Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/4-Charlwood-Problems

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [50]. This is test number [4].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (50)	0.00 (0)
Rubi	96.00 (48)	4.00 (2)
Fricas	96.00 (48)	4.00 (2)
Giac	82.00 (41)	18.00 (9)
Maple	66.00 (33)	34.00 (17)
Maxima	48.00 (24)	52.00 (26)
Sympy	38.00 (19)	62.00 (31)
Mupad	24.00 (12)	76.00 (38)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

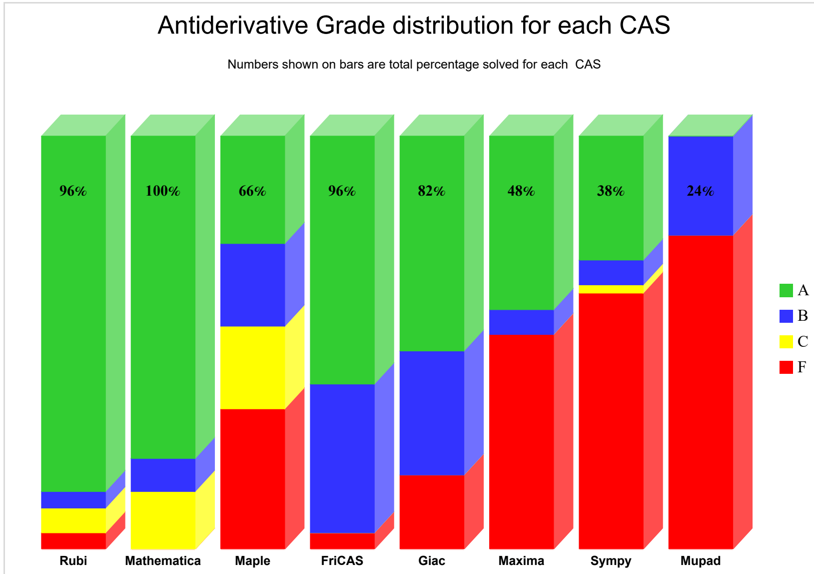
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

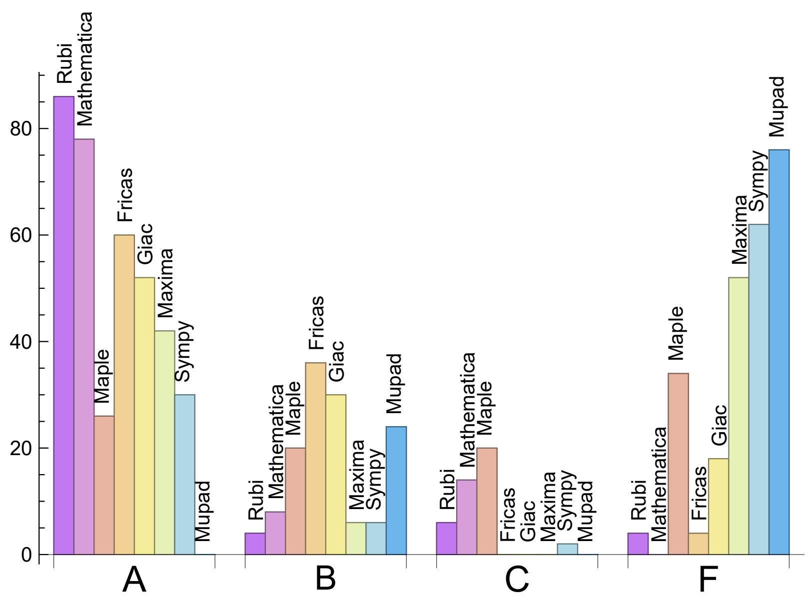
System	% A grade	% B grade	% C grade	% F grade
Rubi	86.00	4.00	6.00	4.00
Mathematica	78.00	8.00	14.00	0.00
Fricas	60.00	36.00	0.00	4.00
Giac	52.00	30.00	0.00	18.00
Maxima	42.00	6.00	0.00	52.00
Sympy	30.00	6.00	2.00	62.00
Maple	26.00	20.00	20.00	34.00
Mupad	N/A	24.00	0.00	76.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	17	100.00 %	0.00 %	0.00 %
Fricas	2	0.00 %	100.00 %	0.00 %
Giac	9	100.00 %	0.00 %	0.00 %
Maxima	26	92.31 %	0.00 %	7.69 %
Sympy	31	70.97 %	22.58 %	6.45 %
Mupad	38	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

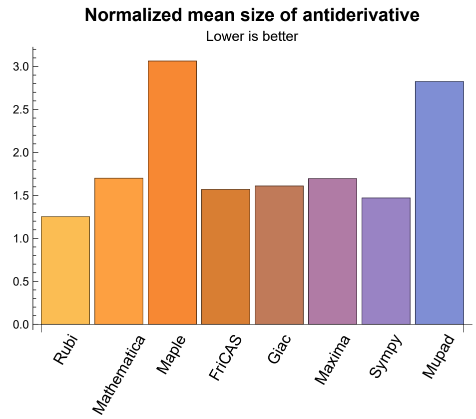
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

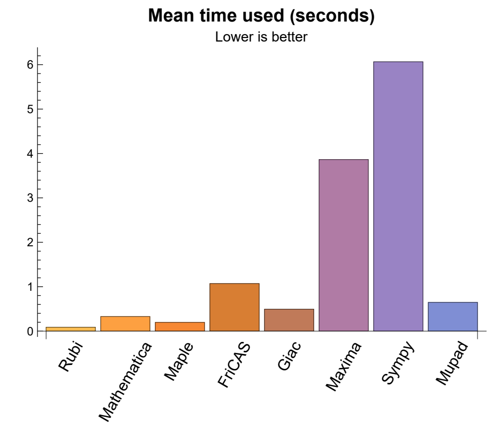
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	66.25	1.25	40.00	1.00
Mathematica	0.33	118.44	1.70	45.50	1.00
Maple	0.20	192.73	3.06	67.00	1.98
Maxima	3.86	53.67	1.69	26.50	0.88
Fricas	1.07	74.44	1.57	48.50	1.35
Sympy	6.07	43.89	1.47	31.00	0.94
Giac	0.49	88.37	1.61	50.00	1.41
Mupad	0.65	223.25	2.82	69.00	1.98

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
6
5
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{5, 48\}$

Mathematica $\{9, 45\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

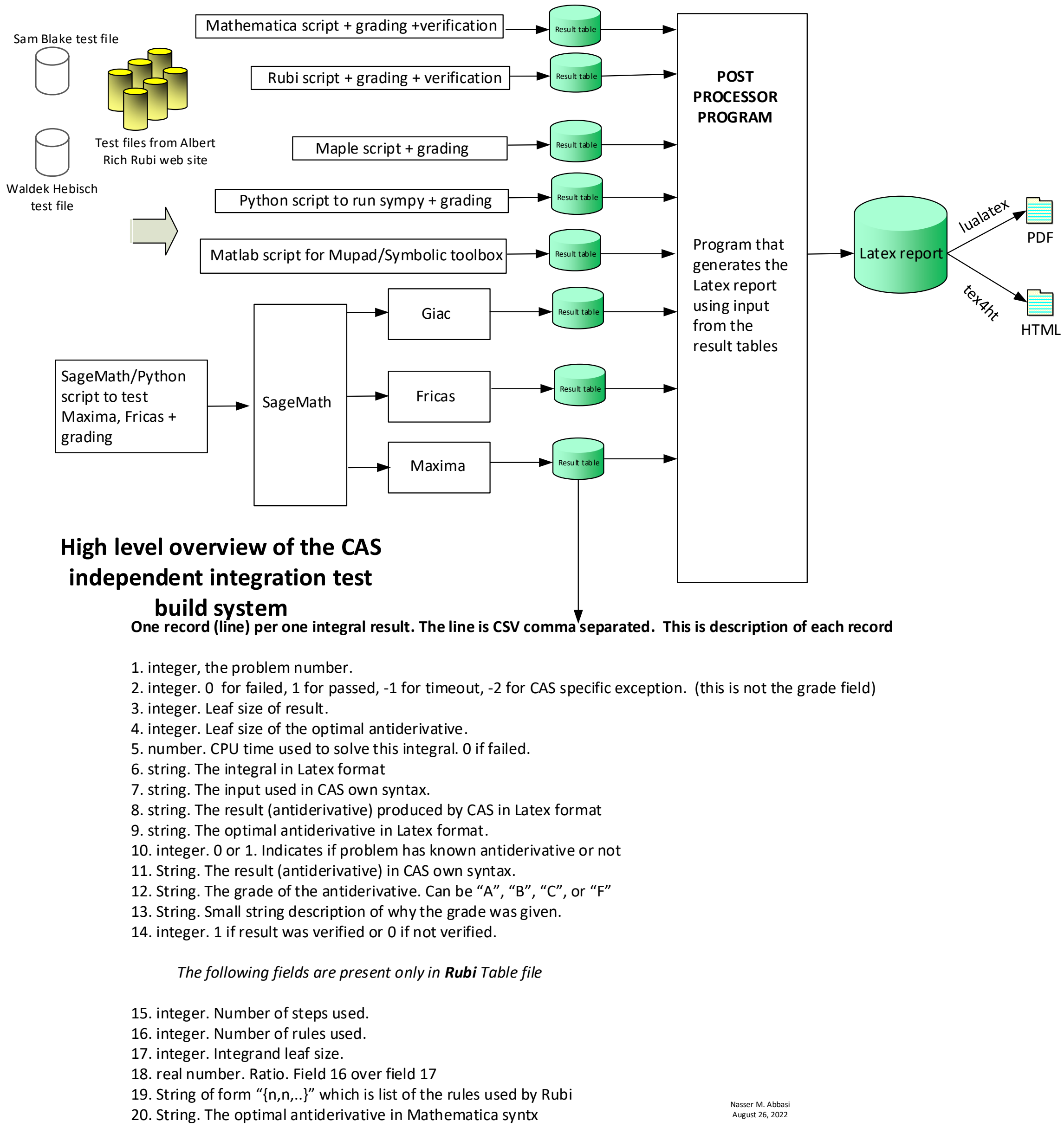
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 48, 49, 50 }

B grade: { 4, 42 }

C grade: { 5, 12, 13 }

F grade: { 3, 45 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 6, 7, 10, 11, 14, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 45, 46, 48, 49, 50 }

B grade: { 15, 17, 21, 41 }

C grade: { 5, 8, 9, 12, 13, 37, 47 }

F grade: { }

2.1.3 Maple

A grade: { 2, 6, 7, 9, 19, 24, 27, 32, 34, 36, 37, 43, 48 }

B grade: { 1, 3, 4, 8, 40, 42, 44, 47, 49, 50 }

C grade: { 5, 12, 18, 28, 31, 33, 35, 38, 39, 46 }

F grade: { 10, 11, 13, 14, 15, 16, 17, 20, 21, 22, 23, 25, 26, 29, 30, 41, 45 }

2.1.4 Maxima

A grade: { 1, 2, 3, 9, 16, 18, 19, 20, 24, 25, 27, 28, 31, 32, 33, 34, 35, 36, 37, 46, 48 }

B grade: { 7, 40, 43 }

C grade: { }

F grade: { 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 17, 21, 22, 23, 26, 29, 30, 38, 39, 41, 42, 44, 45, 47, 49, 50 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 9, 10, 11, 12, 13, 14, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 46, 48 }

B grade: { 4, 6, 7, 15, 16, 17, 21, 22, 37, 38, 40, 41, 42, 43, 44, 47, 49, 50 }

C grade: { }

F grade: { 8, 45 }

2.1.6 Sympy

A grade: { 1, 2, 10, 18, 19, 20, 24, 25, 31, 33, 34, 35, 36, 46, 48 }

B grade: { 28, 37, 40 }

C grade: { 16 }

F grade: { 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 17, 21, 22, 23, 26, 27, 29, 30, 32, 38, 39, 41, 42, 43, 44, 45, 47, 49, 50 }

2.1.7 Giac

A grade: { 2, 6, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 28, 33, 34, 35, 36, 40, 43, 47, 48, 49, 50 }

B grade: { 1, 4, 7, 8, 12, 13, 27, 29, 30, 31, 32, 37, 41, 42, 44 }

C grade: { }

F grade: { 3, 5, 9, 11, 23, 38, 39, 45, 46 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 9, 12, 19, 24, 30, 37, 40, 46, 47, 48, 50 }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 10, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 41, 42, 43, 44, 45, 49 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	93	58	54	102	272	-1
N.S.	1	1.00	1.02	1.82	1.14	1.06	2.00	5.33	-0.02
time (sec)	N/A	0.020	0.017	0.030	4.014	0.661	3.981	0.512	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	12	15	-1
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.71	0.88	-0.06
time (sec)	N/A	0.020	0.005	0.063	2.559	0.688	0.062	0.456	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	B	A	A	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	106	251	4	49	0	0	-1
N.S.	1	0.00	1.54	3.64	0.06	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.715	0.578	4.155	22.130	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	332	188	426	0	402	0	235	666
N.S.	1	3.42	1.94	4.39	0.00	4.14	0.00	2.42	6.87
time (sec)	N/A	0.452	0.337	0.065	0.000	1.036	0.000	0.519	1.573

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F(-2)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	289	159	312	0	33	0	0	-1
N.S.	1	6.42	3.53	6.93	0.00	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.333	1.503	0.404	0.000	0.925	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	74	64	0	88	0	79	-1
N.S.	1	1.00	1.32	1.14	0.00	1.57	0.00	1.41	-0.02
time (sec)	N/A	0.050	0.087	0.060	0.000	0.751	0.000	0.445	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	27	30	0	28	-1
N.S.	1	1.00	1.00	0.80	1.80	2.00	0.00	1.87	-0.07
time (sec)	N/A	0.024	0.012	0.108	2.675	0.618	0.000	0.469	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	99	1604	0	0	0	495	-1
N.S.	1	1.00	0.72	11.71	0.00	0.00	0.00	3.61	-0.01
time (sec)	N/A	0.130	3.665	0.464	0.000	0.000	0.000	0.511	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	283	42	60	32	0	0	60
N.S.	1	1.00	6.90	1.02	1.46	0.78	0.00	0.00	1.46
time (sec)	N/A	0.017	2.922	0.073	2.594	1.058	0.000	0.000	0.417

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	62	38	0	0	28	56	46	-1
N.S.	1	1.41	0.86	0.00	0.00	0.64	1.27	1.05	-0.02
time (sec)	N/A	0.471	0.132	0.014	0.000	0.780	0.379	0.447	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	64	0	0	43	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.038	0.012	0.000	0.869	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	269	307	439	0	191	0	364	661
N.S.	1	1.91	2.18	3.11	0.00	1.35	0.00	2.58	4.69
time (sec)	N/A	0.533	0.323	0.061	0.000	0.821	0.000	0.452	1.373

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	286	2180	0	0	200	0	373	-1
N.S.	1	1.88	14.34	0.00	0.00	1.32	0.00	2.45	-0.01
time (sec)	N/A	0.321	3.248	0.095	0.000	0.933	0.000	0.501	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	51	44	0	0	63	0	57	-1
N.S.	1	1.13	0.98	0.00	0.00	1.40	0.00	1.27	-0.02
time (sec)	N/A	0.083	0.030	180.000	0.000	0.771	0.000	0.470	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	76	0	0	62	0	36	-1
N.S.	1	1.00	2.24	0.00	0.00	1.82	0.00	1.06	-0.03
time (sec)	N/A	0.026	0.085	0.011	0.000	0.678	0.000	0.478	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	18	56	78	18	-1
N.S.	1	1.00	1.00	0.00	0.82	2.55	3.55	0.82	-0.05
time (sec)	N/A	0.014	0.022	0.076	3.798	0.659	18.164	0.465	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	89	0	0	58	0	36	-1
N.S.	1	1.00	2.78	0.00	0.00	1.81	0.00	1.12	-0.03
time (sec)	N/A	0.028	0.068	0.008	0.000	0.509	0.000	0.508	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	89	41	32	34	62	-1
N.S.	1	1.00	1.00	2.07	0.95	0.74	0.79	1.44	-0.02
time (sec)	N/A	0.031	0.019	0.033	3.148	0.480	2.311	0.497	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	34	34	48	35	174
N.S.	1	1.00	1.00	0.75	1.21	1.21	1.71	1.25	6.21
time (sec)	N/A	0.009	0.019	0.125	4.981	0.463	0.556	0.471	0.135

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	22	22	20	22	-1
N.S.	1	1.00	1.00	0.00	0.85	0.85	0.77	0.85	-0.04
time (sec)	N/A	0.023	0.017	0.014	10.711	0.461	3.047	0.452	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	85	0	0	138	0	38	-1
N.S.	1	1.00	2.24	0.00	0.00	3.63	0.00	1.00	-0.03
time (sec)	N/A	0.034	0.060	0.231	0.000	0.497	0.000	0.491	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	94	88	0	0	110	0	52	-1
N.S.	1	1.34	1.26	0.00	0.00	1.57	0.00	0.74	-0.01
time (sec)	N/A	0.091	0.072	0.526	0.000	0.475	0.000	0.476	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	48	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.096	0.028	0.054	0.000	0.467	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	41	37	36	35	31	36	27
N.S.	1	1.00	0.75	0.67	0.65	0.64	0.56	0.65	0.49
time (sec)	N/A	0.038	0.017	0.022	4.160	0.433	3.441	0.452	0.340

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	22	22	20	22	-1
N.S.	1	1.00	1.00	0.00	0.85	0.85	0.77	0.85	-0.04
time (sec)	N/A	0.022	0.016	0.007	3.056	0.415	3.159	0.454	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	119	0	0	115	0	122	-1
N.S.	1	1.00	1.53	0.00	0.00	1.47	0.00	1.56	-0.01
time (sec)	N/A	0.183	0.047	0.012	0.000	0.457	0.000	0.519	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	35	35	39	0	73	-1
N.S.	1	1.00	0.64	0.90	0.90	1.00	0.00	1.87	-0.03
time (sec)	N/A	0.033	0.025	0.028	4.056	0.437	0.000	0.446	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	54	15	23	29	23	-1
N.S.	1	1.00	1.00	3.18	0.88	1.35	1.71	1.35	-0.06
time (sec)	N/A	0.019	0.013	0.108	5.548	0.423	0.528	0.473	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	77	0	0	81	0	104	-1
N.S.	1	1.00	1.35	0.00	0.00	1.42	0.00	1.82	-0.02
time (sec)	N/A	0.053	0.050	0.115	0.000	0.474	0.000	0.467	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	69	0	108	37
N.S.	1	1.00	1.00	0.00	0.00	1.53	0.00	2.40	0.82
time (sec)	N/A	0.028	0.027	0.066	0.000	0.461	0.000	0.474	0.030

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	33	56	22	47	19	54	-1
N.S.	1	1.00	1.14	1.93	0.76	1.62	0.66	1.86	-0.03
time (sec)	N/A	0.032	0.020	0.115	2.944	0.422	4.328	0.498	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	22	0	40	-1
N.S.	1	1.00	1.00	0.95	0.90	1.05	0.00	1.90	-0.05
time (sec)	N/A	0.029	0.012	0.050	2.058	0.454	0.000	0.449	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	119	27	27	29	28	-1
N.S.	1	1.00	0.79	3.50	0.79	0.79	0.85	0.82	-0.03
time (sec)	N/A	0.024	0.017	0.037	3.768	0.425	1.404	0.439	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	21	29	29	33	26	58	-1
N.S.	1	1.00	0.64	0.88	0.88	1.00	0.79	1.76	-0.03
time (sec)	N/A	0.028	0.020	0.025	3.665	0.441	2.776	0.451	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	35	97	15	15	22	22	-1
N.S.	1	1.00	1.40	3.88	0.60	0.60	0.88	0.88	-0.04
time (sec)	N/A	0.018	0.033	0.389	2.439	0.431	16.367	0.456	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	40	39	25	41	41	44	-1
N.S.	1	1.00	1.18	1.15	0.74	1.21	1.21	1.29	-0.03
time (sec)	N/A	0.023	0.014	0.023	3.271	0.410	2.307	0.467	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	14	24	29	46	27	13
N.S.	1	1.00	2.88	0.88	1.50	1.81	2.88	1.69	0.81
time (sec)	N/A	0.013	0.039	0.039	3.953	0.422	8.703	0.489	0.206

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	112	0	42	0	0	-1
N.S.	1	1.00	1.00	4.87	0.00	1.83	0.00	0.00	-0.04
time (sec)	N/A	0.022	0.232	0.185	0.000	0.451	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	112	0	18	0	0	-1
N.S.	1	1.00	1.00	4.87	0.00	0.78	0.00	0.00	-0.04
time (sec)	N/A	0.021	0.221	0.136	0.000	0.487	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	39	54	82	93	105	36	55
N.S.	1	1.00	1.77	2.45	3.73	4.23	4.77	1.64	2.50
time (sec)	N/A	0.051	0.034	0.214	5.226	0.423	0.673	0.480	0.381

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	87	0	0	146	0	90	-1
N.S.	1	1.00	2.07	0.00	0.00	3.48	0.00	2.14	-0.02
time (sec)	N/A	0.108	0.062	0.040	0.000	0.461	0.000	0.487	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	59	45	91	0	54	0	92	-1
N.S.	1	2.11	1.61	3.25	0.00	1.93	0.00	3.29	-0.04
time (sec)	N/A	0.125	0.030	0.154	0.000	0.454	0.000	0.619	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	55	37	565	186	0	50	-1
N.S.	1	1.00	1.62	1.09	16.62	5.47	0.00	1.47	-0.03
time (sec)	N/A	0.033	0.052	0.071	3.712	0.431	0.000	0.506	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	50	65	67	0	63	0	67	-1
N.S.	1	1.28	1.67	1.72	0.00	1.62	0.00	1.72	-0.03
time (sec)	N/A	0.033	0.061	0.261	0.000	0.480	0.000	0.513	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	F(-1)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	337	0	552	0	0	0	0	0	-1
N.S.	1	0.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	1.482	0.110	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	58	1134	67	52	87	0	78
N.S.	1	1.00	0.75	14.73	0.87	0.68	1.13	0.00	1.01
time (sec)	N/A	0.157	0.017	2.349	3.138	0.967	0.483	0.000	0.257

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	95	508	0	287	0	92	413
N.S.	1	1.00	0.79	4.23	0.00	2.39	0.00	0.77	3.44
time (sec)	N/A	0.100	0.107	0.062	0.000	0.927	0.000	0.676	1.086

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	37	31	28	26	22	29	27	40
N.S.	1	1.19	1.00	0.90	0.84	0.71	0.94	0.87	1.29
time (sec)	N/A	0.010	0.077	0.011	3.098	1.467	42.570	0.584	0.797

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	138	0	60	0	34	-1
N.S.	1	1.00	1.00	4.76	0.00	2.07	0.00	1.17	-0.03
time (sec)	N/A	0.018	0.021	0.072	0.000	1.203	0.000	0.605	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	210	0	164	0	111	455
N.S.	1	1.00	1.00	1.98	0.00	1.55	0.00	1.05	4.29
time (sec)	N/A	0.077	0.204	0.046	0.000	0.791	0.000	0.622	1.160

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [29]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.00	5	1.400
2	A	2	2	1.00	15	0.133
3	F	0	0	N/A	0.	N/A
4	B	32	13	3.42	14	0.929
5	C	5	4	6.42	19	0.210
6	A	7	7	1.00	13	0.538
7	A	4	4	1.00	13	0.308
8	A	9	7	1.00	14	0.500
9	A	7	7	1.00	12	0.583
10	A	5	5	1.41	19	0.263
11	A	7	8	1.00	29	0.276
12	C	40	15	1.91	14	1.071
13	C	32	16	1.88	27	0.593
14	A	9	11	1.13	18	0.611
15	A	3	4	1.00	24	0.167
16	A	3	4	1.00	12	0.333
17	A	3	4	1.00	22	0.182
18	A	4	4	1.00	15	0.267
19	A	4	4	1.00	13	0.308
20	A	2	3	1.00	23	0.130
21	A	5	6	1.00	17	0.353
22	A	7	8	1.34	15	0.533
23	A	4	9	1.00	25	0.360
24	A	5	5	1.00	27	0.185
25	A	2	3	1.00	23	0.130

Continued on next page

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	18	11	1.00	27	0.407
27	A	3	3	1.00	17	0.176
28	A	2	2	1.00	13	0.154
29	A	7	6	1.00	17	0.353
30	A	5	5	1.00	15	0.333
31	A	4	4	1.00	15	0.267
32	A	2	2	1.00	17	0.118
33	A	5	5	1.00	13	0.385
34	A	3	3	1.00	15	0.200
35	A	2	2	1.00	13	0.154
36	A	5	5	1.00	13	0.385
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	24	0.083
39	A	2	2	1.00	24	0.083
40	A	4	5	1.00	10	0.500
41	A	6	7	1.00	12	0.583
42	B	5	5	2.11	13	0.385
43	A	4	4	1.00	13	0.308
44	A	4	4	1.28	15	0.267
45	F	0	0	N/A	0.	N/A
46	A	13	10	1.00	12	0.833
47	A	12	8	1.00	12	0.667
48	A	6	6	1.19	18	0.333
49	A	4	4	1.00	14	0.286
50	A	6	6	1.00	14	0.429

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sin^{-1}(x)\log(x)\,dx$

Optimal. Leaf size=51

$$-2\sqrt{1-x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)-x\sin^{-1}(x)(1-\log(x))+\sqrt{1-x^2}\log(x)$$

```
[Out] arctanh((-x^2+1)^(1/2))-x*arcsin(x)*(1-ln(x))-2*(-x^2+1)^(1/2)+ln(x)*(-x^2+
1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.400$, Rules used = {4715, 267, 2434, 272, 52, 65, 212}

$$-x\text{ArcSin}(x)+x\text{ArcSin}(x)\log(x)-2\sqrt{1-x^2}+\sqrt{1-x^2}\log(x)+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]*Log[x],x]
```

```
[Out] -2*Sqrt[1 - x^2] - x*ArcSin[x] + ArcTanh[Sqrt[1 - x^2]] + Sqrt[1 - x^2]*Log
[x] + x*ArcSin[x]*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2434

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(Px_.)*(F_)[(d_.)*((e_.) + (f_.)*
(x_))]^(m_.), x_Symbol] :> With[{u = IntHide[Px*F[d*(e + f*x)]^m, x]}, Dist
[a + b*Log[c*x^n], u, x] - Dist[b*n, Int[Dist[1/x, u, x], x], x]] /; FreeQ[
{a, b, c, d, e, f, n}, x] && PolynomialQ[Px, x] && IGtQ[m, 0] && MemberQ[{A
rcSin, ArcCos, ArcSinh, ArcCosh}, F]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^{-1}(x)\log(x)\,dx &= \sqrt{1-x^2}\,\log(x)+x\sin^{-1}(x)\log(x)-\int\left(\frac{\sqrt{1-x^2}}{x}+\sin^{-1}(x)\right)dx\\
&= \sqrt{1-x^2}\,\log(x)+x\sin^{-1}(x)\log(x)-\int\frac{\sqrt{1-x^2}}{x}\,dx-\int\sin^{-1}(x)\,dx\\
&= -x\sin^{-1}(x)+\sqrt{1-x^2}\,\log(x)+x\sin^{-1}(x)\log(x)-\frac{1}{2}\text{Subst}\left(\int\frac{\sqrt{1-x}}{x}\,dx,x,x^2\right.\\
&= -2\sqrt{1-x^2}-x\sin^{-1}(x)+\sqrt{1-x^2}\,\log(x)+x\sin^{-1}(x)\log(x)-\frac{1}{2}\text{Subst}\left(\int\frac{}{\sqrt{1}}\right.\\
&= -2\sqrt{1-x^2}-x\sin^{-1}(x)+\sqrt{1-x^2}\,\log(x)+x\sin^{-1}(x)\log(x)+\text{Subst}\left(\int\frac{1}{1-x^2}\right.\\
&= -2\sqrt{1-x^2}-x\sin^{-1}(x)+\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sqrt{1-x^2}\,\log(x)+x\sin^{-1}(x)\log
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 1.02

$$-2\sqrt{1-x^2}+x\sin^{-1}(x)(-1+\log(x))+\left(-1+\sqrt{1-x^2}\right)\log(x)+\log\left(1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]*Log[x],x]
```

```
[Out] -2*Sqrt[1 - x^2] + x*ArcSin[x]*(-1 + Log[x]) + (-1 + Sqrt[1 - x^2])*Log[x]
+ Log[1 + Sqrt[1 - x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.
time = 0.03, size = 93, normalized size = 1.82

method	result
default	$\frac{2\arcsin(x)\tan\left(\frac{\arcsin(x)}{2}\right)\ln\left(\frac{2\tan\left(\frac{\arcsin(x)}{2}\right)}{1+\tan^2\left(\frac{\arcsin(x)}{2}\right)}\right)-2\left(\tan^2\left(\frac{\arcsin(x)}{2}\right)\right)\ln\left(\frac{2\tan\left(\frac{\arcsin(x)}{2}\right)}{1+\tan^2\left(\frac{\arcsin(x)}{2}\right)}\right)-2\arcsin(x)\tan\left(\frac{\arcsin(x)}{2}\right)-4}{1+\tan^2\left(\frac{\arcsin(x)}{2}\right)} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(arcsin(x)*tan(1/2*arcsin(x))*ln(2*tan(1/2*arcsin(x))/(1+tan(1/2*arcsin(x
))^2))-tan(1/2*arcsin(x))^2*ln(2*tan(1/2*arcsin(x))/(1+tan(1/2*arcsin(x))^2
))-arcsin(x)*tan(1/2*arcsin(x))-2)/(1+tan(1/2*arcsin(x))^2)-ln(1+tan(1/2*ar
csin(x))^2)
```

Maxima [A]
time = 4.01, size = 58, normalized size = 1.14

$$(x\log(x)-x)\arcsin(x)+\sqrt{-x^2+1}\log(x)-2\sqrt{-x^2+1}+\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*log(x),x, algorithm="maxima")
```

```
[Out] (x*log(x) - x)*arcsin(x) + sqrt(-x^2 + 1)*log(x) - 2*sqrt(-x^2 + 1) + log(2
*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 0.66, size = 54, normalized size = 1.06

$$x\arcsin(x)\log(x)-x\arcsin(x)+\sqrt{-x^2+1}(\log(x)-2)+\frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right)-\frac{1}{2}\log\left(\sqrt{-x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*log(x),x, algorithm="fricas")
```

```
[Out] x*arcsin(x)*log(x) - x*arcsin(x) + sqrt(-x^2 + 1)*(log(x) - 2) + 1/2*log(sq
rt(-x^2 + 1) + 1) - 1/2*log(sqrt(-x^2 + 1) - 1)
```

Sympy [A]
time = 3.98, size = 102, normalized size = 2.00

$$x\log(x)\operatorname{asin}(x) - x\operatorname{asin}(x) + \sqrt{1-x^2}\,\log(x) - \sqrt{1-x^2} - \begin{cases} -\frac{x}{\sqrt{-1+\frac{1}{x^2}}} - \operatorname{acosh}\left(\frac{1}{x}\right) + \frac{1}{x\sqrt{-1+\frac{1}{x^2}}} & \text{for } \frac{1}{|x^2|} > 1 \\ \frac{ix}{\sqrt{1-\frac{1}{x^2}}} + i\operatorname{asin}\left(\frac{1}{x}\right) - \frac{i}{x\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)*ln(x),x)
```

```
[Out] x*log(x)*asin(x) - x*asin(x) + sqrt(1 - x**2)*log(x) - sqrt(1 - x**2) - Pie
cewise((-x/sqrt(-1 + x**(-2)) - acosh(1/x) + 1/(x*sqrt(-1 + x**(-2))), 1/Ab
s(x**2) > 1), (I*x/sqrt(1 - 1/x**2) + I*asin(1/x) - I/(x*sqrt(1 - 1/x**2)),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(42) = 84$.
time = 0.51, size = 272, normalized size = 5.33

$$x\arcsin(x)\log(x) + \sqrt{-x^2+1}\,\log(x) - \frac{2\,x\arcsin(x)}{\left(\sqrt{-x^2+1}+1\right)\left(\frac{x^2}{\left(\sqrt{-x^2+1}+1\right)^2}+1\right)} + \frac{x^2\log\left(\sqrt{-x^2+1}+1\right)}{\left(\sqrt{-x^2+1}+1\right)^2\left(\frac{x^2}{\left(\sqrt{-x^2+1}+1\right)^2}+1\right)} + \frac{\log\left(\sqrt{-x^2+1}+1\right)}{\frac{x^2}{\left(\sqrt{-x^2+1}+1\right)^2}+1} - \frac{x^2\log(|x|)}{\left(\sqrt{-x^2+1}+1\right)^2\left(\frac{x^2}{\left(\sqrt{-x^2+1}+1\right)^2}+1\right)} - \frac{\log(|x|)}{\frac{x^2}{\left(\sqrt{-x^2+1}+1\right)^2}+1} + \frac{2\,x^2}{\left(\sqrt{-x^2+1}+1\right)^2\left(\frac{x^2}{\left(\sqrt{-x^2+1}+1\right)^2}+1\right)} - \frac{2}{\frac{x^2}{\left(\sqrt{-x^2+1}+1\right)^2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*log(x),x, algorithm="giac")
```

```
[Out] x*arcsin(x)*log(x) + sqrt(-x^2 + 1)*log(x) - 2*x*arcsin(x)/((sqrt(-x^2 + 1)
 + 1)*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)) + x^2*log(sqrt(-x^2 + 1) + 1)/((sqr
t(-x^2 + 1) + 1)^2*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)) + log(sqrt(-x^2 + 1) +
 1)/(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1) - x^2*log(abs(x))/((sqrt(-x^2 + 1) + 1
)^2*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)) - log(abs(x))/(x^2/(sqrt(-x^2 + 1) +
1)^2 + 1) + 2*x^2/((sqrt(-x^2 + 1) + 1)^2*(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1))
 - 2/(x^2/(sqrt(-x^2 + 1) + 1)^2 + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \operatorname{asin}(x)\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)*log(x),x)
```

```
[Out] int(asin(x)*log(x), x)
```

3.2 $\int \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=17

$$x-\sqrt{1-x^2}\sin^{-1}(x)$$

```
[Out] x-arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4767, 8}

$$x-\sqrt{1-x^2}\,\text{ArcSin}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] x - Sqrt[1 - x^2]*ArcSin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx &= -\sqrt{1-x^2}\sin^{-1}(x)+\int 1\,dx\\ &= x-\sqrt{1-x^2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$x-\sqrt{1-x^2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] x - Sqrt[1 - x^2]*ArcSin[x]
```

Maple [A]
time = 0.06, size = 16, normalized size = 0.94

method	result	size
default	$x-\arcsin \left(x \right) \sqrt{-x^{2}+1}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x-arcsin(x)*(-x^2+1)^(1/2)
```

Maxima [A]
time = 2.56, size = 15, normalized size = 0.88

$$-\sqrt{-x^{2}+1}\,\arcsin \left(x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)*arcsin(x) + x
```

Fricas [A]
time = 0.69, size = 15, normalized size = 0.88

$$-\sqrt{-x^{2}+1}\,\arcsin \left(x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)*arcsin(x) + x
```

Sympy [A]
time = 0.06, size = 12, normalized size = 0.71

$$x-\sqrt{1-x^{2}}\,\operatorname{asin}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x - sqrt(1 - x**2)*asin(x)
```

Giac [A]
time = 0.46, size = 15, normalized size = 0.88

$$-\sqrt{-x^{2}+1}\,\arcsin \left(x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*arcsin(x) + x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x \operatorname{asin}(x)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*asin(x))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x*asin(x))/(1 - x^2)^(1/2), x)
```

3.3 $\int -\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=69

$$\frac{\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{4\sqrt{2}}-\left(\frac{3}{8}+x\right)\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)$$

```
[Out] -(3/8+x)*arcsin(x^(1/2)-(1+x)^(1/2))+1/8*(x^(1/2)+3*(1+x)^(1/2))*(-x+x^(1/2
)*(1+x)^(1/2))^(1/2)*2^(1/2)
```

Rubi [F]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int -\text{ArcSin}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[-ArcSin[Sqrt[x] - Sqrt[1 + x]],x]
```

```
[Out] -(x*ArcSin[Sqrt[x] - Sqrt[1 + x]]) + Defer[Subst][Defer[Int][Sqrt[1 - x^2 +
 x*Sqrt[-1 + x^2]], x], x, Sqrt[1 + x]]/Sqrt[2]
```

Rubi steps

$$\begin{aligned}
\int -\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx &= -x\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)+\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{2\sqrt{2}\,\sqrt{1+x}}\,dx \\
&= -x\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}}\,dx}{2\sqrt{2}} \\
&= -x\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\text{Subst}\left(\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx, x,\right.}{\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.71, size = 106, normalized size = 1.54

$$\frac{1}{8}\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-2x+2\sqrt{x}\,\sqrt{1+x}}-x\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)-\frac{3}{8}\tan^{-1}\left(\frac{\sqrt{-2x+2\sqrt{x}\,\sqrt{1+x}}}{-\sqrt{x}+\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-ArcSin[Sqrt[x] - Sqrt[1 + x]],x]
```

```
[Out] ((Sqrt[x] + 3*Sqrt[1 + x])*Sqrt[-2*x + 2*Sqrt[x]*Sqrt[1 + x]])/8 - x*ArcSin
[Sqrt[x] - Sqrt[1 + x]] - (3*ArcTan[Sqrt[-2*x + 2*Sqrt[x]*Sqrt[1 + x]]/(-Sq
rt[x] + Sqrt[1 + x])])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(49) = 98$.
time = 0.58, size = 251, normalized size = 3.64

method	result
default	$-\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)\left(\tan^{8}\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)\right)-2\left(\tan^{7}\left(\frac{\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2}\right)\right)+2\arcsin\left(\sqrt{x}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arcsin(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(arcsin(x^(1/2)-(1+x)^(1/2))*tan(1/2*arcsin(x^(1/2)-(1+x)^(1/2)))^8-2
*tan(1/2*arcsin(x^(1/2)-(1+x)^(1/2)))^7+2*arcsin(x^(1/2)-(1+x)^(1/2))*tan(1
/2*arcsin(x^(1/2)-(1+x)^(1/2)))^6-6*tan(1/2*arcsin(x^(1/2)-(1+x)^(1/2)))^5+
18*arcsin(x^(1/2)-(1+x)^(1/2))*tan(1/2*arcsin(x^(1/2)-(1+x)^(1/2)))^4+6*tan
(1/2*arcsin(x^(1/2)-(1+x)^(1/2)))^3+2*arcsin(x^(1/2)-(1+x)^(1/2))*tan(1/2*a
rcsin(x^(1/2)-(1+x)^(1/2)))^2+2*tan(1/2*arcsin(x^(1/2)-(1+x)^(1/2)))+arcsin
(x^(1/2)-(1+x)^(1/2)))/(1+tan(1/2*arcsin(x^(1/2)-(1+x)^(1/2)))^2)^2/tan(1/2
*arcsin(x^(1/2)-(1+x)^(1/2)))^2
```

Maxima [A]
time = 4.16, size = 4, normalized size = 0.06

$$\frac{1}{2}\,\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arcsin(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*pi*x
```

Fricas [A]
time = 22.13, size = 49, normalized size = 0.71

$$\frac{1}{8}\,(8\,x+3)\arcsin\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{8}\,\sqrt{2\,\sqrt{x+1}\,\sqrt{x}-2\,x}\,\left(3\,\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arcsin(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*(8*x + 3)*arcsin(sqrt(x + 1) - sqrt(x)) + 1/8*sqrt(2*sqrt(x + 1)*sqrt(x
) - 2*x)*(3*sqrt(x + 1) + sqrt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \operatorname{asin}\left(\sqrt{x}-\sqrt{x+1}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-asin(x**(1/2)-(1+x)**(1/2)),x)
```

```
[Out] -Integral(asin(sqrt(x) - sqrt(x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arcsin(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(-arcsin(-sqrt(x + 1) + sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{asin}\left(\sqrt{x+1}-\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] int(asin((x + 1)^(1/2) - x^(1/2)), x)
```

3.4 $\int \log \left(1+x\sqrt{1+x^2}\right)\,dx$

Optimal. Leaf size=97

$$-2x+\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{-2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right)-\sqrt{2\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\left(x+\right.\right.$$

```
[Out] -2*x+x*ln(1+x*(x^2+1)^(1/2))-arctanh((x+(x^2+1)^(1/2))*(2+5^(1/2))^(1/2))*(
-2+2*5^(1/2))^(1/2)+arctan((x+(x^2+1)^(1/2))*(-2+5^(1/2))^(1/2))*(2+2*5^(1/
2))^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 332 vs. $2(97) = 194$.
time = 0.45, antiderivative size = 332, normalized size of antiderivative = 3.42, number of steps used = 32, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$,
Rules used = {2628, 6874, 267, 1144, 209, 213, 1265, 838, 721, 1107, 1180, 1261, 713}

$$\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\,\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)+2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)+x\log\left(\sqrt{x^2+1}\,x+1\right)-\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)-2x+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)+2\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x*Sqrt[1 + x^2]],x]
```

```
[Out] -2*x - Sqrt[(1 + Sqrt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] + 2*Sqrt[(2 +
 Sqrt[5])/5]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[2/(5*(-1 + Sqrt[5]))]*A
rcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] + Sqrt[(2*(-1 + Sqrt[5]))/5]*Ar
cTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] + 2*Sqrt[(-2 + Sqrt[5])/5]*ArcTa
nh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1 +
 Sqrt[5])]*x] + Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqr
t[1 + x^2]] - Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[
1 + x^2]] + x*Log[1 + x*Sqrt[1 + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 721

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \log\left(1+x\sqrt{1+x^2}\right)\,dx = x\log\left(1+x\sqrt{1+x^2}\right) - \int \frac{x(1+2x^2)}{x+x^3+\sqrt{1+x^2}}\,dx$$

$$= x\log\left(1+x\sqrt{1+x^2}\right) - \int \left(\frac{x}{x+x^3+\sqrt{1+x^2}} + \frac{2x^3}{x+x^3+\sqrt{1+x^2}}\right) dx$$

$$= x\log\left(1+x\sqrt{1+x^2}\right) - 2\int \frac{x^3}{x+x^3+\sqrt{1+x^2}}\,dx - \int \frac{x}{x+x^3+\sqrt{1+x^2}}$$

$$= x\log\left(1+x\sqrt{1+x^2}\right) - 2\int \left(1-\frac{x}{\sqrt{1+x^2}} + \frac{1-x^2}{-1+x^2+x^4} - \frac{x\sqrt{1+x^2}}{-1+x^2+}\right.$$

$$= -2x + x\log\left(1+x\sqrt{1+x^2}\right) + 2\int \frac{x}{\sqrt{1+x^2}}\,dx - 2\int \frac{1-x^2}{-1+x^2+x^4}\,dx +$$

$$= -2x + \sqrt{1+x^2} + x\log\left(1+x\sqrt{1+x^2}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{1+x}}{-1+x+x^2}\,dx, x,\right.$$

$$= -2x - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}$$

$$= -2x - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}$$

$$= -2x - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}$$

$$= -2x - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}$$

Mathematica [A]
time = 0.34, size = 188, normalized size = 1.94

$$-2x + \frac{\left(5+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}} + \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}+\frac{\sqrt{5}}{2}}\sqrt{1+x^2}\right) - \frac{\left(-5+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}} - \sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{-\frac{1}{2}+\frac{\sqrt{5}}{2}}\sqrt{1+x^2}\right) + x\log\left(1+x\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x*Sqrt[1 + x^2]],x]
```

```
[Out] -2*x + ((5 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5]
)] + Sqrt[(1 + Sqrt[5])/2]*ArcTan[Sqrt[1/2 + Sqrt[5]/2]*Sqrt[1 + x^2]] - ((
-5 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5])] -
```

```
Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[Sqrt[-1/2 + Sqrt[5]/2]*Sqrt[1 + x^2]] + x*Lo
g[1 + x*Sqrt[1 + x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(75) = 150$.
time = 0.06, size = 426, normalized size = 4.39

method	result
default	$x \ln\left(1 + x\sqrt{x^2+1}\right) + \frac{\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}} + \frac{\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}} - \frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{-2+2\sqrt{5}}}\right.}{\sqrt{-2+2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x*(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(1+x*(x^2+1)^(1/2))+1/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2
))+5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(-2+2*5^(1
/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+5^(1/2)/(-2+2*5^(1/2))^(1/2)*a
rctanh(2*x/(-2+2*5^(1/2))^(1/2))-2*x-1/2/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)
^(1/2)-x)/(2+5^(1/2))^(1/2))-1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)
^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x
)/(-2+5^(1/2))^(1/2))-1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-
x)/(-2+5^(1/2))^(1/2))-3/10*5^(1/2)/(2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)
-x)/(2+5^(1/2))^(1/2))-1/2/(2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x)/(2+5^
(1/2))^(1/2))+1/2/(-2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))
^(1/2))-3/10*5^(1/2)/(-2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/
2))^(1/2))+2/5*5^(1/2)*(2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x)/(2+5^(1/2
))^(1/2))-2/5*(-2+5^(1/2))^(1/2)*5^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1
/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x*(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(sqrt(x^2 + 1)*x + 1) - 2*x + arctan(x) + integrate((2*x^2 + 1)/(x^2 +
 (x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(75) = 150$.
time = 1.04, size = 402, normalized size = 4.14

$$-\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{4}\sqrt{2}\sqrt{4x^4+4x^2+\sqrt{5}(2x^2+1)-2\left(2x^3+\sqrt{5}x+x\right)\sqrt{x^2+1}+1}\left(\sqrt{2}x+\sqrt{2}\sqrt{x^2+1}\right)\sqrt{\sqrt{5}+1}-\frac{1}{2}\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}\right)-\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{8}\sqrt{4x^2+2\sqrt{5}+2}\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}-\frac{1}{4}\left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{\sqrt{5}+1}\right)+\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x+\left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x-\left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right)+x\log\left(\sqrt{x^2+1}x+1\right)+\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x+\sqrt{2}\sqrt{\sqrt{5}-1}\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x-\sqrt{2}\sqrt{\sqrt{5}-1}\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x*(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/4*sqrt(2)*sqrt(4*x^4 + 4*x^2 + sqrt(5)*
(2*x^2 + 1) - 2*(2*x^3 + sqrt(5)*x + x)*sqrt(x^2 + 1) + 1)*(sqrt(2)*x + sqr
t(2)*sqrt(x^2 + 1))*sqrt(sqrt(5) + 1) - 1/2*sqrt(2)*sqrt(x^2 + 1)*sqrt(sqrt
(5) + 1)) - sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2
)*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x -
sqrt(2)*x)*sqrt(sqrt(5) + 1)) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(4*x^2 - 4
*sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*sqrt(2) + sq
rt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*
log(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)
*sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + x*log(sqrt(x^2 +
1)*x + 1) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) -
1)) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1)) -
2*x
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x*(x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(75) = 150.
time = 0.52, size = 235, normalized size = 2.42

$$x\log\left(\sqrt{x^2+1}\,x+1\right)+\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2+1}+\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|-x+\sqrt{x^2+1}-\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right|\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x*(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(x^2 + 1)*x + 1) + 1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2
+ 1) + 1/(x - sqrt(x^2 + 1)))/sqrt(2*sqrt(5) - 2)) + 1/2*sqrt(2*sqrt(5) + 2
)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt
(x^2 + 1) + sqrt(2*sqrt(5) + 2) - 1/(x - sqrt(x^2 + 1))) + 1/4*sqrt(2*sqrt(
5) - 2)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log
(abs(x - sqrt(1/2*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + s
qrt(x^2 + 1) - sqrt(2*sqrt(5) + 2) - 1/(x - sqrt(x^2 + 1)))) - 2*x
```

Mupad [**B**]
time = 1.57, size = 666, normalized size = 6.87

$$x\ln\left(x\sqrt{x^2+1}+1\right)-2x+\frac{\ln\left(x-\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x+\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x-\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}+\frac{\ln\left(x+\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}+\frac{\left(\ln\left(x-\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{5}-1}}{2}+\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}+\frac{\left(\ln\left(x+\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}}{2}-\frac{\sqrt{2}\,x\sqrt{\sqrt{5}-1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}-\frac{\left(\ln\left(\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{1-\sqrt{5}}}{2}-\frac{\sqrt{2}\,x\sqrt{-\sqrt{5}-1}}{2}+1\right)-\ln\left(x+\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}-\frac{\left(\ln\left(\frac{\sqrt{2}\,x\sqrt{-\sqrt{5}-1}}{2}+\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{1-\sqrt{5}}}{2}+1\right)-\ln\left(x-\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x*(x^2 + 1)^(1/2) + 1),x)
```

```
[Out] x*log(x*(x^2 + 1)^(1/2) + 1) - 2*x + (log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2))
/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/
2)) - (log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1
/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/2)*(- 5^(1
/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5
^(1/2)/2 - 1/2)^(3/2)) + (log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/
2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) +
((log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log((2^(1/2)*x*(5^(1/2) - 1)^(
1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2 + 1))*((5^(1/2)/2
 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4
*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(1/2)) + ((log(x + (2^(1/2)*(5^
(1/2) - 1)^(1/2))/2) - log((2^(1/2)*(x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2
- (2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(
1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/
2))*(5^(1/2)/2 + 1/2)^(1/2)) - ((log((2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))
^(1/2))/2 - (2^(1/2)*x*(- 5^(1/2) - 1)^(1/2))/2 + 1) - log(x + (2^(1/2)*(-
5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^
(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 -
 5^(1/2)/2)^(1/2)) - ((log((2^(1/2)*x*(- 5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(
x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2 + 1) - log(x - (2^(1/2)*(- 5^(1/2) -
1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((
2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2
)^(1/2))
```

3.5 $\int \frac{\cos^2(x)}{\sqrt{1+\cos^2(x)+\cos^4(x)}}\,dx$

Optimal. Leaf size=45

$$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{\cos(x)\left(1+\cos^2(x)\right)\sin(x)}{1+\cos^2(x)\sqrt{1+\cos^2(x)+\cos^4(x)}}\right)$$

```
[Out] 1/3*x+1/3*arctan(cos(x)*(1+cos(x)^2)*sin(x)/(1+cos(x)^2*(1+cos(x)^2+cos(x)^
4)^(1/2)))
```

Rubi [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.33, antiderivative size = 289, normalized size of antiderivative = 6.42, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6851, 1230, 1117, 1720}

$$\frac{\cos^2(x)\sqrt{\tan^4(x)+3\tan^2(x)+3}\,\mathrm{ArcTan}\left(\frac{\tan(x)}{\sqrt{\tan^4(x)+3\tan^2(x)+3}}\right)}{2\sqrt{\cos^4(x)\left(\tan^4(x)+3\tan^2(x)+3\right)}}-\frac{\left(1+\sqrt{3}\right)\cos^2(x)\left(\tan^2(x)+\sqrt{3}\right)\sqrt{\frac{\tan^4(x)+3\tan^2(x)+3}{\left(\tan^2(x)+\sqrt{3}\right)^2}}\,F\left(2\mathrm{ArcTan}\left(\frac{\tan(x)}{\sqrt[4]{3}}\right)|\frac{1}{4}\left(2-\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{\cos^4(x)\left(\tan^4(x)+3\tan^2(x)+3\right)}}+\frac{\left(2+\sqrt{3}\right)\cos^2(x)\left(\tan^2(x)+\sqrt{3}\right)\sqrt{\frac{\tan^4(x)+3\tan^2(x)+3}{\left(\tan^2(x)+\sqrt{3}\right)^2}}\,\Pi\left(\frac{1}{6}\left(3-2\sqrt{3}\right);2\mathrm{ArcTan}\left(\frac{\tan(x)}{\sqrt[4]{3}}\right)|\frac{1}{4}\left(2-\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{\cos^4(x)\left(\tan^4(x)+3\tan^2(x)+3\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Cos[x]^2/Sqrt[1 + Cos[x]^2 + Cos[x]^4],x]
```

```
[Out] (ArcTan[Tan[x]/Sqrt[3 + 3*Tan[x]^2 + Tan[x]^4]]*Cos[x]^2*Sqrt[3 + 3*Tan[x]^
2 + Tan[x]^4])/(2*Sqrt[Cos[x]^4*(3 + 3*Tan[x]^2 + Tan[x]^4)]) - ((1 + Sqrt[
3])*Cos[x]^2*EllipticF[2*ArcTan[Tan[x]/3^(1/4)], (2 - Sqrt[3])/4]*(Sqrt[3]
+ Tan[x]^2)*Sqrt[(3 + 3*Tan[x]^2 + Tan[x]^4)/(Sqrt[3] + Tan[x]^2)^2])/(4*3^
(1/4)*Sqrt[Cos[x]^4*(3 + 3*Tan[x]^2 + Tan[x]^4)]) + ((2 + Sqrt[3])*Cos[x]^2
*EllipticPi[(3 - 2*Sqrt[3])/6, 2*ArcTan[Tan[x]/3^(1/4)], (2 - Sqrt[3])/4]*(
Sqrt[3] + Tan[x]^2)*Sqrt[(3 + 3*Tan[x]^2 + Tan[x]^4)/(Sqrt[3] + Tan[x]^2)^2
])/(4*3^(1/4)*Sqrt[Cos[x]^4*(3 + 3*Tan[x]^2 + Tan[x]^4)])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\cos^2(x)}{\sqrt{1+\cos^2(x)+\cos^4(x)}}\,dx = \text{Subst}\left(\int \frac{1}{(1+x^2)^2\sqrt{\frac{3+3x^2+x^4}{(1+x^2)^2}}}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\cos^2(x)\sqrt{3+3\tan^2(x)+\tan^4(x)}\right)\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{3+3x^2+x^4}}\,dx, x$$

$$\sqrt{\cos^4(x)\left(3+3\tan^2(x)+\tan^4(x)\right)}$$

$$= \frac{\left(\left(-1-\sqrt{3}\right)\cos^2(x)\sqrt{3+3\tan^2(x)+\tan^4(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3+3x^2+}}\right.}{2\sqrt{\cos^4(x)\left(3+3\tan^2(x)+\tan^4(x)\right)}}$$

$$= \frac{\tan^{-1}\left(\frac{\tan(x)}{\sqrt{3+3\tan^2(x)+\tan^4(x)}}\right)\cos^2(x)\sqrt{3+3\tan^2(x)+\tan^4(x)}}{2\sqrt{\cos^4(x)\left(3+3\tan^2(x)+\tan^4(x)\right)}} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 1.50, size = 159, normalized size = 3.53

$$-\frac{2i\cos^2(x)\Pi\left(\frac{3}{2}+\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{3}}}\tan(x)\right)|\frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right)\sqrt{1-\frac{2i\tan^2(x)}{-3i+\sqrt{3}}}\sqrt{1+\frac{2i\tan^2(x)}{3i+\sqrt{3}}}}{\sqrt{-\frac{i}{-3i+\sqrt{3}}}\sqrt{15+8\cos(2x)+\cos(4x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/Sqrt[1 + Cos[x]^2 + Cos[x]^4],x]
```

```
[Out] ((-2*I)*Cos[x]^2*EllipticPi[3/2 + (I/2)*Sqrt[3], I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[3])]*Tan[x]], (3*I - Sqrt[3])/(3*I + Sqrt[3])]*Sqrt[1 - ((2*I)*Tan
[x]^2)/(-3*I + Sqrt[3])]*Sqrt[1 + ((2*I)*Tan[x]^2)/(3*I + Sqrt[3])])/(Sqrt[
(-I)/(-3*I + Sqrt[3])]*Sqrt[15 + 8*Cos[2*x] + Cos[4*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 312, normalized size = 6.93

method	result
default	$-\frac{2\sqrt{(\cos^2(2x)+4\cos(2x)+7)(\sin^2(2x))}\left(i\sqrt{3}-3\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)(\cos(2x)-1)}{\left(i\sqrt{3}-3\right)(\cos(2x)+1)}}(\cos(2x)+1)^2\sqrt{\frac{\cos(}{(i\sqrt{3}}}}{\left(-1+i\sqrt{3}\right)\sqrt{(\cos(2x)-1)(\cos(2x)+1)(\cos(2x)+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((cos(2*x)^2+4*cos(2*x)+7)*sin(2*x)^2)^(1/2)*(I*3^(1/2)-3)*((-1+I*3^(1/2
))*(cos(2*x)-1)/(I*3^(1/2)-3)/(cos(2*x)+1))^(1/2)*(cos(2*x)+1)^2*((cos(2*x)
+2+I*3^(1/2))/(I*3^(1/2)+3)/(cos(2*x)+1))^(1/2)*((I*3^(1/2)-cos(2*x)-2)/(I*
3^(1/2)-3)/(cos(2*x)+1))^(1/2)*EllipticPi(((-1+I*3^(1/2))*(cos(2*x)-1)/(I*3
^(1/2)-3)/(cos(2*x)+1))^(1/2),(I*3^(1/2)-3)/(-1+I*3^(1/2)),((1+I*3^(1/2))*(
I*3^(1/2)-3)/(I*3^(1/2)+3)/(-1+I*3^(1/2)))^(1/2))/(-1+I*3^(1/2))/((cos(2*x)
-1)*(cos(2*x)+1)*(cos(2*x)+2+I*3^(1/2))*(I*3^(1/2)-cos(2*x)-2))^(1/2)/sin(2
*x)/(cos(2*x)^2+4*cos(2*x)+7)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1), x)
```

Fricas [**A**]
time = 0.92, size = 33, normalized size = 0.73

$$\frac{1}{6}\arctan\left(\frac{2\sqrt{\cos(x)^4+\cos(x)^2+1}\,\cos(x)^3\sin(x)}{2\cos(x)^6-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(2*sqrt(cos(x)^4 + cos(x)^2 + 1)*cos(x)^3*sin(x)/(2*cos(x)^6 - 1)
)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(1+cos(x)**2+cos(x)**4)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(1+cos(x)^2+cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^2/sqrt(cos(x)^4 + cos(x)^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(x)^2}{\sqrt{\cos(x)^4+\cos(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(cos(x)^2 + cos(x)^4 + 1)^(1/2),x)
```

```
[Out] int(cos(x)^2/(cos(x)^2 + cos(x)^4 + 1)^(1/2), x)
```

3.6 $\int \tan(x)\sqrt{1+\tan^4(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{2}\sinh^{-1}\left(\tan^2(x)\right)-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{1+\tan^4(x)}}\right)}{\sqrt{2}}+\frac{1}{2}\sqrt{1+\tan^4(x)}$$

```
[Out] -1/2*arcsinh(tan(x)^2)-1/2*arctanh(1/2*(1-tan(x)^2)*2^(1/2)/(1+tan(x)^4)^(1
/2))*2^(1/2)+1/2*(1+tan(x)^4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3751, 1262, 749, 858, 221, 739, 212}

$$\frac{1}{2}\sqrt{\tan^4(x)+1}-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{\tan^4(x)+1}}\right)}{\sqrt{2}}-\frac{1}{2}\sinh^{-1}\left(\tan^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*Sqrt[1 + Tan[x]^4],x]
```

```
[Out] -1/2*ArcSinh[Tan[x]^2] - ArcTanh[(1 - Tan[x]^2)/(Sqrt[2]*Sqrt[1 + Tan[x]^4]
)]/Sqrt[2] + Sqrt[1 + Tan[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \tan(x)\sqrt{1+\tan^4(x)}\,dx &= \text{Subst}\left(\int \frac{x\sqrt{1+x^4}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\sqrt{1+\tan^4(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1-x}{(1+x)\sqrt{1+x^2}}\,dx, x, \tan^2(x)\right)\\
&= \frac{1}{2}\sqrt{1+\tan^4(x)} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan^2(x)\right) + \text{Subst}\left(\int \frac{}{(1+x}\right.\\
&= -\frac{1}{2}\sinh^{-1}\left(\tan^2(x)\right) + \frac{1}{2}\sqrt{1+\tan^4(x)} - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{1-\tan}{\sqrt{1+\tan}}\right.\\
&= -\frac{1}{2}\sinh^{-1}\left(\tan^2(x)\right) - \frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{1+\tan^4(x)}}\right)}{\sqrt{2}} + \frac{1}{2}\sqrt{1+\tan^4(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 74, normalized size = 1.32

$$
\frac{\left(-2\sqrt{2}\,\sinh^{-1}(\cos(2x))\cos^2(x) - 2\tanh^{-1}\left(\frac{2\sin^2(x)}{\sqrt{3+\cos(4x)}}\right)\cos^2(x) + \sqrt{3+\cos(4x)}\right)\sqrt{1+\tan^4(x)}}{2\sqrt{3+\cos(4x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*Sqrt[1 + Tan[x]^4],x]
```

```
[Out] ((-2*Sqrt[2]*ArcSinh[Cos[2*x]]*Cos[x]^2 - 2*ArcTanh[(2*Sin[x]^2)/Sqrt[3 + C
os[4*x]]]*Cos[x]^2 + Sqrt[3 + Cos[4*x]])*Sqrt[1 + Tan[x]^4])/(2*Sqrt[3 + Co
s[4*x]])
```

Maple [A]
time = 0.06, size = 64, normalized size = 1.14

method	result
derivativedivides	$\frac{\sqrt{\left(1+\tan^2(x)\right)^2-2\left(\tan^2(x)\right)}}{2} - \frac{\operatorname{arcsinh}\left(\tan^2(x)\right)}{2} - \frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-2\left(\tan^2(x)\right)+2\right)\sqrt{}}{4\sqrt{\left(1+\tan^2(x)\right)^2-2}}\right.}{2}$

default	$\frac{\sqrt{(1+\tan^2(x))^2-2(\tan^2(x))}}{2}-\frac{\operatorname{arcsinh}(\tan^2(x))}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-2(\tan^2(x))+2)\sqrt{2}}{4\sqrt{(1+\tan^2(x))^2-2(t}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(x)^4)^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((1+tan(x)^2)^2-2*tan(x)^2)^(1/2)-1/2*arcsinh(tan(x)^2)-1/2*2^(1/2)*arc
tanh(1/4*(-2*tan(x)^2+2)*2^(1/2)/((1+tan(x)^2)^2-2*tan(x)^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^4)^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(tan(x)^4 + 1)*tan(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(43) = 86$.
time = 0.75, size = 88, normalized size = 1.57

$$\frac{1}{4}\sqrt{2}\log\left(\frac{3\tan(x)^4-2\tan(x)^2+2\sqrt{\tan(x)^4+1}\left(\sqrt{2}\tan(x)^2-\sqrt{2}\right)+3}{\tan(x)^4+2\tan(x)^2+1}\right)+\frac{1}{2}\sqrt{\tan(x)^4+1}+\frac{1}{2}\log\left(-\tan(x)^2+\sqrt{\tan(x)^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^4)^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((3*tan(x)^4 - 2*tan(x)^2 + 2*sqrt(tan(x)^4 + 1)*(sqrt(2)*ta
n(x)^2 - sqrt(2)) + 3)/(tan(x)^4 + 2*tan(x)^2 + 1)) + 1/2*sqrt(tan(x)^4 + 1
) + 1/2*log(-tan(x)^2 + sqrt(tan(x)^4 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\tan^4(x)+1}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**4)**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(tan(x)**4 + 1)*tan(x), x)
```

Giac [**A**]
time = 0.44, size = 79, normalized size = 1.41

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\tan(x)^2+\sqrt{2}-\sqrt{\tan(x)^4+1}+1}{\tan(x)^2-\sqrt{2}-\sqrt{\tan(x)^4+1}+1}\right)+\frac{1}{2}\sqrt{\tan(x)^4+1}+\frac{1}{2}\log\left(-\tan(x)^2+\sqrt{\tan(x)^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^4)^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-(tan(x)^2 + sqrt(2) - sqrt(tan(x)^4 + 1) + 1)/(tan(x)^2 -
sqrt(2) - sqrt(tan(x)^4 + 1) + 1)) + 1/2*sqrt(tan(x)^4 + 1) + 1/2*log(-tan(
x)^2 + sqrt(tan(x)^4 + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(x)\,\sqrt{\tan(x)^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(tan(x)^4 + 1)^(1/2),x)
```

```
[Out] int(tan(x)*(tan(x)^4 + 1)^(1/2), x)
```

3.7 $\int \frac{\tan(x)}{\sqrt{1+\sec^3(x)}} \, dx$

Optimal. Leaf size=15

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1+\sec^3(x)}\right)$$

```
[Out] -2/3*arctanh((1+sec(x)^3)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4224, 272, 65, 213}

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{\sec^3(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[1 + Sec[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + Sec[x]^3]])/3
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4224

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a + b*(c*ff*x)^n)^p/x), x],
```

```
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{\sqrt{1+\sec^3(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{1+x^3}}\,dx, x, \sec(x)\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \sec^3(x)\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\sec^3(x)}\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\sqrt{1+\sec^3(x)}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1+\sec^3(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[1 + Sec[x]^3],x]
[Out] (-2*ArcTanh[Sqrt[1 + Sec[x]^3]])/3
```

Maple [A]
time = 0.11, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{2\,\text{arctanh}\left(\sqrt{1+\sec^3(x)}\right)}{3}$	12
default	$-\frac{2\,\text{arctanh}\left(\sqrt{1+\sec^3(x)}\right)}{3}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+sec(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*arctanh((1+sec(x)^3)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.
time = 2.68, size = 27, normalized size = 1.80

$$-\frac{1}{3}\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}+1\right)+\frac{1}{3}\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+sec(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*log(sqrt(1/cos(x)^3 + 1) + 1) + 1/3*log(sqrt(1/cos(x)^3 + 1) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(11) = 22$.
time = 0.62, size = 30, normalized size = 2.00

$$\frac{1}{3}\log\left(2\sqrt{\frac{\cos(x)^3+1}{\cos(x)^3}}\cos(x)^3-2\cos(x)^3-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+sec(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(2*sqrt((cos(x)^3 + 1)/cos(x)^3)*cos(x)^3 - 2*cos(x)^3 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(x)}{\sqrt{(\sec(x)+1)(\sec^2(x)-\sec(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+sec(x)**3)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt((sec(x) + 1)*(sec(x)**2 - sec(x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 0.47, size = 28, normalized size = 1.87

$$-\frac{1}{3}\log\left(\sqrt{\frac{1}{\cos(x)^3}+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{\frac{1}{\cos(x)^3}+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+sec(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(sqrt(1/cos(x)^3 + 1) + 1) + 1/3*log(abs(sqrt(1/cos(x)^3 + 1) - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\tan(x)}{\sqrt{\frac{1}{\cos(x)^3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1/cos(x)^3 + 1)^(1/2),x)
[Out] int(tan(x)/(1/cos(x)^3 + 1)^(1/2), x)
```

3.8 $\int \sqrt{2+2\tan(x)+\tan^2(x)}\,dx$

Optimal. Leaf size=137

$$\sinh^{-1}(1+\tan(x))-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2\tan(x)+\tan^2(x)}}\right)-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right.}$$

```
[Out] arcsinh(1+tan(x))-1/2*arctanh((2*5^(1/2)+(5-5^(1/2))*tan(x))/(-10+10*5^(1/2
))^(1/2)/(2+2*tan(x)+tan(x)^2)^(1/2))*(-2+2*5^(1/2))^(1/2)-1/2*arctan((2*5^
(1/2)-(5+5^(1/2))*tan(x))/(10+10*5^(1/2))^(1/2)/(2+2*tan(x)+tan(x)^2)^(1/2)
)*(2+2*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {1004, 633, 221, 1050, 1044, 213, 209}

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{\tan^2(x)+2\tan(x)+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)\tan(x)+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{\tan^2(x)+2\tan(x)+2}}\right)+\sinh^{-1}(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 2*Tan[x] + Tan[x]^2],x]
```

```
[Out] ArcSinh[1 + Tan[x]] - Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5
])*Tan[x])/(Sqrt[10*(1 + Sqrt[5])]*Sqrt[2 + 2*Tan[x] + Tan[x]^2])] - Sqrt[(
-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5] + (5 - Sqrt[5])*Tan[x])/(Sqrt[10*(-1 +
Sqrt[5])]*Sqrt[2 + 2*Tan[x] + Tan[x]^2])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$\int \sqrt{2+2\tan(x)+\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{2+2x+x^2}}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{\sqrt{2+2x+x^2}}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{-1-2x}{(1+x^2)\sqrt{2+2x+}}\right.$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4}}}\,dx, x, 2+2\tan(x)\right) - \frac{\text{Subst}\left(\int \frac{5-\sqrt{5}-2\sqrt{5}\,x}{(1+x^2)\sqrt{2+2x+}}\right.}{2\sqrt{5}}$$
$$= \sinh^{-1}(1+\tan(x)) - \left(2\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{20\left(1-\sqrt{5}\right)+2x^2}\,dx, x\right.$$
$$= \sinh^{-1}(1+\tan(x)) - \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right.}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2\mathrm{t}}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.67, size = 99, normalized size = 0.72

$$\sinh^{-1}(1+\tan(x)) + \frac{1}{2}i\left(\sqrt{1+2i}\tanh^{-1}\left(\frac{(2+i)+(1+i)\tan(x)}{\sqrt{1+2i}\sqrt{2+2\tan(x)+\tan^2(x)}}\right) - \sqrt{1-2i}\tanh^{-1}\left(\frac{(4-2i)+(2-2i)\tan(x)}{2\sqrt{1-2i}\sqrt{2+2\tan(x)+\tan^2(x)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 2*Tan[x] + Tan[x]^2],x]
[Out] ArcSinh[1 + Tan[x]] + (I/2)*(Sqrt[1 + 2*I]*ArcTanh[((2 + I) + (1 + I)*Tan[x
])/(Sqrt[1 + 2*I]*Sqrt[2 + 2*Tan[x] + Tan[x]^2])] - Sqrt[1 - 2*I]*ArcTanh[(
(4 - 2*I) + (2 - 2*I)*Tan[x])/(2*Sqrt[1 - 2*I]*Sqrt[2 + 2*Tan[x] + Tan[x]^2
])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. 2(105) = 210.
time = 0.46, size = 1604, normalized size = 11.71

method	result	size
derivativedivides	Expression too large to display	1604
default	Expression too large to display	1604

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+2*tan(x)+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] arcsinh(tan(x)+1)-1/10*(10*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-ta
n(x))^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+1
0+2*5^(1/2))^(1/2)*5^(1/2)*(3*(-22+10*5^(1/2))^(1/2)*(-10+10*5^(1/2))^(1/2)
*arctan(1/80*(-22+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+tan(x
))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2
)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+25*(-1/2*5^(1/2)+1/2+tan(x))^2/
(-1/2*5^(1/2)-1/2-tan(x))^2+4*5^(1/2)+10)*(-1/2*5^(1/2)+1/2+tan(x))/(-1/2*5
^(1/2)-1/2-tan(x))*(5^(1/2)-5)/((-1/2*5^(1/2)+1/2+tan(x))^4/(-1/2*5^(1/2)-1
/2-tan(x))^4+3*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+1))*
5^(1/2)+5*(-22+10*5^(1/2))^(1/2)*(-10+10*5^(1/2))^(1/2)*arctan(1/80*(-22+10
*5^(1/2))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1
/2-tan(x))^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/
2*5^(1/2)-1/2-tan(x))^2+25*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-ta
n(x))^2+4*5^(1/2)+10)*(-1/2*5^(1/2)+1/2+tan(x))/(-1/2*5^(1/2)-1/2-tan(x))*(
5^(1/2)-5)/((-1/2*5^(1/2)+1/2+tan(x))^4/(-1/2*5^(1/2)-1/2-tan(x))^4+3*(-1/2
*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+1))-20*arctanh((10*(-1/2
*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-2*5^(1/2)*(-1/2*5^(1/2)+
1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/
2))^(1/2))*5^(1/2)+60*arctanh((10*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)
-1/2-tan(x))^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(
x))^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2)))/(-2*(5^(1/2)*(-1/2*5^(1/
2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-5*(-1/2*5^(1/2)+1/2+tan(x))^2/
(-1/2*5^(1/2)-1/2-tan(x))^2-5^(1/2)-5)/((-1/2*5^(1/2)+1/2+tan(x))/(-1/2*5^(
1/2)-1/2-tan(x))+1)^2)^(1/2)/((-1/2*5^(1/2)+1/2+tan(x))/(-1/2*5^(1/2)-1/2-t
an(x))+1)/(5^(1/2)-5)/(-10+10*5^(1/2))^(1/2)-1/5*(10*(-1/2*5^(1/2)+1/2+tan(
x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1
/2*5^(1/2)-1/2-tan(x))^2+10+2*5^(1/2))^(1/2)*5^(1/2)*((-22+10*5^(1/2))^(1/2
)*(-10+10*5^(1/2))^(1/2)*arctan(1/80*(-22+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2
*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+5^(1/2)+3))^(1/2)*
(11*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+25*(-1/
2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+4*5^(1/2)+10)*(-1/2*5^(
1/2)+1/2+tan(x))/(-1/2*5^(1/2)-1/2-tan(x))*(5^(1/2)-5)/((-1/2*5^(1/2)+1/2+t
an(x))^4/(-1/2*5^(1/2)-1/2-tan(x))^4+3*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^
(1/2)-1/2-tan(x))^2+1))*5^(1/2)+5*(-22+10*5^(1/2))^(1/2)*(-10+10*5^(1/2))^(
1/2)*arctan(1/80*(-22+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2*(-1/2*5^(1/2)+1/2+t
an(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(-1/2*5^
(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+25*(-1/2*5^(1/2)+1/2+tan(x)
)^2/(-1/2*5^(1/2)-1/2-tan(x))^2+4*5^(1/2)+10)*(-1/2*5^(1/2)+1/2+tan(x))/(-1
/2*5^(1/2)-1/2-tan(x))*(5^(1/2)-5)/((-1/2*5^(1/2)+1/2+tan(x))^4/(-1/2*5^(1/
2)-1/2-tan(x))^4+3*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+
1))-20*arctanh((10*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-
2*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2+10+2*5^(1
/2))^(1/2)/(-10+10*5^(1/2))^(1/2))*5^(1/2)+20*arctanh((10*(-1/2*5^(1/2)+1/2
```

```
+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-2*5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^
2/(-1/2*5^(1/2)-1/2-tan(x))^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2)))/
(-2*(5^(1/2)*(-1/2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-5*(-1/
2*5^(1/2)+1/2+tan(x))^2/(-1/2*5^(1/2)-1/2-tan(x))^2-5^(1/2)-5)/((-1/2*5^(1/
2)+1/2+tan(x))/(-1/2*5^(1/2)-1/2-tan(x))+1)^2)^(1/2)/((-1/2*5^(1/2)+1/2+tan
(x))/(-1/2*5^(1/2)-1/2-tan(x))+1)/(5^(1/2)-5)/(-10+10*5^(1/2))^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*tan(x)+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*tan(x)+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tan^{2}(x) + 2\tan(x) + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*tan(x)+tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)**2 + 2*tan(x) + 2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(104) = 208.
time = 0.51, size = 495, normalized size = 3.61

$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(\sqrt{\tan(x)^2+2\tan(x)+2}-\tan(x)\right)+\sqrt{5}\sqrt{\sqrt{5}-2}-\sqrt{5}-2\sqrt{\tan(x)^2+2\tan(x)+2}-2\sqrt{\sqrt{5}-2}+2\tan(x)+2\right)^2+256\left(\sqrt{5}\left(\sqrt{\tan(x)^2+2\tan(x)+2}-\tan(x)\right)+\sqrt{5}-2\sqrt{\tan(x)^2+2\tan(x)+2}+\sqrt{\sqrt{5}-2}+2\tan(x)-2\right)^2\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(\sqrt{\tan(x)^2+2\tan(x)+2}-\tan(x)\right)-\sqrt{5}\sqrt{\sqrt{5}-2}-\sqrt{5}-2\sqrt{\tan(x)^2+2\tan(x)+2}+2\sqrt{\sqrt{5}-2}+2\tan(x)+2\right)^2+256\left(\sqrt{5}\left(\sqrt{\tan(x)^2+2\tan(x)+2}-\tan(x)\right)+\sqrt{5}-2\sqrt{\tan(x)^2+2\tan(x)+2}-\sqrt{\sqrt{5}-2}+2\tan(x)-2\right)^2\right)+\frac{\left(\pi+4\arctan\left(-\frac{1}{2}\left(2\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+4\sqrt{\sqrt{5}-2}+3\right)\left(\sqrt{\tan(x)^2+2\tan(x)+2}-\tan(x)\right)+\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{\sqrt{5}-2}+\frac{1}{2}\right)\right)\sqrt{2\sqrt{5}-2}}{4\left(\sqrt{5}-1\right)}-\frac{\left(\pi+4\arctan\left(\frac{1}{2}\left(2\sqrt{5}\sqrt{\sqrt{5}-2}-\sqrt{5}+4\sqrt{\sqrt{5}-2}-3\right)\left(\sqrt{\tan(x)^2+2\tan(x)+2}-\tan(x)\right)-\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{\sqrt{5}-2}+\frac{1}{2}\right)\right)\sqrt{2\sqrt{5}-2}}{4\left(\sqrt{5}-1\right)}-\log\left(\sqrt{\tan(x)^2+2\tan(x)+2}-\tan(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*tan(x)+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2*sqrt(5) - 2)*log(256*(sqrt(5)*(sqrt(tan(x)^2 + 2*tan(x) + 2) -
tan(x)) + sqrt(5)*sqrt(sqrt(5) - 2) - sqrt(5) - 2*sqrt(tan(x)^2 + 2*tan(x)
+ 2) - 2*sqrt(sqrt(5) - 2) + 2*tan(x) + 2)^2 + 256*(sqrt(5)*(sqrt(tan(x)^2
+ 2*tan(x) + 2) - tan(x)) + sqrt(5) - 2*sqrt(tan(x)^2 + 2*tan(x) + 2) + sqr
t(sqrt(5) - 2) + 2*tan(x) - 2)^2) + 1/4*sqrt(2*sqrt(5) - 2)*log(256*(sqrt(5
)*(sqrt(tan(x)^2 + 2*tan(x) + 2) - tan(x)) - sqrt(5)*sqrt(sqrt(5) - 2) - sq
rt(5) - 2*sqrt(tan(x)^2 + 2*tan(x) + 2) + 2*sqrt(sqrt(5) - 2) + 2*tan(x) +
2)^2 + 256*(sqrt(5)*(sqrt(tan(x)^2 + 2*tan(x) + 2) - tan(x)) + sqrt(5) - 2*
sqrt(tan(x)^2 + 2*tan(x) + 2) - sqrt(sqrt(5) - 2) + 2*tan(x) - 2)^2) + 1/4*
(pi + 4*arctan(-1/2*(2*sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 4*sqrt(sqrt(5)
 - 2) + 3)*(sqrt(tan(x)^2 + 2*tan(x) + 2) - tan(x)) + 3/2*sqrt(5)*sqrt(sqrt
(5) - 2) + 1/2*sqrt(5) + 7/2*sqrt(sqrt(5) - 2) + 3/2))*sqrt(2*sqrt(5) - 2)/
(sqrt(5) - 1) - 1/4*(pi + 4*arctan(1/2*(2*sqrt(5)*sqrt(sqrt(5) - 2) - sqrt(
5) + 4*sqrt(sqrt(5) - 2) - 3)*(sqrt(tan(x)^2 + 2*tan(x) + 2) - tan(x)) - 3/
2*sqrt(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) - 7/2*sqrt(sqrt(5) - 2) + 3/2))*s
qrt(2*sqrt(5) - 2)/(sqrt(5) - 1) - log(sqrt(tan(x)^2 + 2*tan(x) + 2) - tan(
x) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\tan (x)^2 + 2 \tan (x) + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*tan(x) + tan(x)^2 + 2)^(1/2),x)
```

```
[Out] int((2*tan(x) + tan(x)^2 + 2)^(1/2), x)
```

3.9 $\int \tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\sin(x)\,dx$

Optimal. Leaf size=41

$$\frac{1}{2}\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)-\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)+\frac{1}{2}\cos(x)\sqrt{-1+\sec(x)}$$

```
[Out] 1/2*arctan((-1+sec(x))^(1/2))-arctan((-1+sec(x))^(1/2))*cos(x)+1/2*cos(x)*(
-1+sec(x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {4420, 5311, 12, 248, 44, 65, 209}

$$\frac{1}{2}\text{ArcTan}\left(\sqrt{\sec(x)-1}\right)-\cos(x)\text{ArcTan}\left(\sqrt{\sec(x)-1}\right)+\frac{1}{2}\cos(x)\sqrt{\sec(x)-1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[-1 + Sec[x]]]*Sin[x],x]
```

```
[Out] ArcTan[Sqrt[-1 + Sec[x]]]/2 - ArcTan[Sqrt[-1 + Sec[x]]]*Cos[x] + (Cos[x]*Sq
rt[-1 + Sec[x]])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\sin(x)\,dx &= -\text{Subst}\left(\int \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)\,dx, x, \cos(x)\right)\\
&= -\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)+\text{Subst}\left(\int -\frac{1}{2\sqrt{-1+\frac{1}{x}}}\,dx, x\right.\\
&= -\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+\frac{1}{x}}}\,dx, x, \text{c}\right.\\
&= -\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x^2}\,dx, x,\right.\\
&= -\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)+\frac{1}{2}\cos(x)\sqrt{-1+\sec(x)}+\frac{1}{4}\text{Su}\\
&= -\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)+\frac{1}{2}\cos(x)\sqrt{-1+\sec(x)}+\frac{1}{2}\text{Su}\\
&= \frac{1}{2}\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)-\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)+\frac{1}{2}\text{co}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 2.92, size = 283, normalized size = 6.90

$$-\tan^{-1}\left(\sqrt{-1+\sec(x)}\right)\cos(x)+\frac{1}{2}\cos(x)\sqrt{-1+\sec(x)}-\frac{1}{2}\left(-3-2\sqrt{2}\right)\cos^2\left(\frac{x}{4}\right)\left(1-\sqrt{2}+\left(-2+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)\right)\cot\left(\frac{x}{4}\right)\left(F\left(\sin^{-1}\left(\frac{\tan\left(\frac{x}{4}\right)}{\sqrt{3-2\sqrt{2}}}\right)|17-12\sqrt{2}\right)-2\Pi\left(-3+2\sqrt{2};\sin^{-1}\left(\frac{\tan\left(\frac{x}{4}\right)}{\sqrt{3-2\sqrt{2}}}\right)|17-12\sqrt{2}\right)\right)\sqrt{\left(7-5\sqrt{2}+\left(10-7\sqrt{2}\right)\cos\left(\frac{x}{2}\right)\right)\sec^2\left(\frac{x}{4}\right)}\sqrt{\left(-1-\sqrt{2}+\left(2+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)\right)\sec^2\left(\frac{x}{4}\right)}\sqrt{-1+\sec(x)}\sec(x)\sqrt{3-2\sqrt{2}-\tan^2\left(\frac{x}{4}\right)}\sqrt{1+\left(-3+2\sqrt{2}\right)\tan^2\left(\frac{x}{4}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTan[Sqrt[-1 + Sec[x]]]*Sin[x],x]
```

```
[Out] -(ArcTan[Sqrt[-1 + Sec[x]]]*Cos[x]) + (Cos[x]*Sqrt[-1 + Sec[x]])/2 - ((-3 -
 2*Sqrt[2])*Cos[x/4]^2*(1 - Sqrt[2] + (-2 + Sqrt[2])*Cos[x/2])*Cot[x/4]*(El
lipticF[ArcSin[Tan[x/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]] - 2*Elliptic
Pi[-3 + 2*Sqrt[2], ArcSin[Tan[x/4]/Sqrt[3 - 2*Sqrt[2]]], 17 - 12*Sqrt[2]])*
Sqrt[(7 - 5*Sqrt[2] + (10 - 7*Sqrt[2])*Cos[x/2])*Sec[x/4]^2]*Sqrt[(-1 - Sqr
t[2] + (2 + Sqrt[2])*Cos[x/2])*Sec[x/4]^2]*Sqrt[-1 + Sec[x]]*Sec[x]*Sqrt[3
- 2*Sqrt[2] - Tan[x/4]^2*Sqrt[1 + (-3 + 2*Sqrt[2])*Tan[x/4]^2])/2
```

Maple [A]
time = 0.07, size = 42, normalized size = 1.02

method	result	size
derivativedivides	$-\frac{\arctan\left(\sqrt{-\left(\frac{1}{\sec(x)}-1\right)\sec(x)}\right)}{\sec(x)}+\frac{\sqrt{-1+\sec(x)}}{2\sec(x)}+\frac{\arctan\left(\sqrt{-1+\sec(x)}\right)}{2}$	42
default	$-\frac{\arctan\left(\sqrt{-\left(\frac{1}{\sec(x)}-1\right)\sec(x)}\right)}{\sec(x)}+\frac{\sqrt{-1+\sec(x)}}{2\sec(x)}+\frac{\arctan\left(\sqrt{-1+\sec(x)}\right)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan((-1+sec(x))^(1/2))*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/sec(x)*arctan((-(1/sec(x)-1)*sec(x))^(1/2))+1/2*(-1+sec(x))^(1/2)/sec(x)
+1/2*arctan((-1+sec(x))^(1/2))
```

Maxima [A]
time = 2.59, size = 60, normalized size = 1.46

$$-\arctan\left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}}\right)\cos(x)-\frac{\sqrt{-\frac{\cos(x)-1}{\cos(x)}}}{2\left(\frac{\cos(x)-1}{\cos(x)}-1\right)}+\frac{1}{2}\arctan\left(\sqrt{-\frac{\cos(x)-1}{\cos(x)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((-1+sec(x))^(1/2))*sin(x),x, algorithm="maxima")
```

```
[Out] -arctan(sqrt(-(cos(x) - 1)/cos(x)))*cos(x) - 1/2*sqrt(-(cos(x) - 1)/cos(x))
/((cos(x) - 1)/cos(x) - 1) + 1/2*arctan(sqrt(-(cos(x) - 1)/cos(x)))
```

Fricas [A]
time = 1.06, size = 32, normalized size = 0.78

$$-\frac{1}{2}(2\cos(x)-1)\arctan\left(\sqrt{\sec(x)-1}\right)+\frac{1}{2}\sqrt{-\frac{\cos(x)-1}{\cos(x)}}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((-1+sec(x))^(1/2))*sin(x),x, algorithm="fricas")
[Out] -1/2*(2*cos(x) - 1)*arctan(sqrt(sec(x) - 1)) + 1/2*sqrt(-(cos(x) - 1)/cos(x
))*cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin(x)\operatorname{atan}\left(\sqrt{\sec(x)-1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan((-1+sec(x))**(1/2))*sin(x),x)
[Out] Integral(sin(x)*atan(sqrt(sec(x) - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan((-1+sec(x))^(1/2))*sin(x),x, algorithm="giac")
[Out] undef
```

Mupad [B]
time = 0.42, size = 60, normalized size = 1.46

$$-\operatorname{atan}\left(\sqrt{\frac{1}{\cos(x)}-1}\right)\cos(x)-\frac{\cos(x)\left(\frac{3\operatorname{asin}\left(\sqrt{\cos(x)}\right)}{2\cos(x)^{3/2}}-\frac{3\sqrt{1-\cos(x)}}{2\cos(x)}\right)\sqrt{1-\cos(x)}}{3\sqrt{\frac{1}{\cos(x)}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((1/cos(x) - 1)^(1/2))*sin(x),x)
[Out] - atan((1/cos(x) - 1)^(1/2))*cos(x) - (cos(x)*((3*asin(cos(x)^(1/2)))/(2*co
s(x)^(3/2)) - (3*(1 - cos(x))^(1/2))/(2*cos(x)))*(1 - cos(x))^(1/2))/(3*(1/
cos(x) - 1)^(1/2))
```

3.10 $\int \frac{e^{\sin^{-1}(x)}x^3}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=44

$$\frac{1}{10}e^{\sin^{-1}(x)}\left(3x+x^3-3\sqrt{1-x^2}-3x^2\sqrt{1-x^2}\right)$$

```
[Out] 1/10*exp(arcsin(x))*(3*x+x^3-3*(-x^2+1)^(1/2)-3*x^2*(-x^2+1)^(1/2))
```

Rubi [A]
time = 0.47, antiderivative size = 62, normalized size of antiderivative = 1.41, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4920, 6873, 6852, 4519, 4517}

$$\frac{1}{10}x^3e^{\text{ArcSin}(x)}-\frac{3}{10}\sqrt{1-x^2}\,x^2e^{\text{ArcSin}(x)}-\frac{3}{10}\sqrt{1-x^2}\,e^{\text{ArcSin}(x)}+\frac{3}{10}xe^{\text{ArcSin}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcSin[x]*x^3)/Sqrt[1 - x^2],x]
```

```
[Out] (3*E^ArcSin[x]*x)/10 + (E^ArcSin[x]*x^3)/10 - (3*E^ArcSin[x]*Sqrt[1 - x^2])
/10 - (3*E^ArcSin[x]*x^2*Sqrt[1 - x^2])/10
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4519

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]^n/(e^2*n^2 + b^2*c^2*Lo
g[F]^2)), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[e*n*F^(c*(a + b*x))*Cos[d +
 e*x]*(Sin[d + e*x]^(n - 1)/(e^2*n^2 + b^2*c^2*Log[F]^2)), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 4920

```
Int[(u_.)*(f_)^(ArcSin[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Dist[
1/b, Subst[Int[(u /. x -> -a/b + Sin[x]/b)*f^(c*x^n)*Cos[x], x], x, ArcSin[
a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\sin^{-1}(x)} x^3}{\sqrt{1-x^2}} \, dx &= \text{Subst}\left(\int \frac{e^x \cos(x) \sin^3(x)}{\sqrt{1-\sin^2(x)}} \, dx, x, \sin^{-1}(x)\right) \\
&= \text{Subst}\left(\int \frac{e^x \cos(x) \sin^3(x)}{\sqrt{\cos^2(x)}} \, dx, x, \sin^{-1}(x)\right) \\
&= 1\text{Subst}\left(\int e^x \sin^3(x) \, dx, x, \sin^{-1}(x)\right) \\
&= \frac{1}{10} e^{\sin^{-1}(x)} x^3 - \frac{3}{10} e^{\sin^{-1}(x)} x^2 \sqrt{1-x^2} + \frac{3}{5}\text{Subst}\left(\int e^x \sin(x) \, dx, x, \sin^{-1}(x)\right) \\
&= \frac{3}{10} e^{\sin^{-1}(x)} x + \frac{1}{10} e^{\sin^{-1}(x)} x^3 - \frac{3}{10} e^{\sin^{-1}(x)} \sqrt{1-x^2} - \frac{3}{10} e^{\sin^{-1}(x)} x^2 \sqrt{1-x^2}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 38, normalized size = 0.86

$$-\frac{1}{40} e^{\sin^{-1}(x)} \left(15\left(-x+\sqrt{1-x^2}\right) - 3\cos\left(3\sin^{-1}(x)\right) + \sin\left(3\sin^{-1}(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^ArcSin[x]*x^3)/Sqrt[1 - x^2],x]
[Out] -1/40*(E^ArcSin[x]*(15*(-x + Sqrt[1 - x^2]) - 3*Cos[3*ArcSin[x]] + Sin[3*Ar
cSin[x]]))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e^{\arcsin(x)} x^3}{\sqrt{-x^2+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x)
[Out] int(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3*e^arcsin(x)/sqrt(-x^2 + 1), x)
```

Fricas [A]
time = 0.78, size = 28, normalized size = 0.64

$$\frac{1}{10}\left(x^{3}-3\left(x^{2}+1\right)\sqrt{-x^{2}+1}+3\,x\right)e^{\arcsin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/10*(x^3 - 3*(x^2 + 1)*sqrt(-x^2 + 1) + 3*x)*e^arcsin(x)
```

Sympy [A]
time = 0.38, size = 56, normalized size = 1.27

$$\frac{x^{3}e^{\operatorname{asin}(x)}}{10}-\frac{3x^{2}\sqrt{1-x^{2}}\,e^{\operatorname{asin}(x)}}{10}+\frac{3xe^{\operatorname{asin}(x)}}{10}-\frac{3\sqrt{1-x^{2}}\,e^{\operatorname{asin}(x)}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(asin(x))*x**3/(-x**2+1)**(1/2),x)
[Out] x**3*exp(asin(x))/10 - 3*x**2*sqrt(1 - x**2)*exp(asin(x))/10 + 3*x*exp(asin
(x))/10 - 3*sqrt(1 - x**2)*exp(asin(x))/10
```

Giac [A]
time = 0.45, size = 46, normalized size = 1.05

$$\frac{1}{10}\left(x^{2}-1\right)xe^{\arcsin(x)}+\frac{3}{10}\left(-x^{2}+1\right)^{\frac{3}{2}}e^{\arcsin(x)}+\frac{2}{5}\,xe^{\arcsin(x)}-\frac{3}{5}\,\sqrt{-x^{2}+1}\,e^{\arcsin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arcsin(x))*x^3/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/10*(x^2 - 1)*x*e^arcsin(x) + 3/10*(-x^2 + 1)^(3/2)*e^arcsin(x) + 2/5*x*e^
arcsin(x) - 3/5*sqrt(-x^2 + 1)*e^arcsin(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3 \, \mathrm{e}^{\mathrm{asin}(x)}}{\sqrt{1-x^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*exp(asin(x)))/(1 - x^2)^(1/2),x)
[Out] int((x^3*exp(asin(x)))/(1 - x^2)^(1/2), x)
```

3.11 $\int \frac{x \log(1+x^2) \log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=68

$$4x-2\tan^{-1}(x)-x\log\left(1+x^2\right)-2\sqrt{1+x^2}\log\left(x+\sqrt{1+x^2}\right)+\sqrt{1+x^2}\log\left(1+x^2\right)\log\left(x+\sqrt{1+x^2}\right)$$

```
[Out] 4*x-2*arctan(x)-x*ln(x^2+1)-2*ln(x+(x^2+1)^(1/2))*(x^2+1)^(1/2)+ln(x^2+1)*l
n(x+(x^2+1)^(1/2))*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {267, 2634, 8, 2637, 12, 2498, 327, 209}

$$-2\text{ArcTan}(x)+x\left(-\log\left(x^2+1\right)\right)+\sqrt{x^2+1}\log\left(x^2+1\right)\log\left(\sqrt{x^2+1}+x\right)-2\sqrt{x^2+1}\log\left(\sqrt{x^2+1}+x\right)+4x$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[1 + x^2]*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] 4*x - 2*ArcTan[x] - x*Log[1 + x^2] - 2*Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]]
 + Sqrt[1 + x^2]*Log[1 + x^2]*Log[x + Sqrt[1 + x^2]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rubi steps

$$\int \frac{x\log\left(1+x^2\right)\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2}\,\log\left(1+x^2\right)\log\left(x+\sqrt{1+x^2}\right) - \int \log\left(1+x^2\right)\,dx$$

$$= -x\log\left(1+x^2\right) + \sqrt{1+x^2}\,\log\left(1+x^2\right)\log\left(x+\sqrt{1+x^2}\right) + 2$$

$$= 2x - x\log\left(1+x^2\right) - 2\sqrt{1+x^2}\,\log\left(x+\sqrt{1+x^2}\right) + \sqrt{1+x^2}$$

$$= 4x - 2\tan^{-1}(x) - x\log\left(1+x^2\right) - 2\sqrt{1+x^2}\,\log\left(x+\sqrt{1+x^2}\right.$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.94

$$4x - 2\tan^{-1}(x) - 2\sqrt{1+x^2}\,\log\left(x+\sqrt{1+x^2}\right) + \log\left(1+x^2\right)\left(-x+\sqrt{1+x^2}\,\log\left(x+\sqrt{1+x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[1 + x^2]*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] 4*x - 2*ArcTan[x] - 2*Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]] + Log[1 + x^2]*(
-x + Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x \ln \left(x^{2}+1\right) \ln \left(x+\sqrt{x^{2}+1}\right)}{\sqrt{x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^2+1)*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

```
[Out] int(x*ln(x^2+1)*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+1)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] -(2*x^2 - (x^2 + 1)*log(x^2 + 1) + 2)*log(x + sqrt(x^2 + 1))/sqrt(x^2 + 1)
+ integrate((log(x^2 + 1) - 2)/(x^2 + sqrt(x^2 + 1)*x), x) - integrate(-(2*
x^2 - (x^2 + 1)*log(x^2 + 1) + 2)/(sqrt(x^2 + 1)*x), x)
```

Fricas [A]
time = 0.87, size = 43, normalized size = 0.63

$$\sqrt{x^{2}+1}\,\left(\log \left(x^{2}+1\right)-2\right) \log \left(x+\sqrt{x^{2}+1}\right)-x \log \left(x^{2}+1\right)+4\, x-2\, \arctan \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+1)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] sqrt(x^2 + 1)*(log(x^2 + 1) - 2)*log(x + sqrt(x^2 + 1)) - x*log(x^2 + 1) +
4*x - 2*arctan(x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x**2+1)*ln(x+(x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+1)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(x*log(x^2 + 1)*log(x + sqrt(x^2 + 1))/sqrt(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln\left(x^2+1\right)\,\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x^2 + 1)*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
```

```
[Out] int((x*log(x^2 + 1)*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2), x)
```

3.12 $\int \tan^{-1}\left(x+\sqrt{1-x^2}\right)\,dx$

Optimal. Leaf size=141

$$-\frac{1}{2}\sin^{-1}(x)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)+x\tan^{-1}\left(\right.$$

```
[Out] -1/2*arcsin(x)+x*arctan(x+(-x^2+1)^(1/2))-1/4*arctanh(x*(-x^2+1)^(1/2))-1/8
*ln(x^4-x^2+1)-1/4*arctan(1/3*(2*x^2-1)*3^(1/2))*3^(1/2)+1/4*arctan((-1+x*3
^(1/2))/(-x^2+1)^(1/2))*3^(1/2)+1/4*arctan((1+x*3^(1/2))/(-x^2+1)^(1/2))*3^
(1/2)
```

Rubi [C] Result contains complex when optimal does not.
time = 0.53, antiderivative size = 269, normalized size of antiderivative = 1.91, number of steps used = 40, number of rules used = 15, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$,
Rules used = {5311, 12, 6874, 222, 1128, 648, 632, 210, 642, 1188, 399, 385, 211, 1307, 1121}

$$-\frac{\text{ArcSin}(x)}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)+\frac{1}{12}\left(-\sqrt{3}+3i\right)\text{ArcTan}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)+\frac{\text{ArcTan}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{1}{12}\left(\sqrt{3}+3i\right)\text{ArcTan}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)+\frac{\text{ArcTan}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}+x\text{ArcTan}\left(\sqrt{1-x^2}+x\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x + Sqrt[1 - x^2]],x]
```

```
[Out] -1/2*ArcSin[x] + (Sqrt[3]*ArcTan[(1 - 2*x^2)/Sqrt[3]])/4 + ArcTan[x/(Sqrt[-
((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] + ((3*I - Sqrt[3])*A
rcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])])/12 + ArcTan[
(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3] - ((3*I + S
qrt[3])*ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]])/12
+ x*ArcTan[x + Sqrt[1 - x^2]] - Log[1 - x^2 + x^4]/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
```

```
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \tan^{-1}\left(x+\sqrt{1-x^2}\right)\,dx = x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \int \frac{x\left(1-\frac{x}{\sqrt{1-x^2}}\right)}{2\left(1+x\sqrt{1-x^2}\right)}\,dx$$

$$= x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \frac{x\left(1-\frac{x}{\sqrt{1-x^2}}\right)}{1+x\sqrt{1-x^2}}\,dx$$

$$= x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \left(\frac{x^2}{-x+x^3-\sqrt{1-x^2}} + \frac{x}{1+x\sqrt{1-x^2}}\right)$$

$$= x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \frac{x^2}{-x+x^3-\sqrt{1-x^2}}\,dx - \frac{1}{2}\int \frac{x}{1+x\sqrt{1-}}$$

$$= x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \left(\frac{x}{1-x^2+x^4} - \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\right)dx - \frac{1}{2}\int \left(\right.$$

$$= x\tan^{-1}\left(x+\sqrt{1-x^2}\right) + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx - \frac{1}{2}\int \frac{x}{1-x^2+x^4}\,dx - \frac{1}{2}\int$$

$$= \frac{1}{2}\sin^{-1}(x) + x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\sin^{-1}(x) + x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\sin^{-1}(x) + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + x\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{8}\log\left(1-x^2+\right.$$

$$= \frac{1}{2}\sin^{-1}(x) + \frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right) + \frac{1}{12}\left(3i-\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.32, size = 307, normalized size = 2.18

$$x\tan^{-1}\left(x+\sqrt{1-x^2}\right)+\frac{1}{8}\left((-4-4i)\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-2i\tan^{-1}\left(\frac{x+i\left(-1+\sqrt{1-x^2}\right)}{i+x-2ix^2+(i+2x)\sqrt{1-x^2}}\right)-4i\sqrt{3}\tanh^{-1}\left(\frac{1-(1+i)x+(1-i)\sqrt{1-x^2}}{\sqrt{3}}\right)-2\log\left(1+\sqrt{3}+(1+i)x-(1-i)\sqrt{1-x^2}\right)-2i\sqrt{3}\log\left(1+\sqrt{3}+(1+i)x-(1-i)\sqrt{1-x^2}\right)-2\log\left(-1+\sqrt{3}-(1+i)x+(1-i)\sqrt{1-x^2}\right)+2i\sqrt{3}\log\left(-1+\sqrt{3}-(1+i)x+(1-i)\sqrt{1-x^2}\right)-\log\left(\left(-1+ix+x^2\right)\left(-1+2x^2+2ix\sqrt{1-x^2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x + Sqrt[1 - x^2]],x]
```

```
[Out] x*ArcTan[x + Sqrt[1 - x^2]] + ((-4 - 4*I)*ArcTan[x/Sqrt[1 - x^2]] - (2*I)*A
rcTan[(x + I*(-1 + Sqrt[1 - x^2]))/(I + x - (2*I)*x^2 + (I + 2*x)*Sqrt[1 -
x^2])] - (4*I)*Sqrt[3]*ArcTanh[(1 - (1 + I)*x + (1 - I)*Sqrt[1 - x^2])/Sqrt
[3]] - 2*Log[1 + Sqrt[3] + (1 + I)*x - (1 - I)*Sqrt[1 - x^2]] - (2*I)*Sqrt[
3]*Log[1 + Sqrt[3] + (1 + I)*x - (1 - I)*Sqrt[1 - x^2]] - 2*Log[-1 + Sqrt[3
] - (1 + I)*x + (1 - I)*Sqrt[1 - x^2]] + (2*I)*Sqrt[3]*Log[-1 + Sqrt[3] - (
1 + I)*x + (1 - I)*Sqrt[1 - x^2]] - Log[(-1 + I*x + x^2)*(-1 + 2*x^2 + (2*I
)*x*Sqrt[1 - x^2])])/8
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 439, normalized size = 3.11

method	result
default	$x\arctan\left(x+\sqrt{-x^2+1}\right)-\frac{\ln(x^4-x^2+1)}{8}-\frac{\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)\sqrt{3}}{4}-\frac{i\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{(1-}{}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctan(x+(-x^2+1)^(1/2))-1/8*ln(x^4-x^2+1)-1/4*arctan(1/3*(2*x^2-1)*3^(1/
2))*3^(1/2)-1/8*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(1+I*3^(1/2))*((-x^2+
1)^(1/2)-1)/x-1)-1/8*ln(((-x^2+1)^(1/2)-1)^2/x^2+(1+I*3^(1/2))*((-x^2+1)^(1
/2)-1)/x-1)-1/8*ln(((-x^2+1)^(1/2)-1)^2/x^2+(1-I*3^(1/2))*((-x^2+1)^(1/2)-1
)/x-1)+1/8*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(1-I*3^(1/2))*((-x^2+1)^(1
/2)-1)/x-1)-1/8*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-1+I*3^(1/2))*((-x^2
+1)^(1/2)-1)/x-1)+1/8*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-1-I*3^(1/2))*((-x^2+1)^
(1/2)-1)/x-1)+1/8*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-1+I*3^(1/2))*((-x^2+1)^(1/2
)-1)/x-1)+1/8*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(-1-I*3^(1/2))*((-x^2+1
)^(1/2)-1)/x-1)+arctan(((-x^2+1)^(1/2)-1)/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(x + sqrt(x + 1)*sqrt(-x + 1)) - integrate((x^3 + x^2*e^(1/2*log(x
+ 1) + 1/2*log(-x + 1)) - x)/(x^4 + (x^2 - 1)*e^(log(x + 1) + log(-x + 1))
+ 2*(x^3 - x)*e^(1/2*log(x + 1) + 1/2*log(-x + 1)) - 1), x)
```

Fricas [**A**]
time = 0.82, size = 191, normalized size = 1.35

$$x \arctan\left(x + \sqrt{-x^{2} + 1}\right) - \frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 x^{2} - 1\right)\right) - \frac{1}{8} \sqrt{3} \arctan\left(\frac{4 \sqrt{3} \sqrt{-x^{2} + 1} x + \sqrt{3}}{3 \left(2 x^{2} - 1\right)}\right) - \frac{1}{8} \sqrt{3} \arctan\left(\frac{4 \sqrt{3} \sqrt{-x^{2} + 1} x - \sqrt{3}}{3 \left(2 x^{2} - 1\right)}\right) + \frac{1}{2} \arctan\left(\frac{\sqrt{-x^{2} + 1} x}{x^{2} - 1}\right) - \frac{1}{8} \log\left(x^{4} - x^{2} + 1\right) - \frac{1}{16} \log\left(-x^{4} + x^{2} + 2 \sqrt{-x^{2} + 1} x + 1\right) + \frac{1}{16} \log\left(-x^{4} + x^{2} - 2 \sqrt{-x^{2} + 1} x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*arctan(x + sqrt(-x^2 + 1)) - 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1))
- 1/8*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(-x^2 + 1)*x + sqrt(3))/(2*x^2 - 1)
) - 1/8*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(-x^2 + 1)*x - sqrt(3))/(2*x^2 -
1)) + 1/2*arctan(sqrt(-x^2 + 1)*x/(x^2 - 1)) - 1/8*log(x^4 - x^2 + 1) - 1/1
6*log(-x^4 + x^2 + 2*sqrt(-x^2 + 1)*x + 1) + 1/16*log(-x^4 + x^2 - 2*sqrt(-
x^2 + 1)*x + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x+(-x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(110) = 220.
time = 0.45, size = 364, normalized size = 2.58

$$x \arctan\left(x + \sqrt{-x^{2} + 1}\right) - \frac{1}{4} \pi \mathrm{sgn}\left(x\right) + \frac{1}{8} \sqrt{3} \left(\pi \mathrm{sgn}\left(x\right) + 2 \arctan\left(-\frac{\sqrt{3} x \left(\frac{\sqrt{-x^{2} + 1} - 1}{x} + \frac{\left(\sqrt{-x^{2} + 1} - 1\right)^{2}}{x^{2}} - 1\right)}{3 \left(\sqrt{-x^{2} + 1} - 1\right)}\right)\right) + \frac{1}{8} \sqrt{3} \left(\pi \mathrm{sgn}\left(x\right) + 2 \arctan\left(\frac{\sqrt{3} x \left(\frac{\sqrt{-x^{2} + 1} - 1}{x} - \frac{\left(\sqrt{-x^{2} + 1} - 1\right)^{2}}{x^{2}} + 1\right)}{3 \left(\sqrt{-x^{2} + 1} - 1\right)}\right)\right) - \frac{1}{4} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 x^{2} - 1\right)\right) - \frac{1}{2} \arctan\left(-\frac{x \left(\frac{\left(\sqrt{-x^{2} + 1} - 1\right)^{2}}{x^{2}} - 1\right)}{2 \left(\sqrt{-x^{2} + 1} - 1\right)}\right) - \frac{1}{8} \log\left(x^{4} - x^{2} + 1\right) + \frac{1}{8} \log\left(\left(\frac{x}{\sqrt{-x^{2} + 1} - 1} - \frac{\sqrt{-x^{2} + 1} - 1}{x}\right)^{2} + \frac{2 x}{\sqrt{-x^{2} + 1} - 1} - \frac{2 \left(\sqrt{-x^{2} + 1} - 1\right)}{x} + 4\right) - \frac{1}{8} \log\left(\left(\frac{x}{\sqrt{-x^{2} + 1} - 1} - \frac{\sqrt{-x^{2} + 1} - 1}{x}\right)^{2} - \frac{2 x}{\sqrt{-x^{2} + 1} - 1} + \frac{2 \left(\sqrt{-x^{2} + 1} - 1\right)}{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(x + sqrt(-x^2 + 1)) - 1/4*pi*sgn(x) + 1/8*sqrt(3)*(pi*sgn(x) + 2*a
rctan(-1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 -
 1)/(sqrt(-x^2 + 1) - 1))) + 1/8*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*
x*((sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1)
 - 1))) - 1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/2*arctan(-1/2*x*(
(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)) - 1/8*log(x^4 - x^2 +
```

```
 1) + 1/8*log((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 2*x/(sq
rt(-x^2 + 1) - 1) - 2*(sqrt(-x^2 + 1) - 1)/x + 4) - 1/8*log((x/(sqrt(-x^2 +
 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 - 2*x/(sqrt(-x^2 + 1) - 1) + 2*(sqrt(-
x^2 + 1) - 1)/x + 4)
```

Mupad [B]
time = 1.37, size = 661, normalized size = 4.69

$$x\,\mathrm{atan}\left(x+\sqrt{1-x^2}\right)-\frac{\mathrm{asin}(x)}{2}+\frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+\frac{1}{2}\mathrm{i}\right)}{2\sqrt{3}-8\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}}-\frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3-\frac{1}{2}\mathrm{i}\right)}{-2\sqrt{3}+8\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}}-\frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3-\frac{1}{2}\mathrm{i}\right)}{-2\sqrt{3}+8\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}}+\frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+\frac{1}{2}\mathrm{i}\right)}{2\sqrt{3}-8\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)\left(\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2+1\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(2\sqrt{3}-8\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}\right)}-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)\left(\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2+1\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-2\sqrt{3}+8\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}\right)}+\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}}\right)\left(\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2+1\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-2\sqrt{3}+8\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}\right)}-\frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)\left(\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2+1\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(2\sqrt{3}-8\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+2\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x + (1 - x^2)^(1/2)),x)
```

```
[Out] x*atan(x + (1 - x^2)^(1/2)) - asin(x)/2 + (log(x - 3^(1/2)/2 - 1i/2)*(3^(1/
2)/2 + (3^(1/2)/2 + 1i/2)^3 + 1i/2))/(2*3^(1/2) - 8*(3^(1/2)/2 + 1i/2)^3 +
2i) - (log(x - 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + (3^(1/2)/2 - 1i/2)^3 - 1i/2))
/(8*(3^(1/2)/2 - 1i/2)^3 - 2*3^(1/2) + 2i) - (log(x + 3^(1/2)/2 - 1i/2)*(3^
(1/2)/2 + (3^(1/2)/2 - 1i/2)^3 - 1i/2))/(8*(3^(1/2)/2 - 1i/2)^3 - 2*3^(1/2)
 + 2i) + (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + (3^(1/2)/2 + 1i/2)^3 + 1i/
2))/(2*3^(1/2) - 8*(3^(1/2)/2 + 1i/2)^3 + 2i) + (log((((x*(3^(1/2)/2 + 1i/2
) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/
2 - x + 1i/2))*((3^(1/2)/2 + 1i/2)^2 + 1))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2
)*(2*3^(1/2) - 8*(3^(1/2)/2 + 1i/2)^3 + 2i)) - (log((((x*(3^(1/2)/2 - 1i/2)
 - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - 3^(1/
2)/2 + 1i/2))*((3^(1/2)/2 - 1i/2)^2 + 1))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)
*(8*(3^(1/2)/2 - 1i/2)^3 - 2*3^(1/2) + 2i)) + (log((((x*(3^(1/2)/2 - 1i/2)
+ 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2
)/2 - 1i/2))*((3^(1/2)/2 - 1i/2)^2 + 1))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*
(8*(3^(1/2)/2 - 1i/2)^3 - 2*3^(1/2) + 2i)) - (log((((x*(3^(1/2)/2 + 1i/2) +
 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)
/2 + 1i/2))*((3^(1/2)/2 + 1i/2)^2 + 1))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(
2*3^(1/2) - 8*(3^(1/2)/2 + 1i/2)^3 + 2i))
```

3.13 $\int \frac{x \tan^{-1}\left(x+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=152

$$-\frac{1}{2}\sin^{-1}(x)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{-1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt{3}\,x}{\sqrt{1-x^2}}\right)-\frac{1}{4}\sqrt{3}\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)-\sqrt{1-x$$

```
[Out] -1/2*arcsin(x)+1/4*arctanh(x*(-x^2+1)^(1/2))+1/8*ln(x^4-x^2+1)-1/4*arctan(1
/3*(2*x^2-1)*3^(1/2))*3^(1/2)+1/4*arctan((-1+x*3^(1/2))/(-x^2+1)^(1/2))*3^(
1/2)+1/4*arctan((1+x*3^(1/2))/(-x^2+1)^(1/2))*3^(1/2)-arctan(x+(-x^2+1)^(1/
2))*(-x^2+1)^(1/2)
```

Rubi [C] Result contains complex when optimal does not.
time = 0.32, antiderivative size = 286, normalized size of antiderivative = 1.88, number of steps used = 32, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {267, 5315, 12, 6874, 1121, 632, 210, 1307, 222, 1188, 385, 211, 399, 1261, 648, 642}

$$-\frac{\text{ArcSin}(x)}{2}+\frac{1}{4}\sqrt{3}\,\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)-\frac{1}{12}\left(-\sqrt{3}+3i\right)\text{ArcTan}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)+\frac{\text{ArcTan}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{2\sqrt{3}}+\frac{1}{12}\left(\sqrt{3}+3i\right)\text{ArcTan}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)+\frac{\text{ArcTan}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{2\sqrt{3}}-\sqrt{1-x^2}\,\text{ArcTan}\left(\sqrt{1-x^2}+x\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x + Sqrt[1 - x^2]])/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*ArcSin[x] + (Sqrt[3]*ArcTan[(1 - 2*x^2)/Sqrt[3]])/4 + ArcTan[x/(Sqrt[-
((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/(2*Sqrt[3]) - ((3*I - Sqrt[3
])*ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])])/12 + Arc
Tan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/(2*Sqrt[3]) + (
(3*I + Sqrt[3])*ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^
2]])/12 - Sqrt[1 - x^2]*ArcTan[x + Sqrt[1 - x^2]] + Log[1 - x^2 + x^4]/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 5315

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\tan^{-1}\left(x+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \int \frac{x-\sqrt{1-x^2}}{2\left(1+x\sqrt{1-x^2}\right)}\,dx$$

$$= -\sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \frac{x-\sqrt{1-x^2}}{1+x\sqrt{1-x^2}}\,dx$$

$$= -\sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \left(\frac{x}{1+x\sqrt{1-x^2}} - \frac{\sqrt{1-x^2}}{1+x\sqrt{1-x}}\right.$$

$$= -\sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \frac{x}{1+x\sqrt{1-x^2}}\,dx + \frac{1}{2}\int \frac{\sqrt{1}}{1+x\sqrt{}}$$

$$= -\sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \left(\frac{x}{1-x^2+x^4} - \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\right) d$$

$$= -\sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{2}\int \frac{x}{1-x^2+x^4}\,dx + \frac{1}{2}\int \frac{\sqrt{1-x^2}}{1-x^2+}$$

$$= -\sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) - \frac{1}{4}$$

$$= -\frac{1}{2}\sin^{-1}(x) - \sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,d\right.$$

$$= -\frac{1}{2}\sin^{-1}(x) + \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \sqrt{1-x^2}\,\tan^{-1}\left(x+\sqrt{1-x^2}\right) + \frac{1}{8}\log\left(\right.$$

$$= -\frac{1}{2}\sin^{-1}(x) + \frac{1}{4}\sqrt{3}\,\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right) + \frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,\sqrt{1-x^2}}\right)}{2\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.25, size = 2180, normalized size = 14.34

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x + Sqrt[1 - x^2]])/Sqrt[1 - x^2],x]
```

```
[Out] (-24*ArcSin[x] - 48*Sqrt[1 - x^2]*ArcTan[x + Sqrt[1 - x^2]] + (2*(-3*I + Sq
rt[3])*ArcTan[(3 - I*Sqrt[3] + (-3 - I*Sqrt[3])*x^4 + 2*x*(-6*I + 2*Sqrt[3]
 - I*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) - 2*x^3*(6*I + 2*Sqrt[3] + I*Sq
rt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) - (2*I)*Sqrt[3]*x^2*(6 + Sqrt[2 - (2*I
)*Sqrt[3]]*Sqrt[1 - x^2]))/(I - Sqrt[3] + (6*I)*(I + Sqrt[3])*x - 2*(-15*I
+ Sqrt[3])*x^2 + 6*(1 + (3*I)*Sqrt[3])*x^3 + (11*I + 3*Sqrt[3])*x^4)])/Sqrt
[(1 - I*Sqrt[3])/6] - (2*(-3*I + Sqrt[3])*ArcTan[(3 - I*Sqrt[3] + (-3 - I*S
qrt[3])*x^4 + 2*x^3*(6*I + 2*Sqrt[3] + I*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x
^2]) + x*(12*I - 4*Sqrt[3] + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) -
 (2*I)*Sqrt[3]*x^2*(6 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]))/(I - Sqrt[3
] + (6 - (6*I)*Sqrt[3])*x - 2*(-15*I + Sqrt[3])*x^2 + (-6 - (18*I)*Sqrt[3])
*x^3 + (11*I + 3*Sqrt[3])*x^4)])/Sqrt[(1 - I*Sqrt[3])/6] - (2*(3*I + Sqrt[3
])*ArcTan[(-3 - I*Sqrt[3] + (3 - I*Sqrt[3])*x^4 + 2*x^3*(-6*I + 2*Sqrt[3] -
 I*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) - 2*x*(6*I + 2*Sqrt[3] + I*Sqrt[2
 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) - (2*I)*Sqrt[3]*x^2*(6 + Sqrt[2 + (2*I)*Sq
rt[3]]*Sqrt[1 - x^2]))/(-I - Sqrt[3] + (-6 - (6*I)*Sqrt[3])*x - 2*(15*I + S
qrt[3])*x^2 + 6*(1 - (3*I)*Sqrt[3])*x^3 + (-11*I + 3*Sqrt[3])*x^4)])/Sqrt[(
1 + I*Sqrt[3])/6] + (2*(3*I + Sqrt[3])*ArcTan[(-3 - I*Sqrt[3] + (3 - I*Sqrt
[3])*x^4 + 2*x*(6*I + 2*Sqrt[3] + I*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2])
+ x^3*(12*I - 4*Sqrt[3] + (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) - (2
*I)*Sqrt[3]*x^2*(6 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]))/(-I - Sqrt[3]
+ (6 + (6*I)*Sqrt[3])*x - 2*(15*I + Sqrt[3])*x^2 + (-6 + (18*I)*Sqrt[3])*x^
3 + (-11*I + 3*Sqrt[3])*x^4)])/Sqrt[(1 + I*Sqrt[3])/6] + 2*Sqrt[3]*(3*I + S
qrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2] + 2*Sqrt[3]*(-3*I + Sqrt[3])*Log[(I
/2)*(I + Sqrt[3]) + x^2] + ((3 - I*Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2]
)/Sqrt[(1 + I*Sqrt[3])/6] + ((3 + I*Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2
])/Sqrt[(1 - I*Sqrt[3])/6] - (I*(-3*I + Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x
^2)^2])/Sqrt[(1 - I*Sqrt[3])/6] + (I*(3*I + Sqrt[3])*Log[(4 - 4*Sqrt[3]*x +
 4*x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6] - (I*(-3*I + Sqrt[3])*Log[3*I + Sqrt[3]
 - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)
*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (
3*I)*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt
[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 - I*Sqrt[3])/6] + ((3 + I*Sqrt
[3])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]
*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^
3*(3 - (3*I)*Sqrt[3] - I*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I
 + 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 - I*Sqrt[
3])/6] + (I*(3*I + Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*
Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[
3]]*Sqrt[1 - x^2]) + x*(3 - (5*I)*Sqrt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*S
qrt[1 - x^2]) - I*x^3*(-3*I + 3*Sqrt[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 -
x^2])])/Sqrt[(1 + I*Sqrt[3])/6] + ((3 - I*Sqrt[3])*Log[-3*I + Sqrt[3] - (I
+ Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2
```

```
 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*Sqrt
[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 + (6*I
)*Sqrt[3]]*Sqrt[1 - x^2])])/Sqrt[(1 + I*Sqrt[3])/6])/48
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x \arctan\left(x+\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

```
[Out] int(x*arctan(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1)*arctan(x + sqrt(x + 1)*sqrt(-x + 1)) - integrate(
x/(x^2 + 2*x*e^(1/2*log(x + 1) + 1/2*log(-x + 1)) + e^(log(x + 1) + log(-x
+ 1)) + 1), x)
```

Fricas [A]
time = 0.93, size = 200, normalized size = 1.32

$$-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\sqrt{-x^2+1}\arctan\left(x+\sqrt{-x^2+1}\right)-\frac{1}{8}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{-x^2+1}\,x+\sqrt{3}}{3\left(2x^2-1\right)}\right)-\frac{1}{8}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\sqrt{-x^2+1}\,x-\sqrt{3}}{3\left(2x^2-1\right)}\right)+\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2+1}\,x}{x^2-1}\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{16}\log\left(-x^4+x^2+2\sqrt{-x^2+1}\,x+1\right)-\frac{1}{16}\log\left(-x^4+x^2-2\sqrt{-x^2+1}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - sqrt(-x^2 + 1)*arctan(x + sq
rt(-x^2 + 1)) - 1/8*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(-x^2 + 1)*x + sqrt(3
))/(2*x^2 - 1)) - 1/8*sqrt(3)*arctan(1/3*(4*sqrt(3)*sqrt(-x^2 + 1)*x - sqrt
(3))/(2*x^2 - 1)) + 1/2*arctan(sqrt(-x^2 + 1)*x/(x^2 - 1)) + 1/8*log(x^4 -
x^2 + 1) + 1/16*log(-x^4 + x^2 + 2*sqrt(-x^2 + 1)*x + 1) - 1/16*log(-x^4 +
x^2 - 2*sqrt(-x^2 + 1)*x + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x+(-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(119) = 238.
time = 0.50, size = 373, normalized size = 2.45

$$-\frac{1}{4}\pi\operatorname{sgn}(x)+\frac{1}{8}\sqrt{3}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x}+\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)+\frac{1}{8}\sqrt{3}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x}-\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{1}{4}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\sqrt{-x^2+1}\arctan\left(x+\sqrt{-x^2+1}\right)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)^2+\frac{2x}{\sqrt{-x^2+1}-1}-\frac{2\left(\sqrt{-x^2+1}-1\right)}{x}+4\right)+\frac{1}{8}\log\left(\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)^2-\frac{2x}{\sqrt{-x^2+1}-1}+\frac{2\left(\sqrt{-x^2+1}-1\right)}{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) + 1/8*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x
^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) + 1
/8*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (s
qrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) - 1/4*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x^2 - 1)) - sqrt(-x^2 + 1)*arctan(x + sqrt(-x^2 + 1)) - 1/2*a
rctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)) + 1/8*l
og(x^4 - x^2 + 1) - 1/8*log((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/
x)^2 + 2*x/(sqrt(-x^2 + 1) - 1) - 2*(sqrt(-x^2 + 1) - 1)/x + 4) + 1/8*log((
x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 - 2*x/(sqrt(-x^2 + 1) -
1) + 2*(sqrt(-x^2 + 1) - 1)/x + 4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\operatorname{atan}\left(x+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(x + (1 - x^2)^(1/2)))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x*atan(x + (1 - x^2)^(1/2)))/(1 - x^2)^(1/2), x)
```

3.14 $\int \frac{\sin^{-1}(x)}{1+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{x\sin^{-1}(x)}{1+\sqrt{1-x^2}}+\frac{1}{2}\sin^{-1}(x)^2-\log\left(1+\sqrt{1-x^2}\right)$$

```
[Out] 1/2*arcsin(x)^2-ln(1+(-x^2+1)^(1/2))-x*arcsin(x)/(1+(-x^2+1)^(1/2))
```

Rubi [A]
time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6874, 283, 222, 4875, 4723, 272, 65, 212, 4781, 29, 4737}

$$\frac{\sqrt{1-x^2}\,\text{ArcSin}(x)}{x}+\frac{\text{ArcSin}(x)^2}{2}-\frac{\text{ArcSin}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(1 + Sqrt[1 - x^2]),x]
```

```
[Out] -(ArcSin[x]/x) + (Sqrt[1 - x^2]*ArcSin[x])/x + ArcSin[x]^2/2 - ArcTanh[Sqrt
[1 - x^2]] - Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4737

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(a
 + b*ArcSin[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d
+ e, 0] && NeQ[n, -1]
```

Rule 4781

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcS
in[c*x])^n/(f*(m + 1))), x] + (-Dist[b*c*(n/(f*(m + 1)))*Simp[Sqrt[d + e*x^
2]/Sqrt[1 - c^2*x^2]], Int[(f*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1), x], x
] + Dist[(c^2/(f^2*(m + 1)))*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]], Int[(
f*x)^(m + 2)*((a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a
, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 4875

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(Px_.)*((f_) + (g_.)*((d_) + (
e_.)*(x_)^2)^(p_))^(m_.), x_Symbol] :> With[{u = ExpandIntegrand[Px*(f + g*
(d + e*x^2)^p)^m*(a + b*ArcSin[c*x])^n, x]}, Int[u, x] /; SumQ[u]] /; FreeQ
[{a, b, c, d, e, f, g}, x] && PolynomialQ[Px, x] && EqQ[c^2*d + e, 0] && IG
tQ[p + 1/2, 0] && IntegersQ[m, n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^{-1}(x)}{1+\sqrt{1-x^2}}\,dx &= \int \left(\frac{\sin^{-1}(x)}{x^2}-\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x^2}\right)dx\\
&= \int \frac{\sin^{-1}(x)}{x^2}\,dx-\int \frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x^2}\,dx\\
&= -\frac{\sin^{-1}(x)}{x}+\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}-\int \frac{1}{x}\,dx+\int \frac{1}{x\sqrt{1-x^2}}\,dx+\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx\\
&= -\frac{\sin^{-1}(x)}{x}+\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}+\frac{1}{2}\sin^{-1}(x)^2-\log(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x\right.\\
&= -\frac{\sin^{-1}(x)}{x}+\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}+\frac{1}{2}\sin^{-1}(x)^2-\log(x)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1}\right.\\
&= -\frac{\sin^{-1}(x)}{x}+\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}+\frac{1}{2}\sin^{-1}(x)^2-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\log(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.98

$$\frac{\left(-1+\sqrt{1-x^2}\right)\sin^{-1}(x)}{x}+\frac{1}{2}\sin^{-1}(x)^2-\log\left(1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(1 + Sqrt[1 - x^2]),x]
[Out] ((-1 + Sqrt[1 - x^2])*ArcSin[x])/x + ArcSin[x]^2/2 - Log[1 + Sqrt[1 - x^2]]
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{1+\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/(1+(-x^2+1)^(1/2)),x)
[Out] int(arcsin(x)/(1+(-x^2+1)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(arcsin(x)/(sqrt(-x^2 + 1) + 1), x)
```

Fricas [A]
time = 0.77, size = 63, normalized size = 1.40

$$\frac{x \arcsin\left(x\right)^{2} - 2\, x \log\left(x\right) - x \log\left(\sqrt{-x^{2} + 1} + 1\right) + x \log\left(\sqrt{-x^{2} + 1} - 1\right) + 2\, \sqrt{-x^{2} + 1} \arcsin\left(x\right) - 2\, \arcsin\left(x\right)}{2\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(x*arcsin(x)^2 - 2*x*log(x) - x*log(sqrt(-x^2 + 1) + 1) + x*log(sqrt(-x
^2 + 1) - 1) + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*arcsin(x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}{\left(x \right)}}{\sqrt{1 - x^{2}} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/(1+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(asin(x)/(sqrt(1 - x**2) + 1), x)
```

Giac [A]
time = 0.47, size = 57, normalized size = 1.27

$$\frac{1}{2}\, \arcsin\left(x\right)^{2} - \frac{x \arcsin\left(x\right)}{\sqrt{-x^{2} + 1} + 1} - 2\, \log\left(2\right) + \log\left(2\, \sqrt{-x^{2} + 1} + 2\right) - 2\, \log\left(\sqrt{-x^{2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x)^2 - x*arcsin(x)/(sqrt(-x^2 + 1) + 1) - 2*log(2) + log(2*sqrt(
-x^2 + 1) + 2) - 2*log(sqrt(-x^2 + 1) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{asin}(x)}{\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/((1 - x^2)^(1/2) + 1),x)
```

```
[Out] int(asin(x)/((1 - x^2)^(1/2) + 1), x)
```

3.15 $\int \frac{\log\left(x+\sqrt{1+x^2}\right)}{(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2}\sin^{-1}\left(x^2\right)+\frac{x\log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1-x^2}}$$

```
[Out] -1/2*arcsin(x^2)+x*ln(x+(x^2+1)^(1/2))/(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {197, 2634, 281, 222}

$$\frac{x\log\left(\sqrt{x^2+1}+x\right)}{\sqrt{1-x^2}}-\frac{\text{ArcSin}(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x^2]]/(1 - x^2)^(3/2),x]
```

```
[Out] -1/2*ArcSin[x^2] + (x*Log[x + Sqrt[1 + x^2]])/Sqrt[1 - x^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \frac{\log \left(x+\sqrt{1+x^2}\right)}{\left(1-x^2\right)^{3/2}} \, dx = \frac{x \log \left(x+\sqrt{1+x^2}\right)}{\sqrt{1-x^2}} - \int \frac{x}{\sqrt{1-x^4}} \, dx$$
$$= \frac{x \log \left(x+\sqrt{1+x^2}\right)}{\sqrt{1-x^2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, x^2\right)$$
$$= -\frac{1}{2} \sin^{-1}\left(x^2\right) + \frac{x \log \left(x+\sqrt{1+x^2}\right)}{\sqrt{1-x^2}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(34) = 68.
time = 0.08, size = 76, normalized size = 2.24

$$\frac{1}{2}\sqrt{1-x^2}\left(-\frac{\sqrt{1+x^2}\tan^{-1}\left(\frac{x^2}{\sqrt{1-x^4}}\right)}{\sqrt{1-x^4}} - \frac{2x \log \left(x+\sqrt{1+x^2}\right)}{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x^2]]/(1 - x^2)^(3/2),x]
[Out] (Sqrt[1 - x^2]*(-((Sqrt[1 + x^2]*ArcTan[x^2/Sqrt[1 - x^4]])/Sqrt[1 - x^4])
- (2*x*Log[x + Sqrt[1 + x^2]])/(-1 + x^2)))/2
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(x+\sqrt{x^2+1}\right)}{\left(-x^2+1\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x)
[Out] int(ln(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(log(x + sqrt(x^2 + 1))/(-x^2 + 1)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(28) = 56$.
time = 0.68, size = 62, normalized size = 1.82

$$-\frac{\sqrt{-x^2+1}\,x\log\left(x+\sqrt{x^2+1}\right)-(x^2-1)\arctan\left(\frac{\sqrt{x^2+1}\,\sqrt{-x^2+1}-1}{x^2}\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-x^2 + 1)*x*log(x + sqrt(x^2 + 1)) - (x^2 - 1)*arctan((sqrt(x^2 + 1)
*sqrt(-x^2 + 1) - 1)/x^2))/(x^2 - 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2+1)**(1/2))/(-x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 36, normalized size = 1.06

$$-\frac{\sqrt{-x^2+1}\,x\log\left(x+\sqrt{x^2+1}\right)}{x^2-1}-\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2))/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*x*log(x + sqrt(x^2 + 1))/(x^2 - 1) - 1/2*arcsin(x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(x+\sqrt{x^2+1}\right)}{(1-x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x^2 + 1)^(1/2))/(1 - x^2)^(3/2),x)
```

```
[Out] int(log(x + (x^2 + 1)^(1/2))/(1 - x^2)^(3/2), x)
```

3.16 $\int \frac{\sin^{-1}(x)}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{x\sin^{-1}(x)}{\sqrt{1+x^2}}-\frac{1}{2}\sin^{-1}\left(x^2\right)$$

```
[Out] -1/2*arcsin(x^2)+x*arcsin(x)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {197, 4755, 281, 222}

$$\frac{x\text{ArcSin}(x)}{\sqrt{x^2+1}}-\frac{\text{ArcSin}(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(1 + x^2)^(3/2),x]
```

```
[Out] (x*ArcSin[x])/Sqrt[1 + x^2] - ArcSin[x^2]/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4755

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSin[c*x], u, x] -
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0]
)
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^{-1}(x)}{(1+x^2)^{3/2}}\,dx &= \frac{x\sin^{-1}(x)}{\sqrt{1+x^2}} - \int \frac{x}{\sqrt{1-x^4}}\,dx \\
&= \frac{x\sin^{-1}(x)}{\sqrt{1+x^2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^2\right) \\
&= \frac{x\sin^{-1}(x)}{\sqrt{1+x^2}} - \frac{1}{2}\sin^{-1}\left(x^2\right)
\end{aligned}
$$

<u>**Mathematica**</u> [**A**]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{x\sin^{-1}(x)}{\sqrt{1+x^2}} - \frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(1 + x^2)^(3/2),x]
```

```
[Out] (x*ArcSin[x])/Sqrt[1 + x^2] - ArcSin[x^2]/2
```

<u>**Maple**</u> [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\arcsin(x)}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/(x^2+1)^(3/2),x)
```

```
[Out] int(arcsin(x)/(x^2+1)^(3/2),x)
```

<u>**Maxima**</u> [**A**]
time = 3.80, size = 18, normalized size = 0.82

$$\frac{x\arcsin(x)}{\sqrt{x^2+1}} - \frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arcsin(x)/sqrt(x^2 + 1) - 1/2*arcsin(x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.
time = 0.66, size = 56, normalized size = 2.55

$$\frac{2\sqrt{x^2+1}\,x\arcsin\left(x\right)+\left(x^2+1\right)\arctan\left(\frac{\sqrt{x^2+1}\,\sqrt{-x^2+1}\,x^2}{x^4-1}\right)}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*(2*sqrt(x^2 + 1)*x*arcsin(x) + (x^2 + 1)*arctan(sqrt(x^2 + 1)*sqrt(-x^2
 + 1)*x^2/(x^4 - 1)))/(x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.16, size = 78, normalized size = 3.55

$$\frac{x\operatorname{asin}\left(x\right)}{\sqrt{x^2+1}}+\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\,\frac{1}{x^4}\right)}{8\pi^{\frac{3}{2}}}-\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^4}\right)}{8\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/(x**2+1)**(3/2),x)
[Out] x*asin(x)/sqrt(x**2 + 1) + I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1
/4, 1/2, 3/4, 1, 0), ()), x**(-4))/(8*pi**(3/2)) - meijerg(((-1/2, -1/4, 0,
 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/x**4
)/(8*pi**(3/2))
```

Giac [A]
time = 0.46, size = 18, normalized size = 0.82

$$\frac{x\arcsin\left(x\right)}{\sqrt{x^2+1}}-\frac{1}{2}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(x^2+1)^(3/2),x, algorithm="giac")
[Out] x*arcsin(x)/sqrt(x^2 + 1) - 1/2*arcsin(x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{asin}(x)}{\left(x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/(x^2 + 1)^(3/2),x)
[Out] int(asin(x)/(x^2 + 1)^(3/2), x)
```

3.17 $\int \frac{\log\left(x+\sqrt{-1+x^2}\right)}{\left(1+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2}\cosh^{-1}\left(x^2\right)+\frac{x\log\left(x+\sqrt{-1+x^2}\right)}{\sqrt{1+x^2}}$$

```
[Out] -1/2*arccosh(x^2)+x*ln(x+(x^2-1)^(1/2))/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {197, 2634, 282, 54}

$$\frac{x\log\left(\sqrt{x^2-1}+x\right)}{\sqrt{x^2+1}}-\frac{1}{2}\cosh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[-1 + x^2]]/(1 + x^2)^(3/2),x]
```

```
[Out] -1/2*ArcCosh[x^2] + (x*Log[x + Sqrt[-1 + x^2]])/Sqrt[1 + x^2]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 282

```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> With[{k = GCD[m + 1, 2*n]}, Dist[1/k, Subst[Int[x^((m + 1
)/k - 1)*(a1 + b1*x^(n/k))^p*(a2 + b2*x^(n/k))^p, x], x, x^k], x] /; k != 1
] /; FreeQ[{a1, b1, a2, b2, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0]
 && IntegerQ[m]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \frac{\log\left(x+\sqrt{-1+x^2}\right)}{(1+x^2)^{3/2}}\,dx = \frac{x\log\left(x+\sqrt{-1+x^2}\right)}{\sqrt{1+x^2}} - \int \frac{x}{\sqrt{-1+x^2}\,\sqrt{1+x^2}}\,dx$$
$$= \frac{x\log\left(x+\sqrt{-1+x^2}\right)}{\sqrt{1+x^2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx, x, x^2\right)$$
$$= -\frac{1}{2}\cosh^{-1}\left(x^2\right) + \frac{x\log\left(x+\sqrt{-1+x^2}\right)}{\sqrt{1+x^2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(32) = 64.
time = 0.07, size = 89, normalized size = 2.78

$$\frac{4x\log\left(x+\sqrt{-1+x^2}\right) + \frac{\sqrt{-1+x^2}\,(1+x^2)\left(\log\left(1-\frac{x^2}{\sqrt{-1+x^4}}\right)-\log\left(1+\frac{x^2}{\sqrt{-1+x^4}}\right)\right)}{\sqrt{-1+x^4}}}{4\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[-1 + x^2]]/(1 + x^2)^(3/2),x]
[Out] (4*x*Log[x + Sqrt[-1 + x^2]] + (Sqrt[-1 + x^2]*(1 + x^2)*(Log[1 - x^2/Sqrt[
-1 + x^4]] - Log[1 + x^2/Sqrt[-1 + x^4]]))/Sqrt[-1 + x^4])/(4*Sqrt[1 + x^2]
)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(x+\sqrt{x^2-1}\right)}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x)
[Out] int(ln(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(log(x + sqrt(x^2 - 1))/(x^2 + 1)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.
time = 0.51, size = 58, normalized size = 1.81

$$\frac{2\sqrt{x^2+1}\,x\log\left(x+\sqrt{x^2-1}\right)+(x^2+1)\log\left(-x^2+\sqrt{x^2+1}\,\sqrt{x^2-1}\right)}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(x^2 + 1)*x*log(x + sqrt(x^2 - 1)) + (x^2 + 1)*log(-x^2 + sqrt(x
^2 + 1)*sqrt(x^2 - 1)))/(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(x+\sqrt{x^2-1}\right)}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2-1)**(1/2))/(x**2+1)**(3/2),x)
```

```
[Out] Integral(log(x + sqrt(x**2 - 1))/(x**2 + 1)**(3/2), x)
```

Giac [A]
time = 0.51, size = 36, normalized size = 1.12

$$\frac{x\log\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}}+\frac{1}{2}\log\left(x^2-\sqrt{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2))/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] x*log(x + sqrt(x^2 - 1))/sqrt(x^2 + 1) + 1/2*log(x^2 - sqrt(x^4 - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(x+\sqrt{x^2-1}\right)}{\left(x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x^2 - 1)^(1/2))/(x^2 + 1)^(3/2),x)
[Out] int(log(x + (x^2 - 1)^(1/2))/(x^2 + 1)^(3/2), x)
```

3.18 $\int \frac{\log(x)}{x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{-1+x^2}}{x} - \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right) + \frac{\sqrt{-1+x^2}\,\log(x)}{x}$$

```
[Out] -arctanh(x/(x^2-1)^(1/2))+(x^2-1)^(1/2)/x+ln(x)*(x^2-1)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2373, 283, 223, 212}

$$\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2-1}\,\log(x)}{x} - \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] Sqrt[-1 + x^2]/x - ArcTanh[x/Sqrt[-1 + x^2]] + (Sqrt[-1 + x^2]*Log[x])/x
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
```

```
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{x^2\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{-1+x^2}\,\log(x)}{x} - \int \frac{\sqrt{-1+x^2}}{x^2}\,dx \\
&= \frac{\sqrt{-1+x^2}}{x} + \frac{\sqrt{-1+x^2}\,\log(x)}{x} - \int \frac{1}{\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{x} + \frac{\sqrt{-1+x^2}\,\log(x)}{x} - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right) \\
&= \frac{\sqrt{-1+x^2}}{x} - \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right) + \frac{\sqrt{-1+x^2}\,\log(x)}{x}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 43, normalized size = 1.00

$$\frac{\sqrt{-1+x^2}}{x} + \frac{\sqrt{-1+x^2}\,\log(x)}{x} - \log\left(x+\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] Sqrt[-1 + x^2]/x + (Sqrt[-1 + x^2]*Log[x])/x - Log[x + Sqrt[-1 + x^2]]
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 89, normalized size = 2.07

method	result
meijerg	$-\frac{\sqrt{-\mathrm{signum}\left(x^2-1\right)}\,\arcsin(x)}{\sqrt{\mathrm{signum}\left(x^2-1\right)}} + \frac{-\frac{\sqrt{-\mathrm{signum}\left(x^2-1\right)}\,\sqrt{-x^2+1}}{\sqrt{\mathrm{signum}\left(x^2-1\right)}} - \frac{\sqrt{-\mathrm{signum}\left(x^2-1\right)}\,\ln(x)\sqrt{-}}{\sqrt{\mathrm{signum}\left(x^2-1\right)}}}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*arcsin(x)+(-1/signum(x^2-1)^(
1/2)*(-signum(x^2-1))^(1/2)*(-x^2+1)^(1/2)-1/signum(x^2-1)^(1/2)*(-signum(x
^2-1))^(1/2)*ln(x)*(-x^2+1)^(1/2))/x
```

Maxima [A]
time = 3.15, size = 41, normalized size = 0.95

$$\frac{\sqrt{x^2-1}\log(x)}{x}+\frac{\sqrt{x^2-1}}{x}-\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*log(x)/x + sqrt(x^2 - 1)/x - log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A]
time = 0.48, size = 32, normalized size = 0.74

$$\frac{x\log\left(-x+\sqrt{x^2-1}\right)+\sqrt{x^2-1}\left(\log(x)+1\right)+x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*log(-x + sqrt(x^2 - 1)) + sqrt(x^2 - 1)*(log(x) + 1) + x)/x
```

Sympy [A]
time = 2.31, size = 34, normalized size = 0.79

$$\left(\begin{cases}\frac{\sqrt{x^2-1}}{x} & \text{for}\ x>-1\wedge x<1\end{cases}\right)\log(x)-\begin{cases}\text{NaN} & \text{for}\ x<-1\\ \operatorname{acosh}(x)-\frac{\sqrt{x^2-1}}{x} & \text{for}\ x<1\\ \text{NaN} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Piecewise((sqrt(x**2 - 1)/x, (x > -1) & (x < 1)))*log(x) - Piecewise((nan,
x < -1), (acosh(x) - sqrt(x**2 - 1)/x, x < 1), (nan, True))
```

Giac [A]
time = 0.50, size = 62, normalized size = 1.44

$$\frac{2\log(x)}{\left(x-\sqrt{x^2-1}\right)^2+1}+\frac{2}{\left(x-\sqrt{x^2-1}\right)^2+1}+\frac{1}{2}\log\left(\left(x-\sqrt{x^2-1}\right)^2\right)-\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(x)/((x - sqrt(x^2 - 1))^2 + 1) + 2/((x - sqrt(x^2 - 1))^2 + 1) + 1/2*
log((x - sqrt(x^2 - 1))^2) - log(abs(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln(x)}{x^2 \sqrt{x^2-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x^2*(x^2 - 1)^(1/2)),x)
```

```
[Out] int(log(x)/(x^2*(x^2 - 1)^(1/2)), x)
```

3.19 $\int \frac{\sqrt{1+x^3}}{x} \, dx$

Optimal. Leaf size=28

$$\frac{2\sqrt{1+x^3}}{3} - \frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))+2/3*(x^3+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {272, 52, 65, 213}

$$\frac{2\sqrt{x^3+1}}{3} - \frac{2}{3}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^3]/x,x]
```

```
[Out] (2*Sqrt[1 + x^3])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x^3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{1+x^3}}{3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right)\\
&= \frac{2\sqrt{1+x^3}}{3} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right)\\
&= \frac{2\sqrt{1+x^3}}{3} - \frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$\frac{2\sqrt{1+x^3}}{3} - \frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^3]/x,x]
[Out] (2*Sqrt[1 + x^3])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/3
```

Maple [A]
time = 0.12, size = 21, normalized size = 0.75

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\sqrt{x^3+1}}{3}$	21
elliptic	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\sqrt{x^3+1}}{3}$	21
trager	$\frac{2\sqrt{x^3+1}}{3}-\frac{\ln\left(-\frac{x^3+2\sqrt{x^3+1}+2}{x^3}\right)}{3}$	33
meijerg	$-\frac{-2(2-2\ln(2)+3\ln(x))\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{x^3+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)}{6\sqrt{\pi}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*arctanh((x^3+1)^(1/2))+2/3*(x^3+1)^(1/2)
```

Maxima [A]
time = 4.98, size = 34, normalized size = 1.21

$$\frac{2}{3}\sqrt{x^3+1}-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(x^3 + 1) - 1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

Fricas [A]
time = 0.46, size = 34, normalized size = 1.21

$$\frac{2}{3}\sqrt{x^3+1}-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(x^3 + 1) - 1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

Sympy [A]
time = 0.56, size = 48, normalized size = 1.71

$$\frac{2x^{\frac{3}{2}}}{3\sqrt{1+\frac{1}{x^3}}}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2}{3x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)**(1/2)/x,x)
```

```
[Out] 2*x**(3/2)/(3*sqrt(1 + x**(-3))) - 2*asinh(x**(-3/2))/3 + 2/(3*x**(3/2)*sqr
t(1 + x**(-3)))
```

Giac [A]
time = 0.47, size = 35, normalized size = 1.25

$$\frac{2}{3}\sqrt{x^3+1}-\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(x^3 + 1) - 1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(abs(sqrt(x^3 + 1)
- 1))
```

Mupad [B]
time = 0.13, size = 174, normalized size = 6.21

$$\frac{2\sqrt{x^3+1}}{3}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)^(1/2)/x,x)
```

```
[Out] (2*(x^3 + 1)^(1/2))/3 - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/
2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*
1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.20 $\int \frac{x \log\left(x+\sqrt{-1+x^2}\right)}{\sqrt{-1+x^2}} \, dx$

Optimal. Leaf size=26

$$-x+\sqrt{-1+x^2} \log\left(x+\sqrt{-1+x^2}\right)$$

```
[Out] -x+ln(x+(x^2-1)^(1/2))*(x^2-1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {267, 2634, 8}

$$\sqrt{x^2-1} \log\left(\sqrt{x^2-1}+x\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[x + Sqrt[-1 + x^2]])/Sqrt[-1 + x^2],x]
```

```
[Out] -x + Sqrt[-1 + x^2]*Log[x + Sqrt[-1 + x^2]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x \log\left(x+\sqrt{-1+x^2}\right)}{\sqrt{-1+x^2}} \, dx &= \sqrt{-1+x^2} \log\left(x+\sqrt{-1+x^2}\right)-\int 1 \, dx\\ &= -x+\sqrt{-1+x^2} \log\left(x+\sqrt{-1+x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 1.00

$$-x+\sqrt{-1+x^2}\log\left(x+\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x + Sqrt[-1 + x^2]])/Sqrt[-1 + x^2],x]
```

```
[Out] -x + Sqrt[-1 + x^2]*Log[x + Sqrt[-1 + x^2]]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\frac{x\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x+(x^2-1)^(1/2))/(x^2-1)^(1/2),x)
```

```
[Out] int(x*ln(x+(x^2-1)^(1/2))/(x^2-1)^(1/2),x)
```

Maxima [A]
time = 10.71, size = 22, normalized size = 0.85

$$\sqrt{x^2-1}\log\left(x+\sqrt{x^2-1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2-1)^(1/2))/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*log(x + sqrt(x^2 - 1)) - x
```

Fricas [A]
time = 0.46, size = 22, normalized size = 0.85

$$\sqrt{x^2-1}\log\left(x+\sqrt{x^2-1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2-1)^(1/2))/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - 1)*log(x + sqrt(x^2 - 1)) - x
```

Sympy [A]
time = 3.05, size = 20, normalized size = 0.77

$$-x+\sqrt{x^2-1}\log\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x+(x**2-1)**(1/2))/(x**2-1)**(1/2),x)
```

```
[Out] -x + sqrt(x**2 - 1)*log(x + sqrt(x**2 - 1))
```

Giac [A]
time = 0.45, size = 22, normalized size = 0.85

$$\sqrt{x^2-1}\,\log\left(x+\sqrt{x^2-1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2-1)^(1/2))/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 1)*log(x + sqrt(x^2 - 1)) - x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x + (x^2 - 1)^(1/2)))/(x^2 - 1)^(1/2),x)
```

```
[Out] int((x*log(x + (x^2 - 1)^(1/2)))/(x^2 - 1)^(1/2), x)
```

3.21 $\int \frac{x^3 \sin^{-1}(x)}{\sqrt{1-x^4}} \, dx$

Optimal. Leaf size=38

$$\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{2}\sqrt{1-x^4}\,\sin^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)$$

```
[Out] 1/4*arcsinh(x)+1/4*x*(x^2+1)^(1/2)-1/2*arcsin(x)*(-x^4+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {267, 4871, 12, 26, 201, 221}

$$-\frac{1}{2}\sqrt{1-x^4}\,\text{ArcSin}(x)+\frac{1}{4}\sqrt{x^2+1}\,x+\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSin[x])/Sqrt[1 - x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2])/4 - (Sqrt[1 - x^4]*ArcSin[x])/2 + ArcSinh[x]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4871

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcSin[c*x], v, x] - Dist[b*c, Int[SimplifyIntegrand[v/
Sqrt[1 - c^2*x^2], x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b
, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 \sin^{-1}(x)}{\sqrt{1-x^4}}\,dx &= -\frac{1}{2}\sqrt{1-x^4}\,\sin^{-1}(x)-\int -\frac{\sqrt{1-x^4}}{2\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}\sqrt{1-x^4}\,\sin^{-1}(x)+\frac{1}{2}\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}\sqrt{1-x^4}\,\sin^{-1}(x)+\frac{1}{2}\int \sqrt{1+x^2}\,dx\\ &= \frac{1}{4}x\sqrt{1+x^2}-\frac{1}{2}\sqrt{1-x^4}\,\sin^{-1}(x)+\frac{1}{4}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{4}x\sqrt{1+x^2}-\frac{1}{2}\sqrt{1-x^4}\,\sin^{-1}(x)+\frac{1}{4}\sinh^{-1}(x)\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 85 vs. $2(38) = 76$.
time = 0.06, size = 85, normalized size = 2.24

$$\frac{1}{4}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1-x^2}}-2\sqrt{1-x^4}\,\sin^{-1}(x)+\log\left(1-x^2\right)-\log\left(-x+x^3+\sqrt{1-x^2}\,\sqrt{1-x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSin[x])/Sqrt[1 - x^4],x]
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] - 2*Sqrt[1 - x^4]*ArcSin[x] + Log[1 - x^2]
 - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]])/4
```

Maple [**F**]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^3 \arcsin\left(x\right)}{\sqrt{-x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsin(x)/(-x^4+1)^(1/2),x)
```

```
[Out] int(x^3*arcsin(x)/(-x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(x^2 + 1)*sqrt(x + 1)*sqrt(-x + 1)*arctan2(x, sqrt(x + 1)*sqrt(-x
+ 1)) + integrate(1/2*sqrt(x^2 + 1)/(x^2 + e^(log(x + 1) + log(-x + 1))), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(28) = 56$.
time = 0.50, size = 138, normalized size = 3.63

$$-\frac{4\sqrt{-x^4+1}\,(x^2-1)\arcsin(x)+2\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,x+(x^2-1)\log\left(\frac{x^3+\sqrt{-x^4+1}\,\sqrt{-x^2+1}-x}{x^3-x}\right)-(x^2-1)\log\left(-\frac{x^3-\sqrt{-x^4+1}\,\sqrt{-x^2+1}-x}{x^3-x}\right)}{8\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(-x^4 + 1)*(x^2 - 1)*arcsin(x) + 2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1
)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) -
(x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 -
 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{asin}(x)}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asin(x)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(x**3*asin(x)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [A]
time = 0.49, size = 38, normalized size = 1.00

$$\frac{1}{4}\sqrt{x^2+1}\,x-\frac{1}{2}\sqrt{-x^4+1}\,\arcsin(x)-\frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 + 1)*x - 1/2*sqrt(-x^4 + 1)*arcsin(x) - 1/4*log(-x + sqrt(x^2
+ 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3 \operatorname{asin}(x)}{\sqrt{1 - x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*asin(x))/(1 - x^4)^(1/2),x)
```

```
[Out] int((x^3*asin(x))/(1 - x^4)^(1/2), x)
```

3.22 $\int \frac{x^3 \sec^{-1}(x)}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}}\,x}+\frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x)+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{1-\frac{1}{x^2}}\,x}{\sqrt{-1+x^4}}\right)$$

```
[Out] 1/2*arctanh(x*(1-1/x^2)^(1/2)/(x^4-1)^(1/2))+1/2*arcsec(x)*(x^4-1)^(1/2)-1/
2*(x^4-1)^(1/2)/x/(1-1/x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {267, 5354, 12, 1586, 1266, 879, 889, 209}

$$\frac{\sqrt{1-x^2}\,\text{ArcTan}\left(\frac{\sqrt{x^4-1}}{\sqrt{1-x^2}}\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}+\frac{1}{2}\sqrt{x^4-1}\,\sec^{-1}(x)-\frac{\sqrt{x^4-1}}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSec[x])/Sqrt[-1 + x^4],x]
```

```
[Out] -1/2*Sqrt[-1 + x^4]/(Sqrt[1 - x^(-2)]*x) + (Sqrt[-1 + x^4]*ArcSec[x])/2 + (
Sqrt[1 - x^2]*ArcTan[Sqrt[-1 + x^4]/Sqrt[1 - x^2]])/(2*Sqrt[1 - x^(-2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 879

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Simp[(-(d + e*x)^m)*(f + g*x)^(n + 1)*((a + c*x^2)^p/(
g*(m - n - 1))), x] - Dist[c*m*((e*f + d*g)/(e^2*g*(m - n - 1))), Int[(d +
e*x)^(m + 1)*(f + g*x)^n*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e,
f, g, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
 && EqQ[m + p, 0] && GtQ[p, 0] && NeQ[m - n - 1, 0] &&  !IGtQ[n, 0] &&  !(I
ntegerQ[n + p] && LtQ[n + p + 2, 0]) && RationalQ[n]
```

Rule 889

```
Int[Sqrt[(d_) + (e_.)*(x_)]/(((f_.) + (g_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]
), x_Symbol] :> Dist[2*e^2, Subst[Int[1/(c*(e*f + d*g) + e^2*g*x^2), x], x,
 Sqrt[a + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1586

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q]), Int[x^(m + mn*q)*(1 + d*(1/(x^m
n*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && E
qQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 5354

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcSec[c*x], v, x] - Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{x^3 \sec^{-1}(x)}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x) - \int \frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}}\,x^2}\,dx$$

$$= \frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x) - \frac{1}{2}\int \frac{\sqrt{-1+x^4}}{\sqrt{1-\frac{1}{x^2}}\,x^2}\,dx$$

$$= \frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x) - \frac{\sqrt{1-x^2}\int \frac{\sqrt{-1+x^4}}{x\sqrt{1-x^2}}\,dx}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

$$= \frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x) - \frac{\sqrt{1-x^2}\,\text{Subst}\left(\int \frac{\sqrt{-1+x^2}}{\sqrt{1-x}\,x}\,dx, x, x^2\right)}{4\sqrt{1-\frac{1}{x^2}}\,x}$$

$$= -\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}}\,x} + \frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x) + \frac{\sqrt{1-x^2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}}{x\sqrt{-1+x^2}}\,dx, x, x^2\right)}{4\sqrt{1-\frac{1}{x^2}}\,x}$$

$$= -\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}}\,x} + \frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x) + \frac{\sqrt{1-x^2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{-1+x^4}}{\sqrt{1-x^2}}\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

$$= -\frac{\sqrt{-1+x^4}}{2\sqrt{1-\frac{1}{x^2}}\,x} + \frac{1}{2}\sqrt{-1+x^4}\,\sec^{-1}(x) + \frac{\sqrt{1-x^2}\,\tan^{-1}\left(\frac{\sqrt{-1+x^4}}{\sqrt{1-x^2}}\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

Mathematica [A]
time = 0.07, size = 88, normalized size = 1.26

$$\frac{1}{2}\left(-\frac{\sqrt{1-\frac{1}{x^2}}\,x\sqrt{-1+x^4}}{-1+x^2} + \sqrt{-1+x^4}\,\sec^{-1}(x) - \log\left(x-x^3\right) + \log\left(1-x^2-\sqrt{1-\frac{1}{x^2}}\,x\sqrt{-1+x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSec[x])/Sqrt[-1 + x^4],x]
```

```
[Out] (-((Sqrt[1 - x^(-2)]*x*Sqrt[-1 + x^4])/(-1 + x^2)) + Sqrt[-1 + x^4]*ArcSec[
x] - Log[x - x^3] + Log[1 - x^2 - Sqrt[1 - x^(-2)]*x*Sqrt[-1 + x^4]])/2
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{x^3 \text{arcsec}(x)}{\sqrt{x^4-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(x)/(x^4-1)^(1/2),x)
```

```
[Out] int(x^3*arcsec(x)/(x^4-1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 1)*sqrt(x + 1)*sqrt(x - 1)*arctan(sqrt(x + 1)*sqrt(x - 1)) -
 integrate((2*(x^3*e^(3/2*log(x + 1) + 3/2*log(x - 1)) + x^3*e^(1/2*log(x +
 1) + 1/2*log(x - 1)))*sqrt(x^2 + 1)*log(x) + (x^3 + x)*e^(1/2*log(x^2 + 1)
 + 3/2*log(x + 1) + 3/2*log(x - 1)))/((x^2 + 1)*(e^(2*log(x + 1) + 2*log(x
- 1)) + e^(log(x + 1) + log(x - 1)))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(54) = 108$.
time = 0.47, size = 110, normalized size = 1.57

$$\frac{(x^2-1)\log\left(\frac{x^2+\sqrt{x^4-1}\,\sqrt{x^2-1}-1}{x^2-1}\right)-(x^2-1)\log\left(-\frac{x^2-\sqrt{x^4-1}\,\sqrt{x^2-1}-1}{x^2-1}\right)+2\sqrt{x^4-1}\left((x^2-1)\,\text{arcsec}\,(x)-\sqrt{x^2-1}\right)}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log((x^2 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - 1)/(x^2 - 1)) - (x^
2 - 1)*log(-(x^2 - sqrt(x^4 - 1)*sqrt(x^2 - 1) - 1)/(x^2 - 1)) + 2*sqrt(x^4
 - 1)*((x^2 - 1)*arcsec(x) - sqrt(x^2 - 1)))/(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \operatorname{asec}(x)}{\sqrt{(x-1)(x+1)(x^2+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(x)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**3*asec(x)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [A]
time = 0.48, size = 52, normalized size = 0.74

$$\frac{1}{2}\sqrt{x^4-1}\arccos\left(\frac{1}{x}\right)-\frac{2\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)+\log\left(\sqrt{x^2+1}-1\right)}{4\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 - 1)*arccos(1/x) - 1/4*(2*sqrt(x^2 + 1) - log(sqrt(x^2 + 1) +
1) + log(sqrt(x^2 + 1) - 1))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{acos}\left(\frac{1}{x}\right)}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acos(1/x))/(x^4 - 1)^(1/2),x)
```

```
[Out] int((x^3*acos(1/x))/(x^4 - 1)^(1/2), x)
```

3.23 $\int \frac{x \tan^{-1}(x) \log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=58

$$-x\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)+\sqrt{1+x^2}\,\tan^{-1}(x)\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\log^2\left(x+\sqrt{1+x^2}\right)$$

```
[Out] -x*arctan(x)+1/2*ln(x^2+1)-1/2*ln(x+(x^2+1)^(1/2))^2+arctan(x)*ln(x+(x^2+1)
^(1/2))*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5050, 221, 267, 2634, 8, 5320, 6818, 4930, 266}

$$\sqrt{x^2+1}\,\text{ArcTan}(x)\log\left(\sqrt{x^2+1}+x\right)-x\text{ArcTan}(x)-\frac{1}{2}\log^2\left(\sqrt{x^2+1}+x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x]*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] -(x*ArcTan[x]) + Log[1 + x^2]/2 + Sqrt[1 + x^2]*ArcTan[x]*Log[x + Sqrt[1 +
x^2]] - Log[x + Sqrt[1 + x^2]]^2/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 5320

```
Int[ArcTan[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Ar
cTan[v]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/(1 + v^2)
), x], x] - Int[SimplifyIntegrand[z*ArcTan[v]*(D[w, x]/w), x], x]) /; Inver
seFunctionFreeQ[z, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFree
Q[w, x]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x \tan^{-1}(x) \log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2}\,\tan^{-1}(x)\log\left(x+\sqrt{1+x^2}\right) - \int \tan^{-1}(x)\,dx - \int \frac{\log\left(x\right.}{}$$

$$= -x\tan^{-1}(x) + \sqrt{1+x^2}\,\tan^{-1}(x)\log\left(x+\sqrt{1+x^2}\right) - \frac{1}{2}\log^2\left(x+\right.$$

$$= -x\tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right) + \sqrt{1+x^2}\,\tan^{-1}(x)\log\left(x+\sqrt{1+x^2}\right.$$

Mathematica [**A**]

time = 0.03, size = 58, normalized size = 1.00

$$-x\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)+\sqrt{1+x^2}\tan^{-1}(x)\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\log^2\left(x+\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x]*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] -(x*ArcTan[x]) + Log[1 + x^2]/2 + Sqrt[1 + x^2]*ArcTan[x]*Log[x + Sqrt[1 + 
x^2]] - Log[x + Sqrt[1 + x^2]]^2/2
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int\frac{x\arctan(x)\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

```
[Out] int(x*arctan(x)*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(x*arctan(x)*log(x + sqrt(x^2 + 1))/sqrt(x^2 + 1), x)
```

Fricas [A]
time = 0.47, size = 48, normalized size = 0.83

$$\sqrt{x^2+1}\arctan(x)\log\left(x+\sqrt{x^2+1}\right)-x\arctan(x)-\frac{1}{2}\log\left(x+\sqrt{x^2+1}\right)^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] sqrt(x^2 + 1)*arctan(x)*log(x + sqrt(x^2 + 1)) - x*arctan(x) - 1/2*log(x + 
sqrt(x^2 + 1))^2 + 1/2*log(x^2 + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)*ln(x+(x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(x*arctan(x)*log(x + sqrt(x^2 + 1))/sqrt(x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{atan}(x)\,\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(x)*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
```

```
[Out] int((x*atan(x)*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2), x)
```

3.24 $\int \frac{x \log\left(1+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=55

$$\sqrt{1-x^2}-\log\left(1+\sqrt{1-x^2}\right)-\sqrt{1-x^2}\log\left(1+\sqrt{1-x^2}\right)$$

```
[Out] -ln(1+(-x^2+1)^(1/2))+(-x^2+1)^(1/2)-ln(1+(-x^2+1)^(1/2))*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {267, 2634, 1605, 196, 45}

$$\sqrt{1-x^2}-\sqrt{1-x^2}\log\left(\sqrt{1-x^2}+1\right)-\log\left(\sqrt{1-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[1 + Sqrt[1 - x^2]])/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 - x^2] - Log[1 + Sqrt[1 - x^2]] - Sqrt[1 - x^2]*Log[1 + Sqrt[1 - x^2
]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
```

```
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\log\left(1+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx &= -\sqrt{1-x^2}\log\left(1+\sqrt{1-x^2}\right)-\int \frac{x}{1+\sqrt{1-x^2}}\,dx\\
&= -\sqrt{1-x^2}\log\left(1+\sqrt{1-x^2}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1-x^2\right)\\
&= -\sqrt{1-x^2}\log\left(1+\sqrt{1-x^2}\right)+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\sqrt{1-x^2}\log\left(1+\sqrt{1-x^2}\right)+\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx,x,\sqrt{1-x^2}\right)\\
&= \sqrt{1-x^2}-\log\left(1+\sqrt{1-x^2}\right)-\sqrt{1-x^2}\log\left(1+\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 0.75

$$\sqrt{1-x^2}-\left(1+\sqrt{1-x^2}\right)\log\left(1+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[1 + Sqrt[1 - x^2]])/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 - x^2] - (1 + Sqrt[1 - x^2])*Log[1 + Sqrt[1 - x^2]]
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.67

method	result	size
derivativedivides	$-\ln\left(1+\sqrt{-x^2+1}\right)\left(1+\sqrt{-x^2+1}\right)+1+\sqrt{-x^2+1}$	37
default	$-\ln\left(1+\sqrt{-x^2+1}\right)\left(1+\sqrt{-x^2+1}\right)+1+\sqrt{-x^2+1}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1+(-x^2+1)^(1/2))*(1+(-x^2+1)^(1/2))+1+(-x^2+1)^(1/2)
```

Maxima [A]
time = 4.16, size = 36, normalized size = 0.65

$$-\left(\sqrt{-x^2+1}+1\right)\log\left(\sqrt{-x^2+1}+1\right)+\sqrt{-x^2+1}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(-x^2 + 1) + 1)*log(sqrt(-x^2 + 1) + 1) + sqrt(-x^2 + 1) + 1
```

Fricas [A]
time = 0.43, size = 35, normalized size = 0.64

$$-\left(\sqrt{-x^2+1}+1\right)\log\left(\sqrt{-x^2+1}+1\right)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-x^2 + 1) + 1)*log(sqrt(-x^2 + 1) + 1) + sqrt(-x^2 + 1)
```

Sympy [A]
time = 3.44, size = 31, normalized size = 0.56

$$\sqrt{1-x^2}-\left(\sqrt{1-x^2}+1\right)\log\left(\sqrt{1-x^2}+1\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(1+(-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

```
[Out] sqrt(1 - x**2) - (sqrt(1 - x**2) + 1)*log(sqrt(1 - x**2) + 1) + 1
```

Giac [A]
time = 0.45, size = 36, normalized size = 0.65

$$-\left(\sqrt{-x^2+1}+1\right)\log\left(\sqrt{-x^2+1}+1\right)+\sqrt{-x^2+1}+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-x^2 + 1) + 1)*log(sqrt(-x^2 + 1) + 1) + sqrt(-x^2 + 1) + 1
```

Mupad [B]
time = 0.34, size = 27, normalized size = 0.49

$$-\left(\ln\left(\sqrt{1-x^2}+1\right)-1\right)\,\left(\sqrt{1-x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log((1 - x^2)^(1/2) + 1))/(1 - x^2)^(1/2),x)
[Out] -(log((1 - x^2)^(1/2) + 1) - 1)*((1 - x^2)^(1/2) + 1)
```

3.25 $\int \frac{x \log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=26

$$-x+\sqrt{1+x^2}\,\log\left(x+\sqrt{1+x^2}\right)$$

```
[Out] -x+ln(x+(x^2+1)^(1/2))*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {267, 2634, 8}

$$\sqrt{x^2+1}\,\log\left(\sqrt{x^2+1}+x\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] -x + Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x \log\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx &= \sqrt{1+x^2}\,\log\left(x+\sqrt{1+x^2}\right)-\int 1\,dx\\ &= -x+\sqrt{1+x^2}\,\log\left(x+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 1.00

$$-x+\sqrt{1+x^2}\log\left(x+\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x + Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] -x + Sqrt[1 + x^2]*Log[x + Sqrt[1 + x^2]]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

```
[Out] int(x*ln(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

Maxima [A]
time = 3.06, size = 22, normalized size = 0.85

$$\sqrt{x^2+1}\log\left(x+\sqrt{x^2+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*log(x + sqrt(x^2 + 1)) - x
```

Fricas [A]
time = 0.41, size = 22, normalized size = 0.85

$$\sqrt{x^2+1}\log\left(x+\sqrt{x^2+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 1)*log(x + sqrt(x^2 + 1)) - x
```

Sympy [A]
time = 3.16, size = 20, normalized size = 0.77

$$-x+\sqrt{x^2+1}\log\left(x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x+(x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

```
[Out] -x + sqrt(x**2 + 1)*log(x + sqrt(x**2 + 1))
```

Giac [A]
time = 0.45, size = 22, normalized size = 0.85

$$\sqrt{x^2+1}\,\log\left(x+\sqrt{x^2+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1)*log(x + sqrt(x^2 + 1)) - x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
```

```
[Out] int((x*log(x + (x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2), x)
```

3.26 $\int \frac{x \log\left(x+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=78

$$\sqrt{1-x^2}+\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{\sqrt{2}}-\sqrt{1-x^2}\,\log\left(x+\sqrt{1-x^2}\right)$$

```
[Out] 1/2*arctanh(x*2^(1/2))*2^(1/2)-1/2*arctanh(2^(1/2)*(-x^2+1)^(1/2))*2^(1/2)+
(-x^2+1)^(1/2)-ln(x+(-x^2+1)^(1/2))*(-x^2+1)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {267, 2634, 6874, 2132, 327, 212, 455, 52, 65, 213, 396}

$$\sqrt{1-x^2}-\sqrt{1-x^2}\,\log\left(\sqrt{1-x^2}+x\right)-\frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[x + Sqrt[1 - x^2]])/Sqrt[1 - x^2],x]
```

```
[Out] Sqrt[1 - x^2] + ArcTanh[Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[2]*Sqrt[1 - x^2]]
/Sqrt[2] - Sqrt[1 - x^2]*Log[x + Sqrt[1 - x^2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2132

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Dist[-d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\log\left(x+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)-\int \frac{x-\sqrt{1-x^2}}{x+\sqrt{1-x^2}}\,dx$$
$$= -\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)-\int \left(\frac{x}{x+\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}}{x+\sqrt{1-x^2}}\right)dx$$
$$= -\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)-\int \frac{x}{x+\sqrt{1-x^2}}\,dx+\int \frac{\sqrt{1-x^2}}{x+\sqrt{1-x^2}}\,dx$$
$$= -\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)+\int \frac{x^2}{1-2x^2}\,dx-\int \frac{x\sqrt{1-x^2}}{1-2x^2}\,dx+\int \Bigg(\frac{x}{-}$$
$$= -\frac{x}{2}-\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)+\frac{1}{2}\int \frac{1}{1-2x^2}\,dx-\frac{1}{2}\text{Subst}\Bigg(\int \frac{\sqrt{1-x}}{1-2x}$$
$$= \frac{\sqrt{1-x^2}}{2}+\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}}-\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)-\frac{1}{4}\text{Subst}\Bigg(\int$$
$$= \sqrt{1-x^2}+\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}-\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)+\frac{1}{4}\text{Subst}\Bigg(\int$$
$$= \sqrt{1-x^2}+\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{2\sqrt{2}}-\sqrt{1-x^2}\log\left(x+\right.$$
$$= \sqrt{1-x^2}+\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{\sqrt{2}}-\sqrt{1-x^2}\log\left(x+\right.$$

Mathematica [A]
time = 0.05, size = 119, normalized size = 1.53

$$\frac{1}{4}\left(4\sqrt{1-x^2}+2\sqrt{2}\log\left(\sqrt{2}+2x\right)-\sqrt{2}\log\left(2-\sqrt{2}\,x+\sqrt{2-2x^2}\right)-\sqrt{2}\log\left(2+\sqrt{2}\,x+\sqrt{2-2x^2}\right)-4\sqrt{1-x^2}\log\left(x+\sqrt{1-x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x + Sqrt[1 - x^2]])/Sqrt[1 - x^2],x]
```

```
[Out] (4*Sqrt[1 - x^2] + 2*Sqrt[2]*Log[Sqrt[2] + 2*x] - Sqrt[2]*Log[2 - Sqrt[2]*x
 + Sqrt[2 - 2*x^2]] - Sqrt[2]*Log[2 + Sqrt[2]*x + Sqrt[2 - 2*x^2]] - 4*Sqrt
[1 - x^2]*Log[x + Sqrt[1 - x^2]])/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x \ln \left(x+\sqrt{-x^{2}+1}\right)}{\sqrt{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

```
[Out] int(x*ln(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (x^2 - 1)*log(x + sqrt(x + 1)*sqrt(-x + 1))/(sqrt(x + 1)*sqrt(-x + 1)) - in
tegrate((x^2 - 1)*e^(-1/2*log(x + 1) - 1/2*log(-x + 1))/x, x) - integrate(1
/(x^2 + sqrt(x + 1)*x*sqrt(-x + 1)), x)
```

Fricas [A]
time = 0.46, size = 115, normalized size = 1.47

$$-\sqrt{-x^{2}+1} \log \left(x+\sqrt{-x^{2}+1}\right)+\frac{1}{4} \sqrt{2} \log \left(\frac{6 x^{2}-2 \sqrt{2} \left(2 x^{2}-3\right)+2 \sqrt{-x^{2}+1} \left(3 \sqrt{2}-4\right)-9}{2 x^{2}-1}\right)+\frac{1}{4} \sqrt{2} \log \left(\frac{2 x^{2}+2 \sqrt{2} x+1}{2 x^{2}-1}\right)+\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)*log(x + sqrt(-x^2 + 1)) + 1/4*sqrt(2)*log((6*x^2 - 2*sqrt(2
)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2) - 4) - 9)/(2*x^2 - 1)) + 1/4*sq
rt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1)) + sqrt(-x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log \left(x+\sqrt{1-x^{2}}\right)}{\sqrt{-\left(x-1\right) \left(x+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x+(-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(x*log(x + sqrt(1 - x**2))/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A]
time = 0.52, size = 122, normalized size = 1.56

$$-\sqrt{-x^2+1}\log\left(x+\sqrt{-x^2+1}\right)-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right)+\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*log(x + sqrt(-x^2 + 1)) - 1/4*sqrt(2)*log(abs(-4*sqrt(2) +
2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/
x^2 - 6)) + 1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x -
 1/2*sqrt(2))) + sqrt(-x^2 + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln\left(x+\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x + (1 - x^2)^(1/2)))/(1 - x^2)^(1/2),x)
```

```
[Out] int((x*log(x + (1 - x^2)^(1/2)))/(1 - x^2)^(1/2), x)
```

3.27 $\int \frac{\log(x)}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{1-x^2}}{x}-\sin^{-1}(x)-\frac{\sqrt{1-x^2}\,\log(x)}{x}$$

```
[Out] -arcsin(x)-(-x^2+1)^(1/2)/x-ln(x)*(-x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2373, 283, 222}

$$-\text{ArcSin}(x)-\frac{\sqrt{1-x^2}}{x}-\frac{\sqrt{1-x^2}\,\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x) - ArcSin[x] - (Sqrt[1 - x^2]*Log[x])/x
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x)}{x^2\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}\log(x)}{x} + \int \frac{\sqrt{1-x^2}}{x^2}\,dx$$
$$= -\frac{\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}\log(x)}{x} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{\sqrt{1-x^2}}{x} - \sin^{-1}(x) - \frac{\sqrt{1-x^2}\log(x)}{x}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.64

$$-\sin^{-1}(x) - \frac{\sqrt{1-x^2}\,(1+\log(x))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x^2*Sqrt[1 - x^2]),x]
[Out] -ArcSin[x] - (Sqrt[1 - x^2]*(1 + Log[x]))/x
```

Maple [A]
time = 0.03, size = 35, normalized size = 0.90

method	result	size
meijerg	$-\arcsin(x) + \frac{-\ln(x)\sqrt{-x^2+1}-\sqrt{-x^2+1}}{x}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arcsin(x)+(-ln(x)*(-x^2+1)^(1/2)-(-x^2+1)^(1/2))/x
```

Maxima [A]
time = 4.06, size = 35, normalized size = 0.90

$$-\frac{\sqrt{-x^2+1}\,\log(x)}{x} - \frac{\sqrt{-x^2+1}}{x} - \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)*log(x)/x - sqrt(-x^2 + 1)/x - arcsin(x)
```

Fricas [A]
time = 0.44, size = 39, normalized size = 1.00

$$\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}\,(\log\left(x\right)+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] (2*x*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*(log(x) + 1))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(x\right)}{x^2\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**2/(-x**2+1)**(1/2),x)
[Out] Integral(log(x)/(x**2*sqrt(-(x - 1)*(x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.
time = 0.45, size = 73, normalized size = 1.87

$$\frac{1}{2}\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)\log\left(x\right)+\frac{x}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\sqrt{-x^2+1}-1}{2\,x}-\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)*log(x) + 1/2*x/(sqrt(
-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x - arcsin(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\ln\left(x\right)}{x^2\,\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x^2*(1 - x^2)^(1/2)),x)
[Out] int(log(x)/(x^2*(1 - x^2)^(1/2)), x)
```

3.28 $\int \frac{x \tan^{-1}(x)}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=17

$$-\sinh^{-1}(x) + \sqrt{1+x^2} \tan^{-1}(x)$$

```
[Out] -arcsinh(x)+arctan(x)*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5050, 221}

$$\sqrt{x^2+1}\, \text{ArcTan}(x) - \sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x])/Sqrt[1 + x^2],x]
```

```
[Out] -ArcSinh[x] + Sqrt[1 + x^2]*ArcTan[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 5050

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Dist[b*(p/(2*c*(q + 1))), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x \tan^{-1}(x)}{\sqrt{1+x^2}} \, dx &= \sqrt{1+x^2} \tan^{-1}(x) - \int \frac{1}{\sqrt{1+x^2}} \, dx \\ &= -\sinh^{-1}(x) + \sqrt{1+x^2} \tan^{-1}(x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$-\sinh^{-1}(x) + \sqrt{1+x^2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x])/Sqrt[1 + x^2],x]
```

```
[Out] -ArcSinh[x] + Sqrt[1 + x^2]*ArcTan[x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 54, normalized size = 3.18

method	result	size
default	$\sqrt{(x-i)(x+i)}\,\arctan(x)+\ln\left(\frac{ix+1}{\sqrt{x^2+1}}-i\right)-\ln\left(\frac{ix+1}{\sqrt{x^2+1}}+i\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((x-I)*(x+I))^(1/2)*arctan(x)+ln((1+I*x)/(x^2+1)^(1/2)-I)-ln((1+I*x)/(x^2+1
)^(1/2)+I)
```

Maxima [A]
time = 5.55, size = 15, normalized size = 0.88

$$\sqrt{x^2+1}\,\arctan(x)-\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*arctan(x) - arcsinh(x)
```

Fricas [A]
time = 0.42, size = 23, normalized size = 1.35

$$\sqrt{x^2+1}\,\arctan(x)+\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 1)*arctan(x) + log(-x + sqrt(x^2 + 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.
time = 0.53, size = 29, normalized size = 1.71

$$\frac{x^2\operatorname{atan}(x)}{\sqrt{x^2+1}}-\operatorname{asinh}(x)+\frac{\operatorname{atan}(x)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)/(x**2+1)**(1/2),x)
[Out] x**2*atan(x)/sqrt(x**2 + 1) - asinh(x) + atan(x)/sqrt(x**2 + 1)
```

Giac [A]
time = 0.47, size = 23, normalized size = 1.35

$$\sqrt{x^2+1}\arctan(x)+\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 + 1)*arctan(x) + log(-x + sqrt(x^2 + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{x\,\mathrm{atan}(x)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(x))/(x^2 + 1)^(1/2),x)
[Out] int((x*atan(x))/(x^2 + 1)^(1/2), x)
```

3.29 $\int \frac{\tan^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{1-x^2}\tan^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)$$

```
[Out] -arctanh((-x^2+1)^(1/2))+arctanh(1/2*2^(1/2)*(-x^2+1)^(1/2))*2^(1/2)-arctan
(x)*(-x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {270, 5096, 457, 85, 65, 212}

$$-\frac{\sqrt{1-x^2}\,\text{ArcTan}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -((Sqrt[1 - x^2]*ArcTan[x])/x) - ArcTanh[Sqrt[1 - x^2]] + Sqrt[2]*ArcTanh[S
qrt[1 - x^2]/Sqrt[2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5096

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Dis
t[a + b*ArcTan[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&  !(
ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&  !(IL
tQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILtQ[(m
 - 1)/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}\,\tan^{-1}(x)}{x}+\int \frac{\sqrt{1-x^2}}{x\left(1+x^2\right)}\,dx\\
&= -\frac{\sqrt{1-x^2}\,\tan^{-1}(x)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x(1+x)}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-x^2}\,\tan^{-1}(x)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,(1+x)}\,d\right.\\
&= -\frac{\sqrt{1-x^2}\,\tan^{-1}(x)}{x}+2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1-x^2}\right)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{}\right.\\
&= -\frac{\sqrt{1-x^2}\,\tan^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 1.35

$$
-\frac{\sqrt{1-x^2}\,\tan^{-1}(x)}{x}+\log(x)-\frac{\log\left(1+x^2\right)}{\sqrt{2}}+\frac{\log\left(3-x^2+2\sqrt{2-2x^2}\right)}{\sqrt{2}}-\log\left(1+\sqrt{1-x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -((Sqrt[1 - x^2]*ArcTan[x])/x) + Log[x] - Log[1 + x^2]/Sqrt[2] + Log[3 - x^
2 + 2*Sqrt[2 - 2*x^2]]/Sqrt[2] - Log[1 + Sqrt[1 - x^2]]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(x\right)}{x^{2}\sqrt{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/x^2/(-x^2+1)^(1/2),x)
```

```
[Out] int(arctan(x)/x^2/(-x^2+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctan(x)/(sqrt(-x^2 + 1)*x^2), x)
```

Fricas [A]
time = 0.47, size = 81, normalized size = 1.42

$$\frac{\sqrt{2}\,x\log\left(\frac{x^{2}-2\,\sqrt{2}\,\sqrt{-x^{2}+1}-3}{x^{2}+1}\right)-x\log\left(\sqrt{-x^{2}+1}+1\right)+x\log\left(\sqrt{-x^{2}+1}-1\right)-2\,\sqrt{-x^{2}+1}\,\arctan\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*log((x^2 - 2*sqrt(2)*sqrt(-x^2 + 1) - 3)/(x^2 + 1)) - x*log(
sqrt(-x^2 + 1) + 1) + x*log(sqrt(-x^2 + 1) - 1) - 2*sqrt(-x^2 + 1)*arctan(x
))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(x\right)}{x^{2}\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(atan(x)/(x**2*sqrt(-(x - 1)*(x + 1))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(48) = 96.
time = 0.47, size = 104, normalized size = 1.82

$$\frac{1}{2}\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)\arctan(x)-\frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-x^2+1}}{\sqrt{2}+\sqrt{-x^2+1}}\right)-\frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right)+\frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)*arctan(x) - 1/2*sqrt(
2)*log((sqrt(2) - sqrt(-x^2 + 1))/(sqrt(2) + sqrt(-x^2 + 1))) - 1/2*log(sqr
t(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{atan}(x)}{x^2\,\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)/(x^2*(1 - x^2)^(1/2)),x)
```

```
[Out] int(atan(x)/(x^2*(1 - x^2)^(1/2)), x)
```

3.30 $\int \frac{x \tan^{-1}(x)}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=45

$$-\sin^{-1}(x)-\sqrt{1-x^2}\,\tan^{-1}(x)+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$$

```
[Out] -arcsin(x)+arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)-arctan(x)*(-x^2+1)^(1/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5094, 399, 222, 385, 209}

$$-\text{ArcSin}(x)-\sqrt{1-x^2}\,\text{ArcTan}(x)+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x])/Sqrt[1 - x^2],x]
```

```
[Out] -ArcSin[x] - Sqrt[1 - x^2]*ArcTan[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 -
x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
```

```
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 5094

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Dist[b*(c/(2*e*(q + 1))), Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x]
 /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}(x)}{\sqrt{1-x^2}}\,dx &= -\sqrt{1-x^2}\,\tan^{-1}(x)+\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx\\ &= -\sqrt{1-x^2}\,\tan^{-1}(x)+2\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx-\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\sin^{-1}(x)-\sqrt{1-x^2}\,\tan^{-1}(x)+2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\ &= -\sin^{-1}(x)-\sqrt{1-x^2}\,\tan^{-1}(x)+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 1.00

$$-\sin^{-1}(x)-\sqrt{1-x^2}\,\tan^{-1}(x)+\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x])/Sqrt[1 - x^2],x]
[Out] -ArcSin[x] - Sqrt[1 - x^2]*ArcTan[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 -
x^2]]
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x\arctan(x)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)/(-x^2+1)^(1/2),x)
```

```
[Out] int(x*arctan(x)/(-x^2+1)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 0.46, size = 69, normalized size = 1.53

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(3\,x^2-1)\sqrt{-x^2+1}}{4\,(x^3-x)}\right)-\sqrt{-x^2+1}\arctan\left(x\right)+\arctan\left(\frac{\sqrt{-x^2+1}\,x}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*sqrt(2)*(3*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - sqr
t(-x^2 + 1)*arctan(x) + arctan(sqrt(-x^2 + 1)*x/(x^2 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\operatorname{atan}\left(x\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(x*atan(x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(37) = 74.
time = 0.47, size = 108, normalized size = 2.40

$$-\frac{1}{2}\pi\mathrm{sgn}(x)+\frac{1}{2}\sqrt{2}\left(\pi\mathrm{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}\,x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\sqrt{-x^2+1}\arctan\left(x\right)-\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x) + 1/2*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x
^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - sqrt(-x^2 + 1)*arctan(x) -
 arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

Mupad [B]
time = 0.03, size = 37, normalized size = 0.82

$$\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{\sqrt{1-x^2}}\right)-\mathrm{atan}(x)\,\sqrt{1-x^2}-\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*atan(x))/(1 - x^2)^(1/2),x)
```

```
[Out] 2^(1/2)*atan((2^(1/2)*x)/(1 - x^2)^(1/2)) - atan(x)*(1 - x^2)^(1/2) - asin(
x)
```

3.31 $\int \frac{\tan^{-1}(x)}{x^2\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1+x^2}\right)$$

[Out] -arctanh((x^2+1)^(1/2))-arctan(x)*(x^2+1)^(1/2)/x

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5064, 272, 65, 213}

$$-\frac{\sqrt{x^2+1}\,\text{ArcTan}(x)}{x}-\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

[In] Int[ArcTan[x]/(x^2*Sqrt[1 + x^2]),x]

[Out] -((Sqrt[1 + x^2]*ArcTan[x])/x) - ArcTanh[Sqrt[1 + x^2]]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5064

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
```

```
b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Dist[b*c*(p/(f*(m + 1))), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)}{x^2\sqrt{1+x^2}}\,dx &= -\frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{x}+\int \frac{1}{x\sqrt{1+x^2}}\,dx\\
&= -\frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{x}+\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1+x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.14

$$-\frac{\sqrt{1+x^2}\,\tan^{-1}(x)}{x}+\log(x)-\log\left(1+\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(x^2*Sqrt[1 + x^2]),x]
[Out] -((Sqrt[1 + x^2]*ArcTan[x])/x) + Log[x] - Log[1 + Sqrt[1 + x^2]]
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 56, normalized size = 1.93

method	result	size
default	$-\frac{\sqrt{(x-i)(x+i)}\,\arctan(x)}{x}-\ln\left(1+\frac{ix+1}{\sqrt{x^2+1}}\right)+\ln\left(\frac{ix+1}{\sqrt{x^2+1}}-1\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/x^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -((x-I)*(x+I))^(1/2)*arctan(x)/x-ln(1+(1+I*x)/(x^2+1)^(1/2))+ln((1+I*x)/(x^
2+1)^(1/2)-1)
```

Maxima [A]
time = 2.94, size = 22, normalized size = 0.76

$$-\frac{\sqrt{x^2+1}\,\arctan(x)}{x}-\operatorname{arsinh}\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 + 1)*arctan(x)/x - arcsinh(1/abs(x))
```

Fricas [A]
time = 0.42, size = 47, normalized size = 1.62

$$-\frac{x\log\left(-x+\sqrt{x^2+1}+1\right)-x\log\left(-x+\sqrt{x^2+1}-1\right)+\sqrt{x^2+1}\,\arctan(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sqrt(x^2 + 1) - 1) + sqrt(x^2
+ 1)*arctan(x))/x
```

Sympy [A]
time = 4.33, size = 19, normalized size = 0.66

$$-\operatorname{asinh}\left(\frac{1}{x}\right)-\frac{\sqrt{x^2+1}\,\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/x**2/(x**2+1)**(1/2),x)
```

```
[Out] -asinh(1/x) - sqrt(x**2 + 1)*atan(x)/x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.
time = 0.50, size = 54, normalized size = 1.86

$$\frac{2\arctan(x)}{\left(x-\sqrt{x^2+1}\right)^2-1}+\arctan(x)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(x)/((x - sqrt(x^2 + 1))^2 - 1) + arctan(x) - log(abs(-x + sqrt(x^2
 + 1) + 1)) + log(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}(x)}{x^2\,\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x)/(x^2*(x^2 + 1)^(1/2)),x)
```

```
[Out] int(atan(x)/(x^2*(x^2 + 1)^(1/2)), x)
```

3.32 $\int \frac{\sin^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}+\log(x)$$

```
[Out] ln(x)-arcsin(x)*(-x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4771, 29}

$$\log(x)-\frac{\sqrt{1-x^2}\,\text{ArcSin}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -((Sqrt[1 - x^2]*ArcSin[x])/x) + Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 4771

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcSin[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx &= -\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}+\int \frac{1}{x}\,dx\\ &= -\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}+\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$-\frac{\sqrt{1-x^2}\,\sin^{-1}(x)}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(x^2*Sqrt[1 - x^2]),x]
[Out] -((Sqrt[1 - x^2]*ArcSin[x])/x) + Log[x]
```

Maple [A]
time = 0.05, size = 20, normalized size = 0.95

method	result	size
default	$\ln(x)-\frac{\arcsin(x)\sqrt{-x^2+1}}{x}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x)-arcsin(x)*(-x^2+1)^(1/2)/x
```

Maxima [A]
time = 2.06, size = 19, normalized size = 0.90

$$-\frac{\sqrt{-x^2+1}\,\arcsin(x)}{x}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)*arcsin(x)/x + log(x)
```

Fricas [A]
time = 0.45, size = 22, normalized size = 1.05

$$\frac{x\log(x)-\sqrt{-x^2+1}\,\arcsin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] (x*log(x) - sqrt(-x^2 + 1)*arcsin(x))/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{asin}(x)}{x^2\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(asin(x)/(x**2*sqrt(-(x - 1)*(x + 1))), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.
time = 0.45, size = 40, normalized size = 1.90

$$\frac{1}{2}\left(\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1}-1}{x}\right)\arcsin(x) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)*arcsin(x) + log(abs(x
))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\operatorname{asin}(x)}{x^2\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/(x^2*(1 - x^2)^(1/2)),x)
```

```
[Out] int(asin(x)/(x^2*(1 - x^2)^(1/2)), x)
```

3.33 $\int \frac{x \log(x)}{\sqrt{-1+x^2}} \, dx$

Optimal. Leaf size=34

$$-\sqrt{-1+x^2}+\tan^{-1}\left(\sqrt{-1+x^2}\right)+\sqrt{-1+x^2}\log(x)$$

```
[Out] arctan((x^2-1)^(1/2))-(x^2-1)^(1/2)+ln(x)*(x^2-1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2376, 272, 52, 65, 209}

$$\text{ArcTan}\left(\sqrt{x^2-1}\right)-\sqrt{x^2-1}+\sqrt{x^2-1}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[x])/Sqrt[-1 + x^2],x]
```

```
[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx &= \sqrt{-1+x^2}\,\log(x)-\int \frac{\sqrt{-1+x^2}}{x}\,dx\\ &= \sqrt{-1+x^2}\,\log(x)-\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^2\right)\\ &= -\sqrt{-1+x^2}+\sqrt{-1+x^2}\,\log(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx,x,x^2\right)\\ &= -\sqrt{-1+x^2}+\sqrt{-1+x^2}\,\log(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^2}\right)\\ &= -\sqrt{-1+x^2}+\tan^{-1}\left(\sqrt{-1+x^2}\right)+\sqrt{-1+x^2}\,\log(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.79

$$-\tan^{-1}\left(\frac{1}{\sqrt{-1+x^2}}\right)+\sqrt{-1+x^2}\,(-1+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x])/Sqrt[-1 + x^2],x]
```

```
[Out] -ArcTan[1/Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*(-1 + Log[x])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 119, normalized size = 3.50

method	result

meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^2-1)}\left(2-2\sqrt{-x^2+1}\right)}{4\sqrt{\operatorname{signum}(x^2-1)}}+\frac{\sqrt{-\operatorname{signum}(x^2-1)}\ln(x)\left(2-2\sqrt{-x^2+1}\right)}{2\sqrt{\operatorname{signum}(x^2-1)}}+\frac{\sqrt{-\operatorname{sign}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/2))+1/2/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*ln(x)*(2-2*(-x^2+1)^(1/2))+1/32/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(-16+16*(-x^2+1)^(1/2)-32*ln(1/2+1
/2*(-x^2+1)^(1/2)))
```

Maxima [A]
time = 3.77, size = 27, normalized size = 0.79

$$\sqrt{x^2-1}\log(x)-\sqrt{x^2-1}-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) - arcsin(1/abs(x))
```

Fricas [A]
time = 0.42, size = 27, normalized size = 0.79

$$\sqrt{x^2-1}(\log(x)-1)+2\arctan\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - 1)*(log(x) - 1) + 2*arctan(-x + sqrt(x^2 - 1))
```

Sympy [A]
time = 1.40, size = 29, normalized size = 0.85

$$\sqrt{x^2-1}\log(x)-\begin{cases}\sqrt{x^2-1}-\operatorname{acos}\left(\frac{1}{x}\right) & \text{for } x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(x**2 - 1)*log(x) - Piecewise((sqrt(x**2 - 1) - acos(1/x), (x > -1) & (
x < 1)))
```

Giac [**A**]
time = 0.44, size = 28, normalized size = 0.82

$$\sqrt{x^2-1}\,\log(x)-\sqrt{x^2-1}+\arctan\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) + arctan(sqrt(x^2 - 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\ln(x)}{\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x))/(x^2 - 1)^(1/2),x)
[Out] int((x*log(x))/(x^2 - 1)^(1/2), x)
```

3.34 $\int \frac{\log(x)}{x^2\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\sqrt{1+x^2}}{x}+\sinh^{-1}(x)-\frac{\sqrt{1+x^2}\,\log(x)}{x}$$

```
[Out] arcsinh(x)-(x^2+1)^(1/2)/x-ln(x)*(x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {2373, 283, 221}

$$-\frac{\sqrt{x^2+1}}{x}-\frac{\sqrt{x^2+1}\,\log(x)}{x}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x^2*Sqrt[1 + x^2]),x]
```

```
[Out] -(Sqrt[1 + x^2]/x) + ArcSinh[x] - (Sqrt[1 + x^2]*Log[x])/x
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 2373

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Dist[b*(n/(d*(m + 1))), Int[(f*x)^m*(d
+ e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && EqQ
[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x)}{x^2\sqrt{1+x^2}}\,dx = -\frac{\sqrt{1+x^2}\,\log(x)}{x} + \int \frac{\sqrt{1+x^2}}{x^2}\,dx$$
$$= -\frac{\sqrt{1+x^2}}{x} - \frac{\sqrt{1+x^2}\,\log(x)}{x} + \int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= -\frac{\sqrt{1+x^2}}{x} + \sinh^{-1}(x) - \frac{\sqrt{1+x^2}\,\log(x)}{x}$$

Mathematica [**A**]
time = 0.02, size = 21, normalized size = 0.64

$$\sinh^{-1}(x) - \frac{\sqrt{1+x^2}\,(1+\log(x))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x^2*Sqrt[1 + x^2]),x]
[Out] ArcSinh[x] - (Sqrt[1 + x^2]*(1 + Log[x]))/x
```

Maple [**A**]
time = 0.02, size = 29, normalized size = 0.88

method	result	size
meijerg	$\operatorname{arcsinh}(x) + \frac{-\ln(x)\sqrt{x^2+1}-\sqrt{x^2+1}}{x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^2/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(x)+(-ln(x)*(x^2+1)^(1/2)-(x^2+1)^(1/2))/x
```

Maxima [**A**]
time = 3.66, size = 29, normalized size = 0.88

$$-\frac{\sqrt{x^2+1}\,\log(x)}{x} - \frac{\sqrt{x^2+1}}{x} + \operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -sqrt(x^2 + 1)*log(x)/x - sqrt(x^2 + 1)/x + arcsinh(x)
```

Fricas [A]
time = 0.44, size = 33, normalized size = 1.00

$$-\frac{x\log\left(-x+\sqrt{x^2+1}\right)+\sqrt{x^2+1}\,(\log(x)+1)+x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] -(x*log(-x + sqrt(x^2 + 1)) + sqrt(x^2 + 1)*(log(x) + 1) + x)/x
```

Sympy [A]
time = 2.78, size = 26, normalized size = 0.79

$$\operatorname{asinh}(x)-\frac{\sqrt{x^2+1}\,\log(x)}{x}-\frac{\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**2/(x**2+1)**(1/2),x)
[Out] asinh(x) - sqrt(x**2 + 1)*log(x)/x - sqrt(x**2 + 1)/x
```

Giac [A]
time = 0.45, size = 58, normalized size = 1.76

$$\frac{2\log(x)}{\left(x-\sqrt{x^2+1}\right)^2-1}+\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\log\left(-x+\sqrt{x^2+1}\right)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^2/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*log(x)/((x - sqrt(x^2 + 1))^2 - 1) + 2/((x - sqrt(x^2 + 1))^2 - 1) - log(
-x + sqrt(x^2 + 1)) + log(abs(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\ln(x)}{x^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x^2*(x^2 + 1)^(1/2)),x)
[Out] int(log(x)/(x^2*(x^2 + 1)^(1/2)), x)
```

3.35 $\int \frac{x \sec^{-1}(x)}{\sqrt{-1+x^2}} \, dx$

Optimal. Leaf size=25

$$\sqrt{-1+x^2}\sec^{-1}(x)-\frac{x\log(x)}{\sqrt{x^2}}$$

```
[Out] -x*ln(x)/(x^2)^(1/2)+arcsec(x)*(x^2-1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {5344, 29}

$$\sqrt{x^2-1}\sec^{-1}(x)-\frac{x\log(x)}{\sqrt{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSec[x])/Sqrt[-1 + x^2],x]
```

```
[Out] Sqrt[-1 + x^2]*ArcSec[x] - (x*Log[x])/Sqrt[x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5344

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSec[c*x])/(2*e*(p + 1))), x
] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sec^{-1}(x)}{\sqrt{-1+x^2}}\,dx &= \sqrt{-1+x^2}\sec^{-1}(x)-\frac{x\int \frac{1}{x}\,dx}{\sqrt{x^2}}\\ &= \sqrt{-1+x^2}\sec^{-1}(x)-\frac{x\log(x)}{\sqrt{x^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 1.40

$$\frac{(-1+x^2)\sec^{-1}(x)-\sqrt{1-\frac{1}{x^2}}\,x\log(x)}{\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSec[x])/Sqrt[-1 + x^2],x]
[Out] ((-1 + x^2)*ArcSec[x] - Sqrt[1 - x^(-2)]*x*Log[x])/Sqrt[-1 + x^2]
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 97, normalized size = 3.88

method	result	size
default	$-\frac{2i\sqrt{\frac{x^2-1}{x^2}}\,x\,\mathrm{arcsec}(x)}{\sqrt{x^2-1}}+\frac{\left(i\sqrt{\frac{x^2-1}{x^2}}\,x+x^2-1\right)\mathrm{arcsec}(x)}{\sqrt{x^2-1}}+\frac{\sqrt{\frac{x^2-1}{x^2}}\,x\ln\left(\left(\frac{1}{x}+i\sqrt{1-\frac{1}{x^2}}\right)^2+1\right)}{\sqrt{x^2-1}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsec(x)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)+1/(x^2-1)^(1/2)*(I*((x^2
-1)/x^2)^(1/2)*x+x^2-1)*arcsec(x)+1/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(
(1/x+I*(1-1/x^2)^(1/2))^2+1)
```

Maxima [A]
time = 2.44, size = 15, normalized size = 0.60

$$\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)-\log\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] sqrt(x^2 - 1)*arcsec(x) - log(x)
```

Fricas [A]
time = 0.43, size = 15, normalized size = 0.60

$$\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)-\log\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] sqrt(x^2 - 1)*arcsec(x) - log(x)
```

Sympy [A]
time = 16.37, size = 22, normalized size = 0.88

$$\sqrt{x^2-1}\,\mathrm{asec}\,(x)-\begin{cases}-\log\left(\frac{1}{x}\right) & \text{for}\ x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asec(x)/(x**2-1)**(1/2),x)
[Out] sqrt(x**2 - 1)*asec(x) - Piecewise((-log(1/x), (x > -1) & (x < 1)))
```

Giac [A]
time = 0.46, size = 22, normalized size = 0.88

$$\sqrt{x^2-1}\,\arccos\left(\frac{1}{x}\right)-\frac{\log\left(|x|\right)}{\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsec(x)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] sqrt(x^2 - 1)*arccos(1/x) - log(abs(x))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\mathrm{acos}\left(\frac{1}{x}\right)}{\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acos(1/x))/(x^2 - 1)^(1/2),x)
[Out] int((x*acos(1/x))/(x^2 - 1)^(1/2), x)
```

3.36 $\int \frac{x \log(x)}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=34

$$-\sqrt{1+x^2}+\tanh^{-1}\left(\sqrt{1+x^2}\right)+\sqrt{1+x^2}\,\log(x)$$

```
[Out] arctanh((x^2+1)^(1/2))-(x^2+1)^(1/2)+ln(x)*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2376, 272, 52, 65, 213}

$$-\sqrt{x^2+1}+\sqrt{x^2+1}\,\log(x)+\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[x])/Sqrt[1 + x^2],x]
```

```
[Out] -Sqrt[1 + x^2] + ArcTanh[Sqrt[1 + x^2]] + Sqrt[1 + x^2]*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2376

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[f^m*(d + e*x^r)^(q + 1)*((a + b*L
og[c*x^n])^p/(e*r*(q + 1))), x] - Dist[b*f^m*n*(p/(e*r*(q + 1))), Int[(d +
e*x^r)^(q + 1)*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\log(x)}{\sqrt{1+x^2}}\,dx &= \sqrt{1+x^2}\,\log(x) - \int \frac{\sqrt{1+x^2}}{x}\,dx \\
&= \sqrt{1+x^2}\,\log(x) - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, x^2\right) \\
&= -\sqrt{1+x^2} + \sqrt{1+x^2}\,\log(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^2\right) \\
&= -\sqrt{1+x^2} + \sqrt{1+x^2}\,\log(x) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^2}\right) \\
&= -\sqrt{1+x^2} + \tanh^{-1}\left(\sqrt{1+x^2}\right) + \sqrt{1+x^2}\,\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.18

$$-\sqrt{1+x^2} - \log(x) + \sqrt{1+x^2}\,\log(x) + \log\left(1+\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x])/Sqrt[1 + x^2],x]
[Out] -Sqrt[1 + x^2] - Log[x] + Sqrt[1 + x^2]*Log[x] + Log[1 + Sqrt[1 + x^2]]
```

Maple [A]
time = 0.02, size = 39, normalized size = 1.15

method	result	size

meijerg	$1-\sqrt{x^2+1}+\frac{\ln(x)\left(-2+2\sqrt{x^2+1}\right)}{2}+\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1-(x^2+1)^(1/2)+1/2*ln(x)*(-2+2*(x^2+1)^(1/2))+ln(1/2+1/2*(x^2+1)^(1/2))
```

Maxima [A]
time = 3.27, size = 25, normalized size = 0.74

$$\sqrt{x^2+1}\,\log(x)-\sqrt{x^2+1}+\operatorname{arsinh}\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*log(x) - sqrt(x^2 + 1) + arcsinh(1/abs(x))
```

Fricas [A]
time = 0.41, size = 41, normalized size = 1.21

$$\sqrt{x^2+1}\,(\log(x)-1)+\log\left(-x+\sqrt{x^2+1}+1\right)-\log\left(-x+\sqrt{x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 1)*(log(x) - 1) + log(-x + sqrt(x^2 + 1) + 1) - log(-x + sqrt(x^
2 + 1) - 1)
```

Sympy [A]
time = 2.31, size = 41, normalized size = 1.21

$$-\frac{x}{\sqrt{1+\frac{1}{x^2}}}+\sqrt{x^2+1}\,\log(x)+\operatorname{asinh}\left(\frac{1}{x}\right)-\frac{1}{x\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)/(x**2+1)**(1/2),x)
```

```
[Out] -x/sqrt(1 + x**(-2)) + sqrt(x**2 + 1)*log(x) + asinh(1/x) - 1/(x*sqrt(1 + x
**(-2)))
```

Giac [**A**]
time = 0.47, size = 44, normalized size = 1.29

$$\sqrt{x^2+1}\,\log\left(x\right)-\sqrt{x^2+1}+\frac{1}{2}\,\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{2}\,\log\left(\sqrt{x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1)*log(x) - sqrt(x^2 + 1) + 1/2*log(sqrt(x^2 + 1) + 1) - 1/2*log
(sqrt(x^2 + 1) - 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\ln\left(x\right)}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x))/(x^2 + 1)^(1/2),x)
```

```
[Out] int((x*log(x))/(x^2 + 1)^(1/2), x)
```

3.37 $\int \frac{\sin(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=16

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3265, 212}

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Sin[x]^2),x]
```

```
[Out] -(ArcTanh[Cos[x]/Sqrt[2]]/Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{1+\sin^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 46, normalized size = 2.88

$$-\frac{i\left(\tan^{-1}\left(\frac{-i+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\tan^{-1}\left(\frac{i+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Sin[x]^2),x]
[Out] ((-I)*(ArcTan[(-I + Tan[x/2])/Sqrt[2]] - ArcTan[(I + Tan[x/2])/Sqrt[2]]))/S
qrt[2]
```

Maple [A]
time = 0.04, size = 14, normalized size = 0.88

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{\cos(x)\sqrt{2}}{2}\right)\sqrt{2}}{2}$	14
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}\,e^{ix}+1\right)}{4}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+sin(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctanh(1/2*cos(x)*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 3.95, size = 24, normalized size = 1.50

$$\frac{1}{4}\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\cos(x)}{\sqrt{2}+\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - cos(x))/(sqrt(2) + cos(x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.
time = 0.42, size = 29, normalized size = 1.81

$$\frac{1}{4}\sqrt{2}\,\log\left(-\frac{\cos(x)^2-2\sqrt{2}\,\cos(x)+2}{\cos(x)^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(cos(x)^2 - 2*sqrt(2)*cos(x) + 2)/(cos(x)^2 - 2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 8.70, size = 46, normalized size = 2.88

$$\frac{\sqrt{2}\log\left(\tan^2\left(\frac{x}{2}\right)-2\sqrt{2}+3\right)}{4}-\frac{\sqrt{2}\log\left(\tan^2\left(\frac{x}{2}\right)+2\sqrt{2}+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)**2),x)
```

```
[Out] sqrt(2)*log(tan(x/2)**2 - 2*sqrt(2) + 3)/4 - sqrt(2)*log(tan(x/2)**2 + 2*sq
rt(2) + 3)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.
time = 0.49, size = 27, normalized size = 1.69

$$-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}+\cos\left(x\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(sqrt(2) + cos(x)) + 1/4*sqrt(2)*log(sqrt(2) - cos(x))
```

Mupad [B]
time = 0.21, size = 13, normalized size = 0.81

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\cos(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sin(x)^2 + 1),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*cos(x))/2))/2
```

3.38 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1713, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 23, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.18, size = 112, normalized size = 4.87

method	result
elliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)\sqrt{2}}{2}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x-\sqrt{x^4+1}}{(-1+x)(1+x)}\right)}{2}$
default	$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticPi}\left((-1)^{\frac{1}{4}}x,-\right.}{\sqrt{x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(
1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/
4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.
time = 0.45, size = 42, normalized size = 1.83

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}\,x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
+ 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx-\int\frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**
2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2+1}{\left(x^2-1\right)\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] int(-(x^2 + 1)/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

3.39 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1713, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.14, size = 112, normalized size = 4.87

method	result
elliptic	$-\frac{\arctan\left(\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)\sqrt{2}}{2}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x+\sqrt{x^4+1}}{x^2+1}\right)}{2}$
default	$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{2(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x\right.}{\sqrt{x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(
1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Fricas [A]
time = 0.49, size = 18, normalized size = 0.78

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx-\int\left(-\frac{1}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x*
*2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int\frac{x^2-1}{\left(x^2+1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
```

```
[Out] -int((x^2 - 1)/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

3.40 $\int \frac{\log(\sin(x))}{1+\sin(x)}\,dx$

Optimal. Leaf size=22

$$-x-\tanh^{-1}(\cos(x))-\frac{\cos(x)\log(\sin(x))}{1+\sin(x)}$$

```
[Out] -x-arctanh(cos(x))-cos(x)*ln(sin(x))/(1+sin(x))
```

Rubi [A]
time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2727, 2634, 2918, 3855, 8}

$$-x-\tanh^{-1}(\cos(x))-\frac{\cos(x)\log(\sin(x))}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]/(1 + Sin[x]),x]
```

```
[Out] -x - ArcTanh[Cos[x]] - (Cos[x]*Log[Sin[x]])/(1 + Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(\sin(x))}{1+\sin(x)}\,dx &= -\frac{\cos(x)\log(\sin(x))}{1+\sin(x)}+\int \frac{\cos(x)\cot(x)}{1+\sin(x)}\,dx\\ &= -\frac{\cos(x)\log(\sin(x))}{1+\sin(x)}-\int 1\,dx+\int \csc(x)\,dx\\ &= -x-\tanh^{-1}(\cos(x))-\frac{\cos(x)\log(\sin(x))}{1+\sin(x)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.77

$$-x-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{2\log(\sin(x))\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]/(1 + Sin[x]),x]
```

```
[Out] -x - 2*Log[Cos[x/2]] + (2*Log[Sin[x]]*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.
time = 0.21, size = 54, normalized size = 2.45

method	result
norman	$\frac{-x-x\tan\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)}{1+\tan\left(\frac{x}{2}\right)}+\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$
risch	$\frac{2\ln(e^{ix})}{e^{ix}+i}+\frac{-2\ln(e^{2ix}-1)+\ln(1+e^{ix})+\ln(e^{ix}-1)+i\pi-i\ln(1+e^{ix})e^{ix}-2ix+i\pi\,\mathrm{csgn}(\sin(x))\mathrm{csgn}(i\sin(x))^2-i\pi\mathrm{csgn}(\sin(x))^3-\ln(1+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))/(sin(x)+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-x-x*tan(1/2*x)+2*tan(1/2*x)*ln(2*tan(1/2*x)/(1+tan(1/2*x)^2)))/(1+tan(1/2
*x))+ln(1+tan(1/2*x)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(22) = 44$.
time = 5.23, size = 82, normalized size = 3.73

$$-\frac{2\,\log\left(\frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{\frac{\sin(x)}{\cos(x)+1}+1} - 2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) + 2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) - \log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)))/(sin(x)/(cos(
x) + 1) + 1) - 2*arctan(sin(x)/(cos(x) + 1)) + 2*log(sin(x)/(cos(x) + 1)) -
 log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(22) = 44$.
time = 0.42, size = 93, normalized size = 4.23

$$-\frac{4\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\arctan\left(-\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}\right)+4\,x\cos\left(x\right)+\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)-\left(\cos\left(x\right)+\sin\left(x\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+2\left(\cos\left(x\right)-\sin\left(x\right)+1\right)\log\left(\sin\left(x\right)\right)+4\,x\sin\left(x\right)+4\,x}{2\left(\cos\left(x\right)+\sin\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(cos(x) + sin(x) + 1)*arctan(-(cos(x) + sin(x) + 1)/(cos(x) - sin(x
) + 1)) + 4*x*cos(x) + (cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x
) + sin(x) + 1)*log(-1/2*cos(x) + 1/2) + 2*(cos(x) - sin(x) + 1)*log(sin(x)
) + 4*x*sin(x) + 4*x)/(cos(x) + sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(20) = 40$.
time = 0.67, size = 105, normalized size = 4.77

$$-\frac{x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1} - \frac{x}{\tan\left(\frac{x}{2}\right)+1} + \frac{2\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan\left(\frac{x}{2}\right)+1} + \frac{2\log\left(2\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))/(1+sin(x)),x)
```

```
[Out] -x*tan(x/2)/(tan(x/2) + 1) - x/(tan(x/2) + 1) + 2*log(tan(x/2)/(tan(x/2)**2
 + 1))*tan(x/2)/(tan(x/2) + 1) + log(tan(x/2)**2 + 1)*tan(x/2)/(tan(x/2) +
1) + log(tan(x/2)**2 + 1)/(tan(x/2) + 1) + 2*log(2)*tan(x/2)/(tan(x/2) + 1)
```

Giac [A]
time = 0.48, size = 36, normalized size = 1.64

$$-x - \frac{2\,\log\left(\sin\left(x\right)\right)}{\tan\left(\frac{1}{2}\,x\right)+1} - 2\,\log\left(\tan\left(\frac{1}{4}\,x\right)^2+1\right) + 2\,\log\left(\left|\tan\left(\frac{1}{4}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))/(1+sin(x)),x, algorithm="giac")
```

```
[Out] -x - 2*log(sin(x))/(tan(1/2*x) + 1) - 2*log(tan(1/4*x)^2 + 1) + 2*log(abs(t
an(1/4*x)))
```

Mupad [B]
time = 0.38, size = 55, normalized size = 2.50

$$-2x+\ln\left(2\sin\left(x\right)-\cos\left(x\right)\,2\mathrm{i}-2\mathrm{i}\right)\,\left(-1-\mathrm{i}\right)+\ln\left(2\sin\left(x\right)-\cos\left(x\right)\,2\mathrm{i}+2\mathrm{i}\right)\,\left(1-\mathrm{i}\right)-\frac{2\ln\left(\sin\left(x\right)\right)}{\cos\left(x\right)+\sin\left(x\right)\,1\mathrm{i}+1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))/(sin(x) + 1),x)
```

```
[Out] log(2*sin(x) - cos(x)*2i + 2i)*(1 - 1i) - log(2*sin(x) - cos(x)*2i - 2i)*(1
 + 1i) - 2*x - (2*log(sin(x)))/(cos(x) + sin(x)*1i + 1i)
```

3.41 $\int \log(\sin(x))\sqrt{1+\sin(x)}\,dx$

Optimal. Leaf size=42

$$-4\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{1+\sin(x)}}\right)+\frac{4\cos(x)}{\sqrt{1+\sin(x)}}-\frac{2\cos(x)\log(\sin(x))}{\sqrt{1+\sin(x)}}$$

```
[Out] -4*arctanh(cos(x)/(1+sin(x))^(1/2))+4*cos(x)/(1+sin(x))^(1/2)-2*cos(x)*ln(s
in(x))/(1+sin(x))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2725, 2634, 12, 2953, 3060, 2852, 212}

$$\frac{4\cos(x)}{\sqrt{\sin(x)+1}}-\frac{2\cos(x)\log(\sin(x))}{\sqrt{\sin(x)+1}}-4\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{\sin(x)+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sqrt[1 + Sin[x]],x]
```

```
[Out] -4*ArcTanh[Cos[x]/Sqrt[1 + Sin[x]]] + (4*Cos[x])/Sqrt[1 + Sin[x]] - (2*Cos[
x]*Log[Sin[x]])/Sqrt[1 + Sin[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \log(\sin(x))\sqrt{1+\sin(x)}\,dx &= -\frac{2\cos(x)\log(\sin(x))}{\sqrt{1+\sin(x)}} - \int -\frac{2\cos(x)\cot(x)}{\sqrt{1+\sin(x)}}\,dx \\
&= -\frac{2\cos(x)\log(\sin(x))}{\sqrt{1+\sin(x)}} + 2\int \frac{\cos(x)\cot(x)}{\sqrt{1+\sin(x)}}\,dx \\
&= -\frac{2\cos(x)\log(\sin(x))}{\sqrt{1+\sin(x)}} + 2\int \csc(x)(1-\sin(x))\sqrt{1+\sin(x)}\,dx \\
&= \frac{4\cos(x)}{\sqrt{1+\sin(x)}} - \frac{2\cos(x)\log(\sin(x))}{\sqrt{1+\sin(x)}} + 2\int \csc(x)\sqrt{1+\sin(x)}\,dx \\
&= \frac{4\cos(x)}{\sqrt{1+\sin(x)}} - \frac{2\cos(x)\log(\sin(x))}{\sqrt{1+\sin(x)}} - 4\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\cos(x)}{\sqrt{1+\sin(x)}}\right) \\
&= -4\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{1+\sin(x)}}\right) + \frac{4\cos(x)}{\sqrt{1+\sin(x)}} - \frac{2\cos(x)\log(\sin(x))}{\sqrt{1+\sin(x)}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.
time = 0.06, size = 87, normalized size = 2.07

$$\frac{2\left(-\log\left(1+\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(1-\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\cos\left(\frac{x}{2}\right)(-2+\log(\sin(x)))+(-2+\log(\sin(x)))\sin\left(\frac{x}{2}\right)\right)\sqrt{1+\sin(x)}}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(-Log[1 + Cos[x/2] - Sin[x/2]] + Log[1 - Cos[x/2] + Sin[x/2]] - Cos[x/2]
*(-2 + Log[Sin[x]]) + (-2 + Log[Sin[x]])*Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x
/2] + Sin[x/2])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \ln(\sin(x))\sqrt{\sin(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))*(sin(x)+1)^(1/2),x)
```

```
[Out] int(ln(sin(x))*(sin(x)+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*(1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(x) + 1)*log(sin(x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(36) = 72$.
time = 0.46, size = 146, normalized size = 3.48

$$-\frac{(\cos(x)+\sin(x)+1)\log\left(\frac{\cos(x)^2-(\cos(x)-1)\sin(x)+2(\cos(x)-\sin(x)+1)\sqrt{\sin(x)+1}+2\cos(x)+1}{2(\cos(x)+\sin(x)+1)}\right)-(\cos(x)+\sin(x)+1)\log\left(\frac{\cos(x)^2-(\cos(x)-1)\sin(x)-2(\cos(x)-\sin(x)+1)\sqrt{\sin(x)+1}+2\cos(x)+1}{2(\cos(x)+\sin(x)+1)}\right)+2((\cos(x)-\sin(x)+1)\log(\sin(x))-2\cos(x)+2\sin(x)-2)\sqrt{\sin(x)+1}}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*(1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -((cos(x) + sin(x) + 1)*log(1/2*(cos(x)^2 - (cos(x) - 1)*sin(x) + 2*(cos(x)
 - sin(x) + 1)*sqrt(sin(x) + 1) + 2*cos(x) + 1)/(cos(x) + sin(x) + 1)) - (c
os(x) + sin(x) + 1)*log(1/2*(cos(x)^2 - (cos(x) - 1)*sin(x) - 2*(cos(x) - s
```

```
in(x) + 1)*sqrt(sin(x) + 1) + 2*cos(x) + 1)/(cos(x) + sin(x) + 1)) + 2*((co
s(x) - sin(x) + 1)*log(sin(x)) - 2*cos(x) + 2*sin(x) - 2)*sqrt(sin(x) + 1))
/(cos(x) + sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(x)+1}\,\log(\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))*(1+sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x) + 1)*log(sin(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.
time = 0.49, size = 90, normalized size = 2.14

$$\sqrt{2}\left(2\log(\sin(x))\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)+\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{4}\pi-\frac{1}{2}x\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{4}\pi-\frac{1}{2}x\right)\right|}\right)+4\sin\left(\frac{1}{4}\pi-\frac{1}{2}x\right)\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*(1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(2*log(sin(x))*sgn(cos(-1/4*pi + 1/2*x))*sin(-1/4*pi + 1/2*x) + (sq
rt(2)*log(abs(-2*sqrt(2) + 4*sin(1/4*pi - 1/2*x))/abs(2*sqrt(2) + 4*sin(1/4
*pi - 1/2*x))) + 4*sin(1/4*pi - 1/2*x))*sgn(cos(-1/4*pi + 1/2*x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln(\sin(x))\,\sqrt{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*(sin(x) + 1)^(1/2),x)
```

```
[Out] int(log(sin(x))*(sin(x) + 1)^(1/2), x)
```

3.42 $\int \frac{\sec(x)}{\sqrt{-1+\sec^4(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)\cot(x)\sqrt{-1+\sec^4(x)}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*cos(x)*cot(x)*(-1+sec(x)^4)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.
time = 0.13, antiderivative size = 59, normalized size of antiderivative = 2.11, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4233, 6854, 2013, 2033, 212}

$$-\frac{\sqrt{1-\cos^4(x)}\,\sec^2(x)\tanh^{-1}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{2\sin^2(x)-\sin^4(x)}}\right)}{\sqrt{2}\sqrt{\sec^4(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/Sqrt[-1 + Sec[x]^4],x]
```

```
[Out] -((ArcTanh[(Sqrt[2]*Sin[x])/Sqrt[2*Sin[x]^2 - Sin[x]^4]]*Sqrt[1 - Cos[x]^4]
*Sec[x]^2)/(Sqrt[2]*Sqrt[-1 + Sec[x]^4]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2013

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && T
rinomialQ[u, x] &&  !TrinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 4233

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_
))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f,
Subst[Int[(a + b/(1 - ff^2*x^2)^(n/2))^p/(1 - ff^2*x^2)^((m + 1)/2), x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
&& IntegerQ[n/2] &&  !IntegerQ[p]
```

Rule 6854

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(x)}{\sqrt{-1+\sec^4(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{-1+\frac{1}{(1-x^2)^2}}}\,dx, x, \sin(x)\right)\\
&= \frac{\left(\sqrt{1-\cos^4(x)}\,\sec^2(x)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-(1-x^2)^2}}\,dx, x, \sin(x)\right)}{\sqrt{-1+\sec^4(x)}}\\
&= \frac{\left(\sqrt{1-\cos^4(x)}\,\sec^2(x)\right)\text{Subst}\left(\int \frac{1}{\sqrt{2x^2-x^4}}\,dx, x, \sin(x)\right)}{\sqrt{-1+\sec^4(x)}}\\
&= -\frac{\left(\sqrt{1-\cos^4(x)}\,\sec^2(x)\right)\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{\sin(x)}{\sqrt{2\sin^2(x)-\sin^4(x)}}\right)}{\sqrt{-1+\sec^4(x)}}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sin(x)}{\sqrt{2\sin^2(x)-\sin^4(x)}}\right)\sqrt{1-\cos^4(x)}\,\sec^2(x)}{\sqrt{2}\sqrt{-1+\sec^4(x)}}
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 45, normalized size = 1.61

$$-\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{4-2\sin^2(x)}\right)\sqrt{3+\cos(2x)}\,\sec(x)\tan(x)}{2\sqrt{-1+\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/Sqrt[-1 + Sec[x]^4],x]
```

```
[Out] -1/2*(ArcTanh[Sqrt[4 - 2*Sin[x]^2]/2]*Sqrt[3 + Cos[2*x]]*Sec[x]*Tan[x])/Sqr
t[-1 + Sec[x]^4]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(23) = 46$.
time = 0.15, size = 91, normalized size = 3.25

method	result	size
default	$-\frac{\sqrt{8}\,\sqrt{2}\left(\operatorname{arcsinh}\left(\frac{\cos(x)-1}{1+\cos(x)}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{4}}{4\sqrt{\frac{1+\cos^2(x)}{(1+\cos(x))^2}}}\right)\right)(\sin^3(x))\sqrt{\frac{1+\cos^2(x)}{(1+\cos(x))^2}}}{8(\cos(x)-1)\cos(x)^2\sqrt{-\frac{2(\cos^4(x)-1)}{\cos(x)^4}}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(-1+sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*8^(1/2)*2^(1/2)*(arcsinh((cos(x)-1)/(1+cos(x)))-arctanh(1/4*2^(1/2)*4^
(1/2)/((1+cos(x)^2)/(1+cos(x))^2)^(1/2)))*sin(x)^3*((1+cos(x)^2)/(1+cos(x))
^2)^(1/2)/(cos(x)-1)/cos(x)^2/(-2*(cos(x)^4-1)/cos(x)^4)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-1+sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(x)/sqrt(sec(x)^4 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.
time = 0.45, size = 54, normalized size = 1.93

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\left(2\sqrt{2}\sqrt{-\frac{\cos(x)^4-1}{\cos(x)^4}}\cos(x)^2-\left(\cos(x)^2+3\right)\sin(x)\right)}{\left(\cos(x)^2-1\right)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-1+sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-2*(2*sqrt(2)*sqrt(-(cos(x)^4 - 1)/cos(x)^4)*cos(x)^2 - (co
s(x)^2 + 3)*sin(x))/((cos(x)^2 - 1)*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{\sqrt{(\sec(x)-1)(\sec(x)+1)(\sec^2(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-1+sec(x)**4)**(1/2),x)
```

```
[Out] Integral(sec(x)/sqrt((sec(x) - 1)*(sec(x) + 1)*(sec(x)**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(23) = 46.
time = 0.62, size = 92, normalized size = 3.29

$$\frac{\sqrt{2}\left(\log\left(\tan\left(\frac{1}{2}x\right)^2-\sqrt{\tan\left(\frac{1}{2}x\right)^4+1}+1\right)-\log\left(-\tan\left(\frac{1}{2}x\right)^2+\sqrt{\tan\left(\frac{1}{2}x\right)^4+1}+1\right)+\log\left(-\tan\left(\frac{1}{2}x\right)^2+\sqrt{\tan\left(\frac{1}{2}x\right)^4+1}\right)\right)}{4\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^5+2\tan\left(\frac{1}{2}x\right)^3+\tan\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-1+sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(log(tan(1/2*x)^2 - sqrt(tan(1/2*x)^4 + 1) + 1) - log(-tan(1/2*
x)^2 + sqrt(tan(1/2*x)^4 + 1) + 1) + log(-tan(1/2*x)^2 + sqrt(tan(1/2*x)^4
+ 1)))/sgn(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(x)\sqrt{\frac{1}{\cos(x)^4}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(1/cos(x)^4 - 1)^(1/2)),x)
```

```
[Out] int(1/(cos(x)*(1/cos(x)^4 - 1)^(1/2)), x)
```

3.43 $\int \frac{\tan(x)}{\sqrt{1+\tan^4(x)}}\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{1+\tan^4(x)}}\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(1/2*(1-tan(x)^2)*2^(1/2)/(1+tan(x)^4)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3751, 1262, 739, 212}

$$-\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{\tan^4(x)+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[1 + Tan[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(1 - Tan[x]^2)/(Sqrt[2]*Sqrt[1 + Tan[x]^4])]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3751

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[c*(ff/f), Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + ff
^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\sqrt{1+\tan^4(x)}}\,dx &= \operatorname{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{1+x^4}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+x^2}}\,dx, x, \tan^2(x)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{1-\tan^2(x)}{\sqrt{1+\tan^4(x)}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{1-\tan^2(x)}{\sqrt{2}\sqrt{1+\tan^4(x)}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 1.62

$$-\frac{\sqrt{3+\cos(4x)}\log\left(\sqrt{2}\cos(2x)+\sqrt{3+\cos(4x)}\right)\sec^2(x)}{4\sqrt{2}\sqrt{1+\tan^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[1 + Tan[x]^4],x]
```

```
[Out] -1/4*(Sqrt[3 + Cos[4*x]]*Log[Sqrt[2]*Cos[2*x] + Sqrt[3 + Cos[4*x]]]*Sec[x]^
2)/(Sqrt[2]*Sqrt[1 + Tan[x]^4])
```

Maple [A]
time = 0.07, size = 37, normalized size = 1.09

method	result	size

derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-2\left(\tan^2(x)\right)+2\right)\sqrt{2}}{4\sqrt{\left(1+\tan^2(x)\right)^2-2\left(\tan^2(x)\right)}}\right)}{4}$	37
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-2\left(\tan^2(x)\right)+2\right)\sqrt{2}}{4\sqrt{\left(1+\tan^2(x)\right)^2-2\left(\tan^2(x)\right)}}\right)}{4}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+tan(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*arctanh(1/4*(-2*tan(x)^2+2)*2^(1/2)/((1+tan(x)^2)^2-2*tan(x)^2
)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(25) = 50$.
time = 3.71, size = 565, normalized size = 16.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+tan(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*(log(4*sqrt(2*(6*cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 + 36*cos
(4*x)^2 + sin(8*x)^2 + 12*sin(8*x)*sin(4*x) + 36*sin(4*x)^2 + 12*cos(4*x) +
 1)*cos(1/2*arctan2(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))^2 +
4*sqrt(2*(6*cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 + 36*cos(4*x)^2 + sin(8*x)^
2 + 12*sin(8*x)*sin(4*x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)*sin(1/2*arctan2
(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))^2 + 32*(2*(6*cos(4*x) +
 1)*cos(8*x) + cos(8*x)^2 + 36*cos(4*x)^2 + sin(8*x)^2 + 12*sin(8*x)*sin(4*
x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(8*x) + 6*si
n(4*x), cos(8*x) + 6*cos(4*x) + 1)) + 64) + log(4*cos(4*x)^2 + 4*sin(4*x)^2
 + 4*sqrt(2*(6*cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 + 36*cos(4*x)^2 + sin(8*
x)^2 + 12*sin(8*x)*sin(4*x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)*(cos(1/2*arc
tan2(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))^2 + sin(1/2*arctan2
(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))^2) + 8*(2*(6*cos(4*x) +
 1)*cos(8*x) + cos(8*x)^2 + 36*cos(4*x)^2 + sin(8*x)^2 + 12*sin(8*x)*sin(4*
x) + 36*sin(4*x)^2 + 12*cos(4*x) + 1)^(1/4)*((cos(4*x) + 3)*cos(1/2*arctan2
(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1)) + sin(4*x)*sin(1/2*arct
an2(sin(8*x) + 6*sin(4*x), cos(8*x) + 6*cos(4*x) + 1))) + 24*cos(4*x) + 36)
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(25) = 50$.

time = 0.43, size = 186, normalized size = 5.47

$$\frac{1}{32}\sqrt{2}\log\left(\frac{577\tan\left(x\right)^{16}-1912\tan\left(x\right)^{14}+4124\tan\left(x\right)^{12}-6216\tan\left(x\right)^{10}+7110\tan\left(x\right)^{8}-6216\tan\left(x\right)^{6}+4124\tan\left(x\right)^{4}-1912\tan\left(x\right)^{2}+8\left(51\sqrt{2}\tan\left(x\right)^{14}-169\sqrt{2}\tan\left(x\right)^{12}+339\sqrt{2}\tan\left(x\right)^{10}-465\sqrt{2}\tan\left(x\right)^{8}+465\sqrt{2}\tan\left(x\right)^{6}-339\sqrt{2}\tan\left(x\right)^{4}+169\sqrt{2}\tan\left(x\right)^{2}-51\sqrt{2}\right)\sqrt{\tan\left(x\right)^{4}+1}+577}{\tan\left(x\right)^{16}+8\tan\left(x\right)^{14}+28\tan\left(x\right)^{12}+56\tan\left(x\right)^{10}+70\tan\left(x\right)^{8}+56\tan\left(x\right)^{6}+28\tan\left(x\right)^{4}+8\tan\left(x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+tan(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*sqrt(2)*log((577*tan(x)^16 - 1912*tan(x)^14 + 4124*tan(x)^12 - 6216*ta
n(x)^10 + 7110*tan(x)^8 - 6216*tan(x)^6 + 4124*tan(x)^4 - 1912*tan(x)^2 + 8
*(51*sqrt(2)*tan(x)^14 - 169*sqrt(2)*tan(x)^12 + 339*sqrt(2)*tan(x)^10 - 46
5*sqrt(2)*tan(x)^8 + 465*sqrt(2)*tan(x)^6 - 339*sqrt(2)*tan(x)^4 + 169*sqrt
(2)*tan(x)^2 - 51*sqrt(2))*sqrt(tan(x)^4 + 1) + 577)/(tan(x)^16 + 8*tan(x)^
14 + 28*tan(x)^12 + 56*tan(x)^10 + 70*tan(x)^8 + 56*tan(x)^6 + 28*tan(x)^4
+ 8*tan(x)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{\sqrt{\tan^{4}\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+tan(x)**4)**(1/2),x)
```

```
[Out] Integral(tan(x)/sqrt(tan(x)**4 + 1), x)
```

Giac [A]
time = 0.51, size = 50, normalized size = 1.47

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\tan\left(x\right)^{2}+\sqrt{2}-\sqrt{\tan\left(x\right)^{4}+1}+1}{\tan\left(x\right)^{2}-\sqrt{2}-\sqrt{\tan\left(x\right)^{4}+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+tan(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(-(tan(x)^2 + sqrt(2) - sqrt(tan(x)^4 + 1) + 1)/(tan(x)^2 -
sqrt(2) - sqrt(tan(x)^4 + 1) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\tan(x)}{\sqrt{\tan\left(x\right)^{4}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(tan(x)^4 + 1)^(1/2),x)
```

```
[Out] int(tan(x)/(tan(x)^4 + 1)^(1/2), x)
```

3.44 $\int \frac{\sin(x)}{\sqrt{1-\sin^6(x)}}\,dx$

Optimal. Leaf size=39

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}\cos(x)(1+\sin^2(x))}{2\sqrt{1-\sin^6(x)}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctanh(1/2*cos(x)*(1+sin(x)^2)*3^(1/2)/(1-sin(x)^6)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3295, 2021, 1918, 212}

$$\frac{\tanh^{-1}\left(\frac{\cos(x)(6-3\cos^2(x))}{2\sqrt{3}\sqrt{\cos^6(x)-3\cos^4(x)+3\cos^2(x)}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 - Sin[x]^6],x]
```

```
[Out] ArcTanh[(Cos[x]*(6 - 3*Cos[x]^2))/(2*Sqrt[3]*Sqrt[3*Cos[x]^2 - 3*Cos[x]^4 +
 Cos[x]^6])]/(2*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rule 3295

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f
, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(1 - ff^2*x^2)^(n/2))^p, x],
x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1-\sin^6(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx, x, \cos(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{\cos(x)\,(6-3\cos^2(x))}{\sqrt{3\cos^2(x)-3\cos^4(x)+\cos^6(x)}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\cos(x)(6-3\cos^2(x))}{2\sqrt{3}\,\sqrt{3\cos^2(x)-3\cos^4(x)+\cos^6(x)}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 65, normalized size = 1.67

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,(-3+\cos(2x))}{\sqrt{15-8\cos(2x)+\cos(4x)}}\right)\cos(x)\sqrt{15-8\cos(2x)+\cos(4x)}}{4\sqrt{6-6\sin^6(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 - Sin[x]^6],x]
```

```
[Out] -1/4*(ArcTanh[(Sqrt[3/2]*(-3 + Cos[2*x]))/Sqrt[15 - 8*Cos[2*x] + Cos[4*x]]]
*Cos[x]*Sqrt[15 - 8*Cos[2*x] + Cos[4*x]])/Sqrt[6 - 6*Sin[x]^6]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.
time = 0.26, size = 67, normalized size = 1.72

method	result	size

default	$-\frac{\cos(x)\sqrt{\cos^4(x)-3(\cos^2(x))+3}\,\sqrt{3}\,\operatorname{arctanh}\left(\frac{(\cos^2(x)-2)\sqrt{3}}{2\sqrt{\cos^4(x)-3(\cos^2(x))+3}}\right)}{6\sqrt{3(\cos^2(x))-3(\cos^4(x))+\cos^6(x)}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1-sin(x)^6)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(3*cos(x)^2-3*cos(x)^4+cos(x)^6)^(1/2)*cos(x)*(cos(x)^4-3*cos(x)^2+3)^
(1/2)*3^(1/2)*arctanh(1/2*(cos(x)^2-2)*3^(1/2)/(cos(x)^4-3*cos(x)^2+3)^(1/2
))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-sin(x)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)/sqrt(-sin(x)^6 + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.
time = 0.48, size = 63, normalized size = 1.62

$$\frac{1}{12}\sqrt{3}\log\left(\frac{7\cos(x)^5-24\cos(x)^3-4\sqrt{\cos(x)^6-3\cos(x)^4+3\cos(x)^2}\left(\sqrt{3}\cos(x)^2-2\sqrt{3}\right)+24\cos(x)}{\cos(x)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-sin(x)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((7*cos(x)^5 - 24*cos(x)^3 - 4*sqrt(cos(x)^6 - 3*cos(x)^4 +
 3*cos(x)^2)*(sqrt(3)*cos(x)^2 - 2*sqrt(3)) + 24*cos(x))/cos(x)^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-sin(x)**6)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(29) = 58$.
time = 0.51, size = 67, normalized size = 1.72

$$-\frac{\sqrt{3}\log\left(\cos\left(x\right)^{2}+\sqrt{3}-\sqrt{\cos\left(x\right)^{4}-3\cos\left(x\right)^{2}+3}\right)-\sqrt{3}\log\left(-\cos\left(x\right)^{2}+\sqrt{3}+\sqrt{\cos\left(x\right)^{4}-3\cos\left(x\right)^{2}+3}\right)}{6\,\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1-sin(x)^6)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(sqrt(3)*log(cos(x)^2 + sqrt(3) - sqrt(cos(x)^4 - 3*cos(x)^2 + 3)) - s
qrt(3)*log(-cos(x)^2 + sqrt(3) + sqrt(cos(x)^4 - 3*cos(x)^2 + 3)))/sgn(cos(
x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sin\left(x\right)}{\sqrt{1-\sin\left(x\right)^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1 - sin(x)^6)^(1/2),x)
```

```
[Out] int(sin(x)/(1 - sin(x)^6)^(1/2), x)
```

3.45 $\int \sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\,dx$

Optimal. Leaf size=337

$$\sqrt{2}\left(\sqrt{-1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{-2+2\sqrt{2}}\left(-\sqrt{2}-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}\right)}{2\sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}}\right)-\sqrt{1+\sqrt{2}}\,\tan$$

```
[Out] cot(x)*2^(1/2)*(-1+sec(x))^(1/2)*(1+sec(x))^(1/2)*(arctan(1/2*(-2^(1/2)-(-1
+sec(x))^(1/2)+(1+sec(x))^(1/2))*(-2+2*2^(1/2))^(1/2)/(-(-1+sec(x))^(1/2)+(
1+sec(x))^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+arctanh((2+2*2^(1/2))^(1/2)*(-(-1
+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2)/(2^(1/2)-(-1+sec(x))^(1/2)+(1+sec(x)
)^(1/2)))*(2^(1/2)-1)^(1/2)-arctan(1/2*(-2^(1/2)-(-1+sec(x))^(1/2)+(1+sec(x
))^(1/2))*(2+2*2^(1/2))^(1/2)/(-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2))*
(1+2^(1/2))^(1/2)-arctanh((-2+2*2^(1/2))^(1/2)*(-(-1+sec(x))^(1/2)+(1+sec(x
))^(1/2))^(1/2)/(2^(1/2)-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2)))*(1+2^(1/2))^(
1/2))
```

Rubi [F]
time = 0.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]],x]
[Out] Defer[Int][Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]], x]
```

Rubi steps

$$\int \sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\,dx=\int \sqrt{-\sqrt{-1+\sec(x)}+\sqrt{1+\sec(x)}}\,dx$$

Mathematica [A]
time = 1.48, size = 552, normalized size = 1.64

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]],x]
```

```
[Out] (2^(1/4)*Cos[x]*(Sqrt[-1 + Sec[x]] - Sqrt[1 + Sec[x]])^2*(2*ArcTan[Cot[Pi/8
] - (Csc[Pi/8]*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]])/2^(1/4)]*Cos[Pi
/8] - 2*ArcTan[Cot[Pi/8] + (Csc[Pi/8]*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Se
c[x]]])/2^(1/4)]*Cos[Pi/8] + Cos[Pi/8]*Log[2 + Sqrt[2]*(-Sqrt[-1 + Sec[x]]
+ Sqrt[1 + Sec[x]]) - 2*2^(3/4)*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]]
*Sin[Pi/8]] - Cos[Pi/8]*Log[2 + Sqrt[2]*(-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[
x]]) + 2*2^(3/4)*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]]*Sin[Pi/8]] + 2
*ArcTan[(Sec[Pi/8]*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]])/2^(1/4) - T
an[Pi/8]]*Sin[Pi/8] + 2*ArcTan[(Sec[Pi/8]*Sqrt[-Sqrt[-1 + Sec[x]] + Sqrt[1
+ Sec[x]]])/2^(1/4) + Tan[Pi/8]]*Sin[Pi/8] - Log[2 - 2*2^(3/4)*Cos[Pi/8]*Sq
rt[-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]]] + Sqrt[2]*(-Sqrt[-1 + Sec[x]] + S
qrt[1 + Sec[x]])]*Sin[Pi/8] + Log[2 + 2^(1/4)*Csc[Pi/8]*Sqrt[-Sqrt[-1 + Sec
[x]] + Sqrt[1 + Sec[x]]] + Sqrt[2]*(-Sqrt[-1 + Sec[x]] + Sqrt[1 + Sec[x]])]
*Sin[Pi/8])*Sin[x])/(-1 + Cos[2*x] + 2*Cos[x]*Sqrt[-1 + Sec[x]]*Sqrt[1 + Se
c[x]])
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{-\sqrt{-1+\sec \left(x \right)}+\sqrt{1+\sec \left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2),x)
```

```
[Out] int((-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(sqrt(sec(x) + 1) - sqrt(sec(x) - 1)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\sqrt{\sec\left(x\right)-1}+\sqrt{\sec\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+sec(x))**(1/2)+(1+sec(x))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(sec(x) - 1) + sqrt(sec(x) + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+sec(x))^(1/2)+(1+sec(x))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(sec(x) + 1) - sqrt(sec(x) - 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{\sqrt{\frac{1}{\cos\left(x\right)}+1}-\sqrt{\frac{1}{\cos\left(x\right)}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(x) + 1)^(1/2) - (1/cos(x) - 1)^(1/2))^(1/2),x)
```

```
[Out] int(((1/cos(x) + 1)^(1/2) - (1/cos(x) - 1)^(1/2))^(1/2), x)
```

3.46 $\int x\tan^{-1}(x)^2\log(1+x^2)\,dx$

Optimal. Leaf size=77

$$3x\tan^{-1}(x)-\frac{3}{2}\tan^{-1}(x)^2-\frac{1}{2}x^2\tan^{-1}(x)^2-\frac{3}{2}\log(1+x^2)-x\tan^{-1}(x)\log(1+x^2)+\frac{1}{2}(1+x^2)\tan^{-1}(x)^2\log$$

```
[Out] 3*x*arctan(x)-3/2*arctan(x)^2-1/2*x^2*arctan(x)^2-3/2*ln(x^2+1)-x*arctan(x)
*ln(x^2+1)+1/2*(x^2+1)*arctan(x)^2*ln(x^2+1)+1/4*ln(x^2+1)^2
```

Rubi [A]
time = 0.16, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4946, 5036, 4930, 266, 5004, 5143, 5129, 2525, 2437, 2338}

$$-\frac{1}{2}x^2\text{ArcTan}(x)^2+\frac{1}{2}(x^2+1)\text{ArcTan}(x)^2\log(x^2+1)-x\text{ArcTan}(x)\log(x^2+1)-\frac{3\text{ArcTan}(x)^2}{2}+3x\text{ArcTan}(x)+\frac{1}{4}\log^2(x^2+1)-\frac{3}{2}\log(x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x]^2*Log[1 + x^2],x]
```

```
[Out] 3*x*ArcTan[x] - (3*ArcTan[x]^2)/2 - (x^2*ArcTan[x]^2)/2 - (3*Log[1 + x^2])/
2 - x*ArcTan[x]*Log[1 + x^2] + ((1 + x^2)*ArcTan[x]^2*Log[1 + x^2])/2 + Log
[1 + x^2]^2/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
```

```
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5129

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Dist[b*c, Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Dist[2*
e*g, Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

Rule 5143

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2*((d_.) + Log[(f_) + (g_.)*(x_)^2]*
(e_.))*(x_), x_Symbol] :> Simp[(f + g*x^2)*(d + e*Log[f + g*x^2])*((a + b*A
rcTan[c*x])^2/(2*g)), x] + (-Dist[b/c, Int[(d + e*Log[f + g*x^2])*(a + b*Ar
cTan[c*x]), x], x] + Dist[b*c*e, Int[x^2*((a + b*ArcTan[c*x])/(1 + c^2*x^2)
), x], x] - Simp[e*x^2*((a + b*ArcTan[c*x])^2/2), x]) /; FreeQ[{a, b, c, d,
```

```
e, f, g}, x] && EqQ[g, c^2*f]
```

Rubi steps

$$\int x\tan^{-1}(x)^2\log\left(1+x^2\right)\,dx = -\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)^2\log\left(1+x^2\right)+\int\frac{x^2\tan^{-1}(x)}{1+x^2}\,dx -$$
$$= -\frac{1}{2}x^2\tan^{-1}(x)^2-x\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{2}\left(1+x^2\right)\tan^{-1}(x)^2\log\left(1+x\right.$$
$$= x\tan^{-1}(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{1}{2}x^2\tan^{-1}(x)^2-x\tan^{-1}(x)\log\left(1+x^2\right)+\frac{1}{2}(1$$
$$= 3x\tan^{-1}(x)-\frac{3}{2}\tan^{-1}(x)^2-\frac{1}{2}x^2\tan^{-1}(x)^2-\frac{1}{2}\log\left(1+x^2\right)-x\tan^{-1}(x)\mathrm{l}$$
$$= 3x\tan^{-1}(x)-\frac{3}{2}\tan^{-1}(x)^2-\frac{1}{2}x^2\tan^{-1}(x)^2-\frac{3}{2}\log\left(1+x^2\right)-x\tan^{-1}(x)\mathrm{l}$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 0.75

$$\frac{1}{4}\left(-4x\tan^{-1}(x)\left(-3+\log\left(1+x^2\right)\right)+\left(-6+\log\left(1+x^2\right)\right)\log\left(1+x^2\right)+2\tan^{-1}(x)^2\left(-3-x^2+\left(1+x^2\right)\log\left(1+x^2\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x]^2*Log[1 + x^2],x]
```

```
[Out] (-4*x*ArcTan[x]*(-3 + Log[1 + x^2]) + (-6 + Log[1 + x^2])*Log[1 + x^2] + 2*
ArcTan[x]^2*(-3 - x^2 + (1 + x^2)*Log[1 + x^2]))/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.35, size = 1134, normalized size = 14.73

method	result	size
default	Expression too large to display	1134
risch	Expression too large to display	113915

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)^2*ln(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1+I*x)^2/(x^2+1)+1)^2+I*csgn(I/(x^2+1)^(1/2))^2*csgn(I*((1+I*x)^2/(x^2+
1)+1))*ln((1+I*x)^2/(x^2+1)+1)*Pi-1/2*arctan(x)^2-1/2*x^2*arctan(x)^2+3*x*a
rctan(x)+I*csgn(I/(x^2+1)^(1/2))^2*csgn(I/(1+I*x)*(x^2+1)^(1/2))*ln((1+I*x)
^2/(x^2+1)+1)*Pi+3*ln((1+I*x)^2/(x^2+1)+1)-arctan(x)^2*ln((1+I*x)^2/(x^2+1)
+1)-3*I*arctan(x)+ln(2)*arctan(x)^2-arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))*csgn
```

```
(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))-1/2*I*arctan(x)^2*P
i*csgn(I/(x^2+1)^(1/2))^2*csgn(I*((1+I*x)^2/(x^2+1)+1))-1/2*I*arctan(x)^2*P
i*csgn(I/(x^2+1)^(1/2))^2*csgn(I/(1+I*x)*(x^2+1)^(1/2))+1/2*I*arctan(x)^2*P
i*csgn(I/(x^2+1)^(1/2))^3*x^2-I*arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))^3*x-I*cs
gn(I/(x^2+1)^(1/2))^3*ln((1+I*x)^2/(x^2+1)+1)*Pi+1/2*I*arctan(x)^2*Pi*csgn(
I/(x^2+1)^(1/2))^3+arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))^2*csgn(I*((1+I*x)^2/(
x^2+1)+1))+arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))^2*csgn(I/(1+I*x)*(x^2+1)^(1/2
))+2*I*ln(2)*arctan(x)-arctan(x)^2*ln((1+I*x)^2/(x^2+1)+1)*x^2-arctan(x)*Pi
*csgn(I/(x^2+1)^(1/2))^3-2*ln(2)*arctan(x)*x+ln(2)*arctan(x)^2*x^2+2*arctan
(x)*ln((1+I*x)^2/(x^2+1)+1)*x+(x^2*arctan(x)^2+2*I*arctan(x)+arctan(x)^2-2*
x*arctan(x)-2*ln((1+I*x)^2/(x^2+1)+1))*ln((1+I*x)/(x^2+1)^(1/2))+1/2*I*arct
an(x)^2*Pi*csgn(I/(x^2+1)^(1/2))*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*
x)*(x^2+1)^(1/2))*x^2-I*arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))*csgn(I*((1+I*x)^
2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))*x-2*ln((1+I*x)^2/(x^2+1)+1)*ln(
2)-I*Pi*ln((1+I*x)^2/(x^2+1)+1)*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x
)*(x^2+1)^(1/2))*csgn(I/(x^2+1)^(1/2))+1/2*I*arctan(x)^2*Pi*csgn(I/(x^2+1)^
(1/2))*csgn(I*((1+I*x)^2/(x^2+1)+1))*csgn(I/(1+I*x)*(x^2+1)^(1/2))-1/2*I*ar
ctan(x)^2*Pi*csgn(I/(x^2+1)^(1/2))^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*x^2-1/2*
I*arctan(x)^2*Pi*csgn(I/(x^2+1)^(1/2))^2*csgn(I/(1+I*x)*(x^2+1)^(1/2))*x^2+
I*arctan(x)*Pi*csgn(I/(x^2+1)^(1/2))^2*csgn(I*((1+I*x)^2/(x^2+1)+1))*x+I*ar
ctan(x)*Pi*csgn(I/(x^2+1)^(1/2))^2*csgn(I/(1+I*x)*(x^2+1)^(1/2))*x
```

Maxima [A]
time = 3.14, size = 67, normalized size = 0.87

$$-\frac{1}{2}\left(x^{2}-\left(x^{2}+1\right)\log\left(x^{2}+1\right)+1\right)\arctan\left(x\right)^{2}-\left(x\log\left(x^{2}+1\right)-3\,x+2\,\arctan\left(x\right)\right)\arctan\left(x\right)+\arctan\left(x\right)^{2}+\frac{1}{4}\,\log\left(x^{2}+1\right)^{2}-\frac{3}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2*log(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - (x^2 + 1)*log(x^2 + 1) + 1)*arctan(x)^2 - (x*log(x^2 + 1) - 3*x
 + 2*arctan(x))*arctan(x) + arctan(x)^2 + 1/4*log(x^2 + 1)^2 - 3/2*log(x^2
+ 1)
```

Fricas [A]
time = 0.97, size = 52, normalized size = 0.68

$$-\frac{1}{2}\left(x^{2}+3\right)\arctan\left(x\right)^{2}+3\,x\arctan\left(x\right)+\frac{1}{2}\left(\left(x^{2}+1\right)\arctan\left(x\right)^{2}-2\,x\arctan\left(x\right)-3\right)\log\left(x^{2}+1\right)+\frac{1}{4}\,\log\left(x^{2}+1\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2*log(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + 3)*arctan(x)^2 + 3*x*arctan(x) + 1/2*((x^2 + 1)*arctan(x)^2 - 2
*x*arctan(x) - 3)*log(x^2 + 1) + 1/4*log(x^2 + 1)^2
```

Sympy [A]
time = 0.48, size = 87, normalized size = 1.13

$$\frac{x^2\log(x^2+1)\operatorname{atan}^2(x)}{2}-\frac{x^2\operatorname{atan}^2(x)}{2}-x\log(x^2+1)\operatorname{atan}(x)+3x\operatorname{atan}(x)+\frac{\log(x^2+1)^2}{4}+\frac{\log(x^2+1)\operatorname{atan}^2(x)}{2}-\frac{3\log(x^2+1)}{2}-\frac{3\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)**2*ln(x**2+1),x)
```

```
[Out] x**2*log(x**2 + 1)*atan(x)**2/2 - x**2*atan(x)**2/2 - x*log(x**2 + 1)*atan(
x) + 3*x*atan(x) + log(x**2 + 1)**2/4 + log(x**2 + 1)*atan(x)**2/2 - 3*log(
x**2 + 1)/2 - 3*atan(x)**2/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2*log(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x*arctan(x)^2*log(x^2 + 1), x)
```

Mupad [B]
time = 0.26, size = 78, normalized size = 1.01

$$\frac{\ln(x^2+1)^2}{4}-\frac{3\ln(x^2+1)}{2}-\frac{3\operatorname{atan}(x)^2}{2}+\frac{\ln(x^2+1)\operatorname{atan}(x)^2}{2}+x\left(3\operatorname{atan}(x)-\ln(x^2+1)\operatorname{atan}(x)\right)-x^2\left(\frac{\operatorname{atan}(x)^2}{2}-\frac{\ln(x^2+1)\operatorname{atan}(x)^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x^2 + 1)*atan(x)^2,x)
```

```
[Out] log(x^2 + 1)^2/4 - (3*log(x^2 + 1))/2 - (3*atan(x)^2)/2 + (log(x^2 + 1)*ata
n(x)^2)/2 + x*(3*atan(x) - log(x^2 + 1)*atan(x)) - x^2*(atan(x)^2/2 - (log(
x^2 + 1)*atan(x)^2)/2)
```

3.47 $\int \tan^{-1}\left(x\sqrt{1+x^2}\right) dx$

Optimal. Leaf size=120

$$x\tan^{-1}\left(x\sqrt{1+x^2}\right)+\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt{1+x^2}\right)-\frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt{1+x^2}\right)-\frac{1}{4}\sqrt{3}\log\left(2+x^2-\sqrt{3}\right.$$

```
[Out] -1/2*arctan(-3^(1/2)+2*(x^2+1)^(1/2))+x*arctan(x*(x^2+1)^(1/2))-1/2*arctan(
3^(1/2)+2*(x^2+1)^(1/2))-1/4*ln(2+x^2-3^(1/2)*(x^2+1)^(1/2))*3^(1/2)+1/4*ln
(2+x^2+3^(1/2)*(x^2+1)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5311, 1699, 840, 1183, 648, 632, 210, 642}

$$x\text{ArcTan}\left(x\sqrt{x^2+1}\right)+\frac{1}{2}\text{ArcTan}\left(\sqrt{3}-2\sqrt{x^2+1}\right)-\frac{1}{2}\text{ArcTan}\left(2\sqrt{x^2+1}+\sqrt{3}\right)-\frac{1}{4}\sqrt{3}\log\left(x^2-\sqrt{3}\sqrt{x^2+1}+2\right)+\frac{1}{4}\sqrt{3}\log\left(x^2+\sqrt{3}\sqrt{x^2+1}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x*Sqrt[1 + x^2]],x]
```

```
[Out] x*ArcTan[x*Sqrt[1 + x^2]] + ArcTan[Sqrt[3] - 2*Sqrt[1 + x^2]]/2 - ArcTan[Sq
rt[3] + 2*Sqrt[1 + x^2]]/2 - (Sqrt[3]*Log[2 + x^2 - Sqrt[3]*Sqrt[1 + x^2]])
/4 + (Sqrt[3]*Log[2 + x^2 + Sqrt[3]*Sqrt[1 + x^2]])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1699

```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)
^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
&& PolyQ[Px, x^2]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \tan^{-1}\left(x\sqrt{1+x^2}\right) dx = x\tan^{-1}\left(x\sqrt{1+x^2}\right) - \int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)} dx$$
$$= x\tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{\sqrt{1+x}\,(1+x+x^2)} dx, x, x^2\right)$$
$$= x\tan^{-1}\left(x\sqrt{1+x^2}\right) - \text{Subst}\left(\int \frac{-1+2x^2}{1-x^2+x^4} dx, x, \sqrt{1+x^2}\right)$$
$$= x\tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt{3}+3x}{1-\sqrt{3}\,x+x^2} dx, x, \sqrt{1+x^2}\right)}{2\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{-}{1+}\right.}{}$$
$$= x\tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2} dx, x, \sqrt{1+x^2}\right) - \frac{1}{4}\text{Subs}$$
$$= x\tan^{-1}\left(x\sqrt{1+x^2}\right) - \frac{1}{4}\sqrt{3}\log\left(2+x^2-\sqrt{3}\sqrt{1+x^2}\right) + \frac{1}{4}\sqrt{3}\log\left(2+\right.$$
$$= x\tan^{-1}\left(x\sqrt{1+x^2}\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt{1+x^2}\right) - \frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt{1-\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 95, normalized size = 0.79

$$-\frac{1}{2}\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{1+x^2}\right) - \frac{1}{2}\left(1+i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\sqrt{1+x^2}\right) + x\tan^{-1}\left(x\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x*Sqrt[1 + x^2]],x]
```

```
[Out] -1/2*((1 - I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*Sqrt[1 + x^2])/2]) - ((1 + I*
Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*Sqrt[1 + x^2])/2])/2 + x*ArcTan[x*Sqrt[1 +
 x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(92) = 184$.
time = 0.06, size = 508, normalized size = 4.23

method	result
default	$x\arctan\left(x\sqrt{x^2+1}\right) + \frac{\sqrt{2}\,\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\,\sqrt{3}\,\text{arctanh}\left(\frac{\sqrt{\frac{2(1+x)^2}{(1-x)^2}+2}\,\sqrt{3}}{2}\right)}{3\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+1}{\left(\frac{1+x}{1-x}+1\right)^2}}\,\left(\frac{1+x}{1-x}+1\right)} + \frac{\sqrt{2}\,\sqrt{\frac{2(-1+x)^2}{(-1-x)^2}+}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctan(x*(x^2+1)^(1/2))+1/3*2^(1/2)/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^
2)^(1/2)/((1+x)/(1-x)+1)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*3^(1/2)*arctanh(1/2*(2
*(1+x)^2/(1-x)^2+2)^(1/2)*3^(1/2))+1/3*2^(1/2)/(((-1+x)^2/(-1-x)^2+1)/((-1+
x)/(-1-x)+1)^2)^(1/2)/((-1+x)/(-1-x)+1)*(2*(-1+x)^2/(-1-x)^2+2)^(1/2)*3^(1/
2)*arctanh(1/2*(2*(-1+x)^2/(-1-x)^2+2)^(1/2)*3^(1/2))-1/12*2^(1/2)*(2*(1+x)
^2/(1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(1+x)^2/(1-x)^2+2)^(1/2)*3^(1/2
))-3*arctan(1/((1+x)^2/(1-x)^2+1)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(1+x)/(1-x)))
/(((1+x)^2/(1-x)^2+1)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/12*2^(1/2)
*(2*(-1+x)^2/(-1-x)^2+2)^(1/2)*(3^(1/2)*arctanh(1/2*(2*(-1+x)^2/(-1-x)^2+2)
^(1/2)*3^(1/2))-3*arctan(1/((-1+x)^2/(-1-x)^2+1)*(2*(-1+x)^2/(-1-x)^2+2)^(1
/2)*(-1+x)/(-1-x)))/(((-1+x)^2/(-1-x)^2+1)/((-1+x)/(-1-x)+1)^2)^(1/2)/((-1+
x)/(-1-x)+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(x^2 + 1)*x) - integrate((2*x^3 + x)*sqrt(x^2 + 1)/((x^4 + x^2
)*(x^2 + 1) + x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(92) = 184$.
time = 0.93, size = 287, normalized size = 2.39

$$x \arctan\left(\sqrt{x^2+1}\,x\right) - \frac{1}{4}\sqrt{3}\,\log\left(32\,x^4 + 80\,x^2 + 32\,\sqrt{3}\,(x^3+x) - 16\left(2\,x^3 + \sqrt{3}\,(2\,x^2+1) + 4\,x\right)\sqrt{x^2+1} + 32\right) + \frac{1}{4}\sqrt{3}\,\log\left(32\,x^4 + 80\,x^2 - 32\,\sqrt{3}\,(x^3+x) - 16\left(2\,x^3 - \sqrt{3}\,(2\,x^2+1) + 4\,x\right)\sqrt{x^2+1} + 32\right) + \arctan\left(2\,\sqrt{2\,x^4 + 5\,x^2 + 2\,\sqrt{3}\,(x^3+x) - \left(2\,x^3 + \sqrt{3}\,(2\,x^2+1) + 4\,x\right)\sqrt{x^2+1} + 2}\,\left(x + \sqrt{x^2+1}\right) + \sqrt{3} - 2\,\sqrt{x^2+1}\right) + \arctan\left(2\,\sqrt{2\,x^4 + 5\,x^2 - 2\,\sqrt{3}\,(x^3+x) - \left(2\,x^3 - \sqrt{3}\,(2\,x^2+1) + 4\,x\right)\sqrt{x^2+1} + 2}\,\left(x + \sqrt{x^2+1}\right) - \sqrt{3} - 2\,\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*arctan(sqrt(x^2 + 1)*x) - 1/4*sqrt(3)*log(32*x^4 + 80*x^2 + 32*sqrt(3)*(x
^3 + x) - 16*(2*x^3 + sqrt(3)*(2*x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 32) + 1/4*
sqrt(3)*log(32*x^4 + 80*x^2 - 32*sqrt(3)*(x^3 + x) - 16*(2*x^3 - sqrt(3)*(2
*x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 32) + arctan(2*sqrt(2*x^4 + 5*x^2 + 2*sqrt
(3)*(x^3 + x) - (2*x^3 + sqrt(3)*(2*x^2 + 1) + 4*x)*sqrt(x^2 + 1) + 2)*(x +
 sqrt(x^2 + 1)) + sqrt(3) - 2*sqrt(x^2 + 1)) + arctan(2*sqrt(2*x^4 + 5*x^2
- 2*sqrt(3)*(x^3 + x) - (2*x^3 - sqrt(3)*(2*x^2 + 1) + 4*x)*sqrt(x^2 + 1) +
 2)*(x + sqrt(x^2 + 1)) - sqrt(3) - 2*sqrt(x^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}\left(x\sqrt{x^2+1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(x**2+1)**(1/2)),x)
```

```
[Out] Integral(atan(x*sqrt(x**2 + 1)), x)
```

Giac [A]
time = 0.68, size = 92, normalized size = 0.77

$$x\arctan\left(\sqrt{x^2+1}\,x\right)+\frac{1}{4}\sqrt{3}\,\log\left(x^2+\sqrt{3}\,\sqrt{x^2+1}+2\right)-\frac{1}{4}\sqrt{3}\,\log\left(x^2-\sqrt{3}\,\sqrt{x^2+1}+2\right)-\frac{1}{2}\arctan\left(\sqrt{3}+2\sqrt{x^2+1}\right)-\frac{1}{2}\arctan\left(-\sqrt{3}+2\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(sqrt(x^2 + 1)*x) + 1/4*sqrt(3)*log(x^2 + sqrt(3)*sqrt(x^2 + 1) + 2
) - 1/4*sqrt(3)*log(x^2 - sqrt(3)*sqrt(x^2 + 1) + 2) - 1/2*arctan(sqrt(3) +
 2*sqrt(x^2 + 1)) - 1/2*arctan(-sqrt(3) + 2*sqrt(x^2 + 1))
```

Mupad [B]
time = 1.09, size = 413, normalized size = 3.44

$$x\,\mathrm{atan}\left(x\sqrt{x^2+1}\right)-\frac{\left(\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(\frac{x}{2}+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}+1+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\,\left(4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\,\left(4\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\left(\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(\frac{x}{2}+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}+1-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\,\left(4\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-1+\sqrt{3}\,1\mathrm{i}\right)}-\frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\,\left(4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+1+\sqrt{3}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x*(x^2 + 1)^(1/2)),x)
```

```
[Out] x*atan(x*(x^2 + 1)^(1/2)) - ((log(x - (3^(1/2)*1i)/2 - 1/2) - log(x/2 + (3^
(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) + (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 +
2*((3^(1/2)*1i)/2 + 1/2)^3 + 1/2))/((((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)*(3
^(1/2)*1i + 4*((3^(1/2)*1i)/2 + 1/2)^3 + 1)) - ((log(x - (3^(1/2)*1i)/2 + 1
/2) - log((3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1))
*((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2 - 1/2)^3 - 1/2))/((((3^(1/2)*1i)/2 - 1
/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1i)/2 - 1/2)^3 - 1)) - ((log(x +
 (3^(1/2)*1i)/2 - 1/2) - log(x/2 + (3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - (3^
(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2 - 1/2)^3 - 1/2))/((
((3^(1/2)*1i)/2 - 1/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1i)/2 - 1/2)^
3 - 1)) - ((log(x + (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1
)^(1/2) - x/2 - (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2
+ 1/2)^3 + 1/2))/((((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^
(1/2)*1i)/2 + 1/2)^3 + 1))
```

3.48 $\int -\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{x}}{2}-(1+x)\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)$$

[Out] -(1+x)*arctan(x^(1/2)-(1+x)^(1/2))+1/2*x^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {5267, 8, 4930, 52, 65, 209}

$$-\frac{1}{2}x\text{ArcTan}\left(\sqrt{x}\right)-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{\pi x}{4}+\frac{\sqrt{x}}{2}$$

Warning: Unable to verify antiderivative.

[In] Int[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]

[Out] Sqrt[x]/2 + (Pi*x)/4 - ArcTan[Sqrt[x]]/2 - (x*ArcTan[Sqrt[x]])/2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 5267

```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Dist[Pi*(s/4), Int[u,
 x], x] + Dist[1/2, Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2 +
1]
```

Rubi steps

$$\begin{aligned}
\int -\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)\,dx &= -\left(\frac{1}{2}\int \tan^{-1}\left(\sqrt{x}\right)\,dx\right)+\frac{1}{4}\pi\int 1\,dx\\
&= \frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int \frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}x\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 31, normalized size = 1.00

$$\frac{\sqrt{x}}{2}-(1+x)\tan^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
[Out] Sqrt[x]/2 - (1 + x)*ArcTan[Sqrt[x] - Sqrt[1 + x]]
```

Maple [**A**]
time = 0.01, size = 28, normalized size = 0.90

method	result	size
default	$-x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\sqrt{x}}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -x*arctan(x^(1/2)-(1+x)^(1/2))+1/2*x^(1/2)-1/2*arctan(x^(1/2))
```

Maxima [A]
time = 3.10, size = 26, normalized size = 0.84

$$x\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Fricas [A]
time = 1.47, size = 22, normalized size = 0.71

$$(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x)
```

Sympy [A]
time = 42.57, size = 29, normalized size = 0.94

$$\frac{\sqrt{x}}{2}-x\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(x**(1/2)-(1+x)**(1/2)),x)
[Out] sqrt(x)/2 - x*atan(sqrt(x) - sqrt(x + 1)) - atan(sqrt(x))/2
```

Giac [A]
time = 0.58, size = 27, normalized size = 0.87

$$-x\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -x*arctan(-sqrt(x + 1) + sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Mupad [B]
time = 0.80, size = 40, normalized size = 1.29

$$x\,\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan((x + 1)^(1/2) - x^(1/2)),x)
```

```
[Out] x*atan((x + 1)^(1/2) - x^(1/2)) - (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 +
x^(1/2)/2
```

3.49 $\int \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) dx$

Optimal. Leaf size=29

$$x\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\tan^{-1}\left(\sqrt{1-2x^2}\right)$$

```
[Out] x*arcsin(x/(-x^2+1)^(1/2))+arctan((-2*x^2+1)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4924, 455, 65, 209}

$$x\text{ArcSin}\left(\frac{x}{\sqrt{1-x^2}}\right)+\text{ArcTan}\left(\sqrt{1-2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x/Sqrt[1 - x^2]],x]
```

```
[Out] x*ArcSin[x/Sqrt[1 - x^2]] + ArcTan[Sqrt[1 - 2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 4924

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)dx &= x\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\int\frac{x}{\sqrt{1-2x^2}\,(1-x^2)}\,dx\\ &= x\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{\sqrt{1-2x}\,(1-x)}\,dx,x,x^2\right)\\ &= x\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{\frac{1}{2}+\frac{x^2}{2}}\,dx,x,\sqrt{1-2x^2}\right)\\ &= x\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\tan^{-1}\left(\sqrt{1-2x^2}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 29, normalized size = 1.00

$$x\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\tan^{-1}\left(\sqrt{1-2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x/Sqrt[1 - x^2]],x]
[Out] x*ArcSin[x/Sqrt[1 - x^2]] + ArcTan[Sqrt[1 - 2*x^2]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(25) = 50$.
time = 0.07, size = 138, normalized size = 4.76

method	result
default	$x\arcsin\left(\frac{x}{\sqrt{-x^2+1}}\right)+\frac{\sqrt{\frac{2x^2-1}{x^2-1}}\left(\sqrt{-2x^2+1}+\arctan\left(\frac{2x-1}{\sqrt{-2x^2+1}}\right)-\arctan\left(\frac{1+2x}{\sqrt{-2x^2+1}}\right)\right)\sqrt{}}{\sqrt{-2x^2+1}\,\left(2+\sqrt{2}\right)\left(-2+\sqrt{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x/(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*arcsin(x/(-x^2+1)^(1/2))+((2*x^2-1)/(x^2-1))^(1/2)*((-2*x^2+1)^(1/2)+arct
an((2*x-1)/(-2*x^2+1)^(1/2))-arctan((1+2*x)/(-2*x^2+1)^(1/2)))*(-x^2+1)^(1/
```

```
2)/(-2*x^2+1)^(1/2)/(2+2^(1/2))/(-2+2^(1/2))+1/2*((2*x^2-1)/(x^2-1))^(1/2)*
(-x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/(-x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(arcsin(x/sqrt(-x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(25) = 50$.
time = 1.20, size = 60, normalized size = 2.07

$$-x\arcsin\left(\frac{\sqrt{-x^2+1}\,x}{x^2-1}\right)+\arctan\left(\frac{x^2+\sqrt{-x^2+1}\sqrt{\frac{2\,x^2-1}{x^2-1}}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/(-x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -x*arcsin(sqrt(-x^2 + 1)*x/(x^2 - 1)) + arctan((x^2 + sqrt(-x^2 + 1)*sqrt((
2*x^2 - 1)/(x^2 - 1)) - 1)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{asin}\left(\frac{x}{\sqrt{1-x^2}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x/(-x**2+1)**(1/2)),x)
[Out] Integral(asin(x/sqrt(1 - x**2)), x)
```

Giac [A]
time = 0.61, size = 34, normalized size = 1.17

$$x\arcsin\left(\frac{x}{\sqrt{-x^2+1}}\right)+\frac{\arctan\left(\sqrt{-2\,x^2+1}\right)}{\operatorname{sgn}\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*arcsin(x/sqrt(-x^2 + 1)) + arctan(sqrt(-2*x^2 + 1))/sgn(x^2 - 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \operatorname{asin}\left(\frac{x}{\sqrt{1-x^2}}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x/(1 - x^2)^(1/2)),x)
```

```
[Out] int(asin(x/(1 - x^2)^(1/2)), x)
```

3.50 $\int \tan^{-1}\left(x\sqrt{1-x^2}\right)\,dx$

Optimal. Leaf size=106

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{1-x^2}\right)+x\tan^{-1}\left(x\sqrt{1-x^2}\right)+\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\right.$$

```
[Out] x*arctan(x*(-x^2+1)^(1/2))+1/2*arctanh(1/2*(-x^2+1)^(1/2)*(-2+2*5^(1/2))^(1
/2))*(-2+2*5^(1/2))^(1/2)-1/2*arctan(1/2*(-x^2+1)^(1/2)*(2+2*5^(1/2))^(1/2)
)*(2+2*5^(1/2))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5311, 1699, 840, 1180, 210, 212}

$$-\sqrt{\frac{2}{\sqrt{5}-1}}\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{1-x^2}\right)+x\text{ArcTan}\left(x\sqrt{1-x^2}\right)+\sqrt{\frac{2}{1+\sqrt{5}}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x*Sqrt[1 - x^2]],x]
```

```
[Out] -(Sqrt[2/(-1 + Sqrt[5])]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 - x^2]]) + x*
ArcTan[x*Sqrt[1 - x^2]] + Sqrt[2/(1 + Sqrt[5])]*ArcTanh[Sqrt[2/(1 + Sqrt[5]
)]*Sqrt[1 - x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1699

```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)
^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
&& PolyQ[Px, x^2]
```

Rule 5311

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(x\sqrt{1-x^2}\right)dx &= x\tan^{-1}\left(x\sqrt{1-x^2}\right) - \int \frac{x(1-2x^2)}{\sqrt{1-x^2}\,(1+x^2-x^4)}\,dx \\
&= x\tan^{-1}\left(x\sqrt{1-x^2}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1-2x}{\sqrt{1-x}\,(1+x-x^2)}\,dx, x, x^2\right) \\
&= x\tan^{-1}\left(x\sqrt{1-x^2}\right) - \text{Subst}\left(\int \frac{1-2x^2}{1+x^2-x^4}\,dx, x, \sqrt{1-x^2}\right) \\
&= x\tan^{-1}\left(x\sqrt{1-x^2}\right) + \text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}-x^2}\,dx, x, \sqrt{1-x^2}\right) + \text{Subst}\Bigg(\\
&= -\sqrt{\frac{2}{-1+\sqrt{5}}}\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\sqrt{1-x^2}\right) + x\tan^{-1}\left(x\sqrt{1-x^2}\right) + \sqrt{}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 106, normalized size = 1.00

$$x\tan^{-1}\left(x\sqrt{1-x^2}\right) - \frac{\sqrt{1+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{1-x^2}\right) - \sqrt{-1+\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{1-x^2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x*Sqrt[1 - x^2]],x]
```

```
[Out] x*ArcTan[x*Sqrt[1 - x^2]] - (Sqrt[1 + Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]
*Sqrt[1 - x^2]] - Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[1
- x^2]])/Sqrt[2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(79) = 158$.
time = 0.05, size = 210, normalized size = 1.98

method	result
default	$x \arctan\left(x\sqrt{-x^2+1}\right)+\frac{\sqrt{5}\ \mathrm{arctanh}\left(\frac{\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+4+2\sqrt{5}}{4\sqrt{2+\sqrt{5}}}\right)}{5\sqrt{2+\sqrt{5}}}+\frac{\sqrt{5}\ \arctan\left(\frac{\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}}{4\sqrt{-2+\sqrt{5}}}\right.}{5\sqrt{-2+\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x*(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*arctan(x*(-x^2+1)^(1/2))+1/5*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(1/4*(2*((-
x^2+1)^(1/2)-1)^2/x^2+4+2*5^(1/2))/(2+5^(1/2))^(1/2))+1/5*5^(1/2)/(-2+5^(1/
2))^(1/2)*arctan(1/4*(2*((-x^2+1)^(1/2)-1)^2/x^2-2*5^(1/2)+4)/(-2+5^(1/2))^
(1/2))+(1/2+3/10*5^(1/2))/(2+5^(1/2))^(1/2)*arctanh(1/4*(2*((-x^2+1)^(1/2)-
1)^2/x^2+4+2*5^(1/2))/(2+5^(1/2))^(1/2))+(-1/2+3/10*5^(1/2))/(-2+5^(1/2))^(
1/2)*arctan(1/4*(2*((-x^2+1)^(1/2)-1)^2/x^2-2*5^(1/2)+4)/(-2+5^(1/2))^(1/2)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(x + 1)*x*sqrt(-x + 1)) - integrate((2*x^3 - x)*e^(1/2*log(x +
 1) + 1/2*log(-x + 1))/(x^2 + (x^4 - x^2)*e^(log(x + 1) + log(-x + 1)) - 1)
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(79) = 158$.
time = 0.79, size = 164, normalized size = 1.55

$$x \arctan\left(\sqrt{-x^2+1}\,x\right)+\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(-\frac{1}{2}\sqrt{2}\sqrt{-x^2+1}\sqrt{\sqrt{5}+1}+\frac{1}{8}\sqrt{2}\sqrt{-16x^2+8\sqrt{5}+8}\sqrt{\sqrt{5}+1}\right)+\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4\sqrt{-x^2+1}\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{\sqrt{5}-1}+4\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*arctan(sqrt(-x^2 + 1)*x) + sqrt(2)*sqrt(sqrt(5) + 1)*arctan(-1/2*sqrt(2)*
sqrt(-x^2 + 1)*sqrt(sqrt(5) + 1) + 1/8*sqrt(2)*sqrt(-16*x^2 + 8*sqrt(5) + 8
)*sqrt(sqrt(5) + 1)) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log((sqrt(5)*sqrt(2) +
 sqrt(2))*sqrt(sqrt(5) - 1) + 4*sqrt(-x^2 + 1)) - 1/4*sqrt(2)*sqrt(sqrt(5)
- 1)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(sqrt(5) - 1) + 4*sqrt(-x^2 + 1))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x*(-x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 111, normalized size = 1.05

$$x\arctan\left(\sqrt{-x^2+1}\,x\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\,\arctan\left(\frac{\sqrt{-x^2+1}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\,\log\left(\sqrt{-x^2+1}+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\,\log\left(\left|\sqrt{-x^2+1}-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x*(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(sqrt(-x^2 + 1)*x) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(sqrt(-x^2 + 1)/
sqrt(1/2*sqrt(5) - 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(sqrt(-x^2 + 1) + sqr
t(1/2*sqrt(5) + 1/2)) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(sqrt(-x^2 + 1) - sq
rt(1/2*sqrt(5) + 1/2)))
```

Mupad [**B**]
time = 1.16, size = 455, normalized size = 4.29

$$x\,\mathrm{atan}\left(x\sqrt{1-x^2}\right)+\frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-1\right)1i}{\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}-\sqrt{1-x^2}\,1i}{x-\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\left(\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-2\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-4\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-1\right)1i}{\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}-\sqrt{1-x^2}\,1i}{x-\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)\left(\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-2\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-4\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}+1\right)1i}{\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}+\sqrt{1-x^2}\,1i}{x+\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}\right)\left(\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-2\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}-4\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}+1\right)1i}{\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}+\sqrt{1-x^2}\,1i}{x+\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}\right)\left(\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-2\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}-4\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x*(1 - x^2)^(1/2)),x)
```

```
[Out] x*atan(x*(1 - x^2)^(1/2)) + (log((((x*(5^(1/2)/2 + 1/2)^(1/2) - 1)*1i)/(1/2
 - 5^(1/2)/2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (5^(1/2)/2 + 1/2)^(1/2)))*((
5^(1/2)/2 + 1/2)^(1/2) - 2*(5^(1/2)/2 + 1/2)^(3/2)))/((2*(5^(1/2)/2 + 1/2)^
(1/2) - 4*(5^(1/2)/2 + 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)) + (log((((x*(1/
```

```
2 - 5^(1/2)/2)^(1/2) - 1)*1i)/(5^(1/2)/2 + 1/2)^(1/2) - (1 - x^2)^(1/2)*1i)
/(x - (1/2 - 5^(1/2)/2)^(1/2)))*((1/2 - 5^(1/2)/2)^(1/2) - 2*(1/2 - 5^(1/2)
/2)^(3/2)))/((2*(1/2 - 5^(1/2)/2)^(1/2) - 4*(1/2 - 5^(1/2)/2)^(3/2))*(5^(1/
2)/2 + 1/2)^(1/2)) + (log((((x*(5^(1/2)/2 + 1/2)^(1/2) + 1)*1i)/(1/2 - 5^(1
/2)/2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (5^(1/2)/2 + 1/2)^(1/2)))*((5^(1/2)
/2 + 1/2)^(1/2) - 2*(5^(1/2)/2 + 1/2)^(3/2)))/((2*(5^(1/2)/2 + 1/2)^(1/2) -
 4*(5^(1/2)/2 + 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)) + (log((((x*(1/2 - 5^(
1/2)/2)^(1/2) + 1)*1i)/(5^(1/2)/2 + 1/2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (
1/2 - 5^(1/2)/2)^(1/2)))*((1/2 - 5^(1/2)/2)^(1/2) - 2*(1/2 - 5^(1/2)/2)^(3/
2)))/((2*(1/2 - 5^(1/2)/2)^(1/2) - 4*(1/2 - 5^(1/2)/2)^(3/2))*(5^(1/2)/2 +
1/2)^(1/2))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
      ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```