x)+sin(a+x)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3377, 2717}

$$\sin(a+x)-x\cos(a+x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + x],x]
```

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin(a+x)\,dx &= -x\cos(a+x)+\int\cos(a+x)\,dx\\ &= -x\cos(a+x)+\sin(a+x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 1.00

$$-x\cos(a+x)+\sin(a+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + x],x]
```

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Maple [A]
time = 0.02, size = 21, normalized size = 1.75

method	result	size
risch	$-x\cos(a+x)+\sin(a+x)$	13
derivativedivides	$a\cos(a+x)+\sin(a+x)-(a+x)\cos(a+x)$	21
default	$a\cos(a+x)+\sin(a+x)-(a+x)\cos(a+x)$	21
norman	$\frac{x\left(\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)-x+2\tan\left(\frac{a}{2}+\frac{x}{2}\right)}{1+\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)}$	42
meijerg	$2\sin(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)+2\cos(a)\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+x),x,method=_RETURNVERBOSE)
[Out] a*cos(a+x)+sin(a+x)-(a+x)*cos(a+x)
```

Maxima [A]
time = 2.20, size = 20, normalized size = 1.67

$$-(a+x)\cos(a+x)+a\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="maxima")
[Out] -(a + x)*cos(a + x) + a*cos(a + x) + sin(a + x)
```

Fricas [A]
time = 0.89, size = 12, normalized size = 1.00

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="fricas")
[Out] -x*cos(a + x) + sin(a + x)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.83

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x)
```

```
[Out] -x*cos(a + x) + sin(a + x)
```

Giac [**A**]
time = 0.49, size = 12, normalized size = 1.00

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="giac")
```

```
[Out] -x*cos(a + x) + sin(a + x)
```

Mupad [**B**]
time = 0.08, size = 12, normalized size = 1.00

$$\sin(a+x)-x\,\cos(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + x),x)
```

```
[Out] sin(a + x) - x*cos(a + x)
```

3.218 $\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{e^{-x}x}{\log(x)}$$

[Out] x/exp(x)/ln(x)

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2232}

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

[In] Int[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]

[Out] x/(E^x*Log[x])

Rule 2232

```
Int[Log[(d_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((e_) + Log[(d_
.)*(x_)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> Simp[e*x*F^(c*(a + b*x))
*(Log[d*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x]
 && EqQ[e - f*h*(n + 1), 0] && EqQ[g*h*(n + 1) - b*c*e*Log[F], 0] && NeQ[n,
 -1]
```

Rubi steps

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{e^{-x}x}{\log(x)}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]

```
[Out] x/(E^x*Log[x])
```

Maple [A]
time = 0.01, size = 11, normalized size = 1.00

method	result	size
norman	$\frac{x\,\mathrm{e}^{-x}}{\ln(x)}$	11
risch	$\frac{x\,\mathrm{e}^{-x}}{\ln(x)}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(1-x)*ln(x))/exp(x)/ln(x)^2,x,method=_RETURNVERBOSE)
[Out] x/exp(x)/ln(x)
```

Maxima [A]
time = 3.58, size = 10, normalized size = 0.91

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="maxima")
[Out] x*e^(-x)/log(x)
```

Fricas [A]
time = 0.80, size = 10, normalized size = 0.91

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="fricas")
[Out] x*e^(-x)/log(x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.64

$$\frac{xe^{-x}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)
```

```
[Out] x*exp(-x)/log(x)
```

Giac [A]
time = 0.45, size = 10, normalized size = 0.91

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="giac")
```

```
[Out] x*e^(-x)/log(x)
```

Mupad [B]
time = 0.22, size = 10, normalized size = 0.91

$$\frac{x\,\mathrm{e}^{-x}}{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(-x)*(log(x)*(x - 1) + 1))/log(x)^2,x)
```

```
[Out] (x*exp(-x))/log(x)
```

3.219 $\int \frac{x^3}{b+ax^2} \, dx$

Optimal. Leaf size=27

$$\frac{x^2}{2a} - \frac{b\log\left(b+ax^2\right)}{2a^2}$$

```
[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {272, 45}

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b + a*x^2),x]
```

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{b+ax^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{b+ax} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right) \, dx, x, x^2\right)\\
&= \frac{x^2}{2a} - \frac{b\log\left(b+ax^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2a} - \frac{b\log(b + ax^2)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b + a*x^2),x]
```

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.89

method	result	size
default	$\frac{x^2}{2a} - \frac{b\ln(a\,x^2+b)}{2a^2}$	24
norman	$\frac{x^2}{2a} - \frac{b\ln(a\,x^2+b)}{2a^2}$	24
risch	$\frac{x^2}{2a} - \frac{b\ln(a\,x^2+b)}{2a^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

Maxima [A]
time = 5.54, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,a} - \frac{b\log(ax^2 + b)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2
```

Fricas [A]
time = 0.85, size = 22, normalized size = 0.81

$$\frac{ax^2 - b\log(ax^2 + b)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2 - b*log(a*x^2 + b))/a^2
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.74

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**2+b),x)
```

```
[Out] x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)
```

Giac [A]
time = 0.44, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,a} - \frac{b\log\left(|ax^2 + b|\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2
```

Mupad [B]
time = 0.15, size = 22, normalized size = 0.81

$$-\frac{b\,\ln\left(a\,x^2 + b\right) - a\,x^2}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b + a*x^2),x)
```

```
[Out] -(b*log(b + a*x^2) - a*x^2)/(2*a^2)
```

3.220 $\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx$

Optimal. Leaf size=33

$$\frac{2x^{3/2}}{5(1+x)^{5/2}}+\frac{4x^{3/2}}{15(1+x)^{3/2}}$$

```
[Out] 2/5*x^(3/2)/(1+x)^(5/2)+4/15*x^(3/2)/(1+x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {47, 37}

$$\frac{4x^{3/2}}{15(x+1)^{3/2}}+\frac{2x^{3/2}}{5(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x)^(7/2),x]
```

```
[Out] (2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(1+x)^{7/2}} \, dx &= \frac{2x^{3/2}}{5(1+x)^{5/2}}+\frac{2}{5}\int \frac{\sqrt{x}}{(1+x)^{5/2}} \, dx\\ &= \frac{2x^{3/2}}{5(1+x)^{5/2}}+\frac{4x^{3/2}}{15(1+x)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 21, normalized size = 0.64

$$\frac{2x^{3/2}(5+2x)}{15(1+x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x)^(7/2),x]
[Out] (2*x^(3/2)*(5 + 2*x))/(15*(1 + x)^(5/2))
```

Maple [A]
time = 0.03, size = 32, normalized size = 0.97

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
meijerg	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
risch	$\frac{2x^{\frac{3}{2}}(5+2x)}{15(1+x)^{\frac{5}{2}}}$	16
default	$-\frac{2\sqrt{x}}{5(1+x)^{\frac{5}{2}}}+\frac{2\sqrt{x}}{15(1+x)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{15\sqrt{1+x}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x)^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5/(1+x)^(5/2)*x^(1/2)+2/15/(1+x)^(3/2)*x^(1/2)+4/15/(1+x)^(1/2)*x^(1/2)
```

Maxima [A]
time = 1.70, size = 20, normalized size = 0.61

$$\frac{2\,x^{\frac{5}{2}}\left(\frac{5\,(x+1)}{x}-3\right)}{15\,(x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="maxima")
[Out] 2/15*x^(5/2)*(5*(x + 1)/x - 3)/(x + 1)^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(21) = 42.
time = 0.79, size = 50, normalized size = 1.52

$$\frac{2\left(2\,x^3+(2\,x^2+5\,x)\sqrt{x+1}\,\sqrt{x}\,+6\,x^2+6\,x+2\right)}{15\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^3 + (2*x^2 + 5*x)*sqrt(x + 1)*sqrt(x) + 6*x^2 + 6*x + 2)/(x^3 + 3
*x^2 + 3*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.88, size = 167, normalized size = 5.06

$$\begin{cases} \frac{4i\sqrt{-1+\frac{1}{x+1}}\,(x+1)^2}{-15x+15(x+1)^2-15} - \frac{2i\sqrt{-1+\frac{1}{x+1}}\,(x+1)}{-15x+15(x+1)^2-15} - \frac{8i\sqrt{-1+\frac{1}{x+1}}}{-15x+15(x+1)^2-15} + \frac{6i\sqrt{-1+\frac{1}{x+1}}}{(x+1)\left(-15x+15(x+1)^2-15\right)} & \text{for } \frac{1}{|x+1|} > 1 \\ \frac{4\sqrt{1-\frac{1}{x+1}}}{15} + \frac{2\sqrt{1-\frac{1}{x+1}}}{15(x+1)} - \frac{2\sqrt{1-\frac{1}{x+1}}}{5(x+1)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x)**(7/2),x)
```

```
[Out] Piecewise((4*I*sqrt(-1 + 1/(x + 1))*(x + 1)**2/(-15*x + 15*(x + 1)**2 - 15)
 - 2*I*sqrt(-1 + 1/(x + 1))*(x + 1)/(-15*x + 15*(x + 1)**2 - 15) - 8*I*sqrt
(-1 + 1/(x + 1))/(-15*x + 15*(x + 1)**2 - 15) + 6*I*sqrt(-1 + 1/(x + 1))/((
x + 1)*(-15*x + 15*(x + 1)**2 - 15)), 1/Abs(x + 1) > 1), (4*sqrt(1 - 1/(x +
 1))/15 + 2*sqrt(1 - 1/(x + 1))/(15*(x + 1)) - 2*sqrt(1 - 1/(x + 1))/(5*(x
+ 1)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(21) = 42.
time = 0.50, size = 66, normalized size = 2.00

$$\frac{8\left(15\left(\sqrt{x+1}-\sqrt{x}\right)^6-5\left(\sqrt{x+1}-\sqrt{x}\right)^4+5\left(\sqrt{x+1}-\sqrt{x}\right)^2+1\right)}{15\left(\left(\sqrt{x+1}-\sqrt{x}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*(sqrt(x + 1) - sqrt(x))^6 - 5*(sqrt(x + 1) - sqrt(x))^4 + 5*(sqrt(
x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 + 1)^5
```

Mupad [B]
time = 0.30, size = 15, normalized size = 0.45

$$\frac{2\,x^{3/2}\,(2\,x+5)}{15\,(x+1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + 1)^(7/2),x)
```

```
[Out] (2*x^(3/2)*(2*x + 5))/(15*(x + 1)^(5/2))
```

3.221 $\int \frac{1}{x(1+x)} \, dx$

Optimal. Leaf size=9

$$\log(x) - \log(1+x)$$

```
[Out] ln(x)-ln(1+x)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {36, 29, 31}

$$\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)),x]
```

```
[Out] Log[x] - Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1+x)} \, dx &= \int \frac{1}{x} \, dx - \int \frac{1}{1+x} \, dx \\ &= \log(x) - \log(1+x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\log(x) - \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)),x]
```

```
[Out] Log[x] - Log[1 + x]
```

Maple [A]
time = 0.03, size = 10, normalized size = 1.11

method	result	size
default	$\ln(x) - \ln(1 + x)$	10
norman	$\ln(x) - \ln(1 + x)$	10
meijerg	$\ln(x) - \ln(1 + x)$	10
risch	$\ln(x) - \ln(1 + x)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-ln(1+x)
```

Maxima [A]
time = 1.72, size = 9, normalized size = 1.00

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="maxima")
```

```
[Out] -log(x + 1) + log(x)
```

Fricas [A]
time = 0.81, size = 9, normalized size = 1.00

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="fricas")
```

```
[Out] -log(x + 1) + log(x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\log(x) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x)
```

```
[Out] log(x) - log(x + 1)
```

Giac [A]
time = 0.45, size = 11, normalized size = 1.22

$$-\log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="giac")
```

```
[Out] -log(abs(x + 1)) + log(abs(x))
```

Mupad [B]
time = 0.20, size = 8, normalized size = 0.89

$$-\ln\left(\frac{1}{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + 1)),x)
```

```
[Out] -log(1/x + 1)
```

3.222 $\int \frac{1}{\sqrt{x}\,(-1+2x)}\,dx$

Optimal. Leaf size=19

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

```
[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 213}

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-1 + 2*x)),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(-1+2x)}\,dx &= 2\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{x}\right)\\ &= -\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-1 + 2*x)),x]
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Maple [A]
time = 0.07, size = 14, normalized size = 0.74

method	result	size
derivativedivides	$-\operatorname{arctanh}\left(\sqrt{2}\,\sqrt{x}\right)\sqrt{2}$	14
default	$-\operatorname{arctanh}\left(\sqrt{2}\,\sqrt{x}\right)\sqrt{2}$	14
meijerg	$-\operatorname{arctanh}\left(\sqrt{2}\,\sqrt{x}\right)\sqrt{2}$	14
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{x}+\operatorname{RootOf}\left(\_Z^2-2\right)}{2x-1}\right)}{2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(2*x-1),x,method=_RETURNVERBOSE)
[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.
time = 1.60, size = 28, normalized size = 1.47

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{x}}{\sqrt{2}+2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="maxima")
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - 2*sqrt(x))/(sqrt(2) + 2*sqrt(x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.
time = 0.62, size = 28, normalized size = 1.47

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{x}-2x-1}{2x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(x) - 2*x - 1)/(2*x - 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.
time = 0.10, size = 39, normalized size = 2.05

$$\frac{\sqrt{2}\log\left(\sqrt{x}-\frac{\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\log\left(\sqrt{x}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-1+2*x),x)
```

```
[Out] sqrt(2)*log(sqrt(x) - sqrt(2)/2)/2 - sqrt(2)*log(sqrt(x) + sqrt(2)/2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.
time = 0.48, size = 32, normalized size = 1.68

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{1}{2}\sqrt{2}+\sqrt{x}\right)+\frac{1}{2}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(1/2*sqrt(2) + sqrt(x)) + 1/2*sqrt(2)*log(abs(-1/2*sqrt(2)
+ sqrt(x)))
```

Mupad [B]
time = 0.08, size = 13, normalized size = 0.68

$$-\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2*x - 1)),x)
```

```
[Out] -2^(1/2)*atanh((2*x)^(1/2))
```

3.223 $\int \sqrt{x} \left(1+x^2\right) \, dx$

Optimal. Leaf size=19

$$\frac{2x^{3/2}}{3}+\frac{2x^{7/2}}{7}$$

[Out] 2/3*x^(3/2)+2/7*x^(7/2)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{14\}$

$$\frac{2x^{7/2}}{7}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(1 + x^2),x]

[Out] (2*x^(3/2))/3 + (2*x^(7/2))/7

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(1+x^2\right) \, dx &= \int \left(\sqrt{x}+x^{5/2}\right) \, dx\\ &= \frac{2x^{3/2}}{3}+\frac{2x^{7/2}}{7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{2}{21}x^{3/2}\left(7+3x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(1 + x^2),x]

[Out] (2*x^(3/2)*(7 + 3*x^2))/21

Maple [A]
time = 0.01, size = 12, normalized size = 0.63

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{7}{2}}}{7}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{2x^{\frac{7}{2}}}{7}$	12
gosper	$\frac{2x^{\frac{3}{2}}(3x^{2}+7)}{21}$	13
trager	$\frac{2x^{\frac{3}{2}}(3x^{2}+7)}{21}$	13
risch	$\frac{2x^{\frac{3}{2}}(3x^{2}+7)}{21}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)+2/7*x^(7/2)
```

Maxima [A]
time = 3.38, size = 11, normalized size = 0.58

$$\frac{2}{7}x^{\frac{7}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="maxima")
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

Fricas [A]
time = 0.71, size = 14, normalized size = 0.74

$$\frac{2}{21}\left(3\,x^{3}+7\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="fricas")
[Out] 2/21*(3*x^3 + 7*x)*sqrt(x)
```

Sympy [A]
time = 0.51, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{7}{2}}}{7}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(x**2+1),x)
```

```
[Out] 2*x**(7/2)/7 + 2*x**(3/2)/3
```

Giac [A]
time = 0.45, size = 11, normalized size = 0.58

$$\frac{2}{7}\,x^{\frac{7}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="giac")
```

```
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.63

$$\frac{2\,x^{3/2}\,\left(3\,x^2+7\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^2 + 1),x)
```

```
[Out] (2*x^(3/2)*(3*x^2 + 7))/21
```

3.224 $\int \frac{\sqrt[3]{-a+x}}{x}\,dx$

Optimal. Leaf size=88

$$3\sqrt[3]{-a+x}+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+x}}{\sqrt{3}\sqrt[3]{a}}\right)+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)$$

```
[Out] 3*(-a+x)^(1/3)+1/2*a^(1/3)*ln(x)-3/2*a^(1/3)*ln(a^(1/3)+(-a+x)^(1/3))+a^(1/
3)*arctan(1/3*(a^(1/3)-2*(-a+x)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 60, 631, 210, 31}

$$\sqrt{3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{a}}\right)+3\sqrt[3]{x-a}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*(-a + x)^(1/3))/(Sqr
t[3]*a^(1/3))] + (a^(1/3)*Log[x])/2 - (3*a^(1/3)*Log[a^(1/3) + (-a + x)^(1/
3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{-a+x}}{x}\,dx &= 3\sqrt[3]{-a+x}-a\int \frac{1}{x(-a+x)^{2/3}}\,dx\\
&= 3\sqrt[3]{-a+x}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx,x,\sqrt[3]{-a+x}\right)-\frac{1}{2}\left(3a^{2/3}\right)\\
&= 3\sqrt[3]{-a+x}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)-\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right.\\
&= 3\sqrt[3]{-a+x}+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-}\right.
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 112, normalized size = 1.27

$$3\sqrt[3]{-a+x}+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)+\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+x}+(-a+x)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*(-a + x)^(1/3))/a^(1/3))/
Sqrt[3]] - a^(1/3)*Log[a^(1/3) + (-a + x)^(1/3)] + (a^(1/3)*Log[a^(2/3) - a
^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)])/2
```

Maple [A]
time = 0.04, size = 89, normalized size = 1.01

method	result
derivativedivides	$3(-a+x)^{\frac{1}{3}}-3\left(\frac{\ln\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((-a+x)^{\frac{2}{3}}-a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(-a+x)}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$
default	$3(-a+x)^{\frac{1}{3}}-3\left(\frac{\ln\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((-a+x)^{\frac{2}{3}}-a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(-a+x)}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(-a+x)^(1/3)-3*(1/3/a^(2/3)*ln(a^(1/3)+(-a+x)^(1/3))-1/6/a^(2/3)*ln((-a+x
)^(2/3)-a^(1/3)*(-a+x)^(1/3)+a^(2/3))+1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/a^(1/3)*(-a+x)^(1/3)-1)))*a
```

Maxima [A]
time = 3.13, size = 86, normalized size = 0.98

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,\left(a^{\frac{1}{3}}-2\,(-a+x)^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+\frac{1}{2}\,a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)-a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)+3\,(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(-1/3*sqrt(3)*(a^(1/3) - 2*(-a + x)^(1/3))/a^(1/3))
+ 1/2*a^(1/3)*log(a^(2/3) - a^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)) - a^(1
/3)*log(a^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/3)
```

Fricas [A]
time = 0.70, size = 104, normalized size = 1.18

$$\sqrt{3}\,(-a)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,a-2\,\sqrt{3}\,(-a)^{\frac{2}{3}}\,(-a+x)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{2}\,(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)+(-a)^{\frac{1}{3}}\log\left(-(-a)^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right)+3\,(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(-a)^(1/3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-a)^(2/3)*(-a + x)^(
1/3))/a) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a
+ x)^(2/3)) + (-a)^(1/3)*log(-(-a)^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/
3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 153, normalized size = 1.74

$$\frac{4\sqrt[3]{a}\,e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}\,e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{-a+x}\,e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\,e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}\,e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-a+x}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*exp(-I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))
*gamma(4/3)/(3*gamma(7/3)) - 4*a**(1/3)*log(1 - (-a + x)**(1/3)*exp_polar(I
*pi)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(I*pi/3)*log(1 - (
-a + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*
(-a + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

Giac [A]
time = 0.68, size = 103, normalized size = 1.17

$$-\sqrt{3}\,(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left((-a)^{\frac{1}{3}}+2\,(-a+x)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)-\frac{1}{2}\,(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)+(-a)^{\frac{1}{3}}\log\left(\left|-(-a)^{\frac{1}{3}}+(-a+x)^{\frac{1}{3}}\right|\right)+3\,(-a+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(1/3))/(-a)
^(1/3)) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a +
 x)^(2/3)) + (-a)^(1/3)*log(abs(-(-a)^(1/3) + (-a + x)^(1/3))) + 3*(-a + x)
^(1/3)
```

Mupad [B]
time = 0.18, size = 119, normalized size = 1.35

$$(-a)^{1/3}\ln\left(-9\,(-a)^{4/3}-9\,a\,(x-a)^{1/3}\right)+3\,(x-a)^{1/3}+\frac{(-a)^{1/3}\ln\left(\frac{9\,(-a)^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}+9\,a\,(x-a)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{(-a)^{1/3}\ln\left(\frac{9\,(-a)^{4/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}-9\,a\,(x-a)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - a)^(1/3)/x,x)
```

```
[Out] (-a)^(1/3)*log(- 9*(-a)^(4/3) - 9*a*(x - a)^(1/3)) + 3*(x - a)^(1/3) + ((-a
)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i - 1))/2 + 9*a*(x - a)^(1/3))*(3^(1/2)
*1i - 1))/2 - ((-a)^(1/3)*log((9*(-a)^(4/3)*(3^(1/2)*1i + 1))/2 - 9*a*(x -
a)^(1/3))*(3^(1/2)*1i + 1))/2
```

3.225 $\int x \sinh(x) \, dx$

Optimal. Leaf size=9

$$x \cosh(x) - \sinh(x)$$

```
[Out] x*cosh(x)-sinh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2717}

$$x \cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[x],x]
```

```
[Out] x*Cosh[x] - Sinh[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x \sinh(x) \, dx &= x \cosh(x) - \int \cosh(x) \, dx \\ &= x \cosh(x) - \sinh(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$x \cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[x],x]
```

```
[Out] x*Cosh[x] - Sinh[x]
```

Maple [A]
time = 0.02, size = 10, normalized size = 1.11

method	result	size
default	$x\cosh\left(x\right)-\sinh\left(x\right)$	10
meijerg	$x\cosh\left(x\right)-\sinh\left(x\right)$	10
risch	$\left(-\frac{1}{2}+\frac{x}{2}\right){\mathrm e}^{x}+\left(\frac{1}{2}+\frac{x}{2}\right){\mathrm e}^{-x}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x),x,method=_RETURNVERBOSE)
[Out] x*cosh(x)-sinh(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(9) = 18$.
time = 2.11, size = 34, normalized size = 3.78

$$\frac{1}{2}\,x^{2}\sinh\left(x\right)+\frac{1}{4}\,\left(x^{2}+2\,x+2\right)e^{\left(-x\right)}-\frac{1}{4}\,\left(x^{2}-2\,x+2\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="maxima")
[Out] 1/2*x^2*sinh(x) + 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

Fricas [A]
time = 0.62, size = 9, normalized size = 1.00

$$x\cosh\left(x\right)-\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="fricas")
[Out] x*cosh(x) - sinh(x)
```

Sympy [A]
time = 0.06, size = 7, normalized size = 0.78

$$x\cosh\left(x\right)-\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x)
[Out] x*cosh(x) - sinh(x)
```

Giac [A]
time = 0.47, size = 17, normalized size = 1.89

$$\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

Mupad [B]
time = 0.02, size = 9, normalized size = 1.00

$$x\cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x),x)
```

```
[Out] x*cosh(x) - sinh(x)
```

3.226 $\int x \cosh(x)\,dx$

Optimal. Leaf size=9

$$-\cosh(x)+x\sinh(x)$$

```
[Out] -cosh(x)+x*sinh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2718}

$$x\sinh(x)-\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x],x]
```

```
[Out] -Cosh[x] + x*Sinh[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cosh(x)\,dx &= x\sinh(x)-\int \sinh(x)\,dx\\ &= -\cosh(x)+x\sinh(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\cosh(x)+x\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x],x]
```

```
[Out] -Cosh[x] + x*Sinh[x]
```

Maple [A]
time = 0.02, size = 10, normalized size = 1.11

method	result	size
default	$-\cosh\left(x\right)+x\sinh\left(x\right)$	10
risch	$\left(-\frac{1}{2}+\frac{x}{2}\right){\mathrm e}^{x}+\left(-\frac{1}{2}-\frac{x}{2}\right){\mathrm e}^{-x}$	20
meijerg	$-2\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh\left(x\right)}{2\sqrt{\pi}}-\frac{x\sinh\left(x\right)}{2\sqrt{\pi}}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x),x,method=_RETURNVERBOSE)
[Out] -cosh(x)+x*sinh(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.
time = 1.31, size = 34, normalized size = 3.78

$$\frac{1}{2}\,x^{2}\cosh\left(x\right)-\frac{1}{4}\,\left(x^{2}+2\,x+2\right)e^{\left(-x\right)}-\frac{1}{4}\,\left(x^{2}-2\,x+2\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="maxima")
[Out] 1/2*x^2*cosh(x) - 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

Fricas [A]
time = 0.63, size = 9, normalized size = 1.00

$$x\sinh\left(x\right)-\cosh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="fricas")
[Out] x*sinh(x) - cosh(x)
```

Sympy [A]
time = 0.06, size = 7, normalized size = 0.78

$$x\sinh\left(x\right)-\cosh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x)
```

```
[Out] x*sinh(x) - cosh(x)
```

Giac [A]
time = 0.54, size = 17, normalized size = 1.89

$$-\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

Mupad [B]
time = 0.03, size = 9, normalized size = 1.00

$$x\sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x),x)
```

```
[Out] x*sinh(x) - cosh(x)
```

3.227 $\int \tanh(2x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log(\cosh(2x))$$

```
[Out] 1/2*ln(cosh(2*x))
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3556}

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[2*x],x]
[Out] Log[Cosh[2*x]]/2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(2x)\,dx = \frac{1}{2}\log(\cosh(2x))$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[2*x],x]
[Out] Log[Cosh[2*x]]/2
```

Maple [A]
time = 0.01, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln(\cosh(2x))}{2}$	8
default	$\frac{\ln(\cosh(2x))}{2}$	8
risch	$-x+\frac{\ln(e^{4x}+1)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)/cosh(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(cosh(2*x))
```

Maxima [A]
time = 5.81, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\cosh\left(2\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(2*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(7) = 14$.
time = 0.57, size = 26, normalized size = 2.89

$$-x+\frac{1}{2}\log\left(\frac{2\cosh\left(2\,x\right)}{\cosh\left(2\,x\right)-\sinh\left(2\,x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(2*cosh(2*x)/(cosh(2*x) - sinh(2*x)))
```

Sympy [A]
time = 0.04, size = 7, normalized size = 0.78

$$\frac{\log\left(\cosh\left(2x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x)
```

```
[Out] log(cosh(2*x))/2
```

Giac [A]
time = 0.52, size = 13, normalized size = 1.44

$$-x + \frac{1}{2}\,\log\left(e^{(4\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="giac")
```

```
[Out] -x + 1/2*log(e^(4*x) + 1)
```

Mupad [B]
time = 0.19, size = 7, normalized size = 0.78

$$\frac{\ln\left(\cosh(2\,x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)/cosh(2*x),x)
```

```
[Out] log(cosh(2*x))/2
```

3.228 $\int \frac{-1+i\mathbf{eps}\sinh(x)}{ia-x+i\mathbf{eps}\cosh(x)}\,dx$

Optimal. Leaf size=12

$$\log(a+ix+\text{eps}\cosh(x))$$

```
[Out] ln(a+I*x+eps*cosh(x))
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{6816\}$

$$\log(a+\text{eps}\cosh(x)+ix)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]
```

```
[Out] Log[a + I*x + eps*Cosh[x]]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\int \frac{-1+i\text{eps}\sinh(x)}{ia-x+i\text{eps}\cosh(x)}\,dx=\log(a+ix+\text{eps}\cosh(x))$$

Mathematica [A]
time = 0.04, size = 12, normalized size = 1.00

$$\log(a+ix+\text{eps}\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]
```

```
[Out] Log[a + I*x + eps*Cosh[x]]
```

Maple [A]
time = 0.03, size = 16, normalized size = 1.33

---

method	result	size
derivativedivides	$\ln(ia - x + i\mathit{eps}\cosh(x))$	16
default	$\ln(ia - x + i\mathit{eps}\cosh(x))$	16
risch	$-x + \ln\left(1 + \frac{2(ix+a)e^{x}}{\mathit{eps}} + e^{2x}\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(I*a-x+I*eps*cosh(x))
```

Maxima [A]
time = 1.70, size = 13, normalized size = 1.08

$$\log\left(i\ \mathit{eps}\cosh(x) + i\,a - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(I*eps*cosh(x) + I*a - x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.
time = 0.83, size = 26, normalized size = 2.17

$$-x + \log\left(\frac{\mathit{eps}e^{(2x)} + 2(a + ix)e^{x} + \mathit{eps}}{\mathit{eps}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="fricas")
```

```
[Out] -x + log((eps*e^(2*x) + 2*(a + I*x)*e^x + eps)/eps)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.
time = 0.15, size = 22, normalized size = 1.83

$$-x + \log\left(e^{2x} + 1 + \frac{(2a + 2ix)e^{x}}{\mathit{eps}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)
```

```
[Out] -x + log(exp(2*x) + 1 + (2*a + 2*I*x)*exp(x)/eps)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.49, size = 23, normalized size = 1.92

$$-x + \log\left(epse^{(2x)} + 2\,ae^{x} + 2i\,xe^{x} + eps\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="giac")
[Out] -x + log(eps*e^(2*x) + 2*a*e^x + 2*I*x*e^x + eps)
```

Mupad [**B**]
time = 0.32, size = 13, normalized size = 1.08

$$\ln\left(x - a\,1\mathrm{i} - \mathrm{eps}\cosh(x)\ 1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((eps*sinh(x)*1i - 1)/(a*1i - x + eps*cosh(x)*1i),x)
[Out] log(x - a*1i - eps*cosh(x)*1i)
```

3.229 $\int \cos^2(x)\sin(3+2x)\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4}\cos(3+2x)-\frac{1}{16}\cos(3+4x)+\frac{1}{4}x\sin(3)$$

```
[Out] -1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4670, 2718}

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[3 + 2*x],x]
```

```
[Out] -1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\int \cos^2(x)\sin(3+2x)\,dx &= \int \left(\frac{\sin(3)}{4}+\frac{1}{2}\sin(3+2x)+\frac{1}{4}\sin(3+4x)\right)\,dx\\
&= \frac{1}{4}x\sin(3)+\frac{1}{4}\int \sin(3+4x)\,dx+\frac{1}{2}\int \sin(3+2x)\,dx\\
&= -\frac{1}{4}\cos(3+2x)-\frac{1}{16}\cos(3+4x)+\frac{1}{4}x\sin(3)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{1}{4}\cos(3+2x)-\frac{1}{16}\cos(3+4x)+\frac{1}{4}x\sin(3)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[3 + 2*x],x]
```

```
[Out] -1/4*Cos[3 + 2*x] - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Maple [A]
time = 0.05, size = 23, normalized size = 0.82

method	result
default	$-\frac{\cos(3+2x)}{4}-\frac{\cos(3+4x)}{16}+\frac{x\sin(3)}{4}$
risch	$-\frac{\cos(3+2x)}{4}-\frac{\cos(3+4x)}{16}+\frac{x\sin(3)}{4}$
norman	$\frac{x\left(\tan^3\left(\frac{x}{2}\right)\right)-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}+\frac{\left(\tan^2\left(\frac{3}{2}+x\right)\right)}{3}+x\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{3}{2}+x\right)\right)+\frac{\tan\left(\frac{x}{2}\right)\tan\left(\frac{3}{2}+x\right)}{3}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)\tan\left(\frac{3}{2}+x\right)}{3}+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)\left(\tan^2\left(\frac{3}{2}+x\right)\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(1+\tan^2\left(\frac{3}{2}+x\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(3+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)
```

Maxima [A]
time = 1.25, size = 22, normalized size = 0.79

$$\frac{1}{4}\,x\sin\left(3\right)-\frac{1}{16}\,\cos\left(4\,x+3\right)-\frac{1}{4}\,\cos\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="maxima")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

Fricas [A]
time = 1.10, size = 32, normalized size = 1.14

$$-\frac{1}{2}\,\cos\left(3\right)\cos\left(x\right)^{4}+\frac{1}{4}\,x\sin\left(3\right)+\frac{1}{4}\,\left(2\,\cos\left(x\right)^{3}\sin\left(3\right)+\cos\left(x\right)\sin\left(3\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(3)*cos(x)^4 + 1/4*x*sin(3) + 1/4*(2*cos(x)^3*sin(3) + cos(x)*sin(3
))*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(22) = 44.
time = 0.33, size = 76, normalized size = 2.71

$$-\frac{x\sin^{2}\left(x\right)\sin\left(2x+3\right)}{4}-\frac{x\sin\left(x\right)\cos\left(x\right)\cos\left(2x+3\right)}{2}+\frac{x\sin\left(2x+3\right)\cos^{2}\left(x\right)}{4}-\frac{\sin^{2}\left(x\right)\cos\left(2x+3\right)}{2}+\frac{3\sin\left(x\right)\sin\left(2x+3\right)\cos\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(3+2*x),x)
```

```
[Out] -x*sin(x)**2*sin(2*x + 3)/4 - x*sin(x)*cos(x)*cos(2*x + 3)/2 + x*sin(2*x +
3)*cos(x)**2/4 - sin(x)**2*cos(2*x + 3)/2 + 3*sin(x)*sin(2*x + 3)*cos(x)/4
```

Giac [A]
time = 0.49, size = 22, normalized size = 0.79

$$\frac{1}{4}\,x\sin\left(3\right)-\frac{1}{16}\,\cos\left(4\,x+3\right)-\frac{1}{4}\,\cos\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="giac")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

Mupad [B]
time = 0.33, size = 22, normalized size = 0.79

$$\frac{x\,\sin\left(3\right)}{4}-\frac{\cos\left(4\,x+3\right)}{16}-\frac{\cos\left(2\,x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x + 3)*cos(x)^2,x)
```

```
[Out] (x*sin(3))/4 - cos(4*x + 3)/16 - cos(2*x + 3)/4
```

3.230 $\int x \tan^{-1}(x)\, dx$

Optimal. Leaf size=21

$$-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}x^2\tan^{-1}(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4946, 327, 209}

$$\frac{1}{2}x^2\text{ArcTan}(x)+\frac{\text{ArcTan}(x)}{2}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(x)\,dx &= \frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\int\frac{1}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}x^2\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}x^2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A]
time = 0.00, size = 16, normalized size = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^2+3)\arctan(x)}{6}$	16
risch	$-\frac{ix^2\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A]
time = 2.30, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A]
time = 1.01, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^{2}+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.71

$$\frac{x^{2}\operatorname{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Giac [A]
time = 0.48, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^{2}\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.67

$$\operatorname{atan}(x)\,\left(\frac{x^{2}}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x),x)
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

3.231 $\int x \cot^{-1}(x)\, dx$

Optimal. Leaf size=21

$$\frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4947, 327, 209}

$$-\frac{\text{ArcTan}(x)}{2}+\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[x],x]
```

```
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4947

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcCot[c*x^n])^p/(m + 1)), x] + Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcCot[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x\cot^{-1}(x)\,dx &= \frac{1}{2}x^2\cot^{-1}(x) + \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx \\
&= \frac{x}{2} + \frac{1}{2}x^2\cot^{-1}(x) - \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x}{2} + \frac{1}{2}x^2\cot^{-1}(x) - \frac{1}{2}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{x}{2} + \frac{1}{2}x^2\cot^{-1}(x) - \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[x],x]
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.76

method	result	size
default	$\frac{x}{2}+\frac{x^2\,\mathrm{arccot}(x)}{2}-\frac{\arctan(x)}{2}$	16
risch	$\frac{i x^2\ln(ix+1)}{4}-\frac{i x^2\ln(-ix+1)}{4}+\frac{\pi x^2}{4}+\frac{x}{2}-\frac{\arctan(x)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(x),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

Maxima [A]
time = 1.68, size = 15, normalized size = 0.71

$$\frac{1}{2}x^2\,\mathrm{arccot}\,(x) + \frac{1}{2}x - \frac{1}{2}\arctan\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="maxima")
[Out] 1/2*x^2*arccot(x) + 1/2*x - 1/2*arctan(x)
```

Fricas [A]
time = 1.03, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^2+1\right)\operatorname{arccot}\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arccot(x) + 1/2*x
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{acot}\left(x\right)}{2}+\frac{x}{2}+\frac{\operatorname{acot}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(x),x)
[Out] x**2*acot(x)/2 + x/2 + acot(x)/2
```

Giac [A]
time = 0.47, size = 19, normalized size = 0.90

$$\frac{1}{2}\,x^2\arctan\left(\frac{1}{x}\right)+\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(1/x) + 1/2*x + 1/2*arctan(1/x)
```

Mupad [B]
time = 0.05, size = 15, normalized size = 0.71

$$\frac{x}{2}-\frac{\operatorname{atan}(x)}{2}+\frac{x^2\operatorname{acot}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*acot(x),x)
[Out] x/2 - atan(x)/2 + (x^2*acot(x))/2
```

3.232 $\int x \log(a + x^2)\, dx$

Optimal. Leaf size=23

$$-\frac{x^2}{2} + \frac{1}{2}\left(a + x^2\right)\log\left(a + x^2\right)$$

```
[Out] -1/2*x^2+1/2*(x^2+a)*ln(x^2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2504, 2436, 2332}

$$\frac{1}{2}\left(a + x^2\right)\log\left(a + x^2\right) - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a + x^2],x]
```

```
[Out] -1/2*x^2 + ((a + x^2)*Log[a + x^2])/2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log\left(a+x^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int \log(a+x)\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \log(x)\,dx, x, a+x^2\right)$$
$$= -\frac{x^2}{2}+\frac{1}{2}\left(a+x^2\right)\log\left(a+x^2\right)$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.96

$$\frac{1}{2}\left(-x^2+\left(a+x^2\right)\log\left(a+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a + x^2],x]
```

```
[Out] (-x^2 + (a + x^2)*Log[a + x^2])/2
```

Maple [A]
time = 0.01, size = 23, normalized size = 1.00

method	result	size
derivativedivides	$\frac{(x^2+a)\ln(x^2+a)}{2}-\frac{x^2}{2}-\frac{a}{2}$	23
default	$\frac{(x^2+a)\ln(x^2+a)}{2}-\frac{x^2}{2}-\frac{a}{2}$	23
norman	$-\frac{x^2}{2}+\frac{\ln(x^2+a)a}{2}+\frac{\ln(x^2+a)x^2}{2}$	27
risch	$-\frac{x^2}{2}+\frac{\ln(x^2+a)a}{2}+\frac{\ln(x^2+a)x^2}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^2+a)*ln(x^2+a)-1/2*x^2-1/2*a
```

Maxima [A]
time = 1.23, size = 22, normalized size = 0.96

$$-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(x^2+a\right)\log\left(x^2+a\right)-\frac{1}{2}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

Fricas [A]
time = 0.56, size = 19, normalized size = 0.83

$$-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(x^2+a\right)\log\left(x^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="fricas")
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a)
```

Sympy [A]
time = 0.06, size = 26, normalized size = 1.13

$$\frac{a\log\left(a+x^2\right)}{2}+\frac{x^2\log\left(a+x^2\right)}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x**2+a),x)
[Out] a*log(a + x**2)/2 + x**2*log(a + x**2)/2 - x**2/2
```

Giac [A]
time = 0.51, size = 22, normalized size = 0.96

$$-\frac{1}{2}\,x^2+\frac{1}{2}\,\left(x^2+a\right)\log\left(x^2+a\right)-\frac{1}{2}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="giac")
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

Mupad [B]
time = 0.14, size = 41, normalized size = 1.78

$$\frac{a\,\ln\left(x+\sqrt{-a}\,\right)}{2}+\frac{x^2\,\ln\left(x^2+a\right)}{2}+\frac{a\,\ln\left(x-\sqrt{-a}\,\right)}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a + x^2),x)
[Out] (a*log(x + (-a)^(1/2)))/2 + (x^2*log(a + x^2))/2 + (a*log(x - (-a)^(1/2)))/
2 - x^2/2
```

3.233 $\int \cos(x)\sin(a+x)\,dx$

Optimal. Leaf size=18

$$-\frac{1}{4}\cos(a+2x)+\frac{1}{2}x\sin(a)$$

```
[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4670, 2718}

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[a + x],x]
```

```
[Out] -1/4*Cos[a + 2*x] + (x*Sin[a])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(a+x)\,dx &= \int \left(\frac{\sin(a)}{2}+\frac{1}{2}\sin(a+2x)\right)dx\\ &= \frac{1}{2}x\sin(a)+\frac{1}{2}\int \sin(a+2x)\,dx\\ &= -\frac{1}{4}\cos(a+2x)+\frac{1}{2}x\sin(a)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{1}{4}(-\cos(a+2x)+2x\sin(a))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[a + x],x]
[Out] (-Cos[a + 2*x] + 2*x*Sin[a])/4
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.83

method	result	size
default	$-\frac{\cos(a+2x)}{4}+\frac{x\sin(a)}{2}$	15
risch	$-\frac{\cos(a+2x)}{4}+\frac{x\sin(a)}{2}$	15
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)\tan\left(\frac{a}{2}+\frac{x}{2}\right)-x\tan\left(\frac{x}{2}\right)-x\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{a}{2}+\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{a}{2}+\frac{x}{2}\right)\right)}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(a+x),x,method=_RETURNVERBOSE)
[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)
```

Maxima [A]
time = 1.56, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x\sin\left(a\right)-\frac{1}{4}\,\cos\left(a+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="maxima")
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Fricas [A]
time = 0.94, size = 28, normalized size = 1.56

$$-\frac{1}{2}\,\cos\left(a+x\right)^2\cos\left(a\right)-\frac{1}{2}\,\cos\left(a+x\right)\sin\left(a+x\right)\sin\left(a\right)+\frac{1}{2}\,x\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="fricas")
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) + 1/2*x*sin(a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.
time = 0.13, size = 32, normalized size = 1.78

$$-\frac{x\sin\left(x\right)\cos\left(a+x\right)}{2}+\frac{x\sin\left(a+x\right)\cos\left(x\right)}{2}-\frac{\cos\left(x\right)\cos\left(a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x)
```

```
[Out] -x*sin(x)*cos(a + x)/2 + x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x)/2
```

Giac [A]
time = 0.46, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x\sin\left(a\right)-\frac{1}{4}\,\cos\left(a+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="giac")
```

```
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.78

$$\frac{x\,\sin\left(a\right)}{2}-\frac{\cos\left(a+2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + x)*cos(x),x)
```

```
[Out] (x*sin(a))/2 - cos(a + 2*x)/4
```

3.234 $\int \cos(a+x)\sin(x)\,dx$

Optimal. Leaf size=18

$$-\frac{1}{4}\cos(a+2x)-\frac{1}{2}x\sin(a)$$

```
[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4670, 2718}

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + x]*Sin[x],x]
```

```
[Out] -1/4*Cos[a + 2*x] - (x*Sin[a])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(a+x)\sin(x)\,dx &= \int \left(-\frac{\sin(a)}{2}+\frac{1}{2}\sin(a+2x)\right)dx\\ &= -\frac{1}{2}x\sin(a)+\frac{1}{2}\int \sin(a+2x)\,dx\\ &= -\frac{1}{4}\cos(a+2x)-\frac{1}{2}x\sin(a)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{1}{4}(-\cos(a+2x)-2x\sin(a))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + x]*Sin[x],x]
[Out] (-Cos[a + 2*x] - 2*x*Sin[a])/4
```

Maple [A]
time = 0.05, size = 15, normalized size = 0.83

method	result	size
default	$-\frac{\cos(a+2x)}{4}-\frac{x\sin(a)}{2}$	15
risch	$-\frac{\cos(a+2x)}{4}-\frac{x\sin(a)}{2}$	15
meijerg	$\frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}-\frac{\sin(a)\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	45
norman	$\frac{x\tan(\frac{x}{2})+x(\tan^2(\frac{x}{2}))\tan(\frac{a}{2}+\frac{x}{2})+2\tan(\frac{x}{2})\tan(\frac{a}{2}+\frac{x}{2})-x\tan(\frac{a}{2}+\frac{x}{2})-x\tan(\frac{x}{2})(\tan^2(\frac{a}{2}+\frac{x}{2}))}{(1+\tan^2(\frac{a}{2}+\frac{x}{2}))(1+\tan^2(\frac{x}{2}))}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+x)*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)
```

Maxima [A]
time = 2.22, size = 14, normalized size = 0.78

$$-\frac{1}{2}\,x\sin\left(a\right)-\frac{1}{4}\,\cos\left(a+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="maxima")
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Fricas [A]
time = 0.99, size = 28, normalized size = 1.56

$$-\frac{1}{2}\,\cos\left(a+x\right)^{2}\cos\left(a\right)-\frac{1}{2}\,\cos\left(a+x\right)\sin\left(a+x\right)\sin\left(a\right)-\frac{1}{2}\,x\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) - 1/2*x*sin(a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.
time = 0.13, size = 32, normalized size = 1.78

$$\frac{x\sin\left(x\right)\cos\left(a+x\right)}{2}-\frac{x\sin\left(a+x\right)\cos\left(x\right)}{2}-\frac{\cos\left(x\right)\cos\left(a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x)
```

```
[Out] x*sin(x)*cos(a + x)/2 - x*sin(a + x)*cos(x)/2 - cos(x)*cos(a + x)/2
```

Giac [A]
time = 0.46, size = 14, normalized size = 0.78

$$-\frac{1}{2}\,x\sin\left(a\right) - \frac{1}{4}\,\cos\left(a + 2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.78

$$-\frac{\cos\left(a+2\,x\right)}{4} - \frac{x\,\sin\left(a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + x)*sin(x),x)
```

```
[Out] - cos(a + 2*x)/4 - (x*sin(a))/2
```

3.235 $\int \sqrt{1+\sin(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

```
[Out] -2*cos(x)/(1+sin(x))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2725}

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sin[x]],x]
```

```
[Out] (-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\sin(x)}\,dx = -\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(12) = 24.
time = 0.01, size = 40, normalized size = 3.33

$$\frac{2\left(-\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{1+\sin(x)}}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.06, size = 17, normalized size = 1.42

method	result	size
default	$\frac{2(\sin(x)-1)\sqrt{\sin(x)+1}}{\cos(x)}$	17
risch	$-\frac{i\sqrt{2}\,\sqrt{2+2\sin(x)}\,(e^{ix}-i)(e^{ix}+i)}{e^{2ix}+2ie^{ix}-1}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(sin(x)-1)*(sin(x)+1)^(1/2)/cos(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sin(x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 1.11, size = 24, normalized size = 2.00

$$-\frac{2(\cos(x)-\sin(x)+1)\sqrt{\sin(x)+1}}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="fricas")
[Out] -2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))**(1/2),x)
[Out] Integral(sqrt(sin(x) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.45, size = 22, normalized size = 1.83

$$2\sqrt{2}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2)*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x)
```

Mupad [B]
time = 0.14, size = 16, normalized size = 1.33

$$\frac{2\left(\sin\left(x\right)-1\right)\,\sqrt{\sin\left(x\right)+1}}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + 1)^(1/2),x)
[Out] (2*(sin(x) - 1)*(sin(x) + 1)^(1/2))/cos(x)
```

3.236 $\int \sqrt{1-\sin(x)}\,dx$

Optimal. Leaf size=14

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

```
[Out] 2*cos(x)/(1-sin(x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2725}

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sin[x]],x]
```

```
[Out] (2*Cos[x])/Sqrt[1 - Sin[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\sin(x)}\,dx = \frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.
time = 0.01, size = 42, normalized size = 3.00

$$\frac{2\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\sqrt{1-\sin(x)}}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sin[x]],x]
```

```
[Out] (2*(Cos[x/2] + Sin[x/2])*Sqrt[1 - Sin[x]])/(Cos[x/2] - Sin[x/2])
```

Maple [A]
time = 0.09, size = 23, normalized size = 1.64

method	result	size
default	$-\frac{2(\sin(x)-1)(\sin(x)+1)}{\cos(x)\sqrt{1-\sin(x)}}$	23
risch	$\frac{i\sqrt{2-2\sin(x)}\,\sqrt{-i\left(2i\,\mathrm{e}^{2ix}-\mathrm{e}^{3ix}+\mathrm{e}^{ix}\right)}\,\sqrt{2}\,(\mathrm{e}^{ix}-i)(\mathrm{e}^{ix}+i)}{(2i\,\mathrm{e}^{ix}-\mathrm{e}^{2ix}+1)\sqrt{i\left(\mathrm{e}^{3ix}-2i\,\mathrm{e}^{2ix}-\mathrm{e}^{ix}\right)}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(sin(x)-1)*(sin(x)+1)/cos(x)/(1-sin(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sin(x) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 1.27, size = 26, normalized size = 1.86

$$\frac{2\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\sqrt{-\sin\left(x\right)+1}}{\cos\left(x\right)-\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(cos(x) + sin(x) + 1)*sqrt(-sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{1-\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sin(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.
time = 0.46, size = 35, normalized size = 2.50

$$-2\sqrt{2}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)-\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(cos(-1/4*pi + 1/2*x)*sgn(sin(-1/4*pi + 1/2*x)) - sgn(sin(-1/4*p
i + 1/2*x)))
```

Mupad [B]
time = 0.14, size = 18, normalized size = 1.29

$$\frac{2\sqrt{1-\sin(x)}\,(\sin(x)+1)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sin(x))^(1/2),x)
```

```
[Out] (2*(1 - sin(x))^(1/2)*(sin(x) + 1))/cos(x)
```

3.237 $\int \sqrt{1+\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{2\sin(x)}{\sqrt{1+\cos(x)}}$$

[Out] 2*sin(x)/(cos(x)+1)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2725\}$

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Cos[x]],x]

[Out] (2*Sin[x])/Sqrt[1 + Cos[x]]

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\cos(x)}\,dx = \frac{2\sin(x)}{\sqrt{1+\cos(x)}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.33

$$2\sqrt{1+\cos(x)}\,\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Cos[x]],x]

[Out] 2*Sqrt[1 + Cos[x]]*Tan[x/2]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.04, size = 22, normalized size = 1.83

method	result	size
default	$\frac{4\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{2\cos\left(x\right)+2}}$	22
risch	$-\frac{i\sqrt{2}\,\sqrt{\left(1+\mathrm{e}^{ix}\right)^{2}\mathrm{e}^{-ix}}\,\left(\mathrm{e}^{ix}-1\right)}{1+\mathrm{e}^{ix}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 2.63, size = 9, normalized size = 0.75

$$2\sqrt{2}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sin(1/2*x)
```

Fricas [A]
time = 0.82, size = 10, normalized size = 0.83

$$\frac{2\sin\left(x\right)}{\sqrt{\cos\left(x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sin(x)/sqrt(cos(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.
time = 0.36, size = 36, normalized size = 3.00

$$2\sqrt{1-\frac{\tan^{2}\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))**(1/2),x)
```

```
[Out] 2*sqrt(1 - tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)
```

Giac [A]
time = 0.45, size = 14, normalized size = 1.17

$$2\sqrt{2}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*sgn(cos(1/2*x))*sin(1/2*x)
```

Mupad [B]
time = 0.15, size = 10, normalized size = 0.83

$$\frac{2\sin\left(x\right)}{\sqrt{\cos\left(x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)^(1/2),x)
```

```
[Out] (2*sin(x))/(cos(x) + 1)^(1/2)
```

3.238 $\int \sqrt{1-\cos(x)}\,dx$

Optimal. Leaf size=14

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

```
[Out] -2*sin(x)/(1-cos(x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2725\}$

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]],x]
[Out] (-2*Sin[x])/Sqrt[1 - Cos[x]]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\cos(x)}\,dx = -\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.29

$$-2\sqrt{1-\cos(x)}\,\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]],x]
[Out] -2*Sqrt[1 - Cos[x]]*Cot[x/2]
```

Maple [A]
time = 0.05, size = 22, normalized size = 1.57

method	result	size
default	$-\frac{4 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) \sqrt{2}}{\sqrt{2-2 \cos \left(x\right)}}$	22
risch	$-\frac{i \sqrt{2}\, \sqrt{-\left({\mathrm e}^{i x}-1\right)^{2} {\mathrm e}^{-i x}}\, \left(1+{\mathrm e}^{i x}\right)}{{\mathrm e}^{i x}-1}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*sin(1/2*x)*cos(1/2*x)*2^(1/2)/(sin(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 1.93, size = 20, normalized size = 1.43

$$-\frac{2 \sqrt{2}}{\sqrt{\frac{\sin \left(x\right)^{2}}{{\left(\cos \left(x\right) + 1\right)}^{2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 0.82, size = 18, normalized size = 1.29

$$-\frac{2 \, {\left(\cos \left(x\right) + 1\right)} \sqrt{-\cos \left(x\right) + 1}}{\sin \left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(cos(x) + 1)*sqrt(-cos(x) + 1)/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \cos{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(x)), x)
```

Giac [A]
time = 0.48, size = 23, normalized size = 1.64

$$-2\sqrt{2}\left(\cos\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.86

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cos(x))^(1/2),x)
```

```
[Out] -(2*sin(x))/(1 - cos(x))^(1/2)
```

3.239 $\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

```
[Out] 2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2131, 30, 32}

$$\frac{2x^{3/2}}{3}+\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx &= \int \sqrt{-1+x}\,dx+\int \sqrt{x}\,dx\\ &= \frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 17, normalized size = 0.81

$$\frac{2}{3}\left((-1+x)^{3/2}+x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (2*((-1 + x)^(3/2) + x^(3/2)))/3
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.67

method	result	size
default	$\frac{2(-1+x)^{\frac{3}{2}}}{3}+\frac{2x^{\frac{3}{2}}}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(-1+x)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(x - 1) - sqrt(x)), x)
```

Fricas [A]
time = 0.77, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.
time = 0.17, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\,\sqrt{x-1}}{-3\sqrt{x}+3\sqrt{x-1}}-\frac{4x}{-3\sqrt{x}+3\sqrt{x-1}}+\frac{2}{-3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(-3*sqrt(x) + 3*sqrt(x - 1)) - 4*x/(-3*sqrt(x) + 3*sq
rt(x - 1)) + 2/(-3*sqrt(x) + 3*sqrt(x - 1))
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.62

$$\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Mupad [B]
time = 0.19, size = 21, normalized size = 1.00

$$\frac{2x\sqrt{x-1}}{3}-\frac{2\sqrt{x-1}}{3}+\frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x - 1)^(1/2) - x^(1/2)),x)
```

```
[Out] (2*x*(x - 1)^(1/2))/3 - (2*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

3.240 $\int \frac{1}{1-\sqrt{1+x}}\,dx$

Optimal. Leaf size=24

$$-2\sqrt{1+x}-2\log\left(1-\sqrt{1+x}\right)$$

```
[Out] -2*ln(1-(1+x)^(1/2))-2*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 196, 45}

$$-2\sqrt{x+1}-2\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[1 + x])^(-1),x]
```

```
[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1-\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{1-\sqrt{x}}\,dx, x, 1+x\right) \\
&= 2\text{Subst}\left(\int \frac{x}{1-x}\,dx, x, \sqrt{1+x}\right) \\
&= 2\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)\,dx, x, \sqrt{1+x}\right) \\
&= -2\sqrt{1+x} - 2\log\left(1-\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.92

$$-2\sqrt{1+x} - 2\log\left(-1+\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[1 + x])^(-1),x]
```

```
[Out] -2*Sqrt[1 + x] - 2*Log[-1 + Sqrt[1 + x]]
```

Maple [A]
time = 0.04, size = 31, normalized size = 1.29

method	result	size
derivativedivides	$-2\sqrt{1+x}-2\ln\left(-1+\sqrt{1+x}\right)$	19
trager	$-2\sqrt{1+x}-\ln\left(2\sqrt{1+x}-2-x\right)$	24
default	$-\ln\left(x\right)-2\sqrt{1+x}-\ln\left(-1+\sqrt{1+x}\right)+\ln\left(1+\sqrt{1+x}\right)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)-2*(1+x)^(1/2)-ln(-1+(1+x)^(1/2))+ln(1+(1+x)^(1/2))
```

Maxima [A]
time = 2.46, size = 18, normalized size = 0.75

$$-2\sqrt{x+1} - 2\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="maxima")
```

[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)

Fricas [A]
time = 0.59, size = 18, normalized size = 0.75

$$-2\sqrt{x+1}-2\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="fricas")
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.83

$$-2\sqrt{x+1}-2\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**(1/2)),x)
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

Giac [A]
time = 0.43, size = 19, normalized size = 0.79

$$-2\sqrt{x+1}-2\log\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="giac")
[Out] -2*sqrt(x + 1) - 2*log(abs(sqrt(x + 1) - 1))
```

Mupad [B]
time = 0.11, size = 18, normalized size = 0.75

$$-2\ln\left(\sqrt{x+1}-1\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^(1/2) - 1),x)
[Out] - 2*log((x + 1)^(1/2) - 1) - 2*(x + 1)^(1/2)
```

3.241 $\int \frac{x}{\sqrt{36+x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

[Out] 1/2*arcsinh(1/6*x^2)

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {281, 221}

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[36 + x^4],x]

[Out] ArcSinh[x^2/6]/2

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{36+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{36+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 18, normalized size = 1.50

$$\frac{1}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{36+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[36 + x^4],x]
```

```
[Out] ArcTanh[x^2/Sqrt[36 + x^4]]/2
```

Maple [A]
time = 0.08, size = 9, normalized size = 0.75

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
elliptic	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{6}\right)}{2}$	9
trager	$-\frac{\ln\left(x^2-\sqrt{x^4+36}\right)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+36)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(1/6*x^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(8) = 16$.
time = 1.77, size = 33, normalized size = 2.75

$$\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log(sqrt(x^4 + 36)/x^2 + 1) - 1/4*log(sqrt(x^4 + 36)/x^2 - 1)
```

Fricas [A]
time = 0.70, size = 16, normalized size = 1.33

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))
```

Sympy [A]
time = 0.40, size = 7, normalized size = 0.58

$$\frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+36)**(1/2),x)
```

```
[Out] asinh(x**2/6)/2
```

Giac [A]
time = 0.42, size = 16, normalized size = 1.33

$$-\frac{1}{2} \log \left(-x^2 + \sqrt{x^4 + 36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))
```

Mupad [B]
time = 0.04, size = 8, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + 36)^(1/2),x)
```

```
[Out] asinh(x^2/6)/2
```

3.242 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} \, dx$

Optimal. Leaf size=32

$$6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 45}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx$$
$$= 6\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.03

$$\left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] (6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(24) = 48.
time = 0.03, size = 92, normalized size = 2.88

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$2\ln\left(-1+x^{\frac{1}{6}}\right)-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+2\sqrt{x}+\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(-1+x^(1/6))-ln(x^(1/3)+x^(1/6)+1)-2*ln(1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1
)+2*x^(1/2)+ln(x^(1/2)-1)-ln(1+x^(1/2))+6*x^(1/6)-ln(-1+x)-2*ln(-1+x^(1/3))
+ln(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

Maxima [A]
time = 2.44, size = 24, normalized size = 0.75

$$2\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A]
time = 0.61, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Giac [A]
time = 0.44, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6\ln\left(x^{1/6} + 1\right) - 3x^{1/3} + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
```

```
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.243 $\int \log(2+3x^2)\,dx$

Optimal. Leaf size=33

$$-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)+x\log(2+3x^2)$$

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2498, 327, 209}

$$2\sqrt{\frac{2}{3}}\text{ArcTan}\left(\sqrt{\frac{3}{2}}x\right)+x\log(3x^2+2)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(2+3x^2\right)\,dx &= x\log\left(2+3x^2\right)-6\int\frac{x^2}{2+3x^2}\,dx\\ &= -2x+x\log\left(2+3x^2\right)+4\int\frac{1}{2+3x^2}\,dx\\ &= -2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)+x\log\left(2+3x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)+x\log\left(2+3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + 3*x^2],x]
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Maple [A]
time = 0.02, size = 27, normalized size = 0.82

method	result	size
default	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27
risch	$-2x+x\ln\left(3x^2+2\right)+\frac{2\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{3}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(3*x^2+2),x,method=_RETURNVERBOSE)
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A]
time = 2.17, size = 26, normalized size = 0.79

$$x\log\left(3\,x^2+2\right)+\frac{2}{3}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="maxima")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Fricas [A]
time = 0.66, size = 32, normalized size = 0.97

$$\frac{2}{3}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{3}\,\sqrt{2}\,x\right)+x\log\left(3\,x^{2}+2\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

Sympy [A]
time = 0.05, size = 31, normalized size = 0.94

$$x\log\left(3x^{2}+2\right)-2x+\frac{2\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(3*x**2+2),x)
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

Giac [A]
time = 0.44, size = 26, normalized size = 0.79

$$x\log\left(3\,x^{2}+2\right)+\frac{2}{3}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="giac")
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Mupad [B]
time = 0.06, size = 26, normalized size = 0.79

$$\frac{2\,\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}-2\,x+x\,\ln\left(3\,x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(3*x^2 + 2),x)
[Out] (2*6^(1/2)*atan((6^(1/2)*x)/2))/3 - 2*x + x*log(3*x^2 + 2)
```

3.244 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

[Out] ln(sin(x))

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cot[x],x]

[Out] Log[Sin[x]]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x],x]

[Out] Log[Sin[x]]

Maple [A]
time = 0.00, size = 4, normalized size = 1.33

method	result	size

lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(\cot^2(x)+1)}{2}$	10
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
risch	$-ix+\ln(e^{2ix}-1)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sin(x))
```

Maxima [A]
time = 1.38, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.
time = 0.56, size = 11, normalized size = 3.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A]
time = 0.03, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

Giac [**A**]
time = 0.45, size = 4, normalized size = 1.33

$$\log\left(|\sin\left(x\right)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="giac")
```

```
[Out] log(abs(sin(x)))
```

Mupad [**B**]
time = 0.02, size = 3, normalized size = 1.00

$$\ln\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
```

```
[Out] log(sin(x))
```

3.245 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x+\cot(x)-\frac{\cot^3(x)}{3}$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\ &= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\ &= x+\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.50

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A]
time = 0.00, size = 16, normalized size = 1.33

method	result	size
derivativedivides	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\mathrm{arccot}\left(\cot\left(x\right)\right)$	16
default	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\mathrm{arccot}\left(\cot\left(x\right)\right)$	16
norman	$\frac{-\frac{1}{3}+\tan^{2}\left(x\right)+x\left(\tan^{3}\left(x\right)\right)}{\tan\left(x\right)^{3}}$	18
risch	$x+\frac{4i\left(3\,{\mathrm e}^{4ix}-3\,{\mathrm e}^{2ix}+2\right)}{3\left({\mathrm e}^{2ix}-1\right)^{3}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+arccot(cot(x))
```

Maxima [A]
time = 1.66, size = 16, normalized size = 1.33

$$x+\frac{3\,\tan\left(x\right)^{2}-1}{3\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(10) = 20.
time = 0.61, size = 48, normalized size = 4.00

$$\frac{4\,\cos\left(2\,x\right)^{2}+3\left(x\cos\left(2\,x\right)-x\right)\sin\left(2\,x\right)+2\,\cos\left(2\,x\right)-2}{3\left(\cos\left(2\,x\right)-1\right)\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A]
time = 0.02, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.
time = 0.44, size = 34, normalized size = 2.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.83

$$-\frac{\cot(x)^3}{3} + \cot(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
[Out] x + cot(x) - cot(x)^3/3
```

3.246 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

```
[Out] ln(cosh(x))
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size

lookup	$\ln(\cosh(x))$	4
default	$\ln(\cosh(x))$	4
risch	$-x+\ln(1+e^{2x})$	12
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{2}-\frac{\ln(\tanh(x)+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x,method=_RETURNVERBOSE)
[Out] ln(cosh(x))
```

Maxima [A]
time = 1.33, size = 3, normalized size = 1.00

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="maxima")
[Out] log(cosh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.
time = 0.63, size = 18, normalized size = 6.00

$$-x+\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="fricas")
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.
time = 0.04, size = 7, normalized size = 2.33

$$x-\log(\tanh(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x)
[Out] x - log(tanh(x) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.
time = 0.44, size = 11, normalized size = 3.67

$$-x+\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + log(e^(2*x) + 1)
```

Mupad [B]
time = 0.01, size = 3, normalized size = 1.00

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x)
```

```
[Out] log(cosh(x))
```

3.247 $\int \coth(x)\,dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

[Out] ln(sinh(x))

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = $\{3556\}$

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Int[Coth[x],x]

[Out] Log[Sinh[x]]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(x)\,dx = \log(\sinh(x))$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x],x]

[Out] Log[Sinh[x]]

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size

lookup	$\ln(\sinh(x))$	4
default	$\ln(\sinh(x))$	4
risch	$-x+\ln(\mathrm{e}^{2x}-1)$	12
derivativedivides	$-\frac{\ln(\coth(x)-1)}{2}-\frac{\ln(\coth(x)+1)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x,method=_RETURNVERBOSE)
[Out] ln(sinh(x))
```

Maxima [A]
time = 2.45, size = 3, normalized size = 1.00

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="maxima")
[Out] log(sinh(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(3) = 6.
time = 0.62, size = 18, normalized size = 6.00

$$-x+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="fricas")
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(3) = 6.
time = 0.13, size = 12, normalized size = 4.00

$$x-\log(\tanh(x)+1)+\log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x)
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(3) = 6.
time = 0.45, size = 12, normalized size = 4.00

$$-x+\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="giac")
```

```
[Out] -x + log(abs(e^(2*x) - 1))
```

Mupad [B]
time = 0.14, size = 3, normalized size = 1.00

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x)
```

```
[Out] log(sinh(x))
```

3.248 $\int b^x \, dx$

Optimal. Leaf size=8

$$\frac{b^x}{\log(b)}$$

```
[Out] b^x/ln(b)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2225}

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[b^x,x]
```

```
[Out] b^x/Log[b]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int b^x \, dx = \frac{b^x}{\log(b)}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[b^x,x]
```

```
[Out] b^x/Log[b]
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size
gosper	$\frac{b^x}{\ln(b)}$	9
derivativedivides	$\frac{b^x}{\ln(b)}$	9
default	$\frac{b^x}{\ln(b)}$	9
risch	$\frac{b^x}{\ln(b)}$	9
norman	$\frac{e^{x\ln(b)}}{\ln(b)}$	11
meijerg	$-\frac{1-e^{x\ln(b)}}{\ln(b)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^x,x,method=_RETURNVERBOSE)
```

```
[Out] b^x/ln(b)
```

Maxima [A]
time = 3.45, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="maxima")
```

```
[Out] b^x/log(b)
```

Fricas [A]
time = 1.00, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="fricas")
```

```
[Out] b^x/log(b)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 1.00

$$\begin{cases} \frac{b^x}{\log(b)} & \text{for}\ \log(b) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**x,x)
```

```
[Out] Piecewise((b**x/log(b), Ne(log(b), 0)), (x, True))
```

Giac [A]
time = 0.47, size = 8, normalized size = 1.00

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="giac")
```

```
[Out] b^x/log(b)
```

Mupad [B]
time = 0.16, size = 8, normalized size = 1.00

$$\frac{b^x}{\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^x,x)
```

```
[Out] b^x/log(b)
```

3.249 $\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx$

Optimal. Leaf size=49

$$-\frac{x\sqrt{2+\frac{1}{x^4}+x^4}}{1+x^4}+\frac{x^5\sqrt{2+\frac{1}{x^4}+x^4}}{3\,(1+x^4)}$$

```
[Out] -x*(2+1/x^4+x^4)^(1/2)/(x^4+1)+1/3*x^5*(2+1/x^4+x^4)^(1/2)/(x^4+1)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1365, 1369, 14}

$$\frac{x^5\sqrt{x^4+\frac{1}{x^4}+2}}{3\,(x^4+1)}-\frac{x\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^(-4) + x^4],x]
```

```
[Out] -((x*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)) + (x^5*Sqrt[2 + x^(-4) + x^4])/(3*(
1 + x^4))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1365

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_), x_Symbol] :> Dist[x^
(n*FracPart[p])*((a + b/x^n + c*x^n)^FracPart[p]/(b + a*x^n + c*x^(2*n))^Fr
acPart[p]), Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x], x] /; FreeQ[{a, b, c
, n, p}, x] && EqQ[mn, -n] &&  !IntegerQ[p] && PosQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx = \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \frac{\sqrt{1+2x^4+x^8}}{x^2}\,dx}{\sqrt{1+2x^4+x^8}}$$
$$= \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \frac{1+x^4}{x^2}\,dx}{1+x^4}$$
$$= \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \left(\frac{1}{x^2}+x^2\right)\,dx}{1+x^4}$$
$$= -\frac{x\sqrt{2+\frac{1}{x^4}+x^4}}{1+x^4}+\frac{x^5\sqrt{2+\frac{1}{x^4}+x^4}}{3(1+x^4)}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.59

$$\frac{x(-3+x^4)\sqrt{2+\frac{1}{x^4}+x^4}}{3(1+x^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^(-4) + x^4],x]
```

```
[Out] (x*(-3 + x^4)*Sqrt[2 + x^(-4) + x^4])/(3*(1 + x^4))
```

Maple [A]
time = 0.01, size = 32, normalized size = 0.65

method	result	size
gosper	$\frac{x(x^4-3)\sqrt{\frac{x^8+2x^4+1}{x^4}}}{3x^4+3}$	32
default	$\frac{x(x^4-3)\sqrt{\frac{x^8+2x^4+1}{x^4}}}{3x^4+3}$	32
risch	$\frac{\sqrt{\frac{(x^4+1)^2}{x^4}}\,x^5}{3x^4+3}-\frac{\sqrt{\frac{(x^4+1)^2}{x^4}}\,x}{x^4+1}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+1/x^4+x^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(x^4-3)*((x^8+2*x^4+1)/x^4)^(1/2)/(x^4+1)
```

Maxima [**A**]
time = 3.63, size = 10, normalized size = 0.20

$$\frac{x^4-3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^4 - 3)/x
```

Fricas [**A**]
time = 0.75, size = 10, normalized size = 0.20

$$\frac{x^4-3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^4 - 3)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^4+2+\frac{1}{x^4}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x**4+x**4)**(1/2),x)
[Out] Integral(sqrt(x**4 + 2 + x**(-4)), x)
```

Giac [**A**]
time = 0.48, size = 11, normalized size = 0.22

$$\frac{1}{3}\,x^3-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="giac")
[Out] 1/3*x^3 - 1/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{1}{x^4} + x^4 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^4 + x^4 + 2)^(1/2),x)
```

```
[Out] int((1/x^4 + x^4 + 2)^(1/2), x)
```

3.250 $\int \frac{1+2x}{2+3x} \, dx$

Optimal. Leaf size=16

$$\frac{2x}{3} - \frac{1}{9}\log(2+3x)$$

[Out] 2/3*x-1/9*ln(2+3*x)

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{2x}{3} - \frac{1}{9}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/(2 + 3*x),x]

[Out] (2*x)/3 - Log[2 + 3*x]/9

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{2+3x} \, dx &= \int \left(\frac{2}{3} - \frac{1}{3(2+3x)}\right) dx \\ &= \frac{2x}{3} - \frac{1}{9}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.06

$$\frac{1}{9}(4+6x-\log(2+3x))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/(2 + 3*x),x]

[Out] (4 + 6*x - Log[2 + 3*x])/9

Maple [A]
time = 0.03, size = 13, normalized size = 0.81

method	result	size
default	$\frac{2x}{3}-\frac{\ln(2+3x)}{9}$	13
norman	$\frac{2x}{3}-\frac{\ln(2+3x)}{9}$	13
meijerg	$-\frac{\ln(1+\frac{3x}{2})}{9}+\frac{2x}{3}$	13
risch	$\frac{2x}{3}-\frac{\ln(2+3x)}{9}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(2+3*x),x,method=_RETURNVERBOSE)
[Out] 2/3*x-1/9*ln(2+3*x)
```

Maxima [A]
time = 2.95, size = 12, normalized size = 0.75

$$\frac{2}{3}\,x-\frac{1}{9}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="maxima")
[Out] 2/3*x - 1/9*log(3*x + 2)
```

Fricas [A]
time = 0.78, size = 12, normalized size = 0.75

$$\frac{2}{3}\,x-\frac{1}{9}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="fricas")
[Out] 2/3*x - 1/9*log(3*x + 2)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2x}{3}-\frac{\log\left(3x+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x)
```

```
[Out] 2*x/3 - log(3*x + 2)/9
```

Giac [A]
time = 0.44, size = 13, normalized size = 0.81

$$\frac{2}{3}\,x - \frac{1}{9}\,\log\left(|3\,x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="giac")
[Out] 2/3*x - 1/9*log(abs(3*x + 2))
```

Mupad [B]
time = 0.14, size = 10, normalized size = 0.62

$$\frac{2\,x}{3} - \frac{\ln\left(x + \frac{2}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(3*x + 2),x)
[Out] (2*x)/3 - log(x + 2/3)/9
```

3.251 $\int x \log \left(x+\sqrt{1+x^2}\right) \, dx$

Optimal. Leaf size=40

$$-\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{4}\sinh^{-1}(x)+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)$$

```
[Out] 1/4*arcsinh(x)+1/2*x^2*ln(x+(x^2+1)^(1/2))-1/4*x*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2616, 327, 221}

$$-\frac{1}{4}\sqrt{x^2+1}\,x+\frac{1}{2}x^2\log\left(\sqrt{x^2+1}+x\right)+\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -1/4*(x*Sqrt[1 + x^2]) + ArcSinh[x]/4 + (x^2*Log[x + Sqrt[1 + x^2]])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2616

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]]*((g_.)*(x_))
^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[a + c*x^2]]/(
g*(m + 1))), x] - Dist[a*c*(f^2/(g*(m + 1))), Int[(g*x)^(m + 1)/(d*e*(a + c
*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g
, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int x\log\left(x+\sqrt{1+x^2}\right)\,dx=\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\int\frac{x^2}{\sqrt{1+x^2}}\,dx$$
$$=-\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)+\frac{1}{4}\int\frac{1}{\sqrt{1+x^2}}\,dx$$
$$=-\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{4}\sinh^{-1}(x)+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.90

$$\frac{1}{4}\left(-x\sqrt{1+x^2}+\sinh^{-1}(x)+2x^2\log\left(x+\sqrt{1+x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x + Sqrt[1 + x^2]],x]
[Out] (-(x*Sqrt[1 + x^2]) + ArcSinh[x] + 2*x^2*Log[x + Sqrt[1 + x^2]])/4
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\ln\left(x+\sqrt{x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x+(x^2+1)^(1/2)),x)
[Out] int(x*ln(x+(x^2+1)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*x^2 - integrate(1/2*x^2/(x^3 + (x^2 +
1)^(3/2) + x), x) + 1/4*log(x^2 + 1)
```

Fricas [A]
time = 0.73, size = 30, normalized size = 0.75

$$\frac{1}{4}\left(2\,x^2+1\right)\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\sqrt{x^2+1}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^2 + 1)*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \log \left(x + \sqrt{x^2 + 1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral(x*log(x + sqrt(x**2 + 1)), x)
```

Giac [A]
time = 0.47, size = 40, normalized size = 1.00

$$\frac{1}{2} x^2 \log \left(x + \sqrt{x^2 + 1}\right) - \frac{1}{4} \sqrt{x^2 + 1}\, x - \frac{1}{4} \log \left(-x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x - 1/4*log(-x + sqrt(x^
2 + 1))
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.80

$$x \ln \left(x + \sqrt{x^2 + 1}\right) \left(\frac{x}{2} + \frac{1}{4\,x}\right) - \frac{x\,\sqrt{x^2 + 1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x + (x^2 + 1)^(1/2)),x)
```

```
[Out] x*log(x + (x^2 + 1)^(1/2))*(x/2 + 1/(4*x)) - (x*(x^2 + 1)^(1/2))/4
```

3.252 $\int x(1+e^x\sin(x))^2\,dx$

Optimal. Leaf size=128

$$-\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)-\frac{1}{32}e^{2x}\cos(2x)+e^x x\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin$$

```
[Out] -3/32*exp(2*x)+1/8*exp(2*x)*x+1/2*x^2+exp(x)*cos(x)-exp(x)*x*cos(x)-1/32*ex
p(2*x)*cos(2*x)+exp(x)*x*sin(x)+1/16*exp(2*x)*cos(x)*sin(x)-1/4*exp(2*x)*x*
cos(x)*sin(x)-1/16*exp(2*x)*sin(x)^2+1/4*exp(2*x)*x*sin(x)^2+1/32*exp(2*x)*
sin(2*x)
```

Rubi [A]
time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6874, 4517, 4553, 4518, 4519, 2225, 4557, 12}

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^x x\sin(x)+\frac{1}{32}e^{2x}\sin(2x)-e^x x\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x)-\frac{1}{4}e^{2x}x\sin(x)\cos(x)+\frac{1}{16}e^{2x}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[x] - (E^(2
*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x])/16 - (E^(2*x)*x*C
os[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^(2*x)*x*Sin[x]^2)/4 + (E^(2*x)
*Sin[2*x])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
```

```
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 4553

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4557

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x(1+e^x\sin(x))^2\,dx &= \int \left(x+2e^xx\sin(x)+e^{2x}x\sin^2(x)\right)\,dx\\ &= \frac{x^2}{2}+2\int e^xx\sin(x)\,dx+\int e^{2x}x\sin^2(x)\,dx\\ &= \frac{1}{8}e^{2x}x+\frac{x^2}{2}-e^xx\cos(x)+e^xx\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin(x)+\frac{1}{4}e^{2x}x\sin^2(x)-2\\ &= \frac{1}{8}e^{2x}x+\frac{x^2}{2}-e^xx\cos(x)+e^xx\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin(x)+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{8}\\ &= -\frac{e^{2x}}{16}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^xx\cos(x)+e^xx\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)\\ &= -\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^xx\cos(x)+e^xx\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)\\ &= -\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^xx\cos(x)-\frac{1}{32}e^{2x}\cos(2x)+e^xx\sin(x)+\frac{1}{16}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 67, normalized size = 0.52

$$\frac{1}{8}\left(4x^2+e^{2x}(-1+2x)-8e^x(-1+x)\cos(x)-e^{2x}x\cos(2x)+8e^xx\sin(x)-e^{2x}(-1+2x)\cos(x)\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (4*x^2 + E^(2*x)*(-1 + 2*x) - 8*E^x*(-1 + x)*Cos[x] - E^(2*x)*x*Cos[2*x] +
8*E^x*x*Sin[x] - E^(2*x)*(-1 + 2*x)*Cos[x]*Sin[x])/8
```

Maple [A]
time = 0.04, size = 63, normalized size = 0.49

method	result
default	$\frac{x^2}{2}+2\left(-\frac{x}{2}+\frac{1}{2}\right)e^x\cos(x)+e^xx\sin(x)+\frac{e^{2x}x}{4}-\frac{e^{2x}}{8}-\frac{x\,e^{2x}\cos(2x)}{8}+\frac{\left(-\frac{x}{4}+\frac{1}{8}\right)e^{2x}\sin(2x)}{2}$
risch	$\frac{x^2}{2}+\left(-\frac{1}{8}+\frac{x}{4}\right)e^{2x}+\left(-\frac{1}{64}+\frac{i}{64}\right)(-1+i+4x)\,e^{(2+2i)x}+\left(-\frac{1}{4}-\frac{i}{4}\right)(-1+i+2x)\,e^{(1+i)x}+\left(-\frac{1}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+exp(x)*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+2*(-1/2*x+1/2)*exp(x)*cos(x)+exp(x)*x*sin(x)+1/4*exp(x)^2*x-1/8*exp
(x)^2-1/8*x*exp(2*x)*cos(2*x)+1/2*(-1/4*x+1/8)*exp(2*x)*sin(2*x)
```

Maxima [A]
time = 2.08, size = 58, normalized size = 0.45

$$-\frac{1}{8}\,x\cos\left(2\,x\right)e^{\left(2\,x\right)}-\left(x-1\right)\cos\left(x\right)e^{x}-\frac{1}{16}\left(2\,x-1\right)e^{\left(2\,x\right)}\sin\left(2\,x\right)+xe^{x}\sin\left(x\right)+\frac{1}{2}\,x^{2}+\frac{1}{8}\left(2\,x-1\right)e^{\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*x*cos(2*x)*e^(2*x) - (x - 1)*cos(x)*e^x - 1/16*(2*x - 1)*e^(2*x)*sin(2
*x) + x*e^x*sin(x) + 1/2*x^2 + 1/8*(2*x - 1)*e^(2*x)
```

Fricas [A]
time = 0.88, size = 55, normalized size = 0.43

$$-\left(x-1\right)\cos\left(x\right)e^{x}+\frac{1}{2}\,x^{2}-\frac{1}{8}\left(2\,x\cos\left(x\right)^{2}-3\,x+1\right)e^{\left(2\,x\right)}-\frac{1}{8}\left(\left(2\,x-1\right)\cos\left(x\right)e^{\left(2\,x\right)}-8\,xe^{x}\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(x - 1)*cos(x)*e^x + 1/2*x^2 - 1/8*(2*x*cos(x)^2 - 3*x + 1)*e^(2*x) - 1/8*
((2*x - 1)*cos(x)*e^(2*x) - 8*x*e^x)*sin(x)
```

Sympy [A]
time = 0.70, size = 109, normalized size = 0.85

$$\frac{x^{2}}{2}+\frac{3xe^{2x}\sin^{2}\left(x\right)}{8}-\frac{xe^{2x}\sin\left(x\right)\cos\left(x\right)}{4}+\frac{xe^{2x}\cos^{2}\left(x\right)}{8}+xe^{x}\sin\left(x\right)-xe^{x}\cos\left(x\right)-\frac{e^{2x}\sin^{2}\left(x\right)}{8}+\frac{e^{2x}\sin\left(x\right)\cos\left(x\right)}{8}-\frac{e^{2x}\cos^{2}\left(x\right)}{8}+e^{x}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))**2,x)
```

```
[Out] x**2/2 + 3*x*exp(2*x)*sin(x)**2/8 - x*exp(2*x)*sin(x)*cos(x)/4 + x*exp(2*x)
*cos(x)**2/8 + x*exp(x)*sin(x) - x*exp(x)*cos(x) - exp(2*x)*sin(x)**2/8 + e
xp(2*x)*sin(x)*cos(x)/8 - exp(2*x)*cos(x)**2/8 + exp(x)*cos(x)
```

Giac [A]
time = 0.46, size = 57, normalized size = 0.45

$$\frac{1}{2}\,x^{2}-\frac{1}{16}\left(2\,x\cos\left(2\,x\right)+\left(2\,x-1\right)\sin\left(2\,x\right)\right)e^{\left(2\,x\right)}+\frac{1}{8}\left(2\,x-1\right)e^{\left(2\,x\right)}-\left(\left(x-1\right)\cos\left(x\right)-x\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/16*(2*x*cos(2*x) + (2*x - 1)*sin(2*x))*e^(2*x) + 1/8*(2*x - 1)*
e^(2*x) - ((x - 1)*cos(x) - x*sin(x))*e^x
```

Mupad [B]
time = 0.28, size = 69, normalized size = 0.54

$$\frac{3\,x\,{\mathrm{e}}^{2\,x}}{8}-\frac{{\mathrm{e}}^{2\,x}}{8}+{\mathrm{e}}^x\,\cos\left(x\right)+\frac{x^2}{2}-\frac{x\,{\mathrm{e}}^{2\,x}\,{\cos\left(x\right)}^2}{4}+\frac{{\mathrm{e}}^{2\,x}\,\cos\left(x\right)\,\sin\left(x\right)}{8}-x\,{\mathrm{e}}^x\,\cos\left(x\right)+x\,{\mathrm{e}}^x\,\sin\left(x\right)-\frac{x\,{\mathrm{e}}^{2\,x}\,\cos\left(x\right)\,\sin\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(exp(x)*sin(x) + 1)^2,x)
```

```
[Out] (3*x*exp(2*x))/8 - exp(2*x)/8 + exp(x)*cos(x) + x^2/2 - (x*exp(2*x)*cos(x)^
2)/4 + (exp(2*x)*cos(x)*sin(x))/8 - x*exp(x)*cos(x) + x*exp(x)*sin(x) - (x*
exp(2*x)*cos(x)*sin(x))/4
```

3.253 $\int e^x x \cos(x) \, dx$

Optimal. Leaf size=30

$$\frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)$$

```
[Out] 1/2*exp(x)*x*cos(x)-1/2*exp(x)*sin(x)+1/2*exp(x)*x*sin(x)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4518, 4554, 4517}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4554

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int e^x x \cos(x)\,dx = \frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x) - \int \left(\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx$$
$$= \frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x) - \frac{1}{2}\int e^x\cos(x)\,dx - \frac{1}{2}\int e^x\sin(x)\,dx$$
$$= \frac{1}{2}e^x x\cos(x) - \frac{1}{2}e^x\sin(x) + \frac{1}{2}e^x x\sin(x)$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.60

$$\frac{1}{2}e^x(x\cos(x) + (-1+x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Cos[x],x]
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.67

method	result	size
default	$\frac{e^x x\cos(x)}{2} - \left(-\frac{x}{2}+\frac{1}{2}\right)e^x\sin(x)$	20
risch	$\left(\frac{1}{8}-\frac{i}{8}\right)(-1+i+2x)\,e^{(1+i)x} + \left(\frac{1}{8}+\frac{i}{8}\right)(-1-i+2x)\,e^{(1-i)x}$	36
norman	$\frac{e^x x\tan\left(\frac{x}{2}\right)+\frac{e^x x}{2}-e^x\tan\left(\frac{x}{2}\right)-\frac{e^x x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

Maxima [A]
time = 2.46, size = 17, normalized size = 0.57

$$\frac{1}{2}\,x\cos(x)\,e^x + \frac{1}{2}\,(x-1)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Fricas [A]
time = 1.12, size = 17, normalized size = 0.57

$$\frac{1}{2}\,x\cos\left(x\right)e^{x}+\frac{1}{2}\,(x-1)e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A]
time = 0.17, size = 27, normalized size = 0.90

$$\frac{xe^{x}\sin\left(x\right)}{2}+\frac{xe^{x}\cos\left(x\right)}{2}-\frac{e^{x}\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x)
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Giac [A]
time = 0.46, size = 15, normalized size = 0.50

$$\frac{1}{2}\,(x\cos\left(x\right)+(x-1)\sin\left(x\right))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

Mupad [B]
time = 0.17, size = 17, normalized size = 0.57

$$\frac{\mathrm{e}^{x}\,\left(x\,\cos\left(x\right)-\sin\left(x\right)+x\,\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x)*cos(x),x)
[Out] (exp(x)*(x*cos(x) - sin(x) + x*sin(x)))/2
```

3.254 $\int \frac{1}{(-3+x)^4} \, dx$

Optimal. Leaf size=11

$$\frac{1}{3(3-x)^3}$$

[Out] 1/3/(3-x)^3

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {32}

$$\frac{1}{3(3-x)^3}$$

Antiderivative was successfully verified.

[In] Int[(-3 + x)^(-4),x]

[Out] 1/(3*(3 - x)^3)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(-3+x)^4} \, dx = \frac{1}{3(3-x)^3}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.82

$$-\frac{1}{3(-3+x)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x)^(-4),x]

[Out] -1/3*1/(-3 + x)^3

Maple [A]
time = 0.04, size = 8, normalized size = 0.73

method	result	size
gosper	$-\frac{1}{3(-3+x)^3}$	8
default	$-\frac{1}{3(-3+x)^3}$	8
norman	$-\frac{1}{3(-3+x)^3}$	8
risch	$-\frac{1}{3(-3+x)^3}$	8
meijerg	$\frac{x\left(\frac{1}{9}x^2-x+3\right)}{243\left(1-\frac{x}{3}\right)^3}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3+x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(-3+x)^3
```

Maxima [A]
time = 5.06, size = 7, normalized size = 0.64

$$-\frac{1}{3\left(x-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(x - 3)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.
time = 1.42, size = 17, normalized size = 1.55

$$-\frac{1}{3\left(x^3-9\,x^2+27\,x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(x^3 - 9*x^2 + 27*x - 27)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.
time = 0.03, size = 17, normalized size = 1.55

$$-\frac{1}{3x^3-27x^2+81x-81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)**4,x)
```

```
[Out] -1/(3*x**3 - 27*x**2 + 81*x - 81)
```

Giac [A]
time = 0.45, size = 7, normalized size = 0.64

$$-\frac{1}{3\left(x-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="giac")
```

```
[Out] -1/3/(x - 3)^3
```

Mupad [B]
time = 0.07, size = 7, normalized size = 0.64

$$-\frac{1}{3\left(x-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - 3)^4,x)
```

```
[Out] -1/(3*(x - 3)^3)
```

3.255 $\int \frac{x}{-1+x^3} dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] 1/3*ln(1-x)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {298, 31, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^3),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{-1+x^3}\,dx &= \frac{1}{3}\int \frac{1}{-1+x}\,dx - \frac{1}{3}\int \frac{-1+x}{1+x+x^2}\,dx \\
&= \frac{1}{3}\log(1-x) - \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 40, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^3),x]
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Maple [**A**]
time = 0.04, size = 33, normalized size = 0.82

method	result	size
default	$\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	33
risch	$-\frac{\ln(4x^2+4x+4)}{6}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(-1+x)}{3}$	37
meijerg	$\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 2.41, size = 32, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
x - 1)
```

Fricas [A]
time = 1.06, size = 32, normalized size = 0.80

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
x - 1)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.02

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

Giac [A]
time = 0.48, size = 33, normalized size = 0.82

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
abs(x - 1))
```

Mupad [B]
time = 0.11, size = 46, normalized size = 1.15

$$\frac{\ln(x-1)}{3}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 - 1),x)
```

```
[Out] log(x - 1)/3 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6)
```

3.256 $\int \frac{x}{-1+x^4} \, dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

```
[Out] -1/2*arctanh(x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {281, 213}

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^4),x]
```

```
[Out] -1/2*ArcTanh[x^2]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{-1+x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, x^2\right) \\ &= -\frac{1}{2}\tanh^{-1}\left(x^2\right) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(8) = 16.
time = 0.00, size = 23, normalized size = 2.88

$$\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^4),x]
```

```
[Out] Log[1 - x^2]/4 - Log[1 + x^2]/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(6) = 12$.
time = 0.03, size = 22, normalized size = 2.75

method	result	size
meijerg	$-\frac{\operatorname{arctanh}\left(x^{2}\right)}{2}$	7
risch	$-\frac{\ln\left(x^{2}+1\right)}{4}+\frac{\ln\left(x^{2}-1\right)}{4}$	18
default	$\frac{\ln\left(-1+x\right)}{4}+\frac{\ln\left(1+x\right)}{4}-\frac{\ln\left(x^{2}+1\right)}{4}$	22
norman	$\frac{\ln\left(-1+x\right)}{4}+\frac{\ln\left(1+x\right)}{4}-\frac{\ln\left(x^{2}+1\right)}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(-1+x)+1/4*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 4.49, size = 17, normalized size = 2.12

$$-\frac{1}{4}\log\left(x^{2}+1\right)+\frac{1}{4}\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="maxima")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.
time = 0.94, size = 17, normalized size = 2.12

$$-\frac{1}{4}\log\left(x^{2}+1\right)+\frac{1}{4}\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.
time = 0.03, size = 15, normalized size = 1.88

$$\frac{\log(x^2 - 1)}{4} - \frac{\log(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-1),x)
```

```
[Out] log(x**2 - 1)/4 - log(x**2 + 1)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.
time = 0.46, size = 18, normalized size = 2.25

$$-\frac{1}{4}\log(x^2 + 1) + \frac{1}{4}\log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.07, size = 6, normalized size = 0.75

$$-\frac{\operatorname{atanh}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 - 1),x)
```

```
[Out] -atanh(x^2)/2
```

3.257 $\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx$

Optimal. Leaf size=227

$$\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+\sqrt[4]{-2}\right)\mathrm{lo}$$

```
[Out] 1/8*(2+I*(-2)^(1/4))*ln(x)*ln(1-(1/2+1/2*I)*x*2^(1/4))+1/16*(4+(1-I)*2^(3/4
))*ln(x)*ln(1+(1/2+1/2*I)*x*2^(1/4))+1/8*(2+(-2)^(1/4))*ln(x)*ln(1-1/2*(-1)
^(3/4)*x*2^(3/4))+1/8*(2-(-2)^(1/4))*ln(x)*ln(1+1/2*(-1)^(3/4)*x*2^(3/4))+1
/16*(4+(1-I)*2^(3/4))*polylog(2,(-1/2-1/2*I)*x*2^(1/4))+1/8*(2+I*(-2)^(1/4)
)*polylog(2,(1/2+1/2*I)*x*2^(1/4))+1/8*(2-(-2)^(1/4))*polylog(2,-1/2*(-1)^(
3/4)*x*2^(3/4))+1/8*(2+(-2)^(1/4))*polylog(2,1/2*(-1)^(3/4)*x*2^(3/4))
```

Rubi [A]
time = 0.14, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2404, 2354, 2438}

$$\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}\left(2+\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(-1)^{3/4}x}{\sqrt[4]{2}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\log(x)\log\left(\frac{(-1)^{3/4}x}{\sqrt[4]{2}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)*Log[x])/(2 + x^4),x]
```

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)])/8 + ((4 + (1 - I)*
2^(3/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)])/16 + ((2 + (-2)^(1/4))*Log[x]
*Log[1 - ((-1)^(3/4)*x)/2^(1/4)])/8 + ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1
)^(3/4)*x)/2^(1/4)])/8 + ((4 + (1 - I)*2^(3/4))*PolyLog[2, ((-1 - I)*x)/2^(
3/4)])/16 + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8 + ((2 -
(-2)^(1/4))*PolyLog[2, -(((-1)^(3/4)*x)/2^(1/4))])/8 + ((2 + (-2)^(1/4))*Po
lyLog[2, ((-1)^(3/4)*x)/2^(1/4)])/8
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \left(\frac{\left(-2+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}-x\right)}+\frac{\left(-2i+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}-ix\right)}+\frac{\left(2i+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}+ix\right)}+\frac{\left(2+\sqrt[4]{}\right.}{8\left(\sqrt[4]{}\right.}\right.$$

$$=\frac{1}{8}\left(-2+\sqrt[4]{-2}\right)\int\frac{\log(x)}{\sqrt[4]{-2}-x}\,dx+\frac{1}{8}\left(-2i+\sqrt[4]{-2}\right)\int\frac{\log(x)}{\sqrt[4]{-2}-ix}\,dx+\frac{1}{8}\left(2i+\sqrt[4]{-}\right.$$

$$=\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-i\sqrt[4]{-2}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right.$$

$$=\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-i\sqrt[4]{-2}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right.$$

Mathematica [A]
time = 0.19, size = 194, normalized size = 0.85

$$\frac{1}{8}\left(\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\sqrt[4]{-\frac{1}{2}}\,x\right)+\left(2+\frac{1-i}{\sqrt[4]{2}}\right)\log(x)\log\left(1+\sqrt[4]{-\frac{1}{2}}\,x\right)-\left(-2+\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1-i)x}{2^{3/4}}\right)+\left(2+\sqrt[4]{-2}\right)\log(x)\log\left(1+\frac{(1-i)x}{2^{3/4}}\right)+\left(2+\frac{1-i}{\sqrt[4]{2}}\right)\text{Li}_2\left(-\frac{(1+i)x}{2^{3/4}}\right)+\left(2+\sqrt[4]{-2}\right)\text{Li}_2\left(-\frac{(1-i)x}{2^{3/4}}\right)-\left(-2+\sqrt[4]{-2}\right)\text{Li}_2\left(\frac{(1-i)x}{2^{3/4}}\right)+\left(2+i\sqrt[4]{-2}\right)\text{Li}_2\left(\frac{(1+i)x}{2^{3/4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Log[x])/(2 + x^4),x]
```

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - (-1/2)^(1/4)*x] + (2 + (1 - I)/2^(1/4))*
Log[x]*Log[1 + (-1/2)^(1/4)*x] - (-2 + (-2)^(1/4))*Log[x]*Log[1 - ((1 - I)*
x)/2^(3/4)] + (2 + (-2)^(1/4))*Log[x]*Log[1 + ((1 - I)*x)/2^(3/4)] + (2 + (
1 - I)/2^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*PolyLog
[2, ((-1 + I)*x)/2^(3/4)] - (-2 + (-2)^(1/4))*PolyLog[2, ((1 - I)*x)/2^(3/4
)] + (2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(171) = 342$.
time = 0.04, size = 394, normalized size = 1.74

method	result
default	$\frac{\left(\left(\frac{2^{\frac{3}{4}}}{2}+\frac{i2^{\frac{3}{4}}}{2}\right)^{3}+1\right)\left(\ln(x)\ln\left(\frac{\frac{2^{\frac{3}{4}}}{2}+\frac{i2^{\frac{3}{4}}}{2}-x}{\frac{2^{\frac{3}{4}}}{2}+\frac{i2^{\frac{3}{4}}}{2}}\right)+\text{dilog}\left(\frac{\frac{2^{\frac{3}{4}}}{2}+\frac{i2^{\frac{3}{4}}}{2}-x}{\frac{2^{\frac{3}{4}}}{2}+\frac{i2^{\frac{3}{4}}}{2}}\right)\right)}{4\left(\frac{2^{\frac{3}{4}}}{2}+\frac{i2^{\frac{3}{4}}}{2}\right)^{3}}+\frac{\left(\left(\frac{i2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}\right)^{3}+1\right)\left(\ln(x)\ln\left(\frac{\frac{i2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}-x}{\frac{i2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}}\right)+\text{dilog}\right.}{4\left(\frac{i2^{\frac{3}{4}}}{2}-\frac{2^{\frac{3}{4}}}{2}\right)^{3}}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*ln(x)/(x^4+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((1/2*2^(3/4)+1/2*I*2^(3/4))^3+1)/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*(ln(x)*
ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))+dilog((1/2*2^
(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4))))+1/4*((1/2*I*2^(3/4)-1/
2*2^(3/4))^3+1)/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*(ln(x)*ln((1/2*I*2^(3/4)-1/2*
2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+dilog((1/2*I*2^(3/4)-1/2*2^(3/4)-x)
/(1/2*I*2^(3/4)-1/2*2^(3/4))))+1/4*((-1/2*2^(3/4)-1/2*I*2^(3/4))^3+1)/(-1/2
*2^(3/4)-1/2*I*2^(3/4))^3*(ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^
(3/4)-1/2*I*2^(3/4)))+dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/
2*I*2^(3/4))))+1/4*((-1/2*I*2^(3/4)+1/2*2^(3/4))^3+1)/(-1/2*I*2^(3/4)+1/2*2
^(3/4))^3*(ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3
/4)))+dilog((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*log(x)/(x^4 + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)(x^2-x+1)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*ln(x)/(x**4+2),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)*log(x)/(x**4 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="giac")
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln(x)\,\left(x^3+1\right)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)*(x^3 + 1))/(x^4 + 2),x)
[Out] int((log(x)*(x^3 + 1))/(x^4 + 2), x)
```

3.258 $\int (\log(x)+\log(1+x)+\log(2+x))\,dx$

Optimal. Leaf size=24

$$-3x+x\log(x)+(1+x)\log(1+x)+(2+x)\log(2+x)$$

```
[Out] -3*x+x*ln(x)+(1+x)*ln(1+x)+(2+x)*ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2332, 2436}

$$-3x+x\log(x)+(x+1)\log(x+1)+(x+2)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x] + Log[1 + x] + Log[2 + x],x]
```

```
[Out] -3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int (\log(x)+\log(1+x)+\log(2+x))\,dx &= \int \log(x)\,dx+\int \log(1+x)\,dx+\int \log(2+x)\,dx\\ &= -x+x\log(x)+\text{Subst}\left(\int \log(x)\,dx,x,1+x\right)+\text{Subst}\left(\int \log(x)\right.\\ &= -3x+x\log(x)+(1+x)\log(1+x)+(2+x)\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.25

$$-3x+x\log(x)+\log(1+x)+x\log(1+x)+2\log(2+x)+x\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x] + Log[1 + x] + Log[2 + x],x]
[Out] -3*x + x*Log[x] + Log[1 + x] + x*Log[1 + x] + 2*Log[2 + x] + x*Log[2 + x]
```

Maple [A]
time = 0.01, size = 26, normalized size = 1.08

method	result	size
default	$-3x + x\ln(x) + (1+x)\ln(1+x) - 3 + (2+x)\ln(2+x)$	26
risch	$-3x + x\ln(x) + \ln(1+x)\,x + \ln(1+x) + \ln(2+x)\,x + 2\ln(2+x)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)+ln(1+x)+ln(2+x),x,method=_RETURNVERBOSE)
[Out] -3*x+x*ln(x)+(1+x)*ln(1+x)-3+(2+x)*ln(2+x)
```

Maxima [A]
time = 2.42, size = 25, normalized size = 1.04

$$(x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3\,x - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="maxima")
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

Fricas [A]
time = 0.77, size = 24, normalized size = 1.00

$$(x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="fricas")
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x
```

Sympy [A]
time = 0.69, size = 37, normalized size = 1.54

$$x\log(x) - 3x + \left(x + \frac{1}{2}\right)\log(x+1) + (x+1)\log(x+2) + \frac{\log(x+1)}{2} + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)+ln(1+x)+ln(2+x),x)
```

```
[Out] x*log(x) - 3*x + (x + 1/2)*log(x + 1) + (x + 1)*log(x + 2) + log(x + 1)/2 +
 log(x + 2)
```

Giac [A]
time = 0.46, size = 25, normalized size = 1.04

$$(x+2)\log(x+2)+(x+1)\log(x+1)+x\log(x)-3x-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="giac")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

Mupad [B]
time = 0.44, size = 51, normalized size = 2.12

$$\ln(x+1)-3x+2\ln(x+2)+x\ln(x+1)+x\ln(x)+\frac{\ln(x+2)\left(x^3+3x^2+2x\right)}{(x+1)\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + 1) + log(x + 2) + log(x),x)
```

```
[Out] log(x + 1) - 3*x + 2*log(x + 2) + x*log(x + 1) + x*log(x) + (log(x + 2)*(2*
x + 3*x^2 + x^3))/((x + 1)*(x + 2))
```

3.259 $\int \frac{1}{5+x^3} \, dx$

Optimal. Leaf size=78

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}\,5^{2/3}}+\frac{\log\left(\sqrt[3]{5}+x\right)}{3\,5^{2/3}}-\frac{\log\left(5^{2/3}-\sqrt[3]{5}\,x+x^2\right)}{6\,5^{2/3}}$$

```
[Out] 1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)-1/15*arct
an(1/15*(5^(1/3)-2*x)*5^(2/3)*3^(1/2))*5^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}\,5^{2/3}}-\frac{\log\left(x^2-\sqrt[3]{5}\,x+5^{2/3}\right)}{6\,5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\,5^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(5^(1/3) - 2*x)/(Sqrt[3]*5^(1/3))]/(Sqrt[3]*5^(2/3))) + Log[5^(1/3
) + x]/(3*5^(2/3)) - Log[5^(2/3) - 5^(1/3)*x + x^2]/(6*5^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{5+x^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{5}+x}\,dx}{3\ 5^{2/3}}+\frac{\int \frac{2\sqrt[3]{5}-x}{5^{2/3}-\sqrt[3]{5}\,x+x^2}\,dx}{3\ 5^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{5}+x\right)}{3\ 5^{2/3}}-\frac{\int \frac{-\sqrt[3]{5}+2x}{5^{2/3}-\sqrt[3]{5}\,x+x^2}\,dx}{6\ 5^{2/3}}+\frac{\int \frac{1}{5^{2/3}-\sqrt[3]{5}\,x+x^2}\,dx}{2\sqrt[3]{5}}\\
&= \frac{\log\left(\sqrt[3]{5}+x\right)}{3\ 5^{2/3}}-\frac{\log\left(5^{2/3}-\sqrt[3]{5}\,x+x^2\right)}{6\ 5^{2/3}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{5}}\right)}{5^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\,\sqrt[3]{5}}\right)}{\sqrt{3}\,5^{2/3}}+\frac{\log\left(\sqrt[3]{5}+x\right)}{3\ 5^{2/3}}-\frac{\log\left(5^{2/3}-\sqrt[3]{5}\,x+x^2\right)}{6\ 5^{2/3}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.91

$$\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{-5+2\ 5^{2/3}x}{5\sqrt{3}}\right)+2\log\left(5+5^{2/3}x\right)-\log\left(5-5^{2/3}x+\sqrt[3]{5}\,x^2\right)}{6\ 5^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)^(-1),x]
```

[Out] (2*Sqrt[3]*ArcTan[(-5 + 2*5^(2/3)*x)/(5*Sqrt[3])] + 2*Log[5 + 5^(2/3)*x] - Log[5 - 5^(2/3)*x + 5^(1/3)*x^2])/(6*5^(2/3))

Maple [A]
time = 0.03, size = 54, normalized size = 0.69

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}+5\right)}\frac{\ln\left(-\_R+x\right)}{\_R^{2}}\right)}{3}$	22
default	$\frac{\ln\left(5^{\frac{1}{3}}+x\right)5^{\frac{1}{3}}}{15}-\frac{\ln\left(5^{\frac{2}{3}}-5^{\frac{1}{3}}x+x^{2}\right)5^{\frac{1}{3}}}{30}+\frac{5^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2\,5^{\frac{2}{3}}x}{5}-1\right)}{3}\right)}{15}$	54
meijerg	$\frac{5^{\frac{1}{3}}\left(\frac{x\ln\left(1+\frac{5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{5}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}-\frac{x\ln\left(1-\frac{5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{5}+\frac{5^{\frac{1}{3}}\left(x^{3}\right)^{\frac{2}{3}}}{5}\right)}{2\left(x^{3}\right)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}{10-5^{\frac{2}{3}}\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}\right)}{15}$	96

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3+5),x,method=_RETURNVERBOSE)

[Out] 1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)+1/15*5^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/5*5^(2/3)*x-1))

Maxima [A]
time = 3.61, size = 57, normalized size = 0.73

$$\frac{1}{15}\cdot 5^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{1}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(2\,x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot 5^{\frac{1}{3}}\log\left(x^{2}-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot 5^{\frac{1}{3}}\log\left(x+5^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+5),x, algorithm="maxima")

[Out] 1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5^(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(x + 5^(1/3))

Fricas [A]
time = 0.81, size = 69, normalized size = 0.88

$$\frac{1}{15}\cdot 25^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{1}{75}\cdot 25^{\frac{1}{6}}\left(2\cdot 25^{\frac{2}{3}}\sqrt{3}\,x-5\cdot 25^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{150}\cdot 25^{\frac{2}{3}}\log\left(5\,x^{2}-25^{\frac{2}{3}}x+5\cdot 25^{\frac{1}{3}}\right)+\frac{1}{75}\cdot 25^{\frac{2}{3}}\log\left(5\,x+25^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+5),x, algorithm="fricas")

```
[Out] 1/15*25^(1/6)*sqrt(3)*arctan(1/75*25^(1/6)*(2*25^(2/3)*sqrt(3)*x - 5*25^(1/
3)*sqrt(3))) - 1/150*25^(2/3)*log(5*x^2 - 25^(2/3)*x + 5*25^(1/3)) + 1/75*2
5^(2/3)*log(5*x + 25^(2/3))
```

Sympy [A]
time = 0.13, size = 73, normalized size = 0.94

$$\frac{\sqrt[3]{5}\log\left(x+\sqrt[3]{5}\right)}{15}-\frac{\sqrt[3]{5}\log\left(x^2-\sqrt[3]{5}\,x+5^{\frac{2}{3}}\right)}{30}+\frac{\sqrt{3}\cdot\sqrt[3]{5}\operatorname{atan}\left(\frac{2\sqrt{3}\cdot5^{\frac{2}{3}}x}{15}-\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+5),x)
```

```
[Out] 5**(1/3)*log(x + 5**(1/3))/15 - 5**(1/3)*log(x**2 - 5**(1/3)*x + 5**(2/3))/
30 + sqrt(3)*5**(1/3)*atan(2*sqrt(3)*5**(2/3)*x/15 - sqrt(3)/3)/15
```

Giac [A]
time = 0.45, size = 58, normalized size = 0.74

$$\frac{1}{15}\cdot5^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{1}{15}\cdot5^{\frac{2}{3}}\sqrt{3}\left(2\,x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot5^{\frac{1}{3}}\log\left(x^2-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot5^{\frac{1}{3}}\log\left(\left|x+5^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+5),x, algorithm="giac")
```

```
[Out] 1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5^
(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(abs(x + 5^(1/3)))
```

Mupad [B]
time = 0.27, size = 70, normalized size = 0.90

$$\frac{5^{1/3}\ln\left(x+5^{1/3}\right)}{15}+\frac{5^{1/3}\ln\left(x+\frac{5^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{30}-\frac{5^{1/3}\ln\left(x-\frac{5^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + 5),x)
```

```
[Out] (5^(1/3)*log(x + 5^(1/3)))/15 + (5^(1/3)*log(x + (5^(1/3)*(3^(1/2)*1i - 1))
/2)*(3^(1/2)*1i - 1))/30 - (5^(1/3)*log(x - (5^(1/3)*(3^(1/2)*1i + 1))/2)*(
3^(1/2)*1i + 1))/30
```

3.260 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12 vs. 2(2) = 4.
time = 0.00, size = 12, normalized size = 6.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[1 + x^2]]
```

Maple [A]
time = 0.03, size = 3, normalized size = 1.50

method	result	size
default	$\operatorname{arcsinh}(x)$	3
meijerg	$\operatorname{arcsinh}(x)$	3
trager	$\ln\left(x+\sqrt{x^2+1}\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)
```

Maxima [A]
time = 4.33, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(2) = 4.
time = 0.66, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A]
time = 0.04, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(2) = 4.
time = 0.45, size = 25, normalized size = 12.50

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B]
time = 0.00, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
```

```
[Out] asinh(x)
```

3.261 $\int \sqrt{3+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\sqrt{3+x^2}+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] 3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {201, 221}

$$\frac{1}{2}\sqrt{x^2+3}\,x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + x^2],x]
```

```
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+x^2}\,dx &= \frac{1}{2}x\sqrt{3+x^2}+\frac{3}{2}\int \frac{1}{\sqrt{3+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{3+x^2}+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.15

$$\frac{1}{2}x\sqrt{3+x^2}+\frac{3}{2}\tanh^{-1}\left(\frac{x}{\sqrt{3+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + x^2],x]
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcTanh[x/Sqrt[3 + x^2]])/2
```

Maple [A]
time = 0.05, size = 21, normalized size = 0.78

method	result	size
default	$\frac{3\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2}+\frac{x\sqrt{x^2+3}}{2}$	21
risch	$\frac{3\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)}{2}+\frac{x\sqrt{x^2+3}}{2}$	21
trager	$\frac{x\sqrt{x^2+3}}{2}-\frac{3\ln\left(x-\sqrt{x^2+3}\right)}{2}$	26
meijerg	$-\frac{3\left(-\frac{2\sqrt{\pi}\,x\sqrt{3}\,\sqrt{\frac{x^2}{3}+1}}{3}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{3}}{3}\right)\right)}{4\sqrt{\pi}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)
```

Maxima [A]
time = 3.82, size = 20, normalized size = 0.74

$$\frac{1}{2}\sqrt{x^2+3}\,x+\frac{3}{2}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 3)*x + 3/2*arcsinh(1/3*sqrt(3)*x)
```

Fricas [A]
time = 0.62, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+3}\,x-\frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

Sympy [A]
time = 0.07, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+3}}{2}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)**(1/2),x)
[Out] x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2
```

Giac [A]
time = 0.45, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+3}\,x-\frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

Mupad [B]
time = 0.05, size = 20, normalized size = 0.74

$$\frac{3\operatorname{asinh}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}+\frac{x\sqrt{x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)^(1/2),x)
[Out] (3*asinh((3^(1/2)*x)/3))/2 + (x*(x^2 + 3)^(1/2))/2
```

3.262 $\int \frac{x}{(1+x)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{1+x}+\log(1+x)$$

```
[Out] 1/(1+x)+ln(1+x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {45}

$$\frac{1}{x+1}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)^2} \, dx &= \int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right) \, dx \\ &= \frac{1}{1+x}+\log(1+x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{1+x}+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Maple [A]
time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
norman	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
risch	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
meijerg	$-\frac{x}{1+x}+\ln\left(1+x\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^2,x,method=_RETURNVERBOSE)
[Out] 1/(1+x)+ln(1+x)
```

Maxima [A]
time = 1.82, size = 10, normalized size = 1.00

$$\frac{1}{x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="maxima")
[Out] 1/(x + 1) + log(x + 1)
```

Fricas [A]
time = 0.58, size = 16, normalized size = 1.60

$$\frac{\left(x+1\right)\log\left(x+1\right)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="fricas")
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.80

$$\log\left(x+1\right)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

Giac [A]
time = 0.45, size = 11, normalized size = 1.10

$$\frac{1}{x+1} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1))
```

Mupad [B]
time = 0.03, size = 10, normalized size = 1.00

$$\ln(x+1) + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 1)^2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

3.263 $\int \sin^{-1}(x)\,dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4715, 267}

$$x\text{ArcSin}(x)+\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)\,dx &= x\sin^{-1}(x)-\int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x^2}+x\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x],x]
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Maple [A]
time = 0.00, size = 15, normalized size = 0.94

method	result	size
lookup	$\arcsin\left(x\right)x+\sqrt{-x^{2}+1}$	15
default	$\arcsin\left(x\right)x+\sqrt{-x^{2}+1}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x),x,method=_RETURNVERBOSE)
[Out] arcsin(x)*x+(-x^2+1)^(1/2)
```

Maxima [A]
time = 2.43, size = 14, normalized size = 0.88

$$x\arcsin\left(x\right)+\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="maxima")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Fricas [A]
time = 0.92, size = 14, normalized size = 0.88

$$x\arcsin\left(x\right)+\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="fricas")
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.75

$$x\operatorname{asin}\left(x\right)+\sqrt{1-x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x),x)
[Out] x*asin(x) + sqrt(1 - x**2)
```

Giac [A]
time = 0.45, size = 14, normalized size = 0.88

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="giac")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Mupad [B]
time = 0.22, size = 14, normalized size = 0.88

$$x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x),x)
```

```
[Out] x*asin(x) + (1 - x^2)^(1/2)
```

3.264 $\int x^2 \sin^{-1}(x)\, dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-x^2}}{3}-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\sin^{-1}(x)$$

```
[Out] -1/9*(-x^2+1)^(3/2)+1/3*x^3*arcsin(x)+1/3*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4723, 272, 45}

$$\frac{1}{3}x^3\text{ArcSin}(x)-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sin^{-1}(x)\,dx &= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}\,dx \\
&= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{6}\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right)dx, x, x^2\right) \\
&= \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(\sqrt{1-x^2}\left(2+x^2\right) + 3x^3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x],x]
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

Maple [A]
time = 0.00, size = 34, normalized size = 0.85

method	result	size
default	$\frac{x^3\arcsin(x)}{3} + \frac{x^2\sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

Maxima [A]
time = 4.27, size = 33, normalized size = 0.82

$$\frac{1}{3}x^3\arcsin(x) + \frac{1}{9}\sqrt{-x^2+1}\,x^2 + \frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="maxima")
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

Fricas [A]
time = 0.93, size = 24, normalized size = 0.60

$$\frac{1}{3}\,x^3\arcsin\left(x\right)+\frac{1}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A]
time = 0.09, size = 32, normalized size = 0.80

$$\frac{x^3\operatorname{asin}\left(x\right)}{3}+\frac{x^2\sqrt{1-x^2}}{9}+\frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

Giac [A]
time = 0.50, size = 38, normalized size = 0.95

$$\frac{1}{3}\,\left(x^2-1\right)x\arcsin\left(x\right)+\frac{1}{3}\,x\arcsin\left(x\right)-\frac{1}{9}\,\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{3}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sq
rt(-x^2 + 1)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.60

$$\frac{x^3\,\mathrm{asin}(x)}{3}+\frac{\sqrt{1-x^2}\,\left(x^2+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*asin(x),x)
```

```
[Out] (x^3*asin(x))/3 + ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.265 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

Optimal. Leaf size=21

$$-\log(\cos(x)-\sin(x))+\log(2\cos(x)-\sin(x))$$

```
[Out] -ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))
```

Rubi [A]
time = 0.08, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {630, 31}

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{2-3x+x^2}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \tan(x)\right)-\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \tan(x)\right)\\ &= -\log(1-\tan(x))+\log(2-\tan(x))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.38

$$2\left(-\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{1}{2}\log(2\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] 2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)
```

Maple [A]
time = 0.10, size = 14, normalized size = 0.67

method	result	size
default	$\ln\left(\tan\left(x\right)-2\right)-\ln\left(\tan\left(x\right)-1\right)$	14
risch	$-\ln\left({e}^{2 i x}-i\right)+\ln\left({e}^{2 i x}+\frac{3}{5}-\frac{4 i}{5}\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tan(x)-2)-ln(tan(x)-1)
```

Maxima [A]
time = 3.12, size = 13, normalized size = 0.62

$$-\log\left(\tan\left(x\right)-1\right)+\log\left(\tan\left(x\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

Fricas [A]
time = 0.86, size = 29, normalized size = 1.38

$$\frac{1}{2}\,\log\left(\frac{3}{4}\,\cos\left(x\right)^{2}-\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\right)-\frac{1}{2}\,\log\left(-2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(x\right)}{-3\tan\left(x\right)+\sec^{2}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

Giac [A]
time = 0.44, size = 15, normalized size = 0.71

$$-\log\left(|\tan\left(x\right) - 1|\right) + \log\left(|\tan\left(x\right) - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
[Out] -log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

Mupad [B]
time = 0.81, size = 9, normalized size = 0.43

$$-2\,\mathrm{atanh}(2\tan(x) - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)
[Out] -2*atanh(2*tan(x) - 3)
```

3.266 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A]
time = 1.49, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A]
time = 0.98, size = 10, normalized size = 0.71

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A]
time = 0.45, size = 16, normalized size = 1.14

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Mupad [B]
time = 0.00, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] x/2 + sin(2*x)/4
```

3.267 $\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

```
[Out] -1/x+3*ln(2-x)+2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {907}

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

```
[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx &= \int \left(\frac{3}{-2+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx\\ &= -\frac{1}{x}+3\log(2-x)+2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Maple [A]
time = 0.04, size = 17, normalized size = 0.94

method	result	size
default	$-\frac{1}{x}+2\ln\left(x\right)+3\ln\left(-2+x\right)$	17
norman	$-\frac{1}{x}+2\ln\left(x\right)+3\ln\left(-2+x\right)$	17
risch	$-\frac{1}{x}+2\ln\left(x\right)+3\ln\left(-2+x\right)$	17
meijerg	$-\frac{1}{x}+2\ln\left(x\right)-2\ln\left(2\right)+2i\pi+3\ln\left(1-\frac{x}{2}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2-3*x-2)/(-2+x)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x+2*ln(x)+3*ln(-2+x)
```

Maxima [A]
time = 2.54, size = 16, normalized size = 0.89

$$-\frac{1}{x}+3\,\log\left(x-2\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="maxima")
[Out] -1/x + 3*log(x - 2) + 2*log(x)
```

Fricas [A]
time = 1.46, size = 18, normalized size = 1.00

$$\frac{3\,x\log\left(x-2\right)+2\,x\log\left(x\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="fricas")
[Out] (3*x*log(x - 2) + 2*x*log(x) - 1)/x
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.78

$$2\log\left(x\right)+3\log\left(x-2\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)
[Out] 2*log(x) + 3*log(x - 2) - 1/x
```

Giac [A]
time = 0.45, size = 18, normalized size = 1.00

$$-\frac{1}{x} + 3 \log(|x - 2|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="giac")
[Out] -1/x + 3*log(abs(x - 2)) + 2*log(abs(x))
```

Mupad [B]
time = 0.15, size = 16, normalized size = 0.89

$$3 \ln(x - 2) + 2 \ln(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 5*x^2 + 2)/(x^2*(x - 2)),x)
[Out] 3*log(x - 2) + 2*log(x) - 1/x
```

3.268 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

[Out] 1/2*arcsinh(2/3*x)

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {221}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[9 + 4*x^2],x]

[Out] ArcSinh[(2*x)/3]/2

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 2.00

$$-\frac{1}{2}\log\left(-2x+\sqrt{9+4x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[9 + 4*x^2],x]

[Out] -1/2*Log[-2*x + Sqrt[9 + 4*x^2]]

Maple [A]
time = 0.05, size = 7, normalized size = 0.70

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
trager	$\frac{\ln\left(2x+\sqrt{4x^{2}+9}\right)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Maxima [A]
time = 2.99, size = 6, normalized size = 0.60

$$\frac{1}{2}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.
time = 1.11, size = 16, normalized size = 1.60

$$-\frac{1}{2}\,\log\left(-2\,x+\sqrt{4\,x^{2}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.70

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2+9)**(1/2),x)
```

```
[Out] asinh(2*x/3)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.
time = 0.46, size = 29, normalized size = 2.90

$$\frac{1}{2}\sqrt{4x^2+9}\,x - \frac{9}{4}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B]
time = 0.04, size = 6, normalized size = 0.60

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 9)^(1/2),x)
```

```
[Out] asinh((2*x)/3)/2
```

3.269 $\int \frac{1}{\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] arcsinh(1/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {221}

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + x^2],x]
```

```
[Out] ArcSinh[x/2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{2}\right)$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 2.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{4+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[4 + x^2]]
```

Maple [A]
time = 0.05, size = 5, normalized size = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{x}{2}\right)$	5
meijerg	$\operatorname{arcsinh}\left(\frac{x}{2}\right)$	5
trager	$\ln\left(x+\sqrt{x^2+4}\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(1/2*x)
```

Maxima [A]
time = 3.49, size = 4, normalized size = 0.67

$$\operatorname{arsinh}\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.
time = 1.03, size = 14, normalized size = 2.33

$$-\log\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4))
```

Sympy [A]
time = 0.05, size = 3, normalized size = 0.50

$$\operatorname{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)**(1/2),x)
```

```
[Out] asinh(x/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(4) = 8.
time = 0.44, size = 25, normalized size = 4.17

$$\frac{1}{2}\sqrt{x^2+4}\,x-2\log\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 4)*x - 2*log(-x + sqrt(x^2 + 4))
```

Mupad [B]
time = 0.03, size = 4, normalized size = 0.67

$$\operatorname{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 4)^(1/2),x)
```

```
[Out] asinh(x/2)
```

3.270 $\int \frac{1}{10-12x+9x^2}\,dx$

Optimal. Leaf size=21

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

```
[Out] -1/18*arctan(1/6*(2-3*x)*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {632, 210}

$$-\frac{\text{ArcTan}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(10 - 12*x + 9*x^2)^(-1),x]
```

```
[Out] -1/3*ArcTan[(2 - 3*x)/Sqrt[6]]/Sqrt[6]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{10-12x+9x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-216-x^2}\,dx, x, -12+18x\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-2+3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(10 - 12*x + 9*x^2)^(-1),x]
[Out] ArcTan[(-2 + 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Maple [A]
time = 0.16, size = 17, normalized size = 0.81

method	result	size
default	$\frac{\sqrt{6}\,\arctan\left(\frac{(18x-12)\sqrt{6}}{36}\right)}{18}$	17
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{(-2+3x)\sqrt{6}}{6}\right)}{18}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2-12*x+10),x,method=_RETURNVERBOSE)
[Out] 1/18*6^(1/2)*arctan(1/36*(18*x-12)*6^(1/2))
```

Maxima [A]
time = 2.65, size = 16, normalized size = 0.76

$$\frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="maxima")
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

Fricas [A]
time = 0.93, size = 16, normalized size = 0.76

$$\frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

Sympy [A]
time = 0.04, size = 22, normalized size = 1.05

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}-\frac{\sqrt{6}}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2-12*x+10),x)
```

```
[Out] sqrt(6)*atan(sqrt(6)*x/2 - sqrt(6)/3)/18
```

Giac [A]
time = 0.44, size = 16, normalized size = 0.76

$$\frac{1}{18}\,\sqrt{6}\,\arctan\left(\frac{1}{6}\,\sqrt{6}\,(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

Mupad [B]
time = 0.14, size = 16, normalized size = 0.76

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,(3\,x-2)}{6}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2 - 12*x + 10),x)
```

```
[Out] (6^(1/2)*atan((6^(1/2)*(3*x - 2))/6))/18
```

3.271 $\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx$

Optimal. Leaf size=53

$$\frac{1}{2(1-x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{4}\log\left(1+x^2\right)$$

```
[Out] 1/2/(1-x)-1/3/x^3-1/x^2-2/x-5/2*ln(1-x)+2*ln(x)+1/4*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {2083, 266}

$$-\frac{1}{3x^3}-\frac{1}{x^2}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Lo
g[1 + x^2]/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx &= \int \left(\frac{1}{2(-1+x)^2}-\frac{5}{2(-1+x)}+\frac{1}{x^4}+\frac{2}{x^3}+\frac{2}{x^2}+\frac{2}{x}+\frac{x}{2\left(1+x^2\right)}\right)dx\\
&= \frac{1}{2(1-x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{2(1-x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.96

$$-\frac{1}{2(-1+x)}-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)+\frac{1}{4}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] -1/2*1/(-1 + x) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] +
Log[1 + x^2]/4
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.79

method	result	size
default	$-\frac{1}{3x^3}-\frac{1}{x^2}-\frac{2}{x}+2\ln(x)-\frac{1}{2(-1+x)}-\frac{5\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}$	42
norman	$\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)}+2\ln(x)-\frac{5\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}$	42
risch	$\frac{\frac{1}{3}+x^2-\frac{5}{2}x^3+\frac{2}{3}x}{x^3(-1+x)}+2\ln(x)-\frac{5\ln(-1+x)}{2}+\frac{\ln(x^2+1)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3-1/x^2-2/x+2*ln(x)-1/2/(-1+x)-5/2*ln(-1+x)+1/4*ln(x^2+1)
```

Maxima [A]
time = 2.90, size = 47, normalized size = 0.89

$$-\frac{15\,x^3-6\,x^2-4\,x-2}{6\,(x^4-x^3)}+\frac{1}{4}\,\log\left(x^2+1\right)-\frac{5}{2}\,\log\left(x-1\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="maxima")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/(x^4 - x^3) + 1/4*log(x^2 + 1) - 5/2*log(x
- 1) + 2*log(x)
```

Fricas [A]
time = 1.14, size = 73, normalized size = 1.38

$$-\frac{30\,x^3-12\,x^2-3\,(x^4-x^3)\log\left(x^2+1\right)+30\,(x^4-x^3)\log\left(x-1\right)-24\,(x^4-x^3)\log\left(x\right)-8\,x-4}{12\,(x^4-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="fricas")
```

```
[Out] -1/12*(30*x^3 - 12*x^2 - 3*(x^4 - x^3)*log(x^2 + 1) + 30*(x^4 - x^3)*log(x
- 1) - 24*(x^4 - x^3)*log(x) - 8*x - 4)/(x^4 - x^3)
```

Sympy [A]
time = 0.07, size = 46, normalized size = 0.87

$$2\log(x) - \frac{5\log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{-15x^3+6x^2+4x+2}{6x^4-6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/2 + log(x**2 + 1)/4 + (-15*x**3 + 6*x**2 + 4*x + 2)
/(6*x**4 - 6*x**3)
```

Giac [A]
time = 0.46, size = 46, normalized size = 0.87

$$-\frac{15\,x^3-6\,x^2-4\,x-2}{6\,(x-1)x^3} + \frac{1}{4}\,\log\left(x^2+1\right) - \frac{5}{2}\,\log\left(|x-1|\right) + 2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="giac")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/((x - 1)*x^3) + 1/4*log(x^2 + 1) - 5/2*log(
abs(x - 1)) + 2*log(abs(x))
```

Mupad [B]
time = 0.06, size = 45, normalized size = 0.85

$$\frac{\ln\left(x^2+1\right)}{4} - \frac{5\,\ln\left(x-1\right)}{2} + 2\,\ln\left(x\right) - \frac{-\frac{5\,x^3}{2}+x^2+\frac{2\,x}{3}+\frac{1}{3}}{x^3-x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8),x)
```

```
[Out] log(x^2 + 1)/4 - (5*log(x - 1))/2 + 2*log(x) - ((2*x)/3 + x^2 - (5*x^3)/2 +
 1/3)/(x^3 - x^4)
```

3.272 $\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx$

Optimal. Leaf size=49

$$ax+\frac{1}{12}(27a+9b+3c+d)\log(3-x)-\frac{1}{3}d\log(x)-\frac{1}{4}(a-b+c-d)\log(1+x)$$

```
[Out] a*x+1/12*(27*a+9*b+3*c+d)*ln(3-x)-1/3*d*ln(x)-1/4*(a-b+c-d)*ln(1+x)
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.034$, Rules used = $\{1626\}$

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a - b + c -
 d)*Log[1 + x])/4
```

Rule 1626

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx &= \int \left(a+\frac{27a+9b+3c+d}{12(-3+x)}-\frac{d}{3x}+\frac{-a+b-c+d}{4(1+x)}\right)dx\\ &= ax+\frac{1}{12}(27a+9b+3c+d)\log(3-x)-\frac{1}{3}d\log(x)-\frac{1}{4}(a-b+c-d)\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.00

$$ax+\frac{1}{12}(27a+9b+3c+d)\log(3-x)-\frac{1}{3}d\log(x)+\frac{1}{4}(-a+b-c+d)\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 + ((-a + b - c
+ d)*Log[1 + x])/4
```

Maple [A]
time = 0.04, size = 46, normalized size = 0.94

method	result
default	$ax+\left(\frac{9a}{4}+\frac{3b}{4}+\frac{c}{4}+\frac{d}{12}\right)\ln\left(-3+x\right)-\frac{d\ln\left(x\right)}{3}+\left(-\frac{a}{4}+\frac{b}{4}-\frac{c}{4}+\frac{d}{4}\right)\ln\left(1+x\right)$
norman	$ax+\left(\frac{9a}{4}+\frac{3b}{4}+\frac{c}{4}+\frac{d}{12}\right)\ln\left(-3+x\right)-\frac{d\ln\left(x\right)}{3}+\left(-\frac{a}{4}+\frac{b}{4}-\frac{c}{4}+\frac{d}{4}\right)\ln\left(1+x\right)$
risch	$ax+\frac{9\ln(3-x)a}{4}+\frac{3\ln(3-x)b}{4}+\frac{\ln(3-x)c}{4}+\frac{\ln(3-x)d}{12}-\frac{d\ln(x)}{3}-\frac{\ln(-1-x)a}{4}+\frac{\ln(-1-x)b}{4}-\frac{\ln(-1-x)c}{4}+\frac{\ln(-1-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+(9/4*a+3/4*b+1/4*c+1/12*d)*ln(-3+x)-1/3*d*ln(x)+(-1/4*a+1/4*b-1/4*c+1/4
*d)*ln(1+x)
```

Maxima [A]
time = 2.25, size = 41, normalized size = 0.84

$$ax-\frac{1}{4}\left(a-b+c-d\right)\log\left(x+1\right)+\frac{1}{12}\left(27\,a+9\,b+3\,c+d\right)\log\left(x-3\right)-\frac{1}{3}\,d\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="maxima")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
3) - 1/3*d*log(x)
```

Fricas [A]
time = 1.28, size = 41, normalized size = 0.84

$$ax-\frac{1}{4}\left(a-b+c-d\right)\log\left(x+1\right)+\frac{1}{12}\left(27\,a+9\,b+3\,c+d\right)\log\left(x-3\right)-\frac{1}{3}\,d\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="fricas")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
3) - 1/3*d*log(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(49) = 98$.
time = 56.79, size = 762, normalized size = 15.55

$$ax-\frac{d\log\left(x\right)}{3}-\frac{\left(a-b+c-d\right)\log\left(x+\frac{-1512a^2d+1134a^2(a-b+c-d)-864abd+648ab(a-b+c-d)-432acd+324ac(a-b+c-d)-144ad^2+81a(a-b+c-d)^2-216b^2d+162b^2(a-b+c-d)-288bd^2+108bd(a-b+c-d)+81b(a-b+c-d)^2-72c^2d+54c^2(a-b+c-d)+144cd^2-72cd(a-b+c-d)-27c(a-b+c-d)^2-136d^3-54d^2(a-b+c-d)+117d(a-b+c-d)^2}{1215a^3-567a^2b+1593a^2c-2691a^2d-567ab^2+378abc-1638abd+405ac^2-702acd-351ad^2-81b^3-27b^2c-207b^2d+81bc^2-270bcd-27bd^2+27c^3-27c^2d-99cd^2+35d^3}\right)}{4}+\frac{\left(27a+9b+3c+d\right)\log\left(x+\frac{-1512a^2d-378a^2\cdot(27a+9b+3c+d)-864abd-216ab(27a+9b+3c+d)-432acd-108ac(27a+9b+3c+d)-144ad^2+9a(27a+9b+3c+d)^2-216b^2d-54b^2\cdot(27a+9b+3c+d)-288bd^2-36bd(27a+9b+3c+d)+9b(27a+9b+3c+d)^2-72c^2d-18c^2\cdot(27a+9b+3c+d)+144cd^2+24cd(27a+9b+3c+d)-3c(27a+9b+3c+d)^2-136d^3+18d^2\cdot(27a+9b+3c+d)+13d(27a+9b+3c+d)^2}{1215a^3-567a^2b+1593a^2c-2691a^2d-567ab^2+378abc-1638abd+405ac^2-702acd-351ad^2-81b^3-27b^2c-207b^2d+81bc^2-270bcd-27bd^2+27c^3-27c^2d-99cd^2+35d^3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] a*x - d*log(x)/3 - (a - b + c - d)*log(x + (-1512*a**2*d + 1134*a**2*(a - b
 + c - d) - 864*a*b*d + 648*a*b*(a - b + c - d) - 432*a*c*d + 324*a*c*(a -
b + c - d) - 144*a*d**2 + 81*a*(a - b + c - d)**2 - 216*b**2*d + 162*b**2*(
a - b + c - d) - 288*b*d**2 + 108*b*d*(a - b + c - d) + 81*b*(a - b + c - d
)**2 - 72*c**2*d + 54*c**2*(a - b + c - d) + 144*c*d**2 - 72*c*d*(a - b + c
 - d) - 27*c*(a - b + c - d)**2 - 136*d**3 - 54*d**2*(a - b + c - d) + 117*
d*(a - b + c - d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d -
 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2
- 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27
*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3))/4 + (27*a + 9*b + 3*c + d)*log(x
+ (-1512*a**2*d - 378*a**2*(27*a + 9*b + 3*c + d) - 864*a*b*d - 216*a*b*(27
*a + 9*b + 3*c + d) - 432*a*c*d - 108*a*c*(27*a + 9*b + 3*c + d) - 144*a*d*
*2 + 9*a*(27*a + 9*b + 3*c + d)**2 - 216*b**2*d - 54*b**2*(27*a + 9*b + 3*c
 + d) - 288*b*d**2 - 36*b*d*(27*a + 9*b + 3*c + d) + 9*b*(27*a + 9*b + 3*c
+ d)**2 - 72*c**2*d - 18*c**2*(27*a + 9*b + 3*c + d) + 144*c*d**2 + 24*c*d*
(27*a + 9*b + 3*c + d) - 3*c*(27*a + 9*b + 3*c + d)**2 - 136*d**3 + 18*d**2
*(27*a + 9*b + 3*c + d) + 13*d*(27*a + 9*b + 3*c + d)**2)/(1215*a**3 - 567*
a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d +
405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81
*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3
))/12
```

Giac [A]
time = 0.47, size = 44, normalized size = 0.90

$$ax-\frac{1}{4}\left(a-b+c-d\right)\log\left(|x+1|\right)+\frac{1}{12}\left(27\,a+9\,b+3\,c+d\right)\log\left(|x-3|\right)-\frac{1}{3}\,d\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="giac")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(abs(x + 1)) + 1/12*(27*a + 9*b + 3*c + d)*log
(abs(x - 3)) - 1/3*d*log(abs(x))
```

Mupad [B]
time = 0.21, size = 46, normalized size = 0.94

$$\ln\left(x-3\right)\,\left(\frac{9\,a}{4}+\frac{3\,b}{4}+\frac{c}{4}+\frac{d}{12}\right)-\ln\left(x+1\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{c}{4}-\frac{d}{4}\right)+a\,x-\frac{d\,\ln\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + c*x + a*x^3 + b*x^2)/(x*(x + 1)*(x - 3)),x)
```

```
[Out] log(x - 3)*((9*a)/4 + (3*b)/4 + c/4 + d/12) - log(x + 1)*(a/4 - b/4 + c/4 -
 d/4) + a*x - (d*log(x))/3
```

3.273 $\int \frac{1}{(2-\log(1+x^2))^5} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{(2-\log(1+x^2))^5},x\right)$$

```
[Out] Unintegrable(1/(2-ln(x^2+1))^5,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(2-\log(1+x^2))^5} dx$$

Verification is not applicable to the result.

```
[In] Int[(2 - Log[1 + x^2])^(-5),x]
[Out] Defer[Int][(2 - Log[1 + x^2])^(-5), x]
```

Rubi steps

$$\int \frac{1}{(2-\log(1+x^2))^5} dx = \int \frac{1}{(2-\log(1+x^2))^5} dx$$

Mathematica [A]
time = 2.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(2-\log(1+x^2))^5} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 - Log[1 + x^2])^(-5),x]
[Out] Integrate[(2 - Log[1 + x^2])^(-5), x]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(2-\ln(x^2+1))^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-ln(x^2+1))^5,x)
```

```
[Out] int(1/(2-ln(x^2+1))^5,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="maxima")
```

```
[Out] 1/384*(32*x^8 + 56*x^6 + 120*x^4 + (x^8 - 10*x^4 - 24*x^2 - 15)*log(x^2 + 1
)^3 - 2*(2*x^8 - x^6 - 33*x^4 - 75*x^2 - 45)*log(x^2 + 1)^2 + 216*x^2 + 4*(
3*x^8 - 2*x^6 - 38*x^4 - 78*x^2 - 45)*log(x^2 + 1) + 120)/(x^7*log(x^2 + 1)
^4 - 8*x^7*log(x^2 + 1)^3 + 24*x^7*log(x^2 + 1)^2 - 32*x^7*log(x^2 + 1) + 1
6*x^7) - integrate(1/384*(x^8 + 30*x^4 + 120*x^2 + 105)/(x^8*log(x^2 + 1) -
 2*x^8), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="fricas")
```

```
[Out] integral(-1/(log(x^2 + 1)^5 - 10*log(x^2 + 1)^4 + 40*log(x^2 + 1)^3 - 80*lo
g(x^2 + 1)^2 + 80*log(x^2 + 1) - 32), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{120x^2}{x^8\log(x^2+1)-2x^8}\,dx+\int \frac{30x^4}{x^8\log(x^2+1)-2x^8}\,dx+\int \frac{x^8}{x^8\log(x^2+1)-2x^8}\,dx+\int \frac{105}{x^8\log(x^2+1)-2x^8}\,dx}{384}+\frac{\frac{2x^8}{3}+\frac{7x^6}{6}+\frac{5x^4}{2}+\frac{9x^2}{2}+\left(\frac{x^8}{48}-\frac{5x^4}{24}-\frac{x^2}{2}-\frac{5}{16}\right)\log(x^2+1)^3+\left(-\frac{x^8}{12}+\frac{x^6}{24}+\frac{11x^4}{8}+\frac{25x^2}{8}+\frac{15}{8}\right)\log(x^2+1)^2+\left(\frac{x^8}{4}-\frac{x^6}{6}-\frac{19x^4}{6}-\frac{13x^2}{2}-\frac{15}{4}\right)\log(x^2+1)+\frac{5}{2}}{8x^7\log(x^2+1)^4-64x^7\log(x^2+1)^3+192x^7\log(x^2+1)^2-256x^7\log(x^2+1)+128x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-ln(x**2+1))**5,x)
```

```
[Out] -(Integral(120*x**2/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(30*x**4/(x
**8*log(x**2 + 1) - 2*x**8), x) + Integral(x**8/(x**8*log(x**2 + 1) - 2*x**
8), x) + Integral(105/(x**8*log(x**2 + 1) - 2*x**8), x))/384 + (2*x**8/3 +
7*x**6/6 + 5*x**4/2 + 9*x**2/2 + (x**8/48 - 5*x**4/24 - x**2/2 - 5/16)*log(
x**2 + 1)**3 + (-x**8/12 + x**6/24 + 11*x**4/8 + 25*x**2/8 + 15/8)*log(x**2
 + 1)**2 + (x**8/4 - x**6/6 - 19*x**4/6 - 13*x**2/2 - 15/4)*log(x**2 + 1) +
 5/2)/(8*x**7*log(x**2 + 1)**4 - 64*x**7*log(x**2 + 1)**3 + 192*x**7*log(x*
*2 + 1)**2 - 256*x**7*log(x**2 + 1) + 128*x**7)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="giac")
[Out] integrate(-1/(log(x^2 + 1) - 2)^5, x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.07

$$\int -\frac{1}{\left(\ln\left(x^2+1\right)-2\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(log(x^2 + 1) - 2)^5,x)
[Out] int(-1/(log(x^2 + 1) - 2)^5, x)
```

3.274 $\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)\,dx$

Optimal. Leaf size=28

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

```
[Out] exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

Rubi [A]
time = 0.15, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {2241, 2240, 2634, 12, 2627, 6874, 2618}

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2241

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2618

```
Int[Log[(c_.)*(x_)^(n_.)]^(r_.)/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (
a_.)*(x_)^(m_.))), x_Symbol] :> Simp[Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q), x
```

```
] - Dist[a*(m/(b*n*q)), Int[x^(m - 1)/(a*x^m + b*Log[c*x^n]^q), x], x] /; F
reeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, q - 1]
```

Rule 2627

```
Int[(Log[(c_.)*(x_)^(n_.)]*(e_.) + (d_))/(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.)
+ (a_.)*(x_))^2, x_Symbol] :> Simp[(-e)*(Log[c*x^n]/(a*(a*x + b*Log[c*x^n]^
q))), x] + Dist[(d + e*n)/a, Int[1/(x*(a*x + b*Log[c*x^n]^q)), x], x] /; Fr
eeQ[{a, b, c, d, e, n, q}, x] && EqQ[d + e*n*q, 0]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx = 2\int e^{x^2}x\log(x)\,dx+\int\frac{e^{x^2}}{x}\,dx+\int\frac{-2}{(x+\cdots}$$

$$=\frac{\mathrm{Ei}(x^2)}{2}+e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}-2\int\cdots$$

$$=\frac{\mathrm{Ei}(x^2)}{2}+e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+2\int\cdots$$

$$=e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2\cdots\right.$$

Mathematica [A]
time = 40.00, size = 28, normalized size = 1.00

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1
+ x^(-1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Maple [A]
time = 0.19, size = 28, normalized size = 1.00

method	result	size
default	$e^{x^2}\ln\left(x\right)-\frac{\ln\left(x\right)}{x+\ln\left(x\right)^{2}}+\ln\left(x+\ln\left(x\right)^{2}\right)$	28
risch	$e^{x^2}\ln\left(x\right)-\frac{\ln\left(x\right)}{x+\ln\left(x\right)^{2}}+\ln\left(x+\ln\left(x\right)^{2}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)/x+2*exp(x^2)*x*ln(x)+(-2+ln(x))/(x+ln(x)^2)^2+(1+1/x+2*ln(x)/x
)/(x+ln(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

Maxima [A]
time = 3.93, size = 27, normalized size = 0.96

$$e^{\left(x^{2}\right)}\log\left(x\right)-\frac{\log\left(x\right)}{\log\left(x\right)^{2}+x}+\log\left(\log\left(x\right)^{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="maxima")
```

```
[Out] e^(x^2)*log(x) - log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

Fricas [A]
time = 0.85, size = 44, normalized size = 1.57

$$\frac{e^{\left(x^{2}\right)}\log\left(x\right)^{3}+\left(\log\left(x\right)^{2}+x\right)\log\left(\log\left(x\right)^{2}+x\right)+\left(xe^{\left(x^{2}\right)}-1\right)\log\left(x\right)}{\log\left(x\right)^{2}+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="fricas")
```

```
[Out] (e^(x^2)*log(x)^3 + (log(x)^2 + x)*log(log(x)^2 + x) + (x*e^(x^2) - 1)*log(
x))/(log(x)^2 + x)
```

Sympy [A]
time = 0.16, size = 26, normalized size = 0.93

$$e^{x^{2}}\log\left(x\right)+\log\left(x+\log\left(x\right)^{2}\right)-\frac{\log\left(x\right)}{x+\log\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1/x
+2*ln(x)/x)/(x+ln(x)**2),x)
```

```
[Out] exp(x**2)*log(x) + log(x + log(x)**2) - log(x)/(x + log(x)**2)
```

Giac [A]
time = 0.44, size = 27, normalized size = 0.96

$$e^{(x^2)}\log(x) - \frac{3\log(x)}{\log(x)^2 + x} + \log\left(\log(x)^2 + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="giac")
```

```
[Out] e^(x^2)*log(x) - 3*log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

Mupad [B]
time = 0.34, size = 27, normalized size = 0.96

$$\ln\left(\ln(x)^2 + x\right) + e^{x^2}\,\ln(x) - \frac{\ln(x)}{\ln(x)^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) - 2)/(x + log(x)^2)^2 + ((2*log(x))/x + 1/x + 1)/(x + log(x)^2)
 + exp(x^2)/x + 2*x*exp(x^2)*log(x),x)
```

```
[Out] log(x + log(x)^2) + exp(x^2)*log(x) - log(x)/(x + log(x)^2)
```

3.275 $\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$

Optimal. Leaf size=199

$$\frac{24e^{\frac{x}{2}+xz}\pi^4x^3}{64\pi^4+20\pi^2x^2+x^4}-\frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4}+\frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4}-\frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi z)}{16\pi^2+x^2}+\frac{e^{\frac{x}{2}}}{}$$

```
[Out] 24*exp(1/2*x+x*z)*Pi^4*x^3/(64*Pi^4+20*Pi^2*x^2+x^4)-24*exp(1/2*x+x*z)*Pi^3
*x^4*cos(Pi*z)*sin(Pi*z)/(64*Pi^4+20*Pi^2*x^2+x^4)+12*exp(1/2*x+x*z)*Pi^2*x
^5*sin(Pi*z)^2/(64*Pi^4+20*Pi^2*x^2+x^4)-4*exp(1/2*x+x*z)*Pi*x^4*cos(Pi*z)*
sin(Pi*z)^3/(16*Pi^2+x^2)+exp(1/2*x+x*z)*x^5*sin(Pi*z)^4/(16*Pi^2+x^2)
```

Rubi [A]
time = 0.07, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 4519, 2225}

$$\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\sin(\pi z)\cos(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4}+\frac{24\pi^4x^3e^{xz+\frac{x}{2}}}{x^4+20\pi^2x^2+64\pi^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

```
[Out] (24*E^(x/2 + x*z)*Pi^4*x^3)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (24*E^(x/2 + x*
z)*Pi^3*x^4*Cos[Pi*z]*Sin[Pi*z])/(64*Pi^4 + 20*Pi^2*x^2 + x^4) + (12*E^(x/2
 + x*z)*Pi^2*x^5*Sin[Pi*z]^2)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (4*E^(x/2 + x
*z)*Pi*x^4*Cos[Pi*z]*Sin[Pi*z]^3)/(16*Pi^2 + x^2) + (E^(x/2 + x*z)*x^5*Sin[
Pi*z]^4)/(16*Pi^2 + x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int e^{\frac{x}{2}+xz} x^4 \sin^4(\pi z)\,dz &= x^4 \int e^{\frac{x}{2}+xz} \sin^4(\pi z)\,dz \\
&= -\frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi z)}{16\pi^2+x^2} + \frac{e^{\frac{x}{2}+xz}x^5\sin^4(\pi z)}{16\pi^2+x^2} + \frac{(12\pi^2x^4)\int e^{\frac{x}{2}+xz}\sin^2(\pi z)\,dz}{16\pi^2+x^2} \\
&= -\frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4} + \frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4} - \frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3}{16\pi^2+x^2} \\
&= \frac{24e^{\frac{x}{2}+xz}\pi^4x^3}{64\pi^4+20\pi^2x^2+x^4} - \frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4} + \frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4} - \frac{4e}{}
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 136, normalized size = 0.68

$$
\frac{e^{x\left(\frac{1}{2}+z\right)}x^4(192\pi^4+60\pi^2x^2+3x^4-4x^2(16\pi^2+x^2)\cos(2\pi z)+x^2(4\pi^2+x^2)\cos(4\pi z)-128\pi^3x\sin(2\pi z)-8\pi x^3\sin(2\pi z)+16\pi^3x\sin(4\pi z)+4\pi x^3\sin(4\pi z))}{8\,(64\pi^4x+20\pi^2x^3+x^5)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

```
[Out] (E^(x*(1/2 + z))*x^4*(192*Pi^4 + 60*Pi^2*x^2 + 3*x^4 - 4*x^2*(16*Pi^2 + x^2
)*Cos[2*Pi*z] + x^2*(4*Pi^2 + x^2)*Cos[4*Pi*z] - 128*Pi^3*x*Sin[2*Pi*z] - 8
*Pi*x^3*Sin[2*Pi*z] + 16*Pi^3*x*Sin[4*Pi*z] + 4*Pi*x^3*Sin[4*Pi*z]))/(8*(64
*Pi^4*x + 20*Pi^2*x^3 + x^5))
```

Maple [A]
time = 0.18, size = 128, normalized size = 0.64

method	result
default	$-\frac{x^4\left(-\frac{3\,e^{\frac{1}{2}x+xz}}{x}-\frac{x\,e^{\frac{1}{2}x+xz}\cos(4\pi z)}{16\pi^2+x^2}-\frac{4\pi\,e^{\frac{1}{2}x+xz}\sin(4\pi z)}{16\pi^2+x^2}+\frac{4x\,e^{\frac{1}{2}x+xz}\cos(2\pi z)}{4\pi^2+x^2}+\frac{8\pi\,e^{\frac{1}{2}x+xz}\sin(2\pi z)}{4\pi^2+x^2}\right)}{8}$
risch	$\frac{3x^3\mathrm{e}^{\frac{x(1+2z)}{2}}}{8}+\frac{x^5\mathrm{e}^{\frac{1}{2}x+xz}\cos(4\pi z)}{128\pi^2+8x^2}+\frac{x^4\mathrm{e}^{\frac{1}{2}x+xz}\pi\sin(4\pi z)}{32\pi^2+2x^2}-\frac{x^5\mathrm{e}^{\frac{1}{2}x+xz}\cos(2\pi z)}{2(4\pi^2+x^2)}-\frac{x^4\mathrm{e}^{\frac{1}{2}x+xz}\pi\sin(2\pi z)}{4\pi^2+x^2}$
norman	$\frac{\frac{24\,e^{\frac{1}{2}x+xz}\pi^4x^3}{64\pi^4+20\pi^2x^2+x^4}-\frac{48\pi^3x^4e^{\frac{1}{2}x+xz}\tan\left(\frac{\pi z}{2}\right)}{64\pi^4+20\pi^2x^2+x^4}+\frac{48\pi^3x^4e^{\frac{1}{2}x+xz}\left(\tan^7\left(\frac{\pi z}{2}\right)\right)}{64\pi^4+20\pi^2x^2+x^4}+\frac{16\left(9\pi^4+10\pi^2x^2+x^4\right)x^3e^{\frac{1}{2}x+xz}\left(\tan^4\left(\frac{\pi z}{2}\right)\right)}{64\pi^4+20\pi^2x^2+x^4}+\frac{24\,e^{\frac{1}{2}x+xz}\pi^4x^3}{64\pi^4+20\pi}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(1/2*x+x*z)*sin(Pi*z)^4,z,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^4*(-3*exp(1/2*x+x*z)/x-x/(16*Pi^2+x^2)*exp(1/2*x+x*z)*cos(4*Pi*z)-4*
Pi/(16*Pi^2+x^2)*exp(1/2*x+x*z)*sin(4*Pi*z)+4*x/(4*Pi^2+x^2)*exp(1/2*x+x*z)
*cos(2*Pi*z)+8*Pi/(4*Pi^2+x^2)*exp(1/2*x+x*z)*sin(2*Pi*z))
```

Maxima [A]
time = 5.23, size = 160, normalized size = 0.80

$$\frac{\left((4\,\pi^2x^2+x^4)\cos(4\,\pi z)\,e^{\left(xz+\frac{1}{2}\,x\right)}-4\,(16\,\pi^2x^2+x^4)\cos(2\,\pi z)\,e^{\left(xz+\frac{1}{2}\,x\right)}+4\,(4\,\pi^3x+\pi x^3)e^{\left(xz+\frac{1}{2}\,x\right)}\sin(4\,\pi z)-8\,(16\,\pi^3x+\pi x^3)e^{\left(xz+\frac{1}{2}\,x\right)}\sin(2\,\pi z)+3\,(64\,\pi^4+20\,\pi^2x^2+x^4)e^{\left(xz+\frac{1}{2}\,x\right)}\right)x^4}{8\,(64\,\pi^4x+20\,\pi^2x^3+x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="maxima")
```

```
[Out] 1/8*((4*pi^2*x^2 + x^4)*cos(4*pi*z)*e^(x*z + 1/2*x) - 4*(16*pi^2*x^2 + x^4)
*cos(2*pi*z)*e^(x*z + 1/2*x) + 4*(4*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(4*
pi*z) - 8*(16*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(2*pi*z) + 3*(64*pi^4 + 2
0*pi^2*x^2 + x^4)*e^(x*z + 1/2*x))*x^4/(64*pi^4*x + 20*pi^2*x^3 + x^5)
```

Fricas [A]
time = 0.76, size = 145, normalized size = 0.73

$$\frac{4\left((4\,\pi^3x^4+\pi x^6)\cos(\pi z)^3-(10\,\pi^3x^4+\pi x^6)\cos(\pi z)\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin(\pi z)+\left(24\,\pi^4x^3+16\,\pi^2x^5+x^7+(4\,\pi^2x^5+x^7)\cos(\pi z)^4-2\,(10\,\pi^2x^5+x^7)\cos(\pi z)^2\right)e^{\left(xz+\frac{1}{2}\,x\right)}}{64\,\pi^4+20\,\pi^2x^2+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="fricas")
```

```
[Out] (4*((4*pi^3*x^4 + pi*x^6)*cos(pi*z)^3 - (10*pi^3*x^4 + pi*x^6)*cos(pi*z))*e
^(x*z + 1/2*x)*sin(pi*z) + (24*pi^4*x^3 + 16*pi^2*x^5 + x^7 + (4*pi^2*x^5 +
 x^7)*cos(pi*z)^4 - 2*(10*pi^2*x^5 + x^7)*cos(pi*z)^2)*e^(x*z + 1/2*x))/(64
*pi^4 + 20*pi^2*x^2 + x^4)
```

Sympy [A]
time = 147.27, size = 1277, normalized size = 6.42

$$x^4\left(\begin{cases}\frac{3z\sin^4(\pi z)}{8}+\frac{3z\sin^2(\pi z)\cos^2(\pi z)}{4}+\frac{3z\cos^4(\pi z)}{8}-\frac{5\sin^3(\pi z)\cos(\pi z)}{8\pi}-\frac{3\sin(\pi z)\cos^3(\pi z)}{8\pi} & \text{for}\ x=0\\ \frac{ze^{-4i\pi z}\sin^4(\pi z)}{16}-\frac{ize^{-4i\pi z}\sin^3(\pi z)\cos(\pi z)}{4}-\frac{3ze^{-4i\pi z}\sin^2(\pi z)\cos^2(\pi z)}{8}+\frac{ize^{-4i\pi z}\sin(\pi z)\cos^3(\pi z)}{4}+\frac{ze^{-4i\pi z}\cos^4(\pi z)}{16}+\frac{7ie^{-4i\pi z}\sin^4(\pi z)}{24\pi}+\frac{11e^{-4i\pi z}\sin^3(\pi z)\cos(\pi z)}{48\pi}+\frac{5e^{-4i\pi z}\sin(\pi z)\cos^3(\pi z)}{48\pi}-\frac{ie^{-4i\pi z}\cos^4(\pi z)}{24\pi} & \text{for}\ x=-4i\pi\\ -\frac{ze^{-2i\pi z}\sin^4(\pi z)}{4}+\frac{ize^{-2i\pi z}\sin^3(\pi z)\cos(\pi z)}{2}+\frac{ize^{-2i\pi z}\sin(\pi z)\cos^3(\pi z)}{2}+\frac{ze^{-2i\pi z}\cos^4(\pi z)}{4}-\frac{5ie^{-2i\pi z}\sin^4(\pi z)}{24\pi}+\frac{e^{-2i\pi z}\sin^3(\pi z)\cos(\pi z)}{3\pi}-\frac{ie^{-2i\pi z}\sin^2(\pi z)\cos^2(\pi z)}{2\pi}-\frac{ie^{-2i\pi z}\cos^4(\pi z)}{8\pi} & \text{for}\ x=-2i\pi\\ -\frac{ze^{2i\pi z}\sin^4(\pi z)}{4}-\frac{ize^{2i\pi z}\sin^3(\pi z)\cos(\pi z)}{2}-\frac{ize^{2i\pi z}\sin(\pi z)\cos^3(\pi z)}{2}+\frac{ze^{2i\pi z}\cos^4(\pi z)}{4}+\frac{5ie^{2i\pi z}\sin^4(\pi z)}{24\pi}+\frac{e^{2i\pi z}\sin^3(\pi z)\cos(\pi z)}{3\pi}+\frac{ie^{2i\pi z}\sin^2(\pi z)\cos^2(\pi z)}{2\pi}+\frac{ie^{2i\pi z}\cos^4(\pi z)}{8\pi} & \text{for}\ x=2i\pi\\ \frac{ze^{4i\pi z}\sin^4(\pi z)}{16}+\frac{ize^{4i\pi z}\sin^3(\pi z)\cos(\pi z)}{4}-\frac{3ze^{4i\pi z}\sin^2(\pi z)\cos^2(\pi z)}{8}-\frac{ize^{4i\pi z}\sin(\pi z)\cos^3(\pi z)}{4}+\frac{ze^{4i\pi z}\cos^4(\pi z)}{16}-\frac{7ie^{4i\pi z}\sin^4(\pi z)}{24\pi}+\frac{11e^{4i\pi z}\sin^3(\pi z)\cos(\pi z)}{48\pi}+\frac{5e^{4i\pi z}\sin(\pi z)\cos^3(\pi z)}{48\pi}+\frac{ie^{4i\pi z}\cos^4(\pi z)}{24\pi} & \text{for}\ x=4i\pi\\ \frac{x^4e^{\frac{x}{2}}e^{xz}\sin^4(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}-\frac{4\pi x^3e^{\frac{x}{2}}e^{xz}\sin^3(\pi z)\cos(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}+\frac{16\pi^2x^2e^{\frac{x}{2}}e^{xz}\sin^4(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}+\frac{12\pi^2x^2e^{\frac{x}{2}}e^{xz}\sin^2(\pi z)\cos^2(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}-\frac{40\pi^3xe^{\frac{x}{2}}e^{xz}\sin^3(\pi z)\cos(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}-\frac{24\pi^3xe^{\frac{x}{2}}e^{xz}\sin(\pi z)\cos^3(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}+\frac{24\pi^4e^{\frac{x}{2}}e^{xz}\sin^4(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}+\frac{48\pi^4e^{\frac{x}{2}}e^{xz}\sin^2(\pi z)\cos^2(\pi z)}{x^5+20\pi^2x^3+64\pi^4x}+\frac{24\pi^4e^{\frac{x}{2}}e^{xz}\cos^4(\pi z)}{x^5+20\pi^2x^3+64\pi^4x} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)
```

```
[Out] x**4*Piecewise((3*z*sin(pi*z)**4/8 + 3*z*sin(pi*z)**2*cos(pi*z)**2/4 + 3*z*
cos(pi*z)**4/8 - 5*sin(pi*z)**3*cos(pi*z)/(8*pi) - 3*sin(pi*z)*cos(pi*z)**3
/(8*pi), Eq(x, 0)), (z*exp(-4*I*pi*z)*sin(pi*z)**4/16 - I*z*exp(-4*I*pi*z)*
sin(pi*z)**3*cos(pi*z)/4 - 3*z*exp(-4*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/8 +
 I*z*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(-4*I*pi*z)*cos(pi*z)**
4/16 + 7*I*exp(-4*I*pi*z)*sin(pi*z)**4/(24*pi) + 11*exp(-4*I*pi*z)*sin(pi*z
)**3*cos(pi*z)/(48*pi) + 5*exp(-4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/(48*pi) -
I*exp(-4*I*pi*z)*cos(pi*z)**4/(24*pi), Eq(x, -4*I*pi)), (-z*exp(-2*I*pi*z)*
```

```
sin(pi*z)**4/4 + I*z*exp(-2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 + I*z*exp(-2*I
*pi*z)*sin(pi*z)*cos(pi*z)**3/2 + z*exp(-2*I*pi*z)*cos(pi*z)**4/4 - 5*I*exp
(-2*I*pi*z)*sin(pi*z)**4/(24*pi) + exp(-2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/(3
*pi) - I*exp(-2*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/(2*pi) - I*exp(-2*I*pi*z)
*cos(pi*z)**4/(8*pi), Eq(x, -2*I*pi)), (-z*exp(2*I*pi*z)*sin(pi*z)**4/4 - I
*z*exp(2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/2 - I*z*exp(2*I*pi*z)*sin(pi*z)*cos
(pi*z)**3/2 + z*exp(2*I*pi*z)*cos(pi*z)**4/4 + 5*I*exp(2*I*pi*z)*sin(pi*z)*
*4/(24*pi) + exp(2*I*pi*z)*sin(pi*z)**3*cos(pi*z)/(3*pi) + I*exp(2*I*pi*z)*
sin(pi*z)**2*cos(pi*z)**2/(2*pi) + I*exp(2*I*pi*z)*cos(pi*z)**4/(8*pi), Eq(
x, 2*I*pi)), (z*exp(4*I*pi*z)*sin(pi*z)**4/16 + I*z*exp(4*I*pi*z)*sin(pi*z)
**3*cos(pi*z)/4 - 3*z*exp(4*I*pi*z)*sin(pi*z)**2*cos(pi*z)**2/8 - I*z*exp(4
*I*pi*z)*sin(pi*z)*cos(pi*z)**3/4 + z*exp(4*I*pi*z)*cos(pi*z)**4/16 - 7*I*e
xp(4*I*pi*z)*sin(pi*z)**4/(24*pi) + 11*exp(4*I*pi*z)*sin(pi*z)**3*cos(pi*z)
/(48*pi) + 5*exp(4*I*pi*z)*sin(pi*z)*cos(pi*z)**3/(48*pi) + I*exp(4*I*pi*z)
*cos(pi*z)**4/(24*pi), Eq(x, 4*I*pi)), (x**4*exp(x/2)*exp(x*z)*sin(pi*z)**4
/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) - 4*pi*x**3*exp(x/2)*exp(x*z)*sin(pi*z
)**3*cos(pi*z)/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 16*pi**2*x**2*exp(x/2)
*exp(x*z)*sin(pi*z)**4/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 12*pi**2*x**2*
exp(x/2)*exp(x*z)*sin(pi*z)**2*cos(pi*z)**2/(x**5 + 20*pi**2*x**3 + 64*pi**
4*x) - 40*pi**3*x*exp(x/2)*exp(x*z)*sin(pi*z)**3*cos(pi*z)/(x**5 + 20*pi**2
*x**3 + 64*pi**4*x) - 24*pi**3*x*exp(x/2)*exp(x*z)*sin(pi*z)*cos(pi*z)**3/(
x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 24*pi**4*exp(x/2)*exp(x*z)*sin(pi*z)**
4/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 48*pi**4*exp(x/2)*exp(x*z)*sin(pi*z
)**2*cos(pi*z)**2/(x**5 + 20*pi**2*x**3 + 64*pi**4*x) + 24*pi**4*exp(x/2)*e
xp(x*z)*cos(pi*z)**4/(x**5 + 20*pi**2*x**3 + 64*pi**4*x), True))
```

Giac [A]
time = 0.45, size = 114, normalized size = 0.57

$$\frac{1}{8}\left(\left(\frac{x\cos\left(4\,\pi z\right)}{16\,\pi^2+x^2}+\frac{4\,\pi\sin\left(4\,\pi z\right)}{16\,\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}\,x\right)}-4\left(\frac{x\cos\left(2\,\pi z\right)}{4\,\pi^2+x^2}+\frac{2\,\pi\sin\left(2\,\pi z\right)}{4\,\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}\,x\right)}+\frac{3\,e^{\left(xz+\frac{1}{2}\,x\right)}}{x}\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="giac")
```

```
[Out] 1/8*((x*cos(4*pi*z)/(16*pi^2 + x^2) + 4*pi*sin(4*pi*z)/(16*pi^2 + x^2))*e^(
x*z + 1/2*x) - 4*(x*cos(2*pi*z)/(4*pi^2 + x^2) + 2*pi*sin(2*pi*z)/(4*pi^2 +
 x^2))*e^(x*z + 1/2*x) + 3*e^(x*z + 1/2*x)/x)*x^4
```

Mupad [B]
time = 1.11, size = 140, normalized size = 0.70

$$\frac{x^3\,\mathrm{e}^{\frac{x}{2}+x\,z}\,\left(24\,\Pi^4-\frac{x^4\,\cos\left(2\,\Pi\,z\right)}{2}+\frac{x^4\,\cos\left(4\,\Pi\,z\right)}{8}+\frac{3\,x^4}{8}+\frac{15\,\Pi^2\,x^2}{2}-\Pi\,x^3\,\sin\left(2\,\Pi\,z\right)-16\,\Pi^3\,x\,\sin\left(2\,\Pi\,z\right)+\frac{\Pi\,x^3\,\sin\left(4\,\Pi\,z\right)}{2}+2\,\Pi^3\,x\,\sin\left(4\,\Pi\,z\right)-8\,\Pi^2\,x^2\,\cos\left(2\,\Pi\,z\right)+\frac{\Pi^2\,x^2\,\cos\left(4\,\Pi\,z\right)}{2}\right)}{64\,\Pi^4+20\,\Pi^2\,x^2+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(x/2 + x*z)*sin(Pi*z)^4,z)
```

```
[Out] (x^3*exp(x/2 + x*z)*(24*Pi^4 - (x^4*cos(2*Pi*z))/2 + (x^4*cos(4*Pi*z))/8 +
(3*x^4)/8 + (15*Pi^2*x^2)/2 - Pi*x^3*sin(2*Pi*z) - 16*Pi^3*x*sin(2*Pi*z) +
(Pi*x^3*sin(4*Pi*z))/2 + 2*Pi^3*x*sin(4*Pi*z) - 8*Pi^2*x^2*cos(2*Pi*z) + (P
i^2*x^2*cos(4*Pi*z))/2))/(64*Pi^4 + x^4 + 20*Pi^2*x^2)
```

3.276 $\int \operatorname{erf}(x) \, dx$

Optimal. Leaf size=18

$$\frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

```
[Out] x*erf(x)+1/exp(x^2)/Pi^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6484}

$$x\operatorname{Erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[x],x]
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Rule 6484

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b),
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \operatorname{erf}(x) \, dx = \frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{e^{-x^2}}{\sqrt{\pi}} + x\operatorname{erf}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[x],x]
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Maple [A]
time = 0.01, size = 16, normalized size = 0.89

method	result	size
default	$x\,\mathrm{erf}\,(x)+\frac{\mathrm{e}^{-x^2}}{\sqrt{\pi}}$	16
meijerg	$\frac{-2+2\,\mathrm{e}^{-x^2}+2x\sqrt{\pi}\,\,\mathrm{erf}(x)}{2\sqrt{\pi}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*erf(x)+1/Pi^(1/2)*exp(-x^2)
```

Maxima [A]
time = 3.16, size = 15, normalized size = 0.83

$$x\,\mathrm{erf}\,(x)+\frac{e^{(-x^2)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="maxima")
```

```
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

Fricas [A]
time = 0.95, size = 20, normalized size = 1.11

$$\frac{\pi x\,\mathrm{erf}\,(x)+\sqrt{\pi}\,e^{(-x^2)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="fricas")
```

```
[Out] (pi*x*erf(x) + sqrt(pi)*e^(-x^2))/pi
```

Sympy [A]
time = 0.13, size = 15, normalized size = 0.83

$$x\,\mathrm{erf}\,(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x)
```

```
[Out] x*erf(x) + exp(-x**2)/sqrt(pi)
```

Giac [A]
time = 0.44, size = 15, normalized size = 0.83

$$x \operatorname{erf}(x) + \frac{e^{\left(-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="giac")
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

Mupad [B]
time = 0.15, size = 15, normalized size = 0.83

$$\frac{\mathrm{e}^{-x^2}}{\sqrt{\pi}} + x \operatorname{erf}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(x),x)
[Out] exp(-x^2)/pi^(1/2) + x*erf(x)
```

3.277 $\int \mathrm{erf}(a+x)\,dx$

Optimal. Leaf size=24

$$\frac{e^{-(a+x)^2}}{\sqrt{\pi}}+(a+x)\mathrm{erf}(a+x)$$

```
[Out] (a+x)*erf(a+x)+1/exp((a+x)^2)/Pi^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = $\{6484\}$

$$(a+x)\mathrm{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[a + x],x]
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Rule 6484

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[(a + b*x)*(Erf[a + b*x]/b), x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \mathrm{erf}(a+x)\,dx=\frac{e^{-(a+x)^2}}{\sqrt{\pi}}+(a+x)\mathrm{erf}(a+x)$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{e^{-(a+x)^2}}{\sqrt{\pi}}+(a+x)\mathrm{erf}(a+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[a + x],x]
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.92

method	result	size
derivativedivides	$(a+x)\operatorname{erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$	22
default	$(a+x)\operatorname{erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(a+x),x,method=_RETURNVERBOSE)
```

```
[Out] (a+x)*erf(a+x)+1/Pi^(1/2)*exp(-(a+x)^2)
```

Maxima [A]
time = 2.99, size = 21, normalized size = 0.88

$$(a+x)\operatorname{erf}(a+x)+\frac{e^{\left(-(a+x)^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="maxima")
```

```
[Out] (a + x)*erf(a + x) + e^(-(a + x)^2)/sqrt(pi)
```

Fricas [A]
time = 0.82, size = 37, normalized size = 1.54

$$\frac{(\pi a+\pi x)\operatorname{erf}(a+x)+\sqrt{\pi}\,e^{\left(-a^2-2\,ax-x^2\right)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="fricas")
```

```
[Out] ((pi*a + pi*x)*erf(a + x) + sqrt(pi)*e^(-a^2 - 2*a*x - x^2))/pi
```

Sympy [A]
time = 0.22, size = 36, normalized size = 1.50

$$a\operatorname{erf}(a+x)+x\operatorname{erf}(a+x)+\frac{e^{-a^2}e^{-x^2}e^{-2ax}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x)
```

```
[Out] a*erf(a + x) + x*erf(a + x) + exp(-a**2)*exp(-x**2)*exp(-2*a*x)/sqrt(pi)
```

Giac [A]
time = 0.49, size = 37, normalized size = 1.54

$$x\,\mathrm{erf}\,(a+x)+\frac{\sqrt{\pi}\,a\,\mathrm{erf}\,(a+x)+e^{\left(-a^2-2\,ax-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="giac")
[Out] x*erf(a + x) + (sqrt(pi)*a*erf(a + x) + e^(-a^2 - 2*a*x - x^2))/sqrt(pi)
```

Mupad [B]
time = 0.07, size = 21, normalized size = 0.88

$$\mathrm{erf}(a+x)\ (a+x)+\frac{\mathrm{e}^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(a + x),x)
[Out] erf(a + x)*(a + x) + exp(-(a + x)^2)/pi^(1/2)
```

3.278 $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$

Optimal. Leaf size=94

$$\frac{(1+2x)\sqrt{1+2x^2+4x^3+x^4}}{2(-1+2x^2)}-\tanh^{-1}\left(\frac{x(2+x)(7-x+27x^2+33x^3)}{(2+37x^2+31x^3)\sqrt{1+2x^2+4x^3+x^4}}\right)$$

```
[Out] -arctanh(x*(2+x)*(33*x^3+27*x^2-x+7)/(31*x^3+37*x^2+2)/(x^4+4*x^3+2*x^2+1)^
(1/2))+1/2*(1+2*x)*(x^4+4*x^3+2*x^2+1)^(1/2)/(2*x^2-1)
```

Rubi [F]
time = 1.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*Sqrt[1
 + 2*x^2 + 4*x^3 + x^4]),x]
```

```
[Out] (9*Defer[Int][1/Sqrt[1 + 2*x^2 + 4*x^3 + x^4], x])/4 - (13*Defer[Int][1/((S
qrt[2] - 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/4 + Defer[Int][x/Sqrt[1
 + 2*x^2 + 4*x^3 + x^4], x] + Defer[Int][x^2/Sqrt[1 + 2*x^2 + 4*x^3 + x^4],
 x]/2 - (13*Defer[Int][1/((Sqrt[2] + 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),
 x])/4 - (13*Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),
x])/8 - ((15 + Sqrt[2])*Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^
3 + x^4]), x])/8 - (13*Defer[Int][1/((1 + Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3
 + x^4]), x])/8 - ((15 - Sqrt[2])*Defer[Int][1/((1 + Sqrt[2]*x)*Sqrt[1 + 2*
x^2 + 4*x^3 + x^4]), x])/8 - (17*Defer[Int][x/((-1 + 2*x^2)^2*Sqrt[1 + 2*x^
2 + 4*x^3 + x^4]), x])/2
```

Rubi steps

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \int \left(\frac{9}{4\sqrt{1+2x^2+4x^3+x^4}} + \frac{x}{\sqrt{1+2x^2+4x^3+x^4}} + \frac{}{2\surd}\right.$$

$$= \frac{1}{4}\int \frac{15+2x}{(-1+2x^2)\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{1}{2}\int \frac{}{\sqrt{1+2x^2}}$$

$$= \frac{1}{4}\int \left(-\frac{15+\sqrt{2}}{2\left(1-\sqrt{2}\,x\right)\sqrt{1+2x^2+4x^3+x^4}} - \frac{}{2\left(1+\sqrt{2}\right.}\right.$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx - \frac{13}{8}\int \frac{1}{\left(1-\sqrt{2}\,x\right)\sqrt{1+2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 16.13, size = 5137, normalized size = 54.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*
Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]
```

```
[Out] Result too large to show
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.34, size = 1197351, normalized size = 12737.78

method	result

trager	$\frac{(1+2x)\sqrt{x^4+4x^3+2x^2+1}}{4x^2-2}+\frac{\ln\left(-\frac{-1025x^{10}+1023\sqrt{x^4+4x^3+2x^2+1}\,x^8-6138x^9+4104\sqrt{x^4+4x^3+2x}}{}\right.}{}$
risch	Expression too large to display
elliptic	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/
2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3
 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(88) = 176$.
time = 1.47, size = 179, normalized size = 1.90

$$\frac{(2x^2-1)\log\left(\frac{1025x^{10}+6138x^9+12307x^8+10188x^7+4503x^6+3134x^5+1589x^4+140x^3+176x^2-(1023x^8+4104x^7+5084x^6+2182x^5+805x^4+624x^3+10x^2+28x)\sqrt{x^4+4x^3+2x^2+1}+2}{32x^{10}-80x^8+80x^6-40x^4+10x^2-1}\right)+\sqrt{x^4+4x^3+2x^2+1}\,(2x+1)}{2(2x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*x^2 - 1)*log((1025*x^10 + 6138*x^9 + 12307*x^8 + 10188*x^7 + 4503*x
^6 + 3134*x^5 + 1589*x^4 + 140*x^3 + 176*x^2 - (1023*x^8 + 4104*x^7 + 5084*
x^6 + 2182*x^5 + 805*x^4 + 624*x^3 + 10*x^2 + 28*x)*sqrt(x^4 + 4*x^3 + 2*x^
2 + 1) + 2)/(32*x^10 - 80*x^8 + 80*x^6 - 40*x^4 + 10*x^2 - 1)) + sqrt(x^4 +
 4*x^3 + 2*x^2 + 1)*(2*x + 1))/(2*x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^6+4x^5+7x^4-3x^3-x^2-8x-8}{\sqrt{(x+1)(x^3+3x^2-x+1)}\,(2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-1)**2/(x**4+4*x*
*3+2*x**2+1)**(1/2),x)
```

```
[Out] Integral((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sqrt((x + 1)
*(x**3 + 3*x**2 - x + 1))*(2*x**2 - 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3
 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{-2\,x^6-4\,x^5-7\,x^4+3\,x^3+x^2+8\,x+8}{{\left(2\,x^2-1\right)}^2\,\sqrt{x^4+4\,x^3+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2
+ 4*x^3 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(-(8*x + x^2 + 3*x^3 - 7*x^4 - 4*x^5 - 2*x^6 + 8)/((2*x^2 - 1)^2*(2*x^2
+ 4*x^3 + x^4 + 1)^(1/2)), x)
```

3.279 $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$

Optimal. Leaf size=142

$$-\frac{1}{4}\tanh^{-1}\left(\frac{(1-3y)\sqrt{1-5y-5y^2}}{(1-5y)\sqrt{1-y-y^2}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{(4+3y)\sqrt{1-5y-5y^2}}{(6+5y)\sqrt{1-y-y^2}}\right)+\frac{9}{4}\tanh^{-1}\left(\frac{(11+7y)\sqrt{1}}{3(7+5y)\sqrt{}}\right.$$

```
[Out] -1/4*arctanh((1-3*y)*(-5*y^2-5*y+1)^(1/2)/(1-5*y)/(-y^2-y+1)^(1/2))-1/2*arc
tanh((4+3*y)*(-5*y^2-5*y+1)^(1/2)/(6+5*y)/(-y^2-y+1)^(1/2))+9/4*arctanh(1/3
*(11+7*y)*(-5*y^2-5*y+1)^(1/2)/(7+5*y)/(-y^2-y+1)^(1/2))
```

Rubi [F]
time = 2.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$$

Verification is not applicable to the result.

```
[In] Int[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2])
,y]
```

```
[Out] Defer[Int][Sqrt[1 - 5*y - 5*y^2]/(y*Sqrt[1 - y - y^2]), y]/2 + Defer[Int][S
qrt[1 - 5*y - 5*y^2]/((1 + y)*Sqrt[1 - y - y^2]), y] - (3*Defer[Int][Sqrt[1
 - 5*y - 5*y^2]/((2 + y)*Sqrt[1 - y - y^2]), y])/2
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy &= \int \left(\frac{\sqrt{1-5y-5y^2}}{2y\sqrt{1-y-y^2}}+\frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}}-\frac{3\sqrt{1-5y-5y^2}}{2(2+y)\sqrt{1-y-}}\right.\\ &= \frac{1}{2}\int \frac{\sqrt{1-5y-5y^2}}{y\sqrt{1-y-y^2}}\,dy-\frac{3}{2}\int \frac{\sqrt{1-5y-5y^2}}{(2+y)\sqrt{1-y-y^2}}\,dy+\int \frac{\sqrt{1}}{(1+y}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 3.63, size = 630, normalized size = 4.44

$$\frac{\left(-1-\frac{2}{\sqrt{5}}\right)\left(1+\sqrt{5}+2y\right)^2\sqrt{\frac{5+3\sqrt{5}+10y}{5+5\sqrt{5}+10y}}\left(20\left(-4\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}+\sqrt{5}\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}+5\sqrt{-\frac{-5+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}\sqrt{-\frac{-3+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}-2\sqrt{5}\sqrt{-\frac{-5+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}\sqrt{-\frac{-3+\sqrt{5}+2\sqrt{5}\,y}{1+\sqrt{5}+2y}}\right)F\left(\sin^{-1}\left(\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right)|\frac{15}{16}\right)+\sqrt{\frac{-5+3\sqrt{5}-10y}{1+\sqrt{5}+2y}}\sqrt{\frac{-1+\sqrt{5}-2y}{1+\sqrt{5}+2y}}\left(9\sqrt{5}\,\Pi\left(\frac{5}{8}-\frac{\sqrt{5}}{8};\sin^{-1}\left(\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right)|\frac{15}{16}\right)+\left(-20+9\sqrt{5}\right)\Pi\left(-\frac{3}{8}\left(-5+\sqrt{5}\right);\sin^{-1}\left(\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right)|\frac{15}{16}\right)+2\sqrt{5}\,\Pi\left(\frac{3}{8}\left(5+\sqrt{5}\right);\sin^{-1}\left(\frac{2\sqrt{\frac{5+3\sqrt{5}+10y}{1+\sqrt{5}+2y}}}{\sqrt{15}}\right)|\frac{15}{16}\right)\right)\right)}{16\sqrt{1-5y-5y^2}\sqrt{1-y-y^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y -
 y^2]),y]
```

```
[Out] ((-1 - 2/Sqrt[5])*(1 + Sqrt[5] + 2*y)^2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(5 + 5*
Sqrt[5] + 10*y)]*(20*(-4*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*
Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + Sqrt[5]*Sqrt[(-5 + 3*Sqrt[
5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*
y)] + 5*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-
3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] - 2*Sqrt[5]*Sqrt[-((-5 + S
qrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2*Sqrt[5
]*y)/(1 + Sqrt[5] + 2*y))])*EllipticF[ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)
/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16] + Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1
+ Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)]*(9*Sqrt[5]
*EllipticPi[5/8 - Sqrt[5]/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqr
t[5] + 2*y)])/Sqrt[15]], 15/16] + (-20 + 9*Sqrt[5])*EllipticPi[(-3*(-5 + Sq
rt[5]))/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt
[15]], 15/16] + 2*Sqrt[5]*EllipticPi[(3*(5 + Sqrt[5]))/8, ArcSin[(2*Sqrt[(5
 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16])))/(16*Sqrt[1
- 5*y - 5*y^2]*Sqrt[1 - y - y^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.34, size = 354, normalized size = 2.49

method	result
default	$\frac{300\sqrt{-5y^2-5y+1}\,\sqrt{-y^2-y+1}\,\sqrt{-\frac{10y+5+3\sqrt{5}}{-10y-5+3\sqrt{5}}}\left(-10y-5+3\sqrt{5}\right)^2\sqrt{\frac{-2y+\sqrt{5}-1}{-10y-5+3\sqrt{5}}}\,\sqrt{5}\,\sqrt{\frac{2y\cdots}{-10y\cdots}}\cdots}{\sqrt{5y^4+10y^3-y^2-6y+1}\,\sqrt{(10y+5+3\sqrt{5}\cdots}}$
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y,method=_R
ETURNVERBOSE)
```

```
[Out] 300*(-5*y^2-5*y+1)^(1/2)*(-y^2-y+1)^(1/2)*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5
^(1/2)))^(1/2)*(-10*y-5+3*5^(1/2))^2*((-2*y+5^(1/2)-1)/(-10*y-5+3*5^(1/2)))
^(1/2)*5^(1/2)*((2*y+1+5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2)*(3*EllipticPi(2*
(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(5+5^(1/2))/(5^(1/2)-5
),1/4)-EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(3
*5^(1/2)-5)/(5+3*5^(1/2)),1/4)-2*EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5
+3*5^(1/2)))^(1/2),-1/4*(5+3*5^(1/2))/(3*5^(1/2)-5),1/4))/(5*y^4+10*y^3-y^2
-6*y+1)^(1/2)/((10*y+5+3*5^(1/2))*(-10*y-5+3*5^(1/2))*(-2*y+5^(1/2)-1)*(2*y
+1+5^(1/2)))^(1/2)/(5+5^(1/2))/(5^(1/2)-5)/(3*5^(1/2)-5)/(5+3*5^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="maxima")
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y +
 1)*y), y)
```

Fricas [A]
time = 1.36, size = 223, normalized size = 1.57

$$\frac{9}{8}\log\left(-\frac{235y^4+935y^3-3(35y^2+104y+77)\sqrt{-y^2-y+1}\sqrt{-5y^2-5y+1}+1086y^2+131y-281}{y^4+8y^3+24y^2+32y+16}\right)+\frac{1}{4}\log\left(\frac{35y^4+125y^3+(15y^2+38y+24)\sqrt{-y^2-y+1}\sqrt{-5y^2-5y+1}+131y^2+16y-26}{y^4+4y^3+6y^2+4y+1}\right)+\frac{1}{8}\log\left(\frac{35y^4+15y^3+(15y^2-8y+1)\sqrt{-y^2-y+1}\sqrt{-5y^2-5y+1}-34y^2+11y-1}{y^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="fricas")
```

```
[Out] 9/8*log(-(235*y^4 + 935*y^3 - 3*(35*y^2 + 104*y + 77)*sqrt(-y^2 - y + 1)*sq
rt(-5*y^2 - 5*y + 1) + 1086*y^2 + 131*y - 281)/(y^4 + 8*y^3 + 24*y^2 + 32*y
 + 16)) + 1/4*log((35*y^4 + 125*y^3 + (15*y^2 + 38*y + 24)*sqrt(-y^2 - y +
1)*sqrt(-5*y^2 - 5*y + 1) + 131*y^2 + 16*y - 26)/(y^4 + 4*y^3 + 6*y^2 + 4*y
 + 1)) + 1/8*log((35*y^4 + 15*y^3 + (15*y^2 - 8*y + 1)*sqrt(-y^2 - y + 1)*s
qrt(-5*y^2 - 5*y + 1) - 34*y^2 + 11*y - 1)/y^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2y+1)\sqrt{-5y^2-5y+1}}{y(y+1)(y+2)\sqrt{-y^2-y+1}}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-y+1)**(1/2),y
)
```

```
[Out] Integral((2*y + 1)*sqrt(-5*y**2 - 5*y + 1)/(y*(y + 1)*(y + 2)*sqrt(-y**2 -
y + 1)), y)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="giac")
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y +
 1)*y), y)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,y+1)\,\sqrt{-5\,y^2-5\,y+1}}{y\,(y+1)\,(y+2)\,\sqrt{-y^2-y+1}}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1
/2)),y)
```

```
[Out] int(((2*y + 1)*(1 - 5*y^2 - 5*y)^(1/2))/(y*(y + 1)*(y + 2)*(1 - y^2 - y)^(1
/2)), y)
```

3.280 $\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-\right.}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}$

Optimal. Leaf size=21

$$\log\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)$$

```
[Out] ln(1+(x^2-4)^(1/2)+(x^2-1)^(1/2))
```

Rubi [A]
time = 0.19, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 85, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = {6816}

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt[
-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])),x]
```

```
[Out] Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{(4-5x^2+x^4)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx = \log\left(1+\sqrt{-4+x^2}+\sqrt{-1+x\right.$$

Mathematica [A]
time = 0.62, size = 31, normalized size = 1.48

$$2\tanh^{-1}\left(1-\frac{2}{3}\sqrt{-4+x^2}+\frac{2}{3}\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2
*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]))
,x]
```

```
[Out] 2*ArcTanh[1 - (2*Sqrt[-4 + x^2])/3 + (2*Sqrt[-1 + x^2])/3]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(17) = 34$.
time = 0.20, size = 1088, normalized size = 51.81

method	result
elliptic	$\frac{\sqrt{(x^2-4)(x^2-1)}\left(\frac{\ln(x^2-5)}{4}+\frac{\ln\left(-\frac{5}{2}+x^2+\sqrt{x^4-5x^2+4}\right)}{4}+\frac{\operatorname{arctanh}\left(\frac{5x^2-17}{4\sqrt{(x^2-5)^2+5x^2-21}}\right)}{4}+\frac{\operatorname{arctanh}}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/
(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(2+5^(1/2))/(-2+5^(1/2))*((-2+x)^2+4*x-8)^(1/2)-1/2/(2+5^(1/2))/(-2+5^
(1/2))*ln(x+((-2+x)^2+4*x-8)^(1/2))-1/4/(2+5^(1/2))/(-2+5^(1/2))*((2+x)^2-4
*x-8)^(1/2)+1/2/(2+5^(1/2))/(-2+5^(1/2))*ln(x+((2+x)^2-4*x-8)^(1/2))+1/2/(5
^(1/2)+1)/(5^(1/2)-1)*((1+x)^2-2*x-2)^(1/2)-1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(
x+((1+x)^2-2*x-2)^(1/2))+1/2/(5^(1/2)+1)/(5^(1/2)-1)*((-1+x)^2-2+2*x)^(1/2)
+1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((-1+x)^2-2+2*x)^(1/2))+1/(5^(1/2)+1)/(2+
5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(1/2)*(x-
5^(1/2))+1)^(1/2))-1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1/
2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))-1/2/(5^(1/2)+1)/(2+5
^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(1/2)*(x-5
^(1/2))+4)^(1/2))+1/2/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*5^(1
/2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1/2))+1/4*ln(x^2-5)+1/(5^
(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1/4*(8-2*5^(1/2)*(x+5
^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1/2))-1/2/(5^(1/2)+1)/(2+
5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(
1/2)+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1/4*(8+2*5^
(1/2)*(x-5^(1/2)))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+4)^(1/2))-1/2/(5^(1
/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x-5^(1/2))^2+2*5^(1/2)*(x-5^(
1/2))+4)^(1/2)-1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*arctanh(1
/2*(2-2*5^(1/2)*(x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))
+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x+5^(1/2))^2-2*5^(1/2
```

```
)*(x+5^(1/2))+1)^(1/2)-1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*a
rctanh(1/2*(2+2*5^(1/2)*(x-5^(1/2)))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1
)^(1/2))+1/(5^(1/2)+1)/(2+5^(1/2))/(-2+5^(1/2))/(5^(1/2)-1)*((x-5^(1/2))^2+
2*5^(1/2)*(x-5^(1/2))+1)^(1/2)+7/8*(x^2-4)^(1/2)*(x^2-1)^(1/2)/(x^4-5*x^2+4
)^(1/2)*arctanh(1/4*(5*x^2-17)/(x^4-5*x^2+4)^(1/2))+1/8*(x^2-4)^(1/2)*(x^2-
1)^(1/2)*(2*ln(-5/2+x^2+(x^4-5*x^2+4)^(1/2))-5*arctanh(1/4*(5*x^2-17)/(x^4-
5*x^2+4)^(1/2)))/(x^4-5*x^2+4)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(17) = 34$.
time = 2.53, size = 171, normalized size = 8.14

$$\frac{1}{4}\log\left(x+1\right)+\frac{3}{8}\log\left(x-1\right)+\frac{1}{8}\log\left(x-2\right)+\frac{1}{4}\log\left(\frac{2\,x^4+4\,(x^2-3)\sqrt{x+1}\,\sqrt{x-1}-7\,x^2+2\left((x^2-1)\sqrt{x+1}\,\sqrt{x-1}\,\sqrt{x-2}+(2\,x^2-3)\sqrt{x-2}\right)\sqrt{x+2}+3}{2\left((x^2-1)\sqrt{x+1}\,\sqrt{x-1}\,\sqrt{x-2}+(2\,x^2-3)\sqrt{x-2}\right)}\right)+\frac{1}{4}\log\left(\frac{(x^2-1)\sqrt{x+1}\,\sqrt{x-1}+2\,x^2-3}{(x^2-1)\sqrt{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*log(x + 1) + 3/8*log(x - 1) + 1/8*log(x - 2) + 1/4*log(1/2*(2*x^4 + 4*(
x^2 - 3)*sqrt(x + 1)*sqrt(x - 1) - 7*x^2 + 2*((x^2 - 1)*sqrt(x + 1)*sqrt(x
- 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))*sqrt(x + 2) + 3)/((x^2 - 1)*sqr
t(x + 1)*sqrt(x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))) + 1/4*log(((x^
2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 2*x^2 - 3)/((x^2 - 1)*sqrt(x - 1)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(17) = 34$.
time = 0.83, size = 162, normalized size = 7.71

$$-\frac{1}{4}\log\left(4\,x^4-(4\,x^2-11)\sqrt{x^2-1}\,\sqrt{x^2-4}-21\,x^2+23\right)-\frac{1}{4}\log\left(x^2-\sqrt{x^2-1}\,(x+2)+2\,x-1\right)+\frac{1}{4}\log\left(x^2-\sqrt{x^2-4}\,(x+1)+x-4\right)-\frac{1}{4}\log\left(x^2-\sqrt{x^2-4}\,(x-1)-x-4\right)+\frac{1}{4}\log\left(x^2-\sqrt{x^2-1}\,(x-2)-2\,x-1\right)+\frac{1}{4}\log\left(x^2-5\right)+\frac{1}{4}\log\left(-x^2+\sqrt{x^2-1}\,\sqrt{x^2-4}+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*log(4*x^4 - (4*x^2 - 11)*sqrt(x^2 - 1)*sqrt(x^2 - 4) - 21*x^2 + 23) -
1/4*log(x^2 - sqrt(x^2 - 1)*(x + 2) + 2*x - 1) + 1/4*log(x^2 - sqrt(x^2 - 4
)*(x + 1) + x - 4) - 1/4*log(x^2 - sqrt(x^2 - 4)*(x - 1) - x - 4) + 1/4*log
(x^2 - sqrt(x^2 - 1)*(x - 2) - 2*x - 1) + 1/4*log(x^2 - 5) + 1/4*log(-x^2 +
 sqrt(x^2 - 1)*sqrt(x^2 - 4) + 7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-1)**(1/2)+x**2*(
x**2-1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(17) = 34$.
time = 0.44, size = 76, normalized size = 3.62

$$-\frac{1}{2}\log\left(\sqrt{x^2-1}-\sqrt{x^2-4}+1\right)-\frac{1}{2}\log\left(\sqrt{x^2-1}-\sqrt{x^2-4}\right)+\frac{1}{2}\log\left(\sqrt{x^2-1}+2\right)+\frac{1}{2}\log\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-4}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x^2 - 1) - sqrt(x^2 - 4) + 1) - 1/2*log(sqrt(x^2 - 1) - sqrt(
x^2 - 4)) + 1/2*log(sqrt(x^2 - 1) + 2) + 1/2*log(abs(-sqrt(x^2 - 1) + sqrt(
x^2 - 4) - 3))
```

Mupad [B]
time = 2.37, size = 172, normalized size = 8.19

$$\frac{\ln\left(x-\sqrt{5}\right)}{4}-\operatorname{atanh}\left(\frac{\sqrt{3}-\sqrt{x^2-1}}{\sqrt{x^2-4}}\right)+\frac{\operatorname{atanh}\left(\frac{\sqrt{x^2-1}}{2}\right)}{2}+\frac{\ln\left(x+\sqrt{5}\right)}{4}-\frac{7\operatorname{atanh}\left(\frac{4\left(\sqrt{3}-\sqrt{x^2-1}\right)}{\sqrt{x^2-4}\left(\frac{\left(\sqrt{3}-\sqrt{x^2-1}\right)^2}{x^2-4}+1\right)}\right)}{4}+\frac{5\operatorname{atanh}\left(\frac{12150\left(\sqrt{3}-\sqrt{x^2-1}\right)}{\sqrt{x^2-4}\left(\frac{6075\left(\sqrt{3}-\sqrt{x^2-1}\right)^2}{2\left(x^2-4\right)}+\frac{6075}{2}\right)}\right)}{4}-\frac{\operatorname{atanh}\left(\sqrt{x^2-4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(4*(x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) - x^2*(x^2 - 1)^(1/2) - x^2*(x
^2 - 4)^(1/2)))/((x^4 - 5*x^2 + 4)*((x^2 - 1)^(1/2) + (x^2 - 4)^(1/2) + 1))
,x)
```

```
[Out] log(x - 5^(1/2))/4 - atanh((3^(1/2) - (x^2 - 1)^(1/2))/(x^2 - 4)^(1/2)) + a
tanh((x^2 - 1)^(1/2)/2)/2 + log(x + 5^(1/2))/4 - (7*atanh((4*(3^(1/2) - (x^
2 - 1)^(1/2)))/((x^2 - 4)^(1/2)*((3^(1/2) - (x^2 - 1)^(1/2))^2/(x^2 - 4) +
1))))/4 + (5*atanh((12150*(3^(1/2) - (x^2 - 1)^(1/2)))/((x^2 - 4)^(1/2)*((6
075*(3^(1/2) - (x^2 - 1)^(1/2))^2)/(2*(x^2 - 4)) + 6075/2))))/4 - atanh((x^
2 - 4)^(1/2))/2
```

3.281 $\int \left(\sqrt{9-4\sqrt{2}}\, x - \sqrt{2}\, \sqrt{1+4x+2x^2+x^4} \right) dx$

Optimal. Leaf size=4030

result too large to display

```
[Out] 1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(-1/3*(x^4+2*x^2+4*x+1)^(1/2)+1/3*(1+x)*(x^4
+2*x^2+4*x+1)^(1/2)+4*I*(-13+3*33^(1/2))^(1/3)*(x^4+2*x^2+4*x+1)^(1/2)/(4*2
^(2/3)*(-I+3^(1/2))-2*I*(-13+3*33^(1/2))^(1/3)+6*I*x*(-13+3*33^(1/2))^(1/3)
+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))-8*2^(2/3)*EllipticE((26-6*33^(
1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2))*(-26+6
*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/2)/((39+13*I*3^
(1/2)-9*I*11^(1/2)-9*33^(1/2)+4*(3-I*3^(1/2))*(-26+6*33^(1/2))^(1/3))/(39-1
3*I*3^(1/2)+9*I*11^(1/2)-9*33^(1/2)+4*(3+I*3^(1/2))*(-26+6*33^(1/2))^(1/3))
)^(1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/2)-9*I*11^(1/2)+
3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3))
^(1/2),((84+28*I*3^(1/2)-12*I*11^(1/2)-12*33^(1/2)+(3-I*3^(1/2)-3*I*11^(1/2
)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3))/(84-28*I*3^(1/2)+12*I*11^(1/2)-12*33^
(1/2)+(3+I*3^(1/2)+3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)))^(1/2))
*(x^4+2*x^2+4*x+1)^(1/2)*3^(1/2)/(-13+3*33^(1/2)+4*(-26+6*33^(1/2))^(1/3))^
(1/2)*(I*(-19899+x*(59697-10335*33^(1/2))+3445*33^(1/2)+(-26+6*33^(1/2))^(2
/3)*(-2574+466*33^(1/2))+(-26+6*33^(1/2))^(1/3)*(-19899+3445*33^(1/2)))/(-3
9-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I*(3*I+3^(1/2))*(-26+6*33^(1/2))^(
1/3))/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/2)-9*I*11^(1/2)+3*
33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3)))^
(1/2)/(4*2^(2/3)-(-13+3*33^(1/2))^(1/3)+3*x*(-13+3*33^(1/2))^(1/3)-2^(1/3)*
(-13+3*33^(1/2))^(2/3))/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13*I*3^(1/
2)-9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*
33^(1/2))^(2/3))^(1/2)/(I*(1+x)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13+13
*I*3^(1/2)-9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))
*(-26+6*33^(1/2))^(2/3))/(104-24*33^(1/2)+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*
33^(1/2))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3)))^(
1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*3
3^(1/2))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/
2)+1/6*EllipticPi(1/6*(13-3*33^(1/2)-2^(1/3)*(-13+3*33^(1/2))^(4/3)+4*(-26+
6*33^(1/2))^(2/3)+x*(-39+9*33^(1/2)))^(1/2)*2^(5/6)*3^(1/2)/(-13+3*33^(1/2)
)^(2/3)/(1+x)^(1/2)/((-39+13*I*3^(1/2)-9*I*11^(1/2)+9*33^(1/2)-4*I*(-3*I+3^
(1/2))*(-26+6*33^(1/2))^(1/3))/(104-24*33^(1/2)+(-13+13*I*3^(1/2)-9*I*11^(1
/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2
/3)))^(1/2),2^(1/3)*(4*2^(1/3)*(-3*I+3^(1/2))+(3*I+3^(1/2))*(-13+3*33^(1/2)
)^(2/3))/(4*2^(2/3)*(-I+3^(1/2))-8*I*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(3^(1/2
)+I)*(-13+3*33^(1/2))^(2/3)),((84-28*I*3^(1/2)+12*I*11^(1/2)-12*33^(1/2)+(3
+I*3^(1/2)+3*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3))/(84+28*I*3^(1/2
)-12*I*11^(1/2)-12*33^(1/2)+(3-I*3^(1/2)-3*I*11^(1/2)+3*33^(1/2))*(-26+6*33
^(1/2))^(1/3)))^(1/2))*(4*2^(2/3)+2*(-13+3*33^(1/2))^(1/3)-2^(1/3)*(-13+3*3
```

3^(1/2))^(2/3))*(4*2^(2/3)*(3^(1/2)+I)-4*I*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(
-I+3^(1/2))*(-13+3*33^(1/2))^(2/3))*(4*2^(2/3)*(-I+3^(1/2))+4*I*(-13+3*33^(
1/2))^(1/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))*(1+x)^(1/2)*(x^4+2*
x^2+4*x+1)^(1/2)*((-39+13*I*3^(1/2)-9*I*11^(1/2)+9*33^(1/2)-4*I*(-3*I+3^(1/
2))*(-26+6*33^(1/2))^(1/3))/(104-24*33^(1/2)+(-13+13*I*3^(1/2)-9*I*11^(1/2)
+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-4-4*I*3^(1/2))*(-26+6*33^(1/2))^(2/3)
))^(1/2)*((104-24*33^(1/2)+2*(1+14*I*3^(1/2)-6*I*11^(1/2)+33^(1/2))*(-26+6*
33^(1/2))^(1/3)+(-7-I*3^(1/2)-3*I*11^(1/2)+33^(1/2))*(-26+6*33^(1/2))^(2/3)
+2*x*(-52+12*33^(1/2)+2^(1/3)*(-13+3*33^(1/2))^(4/3)-4*(-26+6*33^(1/2))^(2/
3)))/(1+x)/(-39+13*I*3^(1/2)-9*I*11^(1/2)+9*33^(1/2)-4*I*(-3*I+3^(1/2))*(-2
6+6*33^(1/2))^(1/3)))^(1/2)*((104-24*33^(1/2)+2*(1-14*I*3^(1/2)+6*I*11^(1/2
)+33^(1/2))*(-26+6*33^(1/2))^(1/3)+(-7+I*3^(1/2)+3*I*11^(1/2)+33^(1/2))*(-2
6+6*33^(1/2))^(2/3)+2*x*(-52+12*33^(1/2)+2^(1/3)*(-13+3*33^(1/2))^(4/3)-4*(
-26+6*33^(1/2))^(2/3)))/(1+x)/(-39-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I
*(3*I+3^(1/2))*(-26+6*33^(1/2))^(1/3)))^(1/2)*2^(5/6)*3^(1/2)/(4*2^(2/3)*(3
^(1/2)+I)+2*I*(-13+3*33^(1/2))^(1/3)-6*I*x*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(
-I+3^(1/2))*(-13+3*33^(1/2))^(2/3))/(4*2^(2/3)*(-I+3^(1/2))-2*I*(-13+3*33^(
1/2))^(1/3)+6*I*x*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2
))^(2/3))/(13-3*33^(1/2)-2^(1/3)*(-13+3*33^(1/2))^(4/3)+4*(-26+6*33^(1/2))^
(2/3)+x*(-39+9*33^(1/2)))^(1/2)+1/18*I*EllipticF(1/6*I*(-52+12*33^(1/2)+2^(
1/3)*(-13+3*33^(1/2))^(4/3)-4*(-26+6*33^(1/2))^(2/3))^(1/2)*(26-6*33^(1/2)+
6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(
1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3))^(1/2)*2^(5/6)*3^(1/2)/(
-13+3*33^(1/2))^(2/3)/(1+x)^(1/2)/(39+13*I*3^(1/2)-9*I*11^(1/2)-9*33^(1/2)+
4*(3-I*3^(1/2))*(-26+6*33^(1/2))^(1/3))^(1/2),((84*I-28*3^(1/2)+12*11^(1/2)
-12*I*33^(1/2)+(3*I+3^(1/2)+3*11^(1/2)+3*I*33^(1/2))*(-26+6*33^(1/2))^(1/3)
)/(-56*3^(1/2)+24*11^(1/2)+2*(-26+6*33^(1/2))^(1/3)*(3^(1/2)+3*11^(1/2))))^
(1/2))*(2^(1/3)*(13-13*I*3^(1/2)+9*I*11^(1/2)-3*33^(1/2))+4*2^(2/3)*(1+I*3^
(1/2))*(-13+3*33^(1/2))^(1/3)+20*(-13+3*33^(1/2))^(2/3))*(4*2^(2/3)*(3^(1/2
)+I)+8*I*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(-I+3^(1/2))*(-13+3*33^(1/2))^(2/3)
)*(x^4+2*x^2+4*x+1)^(1/2)*(1/(-13+3*33^(1/2)+4*(-26+6*33^(1/2))^(1/3))*(-52
+12*33^(1/2)+2^(1/3)*(-13+3*33^(1/2))^(4/3)-4*(-26+6*33^(1/2))^(2/3)))^(1/2
)*((-8*I*(-13+3*33^(1/2))+(-43*I-13*3^(1/2)+9*11^(1/2)+5*I*33^(1/2))*(-26+6
*33^(1/2))^(1/3)+(2*I+4*3^(1/2)-2*I*33^(1/2))*(-26+6*33^(1/2))^(2/3)+x*(8*I
*(-13+3*33^(1/2))+(13*I-13*3^(1/2)+9*11^(1/2)-3*I*33^(1/2))*(-26+6*33^(1/2)
)^(1/3)+4*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3)))/(1+x))^(1/2)*2^(1/3)*3^(1/4)
/(-13+3*33^(1/2))^(1/3)/(4*2^(2/3)*(-I+3^(1/2))-2*I*(-13+3*33^(1/2))^(1/3)+
6*I*x*(-13+3*33^(1/2))^(1/3)+2^(1/3)*(3^(1/2)+I)*(-13+3*33^(1/2))^(2/3))/(1
+x)^(1/2)/(39+13*I*3^(1/2)-9*I*11^(1/2)-9*33^(1/2)+4*(3-I*3^(1/2))*(-26+6*3
3^(1/2))^(1/3))^(1/2)/(26-6*33^(1/2)+6*x*(-13+3*33^(1/2))+(-13-13*I*3^(1/2)
+9*I*11^(1/2)+3*33^(1/2))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^
(1/2))^(2/3))^(1/2)/((-104+24*33^(1/2)-(5-3*I*3^(1/2)+3*I*11^(1/2)+33^(1/2)
)*(-26+6*33^(1/2))^(2/3)+(-26+6*33^(1/2))^(1/3)*(-41+15*I*3^(1/2)-3*I*11^(1
/2)+7*33^(1/2))+x*(104-24*33^(1/2)+(-13-13*I*3^(1/2)+9*I*11^(1/2)+3*33^(1/2
))*(-26+6*33^(1/2))^(1/3)+4*I*(3^(1/2)+I)*(-26+6*33^(1/2))^(2/3)))/(1+x)/(-

```
39-13*I*3^(1/2)+9*I*11^(1/2)+9*33^(1/2)+4*I*(3*I+3^(1/2))*(-26+6*33^(1/2))^
(1/3)))^(1/2))
```

Rubi [F]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\sqrt{9-4\sqrt{2}}\,x-\sqrt{2}\,\sqrt{1+4x+2x^2+x^4}\right)dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - Sqrt[2]*Defer[Int][Sqrt[1 + 4*x + 2*x^2 + x^4
], x]
```

Rubi steps

$$\int \left(\sqrt{9-4\sqrt{2}}\,x-\sqrt{2}\,\sqrt{1+4x+2x^2+x^4}\right)dx = \frac{1}{2}\sqrt{9-4\sqrt{2}}\,x^2-\sqrt{2}\int \sqrt{1+4x+2x^2+x^4}\,dx$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 16.69, size = 3168, normalized size = 0.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - (Sqrt[2]*x*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 - (
2*Sqrt[2]*((6*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])^2*(-(EllipticF[Ar
cSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1
+ Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1^3
 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1
+ 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*R
oot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) + EllipticPi[(1 + Root[1 + 3*#1 - #1^
2 + #1^3 & , 3, 0])/(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1
```

```
 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]],
((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2
, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]))]*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]))*Sq
rt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2 +
 #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*(-1 - Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & ,
3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 3, 0]))]*Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))])/(Sqrt[1 + 4*
x + 2*x^2 + x^4]*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #
1^2 + #1^3 & , 3, 0])) + (2*EllipticF[ArcSin[Sqrt[((1 + x)*(-Root[1 + 3*#1
- #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
 0]))]], ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #
1^3 & , 2, 0])*(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((-1 - Root[1
+ 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*(x - Root[1 + 3*#1 - #1^2 + #1^3 & ,
 1, 0])^2*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*
(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + 3*#1 - #1^2
 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[((1 + x)*(-Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1
 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))])/(S
qrt[1 + 4*x + 2*x^2 + x^4]*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0])) + ((1 + x)*(x - Root[1 + 3*#1 - #1^2 + #1
^3 & , 2, 0])*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]) + (x - Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0])^2*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*
Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2
 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*Sqrt[(x -
Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 &
, 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[-(((1 + x)*(Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])
)/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 +
#1^3 & , 3, 0])))]*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*((EllipticE[
ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1
^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]
*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))/(1 + Root[1 + 3*#1 - #1^2 + #
```

```
1^3 & , 1, 0]) - (EllipticPi[(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/(-
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2
 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]))]*(1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #
1^3 & , 2, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & ,...
```

Maple [A]
time = 0.87, size = 4640, normalized size = 1.15

method	result	size
default	Expression too large to display	4640
elliptic	Expression too large to display	4646

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(1/3*x*(x^4+2*x^2+4*x+1)^(1/2)+4/3*(-4/3-1/6
*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*
33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(1/3)-4/(
26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33
^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+
4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(
x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1/3*(2
6+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*33^(1/2))
^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+
6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3+1
/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3
*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+
4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3
/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/
3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))
)/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)/(1/2*(
26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(
1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26
+6*33^(1/2))^(1/3)+4/3)/((1+x)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1
/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-
1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/
```

6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(
26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2)*EllipticF(((1/2*(26
+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/
2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/
(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26
+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
-1/3))^(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(
1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+
6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1
/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/
(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*
3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+4*(
-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(
1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/
(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2)
)^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(
1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(
x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+
4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3
/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/
3)+4/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))
)/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3
*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2)
)^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(
1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1
/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2
)/(1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(2
6+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)
+8/3/(26+6*33^(1/2))^(1/3)+4/3)/((1+x)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26
+6*33^(1/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1
/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
))*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)
(-1/3(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2)*((-1/3*(26+
6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)*EllipticF(((1/2*(26+6*33^(
1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/
3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*3
3^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(
1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^
(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-
1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="m
axima")
```

```
[Out] 1/2*x^2*(2*sqrt(2) - 1) - sqrt(2)*integrate(sqrt(x^3 - x^2 + 3*x + 1)*sqrt(
x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(2*sqrt(2)*x - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1) - x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x\left(-1+2\sqrt{2}\right)-\sqrt{2}\,\sqrt{x^4+2x^2+4x+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)
```

```
[Out] Integral(x*(-1 + 2*sqrt(2)) - sqrt(2)*sqrt(x**4 + 2*x**2 + 4*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\,\sqrt{x^4+2\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2),x)
[Out] int(x*(2*2^(1/2) - 1) - 2^(1/2)*(4*x + 2*x^2 + x^4 + 1)^(1/2), x)
```

3.282 $\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\mathbf{mc}^8+4\mathbf{mc}^9+24\mathbf{mc}^6x-48\mathbf{mc}^7x-144\mathbf{mc}^5x^2-24\mathbf{r}\right.\right.}{}$

Optimal. Leaf size=330

$$\frac{e^{-\frac{x}{y}}(3-4\text{mc})\text{mc}^8\pi^2}{384x}+\frac{3}{8}e^{-\frac{x}{y}}\text{mc}^5\pi^2y+\frac{1}{48}e^{-\frac{x}{y}}(3-22\text{mc})\text{mc}^2\pi^2xy-\frac{1}{128}e^{-\frac{x}{y}}(1+4\text{mc})\pi^2x^2y+\frac{1}{48}e^{-\frac{x}{y}}(3-22\text{mc})\text{r}$$

```
[Out] 1/384*(3-4*mc)*mc^8*Pi^2/exp(x/y)/x+3/8*mc^5*Pi^2*y/exp(x/y)+1/48*(3-22*mc)
*mc^2*Pi^2*x*y/exp(x/y)-1/128*(1+4*mc)*Pi^2*x^2*y/exp(x/y)+1/48*(3-22*mc)*m
c^2*Pi^2*y^2/exp(x/y)+1/4*mc^3*Pi^2*y^2/exp(x/y)-1/64*(1+4*mc)*Pi^2*x*y^2/e
xp(x/y)-1/64*(1+4*mc)*Pi^2*y^3/exp(x/y)+1/16*(1-2*mc)*mc^6*Pi^2*Ei(-x/y)+1/
384*(3-4*mc)*mc^8*Pi^2*Ei(-x/y)/y+1/32*mc^3*Pi^2*(-12*mc^2+3*mc-8*y)*y*Ei(-
x/y)-1/32*mc^3*Pi^2*(3*(1-4*mc)*mc-8*x)*y*ln(x/mc^2)/exp(x/y)+1/4*mc^3*Pi^2
*y^2*ln(x/mc^2)/exp(x/y)
```

Rubi [A]
time = 0.63, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 107, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {12, 6874, 2230, 2225, 2208, 2209, 2207, 2634}

$$\frac{\pi^2(3-4\text{mc})\text{mc}^8\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}+\frac{1}{16}\pi^2(1-2\text{mc})\text{mc}^6\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{1}{32}\pi^2\text{mc}^3y\left(-12\text{mc}^2+3\text{mc}-8y\right)\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{\pi^2(3-4\text{mc})\text{mc}^8e^{-\frac{x}{y}}}{384x}+\frac{3}{8}\pi^2\text{mc}^5ye^{-\frac{x}{y}}+\frac{1}{4}\pi^2\text{mc}^3y^2e^{-\frac{x}{y}}+\frac{1}{48}\pi^2(3-22\text{mc})\text{mc}^2y^2e^{-\frac{x}{y}}+\frac{1}{48}\pi^2(3-22\text{mc})\text{mc}^2xye^{-\frac{x}{y}}+\frac{1}{4}\pi^2\text{mc}^3y^2e^{-\frac{x}{y}}\log\left(\frac{x}{\text{mc}^2}\right)-\frac{1}{32}\pi^2\text{mc}^3y(3(1-4\text{mc})\text{mc}-8x)e^{-\frac{x}{y}}\log\left(\frac{x}{\text{mc}^2}\right)-\frac{1}{128}\pi^2(4\text{mc}+1)x^2ye^{-\frac{x}{y}}-\frac{1}{64}\pi^2(4\text{mc}+1)y^3e^{-\frac{x}{y}}-\frac{1}{64}\pi^2(4\text{mc}+1)xy^2e^{-\frac{x}{y}}$$

Antiderivative was successfully verified.

```
[In] Int[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^
2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2 -
8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

```
[Out] ((3 - 4*mc)*mc^8*Pi^2)/(384*E^(x/y)*x) + (3*mc^5*Pi^2*y)/(8*E^(x/y)) + ((3
- 22*mc)*mc^2*Pi^2*x*y)/(48*E^(x/y)) - ((1 + 4*mc)*Pi^2*x^2*y)/(128*E^(x/y)
) + ((3 - 22*mc)*mc^2*Pi^2*y^2)/(48*E^(x/y)) + (mc^3*Pi^2*y^2)/(4*E^(x/y))
- ((1 + 4*mc)*Pi^2*x*y^2)/(64*E^(x/y)) - ((1 + 4*mc)*Pi^2*y^3)/(64*E^(x/y))
 + ((1 - 2*mc)*mc^6*Pi^2*ExpIntegralEi[-(x/y)])/16 + ((3 - 4*mc)*mc^8*Pi^2*
ExpIntegralEi[-(x/y)])/(384*y) + (mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpInt
egralEi[-(x/y)])/32 - (mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2])/(32
*E^(x/y)) + (mc^3*Pi^2*y^2*Log[x/mc^2])/(4*E^(x/y))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
```

```
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !TrueQ[$UseGamma]
```

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] && !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2230

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] && !TrueQ[$UseGamma]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{e^{-\frac{x}{y}} \left(\pi^2(-3\text{mc}^8 + 4\text{mc}^9 + 24\text{mc}^6 x - 48\text{mc}^7 x - 144\text{mc}^5 x^2 - 24\text{mc}^2 x^3 + 176\text{mc}^3 x^3 + 3x^4 + 12\text{mc}x^4) + 12\right.}{384x^2}$$

Mathematica [A]
time = 0.10, size = 181, normalized size = 0.55

$$\frac{1}{384}\pi^2\left(-\frac{\text{mc}^3(-3\text{mc}^5 + 4\text{mc}^6 - 24\text{mc}^3 y + 48\text{mc}^4 y - 36\text{mc}y^2 + 144\text{mc}^2 y^2 + 96y^3)\,\text{Ei}\left(-\frac{x}{y}\right)}{y} + \frac{e^{-\frac{x}{y}}\left(3\text{mc}^8 - 4\text{mc}^9 + 144\text{mc}^5 xy + 24\text{mc}^2 xy(x+y) - 16\text{mc}^3 xy(11x+5y) - 3xy(x^2+2xy+2y^2) - 12\text{mc}xy(x^2+2xy+2y^2) + 12\text{mc}^3 xy(-3\text{mc} + 12\text{mc}^2 + 8(x+y))\log\left(\frac{x}{\text{mc}^2}\right)\right)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 -
24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*m
c^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

```
[Out] (Pi^2*(-((mc^3*(-3*mc^5 + 4*mc^6 - 24*mc^3*y + 48*mc^4*y - 36*mc*y^2 + 144*
mc^2*y^2 + 96*y^3)*ExpIntegralEi[-(x/y)])/y) + (3*mc^8 - 4*mc^9 + 144*mc^5*
x*y + 24*mc^2*x*y*(x + y) - 16*mc^3*x*y*(11*x + 5*y) - 3*x*y*(x^2 + 2*x*y +
 2*y^2) - 12*mc*x*y*(x^2 + 2*x*y + 2*y^2) + 12*mc^3*x*y*(-3*mc + 12*mc^2 +
8*(x + y))*Log[x/mc^2])/(E^(x/y)*x)))/384
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 1356, normalized size = 4.11

method	result	size
risch	Expression too large to display	1356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/384*(Pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^
3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*Pi^2*(-12*mc^2+3*mc-8*x)*x^2*ln(x/mc
^2))/exp(x/y)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*y*Pi^2*ln(mc)*mc^3*x*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc^2)^3*exp(
-x/y)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc^2)^3+3/64*I*y*Pi^3*exp(-x/y)*m
c^4*csgn(I/mc^2*x)^3+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*mc^2)^3-3/16*I*y*P
i^3*exp(-x/y)*mc^5*csgn(I/mc^2*x)^3-1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2*x)^3*ex
p(-x/y)-1/8*I*y*Pi^3*mc^3*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)*x*exp(-x/y)
+1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x)^2*x*exp(-x/y)-1/8*I*y^2*Pi^3
*mc^3*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csg
n(I*x)*csgn(I/mc^2*x)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*m
c^2)*x*exp(-x/y)-1/4*I*y*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*x*exp(-x/y)+3/
64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)-3/16*I*y*P
i^3*exp(-x/y)*mc^5*csgn(I*x)*csgn(I/mc^2)*csgn(I/mc^2*x)-1/16*y^3*Pi^2*mc*e
xp(-x/y)+1/16*y^2*Pi^2*mc^2*exp(-x/y)-1/128*y*Pi^2*x^2*exp(-x/y)-1/64*y^2*P
i^2*x*exp(-x/y)+3/8*y*Pi^2*exp(-x/y)*mc^5-1/128/y*Pi^2*mc^8*Ei(1,x/y)+1/96/
y*Pi^2*mc^9*Ei(1,x/y)+1/128*Pi^2*mc^8/x*exp(-x/y)-1/96*Pi^2*mc^9/x*exp(-x/y
)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc)^2*csgn(I*mc^2)+1/8*I*y^2*Pi^3*mc^
3*csgn(I*x)*csgn(I/mc^2*x)^2*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csg
n(I/mc^2*x)^2*exp(-x/y)-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2*x)^3*x*exp(-x/y)+1/8*
I*y*Pi^3*mc^3*csgn(I*mc^2)^3*x*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc)^2*c
sgn(I*mc^2)*exp(-x/y)-1/4*I*y^2*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*exp(-x/
y)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x)^2+3/16*I*y*Pi^3
*exp(-x/y)*mc^5*csgn(I*mc)^2*csgn(I*mc^2)-3/8*I*y*Pi^3*exp(-x/y)*mc^5*csgn(
I*mc)*csgn(I*mc^2)^2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*x)*csgn(I/mc^2*x)^
2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I/mc^2*x)^2+3/32*I*y*Pi^3*
exp(-x/y)*mc^4*csgn(I*mc)*csgn(I*mc^2)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(
I*x)*csgn(I/mc^2*x)^2+1/384*(144*Pi^2*mc^5*y-36*Pi^2*mc^4*y+96*Pi^2*mc^3*x*
y+96*Pi^2*mc^3*y^2)*exp(-x/y)*ln(x)+1/4*Pi^2*mc^3*y^2*Ei(1,x/y)-3/32*Pi^2*m
c^4*y*Ei(1,x/y)+3/8*Pi^2*mc^5*y*Ei(1,x/y)-5/24*mc^3*Pi^2*y^2*exp(-x/y)+1/8*
Pi^2*mc^7*Ei(1,x/y)-1/16*Pi^2*mc^6*Ei(1,x/y)-1/64*y^3*Pi^2*exp(-x/y)-11/24*
y*Pi^2*mc^3*x*exp(-x/y)-1/32*y*Pi^2*mc*x^2*exp(-x/y)-1/16*y^2*Pi^2*mc*x*exp
(-x/y)+1/16*y*Pi^2*mc^2*x*exp(-x/y)-1/2*y^2*Pi^2*ln(mc)*mc^3*exp(-x/y)-3/4*
y*Pi^2*exp(-x/y)*ln(mc)*mc^5+3/16*y*Pi^2*exp(-x/y)*ln(mc)*mc^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="maxima")
```

```
[Out] -1/96*pi^2*mc^9*gamma(-1, x/y)/y - 1/8*pi^2*mc^7*Ei(-x/y) + 1/128*pi^2*mc^8
*gamma(-1, x/y)/y + 3/8*pi^2*mc^5*y*e^(-x/y)*log(x/mc^2) + 1/16*pi^2*mc^6*E
i(-x/y) - 3/8*pi^2*mc^5*y*Ei(-x/y) + 3/8*pi^2*mc^5*y*e^(-x/y) - 3/32*pi^2*m
c^4*y*e^(-x/y)*log(x/mc^2) + 3/32*pi^2*mc^4*y*Ei(-x/y) - 11/24*pi^2*(x*y +
y^2)*mc^3*e^(-x/y) + 1/4*pi^2*((x*y + y^2)*e^(-x/y)*log(x) + integrate((2*x
^2*log(mc) - x*y - y^2)*e^(-x/y)/x, x))*mc^3 + 1/16*pi^2*(x*y + y^2)*mc^2*e
^(-x/y) - 1/32*pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*mc*e^(-x/y) - 1/128*pi^2*(x^2
*y + 2*x*y^2 + 2*y^3)*e^(-x/y)
```

Fricas [A]
time = 0.65, size = 269, normalized size = 0.82

$$\frac{12\,(8\,\pi^2 mc^3 x y^3 + (8\,\pi^2 mc^3 x^2 + 3\,\pi^2(4\,mc^5 - mc^4)x)y^2)e^{\left(-\frac{x}{y}\right)}\log\left(\frac{x}{mc^2}\right) - (96\,\pi^2 mc^3 x y^3 + 36\,\pi^2(4\,mc^5 - mc^4)xy^2 + 24\,\pi^2(2\,mc^7 - mc^6)xy + \pi^2(4\,mc^9 - 3\,mc^8)x)\mathrm{Ei}\left(-\frac{x}{y}\right) - (6\,\pi^2(4\,mc + 1)xy^4 + \pi^2(4\,mc^9 - 3\,mc^8)y + 2\,(3\,\pi^2(4\,mc + 1)x^2 + 4\,\pi^2(10\,mc^3 - 3\,mc^2)x)y^3 - (144\,\pi^2 mc^5 x - 3\,\pi^2(4\,mc + 1)x^3 - 8\,\pi^2(22\,mc^3 - 3\,mc^2)x^2)y^2)e^{\left(-\frac{x}{y}\right)}}{384\,xy}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="fricas")
```

```
[Out] 1/384*(12*(8*pi^2*mc^3*x*y^3 + (8*pi^2*mc^3*x^2 + 3*pi^2*(4*mc^5 - mc^4)*x)
*y^2)*e^(-x/y)*log(x/mc^2) - (96*pi^2*mc^3*x*y^3 + 36*pi^2*(4*mc^5 - mc^4)*
x*y^2 + 24*pi^2*(2*mc^7 - mc^6)*x*y + pi^2*(4*mc^9 - 3*mc^8)*x)*Ei(-x/y) -
(6*pi^2*(4*mc + 1)*x*y^4 + pi^2*(4*mc^9 - 3*mc^8)*y + 2*(3*pi^2*(4*mc + 1)*
x^2 + 4*pi^2*(10*mc^3 - 3*mc^2)*x)*y^3 - (144*pi^2*mc^5*x - 3*pi^2*(4*mc +
1)*x^3 - 8*pi^2*(22*mc^3 - 3*mc^2)*x^2)*y^2)*e^(-x/y))/(x*y)
```

Sympy [A]
time = 9.75, size = 330, normalized size = 1.00

$$-\frac{\pi^2 mc^9 \operatorname{E}_2\left(\frac{x}{y}\right)}{96x} + \frac{\pi^2 mc^8 \operatorname{E}_2\left(\frac{x}{y}\right)}{128x} - \frac{\pi^2 mc^7 \operatorname{Ei}\left(-\frac{x}{y}\right)}{8} + \frac{\pi^2 mc^6 \operatorname{Ei}\left(-\frac{x}{y}\right)}{16} + \frac{3\pi^2 mc^5 y e^{-\frac{x}{y}}}{8} - \frac{3\pi^2 mc^5 \left(y \operatorname{Ei}\left(-\frac{x}{y}\right) - y e^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right)\right)}{8} + \frac{3\pi^2 mc^4 \left(y \operatorname{Ei}\left(-\frac{x}{y}\right) - y e^{-\frac{x}{y}} \log\left(\frac{x}{mc^2}\right)\right)}{32} + \frac{11\pi^2 mc^3 \left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)}{24} - \frac{\pi^2 mc^3 \left(y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) - y^2 e^{-\frac{x}{y}} + \left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)\log\left(\frac{x}{mc^2}\right)\right)}{4} - \frac{\pi^2 mc^2 \left(-xye^{-\frac{x}{y}} - y^2 e^{-\frac{x}{y}}\right)}{16} + \frac{\pi^2 mc \left(-x^2 y e^{-\frac{x}{y}} - 2xy^2 e^{-\frac{x}{y}} - 2y^3 e^{-\frac{x}{y}}\right)}{32} + \frac{\pi^2 \left(-x^2 y e^{-\frac{x}{y}} - 2xy^2 e^{-\frac{x}{y}} - 2y^3 e^{-\frac{x}{y}}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-144*mc**5*x**
2+176*mc**3*x**3-24*mc**2*x**3+12*mc*x**4+3*x**4)+12*mc**3*pi**2*(-12*mc**2
+3*mc-8*x)*x**2*ln(x/mc**2))/exp(x/y)/x**2,x)
```

```
[Out] -pi**2*mc**9*expint(2, x/y)/(96*x) + pi**2*mc**8*expint(2, x/y)/(128*x) - p
i**2*mc**7*Ei(-x/y)/8 + pi**2*mc**6*Ei(-x/y)/16 + 3*pi**2*mc**5*y*exp(-x/y)
/8 - 3*pi**2*mc**5*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/8 + 3*pi**2*mc**
4*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/32 + 11*pi**2*mc**3*(-x*y*exp(-x/
```

```
y) - y**2*exp(-x/y))/24 - pi**2*mc**3*(y**2*Ei(-x/y) - y**2*exp(-x/y) + (-x
*y*exp(-x/y) - y**2*exp(-x/y))*log(x/mc**2))/4 - pi**2*mc**2*(-x*y*exp(-x/y
) - y**2*exp(-x/y))/16 + pi**2*mc*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) -
 2*y**3*exp(-x/y))/32 + pi**2*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) - 2*y
**3*exp(-x/y))/128
```

Giac [A]
time = 0.61, size = 472, normalized size = 1.43

$$-\frac{4\,\pi^2 mc^9 x \operatorname{Ei}\left(-\frac{x}{y}\right) + 4\,\pi^2 mc^9 y e^{\left(-\frac{x}{y}\right)} - 3\,\pi^2 mc^8 x \operatorname{Ei}\left(-\frac{x}{y}\right) + 48\,\pi^2 mc^7 x y \operatorname{Ei}\left(-\frac{x}{y}\right) - 3\,\pi^2 mc^8 y e^{\left(-\frac{x}{y}\right)} - 144\,\pi^2 mc^5 x y^2 e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{mc^2}\right) - 24\,\pi^2 mc^6 x y \operatorname{Ei}\left(-\frac{x}{y}\right) + 144\,\pi^2 mc^5 x y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) - 144\,\pi^2 mc^5 x y^2 e^{\left(-\frac{x}{y}\right)} + 36\,\pi^2 mc^4 x y^2 e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{mc^2}\right) - 96\,\pi^2 mc^3 x^2 y^2 e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{mc^2}\right) - 96\,\pi^2 mc^3 x y^3 e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{mc^2}\right) - 36\,\pi^2 mc^4 x y^2 \operatorname{Ei}\left(-\frac{x}{y}\right) + 96\,\pi^2 mc^3 x y^3 \operatorname{Ei}\left(-\frac{x}{y}\right) + 176\,\pi^2 mc^3 x^2 y^2 e^{\left(-\frac{x}{y}\right)} + 80\,\pi^2 mc^3 x y^3 e^{\left(-\frac{x}{y}\right)} - 24\,\pi^2 mc^2 x^2 y^2 e^{\left(-\frac{x}{y}\right)} + 12\,\pi^2 mc x^3 y^2 e^{\left(-\frac{x}{y}\right)} - 24\,\pi^2 mc^2 x y^3 e^{\left(-\frac{x}{y}\right)} + 24\,\pi^2 mc x^2 y^3 e^{\left(-\frac{x}{y}\right)} + 24\,\pi^2 mc x y^4 e^{\left(-\frac{x}{y}\right)} + 3\,\pi^2 x^3 y^2 e^{\left(-\frac{x}{y}\right)} + 6\,\pi^2 x^2 y^3 e^{\left(-\frac{x}{y}\right)} + 6\,\pi^2 x y^4 e^{\left(-\frac{x}{y}\right)}}{384\,x y}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="giac")
```

```
[Out] -1/384*(4*pi^2*mc^9*x*Ei(-x/y) + 4*pi^2*mc^9*y*e^(-x/y) - 3*pi^2*mc^8*x*Ei(
-x/y) + 48*pi^2*mc^7*x*y*Ei(-x/y) - 3*pi^2*mc^8*y*e^(-x/y) - 144*pi^2*mc^5*
x*y^2*e^(-x/y)*log(x/mc^2) - 24*pi^2*mc^6*x*y*Ei(-x/y) + 144*pi^2*mc^5*x*y^
2*Ei(-x/y) - 144*pi^2*mc^5*x*y^2*e^(-x/y) + 36*pi^2*mc^4*x*y^2*e^(-x/y)*log
(x/mc^2) - 96*pi^2*mc^3*x^2*y^2*e^(-x/y)*log(x/mc^2) - 96*pi^2*mc^3*x*y^3*e
^(-x/y)*log(x/mc^2) - 36*pi^2*mc^4*x*y^2*Ei(-x/y) + 96*pi^2*mc^3*x*y^3*Ei(-
x/y) + 176*pi^2*mc^3*x^2*y^2*e^(-x/y) + 80*pi^2*mc^3*x*y^3*e^(-x/y) - 24*pi
^2*mc^2*x^2*y^2*e^(-x/y) + 12*pi^2*mc*x^3*y^2*e^(-x/y) - 24*pi^2*mc^2*x*y^3
*e^(-x/y) + 24*pi^2*mc*x^2*y^3*e^(-x/y) + 24*pi^2*mc*x*y^4*e^(-x/y) + 3*pi^
2*x^3*y^2*e^(-x/y) + 6*pi^2*x^2*y^3*e^(-x/y) + 6*pi^2*x*y^4*e^(-x/y))/(x*y)
```

Mupad [B]
time = 0.77, size = 265, normalized size = 0.80

$$\operatorname{ei}\left(-\frac{x}{y}\right)\left(\frac{\frac{\Pi^2\,\mathrm{mc}^8}{128}-\frac{\Pi^2\,\mathrm{mc}^9}{96}}{y}+\frac{\Pi^2\,\mathrm{mc}^6}{16}-\frac{\Pi^2\,\mathrm{mc}^7}{8}+y\left(\frac{3\,\Pi^2\,\mathrm{mc}^4}{32}-\frac{3\,\Pi^2\,\mathrm{mc}^5}{8}\right)-\frac{\Pi^2\,\mathrm{mc}^3\,y^2}{4}\right)-\frac{2\,\Pi^2\,x^2\,y\,\mathrm{e}^{-\frac{x}{y}}\,\left(-72\,\mathrm{mc}^5+40\,\mathrm{mc}^3\,y-12\,\mathrm{mc}^2\,y+12\,\mathrm{mc}\,y^2+3\,y^2\right)+2\,\Pi^2\,x^3\,y\,\mathrm{e}^{-\frac{x}{y}}\,\left(88\,\mathrm{mc}^3-12\,\mathrm{mc}^2+12\,y\,\mathrm{mc}+3\,y\right)+\Pi^2\,\mathrm{mc}^8\,x\,\mathrm{e}^{-\frac{x}{y}}\,\left(4\,\mathrm{mc}-3\right)+3\,\Pi^2\,x^4\,y\,\mathrm{e}^{-\frac{x}{y}}\,\left(4\,\mathrm{mc}+1\right)-96\,\Pi^2\,\mathrm{mc}^3\,x^3\,y\,\ln\left(\frac{x}{\mathrm{mc}^2}\right)\,\mathrm{e}^{-\frac{x}{y}}-12\,\Pi^2\,\mathrm{mc}^3\,x^2\,y\,\ln\left(\frac{x}{\mathrm{mc}^2}\right)\,\mathrm{e}^{-\frac{x}{y}}\,\left(12\,\mathrm{mc}^2-3\,\mathrm{mc}+8\,y\right)}{384\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(-x/y)*((Pi^2*(176*mc^3*x^3 - 24*mc^2*x^3 - 144*mc^5*x^2 + 12*mc*x^
4 + 24*mc^6*x - 48*mc^7*x - 3*mc^8 + 4*mc^9 + 3*x^4))/384 - (Pi^2*mc^3*x^2*
log(x/mc^2)*(8*x - 3*mc + 12*mc^2))/32))/x^2,x)
```

```
[Out] ei(-x/y)*(((Pi^2*mc^8)/128 - (Pi^2*mc^9)/96)/y + (Pi^2*mc^6)/16 - (Pi^2*mc^
7)/8 + y*((3*Pi^2*mc^4)/32 - (3*Pi^2*mc^5)/8) - (Pi^2*mc^3*y^2)/4) - (2*Pi^
2*x^2*y*exp(-x/y)*(12*mc*y^2 - 12*mc^2*y + 40*mc^3*y - 72*mc^5 + 3*y^2) + 2
*Pi^2*x^3*y*exp(-x/y)*(3*y + 12*mc*y - 12*mc^2 + 88*mc^3) + Pi^2*mc^8*x*exp
(-x/y)*(4*mc - 3) + 3*Pi^2*x^4*y*exp(-x/y)*(4*mc + 1) - 96*Pi^2*mc^3*x^3*y*
log(x/mc^2)*exp(-x/y) - 12*Pi^2*mc^3*x^2*y*log(x/mc^2)*exp(-x/y)*(8*y - 3*m
c + 12*mc^2))/(384*x^2)
```

3.283 $\int \sec(x)\sin(2x)\,dx$

Optimal. Leaf size=4

$$-2\cos(x)$$

```
[Out] -2*cos(x)
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4372, 2718}

$$-2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sin[2*x],x]
```

```
[Out] -2*Cos[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \sec(x)\sin(2x)\,dx &= 2\int \sin(x)\,dx \\ &= -2\cos(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sin[2*x],x]
```

```
[Out] -2*Cos[x]
```

Maple [A]
time = 0.03, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$-2\cos(x)$	5
default	$-2\cos(x)$	5
risch	$-2\cos(x)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] -2*cos(x)
```

Maxima [A]
time = 2.66, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="maxima")
```

```
[Out] -2*cos(x)
```

Fricas [A]
time = 0.64, size = 4, normalized size = 1.00

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="fricas")
```

```
[Out] -2*cos(x)
```

Sympy [A]
time = 0.38, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

Giac [**A**]
time = 0.50, size = 4, normalized size = 1.00

$$-2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="giac")
```

```
[Out] -2*cos(x)
```

Mupad [**B**]
time = 0.02, size = 4, normalized size = 1.00

$$-2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

3.284 $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$

Optimal. Leaf size=71

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(1+x+\sqrt{2}\,x+\sqrt{2}\,x^2-x^7\right)-\left(-1+\sqrt{2}\right)\log\left(-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}\,x^2+x^7\right)\right)$$

```
[Out] -1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))*(2^(1/2)-1)+1/2*ln(1+x-x^7+x*2^(1
/2)+x^2*2^(1/2))*(1+2^(1/2))
```

Rubi [F]
time = 0.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x -
x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

```
[Out] Log[1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14]/2 + 2*Defer[Int][
(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14)^(-1), x] + 4*Defer[I
nt][x/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 2*Defer[
Int][x^2/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 12*De
fer[Int][x^7/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 1
0*Defer[Int][x^8/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x]
```

Rubi steps

$$\begin{aligned}\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx &= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)\\ &= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)\\ &= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)\\ &= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)\\ &= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.00

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(1+x+\sqrt{2}\,x+\sqrt{2}\,x^2-x^7\right)-\left(-1+\sqrt{2}\right)\log\left(-1+\left(-1+\sqrt{2}\right)x+\sqrt{2}\,x^2+x^7\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 +
2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

```
[Out] ((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 + Sqrt[2])*
Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2
```

Maple [A]
time = 0.04, size = 61, normalized size = 0.86

method	result
default	$\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\ln\left(x^7-x^2\sqrt{2}+\left(-1-\sqrt{2}\right)x-1\right)+\left(-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\ln\left(-1+x^7+x\left(\sqrt{2}-1\right)+\right.$
risch	$\frac{\ln\left(x^7-x^2\sqrt{2}+\left(-1-\sqrt{2}\right)x-1\right)}{2}+\frac{\ln\left(x^7-x^2\sqrt{2}+\left(-1-\sqrt{2}\right)x-1\right)\sqrt{2}}{2}-\frac{\ln\left(-1+x^7+x\left(\sqrt{2}-1\right)+x^2\sqrt{2}\right)\sqrt{}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4
*x^3-x^2+2*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2+1/2*2^(1/2))*ln(x^7-x^2*2^(1/2)+(-1-2^(1/2))*x-1)+(-1/2*2^(1/2)+1/2)*l
n(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14
 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(56) = 112$.
time = 0.59, size = 137, normalized size = 1.93

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^{14}-2\,x^8-2\,x^7+2\,x^4+4\,x^3+3\,x^2-2\,\sqrt{2}\,(x^9+x^8-x^3-2\,x^2-x)+2\,x+1}{x^{14}-2\,x^8-2\,x^7-2\,x^4-4\,x^3-x^2+2\,x+1}\right)+\frac{1}{2}\log\left(x^{14}-2\,x^8-2\,x^7-2\,x^4-4\,x^3-x^2+2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((x^14 - 2*x^8 - 2*x^7 + 2*x^4 + 4*x^3 + 3*x^2 - 2*sqrt(2)*(
x^9 + x^8 - x^3 - 2*x^2 - x) + 2*x + 1)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x
^3 - x^2 + 2*x + 1)) + 1/2*log(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 +
 2*x + 1)
```

Sympy [A]
time = 0.09, size = 76, normalized size = 1.07

$$\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^7-\sqrt{2}\,x^2-2x\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)-1\right)+\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)\log\left(x^7+\sqrt{2}\,x^2-2x\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-2*x**8
-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)
```

```
[Out] (1/2 + sqrt(2)/2)*log(x**7 - sqrt(2)*x**2 - 2*x*(1/2 + sqrt(2)/2) - 1) + (1
/2 - sqrt(2)/2)*log(x**7 + sqrt(2)*x**2 - 2*x*(1/2 - sqrt(2)/2) - 1)
```

Giac [A]
time = 0.46, size = 94, normalized size = 1.32

$$-\frac{1}{2}\sqrt{2}\log\left(\left|x^7+\sqrt{2}\,x^2+\sqrt{2}\,x-x-1\right|\right)+\frac{1}{2}\sqrt{2}\log\left(\left|x^7-\sqrt{2}\,x^2-\sqrt{2}\,x-x-1\right|\right)+\frac{1}{2}\log\left(\left|x^{14}-2\,x^8-2\,x^7-2\,x^4-4\,x^3-x^2+2\,x+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(x^7 + sqrt(2)*x^2 + sqrt(2)*x - x - 1)) + 1/2*sqrt(2)*
log(abs(x^7 - sqrt(2)*x^2 - sqrt(2)*x - x - 1)) + 1/2*log(abs(x^14 - 2*x^8
- 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1))
```

Mupad [B]
time = 0.28, size = 103, normalized size = 1.45

$$\frac{\ln\left(\sqrt{2}\,x-x+\sqrt{2}\,x^2+x^7-1\right)}{2}+\frac{\ln\left(x^7-\sqrt{2}\,x-\sqrt{2}\,x^2-x-1\right)}{2}-\frac{\sqrt{2}\,\ln\left(\sqrt{2}\,x-x+\sqrt{2}\,x^2+x^7-1\right)}{2}+\frac{\sqrt{2}\,\ln\left(x^7-\sqrt{2}\,x-\sqrt{2}\,x^2-x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13 + 3)/(x^2 - 2*x
 + 4*x^3 + 2*x^4 + 2*x^7 + 2*x^8 - x^14 - 1),x)
```

```
[Out] log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1)/2 + log(x^7 - 2^(1/2)*x - 2^(1/2
)*x^2 - x - 1)/2 - (2^(1/2)*log(2^(1/2)*x - x + 2^(1/2)*x^2 + x^7 - 1))/2 +
 (2^(1/2)*log(x^7 - 2^(1/2)*x - 2^(1/2)*x^2 - x - 1))/2
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)  #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(5,m1)  #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(6,m1)  #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```