Computer algebra independent integration tests

Summer 2022 edition

0-Independent-test-suites/1-Apostol-Problems

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [175]. This is test number [1].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (175)	0.00 (0)
Mathematica	100.00 (175)	0.00 (0)
Fricas	99.43 (174)	0.57 (1)
Maple	98.86 (173)	1.14 (2)
Giac	97.14 (170)	2.86 (5)
Mupad	96.57 (169)	3.43 (6)
Maxima	94.86 (166)	5.14 (9)
Sympy	91.43 (160)	8.57 (15)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

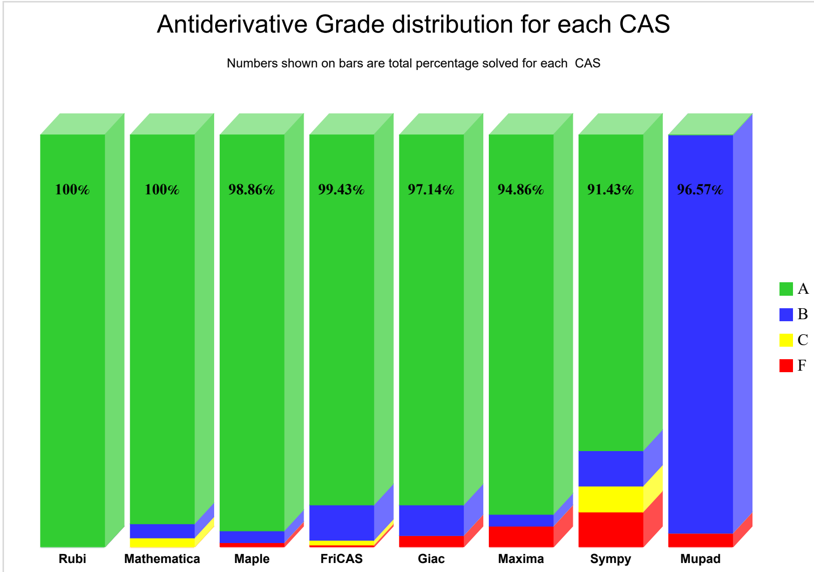
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

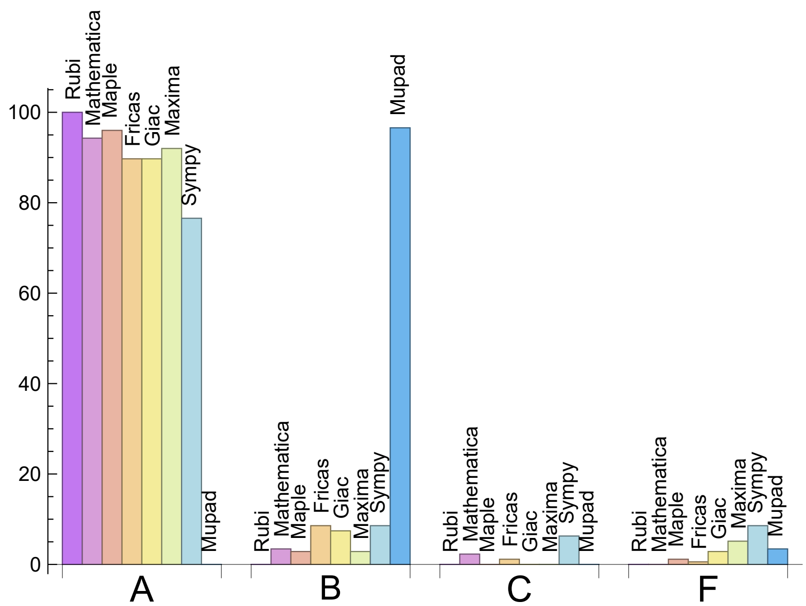
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	96.00	2.86	0.00	1.14
Mathematica	94.29	3.43	2.29	0.00
Maxima	92.00	2.86	0.00	5.14
Fricas	89.71	8.57	1.14	0.57
Giac	89.71	7.43	0.00	2.86
Sympy	76.57	8.57	6.29	8.57
Mupad	N/A	96.57	0.00	3.43

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Fricas	1	100.00 %	0.00 %	0.00 %
Giac	5	100.00 %	0.00 %	0.00 %
Maxima	9	66.67 %	0.00 %	33.33 %
Sympy	15	100.00 %	0.00 %	0.00 %
Mupad	6	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

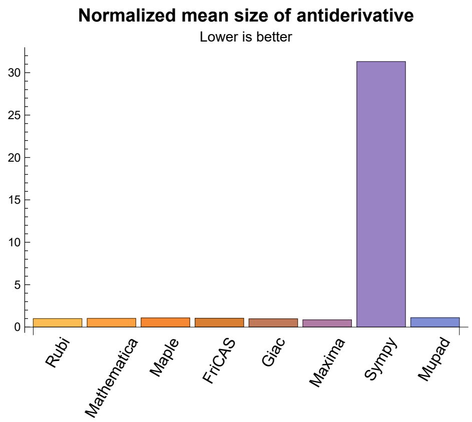
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

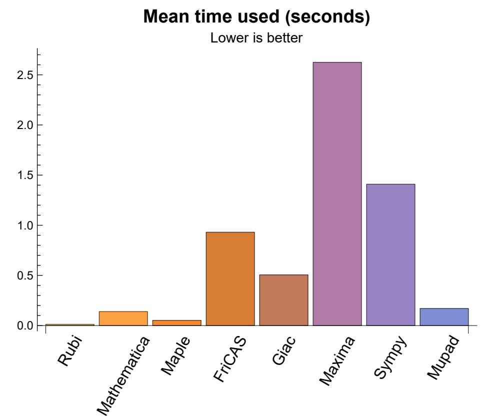
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.01	23.09	1.00	19.00	1.00
Mathematica	0.14	21.68	1.02	18.00	1.00
Maple	0.05	22.65	1.08	16.00	0.90
Maxima	2.62	18.24	0.86	14.50	0.81
Fricas	0.93	21.95	1.04	17.00	0.85
Sympy	1.41	480.19	31.31	17.00	0.86
Giac	0.51	21.03	0.98	15.50	0.83
Mupad	0.17	31.83	1.11	14.00	0.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
25
30
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {98}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

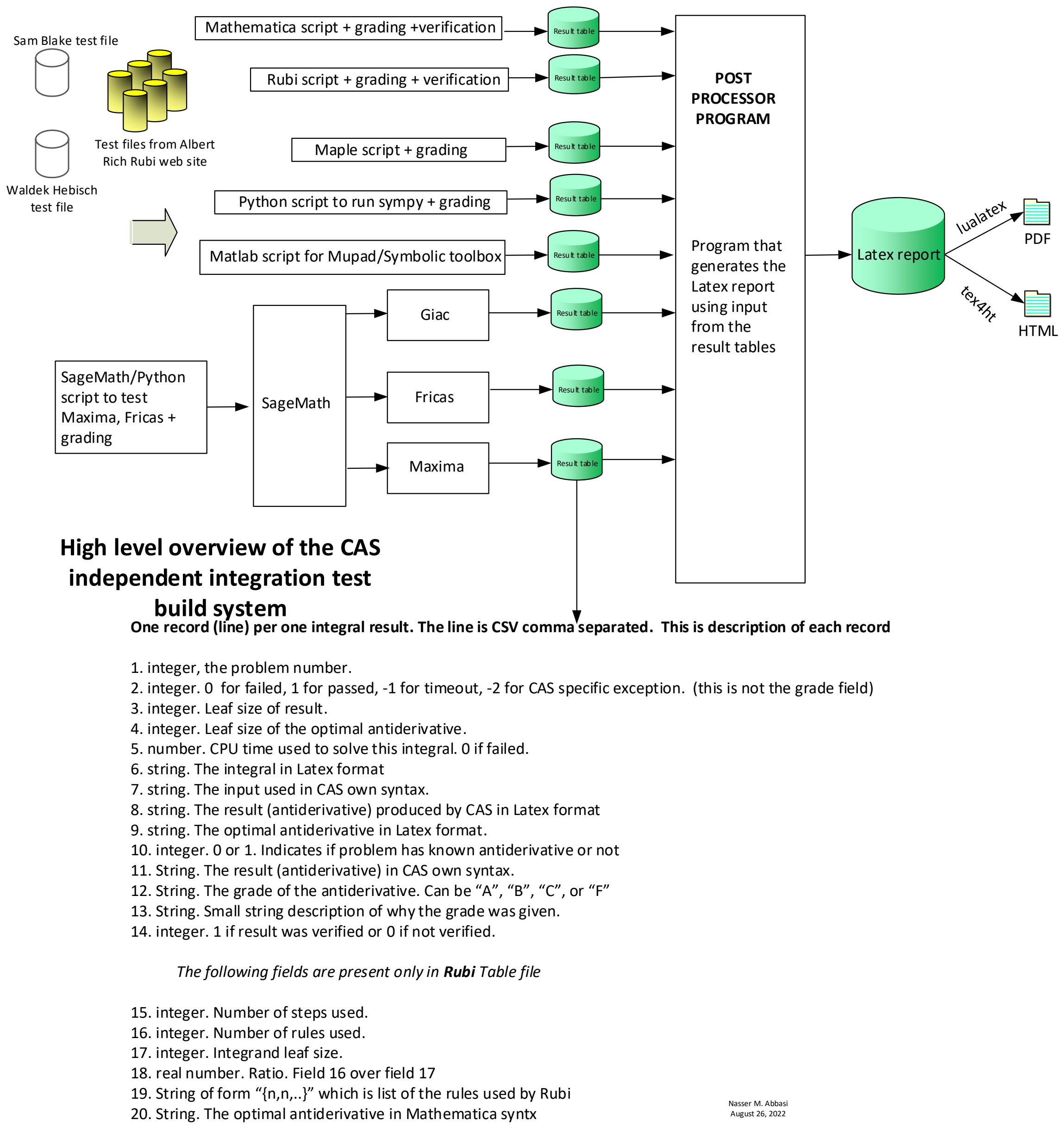
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 50, 51, 83, 84, 88, 154 }

C grade: { 41, 98, 113, 175 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 173, 174, 175 }

B grade: { 51, 114, 139, 158, 170 }

C grade: { }

F grade: { 19, 172 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173 }

B grade: { 51, 83, 84, 113, 169 }

C grade: { }

F grade: { 19, 41, 98, 99, 104, 105, 141, 174, 175 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 16, 44, 45, 48, 50, 51, 61, 84, 88, 113, 114, 124, 131, 145, 146 }

C grade: { 41, 175 }

F grade: { 156 }

2.1.6 Sympy

A grade: { 1, 2, 3, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 43, 44, 45, 46, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 84, 85, 86, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 147, 148, 149, 152, 157, 158, 159, 160, 161, 166, 167, 168, 170, 171, 172, 175 }

B grade: { 9, 17, 42, 47, 48, 50, 51, 62, 90, 101, 114, 141, 144, 145, 146 }

C grade: { 4, 7, 39, 80, 81, 83, 87, 89, 150, 156, 169 }

F grade: { 19, 88, 103, 104, 105, 151, 153, 154, 155, 162, 163, 164, 165, 173, 174 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 173, 174 }

B grade: { 44, 45, 51, 83, 84, 88, 98, 105, 113, 136, 154, 155, 164 }

C grade: { }

F grade: { 41, 62, 156, 172, 175 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175 }

C grade: { }

F grade: { 98, 99, 104, 105, 165, 174 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	0.69
time (sec)	N/A	0.006	0.030	0.098	2.754	0.583	0.007	0.722	0.321

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	20	19	19	39	19	14
N.S.	1	1.00	0.67	0.74	0.70	0.70	1.44	0.70	0.52
time (sec)	N/A	0.004	0.007	0.063	3.710	0.586	0.488	0.682	0.085

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	23	22	22	48	22	19
N.S.	1	1.00	0.62	0.68	0.65	0.65	1.41	0.65	0.56
time (sec)	N/A	0.003	0.050	0.069	4.791	0.606	0.646	0.790	0.069

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	20	19	14	60	19	14
N.S.	1	1.00	0.67	0.74	0.70	0.52	2.22	0.70	0.52
time (sec)	N/A	0.003	0.007	0.071	3.825	0.509	0.469	0.721	0.057

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	22	22	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.57	1.57	0.86	0.86
time (sec)	N/A	0.014	0.018	0.129	2.340	0.589	0.048	0.717	0.065

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.056	0.005	0.089	2.769	0.611	0.010	0.822	0.158

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	16	15	17	92	15	12
N.S.	1	1.00	0.78	0.70	0.65	0.74	4.00	0.65	0.52
time (sec)	N/A	0.003	0.010	0.078	2.648	0.873	0.475	0.810	0.031

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	10	8	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.25	1.00	0.75	0.75
time (sec)	N/A	0.009	0.036	0.019	4.892	0.697	0.027	0.690	0.418

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	18	29	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.12	1.81	0.75	0.75
time (sec)	N/A	0.015	0.015	0.072	4.348	0.703	0.093	0.828	0.171

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.011	0.012	0.023	3.615	0.541	0.164	0.811	0.043

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	6	9
N.S.	1	1.00	1.00	0.92	0.83	1.00	1.00	0.50	0.75
time (sec)	N/A	0.036	0.090	0.051	3.178	0.494	0.243	0.762	0.199

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.018	0.018	0.056	3.236	0.445	0.092	0.746	0.171

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.007	0.014	0.055	4.168	0.473	3.520	0.759	0.207

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.73
time (sec)	N/A	0.002	0.009	0.067	3.777	0.407	0.079	0.868	0.354

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	16	15	15	34	15	12
N.S.	1	1.00	0.78	0.70	0.65	0.65	1.48	0.65	0.52
time (sec)	N/A	0.002	0.009	0.063	11.262	0.588	0.479	0.799	0.034

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	22	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	2.00	0.73	0.82	0.82
time (sec)	N/A	0.001	0.023	0.062	2.584	0.793	0.343	0.699	0.072

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	0.73
time (sec)	N/A	0.002	0.010	0.072	5.546	0.732	0.087	1.078	0.185

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	15
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	1.00
time (sec)	N/A	0.018	0.038	0.076	4.389	3.361	0.098	0.745	0.239

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	23	0	15	47
N.S.	1	1.00	1.00	0.00	0.00	0.72	0.00	0.47	1.47
time (sec)	N/A	0.175	1.689	0.019	0.000	0.669	0.000	0.701	0.589

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.071	0.025	0.061	3.776	0.494	0.105	0.585	0.210

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	10	7	12	15	12	9
N.S.	1	1.00	0.81	0.62	0.44	0.75	0.94	0.75	0.56
time (sec)	N/A	0.005	0.008	0.084	3.983	0.418	0.618	0.609	0.156

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.005	0.002	0.018	3.606	0.465	0.052	0.593	0.064

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	15	15	17	15	15
N.S.	1	1.00	0.88	1.06	0.88	0.88	1.00	0.88	0.88
time (sec)	N/A	0.013	0.013	0.017	5.855	0.458	0.081	0.543	0.033

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	24	20	20	26	20	24
N.S.	1	1.00	0.83	1.04	0.87	0.87	1.13	0.87	1.04
time (sec)	N/A	0.023	0.012	0.022	4.803	0.407	0.128	0.546	0.030

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	25	21	21	26	21	23
N.S.	1	1.00	0.83	1.04	0.88	0.88	1.08	0.88	0.96
time (sec)	N/A	0.023	0.011	0.023	1.920	0.426	0.127	0.537	0.030

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.005	0.001	0.012	4.396	0.451	0.024	0.499	0.019

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	14	17	24	14	18
N.S.	1	1.00	0.78	0.78	0.61	0.74	1.04	0.61	0.78
time (sec)	N/A	0.008	0.002	0.013	5.412	0.519	0.082	0.455	0.095

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.003	0.002	0.032	3.146	0.437	0.009	0.494	0.032

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	11	11	8	11	10
N.S.	1	1.00	1.15	0.85	0.85	0.85	0.62	0.85	0.77
time (sec)	N/A	0.004	0.002	0.000	3.095	0.456	0.010	0.469	0.002

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	16	19	24	16	16
N.S.	1	1.00	0.92	0.75	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.006	0.002	0.082	2.747	2.666	0.009	0.467	0.036

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	17	17	17	17	17
N.S.	1	1.00	1.10	0.81	0.81	0.81	0.81	0.81	0.81
time (sec)	N/A	0.004	0.002	0.101	4.231	1.390	0.012	0.468	0.043

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	24	25	36	22	22
N.S.	1	1.00	0.88	0.71	0.71	0.74	1.06	0.65	0.65
time (sec)	N/A	0.010	0.002	0.083	3.179	4.798	0.009	0.470	0.039

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	19	19	36	19	19
N.S.	1	1.00	1.00	1.00	0.76	0.76	1.44	0.76	0.76
time (sec)	N/A	0.009	0.009	0.018	2.744	1.922	0.085	0.493	0.093

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	23	23	39	23	25
N.S.	1	1.00	0.94	0.70	0.70	0.70	1.18	0.70	0.76
time (sec)	N/A	0.014	0.007	0.038	3.247	1.544	0.131	0.484	0.119

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	26	29	56	26	28
N.S.	1	1.00	0.71	0.90	0.63	0.71	1.37	0.63	0.68
time (sec)	N/A	0.018	0.024	0.043	2.515	1.568	0.130	0.508	0.063

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.004	0.002	0.016	2.257	1.252	0.008	0.504	0.026

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	9	10	8	9	9
N.S.	1	1.00	1.36	1.00	0.82	0.91	0.73	0.82	0.82
time (sec)	N/A	0.004	0.002	0.074	4.279	1.534	0.010	0.530	0.033

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	16	19	24	16	16
N.S.	1	1.00	0.92	0.75	0.67	0.79	1.00	0.67	0.67
time (sec)	N/A	0.008	0.002	0.101	4.496	1.056	0.008	0.461	0.029

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	75	60	60	180	50	37
N.S.	1	1.00	0.73	0.89	0.71	0.71	2.14	0.60	0.44
time (sec)	N/A	0.011	0.071	0.073	3.438	0.701	2.328	0.495	0.215

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	35	34	21	39	28	20
N.S.	1	1.00	0.66	0.92	0.89	0.55	1.03	0.74	0.53
time (sec)	N/A	0.009	0.012	0.074	2.895	0.612	0.229	0.509	0.029

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	34	168	0	17	31	0	301
N.S.	1	1.00	0.20	0.98	0.00	0.10	0.18	0.00	1.75
time (sec)	N/A	0.047	10.034	0.105	0.000	0.311	0.340	0.000	0.075

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	9	6	6	7	6	6
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.003	0.006	0.010	3.168	0.794	0.012	0.497	0.070

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	12	12	19	12	12
N.S.	1	1.00	1.29	1.07	0.86	0.86	1.36	0.86	0.86
time (sec)	N/A	0.005	0.003	0.013	3.183	1.084	0.014	0.444	0.034

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	14	10	20	8	18	8
N.S.	1	1.00	1.00	1.75	1.25	2.50	1.00	2.25	1.00
time (sec)	N/A	0.004	0.005	0.011	2.442	1.213	0.012	0.462	0.057

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	16	16	48	19	34	10
N.S.	1	1.00	1.50	1.33	1.33	4.00	1.58	2.83	0.83
time (sec)	N/A	0.007	0.003	0.028	1.945	1.719	0.015	0.490	0.022

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	20	18	26	18	20
N.S.	1	1.00	1.18	0.95	0.91	0.82	1.18	0.82	0.91
time (sec)	N/A	0.007	0.014	0.023	1.065	2.308	0.057	0.506	0.103

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	22	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.69	0.69	0.69
time (sec)	N/A	0.001	0.001	0.059	2.197	1.652	0.058	0.501	0.025

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	51	58	9	9
N.S.	1	1.00	0.85	0.77	0.69	3.92	4.46	0.69	0.69
time (sec)	N/A	0.001	0.001	0.064	2.561	0.958	0.009	0.454	0.097

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	20	19	19	15	14	14
N.S.	1	1.00	0.89	1.11	1.06	1.06	0.83	0.78	0.78
time (sec)	N/A	0.001	0.003	0.064	1.343	2.029	0.030	0.477	0.042

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	43	10	9	31	31	9	31
N.S.	1	1.00	3.91	0.91	0.82	2.82	2.82	0.82	2.82
time (sec)	N/A	0.001	0.001	0.056	1.296	1.114	0.007	0.436	0.025

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	140	107	106	106	131	106	106
N.S.	1	1.00	2.50	1.91	1.89	1.89	2.34	1.89	1.89
time (sec)	N/A	0.023	0.002	0.066	0.772	0.925	0.017	0.458	0.464

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.006	0.008	0.014	1.740	0.694	0.166	0.456	0.096

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	49	37	37	60	37	41
N.S.	1	1.00	0.74	0.79	0.60	0.60	0.97	0.60	0.66
time (sec)	N/A	0.027	0.023	0.012	0.888	1.157	0.350	0.424	0.226

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.005	0.002	0.018	0.946	0.927	0.079	0.444	0.050

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.038	0.010	0.038	0.830	1.193	0.983	0.443	0.193

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	9	6
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60
time (sec)	N/A	0.001	0.001	0.061	2.921	0.927	0.006	0.520	0.074

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	12	15	15	15	12
N.S.	1	1.00	1.00	1.07	0.80	1.00	1.00	1.00	0.80
time (sec)	N/A	0.002	0.002	0.018	3.278	1.198	0.028	0.444	0.079

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	9
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.53
time (sec)	N/A	0.002	0.001	0.004	2.443	1.111	0.024	0.470	0.032

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	22	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.79	0.79	0.61
time (sec)	N/A	0.006	0.002	0.005	1.793	1.682	0.034	0.458	0.030

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	5	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.25	1.00
time (sec)	N/A	0.000	0.001	0.058	2.998	1.060	0.005	0.452	0.015

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	11	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	3.67	1.00	1.33	1.00
time (sec)	N/A	0.002	0.002	0.012	3.586	1.259	0.027	0.440	0.023

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	36	28	32	61	0	38
N.S.	1	1.00	0.75	1.29	1.00	1.14	2.18	0.00	1.36
time (sec)	N/A	0.007	0.005	0.037	0.294	0.935	0.201	0.000	0.205

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	17	22	26	22	17
N.S.	1	1.00	1.00	0.82	0.61	0.79	0.93	0.79	0.61
time (sec)	N/A	0.011	0.002	0.005	4.004	0.980	0.035	0.486	0.029

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	4	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.33	1.00
time (sec)	N/A	0.008	0.004	0.021	7.557	0.951	0.027	0.436	0.070

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.012	0.003	0.066	4.527	1.083	0.026	0.426	0.346

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	13
N.S.	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.57
time (sec)	N/A	0.026	0.018	0.059	5.802	0.905	2.413	0.477	0.168

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	23	31	37	31	23
N.S.	1	1.00	1.00	0.82	0.59	0.79	0.95	0.79	0.59
time (sec)	N/A	0.019	0.002	0.008	3.240	0.964	0.047	0.464	0.036

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.67
time (sec)	N/A	0.008	0.010	0.010	2.981	0.898	0.023	0.493	0.076

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	12	11	10	11	11
N.S.	1	1.00	1.00	0.86	0.86	0.79	0.71	0.79	0.79
time (sec)	N/A	0.008	0.003	0.019	1.534	1.322	0.047	0.518	0.105

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.70	0.70	0.70
time (sec)	N/A	0.006	0.006	0.026	2.150	1.345	0.107	0.512	0.101

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	11	13	15	11	11
N.S.	1	1.00	0.74	0.74	0.58	0.68	0.79	0.58	0.58
time (sec)	N/A	0.005	0.010	0.021	2.873	1.397	0.084	0.425	0.018

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	14	9	13	15	9	9
N.S.	1	1.00	0.63	0.74	0.47	0.68	0.79	0.47	0.47
time (sec)	N/A	0.004	0.007	0.023	4.390	1.056	0.081	0.500	0.019

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	9	9	7	9	9
N.S.	1	1.00	1.00	1.20	0.90	0.90	0.70	0.90	0.90
time (sec)	N/A	0.004	0.009	0.010	1.473	0.847	0.018	0.471	0.045

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	10	6	6	5	6	6
N.S.	1	1.00	0.64	0.91	0.55	0.55	0.45	0.55	0.55
time (sec)	N/A	0.004	0.005	0.023	2.480	0.901	0.021	0.431	0.018

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	15	9	9	7	9	9
N.S.	1	1.00	0.69	0.94	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.004	0.009	0.010	3.358	0.698	0.022	0.439	0.021

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	17	11	11	10	11	11
N.S.	1	1.00	0.63	0.89	0.58	0.58	0.53	0.58	0.58
time (sec)	N/A	0.009	0.010	0.009	1.425	0.775	0.022	0.452	0.022

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	19	30	16	16	17	16	16
N.S.	1	1.00	0.59	0.94	0.50	0.50	0.53	0.50	0.50
time (sec)	N/A	0.011	0.012	0.013	2.493	0.782	0.023	0.445	0.075

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	11	11	20	11	11
N.S.	1	1.00	0.67	0.71	0.46	0.46	0.83	0.46	0.46
time (sec)	N/A	0.005	0.006	0.005	1.807	0.881	0.061	0.509	0.024

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	21	13	13	12	13	13
N.S.	1	1.00	0.62	0.81	0.50	0.50	0.46	0.50	0.50
time (sec)	N/A	0.013	0.017	0.032	2.141	0.712	0.023	0.643	0.109

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	40	27	31	136	36	27
N.S.	1	1.00	0.68	0.98	0.66	0.76	3.32	0.88	0.66
time (sec)	N/A	0.010	0.023	0.049	2.303	0.735	0.266	0.574	0.032

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	41	29	33	139	38	29
N.S.	1	1.00	0.69	0.98	0.69	0.79	3.31	0.90	0.69
time (sec)	N/A	0.007	0.023	0.022	1.582	0.821	0.273	0.548	0.021

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	21	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.40	0.87
time (sec)	N/A	0.002	0.002	0.022	3.448	0.997	0.062	0.524	0.105

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	64	22	35	33	17	37	21
N.S.	1	1.00	3.37	1.16	1.84	1.74	0.89	1.95	1.11
time (sec)	N/A	0.006	0.050	0.006	4.812	0.806	1.105	0.498	0.609

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	20	35	35	17	37	20
N.S.	1	1.00	3.76	1.18	2.06	2.06	1.00	2.18	1.18
time (sec)	N/A	0.006	0.027	0.003	2.946	0.901	1.119	0.481	0.209

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	23	22	23	22
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.88	0.92	0.88
time (sec)	N/A	0.021	0.006	0.030	3.873	0.941	0.060	0.474	0.032

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	33	39	22	38	20
N.S.	1	1.00	1.00	0.95	1.50	1.77	1.00	1.73	0.91
time (sec)	N/A	0.010	0.003	0.005	2.406	0.910	0.983	0.474	0.021

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	6	23	19	28	14
N.S.	1	1.00	1.00	0.94	0.38	1.44	1.19	1.75	0.88
time (sec)	N/A	0.001	0.003	0.076	3.264	0.992	0.442	0.478	0.163

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	10	11	21	0	27	11
N.S.	1	1.00	2.30	1.00	1.10	2.10	0.00	2.70	1.10
time (sec)	N/A	0.009	0.055	0.117	1.239	0.992	0.000	0.460	0.090

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00
time (sec)	N/A	0.083	0.002	0.074	2.190	0.926	0.036	0.461	0.036

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.009	0.004	0.077	1.573	0.718	0.048	0.427	0.101

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84
time (sec)	N/A	0.009	0.005	0.155	1.430	1.194	0.034	0.434	0.076

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	15	13	15	15	14
N.S.	1	1.00	1.00	0.76	0.71	0.62	0.71	0.71	0.67
time (sec)	N/A	0.005	0.003	0.020	2.099	1.008	0.073	0.464	0.023

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	34	33	24	32	33	24
N.S.	1	1.00	0.75	0.85	0.82	0.60	0.80	0.82	0.60
time (sec)	N/A	0.014	0.009	0.013	1.906	1.061	0.093	0.463	0.029

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	34	25	29	29	29
N.S.	1	1.00	0.74	0.86	0.97	0.71	0.83	0.83	0.83
time (sec)	N/A	0.035	0.007	0.036	1.503	0.818	0.094	0.461	0.124

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	16	14	19	16	16
N.S.	1	1.00	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.004	0.012	0.007	3.529	0.936	0.718	0.519	0.057

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.023	0.006	0.055	1.890	0.958	0.425	0.498	0.661

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.002	0.035	0.072	1.505	0.839	0.063	0.467	0.078

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	37	16	0	15	31	57	-1
N.S.	1	1.00	1.68	0.73	0.00	0.68	1.41	2.59	-0.05
time (sec)	N/A	0.026	0.008	0.071	0.000	0.946	11.595	0.476	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	15	31	22	-1
N.S.	1	1.00	1.00	0.80	0.00	0.75	1.55	1.10	-0.05
time (sec)	N/A	0.014	0.005	0.066	0.000	0.896	11.050	0.456	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89
time (sec)	N/A	0.002	0.006	0.084	1.084	0.992	0.032	0.441	0.028

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	3.75	0.75	0.75
time (sec)	N/A	0.010	0.013	0.015	1.047	1.117	0.037	0.459	0.100

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	19	28	19	21	22
N.S.	1	1.00	1.00	0.93	0.70	1.04	0.70	0.78	0.81
time (sec)	N/A	0.013	0.013	0.032	1.122	1.055	2.173	0.444	0.136

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	67	61	49	38	0	36	49
N.S.	1	1.00	1.60	1.45	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.010	0.086	0.040	0.987	0.840	0.000	0.454	0.070

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	76	68	0	80	0	61	-1
N.S.	1	1.00	1.07	0.96	0.00	1.13	0.00	0.86	-0.01
time (sec)	N/A	0.015	0.099	0.105	0.000	0.946	0.000	0.445	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	55	28	0	43	0	61	-1
N.S.	1	1.00	1.72	0.88	0.00	1.34	0.00	1.91	-0.03
time (sec)	N/A	0.008	0.041	0.079	0.000	0.966	0.000	0.448	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87
time (sec)	N/A	0.003	0.003	0.078	0.980	1.234	0.032	0.454	0.051

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	13	14	15	13
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.74	0.79	0.68
time (sec)	N/A	0.003	0.003	0.072	1.705	1.029	0.034	0.462	0.038

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	19	21	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.91	0.83
time (sec)	N/A	0.018	0.004	0.086	0.964	0.821	0.037	0.439	0.041

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	20	19
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.83
time (sec)	N/A	0.024	0.004	0.015	1.014	0.748	0.049	0.448	0.189

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	19	18	26	19	24	18
N.S.	1	1.00	0.92	0.79	0.75	1.08	0.79	1.00	0.75
time (sec)	N/A	0.011	0.008	0.067	6.607	0.809	0.039	0.475	0.050

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	28	3	29	57
N.S.	1	1.00	1.00	1.04	1.00	1.00	0.11	1.04	2.04
time (sec)	N/A	0.020	0.022	0.081	3.470	0.868	0.049	0.454	0.186

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	61	37	36	51	14	37	53
N.S.	1	1.00	1.24	0.76	0.73	1.04	0.29	0.76	1.08
time (sec)	N/A	0.045	0.023	0.076	3.445	0.644	0.068	0.475	0.101

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	39	38	39	37	21
N.S.	1	1.00	1.14	0.90	1.86	1.81	1.86	1.76	1.00
time (sec)	N/A	0.006	0.016	0.056	10.999	0.973	0.210	0.476	0.326

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	271	14	55	44	14	87
N.S.	1	1.00	1.00	16.94	0.88	3.44	2.75	0.88	5.44
time (sec)	N/A	0.030	0.014	0.070	4.418	0.783	2.105	0.449	0.157

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	9	9	8	11	9
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82
time (sec)	N/A	0.003	0.003	0.065	3.332	1.055	0.024	0.438	0.047

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83
time (sec)	N/A	0.007	0.004	0.073	3.609	1.296	0.045	0.448	0.111

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	21	20	27	22	22	18
N.S.	1	1.00	0.93	0.70	0.67	0.90	0.73	0.73	0.60
time (sec)	N/A	0.012	0.007	0.015	3.108	1.032	0.031	0.454	0.041

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	23	20	26	30
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.74	0.96	1.11
time (sec)	N/A	0.023	0.004	0.027	1.610	0.924	0.046	0.440	0.095

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	24	20	32	17	16	15
N.S.	1	1.00	1.04	1.04	0.87	1.39	0.74	0.70	0.65
time (sec)	N/A	0.018	0.008	0.077	0.903	0.755	0.035	0.434	0.086

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	15	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	0.88
time (sec)	N/A	0.011	0.003	0.072	1.311	0.899	0.028	0.460	0.044

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	12	14	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67
time (sec)	N/A	0.009	0.005	0.022	1.571	1.072	0.055	0.483	0.038

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	1.00
time (sec)	N/A	0.004	0.002	0.067	0.861	0.825	0.030	0.470	0.103

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.008	0.007	0.070	0.826	0.764	0.034	0.440	0.035

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	46	83	46	52	45
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98
time (sec)	N/A	0.014	0.014	0.070	1.201	0.918	0.077	0.454	0.088

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	16	8	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.60	0.80	1.10	1.00
time (sec)	N/A	0.003	0.003	0.066	0.740	1.038	0.019	0.467	0.028

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	13	10	16	13
N.S.	1	1.00	1.00	1.06	1.00	0.76	0.59	0.94	0.76
time (sec)	N/A	0.005	0.002	0.072	1.338	0.795	0.028	0.460	0.110

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	14	17	16	14
N.S.	1	1.00	1.00	0.75	0.70	0.70	0.85	0.80	0.70
time (sec)	N/A	0.005	0.003	0.078	0.748	0.768	0.033	0.443	0.045

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	12	16	8	13	12
N.S.	1	1.00	0.75	0.81	0.75	1.00	0.50	0.81	0.75
time (sec)	N/A	0.004	0.003	0.079	2.949	0.735	0.022	0.457	0.037

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	17	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00
time (sec)	N/A	0.007	0.006	0.145	1.269	0.736	0.045	0.446	0.082

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81
time (sec)	N/A	0.013	0.004	0.014	1.246	0.824	0.048	0.486	0.063

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	23	34	20	25	17
N.S.	1	1.00	1.29	1.33	1.10	1.62	0.95	1.19	0.81
time (sec)	N/A	0.002	0.006	0.071	0.677	0.745	0.036	0.468	0.087

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73
time (sec)	N/A	0.007	0.003	0.068	1.029	0.796	0.026	0.477	0.159

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	18	8	14	10
N.S.	1	1.00	1.00	1.10	1.40	1.80	0.80	1.40	1.00
time (sec)	N/A	0.010	0.004	0.065	0.599	1.332	0.028	0.474	0.030

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	19	25	19	21	16
N.S.	1	1.00	1.00	0.71	0.61	0.81	0.61	0.68	0.52
time (sec)	N/A	0.007	0.002	0.072	0.982	1.053	0.038	0.478	0.040

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	24
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	1.33
time (sec)	N/A	0.017	0.004	0.075	0.827	0.829	0.050	0.455	0.039

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	17	17	17	19	9
N.S.	1	1.00	1.92	0.77	1.31	1.31	1.31	1.46	0.69
time (sec)	N/A	0.002	0.003	0.074	1.033	1.034	0.047	0.445	0.029

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	52	72	100	73	72	33
N.S.	1	1.00	0.75	0.61	0.85	1.18	0.86	0.85	0.39
time (sec)	N/A	0.028	0.013	0.065	1.407	1.299	0.053	0.439	0.115

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	15	16	15	26	14	15	15
N.S.	1	1.00	0.65	0.70	0.65	1.13	0.61	0.65	0.65
time (sec)	N/A	0.005	0.006	0.095	0.814	1.051	0.035	0.461	0.081

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	11	11	8	11	11
N.S.	1	1.00	1.00	3.73	1.00	1.00	0.73	1.00	1.00
time (sec)	N/A	0.005	0.005	0.016	1.095	1.044	0.035	0.430	0.062

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	23	20	23	36	39	47	21
N.S.	1	1.00	0.51	0.44	0.51	0.80	0.87	1.04	0.47
time (sec)	N/A	0.032	0.020	0.095	0.989	0.920	0.181	0.437	0.100

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	30	0	111	110	53	28
N.S.	1	1.00	0.84	0.81	0.00	3.00	2.97	1.43	0.76
time (sec)	N/A	0.024	0.018	0.039	0.000	0.691	1.410	0.461	0.319

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	20	16	37	50	36	35	15
N.S.	1	1.00	0.36	0.29	0.66	0.89	0.64	0.62	0.27
time (sec)	N/A	0.013	0.011	0.025	1.397	0.639	0.127	0.477	0.239

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	16	19	23	32	40	32
N.S.	1	1.00	0.65	0.52	0.61	0.74	1.03	1.29	1.03
time (sec)	N/A	0.010	0.009	0.033	1.620	0.608	0.111	0.459	0.219

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	18	17	14	33	248	48	26
N.S.	1	1.00	0.50	0.47	0.39	0.92	6.89	1.33	0.72
time (sec)	N/A	0.026	0.023	0.060	1.482	1.056	23.520	0.482	0.208

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	71839	26	15
N.S.	1	1.00	1.00	1.07	1.00	2.87	4789.27	1.73	1.00
time (sec)	N/A	0.016	0.033	0.089	1.663	1.012	17.724	0.456	0.455

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	39	595	13	29
N.S.	1	1.00	1.00	0.82	0.82	2.29	35.00	0.76	1.71
time (sec)	N/A	0.009	0.022	0.104	1.104	0.950	124.870	0.440	0.634

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	27	41	11	22	25	34
N.S.	1	1.00	0.73	0.90	1.37	0.37	0.73	0.83	1.13
time (sec)	N/A	0.021	0.027	0.058	1.405	0.884	0.124	0.473	0.314

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	23	22	29	24	22	22
N.S.	1	1.00	1.21	0.79	0.76	1.00	0.83	0.76	0.76
time (sec)	N/A	0.002	0.023	0.085	1.783	0.796	0.069	0.426	0.037

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	8	11	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.62	0.85	0.85
time (sec)	N/A	0.001	0.001	0.065	1.211	0.667	0.051	0.466	0.169

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	30	41	40	87	47	35
N.S.	1	1.00	0.89	0.81	1.11	1.08	2.35	1.27	0.95
time (sec)	N/A	0.012	0.036	0.086	1.505	0.630	0.668	0.461	0.209

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	37	22	25	25	0	26	21
N.S.	1	1.00	1.68	1.00	1.14	1.14	0.00	1.18	0.95
time (sec)	N/A	0.005	0.041	0.066	1.435	0.567	0.000	0.457	0.085

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	21	20	25	24	25	20
N.S.	1	1.00	1.15	0.78	0.74	0.93	0.89	0.93	0.74
time (sec)	N/A	0.002	0.019	0.067	1.426	0.568	0.066	0.463	0.088

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	21	22	27	0	27	23
N.S.	1	1.00	1.22	0.78	0.81	1.00	0.00	1.00	0.85
time (sec)	N/A	0.007	0.046	0.166	4.101	0.526	0.000	0.446	0.053

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	37	12	15	17	0	33	11
N.S.	1	1.00	2.64	0.86	1.07	1.21	0.00	2.36	0.79
time (sec)	N/A	0.002	0.023	0.061	6.857	0.522	0.000	0.446	0.171

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	88	59	92	0	168	73
N.S.	1	1.00	1.12	1.29	0.87	1.35	0.00	2.47	1.07
time (sec)	N/A	0.023	0.111	0.237	4.863	0.815	0.000	0.482	0.085

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	12	0	73	0	13
N.S.	1	1.00	1.00	1.00	0.92	0.00	5.62	0.00	1.00
time (sec)	N/A	0.010	0.003	0.066	4.857	0.000	0.849	0.000	0.031

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	2	2	15	2	2
N.S.	1	1.00	1.00	1.00	0.13	0.13	1.00	0.13	0.13
time (sec)	N/A	0.007	0.008	0.043	3.052	1.219	0.068	0.459	0.099

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	2	2	2	2	2
N.S.	1	1.00	1.00	4.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.006	0.009	0.014	4.657	0.872	0.310	0.452	0.007

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	4	4	3	4	4
N.S.	1	1.00	1.00	2.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.008	0.010	0.015	6.971	0.858	0.339	0.452	0.014

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	16	5	13	7	13	14
N.S.	1	1.00	1.00	1.45	0.45	1.18	0.64	1.18	1.27
time (sec)	N/A	0.013	0.011	0.013	1.646	0.883	0.416	0.440	0.024

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	9	13	10	18	9
N.S.	1	1.00	1.00	1.07	0.64	0.93	0.71	1.29	0.64
time (sec)	N/A	0.008	0.002	0.016	1.295	0.807	0.468	0.462	0.017

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	16	14	0	14	14
N.S.	1	1.00	1.00	1.13	1.07	0.93	0.00	0.93	0.93
time (sec)	N/A	0.012	0.028	0.074	0.929	0.708	0.000	0.454	0.028

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	10	0	10	10
N.S.	1	1.00	1.00	1.08	1.00	0.77	0.00	0.77	0.77
time (sec)	N/A	0.055	0.048	0.060	1.710	0.777	0.000	0.489	0.118

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	16	23	0	80	17
N.S.	1	1.00	1.00	1.16	0.84	1.21	0.00	4.21	0.89
time (sec)	N/A	0.015	0.038	0.057	1.809	0.643	0.000	0.467	0.126

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	19	0	16	-1
N.S.	1	1.00	1.00	1.06	1.00	1.06	0.00	0.89	-0.06
time (sec)	N/A	0.014	0.008	0.025	3.453	0.826	0.000	0.457	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	15	9	9	7	9	9
N.S.	1	1.00	0.69	0.94	0.56	0.56	0.44	0.56	0.56
time (sec)	N/A	0.006	0.009	0.009	0.928	0.667	0.021	0.434	0.017

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	24	14	14	12	14	14
N.S.	1	1.00	0.62	0.92	0.54	0.54	0.46	0.54	0.54
time (sec)	N/A	0.013	0.011	0.007	1.956	0.769	0.023	0.413	0.030

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	33	19	19	17	19	19
N.S.	1	1.00	0.58	0.92	0.53	0.53	0.47	0.53	0.53
time (sec)	N/A	0.021	0.012	0.010	1.542	0.721	0.024	0.450	0.023

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	39	66	181	60	360	77	2034
N.S.	1	1.00	0.81	1.38	3.77	1.25	7.50	1.60	42.38
time (sec)	N/A	0.023	0.089	0.099	1.389	0.692	0.394	0.477	10.613

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	3	2	2	3	2
N.S.	1	1.00	1.00	4.50	1.50	1.00	1.00	1.50	1.00
time (sec)	N/A	0.001	0.056	0.005	1.239	0.866	0.233	0.494	0.009

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	6	14	7	11	10
N.S.	1	1.00	1.00	1.70	0.60	1.40	0.70	1.10	1.00
time (sec)	N/A	0.003	0.002	0.021	2.013	0.655	0.226	0.511	0.034

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	22	15	24	0	22
N.S.	1	1.00	1.00	0.00	1.00	0.68	1.09	0.00	1.00
time (sec)	N/A	0.010	0.019	0.005	0.122	0.151	1.425	0.000	0.062

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	12	11	9	0	9	9
N.S.	1	1.00	0.83	1.00	0.92	0.75	0.00	0.75	0.75
time (sec)	N/A	0.009	0.038	0.065	1.104	0.865	0.000	0.484	0.015

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	20	0	20	-1
N.S.	1	1.00	1.00	1.05	0.00	0.91	0.00	0.91	-0.05
time (sec)	N/A	0.037	0.089	0.086	0.000	0.584	0.000	0.481	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	17	116	0	6	27	0	155
N.S.	1	1.00	0.17	1.13	0.00	0.06	0.26	0.00	1.50
time (sec)	N/A	0.010	10.031	0.103	0.000	0.109	0.300	0.000	0.285

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [112] had the largest ratio of [31]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	9	0.111
2	A	2	1	1.00	11	0.091
3	A	2	1	1.00	11	0.091
4	A	2	1	1.00	11	0.091
5	A	1	1	1.00	14	0.071
6	A	2	1	1.00	4	0.250
7	A	2	1	1.00	9	0.111
8	A	2	2	1.00	7	0.286
9	A	2	2	1.00	17	0.118
10	A	2	2	1.00	9	0.222
11	A	3	3	1.00	11	0.273
12	A	3	3	1.00	16	0.188
13	A	2	2	1.00	10	0.200
14	A	1	1	1.00	15	0.067
15	A	2	1	1.00	9	0.111
16	A	1	1	1.00	9	0.111
17	A	1	1	1.00	15	0.067
18	A	1	1	1.00	17	0.059
19	A	3	2	1.00	20	0.100
20	A	1	1	1.00	26	0.038
21	A	2	2	1.00	20	0.100
22	A	2	2	1.00	4	0.500
23	A	3	2	1.00	6	0.333
24	A	4	2	1.00	6	0.333
25	A	4	2	1.00	6	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	5	0.400
27	A	3	3	1.00	6	0.500
28	A	2	2	1.00	4	0.500
29	A	2	1	1.00	4	0.250
30	A	3	2	1.00	4	0.500
31	A	2	1	1.00	4	0.250
32	A	4	2	1.00	4	0.500
33	A	2	2	1.00	6	0.333
34	A	3	3	1.00	6	0.500
35	A	4	4	1.00	8	0.500
36	A	2	2	1.00	4	0.500
37	A	2	1	1.00	4	0.250
38	A	3	2	1.00	4	0.500
39	A	5	3	1.00	13	0.231
40	A	3	2	1.00	13	0.154
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	4	0.500
43	A	3	2	1.00	4	0.500
44	A	2	2	1.00	4	0.500
45	A	3	2	1.00	4	0.500
46	A	2	2	1.00	10	0.200
47	A	1	1	1.00	11	0.091
48	A	1	1	1.00	9	0.111
49	A	1	1	1.00	13	0.077
50	A	1	1	1.00	11	0.091
51	A	2	1	1.00	11	0.091
52	A	2	2	1.00	8	0.250
53	A	5	3	1.00	8	0.375
54	A	1	1	1.00	10	0.100
55	A	3	2	1.00	17	0.118
56	A	1	1	1.00	7	0.143
57	A	2	2	1.00	4	0.500
58	A	1	1	1.00	4	0.250
59	A	2	2	1.00	6	0.333
60	A	1	1	1.00	5	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	1	1	1.00	2	0.500
62	A	1	1	1.00	8	0.125
63	A	2	2	1.00	8	0.250
64	A	2	2	1.00	8	0.250
65	A	2	2	1.00	14	0.143
66	A	3	2	1.00	14	0.143
67	A	3	2	1.00	8	0.250
68	A	1	1	1.00	9	0.111
69	A	1	1	1.00	13	0.077
70	A	2	2	1.00	9	0.222
71	A	1	1	1.00	6	0.167
72	A	1	1	1.00	6	0.167
73	A	4	4	1.00	7	0.571
74	A	2	2	1.00	5	0.400
75	A	2	2	1.00	7	0.286
76	A	3	2	1.00	7	0.286
77	A	3	2	1.00	9	0.222
78	A	3	3	1.00	7	0.429
79	A	2	2	1.00	11	0.182
80	A	1	1	1.00	10	0.100
81	A	1	1	1.00	10	0.100
82	A	2	2	1.00	2	1.000
83	A	4	4	1.00	2	2.000
84	A	4	4	1.00	2	2.000
85	A	3	3	1.00	4	0.750
86	A	4	4	1.00	6	0.667
87	A	2	2	1.00	13	0.154
88	A	2	2	1.00	14	0.143
89	A	1	1	1.00	9	0.111
90	A	1	1	1.00	9	0.111
91	A	2	2	1.00	10	0.200
92	A	3	3	1.00	4	0.750
93	A	4	3	1.00	6	0.500
94	A	5	5	1.00	6	0.833
95	A	4	4	1.00	6	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	1	3	1.00	17	0.176
97	A	2	2	1.00	11	0.182
98	A	1	1	1.00	15	0.067
99	A	1	1	1.00	14	0.071
100	A	2	2	1.00	11	0.182
101	A	2	2	1.00	13	0.154
102	A	5	6	1.00	10	0.600
103	A	3	3	1.00	15	0.200
104	A	4	4	1.00	15	0.267
105	A	3	3	1.00	15	0.200
106	A	3	2	1.00	16	0.125
107	A	3	2	1.00	16	0.125
108	A	6	4	1.00	18	0.222
109	A	3	2	1.00	23	0.087
110	A	2	1	1.00	19	0.053
111	A	5	5	1.00	18	0.278
112	A	6	5	1.00	31	0.161
113	A	2	2	1.00	7	0.286
114	A	3	2	1.00	22	0.091
115	A	2	1	1.00	16	0.062
116	A	2	1	1.00	17	0.059
117	A	2	1	1.00	12	0.083
118	A	3	2	1.00	21	0.095
119	A	2	1	1.00	20	0.050
120	A	3	3	1.00	16	0.188
121	A	4	3	1.00	16	0.188
122	A	2	1	1.00	11	0.091
123	A	3	2	1.00	11	0.182
124	A	2	1	1.00	16	0.062
125	A	2	1	1.00	7	0.143
126	A	5	5	1.00	9	0.556
127	A	4	3	1.00	12	0.250
128	A	3	2	1.00	14	0.143
129	A	4	4	1.00	21	0.190
130	A	3	2	1.00	18	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	7	0.286
132	A	3	3	1.00	11	0.273
133	A	3	2	1.00	16	0.125
134	A	3	2	1.00	11	0.182
135	A	5	4	1.00	18	0.222
136	A	3	3	1.00	7	0.429
137	A	9	6	1.00	7	0.857
138	A	3	3	1.00	14	0.214
139	A	1	1	1.00	16	0.062
140	A	3	3	1.00	12	0.250
141	A	2	2	1.00	8	0.250
142	A	2	2	1.00	8	0.250
143	A	1	1	1.00	10	0.100
144	A	3	3	1.00	13	0.231
145	A	2	1	1.00	19	0.053
146	A	1	1	1.00	11	0.091
147	A	3	3	1.00	11	0.273
148	A	2	2	1.00	11	0.182
149	A	1	1	1.00	13	0.077
150	A	4	4	1.00	15	0.267
151	A	3	3	1.00	13	0.231
152	A	2	2	1.00	9	0.222
153	A	3	3	1.00	12	0.250
154	A	2	2	1.00	9	0.222
155	A	6	6	1.00	18	0.333
156	A	2	2	1.00	8	0.250
157	A	3	3	1.00	5	0.600
158	A	1	1	1.00	7	0.143
159	A	1	1	1.00	9	0.111
160	A	2	2	1.00	7	0.286
161	A	2	2	1.00	5	0.400
162	A	1	1	1.00	14	0.071
163	A	2	2	1.00	14	0.143
164	A	2	2	1.00	9	0.222
165	A	2	3	1.00	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	2	1.00	7	0.286
167	A	3	2	1.00	9	0.222
168	A	4	2	1.00	9	0.222
169	A	1	1	1.00	21	0.048
170	A	1	1	1.00	4	0.250
171	A	2	2	1.00	4	0.500
172	A	2	2	1.00	8	0.250
173	A	1	1	1.00	11	0.091
174	A	4	2	1.00	16	0.125
175	A	1	1	1.00	9	0.111

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sqrt{1+2x}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}(1+2x)^{3/2}$$

```
[Out] 1/3*(1+2*x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{1}{3}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x],x]
```

```
[Out] (1 + 2*x)^(3/2)/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(1+2x)^{3/2}$$

Mathematica [A]
time = 0.03, size = 13, normalized size = 1.00

$$\frac{1}{3}(1+2x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x],x]
```

```
[Out] (1 + 2*x)^(3/2)/3
```

Maple [A]
time = 0.10, size = 10, normalized size = 0.77

method	result	size
gosper	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
default	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(1+2x)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{1}{3}+\frac{2x}{3}\right)\sqrt{1+2x}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2+4x)\sqrt{1+2x}}{3}}{4\sqrt{\pi}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+2*x)^(3/2)
```

Maxima [A]
time = 2.75, size = 9, normalized size = 0.69

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

Fricas [A]
time = 0.58, size = 9, normalized size = 0.69

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.62

$$\frac{(2x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(1/2),x)
```

```
[Out] (2*x + 1)**(3/2)/3
```

Giac [A]
time = 0.72, size = 9, normalized size = 0.69

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

Mupad [B]
time = 0.32, size = 9, normalized size = 0.69

$$\frac{(2\,x+1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^(1/2),x)
```

```
[Out] (2*x + 1)^(3/2)/3
```

3.2 $\int x\sqrt{1+3x}\,dx$

Optimal. Leaf size=27

$$-\frac{2}{27}(1+3x)^{3/2}+\frac{2}{45}(1+3x)^{5/2}$$

```
[Out] -2/27*(1+3*x)^(3/2)+2/45*(1+3*x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + 3*x],x]
[Out] (-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+3x}\,dx &= \int\left(-\frac{1}{3}\sqrt{1+3x}+\frac{1}{3}(1+3x)^{3/2}\right)dx\\ &= -\frac{2}{27}(1+3x)^{3/2}+\frac{2}{45}(1+3x)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.67

$$\frac{2}{135}(1+3x)^{3/2}(-2+9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + 3*x],x]
[Out] (2*(1 + 3*x)^(3/2)*(-2 + 9*x))/135
```

Maple [A]
time = 0.06, size = 20, normalized size = 0.74

method	result	size
gosper	$\frac{2(1+3x)^{\frac{3}{2}}(9x-2)}{135}$	15
trager	$\left(\frac{2}{5}x^2+\frac{2}{45}x-\frac{4}{135}\right)\sqrt{1+3x}$	19
derivativedivides	$-\frac{2(1+3x)^{\frac{3}{2}}}{27}+\frac{2(1+3x)^{\frac{5}{2}}}{45}$	20
default	$-\frac{2(1+3x)^{\frac{3}{2}}}{27}+\frac{2(1+3x)^{\frac{5}{2}}}{45}$	20
risch	$\frac{2(27x^2+3x-2)\sqrt{1+3x}}{135}$	20
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+3x)^{\frac{3}{2}}(-9x+2)}{15}}{18\sqrt{\pi}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/27*(1+3*x)^(3/2)+2/45*(1+3*x)^(5/2)
```

Maxima [A]
time = 3.71, size = 19, normalized size = 0.70

$$\frac{2}{45}\,(3\,x+1)^{\frac{5}{2}}-\frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

Fricas [A]
time = 0.59, size = 19, normalized size = 0.70

$$\frac{2}{135}\,\left(27\,x^2+3\,x-2\right)\sqrt{3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/135*(27*x^2 + 3*x - 2)*sqrt(3*x + 1)
```

Sympy [A]
time = 0.49, size = 39, normalized size = 1.44

$$\frac{2x^2\sqrt{3x+1}}{5}+\frac{2x\sqrt{3x+1}}{45}-\frac{4\sqrt{3x+1}}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(3*x + 1)/5 + 2*x*sqrt(3*x + 1)/45 - 4*sqrt(3*x + 1)/135
```

Giac [A]
time = 0.68, size = 19, normalized size = 0.70

$$\frac{2}{45}\left(3\,x+1\right)^{\frac{5}{2}}-\frac{2}{27}\left(3\,x+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

Mupad [B]
time = 0.09, size = 14, normalized size = 0.52

$$\frac{2\left(3\,x+1\right)^{3/2}\left(9\,x-2\right)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x + 1)^(1/2),x)
```

```
[Out] (2*(3*x + 1)^(3/2)*(9*x - 2))/135
```

3.3 $\int x^2 \sqrt{1+x}\, dx$

Optimal. Leaf size=34

$$\frac{2}{3}(1+x)^{3/2}-\frac{4}{5}(1+x)^{5/2}+\frac{2}{7}(1+x)^{7/2}$$

```
[Out] 2/3*(1+x)^(3/2)-4/5*(1+x)^(5/2)+2/7*(1+x)^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x],x]
[Out] (2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{1+x}\, dx &= \int \left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right) dx\\ &= \frac{2}{3}(1+x)^{3/2}-\frac{4}{5}(1+x)^{5/2}+\frac{2}{7}(1+x)^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 21, normalized size = 0.62

$$\frac{2}{105}(1+x)^{3/2}\left(8-12x+15x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(8 - 12*x + 15*x^2))/105
```

Maple [A]
time = 0.07, size = 23, normalized size = 0.68

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}(15x^2-12x+8)}{105}$	18
trager	$\left(\frac{2}{7}x^3+\frac{2}{35}x^2-\frac{8}{105}x+\frac{16}{105}\right)\sqrt{1+x}$	22
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{7}{2}}}{7}$	23
default	$\frac{2(1+x)^{\frac{3}{2}}}{3}-\frac{4(1+x)^{\frac{5}{2}}}{5}+\frac{2(1+x)^{\frac{7}{2}}}{7}$	23
risch	$\frac{2(15x^3+3x^2-4x+8)\sqrt{1+x}}{105}$	23
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(15x^2-12x+8)}{105}}{2\sqrt{\pi}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+x)^(3/2)-4/5*(1+x)^(5/2)+2/7*(1+x)^(7/2)
```

Maxima [A]
time = 4.79, size = 22, normalized size = 0.65

$$\frac{2}{7}(x+1)^{\frac{7}{2}}-\frac{4}{5}(x+1)^{\frac{5}{2}}+\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

Fricas [A]
time = 0.61, size = 22, normalized size = 0.65

$$\frac{2}{105}\left(15\,x^3+3\,x^2-4\,x+8\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*x^3 + 3*x^2 - 4*x + 8)*sqrt(x + 1)
```

Sympy [A]
time = 0.65, size = 48, normalized size = 1.41

$$\frac{2x^3\sqrt{x+1}}{7}+\frac{2x^2\sqrt{x+1}}{35}-\frac{8x\sqrt{x+1}}{105}+\frac{16\sqrt{x+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(1/2),x)
```

```
[Out] 2*x**3*sqrt(x + 1)/7 + 2*x**2*sqrt(x + 1)/35 - 8*x*sqrt(x + 1)/105 + 16*sqr
t(x + 1)/105
```

Giac [A]
time = 0.79, size = 22, normalized size = 0.65

$$\frac{2}{7}(x+1)^{\frac{7}{2}}-\frac{4}{5}(x+1)^{\frac{5}{2}}+\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

Mupad [B]
time = 0.07, size = 19, normalized size = 0.56

$$-\frac{2\,{\left(x+1\right)}^{3/2}\,\left(42\,x-15\,{\left(x+1\right)}^2+7\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x + 1)^(1/2),x)
```

```
[Out] -(2*(x + 1)^(3/2)*(42*x - 15*(x + 1)^2 + 7))/105
```

3.4 $\int \frac{x}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=27

$$-\frac{4}{9}\sqrt{2-3x}+\frac{2}{27}(2-3x)^{3/2}$$

```
[Out] 2/27*(2-3*x)^(3/2)-4/9*(2-3*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 - 3*x],x]
```

```
[Out] (-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2-3x}}\,dx &= \int \left(\frac{2}{3\sqrt{2-3x}}-\frac{1}{3}\sqrt{2-3x}\right)dx\\ &= -\frac{4}{9}\sqrt{2-3x}+\frac{2}{27}(2-3x)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.67

$$-\frac{2}{27}\sqrt{2-3x}\,(4+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 - 3*x],x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*(4 + 3*x))/27
```

Maple [A]
time = 0.07, size = 20, normalized size = 0.74

method	result	size
trager	$\left(-\frac{2x}{9}-\frac{8}{27}\right)\sqrt{2-3x}$	14
gosper	$-\frac{2(3x+4)\sqrt{2-3x}}{27}$	15
derivativedivides	$\frac{2(2-3x)^{\frac{3}{2}}}{27}-\frac{4\sqrt{2-3x}}{9}$	20
default	$\frac{2(2-3x)^{\frac{3}{2}}}{27}-\frac{4\sqrt{2-3x}}{9}$	20
risch	$\frac{2(-2+3x)(3x+4)}{27\sqrt{2-3x}}$	20
meijerg	$\frac{2\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(6x+8)\sqrt{1-\frac{3x}{2}}}{6}\right)}{9\sqrt{\pi}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/27*(2-3*x)^(3/2)-4/9*(2-3*x)^(1/2)
```

Maxima [A]
time = 3.82, size = 19, normalized size = 0.70

$$\frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}}-\frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="maxima")
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

Fricas [A]
time = 0.51, size = 14, normalized size = 0.52

$$-\frac{2}{27}\,(3\,x+4)\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="fricas")
[Out] -2/27*(3*x + 4)*sqrt(-3*x + 2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 60, normalized size = 2.22

$$\begin{cases} -\frac{2ix\sqrt{3x-2}}{9} - \frac{8i\sqrt{3x-2}}{27} & \text{for } |x| > \frac{2}{3} \\ -\frac{2x\sqrt{2-3x}}{9} - \frac{8\sqrt{2-3x}}{27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*x*sqrt(3*x - 2)/9 - 8*I*sqrt(3*x - 2)/27, Abs(x) > 2/3), (-
2*x*sqrt(2 - 3*x)/9 - 8*sqrt(2 - 3*x)/27, True))
```

Giac [A]
time = 0.72, size = 19, normalized size = 0.70

$$\frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}} - \frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

Mupad [B]
time = 0.06, size = 14, normalized size = 0.52

$$-\frac{2\,\sqrt{2-3\,x}\,\left(3\,x+4\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2 - 3*x)^(1/2),x)
```

```
[Out] -(2*(2 - 3*x)^(1/2)*(3*x + 4))/27
```

3.5 $\int \frac{1+x}{(2+2x+x^2)^3} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{4(2+2x+x^2)^2}$$

```
[Out] -1/4/(x^2+2*x+2)^2
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {643}

$$-\frac{1}{4(x^2+2x+2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(2 + 2*x + x^2)^3,x]
```

```
[Out] -1/4*1/(2 + 2*x + x^2)^2
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+x}{(2+2x+x^2)^3} \, dx = -\frac{1}{4(2+2x+x^2)^2}$$

Mathematica [A]
time = 0.02, size = 14, normalized size = 1.00

$$-\frac{1}{4(2+2x+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(2 + 2*x + x^2)^3,x]
```

```
[Out] -1/4*1/(2 + 2*x + x^2)^2
```

Maple [A]
time = 0.13, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{4(x^2+2x+2)^2}$	13
default	$-\frac{1}{4(x^2+2x+2)^2}$	13
norman	$-\frac{1}{4(x^2+2x+2)^2}$	13
risch	$-\frac{1}{4(x^2+2x+2)^2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+2*x+2)^3,x,method=_RETURNVERBOSE)
[Out] -1/4/(x^2+2*x+2)^2
```

Maxima [A]
time = 2.34, size = 12, normalized size = 0.86

$$-\frac{1}{4\,(x^2+2\,x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="maxima")
[Out] -1/4/(x^2 + 2*x + 2)^2
```

Fricas [A]
time = 0.59, size = 22, normalized size = 1.57

$$-\frac{1}{4\,(x^4+4\,x^3+8\,x^2+8\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="fricas")
[Out] -1/4/(x^4 + 4*x^3 + 8*x^2 + 8*x + 4)
```

Sympy [A]
time = 0.05, size = 22, normalized size = 1.57

$$-\frac{1}{4x^4+16x^3+32x^2+32x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x+2)**3,x)
```

```
[Out] -1/(4*x**4 + 16*x**3 + 32*x**2 + 32*x + 16)
```

Giac [A]
time = 0.72, size = 12, normalized size = 0.86

$$-\frac{1}{4\,(x^2+2\,x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="giac")
[Out] -1/4/(x^2 + 2*x + 2)^2
```

Mupad [B]
time = 0.06, size = 12, normalized size = 0.86

$$-\frac{1}{4\,(x^2+2\,x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(2*x + x^2 + 2)^3,x)
[Out] -1/(4*(2*x + x^2 + 2)^2)
```

3.6 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$-\cos(x)+\frac{\cos^3(x)}{3}$$

```
[Out] -cos(x)+1/3*cos(x)^3
```

Rubi [A]
time = 0.06, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.15

$$-\frac{3\cos(x)}{4}+\frac{1}{12}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A]
time = 0.09, size = 11, normalized size = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
norman	$\frac{-4(\tan^2(\frac{x}{2}))-\frac{4}{3}}{(1+\tan^2(\frac{x}{2}))^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A]
time = 2.77, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A]
time = 0.61, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

Giac [A]
time = 0.82, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos\left(x\right)^{3} - \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Mupad [B]
time = 0.16, size = 10, normalized size = 0.77

$$\frac{\cos\left(x\right)\,\left(\cos\left(x\right)^{2}-3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

3.7 $\int \sqrt[3]{-1+z}\, z\, dz$

Optimal. Leaf size=23

$$\frac{3}{4}(-1+z)^{4/3}+\frac{3}{7}(-1+z)^{7/3}$$

```
[Out] 3/4*(-1+z)^(4/3)+3/7*(-1+z)^(7/3)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{-1+z}\, z\, dz &= \int \left(\sqrt[3]{-1+z}+(-1+z)^{4/3}\right) dz\\ &= \frac{3}{4}(-1+z)^{4/3}+\frac{3}{7}(-1+z)^{7/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$\frac{3}{28}(7+4(-1+z))(-1+z)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(7 + 4*(-1 + z))*(-1 + z)^(4/3))/28
```

Maple [A]
time = 0.08, size = 16, normalized size = 0.70

method	result	size
gosper	$\frac{3(-1+z)^{\frac{4}{3}}(4z+3)}{28}$	13
derivativedivides	$\frac{3(-1+z)^{\frac{4}{3}}}{4}+\frac{3(-1+z)^{\frac{7}{3}}}{7}$	16
default	$\frac{3(-1+z)^{\frac{4}{3}}}{4}+\frac{3(-1+z)^{\frac{7}{3}}}{7}$	16
trager	$\left(\frac{3}{7}z^2-\frac{3}{28}z-\frac{9}{28}\right)(-1+z)^{\frac{1}{3}}$	17
risch	$\frac{3(-1+z)^{\frac{1}{3}}(4z^2-z-3)}{28}$	18
meijerg	$\frac{\mathrm{signum}(-1+z)^{\frac{1}{3}}z^2\,\mathrm{hypergeom}([-\frac{1}{3},2],[3],z)}{2(-\mathrm{signum}(-1+z))^{\frac{1}{3}}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+z)^(1/3)*z,z,method=_RETURNVERBOSE)
[Out] 3/4*(-1+z)^(4/3)+3/7*(-1+z)^(7/3)
```

Maxima [A]
time = 2.65, size = 15, normalized size = 0.65

$$\frac{3}{7}(z-1)^{\frac{7}{3}}+\frac{3}{4}(z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="maxima")
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

Fricas [A]
time = 0.87, size = 17, normalized size = 0.74

$$\frac{3}{28}\left(4z^2-z-3\right)(z-1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="fricas")
[Out] 3/28*(4*z^2 - z - 3)*(z - 1)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 92, normalized size = 4.00

$$\begin{cases} \frac{3z^2\sqrt[3]{z-1}}{7}-\frac{3z\sqrt[3]{z-1}}{28}-\frac{9\sqrt[3]{z-1}}{28} & \text{for } |z|>1 \\ \frac{3z^2\sqrt[3]{1-z}\,e^{\frac{i\pi}{3}}}{7}-\frac{3z\sqrt[3]{1-z}\,e^{\frac{i\pi}{3}}}{28}-\frac{9\sqrt[3]{1-z}\,e^{\frac{i\pi}{3}}}{28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)**(1/3)*z,z)
```

```
[Out] Piecewise((3*z**2*(z - 1)**(1/3)/7 - 3*z*(z - 1)**(1/3)/28 - 9*(z - 1)**(1/
3)/28, Abs(z) > 1), (3*z**2*(1 - z)**(1/3)*exp(I*pi/3)/7 - 3*z*(1 - z)**(1/
3)*exp(I*pi/3)/28 - 9*(1 - z)**(1/3)*exp(I*pi/3)/28, True))
```

Giac [A]
time = 0.81, size = 15, normalized size = 0.65

$$\frac{3}{7}\,(z-1)^{\frac{7}{3}}+\frac{3}{4}\,(z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="giac")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.52

$$\frac{3\,(4\,z+3)\,\left(z-1\right)^{4/3}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z*(z - 1)^(1/3),z)
```

```
[Out] (3*(4*z + 3)*(z - 1)^(4/3))/28
```

3.8 $\int \cot(x)\csc^2(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\csc^2(x)$$

[Out] -1/2*csc(x)^2

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2686, 30}

$$-\frac{1}{2}\csc^2(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]*Csc[x]^2,x]

[Out] -1/2*Csc[x]^2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc^2(x)\,dx &= -\text{Subst}\left(\int x\,dx, x, \csc(x)\right)\\ &= -\frac{1}{2}\csc^2(x)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 8, normalized size = 1.00

$$-\frac{1}{2}\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x]^2,x]
```

```
[Out] -1/2*Csc[x]^2
```

Maple [A]
time = 0.02, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{1}{2\sin(x)^2}$	7
default	$-\frac{1}{2\sin(x)^2}$	7
risch	$\frac{2\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}-1)^2}$	17
norman	$\frac{-\frac{1}{8}-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}}{\tan\left(\frac{x}{2}\right)^2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/sin(x)^2
```

Maxima [A]
time = 4.89, size = 6, normalized size = 0.75

$$-\frac{1}{2\,\sin\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/sin(x)^2
```

Fricas [A]
time = 0.70, size = 10, normalized size = 1.25

$$\frac{1}{2\left(\cos\left(x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(cos(x)^2 - 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 1.00

$$-\frac{1}{2\sin^2\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)**3,x)
```

```
[Out] -1/(2*sin(x)**2)
```

Giac [A]
time = 0.69, size = 6, normalized size = 0.75

$$-\frac{1}{2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/2/sin(x)^2
```

Mupad [B]
time = 0.42, size = 6, normalized size = 0.75

$$-\frac{\cot(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)^3,x)
```

```
[Out] -cot(x)^2/2
```

3.9 $\int \cos(2x)\sqrt{4-\sin(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

```
[Out] -1/3*(4-sin(2*x))^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2747, 32}

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

```
[Out] -1/3*(4 - Sin[2*x])^(3/2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\sqrt{4-\sin(2x)}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \sqrt{4+x}\,dx, x, -\sin(2x)\right)\right)\\ &= -\frac{1}{3}(4-\sin(2x))^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
[Out] -1/3*(4 - Sin[2*x])^(3/2)
```

Maple [A]
time = 0.07, size = 13, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{(4-\sin(2x))^{\frac{3}{2}}}{3}$	13
default	$-\frac{(4-\sin(2x))^{\frac{3}{2}}}{3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(4-sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(4-sin(2*x))^(3/2)
```

Maxima [A]
time = 4.35, size = 12, normalized size = 0.75

$$-\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="maxima")
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

Fricas [A]
time = 0.70, size = 18, normalized size = 1.12

$$\frac{1}{3}\left(\sin\left(2\,x\right)-4\right)\sqrt{-\sin\left(2\,x\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="fricas")
[Out] 1/3*(sin(2*x) - 4)*sqrt(-sin(2*x) + 4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.09, size = 29, normalized size = 1.81

$$\frac{\sqrt{4-\sin\left(2x\right)}\,\sin\left(2x\right)}{3}-\frac{4\sqrt{4-\sin\left(2x\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))**(1/2),x)
```

```
[Out] sqrt(4 - sin(2*x))*sin(2*x)/3 - 4*sqrt(4 - sin(2*x))/3
```

Giac [A]
time = 0.83, size = 12, normalized size = 0.75

$$-\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

Mupad [B]
time = 0.17, size = 12, normalized size = 0.75

$$-\frac{\left(4-\sin\left(2\,x\right)\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(4 - sin(2*x))^(1/2),x)
```

```
[Out] -(4 - sin(2*x))^(3/2)/3
```

3.10 $\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx$

Optimal. Leaf size=6

$$\frac{1}{3+\cos(x)}$$

```
[Out] 1/(3+cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2747, 32}

$$\frac{1}{\cos(x)+3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(3 + Cos[x])^2,x]
```

```
[Out] (3 + Cos[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx &= -\text{Subst}\left(\int \frac{1}{(3+x)^2}\,dx, x, \cos(x)\right)\\ &= \frac{1}{3+\cos(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$\frac{1}{3+\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(3 + Cos[x])^2,x]
```

```
[Out] (3 + Cos[x])^(-1)
```

Maple [A]
time = 0.02, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$\frac{1}{3+\cos(x)}$	7
default	$\frac{1}{3+\cos(x)}$	7
risch	$\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+6\,\mathrm{e}^{ix}+1}$	24
norman	$\frac{-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2\left(\frac{x}{2}\right)+2\right)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(3+cos(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(3+cos(x))
```

Maxima [A]
time = 3.61, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/(cos(x) + 3)
```

Fricas [A]
time = 0.54, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/(cos(x) + 3)
```

Sympy [A]
time = 0.16, size = 5, normalized size = 0.83

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))**2,x)
```

```
[Out] 1/(cos(x) + 3)
```

Giac [A]
time = 0.81, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/(cos(x) + 3)
```

Mupad [B]
time = 0.04, size = 6, normalized size = 1.00

$$\frac{1}{\cos(x) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + 3)^2,x)
```

```
[Out] 1/(cos(x) + 3)
```

3.11 $\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx$

Optimal. Leaf size=12

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

[Out] 2*cos(x)/(cos(x)^3)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3286, 2645, 30}

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/Sqrt[Cos[x]^3],x]

[Out] (2*Cos[x])/Sqrt[Cos[x]^3]

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx = \frac{\cos^{\frac{3}{2}}(x)\int \frac{\sin(x)}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{\cos^3(x)}}$$
$$= -\frac{\cos^{\frac{3}{2}}(x)\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \cos(x)\right)}{\sqrt{\cos^3(x)}}$$
$$= \frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Mathematica [A]
time = 0.09, size = 12, normalized size = 1.00

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[Cos[x]^3],x]
```

```
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Maple [A]
time = 0.05, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\frac{4\cos(x)}{\sqrt{\cos(3x)+3\cos(x)}}$	11
default	$\frac{4\cos(x)}{\sqrt{\cos(3x)+3\cos(x)}}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*cos(x)/(cos(x)^3)^(1/2)
```

Maxima [A]
time = 3.18, size = 10, normalized size = 0.83

$$\frac{2\cos(x)}{\sqrt{\cos(x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*cos(x)/sqrt(cos(x)^3)
```

Fricas [A]
time = 0.49, size = 12, normalized size = 1.00

$$\frac{2\sqrt{\cos(x)^3}}{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(cos(x)^3)/cos(x)^2
```

Sympy [A]
time = 0.24, size = 12, normalized size = 1.00

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)**3)**(1/2),x)
[Out] 2*cos(x)/sqrt(cos(x)**3)
```

Giac [A]
time = 0.76, size = 6, normalized size = 0.50

$$\frac{2}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="giac")
[Out] 2/sqrt(cos(x))
```

Mupad [B]
time = 0.20, size = 9, normalized size = 0.75

$$\frac{2\,|\cos(x)|}{\cos(x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3)^(1/2),x)
[Out] (2*abs(cos(x)))/cos(x)^(3/2)
```

3.12 $\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=10

$$-2\cos\left(\sqrt{1+x}\right)$$

```
[Out] -2*cos((1+x)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3512, 15, 2718}

$$-2\cos\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -2*Cos[Sqrt[1 + x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx = 2\text{Subst}\left(\int \frac{x\sin(x)}{\sqrt{x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{1+x}\right)$$
$$= -2\cos\left(\sqrt{1+x}\right)$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 1.00

$$-2\cos\left(\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
[Out] -2*Cos[Sqrt[1 + x]]
```

Maple [A]
time = 0.06, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-2\cos\left(\sqrt{1+x}\right)$	9
default	$-2\cos\left(\sqrt{1+x}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((1+x)^(1/2))/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*cos((1+x)^(1/2))
```

Maxima [A]
time = 3.24, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="maxima")
[Out] -2*cos(sqrt(x + 1))
```

Fricas [A]
time = 0.45, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="fricas")
[Out] -2*cos(sqrt(x + 1))
```

Sympy [A]
time = 0.09, size = 10, normalized size = 1.00

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)**(1/2))/(1+x)**(1/2),x)
[Out] -2*cos(sqrt(x + 1))
```

Giac [A]
time = 0.75, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="giac")
[Out] -2*cos(sqrt(x + 1))
```

Mupad [B]
time = 0.17, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x + 1)^(1/2))/(x + 1)^(1/2),x)
[Out] -2*cos((x + 1)^(1/2))
```

3.13 $\int x^{-1+n} \sin(x^n)\, dx$

Optimal. Leaf size=9

$$-\frac{\cos(x^n)}{n}$$

```
[Out] -cos(x^n)/n
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3460, 2718}

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*Sin[x^n],x]
```

```
[Out] -(Cos[x^n]/n)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^{-1+n} \sin(x^n)\, dx &= \frac{\text{Subst}\left(\int \sin(x)\, dx, x, x^n\right)}{n}\\ &= -\frac{\cos(x^n)}{n}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*Sin[x^n],x]
```

```
[Out] -(Cos[x^n]/n)
```

Maple [A]
time = 0.06, size = 10, normalized size = 1.11

method	result	size
default	$-\frac{\cos\left(x^{n}\right)}{n}$	10
risch	$-\frac{\cos\left(x^{n}\right)}{n}$	10
norman	$\frac{2\left(\tan^{2}\left(\frac{{\mathrm e}^{n \ln\left(x\right)}}{2}\right)\right)}{n\left(1+\tan^{2}\left(\frac{{\mathrm e}^{n \ln\left(x\right)}}{2}\right)\right)}$	30
meijerg	$\frac{\sqrt{\pi}\, \left(\frac{2^{1-\frac{-1+n}{n}-\frac{1}{n}} \left(-1\right)^{\frac{1}{2}-\frac{-1+n}{2 n}-\frac{1}{2 n}}}{\sqrt{\pi}\, \Gamma\left(3-\frac{-1+n}{n}-\frac{1}{n}\right)}-\frac{\left(-1\right)^{\frac{1}{2}-\frac{-1+n}{2 n}-\frac{1}{2 n}} 2^{1-\frac{-1+n}{n}-\frac{1}{n}} \cos\left(x^{n}\right)}{\sqrt{\pi}\, \Gamma\left(3-\frac{-1+n}{n}-\frac{1}{n}\right)}\right)}{n}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*sin(x^n),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x^n)/n
```

Maxima [A]
time = 4.17, size = 9, normalized size = 1.00

$$-\frac{\cos\left(x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="maxima")
```

```
[Out] -cos(x^n)/n
```

Fricas [A]
time = 0.47, size = 9, normalized size = 1.00

$$-\frac{\cos\left(x^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="fricas")
```

```
[Out] -cos(x^n)/n
```

Sympy [A]
time = 3.52, size = 7, normalized size = 0.78

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*sin(x**n),x)
[Out] -cos(x**n)/n
```

Giac [A]
time = 0.76, size = 9, normalized size = 1.00

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="giac")
[Out] -cos(x^n)/n
```

Mupad [B]
time = 0.21, size = 9, normalized size = 1.00

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*sin(x^n),x)
[Out] -cos(x^n)/n
```

3.14 $\int \frac{x^5}{\sqrt{1-x^6}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\sqrt{1-x^6}$$

```
[Out] -1/3*(-x^6+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {267}

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 - x^6],x]
[Out] -1/3*Sqrt[1 - x^6]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1-x^6}}\,dx = -\frac{1}{3}\sqrt{1-x^6}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 - x^6],x]
[Out] -1/3*Sqrt[1 - x^6]
```

Maple [A]
time = 0.07, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{-x^6+1}}{3}$	12
default	$-\frac{\sqrt{-x^6+1}}{3}$	12
trager	$-\frac{\sqrt{-x^6+1}}{3}$	12
risch	$\frac{x^6-1}{3\sqrt{-x^6+1}}$	17
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^6+1}}{6\sqrt{\pi}}$	26
gosper	$\frac{(-1+x)(1+x)(x^2+x+1)(x^2-x+1)}{3\sqrt{-x^6+1}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^6+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-x^6+1)^(1/2)
```

Maxima [A]
time = 3.78, size = 11, normalized size = 0.73

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

Fricas [A]
time = 0.41, size = 11, normalized size = 0.73

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

Sympy [A]
time = 0.08, size = 10, normalized size = 0.67

$$-\frac{\sqrt{1-x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**6+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**6)/3
```

Giac [A]
time = 0.87, size = 11, normalized size = 0.73

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

Mupad [B]
time = 0.35, size = 11, normalized size = 0.73

$$-\frac{\sqrt{1-x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(1 - x^6)^(1/2),x)
```

```
[Out] -(1 - x^6)^(1/2)/3
```

3.15 $\int t\sqrt[4]{1+t}\, dt$

Optimal. Leaf size=23

$$-\frac{4}{5}(1+t)^{5/4}+\frac{4}{9}(1+t)^{9/4}$$

```
[Out] -4/5*(1+t)^(5/4)+4/9*(1+t)^(9/4)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[t*(1 + t)^(1/4),t]

[Out] (-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int t\sqrt[4]{1+t}\, dt &= \int \left(-\sqrt[4]{1+t}+(1+t)^{5/4}\right) dt\\ &= -\frac{4}{5}(1+t)^{5/4}+\frac{4}{9}(1+t)^{9/4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.78

$$\frac{4}{45}(1+t)^{5/4}(-9+5(1+t))$$

Antiderivative was successfully verified.

```
[In] Integrate[t*(1 + t)^(1/4),t]

[Out] (4*(1 + t)^(5/4)*(-9 + 5*(1 + t)))/45
```

Maple [A]
time = 0.06, size = 16, normalized size = 0.70

method	result	size
gosper	$\frac{4(1+t)^{\frac{5}{4}}(5t-4)}{45}$	13
meijerg	$\frac{t^2 \operatorname{hypergeom}\left(\left[-\frac{1}{4},2\right],[3],-t\right)}{2}$	15
derivativedivides	$-\frac{4(1+t)^{\frac{5}{4}}}{5}+\frac{4(1+t)^{\frac{9}{4}}}{9}$	16
default	$-\frac{4(1+t)^{\frac{5}{4}}}{5}+\frac{4(1+t)^{\frac{9}{4}}}{9}$	16
risch	$\frac{4(1+t)^{\frac{1}{4}}(5t^2+t-4)}{45}$	16
trager	$\left(\frac{4}{9}t^2+\frac{4}{45}t-\frac{16}{45}\right)(1+t)^{\frac{1}{4}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*(1+t)^(1/4),t,method=_RETURNVERBOSE)
```

```
[Out] -4/5*(1+t)^(5/4)+4/9*(1+t)^(9/4)
```

Maxima [A]
time = 11.26, size = 15, normalized size = 0.65

$$\frac{4}{9}(t+1)^{\frac{9}{4}}-\frac{4}{5}(t+1)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="maxima")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

Fricas [A]
time = 0.59, size = 15, normalized size = 0.65

$$\frac{4}{45}\left(5t^2+t-4\right)(t+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="fricas")
```

```
[Out] 4/45*(5*t^2 + t - 4)*(t + 1)^(1/4)
```

Sympy [A]
time = 0.48, size = 34, normalized size = 1.48

$$\frac{4t^2\sqrt[4]{t+1}}{9}+\frac{4t\sqrt[4]{t+1}}{45}-\frac{16\sqrt[4]{t+1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)**(1/4),t)
```

```
[Out] 4*t**2*(t + 1)**(1/4)/9 + 4*t*(t + 1)**(1/4)/45 - 16*(t + 1)**(1/4)/45
```

Giac [A]
time = 0.80, size = 15, normalized size = 0.65

$$\frac{4}{9}(t+1)^{\frac{9}{4}}-\frac{4}{5}(t+1)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="giac")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.52

$$\frac{4\,(5\,t-4)\,(t+1)^{5/4}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*(t + 1)^(1/4),t)
```

```
[Out] (4*(5*t - 4)*(t + 1)^(5/4))/45
```

3.16 $\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=11

$$\frac{x}{\sqrt{1+x^2}}$$

```
[Out] x/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {197}

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-3/2),x]
```

```
[Out] x/Sqrt[1 + x^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx = \frac{x}{\sqrt{1+x^2}}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$\frac{x}{\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-3/2),x]
```

```
[Out] x/Sqrt[1 + x^2]
```

Maple [A]
time = 0.06, size = 10, normalized size = 0.91

method	result	size
gosper	$\frac{x}{\sqrt{x^2+1}}$	10
default	$\frac{x}{\sqrt{x^2+1}}$	10
trager	$\frac{x}{\sqrt{x^2+1}}$	10
meijerg	$\frac{x}{\sqrt{x^2+1}}$	10
risch	$\frac{x}{\sqrt{x^2+1}}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/(x^2+1)^(1/2)
```

Maxima [A]
time = 2.58, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.
time = 0.79, size = 22, normalized size = 2.00

$$\frac{x^2+\sqrt{x^2+1}\,x+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (x^2 + sqrt(x^2 + 1)*x + 1)/(x^2 + 1)
```

Sympy [A]
time = 0.34, size = 8, normalized size = 0.73

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(3/2),x)
```

```
[Out] x/sqrt(x**2 + 1)
```

Giac [A]
time = 0.70, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] x/sqrt(x^2 + 1)
```

Mupad [B]
time = 0.07, size = 9, normalized size = 0.82

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(3/2),x)
```

```
[Out] x/(x^2 + 1)^(1/2)
```

3.17 $\int x^2(27+8x^3)^{2/3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{40}\left(27+8x^3\right)^{5/3}$$

```
[Out] 1/40*(8*x^3+27)^(5/3)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {267}

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(27 + 8*x^3)^(2/3),x]
```

```
[Out] (27 + 8*x^3)^(5/3)/40
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2\left(27+8x^3\right)^{2/3}\,dx=\frac{1}{40}\left(27+8x^3\right)^{5/3}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{40}\left(27+8x^3\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(27 + 8*x^3)^(2/3),x]
```

```
[Out] (27 + 8*x^3)^(5/3)/40
```

Maple [A]
time = 0.07, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
default	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
risch	$\frac{(8x^3+27)^{\frac{5}{3}}}{40}$	12
meijerg	$3x^3 \operatorname{hypergeom}\left(\left[-\frac{2}{3},1\right],[2],-\frac{8x^3}{27}\right)$	17
trager	$\left(\frac{x^3}{5}+\frac{27}{40}\right)(8x^3+27)^{\frac{2}{3}}$	18
gosper	$\frac{(3+2x)(4x^2-6x+9)(8x^3+27)^{\frac{2}{3}}}{40}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(8*x^3+27)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(8*x^3+27)^(5/3)
```

Maxima [A]
time = 5.55, size = 11, normalized size = 0.73

$$\frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Fricas [A]
time = 0.73, size = 11, normalized size = 0.73

$$\frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.09, size = 27, normalized size = 1.80

$$\frac{x^3(8x^3+27)^{\frac{2}{3}}}{5}+\frac{27(8x^3+27)^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(8*x**3+27)**(2/3),x)
```

```
[Out] x**3*(8*x**3 + 27)**(2/3)/5 + 27*(8*x**3 + 27)**(2/3)/40
```

Giac [A]
time = 1.08, size = 11, normalized size = 0.73

$$\frac{1}{40} \left(8 x^3 + 27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="giac")
```

```
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Mupad [B]
time = 0.19, size = 11, normalized size = 0.73

$$\frac{\left(8 x^3 + 27\right)^{5/3}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(8*x^3 + 27)^(2/3),x)
```

```
[Out] (8*x^3 + 27)^(5/3)/40
```

3.18 $\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

```
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3224}

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Rule 3224

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[(c*B - b*C)*((b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(b
^2 + c^2))), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx = \frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

Mathematica [A]
time = 0.04, size = 15, normalized size = 1.00

$$\frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Maple [A]
time = 0.08, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$\frac{3(-\cos(x)+\sin(x))^{\frac{2}{3}}}{2}$	12
default	$\frac{3(-\cos(x)+\sin(x))^{\frac{2}{3}}}{2}$	12
risch	$\frac{\left(-\frac{3}{2}-\frac{3i}{2}\right)\left((1+i)\left(-\mathrm{e}^{4ix}+i\mathrm{e}^{2ix}\right)\right)^{\frac{1}{3}}\left(\mathrm{e}^{ix}-i\mathrm{e}^{-ix}\right)}{\left(-8\cos(x)+8\sin(x)\right)^{\frac{1}{3}}\left((-1-i)\left(\mathrm{e}^{4ix}-i\mathrm{e}^{2ix}\right)\right)^{\frac{1}{3}}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

Maxima [A]
time = 4.39, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="maxima")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Fricas [A]
time = 3.36, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="fricas")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Sympy [A]
time = 0.10, size = 12, normalized size = 0.80

$$\frac{3\left(\sin\left(x\right)-\cos\left(x\right)\right)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))**(1/3),x)
[Out] 3*(sin(x) - cos(x))**(2/3)/2
```

Giac [A]
time = 0.75, size = 11, normalized size = 0.73

$$\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="giac")
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Mupad [B]
time = 0.24, size = 15, normalized size = 1.00

$$\frac{3\,2^{1/3}\,{\left(-\cos\left(x+\frac{\pi}{4}\right)\right)}^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(sin(x) - cos(x))^(1/3),x)
[Out] (3*2^(1/3)*(-cos(x + pi/4))^(2/3))/2
```

3.19 $\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}}\,dx$

Optimal. Leaf size=32

$$\frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}$$

```
[Out] 2*((x^2+1)*(1+(x^2+1)^(1/2)))^(1/2)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6847, 1602}

$$\frac{2\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
```

```
[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+(1+x)^{3/2}}}\,dx, x, x^2\right)$$
$$= \text{Subst}\left(\int \frac{x}{\sqrt{x^2(1+x)}}\,dx, x, \sqrt{1+x^2}\right)$$
$$= \frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}$$

Mathematica [**A**]
time = 1.69, size = 32, normalized size = 1.00

$$\frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{1+x^2+\left(x^2+1\right)^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
[Out] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1), x)
```

Fricas [A]
time = 0.67, size = 23, normalized size = 0.72

$$\frac{2\sqrt{x^2 + (x^2+1)^{\frac{3}{2}} + 1}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x^2 + (x^2 + 1)^(3/2) + 1)/sqrt(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x^2+1)\left(\sqrt{x^2+1}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x**2+(x**2+1)**(3/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt((x**2 + 1)*(sqrt(x**2 + 1) + 1)), x)
```

Giac [A]
time = 0.70, size = 15, normalized size = 0.47

$$2\sqrt{\sqrt{x^2+1}+1} - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1) - 2
```

Mupad [B]
time = 0.59, size = 47, normalized size = 1.47

$$\frac{2\,(x^2+1)\,\sqrt{\sqrt{x^2+1}+1}}{\left(\sqrt{\sqrt{x^2+1}+1}+1\right)\sqrt{(x^2+1)^{3/2}+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)^(3/2) + x^2 + 1)^(1/2),x)
```

```
[Out] (2*(x^2 + 1)*((x^2 + 1)^(1/2) + 1)^(1/2))/((((x^2 + 1)^(1/2) + 1)^(1/2) + 1
)*((x^2 + 1)^(3/2) + x^2 + 1)^(1/2))
```

3.20 $\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=17

$$2\sqrt{1+\sqrt{1+x^2}}$$

```
[Out] 2*(1+(x^2+1)^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6818}

$$2\sqrt{\sqrt{x^2+1}+1}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx = 2\sqrt{1+\sqrt{1+x^2}}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$2\sqrt{1+\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Maple [A]
time = 0.06, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$2\sqrt{1+\sqrt{x^2+1}}$	14
default	$2\sqrt{1+\sqrt{x^2+1}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+(x^2+1)^(1/2))^(1/2)
```

Maxima [A]
time = 3.78, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Fricas [A]
time = 0.49, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Sympy [A]
time = 0.11, size = 14, normalized size = 0.82

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/2)/(1+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] 2*sqrt(sqrt(x**2 + 1) + 1)
```

Giac [A]
time = 0.59, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/(1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Mupad [B]
time = 0.21, size = 13, normalized size = 0.76

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)^(1/2)*((x^2 + 1)^(1/2) + 1)^(1/2)),x)
```

```
[Out] 2*((x^2 + 1)^(1/2) + 1)^(1/2)
```

3.21 $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} \, dx$

Optimal. Leaf size=16

$$-\frac{5}{2}\sqrt[5]{1-2x+x^2}$$

```
[Out] -5/2*(x^2-2*x+1)^(1/5)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {657, 643}

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

```
[Out] (-5*(1 - 2*x + x^2)^(1/5))/2
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 657

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[5]{1-2x+x^2}}{1-x} \, dx &= \int \frac{1-x}{\left(1-2x+x^2\right)^{4/5}} \, dx\\ &= -\frac{5}{2}\sqrt[5]{1-2x+x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 0.81

$$-\frac{5}{2}\sqrt[5]{(-1+x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

```
[Out] (-5*((-1 + x)^2)^(1/5))/2
```

Maple [A]
time = 0.08, size = 10, normalized size = 0.62

method	result	size
risch	$-\frac{5\left(\left(-1+x\right)^{2}\right)^{\frac{1}{5}}}{2}$	10
gosper	$-\frac{5\left(x^{2}-2x+1\right)^{\frac{1}{5}}}{2}$	13
trager	$-\frac{5\left(x^{2}-2x+1\right)^{\frac{1}{5}}}{2}$	13
meijerg	$\frac{\mathrm{signum}\left(-1+x\right)^{\frac{2}{5}} x\, \mathrm{hypergeom}\left(\left[\frac{3}{5},1\right],[2],x\right)}{\left(-\mathrm{signum}\left(-1+x\right)\right)^{\frac{2}{5}}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x+1)^(1/5)/(1-x),x,method=_RETURNVERBOSE)
```

```
[Out] -5/2*((-1+x)^2)^(1/5)
```

Maxima [A]
time = 3.98, size = 7, normalized size = 0.44

$$-\frac{5}{2}\,\left(x-1\right)^{\frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="maxima")
```

```
[Out] -5/2*(x - 1)^(2/5)
```

Fricas [A]
time = 0.42, size = 12, normalized size = 0.75

$$-\frac{5}{2}\,\left(x^{2}-2\,x+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="fricas")
```

```
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

Sympy [A]
time = 0.62, size = 15, normalized size = 0.94

$$-\frac{5\sqrt[5]{x^2-2x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x+1)**(1/5)/(1-x),x)
```

```
[Out] -5*(x**2 - 2*x + 1)**(1/5)/2
```

Giac [A]
time = 0.61, size = 12, normalized size = 0.75

$$-\frac{5}{2}\left(x^2-2x+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="giac")
```

```
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

Mupad [B]
time = 0.16, size = 9, normalized size = 0.56

$$-\frac{5\left((x-1)^2\right)^{1/5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2*x + 1)^(1/5)/(x - 1),x)
```

```
[Out] -(5*((x - 1)^2)^(1/5))/2
```

3.22 $\int x \sin(x)\, dx$

Optimal. Leaf size=8

$$-x\cos(x)+\sin(x)$$

```
[Out] -x*cos(x)+sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3377, 2717}

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sin(x)\,dx &= -x\cos(x)+\int \cos(x)\,dx\\ &= -x\cos(x)+\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-x\cos(x)+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A]
time = 0.02, size = 9, normalized size = 1.12

method	result	size
default	$-x \cos(x) + \sin(x)$	9
risch	$-x \cos(x) + \sin(x)$	9
meijerg	$2\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}} + \frac{\sin(x)}{2\sqrt{\pi}}\right)$	22
norman	$\frac{x(\tan^2(\frac{x}{2})) - x + 2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -x*cos(x)+sin(x)
```

Maxima [A]
time = 3.61, size = 8, normalized size = 1.00

$$-x \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

Fricas [A]
time = 0.46, size = 8, normalized size = 1.00

$$-x \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
```

```
[Out] -x*cos(x) + sin(x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.88

$$-x \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

Giac [**A**]
time = 0.59, size = 8, normalized size = 1.00

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

Mupad [**B**]
time = 0.06, size = 8, normalized size = 1.00

$$\sin(x) - x\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] sin(x) - x*cos(x)
```

3.23 $\int x^2 \sin(x)\,dx$

Optimal. Leaf size=17

$$2\cos(x) - x^2\cos(x) + 2x\sin(x)$$

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2718}

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x],x]
```

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\sin(x)\,dx &= -x^2\cos(x) + 2\int x\cos(x)\,dx \\
&= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\,dx \\
&= 2\cos(x) - x^2\cos(x) + 2x\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.88

$$-\left(\left(-2+x^2\right)\cos(x)\right) + 2x\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x],x]
[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

Maple [A]
time = 0.02, size = 18, normalized size = 1.06

method	result	size
risch	$(-x^2+2)\cos(x)+2x\sin(x)$	17
default	$2\cos(x)-x^2\cos(x)+2x\sin(x)$	18
meijerg	$4\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2}{2}+1\right)\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)$	34
norman	$\frac{x^2(\tan^2(\frac{x}{2}))-x^2+4x\tan(\frac{x}{2})+4}{1+\tan^2(\frac{x}{2})}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x,method=_RETURNVERBOSE)
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Maxima [A]
time = 5.86, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos(x)+2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="maxima")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Fricas [A]
time = 0.46, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos(x)+2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="fricas")
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Sympy [A]
time = 0.08, size = 17, normalized size = 1.00

$$-x^2\cos(x)+2x\sin(x)+2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x),x)
```

```
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

Giac [A]
time = 0.54, size = 15, normalized size = 0.88

$$-\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="giac")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.88

$$2\,x\,\sin\left(x\right)-\cos\left(x\right)\,\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
```

```
[Out] 2*x*sin(x) - cos(x)*(x^2 - 2)
```

3.24 $\int x^3 \cos(x) \, dx$

Optimal. Leaf size=23

$$-6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)$$

```
[Out] -6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2718}

$$x^3\sin(x) + 3x^2\cos(x) - 6x\sin(x) - 6\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x],x]
[Out] -6*Cos[x] + 3*x^2*Cos[x] - 6*x*Sin[x] + x^3*Sin[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\cos(x)\,dx &= x^3\sin(x) - 3\int x^2\sin(x)\,dx \\
&= 3x^2\cos(x) + x^3\sin(x) - 6\int x\cos(x)\,dx \\
&= 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x) + 6\int \sin(x)\,dx \\
&= -6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.83

$$3\left(-2+x^2\right)\cos(x) + x\left(-6+x^2\right)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x],x]
[Out] 3*(-2 + x^2)*Cos[x] + x*(-6 + x^2)*Sin[x]
```

Maple [A]
time = 0.02, size = 24, normalized size = 1.04

method	result	size
risch	$3(x^2-2)\cos(x)+x(x^2-6)\sin(x)$	20
default	$-6\cos(x)+3x^2\cos(x)-6x\sin(x)+x^3\sin(x)$	24
meijerg	$8\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2}{2}+3\right)\cos(x)}{4\sqrt{\pi}}-\frac{x\left(-\frac{x^2}{2}+3\right)\sin(x)}{4\sqrt{\pi}}\right)$	41
norman	$\frac{3x^2-12x\tan(\frac{x}{2})-3x^2(\tan^2(\frac{x}{2}))+2x^3\tan(\frac{x}{2})-12}{1+\tan^2(\frac{x}{2})}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x),x,method=_RETURNVERBOSE)
[Out] -6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

Maxima [A]
time = 4.80, size = 20, normalized size = 0.87

$$3\left(x^2-2\right)\cos(x)+\left(x^3-6x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="maxima")
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Fricas [A]
time = 0.41, size = 20, normalized size = 0.87

$$3\left(x^2-2\right)\cos(x)+\left(x^3-6x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="fricas")
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Sympy [A]
time = 0.13, size = 26, normalized size = 1.13

$$x^3\sin(x)+3x^2\cos(x)-6x\sin(x)-6\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x),x)
```

```
[Out] x**3*sin(x) + 3*x**2*cos(x) - 6*x*sin(x) - 6*cos(x)
```

Giac [A]
time = 0.55, size = 20, normalized size = 0.87

$$3\left(x^2-2\right)\cos\left(x\right)+\left(x^3-6\,x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="giac")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 1.04

$$\cos\left(x\right)\,\left(3\,x^2-6\right)-\sin\left(x\right)\,\left(6\,x-x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x),x)
```

```
[Out] cos(x)*(3*x^2 - 6) - sin(x)*(6*x - x^3)
```

3.25 $\int x^3 \sin(x)\, dx$

Optimal. Leaf size=24

$$6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)$$

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3377, 2717}

$$x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x],x]
```

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\sin(x)\,dx &= -x^3\cos(x) + 3\int x^2\cos(x)\,dx \\
&= -x^3\cos(x) + 3x^2\sin(x) - 6\int x\sin(x)\,dx \\
&= 6x\cos(x) - x^3\cos(x) + 3x^2\sin(x) - 6\int \cos(x)\,dx \\
&= 6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.83

$$-x\left(-6+x^2\right)\cos(x) + 3\left(-2+x^2\right)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x],x]
```

```
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

Maple [A]
time = 0.02, size = 25, normalized size = 1.04

method	result	size
risch	$(-x^3+6x)\cos(x)+3(x^2-2)\sin(x)$	23
default	$6x\cos(x)-x^3\cos(x)-6\sin(x)+3x^2\sin(x)$	25
meijerg	$8\sqrt{\pi}\left(\frac{x\left(-\frac{5x^2}{2}+15\right)\cos(x)}{20\sqrt{\pi}}-\frac{\left(-\frac{15x^2}{2}+15\right)\sin(x)}{20\sqrt{\pi}}\right)$	36
norman	$\frac{x^3\left(\tan^2\left(\frac{x}{2}\right)\right)+6x-x^3-6x\left(\tan^2\left(\frac{x}{2}\right)\right)+6x^2\tan\left(\frac{x}{2}\right)-12\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Maxima [A]
time = 1.92, size = 21, normalized size = 0.88

$$-\left(x^3-6\,x\right)\cos\left(x\right)+3\,\left(x^2-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Fricas [A]
time = 0.43, size = 21, normalized size = 0.88

$$-\left(x^3-6\,x\right)\cos\left(x\right)+3\,\left(x^2-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Sympy [A]
time = 0.13, size = 26, normalized size = 1.08

$$-x^3\cos\left(x\right)+3x^2\sin\left(x\right)+6x\cos\left(x\right)-6\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x),x)
```

```
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

Giac [A]
time = 0.54, size = 21, normalized size = 0.88

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\,\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="giac")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.96

$$\cos\left(x\right)\,\left(6\,x-x^{3}\right)+\sin\left(x\right)\,\left(3\,x^{2}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x)
```

```
[Out] cos(x)*(6*x - x^3) + sin(x)*(3*x^2 - 6)
```

3.26 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] 1/2*sin(x)^2
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2644, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]
```

```
[Out] -1/2*Cos[x]^2
```

Maple [A]
time = 0.01, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\sin^2(x))}{2}$	7
default	$\frac{(\sin^2(x))}{2}$	7
risch	$-\frac{\cos(2x)}{4}$	7
norman	$\frac{2(\tan^2(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2}$	19
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)^2
```

Maxima [A]
time = 4.40, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A]
time = 0.45, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

Giac [A]
time = 0.50, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.75

$$\frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
```

```
[Out] sin(x)^2/2
```

3.27 $\int x\cos(x)\sin(x)\,dx$

Optimal. Leaf size=23

$$-\frac{x}{4}+\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)$$

```
[Out] -1/4*x+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3524, 2715, 8}

$$-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]*Sin[x],x]
```

```
[Out] -1/4*x + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\sin(x)\,dx&=\frac{1}{2}x\sin^2(x)-\frac{1}{2}\int\sin^2(x)\,dx\\&=\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)-\frac{\int 1\,dx}{4}\\&=-\frac{x}{4}+\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 0.78

$$-\frac{1}{4}x\cos(2x)+\frac{1}{8}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]*Sin[x],x]
```

```
[Out] -1/4*(x*Cos[2*x]) + Sin[2*x]/8
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.78

method	result	size
risch	$-\frac{x\cos(2x)}{4}+\frac{\sin(2x)}{8}$	15
default	$-\frac{x\left(\cos^2(x)\right)}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}$	18
meijerg	$\frac{\sqrt{\pi}\left(-\frac{x\cos(2x)}{\sqrt{\pi}}+\frac{\sin(2x)}{2\sqrt{\pi}}\right)}{4}$	26
norman	$\frac{-\frac{x}{4}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x
```

Maxima [A]
time = 5.41, size = 14, normalized size = 0.61

$$-\frac{1}{4}x\cos\left(2x\right)+\frac{1}{8}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

Fricas [A]
time = 0.52, size = 17, normalized size = 0.74

$$-\frac{1}{2}x\cos\left(x\right)^2+\frac{1}{4}\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)^2 + 1/4*cos(x)*sin(x) + 1/4*x
```

Sympy [A]
time = 0.08, size = 24, normalized size = 1.04

$$\frac{x\sin^2(x)}{4} - \frac{x\cos^2(x)}{4} + \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x)
```

```
[Out] x*sin(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Giac [A]
time = 0.46, size = 14, normalized size = 0.61

$$-\frac{1}{4}x\cos(2x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

Mupad [B]
time = 0.09, size = 18, normalized size = 0.78

$$\frac{\sin(2x)}{8} + \frac{x\left(2\sin(x)^2 - 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*sin(x),x)
```

```
[Out] sin(2*x)/8 + (x*(2*sin(x)^2 - 1))/4
```

3.28 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)$$

[Out] 1/2*x-1/2*cos(x)*sin(x)

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {2715, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
[Out] x/2 - Sin[2*x]/4
```

Maple [A]
time = 0.03, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}-\frac{\sin(2x)}{4}$	11
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	22
norman	$\frac{\tan^{3}\left(\frac{x}{2}\right)+x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}-\tan\left(\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A]
time = 3.15, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x-\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A]
time = 0.44, size = 10, normalized size = 0.71

$$-\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

Giac [A]
time = 0.49, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin\left(2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] x/2 - sin(2*x)/4
```

3.29 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$-\cos(x)+\frac{\cos^3(x)}{3}$$

[Out] -cos(x)+1/3*cos(x)^3

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^3,x]

[Out] -Cos[x] + Cos[x]^3/3

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.15

$$-\frac{3\cos(x)}{4}+\frac{1}{12}\cos(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^3,x]

[Out] (-3*Cos[x])/4 + Cos[3*x]/12

Maple [A]
time = 0.00, size = 11, normalized size = 0.85

method	result	size
default	$-\frac{(2+\sin^2(x))\cos(x)}{3}$	11
risch	$-\frac{3\cos(x)}{4}+\frac{\cos(3x)}{12}$	12
norman	$\frac{-4\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{4}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A]
time = 3.09, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A]
time = 0.46, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
[Out] cos(x)**3/3 - cos(x)
```

Giac [A]
time = 0.47, size = 11, normalized size = 0.85

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
[Out] 1/3*cos(x)^3 - cos(x)
```

Mupad [B]
time = 0.00, size = 10, normalized size = 0.77

$$\frac{\cos(x)\,\left(\cos(x)^2 - 3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
[Out] (cos(x)*(cos(x)^2 - 3))/3
```

3.30 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)$$

```
[Out] 3/8*x-3/8*cos(x)*sin(x)-1/4*cos(x)*sin(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \sin^4(x)\,dx &= -\frac{1}{4}\cos(x)\sin^3(x)+\frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.75

method	result	size
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}-\frac{\sin(2x)}{4}$	17
default	$-\frac{\left(\sin^{3}(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{11\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{4}+\frac{11\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{8}-\frac{3\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{4}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

Maxima [A]
time = 2.75, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x+\frac{1}{32}\,\sin\left(4\,x\right)-\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A]
time = 2.67, size = 19, normalized size = 0.79

$$\frac{1}{8}\,\left(2\,\cos\left(x\right)^{3}-5\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{3}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{3x}{8}-\frac{\sin^{3}\left(x\right)\cos\left(x\right)}{4}-\frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Giac [A]
time = 0.47, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x + \frac{1}{32}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Mupad [B]
time = 0.04, size = 16, normalized size = 0.67

$$\frac{3\,x}{8} - \frac{\sin\left(2\,x\right)}{4} + \frac{\sin\left(4\,x\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
```

```
[Out] (3*x)/8 - sin(2*x)/4 + sin(4*x)/32
```

3.31 $\int \sin^5(x)\,dx$

Optimal. Leaf size=21

$$-\cos(x)+\frac{2\cos^3(x)}{3}-\frac{\cos^5(x)}{5}$$

```
[Out] -cos(x)+2/3*cos(x)^3-1/5*cos(x)^5
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5,x]
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{2\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.10

$$-\frac{5\cos(x)}{8}+\frac{5}{48}\cos(3x)-\frac{1}{80}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5,x]
[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

Maple [A]
time = 0.10, size = 17, normalized size = 0.81

method	result	size
default	$-\frac{\left(\frac{8}{3}+\sin^4(x)+\frac{4(\sin^2(x))}{3}\right)\cos(x)}{5}$	17
risch	$-\frac{5\cos(x)}{8}-\frac{\cos(5x)}{80}+\frac{5\cos(3x)}{48}$	18
norman	$\frac{\frac{16\left(\tan^8\left(\frac{x}{2}\right)\right)}{3}+\frac{16\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{15}+\frac{32\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^5}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x,method=_RETURNVERBOSE)
[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A]
time = 4.23, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="maxima")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Fricas [A]
time = 1.39, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="fricas")
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Sympy [A]
time = 0.01, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5}+\frac{2\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5,x)
```

```
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

Giac [A]
time = 0.47, size = 17, normalized size = 0.81

$$-\frac{1}{5}\cos\left(x\right)^{5}+\frac{2}{3}\cos\left(x\right)^{3}-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Mupad [B]
time = 0.04, size = 17, normalized size = 0.81

$$-\frac{\cos\left(x\right)^{5}}{5}+\frac{2\cos\left(x\right)^{3}}{3}-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x)
```

```
[Out] (2*cos(x)^3)/3 - cos(x) - cos(x)^5/5
```

3.32 $\int \sin^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}-\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)$$

```
[Out] 5/16*x-5/16*cos(x)*sin(x)-5/24*cos(x)*sin(x)^3-1/6*cos(x)*sin(x)^5
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2715, 8}

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \sin^6(x)\,dx &= -\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{6}\int \sin^4(x)\,dx\\
&= -\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{8}\int \sin^2(x)\,dx\\
&= -\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5\int 1\,dx}{16}\\
&= \frac{5x}{16}-\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 0.88

$$\frac{5x}{16} - \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6,x]
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

Maple [A]
time = 0.08, size = 24, normalized size = 0.71

method	result
risch	$\frac{5x}{16}-\frac{\sin(6x)}{192}+\frac{3\sin(4x)}{64}-\frac{15\sin(2x)}{64}$
default	$-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}$
norman	$\frac{\frac{5x}{16}-\frac{85\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{33\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{33\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{85\left(\tan^9\left(\frac{x}{2}\right)\right)}{24}+\frac{5\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{8}+\frac{15x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{75x\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{25x\left(\tan^6\left(\frac{x}{2}\right)\right)}{4}+\frac{75x}{}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x,method=_RETURNVERBOSE)
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x
```

Maxima [A]
time = 3.18, size = 24, normalized size = 0.71

$$\frac{1}{48}\sin\left(2\,x\right)^3 + \frac{5}{16}\,x + \frac{3}{64}\,\sin\left(4\,x\right) - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="maxima")
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A]
time = 4.80, size = 25, normalized size = 0.74

$$-\frac{1}{48}\left(8\,\cos\left(x\right)^5 - 26\,\cos\left(x\right)^3 + 33\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{5}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

Sympy [**A**]
time = 0.01, size = 36, normalized size = 1.06

$$\frac{5x}{16} - \frac{\sin^5(x)\cos(x)}{6} - \frac{5\sin^3(x)\cos(x)}{24} - \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6,x)
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

Giac [**A**]
time = 0.47, size = 22, normalized size = 0.65

$$\frac{5}{16}x - \frac{1}{192}\sin(6x) + \frac{3}{64}\sin(4x) - \frac{15}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="giac")
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

Mupad [**B**]
time = 0.04, size = 22, normalized size = 0.65

$$\frac{5x}{16} - \frac{15\sin(2x)}{64} + \frac{3\sin(4x)}{64} - \frac{\sin(6x)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x)
[Out] (5*x)/16 - (15*sin(2*x))/64 + (3*sin(4*x))/64 - sin(6*x)/192
```

3.33 $\int x \sin^2(x) \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4} - \frac{1}{2} x \cos(x) \sin(x) + \frac{\sin^2(x)}{4}$$

```
[Out] 1/4*x^2-1/2*x*cos(x)*sin(x)+1/4*sin(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3391, 30}

$$\frac{x^2}{4} + \frac{\sin^2(x)}{4} - \frac{1}{2} x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned} \int x \sin^2(x) \, dx &= -\frac{1}{2} x \cos(x) \sin(x) + \frac{\sin^2(x)}{4} + \frac{\int x \, dx}{2} \\ &= \frac{x^2}{4} - \frac{1}{2} x \cos(x) \sin(x) + \frac{\sin^2(x)}{4} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{4} - \frac{1}{8} \cos(2x) - \frac{1}{4} x \sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2,x]
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

Maple [A]
time = 0.02, size = 25, normalized size = 1.00

method	result	size
risch	$\frac{x^2}{4}-\frac{\cos(2x)}{8}-\frac{x\sin(2x)}{4}$	20
default	$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin^2(x))}{4}$	25
meijerg	$\frac{\sqrt{\pi}\left(\frac{2x^2+1}{2\sqrt{\pi}}-\frac{\cos(2x)}{2\sqrt{\pi}}-\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{4}$	38
norman	$\frac{x\left(\tan^3\left(\frac{x}{2}\right)\right)+\tan^2\left(\frac{x}{2}\right)+\frac{x^2}{4}-x\tan\left(\frac{x}{2}\right)+\frac{x^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{x^2\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] x*(1/2*x-1/2*cos(x)*sin(x))-1/4*x^2+1/4*sin(x)^2
```

Maxima [A]
time = 2.74, size = 19, normalized size = 0.76

$$\frac{1}{4}\,x^2-\frac{1}{4}\,x\sin\left(2\,x\right)-\frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="maxima")
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Fricas [A]
time = 1.92, size = 19, normalized size = 0.76

$$-\frac{1}{2}\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x^2-\frac{1}{4}\,\cos\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="fricas")
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

Sympy [A]
time = 0.09, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2(x)}{4}+\frac{x^2\cos^2(x)}{4}-\frac{x\sin(x)\cos(x)}{2}-\frac{\cos^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 - cos(x)**2/4
```

Giac [A]
time = 0.49, size = 19, normalized size = 0.76

$$\frac{1}{4}x^2-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Mupad [B]
time = 0.09, size = 19, normalized size = 0.76

$$\frac{\sin(x)^2}{4}-\frac{x\sin(2x)}{4}+\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
```

```
[Out] sin(x)^2/4 - (x*sin(2*x))/4 + x^2/4
```

3.34 $\int x\sin^3(x)\,dx$

Optimal. Leaf size=33

$$-\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}$$

```
[Out] -2/3*x*cos(x)+2/3*sin(x)-1/3*x*cos(x)*sin(x)^2+1/9*sin(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {3391, 3377, 2717}

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\sin^3(x)\,dx &= -\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int x\sin(x)\,dx\\ &= -\frac{2}{3}x\cos(x)-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int\cos(x)\,dx\\ &= -\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.94

$$-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)+\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^3,x]
```

```
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

Maple [A]
time = 0.04, size = 23, normalized size = 0.70

method	result	size
default	$-\frac{x\left(2+\sin^2(x)\right)\cos(x)}{3}+\frac{\left(\sin^3(x)\right)}{9}+\frac{2\sin(x)}{3}$	23
risch	$-\frac{3x\cos(x)}{4}+\frac{3\sin(x)}{4}+\frac{x\cos(3x)}{12}-\frac{\sin(3x)}{36}$	24
norman	$\frac{-\frac{2x}{3}+\frac{32\left(\tan^3\left(\frac{x}{2}\right)\right)}{9}+\frac{4\left(\tan^5\left(\frac{x}{2}\right)\right)}{3}-2x\left(\tan^2\left(\frac{x}{2}\right)\right)+2x\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{2x\left(\tan^6\left(\frac{x}{2}\right)\right)}{3}+\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

Maxima [A]
time = 3.25, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos\left(3\,x\right)-\frac{3}{4}\,x\cos\left(x\right)-\frac{1}{36}\,\sin\left(3\,x\right)+\frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Fricas [A]
time = 1.54, size = 23, normalized size = 0.70

$$\frac{1}{3}\,x\cos\left(x\right)^{3}-x\cos\left(x\right)-\frac{1}{9}\,\left(\cos\left(x\right)^{2}-7\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="fricas")
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

Sympy [A]
time = 0.13, size = 39, normalized size = 1.18

$$-x\sin^{2}\left(x\right)\cos\left(x\right)-\frac{2x\cos^{3}\left(x\right)}{3}+\frac{7\sin^{3}\left(x\right)}{9}+\frac{2\sin\left(x\right)\cos^{2}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3,x)
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/
3
```

Giac [A]
time = 0.48, size = 23, normalized size = 0.70

$$\frac{1}{12}\,x\cos\left(3\,x\right)-\frac{3}{4}\,x\cos\left(x\right)-\frac{1}{36}\,\sin\left(3\,x\right)+\frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="giac")
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Mupad [B]
time = 0.12, size = 25, normalized size = 0.76

$$\frac{x\cos\left(x\right)^3}{3}-\frac{\sin\left(x\right)\,\cos\left(x\right)^2}{9}-x\,\cos\left(x\right)+\frac{7\,\sin\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
[Out] (7*sin(x))/9 + (x*cos(x)^3)/3 - (cos(x)^2*sin(x))/9 - x*cos(x)
```

3.35 $\int x^2 \sin^2(x)\, dx$

Optimal. Leaf size=41

$$-\frac{x}{4}+\frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)$$

```
[Out] -1/4*x+1/6*x^3+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3392, 30, 2715, 8}

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^2,x]
```

```
[Out] -1/4*x + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sin^2(x)\,dx &= -\frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) + \frac{\int x^2\,dx}{2} - \frac{1}{2}\int \sin^2(x)\,dx \\
&= \frac{x^3}{6} + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4} \\
&= -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3 - 6x\cos(2x) + \left(3-6x^2\right)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^2,x]
```

```
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

Maple [A]
time = 0.04, size = 37, normalized size = 0.90

method	result	size
meijerg	$\frac{x^5 \operatorname{hypergeom}\left(\left[1,\frac{5}{2}\right],\left[\frac{3}{2},2,\frac{7}{2}\right],-x^2\right)}{5}$	19
risch	$\frac{x^3}{6} - \frac{x\cos(2x)}{4} - \frac{\left(2x^2-1\right)\sin(2x)}{8}$	27
default	$x^2\left(\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}\right) - \frac{x\left(\cos^2(x)\right)}{2} + \frac{\cos(x)\sin(x)}{4} + \frac{x}{4} - \frac{x^3}{3}$	37
norman	$\frac{x^2\left(\tan^3\left(\frac{x}{2}\right)\right) - \frac{x}{4} + \frac{x^3}{6} - \frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{2} + \frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2} - \frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4} - x^2\tan\left(\frac{x}{2}\right) + \frac{x^3\left(\tan^2\left(\frac{x}{2}\right)\right)}{3} + \frac{x^3\left(\tan^4\left(\frac{x}{2}\right)\right)}{6} + \frac{\tan\left(\frac{x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(1/2*x-1/2*cos(x)*sin(x))-1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x-1/3*x^
3
```

Maxima [A]
time = 2.51, size = 26, normalized size = 0.63

$$\frac{1}{6}\,x^3 - \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{8}\,\left(2\,x^2-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A]
time = 1.57, size = 29, normalized size = 0.71

$$\frac{1}{6}x^3 - \frac{1}{2}x\cos(x)^2 - \frac{1}{4}\left(2x^2 - 1\right)\cos(x)\sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

Sympy [A]
time = 0.13, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6} + \frac{x^3\cos^2(x)}{6} - \frac{x^2\sin(x)\cos(x)}{2} + \frac{x\sin^2(x)}{4} - \frac{x\cos^2(x)}{4} + \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
- x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Giac [A]
time = 0.51, size = 26, normalized size = 0.63

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Mupad [B]
time = 0.06, size = 28, normalized size = 0.68

$$\frac{\sin(2x)}{8} - \frac{x\cos(2x)}{4} - \frac{x^2\sin(2x)}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] sin(2*x)/8 - (x*cos(2*x))/4 - (x^2*sin(2*x))/4 + x^3/6
```

3.36 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2715, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
[Out] x/2 + Sin[2*x]/4
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.79

method	result	size
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$	11
risch	$\frac{x}{2}+\frac{\sin(2x)}{4}$	11
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{x}{2}-\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A]
time = 2.26, size = 10, normalized size = 0.71

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A]
time = 1.25, size = 10, normalized size = 0.71

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [**A**]
time = 0.50, size = 10, normalized size = 0.71

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
[Out] 1/2*x + 1/4*sin(2*x)
```

Mupad [**B**]
time = 0.03, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
[Out] x/2 + sin(2*x)/4
```

3.37 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x) - \frac{\sin^3(x)}{3}$$

```
[Out] sin(x)-1/3*sin(x)^3
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2713}

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x) - \frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4} + \frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

Maple [A]
time = 0.07, size = 11, normalized size = 1.00

method	result	size
default	$\frac{(2+\cos^2(x))\sin(x)}{3}$	11
risch	$\frac{3\sin(x)}{4}+\frac{\sin(3x)}{12}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A]
time = 4.28, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A]
time = 1.53, size = 10, normalized size = 0.91

$$\frac{1}{3}\left(\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
[Out] -sin(x)**3/3 + sin(x)
```

Giac [A]
time = 0.53, size = 9, normalized size = 0.82

$$-\frac{1}{3}\sin\left(x\right)^{3}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.82

$$\sin\left(x\right)-\frac{\sin\left(x\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] sin(x) - sin(x)^3/3
```

3.38 $\int \cos^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}+\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)$$

```
[Out] 3/8*x+3/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {2715, 8}

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\,dx&=\frac{1}{4}\cos^3(x)\sin(x)+\frac{3}{4}\int \cos^2(x)\,dx\\&=\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+\frac{3\int 1\,dx}{8}\\&=\frac{3x}{8}+\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.92

$$\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4,x]
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A]
time = 0.10, size = 18, normalized size = 0.75

method	result	size
risch	$\frac{3x}{8}+\frac{\sin(4x)}{32}+\frac{\sin(2x)}{4}$	17
default	$\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}$	18
norman	$\frac{\frac{3x}{8}-\frac{3\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{3\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}-\frac{5\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}+\frac{5\tan\left(\frac{x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x,method=_RETURNVERBOSE)
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x
```

Maxima [A]
time = 4.50, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x+\frac{1}{32}\,\sin\left(4\,x\right)+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="maxima")
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A]
time = 1.06, size = 19, normalized size = 0.79

$$\frac{1}{8}\left(2\,\cos\left(x\right)^3+3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{3}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

Sympy [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{3x}{8}+\frac{\sin\left(x\right)\cos^3\left(x\right)}{4}+\frac{3\sin\left(x\right)\cos\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

Giac [A]
time = 0.46, size = 16, normalized size = 0.67

$$\frac{3}{8}\,x+\frac{1}{32}\,\sin\left(4\,x\right)+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.67

$$\frac{3\,x}{8}+\frac{\sin\left(2\,x\right)}{4}+\frac{\sin\left(4\,x\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x)
```

```
[Out] (3*x)/8 + sin(2*x)/4 + sin(4*x)/32
```

3.39 $\int (a^2-x^2)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] 5/24*a^2*x*(a^2-x^2)^(3/2)+1/6*x*(a^2-x^2)^(5/2)+5/16*a^6*arctan(x/(a^2-x^2
)^(1/2))+5/16*a^4*x*(a^2-x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {201, 223, 209}

$$\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\text{ArcTan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{5}{16}a^4x\sqrt{a^2-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(5/2),x]
```

```
[Out] (5*a^4*x*Sqrt[a^2 - x^2])/16 + (5*a^2*x*(a^2 - x^2)^(3/2))/24 + (x*(a^2 - x
^2)^(5/2))/6 + (5*a^6*ArcTan[x/Sqrt[a^2 - x^2]])/16
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a^2-x^2\right)^{5/2} dx &= \frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{6}\left(5a^2\right)\int \left(a^2-x^2\right)^{3/2} dx \\
&= \frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{8}\left(5a^4\right)\int \sqrt{a^2-x^2}\, dx \\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{16}\left(5a^6\right)\int \frac{1}{\sqrt{a^2-x^2}}\, dx \\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{16}\left(5a^6\right)\text{Subst}\left(\int \frac{1}{1+x^2} d\right. \\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 0.73

$$\frac{1}{48}\sqrt{a^2-x^2}\left(33a^4x-26a^2x^3+8x^5\right)+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - x^2)^(5/2),x]
```

```
[Out] (Sqrt[a^2 - x^2]*(33*a^4*x - 26*a^2*x^3 + 8*x^5))/48 + (5*a^6*ArcTan[x/Sqrt
[a^2 - x^2]])/16
```

Maple [A]
time = 0.07, size = 75, normalized size = 0.89

method	result	size
risch	$\frac{x\left(33a^4-26a^2x^2+8x^4\right)\sqrt{a^2-x^2}}{48}+\frac{5a^6\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{16}$	54
default	$\frac{x\left(a^2-x^2\right)^{\frac{5}{2}}}{6}+\frac{5a^2\left(\frac{\left(a^2-x^2\right)^{\frac{3}{2}}x}{4}+\frac{3a^2\left(\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{2}\right)}{4}\right)}{6}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(a^2-x^2)^(5/2)+5/6*a^2*(1/4*(a^2-x^2)^(3/2)*x+3/4*a^2*(1/2*x*(a^2-x^
2)^(1/2)+1/2*a^2*arctan(x/(a^2-x^2)^(1/2))))
```

Maxima [A]
time = 3.44, size = 60, normalized size = 0.71

$$\frac{5}{16}a^6\arcsin\left(\frac{x}{a}\right)+\frac{5}{16}\sqrt{a^2-x^2}\,a^4x+\frac{5}{24}\left(a^2-x^2\right)^{\frac{3}{2}}a^2x+\frac{1}{6}\left(a^2-x^2\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*arcsin(x/a) + 5/16*sqrt(a^2 - x^2)*a^4*x + 5/24*(a^2 - x^2)^(3/2)*
a^2*x + 1/6*(a^2 - x^2)^(5/2)*x
```

Fricas [A]
time = 0.70, size = 60, normalized size = 0.71

$$-\frac{5}{8}a^6\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\frac{1}{48}\left(33\,a^4x-26\,a^2x^3+8\,x^5\right)\sqrt{a^2-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -5/8*a^6*arctan(-(a - sqrt(a^2 - x^2))/x) + 1/48*(33*a^4*x - 26*a^2*x^3 + 8
*x^5)*sqrt(a^2 - x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.33, size = 180, normalized size = 2.14

$$\begin{cases} -\frac{5ia^6\operatorname{acosh}\left(\frac{x}{a}\right)}{16}-\frac{11ia^5x}{16\sqrt{-1+\frac{x^2}{a^2}}}+\frac{59ia^3x^3}{48\sqrt{-1+\frac{x^2}{a^2}}}-\frac{17iax^5}{24\sqrt{-1+\frac{x^2}{a^2}}}+\frac{ix^7}{6a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right|>1 \\ \frac{5a^6\operatorname{asin}\left(\frac{x}{a}\right)}{16}+\frac{11a^5x\sqrt{1-\frac{x^2}{a^2}}}{16}-\frac{13a^3x^3\sqrt{1-\frac{x^2}{a^2}}}{24}+\frac{ax^5\sqrt{1-\frac{x^2}{a^2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(5/2),x)
```

```
[Out] Piecewise((-5*I*a**6*acosh(x/a)/16 - 11*I*a**5*x/(16*sqrt(-1 + x**2/a**2))
+ 59*I*a**3*x**3/(48*sqrt(-1 + x**2/a**2)) - 17*I*a*x**5/(24*sqrt(-1 + x**2
/a**2)) + I*x**7/(6*a*sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1), (5*a**6*a
sin(x/a)/16 + 11*a**5*x*sqrt(1 - x**2/a**2)/16 - 13*a**3*x**3*sqrt(1 - x**2
/a**2)/24 + a*x**5*sqrt(1 - x**2/a**2)/6, True))
```

Giac [A]
time = 0.49, size = 50, normalized size = 0.60

$$\frac{5}{16}a^6\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)+\frac{1}{48}\left(33\,a^4-2\left(13\,a^2-4\,x^2\right)x^2\right)\sqrt{a^2-x^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/16*a^6*arcsin(x/a)*sgn(a) + 1/48*(33*a^4 - 2*(13*a^2 - 4*x^2)*x^2)*sqrt(a
^2 - x^2)*x
```

Mupad [B]
time = 0.21, size = 37, normalized size = 0.44

$$\frac{x\,\left(a^2-x^2\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{x^2}{a^2}\right)}{\left(1-\frac{x^2}{a^2}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 - x^2)^(5/2),x)
```

```
[Out] (x*(a^2 - x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, x^2/a^2))/(1 - x^2/a^2)^(5
/2)
```

3.40 $\int \frac{x^5}{\sqrt{5+x^2}}\,dx$

Optimal. Leaf size=38

$$25\sqrt{5+x^2}-\frac{10}{3}\left(5+x^2\right)^{3/2}+\frac{1}{5}\left(5+x^2\right)^{5/2}$$

```
[Out] -10/3*(x^2+5)^(3/2)+1/5*(x^2+5)^(5/2)+25*(x^2+5)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {272, 45}

$$\frac{1}{5}\left(x^2+5\right)^{5/2}-\frac{10}{3}\left(x^2+5\right)^{3/2}+25\sqrt{x^2+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[5 + x^2],x]
```

```
[Out] 25*Sqrt[5 + x^2] - (10*(5 + x^2)^(3/2))/3 + (5 + x^2)^(5/2)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{5+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\sqrt{5+x}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{25}{\sqrt{5+x}}-10\sqrt{5+x}+(5+x)^{3/2}\right)\,dx,x,x^2\right)\\
&= 25\sqrt{5+x^2}-\frac{10}{3}\left(5+x^2\right)^{3/2}+\frac{1}{5}\left(5+x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.66

$$\frac{1}{15}\sqrt{5+x^2}\left(200-20x^2+3x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[5 + x^2],x]
[Out] (Sqrt[5 + x^2]*(200 - 20*x^2 + 3*x^4))/15
```

Maple [A]
time = 0.07, size = 35, normalized size = 0.92

method	result	size
trager	$\sqrt{x^2+5}\,\left(\frac{1}{5}x^4-\frac{4}{3}x^2+\frac{40}{3}\right)$	21
gosper	$\frac{\sqrt{x^2+5}\,\left(3x^4-20x^2+200\right)}{15}$	22
risch	$\frac{\sqrt{x^2+5}\,\left(3x^4-20x^2+200\right)}{15}$	22
default	$\frac{x^4\sqrt{x^2+5}}{5}-\frac{4x^2\sqrt{x^2+5}}{3}+\frac{40\sqrt{x^2+5}}{3}$	35
meijerg	$\frac{25\sqrt{5}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,\left(\frac{6}{25}x^4-\frac{8}{5}x^2+16\right)\sqrt{1+\frac{x^2}{5}}}{15}\right)}{2\sqrt{\pi}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*x^4*(x^2+5)^(1/2)-4/3*x^2*(x^2+5)^(1/2)+40/3*(x^2+5)^(1/2)
```

Maxima [A]
time = 2.90, size = 34, normalized size = 0.89

$$\frac{1}{5}\sqrt{x^2+5}\,x^4-\frac{4}{3}\sqrt{x^2+5}\,x^2+\frac{40}{3}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="maxima")
[Out] 1/5*sqrt(x^2 + 5)*x^4 - 4/3*sqrt(x^2 + 5)*x^2 + 40/3*sqrt(x^2 + 5)
```

Fricas [A]
time = 0.61, size = 21, normalized size = 0.55

$$\frac{1}{15}\left(3\,x^4-20\,x^2+200\right)\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 - 20*x^2 + 200)*sqrt(x^2 + 5)
```

Sympy [A]
time = 0.23, size = 39, normalized size = 1.03

$$\frac{x^4\sqrt{x^2+5}}{5} - \frac{4x^2\sqrt{x^2+5}}{3} + \frac{40\sqrt{x^2+5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**2+5)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 5)/5 - 4*x**2*sqrt(x**2 + 5)/3 + 40*sqrt(x**2 + 5)/3
```

Giac [A]
time = 0.51, size = 28, normalized size = 0.74

$$\frac{1}{5}\left(x^2+5\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2+5\right)^{\frac{3}{2}} + 25\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^2 + 5)^(5/2) - 10/3*(x^2 + 5)^(3/2) + 25*sqrt(x^2 + 5)
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.53

$$\sqrt{x^2+5}\,\left(\frac{x^4}{5}-\frac{4\,x^2}{3}+\frac{40}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2 + 5)^(1/2),x)
```

```
[Out] (x^2 + 5)^(1/2)*(x^4/5 - (4*x^2)/3 + 40/3)
```

3.41 $\int \frac{t^3}{\sqrt{4+t^3}}\,dt$

Optimal. Leaf size=172

$$\frac{2}{5}t\sqrt{4+t^3}-\frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\ (2^{2/3}+t)\sqrt{\frac{2\sqrt[3]{2}-2^{2/3}t+t^2}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\ F\left(\sin^{-1}\left(\frac{2^{2/3}\left(1-\sqrt{3}\right)+t}{2^{2/3}\left(1+\sqrt{3}\right)+t}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\ \sqrt{4+t^3}}$$

```
[Out] 2/5*t*(t^3+4)^(1/2)-8/15*2^(2/3)*(2^(2/3)+t)*EllipticF((t+2^(2/3)*(1-3^(1/2
)))/(t+2^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((2*2^
(1/3)-2^(2/3)*t+t^2)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(t^3+4)^(1/2)
/((2^(2/3)+t)/(t+2^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {327, 224}

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\ (t+2^{2/3})\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\ \text{EllipticF}\left(\text{ArcSin}\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\ \sqrt{t^3+4}}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t)*Sqrt[(2*
2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*EllipticF[ArcSin[
(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)], -7 - 4*Sqrt[3]])/
(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}t\sqrt{4+t^3} - \frac{8}{5}\int \frac{1}{\sqrt{4+t^3}}\,dt$$

$$= \frac{2}{5}t\sqrt{4+t^3} - \frac{8\,2^{2/3}\sqrt{2+\sqrt{3}}\,\left(2^{2/3}+t\right)\sqrt{\frac{2\sqrt[3]{2}-2^{2/3}t+t^2}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\,F\left(\sin^{-1}\left(\frac{2^{2/3}\left(1-\sqrt{}\right.}{2^{2/3}\left(1+\sqrt{}\right.}\right.\right.}{5\sqrt[4]{3}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}\left(1+\sqrt{3}\right)+t\right)^2}}\sqrt{4+t^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 34, normalized size = 0.20

$$\frac{2}{5}t\left(\sqrt{4+t^3} - 2\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{t^3}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*(Sqrt[4 + t^3] - 2*Hypergeometric2F1[1/3, 1/2, 4/3, -1/4*t^3]))/5
```

Maple [A]
time = 0.10, size = 168, normalized size = 0.98

method	result
meijerg	$\frac{t^4\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{4}{3}\right],\left[\frac{7}{3}\right],-\frac{t^3}{4}\right)}{8}$

default	$\frac{2t\sqrt{t^3+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)}}{15\sqrt{t^3+4}}$
risch	$\frac{2t\sqrt{t^3+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)}}{15\sqrt{t^3+4}}$
elliptic	$\frac{2t\sqrt{t^3+4}}{5}+\frac{8i\sqrt{3}\,2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)\sqrt{3}\,2^{\frac{1}{3}}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i\sqrt{3}\,2^{\frac{2}{3}}}{2}\right)}}{15\sqrt{t^3+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/(t^3+4)^(1/2),t,method=_RETURNVERBOSE)
```

```
[Out] 2/5*t*(t^3+4)^(1/2)+8/15*I*3^(1/2)*2^(2/3)*(I*(t-1/2*2^(2/3)-1/2*I*3^(1/2)*
2^(2/3))*3^(1/2)*2^(1/3))^(1/2)*((2^(2/3)+t)/(3/2*2^(2/3)+1/2*I*3^(1/2)*2^(
2/3)))^(1/2)*(-I*(t-1/2*2^(2/3)+1/2*I*3^(1/2)*2^(2/3))*3^(1/2)*2^(1/3))^(1/
2)/(t^3+4)^(1/2)*EllipticF(1/6*6^(1/2)*(I*(t-1/2*2^(2/3)-1/2*I*3^(1/2)*2^(2
/3))*3^(1/2)*2^(1/3))^(1/2),(I*3^(1/2)*2^(2/3)/(3/2*2^(2/3)+1/2*I*3^(1/2)*2
^(2/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="maxima")
```

```
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 17, normalized size = 0.10

$$\frac{2}{5}\sqrt{t^3+4}\,t-\frac{16}{5}\,\text{weierstrassPInverse}(0,-16,t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="fricas")
```

```
[Out] 2/5*sqrt(t^3 + 4)*t - 16/5*weierstrassPInverse(0, -16, t)
```

Sympy [A]
time = 0.34, size = 31, normalized size = 0.18

$$\frac{t^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{t^3e^{i\pi}}{4}\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/(t**3+4)**(1/2),t)
```

```
[Out] t**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), t**3*exp_polar(I*pi)/4)/(6*gamma(
7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="giac")
```

```
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

Mupad [B]
time = 0.08, size = 301, normalized size = 1.75

$$\frac{2\,t\,\sqrt{t^3+4}}{5}-\frac{16\,\sqrt{-\frac{t-2^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2^{2/3}+2^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\,\sqrt{-\frac{t+2^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2^{2/3}-2^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\,\sqrt{\frac{t+2^{2/3}}{2^{2/3}+2^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\,\left(2^{2/3}+2^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{t+2^{2/3}}{2^{2/3}+2^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\right)\middle|\frac{2^{2/3}+2^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2^{2/3}-2^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}\right)}{5\,\sqrt{t^3+\left(2^{2/3}+2^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-2^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,t^2+\left(2\,2^{1/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-2\,2^{1/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-2\,2^{1/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,t-4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/(t^3 + 4)^(1/2),t)
```

```
[Out] (2*t*(t^3 + 4)^(1/2))/5 - (16*(-(t - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^(2/
3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(-(t + 2^(2/3)*((3^(1/2)*1i)/2
- 1/2))/(2^(2/3) - 2^(2/3)*((3^(1/2)*1i)/2 - 1/2)))^(1/2)*((t + 2^(2/3))/(2
^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)))^(1/2)*(2^(2/3) + 2^(2/3)*((3^(1/2
)*1i)/2 + 1/2))*ellipticF(asin(((t + 2^(2/3))/(2^(2/3) + 2^(2/3)*((3^(1/2)*
1i)/2 + 1/2)))^(1/2)), (2^(2/3) + 2^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(2^(2/3)
```

```
- 2^(2/3)*((3^(1/2)*1i)/2 - 1/2))))/(5*(t^2*(2^(2/3) + 2^(2/3)*((3^(1/2)*1i
)/2 - 1/2) - 2^(2/3)*((3^(1/2)*1i)/2 + 1/2)) - 4*((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) + t^3 - t*(2*2^(1/3)*((3^(1/2)*1i)/2 + 1/2) - 2*2^(1/3)
*((3^(1/2)*1i)/2 - 1/2) + 2*2^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2)))^(1/2))
```

3.42 $\int \tan^2(x)\,dx$

Optimal. Leaf size=6

$$-x+\tan(x)$$

```
[Out] -x+tan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$\tan(x)-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\,dx &= \tan(x)-\int 1\,dx\\ &= -x+\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$-x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Maple [A]
time = 0.01, size = 9, normalized size = 1.50

method	result	size
norman	$-x+\tan\left(x\right)$	7
derivativedivides	$\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	9
default	$\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	9
risch	$-x+\frac{2i}{e^{2ix}+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x,method=_RETURNVERBOSE)
[Out] tan(x)-arctan(tan(x))
```

Maxima [A]
time = 3.17, size = 6, normalized size = 1.00

$$-x+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
[Out] -x + tan(x)
```

Fricas [A]
time = 0.79, size = 6, normalized size = 1.00

$$-x+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
[Out] -x + tan(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.
time = 0.01, size = 7, normalized size = 1.17

$$-x+\frac{\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2,x)
```

```
[Out] -x + sin(x)/cos(x)
```

Giac [A]
time = 0.50, size = 6, normalized size = 1.00

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="giac")
[Out] -x + tan(x)
```

Mupad [B]
time = 0.07, size = 6, normalized size = 1.00

$$\tan(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
[Out] tan(x) - x
```

3.43 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x-\tan(x)+\frac{\tan^3(x)}{3}$$

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4,x]
```

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(x)\,dx &= \frac{\tan^3(x)}{3}-\int \tan^2(x)\,dx\\ &= -\tan(x)+\frac{\tan^3(x)}{3}+\int 1\,dx\\ &= x-\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.29

$$x-\frac{4\tan(x)}{3}+\frac{1}{3}\sec^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4,x]
```

```
[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A]
time = 0.01, size = 15, normalized size = 1.07

method	result	size
norman	$x-\tan\left(x\right)+\frac{\left(\tan^{3}\left(x\right)\right)}{3}$	13
derivativedivides	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
default	$\frac{\left(\tan^{3}\left(x\right)\right)}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)$	15
risch	$x-\frac{4i\left(3\,e^{4ix}+3\,e^{2ix}+2\right)}{3\left(e^{2ix}+1\right)^{3}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tan(x)^3-tan(x)+arctan(tan(x))
```

Maxima [A]
time = 3.18, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan\left(x\right)^{3}+x-\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Fricas [A]
time = 1.08, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan\left(x\right)^{3}+x-\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A]
time = 0.01, size = 19, normalized size = 1.36

$$x+\frac{\sin^{3}\left(x\right)}{3\cos^{3}\left(x\right)}-\frac{\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

Giac [A]
time = 0.44, size = 12, normalized size = 0.86

$$\frac{1}{3}\tan\left(x\right)^{3} + x - \tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.86

$$\frac{\tan(x)^3}{3} - \tan(x) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
```

```
[Out] x - tan(x) + tan(x)^3/3
```

3.44 $\int \cot^2(x)\,dx$

Optimal. Leaf size=8

$$-x-\cot(x)$$

```
[Out] -x-cot(x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \int \cot^2(x)\,dx &= -\cot(x)-\int 1\,dx \\ &= -x-\cot(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Maple [A]
time = 0.01, size = 14, normalized size = 1.75

method	result	size
norman	$\frac{-1-x\tan(x)}{\tan(x)}$	13
derivativedivides	$-\cot(x)+\frac{\pi}{2}-\mathrm{arccot}(\cot(x))$	14
default	$-\cot(x)+\frac{\pi}{2}-\mathrm{arccot}(\cot(x))$	14
risch	$-x-\frac{2i}{\mathrm{e}^{2ix}-1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -cot(x)+1/2*Pi-arccot(cot(x))
```

Maxima [A]
time = 2.44, size = 10, normalized size = 1.25

$$-x-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="maxima")
```

```
[Out] -x - 1/tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.
time = 1.21, size = 20, normalized size = 2.50

$$-\frac{x\sin(2x)+\cos(2x)+1}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 1.00

$$-x-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2,x)
```

```
[Out] -x - cos(x)/sin(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.
time = 0.46, size = 18, normalized size = 2.25

$$-x - \frac{1}{2 \tan\left(\frac{1}{2} x\right)} + \frac{1}{2} \tan\left(\frac{1}{2} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

Mupad [B]
time = 0.06, size = 8, normalized size = 1.00

$$-x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)
```

```
[Out] - x - cot(x)
```

3.45 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x+\cot(x)-\frac{\cot^3(x)}{3}$$

```
[Out] x+cot(x)-1/3*cot(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\ &= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\ &= x+\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.50

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A]
time = 0.03, size = 16, normalized size = 1.33

method	result	size
derivativedivides	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
default	$-\frac{\left(\cot^{3}\left(x\right)\right)}{3}+\cot\left(x\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(x\right)\right)$	16
norman	$\frac{-\frac{1}{3}+\tan^{2}\left(x\right)+x\left(\tan^{3}\left(x\right)\right)}{\tan\left(x\right)^{3}}$	18
risch	$x+\frac{4 i \left(3\,{\mathrm e}^{4 i x}-3\,{\mathrm e}^{2 i x}+2\right)}{3 \left({\mathrm e}^{2 i x}-1\right)^{3}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+arccot(cot(x))
```

Maxima [A]
time = 1.95, size = 16, normalized size = 1.33

$$x+\frac{3\,\tan\left(x\right)^{2}-1}{3\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.
time = 1.72, size = 48, normalized size = 4.00

$$\frac{4\,\cos\left(2\,x\right)^{2}+3\,\left(x\cos\left(2\,x\right)-x\right)\sin\left(2\,x\right)+2\,\cos\left(2\,x\right)-2}{3\,\left(\cos\left(2\,x\right)-1\right)\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A]
time = 0.02, size = 19, normalized size = 1.58

$$x+\frac{\cos(x)}{\sin(x)}-\frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(10) = 20.
time = 0.49, size = 34, normalized size = 2.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3+x+\frac{15\tan\left(\frac{1}{2}x\right)^2-1}{24\tan\left(\frac{1}{2}x\right)^3}-\frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$-\frac{\cot(x)^3}{3}+\cot(x)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] x + cot(x) - cot(x)^3/3
```

3.46 $\int (2+3x)\sin(5x)\,dx$

Optimal. Leaf size=22

$$-\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{25}\sin(5x)$$

```
[Out] -1/5*(2+3*x)*cos(5*x)+3/25*sin(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3377, 2717}

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)*Sin[5*x],x]
```

```
[Out] -1/5*((2 + 3*x)*Cos[5*x]) + (3*Sin[5*x])/25
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (2+3x)\sin(5x)\,dx &= -\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{5}\int \cos(5x)\,dx\\ &= -\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{25}\sin(5x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.18

$$-\frac{2}{5}\cos(5x)-\frac{3}{5}x\cos(5x)+\frac{3}{25}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)*Sin[5*x],x]
[Out] (-2*Cos[5*x])/5 - (3*x*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.95

method	result	size
risch	$\left(-\frac{2}{5}-\frac{3x}{5}\right)\cos\left(5x\right)+\frac{3\sin(5x)}{25}$	18
derivativedivides	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3\cos(5x)x}{5}$	21
default	$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3\cos(5x)x}{5}$	21
norman	$\frac{-\frac{3x}{5}+\frac{3x\left(\tan^2\left(\frac{5x}{2}\right)\right)}{5}+\frac{6\tan\left(\frac{5x}{2}\right)}{25}-\frac{4}{5}}{1+\tan^2\left(\frac{5x}{2}\right)}$	32
meijerg	$\frac{2\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(5x)}{\sqrt{\pi}}\right)}{5}+\frac{6\sqrt{\pi}\left(-\frac{5x\cos(5x)}{2\sqrt{\pi}}+\frac{\sin(5x)}{2\sqrt{\pi}}\right)}{25}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*sin(5*x),x,method=_RETURNVERBOSE)
[Out] -2/5*cos(5*x)+3/25*sin(5*x)-3/5*cos(5*x)*x
```

Maxima [A]
time = 1.06, size = 20, normalized size = 0.91

$$-\frac{3}{5}\,x\cos\left(5\,x\right)-\frac{2}{5}\,\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="maxima")
[Out] -3/5*x*cos(5*x) - 2/5*cos(5*x) + 3/25*sin(5*x)
```

Fricas [A]
time = 2.31, size = 18, normalized size = 0.82

$$-\frac{1}{5}\left(3\,x+2\right)\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="fricas")
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

Sympy [A]
time = 0.06, size = 26, normalized size = 1.18

$$-\frac{3x\cos\left(5x\right)}{5}+\frac{3\sin\left(5x\right)}{25}-\frac{2\cos\left(5x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x)
[Out] -3*x*cos(5*x)/5 + 3*sin(5*x)/25 - 2*cos(5*x)/5
```

Giac [A]
time = 0.51, size = 18, normalized size = 0.82

$$-\frac{1}{5}\left(3\,x+2\right)\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="giac")
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

Mupad [B]
time = 0.10, size = 20, normalized size = 0.91

$$\frac{3\,\sin\left(5\,x\right)}{25}-\frac{2\,\cos\left(5\,x\right)}{5}-\frac{3\,x\,\cos\left(5\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)*(3*x + 2),x)
[Out] (3*sin(5*x))/25 - (2*cos(5*x))/5 - (3*x*cos(5*x))/5
```

3.47 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] 1/3*(x^2+1)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{267\}$

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Maple [A]
time = 0.06, size = 10, normalized size = 0.77

method	result	size
gosper	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
derivativedivides	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
default	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
risch	$\frac{(x^2+1)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2+1}$	16
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2x^2+2)\sqrt{x^2+1}}{3}}{4\sqrt{\pi}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A]
time = 2.20, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A]
time = 1.65, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(8) = 16.
time = 0.06, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Giac [A]
time = 0.50, size = 9, normalized size = 0.69

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.69

$$\frac{\left(x^2+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^(1/2),x)
```

```
[Out] (x^2 + 1)^(3/2)/3
```

3.48 $\int x(-1+x^2)^9 \, dx$

Optimal. Leaf size=13

$$\frac{1}{20}(1-x^2)^{10}$$

```
[Out] 1/20*(-x^2+1)^10
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {267}

$$\frac{1}{20}(1-x^2)^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^9,x]
[Out] (1 - x^2)^10/20
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x(-1+x^2)^9 \, dx = \frac{1}{20}(1-x^2)^{10}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{20}(-1+x^2)^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^9,x]
[Out] (-1 + x^2)^10/20
```

Maple [A]
time = 0.06, size = 10, normalized size = 0.77

method	result	size
default	$\frac{(x^2-1)^{10}}{20}$	10
gosper	$\frac{x^2(x^{18}-10x^{16}+45x^{14}-120x^{12}+210x^{10}-252x^8+210x^6-120x^4+45x^2-10)}{20}$	51
norman	$-\frac{1}{2}x^2+\frac{9}{4}x^4-6x^6+\frac{21}{2}x^8-\frac{63}{5}x^{10}+\frac{21}{2}x^{12}-6x^{14}+\frac{9}{4}x^{16}-\frac{1}{2}x^{18}+\frac{1}{20}x^{20}$	52
risch	$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^8-6x^6+\frac{9}{4}x^4-\frac{1}{2}x^2+\frac{1}{20}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^9,x,method=_RETURNVERBOSE)
[Out] 1/20*(x^2-1)^10
```

Maxima [A]
time = 2.56, size = 9, normalized size = 0.69

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="maxima")
[Out] 1/20*(x^2 - 1)^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(9) = 18$.
time = 0.96, size = 51, normalized size = 3.92

$$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^8-6x^6+\frac{9}{4}x^4-\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="fricas")
[Out] 1/20*x^20 - 1/2*x^18 + 9/4*x^16 - 6*x^14 + 21/2*x^12 - 63/5*x^10 + 21/2*x^8
 - 6*x^6 + 9/4*x^4 - 1/2*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(7) = 14$.
time = 0.01, size = 58, normalized size = 4.46

$$\frac{x^{20}}{20}-\frac{x^{18}}{2}+\frac{9x^{16}}{4}-6x^{14}+\frac{21x^{12}}{2}-\frac{63x^{10}}{5}+\frac{21x^8}{2}-6x^6+\frac{9x^4}{4}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**9,x)
```

```
[Out] x**20/20 - x**18/2 + 9*x**16/4 - 6*x**14 + 21*x**12/2 - 63*x**10/5 + 21*x**
8/2 - 6*x**6 + 9*x**4/4 - x**2/2
```

Giac [A]
time = 0.45, size = 9, normalized size = 0.69

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="giac")
```

```
[Out] 1/20*(x^2 - 1)^10
```

Mupad [B]
time = 0.10, size = 9, normalized size = 0.69

$$\frac{\left(x^2-1\right)^{10}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 - 1)^9,x)
```

```
[Out] (x^2 - 1)^10/20
```

3.49 $\int \frac{3+2x}{(7+6x)^3} \, dx$

Optimal. Leaf size=18

$$-\frac{(3+2x)^2}{8(7+6x)^2}$$

[Out] -1/8*(3+2*x)^2/(7+6*x)^2

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {37}

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/(7 + 6*x)^3,x]

[Out] -1/8*(3 + 2*x)^2/(7 + 6*x)^2

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{3+2x}{(7+6x)^3} \, dx = -\frac{(3+2x)^2}{8(7+6x)^2}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.89

$$-\frac{4+3x}{9(7+6x)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/(7 + 6*x)^3,x]

[Out] -1/9*(4 + 3*x)/(7 + 6*x)^2

Maple [A]
time = 0.06, size = 20, normalized size = 1.11

method	result	size
norman	$\frac{-\frac{x}{3}-\frac{4}{9}}{(7+6x)^2}$	14
gosper	$-\frac{3x+4}{9(7+6x)^2}$	15
risch	$\frac{-\frac{x}{3}-\frac{4}{9}}{(7+6x)^2}$	15
default	$-\frac{1}{18(7+6x)}-\frac{1}{18(7+6x)^2}$	20
meijerg	$\frac{3x\left(\frac{6x}{7}+2\right)}{686\left(1+\frac{6x}{7}\right)^2}+\frac{x^2}{343\left(1+\frac{6x}{7}\right)^2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(7+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18/(7+6*x)-1/18/(7+6*x)^2
```

Maxima [A]
time = 1.34, size = 19, normalized size = 1.06

$$-\frac{3x+4}{9(36x^2+84x+49)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Fricas [A]
time = 2.03, size = 19, normalized size = 1.06

$$-\frac{3x+4}{9(36x^2+84x+49)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.83

$$\frac{-3x-4}{324x^2+756x+441}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)**3,x)
```

```
[Out] (-3*x - 4)/(324*x**2 + 756*x + 441)
```

Giac [A]
time = 0.48, size = 14, normalized size = 0.78

$$-\frac{3\,x+4}{9\,{\left(6\,x+7\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/9*(3*x + 4)/(6*x + 7)^2
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.78

$$-\frac{3\,x+4}{9\,{\left(6\,x+7\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/(6*x + 7)^3,x)
```

```
[Out] -(3*x + 4)/(9*(6*x + 7)^2)
```

3.50 $\int x^4(1+x^5)^5\,dx$

Optimal. Leaf size=11

$$\frac{1}{30}(1+x^5)^6$$

```
[Out] 1/30*(x^5+1)^6
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {267}

$$\frac{1}{30}(x^5+1)^6$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^5)^5,x]
```

```
[Out] (1 + x^5)^6/30
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^4(1+x^5)^5\,dx = \frac{1}{30}(1+x^5)^6$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(11) = 22.
time = 0.00, size = 43, normalized size = 3.91

$$\frac{x^5}{5}+\frac{x^{10}}{2}+\frac{2x^{15}}{3}+\frac{x^{20}}{2}+\frac{x^{25}}{5}+\frac{x^{30}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^5)^5,x]
```

```
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

Maple [A]
time = 0.06, size = 10, normalized size = 0.91

method	result	size
default	$\frac{(x^5+1)^6}{30}$	10
gosper	$\frac{x^5(x^{25}+6x^{20}+15x^{15}+20x^{10}+15x^5+6)}{30}$	31
norman	$\frac{1}{5}x^5+\frac{1}{2}x^{10}+\frac{2}{3}x^{15}+\frac{1}{2}x^{20}+\frac{1}{5}x^{25}+\frac{1}{30}x^{30}$	32
risch	$\frac{1}{30}x^{30}+\frac{1}{5}x^{25}+\frac{1}{2}x^{20}+\frac{2}{3}x^{15}+\frac{1}{2}x^{10}+\frac{1}{5}x^5+\frac{1}{30}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*(x^5+1)^6
```

Maxima [A]
time = 1.30, size = 9, normalized size = 0.82

$$\frac{1}{30}\left(x^5+1\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="maxima")
```

```
[Out] 1/30*(x^5 + 1)^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.
time = 1.11, size = 31, normalized size = 2.82

$$\frac{1}{30}x^{30}+\frac{1}{5}x^{25}+\frac{1}{2}x^{20}+\frac{2}{3}x^{15}+\frac{1}{2}x^{10}+\frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="fricas")
```

```
[Out] 1/30*x^30 + 1/5*x^25 + 1/2*x^20 + 2/3*x^15 + 1/2*x^10 + 1/5*x^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(7) = 14$.
time = 0.01, size = 31, normalized size = 2.82

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5+1)**5,x)
```

```
[Out] x**30/30 + x**25/5 + x**20/2 + 2*x**15/3 + x**10/2 + x**5/5
```

Giac [A]
time = 0.44, size = 9, normalized size = 0.82

$$\frac{1}{30}\left(x^{5}+1\right)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="giac")
```

```
[Out] 1/30*(x^5 + 1)^6
```

Mupad [B]
time = 0.03, size = 31, normalized size = 2.82

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2\,x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5 + 1)^5,x)
```

```
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

3.51 $\int (1-x)^{20} x^4 \, dx$

Optimal. Leaf size=56

$$-\frac{1}{21}(1-x)^{21}+\frac{2}{11}(1-x)^{22}-\frac{6}{23}(1-x)^{23}+\frac{1}{6}(1-x)^{24}-\frac{1}{25}(1-x)^{25}$$

```
[Out] -1/21*(1-x)^21+2/11*(1-x)^22-6/23*(1-x)^23+1/6*(1-x)^24-1/25*(1-x)^25
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^20*x^4,x]
```

```
[Out] -1/21*(1 - x)^21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^24/6 - (
1 - x)^25/25
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{20} x^4 \, dx &= \int \left((1-x)^{20}-4(1-x)^{21}+6(1-x)^{22}-4(1-x)^{23}+(1-x)^{24}\right) \, dx \\
&= -\frac{1}{21}(1-x)^{21}+\frac{2}{11}(1-x)^{22}-\frac{6}{23}(1-x)^{23}+\frac{1}{6}(1-x)^{24}-\frac{1}{25}(1-x)^{25}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(56) = 112.
time = 0.00, size = 140, normalized size = 2.50

$$\frac{x^5}{5}-\frac{10x^6}{3}+\frac{190x^7}{7}-\frac{285x^8}{2}+\frac{1615x^9}{3}-\frac{7752x^{10}}{5}+\frac{38760x^{11}}{11}-6460x^{12}+9690x^{13}-\frac{83980x^{14}}{7}+\frac{184756x^{15}}{15}-\frac{20995x^{16}}{2}+7410x^{17}-\frac{12920x^{18}}{3}+2040x^{19}-\frac{3876x^{20}}{5}+\frac{1615x^{21}}{7}-\frac{570x^{22}}{11}+\frac{190x^{23}}{23}-\frac{5x^{24}}{6}+\frac{x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^20*x^4,x]
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10)
/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x^1
5)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*x^2
0)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.
time = 0.07, size = 107, normalized size = 1.91

method	result
gosper	$\frac{x^5(10626x^{20}-221375x^{19}+2194500x^{18}-13765500x^{17}+61289250x^{16}-205931880x^{15}+541926000x^{14}-1144066000x^{13}+1968466500x}{}$
default	$-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}+\frac{184756}{15}x^{15}-\frac{20995}{2}x^{16}+7410x^{17}-\frac{12920}{3}x^{18}+2040x^{19}-\frac{3876}{5}x^{20}$
norman	$-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}+\frac{184756}{15}x^{15}-\frac{20995}{2}x^{16}+7410x^{17}-\frac{12920}{3}x^{18}+2040x^{19}-\frac{3876}{5}x^{20}$
risch	$-6460x^{12}+9690x^{13}-\frac{83980}{7}x^{14}+\frac{184756}{15}x^{15}-\frac{20995}{2}x^{16}+7410x^{17}-\frac{12920}{3}x^{18}+2040x^{19}-\frac{3876}{5}x^{20}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^20*x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -6460*x^12+9690*x^13-83980/7*x^14+184756/15*x^15-20995/2*x^16+7410*x^17-129
20/3*x^18+2040*x^19-3876/5*x^20+1615/7*x^21-570/11*x^22+190/23*x^23-5/6*x^2
4+1/25*x^25+1615/3*x^9-7752/5*x^10+38760/11*x^11-10/3*x^6+190/7*x^7-285/2*x
^8+1/5*x^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.
time = 0.77, size = 106, normalized size = 1.89

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040x^{19}-\frac{12920}{3}x^{18}+7410x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}-\frac{83980}{7}x^{14}+9690x^{13}-6460x^{12}+\frac{38760}{11}x^{11}-\frac{7752}{5}x^{10}+\frac{1615}{3}x^{9}-\frac{285}{2}x^{8}+\frac{190}{7}x^{7}-\frac{10}{3}x^{6}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.
time = 0.93, size = 106, normalized size = 1.89

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040x^{19}-\frac{12920}{3}x^{18}+7410x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}-\frac{83980}{7}x^{14}+9690x^{13}-6460x^{12}+\frac{38760}{11}x^{11}-\frac{7752}{5}x^{10}+\frac{1615}{3}x^{9}-\frac{285}{2}x^{8}+\frac{190}{7}x^{7}-\frac{10}{3}x^{6}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.
time = 0.02, size = 131, normalized size = 2.34

$$\frac{x^{25}}{25} - \frac{5x^{24}}{6} + \frac{190x^{23}}{23} - \frac{570x^{22}}{11} + \frac{1615x^{21}}{7} - \frac{3876x^{20}}{5} + 2040x^{19} - \frac{12920x^{18}}{3} + 7410x^{17} - \frac{20995x^{16}}{2} + \frac{184756x^{15}}{15} - \frac{83980x^{14}}{7} + 9690x^{13} - 6460x^{12} + \frac{38760x^{11}}{11} - \frac{7752x^{10}}{5} + \frac{1615x^{9}}{3} - \frac{285x^{8}}{2} + \frac{190x^{7}}{7} - \frac{10x^{6}}{3} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**20*x**4,x)
```

```
[Out] x**25/25 - 5*x**24/6 + 190*x**23/23 - 570*x**22/11 + 1615*x**21/7 - 3876*x*
*20/5 + 2040*x**19 - 12920*x**18/3 + 7410*x**17 - 20995*x**16/2 + 184756*x*
*15/15 - 83980*x**14/7 + 9690*x**13 - 6460*x**12 + 38760*x**11/11 - 7752*x*
*10/5 + 1615*x**9/3 - 285*x**8/2 + 190*x**7/7 - 10*x**6/3 + x**5/5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.
time = 0.46, size = 106, normalized size = 1.89

$$\frac{1}{25}x^{25} - \frac{5}{6}x^{24} + \frac{190}{23}x^{23} - \frac{570}{11}x^{22} + \frac{1615}{7}x^{21} - \frac{3876}{5}x^{20} + 2040x^{19} - \frac{12920}{3}x^{18} + 7410x^{17} - \frac{20995}{2}x^{16} + \frac{184756}{15}x^{15} - \frac{83980}{7}x^{14} + 9690x^{13} - 6460x^{12} + \frac{38760}{11}x^{11} - \frac{7752}{5}x^{10} + \frac{1615}{3}x^{9} - \frac{285}{2}x^{8} + \frac{190}{7}x^{7} - \frac{10}{3}x^{6} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="giac")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Mupad [B]
time = 0.46, size = 106, normalized size = 1.89

$$\frac{x^{25}}{25} - \frac{5\,x^{24}}{6} + \frac{190\,x^{23}}{23} - \frac{570\,x^{22}}{11} + \frac{1615\,x^{21}}{7} - \frac{3876\,x^{20}}{5} + 2040\,x^{19} - \frac{12920\,x^{18}}{3} + 7410\,x^{17} - \frac{20995\,x^{16}}{2} + \frac{184756\,x^{15}}{15} - \frac{83980\,x^{14}}{7} + 9690\,x^{13} - 6460\,x^{12} + \frac{38760\,x^{11}}{11} - \frac{7752\,x^{10}}{5} + \frac{1615\,x^{9}}{3} - \frac{285\,x^{8}}{2} + \frac{190\,x^{7}}{7} - \frac{10\,x^{6}}{3} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x - 1)^20,x)
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10)
/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x^1
5)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*x^2
0)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

3.52 $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=4

$$\cos\left(\frac{1}{x}\right)$$

```
[Out] cos(1/x)
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3460, 2718}

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(-1)]/x^2,x]
```

```
[Out] Cos[x^(-1)]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \sin(x)\,dx, x, \frac{1}{x}\right)\\ &= \cos\left(\frac{1}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(-1)]/x^2,x]
```

```
[Out] Cos[x^(-1)]
```

Maple [A]
time = 0.01, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$\cos\left(\frac{1}{x}\right)$	5
default	$\cos\left(\frac{1}{x}\right)$	5
risch	$\cos\left(\frac{1}{x}\right)$	5
norman	$\frac{2}{1+\tan^{2}\left(\frac{1}{2x}\right)}$	15
meijerg	$-\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{1}{x}\right)}{\sqrt{\pi}}\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] cos(1/x)
```

Maxima [A]
time = 1.74, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] cos(1/x)
```

Fricas [A]
time = 0.69, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="fricas")
```

```
[Out] cos(1/x)
```

Sympy [A]
time = 0.17, size = 3, normalized size = 0.75

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x**2,x)
```

```
[Out] cos(1/x)
```

Giac [A]
time = 0.46, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="giac")
```

```
[Out] cos(1/x)
```

Mupad [B]
time = 0.10, size = 4, normalized size = 1.00

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/x)/x^2,x)
```

```
[Out] cos(1/x)
```

3.53 $\int \sin\left(\sqrt[4]{-1+x}\right)\,dx$

Optimal. Leaf size=62

$$24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)-4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)-24\sin\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)$$

```
[Out] 24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin((
-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3442, 3377, 2717}

$$12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)-4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] 24*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] - 4*(-1 + x)^(3/4)*Cos[(-1 + x)^(1/4)
] - 24*Sin[(-1 + x)^(1/4)] + 12*Sqrt[-1 + x]*Sin[(-1 + x)^(1/4)]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3442

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \sin\left(\sqrt[4]{-1+x}\right)\,dx = 4\text{Subst}\left(\int x^3\sin(x)\,dx, x, \sqrt[4]{-1+x}\right)$$
$$= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right) + 12\text{Subst}\left(\int x^2\cos(x)\,dx, x, \sqrt[4]{-1+x}\right)$$
$$= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right) + 12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right) - 24\text{Subst}\left(\int x\,\text{si}$$
$$= 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right) - 4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right) + 12\sqrt{-1+x}\sin\left(\right.$$
$$= 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right) - 4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right) - 24\sin\left(\sqrt[4]{-1+x}\right.$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.74

$$-4\left(-6+\sqrt{-1+x}\right)\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right) + 12\left(-2+\sqrt{-1+x}\right)\sin\left(\sqrt[4]{-1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] -4*(-6 + Sqrt[-1 + x])*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] + 12*(-2 + Sqrt[-
1 + x])*Sin[(-1 + x)^(1/4)]
```

Maple [A]
time = 0.01, size = 49, normalized size = 0.79

method	result
derivativedivides	$24(-1+x)^{\frac{1}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right) - 4(-1+x)^{\frac{3}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right) - 24\sin\left((-1+x)^{\frac{1}{4}}\right) +$
default	$24(-1+x)^{\frac{1}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right) - 4(-1+x)^{\frac{3}{4}}\cos\left((-1+x)^{\frac{1}{4}}\right) - 24\sin\left((-1+x)^{\frac{1}{4}}\right) +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-1+x)^(1/4)),x,method=_RETURNVERBOSE)
```

```
[Out] 24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin((
-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

Maxima [A]
time = 0.89, size = 37, normalized size = 0.60

$$-4\left((x-1)^{\frac{3}{4}} - 6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right) + 12\left(\sqrt{x-1} - 2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Fricas [A]
time = 1.16, size = 37, normalized size = 0.60

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Sympy [A]
time = 0.35, size = 60, normalized size = 0.97

$$-4(x-1)^{\frac{3}{4}}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)+12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)**(1/4)),x)
```

```
[Out] -4*(x - 1)**(3/4)*cos((x - 1)**(1/4)) + 24*(x - 1)**(1/4)*cos((x - 1)**(1/4
)) + 12*sqrt(x - 1)*sin((x - 1)**(1/4)) - 24*sin((x - 1)**(1/4))
```

Giac [A]
time = 0.42, size = 37, normalized size = 0.60

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="giac")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Mupad [B]
time = 0.23, size = 41, normalized size = 0.66

$$4\,\cos\left((x-1)^{1/4}\right)\,\left(6\,(x-1)^{1/4}-(x-1)^{3/4}\right)+4\,\sin\left((x-1)^{1/4}\right)\,\left(3\,\sqrt{x-1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x - 1)^(1/4)),x)
```

```
[Out] 4*cos((x - 1)^(1/4))*(6*(x - 1)^(1/4) - (x - 1)^(3/4)) + 4*sin((x - 1)^(1/4
))*(3*(x - 1)^(1/2) - 6)
```

3.54 $\int x \cos(x^2) \sin(x^2)\, dx$

Optimal. Leaf size=10

$$\frac{1}{4} \sin^2(x^2)$$

[Out] 1/4*sin(x^2)^2

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3522}

$$\frac{1}{4} \sin^2(x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x^2]*Sin[x^2],x]

[Out] Sin[x^2]^2/4

Rule 3522

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x \cos(x^2) \sin(x^2)\, dx = \frac{1}{4} \sin^2(x^2)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{4} \cos^2(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x^2]*Sin[x^2],x]

[Out] -1/4*Cos[x^2]^2

Maple [A]
time = 0.02, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-\frac{\left(\cos^{2}\left(x^{2}\right)\right)}{4}$	9
default	$-\frac{\left(\cos^{2}\left(x^{2}\right)\right)}{4}$	9
risch	$-\frac{\cos\left(2x^{2}\right)}{8}$	9
meijerg	$\frac{\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(2x^{2}\right)}{\sqrt{\pi}}\right)}{8}$	21
norman	$\frac{-\frac{\left(\tan^{4}\left(\frac{x^{2}}{2}\right)\right)}{2}-\frac{1}{2}}{\left(1+\tan^{2}\left(\frac{x^{2}}{2}\right)\right)^{2}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)*sin(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(x^2)^2
```

Maxima [A]
time = 0.95, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(x^{2}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/4*cos(x^2)^2
```

Fricas [A]
time = 0.93, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(x^{2}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/4*cos(x^2)^2
```

Sympy [A]
time = 0.08, size = 8, normalized size = 0.80

$$-\frac{\cos^{2}\left(x^{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)*sin(x**2),x)
```

```
[Out] -cos(x**2)**2/4
```

Giac [A]
time = 0.44, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/4*cos(x^2)^2
```

Mupad [B]
time = 0.05, size = 8, normalized size = 0.80

$$\frac{\sin\left(x^2\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)*sin(x^2),x)
```

```
[Out] sin(x^2)^2/4
```

3.55 $\int \sqrt{1+3\cos^2(x)}\,\sin(2x)\,dx$

Optimal. Leaf size=16

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

```
[Out] -2/9*(4-3*sin(x)^2)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {12, 267}

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+3\cos^2(x)}\,\sin(2x)\,dx &= \text{Subst}\left(\int 2x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= 2\text{Subst}\left(\int x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Maple [A]
time = 0.04, size = 13, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{2\left(1+3\left(\cos^{2}\left(x\right)\right)\right)^{\frac{3}{2}}}{9}$	13
default	$-\frac{2\left(1+3\left(\cos^{2}\left(x\right)\right)\right)^{\frac{3}{2}}}{9}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(1+3*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(1+3*cos(x)^2)^(3/2)
```

Maxima [A]
time = 0.83, size = 12, normalized size = 0.75

$$-\frac{2}{9}\left(3\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Fricas [A]
time = 1.19, size = 12, normalized size = 0.75

$$-\frac{2}{9}\left(3\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Sympy [A]
time = 0.98, size = 15, normalized size = 0.94

$$-\frac{2(3\cos^{2}\left(x\right)+1)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)**2)**(1/2),x)
```

```
[Out] -2*(3*cos(x)**2 + 1)**(3/2)/9
```

Giac [A]
time = 0.44, size = 12, normalized size = 0.75

$$-\frac{2}{9}\left(3\,\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Mupad [B]
time = 0.19, size = 12, normalized size = 0.75

$$-\frac{2\left(3\cos\left(x\right)^{2}+1\right)^{3/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(3*cos(x)^2 + 1)^(1/2),x)
```

```
[Out] -(2*(3*cos(x)^2 + 1)^(3/2))/9
```

3.56 $\int \frac{1}{2+3x}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(2+3x)$$

```
[Out] 1/3*ln(2+3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = $\{31\}$

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+3x}\,dx = \frac{1}{3}\log(2+3x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Maple [A]
time = 0.06, size = 9, normalized size = 0.90

method	result	size
default	$\frac{\ln(2+3x)}{3}$	9
norman	$\frac{\ln(2+3x)}{3}$	9
meijerg	$\frac{\ln\left(1+\frac{3x}{2}\right)}{3}$	9
risch	$\frac{\ln(2+3x)}{3}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(2+3*x)
```

Maxima [A]
time = 2.92, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(3*x + 2)
```

Fricas [A]
time = 0.93, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log\left(3x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

Giac [A]
time = 0.52, size = 9, normalized size = 0.90

$$\frac{1}{3}\log\left(|3x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2))
```

Mupad [B]
time = 0.07, size = 6, normalized size = 0.60

$$\frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + 2),x)
```

```
[Out] log(x + 2/3)/3
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x - 2x\log(x) + x\log^2(x)$$

[Out] 2*x-2*x*ln(x)+x*ln(x)^2

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2333, 2332}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

[In] Int[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$2x - 2x\log(x) + x\log^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]^2,x]

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A]
time = 0.02, size = 16, normalized size = 1.07

method	result	size
default	$2x - 2x \ln(x) + x \ln(x)^2$	16
norman	$2x - 2x \ln(x) + x \ln(x)^2$	16
risch	$2x - 2x \ln(x) + x \ln(x)^2$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A]
time = 3.28, size = 12, normalized size = 0.80

$$\left(\log(x)^2 - 2 \log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A]
time = 1.20, size = 15, normalized size = 1.00

$$x \log(x)^2 - 2 x \log(x) + 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="fricas")
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A]
time = 0.03, size = 15, normalized size = 1.00

$$x \log(x)^2 - 2x \log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Giac [**A**]
time = 0.44, size = 15, normalized size = 1.00

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Mupad [**B**]
time = 0.08, size = 12, normalized size = 0.80

$$x\left(\ln(x)^2 - 2\,\ln(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)^2,x)
```

```
[Out] x*(log(x)^2 - 2*log(x) + 2)
```

3.58 $\int x \log(x) \, dx$

Optimal. Leaf size=17

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2341}

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx=-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
[Out] -1/4*x^2 + (x^2*Log[x])/2
```

Maple [A]
time = 0.00, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
norman	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14
risch	$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A]
time = 2.44, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A]
time = 1.11, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.71

$$\frac{x^2\log\left(x\right)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A]
time = 0.47, size = 13, normalized size = 0.76

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.53

$$\frac{x^2\left(\ln\left(x\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x),x)
```

```
[Out] (x^2*(log(x) - 1/2))/2
```

3.59 $\int x \log^2(x) \, dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2342, 2341}

$$\frac{x^2}{4} + \frac{1}{2}x^2\log^2(x) - \frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\log^2(x)\,dx &= \frac{1}{2}x^2\log^2(x) - \int x\log(x)\,dx\\ &= \frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^2}{4} - \frac{1}{2}x^2\log(x) + \frac{1}{2}x^2\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A]
time = 0.00, size = 23, normalized size = 0.82

method	result	size
default	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
norman	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23
risch	$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2\ln(x)^2}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A]
time = 1.79, size = 17, normalized size = 0.61

$$\frac{1}{4}\left(2\,\log\left(x\right)^{2}-2\,\log\left(x\right)+1\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A]
time = 1.68, size = 22, normalized size = 0.79

$$\frac{1}{2}\,x^{2}\log\left(x\right)^{2}-\frac{1}{2}\,x^{2}\log\left(x\right)+\frac{1}{4}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.79

$$\frac{x^{2}\log\left(x\right)^{2}}{2}-\frac{x^{2}\log\left(x\right)}{2}+\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Giac [A]
time = 0.46, size = 22, normalized size = 0.79

$$\frac{1}{2}\,x^2\log\left(x\right)^2-\frac{1}{2}\,x^2\log\left(x\right)+\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.61

$$\frac{x^2\,\left(2\ln\left(x\right)^2-2\,\ln\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x)^2,x)
```

```
[Out] (x^2*(2*log(x)^2 - 2*log(x) + 1))/4
```

3.60 $\int \frac{1}{1+t} \, dt$

Optimal. Leaf size=4

$$\log(1+t)$$

```
[Out] ln(1+t)
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {31}

$$\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + t)^(-1),t]
```

```
[Out] Log[1 + t]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{1+t} \, dt = \log(1+t)$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$\log(1+t)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + t)^(-1),t]
```

```
[Out] Log[1 + t]
```

Maple [A]
time = 0.06, size = 5, normalized size = 1.25

method	result	size

default	$\ln(1+t)$	5
norman	$\ln(1+t)$	5
meijerg	$\ln(1+t)$	5
risch	$\ln(1+t)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+t),t,method=_RETURNVERBOSE)
```

```
[Out] ln(1+t)
```

Maxima [A]
time = 3.00, size = 4, normalized size = 1.00

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t, algorithm="maxima")
```

```
[Out] log(t + 1)
```

Fricas [A]
time = 1.06, size = 4, normalized size = 1.00

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t, algorithm="fricas")
```

```
[Out] log(t + 1)
```

Sympy [A]
time = 0.01, size = 3, normalized size = 0.75

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t)
```

```
[Out] log(t + 1)
```

Giac [A]
time = 0.45, size = 5, normalized size = 1.25

$$\log(|t+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t, algorithm="giac")
```

```
[Out] log(abs(t + 1))
```

Mupad [**B**]
time = 0.02, size = 4, normalized size = 1.00

$$\ln(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t + 1),t)
```

```
[Out] log(t + 1)
```

3.61 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] ln(sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size

lookup	$\ln(\sin(x))$	4
default	$\ln(\sin(x))$	4
derivativedivides	$-\frac{\ln(\cot^2(x)+1)}{2}$	10
norman	$-\frac{\ln(1+\tan^2(x))}{2}+\ln(\tan(x))$	14
risch	$-ix+\ln(e^{2ix}-1)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sin(x))
```

Maxima [A]
time = 3.59, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(3) = 6.
time = 1.26, size = 11, normalized size = 3.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A]
time = 0.03, size = 3, normalized size = 1.00

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

Giac [A]
time = 0.44, size = 4, normalized size = 1.33

$$\log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="giac")
```

```
[Out] log(abs(sin(x)))
```

Mupad [B]
time = 0.02, size = 3, normalized size = 1.00

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
```

```
[Out] log(sin(x))
```

3.62 $\int x^n \log(ax)\, dx$

Optimal. Leaf size=28

$$-\frac{x^{1+n}}{(1+n)^2}+\frac{x^{1+n}\log(ax)}{1+n}$$

```
[Out] -x^(1+n)/(1+n)^2+x^(1+n)*ln(a*x)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*Log[a*x],x]
[Out] -(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^n \log(ax)\, dx = -\frac{x^{1+n}}{(1+n)^2}+\frac{x^{1+n}\log(ax)}{1+n}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.75

$$\frac{x^{1+n}(-1+(1+n)\log(ax))}{(1+n)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*Log[a*x],x]
[Out] (x^(1 + n)*(-1 + (1 + n)*Log[a*x]))/(1 + n)^2
```

Maple [A]
time = 0.04, size = 36, normalized size = 1.29

method	result
norman	$\frac{x\ln(ax)e^{n\ln(x)}}{1+n}-\frac{x\,e^{n\ln(x)}}{n^2+2n+1}$
risch	$\frac{x\left(-i\pi\,\mathrm{csgn}(ia)\mathrm{csgn}(ix)\mathrm{csgn}(iax)n+i\pi\,\mathrm{csgn}(ia)\mathrm{csgn}(iax)^2n+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(iax)^2n-i\pi\mathrm{csgn}(iax)^3n-i\pi\,\mathrm{csgn}(ia)\mathrm{csgn}(ix)\mathrm{csgn}(iax)+\right.}{2(1+n)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*ln(a*x),x,method=_RETURNVERBOSE)
[Out] 1/(1+n)*x*ln(a*x)*exp(n*ln(x))-1/(n^2+2*n+1)*x*exp(n*ln(x))
```

Maxima [A]
time = 0.29, size = 28, normalized size = 1.00

$$\frac{x^{n+1}\log{(ax)}}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="maxima")
[Out] x^(n + 1)*log(a*x)/(n + 1) - x^(n + 1)/(n + 1)^2
```

Fricas [A]
time = 0.94, size = 32, normalized size = 1.14

$$\frac{((n+1)x\log{(a)}+(n+1)x\log{(x)}-x)x^n}{n^2+2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="fricas")
[Out] ((n + 1)*x*log(a) + (n + 1)*x*log(x) - x)*x^n/(n^2 + 2*n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.
time = 0.20, size = 61, normalized size = 2.18

$$\begin{cases}\frac{nxx^n\log{(ax)}}{n^2+2n+1}+\frac{xx^n\log{(ax)}}{n^2+2n+1}-\frac{xx^n}{n^2+2n+1} & \text{for } n\neq -1\\ \frac{\log{(ax)}^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*ln(a*x),x)
[Out] Piecewise((n*x*x**n*log(a*x)/(n**2 + 2*n + 1) + x*x**n*log(a*x)/(n**2 + 2*n
 + 1) - x*x**n/(n**2 + 2*n + 1), Ne(n, -1)), (log(a*x)**2/2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^n*log(a*x), x)
```

Mupad [B]
time = 0.21, size = 38, normalized size = 1.36

$$\begin{cases} \frac{\ln(a\,x)^2}{2} & \text{if } n=-1 \\ \frac{x^{n+1}\,\left(\ln(a\,x)-\frac{1}{n+1}\right)}{n+1} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*log(a*x),x)
```

```
[Out] piecewise(n == -1, log(a*x)^2/2, n ~= -1, (x^(n + 1)*(log(a*x) - 1/(n + 1))
)/(n + 1))
```

3.63 $\int x^2 \log^2(x) \, dx$

Optimal. Leaf size=28

$$\frac{2x^3}{27} - \frac{2}{9}x^3\log(x) + \frac{1}{3}x^3\log^2(x)$$

```
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2342, 2341}

$$\frac{2x^3}{27} + \frac{1}{3}x^3\log^2(x) - \frac{2}{9}x^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[x]^2,x]
```

```
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\log^2(x)\,dx &= \frac{1}{3}x^3\log^2(x) - \frac{2}{3}\int x^2\log(x)\,dx\\ &= \frac{2x^3}{27} - \frac{2}{9}x^3\log(x) + \frac{1}{3}x^3\log^2(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{2x^3}{27} - \frac{2}{9}x^3\log(x) + \frac{1}{3}x^3\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[x]^2,x]
```

```
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Maple [A]
time = 0.00, size = 23, normalized size = 0.82

method	result	size
default	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23
norman	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23
risch	$\frac{2x^3}{27}-\frac{2x^3\ln(x)}{9}+\frac{x^3\ln(x)^2}{3}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

Maxima [A]
time = 4.00, size = 17, normalized size = 0.61

$$\frac{1}{27}\left(9\,\log\left(x\right)^{2}-6\,\log\left(x\right)+2\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/27*(9*log(x)^2 - 6*log(x) + 2)*x^3
```

Fricas [A]
time = 0.98, size = 22, normalized size = 0.79

$$\frac{1}{3}\,x^{3}\log\left(x\right)^{2}-\frac{2}{9}\,x^{3}\log\left(x\right)+\frac{2}{27}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

Sympy [A]
time = 0.04, size = 26, normalized size = 0.93

$$\frac{x^{3}\log\left(x\right)^{2}}{3}-\frac{2x^{3}\log\left(x\right)}{9}+\frac{2x^{3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x)**2,x)
```

```
[Out] x**3*log(x)**2/3 - 2*x**3*log(x)/9 + 2*x**3/27
```

Giac [A]
time = 0.49, size = 22, normalized size = 0.79

$$\frac{1}{3}\,x^{3}\log\left(x\right)^{2}-\frac{2}{9}\,x^{3}\log\left(x\right)+\frac{2}{27}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.61

$$\frac{2\,x^{3}\,\left(\frac{9\ln(x)^{2}}{2}-3\,\ln\left(x\right)+1\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(x)^2,x)
```

```
[Out] (2*x^3*((9*log(x)^2)/2 - 3*log(x) + 1))/27
```

3.64 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] ln(ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2339, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Maple [A]
time = 0.02, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\ln(\ln(x))$	4
default	$\ln(\ln(x))$	4
norman	$\ln(\ln(x))$	4
risch	$\ln(\ln(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(ln(x))
```

Maxima [A]
time = 7.56, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
```

```
[Out] log(log(x))
```

Fricas [A]
time = 0.95, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

Sympy [A]
time = 0.03, size = 3, normalized size = 1.00

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

Giac [**A**]
time = 0.44, size = 4, normalized size = 1.33

$$\log(|\log(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
```

```
[Out] log(abs(log(x)))
```

Mupad [**B**]
time = 0.07, size = 3, normalized size = 1.00

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)),x)
```

```
[Out] log(log(x))
```

3.65 $\int \frac{\log(1-t)}{1-t}\,dt$

Optimal. Leaf size=12

$$-\frac{1}{2}\log^2(1-t)$$

```
[Out] -1/2*ln(1-t)^2
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2437, 2338}

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - t]/(1 - t),t]
```

```
[Out] -1/2*Log[1 - t]^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(1-t)}{1-t}\,dt &= -\text{Subst}\left(\int \frac{\log(t)}{t}\,dt, t, 1-t\right)\\ &= -\frac{1}{2}\log^2(1-t)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - t]/(1 - t),t]
```

```
[Out] -1/2*Log[1 - t]^2
```

Maple [A]
time = 0.07, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{\ln(1-t)^2}{2}$	11
default	$-\frac{\ln(1-t)^2}{2}$	11
norman	$-\frac{\ln(1-t)^2}{2}$	11
risch	$-\frac{\ln(1-t)^2}{2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-t)/(1-t),t,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(1-t)^2
```

Maxima [A]
time = 4.53, size = 10, normalized size = 0.83

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="maxima")
```

```
[Out] -1/2*log(-t + 1)^2
```

Fricas [A]
time = 1.08, size = 10, normalized size = 0.83

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="fricas")
```

```
[Out] -1/2*log(-t + 1)^2
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$-\frac{\log\left(1-t\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-t)/(1-t),t)
```

```
[Out] -log(1 - t)**2/2
```

Giac [A]
time = 0.43, size = 10, normalized size = 0.83

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="giac")
```

```
[Out] -1/2*log(-t + 1)^2
```

Mupad [B]
time = 0.35, size = 10, normalized size = 0.83

$$-\frac{\ln\left(1-t\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - t)/(t - 1),t)
```

```
[Out] -log(1 - t)^2/2
```

3.66 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}$$

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2412, 45}

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx,x,\log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx,x,\log(x)\right)\\ &= -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 0.70

$$\frac{2}{3}(-2+\log(x))\sqrt{1+\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Maple [A]
time = 0.06, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(1+ln(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Maxima [A]
time = 5.80, size = 17, normalized size = 0.74

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Fricas [A]
time = 0.91, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\log(x)+1}(\log(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A]
time = 2.41, size = 20, normalized size = 0.87

$$\frac{2(\log(x)+1)^{\frac{3}{2}}}{3}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
```

```
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

Giac [A]
time = 0.48, size = 17, normalized size = 0.74

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Mupad [B]
time = 0.17, size = 13, normalized size = 0.57

$$\sqrt{\ln(x)+1}\left(\frac{2\ln(x)}{3}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*(log(x) + 1)^(1/2)),x)
```

```
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

3.67 $\int x^3 \log^3(x)\,dx$

Optimal. Leaf size=39

$$-\frac{3x^4}{128}+\frac{3}{32}x^4\log(x)-\frac{3}{16}x^4\log^2(x)+\frac{1}{4}x^4\log^3(x)$$

```
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2342, 2341}

$$-\frac{3x^4}{128}+\frac{1}{4}x^4\log^3(x)-\frac{3}{16}x^4\log^2(x)+\frac{3}{32}x^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^3\log^3(x)\,dx &= \frac{1}{4}x^4\log^3(x)-\frac{3}{4}\int x^3\log^2(x)\,dx\\
&= -\frac{3}{16}x^4\log^2(x)+\frac{1}{4}x^4\log^3(x)+\frac{3}{8}\int x^3\log(x)\,dx\\
&= -\frac{3x^4}{128}+\frac{3}{32}x^4\log(x)-\frac{3}{16}x^4\log^2(x)+\frac{1}{4}x^4\log^3(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.00

$$-\frac{3x^4}{128}+\frac{3}{32}x^4\log(x)-\frac{3}{16}x^4\log^2(x)+\frac{1}{4}x^4\log^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[x]^3,x]
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Maple [A]
time = 0.01, size = 32, normalized size = 0.82

method	result	size
default	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
norman	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32
risch	$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4\ln(x)^2}{16}+\frac{x^4\ln(x)^3}{4}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(x)^3,x,method=_RETURNVERBOSE)
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

Maxima [A]
time = 3.24, size = 23, normalized size = 0.59

$$\frac{1}{128}\left(32\log\left(x\right)^3-24\log\left(x\right)^2+12\log\left(x\right)-3\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="maxima")
[Out] 1/128*(32*log(x)^3 - 24*log(x)^2 + 12*log(x) - 3)*x^4
```

Fricas [A]
time = 0.96, size = 31, normalized size = 0.79

$$\frac{1}{4}x^4\log\left(x\right)^3-\frac{3}{16}x^4\log\left(x\right)^2+\frac{3}{32}x^4\log\left(x\right)-\frac{3}{128}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="fricas")
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.95

$$\frac{x^4\log(x)^3}{4} - \frac{3x^4\log(x)^2}{16} + \frac{3x^4\log(x)}{32} - \frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(x)**3,x)
```

```
[Out] x**4*log(x)**3/4 - 3*x**4*log(x)**2/16 + 3*x**4*log(x)/32 - 3*x**4/128
```

Giac [A]
time = 0.46, size = 31, normalized size = 0.79

$$\frac{1}{4}\,x^4\log(x)^3 - \frac{3}{16}\,x^4\log(x)^2 + \frac{3}{32}\,x^4\log(x) - \frac{3}{128}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.59

$$\frac{3\,x^4\left(\frac{32\ln(x)^3}{3} - 8\ln(x)^2 + 4\,\ln(x) - 1\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x)^3,x)
```

```
[Out] (3*x^4*(4*log(x) - 8*log(x)^2 + (32*log(x)^3)/3 - 1))/128
```

3.68 $\int e^{x^3} x^2 \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^3}}{3}$$

```
[Out] 1/3*exp(x^3)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2240}

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^3*x^2,x]
```

```
[Out] E^x^3/3
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^3*x^2,x]
```

```
[Out] E^x^3/3
```

Maple [A]
time = 0.01, size = 7, normalized size = 0.78

method	result	size
gosper	$\frac{e^{x^3}}{3}$	7
derivativedivides	$\frac{e^{x^3}}{3}$	7
default	$\frac{e^{x^3}}{3}$	7
norman	$\frac{e^{x^3}}{3}$	7
risch	$\frac{e^{x^3}}{3}$	7
meijerg	$-\frac{1}{3}+\frac{e^{x^3}}{3}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^3)*x^2,x,method=_RETURNVERBOSE)
[Out] 1/3*exp(x^3)
```

Maxima [A]
time = 2.98, size = 6, normalized size = 0.67

$$\frac{1}{3}\,e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="maxima")
[Out] 1/3*e^(x^3)
```

Fricas [A]
time = 0.90, size = 6, normalized size = 0.67

$$\frac{1}{3}\,e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="fricas")
[Out] 1/3*e^(x^3)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.56

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**3)*x**2,x)
```

```
[Out] exp(x**3)/3
```

Giac [A]
time = 0.49, size = 6, normalized size = 0.67

$$\frac{1}{3}\,e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*e^(x^3)
```

Mupad [B]
time = 0.08, size = 6, normalized size = 0.67

$$\frac{\mathrm{e}^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x^3),x)
```

```
[Out] exp(x^3)/3
```

3.69 $\int \frac{2^{\sqrt{x}}}{\sqrt{x}} dx$

Optimal. Leaf size=14

$$\frac{2^{1+\sqrt{x}}}{\log(2)}$$

```
[Out] 2^(1+x^(1/2))/ln(2)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2240\}$

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Sqrt[x]/Sqrt[x],x]
```

```
[Out] 2^(1 + Sqrt[x])/Log[2]
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}} dx = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{2^{1+\sqrt{x}}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sqrt[x]/Sqrt[x],x]
```

```
[Out] 2^(1 + Sqrt[x])/Log[2]
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
default	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
meijerg	$-\frac{2\left(1-\mathrm{e}^{\sqrt{x}\,\ln(2)}\right)}{\ln(2)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*2^(x^(1/2))/ln(2)
```

Maxima [A]
time = 1.53, size = 12, normalized size = 0.86

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2^(sqrt(x) + 1)/log(2)
```

Fricas [A]
time = 1.32, size = 11, normalized size = 0.79

$$\frac{2 \cdot 2^{\left(\sqrt{x}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*2^sqrt(x)/log(2)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.71

$$\frac{2 \cdot 2^{\sqrt{x}}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*2**(sqrt(x))/log(2)
```

Giac [A]
time = 0.52, size = 11, normalized size = 0.79

$$\frac{2 \cdot 2^{\left(\sqrt{x}\right)}}{\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*2^sqrt(x)/log(2)
```

Mupad [B]
time = 0.10, size = 11, normalized size = 0.79

$$\frac{2\,2^{\sqrt{x}}}{\ln\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(x^(1/2))/x^(1/2),x)
```

```
[Out] (2*2^(x^(1/2)))/log(2)
```

3.70 $\int e^{2\sin(x)}\cos(x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}e^{2\sin(x)}$$

```
[Out] 1/2*exp(2*sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4419, 2225}

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*Sin[x])*Cos[x],x]
```

```
[Out] E^(2*Sin[x])/2
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{2\sin(x)}\cos(x)\,dx &= \text{Subst}\left(\int e^{2x}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}e^{2\sin(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*Sin[x])*Cos[x],x]
```

```
[Out] E^(2*Sin[x])/2
```

Maple [A]
time = 0.03, size = 8, normalized size = 0.80

method	result	size
derivativedivides	$\frac{e^{2\sin(x)}}{2}$	8
default	$\frac{e^{2\sin(x)}}{2}$	8
risch	$\frac{e^{2\sin(x)}}{2}$	8
norman	$\frac{\frac{\left(\tan^2\left(\frac{x}{2}\right)\right) e^{\frac{4\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}}{2}+\frac{e^{\frac{4\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}}{2}}{1+\tan^2\left(\frac{x}{2}\right)}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*sin(x))*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(2*sin(x))
```

Maxima [A]
time = 2.15, size = 7, normalized size = 0.70

$$\frac{1}{2}\,e^{(2\,\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*sin(x))
```

Fricas [A]
time = 1.35, size = 7, normalized size = 0.70

$$\frac{1}{2}\,e^{(2\,\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*e^(2*sin(x))
```

<u>**Sympy**</u> [**A**]
time = 0.11, size = 7, normalized size = 0.70

$$\frac{e^{2\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x)
```

```
[Out] exp(2*sin(x))/2
```

<u>**Giac**</u> [**A**]
time = 0.51, size = 7, normalized size = 0.70

$$\frac{1}{2}\,e^{(2\,\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(2*sin(x))
```

<u>**Mupad**</u> [**B**]
time = 0.10, size = 7, normalized size = 0.70

$$\frac{\mathrm{e}^{2\,\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*sin(x))*cos(x),x)
```

```
[Out] exp(2*sin(x))/2
```

3.71 $\int e^x \sin(x)\, dx$

Optimal. Leaf size=19

$$-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4517}

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
[Out] -1/2*(E^x*Cos[x]) + (E^x*Sin[x])/2
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(x)\,dx=-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(-\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
default	$-\frac{e^{x}\cos(x)}{2}+\frac{e^{x}\sin(x)}{2}$	14
norman	$\frac{e^{x}\tan\left(\frac{x}{2}\right)+\frac{e^{x}\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$-\frac{e^{(1+i)x}}{4}-\frac{i e^{(1+i)x}}{4}-\frac{e^{(1-i)x}}{4}+\frac{i e^{(1-i)x}}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A]
time = 2.87, size = 11, normalized size = 0.58

$$-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A]
time = 1.40, size = 13, normalized size = 0.68

$$-\frac{1}{2}\cos\left(x\right)e^{x}+\frac{1}{2}e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2}-\frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Giac [A]
time = 0.42, size = 11, normalized size = 0.58

$$-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.58

$$-\frac{\mathrm{e}^{x}\left(\cos\left(x\right)-\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
```

```
[Out] -(exp(x)*(cos(x) - sin(x)))/2
```

3.72 $\int e^x \cos(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

```
[Out] 1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4518}

$$\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[x],x]
[Out] (E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\cos(x)\,dx=\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 0.63

$$\frac{1}{2}e^x(\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[x],x]
[Out] (E^x*(Cos[x] + Sin[x]))/2
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
default	$\frac{e^{x} \cos\left(x\right)}{2}+\frac{e^{x} \sin\left(x\right)}{2}$	14
norman	$\frac{e^{x} \tan\left(\frac{x}{2}\right)-\frac{e^{x} \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{e^{x}}{2}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	34
risch	$\frac{e^{\left(1+i\right) x}}{4}-\frac{i e^{\left(1+i\right) x}}{4}+\frac{e^{\left(1-i\right) x}}{4}+\frac{i e^{\left(1-i\right) x}}{4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A]
time = 4.39, size = 9, normalized size = 0.47

$$\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="maxima")
[Out] 1/2*(cos(x) + sin(x))*e^x
```

Fricas [A]
time = 1.06, size = 13, normalized size = 0.68

$$\frac{1}{2}\cos\left(x\right)e^{x}+\frac{1}{2}e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="fricas")
[Out] 1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2}+\frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x)
[Out] exp(x)*sin(x)/2 + exp(x)*cos(x)/2
```

Giac [A]
time = 0.50, size = 9, normalized size = 0.47

$$\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.47

$$\frac{\mathrm{e}^{x}\left(\cos\left(x\right)+\sin\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x),x)
```

```
[Out] (exp(x)*(cos(x) + sin(x)))/2
```

3.73 $\int \frac{1}{1+e^x}\,dx$

Optimal. Leaf size=10

$$x-\log\left(1+e^x\right)$$

[Out] x-ln(1+exp(x))

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2320, 36, 29, 31}

$$x-\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + E^x)^(-1),x]

[Out] x - Log[1 + E^x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+e^x}\,dx &= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, e^x\right) \\
&= \text{Subst}\left(\int \frac{1}{x}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right) \\
&= x - \log\left(1+e^x\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-2\tanh^{-1}\left(1+2e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)^(-1),x]
```

```
[Out] -2*ArcTanh[1 + 2*E^x]
```

Maple [A]
time = 0.01, size = 12, normalized size = 1.20

method	result	size
norman	$x - \ln\left(1+e^x\right)$	10
risch	$x - \ln\left(1+e^x\right)$	10
derivativedivides	$-\ln\left(1+e^x\right)+\ln\left(e^x\right)$	12
default	$-\ln\left(1+e^x\right)+\ln\left(e^x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1+exp(x))+ln(exp(x))
```

Maxima [A]
time = 1.47, size = 9, normalized size = 0.90

$$x - \log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] x - log(e^x + 1)
```

Fricas [A]
time = 0.85, size = 9, normalized size = 0.90

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="fricas")
[Out] x - log(e^x + 1)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.70

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x)
[Out] x - log(exp(x) + 1)
```

Giac [A]
time = 0.47, size = 9, normalized size = 0.90

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="giac")
[Out] x - log(e^x + 1)
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.90

$$x - \ln(\mathrm{e}^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(exp(x) + 1),x)
[Out] x - log(exp(x) + 1)
```

3.74 $\int e^x x \, dx$

Optimal. Leaf size=11

$$-e^x + e^x x$$

[Out] -exp(x)+exp(x)*x

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2207, 2225}

$$e^x x - e^x$$

Antiderivative was successfully verified.

[In] Int[E^x*x,x]

[Out] -E^x + E^x*x

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^x x \, dx &= e^x x - \int e^x \, dx \\ &= -e^x + e^x x \end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 0.64

$$e^x(-1 + x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x,x]
```

```
[Out] E^x*(-1 + x)
```

Maple [A]
time = 0.02, size = 10, normalized size = 0.91

method	result	size
gosper	$(-1+x)\,\mathrm{e}^{x}$	7
risch	$(-1+x)\,\mathrm{e}^{x}$	7
default	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
norman	$-\mathrm{e}^{x}+\mathrm{e}^{x}x$	10
meijerg	$1-\frac{(-2x+2)\mathrm{e}^{x}}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x,x,method=_RETURNVERBOSE)
```

```
[Out] -exp(x)+exp(x)*x
```

Maxima [A]
time = 2.48, size = 6, normalized size = 0.55

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x
```

Fricas [A]
time = 0.90, size = 6, normalized size = 0.55

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.45

$$\left(x-1\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x)
```

```
[Out] (x - 1)*exp(x)
```

Giac [A]
time = 0.43, size = 6, normalized size = 0.55

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="giac")
```

```
[Out] (x - 1)*e^x
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.55

$$\mathrm{e}^{x}\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(x),x)
```

```
[Out] exp(x)*(x - 1)
```

3.75 $\int e^{-x}x\,dx$

Optimal. Leaf size=16

$$-e^{-x}-e^{-x}x$$

[Out] -1/exp(x)-x/exp(x)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2207, 2225}

$$-e^{-x}x-e^{-x}$$

Antiderivative was successfully verified.

[In] Int[x/E^x,x]

[Out] -E^(-x) - x/E^x

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x}x\,dx&=-e^{-x}x+\int e^{-x}\,dx\\&=-e^{-x}-e^{-x}x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 0.69

$$e^{-x}(-1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^x,x]
[Out] (-1 - x)/E^x
```

Maple [A]
time = 0.01, size = 15, normalized size = 0.94

method	result	size
gosper	$-(1+x)\,\mathrm{e}^{-x}$	10
norman	$(-1-x)\,\mathrm{e}^{-x}$	11
risch	$(-1-x)\,\mathrm{e}^{-x}$	11
meijerg	$1-\frac{(2+2x)\mathrm{e}^{-x}}{2}$	14
default	$-\mathrm{e}^{-x}-x\,\mathrm{e}^{-x}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/exp(x),x,method=_RETURNVERBOSE)
[Out] -1/exp(x)-x/exp(x)
```

Maxima [A]
time = 3.36, size = 9, normalized size = 0.56

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="maxima")
[Out] -(x + 1)*e^(-x)
```

Fricas [A]
time = 0.70, size = 9, normalized size = 0.56

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="fricas")
[Out] -(x + 1)*e^(-x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.44

$$(-x-1)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x)
```

```
[Out] (-x - 1)*exp(-x)
```

Giac [A]
time = 0.44, size = 9, normalized size = 0.56

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x)
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.56

$$-\mathrm{e}^{-x}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*exp(-x),x)
```

```
[Out] -exp(-x)*(x + 1)
```

3.76 $\int e^x x^2 \, dx$

Optimal. Leaf size=19

$$2e^x - 2e^x x + e^x x^2$$

```
[Out] 2*exp(x)-2*exp(x)*x+exp(x)*x^2
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2207, 2225}

$$e^x x^2 - 2e^x x + 2e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^x x^2 \, dx &= e^x x^2 - 2 \int e^x x \, dx \\
&= -2e^x x + e^x x^2 + 2 \int e^x \, dx \\
&= 2e^x - 2e^x x + e^x x^2
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 0.63

$$e^x \left(2 - 2x + x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2,x]
```

```
[Out] E^x*(2 - 2*x + x^2)
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.89

method	result	size
gosper	$\left(x^{2}-2x+2\right) {\mathrm e}^{x}$	12
risch	$\left(x^{2}-2x+2\right) {\mathrm e}^{x}$	12
default	$2\,{\mathrm e}^{x}-2\,{\mathrm e}^{x} x+{\mathrm e}^{x} x^{2}$	17
norman	$2\,{\mathrm e}^{x}-2\,{\mathrm e}^{x} x+{\mathrm e}^{x} x^{2}$	17
meijerg	$-2+\frac{\left(3x^{2}-6x+6\right) {\mathrm e}^{x}}{3}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*exp(x)-2*exp(x)*x+exp(x)*x^2
```

Maxima [A]
time = 1.42, size = 11, normalized size = 0.58

$$\left(x^{2}-2\,x+2\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="maxima")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Fricas [A]
time = 0.77, size = 11, normalized size = 0.58

$$\left(x^{2}-2\,x+2\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="fricas")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.53

$$\left(x^{2}-2x+2\right) e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2,x)
```

```
[Out] (x**2 - 2*x + 2)*exp(x)
```

Giac [A]
time = 0.45, size = 11, normalized size = 0.58

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="giac")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.58

$$\mathrm{e}^x\,\left(x^2 - 2\,x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(x),x)
```

```
[Out] exp(x)*(x^2 - 2*x + 2)
```

3.77 $\int e^{-2x}x^2\,dx$

Optimal. Leaf size=32

$$-\frac{1}{4}e^{-2x}-\frac{1}{2}e^{-2x}x-\frac{1}{2}e^{-2x}x^2$$

```
[Out] -1/4/exp(2*x)-1/2*x/exp(2*x)-1/2*x^2/exp(2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2207, 2225}

$$-\frac{1}{2}e^{-2x}x^2-\frac{1}{2}e^{-2x}x-\frac{e^{-2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*x),x]
```

```
[Out] -1/4*1/E^(2*x) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}x^2\,dx&=-\frac{1}{2}e^{-2x}x^2+\int e^{-2x}x\,dx\\&=-\frac{1}{2}e^{-2x}x-\frac{1}{2}e^{-2x}x^2+\frac{1}{2}\int e^{-2x}\,dx\\&=-\frac{1}{4}e^{-2x}-\frac{1}{2}e^{-2x}x-\frac{1}{2}e^{-2x}x^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.59

$$-\frac{1}{4}e^{-2x}\left(1+2x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*x),x]
[Out] -1/4*(1 + 2*x + 2*x^2)/E^(2*x)
```

Maple [A]
time = 0.01, size = 30, normalized size = 0.94

method	result	size
risch	$\left(-\frac{1}{2}x^2-\frac{1}{2}x-\frac{1}{4}\right)e^{-2x}$	16
norman	$\left(-\frac{1}{2}x^2-\frac{1}{2}x-\frac{1}{4}\right)e^{-2x}$	18
gosper	$-\frac{(2x^2+2x+1)e^{-2x}}{4}$	19
meijerg	$\frac{1}{4}-\frac{(12x^2+12x+6)e^{-2x}}{24}$	19
derivativedivides	$-\frac{e^{-2x}}{4}-\frac{x e^{-2x}}{2}-\frac{x^2e^{-2x}}{2}$	30
default	$-\frac{e^{-2x}}{4}-\frac{x e^{-2x}}{2}-\frac{x^2e^{-2x}}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/exp(2*x),x,method=_RETURNVERBOSE)
[Out] -1/4/exp(2*x)-1/2*x/exp(2*x)-1/2*x^2/exp(2*x)
```

Maxima [A]
time = 2.49, size = 16, normalized size = 0.50

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Fricas [A]
time = 0.78, size = 16, normalized size = 0.50

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="fricas")
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Sympy [A]
time = 0.02, size = 17, normalized size = 0.53

$$\frac{(-2x^2 - 2x - 1)\,e^{-2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/exp(2*x),x)
```

```
[Out] (-2*x**2 - 2*x - 1)*exp(-2*x)/4
```

Giac [A]
time = 0.45, size = 16, normalized size = 0.50

$$-\frac{1}{4}\left(2\,x^2 + 2\,x + 1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Mupad [B]
time = 0.08, size = 16, normalized size = 0.50

$$-\frac{\mathrm{e}^{-2\,x}\,\left(4\,x^2 + 4\,x + 2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(-2*x),x)
```

```
[Out] -(exp(-2*x)*(4*x + 4*x^2 + 2))/8
```

3.78 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$-2e^{\sqrt{x}}+2e^{\sqrt{x}}\sqrt{x}$$

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2238, 2207, 2225}

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2238

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{\sqrt{x}}\,dx &= 2\text{Subst}\left(\int e^x x\,dx,x,\sqrt{x}\right)\\ &= 2e^{\sqrt{x}}\sqrt{x}-2\text{Subst}\left(\int e^x\,dx,x,\sqrt{x}\right)\\ &= -2e^{\sqrt{x}}+2e^{\sqrt{x}}\sqrt{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(-1+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sqrt[x],x]
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A]
time = 0.00, size = 17, normalized size = 0.71

method	result	size
meijerg	$2-\left(-2\sqrt{x}+2\right)e^{\sqrt{x}}$	16
derivativedivides	$-2\,e^{\sqrt{x}}+2\,e^{\sqrt{x}}\sqrt{x}$	17
default	$-2\,e^{\sqrt{x}}+2\,e^{\sqrt{x}}\sqrt{x}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Maxima [A]
time = 1.81, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Fricas [A]
time = 0.88, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.83

$$2\sqrt{x}\,e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Giac [A]
time = 0.51, size = 11, normalized size = 0.46

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.46

$$2\,\mathrm{e}^{\sqrt{x}}\,\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
[Out] 2*exp(x^(1/2))*(x^(1/2) - 1)
```

3.79 $\int e^{-x^2} x^3 \, dx$

Optimal. Leaf size=26

$$-\frac{e^{-x^2}}{2} - \frac{1}{2} e^{-x^2} x^2$$

```
[Out] -1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2243, 2240}

$$-\frac{1}{2} e^{-x^2} x^2 - \frac{e^{-x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^x^2,x]
```

```
[Out] -1/2*1/E^x^2 - x^2/(2*E^x^2)
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2243

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n
)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{-x^2} x^3 \, dx &= -\frac{1}{2} e^{-x^2} x^2 + \int e^{-x^2} x \, dx \\ &= -\frac{e^{-x^2}}{2} - \frac{1}{2} e^{-x^2} x^2\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 0.62

$$-\frac{1}{2}e^{-x^2}\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^x^2,x]
```

```
[Out] -1/2*(1 + x^2)/E^x^2
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.81

method	result	size
gosper	$-\frac{\left(x^{2}+1\right)\mathrm{e}^{-x^{2}}}{2}$	14
norman	$\left(-\frac{x^{2}}{2}-\frac{1}{2}\right)\mathrm{e}^{-x^{2}}$	15
risch	$\left(-\frac{x^{2}}{2}-\frac{1}{2}\right)\mathrm{e}^{-x^{2}}$	15
meijerg	$\frac{1}{2}-\frac{\left(2x^{2}+2\right)\mathrm{e}^{-x^{2}}}{4}$	18
derivativedivides	$-\frac{\mathrm{e}^{-x^{2}}}{2}-\frac{x^{2}\mathrm{e}^{-x^{2}}}{2}$	21
default	$-\frac{\mathrm{e}^{-x^{2}}}{2}-\frac{x^{2}\mathrm{e}^{-x^{2}}}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/exp(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/exp(x^2)-1/2*x^2/exp(x^2)
```

Maxima [A]
time = 2.14, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Fricas [A]
time = 0.71, size = 13, normalized size = 0.50

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="fricas")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.46

$$\frac{(-x^2-1)\,e^{-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/exp(x**2),x)
[Out] (-x**2 - 1)*exp(-x**2)/2
```

Giac [A]
time = 0.64, size = 13, normalized size = 0.50

$$-\frac{1}{2}\,(x^2+1)\,e^{(-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="giac")
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Mupad [B]
time = 0.11, size = 13, normalized size = 0.50

$$-\frac{\mathrm{e}^{-x^2}\,(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*exp(-x^2),x)
[Out] -(exp(-x^2)*(x^2 + 1))/2
```

3.80 $\int e^{ax}\cos(bx)\,dx$

Optimal. Leaf size=41

$$\frac{ae^{ax}\cos(bx)}{a^2+b^2}+\frac{be^{ax}\sin(bx)}{a^2+b^2}$$

```
[Out] a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4518}

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*Cos[b*x],x]
[Out] (a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{ax}\cos(bx)\,dx=\frac{ae^{ax}\cos(bx)}{a^2+b^2}+\frac{be^{ax}\sin(bx)}{a^2+b^2}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.68

$$\frac{e^{ax}(a\cos(bx)+b\sin(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*Cos[b*x],x]
[Out] (E^(a*x)*(a*Cos[b*x] + b*Sin[b*x]))/(a^2 + b^2)
```

Maple [A]
time = 0.05, size = 40, normalized size = 0.98

method	result	size
default	$\frac{a\,\mathrm{e}^{ax}\cos(bx)}{a^2+b^2}+\frac{b\,\mathrm{e}^{ax}\sin(bx)}{a^2+b^2}$	40
risch	$\frac{\mathrm{e}^{x(ib+a)}}{2ib+2a}+\frac{\mathrm{e}^{x(-ib+a)}}{-2ib+2a}$	40
norman	$\frac{\frac{a\,\mathrm{e}^{ax}}{a^2+b^2}-\frac{a\,\mathrm{e}^{ax}\left(\tan^2\left(\frac{bx}{2}\right)\right)}{a^2+b^2}+\frac{2b\,\mathrm{e}^{ax}\tan\left(\frac{bx}{2}\right)}{a^2+b^2}}{1+\tan^2\left(\frac{bx}{2}\right)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*cos(b*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Maxima [A]
time = 2.30, size = 27, normalized size = 0.66

$$\frac{(a\cos(bx)+b\sin(bx))e^{(ax)}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="maxima")
```

```
[Out] (a*cos(b*x) + b*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Fricas [A]
time = 0.73, size = 31, normalized size = 0.76

$$\frac{a\cos(bx)\,e^{(ax)}+be^{(ax)}\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="fricas")
```

```
[Out] (a*cos(b*x)*e^(a*x) + b*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.27, size = 136, normalized size = 3.32

$$\begin{cases} x & \text{for } a=0\wedge b=0 \\ \frac{ixe^{-ibx}\sin(bx)}{2}+\frac{xe^{-ibx}\cos(bx)}{2}+\frac{e^{-ibx}\sin(bx)}{2b} & \text{for } a=-ib \\ -\frac{ixe^{ibx}\sin(bx)}{2}+\frac{xe^{ibx}\cos(bx)}{2}+\frac{e^{ibx}\sin(bx)}{2b} & \text{for } a=ib \\ \frac{ae^{ax}\cos(bx)}{a^2+b^2}+\frac{be^{ax}\sin(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x)
```

```
[Out] Piecewise((x, Eq(a, 0) & Eq(b, 0)), (I*x*exp(-I*b*x)*sin(b*x)/2 + x*exp(-I*
b*x)*cos(b*x)/2 + exp(-I*b*x)*sin(b*x)/(2*b), Eq(a, -I*b)), (-I*x*exp(I*b*x
)*sin(b*x)/2 + x*exp(I*b*x)*cos(b*x)/2 + exp(I*b*x)*sin(b*x)/(2*b), Eq(a, I
*b)), (a*exp(a*x)*cos(b*x)/(a**2 + b**2) + b*exp(a*x)*sin(b*x)/(a**2 + b**2
), True))
```

Giac [A]
time = 0.57, size = 36, normalized size = 0.88

$$\left(\frac{a\cos\left(bx\right)}{a^2+b^2}+\frac{b\sin\left(bx\right)}{a^2+b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="giac")
```

```
[Out] (a*cos(b*x)/(a^2 + b^2) + b*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.66

$$\frac{\mathrm{e}^{a\,x}\,\left(a\,\cos\left(b\,x\right)+b\,\sin\left(b\,x\right)\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*cos(b*x),x)
```

```
[Out] (exp(a*x)*(a*cos(b*x) + b*sin(b*x)))/(a^2 + b^2)
```

3.81 $\int e^{ax} \sin(bx)\, dx$

Optimal. Leaf size=42

$$-\frac{be^{ax}\cos(bx)}{a^2+b^2}+\frac{ae^{ax}\sin(bx)}{a^2+b^2}$$

```
[Out] -b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4517}

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*Sin[b*x],x]
```

```
[Out] -((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{ax}\sin(bx)\,dx=-\frac{be^{ax}\cos(bx)}{a^2+b^2}+\frac{ae^{ax}\sin(bx)}{a^2+b^2}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.69

$$\frac{e^{ax}(-b\cos(bx)+a\sin(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*Sin[b*x],x]
```

```
[Out] (E^(a*x)*(-(b*Cos[b*x]) + a*Sin[b*x]))/(a^2 + b^2)
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.98

method	result	size
default	$-\frac{b\,\mathrm{e}^{ax}\cos(bx)}{a^2+b^2}+\frac{a\,\mathrm{e}^{ax}\sin(bx)}{a^2+b^2}$	41
risch	$-\frac{i\mathrm{e}^{x(ib+a)}}{2(ib+a)}+\frac{i\mathrm{e}^{x(-ib+a)}}{-2ib+2a}$	42
norman	$\frac{\frac{b\,\mathrm{e}^{ax}\left(\tan^2\left(\frac{bx}{2}\right)\right)}{a^2+b^2}-\frac{b\,\mathrm{e}^{ax}}{a^2+b^2}+\frac{2a\,\mathrm{e}^{ax}\tan\left(\frac{bx}{2}\right)}{a^2+b^2}}{1+\tan^2\left(\frac{bx}{2}\right)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*sin(b*x),x,method=_RETURNVERBOSE)
[Out] -b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Maxima [A]
time = 1.58, size = 29, normalized size = 0.69

$$-\frac{(b\cos(bx)-a\sin(bx))e^{(ax)}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="maxima")
[Out] -(b*cos(b*x) - a*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Fricas [A]
time = 0.82, size = 33, normalized size = 0.79

$$-\frac{b\cos(bx)\,e^{(ax)}-ae^{(ax)}\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="fricas")
[Out] -(b*cos(b*x)*e^(a*x) - a*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.27, size = 139, normalized size = 3.31

$$\begin{cases} 0 & \text{for}\: a=0\wedge b=0 \\ \frac{xe^{-ibx}\sin(bx)}{2}-\frac{ixe^{-ibx}\cos(bx)}{2}+\frac{ie^{-ibx}\sin(bx)}{2b} & \text{for}\: a=-ib \\ \frac{xe^{ibx}\sin(bx)}{2}+\frac{ixe^{ibx}\cos(bx)}{2}-\frac{ie^{ibx}\sin(bx)}{2b} & \text{for}\: a=ib \\ \frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*exp(-I*b*x)*sin(b*x)/2 - I*x*exp(-I*
b*x)*cos(b*x)/2 + I*exp(-I*b*x)*sin(b*x)/(2*b), Eq(a, -I*b)), (x*exp(I*b*x)
*sin(b*x)/2 + I*x*exp(I*b*x)*cos(b*x)/2 - I*exp(I*b*x)*sin(b*x)/(2*b), Eq(a
, I*b)), (a*exp(a*x)*sin(b*x)/(a**2 + b**2) - b*exp(a*x)*cos(b*x)/(a**2 + b
**2), True))
```

Giac [A]
time = 0.55, size = 38, normalized size = 0.90

$$-\left(\frac{b\cos\left(bx\right)}{a^2+b^2}-\frac{a\sin\left(bx\right)}{a^2+b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="giac")
```

```
[Out] -(b*cos(b*x)/(a^2 + b^2) - a*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

Mupad [B]
time = 0.02, size = 29, normalized size = 0.69

$$-\frac{\mathrm{e}^{a\,x}\,\left(b\,\cos\left(b\,x\right)-a\,\sin\left(b\,x\right)\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*sin(b*x),x)
```

```
[Out] -(exp(a*x)*(b*cos(b*x) - a*sin(b*x)))/(a^2 + b^2)
```

3.82 $\int \cot^{-1}(x)\,dx$

Optimal. Leaf size=15

$$x\cot^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] x*arccot(x)+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4931, 266}

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x],x]
```

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(x)\,dx &= x\cot^{-1}(x)+\int \frac{x}{1+x^2}\,dx\\ &= x\cot^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$x\cot^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x],x]
```

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.93

method	result	size
lookup	$x \operatorname{arccot}(x) + \frac{\ln(x^2+1)}{2}$	14
default	$x \operatorname{arccot}(x) + \frac{\ln(x^2+1)}{2}$	14
risch	$\frac{ix \ln(ix+1)}{2} - \frac{ix \ln(-ix+1)}{2} + \frac{\pi x}{2} + \frac{\ln(x^2+1)}{2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccot(x)+1/2*ln(x^2+1)
```

Maxima [A]
time = 3.45, size = 13, normalized size = 0.87

$$x \operatorname{arccot}(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="maxima")
```

```
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 1.00, size = 13, normalized size = 0.87

$$x \operatorname{arccot}(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="fricas")
```

```
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.06, size = 12, normalized size = 0.80

$$x \operatorname{acot}(x) + \frac{\log(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x),x)
```

```
[Out] x*acot(x) + log(x**2 + 1)/2
```

Giac [A]
time = 0.52, size = 21, normalized size = 1.40

$$x \arctan\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(\frac{1}{x^2} + 1\right) - \frac{1}{2}\log\left(\frac{1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="giac")
```

```
[Out] x*arctan(1/x) + 1/2*log(1/x^2 + 1) - 1/2*log(x^(-2))
```

Mupad [B]
time = 0.10, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^2+1\right)}{2} + x\,\mathrm{acot}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(x),x)
```

```
[Out] log(x^2 + 1)/2 + x*acot(x)
```

3.83 $\int \sec^{-1}(x)\,dx$

Optimal. Leaf size=19

$$x\sec^{-1}(x)-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] x*arcsec(x)-arctanh((1-1/x^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.000, Rules used = {5322, 272, 65, 212}

$$x\sec^{-1}(x)-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[x],x]
```

```
[Out] x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5322

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^{-1}(x)\,dx &= x\sec^{-1}(x) - \int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx \\
&= x\sec^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x^2}\right) \\
&= x\sec^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= x\sec^{-1}(x) - \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(19) = 38$.
time = 0.05, size = 64, normalized size = 3.37

$$
x\sec^{-1}(x) - \frac{\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x],x]
```

```
[Out] x*ArcSec[x] - (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [**A**]
time = 0.01, size = 22, normalized size = 1.16

method	result	size
lookup	$x\,\text{arcsec}(x) - \ln\left(x + x\sqrt{1-\frac{1}{x^2}}\right)$	22
default	$x\,\text{arcsec}(x) - \ln\left(x + x\sqrt{1-\frac{1}{x^2}}\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*arcsec(x)-ln(x+x*(1-1/x^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 4.81, size = 35, normalized size = 1.84

$$x \operatorname{arcsec}(x) - \frac{1}{2} \log\left(\sqrt{-\frac{1}{x^2} + 1} + 1\right) + \frac{1}{2} \log\left(-\sqrt{-\frac{1}{x^2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="maxima")
```

```
[Out] x*arcsec(x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

Fricas [A]
time = 0.81, size = 33, normalized size = 1.74

$$(x - 2) \operatorname{arcsec}(x) + 4 \arctan\left(-x + \sqrt{x^2 - 1}\right) + \log\left(-x + \sqrt{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="fricas")
```

```
[Out] (x - 2)*arcsec(x) + 4*arctan(-x + sqrt(x^2 - 1)) + log(-x + sqrt(x^2 - 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.11, size = 17, normalized size = 0.89

$$x \operatorname{asec}(x) - \begin{cases} \operatorname{acosh}(x) & \text{for } |x^2| > 1 \\ -i \operatorname{asin}(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x),x)
```

```
[Out] x*asec(x) - Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.50, size = 37, normalized size = 1.95

$$x \arccos\left(\frac{1}{x}\right) - \frac{1}{2} \log\left(\sqrt{-\frac{1}{x^2} + 1} + 1\right) + \frac{1}{2} \log\left(-\sqrt{-\frac{1}{x^2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="giac")
```

```
[Out] x*arccos(1/x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) +
 1)
```

Mupad [B]
time = 0.61, size = 21, normalized size = 1.11

$$x \operatorname{acos}\left(\frac{1}{x}\right) - \ln\left(x + \sqrt{x^2 - 1}\right) \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acos(1/x),x)
[Out] x*acos(1/x) - log(x + (x^2 - 1)^(1/2))*sign(x)
```

3.84 $\int \csc^{-1}(x)\,dx$

Optimal. Leaf size=17

$$x\csc^{-1}(x)+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] x*arccsc(x)+arctanh((1-1/x^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 2.000$, Rules used = {5323, 272, 65, 212}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+x\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5323

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}\int \csc^{-1}(x)\,dx &= x\csc^{-1}(x)+\int \frac{1}{\sqrt{1-\frac{1}{x^2}}\,x}\,dx\\ &= x\csc^{-1}(x)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,\frac{1}{x^2}\right)\\ &= x\csc^{-1}(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-\frac{1}{x^2}}\right)\\ &= x\csc^{-1}(x)+\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.
time = 0.03, size = 64, normalized size = 3.76

$$x\csc^{-1}(x)+\frac{\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [**A**]
time = 0.00, size = 20, normalized size = 1.18

method	result	size
lookup	$x\,\mathrm{arccsc}(x)+\ln\left(x+x\sqrt{1-\frac{1}{x^2}}\right)$	20
default	$x\,\mathrm{arccsc}(x)+\ln\left(x+x\sqrt{1-\frac{1}{x^2}}\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsc(x),x,method=_RETURNVERBOSE)
```

```
[Out] x*arccsc(x)+ln(x+x*(1-1/x^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 2.95, size = 35, normalized size = 2.06

$$x\operatorname{arccsc}(x) + \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="maxima")
```

```
[Out] x*arccsc(x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.90, size = 35, normalized size = 2.06

$$(x-2)\operatorname{arccsc}(x) - 4\arctan\left(-x+\sqrt{x^2-1}\right) - \log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="fricas")
```

```
[Out] (x - 2)*arccsc(x) - 4*arctan(-x + sqrt(x^2 - 1)) - log(-x + sqrt(x^2 - 1))
```

Sympy [A]
time = 1.12, size = 17, normalized size = 1.00

$$x\operatorname{acsc}(x) + \begin{cases} \operatorname{acosh}(x) & \text{for } |x^2| > 1 \\ -i\operatorname{asin}(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsc(x),x)
```

```
[Out] x*acsc(x) + Piecewise((acosh(x), Abs(x**2) > 1), (-I*asin(x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.48, size = 37, normalized size = 2.18

$$x\arcsin\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="giac")
```

```
[Out] x*arcsin(1/x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) +
 1)
```

Mupad [B]
time = 0.21, size = 20, normalized size = 1.18

$$x \operatorname{asin}\left(\frac{1}{x}\right) + \ln\left(x + \sqrt{x^2 - 1}\right) \operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(1/x),x)
[Out] x*asin(1/x) + log(x + (x^2 - 1)^(1/2))*sign(x)
```

3.85 $\int \sin^{-1}(x)^2 \, dx$

Optimal. Leaf size=25

$$-2x+2\sqrt{1-x^2}\,\sin^{-1}(x)+x\sin^{-1}(x)^2$$

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4715, 4767, 8}

$$2\sqrt{1-x^2}\,\text{ArcSin}(x)+x\text{ArcSin}(x)^2-2x$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4767

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])^n/(2*e*(p +
1))), x] + Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \sin^{-1}(x)^2 \, dx &= x\sin^{-1}(x)^2-2\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx\\
&= 2\sqrt{1-x^2}\,\sin^{-1}(x)+x\sin^{-1}(x)^2-2\int 1 \, dx\\
&= -2x+2\sqrt{1-x^2}\,\sin^{-1}(x)+x\sin^{-1}(x)^2
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-2x+2\sqrt{1-x^2}\,\sin^{-1}(x)+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.96

method	result	size
default	$-2x+x\arcsin\left(x\right)^{2}+2\arcsin\left(x\right)\sqrt{-x^{2}+1}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Maxima [A]
time = 3.87, size = 23, normalized size = 0.92

$$x\arcsin\left(x\right)^{2}+2\,\sqrt{-x^{2}+1}\,\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Fricas [A]
time = 0.94, size = 23, normalized size = 0.92

$$x\arcsin\left(x\right)^{2}+2\,\sqrt{-x^{2}+1}\,\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A]
time = 0.06, size = 22, normalized size = 0.88

$$x\operatorname{asin}^{2}\left(x\right)-2x+2\sqrt{1-x^{2}}\,\operatorname{asin}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)**2,x)
```

```
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

Giac [A]
time = 0.47, size = 23, normalized size = 0.92

$$x\arcsin\left(x\right)^{2}+2\sqrt{-x^{2}+1}\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Mupad [B]
time = 0.03, size = 22, normalized size = 0.88

$$2\,\mathrm{asin}(x)\,\sqrt{1-x^2}+x\,\left(\mathrm{asin}(x)^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)^2,x)
```

```
[Out] 2*asin(x)*(1 - x^2)^(1/2) + x*(asin(x)^2 - 2)
```

3.86 $\int \frac{\sin^{-1}(x)}{x^2} dx$

Optimal. Leaf size=22

$$-\frac{\sin^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -arcsin(x)/x-arctanh((-x^2+1)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4723, 272, 65, 212}

$$-\frac{\text{ArcSin}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/x^2,x]
```

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(x)}{x^2}\,dx &= -\frac{\sin^{-1}(x)}{x}+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sin^{-1}(x)}{x}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= -\frac{\sin^{-1}(x)}{x}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sin^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$-\frac{\sin^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/x^2,x]
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A]
time = 0.00, size = 21, normalized size = 0.95

method	result	size
default	$-\frac{\arcsin(x)}{x}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^2,x,method=_RETURNVERBOSE)
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A]
time = 2.41, size = 33, normalized size = 1.50

$$-\frac{\arcsin(x)}{x}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="maxima")
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 0.91, size = 39, normalized size = 1.77

$$-\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\arcsin\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="fricas")
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/
x
```

Sympy [A]
time = 0.98, size = 22, normalized size = 1.00

$$\begin{cases}-\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise}\end{cases}-\frac{\operatorname{asin}\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x**2,x)
[Out] Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

Giac [A]
time = 0.47, size = 38, normalized size = 1.73

$$-\frac{\arcsin\left(x\right)}{x}-\frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right)+\frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="giac")
[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

Mupad [B]
time = 0.02, size = 20, normalized size = 0.91

$$-\operatorname{atanh}\left(\frac{1}{\sqrt{1-x^2}}\right)-\frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(asin(x)/x^2,x)
[Out] - atanh(1/(1 - x^2)^(1/2)) - asin(x)/x
```

3.87 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {223, 209}

$$\text{ArcTan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.94

method	result	size
default	$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Maxima [A]
time = 3.26, size = 6, normalized size = 0.38

$$\arcsin\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(x/a)
```

Fricas [A]
time = 0.99, size = 23, normalized size = 1.44

$$-2 \arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 19, normalized size = 1.19

$$\begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

Giac [A]
time = 0.48, size = 28, normalized size = 1.75

$$\frac{1}{2}\,a^2\arcsin\left(\frac{x}{a}\right)\mathrm{sgn}(a)+\frac{1}{2}\,\sqrt{a^2-x^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arcsin(x/a)*sgn(a) + 1/2*sqrt(a^2 - x^2)*x
```

Mupad [B]
time = 0.16, size = 14, normalized size = 0.88

$$\mathrm{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 - x^2)^(1/2),x)
```

```
[Out] atan(x/(a^2 - x^2)^(1/2))
```

3.88 $\int \frac{1}{\sqrt{1-2x-x^2}}\,dx$

Optimal. Leaf size=10

$$\sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right)$$

```
[Out] arcsin(1/2*(1+x)*2^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\text{ArcSin}\left(\frac{x+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - 2*x - x^2],x]
```

```
[Out] ArcSin[(1 + x)/Sqrt[2]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-2x-x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{8}}}\,dx, x, -2-2x\right)}{2\sqrt{2}}\\ &= \sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.
time = 0.05, size = 23, normalized size = 2.30

$$2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-2x-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - 2*x - x^2],x]
[Out] 2*ArcTan[x/(-1 + Sqrt[1 - 2*x - x^2])]
```

Maple [A]
time = 0.12, size = 10, normalized size = 1.00

method	result	size
default	$\arcsin\left(\frac{(1+x)\sqrt{2}}{2}\right)$	10
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x-\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-2x+1}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-2*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(1/2*(1+x)*2^(1/2))
```

Maxima [A]
time = 1.24, size = 11, normalized size = 1.10

$$-\arcsin\left(-\frac{1}{2}\sqrt{2}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/2*sqrt(2)*(x + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.
time = 0.99, size = 21, normalized size = 2.10

$$-2\arctan\left(\frac{\sqrt{-x^2-2x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 - 2*x + 1) - 1)/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 - 2x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-2*x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 2*x + 1), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(9) = 18$.
time = 0.46, size = 27, normalized size = 2.70

$$\frac{1}{2}\sqrt{-x^2 - 2x + 1}\,(x+1) + \arcsin\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 - 2*x + 1)*(x + 1) + arcsin(1/2*sqrt(2)*(x + 1))
```

Mupad [**B**]
time = 0.09, size = 11, normalized size = 1.10

$$\mathrm{asin}\left(\frac{\sqrt{8}\,(2x+2)}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^2 - 2*x)^(1/2),x)
```

```
[Out] asin((8^(1/2)*(2*x + 2))/8)
```

3.89 $\int \frac{1}{a^2+x^2} dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

```
[Out] arctan(x/a)/a
```

Rubi [A]
time = 0.08, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {209}

$$\frac{\text{ArcTan}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a^2+x^2} dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Maple [A]
time = 0.07, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x/a)/a
```

Maxima [A]
time = 2.19, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a)/a
```

Fricas [A]
time = 0.93, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a)/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.04, size = 20, normalized size = 2.00

$$\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

Giac [A]
time = 0.46, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a)/a
```

Mupad [B]
time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + x^2),x)
```

```
[Out] atan(x/a)/a
```

3.90 $\int \frac{1}{a+bx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{211\}$

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.08, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 1.57, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.72, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.
time = 0.05, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [A]
time = 0.43, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 0.10, size = 16, normalized size = 0.67

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.91 $\int \frac{1}{2-x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -2/7*arctan(1/7*(1-2*x)*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-x+x^2}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, -1+2x\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{-1+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)^(-1),x]
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A]
time = 0.16, size = 17, normalized size = 0.89

method	result	size
default	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17
risch	$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-x+2),x,method=_RETURNVERBOSE)
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Maxima [A]
time = 1.43, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Fricas [A]
time = 1.19, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A]
time = 0.03, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\,\operatorname{atan}\left(\frac{2\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-x+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.84

$$\frac{2}{7}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Mupad [B]
time = 0.08, size = 16, normalized size = 0.84

$$\frac{2\,\sqrt{7}\,\operatorname{atan}\left(\frac{\sqrt{7}\,(2\,x-1)}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x + 2),x)
[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

3.92 $\int x \tan^{-1}(x)\,dx$

Optimal. Leaf size=21

$$-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}x^2\tan^{-1}(x)$$

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4946, 327, 209}

$$\frac{1}{2}x^2\text{ArcTan}(x)+\frac{\text{ArcTan}(x)}{2}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(x)\,dx &= \frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\int\frac{1}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}x^2\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}x^2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
[Out] -1/2*x + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.76

method	result	size
default	$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2}+\frac{(3x^2+3)\arctan(x)}{6}$	16
risch	$-\frac{ix^2\ln(ix+1)}{4}+\frac{ix^2\ln(-ix+1)}{4}-\frac{x}{2}+\frac{\arctan(x)}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A]
time = 2.10, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A]
time = 1.01, size = 13, normalized size = 0.62

$$\frac{1}{2}\left(x^2+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A]
time = 0.07, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Giac [A]
time = 0.46, size = 15, normalized size = 0.71

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.67

$$\operatorname{atan}(x)\,\left(\frac{x^2}{2}+\frac{1}{2}\right)-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x),x)
[Out] atan(x)*(x^2/2 + 1/2) - x/2
```

3.93 $\int x^2 \cos^{-1}(x) \, dx$

Optimal. Leaf size=40

$$-\frac{1}{3}\sqrt{1-x^2}+\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\cos^{-1}(x)$$

```
[Out] 1/9*(-x^2+1)^(3/2)+1/3*x^3*arccos(x)-1/3*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4724, 272, 45}

$$\frac{1}{3}x^3\text{ArcCos}(x)+\frac{1}{9}\left(1-x^2\right)^{3/2}-\frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[x],x]
```

```
[Out] -1/3*Sqrt[1 - x^2] + (1 - x^2)^(3/2)/9 + (x^3*ArcCos[x])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \cos^{-1}(x)\,dx &= \frac{1}{3}x^3\cos^{-1}(x) + \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}\,dx \\
&= \frac{1}{3}x^3\cos^{-1}(x) + \frac{1}{6}\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\cos^{-1}(x) + \frac{1}{6}\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right)\,dx, x, x^2\right) \\
&= -\frac{1}{3}\sqrt{1-x^2} + \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3\cos^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.75

$$-\frac{1}{9}\sqrt{1-x^2}\,\left(2+x^2\right) + \frac{1}{3}x^3\cos^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[x],x]
[Out] -1/9*(Sqrt[1 - x^2]*(2 + x^2)) + (x^3*ArcCos[x])/3
```

Maple [A]
time = 0.01, size = 34, normalized size = 0.85

method	result	size
default	$\frac{x^3\arccos(x)}{3} - \frac{x^2\sqrt{-x^2+1}}{9} - \frac{2\sqrt{-x^2+1}}{9}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(x),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*arccos(x)-1/9*x^2*(-x^2+1)^(1/2)-2/9*(-x^2+1)^(1/2)
```

Maxima [A]
time = 1.91, size = 33, normalized size = 0.82

$$\frac{1}{3}\,x^3\arccos\left(x\right) - \frac{1}{9}\,\sqrt{-x^2+1}\,x^2 - \frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="maxima")
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

Fricas [A]
time = 1.06, size = 24, normalized size = 0.60

$$\frac{1}{3}\,x^3\arccos\left(x\right)-\frac{1}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="fricas")
[Out] 1/3*x^3*arccos(x) - 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A]
time = 0.09, size = 32, normalized size = 0.80

$$\frac{x^3\,\mathrm{acos}\left(x\right)}{3}-\frac{x^2\sqrt{1-x^2}}{9}-\frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(x),x)
[Out] x**3*acos(x)/3 - x**2*sqrt(1 - x**2)/9 - 2*sqrt(1 - x**2)/9
```

Giac [A]
time = 0.46, size = 33, normalized size = 0.82

$$\frac{1}{3}\,x^3\arccos\left(x\right)-\frac{1}{9}\,\sqrt{-x^2+1}\,x^2-\frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="giac")
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.60

$$\frac{x^3\,\mathrm{acos}(x)}{3}-\frac{\sqrt{1-x^2}\,\left(x^2+2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*acos(x),x)
[Out] (x^3*acos(x))/3 - ((1 - x^2)^(1/2)*(x^2 + 2))/9
```

3.94 $\int x \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=35

$$-x \tan^{-1}(x) + \frac{1}{2} \tan^{-1}(x)^2 + \frac{1}{2} x^2 \tan^{-1}(x)^2 + \frac{1}{2} \log \left(1 + x^2\right)$$

```
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {4946, 5036, 4930, 266, 5004}

$$\frac{1}{2} x^2 \text{ArcTan}(x)^2 + \frac{\text{ArcTan}(x)^2}{2} - x \text{ArcTan}(x) + \frac{1}{2} \log \left(x^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x]^2,x]
```

```
[Out] -(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int x\tan^{-1}(x)^2\,dx &= \frac{1}{2}x^2\tan^{-1}(x)^2-\int\frac{x^2\tan^{-1}(x)}{1+x^2}\,dx\\
&= \frac{1}{2}x^2\tan^{-1}(x)^2-\int\tan^{-1}(x)\,dx+\int\frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= -x\tan^{-1}(x)+\frac{1}{2}\tan^{-1}(x)^2+\frac{1}{2}x^2\tan^{-1}(x)^2+\int\frac{x}{1+x^2}\,dx\\
&= -x\tan^{-1}(x)+\frac{1}{2}\tan^{-1}(x)^2+\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.74

$$\frac{1}{2}\left(-2x\tan^{-1}(x)+\left(1+x^2\right)\tan^{-1}(x)^2+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x]^2,x]
[Out] (-2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2 + Log[1 + x^2])/2
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.86

method	result
default	$-x\arctan\left(x\right)+\frac{\arctan(x)^2}{2}+\frac{x^2\arctan(x)^2}{2}+\frac{\ln(x^2+1)}{2}$
risch	$-\frac{\left(\frac{x^2}{2}+\frac{1}{2}\right)\ln(ix+1)^2}{4}-\frac{\left(-x^2\ln(-ix+1)-2ix-\ln(-ix+1)\right)\ln(ix+1)}{4}-\frac{x^2\ln(-ix+1)^2}{8}-\frac{\ln(-ix+1)^2}{8}-\frac{ix\ln(-ix+1)}{2}+$ ln(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)^2,x,method=_RETURNVERBOSE)
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

Maxima [A]
time = 1.50, size = 34, normalized size = 0.97

$$\frac{1}{2}\,x^2\arctan\left(x\right)^2-\left(x-\arctan\left(x\right)\right)\arctan\left(x\right)-\frac{1}{2}\,\arctan\left(x\right)^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x)^2 - (x - arctan(x))*arctan(x) - 1/2*arctan(x)^2 + 1/2*log
(x^2 + 1)
```

Fricas [A]
time = 0.82, size = 25, normalized size = 0.71

$$\frac{1}{2}\,\left(x^2+1\right)\arctan\left(x\right)^2-x\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x)^2 - x*arctan(x) + 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.09, size = 29, normalized size = 0.83

$$\frac{x^2\,\mathrm{atan}^2\left(x\right)}{2}-x\,\mathrm{atan}\left(x\right)+\frac{\log\left(x^2+1\right)}{2}+\frac{\mathrm{atan}^2\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)**2,x)
```

```
[Out] x**2*atan(x)**2/2 - x*atan(x) + log(x**2 + 1)/2 + atan(x)**2/2
```

Giac [A]
time = 0.46, size = 29, normalized size = 0.83

$$\frac{1}{2}\,x^2\arctan\left(x\right)^2-x\arctan\left(x\right)+\frac{1}{2}\,\arctan\left(x\right)^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x)^2 - x*arctan(x) + 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.12, size = 29, normalized size = 0.83

$$\frac{\ln\left(x^2+1\right)}{2}+\frac{\mathrm{atan}(x)^2}{2}+\frac{x^2\,\mathrm{atan}(x)^2}{2}-x\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x)^2,x)
[Out] log(x^2 + 1)/2 + atan(x)^2/2 + (x^2*atan(x)^2)/2 - x*atan(x)
```

3.95 $\int \tan^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$-\sqrt{x}+\tan^{-1}\left(\sqrt{x}\right)+x\tan^{-1}\left(\sqrt{x}\right)$$

[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4930, 52, 65, 209}

$$x\text{ArcTan}\left(\sqrt{x}\right)+\text{ArcTan}\left(\sqrt{x}\right)-\sqrt{x}$$

Antiderivative was successfully verified.

[In] Int[ArcTan[Sqrt[x]],x]

[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(\sqrt{x}\right)\,dx &= x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx\\ &= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\ &= -\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= -\sqrt{x}+\tan^{-1}\left(\sqrt{x}\right)+x\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.82

$$-\sqrt{x}+(1+x)\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]],x]
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
default	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
meijerg	$-\sqrt{x}+\frac{(3x+3)\arctan\left(\sqrt{x}\right)}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Maxima [A]
time = 3.53, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Fricas [A]
time = 0.94, size = 14, normalized size = 0.64

$$(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A]
time = 0.72, size = 19, normalized size = 0.86

$$-\sqrt{x}+x\,\mathrm{atan}\left(\sqrt{x}\right)+\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2)),x)
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Giac [A]
time = 0.52, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.73

$$\mathrm{atan}\left(\sqrt{x}\right)+x\,\mathrm{atan}\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2)),x)
[Out] atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

3.96 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\,(1+x)}\,dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

```
[Out] arctan(x^(1/2))^2
```

Rubi [A]
time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {65, 209, 6818}

$$\text{ArcTan}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\,(1+x)}\,dx = \tan^{-1}\left(\sqrt{x}\right)^2$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

Maple [A]
time = 0.06, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)^2$	7
default	$\arctan\left(\sqrt{x}\right)^2$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/(1+x)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x^(1/2))^2
```

Maxima [A]
time = 1.89, size = 6, normalized size = 0.75

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x))^2
```

Fricas [A]
time = 0.96, size = 6, normalized size = 0.75

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))^2
```

Sympy [A]
time = 0.43, size = 7, normalized size = 0.88

$$\operatorname{atan}^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/(1+x)/x**(1/2),x)
```

```
[Out] atan(sqrt(x))**2
```

Giac [A]
time = 0.50, size = 6, normalized size = 0.75

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x))^2
```

Mupad [B]
time = 0.66, size = 6, normalized size = 0.75

$$\mathrm{atan}\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/(x^(1/2)*(x + 1)),x)
```

```
[Out] atan(x^(1/2))^2
```

3.97 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {201, 222}

$$\frac{\text{ArcSin}(x)}{2}+\frac{1}{2}\sqrt{1-x^2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\,dx&=\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\&=\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 1.61

$$\frac{1}{2}x\sqrt{1-x^2}-\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.07, size = 18, normalized size = 0.78

method	result	size
default	$\frac{\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A]
time = 1.51, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.84, size = 31, normalized size = 1.35

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [**A**]
time = 0.06, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Giac [**A**]
time = 0.47, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [**B**]
time = 0.08, size = 17, normalized size = 0.74

$$\frac{\operatorname{asin}(x)}{2}+\frac{x\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2),x)
```

```
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

3.98 $\int \frac{e^{\tan^{-1}(x)}x}{(1+x^2)^{3/2}} dx$

Optimal. Leaf size=22

$$-\frac{e^{\tan^{-1}(x)}(1-x)}{2\sqrt{1+x^2}}$$

```
[Out] -1/2*exp(arctan(x))*(1-x)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {5185}

$$-\frac{(1-x)e^{\text{ArcTan}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

```
[Out] -1/2*(E^ArcTan[x]*(1 - x))/Sqrt[1 + x^2]
```

Rule 5185

```
Int[(E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> Simp[(-(1 - a*n*x))*(E^(n*ArcTan[a*x])/(d*(n^2 + 1)*Sqrt[c + d*x^2])
), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}x}{(1+x^2)^{3/2}} dx = -\frac{e^{\tan^{-1}(x)}(1-x)}{2\sqrt{1+x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.01, size = 37, normalized size = 1.68

$$\frac{1}{2}(1-ix)^{-\frac{1}{2}+\frac{i}{2}}(1+ix)^{-\frac{1}{2}-\frac{i}{2}}(-1+x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

```
[Out] (-1 + x)/(2*(1 - I*x)^(1/2 - I/2)*(1 + I*x)^(1/2 + I/2))
```

Maple [A]
time = 0.07, size = 16, normalized size = 0.73

method	result	size
gosper	$\frac{(-1+x)e^{\arctan(x)}}{2\sqrt{x^2+1}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))*x/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-1+x)*exp(arctan(x))/(x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Fricas [A]
time = 0.95, size = 15, normalized size = 0.68

$$\frac{(x-1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*(x - 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [A]
time = 11.60, size = 31, normalized size = 1.41

$$\frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}} - \frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))*x/(x**2+1)**(3/2),x)
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) - exp(atan(x))/(2*sqrt(x**2 + 1))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(15) = 30.
time = 0.48, size = 57, normalized size = 2.59

$$-\frac{1}{2}\cos\left(\frac{1}{2}\arctan(x)\right)^4 e^{\arctan(x)} + \cos\left(\frac{1}{2}\arctan(x)\right)^3 e^{\arctan(x)}\sin\left(\frac{1}{2}\arctan(x)\right) + \cos\left(\frac{1}{2}\arctan(x)\right)e^{\arctan(x)}\sin\left(\frac{1}{2}\arctan(x)\right)^3 + \frac{1}{2}e^{\arctan(x)}\sin\left(\frac{1}{2}\arctan(x)\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*cos(1/2*arctan(x))^4*e^arctan(x) + cos(1/2*arctan(x))^3*e^arctan(x)*si
n(1/2*arctan(x)) + cos(1/2*arctan(x))*e^arctan(x)*sin(1/2*arctan(x))^3 + 1/
2*e^arctan(x)*sin(1/2*arctan(x))^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x\,\mathrm{e}^{\mathrm{atan}(x)}}{{\left(x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*exp(atan(x)))/(x^2 + 1)^(3/2),x)
```

```
[Out] int((x*exp(atan(x)))/(x^2 + 1)^(3/2), x)
```

3.99 $\int \frac{e^{\tan^{-1}(x)}}{(1+x^2)^{3/2}} dx$

Optimal. Leaf size=20

$$\frac{e^{\tan^{-1}(x)}(1+x)}{2\sqrt{1+x^2}}$$

```
[Out] 1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5177}

$$\frac{(x+1)e^{\text{ArcTan}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[x]/(1 + x^2)^(3/2),x]
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Rule 5177

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[(n + a*x)*(E^(n*ArcTan[a*x])/(a*c*(n^2 + 1)*Sqrt[c + d*x^2])), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}}{(1+x^2)^{3/2}} dx = \frac{e^{\tan^{-1}(x)}(1+x)}{2\sqrt{1+x^2}}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{e^{\tan^{-1}(x)}(1+x)}{2\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[x]/(1 + x^2)^(3/2),x]
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Maple [A]
time = 0.07, size = 16, normalized size = 0.80

method	result	size
gosper	$\frac{e^{\arctan(x)}(1+x)}{2\sqrt{x^2+1}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Fricas [A]
time = 0.90, size = 15, normalized size = 0.75

$$\frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] 1/2*(x + 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [A]
time = 11.05, size = 31, normalized size = 1.55

$$\frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}+\frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))/(x**2+1)**(3/2),x)
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) + exp(atan(x))/(2*sqrt(x**2 + 1))
```

Giac [A]
time = 0.46, size = 22, normalized size = 1.10

$$\frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}}+\frac{1}{\sqrt{x^2+1}}\right)e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="giac")
[Out] 1/2*(x/sqrt(x^2 + 1) + 1/sqrt(x^2 + 1))*e^arctan(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{e^{\mathrm{atan}(x)}}{(x^2+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(atan(x))/(x^2 + 1)^(3/2),x)
[Out] int(exp(atan(x))/(x^2 + 1)^(3/2), x)
```

3.100 $\int \frac{x^2}{(1+x^2)^2} dx$

Optimal. Leaf size=19

$$-\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {294, 209}

$$\frac{\text{ArcTan}(x)}{2}-\frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+x^2)^2}\,dx &= -\frac{x}{2(1+x^2)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

Maple [A]
time = 0.08, size = 16, normalized size = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A]
time = 1.08, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A]
time = 0.99, size = 21, normalized size = 1.11

$$\frac{(x^2+1)\arctan(x)-x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Giac [A]
time = 0.44, size = 15, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2}-\frac{x}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + 1)^2,x)
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

3.101 $\int \frac{e^x}{1+e^{2x}} dx$

Optimal. Leaf size=4

$$\tan^{-1}(e^x)$$

```
[Out] arctan(exp(x))
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2281, 209}

$$\text{ArcTan}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^(2*x)),x]
```

```
[Out] ArcTan[E^x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2281

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Denom
inator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+e^{2x}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\ &= \tan^{-1}(e^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^(2*x)),x]
```

```
[Out] ArcTan[E^x]
```

Maple [A]
time = 0.02, size = 4, normalized size = 1.00

method	result	size
default	$\arctan\left(\mathrm{e}^{x}\right)$	4
risch	$\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{2}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(exp(x))
```

Maxima [A]
time = 1.05, size = 3, normalized size = 0.75

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")
```

```
[Out] arctan(e^x)
```

Fricas [A]
time = 1.12, size = 3, normalized size = 0.75

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] arctan(e^x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.
time = 0.04, size = 15, normalized size = 3.75

$$\operatorname{RootSum}\left(4z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Giac [**A**]
time = 0.46, size = 3, normalized size = 0.75

$$\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")
```

```
[Out] arctan(e^x)
```

Mupad [**B**]
time = 0.10, size = 3, normalized size = 0.75

$$\mathrm{atan}(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(exp(2*x) + 1),x)
```

```
[Out] atan(exp(x))
```

3.102 $\int e^{-x} \cot^{-1}(e^x) \, dx$

Optimal. Leaf size=27

$$-x - e^{-x} \cot^{-1}(e^x) + \frac{1}{2} \log\left(1 + e^{2x}\right)$$

```
[Out] -x-arccot(exp(x))/exp(x)+1/2*ln(1+exp(2*x))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2225, 5316, 2320, 36, 29, 31}

$$-x + \frac{1}{2} \log\left(e^{2x} + 1\right) - e^{-x} \cot^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[E^x]/E^x,x]
```

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 5316

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[w*(D[u, x]/(1
+ u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rubi steps

$$\begin{aligned}
\int e^{-x}\cot^{-1}\left(e^x\right)\,dx &= -e^{-x}\cot^{-1}\left(e^x\right)-\int\frac{1}{1+e^{2x}}\,dx\\
&= -e^{-x}\cot^{-1}\left(e^x\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\
&= -e^{-x}\cot^{-1}\left(e^x\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{x}\,dx,x,e^{2x}\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x}\,dx,x,e^{2x}\right)\\
&= -x-e^{-x}\cot^{-1}\left(e^x\right)+\frac{1}{2}\log\left(1+e^{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$-x-e^{-x}\cot^{-1}\left(e^x\right)+\frac{1}{2}\log\left(1+e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[E^x]/E^x,x]
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Maple [A]
time = 0.03, size = 25, normalized size = 0.93

method	result	size
derivativedivides	$-\mathrm{arccot}(\mathrm{e}^{x})\,\mathrm{e}^{-x}-\ln\left(\mathrm{e}^{x}\right)+\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{2}$	25
default	$-\mathrm{arccot}(\mathrm{e}^{x})\,\mathrm{e}^{-x}-\ln\left(\mathrm{e}^{x}\right)+\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{2}$	25
risch	$-\frac{i\mathrm{e}^{-x}\ln\left(1+i\mathrm{e}^{x}\right)}{2}-\frac{\left(-i\ln\left(1-i\mathrm{e}^{x}\right)-\ln\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{x}+2\,\mathrm{e}^{x}x+\pi\right)\mathrm{e}^{-x}}{2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(exp(x))/exp(x),x,method=_RETURNVERBOSE)
```

```
[Out] -arccot(exp(x))/exp(x)-ln(exp(x))+1/2*ln(1+exp(x)^2)
```

Maxima [A]
time = 1.12, size = 19, normalized size = 0.70

$$-\operatorname{arccot}\left(e^{x}\right)e^{(-x)}+\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")
```

```
[Out] -arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Fricas [A]
time = 1.05, size = 28, normalized size = 1.04

$$-\frac{1}{2}\left(2xe^{x}-e^{x}\log\left(e^{(2x)}+1\right)+2\operatorname{arccot}\left(e^{x}\right)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

Sympy [A]
time = 2.17, size = 19, normalized size = 0.70

$$-x+\frac{\log\left(e^{2x}+1\right)}{2}-e^{-x}\operatorname{acot}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(exp(x))/exp(x),x)
```

```
[Out] -x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

Giac [A]
time = 0.44, size = 21, normalized size = 0.78

$$-\arctan\left(e^{(-x)}\right)e^{(-x)}+\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
[Out] -arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Mupad [B]
time = 0.14, size = 22, normalized size = 0.81

$$\frac{\ln\left(e^{2x}+1\right)}{2}-x-\mathrm{acot}(e^{x})\;e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acot(exp(x))*exp(-x),x)
[Out] log(exp(2*x) + 1)/2 - x - acot(exp(x))*exp(-x)
```

3.103 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$-\left((a-x)\sqrt{\frac{a+x}{a-x}}\right)+2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

```
[Out] 2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1979, 294, 209}

$$2a\text{ArcTan}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+x}{a-x}}\,dx &= (4a)\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}} + (2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}} + 2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 1.60

$$\frac{\sqrt{\frac{a+x}{a-x}}\left((-a+x)\sqrt{a+x}+2a\sqrt{a-x}\tan^{-1}\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)\right)}{\sqrt{a+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
[Out] (Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + 2*a*Sqrt[a - x]*ArcTan[Sqrt[
a + x]/Sqrt[a - x]]))/Sqrt[a + x]
```

Maple [A]
time = 0.04, size = 61, normalized size = 1.45

method	result	size
default	$\frac{\sqrt{\frac{a+x}{a-x}}\,(a-x)\left(a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)}{\sqrt{(a-x)(a+x)}}$	61
risch	$-\frac{(a-x)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{\sqrt{-(-a+x)(a+x)}}+\frac{a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{a+x}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+x)/(a-x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+x)/(a-x))^(1/2)*(a-x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))/((a
-x)*(a+x))^(1/2)
```

Maxima [A]
time = 0.99, size = 49, normalized size = 1.17

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

Fricas [A]
time = 0.84, size = 38, normalized size = 0.90

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

Giac [A]
time = 0.45, size = 36, normalized size = 0.86

$$a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a)-\sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

Mupad [B]
time = 0.07, size = 49, normalized size = 1.17

$$2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + x)/(a - x))^(1/2),x)
[Out] 2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)/
(a - x) + 1)
```

3.104 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*(
-a*b+(a+b)*x-x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1976, 626, 635, 210}

$$-\frac{1}{8}(a-b)^2\text{ArcTan}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -1/4*((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2]) - ((a - b)^2*ArcTan[(a
+ b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(b-x)(-a+x)}\,dx &= \int \sqrt{-ab+(a+b)x-x^2}\,dx\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-ab+(a+b)x-x^2}\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{4}(a-b)^2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,-\right.\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x}}\right.\end{aligned}$$

Mathematica [A]
time = 0.10, size = 76, normalized size = 1.07

$$\frac{1}{4}\sqrt{(a-x)(-b+x)}\left(-a-b+2x-\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{-a+x}}\right)}{\sqrt{b-x}\,\sqrt{-a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (Sqrt[(a - x)*(-b + x)]*(-a - b + 2*x - ((a - b)^2*ArcTan[Sqrt[b - x]/Sqrt[
-a + x]])/(Sqrt[b - x]*Sqrt[-a + x])))/4
```

Maple [A]
time = 0.10, size = 68, normalized size = 0.96

method	result
default	$-\frac{(a+b-2x)\sqrt{-ab+(a+b)x-x^2}}{4}-\frac{\left(4ab-(a+b)^2\right)\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)}{8}$
risch	$\frac{(b-x)(a-x)(a+b-2x)}{4\sqrt{-(-b+x)(-a+x)}}-\frac{\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)ab}{4}+\frac{\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/8*(4*a*b-(a+b)^2)*arctan((x-1/2*b
-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.95, size = 80, normalized size = 1.13

$$-\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^{2}}\,(a+b-2\,x)}{2\,(ab-(a+b)x+x^{2})}\right)-\frac{1}{4}\,\sqrt{-ab+(a+b)x-x^{2}}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{(-a+x)\,(b-x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

Giac [A]
time = 0.45, size = 61, normalized size = 0.86

$$\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b)-\frac{1}{4}\,\sqrt{-ab+ax+bx-x^{2}}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-\left(a-x\right)\,\left(b-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int((-(a - x)*(b - x))^(1/2), x)
```

3.105 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1976, 635, 210}

$$-\text{ArcTan}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)$$
$$= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 1.72

$$-\frac{2\sqrt{b-x}\sqrt{-a+x}\tan^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{-a+x}}\right)}{\sqrt{(b-x)(-a+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
[Out] (-2*Sqrt[b - x]*Sqrt[-a + x]*ArcTan[Sqrt[b - x]/Sqrt[-a + x]])/Sqrt[(b - x)
*(-a + x)]
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.88

method	result	size
default	$\arctan\left(\frac{x-\frac{b}{2}-\frac{a}{2}}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.97, size = 43, normalized size = 1.34

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(-a+x)\,(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.
time = 0.45, size = 61, normalized size = 1.91

$$\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b)-\frac{1}{4}\sqrt{-ab+ax+bx-x^2}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{-\left(a-x\right)\,\left(b-x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int(1/(-(a - x)*(b - x))^(1/2), x)
```

3.106 $\int \frac{3+5x}{-3+2x+x^2} \, dx$

Optimal. Leaf size=15

$$2\log(1-x)+3\log(3+x)$$

[Out] 2*ln(1-x)+3*ln(3+x)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {646, 31}

$$2\log(1-x)+3\log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(3 + 5*x)/(-3 + 2*x + x^2),x]

[Out] 2*Log[1 - x] + 3*Log[3 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{3+5x}{-3+2x+x^2} \, dx &= 2\int \frac{1}{-1+x} \, dx+3\int \frac{1}{3+x} \, dx\\ &= 2\log(1-x)+3\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$2\log(1-x)+3\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*x)/(-3 + 2*x + x^2),x]
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.93

method	result	size
default	$2\ln(-1+x)+3\ln(3+x)$	14
norman	$2\ln(-1+x)+3\ln(3+x)$	14
risch	$2\ln(-1+x)+3\ln(3+x)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+5*x)/(x^2+2*x-3),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+3*ln(3+x)
```

Maxima [A]
time = 0.98, size = 13, normalized size = 0.87

$$3\log(x+3)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="maxima")
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Fricas [A]
time = 1.23, size = 13, normalized size = 0.87

$$3\log(x+3)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="fricas")
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.80

$$2\log(x-1)+3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x**2+2*x-3),x)
[Out] 2*log(x - 1) + 3*log(x + 3)
```

Giac [**A**]
time = 0.45, size = 15, normalized size = 1.00

$$3 \log(|x+3|) + 2 \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="giac")
```

```
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1))
```

Mupad [**B**]
time = 0.05, size = 13, normalized size = 0.87

$$2 \ln(x-1) + 3 \ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3)/(2*x + x^2 - 3),x)
```

```
[Out] 2*log(x - 1) + 3*log(x + 3)
```

3.107 $\int \frac{5+2x}{-3+2x+x^2} \, dx$

Optimal. Leaf size=19

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(3+x)$$

```
[Out] 7/4*ln(1-x)+1/4*ln(3+x)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {646, 31}

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)/(-3 + 2*x + x^2),x]
```

```
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{5+2x}{-3+2x+x^2}\,dx &= \frac{1}{4}\int \frac{1}{3+x}\,dx+\frac{7}{4}\int \frac{1}{-1+x}\,dx\\ &= \frac{7}{4}\log(1-x)+\frac{1}{4}\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)/(-3 + 2*x + x^2),x]
```

```
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Maple [A]
time = 0.07, size = 14, normalized size = 0.74

method	result	size
default	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
norman	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14
risch	$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)/(x^2+2*x-3),x,method=_RETURNVERBOSE)
```

```
[Out] 7/4*ln(-1+x)+1/4*ln(3+x)
```

Maxima [A]
time = 1.71, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x+3\right)+\frac{7}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="maxima")
```

```
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Fricas [A]
time = 1.03, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x+3\right)+\frac{7}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="fricas")
```

```
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.74

$$\frac{7\log\left(x-1\right)}{4}+\frac{\log\left(x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x**2+2*x-3),x)
```

```
[Out] 7*log(x - 1)/4 + log(x + 3)/4
```

Giac [A]
time = 0.46, size = 15, normalized size = 0.79

$$\frac{1}{4}\log\left(|x+3|\right)+\frac{7}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(x + 3)) + 7/4*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 13, normalized size = 0.68

$$\frac{7\ln\left(x-1\right)}{4}+\frac{\ln\left(x+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)/(2*x + x^2 - 3),x)
```

```
[Out] (7*log(x - 1))/4 + log(x + 3)/4
```

3.108 $\int \frac{3x+x^3}{-3-2x+x^2}\,dx$

Optimal. Leaf size=23

$$2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)$$

```
[Out] 2*x+1/2*x^2+9*ln(3-x)+ln(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1607, 1642, 646, 31}

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x + x^3)/(-3 - 2*x + x^2),x]
```

```
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{3x+x^3}{-3-2x+x^2}\,dx &= \int \frac{x(3+x^2)}{-3-2x+x^2}\,dx\\ &= \int \left(2+x+\frac{2(3+5x)}{-3-2x+x^2}\right)dx\\ &= 2x+\frac{x^2}{2}+2\int \frac{3+5x}{-3-2x+x^2}\,dx\\ &= 2x+\frac{x^2}{2}+9\int \frac{1}{-3+x}\,dx+\int \frac{1}{1+x}\,dx\\ &= 2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x + x^3)/(-3 - 2*x + x^2),x]
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.87

method	result	size
default	$2x+\frac{x^2}{2}+9\ln(-3+x)+\ln(1+x)$	20
norman	$2x+\frac{x^2}{2}+9\ln(-3+x)+\ln(1+x)$	20
risch	$2x+\frac{x^2}{2}+9\ln(-3+x)+\ln(1+x)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x)/(x^2-2*x-3),x,method=_RETURNVERBOSE)
[Out] 2*x+1/2*x^2+9*ln(-3+x)+ln(1+x)
```

Maxima [A]
time = 0.96, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2+2\,x+\log(x+1)+9\,\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Fricas [A]
time = 0.82, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2 + 2\,x + \log\left(x+1\right) + 9\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + 2x + 9\log\left(x-3\right) + \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x)/(x**2-2*x-3),x)
```

```
[Out] x**2/2 + 2*x + 9*log(x - 3) + log(x + 1)
```

Giac [A]
time = 0.44, size = 21, normalized size = 0.91

$$\frac{1}{2}\,x^2 + 2\,x + \log\left(|x+1|\right) + 9\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2*x + log(abs(x + 1)) + 9*log(abs(x - 3))
```

Mupad [B]
time = 0.04, size = 19, normalized size = 0.83

$$2\,x + \ln\left(x+1\right) + 9\,\ln\left(x-3\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + x^3)/(2*x - x^2 + 3),x)
```

```
[Out] 2*x + log(x + 1) + 9*log(x - 3) + x^2/2
```

3.109 $\int \frac{-1+5x+2x^2}{-2x+x^2+x^3} dx$

Optimal. Leaf size=23

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)$$

```
[Out] 2*ln(1-x)+1/2*ln(x)-1/2*ln(2+x)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1608, 1642}

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx &= \int \frac{-1+5x+2x^2}{x\left(-2+x+x^2\right)}\,dx\\ &= \int \left(\frac{2}{-1+x}+\frac{1}{2x}-\frac{1}{2(2+x)}\right)dx\\ &= 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.78

method	result	size
default	$\frac{\ln(x)}{2}+2\ln(-1+x)-\frac{\ln(2+x)}{2}$	18
norman	$\frac{\ln(x)}{2}+2\ln(-1+x)-\frac{\ln(2+x)}{2}$	18
risch	$\frac{\ln(x)}{2}+2\ln(-1+x)-\frac{\ln(2+x)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+5*x-1)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)+2*ln(-1+x)-1/2*ln(2+x)
```

Maxima [A]
time = 1.01, size = 17, normalized size = 0.74

$$-\frac{1}{2}\log(x+2)+2\log(x-1)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Fricas [A]
time = 0.75, size = 17, normalized size = 0.74

$$-\frac{1}{2}\log(x+2)+2\log(x-1)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.74

$$\frac{\log\left(x\right)}{2} + 2\log\left(x - 1\right) - \frac{\log\left(x + 2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+5*x-1)/(x**3+x**2-2*x),x)
[Out] log(x)/2 + 2*log(x - 1) - log(x + 2)/2
```

Giac [A]
time = 0.45, size = 20, normalized size = 0.87

$$-\frac{1}{2}\,\log\left(|x + 2|\right) + 2\,\log\left(|x - 1|\right) + \frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] -1/2*log(abs(x + 2)) + 2*log(abs(x - 1)) + 1/2*log(abs(x))
```

Mupad [B]
time = 0.19, size = 19, normalized size = 0.83

$$2\,\ln\left(x - 1\right) + \mathrm{atanh}\left(\frac{135}{11\,\left(11\,x - 5\right)} + \frac{16}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 2*x^2 - 1)/(x^2 - 2*x + x^3),x)
[Out] 2*log(x - 1) + atanh(135/(11*(11*x - 5)) + 16/11)
```

3.110 $\int \frac{3+2x+x^2}{(-1+x)(1+x)^2} dx$

Optimal. Leaf size=24

$$\frac{1}{1+x}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(1+x)$$

```
[Out] 1/(1+x)+3/2*ln(1-x)-1/2*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {907}

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

```
[Out] (1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx &= \int \left(\frac{3}{2(-1+x)}-\frac{1}{(1+x)^2}-\frac{1}{2(1+x)}\right) dx\\ &= \frac{1}{1+x}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.92

$$\frac{1}{1+x}+\frac{3}{2}\log(-1+x)-\frac{1}{2}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

```
[Out] (1 + x)^(-1) + (3*Log[-1 + x])/2 - Log[1 + x]/2
```

Maple [A]
time = 0.07, size = 19, normalized size = 0.79

method	result	size
default	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
norman	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19
risch	$\frac{3\ln(-1+x)}{2}+\frac{1}{1+x}-\frac{\ln(1+x)}{2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+3)/(-1+x)/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*ln(-1+x)+1/(1+x)-1/2*ln(1+x)
```

Maxima [A]
time = 6.61, size = 18, normalized size = 0.75

$$\frac{1}{x+1}-\frac{1}{2}\log\left(x+1\right)+\frac{3}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) - 1/2*log(x + 1) + 3/2*log(x - 1)
```

Fricas [A]
time = 0.81, size = 26, normalized size = 1.08

$$-\frac{(x+1)\log\left(x+1\right)-3\,(x+1)\log\left(x-1\right)-2}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x + 1)*log(x + 1) - 3*(x + 1)*log(x - 1) - 2)/(x + 1)
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.79

$$\frac{3\log\left(x-1\right)}{2}-\frac{\log\left(x+1\right)}{2}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+3)/(-1+x)/(1+x)**2,x)
```

```
[Out] 3*log(x - 1)/2 - log(x + 1)/2 + 1/(x + 1)
```

Giac [A]
time = 0.48, size = 24, normalized size = 1.00

$$\frac{1}{x+1} + \log\left(|x+1|\right) + \frac{3}{2}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1)) + 3/2*log(abs(-2/(x + 1) + 1))
```

Mupad [B]
time = 0.05, size = 18, normalized size = 0.75

$$\frac{3\ln\left(x-1\right)}{2} - \frac{\ln\left(x+1\right)}{2} + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 3)/((x - 1)*(x + 1)^2),x)
```

```
[Out] (3*log(x - 1))/2 - log(x + 1)/2 + 1/(x + 1)
```

3.111 $\int \frac{-2+2x+3x^2}{-1+x^3} \, dx$

Optimal. Leaf size=28

$$\frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(1-x^3\right)$$

```
[Out] ln(-x^3+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1885, 1600, 632, 210, 266}

$$\frac{4\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+2x+3x^2}{-1+x^3}\,dx &= 3\int \frac{x^2}{-1+x^3}\,dx+\int \frac{-2+2x}{-1+x^3}\,dx\\ &= \log\left(1-x^3\right)+\int \frac{1}{\frac{1}{2}+\frac{x}{2}+\frac{x^2}{2}}\,dx\\ &= \log\left(1-x^3\right)-2\mathrm{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx,x,\frac{1}{2}+x\right)\\ &= \frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(1-x^3\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$\frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Maple [A]
time = 0.08, size = 29, normalized size = 1.04

method	result
default	$\ln(-1+x)+\ln(x^2+x+1)+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\ln(-1+x)+\ln(16x^2+16x+16)+\frac{4\sqrt{3}\,\arctan\left(\frac{(2+4x)\sqrt{3}}{6}\right)}{3}$

meijerg	$-\frac{2x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\ln\left(-x^3+1\right)+\frac{2x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x-2)/(x^3-1),x,method=_RETURNVERBOSE)
[Out] ln(-1+x)+ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 3.47, size = 28, normalized size = 1.00

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

Fricas [A]
time = 0.87, size = 28, normalized size = 1.00

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

Sympy [A]
time = 0.05, size = 3, normalized size = 0.11

$$\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x-2)/(x**3-1),x)
[Out] log(x - 1)
```

Giac [A]
time = 0.45, size = 29, normalized size = 1.04

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x -
1))
```

Mupad [B]
time = 0.19, size = 57, normalized size = 2.04

$$\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)+\ln\left(x-1\right)-\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\ 2\mathrm{i}}{3}+\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\ 2\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 - 2)/(x^3 - 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x - 1)
- (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*2i)/3 + (3^(1/2)*log(x + (3^(1/2)*
1i)/2 + 1/2)*2i)/3
```

3.112 $\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2} \, dx$

Optimal. Leaf size=49

$$\frac{1}{2\left(2+x^2\right)}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{3}\log(1-x)+\frac{1}{3}\log\left(2+x^2\right)$$

[Out] 1/2/(x^2+2)+1/3*ln(1-x)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1661, 1643, 649, 209, 266}

$$-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 + Log[2 + x^
2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx &= \frac{1}{2\left(2+x^2\right)}-\frac{1}{4}\int \frac{-4+4x-4x^2}{(-1+x)\left(2+x^2\right)}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}-\frac{1}{4}\int \left(-\frac{4}{3(-1+x)}-\frac{4(-1+2x)}{3\left(2+x^2\right)}\right)dx\\
&= \frac{1}{2\left(2+x^2\right)}+\frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{-1+2x}{2+x^2}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}+\frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{1}{2+x^2}\,dx+\frac{2}{3}\int \frac{x}{2+x^2}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{3}\log(1-x)+\frac{1}{3}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 1.24

$$\frac{1}{2\left(3+2(-1+x)+(-1+x)^2\right)}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{3}\log\left(3+2(-1+x)+(-1+x)^2\right)+\frac{1}{3}\log(-1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(3 + 2*(-1 + x) + (-1 + x)^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[3
 + 2*(-1 + x) + (-1 + x)^2]/3 + Log[-1 + x]/3
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.76

method	result	size
default	$\frac{1}{2x^2+4}+\frac{\ln(x^2+2)}{3}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{6}+\frac{\ln(-1+x)}{3}$	37
risch	$\frac{1}{2x^2+4}+\frac{\ln(x^2+2)}{3}-\frac{\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}}{6}+\frac{\ln(-1+x)}{3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(x^2+2)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)+1/3*ln(-1+x)
```

Maxima [A]
time = 3.45, size = 36, normalized size = 0.73

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{1}{2\,(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(x - 1)
```

Fricas [A]
time = 0.64, size = 51, normalized size = 1.04

$$-\frac{\sqrt{2}\,(x^2+2)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-2\,(x^2+2)\log\left(x^2+2\right)-2\,(x^2+2)\log\left(x-1\right)-3}{6\,(x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) - 2*(x^2 + 2)*log(x^2 + 2) -
2*(x^2 + 2)*log(x - 1) - 3)/(x^2 + 2)
```

Sympy [A]
time = 0.07, size = 14, normalized size = 0.29

$$\frac{\log\left(x-1\right)}{3}+\frac{1}{2x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+2*x**2-x+2)/(-1+x)/(x**2+2)**2,x)
```

```
[Out] log(x - 1)/3 + 1/(2*x**2 + 4)
```

Giac [**A**]
time = 0.47, size = 37, normalized size = 0.76

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{1}{2\,(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(abs(x - 1))
```

Mupad [**B**]
time = 0.10, size = 53, normalized size = 1.08

$$\frac{\ln(x-1)}{3}+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-\frac{1}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{12}\right)+\frac{1}{2\,(x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x - x^3 + x^4 + 2)/((x^2 + 2)^2*(x - 1)),x)
```

```
[Out] log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/12 + 1/3) - log(x + 2^(1/2
)*1i)*((2^(1/2)*1i)/12 - 1/3) + 1/(2*(x^2 + 2))
```

3.113 $\int \frac{1}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(cos(x)-sin(x))*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3153, 212}

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos(x)+\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)-\sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 24, normalized size = 1.14

$$(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])^(-1),x]
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

Maple [A]
time = 0.06, size = 19, normalized size = 0.90

method	result	size
default	$\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)$	19
risch	$\frac{\sqrt{2}\ \ln\left(\mathrm{e}^{ix}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\ \ln\left(\mathrm{e}^{ix}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.
time = 11.00, size = 39, normalized size = 1.86

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.97, size = 38, normalized size = 1.81

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos\left(x\right)\right)\sin\left(x\right)-2\sqrt{2}\cos\left(x\right)+3}{2\cos\left(x\right)\sin\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) + 1))
```

Sympy [A]
time = 0.21, size = 39, normalized size = 1.86

$$\frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{2}-\frac{\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/2 - sqrt(2)*log(tan(x/2) - sqrt(2) - 1)
/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.
time = 0.48, size = 37, normalized size = 1.76

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2))
```

Mupad [B]
time = 0.33, size = 21, normalized size = 1.00

$$-\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) + sin(x)),x)
```

```
[Out] -2^(1/2)*atanh(2^(1/2)/2 - (2^(1/2)*tan(x/2))/2)
```

3.114 $\int \frac{x}{4-x^2+\sqrt{4-x^2}} \, dx$

Optimal. Leaf size=16

$$-\log \left(1+\sqrt{4-x^2}\right)$$

```
[Out] -ln(1+(-x^2+4)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2186, 31}

$$-\log \left(\sqrt{4-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{4-x^2+\sqrt{4-x^2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+\sqrt{4-x}-x} \, dx, x, x^2\right) \\ &= -\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, \sqrt{4-x^2}\right) \\ &= -\log \left(1+\sqrt{4-x^2}\right) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\log \left(1+\sqrt{4-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(14) = 28$.
time = 0.07, size = 271, normalized size = 16.94

method	result
trager	$-\ln\left(-1-\sqrt{-x^{2}+4}\right)$
default	$-\frac{\ln\left(x^{2}-3\right)}{2}+\frac{\sqrt{-\left(-2+x\right)^{2}-4x+8}-2\arcsin\left(\frac{x}{2}\right)}{2\left(2+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}-\frac{\sqrt{-\left(x+\sqrt{3}\right)^{2}+2\sqrt{3}\,\left(x+\sqrt{3}\right)+1}+\sqrt{3}}{2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4-x^2+(-x^2+4)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x^2-3)+1/2/(2+3^(1/2))/(-2+3^(1/2))*((-(-2+x)^2-4*x+8)^(1/2)-2*arcs
in(1/2*x))-1/2/(2+3^(1/2))/(-2+3^(1/2))*((-(x+3^(1/2))^2+2*3^(1/2)*(x+3^(1/
2))+1)^(1/2)+3^(1/2)*arcsin(1/2*x)-arctanh(1/2*(2+2*3^(1/2)*(x+3^(1/2)))/(-
(x+3^(1/2))^2+2*3^(1/2)*(x+3^(1/2))+1)^(1/2)))-1/2/(2+3^(1/2))/(-2+3^(1/2))
*((-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^(1/2)-3^(1/2)*arcsin(1/2*x)-arct
anh(1/2*(2-2*3^(1/2)*(x-3^(1/2)))/(-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^
(1/2)))+1/2/(2+3^(1/2))/(-2+3^(1/2))*((-(2+x)^2+4*x+8)^(1/2)+2*arcsin(1/2*x
))
```

Maxima [A]
time = 4.42, size = 14, normalized size = 0.88

$$-\log\left(\sqrt{-x^{2}+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log(sqrt(-x^2 + 4) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(14) = 28$.
time = 0.78, size = 55, normalized size = 3.44

$$-\frac{1}{2}\log\left(x^{2}-3\right)+\frac{1}{2}\log\left(-\frac{x^{2}+3\sqrt{-x^{2}+4}-6}{x^{2}}\right)-\frac{1}{2}\log\left(-\frac{x^{2}+\sqrt{-x^{2}+4}-2}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 3) + 1/2*log(-(x^2 + 3*sqrt(-x^2 + 4) - 6)/x^2) - 1/2*log(-(
x^2 + sqrt(-x^2 + 4) - 2)/x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.
time = 2.10, size = 44, normalized size = 2.75

$$\frac{\log\left(2\sqrt{4-x^2}\right)}{2} - \frac{\log\left(2\sqrt{4-x^2}+2\right)}{2} - \frac{\log\left(x^2-\sqrt{4-x^2}-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x**2+(-x**2+4)**(1/2)),x)
```

```
[Out] log(2*sqrt(4 - x**2))/2 - log(2*sqrt(4 - x**2) + 2)/2 - log(x**2 - sqrt(4 -
 x**2) - 4)/2
```

Giac [A]
time = 0.45, size = 14, normalized size = 0.88

$$-\log\left(\sqrt{-x^2+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(-x^2 + 4) + 1)
```

Mupad [B]
time = 0.16, size = 87, normalized size = 5.44

$$-\frac{\ln\left(x-\sqrt{3}\right)}{2} - \frac{\ln\left(\frac{\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x+\sqrt{3}}\right)}{2} - \frac{\ln\left(x+\sqrt{3}\right)}{2} - \frac{\ln\left(\frac{-\sqrt{3}\,x\,1\mathrm{i}+\sqrt{4-x^2}\,1\mathrm{i}+4\mathrm{i}}{x-\sqrt{3}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((4 - x^2)^(1/2) - x^2 + 4),x)
```

```
[Out] - log(x - 3^(1/2))/2 - log((3^(1/2)*x*1i + (4 - x^2)^(1/2)*1i + 4i)/(x + 3^
(1/2)))/2 - log(x + 3^(1/2))/2 - log(((4 - x^2)^(1/2)*1i - 3^(1/2)*x*1i + 4
i)/(x - 3^(1/2)))/2
```

3.115 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(5+x)$$

```
[Out] ln(2-x)+ln(5+x)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{78\}$

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[2 - x] + Log[5 + x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(-2+x)(5+x)}\,dx &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right)dx\\ &= \log(2-x)+\log(5+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.82

$$\log(-2+x)+\log(5+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
```

```
[Out] Log[-2 + x] + Log[5 + x]
```

Maple [A]
time = 0.06, size = 9, normalized size = 0.82

method	result	size
default	$\ln \left(\left(-2+x\right) \left(5+x\right)\right)$	9
norman	$\ln \left(-2+x\right)+\ln \left(5+x\right)$	10
risch	$\ln \left(x^{2}+3 x-10\right)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(-2+x)/(5+x),x,method=_RETURNVERBOSE)
[Out] ln((-2+x)*(5+x))
```

Maxima [A]
time = 3.33, size = 9, normalized size = 0.82

$$\log\left(x+5\right)+\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
[Out] log(x + 5) + log(x - 2)
```

Fricas [A]
time = 1.06, size = 9, normalized size = 0.82

$$\log\left(x^{2}+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
[Out] log(x^2 + 3*x - 10)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.73

$$\log\left(x^{2}+3x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
[Out] log(x**2 + 3*x - 10)
```

Giac [A]
time = 0.44, size = 11, normalized size = 1.00

$$\log\left(|x+5|\right)+\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

```
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

Mupad [B]
time = 0.05, size = 9, normalized size = 0.82

$$\ln\left(x^2+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
```

```
[Out] log(3*x + x^2 - 10)
```

3.116 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)$$

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{153\}$

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)(2+x)(3+x)} \, dx &= \int \left(-\frac{1}{2(1+x)}+\frac{2}{2+x}-\frac{3}{2(3+x)}\right) dx\\ &= -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Maple [A]
time = 0.07, size = 20, normalized size = 0.87

method	result	size
default	$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$	20
norman	$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$	20
risch	$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(2+x)/(3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Maxima [A]
time = 3.61, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log(x+3)+2\log(x+2)-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Fricas [A]
time = 1.30, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log(x+3)+2\log(x+2)-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.87

$$-\frac{\log(x+1)}{2}+2\log(x+2)-\frac{3\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Giac [A]
time = 0.45, size = 22, normalized size = 0.96

$$-\frac{3}{2}\,\log\left(|x+3|\right)+2\,\log\left(|x+2|\right)-\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

Mupad [B]
time = 0.11, size = 19, normalized size = 0.83

$$2\,\ln\left(x+2\right)-\frac{\ln\left(x+1\right)}{2}-\frac{3\,\ln\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(x + 2)*(x + 3)),x)
[Out] 2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

3.117 $\int \frac{x}{2-3x+x^3} \, dx$

Optimal. Leaf size=30

$$\frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(2+x)$$

```
[Out] 1/3/(1-x)+2/9*ln(1-x)-2/9*ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2099}

$$\frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - 3*x + x^3),x]
```

```
[Out] 1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2-3x+x^3} \, dx &= \int \left(\frac{1}{3(-1+x)^2} + \frac{2}{9(-1+x)} - \frac{2}{9(2+x)}\right) dx \\ &= \frac{1}{3(1-x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.93

$$-\frac{1}{3(-1+x)} + \frac{2}{9}\log(1-x) - \frac{2}{9}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - 3*x + x^3),x]
```

```
[Out] -1/3*1/(-1 + x) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.70

method	result	size
default	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
norman	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21
risch	$-\frac{1}{3(-1+x)}+\frac{2\ln(-1+x)}{9}-\frac{2\ln(2+x)}{9}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-3*x+2),x,method=_RETURNVERBOSE)
[Out] -1/3/(-1+x)+2/9*ln(-1+x)-2/9*ln(2+x)
```

Maxima [A]
time = 3.11, size = 20, normalized size = 0.67

$$-\frac{1}{3\,(x-1)}-\frac{2}{9}\,\log\left(x+2\right)+\frac{2}{9}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="maxima")
[Out] -1/3/(x - 1) - 2/9*log(x + 2) + 2/9*log(x - 1)
```

Fricas [A]
time = 1.03, size = 27, normalized size = 0.90

$$-\frac{2\,(x-1)\log\left(x+2\right)-2\,(x-1)\log\left(x-1\right)+3}{9\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="fricas")
[Out] -1/9*(2*(x - 1)*log(x + 2) - 2*(x - 1)*log(x - 1) + 3)/(x - 1)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.73

$$\frac{2\log\left(x-1\right)}{9}-\frac{2\log\left(x+2\right)}{9}-\frac{1}{3x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-3*x+2),x)
```

```
[Out] 2*log(x - 1)/9 - 2*log(x + 2)/9 - 1/(3*x - 3)
```

Giac [**A**]
time = 0.45, size = 22, normalized size = 0.73

$$-\frac{1}{3(x-1)}-\frac{2}{9}\log(|x+2|)+\frac{2}{9}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="giac")
```

```
[Out] -1/3/(x - 1) - 2/9*log(abs(x + 2)) + 2/9*log(abs(x - 1))
```

Mupad [**B**]
time = 0.04, size = 18, normalized size = 0.60

$$-\frac{4\,\mathrm{atanh}\left(\frac{2x}{3}+\frac{1}{3}\right)}{9}-\frac{1}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3 - 3*x + 2),x)
```

```
[Out] - (4*atanh((2*x)/3 + 1/3))/9 - 1/(3*(x - 1))
```

3.118 $\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx$

Optimal. Leaf size=27

$$-x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)$$

```
[Out] -x+1/2*x^2-ln(1-x)+3*ln(x)+ln(2+x)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1608, 1642}

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx &= \int \frac{-6+2x+x^4}{x\left(-2+x+x^2\right)} \, dx \\
&= \int \left(-1+\frac{1}{1-x}+\frac{3}{x}+x+\frac{1}{2+x}\right) \, dx \\
&= -x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$-x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.89

method	result	size
default	$-x+\frac{x^2}{2}+3\ln\left(x\right)-\ln\left(-1+x\right)+\ln\left(2+x\right)$	24
norman	$-x+\frac{x^2}{2}+3\ln\left(x\right)-\ln\left(-1+x\right)+\ln\left(2+x\right)$	24
risch	$-x+\frac{x^2}{2}+3\ln\left(x\right)-\ln\left(-1+x\right)+\ln\left(2+x\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x-6)/(x^3+x^2-2*x),x,method=_RETURNVERBOSE)
[Out] -x+1/2*x^2+3*ln(x)-ln(-1+x)+ln(2+x)
```

Maxima [A]
time = 1.61, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x^2-x+\log\left(x+2\right)-\log\left(x-1\right)+3\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Fricas [A]
time = 0.92, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x^2-x+\log\left(x+2\right)-\log\left(x-1\right)+3\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.74

$$\frac{x^2}{2} - x + 3\log(x) - \log(x-1) + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x-6)/(x**3+x**2-2*x),x)
[Out] x**2/2 - x + 3*log(x) - log(x - 1) + log(x + 2)
```

Giac [A]
time = 0.44, size = 26, normalized size = 0.96

$$\frac{1}{2}x^2 - x + \log(|x+2|) - \log(|x-1|) + 3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] 1/2*x^2 - x + log(abs(x + 2)) - log(abs(x - 1)) + 3*log(abs(x))
```

Mupad [B]
time = 0.10, size = 30, normalized size = 1.11

$$3\ln(x) - x + \frac{x^2}{2} + \operatorname{atan}\left(\frac{192\mathrm{i}}{7(28x-40)} + \frac{9}{7}\mathrm{i}\right) 2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4 - 6)/(x^2 - 2*x + x^3),x)
[Out] atan(192i/(7*(28*x - 40)) + 9i/7)*2i - x + 3*log(x) + x^2/2
```

3.119 $\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx$

Optimal. Leaf size=23

$$-\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\log(1+x)$$

```
[Out] -3/(1+2*x)^2+3/(1+2*x)+ln(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1634}

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

```
[Out] -3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx &= \int \left(\frac{1}{1+x}+\frac{12}{(1+2x)^3}-\frac{6}{(1+2x)^2}\right)dx\\ &= -\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.04

$$\frac{6x+(1+2x)^2\log(1+x)}{(1+2x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

```
[Out] (6*x + (1 + 2*x)^2*Log[1 + x])/(1 + 2*x)^2
```

Maple [A]
time = 0.08, size = 24, normalized size = 1.04

method	result	size
norman	$\frac{6x}{(1+2x)^2}+\ln(1+x)$	16
risch	$\frac{6x}{(1+2x)^2}+\ln(1+x)$	16
default	$-\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\ln(1+x)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+7)/(1+x)/(1+2*x)^3,x,method=_RETURNVERBOSE)
[Out] -3/(1+2*x)^2+3/(1+2*x)+ln(1+x)
```

Maxima [A]
time = 0.90, size = 20, normalized size = 0.87

$$\frac{6\,x}{4\,x^2+4\,x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="maxima")
[Out] 6*x/(4*x^2 + 4*x + 1) + log(x + 1)
```

Fricas [A]
time = 0.76, size = 32, normalized size = 1.39

$$\frac{(4\,x^2+4\,x+1)\log\left(x+1\right)+6\,x}{4\,x^2+4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="fricas")
[Out] ((4*x^2 + 4*x + 1)*log(x + 1) + 6*x)/(4*x^2 + 4*x + 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.74

$$\frac{6x}{4x^2+4x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+7)/(1+x)/(1+2*x)**3,x)
```

```
[Out] 6*x/(4*x**2 + 4*x + 1) + log(x + 1)
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.70

$$\frac{6x}{(2x+1)^2} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] 6*x/(2*x + 1)^2 + log(abs(x + 1))
```

Mupad [B]
time = 0.09, size = 15, normalized size = 0.65

$$\ln(x+1) + \frac{6x}{(2x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3 + 7)/((2*x + 1)^3*(x + 1)),x)
```

```
[Out] log(x + 1) + (6*x)/(2*x + 1)^2
```

3.120 $\int \frac{1+x+4x^2}{-1+x^3} \, dx$

Optimal. Leaf size=16

$$2\log(1-x)+\log\left(1+x+x^2\right)$$

[Out] 2*ln(1-x)+ln(x^2+x+1)

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1889, 31, 642}

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(-1 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1889

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Dist[q*((A + B*q + C*q^2)/(3*a)), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && LtQ[a/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+4x^2}{-1+x^3} \, dx &= -\left(\frac{1}{3}\int \frac{-3-6x}{1+x+x^2} \, dx\right)-2\int \frac{1}{1-x} \, dx\\ &= 2\log(1-x)+\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$2\log(1-x)+\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(-1 + x^3),x]
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.94

method	result
default	$2\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)$
norman	$2\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)$
risch	$2\ln\left(-1+x\right)+\ln\left(x^{2}+x+1\right)$
meijerg	$\frac{x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\frac{4\ln\left(-x^{3}+1\right)}{3}+\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right.}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(x^3-1),x,method=_RETURNVERBOSE)
[Out] 2*ln(-1+x)+ln(x^2+x+1)
```

Maxima [A]
time = 1.31, size = 14, normalized size = 0.88

$$\log\left(x^2+x+1\right)+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="maxima")
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Fricas [A]
time = 0.90, size = 14, normalized size = 0.88

$$\log\left(x^2+x+1\right)+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="fricas")
```

```
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.88

$$2\log{(x-1)}+\log{(x^2+x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(x**3-1),x)
[Out] 2*log(x - 1) + log(x**2 + x + 1)
```

Giac [A]
time = 0.46, size = 15, normalized size = 0.94

$$\log\left(x^2+x+1\right)+2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="giac")
[Out] log(x^2 + x + 1) + 2*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.88

$$\ln\left(x^2+x+1\right)+2\,\ln\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^2 + 1)/(x^3 - 1),x)
[Out] log(x + x^2 + 1) + 2*log(x - 1)
```

3.121 $\int \frac{x^4}{4+5x^2+x^4}\,dx$

Optimal. Leaf size=18

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1136, 1180, 209}

$$-\frac{8}{3}\text{ArcTan}\left(\frac{x}{2}\right)+\frac{\text{ArcTan}(x)}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{4+5x^2+x^4}\,dx = x - \int \frac{4+5x^2}{4+5x^2+x^4}\,dx$$
$$= x + \frac{1}{3}\int \frac{1}{1+x^2}\,dx - \frac{16}{3}\int \frac{1}{4+x^2}\,dx$$
$$= x - \frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$x + \frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A]
time = 0.02, size = 13, normalized size = 0.72

method	result	size
default	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13
risch	$x-\frac{8\arctan\left(\frac{x}{2}\right)}{3}+\frac{\arctan(x)}{3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+5*x^2+4),x,method=_RETURNVERBOSE)
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A]
time = 1.57, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A]
time = 1.07, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.78

$$x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Giac [A]
time = 0.48, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Mupad [B]
time = 0.04, size = 12, normalized size = 0.67

$$x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(5*x^2 + x^4 + 4),x)
[Out] x - (8*atan(x/2))/3 + atan(x)/3
```

3.122 $\int \frac{2+x}{x+x^2} dx$

Optimal. Leaf size=11

$$2\log(x) - \log(1+x)$$

[Out] 2*ln(x)-ln(1+x)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {645}

$$2\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/(x + x^2),x]

[Out] 2*Log[x] - Log[1 + x]

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{x+x^2} dx &= \int \left(\frac{1}{-1-x} + \frac{2}{x}\right) dx \\ &= 2\log(x) - \log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$2\log(x) - \log(1+x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/(x + x^2),x]

[Out] 2*Log[x] - Log[1 + x]

Maple [A]
time = 0.07, size = 12, normalized size = 1.09

method	result	size
default	$2\ln(x) - \ln(1+x)$	12
norman	$2\ln(x) - \ln(1+x)$	12
meijerg	$2\ln(x) - \ln(1+x)$	12
risch	$2\ln(x) - \ln(1+x)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(x)-ln(1+x)
```

Maxima [A]
time = 0.86, size = 11, normalized size = 1.00

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="maxima")
```

```
[Out] -log(x + 1) + 2*log(x)
```

Fricas [A]
time = 0.82, size = 11, normalized size = 1.00

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="fricas")
```

```
[Out] -log(x + 1) + 2*log(x)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$2\log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x),x)
```

```
[Out] 2*log(x) - log(x + 1)
```

Giac [A]
time = 0.47, size = 13, normalized size = 1.18

$$-\log(|x+1|) + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="giac")
```

```
[Out] -log(abs(x + 1)) + 2*log(abs(x))
```

Mupad [B]
time = 0.10, size = 11, normalized size = 1.00

$$2 \ln(x) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x + x^2),x)
```

```
[Out] 2*log(x) - log(x + 1)
```

3.123 $\int \frac{1}{x(1+x^2)^2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2(1+x^2)}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {272, 46}

$$\frac{1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)^2),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+x^2)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{1}{x}-\frac{1}{(1+x)^2}\right)\,dx,x,x^2\right)\\ &= \frac{1}{2(1+x^2)}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{2(1+x^2)}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)^2),x]
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.88

method	result	size
default	$\frac{1}{2x^2+2}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$	21
norman	$\frac{1}{2x^2+2}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$	21
risch	$\frac{1}{2x^2+2}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$	21
meijerg	$\frac{1}{2}+\ln\left(x\right)-\frac{x^2}{2x^2+2}-\frac{\ln\left(x^2+1\right)}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.83, size = 24, normalized size = 1.00

$$\frac{1}{2(x^2+1)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/2/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A]
time = 0.76, size = 32, normalized size = 1.33

$$-\frac{(x^2+1)\log(x^2+1)-2(x^2+1)\log(x)-1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) - 1)/(x^2 + 1)
```

Sympy [**A**]
time = 0.03, size = 19, normalized size = 0.79

$$\log(x) - \frac{\log(x^2+1)}{2} + \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)**2,x)
[Out] log(x) - log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

Giac [**A**]
time = 0.44, size = 29, normalized size = 1.21

$$\frac{x^2+2}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="giac")
[Out] 1/2*(x^2 + 2)/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Mupad [**B**]
time = 0.03, size = 20, normalized size = 0.83

$$\ln(x) - \frac{\ln(x^2+1)}{2} + \frac{1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 + 1)^2),x)
[Out] log(x) - log(x^2 + 1)/2 + 1/(2*(x^2 + 1))
```

3.124 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)$$

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {90}

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx &= \int \left(\frac{1}{8(1+x)}-\frac{1}{(2+x)^2}+\frac{2}{2+x}-\frac{1}{2(3+x)^3}-\frac{5}{4(3+x)^2}-\frac{17}{8(3+x)}\right)dx\\ &= \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{2+x}+\frac{2}{(3+x)^2}+\frac{10}{3+x}+\log(-1-x)+16\log(2+x)-17\log(3+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

Maple [A]
time = 0.07, size = 39, normalized size = 0.85

method	result	size
default	$\frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{\ln(1+x)}{8}+2\ln\left(2+x\right)-\frac{17\ln(3+x)}{8}$	39
norman	$\frac{\frac{9}{4}x^2+\frac{25}{2}x+17}{(2+x)(3+x)^2}+\frac{\ln(1+x)}{8}+2\ln\left(2+x\right)-\frac{17\ln(3+x)}{8}$	41
risch	$\frac{\frac{9}{4}x^2+\frac{25}{2}x+17}{(2+x)(3+x)^2}+\frac{\ln(1+x)}{8}+2\ln\left(2+x\right)-\frac{17\ln(3+x)}{8}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(2+x)^2/(3+x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Maxima [A]
time = 1.20, size = 46, normalized size = 1.00

$$\frac{9\,x^2+50\,x+68}{4\,(x^3+8\,x^2+21\,x+18)}-\frac{17}{8}\,\log\left(x+3\right)+2\,\log\left(x+2\right)+\frac{1}{8}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.
time = 0.92, size = 83, normalized size = 1.80

$$\frac{18\,x^2-17\,(x^3+8\,x^2+21\,x+18)\log\left(x+3\right)+16\,(x^3+8\,x^2+21\,x+18)\log\left(x+2\right)+(x^3+8\,x^2+21\,x+18)\log\left(x+1\right)+100\,x+136}{8\,(x^3+8\,x^2+21\,x+18)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A]
time = 0.08, size = 46, normalized size = 1.00

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72}+\frac{\log{(x+1)}}{8}+2\log{(x+2)}-\frac{17\log{(x+3)}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

Giac [A]
time = 0.45, size = 52, normalized size = 1.13

$$\frac{1}{x+2}-\frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2}+\frac{1}{8}\log\left(\left|-\frac{1}{x+2}+1\right|\right)-\frac{17}{8}\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

Mupad [B]
time = 0.09, size = 45, normalized size = 0.98

$$\frac{\ln\left(x+1\right)}{8}+2\,\ln\left(x+2\right)-\frac{17\,\ln\left(x+3\right)}{8}+\frac{\frac{9\,x^2}{4}+\frac{25\,x}{2}+17}{x^3+8\,x^2+21\,x+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 1
7)/(21*x + 8*x^2 + x^3 + 18)
```

3.125 $\int \frac{x}{(1+x)^2} dx$

Optimal. Leaf size=10

$$\frac{1}{1+x} + \log(1+x)$$

[Out] 1/(1+x)+ln(1+x)

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {45}

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x)^2,x]

[Out] (1 + x)^(-1) + Log[1 + x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)^2} dx &= \int \left(-\frac{1}{(1+x)^2} + \frac{1}{1+x}\right) dx \\ &= \frac{1}{1+x} + \log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{1+x} + \log(1+x)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x)^2,x]

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Maple [A]
time = 0.07, size = 11, normalized size = 1.10

method	result	size
default	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
norman	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
risch	$\frac{1}{1+x}+\ln\left(1+x\right)$	11
meijerg	$-\frac{x}{1+x}+\ln\left(1+x\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+x)+ln(1+x)
```

Maxima [A]
time = 0.74, size = 10, normalized size = 1.00

$$\frac{1}{x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) + log(x + 1)
```

Fricas [A]
time = 1.04, size = 16, normalized size = 1.60

$$\frac{\left(x+1\right)\log\left(x+1\right)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.80

$$\log\left(x+1\right)+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

Giac [**A**]
time = 0.47, size = 11, normalized size = 1.10

$$\frac{1}{x+1} + \log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1))
```

Mupad [**B**]
time = 0.03, size = 10, normalized size = 1.00

$$\ln\left(x+1\right) + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 1)^2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

3.126 $\int \frac{1}{-x+x^3} dx$

Optimal. Leaf size=17

$$-\log(x) + \frac{1}{2}\log\left(1 - x^2\right)$$

```
[Out] -ln(x)+1/2*ln(-x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1607, 272, 36, 31, 29}

$$\frac{1}{2}\log\left(1 - x^2\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-x+x^3}\,dx &= \int \frac{1}{x\left(-1+x^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)\\
&= -\log(x)+\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\log(x)+\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)^(-1),x]
[Out] -Log[x] + Log[1 - x^2]/2
```

Maple [A]
time = 0.07, size = 18, normalized size = 1.06

method	result	size
risch	$-\ln\left(x\right)+\frac{\ln\left(x^2-1\right)}{2}$	14
default	$-\ln\left(x\right)+\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	18
norman	$-\ln\left(x\right)+\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	18
meijerg	$-\ln\left(x\right)-\frac{i\pi}{2}+\frac{\ln\left(-x^2+1\right)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-x),x,method=_RETURNVERBOSE)
[Out] -ln(x)+1/2*ln(-1+x)+1/2*ln(1+x)
```

Maxima [A]
time = 1.34, size = 17, normalized size = 1.00

$$\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [A]
time = 0.80, size = 13, normalized size = 0.76

$$\frac{1}{2}\log\left(x^2-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1) - log(x)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.59

$$-\log\left(x\right)+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1)/2
```

Giac [A]
time = 0.46, size = 16, normalized size = 0.94

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.11, size = 13, normalized size = 0.76

$$\frac{\ln\left(x^2-1\right)}{2}-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x - x^3),x)
```

```
[Out] log(x^2 - 1)/2 - log(x)
```

3.127 $\int \frac{x^2}{-6+x+x^2} \, dx$

Optimal. Leaf size=20

$$x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(3+x)$$

```
[Out] x+4/5*ln(2-x)-9/5*ln(3+x)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {717, 646, 31}

$$x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{-6+x+x^2}\,dx &= x+\int \frac{6-x}{-6+x+x^2}\,dx \\
&= x+\frac{4}{5}\int \frac{1}{-2+x}\,dx-\frac{9}{5}\int \frac{1}{3+x}\,dx \\
&= x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(3+x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(3+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.75

method	result	size
default	$x+\frac{4\ln(-2+x)}{5}-\frac{9\ln(3+x)}{5}$	15
norman	$x+\frac{4\ln(-2+x)}{5}-\frac{9\ln(3+x)}{5}$	15
risch	$x+\frac{4\ln(-2+x)}{5}-\frac{9\ln(3+x)}{5}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+x-6),x,method=_RETURNVERBOSE)
```

```
[Out] x+4/5*ln(-2+x)-9/5*ln(3+x)
```

Maxima [A]
time = 0.75, size = 14, normalized size = 0.70

$$x-\frac{9}{5}\log\left(x+3\right)+\frac{4}{5}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="maxima")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Fricas [A]
time = 0.77, size = 14, normalized size = 0.70

$$x - \frac{9}{5} \log\left(x + 3\right) + \frac{4}{5} \log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="fricas")
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.85

$$x + \frac{4 \log\left(x - 2\right)}{5} - \frac{9 \log\left(x + 3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+x-6),x)
[Out] x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

Giac [A]
time = 0.44, size = 16, normalized size = 0.80

$$x - \frac{9}{5} \log\left(|x + 3|\right) + \frac{4}{5} \log\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="giac")
[Out] x - 9/5*log(abs(x + 3)) + 4/5*log(abs(x - 2))
```

Mupad [B]
time = 0.04, size = 14, normalized size = 0.70

$$x + \frac{4\ \ln\left(x - 2\right)}{5} - \frac{9\ \ln\left(x + 3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x + x^2 - 6),x)
[Out] x + (4*log(x - 2))/5 - (9*log(x + 3))/5
```

3.128 $\int \frac{2+x}{4-4x+x^2}\,dx$

Optimal. Leaf size=16

$$\frac{4}{2-x}+\log(2-x)$$

[Out] 4/(2-x)+ln(2-x)

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {27, 45}

$$\frac{4}{2-x}+\log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/(4 - 4*x + x^2),x]

[Out] 4/(2 - x) + Log[2 - x]

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{4-4x+x^2}\,dx &= \int \frac{2+x}{(-2+x)^2}\,dx\\ &= \int \left(\frac{4}{(-2+x)^2}+\frac{1}{-2+x}\right)dx\\ &= \frac{4}{2-x}+\log(2-x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 0.75

$$-\frac{4}{-2+x}+\log(-2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(4 - 4*x + x^2),x]
[Out] -4/(-2 + x) + Log[-2 + x]
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.81

method	result	size
default	$\ln\left(-2+x\right)-\frac{4}{-2+x}$	13
norman	$\ln\left(-2+x\right)-\frac{4}{-2+x}$	13
risch	$\ln\left(-2+x\right)-\frac{4}{-2+x}$	13
meijerg	$\frac{x}{1-\frac{x}{2}}+\ln\left(1-\frac{x}{2}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2-4*x+4),x,method=_RETURNVERBOSE)
[Out] ln(-2+x)-4/(-2+x)
```

Maxima [A]
time = 2.95, size = 12, normalized size = 0.75

$$-\frac{4}{x-2}+\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="maxima")
[Out] -4/(x - 2) + log(x - 2)
```

Fricas [A]
time = 0.73, size = 16, normalized size = 1.00

$$\frac{(x-2)\log\left(x-2\right)-4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="fricas")
```

```
[Out] ((x - 2)*log(x - 2) - 4)/(x - 2)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.50

$$\log(x-2) - \frac{4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2-4*x+4),x)
[Out] log(x - 2) - 4/(x - 2)
```

Giac [A]
time = 0.46, size = 13, normalized size = 0.81

$$-\frac{4}{x-2} + \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="giac")
[Out] -4/(x - 2) + log(abs(x - 2))
```

Mupad [B]
time = 0.04, size = 12, normalized size = 0.75

$$\ln(x-2) - \frac{4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x^2 - 4*x + 4),x)
[Out] log(x - 2) - 4/(x - 2)
```

3.129 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

[Out] 1/(2-x)-arctan(-2+x)

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {27, 707, 632, 210}

$$\text{ArcTan}(2-x)+\frac{1}{2-x}$$

Antiderivative was successfully verified.

[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] (2 - x)^(-1) + ArcTan[2 - x]

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 707

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[-2*b*d*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(d^2*(m
+ 1)*(b^2 - 4*a*c))), x] + Dist[b^2*((m + 2*p + 3)/(d^2*(m + 1)*(b^2 - 4*a*
c))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
```

```
IntegerQ[(m + 2*p + 3)/2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx &= \int \frac{1}{(-2+x)^2(5-4x+x^2)}\,dx \\
&= \frac{1}{2-x} - \int \frac{1}{5-4x+x^2}\,dx \\
&= \frac{1}{2-x} + 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -4+2x\right) \\
&= \frac{1}{2-x} + \tan^{-1}(2-x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{-2+x} + \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A]
time = 0.14, size = 15, normalized size = 1.07

method	result	size
default	$-\frac{1}{-2+x} - \arctan(-2+x)$	15
risch	$-\frac{1}{-2+x} - \arctan(-2+x)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x,method=_RETURNVERBOSE)
[Out] -1/(-2+x)-arctan(-2+x)
```

Maxima [A]
time = 1.27, size = 14, normalized size = 1.00

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Fricas [A]
time = 0.74, size = 17, normalized size = 1.21

$$-\frac{(x-2)\arctan(x-2)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
```

```
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

Giac [A]
time = 0.45, size = 14, normalized size = 1.00

$$-\frac{1}{x-2}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Mupad [B]
time = 0.08, size = 14, normalized size = 1.00

$$-\mathrm{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
```

```
[Out] - atan(x - 2) - 1/(x - 2)
```

3.130 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1608, 814}

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+x}{2x+3x^2+x^3}\,dx &= \int \frac{-3+x}{x\left(2+3x+x^2\right)}\,dx \\
&= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)dx \\
&= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.86

method	result	size
default	$-\frac{3\ln(x)}{2}+4\ln\left(1+x\right)-\frac{5\ln(2+x)}{2}$	18
norman	$-\frac{3\ln(x)}{2}+4\ln\left(1+x\right)-\frac{5\ln(2+x)}{2}$	18
risch	$-\frac{3\ln(x)}{2}+4\ln\left(1+x\right)-\frac{5\ln(2+x)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)/(x^3+3*x^2+2*x),x,method=_RETURNVERBOSE)
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Maxima [A]
time = 1.25, size = 17, normalized size = 0.81

$$-\frac{5}{2}\log\left(x+2\right)+4\log\left(x+1\right)-\frac{3}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Fricas [A]
time = 0.82, size = 17, normalized size = 0.81

$$-\frac{5}{2}\log\left(x+2\right)+4\log\left(x+1\right)-\frac{3}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.95

$$-\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Giac [A]
time = 0.49, size = 20, normalized size = 0.95

$$-\frac{5}{2}\log(|x+2|) + 4\log(|x+1|) - \frac{3}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.81

$$4\ln(x+1) - \frac{5\ln(x+2)}{2} - \frac{3\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)
[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

3.131 $\int \frac{1}{(-1+x^2)^2} dx$

Optimal. Leaf size=21

$$\frac{x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x)$$

```
[Out] 1/2*x/(-x^2+1)+1/2*arctanh(x)
```

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {205, 213}

$$\frac{x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x^2)^2} dx &= \frac{x}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{-1+x^2} dx \\ &= \frac{x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{-1+x^2}-\log(1-x)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(-2),x]
```

```
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A]
time = 0.07, size = 28, normalized size = 1.33

method	result	size
meijerg	$-\frac{i\left(\frac{2ix}{-2x^2+2}+i\,\mathrm{arctanh}(x)\right)}{2}$	23
norman	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
risch	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
default	$-\frac{1}{4(-1+x)}-\frac{\ln(-1+x)}{4}-\frac{1}{4(1+x)}+\frac{\ln(1+x)}{4}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-1+x)-1/4*ln(-1+x)-1/4/(1+x)+1/4*ln(1+x)
```

Maxima [A]
time = 0.68, size = 23, normalized size = 1.10

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.
time = 0.75, size = 34, normalized size = 1.62

$$\frac{\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-2\,x}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2}-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2,x)
```

```
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Giac [A]
time = 0.47, size = 25, normalized size = 1.19

$$-\frac{x}{2(x^2-1)}+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}(x)}{2}-\frac{x}{2\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^2,x)
```

```
[Out] atanh(x)/2 - x/(2*(x^2 - 1))
```

3.132 $\int \frac{1+x}{-1+x^3} \, dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(1-x) - \frac{1}{3}\log\left(1+x+x^2\right)$$

```
[Out] 2/3*ln(1-x)-1/3*ln(x^2+x+1)
```

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1875, 31, 642}

$$\frac{2}{3}\log(1-x) - \frac{1}{3}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-1 + x^3),x]
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1875

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Dist[r*((B*r + A*s)/(3*a*
s)), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s*(B*
r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && N
eQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{-1+x^3} \, dx &= \frac{1}{3}\int \frac{-1-2x}{1+x+x^2} \, dx - \frac{2}{3}\int \frac{1}{1-x} \, dx \\ &= \frac{2}{3}\log(1-x) - \frac{1}{3}\log\left(1+x+x^2\right) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-1 + x^3),x]
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Maple [A]
time = 0.07, size = 17, normalized size = 0.77

method	result
default	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
norman	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
risch	$\frac{2\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{3}$
meijerg	$\frac{x\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^3\right)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\right.}{3\left(x^3\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^3-1),x,method=_RETURNVERBOSE)
[Out] 2/3*ln(-1+x)-1/3*ln(x^2+x+1)
```

Maxima [A]
time = 1.03, size = 16, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="maxima")
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Fricas [A]
time = 0.80, size = 16, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\log\left(x-1\right)}{3}-\frac{\log\left(x^{2}+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3-1),x)
```

```
[Out] 2*log(x - 1)/3 - log(x**2 + x + 1)/3
```

Giac [A]
time = 0.48, size = 17, normalized size = 0.77

$$-\frac{1}{3}\log\left(x^{2}+x+1\right)+\frac{2}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="giac")
```

```
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(abs(x - 1))
```

Mupad [B]
time = 0.16, size = 16, normalized size = 0.73

$$\frac{2\,\ln\left(x-1\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^3 - 1),x)
```

```
[Out] (2*log(x - 1))/3 - log(x + x^2 + 1)/3
```

3.133 $\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{1+x^2}+\log(x)$$

[Out] 1/(x^2+1)+ln(x)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1266, 908}

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(x*(1 + x^2)^2),x]

[Out] (1 + x^2)^(-1) + Log[x]

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x(1+x)^2} \, dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right) \, dx,x,x^2\right)\\
&= \frac{1}{1+x^2}+\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{1+x^2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
[Out] (1 + x^2)^(-1) + Log[x]
```

Maple [A]
time = 0.06, size = 11, normalized size = 1.10

method	result	size
default	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
norman	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
risch	$\frac{1}{x^2+1}+\ln\left(x\right)$	11
meijerg	$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}+\ln\left(x\right)-\frac{x^2}{2x^2+2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/x/(x^2+1)^2,x,method=_RETURNVERBOSE)
[Out] 1/(x^2+1)+ln(x)
```

Maxima [A]
time = 0.60, size = 14, normalized size = 1.40

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A]
time = 1.33, size = 18, normalized size = 1.80

$$\frac{(x^2+1)\log\left(x\right)+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.80

$$\log{(x)} + \frac{1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/x/(x**2+1)**2,x)
```

```
[Out] log(x) + 1/(x**2 + 1)
```

Giac [A]
time = 0.47, size = 14, normalized size = 1.40

$$\frac{1}{x^2 + 1} + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Mupad [B]
time = 0.03, size = 10, normalized size = 1.00

$$\ln{(x)} + \frac{1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) + 1/(x^2 + 1)
```

3.134 $\int \frac{1}{-2x^3+x^4} \, dx$

Optimal. Leaf size=31

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

[Out] 1/4/x^2+1/4/x+1/8*ln(2-x)-1/8*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 46}

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

[In] Int[(-2*x^3 + x^4)^(-1),x]

[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-2x^3+x^4} \, dx &= \int \frac{1}{(-2+x)x^3} \, dx \\
&= \int \left(\frac{1}{8(-2+x)}-\frac{1}{2x^3}-\frac{1}{4x^2}-\frac{1}{8x}\right) dx \\
&= \frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x^3 + x^4)^(-1),x]
```

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Maple [A]
time = 0.07, size = 22, normalized size = 0.71

method	result	size
norman	$\frac{\frac{1}{4}+\frac{x}{4}}{x^2}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$	21
risch	$\frac{\frac{1}{4}+\frac{x}{4}}{x^2}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$	21
default	$\frac{1}{4x^2}+\frac{1}{4x}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$	22
meijerg	$\frac{1}{4x^2}+\frac{1}{4x}-\frac{\ln(x)}{8}+\frac{\ln(2)}{8}-\frac{i\pi}{8}+\frac{\ln(1-\frac{x}{2})}{8}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-2*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/x^2+1/4/x-1/8*ln(x)+1/8*ln(-2+x)
```

Maxima [A]
time = 0.98, size = 19, normalized size = 0.61

$$\frac{x+1}{4\,x^2}+\frac{1}{8}\,\log\left(x-2\right)-\frac{1}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(x + 1)/x^2 + 1/8*log(x - 2) - 1/8*log(x)
```

Fricas [A]
time = 1.05, size = 25, normalized size = 0.81

$$\frac{x^2\log\left(x-2\right)-x^2\log\left(x\right)+2\,x+2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="fricas")
```

```
[Out] 1/8*(x^2*log(x - 2) - x^2*log(x) + 2*x + 2)/x^2
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.61

$$-\frac{\log(x)}{8} + \frac{\log(x-2)}{8} + \frac{x+1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-2*x**3),x)
[Out] -log(x)/8 + log(x - 2)/8 + (x + 1)/(4*x**2)
```

Giac [A]
time = 0.48, size = 21, normalized size = 0.68

$$\frac{x+1}{4\,x^2} + \frac{1}{8}\,\log(|x-2|) - \frac{1}{8}\,\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="giac")
[Out] 1/4*(x + 1)/x^2 + 1/8*log(abs(x - 2)) - 1/8*log(abs(x))
```

Mupad [B]
time = 0.04, size = 16, normalized size = 0.52

$$\frac{\frac{x}{4}+\frac{1}{4}}{x^2} - \frac{\operatorname{atanh}(x-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(2*x^3 - x^4),x)
[Out] (x/4 + 1/4)/x^2 - atanh(x - 1)/4
```

3.135 $\int \frac{1-x^3}{x(1+x^2)}\,dx$

Optimal. Leaf size=18

$$-x+\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1816, 649, 209, 266}

$$\text{ArcTan}(x)-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x*(1 + x^2)),x]
```

```
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1-x^3}{x(1+x^2)}\,dx = \int \left(-1+\frac{1}{x}+\frac{1-x}{1+x^2}\right)dx$$
$$= -x+\log(x)+\int \frac{1-x}{1+x^2}\,dx$$
$$= -x+\log(x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx$$
$$= -x+\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-x+\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x*(1 + x^2)),x]
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.08, size = 17, normalized size = 0.94

method	result	size
default	$-x+\arctan(x)+\ln(x)-\frac{\ln(x^2+1)}{2}$	17
meijerg	$-x+\arctan(x)+\ln(x)-\frac{\ln(x^2+1)}{2}$	17
risch	$-x+\arctan(x)+\ln(x)-\frac{\ln(x^2+1)}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.83, size = 16, normalized size = 0.89

$$-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="maxima")
```

```
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A]
time = 0.83, size = 16, normalized size = 0.89

$$-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="fricas")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.83

$$-x+\log\left(x\right)-\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x/(x**2+1),x)
[Out] -x + log(x) - log(x**2 + 1)/2 + atan(x)
```

Giac [A]
time = 0.45, size = 17, normalized size = 0.94

$$-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="giac")
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

Mupad [B]
time = 0.04, size = 24, normalized size = 1.33

$$\ln\left(x\right)-x+\ln\left(x-\mathrm{i}\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x*(x^2 + 1)),x)
[Out] log(x) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) - x
```

3.136 $\int \frac{1}{-1+x^4}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

[Out] -1/2*arctan(x)-1/2*arctanh(x)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {218, 212, 209}

$$-\frac{\text{ArcTan}(x)}{2}-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^4)^(-1),x]

[Out] -1/2*ArcTan[x] - ArcTanh[x]/2

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.92

$$-\frac{1}{2}\tan^{-1}(x)+\frac{1}{4}\log(1-x)-\frac{1}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[x] + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A]
time = 0.07, size = 10, normalized size = 0.77

method	result	size
default	$-\frac{\arctan(x)}{2}-\frac{\operatorname{arctanh}(x)}{2}$	10
risch	$\frac{\ln(-1+x)}{4}-\frac{\arctan(x)}{2}-\frac{\ln(1+x)}{4}$	18
meijerg	$\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Maxima [A]
time = 1.03, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A]
time = 1.03, size = 17, normalized size = 1.31

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 1.31

$$\frac{\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1),x)
```

```
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.
time = 0.44, size = 19, normalized size = 1.46

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(|x+1|) + \frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.69

$$-\frac{\operatorname{atan}(x)}{2} - \frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 - 1),x)
```

```
[Out] - atan(x)/2 - atanh(x)/2
```

3.137 $\int \frac{1}{1+x^4}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^(1/2))*2^(1/2)-1/8*ln(1+x
^2-x*2^(1/2))*2^(1/2)+1/8*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+x^4}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx \\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}\,x-x^2}\,dx}{4\sqrt{2}} \\
&= -\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x\right)}{2\sqrt{2}} - \text{Subst} \\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{2\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.75

$$
\frac{-2\tan^{-1}\left(1-\sqrt{2}\,x\right)+2\tan^{-1}\left(1+\sqrt{2}\,x\right)-\log\left(1-\sqrt{2}\,x+x^2\right)+\log\left(1+\sqrt{2}\,x+x^2\right)}{4\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A]
time = 0.06, size = 52, normalized size = 0.61

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(-\_R+x\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}$
meijerg	$-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{1}{4}}}+x\sqrt{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))+2*arctan(1+x*2^(1/2))+
2*arctan(-1+x*2^(1/2)))
```

Maxima [A]
time = 1.41, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A]
time = 1.30, size = 100, normalized size = 1.18

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(4x^2+4\sqrt{2}x+4\right)-\frac{1}{8}\sqrt{2}\log\left(4x^2-4\sqrt{2}x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
```

```
*sqrt(2)*log(4*x^2 + 4*sqrt(2)*x + 4) - 1/8*sqrt(2)*log(4*x^2 - 4*sqrt(2)*x
 + 4)
```

Sympy [A]
time = 0.05, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Giac [A]
time = 0.44, size = 72, normalized size = 0.85

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B]
time = 0.12, size = 33, normalized size = 0.39

$$\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/4 + 1i/4) + 2^(1/2)*atan(2^(1/2)*x*
(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.138 $\int \frac{x^2}{(2+2x+x^2)^2} dx$

Optimal. Leaf size=23

$$-\frac{x(2+x)}{2(2+2x+x^2)} + \tan^{-1}(1+x)$$

```
[Out] -1/2*x*(2+x)/(x^2+2*x+2)+arctan(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {736, 631, 210}

$$\text{ArcTan}(x+1) - \frac{x(x+2)}{2(x^2+2x+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 2*x + x^2)^2,x]
```

```
[Out] -1/2*(x*(2 + x))/(2 + 2*x + x^2) + ArcTan[1 + x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(2+2x+x^2)^2}\,dx &= -\frac{x(2+x)}{2\,(2+2x+x^2)} + \int \frac{1}{2+2x+x^2}\,dx \\
&= -\frac{x(2+x)}{2\,(2+2x+x^2)} - \operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+x\right) \\
&= -\frac{x(2+x)}{2\,(2+2x+x^2)} + \tan^{-1}(1+x)
\end{aligned}
$$

<u>**Mathematica**</u> [A]
time = 0.01, size = 15, normalized size = 0.65

$$\frac{1}{2+2x+x^2} + \tan^{-1}(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 2*x + x^2)^2,x]
[Out] (2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

<u>**Maple**</u> [A]
time = 0.10, size = 16, normalized size = 0.70

method	result	size
default	$\frac{1}{x^2+2x+2} + \arctan(1+x)$	16
risch	$\frac{1}{x^2+2x+2} + \arctan(1+x)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+2*x+2)^2,x,method=_RETURNVERBOSE)
[Out] 1/(x^2+2*x+2)+arctan(1+x)
```

<u>**Maxima**</u> [A]
time = 0.81, size = 15, normalized size = 0.65

$$\frac{1}{x^2+2\,x+2} + \arctan(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="maxima")
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Fricas [A]
time = 1.05, size = 26, normalized size = 1.13

$$\frac{(x^2+2\,x+2)\arctan\left(x+1\right)+1}{x^2+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.61

$$\operatorname{atan}\left(x+1\right)+\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+2*x+2)**2,x)
```

```
[Out] atan(x + 1) + 1/(x**2 + 2*x + 2)
```

Giac [A]
time = 0.46, size = 15, normalized size = 0.65

$$\frac{1}{x^2+2\,x+2}+\arctan\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Mupad [B]
time = 0.08, size = 15, normalized size = 0.65

$$\operatorname{atan}(x+1)+\frac{1}{x^2+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x + x^2 + 2)^2,x)
```

```
[Out] atan(x + 1) + 1/(2*x + x^2 + 2)
```

3.139 $\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx$

Optimal. Leaf size=11

$$-\frac{x}{1+x+x^5}$$

[Out] -x/(x^5+x+1)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1602\}$

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx = -\frac{x}{1+x+x^5}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{x}{1+x+x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.
time = 0.02, size = 41, normalized size = 3.73

method	result	size
gosper	$-\frac{x}{x^5+x+1}$	12
norman	$-\frac{x}{x^5+x+1}$	12
risch	$-\frac{x}{x^5+x+1}$	12
default	$-\frac{-3x^2+5x-1}{7(x^3-x^2+1)}+\frac{-3x-1}{7x^2+7x+7}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x,method=_RETURNVERBOSE)
[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)
```

Maxima [A]
time = 1.10, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
[Out] -x/(x^5 + x + 1)
```

Fricas [A]
time = 1.04, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
[Out] -x/(x^5 + x + 1)
```

Sympy [A]
time = 0.04, size = 8, normalized size = 0.73

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

Giac [A]
time = 0.43, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

Mupad [B]
time = 0.06, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

```
[Out] -x/(x + x^5 + 1)
```

3.140 $\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{2\cos(x)+\sin(x)}{5+2\sqrt{5}-\cos(x)+2\sin(x)}\right)}{\sqrt{5}}$$

```
[Out] 1/10*x*5^(1/2)+1/5*arctan((2*cos(x)+sin(x))/(5-cos(x)+2*sin(x)+2*5^(1/2)))*
5^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3203, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}+\frac{x}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[5]) + ArcTan[(2*Cos[x] + Sin[x])/(5 + 2*Sqrt[5] - Cos[x] + 2*Sin[
x])]/Sqrt[5]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx &= 2\text{Subst}\left(\int \frac{1}{4+4x+6x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\text{Subst}\left(\int \frac{1}{-80-x^2}\,dx, x, 4+12\tan\left(\frac{x}{2}\right)\right)\right)\\ &= \frac{x}{2\sqrt{5}} + \frac{\tan^{-1}\left(\frac{2\cos(x)+\sin(x)}{5+2\sqrt{5}-\cos(x)+2\sin(x)}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.51

$$\frac{\tan^{-1}\left(\frac{1+3\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] ArcTan[(1 + 3*Tan[x/2])/Sqrt[5]]/Sqrt[5]
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.44

method	result	size
default	$\frac{\sqrt{5}\,\arctan\left(\frac{\left(6\tan\left(\frac{x}{2}\right)+2\right)\sqrt{5}}{10}\right)}{5}$	20
risch	$\frac{i\sqrt{5}\,\ln\left(e^{ix}-1+2i+\frac{4i\sqrt{5}}{5}-\frac{2\sqrt{5}}{5}\right)}{10}-\frac{i\sqrt{5}\,\ln\left(e^{ix}-1+2i-\frac{4i\sqrt{5}}{5}+\frac{2\sqrt{5}}{5}\right)}{10}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5-cos(x)+2*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*5^(1/2)*arctan(1/10*(6*tan(1/2*x)+2)*5^(1/2))
```

Maxima [A]
time = 0.99, size = 23, normalized size = 0.51

$$\frac{1}{5}\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="maxima")
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(3*sin(x)/(cos(x) + 1) + 1))
```

Fricas [A]
time = 0.92, size = 36, normalized size = 0.80

$$\frac{1}{10}\,\sqrt{5}\,\arctan\left(-\frac{\sqrt{5}\,\cos\left(x\right)-2\,\sqrt{5}\,\sin\left(x\right)-\sqrt{5}}{2\,\left(2\,\cos\left(x\right)+\sin\left(x\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="fricas")
[Out] 1/10*sqrt(5)*arctan(-1/2*(sqrt(5)*cos(x) - 2*sqrt(5)*sin(x) - sqrt(5))/(2*c
os(x) + sin(x)))
```

Sympy [A]
time = 0.18, size = 39, normalized size = 0.87

$$\frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{3\sqrt{5}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{5}}{5}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x)
[Out] sqrt(5)*(atan(3*sqrt(5)*tan(x/2)/5 + sqrt(5)/5) + pi*floor((x/2 - pi/2)/pi)
)/5
```

Giac [A]
time = 0.44, size = 47, normalized size = 1.04

$$\frac{1}{10}\,\sqrt{5}\left(x+2\,\arctan\left(-\frac{\sqrt{5}\,\sin\left(x\right)-\cos\left(x\right)-3\,\sin\left(x\right)-1}{\sqrt{5}\,\cos\left(x\right)+\sqrt{5}-3\,\cos\left(x\right)+\sin\left(x\right)+3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="giac")
[Out] 1/10*sqrt(5)*(x + 2*arctan(-(sqrt(5)*sin(x) - cos(x) - 3*sin(x) - 1)/(sqrt(
5)*cos(x) + sqrt(5) - 3*cos(x) + sin(x) + 3)))
```

Mupad [B]
time = 0.10, size = 21, normalized size = 0.47

$$\frac{\sqrt{20}\,\operatorname{atan}\left(\frac{3\,\sqrt{20}\,\tan\left(\frac{x}{2}\right)}{10}+\frac{\sqrt{20}}{10}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x) - cos(x) + 5),x)
[Out] (20^(1/2)*atan((3*20^(1/2)*tan(x/2))/10 + 20^(1/2)/10))/10
```

3.141 $\int \frac{1}{1+a\cos(x)}\,dx$

Optimal. Leaf size=37

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{1+a}}\right)}{\sqrt{1-a^2}}$$

```
[Out] 2*arctan((1-a)^(1/2)*tan(1/2*x)/(1+a)^(1/2))/(-a^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2738, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{1-a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+a\cos(x)}\,dx &= 2\text{Subst}\left(\int \frac{1}{1+a+(1-a)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\,\tan\left(\frac{x}{2}\right)}{\sqrt{1+a}}\right)}{\sqrt{1-a^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.84

$$\frac{2\tanh^{-1}\left(\frac{(-1+a)\tan\left(\frac{x}{2}\right)}{\sqrt{-1+a^2}}\right)}{\sqrt{-1+a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*Cos[x])^(-1),x]
[Out] (2*ArcTanh[((-1 + a)*Tan[x/2])/Sqrt[-1 + a^2]])/Sqrt[-1 + a^2]
```

Maple [A]
time = 0.04, size = 30, normalized size = 0.81

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{(1+a)(a-1)}}\right)}{\sqrt{(1+a)(a-1)}}$	30
risch	$\frac{\ln\left(\mathrm{e}^{ix}+\frac{i a^2-i+\sqrt{a^2-1}}{a\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}-\frac{\ln\left(\mathrm{e}^{ix}-\frac{i a^2-i-\sqrt{a^2-1}}{a\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+a*cos(x)),x,method=_RETURNVERBOSE)
[Out] 2/((1+a)*(a-1))^(1/2)*arctanh((a-1)*tan(1/2*x)/((1+a)*(a-1))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a^2-1.0>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.69, size = 111, normalized size = 3.00

$$\left[\frac{\log\left(-\frac{(a^2-2)\cos(x)^2-2\sqrt{a^2-1}\,(a+\cos(x))\sin(x)-2a^2-2a\cos(x)+1}{a^2\cos(x)^2+2a\cos(x)+1}\right)}{2\sqrt{a^2-1}}, -\frac{\sqrt{-a^2+1}\,\arctan\left(\frac{\sqrt{-a^2+1}\,(a+\cos(x))}{(a^2-1)\sin(x)}\right)}{a^2-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a^2 - 2)*cos(x)^2 - 2*sqrt(a^2 - 1)*(a + cos(x))*sin(x) - 2*a^2
 - 2*a*cos(x) + 1)/(a^2*cos(x)^2 + 2*a*cos(x) + 1))/sqrt(a^2 - 1), -sqrt(-a
^2 + 1)*arctan(sqrt(-a^2 + 1)*(a + cos(x))/((a^2 - 1)*sin(x)))/(a^2 - 1)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(29) = 58$.
time = 1.41, size = 110, normalized size = 2.97

$$\begin{cases} -\frac{1}{\tan\left(\frac{x}{2}\right)} & \text{for } a = -1 \\ \tan\left(\frac{x}{2}\right) & \text{for } a = 1 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} + \frac{\log\left(\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x)
```

```
[Out] Piecewise((-1/tan(x/2), Eq(a, -1)), (tan(x/2), Eq(a, 1)), (-log(-sqrt(a/(a
- 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a - 1) + 1/(a - 1)) - sqrt(a/(a -
1) + 1/(a - 1))) + log(sqrt(a/(a - 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a
 - 1) + 1/(a - 1)) - sqrt(a/(a - 1) + 1/(a - 1))), True))
```

Giac [A]
time = 0.46, size = 53, normalized size = 1.43

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)-\tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+1}}\right)\right)}{\sqrt{-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) - tan(1/2*
x))/sqrt(-a^2 + 1)))/sqrt(-a^2 + 1)
```

Mupad [B]
time = 0.32, size = 28, normalized size = 0.76

$$\frac{2\operatorname{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{a-1}}{\sqrt{a+1}}\right)}{\sqrt{a-1}\sqrt{a+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + 1),x)
```

```
[Out] (2*atanh((tan(x/2)*(a - 1)^(1/2))/(a + 1)^(1/2)))/((a - 1)^(1/2)*(a + 1)^(1
/2))
```

3.142 $\int \frac{1}{1+2\cos(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

```
[Out] -1/3*ln(-sin(1/2*x)+cos(1/2*x)*3^(1/2))*3^(1/2)+1/3*ln(sin(1/2*x)+cos(1/2*x
)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2738, 212}

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] -(Log[Sqrt[3]*Cos[x/2] - Sin[x/2]]/Sqrt[3]) + Log[Sqrt[3]*Cos[x/2] + Sin[x/
2]]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+2\cos(x)}\,dx &= 2\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.36

$$\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Cos[x])^(-1),x]
[Out] (2*ArcTanh[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.29

method	result	size
default	$\frac{2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{\sqrt{3}\,\ln\left(e^{ix}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{3}-\frac{\sqrt{3}\,\ln\left(e^{ix}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*cos(x)),x,method=_RETURNVERBOSE)
[Out] 2/3*3^(1/2)*arctanh(1/3*tan(1/2*x)*3^(1/2))
```

Maxima [A]
time = 1.40, size = 37, normalized size = 0.66

$$-\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{3}+\frac{\sin(x)}{\cos(x)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*log(-(sqrt(3) - sin(x)/(cos(x) + 1))/(sqrt(3) + sin(x)/(cos(x)
 + 1)))
```

Fricas [A]
time = 0.64, size = 50, normalized size = 0.89

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{2\cos\left(x\right)^{2}-2\left(\sqrt{3}\cos\left(x\right)+2\sqrt{3}\right)\sin\left(x\right)-4\cos\left(x\right)-7}{4\cos\left(x\right)^{2}+4\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(2*cos(x)^2 - 2*(sqrt(3)*cos(x) + 2*sqrt(3))*sin(x) - 4*co
s(x) - 7)/(4*cos(x)^2 + 4*cos(x) + 1))
```

Sympy [A]
time = 0.13, size = 36, normalized size = 0.64

$$-\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3}+\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x)
```

```
[Out] -sqrt(3)*log(tan(x/2) - sqrt(3))/3 + sqrt(3)*log(tan(x/2) + sqrt(3))/3
```

Giac [A]
time = 0.48, size = 35, normalized size = 0.62

$$-\frac{1}{3}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x))/abs(2*sqrt(3) + 2*tan(1/2*x
)))
```

Mupad [B]
time = 0.24, size = 15, normalized size = 0.27

$$\frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(x) + 1),x)
```

```
[Out] (2*3^(1/2)*atanh((3^(1/2)*tan(x/2))/3))/3
```

3.143 $\int \frac{1}{1+\frac{\cos(x)}{2}} \, dx$

Optimal. Leaf size=31

$$\frac{2x}{\sqrt{3}} - \frac{4\tan^{-1}\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

```
[Out] 2/3*x*3^(1/2)-4/3*arctan(sin(x)/(2+cos(x)+3^(1/2)))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2736}

$$\frac{2x}{\sqrt{3}} - \frac{4\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]/2)^(-1),x]
[Out] (2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{1+\frac{\cos(x)}{2}} \, dx = \frac{2x}{\sqrt{3}} - \frac{4\tan^{-1}\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.65

$$\frac{4\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]/2)^(-1),x]
[Out] (4*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.03, size = 16, normalized size = 0.52

method	result	size
default	$\frac{4\sqrt{3}\,\arctan\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{3}}{3}\right)}{3}$	16
risch	$\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{ix}+\sqrt{3}+2\right)}{3}-\frac{2i\sqrt{3}\,\ln\left(\mathrm{e}^{ix}-\sqrt{3}+2\right)}{3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+1/2*cos(x)),x,method=_RETURNVERBOSE)
[Out] 4/3*3^(1/2)*arctan(1/3*tan(1/2*x)*3^(1/2))
```

Maxima [A]
time = 1.62, size = 19, normalized size = 0.61

$$\frac{4}{3}\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sin\left(x\right)}{3\,\left(\cos\left(x\right)+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.61, size = 23, normalized size = 0.74

$$-\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{2\,\sqrt{3}\,\cos\left(x\right)+\sqrt{3}}{3\,\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

Sympy [A]
time = 0.11, size = 32, normalized size = 1.03

$$\frac{4\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\,\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x)
```

```
[Out] 4*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/3
```

Giac [A]
time = 0.46, size = 40, normalized size = 1.29

$$\frac{2}{3}\sqrt{3}\left(x+2\,\arctan\left(-\frac{\sqrt{3}\,\sin\left(x\right)-\sin\left(x\right)}{\sqrt{3}\,\cos\left(x\right)+\sqrt{3}-\cos\left(x\right)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos(x) + sqrt
(3) - cos(x) + 1)))
```

Mupad [B]
time = 0.22, size = 32, normalized size = 1.03

$$\frac{4\sqrt{3}\,\left(\frac{x}{2}-\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)\right)\right)}{3}+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)/2 + 1),x)
```

```
[Out] (4*3^(1/2)*(x/2 - atan(tan(x/2))))/3 + (4*3^(1/2)*atan((3^(1/2)*tan(x/2))/3
))/3
```

3.144 $\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$x-\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}$$

```
[Out] x-1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3250, 3260, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}-\frac{x}{\sqrt{2}}+x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx &= x-\int \frac{1}{1+\sin^2(x)}\,dx\\ &= x-\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= x-\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.50

$$x-\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]
```

Maple [A]
time = 0.06, size = 17, normalized size = 0.47

method	result	size
default	$\arctan\left(\tan\left(x\right)\right)-\frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{2}$	17
risch	$x-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{4}+\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{4}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(1+sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(tan(x))-1/2*2^(1/2)*arctan(tan(x)*2^(1/2))
```

Maxima [A]
time = 1.48, size = 14, normalized size = 0.39

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(x\right)\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x
```

Fricas [A]
time = 1.06, size = 33, normalized size = 0.92

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-2\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) +
x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(36)=72$.
time = 23.52, size = 248, normalized size = 6.89

$$\frac{31988856\sqrt{2}x}{31988856\sqrt{2}+45239074}+\frac{45239074x}{31988856\sqrt{2}+45239074}-\frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}-\frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}-\frac{13250218\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}-\frac{9369319\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}+45239074}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(1+sin(x)**2),x)
```

```
[Out] 31988856*sqrt(2)*x/(31988856*sqrt(2) + 45239074) + 45239074*x/(31988856*sqr
t(2) + 45239074) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*s
qrt(2))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 54608
393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*fl
oor((x/2 - pi/2)/pi))/(31988856*sqrt(2) + 45239074) - 13250218*sqrt(2*sqrt(
2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(3
1988856*sqrt(2) + 45239074) - 9369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan
(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2) +
 45239074)
```

Giac [A]
time = 0.48, size = 48, normalized size = 1.33

$$-\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-2\cos\left(2x\right)+2}\right)\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2))) + x
```

Mupad [B]
time = 0.21, size = 26, normalized size = 0.72

$$x - \frac{\sqrt{2}\,\left(x - \mathrm{atan}(\tan(x))\right)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(sin(x)^2 + 1),x)
[Out] x - (2^(1/2)*(x - atan(tan(x))))/2 - (2^(1/2)*atan(2^(1/2)*tan(x)))/2
```

3.145 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] arctan(a*tan(x)/b)/a/b
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{211\}$

$$\frac{\text{ArcTan}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 15, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A]
time = 0.09, size = 16, normalized size = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$	16
risch	$\frac{i\ln\left(e^{2ix}-\frac{a+b}{a-b}\right)}{2ab}-\frac{i\ln\left(e^{2ix}-\frac{a-b}{a+b}\right)}{2ab}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(a*tan(x)/b)/a/b
```

Maxima [A]
time = 1.66, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.
time = 1.01, size = 43, normalized size = 2.87

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71839 vs. $2(10) = 20$.
time = 17.72, size = 71839, normalized size = 4789.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)

[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-2*tan(x/
2)/(b**2*(tan(x/2)**2 - 1)), Eq(a, 0)), (x/(b**2*sin(x)**2 + b**2*cos(x)**2
), Eq(a, b) | Eq(a, -b)), ((tan(x/2)/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (
8192*a**15*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**15*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
```

```
+ 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**...
```

Giac [A]
time = 0.46, size = 26, normalized size = 1.73

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

Mupad [B]
time = 0.46, size = 15, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{a\tan(x)}{b}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
[Out] atan((a*tan(x))/b)/(a*b)
```

3.146 $\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

```
[Out] sin(x)/b/(b*cos(x)+a*sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{3154\}$

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[x] + a*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx = \frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[x] + a*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Maple [A]
time = 0.10, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{1}{a(a\tan(x)+b)}$	14
norman	$\frac{-\frac{1}{a}+\frac{\tan^2\left(\frac{x}{2}\right)}{a}}{-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b}$	38
risch	$-\frac{2i}{(a\,e^{2ix}+ib\,e^{2ix}-a+ib)(ib+a)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x)+a*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/(a*tan(x)+b)
```

Maxima [A]
time = 1.10, size = 14, normalized size = 0.82

$$-\frac{1}{a^2\tan(x)+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(a^2*tan(x) + a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.
time = 0.95, size = 39, normalized size = 2.29

$$-\frac{a\cos(x)-b\sin(x)}{(a^2b+b^3)\cos(x)+(a^3+ab^2)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(a*cos(x) - b*sin(x))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)*sin(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(14) = 28$.
time = 124.87, size = 595, normalized size = 35.00

$$\begin{cases} \frac{\tilde{\infty}\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} & \text{for } a=0\wedge b=0 \\ \frac{x\tan^4\left(\frac{x}{2}\right)}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right)-4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right)+2b^2\sin^2(x)+8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right)-8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right)+8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)}+\frac{2x\tan^2\left(\frac{x}{2}\right)}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right)-4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right)+2b^2\sin^2(x)+8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right)-8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right)+8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)}+\frac{x}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right)-4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right)+2b^2\sin^2(x)+8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right)-8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right)+8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)}+\frac{2\tan^3\left(\frac{x}{2}\right)}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right)-4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right)+2b^2\sin^2(x)+8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right)-8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right)+8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)}-\frac{2\tan\left(\frac{x}{2}\right)}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right)-4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right)+2b^2\sin^2(x)+8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right)-8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right)+8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)} & \text{for } a=\frac{b\left(\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}\right)}{2} \\ \frac{\frac{\tan\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tan\left(\frac{x}{2}\right)}}{a^2} & \text{for } b=0 \\ \frac{2\tan\left(\frac{x}{2}\right)}{2ab\tan\left(\frac{x}{2}\right)-b^2\tan^2\left(\frac{x}{2}\right)+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))**2,x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (x*tan(x/2
)**4/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*
sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2
) + 8*b**2*cos(x)**2*tan(x/2)**2) + 2*x*tan(x/2)**2/(2*b**2*sin(x)**2*tan(x
/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*co
s(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2
)**2) + x/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*
b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*ta
n(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2) + 2*tan(x/2)**3/(2*b**2*sin(x)**2*ta
n(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)
*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(
x/2)**2) - 2*tan(x/2)/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(
x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(
x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2), Eq(a, b*(tan(x/2) - 1/t
an(x/2))/2)), ((tan(x/2)/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (2*tan(x/2)/(
2*a*b*tan(x/2) - b**2*tan(x/2)**2 + b**2), True))
```

Giac [A]
time = 0.44, size = 13, normalized size = 0.76

$$-\frac{1}{(a\tan(x)+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -1/((a*tan(x) + b)*a)
```

Mupad [B]
time = 0.63, size = 29, normalized size = 1.71

$$\frac{2\tan\left(\frac{x}{2}\right)}{b\,\left(-b\tan\left(\frac{x}{2}\right)^2+2\,a\tan\left(\frac{x}{2}\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x) + a*sin(x))^2,x)
```

```
[Out] (2*tan(x/2))/(b*(b + 2*a*tan(x/2) - b*tan(x/2)^2))
```

3.147 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=30

$$\frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] 1/2*x-1/2*ln(1+cos(x)+sin(x))-1/2*ln(1+tan(1/2*x))
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3216, 3203, 31}

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx &= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\int \frac{1}{1+\cos(x)+\sin(x)}\,dx \\
&= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\text{Subst}\left(\int \frac{1}{2+2x}\,dx,x,\tan\left(\frac{x}{2}\right)\right) \\
&= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.73

$$\frac{x}{2}-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.06, size = 27, normalized size = 0.90

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\ln\left(e^{ix}+i\right)$	20
default	$-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	27
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] -ln(1+tan(1/2*x))+1/2*ln(1+tan(1/2*x)^2)+arctan(tan(1/2*x))
```

Maxima [A]
time = 1.41, size = 41, normalized size = 1.37

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)
^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 0.88, size = 11, normalized size = 0.37

$$\frac{1}{2}\,x - \frac{1}{2}\,\log\left(\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

Sympy [A]
time = 0.12, size = 22, normalized size = 0.73

$$\frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right) + 1\right) + \frac{\log\left(\tan^{2}\left(\frac{x}{2}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

Giac [A]
time = 0.47, size = 25, normalized size = 0.83

$$\frac{1}{2}\,x + \frac{1}{2}\,\log\left(\tan\left(\frac{1}{2}\,x\right)^{2} + 1\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

Mupad [B]
time = 0.31, size = 34, normalized size = 1.13

$$-\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right)\,\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + sin(x) + 1),x)
```

```
[Out] log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(1
/2 + 1i/2)
```

3.148 $\int \sqrt{3-x^2}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 222}

$$\frac{3}{2}\text{ArcSin}\left(\frac{x}{\sqrt{3}}\right)+\frac{1}{2}\sqrt{3-x^2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x^2],x]
```

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-x^2}\,dx &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{3-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.21

$$\frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\tan^{-1}\left(\frac{x}{\sqrt{3-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x^2],x]
```

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcTan[x/Sqrt[3 - x^2]])/2
```

Maple [A]
time = 0.08, size = 23, normalized size = 0.79

method	result	size
default	$\frac{3\arcsin\left(\frac{x\sqrt{3}}{3}\right)}{2}+\frac{x\sqrt{-x^{2}+3}}{2}$	23
risch	$-\frac{x\left(x^{2}-3\right)}{2\sqrt{-x^{2}+3}}+\frac{3\arcsin\left(\frac{x\sqrt{3}}{3}\right)}{2}$	28
meijerg	$\frac{3i\left(-\frac{2i\sqrt{\pi}\,x\sqrt{3}\,\sqrt{-\frac{x^{2}}{3}+1}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{3}}{3}\right)\right)}{4\sqrt{\pi}}$	40
trager	$\frac{x\sqrt{-x^{2}+3}}{2}+\frac{3\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+3}+x\right)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

Maxima [A]
time = 1.78, size = 22, normalized size = 0.76

$$\frac{1}{2}\,\sqrt{-x^{2}+3}\,x+\frac{3}{2}\,\arcsin\left(\frac{1}{3}\,\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

Fricas [A]
time = 0.80, size = 29, normalized size = 1.00

$$\frac{1}{2}\,\sqrt{-x^{2}+3}\,x-\frac{3}{2}\,\arctan\left(\frac{\sqrt{-x^{2}+3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x - 3/2*arctan(sqrt(-x^2 + 3)/x)
```

Sympy [A]
time = 0.07, size = 24, normalized size = 0.83

$$\frac{x\sqrt{3-x^2}}{2}+\frac{3\operatorname{asin}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(3 - x**2)/2 + 3*asin(sqrt(3)*x/3)/2
```

Giac [A]
time = 0.43, size = 22, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+3}\,x+\frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.76

$$\frac{3\operatorname{asin}\left(\frac{\sqrt{3}\,x}{3}\right)}{2}+\frac{x\,\sqrt{3-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - x^2)^(1/2),x)
```

```
[Out] (3*asin((3^(1/2)*x)/3))/2 + (x*(3 - x^2)^(1/2))/2
```

3.149 $\int \frac{x}{\sqrt{3-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{3-x^2}$$

[Out] -(-x^2+3)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {267}

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[3 - x^2],x]

[Out] -Sqrt[3 - x^2]

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = -\sqrt{3-x^2}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[3 - x^2],x]

[Out] -Sqrt[3 - x^2]

Maple [A]
time = 0.06, size = 12, normalized size = 0.92

method	result	size
gosper	$-\sqrt{-x^2+3}$	12
derivativedivides	$-\sqrt{-x^2+3}$	12
default	$-\sqrt{-x^2+3}$	12
trager	$-\sqrt{-x^2+3}$	12
risch	$\frac{x^2-3}{\sqrt{-x^2+3}}$	16
meijerg	$-\frac{\sqrt{3}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-\frac{x^2}{3}+1}\right)}{2\sqrt{\pi}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+3)^(1/2)
```

Maxima [A]
time = 1.21, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 3)
```

Fricas [A]
time = 0.67, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 3)
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.62

$$-\sqrt{3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3 - x**2)
```

Giac [A]
time = 0.47, size = 11, normalized size = 0.85

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 3)
```

Mupad [B]
time = 0.17, size = 11, normalized size = 0.85

$$-\sqrt{3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3 - x^2)^(1/2),x)
```

```
[Out] -(3 - x^2)^(1/2)
```

3.150 $\int \frac{\sqrt{3-x^2}}{x} dx$

Optimal. Leaf size=37

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

```
[Out] -arctanh(1/3*(-x^2+3)^(1/2)*3^(1/2))*3^(1/2)+(-x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {272, 52, 65, 212}

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x^2]/x,x]
```

```
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{3-x^2}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{3-x}}{x}\,dx, x, x^2\right)\\
&= \sqrt{3-x^2} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3-x}\,x}\,dx, x, x^2\right)\\
&= \sqrt{3-x^2} - 3\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \sqrt{3-x^2}\right)\\
&= \sqrt{3-x^2} - \sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 0.89

$$\sqrt{3-x^2} - \sqrt{3}\tanh^{-1}\left(\sqrt{1-\frac{x^2}{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x^2]/x,x]
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[1 - x^2/3]]
```

Maple [A]
time = 0.09, size = 30, normalized size = 0.81

method	result	size
default	$\sqrt{-x^2+3} - \sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3}}{\sqrt{-x^2+3}}\right)$	30
trager	$\sqrt{-x^2+3} - \text{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\sqrt{-x^2+3}+\text{RootOf}\left(\_Z^2-3\right)}{x}\right)$	40

meijerg	$-\frac{\sqrt{3}\left(-2(2-2\ln(2)+2\ln(x)-\ln(3)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-\frac{x^2}{3}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{x^2}{3}+1}}{2}\right)\right)}{4\sqrt{\pi}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-x^2+3)^(1/2)-3^(1/2)*arctanh(3^(1/2)/(-x^2+3)^(1/2))
```

Maxima [A]
time = 1.51, size = 41, normalized size = 1.11

$$-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+3}}{|x|}+\frac{6}{|x|}\right)+\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 3)/abs(x) + 6/abs(x)) + sqrt(-x^2 + 3)
```

Fricas [A]
time = 0.63, size = 40, normalized size = 1.08

$$\frac{1}{2}\sqrt{3}\log\left(-\frac{x^2+2\sqrt{3}\sqrt{-x^2+3}-6}{x^2}\right)+\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(-(x^2 + 2*sqrt(3)*sqrt(-x^2 + 3) - 6)/x^2) + sqrt(-x^2 + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 87, normalized size = 2.35

$$\begin{cases} i\sqrt{x^2-3}-\sqrt{3}\log(x)+\frac{\sqrt{3}\log(x^2)}{2}+\sqrt{3}\,i\operatorname{asin}\left(\frac{\sqrt{3}}{x}\right) & \text{for } |x^2|>3 \\ \sqrt{3-x^2}+\frac{\sqrt{3}\log(x^2)}{2}-\sqrt{3}\log\left(\sqrt{1-\frac{x^2}{3}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(x**2 - 3) - sqrt(3)*log(x) + sqrt(3)*log(x**2)/2 + sqrt(3
)*I*asin(sqrt(3)/x), Abs(x**2) > 3), (sqrt(3 - x**2) + sqrt(3)*log(x**2)/2
- sqrt(3)*log(sqrt(1 - x**2/3) + 1), True))
```

Giac [A]
time = 0.46, size = 47, normalized size = 1.27

$$\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt{3}-\sqrt{-x^2+3}}{\sqrt{3}+\sqrt{-x^2+3}}\right)+\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="giac")
[Out] 1/2*sqrt(3)*log((sqrt(3) - sqrt(-x^2 + 3))/(sqrt(3) + sqrt(-x^2 + 3))) + sq
rt(-x^2 + 3)
```

Mupad [B]
time = 0.21, size = 35, normalized size = 0.95

$$\sqrt{3}\,\ln\left(\sqrt{\frac{3}{x^2}-1}-\sqrt{3}\,\sqrt{\frac{1}{x^2}}\right)+\sqrt{3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - x^2)^(1/2)/x,x)
[Out] 3^(1/2)*log((3/x^2 - 1)^(1/2) - 3^(1/2)*(1/x^2)^(1/2)) + (3 - x^2)^(1/2)
```

3.151 $\int \frac{\sqrt{x+x^2}}{x}\,dx$

Optimal. Leaf size=22

$$\sqrt{x+x^2}+\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

```
[Out] arctanh(x/(x^2+x)^(1/2))+(x^2+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {678, 634, 212}

$$\sqrt{x^2+x}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^2]/x,x]
```

```
[Out] Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sqrt{x+x^2}}{x}\,dx = \sqrt{x+x^2} + \frac{1}{2}\int \frac{1}{\sqrt{x+x^2}}\,dx$$
$$= \sqrt{x+x^2} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right)$$
$$= \sqrt{x+x^2} + \tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 1.68

$$\sqrt{x(1+x)}\left(1+\frac{\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)}{\sqrt{x}\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^2]/x,x]
[Out] Sqrt[x*(1 + x)]*(1 + ArcTanh[Sqrt[x/(1 + x)]]/(Sqrt[x]*Sqrt[1 + x]))
```

Maple [A]
time = 0.07, size = 22, normalized size = 1.00

method	result	size
default	$\sqrt{x^2+x}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2}$	22
trager	$\sqrt{x^2+x}-\frac{\ln\left(2\sqrt{x^2+x}-1-2x\right)}{2}$	26
risch	$\frac{x(1+x)}{\sqrt{x\,(1+x)}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2}$	27
meijerg	$-\frac{-2\sqrt{\pi}\,\sqrt{x}\,\sqrt{1+x}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{2\sqrt{\pi}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (x^2+x)^(1/2)+1/2*ln(x+1/2+(x^2+x)^(1/2))
```

Maxima [A]
time = 1.44, size = 25, normalized size = 1.14

$$\sqrt{x^2+x}+\frac{1}{2}\log\left(2\,x+2\,\sqrt{x^2+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="maxima")
[Out] sqrt(x^2 + x) + 1/2*log(2*x + 2*sqrt(x^2 + x) + 1)
```

Fricas [A]
time = 0.57, size = 25, normalized size = 1.14

$$\sqrt{x^2+x}-\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="fricas")
[Out] sqrt(x^2 + x) - 1/2*log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x\,(x+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(1/2)/x,x)
[Out] Integral(sqrt(x*(x + 1))/x, x)
```

Giac [A]
time = 0.46, size = 26, normalized size = 1.18

$$\sqrt{x^2+x}-\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="giac")
[Out] sqrt(x^2 + x) - 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

Mupad [B]
time = 0.08, size = 21, normalized size = 0.95

$$\frac{\ln\left(x+\sqrt{x\ (x+1)}+\frac{1}{2}\right)}{2}+\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)^(1/2)/x,x)
[Out] log(x + (x*(x + 1))^(1/2) + 1/2)/2 + (x + x^2)^(1/2)
```

3.152 $\int \sqrt{5+x^2}\, dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {201, 221}

$$\frac{1}{2}\sqrt{x^2+5}\,x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + x^2],x]
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{5+x^2}\, dx &= \frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\int \frac{1}{\sqrt{5+x^2}}\, dx\\ &= \frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.15

$$\frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\tanh^{-1}\left(\frac{x}{\sqrt{5+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + x^2],x]
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcTanh[x/Sqrt[5 + x^2]])/2
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.78

method	result	size
default	$\frac{5\,\mathrm{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)}{2}+\frac{x\sqrt{x^2+5}}{2}$	21
risch	$\frac{5\,\mathrm{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)}{2}+\frac{x\sqrt{x^2+5}}{2}$	21
trager	$\frac{x\sqrt{x^2+5}}{2}+\frac{5\ln\left(x+\sqrt{x^2+5}\right)}{2}$	24
meijerg	$-\frac{5\left(-\frac{2\sqrt{\pi}\,x\sqrt{5}\,\sqrt{1+\frac{x^2}{5}}}{5}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x\sqrt{5}}{5}\right)\right)}{4\sqrt{\pi}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

Maxima [A]
time = 1.43, size = 20, normalized size = 0.74

$$\frac{1}{2}\sqrt{x^2+5}\,x+\frac{5}{2}\,\mathrm{arsinh}\left(\frac{1}{5}\sqrt{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 5)*x + 5/2*arcsinh(1/5*sqrt(5)*x)
```

Fricas [A]
time = 0.57, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+5}\,x-\frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

Sympy [A]
time = 0.07, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+5}}{2}+\frac{5\,\text{asinh}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 5)/2 + 5*asinh(sqrt(5)*x/5)/2
```

Giac [A]
time = 0.46, size = 25, normalized size = 0.93

$$\frac{1}{2}\sqrt{x^2+5}\,x-\frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

Mupad [B]
time = 0.09, size = 20, normalized size = 0.74

$$\frac{5\,\text{asinh}\left(\frac{\sqrt{5}\,x}{5}\right)}{2}+\frac{x\,\sqrt{x^2+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 5)^(1/2),x)
```

```
[Out] (5*asinh((5^(1/2)*x)/5))/2 + (x*(x^2 + 5)^(1/2))/2
```

3.153 $\int \frac{x}{\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=27

$$\sqrt{1+x+x^2}-\frac{1}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

```
[Out] -1/2*arcsinh(1/3*(1+2*x)*3^(1/2))+(x^2+x+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {654, 633, 221}

$$\sqrt{x^2+x+1}-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x + x^2],x]
```

```
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x+x^2}}\,dx = \sqrt{1+x+x^2} - \frac{1}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx$$
$$= \sqrt{1+x+x^2} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x\right)}{2\sqrt{3}}$$
$$= \sqrt{1+x+x^2} - \frac{1}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 1.22

$$\sqrt{1+x+x^2} + \frac{1}{2}\log\left(-1-2x+2\sqrt{1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x + x^2],x]
[Out] Sqrt[1 + x + x^2] + Log[-1 - 2*x + 2*Sqrt[1 + x + x^2]]/2
```

Maple [A]
time = 0.17, size = 21, normalized size = 0.78

method	result	size
default	$\sqrt{x^2+x+1} - \frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	21
risch	$\sqrt{x^2+x+1} - \frac{\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{2}$	21
trager	$\sqrt{x^2+x+1} - \frac{\ln\left(2x+1+2\sqrt{x^2+x+1}\right)}{2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x^2+x+1)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A]
time = 4.10, size = 22, normalized size = 0.81

$$\sqrt{x^2+x+1} - \frac{1}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + x + 1) - 1/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A]
time = 0.53, size = 27, normalized size = 1.00

$$\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2 + x + 1), x)
```

Giac [A]
time = 0.45, size = 27, normalized size = 1.00

$$\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Mupad [B]
time = 0.05, size = 23, normalized size = 0.85

$$\sqrt{x^2+x+1}-\frac{\ln\left(x+\sqrt{x^2+x+1}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^2 + 1)^(1/2),x)
```

```
[Out] (x + x^2 + 1)^(1/2) - log(x + (x + x^2 + 1)^(1/2) + 1/2)/2
```

3.154 $\int \frac{1}{\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=14

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

```
[Out] 2*arctanh(x/(x^2+x)^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {634, 212}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x+x^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right)\\ &= 2\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(14) = 28.
time = 0.02, size = 37, normalized size = 2.64

$$\frac{2\sqrt{x}\,\sqrt{1+x}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)}{\sqrt{x(1+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + x^2],x]
[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcTanh[Sqrt[x]/Sqrt[1 + x]])/Sqrt[x*(1 + x)]
```

Maple [A]
time = 0.06, size = 12, normalized size = 0.86

method	result	size
meijerg	$2\,\mathrm{arcsinh}\left(\sqrt{x}\right)$	7
default	$\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)$	12
trager	$-\ln\left(2\sqrt{x^{2}+x}-1-2x\right)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x+1/2+(x^2+x)^(1/2))
```

Maxima [A]
time = 6.86, size = 15, normalized size = 1.07

$$\log\left(2\,x+2\,\sqrt{x^{2}+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 + x) + 1)
```

Fricas [A]
time = 0.52, size = 17, normalized size = 1.21

$$-\log\left(-2\,x+2\,\sqrt{x^{2}+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="fricas")
[Out] -log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^{2}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 0.45, size = 33, normalized size = 2.36

$$\frac{1}{4}\sqrt{x^2+x}\,(2x+1)+\frac{1}{8}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 + x)*(2*x + 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

Mupad [B]
time = 0.17, size = 11, normalized size = 0.79

$$\ln\left(x+\sqrt{x\,(x+1)}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2)^(1/2),x)
```

```
[Out] log(x + (x*(x + 1))^(1/2) + 1/2)
```

3.155 $\int \frac{\sqrt{2-x-x^2}}{x^2} \, dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{2-x-x^2}}{x}+\sin^{-1}\left(\frac{1}{3}(-1-2x)\right)+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}$$

```
[Out] -arcsin(1/3+2/3*x)+1/4*arctanh(1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)-
(-x^2-x+2)^(1/2)/x
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {746, 857, 633, 222, 738, 212}

$$\text{ArcSin}\left(\frac{1}{3}(-2x-1)\right)-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{2}\int \frac{-1-2x}{x\sqrt{2-x-x^2}}\,dx\\ &= -\frac{\sqrt{2-x-x^2}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{2-x-x^2}}\,dx-\int \frac{1}{\sqrt{2-x-x^2}}\,dx\\ &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,-1-2x\right)+\text{Subst}\left(\int \frac{1}{8-x^2}\,dx,x\right.\\ &= -\frac{\sqrt{2-x-x^2}}{x}+\sin^{-1}\left(\frac{1}{3}(-1-2x)\right)+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 76, normalized size = 1.12

$$-\frac{\sqrt{2-x-x^2}}{x}+2\tan^{-1}\left(\frac{\sqrt{2-x-x^2}}{2+x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2-x-x^2}}{\sqrt{2}\,(-1+x)}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + 2*ArcTan[Sqrt[2 - x - x^2]/(2 + x)] - ArcTanh[Sqrt
[2 - x - x^2]/(Sqrt[2]*(-1 + x))]/Sqrt[2]
```

Maple [A]
time = 0.24, size = 88, normalized size = 1.29

method	result
risch	$\frac{x^2+x-2}{x\sqrt{-x^2-x+2}} - \arcsin\left(\frac{1}{3}+\frac{2x}{3}\right) + \frac{\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)\sqrt{2}}{4}$
default	$-\frac{(-x^2-x+2)^{\frac{3}{2}}}{2x} - \frac{\sqrt{-x^2-x+2}}{4} - \arcsin\left(\frac{1}{3}+\frac{2x}{3}\right) + \frac{\operatorname{arctanh}\left(\frac{(4-x)\sqrt{2}}{4\sqrt{-x^2-x+2}}\right)\sqrt{2}}{4} + \frac{(-2x-1)\sqrt{-x}}{4}$
trager	$-\frac{\sqrt{-x^2-x+2}}{x} - \frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{-x^2-x+2}-4\operatorname{RootOf}\left(\_Z^2-2\right)}{x}\right)}{4} + \operatorname{RootOf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-x+2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x*(-x^2-x+2)^(3/2)-1/4*(-x^2-x+2)^(1/2)-arcsin(1/3+2/3*x)+1/4*arctanh(
1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)+1/4*(-2*x-1)*(-x^2-x+2)^(1/2)
```

Maxima [A]
time = 4.86, size = 59, normalized size = 0.87

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right)-\frac{\sqrt{-x^2-x+2}}{x}+\arcsin\left(-\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) - 1) - sqrt(
-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

Fricas [A]
time = 0.82, size = 92, normalized size = 1.35

$$\frac{\sqrt{2}\,x\log\left(-\frac{4\sqrt{2}\sqrt{-x^2-x+2}(x-4)+7x^2+16x-32}{x^2}\right)+8x\arctan\left(\frac{\sqrt{-x^2-x+2}(2x+1)}{2(x^2+x-2)}\right)-8\sqrt{-x^2-x+2}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="fricas")

[Out] 1/8*(sqrt(2)*x*log(-(4*sqrt(2)*sqrt(-x^2 - x + 2)*(x - 4) + 7*x^2 + 16*x - 32)/x^2) + 8*x*arctan(1/2*sqrt(-x^2 - x + 2)*(2*x + 1)/(x^2 + x - 2)) - 8*sqrt(-x^2 - x + 2))/x

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2-x+2)**(1/2)/x**2,x)

[Out] Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(52) = 104.
time = 0.48, size = 168, normalized size = 2.47

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}\right)+\frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2x+1)^2}+1}-\arcsin\left(\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="giac")

[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)) + 6*(3*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2*x + 1)^2 + 1) - arcsin(2/3*x + 1/3)

Mupad [**B**]
time = 0.08, size = 73, normalized size = 1.07

$$\frac{\sqrt{2}\,\ln\left(\frac{2}{x}+\frac{\sqrt{2}\,\sqrt{-x^2-x+2}}{x}-\frac{1}{2}\right)}{4}-\frac{\sqrt{-x^2-x+2}}{x}+\ln\left(x\,1\mathrm{i}+\sqrt{-x^2-x+2}+\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2 - x^2 - x)^(1/2)/x^2,x)

[Out] log(x*1i + (2 - x^2 - x)^(1/2) + 1i/2)*1i - (2 - x^2 - x)^(1/2)/x + (2^(1/2)*log(2/x + (2^(1/2)*(2 - x^2 - x)^(1/2))/x - 1/2))/4

3.156 $\int \frac{\log(t)}{1+t}\,dt$

Optimal. Leaf size=13

$$\log(t)\log(1+t)+\text{Li}_2(-t)$$

```
[Out] ln(t)*ln(1+t)+polylog(2,-t)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2354, 2438}

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]/(1 + t),t]
```

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(t)}{1+t}\,dt &= \log(t)\log(1+t)-\int \frac{\log(1+t)}{t}\,dt\\ &= \log(t)\log(1+t)+\text{Li}_2(-t)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\log(t)\log(1+t)+\text{Li}_2(-t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]/(1 + t),t]
```

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Maple [A]
time = 0.07, size = 13, normalized size = 1.00

method	result	size
default	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13
risch	$\operatorname{dilog}(1+t)+\ln(t)\ln(1+t)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)/(1+t),t,method=_RETURNVERBOSE)
```

```
[Out] dilog(1+t)+ln(t)*ln(1+t)
```

Maxima [A]
time = 4.86, size = 12, normalized size = 0.92

$$\log(t+1)\log(t)+\operatorname{Li}_2(-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="maxima")
```

```
[Out] log(t + 1)*log(t) + dilog(-t)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="fricas")
```

```
[Out] integral(log(t)/(t + 1), t)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.85, size = 73, normalized size = 5.62

$$\begin{cases} -\operatorname{Li}_2(t+1) & \text{for } \frac{1}{|t+1|}<1 \wedge |t+1|<1 \\ i\pi\log(t+1)-\operatorname{Li}_2(t+1) & \text{for } |t+1|<1 \\ -i\pi\log\left(\frac{1}{t+1}\right)-\operatorname{Li}_2(t+1) & \text{for } \frac{1}{|t+1|}<1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| t+1\right)+i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| t+1\right)-\operatorname{Li}_2(t+1) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)/(1+t),t)
```

```
[Out] Piecewise((-polylog(2, t + 1), (Abs(t + 1) < 1) & (1/Abs(t + 1) < 1)), (I*p
i*log(t + 1) - polylog(2, t + 1), Abs(t + 1) < 1), (-I*pi*log(1/(t + 1)) -
polylog(2, t + 1), 1/Abs(t + 1) < 1), (-I*pi*meijerg(((), (1, 1)), ((0, 0),
 ()), t + 1) + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), t + 1) - polylog(2,
 t + 1), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="giac")
```

```
[Out] integrate(log(t)/(t + 1), t)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 1.00

$$\mathrm{polylog}(2,-t)+\ln\left(t+1\right)\,\ln\left(t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(t)/(t + 1),t)
```

```
[Out] polylog(2, -t) + log(t + 1)*log(t)
```

3.157 $\int \log \left(e^{\cos(x)}\right)\, dx$

Optimal. Leaf size=15

$$-x \cos(x) + x \log \left(e^{\cos(x)}\right) + \sin(x)$$

```
[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

Rubi [A]

time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2628, 3377, 2717}

$$\sin(x) - x \cos(x) + x \log \left(e^{\cos(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^Cos[x]],x]
```

```
[Out] -(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log \left(e^{\cos(x)}\right)\, dx &= x \log \left(e^{\cos(x)}\right) + \int x \sin(x)\, dx \\
&= -x \cos(x) + x \log \left(e^{\cos(x)}\right) + \int \cos(x)\, dx \\
&= -x \cos(x) + x \log \left(e^{\cos(x)}\right) + \sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$x\left(-\cos(x)+\log\left(e^{\cos(x)}\right)\right)+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^Cos[x]],x]
[Out] x*(-Cos[x] + Log[E^Cos[x]]) + Sin[x]
```

Maple [A]
time = 0.04, size = 15, normalized size = 1.00

method	result	size
default	$-x\cos\left(x\right)+x\ln\left(\mathrm{e}^{\cos(x)}\right)+\sin\left(x\right)$	15
risch	$-x\cos\left(x\right)+x\ln\left(\mathrm{e}^{\cos(x)}\right)+\sin\left(x\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(cos(x))),x,method=_RETURNVERBOSE)
[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

Maxima [A]
time = 3.05, size = 2, normalized size = 0.13

$$\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="maxima")
[Out] sin(x)
```

Fricas [A]
time = 1.22, size = 2, normalized size = 0.13

$$\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="fricas")
[Out] sin(x)
```

Sympy [A]
time = 0.07, size = 15, normalized size = 1.00

$$x\log\left(e^{\cos\left(x\right)}\right)-x\cos\left(x\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(cos(x))),x)
```

```
[Out] x*log(exp(cos(x))) - x*cos(x) + sin(x)
```

Giac [A]
time = 0.46, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="giac")
```

```
[Out] sin(x)
```

Mupad [B]
time = 0.10, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(cos(x))),x)
```

```
[Out] sin(x)
```

3.158 $\int \frac{e^t}{t} \, dt$

Optimal. Leaf size=2

$$\text{Ei}(t)$$

```
[Out] Ei(t)
```

Rubi [A]
time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2209}

$$\text{ExpIntegralEi}(t)$$

Antiderivative was successfully verified.

```
[In] Int[E^t/t,t]
```

```
[Out] ExpIntegralEi[t]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\int \frac{e^t}{t} \, dt = \text{Ei}(t)$$

Mathematica [A]
time = 0.01, size = 2, normalized size = 1.00

$$\text{Ei}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/t,t]
```

```
[Out] ExpIntegralEi[t]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(2) = 4.
time = 0.01, size = 8, normalized size = 4.00

method	result	size
default	$-\operatorname{expIntegral}(1,-t)$	8
risch	$-\operatorname{expIntegral}(1,-t)$	8
meijerg	$\ln(t)+i\pi-\ln(-t)-\operatorname{expIntegral}(1,-t)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t,t,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-t)
```

Maxima [A]
time = 4.66, size = 2, normalized size = 1.00

$$\operatorname{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="maxima")
```

```
[Out] Ei(t)
```

Fricas [A]
time = 0.87, size = 2, normalized size = 1.00

$$\operatorname{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="fricas")
```

```
[Out] Ei(t)
```

Sympy [A]
time = 0.31, size = 2, normalized size = 1.00

$$\operatorname{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t)
```

```
[Out] Ei(t)
```

Giac [A]
time = 0.45, size = 2, normalized size = 1.00

$$\operatorname{Ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="giac")
```

```
[Out] Ei(t)
```

Mupad [B]
time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{ei}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t,t)
```

```
[Out] ei(t)
```

3.159 $\int \frac{e^{at}}{t}\,dt$

Optimal. Leaf size=4

$$\text{Ei}(at)$$

[Out] Ei(a*t)

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2209}

$$\text{ExpIntegralEi}(at)$$

Antiderivative was successfully verified.

[In] Int[E^(a*t)/t,t]

[Out] ExpIntegralEi[a*t]

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\int \frac{e^{at}}{t}\,dt = \text{Ei}(at)$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$\text{Ei}(at)$$

Antiderivative was successfully verified.

[In] Integrate[E^(a*t)/t,t]

[Out] ExpIntegralEi[a*t]

Maple [A]
time = 0.02, size = 9, normalized size = 2.25

method	result	size
derivativedivides	$-\operatorname{expIntegral}(1, -at)$	9
default	$-\operatorname{expIntegral}(1, -at)$	9
risch	$-\operatorname{expIntegral}(1, -at)$	9
meijerg	$\ln(t) + \ln(-a) - \ln(-at) - \operatorname{expIntegral}(1, -at)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*t)/t,t,method=_RETURNVERBOSE)
[Out] -Ei(1,-a*t)
```

Maxima [A]
time = 6.97, size = 4, normalized size = 1.00

$$\mathrm{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="maxima")
[Out] Ei(a*t)
```

Fricas [A]
time = 0.86, size = 4, normalized size = 1.00

$$\mathrm{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="fricas")
[Out] Ei(a*t)
```

Sympy [A]
time = 0.34, size = 3, normalized size = 0.75

$$\mathrm{Ei}\left(at\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t)
[Out] Ei(a*t)
```

Giac [A]
time = 0.45, size = 4, normalized size = 1.00

$$\mathrm{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="giac")
```

```
[Out] Ei(a*t)
```

Mupad [**B**]
time = 0.01, size = 4, normalized size = 1.00

$$\mathrm{ei}(a\,t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*t)/t,t)
```

```
[Out] ei(a*t)
```

3.160 $\int \frac{e^t}{t^2}\,dt$

Optimal. Leaf size=11

$$-\frac{e^t}{t}+\text{Ei}(t)$$

[Out] -exp(t)/t+Ei(t)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2208, 2209}

$$\text{ExpIntegralEi}(t)-\frac{e^t}{t}$$

Antiderivative was successfully verified.

[In] Int[E^t/t^2,t]

[Out] -(E^t/t) + ExpIntegralEi[t]

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}\int \frac{e^t}{t^2}\,dt &= -\frac{e^t}{t}+\int \frac{e^t}{t}\,dt\\ &= -\frac{e^t}{t}+\text{Ei}(t)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{e^t}{t}+\text{Ei}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/t^2,t]
```

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Maple [A]
time = 0.01, size = 16, normalized size = 1.45

method	result	size
default	$-\frac{e^{t}}{t}-\operatorname{expIntegral}\left(1,-t\right)$	16
risch	$-\frac{e^{t}}{t}-\operatorname{expIntegral}\left(1,-t\right)$	16
meijerg	$-\frac{1}{t}-1+\ln\left(t\right)+i\pi+\frac{2t+2}{2t}-\frac{e^{t}}{t}-\ln\left(-t\right)-\operatorname{expIntegral}\left(1,-t\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t^2,t,method=_RETURNVERBOSE)
```

```
[Out] -exp(t)/t-Ei(1,-t)
```

Maxima [A]
time = 1.65, size = 5, normalized size = 0.45

$$\Gamma(-1,-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="maxima")
```

```
[Out] gamma(-1, -t)
```

Fricas [A]
time = 0.88, size = 13, normalized size = 1.18

$$\frac{t\mathrm{Ei}(t)-e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="fricas")
```

```
[Out] (t*Ei(t) - e^t)/t
```

Sympy [A]
time = 0.42, size = 7, normalized size = 0.64

$$\operatorname{Ei}\left(t\right)-\frac{e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t**2,t)
[Out] Ei(t) - exp(t)/t
```

Giac [**A**]
time = 0.44, size = 13, normalized size = 1.18

$$\frac{t\mathrm{Ei}(t) - e^t}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="giac")
[Out] (t*Ei(t) - e^t)/t
```

Mupad [**B**]
time = 0.02, size = 14, normalized size = 1.27

$$-\frac{\mathrm{e}^t}{t} - \mathrm{expint}(-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t^2,t)
[Out] - exp(t)/t - expint(-t)
```

3.161 $\int e^{\frac{1}{t}} dt$

Optimal. Leaf size=14

$$e^{\frac{1}{t}}t - \text{Ei}\left(\frac{1}{t}\right)$$

```
[Out] exp(1/t)*t-Ei(1/t)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2237, 2241}

$$e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^t^(-1),t]
```

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Rule 2237

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Dist[b*n*Log[F], Int[(c + d*x)^n*F^(a
+ b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && I
LtQ[n, 0]
```

Rule 2241

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned} \int e^{\frac{1}{t}} dt &= e^{\frac{1}{t}}t + \int \frac{e^{\frac{1}{t}}}{t} dt \\ &= e^{\frac{1}{t}}t - \text{Ei}\left(\frac{1}{t}\right) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$e^{\frac{1}{t}}t - \text{Ei}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t^(-1),t]
```

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Maple [A]
time = 0.02, size = 15, normalized size = 1.07

method	result	size
derivativedivides	$e^{\frac{1}{t}} t+\mathrm{expIntegral}\left(1,-\frac{1}{t}\right)$	15
default	$e^{\frac{1}{t}} t+\mathrm{expIntegral}\left(1,-\frac{1}{t}\right)$	15
risch	$e^{\frac{1}{t}} t+\mathrm{expIntegral}\left(1,-\frac{1}{t}\right)$	15
meijerg	$t+1+\ln\left(t\right)-i\pi-\frac{t\left(\frac{2}{t}+2\right)}{2}+e^{\frac{1}{t}} t+\ln\left(-\frac{1}{t}\right)+\mathrm{expIntegral}\left(1,-\frac{1}{t}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/t),t,method=_RETURNVERBOSE)
```

```
[Out] exp(1/t)*t+Ei(1,-1/t)
```

Maxima [A]
time = 1.30, size = 9, normalized size = 0.64

$$-\Gamma\left(-1,-\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="maxima")
```

```
[Out] -gamma(-1, -1/t)
```

Fricas [A]
time = 0.81, size = 13, normalized size = 0.93

$$te^{\frac{1}{t}}-\mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="fricas")
```

```
[Out] t*e^(1/t) - Ei(1/t)
```

Sympy [A]
time = 0.47, size = 10, normalized size = 0.71

$$te^{\frac{1}{t}}-\mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t)
```

```
[Out] t*exp(1/t) - Ei(1/t)
```

Giac [A]
time = 0.46, size = 18, normalized size = 1.29

$$-t\left(\frac{\mathrm{Ei}\left(\frac{1}{t}\right)}{t}-e^{\frac{1}{t}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="giac")
```

```
[Out] -t*(Ei(1/t)/t - e^(1/t))
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.64

$$t\,\mathrm{expint}\left(2,-\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/t),t)
```

```
[Out] t*expint(2, -1/t)
```

3.162 $\int \frac{e^{-t}}{-1-a+t} \, dt$

Optimal. Leaf size=15

$$e^{-1-a}\text{Ei}(1+a-t)$$

```
[Out] exp(-1-a)*Ei(1+a-t)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2209}

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^t*(-1 - a + t)),t]
```

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\int \frac{e^{-t}}{-1-a+t} \, dt = e^{-1-a}\text{Ei}(1+a-t)$$

Mathematica [A]
time = 0.03, size = 15, normalized size = 1.00

$$e^{-1-a}\text{Ei}(1+a-t)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^t*(-1 - a + t)),t]
```

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Maple [A]
time = 0.07, size = 17, normalized size = 1.13

method	result	size
default	$-e^{-1-a}\,\mathrm{expIntegral}\,(1,-1-a+t)$	17
risch	$-e^{-1-a}\,\mathrm{expIntegral}\,(1,-1-a+t)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(t)/(-1-a+t),t,method=_RETURNVERBOSE)
```

```
[Out] -exp(-1-a)*Ei(1,-1-a+t)
```

Maxima [A]
time = 0.93, size = 16, normalized size = 1.07

$$-e^{(-a-1)}E_1(-a+t-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="maxima")
```

```
[Out] -e^(-a - 1)*exp_integral_e(1, -a + t - 1)
```

Fricas [A]
time = 0.71, size = 14, normalized size = 0.93

$$\mathrm{Ei}(a-t+1)\,e^{(-a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="fricas")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{-t}}{-a+t-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t)
```

```
[Out] Integral(exp(-t)/(-a + t - 1), t)
```

Giac [A]
time = 0.45, size = 14, normalized size = 0.93

$$\mathrm{Ei}(a-t+1)\,e^{(-a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="giac")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.93

$$e^{-a-1}\,\mathrm{ei}(a-t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-exp(-t)/(a - t + 1),t)
```

```
[Out] exp(- a - 1)*ei(a - t + 1)
```

3.163 $\int \frac{e^{t^2} t}{1+t^2} \, dt$

Optimal. Leaf size=13

$$\frac{\text{Ei}(1+t^2)}{2e}$$

```
[Out] 1/2*Ei(t^2+1)/exp(1)
```

Rubi [A]
time = 0.06, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6847, 2209}

$$\frac{\text{ExpIntegralEi}(t^2+1)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(E^t^2*t)/(1 + t^2),t]
```

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{t^2} t}{1+t^2} \, dt &= \frac{1}{2}\text{Subst}\left(\int \frac{e^t}{1+t} \, dt, t, t^2\right)\\ &= \frac{\text{Ei}(1+t^2)}{2e}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(1+t^2)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^t^2*t)/(1 + t^2),t]
```

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Maple [A]
time = 0.06, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{e^{-1} \operatorname{expIntegral}(1,-t^2-1)}{2}$	14
default	$-\frac{e^{-1} \operatorname{expIntegral}(1,-t^2-1)}{2}$	14
risch	$-\frac{e^{-1} \operatorname{expIntegral}(1,-t^2-1)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t^2)*t/(t^2+1),t,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-1)*Ei(1,-t^2-1)
```

Maxima [A]
time = 1.71, size = 13, normalized size = 1.00

$$-\frac{1}{2}\,e^{(-1)}E_1\left(-t^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="maxima")
```

```
[Out] -1/2*e^(-1)*exp_integral_e(1, -t^2 - 1)
```

Fricas [A]
time = 0.78, size = 10, normalized size = 0.77

$$\frac{1}{2}\,\operatorname{Ei}\left(t^2+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="fricas")
```

```
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{te^{t^2}}{t^2+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t**2)*t/(t**2+1),t)
```

```
[Out] Integral(t*exp(t**2)/(t**2 + 1), t)
```

Giac [A]
time = 0.49, size = 10, normalized size = 0.77

$$\frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="giac")
```

```
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

Mupad [B]
time = 0.12, size = 10, normalized size = 0.77

$$\frac{\mathrm{e}^{-1}\,\mathrm{ei}\left(t^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((t*exp(t^2))/(t^2 + 1),t)
```

```
[Out] (exp(-1)*ei(t^2 + 1))/2
```

3.164 $\int \frac{e^t}{(1+t)^2}\,dt$

Optimal. Leaf size=19

$$-\frac{e^t}{1+t}+\frac{\text{Ei}(1+t)}{e}$$

[Out] -exp(t)/(1+t)+Ei(1+t)/exp(1)

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2208, 2209}

$$\frac{\text{ExpIntegralEi}(t+1)}{e}-\frac{e^t}{t+1}$$

Antiderivative was successfully verified.

[In] Int[E^t/(1 + t)^2,t]

[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E

Rule 2208

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Dist[f*g*n*(Log[F]/(d*(m + 1))), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !TrueQ[$UseGamma]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\begin{aligned}\int \frac{e^t}{(1+t)^2}\,dt &= -\frac{e^t}{1+t}+\int \frac{e^t}{1+t}\,dt\\ &= -\frac{e^t}{1+t}+\frac{\text{Ei}(1+t)}{e}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 1.00

$$-\frac{e^t}{1+t}+\frac{\text{Ei}(1+t)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/(1 + t)^2,t]
```

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Maple [A]
time = 0.06, size = 22, normalized size = 1.16

method	result	size
default	$-\frac{e^{t}}{1+t}-\mathrm{e}^{-1}\,\mathrm{expIntegral}\left(1,-1-t\right)$	22
risch	$-\frac{e^{t}}{1+t}-\mathrm{e}^{-1}\,\mathrm{expIntegral}\left(1,-1-t\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/(1+t)^2,t,method=_RETURNVERBOSE)
```

```
[Out] -exp(t)/(1+t)-exp(-1)*Ei(1,-1-t)
```

Maxima [A]
time = 1.81, size = 16, normalized size = 0.84

$$-\frac{e^{(-1)}E_2(-t-1)}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="maxima")
```

```
[Out] -e^(-1)*exp_integral_e(2, -t - 1)/(t + 1)
```

Fricas [A]
time = 0.64, size = 23, normalized size = 1.21

$$\frac{\left((t+1)\mathrm{Ei}(t+1)-e^{(t+1)}\right)e^{(-1)}}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="fricas")
```

```
[Out] ((t + 1)*Ei(t + 1) - e^(t + 1))*e^(-1)/(t + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e^{t}}{(t+1)^{2}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)**2,t)
```

```
[Out] Integral(exp(t)/(t + 1)**2, t)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(17) = 34.
time = 0.47, size = 80, normalized size = 4.21

$$\frac{(t+1)\left(\frac{1}{t+1}-1\right)\mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)-\mathrm{Ei}\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)+e^{\left(-(t+1)\left(\frac{1}{t+1}-1\right)+1\right)}}{(t+1)\left(\frac{1}{t+1}-1\right)e-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="giac")
```

```
[Out] ((t + 1)*(1/(t + 1) - 1)*Ei(-(t + 1)*(1/(t + 1) - 1) + 1) - Ei(-(t + 1)*(1/
(t + 1) - 1) + 1) + e^(-(t + 1)*(1/(t + 1) - 1) + 1))/((t + 1)*(1/(t + 1) -
 1)*e - e)
```

Mupad [B]
time = 0.13, size = 17, normalized size = 0.89

$$\mathrm{ei}(t+1)\ \mathrm{e}^{-1}-\frac{\mathrm{e}^{t}}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/(t + 1)^2,t)
```

```
[Out] ei(t + 1)*exp(-1) - exp(t)/(t + 1)
```

3.165 $\int e^t \log(1+t)\,dt$

Optimal. Leaf size=18

$$-\frac{\text{Ei}(1+t)}{e}+e^t\log(1+t)$$

```
[Out] -Ei(1+t)/exp(1)+exp(t)*ln(1+t)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2225, 2634, 2209}

$$e^t\log(t+1)-\frac{\text{ExpIntegralEi}(t+1)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^t\log(1+t)\,dt&=e^t\log(1+t)-\int\frac{e^t}{1+t}\,dt\\&=-\frac{\text{Ei}(1+t)}{e}+e^t\log(1+t)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{\text{Ei}(1+t)}{e}+e^t\log(1+t)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.06

method	result	size
risch	$\mathrm{e}^{t} \ln \left(1+t\right)+\mathrm{e}^{-1} \operatorname{expIntegral}\left(1,-1-t\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*ln(1+t),t,method=_RETURNVERBOSE)
```

```
[Out] exp(t)*ln(1+t)+exp(-1)*Ei(1,-1-t)
```

Maxima [A]
time = 3.45, size = 18, normalized size = 1.00

$$e^{(-1)}E_1(-t-1)+e^t\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="maxima")
```

```
[Out] e^(-1)*exp_integral_e(1, -t - 1) + e^t*log(t + 1)
```

Fricas [A]
time = 0.83, size = 19, normalized size = 1.06

$$\left(e^{(t+1)}\log(t+1)-\text{Ei}(t+1)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="fricas")
```

```
[Out] (e^(t + 1)*log(t + 1) - Ei(t + 1))*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^t\log(t+1)\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*ln(1+t),t)
```

```
[Out] Integral(exp(t)*log(t + 1), t)
```

Giac [A]
time = 0.46, size = 16, normalized size = 0.89

$$-\mathrm{Ei}(t+1)\,e^{(-1)}+e^{t}\log\left(t+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="giac")
```

```
[Out] -Ei(t + 1)*e^(-1) + e^t*log(t + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \ln\left(t+1\right)\,\mathrm{e}^{t}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(t + 1)*exp(t),t)
```

```
[Out] int(log(t + 1)*exp(t), t)
```

3.166 $\int e^{-t}t\,dt$

Optimal. Leaf size=16

$$-e^{-t}-e^{-t}t$$

```
[Out] -1/exp(t)-t/exp(t)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {2207, 2225}

$$-e^{-t}t-e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-t}t\,dt&=-e^{-t}t+\int e^{-t}\,dt\\&=-e^{-t}-e^{-t}t\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 0.69

$$e^{-t}(-1-t)$$

Antiderivative was successfully verified.

```
[In] Integrate[t/E^t,t]
[Out] (-1 - t)/E^t
```

Maple [A]
time = 0.01, size = 15, normalized size = 0.94

method	result	size
gosper	$-(1+t)\,\mathrm{e}^{-t}$	10
norman	$(-1-t)\,\mathrm{e}^{-t}$	11
risch	$(-1-t)\,\mathrm{e}^{-t}$	11
meijerg	$1-\frac{(2t+2)\mathrm{e}^{-t}}{2}$	14
default	$-\mathrm{e}^{-t}-t\,\mathrm{e}^{-t}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t/exp(t),t,method=_RETURNVERBOSE)
[Out] -1/exp(t)-t/exp(t)
```

Maxima [A]
time = 0.93, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="maxima")
[Out] -(t + 1)*e^(-t)
```

Fricas [A]
time = 0.67, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="fricas")
[Out] -(t + 1)*e^(-t)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.44

$$(-t-1)\,e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t)
```

```
[Out] (-t - 1)*exp(-t)
```

Giac [**A**]
time = 0.43, size = 9, normalized size = 0.56

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="giac")
```

```
[Out] -(t + 1)*e^(-t)
```

Mupad [**B**]
time = 0.02, size = 9, normalized size = 0.56

$$-\mathrm{e}^{-t}\,(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*exp(-t),t)
```

```
[Out] -exp(-t)*(t + 1)
```

3.167 $\int e^{-t}t^2\,dt$

Optimal. Leaf size=26

$$-2e^{-t} - 2e^{-t}t - e^{-t}t^2$$

```
[Out] -2/exp(t)-2*t/exp(t)-t^2/exp(t)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2207, 2225}

$$-e^{-t}t^2 - 2e^{-t}t - 2e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t^2/E^t,t]
```

```
[Out] -2/E^t - (2*t)/E^t - t^2/E^t
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-t}t^2\,dt &= -e^{-t}t^2 + 2\int e^{-t}t\,dt \\
&= -2e^{-t}t - e^{-t}t^2 + 2\int e^{-t}\,dt \\
&= -2e^{-t} - 2e^{-t}t - e^{-t}t^2
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.62

$$e^{-t}\left(-2 - 2t - t^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^2/E^t,t]
```

```
[Out] (-2 - 2*t - t^2)/E^t
```

Maple [A]
time = 0.01, size = 24, normalized size = 0.92

method	result	size
gosper	$-\left(t^{2}+2t+2\right){\mathrm e}^{-t}$	15
norman	$\left(-t^{2}-2t-2\right){\mathrm e}^{-t}$	16
risch	$\left(-t^{2}-2t-2\right){\mathrm e}^{-t}$	16
meijerg	$2-\frac{\left(3t^{2}+6t+6\right){\mathrm e}^{-t}}{3}$	19
default	$-2\,{\mathrm e}^{-t}-2t\,{\mathrm e}^{-t}-t^{2}{\mathrm e}^{-t}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2/exp(t),t,method=_RETURNVERBOSE)
```

```
[Out] -2/exp(t)-2*t/exp(t)-t^2/exp(t)
```

Maxima [A]
time = 1.96, size = 14, normalized size = 0.54

$$-\left(t^{2}+2\,t+2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="maxima")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Fricas [A]
time = 0.77, size = 14, normalized size = 0.54

$$-\left(t^{2}+2\,t+2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="fricas")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.46

$$\left(-t^{2}-2t-2\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**2/exp(t),t)
```

```
[Out] (-t**2 - 2*t - 2)*exp(-t)
```

Giac [A]
time = 0.41, size = 14, normalized size = 0.54

$$-\left(t^2+2\,t+2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="giac")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.54

$$-\mathrm{e}^{-t}\,\left(t^2+2\,t+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2*exp(-t),t)
```

```
[Out] -exp(-t)*(2*t + t^2 + 2)
```

3.168 $\int e^{-t}t^3\,dt$

Optimal. Leaf size=36

$$-6e^{-t}-6e^{-t}t-3e^{-t}t^2-e^{-t}t^3$$

```
[Out] -6/exp(t)-6*t/exp(t)-3*t^2/exp(t)-t^3/exp(t)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2207, 2225}

$$-e^{-t}t^3-3e^{-t}t^2-6e^{-t}t-6e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/E^t,t]
```

```
[Out] -6/E^t - (6*t)/E^t - (3*t^2)/E^t - t^3/E^t
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-t}t^3\,dt &= -e^{-t}t^3+3\int e^{-t}t^2\,dt\\
&= -3e^{-t}t^2-e^{-t}t^3+6\int e^{-t}t\,dt\\
&= -6e^{-t}t-3e^{-t}t^2-e^{-t}t^3+6\int e^{-t}\,dt\\
&= -6e^{-t}-6e^{-t}t-3e^{-t}t^2-e^{-t}t^3
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.58

$$e^{-t}\left(-6-6t-3t^2-t^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/E^t,t]
```

```
[Out] (-6 - 6*t - 3*t^2 - t^3)/E^t
```

Maple [A]
time = 0.01, size = 33, normalized size = 0.92

method	result	size
gosper	$-\left(t^{3}+3 t^{2}+6 t+6\right) \mathrm{e}^{-t}$	20
norman	$\left(-t^{3}-3 t^{2}-6 t-6\right) \mathrm{e}^{-t}$	21
risch	$\left(-t^{3}-3 t^{2}-6 t-6\right) \mathrm{e}^{-t}$	21
meijerg	$6-\frac{\left(4 t^{3}+12 t^{2}+24 t+24\right) \mathrm{e}^{-t}}{4}$	24
default	$-6 \,\mathrm{e}^{-t}-6 t \,\mathrm{e}^{-t}-3 t^{2} \mathrm{e}^{-t}-t^{3} \mathrm{e}^{-t}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/exp(t),t,method=_RETURNVERBOSE)
```

```
[Out] -6/exp(t)-6*t/exp(t)-3*t^2/exp(t)-t^3/exp(t)
```

Maxima [A]
time = 1.54, size = 19, normalized size = 0.53

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{\left(-t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="maxima")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Fricas [A]
time = 0.72, size = 19, normalized size = 0.53

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{\left(-t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="fricas")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Sympy [A]
time = 0.02, size = 17, normalized size = 0.47

$$\left(-t^{3}-3t^{2}-6t-6\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/exp(t),t)
```

```
[Out] (-t**3 - 3*t**2 - 6*t - 6)*exp(-t)
```

Giac [A]
time = 0.45, size = 19, normalized size = 0.53

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="giac")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Mupad [B]
time = 0.02, size = 19, normalized size = 0.53

$$-\mathrm{e}^{-t}\,\left(t^{3}+3\,t^{2}+6\,t+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3*exp(-t),t)
```

```
[Out] -exp(-t)*(6*t + 3*t^2 + t^3 + 6)
```

3.169 $\int \frac{\mathbf{b1}\cos(x)+\mathbf{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=48

$$\frac{(a\text{a1}+b\text{b1})x}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

```
[Out] (a*a1+b*b1)*x/(a^2+b^2)-(-a*b1+a1*b)*ln(b*cos(x)+a*sin(x))/(a^2+b^2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3212}

$$\frac{x(a\text{a1}+b\text{b1})}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a
^2 + b^2)
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\text{b1}\cos(x)+\text{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{(a\text{a1}+b\text{b1})x}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A]
time = 0.09, size = 39, normalized size = 0.81

$$\frac{(a\text{a1}+b\text{b1})x+(-\text{a1}b+a\text{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
[Out] ((a*a1 + b*b1)*x + (-(a1*b) + a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

Maple [A]
time = 0.10, size = 66, normalized size = 1.38

method	result	size
default	$\frac{\frac{(-ab1+a1b)\ln\left(1+\tan^2(x)\right)}{2}+(aa1+bb1)\arctan(\tan(x))}{a^2+b^2}+\frac{(ab1-a1b)\ln(a\tan(x)+b)}{a^2+b^2}$	66
norman	$\frac{\frac{(aa1+bb1)x}{a^2+b^2}+\frac{(aa1+bb1)x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{(ab1-a1b)\ln\left(-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b\right)}{a^2+b^2}-\frac{(ab1-a1b)\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}$	121
risch	$\frac{ixb1}{ib+a}+\frac{xa1}{ib+a}-\frac{2ixab1}{a^2+b^2}+\frac{2ixa1b}{a^2+b^2}+\frac{\ln\left(e^{2ix}+\frac{ib-a}{ib+a}\right)ab1}{a^2+b^2}-\frac{\ln\left(e^{2ix}+\frac{ib-a}{ib+a}\right)a1b}{a^2+b^2}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/(a^2+b^2)*(1/2*(-a*b1+a1*b)*ln(1+tan(x)^2)+(a*a1+b*b1)*arctan(tan(x)))+(a
*b1-a1*b)/(a^2+b^2)*ln(a*tan(x)+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(48) = 96.
time = 1.39, size = 181, normalized size = 3.77

$$a_1\left(\frac{2\,a\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}-\frac{b\log\left(-b-\frac{2\,a\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2}+\frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)+b_1\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}+\frac{a\log\left(-b-\frac{2\,a\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2}-\frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
[Out] a1*(2*a*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - b*log(-b - 2*a*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + b*log(sin(x)^2/(cos(x)
 + 1)^2 + 1)/(a^2 + b^2)) + b1*(2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2)
 + a*log(-b - 2*a*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b
^2) - a*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^2 + b^2))
```

Fricas [A]
time = 0.69, size = 60, normalized size = 1.25

$$\frac{2\,(aa_1+bb_1)x-(a_1b-ab_1)\log\left(2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2+a^2\right)}{2\,(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*a1 + b*b1)*x - (a1*b - a*b1)*log(2*a*b*cos(x)*sin(x) - (a^2 - b^2
)*cos(x)^2 + a^2))/(a^2 + b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 360, normalized size = 7.50

$$\begin{cases} \tilde{\infty}(-a_1 \log(\cos(x)) + b_1 x) & \text{for } a = 0 \wedge b = 0 \\ \frac{a_1 x \sin(x)}{-2ib\sin(x)+2b\cos(x)} + \frac{ia_1 x \cos(x)}{-2ib\sin(x)+2b\cos(x)} - \frac{a_1 \cos(x)}{-2ib\sin(x)+2b\cos(x)} - \frac{ib_1 x \sin(x)}{-2ib\sin(x)+2b\cos(x)} + \frac{b_1 x \cos(x)}{-2ib\sin(x)+2b\cos(x)} - \frac{ib_1 \cos(x)}{-2ib\sin(x)+2b\cos(x)} & \text{for } a = -ib \\ \frac{a_1 x \sin(x)}{2ib\sin(x)+2b\cos(x)} - \frac{ia_1 x \cos(x)}{2ib\sin(x)+2b\cos(x)} - \frac{a_1 \cos(x)}{2ib\sin(x)+2b\cos(x)} + \frac{ib_1 x \sin(x)}{2ib\sin(x)+2b\cos(x)} + \frac{b_1 x \cos(x)}{2ib\sin(x)+2b\cos(x)} + \frac{ib_1 \cos(x)}{2ib\sin(x)+2b\cos(x)} & \text{for } a = ib \\ \frac{a_1 x + b_1 \log(\sin(x))}{a} & \text{for } b = 0 \\ \frac{aa_1 x}{a^2+b^2} + \frac{ab_1 \log\left(\frac{a\sin(x)}{b} + \cos(x)\right)}{a^2+b^2} - \frac{a_1 b \log\left(\frac{a\sin(x)}{b} + \cos(x)\right)}{a^2+b^2} + \frac{bb_1 x}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Piecewise((zoo*(-a1*log(cos(x)) + b1*x), Eq(a, 0) & Eq(b, 0)), (a1*x*sin(x)
/(-2*I*b*sin(x) + 2*b*cos(x)) + I*a1*x*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x))
- a1*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)) - I*b1*x*sin(x)/(-2*I*b*sin(x) + 2
*b*cos(x)) + b1*x*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)) - I*b1*cos(x)/(-2*I*b
*sin(x) + 2*b*cos(x)), Eq(a, -I*b)), (a1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x
)) - I*a1*x*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) - a1*cos(x)/(2*I*b*sin(x) +
2*b*cos(x)) + I*b1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)) + b1*x*cos(x)/(2*I*
b*sin(x) + 2*b*cos(x)) + I*b1*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)), Eq(a, I*b
)), ((a1*x + b1*log(sin(x)))/a, Eq(b, 0)), (a*a1*x/(a**2 + b**2) + a*b1*log
(a*sin(x)/b + cos(x))/(a**2 + b**2) - a1*b*log(a*sin(x)/b + cos(x))/(a**2 +
 b**2) + b*b1*x/(a**2 + b**2), True))
```

Giac [A]
time = 0.48, size = 77, normalized size = 1.60

$$\frac{(aa_1 + bb_1)x}{a^2 + b^2} + \frac{(a_1 b - ab_1)\log\left(\tan(x)^2 + 1\right)}{2(a^2 + b^2)} - \frac{(aa_1 b - a^2 b_1)\log(|a\tan(x) + b|)}{a^3 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] (a*a1 + b*b1)*x/(a^2 + b^2) + 1/2*(a1*b - a*b1)*log(tan(x)^2 + 1)/(a^2 + b^
2) - (a*a1*b - a^2*b1)*log(abs(a*tan(x) + b))/(a^3 + a*b^2)
```

Mupad [B]
time = 10.61, size = 2034, normalized size = 42.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1*cos(x) + a1*sin(x))/(b*cos(x) + a*sin(x)),x)
```

```
[Out] (2*atan((tan(x/2)*(((((a*a1 + b*b1)^3*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^
3 + ((((a*a1 + b*b1)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2
*b^3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 + b^
2) - ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^
2)^2))*(2*a*b1 - 2*a1*b))/(2*(a^2 + b^2)) - ((a*a1 + b*b1)*(32*b^3*b1^2 - (
(2*a*b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^
3))/(2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(2*(a^2 + b^
2)) + 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(a^2 + b^2))*(a^4*a
1^2 + 4*a1^2*b^4 - 4*a^4*b1^2 - b^4*b1^2 - 13*a^2*a1^2*b^2 + 13*a^2*b^2*b1^
2 - 18*a*a1*b^3*b1 + 18*a^3*a1*b*b1))/((a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2
 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1*b*b1)^2) - ((((((a*a1 + b*b1)*(32*a^2*a1*
b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2)) - 64*a1*b
^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(a^2 + b^2) - ((a*a1 + b*b1)*(2*a*b1 - 2*
a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2 + b^2)^2))*(a*a1 + b*b1))/(a^2 + b^2
) - 32*a1*b^2*b1^2 - 64*a1^3*b^2 + ((2*a*b1 - 2*a1*b)*(32*b^3*b1^2 - ((2*a*
b1 - 2*a1*b)*(32*a^2*a1*b^2 - ((2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(
2*(a^2 + b^2)) - 64*a1*b^4 + 128*a*b^3*b1 + 32*a^3*b*b1))/(2*(a^2 + b^2)) +
 64*a^2*a1^2*b - 96*a^2*b*b1^2 + 192*a*a1*b^2*b1))/(2*(a^2 + b^2)) + 32*a*b
*b1^3 - ((a*a1 + b*b1)^2*(2*a*b1 - 2*a1*b)*(96*a^4*b + 96*a^2*b^3))/(2*(a^2
 + b^2)^3) + 64*a*a1^2*b*b1)*(12*a*a1^2*b^3 - 6*a^3*a1^2*b - 6*a*b^3*b1^2 +
 12*a^3*b*b1^2 + 4*a^4*a1*b1 + 4*a1*b^4*b1 - 28*a^2*a1*b^2*b1))/((a^2 + b^2
)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1*b*b1)^2))*(a^4
+ b^4 + 2*a^2*b^2))/(32*b^2*b1 + 32*a*a1*b) + ((a^4 + b^4 + 2*a^2*b^2)*(32*
a1^2*b^2*b1 + ((((a*a1 + b*b1)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))
/(2*(a^2 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/
(a^2 + b^2) + ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*
(a^2 + b^2)^2))*(a*a1 + b*b1))/(a^2 + b^2) - ((2*a*b1 - 2*a1*b)*(((2*a*b1 -
 2*a1*b)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)) - 32*
b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/(2*(a^2 + b^2)) - 32*a
*a1^2*b^2 - 32*a*b^2*b1^2 + 64*a1*b^3*b1 + 64*a^2*a1*b*b1))/(2*(a^2 + b^2))
 + ((a*a1 + b*b1)^2*(2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^
2)^3) - 32*a*a1*b*b1^2)*(12*a*a1^2*b^3 - 6*a^3*a1^2*b - 6*a*b^3*b1^2 + 12*a
^3*b*b1^2 + 4*a^4*a1*b1 + 4*a1*b^4*b1 - 28*a^2*a1*b^2*b1))/((32*b^2*b1 + 32
*a*a1*b)*(a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a
*a1*b*b1)^2) - ((a^4 + b^4 + 2*a^2*b^2)*(((((a*a1 + b*b1)*(((2*a*b1 - 2*a1*
b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 6
4*a*a1*b^3 + 32*a^3*a1*b))/(a^2 + b^2) + ((a*a1 + b*b1)*(2*a*b1 - 2*a1*b)*(
96*a*b^4 + 96*a^3*b^2))/(2*(a^2 + b^2)^2))*(2*a*b1 - 2*a1*b))/(2*(a^2 + b^2
)) - ((a*a1 + b*b1)^3*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + ((a*a1 + b*b
1)*(((2*a*b1 - 2*a1*b)*(((2*a*b1 - 2*a1*b)*(96*a*b^4 + 96*a^3*b^2))/(2*(a^2
 + b^2)) - 32*b^4*b1 + 64*a^2*b^2*b1 - 64*a*a1*b^3 + 32*a^3*a1*b))/(2*(a^2
+ b^2)) - 32*a*a1^2*b^2 - 32*a*b^2*b1^2 + 64*a1*b^3*b1 + 64*a^2*a1*b*b1))/(
a^2 + b^2))*(a^4*a1^2 + 4*a1^2*b^4 - 4*a^4*b1^2 - b^4*b1^2 - 13*a^2*a1^2*b^
2 + 13*a^2*b^2*b1^2 - 18*a*a1*b^3*b1 + 18*a^3*a1*b*b1))/((32*b^2*b1 + 32*a*
a1*b)*(a^2 + b^2)^2*(a^2*a1^2 + 4*a^2*b1^2 + 4*a1^2*b^2 + b^2*b1^2 - 6*a*a1
```

```
*b*b1)^2))*(a*a1 + b*b1))/(a^2 + b^2) - (log(1/(cos(x) + 1))*(2*a*b1 - 2*a1
*b))/(2*(a^2 + b^2)) + (log(b + 2*a*tan(x/2) - b*tan(x/2)^2)*(a*b1 - a1*b))
/(a^2 + b^2)
```

3.170 $\int \frac{1}{\log(t)}\,dt$

Optimal. Leaf size=2

$$\mathrm{li}(t)$$

```
[Out] Li(t)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2335}

$$\mathrm{LogIntegral}(t)$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-1),t]
```

```
[Out] LogIntegral[t]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(t)}\,dt = \mathrm{li}(t)$$

Mathematica [A]
time = 0.06, size = 2, normalized size = 1.00

$$\mathrm{li}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-1),t]
```

```
[Out] LogIntegral[t]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(2) = 4.
time = 0.00, size = 9, normalized size = 4.50

method	result	size

default	$-\text{expIntegral}(1, -\ln(t))$	9
risch	$-\text{expIntegral}(1, -\ln(t))$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(t),t,method=_RETURNVERBOSE)
```

```
[Out] -Ei(1,-ln(t))
```

Maxima [A]
time = 1.24, size = 3, normalized size = 1.50

$$\text{Ei}(\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="maxima")
```

```
[Out] Ei(log(t))
```

Fricas [A]
time = 0.87, size = 2, normalized size = 1.00

$$\text{log\_integral}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="fricas")
```

```
[Out] log_integral(t)
```

Sympy [A]
time = 0.23, size = 2, normalized size = 1.00

$$\text{li}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(t),t)
```

```
[Out] li(t)
```

Giac [A]
time = 0.49, size = 3, normalized size = 1.50

$$\text{Ei}(\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="giac")
```

```
[Out] Ei(log(t))
```

Mupad [**B**]
time = 0.01, size = 2, normalized size = 1.00

$$\mathrm{logint}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(t),t)
```

```
[Out] logint(t)
```

3.171 $\int \frac{1}{\log^2(t)} \, dt$

Optimal. Leaf size=10

$$-\frac{t}{\log(t)} + \text{li}(t)$$

```
[Out] Li(t)-t/ln(t)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2334, 2335}

$$\text{LogIntegral}(t) - \frac{t}{\log(t)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-2),t]
```

```
[Out] -(t/Log[t]) + LogIntegral[t]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\log^2(t)} \, dt &= -\frac{t}{\log(t)} + \int \frac{1}{\log(t)} \, dt \\
&= -\frac{t}{\log(t)} + \text{li}(t)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{t}{\log(t)} + \text{li}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-2),t]
```

```
[Out] -(t/Log[t]) + LogIntegral[t]
```

Maple [A]
time = 0.02, size = 17, normalized size = 1.70

method	result	size
default	$-\frac{t}{\ln(t)} - \mathrm{expIntegral}\left(1, -\ln\left(t\right)\right)$	17
risch	$-\frac{t}{\ln(t)} - \mathrm{expIntegral}\left(1, -\ln\left(t\right)\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(t)^2,t,method=_RETURNVERBOSE)
```

```
[Out] -t/ln(t)-Ei(1,-ln(t))
```

Maxima [A]
time = 2.01, size = 6, normalized size = 0.60

$$\Gamma\left(-1, -\log\left(t\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="maxima")
```

```
[Out] gamma(-1, -log(t))
```

Fricas [A]
time = 0.65, size = 14, normalized size = 1.40

$$\frac{\log\left(t\right)\mathrm{log\_integral}\left(t\right) - t}{\log\left(t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="fricas")
```

```
[Out] (log(t)*log_integral(t) - t)/log(t)
```

Sympy [A]
time = 0.23, size = 7, normalized size = 0.70

$$-\frac{t}{\log\left(t\right)} + \mathrm{li}\left(t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(t)**2,t)
```

```
[Out] -t/log(t) + li(t)
```

Giac [**A**]
time = 0.51, size = 11, normalized size = 1.10

$$-\frac{t}{\log(t)} + \mathrm{Ei}(\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="giac")
```

```
[Out] -t/log(t) + Ei(log(t))
```

Mupad [**B**]
time = 0.03, size = 10, normalized size = 1.00

$$\mathrm{logint}(t) - \frac{t}{\ln(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(t)^2,t)
```

```
[Out] logint(t) - t/log(t)
```

3.172 $\int \log^{-1-n}(t)\,dt$

Optimal. Leaf size=22

$$-\Gamma(-n,-\log(t))(-\log(t))^n\log^{-n}(t)$$

```
[Out] -GAMMA(-n,-ln(t))*(-ln(t))^n/(ln(t)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2336, 2212}

$$(-\log(t))^n\log^{-n}(t)(-\text{Gamma}(-n,-\log(t)))$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-1 - n),t]
```

```
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int \log^{-1-n}(t)\,dt &= \text{Subst}\left(\int e^t t^{-1-n}\,dt, t, \log(t)\right)\\ &= -\Gamma(-n,-\log(t))(-\log(t))^n\log^{-n}(t)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$-\Gamma(-n,-\log(t))(-\log(t))^n\log^{-n}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-1 - n),t]
```

```
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \ln(t)^{-1-n}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)^(-1-n),t)
```

```
[Out] int(ln(t)^(-1-n),t)
```

Maxima [A]
time = 0.12, size = 22, normalized size = 1.00

$$-(-\log(t))^{n}\log(t)^{-n}\,\Gamma(-n, -\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="maxima")
```

```
[Out] -(-log(t))^n*log(t)^(-n)*gamma(-n, -log(t))
```

Fricas [A]
time = 0.15, size = 15, normalized size = 0.68

$$\cos(\pi + \pi n)\,\Gamma(-n, -\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="fricas")
```

```
[Out] cos(pi + pi*n)*gamma(-n, -log(t))
```

Sympy [A]
time = 1.42, size = 24, normalized size = 1.09

$$(-\log(t))^{n+1}\log(t)^{-n-1}\Gamma(-n, -\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)**(-1-n),t)
```

```
[Out] (-log(t))**(n + 1)*log(t)**(-n - 1)*uppergamma(-n, -log(t))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="giac")
[Out] integrate(log(t)^(-n - 1), t)
```

Mupad [**B**]
time = 0.06, size = 22, normalized size = 1.00

$$-\frac{(-\ln(t))^n\,\Gamma(-n,-\ln(t))}{\ln(t)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/log(t)^(n + 1),t)
[Out] -((-log(t))^n*igamma(-n, -log(t)))/log(t)^n
```

3.173 $\int \frac{e^{2t}}{-1+t} \, dt$

Optimal. Leaf size=12

$$e^2\text{Ei}(-2(1-t))$$

```
[Out] exp(2)*Ei(-2+2*t)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2209}

$$e^2\text{ExpIntegralEi}(-2(1-t))$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*t)/(-1 + t),t]
```

```
[Out] E^2*ExpIntegralEi[-2*(1 - t)]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rubi steps

$$\int \frac{e^{2t}}{-1+t} \, dt = e^2\text{Ei}(-2(1-t))$$

Mathematica [A]
time = 0.04, size = 10, normalized size = 0.83

$$e^2\text{Ei}(2(-1+t))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*t)/(-1 + t),t]
```

```
[Out] E^2*ExpIntegralEi[2*(-1 + t)]
```

Maple [A]
time = 0.06, size = 12, normalized size = 1.00

method	result	size
derivativedivides	$-e^2 \operatorname{expIntegral}(1, -2t + 2)$	12
default	$-e^2 \operatorname{expIntegral}(1, -2t + 2)$	12
risch	$-e^2 \operatorname{expIntegral}(1, -2t + 2)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*t)/(-1+t),t,method=_RETURNVERBOSE)
```

```
[Out] -exp(2)*Ei(1,-2*t+2)
```

Maxima [A]
time = 1.10, size = 11, normalized size = 0.92

$$-e^2 E_1(-2t + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="maxima")
```

```
[Out] -e^2*exp_integral_e(1, -2*t + 2)
```

Fricas [A]
time = 0.86, size = 9, normalized size = 0.75

$$\operatorname{Ei}(2t - 2) e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="fricas")
```

```
[Out] Ei(2*t - 2)*e^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2t}}{t - 1}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t)
```

```
[Out] Integral(exp(2*t)/(t - 1), t)
```

Giac [A]
time = 0.48, size = 9, normalized size = 0.75

$$\operatorname{Ei}(2t - 2) e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="giac")
```

```
[Out] Ei(2*t - 2)*e^2
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.75

$$e^2 \operatorname{ei}(2t - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*t)/(t - 1),t)
```

```
[Out] exp(2)*ei(2*t - 2)
```

3.174 $\int \frac{e^{2x}}{2-3x+x^2} dx$

Optimal. Leaf size=22

$$e^4\text{Ei}(-4+2x)-e^2\text{Ei}(-2+2x)$$

```
[Out] exp(4)*Ei(-4+2*x)-exp(2)*Ei(-2+2*x)
```

Rubi [A]
time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2300, 2209}

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(2 - 3*x + x^2),x]
```

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2300

```
Int[(F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + b*x + c*x^
2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2x}}{2-3x+x^2}\,dx &= \int \left(-\frac{2e^{2x}}{4-2x}-\frac{2e^{2x}}{-2+2x}\right)dx\\ &= -\left(2\int \frac{e^{2x}}{4-2x}\,dx\right)-2\int \frac{e^{2x}}{-2+2x}\,dx\\ &= e^4\text{Ei}(-4+2x)-e^2\text{Ei}(-2+2x)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 22, normalized size = 1.00

$$e^4\text{Ei}(-4+2x)-e^2\text{Ei}(-2+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(2 - 3*x + x^2),x]
```

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Maple [A]
time = 0.09, size = 23, normalized size = 1.05

method	result	size
derivativedivides	$-e^4\,\mathrm{expIntegral}\left(1,-2x+4\right)+e^2\,\mathrm{expIntegral}\left(1,-2x+2\right)$	23
default	$-e^4\,\mathrm{expIntegral}\left(1,-2x+4\right)+e^2\,\mathrm{expIntegral}\left(1,-2x+2\right)$	23
risch	$-e^4\,\mathrm{expIntegral}\left(1,-2x+4\right)+e^2\,\mathrm{expIntegral}\left(1,-2x+2\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(x^2-3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(4)*Ei(1,-2*x+4)+exp(2)*Ei(1,-2*x+2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*x)/(x^2 - 3*x + 2), x)
```

Fricas [A]
time = 0.58, size = 20, normalized size = 0.91

$$\mathrm{Ei}(2\,x-4)\,e^4-\mathrm{Ei}(2\,x-2)\,e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="fricas")
```

```
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e^{2x}}{\left(x-2\right)\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x**2-3*x+2),x)
[Out] Integral(exp(2*x)/((x - 2)*(x - 1)), x)
```

Giac [A]
time = 0.48, size = 20, normalized size = 0.91

$$\mathrm{Ei}(2\,x-4)\,e^{4}-\mathrm{Ei}(2\,x-2)\,e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="giac")
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\mathrm{e}^{2\,x}}{x^2-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(x^2 - 3*x + 2),x)
[Out] int(exp(2*x)/(x^2 - 3*x + 2), x)
```

3.175 $\int \frac{1}{\sqrt{1+t^3}}\,dt$

Optimal. Leaf size=103

$$\frac{2\sqrt{2+\sqrt{3}}\,(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\sqrt{1+t^3}}$$

```
[Out] 2/3*(1+t)*EllipticF((1+t-3^(1/2))/(1+t+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((t^2-t+1)/(1+t+3^(1/2))^2)^(1/2)*3^(3/4)/(t^3+1)^(1/2)/((1+t
)/(1+t+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {224}

$$\frac{2\sqrt{2+\sqrt{3}}\,(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\text{ArcSin}\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + t^3],t]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+t^3}}\,dt = \frac{2\sqrt{2+\sqrt{3}}\,(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\,\sqrt{1+t^3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 17, normalized size = 0.17

$$t\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-t^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + t^3],t]
```

```
[Out] t*Hypergeometric2F1[1/3, 1/2, 4/3, -t^3]
```

Maple [A]
time = 0.10, size = 116, normalized size = 1.13

method	result	size
meijerg	$t\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-t^3\right)$	14
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{t^3+1}}$	116
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{t-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+t}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{t^3+1}}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t^3+1)^(1/2),t,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+t)/(3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(t^3+1)^(1/2)*EllipticF(((1+t)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(t^3 + 1), t)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 6, normalized size = 0.06

$$2\,\mathrm{weierstrassPInverse}(0,-4,t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="fricas")
```

```
[Out] 2*weierstrassPInverse(0, -4, t)
```

Sympy [A]
time = 0.30, size = 27, normalized size = 0.26

$$\frac{t\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|t^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t**3+1)**(1/2),t)
```

```
[Out] t*gamma(1/3)*hyper((1/3, 1/2), (4/3,), t**3*exp_polar(I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="giac")
```

```
[Out] integrate(1/sqrt(t^3 + 1), t)
```

Mupad [B]
time = 0.29, size = 155, normalized size = 1.50

$$\frac{\left(3+\sqrt{3}\ 1\mathrm{i}\right)\sqrt{\frac{t-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\ \sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\ \sqrt{\frac{\frac{1}{2}-t+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\ \mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{t+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}\right)}{\sqrt{t^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)-1\right)t-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t^3 + 1)^(1/2),t)
```

```
[Out] ((3^(1/2)*1i + 3)*((t + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - t + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((t + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(t^3 - t*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
       return False

def expnType(expn):
    debug=False
    if debug:
       print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
          return 1
       else:
          return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
       if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
          return expnType(expn.args[0])   #ExpnType(op(1,expn))
       elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
          if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
             return 1
          else:
             return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
       else:
          return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
       m1 = expnType(expn.args[0])
       m2 = expnType(list(expn.args[1:]))
       return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
       return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
       m1 = max(map(expnType, list(expn.args)))
       return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
       m1 = max(map(expnType, list(expn.args)))
       return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
       m1 = max(map(expnType, list(expn.args)))
       return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
       m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
       return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
     m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
     m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
     return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
     return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
     return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
     print ("Enter grade_antiderivative for sagemath")
     print("Enter grade_antiderivative, result=",result)
     print("Enter grade_antiderivative, optimal=",optimal)
     print("type(anti)=",type(result))
     print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

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if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```