# Computer algebra independent integration tests

Summer 2022 edition

## 0-Independent-test-suites/11-Welz-Problems

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# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 116 ]. This is test number [ 11 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 97.41 ( 113 ) | 2.59 ( 3 ) |
| Mathematica | 87.07 ( 101 ) | 12.93 ( 15 ) |
| Fricas | 77.59 ( 90 ) | 22.41 ( 26 ) |
| Maple | 68.10 ( 79 ) | 31.90 ( 37 ) |
| Mupad | 31.90 ( 37 ) | 68.10 ( 79 ) |
| Giac | 30.17 ( 35 ) | 69.83 ( 81 ) |
| Sympy | 25.00 ( 29 ) | 75.00 ( 87 ) |
| Maxima | 17.24 ( 20 ) | 82.76 ( 96 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

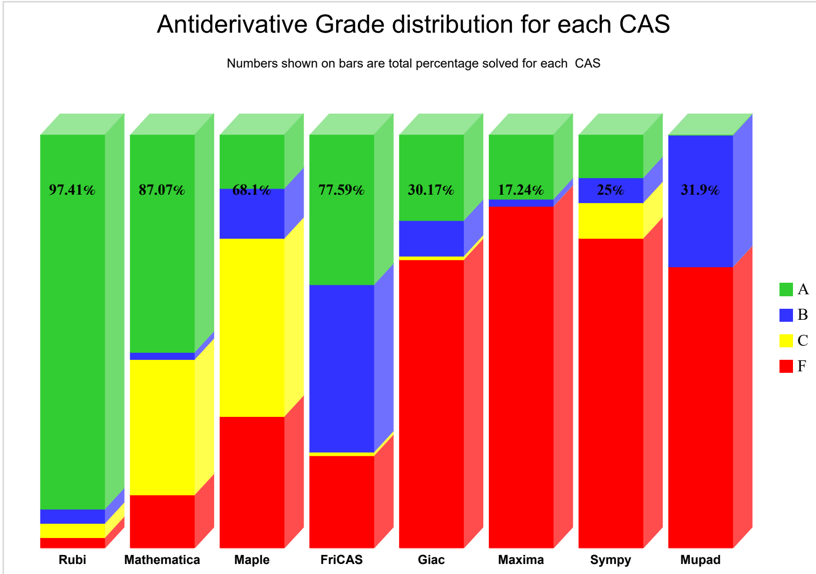
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

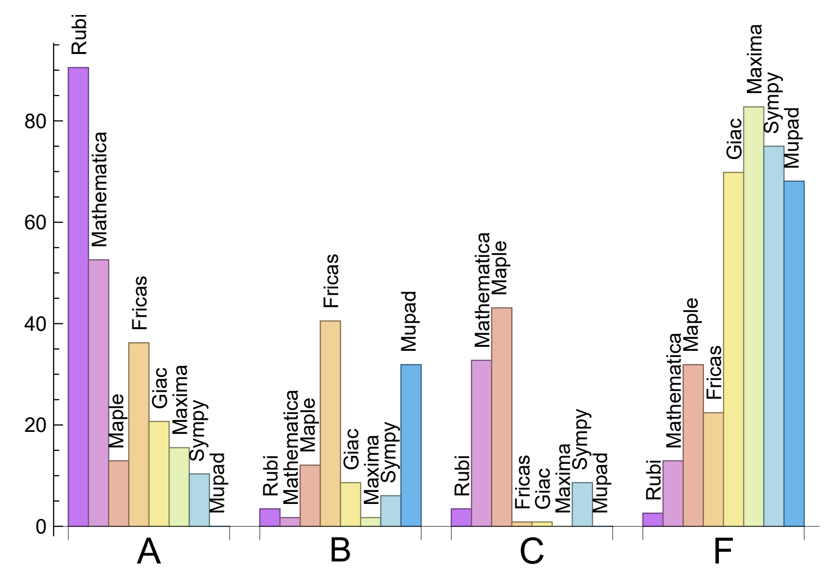
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 90.52 | 3.45 | 3.45 | 2.59 |
| Mathematica | 52.59 | 1.72 | 32.76 | 12.93 |
| Fricas | 36.21 | 40.52 | 0.86 | 22.41 |
| Giac | 20.69 | 8.62 | 0.86 | 69.83 |
| Maxima | 15.52 | 1.72 | 0.00 | 82.76 |
| Maple | 12.93 | 12.07 | 43.10 | 31.90 |
| Sympy | 10.34 | 6.03 | 8.62 | 75.00 |
| Mupad | N/A | 31.90 | 0.00 | 68.10 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 3 | 100.00 % | 0.00 % | 0.00 % |
| Mathematica | 15 | 100.00 % | 0.00 % | 0.00 % |
| Maple | 37 | 100.00 % | 0.00 % | 0.00 % |
| Fricas | 26 | 42.31 % | 26.92 % | 30.77 % |
| Giac | 81 | 97.53 % | 1.23 % | 1.23 % |
| Maxima | 96 | 98.96 % | 0.00 % | 1.04 % |
| Sympy | 87 | 90.80 % | 6.90 % | 2.30 % |
| Mupad | 79 | 98.73 % | 1.27 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

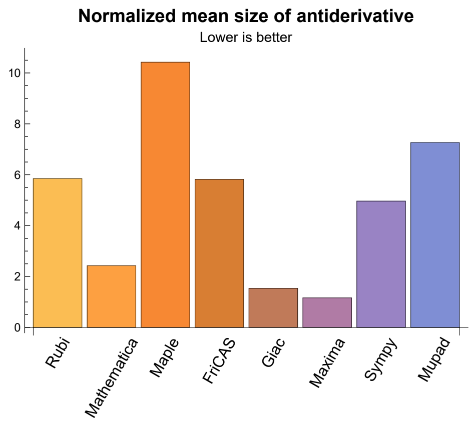
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

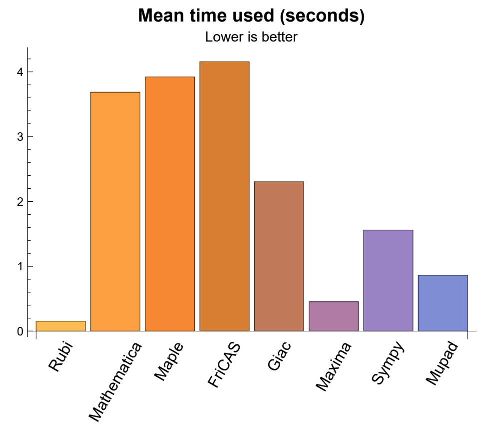
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.15 | 162.17 | 5.85 | 88.00 | 1.00 |
| Mathematica | 3.69 | 131.26 | 2.42 | 86.00 | 1.06 |
| Maple | 3.92 | 1793.58 | 10.42 | 317.00 | 2.34 |
| Maxima | 0.45 | 81.80 | 1.16 | 62.00 | 1.05 |
| Fricas | 4.16 | 817.48 | 5.82 | 194.50 | 1.92 |
| Sympy | 1.56 | 202.10 | 4.96 | 37.00 | 0.80 |
| Giac | 2.30 | 251.17 | 1.53 | 67.00 | 1.10 |
| Mupad | 0.86 | 191.78 | 7.26 | 76.00 | 1.11 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** $\{82\}$

**Mathematica** $\{53, 54, 69, 70, 71, 72, 77, 78, 79, 80, 115\}$

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

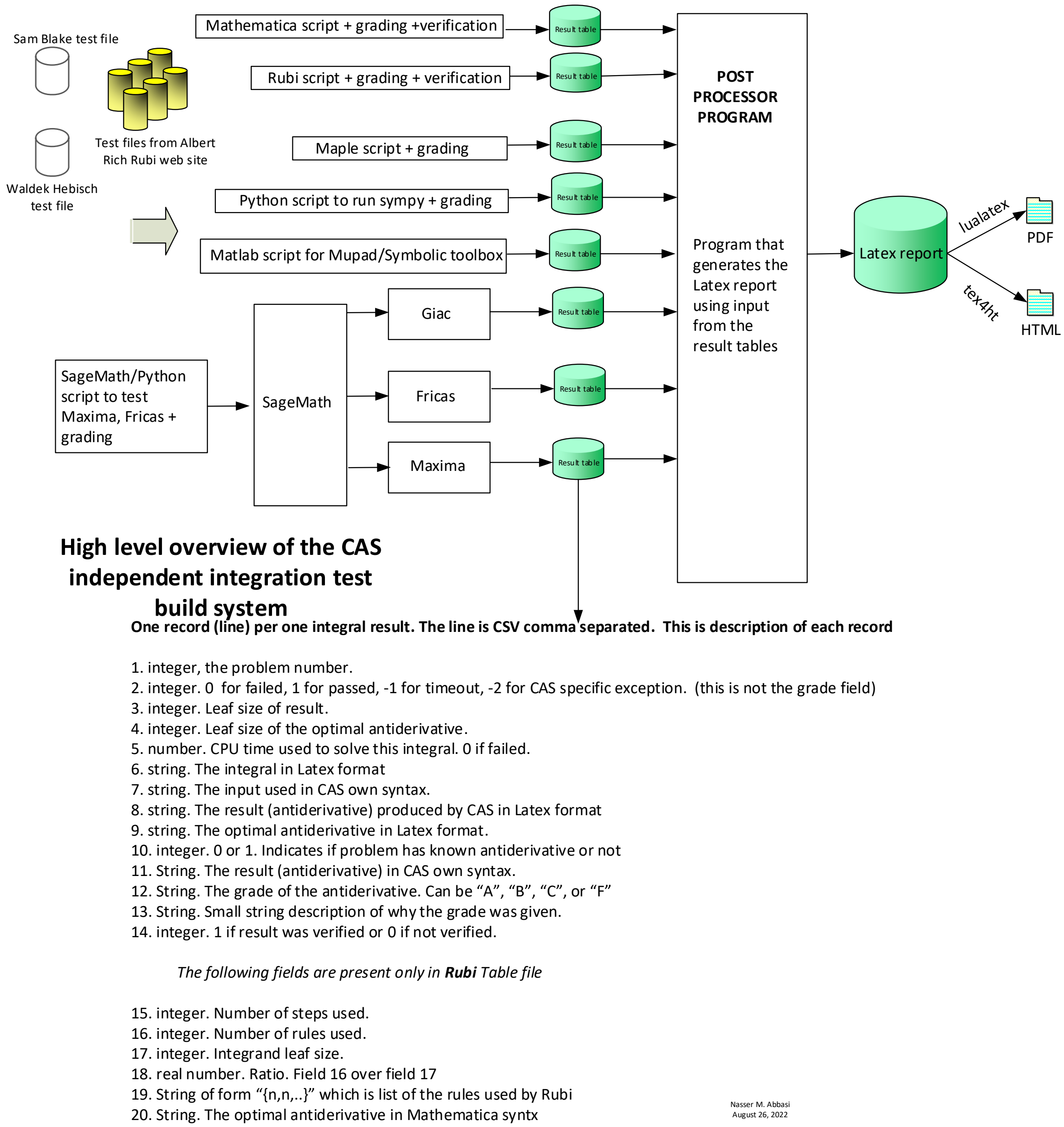
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

# 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

### 2.1.1 Rubi

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

B grade: { 10, 100, 101, 102 }

C grade: { 2, 52, 82, 83 }

F grade: { 43, 44, 45 }

### 2.1.2 Mathematica

A grade: { 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 43, 52, 55, 56, 57, 62, 66, 67, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 113, 116 }

B grade: { 12, 13 }

C grade: { 2, 24, 40, 47, 48, 49, 50, 51, 53, 54, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 94, 96, 106, 114, 115 }

F grade: { 38, 44, 45, 46, 58, 59, 60, 61, 93, 95, 108, 109, 110, 111, 112 }

### 2.1.3 Maple

A grade: { 1, 7, 8, 16, 21, 22, 23, 32, 47, 48, 49, 62, 103, 104, 105 }

B grade: { 3, 4, 5, 6, 9, 10, 11, 17, 24, 31, 50, 51, 65, 68 }

C grade: { 2, 15, 28, 33, 34, 35, 36, 37, 38, 39, 40, 52, 55, 56, 57, 58, 59, 63, 64, 66, 67, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 106, 107, 110, 116 }

F grade: { 12, 13, 14, 18, 19, 20, 25, 26, 27, 29, 30, 41, 42, 43, 44, 45, 46, 53, 54, 60, 61, 69, 70, 71, 72, 80, 94, 95, 96, 98, 108, 109, 111, 112, 113, 114, 115 }

### 2.1.4 Maxima

A grade: { 1, 6, 16, 21, 22, 23, 29, 32, 33, 34, 35, 36, 56, 57, 62, 96, 99, 107 }

B grade: { 2, 106 }

C grade: { }

F grade: { 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 24, 25, 26, 27, 28, 30, 31, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

### 2.1.5 FriCAS

A grade: { 1, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 30, 31, 32, 33, 34, 36, 40, 52, 56, 57, 62, 63, 64, 65, 66, 67, 81, 83, 99, 106, 107, 113, 116 }

B grade: { 4, 7, 9, 10, 12, 13, 14, 24, 35, 37, 39, 41, 42, 43, 45, 47, 48, 49, 50, 51, 55, 59, 68, 73, 74, 75, 76, 77, 78, 79, 80, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 100, 101, 102, 110 }

C grade: { 82 }

F grade: { 2, 29, 38, 44, 46, 53, 54, 58, 60, 61, 69, 70, 71, 72, 84, 92, 95, 103, 104, 105, 108, 109, 111, 112, 114, 115 }

### 2.1.6 Sympy

A grade: { 1, 2, 14, 15, 21, 22, 23, 25, 26, 62, 63, 64 }

B grade: { 8, 16, 17, 19, 20, 30, 31 }

C grade: { 27, 28, 33, 34, 35, 36, 56, 57, 106, 107 }

F grade: { 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 18, 24, 29, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

### 2.1.7 Giac

A grade: { 1, 6, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 36, 41, 49, 57, 62, 63, 64, 96, 99, 107 }

B grade: { 2, 3, 4, 7, 9, 10, 11, 24, 47, 48 }

C grade: { 50 }

F grade: { 5, 12, 13, 14, 15, 17, 18, 27, 28, 29, 31, 32, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

### 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 5, 8, 16, 19, 20, 21, 22, 23, 25, 26, 30, 33, 34, 35, 36, 41, 47, 48, 49, 57, 62, 63, 64, 73, 74, 75, 81, 82, 84, 85, 96, 99, 106, 107 }

C grade: { }

F grade: { 4, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 24, 27, 28, 29, 31, 32, 37, 38, 39, 40, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 13 | 13 | 12 | 13 | 13 |
| N.S. | 1 | 1.00 | 1.00 | 0.93 | 0.87 | 0.87 | 0.80 | 0.87 | 0.87 |
| time (sec) | N/A | 0.005 | 0.007 | 0.088 | 0.271 | 0.394 | 0.008 | 1.110 | 0.027 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | B | F | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 52 | 37 | 42 | 41 | 0 | 42 | 41 | 43 |
| N.S. | 1 | 3.47 | 2.47 | 2.80 | 2.73 | 0.00 | 2.80 | 2.73 | 2.87 |
| time (sec) | N/A | 0.200 | 0.019 | 0.102 | 0.264 | 0.000 | 29.281 | 0.759 | 0.515 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 60 | 370 | 0 | 100 | 0 | 177 | 204 |
| N.S. | 1 | 1.00 | 0.73 | 4.51 | 0.00 | 1.22 | 0.00 | 2.16 | 2.49 |
| time (sec) | N/A | 0.048 | 0.213 | 0.165 | 0.000 | 0.436 | 0.000 | 0.876 | 0.569 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 71 | 172 | 0 | 80 | 0 | 94 | -1 |
| N.S. | 1 | 1.00 | 1.65 | 4.00 | 0.00 | 1.86 | 0.00 | 2.19 | -0.02 |
| time (sec) | N/A | 0.007 | 0.091 | 0.185 | 0.000 | 0.542 | 0.000 | 0.740 | 0.000 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 69 | 115 | 0 | 105 | 0 | 0 | 82 |
| N.S. | 1 | 1.00 | 0.93 | 1.55 | 0.00 | 1.42 | 0.00 | 0.00 | 1.11 |
| time (sec) | N/A | 0.033 | 0.082 | 0.015 | 0.000 | 0.678 | 0.000 | 0.000 | 1.707 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 66 | 150 | 53 | 89 | 0 | 84 | -1 |
| N.S. | 1 | 1.00 | 1.03 | 2.34 | 0.83 | 1.39 | 0.00 | 1.31 | -0.02 |
| time (sec) | N/A | 0.016 | 0.196 | 0.125 | 0.497 | 0.863 | 0.000 | 1.160 | 0.000 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 55 | 45 | 0 | 83 | 0 | 101 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 0.94 | 0.00 | 1.73 | 0.00 | 2.10 | -0.02 |
| time (sec) | N/A | 0.008 | 0.110 | 0.138 | 0.000 | 0.726 | 0.000 | 0.976 | 0.000 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 26 | 21 | 0 | 18 | 53 | 20 | 21 |
| N.S. | 1 | 1.00 | 0.87 | 0.70 | 0.00 | 0.60 | 1.77 | 0.67 | 0.70 |
| time (sec) | N/A | 0.047 | 0.190 | 0.082 | 0.000 | 0.809 | 0.272 | 1.170 | 0.381 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 365 | 195 | 902 | 0 | 424 | 0 | 367 | -1 |
| N.S. | 1 | 1.66 | 0.89 | 4.10 | 0.00 | 1.93 | 0.00 | 1.67 | -0.00 |
| time (sec) | N/A | 0.332 | 6.476 | 0.121 | 0.000 | 1.126 | 0.000 | 2.039 | 0.000 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | B | F | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 541 | 195 | 1637 | 0 | 424 | 0 | 358 | -1 |
| N.S. | 1 | 2.46 | 0.89 | 7.44 | 0.00 | 1.93 | 0.00 | 1.63 | -0.00 |
| time (sec) | N/A | 0.396 | 6.592 | 0.251 | 0.000 | 0.903 | 0.000 | 1.881 | 0.000 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-2) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 126 | 278 | 0 | 161 | 0 | 547 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 2.01 | 0.00 | 1.17 | 0.00 | 3.96 | -0.01 |
| time (sec) | N/A | 0.045 | 3.744 | 0.123 | 0.000 | 0.770 | 0.000 | 0.881 | 0.000 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 272 | 0 | 0 | 394 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.18 | 0.00 | 0.00 | 3.15 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.103 | 2.310 | 0.033 | 0.000 | 2.283 | 0.000 | 0.000 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 205 | 0 | 0 | 369 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.53 | 0.00 | 0.00 | 4.56 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.090 | 0.906 | 0.035 | 0.000 | 5.346 | 0.000 | 0.000 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 0 | 0 | 60 | 15 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 1.94 | 0.48 | 0.00 | -0.03 |
| time (sec) | N/A | 0.030 | 0.150 | 0.020 | 0.000 | 0.957 | 0.909 | 0.000 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 22 | 0 | 29 | 15 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.67 | 0.00 | 0.88 | 0.45 | 0.00 | -0.03 |
| time (sec) | N/A | 0.032 | 0.147 | 0.102 | 0.000 | 0.802 | 0.400 | 0.000 | 0.000 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 16 | 15 | 28 | 56 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.84 | 0.79 | 1.47 | 2.95 | 0.79 | 0.79 |
| time (sec) | N/A | 0.244 | 0.020 | 0.160 | 0.294 | 0.649 | 1.065 | 0.695 | 0.400 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 43 | 120 | 0 | 32 | 2147 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 2.31 | 0.00 | 0.62 | 41.29 | 0.00 | -0.02 |
| time (sec) | N/A | 0.014 | 0.152 | 0.032 | 0.000 | 0.659 | 1.559 | 0.000 | 0.000 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 50 | 0 | 0 | 33 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.59 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.014 | 0.151 | 0.020 | 0.000 | 0.727 | 0.000 | 0.000 | 0.000 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 0 | 0 | 15 | 311 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.88 | 18.29 | 0.88 | 0.88 |
| time (sec) | N/A | 0.031 | 0.021 | 0.023 | 0.000 | 0.650 | 1.375 | 0.689 | 0.297 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 0 | 0 | 18 | 36 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.90 | 1.80 | 0.90 | 0.90 |
| time (sec) | N/A | 0.032 | 0.022 | 0.040 | 0.000 | 0.840 | 0.904 | 0.862 | 0.296 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 39 | 43 | 40 | 52 | 36 | 47 | 58 |
| N.S. | 1 | 1.00 | 0.93 | 1.02 | 0.95 | 1.24 | 0.86 | 1.12 | 1.38 |
| time (sec) | N/A | 0.019 | 0.048 | 0.021 | 0.351 | 0.717 | 0.055 | 1.041 | 0.413 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 21 | 20 | 19 | 22 | 19 | 24 |
| N.S. | 1 | 1.00 | 1.00 | 0.95 | 0.91 | 0.86 | 1.00 | 0.86 | 1.09 |
| time (sec) | N/A | 0.014 | 0.028 | 0.015 | 0.301 | 0.660 | 0.037 | 0.670 | 0.385 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 49 | 50 | 51 | 58 | 49 | 74 | 47 |
| N.S. | 1 | 1.00 | 0.79 | 0.81 | 0.82 | 0.94 | 0.79 | 1.19 | 0.76 |
| time (sec) | N/A | 0.056 | 0.070 | 0.059 | 0.356 | 0.823 | 0.072 | 0.814 | 0.406 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 239 | 455 | 0 | 358 | 0 | 304 | -1 |
| N.S. | 1 | 1.00 | 2.78 | 5.29 | 0.00 | 4.16 | 0.00 | 3.53 | -0.01 |
| time (sec) | N/A | 0.053 | 0.319 | 0.736 | 0.000 | 1.061 | 0.000 | 0.848 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 19 | 0 | 0 | 15 | 15 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.79 | 0.79 | 0.79 | 0.79 |
| time (sec) | N/A | 0.037 | 0.031 | 0.023 | 0.000 | 0.972 | 0.071 | 0.863 | 0.423 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 0 | 0 | 22 | 27 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.85 | 1.04 | 0.85 | 0.85 |
| time (sec) | N/A | 0.057 | 0.045 | 180.000 | 0.000 | 1.035 | 0.467 | 1.186 | 0.506 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | C | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 56 | 0 | 0 | 198 | 46 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 3.14 | 0.73 | 0.00 | -0.02 |
| time (sec) | N/A | 0.130 | 0.070 | 0.023 | 0.000 | 0.992 | 0.999 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | C | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 82 | 25 | 0 | 216 | 51 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.30 | 0.00 | 2.63 | 0.62 | 0.00 | -0.01 |
| time (sec) | N/A | 0.042 | 0.010 | 0.020 | 0.000 | 1.122 | 1.690 | 0.000 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 606 | 606 | 412 | 0 | 518 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.68 | 0.00 | 0.85 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.780 | 0.226 | 0.092 | 0.495 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 0 | 0 | 15 | 311 | 15 | 15 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.88 | 18.29 | 0.88 | 0.88 |
| time (sec) | N/A | 0.031 | 0.026 | 0.025 | 0.000 | 0.558 | 1.202 | 0.936 | 0.314 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | B | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 46 | 120 | 0 | 32 | 2147 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.88 | 2.31 | 0.00 | 0.62 | 41.29 | 0.00 | -0.02 |
| time (sec) | N/A | 0.015 | 0.151 | 0.026 | 0.000 | 0.730 | 1.186 | 0.000 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 33 | 44 | 48 | 48 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.97 | 1.29 | 1.41 | 1.41 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.025 | 0.097 | 0.036 | 0.608 | 0.717 | 0.000 | 0.000 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 77 | 65 | 62 | 64 | 36 | 64 | 86 |
| N.S. | 1 | 1.00 | 1.33 | 1.12 | 1.07 | 1.10 | 0.62 | 1.10 | 1.48 |
| time (sec) | N/A | 0.023 | 0.049 | 1.092 | 0.486 | 0.842 | 0.391 | 0.714 | 0.543 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 77 | 48 | 62 | 64 | 37 | 64 | 76 |
| N.S. | 1 | 1.00 | 1.33 | 0.83 | 1.07 | 1.10 | 0.64 | 1.10 | 1.31 |
| time (sec) | N/A | 0.021 | 0.036 | 1.089 | 0.495 | 0.643 | 0.408 | 0.680 | 0.457 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | B | C | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 86 | 12 | 78 | 82 | 29 | 0 | 10 |
| N.S. | 1 | 1.00 | 1.76 | 0.24 | 1.59 | 1.67 | 0.59 | 0.00 | 0.20 |
| time (sec) | N/A | 0.003 | 0.027 | 1.203 | 0.508 | 0.600 | 0.375 | 0.000 | 0.334 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 79 | 65 | 62 | 64 | 32 | 63 | 80 |
| N.S. | 1 | 1.00 | 1.44 | 1.18 | 1.13 | 1.16 | 0.58 | 1.15 | 1.45 |
| time (sec) | N/A | 0.020 | 0.036 | 1.930 | 0.494 | 0.702 | 0.387 | 2.294 | 0.514 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 148 | 1163 | 0 | 301 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.53 | 11.99 | 0.00 | 3.10 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.026 | 0.780 | 5.241 | 0.000 | 3.053 | 0.000 | 0.000 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 145 | 0 | 1790 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 12.34 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.061 | 5.532 | 12.289 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 176 | 162 | 1069 | 0 | 277 | 0 | 0 | -1 |
| N.S. | 1 | 1.60 | 1.47 | 9.72 | 0.00 | 2.52 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.014 | 0.190 | 5.380 | 0.000 | 2.298 | 0.000 | 0.000 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 131 | 85 | 433 | 0 | 120 | 0 | 0 | -1 |
| N.S. | 1 | 1.62 | 1.05 | 5.35 | 0.00 | 1.48 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.049 | 0.011 | 0.895 | 0.000 | 1.102 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 117 | 130 | 0 | 0 | 415 | 0 | 67 | 37 |
| N.S. | 1 | 1.77 | 1.97 | 0.00 | 0.00 | 6.29 | 0.00 | 1.02 | 0.56 |
| time (sec) | N/A | 0.036 | 0.622 | 0.043 | 0.000 | 1.779 | 0.000 | 0.845 | 0.394 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 145 | 145 | 0 | 0 | 665 | 0 | 0 | -1 |
| N.S. | 1 | 1.84 | 1.84 | 0.00 | 0.00 | 8.42 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.064 | 0.730 | 0.005 | 0.000 | 1.416 | 0.000 | 0.000 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | A | F | F | B | F | F | F |
| verified | N/A | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 0 | 189 | 0 | 0 | 1496 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 1.60 | 0.00 | 0.00 | 12.68 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 10.224 | 2.230 | 0.009 | 0.000 | 10.682 | 0.000 | 0.000 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | F(-1) | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.349 | 41.159 | 0.027 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | F | F | F | F | B | F | F | F |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 0 | 0 | 0 | 0 | 932 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 5.30 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.282 | 10.461 | 0.023 | 0.000 | 36.697 | 0.000 | 0.000 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 493 | 570 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.279 | 10.254 | 0.086 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 178 | 415 | 0 | 957 | 0 | 795 | 343 |
| N.S. | 1 | 1.00 | 0.44 | 1.02 | 0.00 | 2.35 | 0.00 | 1.95 | 0.84 |
| time (sec) | N/A | 0.395 | 4.409 | 0.888 | 0.000 | 1.323 | 0.000 | 1.281 | 0.474 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 648 | 648 | 253 | 550 | 0 | 1563 | 0 | 1000 | 567 |
| N.S. | 1 | 1.00 | 0.39 | 0.85 | 0.00 | 2.41 | 0.00 | 1.54 | 0.88 |
| time (sec) | N/A | 0.710 | 7.147 | 0.907 | 0.000 | 1.726 | 0.000 | 1.749 | 0.561 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | B | F(-2) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1058 | 1058 | 1100 | 820 | 0 | 2763 | 0 | 1410 | 2500 |
| N.S. | 1 | 1.00 | 1.04 | 0.78 | 0.00 | 2.61 | 0.00 | 1.33 | 2.36 |
| time (sec) | N/A | 1.626 | 16.116 | 1.181 | 0.000 | 39.484 | 0.000 | 2.528 | 0.972 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | C | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 378 | 378 | 733 | 21028 | 0 | 1873 | 0 | 2509 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 55.63 | 0.00 | 4.96 | 0.00 | 6.64 | -0.00 |
| time (sec) | N/A | 0.485 | 2.674 | 1.516 | 0.000 | 1.130 | 0.000 | 34.882 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 638 | 638 | 1431 | 86793 | 0 | 2775 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.24 | 136.04 | 0.00 | 4.35 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.869 | 16.853 | 4.553 | 0.000 | 1.289 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | A | C | F | A | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 204 | 99 | 787 | 0 | 546 | 0 | 0 | -1 |
| N.S. | 1 | 3.09 | 1.50 | 11.92 | 0.00 | 8.27 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.752 | 1.280 | 0.171 | 0.000 | 1.410 | 0.000 | 0.000 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 145 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.062 | 10.170 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 153 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.054 | 10.178 | 0.043 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 168 | 1592 | 0 | 171 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.73 | 16.41 | 0.00 | 1.76 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.008 | 0.212 | 8.731 | 0.000 | 3.670 | 0.000 | 0.000 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | C | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 99 | 15 | 105 | 96 | 32 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.36 | 0.21 | 1.44 | 1.32 | 0.44 | 0.00 | -0.01 |
| time (sec) | N/A | 0.008 | 0.148 | 1.000 | 0.486 | 2.037 | 0.487 | 0.000 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 88 | 49 | 71 | 73 | 37 | 72 | 83 |
| N.S. | 1 | 1.00 | 1.31 | 0.73 | 1.06 | 1.09 | 0.55 | 1.07 | 1.24 |
| time (sec) | N/A | 0.024 | 0.047 | 1.661 | 0.675 | 0.983 | 0.470 | 1.856 | 0.366 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 482 | 482 | 0 | 2990 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 6.20 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.198 | 45.622 | 12.523 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | C | F | B | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 280 | 408 | 0 | 925 | 0 | 3085 | 0 | 0 | -1 |
| N.S. | 1 | 1.46 | 0.00 | 3.30 | 0.00 | 11.02 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.152 | 9.232 | 7.064 | 0.000 | 5.424 | 0.000 | 0.000 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 232 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.119 | 45.667 | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.098 | 10.100 | 0.083 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 24 | 23 | 23 | 22 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.00 | 0.96 | 0.92 | 0.92 | 0.88 | 0.92 | 0.92 |
| time (sec) | N/A | 0.013 | 0.005 | 0.013 | 0.288 | 0.968 | 0.029 | 1.230 | 0.074 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 86 | 62 | 0 | 43 | 73 | 40 | 53 |
| N.S. | 1 | 1.00 | 1.46 | 1.05 | 0.00 | 0.73 | 1.24 | 0.68 | 0.90 |
| time (sec) | N/A | 0.020 | 0.013 | 0.026 | 0.000 | 0.963 | 0.058 | 1.289 | 0.335 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 99 | 70 | 0 | 66 | 100 | 64 | 76 |
| N.S. | 1 | 1.00 | 1.27 | 0.90 | 0.00 | 0.85 | 1.28 | 0.82 | 0.97 |
| time (sec) | N/A | 0.043 | 0.014 | 0.029 | 0.000 | 0.758 | 0.067 | 1.661 | 0.090 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 57 | 113 | 0 | 56 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.16 | 2.31 | 0.00 | 1.14 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.005 | 0.132 | 0.417 | 0.000 | 1.050 | 0.000 | 0.000 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 44 | 365 | 0 | 61 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.83 | 6.89 | 0.00 | 1.15 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.008 | 0.165 | 0.283 | 0.000 | 1.465 | 0.000 | 0.000 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 81 | 1512 | 0 | 359 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.08 | 20.16 | 0.00 | 4.79 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.060 | 0.522 | 0.107 | 0.000 | 1.058 | 0.000 | 0.000 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 92 | 625 | 0 | 2667 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.54 | 3.65 | 0.00 | 15.60 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.047 | 0.255 | 0.111 | 0.000 | 4.370 | 0.000 | 0.000 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 157 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.025 | 10.191 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 162 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.029 | 10.195 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 144 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.028 | 10.176 | 0.044 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 152 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.022 | 10.143 | 0.035 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 28 | 164 | 0 | 1191 | 0 | 0 | 653 |
| N.S. | 1 | 1.00 | 0.22 | 1.29 | 0.00 | 9.38 | 0.00 | 0.00 | 5.14 |
| time (sec) | N/A | 0.013 | 10.032 | 5.863 | 0.000 | 1.598 | 0.000 | 0.000 | 0.449 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 54 | 240 | 0 | 1666 | 0 | 0 | 331 |
| N.S. | 1 | 1.00 | 0.34 | 1.53 | 0.00 | 10.61 | 0.00 | 0.00 | 2.11 |
| time (sec) | N/A | 0.020 | 10.041 | 0.232 | 0.000 | 1.212 | 0.000 | 0.000 | 15.034 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 48 | 421 | 0 | 547 | 0 | 0 | 533 |
| N.S. | 1 | 1.00 | 0.65 | 5.69 | 0.00 | 7.39 | 0.00 | 0.00 | 7.20 |
| time (sec) | N/A | 0.092 | 10.027 | 2.232 | 0.000 | 1.447 | 0.000 | 0.000 | 0.211 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 32 | 383 | 0 | 497 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.31 | 3.72 | 0.00 | 4.83 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.177 | 10.034 | 0.392 | 0.000 | 1.733 | 0.000 | 0.000 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 126 | 651 | 0 | 1792 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.56 | 8.04 | 0.00 | 22.12 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.007 | 4.950 | 3.617 | 0.000 | 2.214 | 0.000 | 0.000 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 126 | 239 | 0 | 345 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.56 | 2.95 | 0.00 | 4.26 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.007 | 4.614 | 1.178 | 0.000 | 1.446 | 0.000 | 0.000 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 118 | 938 | 0 | 1943 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.04 | 8.30 | 0.00 | 17.19 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.008 | 4.575 | 11.552 | 0.000 | 2.088 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 124 | 0 | 0 | 1685 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.14 | 0.00 | 0.00 | 15.46 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.008 | 4.642 | 0.048 | 0.000 | 2.191 | 0.000 | 0.000 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | A | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 159 | 206 | 0 | 85 | 0 | 0 | 217 |
| N.S. | 1 | 1.00 | 1.83 | 2.37 | 0.00 | 0.98 | 0.00 | 0.00 | 2.49 |
| time (sec) | N/A | 0.536 | 18.691 | 0.101 | 0.000 | 1.217 | 0.000 | 0.000 | 0.169 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | C | F | F | B |
| verified | N/A | NO | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1 | 529 | 127 | 317 | 0 | 70 | 0 | 0 | 207 |
| N.S. | 1 | 529.00 | 127.00 | 317.00 | 0.00 | 70.00 | 0.00 | 0.00 | 207.00 |
| time (sec) | N/A | 1.033 | 18.318 | 0.096 | 0.000 | 0.693 | 0.000 | 0.000 | 0.478 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | C | C | C | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 180 | 133 | 536 | 0 | 63 | 0 | 0 | -1 |
| N.S. | 1 | 3.91 | 2.89 | 11.65 | 0.00 | 1.37 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.963 | 20.771 | 0.105 | 0.000 | 0.697 | 0.000 | 0.000 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-2) | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 34 | 258 | 0 | 0 | 0 | 0 | 67 |
| N.S. | 1 | 1.00 | 1.06 | 8.06 | 0.00 | 0.00 | 0.00 | 0.00 | 2.09 |
| time (sec) | N/A | 0.059 | 1.033 | 1.315 | 0.000 | 0.000 | 0.000 | 0.000 | 1.686 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 31 | 240 | 0 | 44 | 0 | 0 | 204 |
| N.S. | 1 | 1.00 | 1.35 | 10.43 | 0.00 | 1.91 | 0.00 | 0.00 | 8.87 |
| time (sec) | N/A | 0.032 | 0.658 | 0.273 | 0.000 | 0.729 | 0.000 | 0.000 | 0.219 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 218 | 218 | 47 | 353 | 0 | 7739 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.22 | 1.62 | 0.00 | 35.50 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.029 | 10.046 | 37.470 | 0.000 | 6.261 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 210 | 50 | 350 | 0 | 8237 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.24 | 1.67 | 0.00 | 39.22 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.022 | 10.069 | 37.867 | 0.000 | 5.484 | 0.000 | 0.000 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 222 | 65 | 349 | 0 | 7910 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.29 | 1.57 | 0.00 | 35.63 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.019 | 10.054 | 37.967 | 0.000 | 4.937 | 0.000 | 0.000 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 214 | 214 | 68 | 350 | 0 | 8105 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.32 | 1.64 | 0.00 | 37.87 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.019 | 10.049 | 37.459 | 0.000 | 4.901 | 0.000 | 0.000 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 77 | 327 | 0 | 323 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.18 | 5.03 | 0.00 | 4.97 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.083 | 8.315 | 0.422 | 0.000 | 1.118 | 0.000 | 0.000 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 77 | 311 | 0 | 112 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 4.94 | 0.00 | 1.78 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.084 | 8.196 | 0.434 | 0.000 | 1.249 | 0.000 | 0.000 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 92 | 544 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.74 | 10.26 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.039 | 0.607 | 2.296 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | C | F | B | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 0 | 1421 | 0 | 267 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 13.16 | 0.00 | 2.47 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.068 | 11.483 | 4.135 | 0.000 | 1.482 | 0.000 | 0.000 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 189 | 0 | 0 | 1252 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.93 | 0.00 | 0.00 | 12.78 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.015 | 0.900 | 0.045 | 0.000 | 134.078 | 0.000 | 0.000 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 154 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.130 | 10.153 | 0.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | A | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 182 | 0 | 110 | 387 | 0 | 113 | 157 |
| N.S. | 1 | 1.00 | 1.90 | 0.00 | 1.15 | 4.03 | 0.00 | 1.18 | 1.64 |
| time (sec) | N/A | 0.049 | 0.343 | 0.046 | 0.625 | 1.090 | 0.000 | 7.254 | 0.590 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 114 | 907 | 0 | 253 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.30 | 10.31 | 0.00 | 2.88 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.007 | 0.237 | 1.590 | 0.000 | 3.050 | 0.000 | 0.000 | 0.000 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 283 | 0 | 0 | 373 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.21 | 0.00 | 0.00 | 1.60 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.063 | 0.799 | 0.045 | 0.000 | 2.458 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 104 | 657 | 86 | 90 | 0 | 87 | 100 |
| N.S. | 1 | 1.00 | 1.27 | 8.01 | 1.05 | 1.10 | 0.00 | 1.06 | 1.22 |
| time (sec) | N/A | 0.034 | 0.100 | 2.311 | 0.497 | 0.744 | 0.000 | 1.116 | 0.548 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 383 | 145 | 720 | 0 | 318 | 0 | 0 | -1 |
| N.S. | 1 | 2.84 | 1.07 | 5.33 | 0.00 | 2.36 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.177 | 0.917 | 5.210 | 0.000 | 6.903 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 383 | 145 | 737 | 0 | 318 | 0 | 0 | -1 |
| N.S. | 1 | 2.84 | 1.07 | 5.46 | 0.00 | 2.36 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.186 | 0.906 | 5.145 | 0.000 | 6.846 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | B | A | C | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 357 | 139 | 714 | 0 | 268 | 0 | 0 | -1 |
| N.S. | 1 | 3.00 | 1.17 | 6.00 | 0.00 | 2.25 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.179 | 0.899 | 5.451 | 0.000 | 8.219 | 0.000 | 0.000 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 43 | 34 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.032 | 10.056 | 0.211 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 43 | 34 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.050 | 10.065 | 0.207 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 43 | 34 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.018 | 10.017 | 0.127 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | B | A | C | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 101 | 12 | 105 | 94 | 31 | 0 | 10 |
| N.S. | 1 | 1.00 | 1.51 | 0.18 | 1.57 | 1.40 | 0.46 | 0.00 | 0.15 |
| time (sec) | N/A | 0.008 | 0.088 | 1.026 | 0.482 | 1.040 | 0.492 | 0.000 | 0.342 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 90 | 66 | 73 | 75 | 41 | 74 | 91 |
| N.S. | 1 | 1.00 | 1.29 | 0.94 | 1.04 | 1.07 | 0.59 | 1.06 | 1.30 |
| time (sec) | N/A | 0.023 | 0.054 | 1.707 | 0.592 | 1.231 | 0.492 | 1.059 | 0.403 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F(-1) | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 384 | 384 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.277 | 26.634 | 0.004 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.101 | 30.226 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | C | F | B | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 250 | 0 | 457 | 0 | 1827 | 0 | 0 | -1 |
| N.S. | 1 | 1.26 | 0.00 | 2.30 | 0.00 | 9.18 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.168 | 25.496 | 24.621 | 0.000 | 2.496 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.088 | 46.200 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.099 | 39.093 | 0.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 204 | 0 | 0 | 191 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.55 | 0.00 | 0.00 | 1.45 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.016 | 0.338 | 0.043 | 0.000 | 1.154 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 26 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.088 | 10.012 | 0.038 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 648 | 138 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.69 | 0.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.581 | 10.165 | 0.042 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 272 | 272 | 283 | 681 | 0 | 341 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.04 | 2.50 | 0.00 | 1.25 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.084 | 2.138 | 4.348 | 0.000 | 3.507 | 0.000 | 0.000 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [51]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 1 | 1 | 1.00 | 10 | 0.100 |
| 2 | C | 5 | 2 | 3.47 | 37 | 0.054 |
| 3 | A | 9 | 6 | 1.00 | 15 | 0.400 |
| 4 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 5 | A | 8 | 7 | 1.00 | 17 | 0.412 |
| 6 | A | 6 | 6 | 1.00 | 17 | 0.353 |
| 7 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 8 | A | 4 | 2 | 1.00 | 23 | 0.087 |
| 9 | A | 18 | 12 | 1.66 | 27 | 0.444 |
| 10 | B | 25 | 13 | 2.46 | 39 | 0.333 |
| 11 | A | 7 | 3 | 1.00 | 45 | 0.067 |
| 12 | A | 7 | 4 | 1.00 | 32 | 0.125 |
| 13 | A | 5 | 3 | 1.00 | 32 | 0.094 |
| 14 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 15 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 16 | A | 2 | 1 | 1.00 | 30 | 0.033 |
| 17 | A | 3 | 2 | 1.00 | 13 | 0.154 |
| 18 | A | 3 | 2 | 1.00 | 15 | 0.133 |
| 19 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 20 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 21 | A | 3 | 2 | 1.00 | 11 | 0.182 |
| 22 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 23 | A | 6 | 6 | 1.00 | 20 | 0.300 |
| 24 | A | 6 | 5 | 1.00 | 31 | 0.161 |
| 25 | A | 2 | 2 | 1.00 | 29 | 0.069 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 26 | A | 2 | 2 | 1.00 | 35 | 0.057 |
| 27 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 28 | A | 6 | 6 | 1.00 | 21 | 0.286 |
| 29 | A | 359 | 30 | 1.00 | 14 | 2.143 |
| 30 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 31 | A | 3 | 2 | 1.00 | 13 | 0.154 |
| 32 | A | 2 | 2 | 1.00 | 33 | 0.061 |
| 33 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 34 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 35 | A | 1 | 1 | 1.00 | 11 | 0.091 |
| 36 | A | 5 | 5 | 1.00 | 15 | 0.333 |
| 37 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 38 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 39 | A | 2 | 2 | 1.60 | 16 | 0.125 |
| 40 | A | 5 | 5 | 1.62 | 17 | 0.294 |
| 41 | A | 5 | 5 | 1.77 | 13 | 0.385 |
| 42 | A | 5 | 5 | 1.84 | 16 | 0.312 |
| 43 | F | 0 | 0 | N/A | 0. | N/A |
| 44 | F | 0 | 0 | N/A | 0. | N/A |
| 45 | F | 0 | 0 | N/A | 0. | N/A |
| 46 | A | 19 | 14 | 1.16 | 32 | 0.438 |
| 47 | A | 19 | 9 | 1.00 | 20 | 0.450 |
| 48 | A | 29 | 9 | 1.00 | 20 | 0.450 |
| 49 | A | 49 | 9 | 1.00 | 20 | 0.450 |
| 50 | A | 14 | 6 | 1.00 | 23 | 0.261 |
| 51 | A | 24 | 6 | 1.00 | 23 | 0.261 |
| 52 | C | 9 | 8 | 3.09 | 48 | 0.167 |
| 53 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 54 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 55 | A | 1 | 1 | 1.00 | 18 | 0.056 |
| 56 | A | 2 | 2 | 1.00 | 13 | 0.154 |
| 57 | A | 6 | 6 | 1.00 | 15 | 0.400 |
| 58 | A | 25 | 15 | 1.00 | 17 | 0.882 |
| 59 | A | 19 | 12 | 1.46 | 22 | 0.546 |
| 60 | A | 12 | 11 | 1.00 | 17 | 0.647 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 9 | 8 | 1.00 | 21 | 0.381 |
| 62 | A | 1 | 1 | 1.00 | 38 | 0.026 |
| 63 | A | 1 | 1 | 1.00 | 33 | 0.030 |
| 64 | A | 2 | 2 | 1.00 | 38 | 0.053 |
| 65 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 66 | A | 4 | 4 | 1.00 | 19 | 0.210 |
| 67 | A | 4 | 4 | 1.00 | 24 | 0.167 |
| 68 | A | 1 | 1 | 1.00 | 24 | 0.042 |
| 69 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 70 | A | 7 | 7 | 1.00 | 24 | 0.292 |
| 71 | A | 3 | 3 | 1.00 | 26 | 0.115 |
| 72 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 73 | A | 1 | 1 | 1.00 | 22 | 0.045 |
| 74 | A | 1 | 1 | 1.00 | 23 | 0.043 |
| 75 | A | 8 | 8 | 1.00 | 18 | 0.444 |
| 76 | A | 8 | 7 | 1.00 | 23 | 0.304 |
| 77 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 78 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 79 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 80 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 81 | A | 4 | 4 | 1.00 | 34 | 0.118 |
| 82 | C | 5 | 5 | 529.00 | 40 | 0.125 |
| 83 | C | 7 | 7 | 3.91 | 51 | 0.137 |
| 84 | A | 2 | 2 | 1.00 | 29 | 0.069 |
| 85 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 86 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 87 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 88 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 89 | A | 1 | 1 | 1.00 | 25 | 0.040 |
| 90 | A | 2 | 2 | 1.00 | 40 | 0.050 |
| 91 | A | 2 | 2 | 1.00 | 40 | 0.050 |
| 92 | A | 1 | 1 | 1.00 | 18 | 0.056 |
| 93 | A | 3 | 3 | 1.00 | 15 | 0.200 |
| 94 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 95 | A | 8 | 7 | 1.00 | 23 | 0.304 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 96 | A | 5 | 5 | 1.00 | 24 | 0.208 |
| 97 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 98 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 99 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 100 | B | 16 | 12 | 2.84 | 25 | 0.480 |
| 101 | B | 17 | 13 | 2.84 | 24 | 0.542 |
| 102 | B | 16 | 12 | 3.00 | 23 | 0.522 |
| 103 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 104 | A | 5 | 4 | 1.00 | 25 | 0.160 |
| 105 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 106 | A | 2 | 2 | 1.00 | 11 | 0.182 |
| 107 | A | 6 | 6 | 1.00 | 15 | 0.400 |
| 108 | A | 13 | 12 | 1.00 | 19 | 0.632 |
| 109 | A | 13 | 12 | 1.00 | 22 | 0.546 |
| 110 | A | 14 | 12 | 1.26 | 27 | 0.444 |
| 111 | A | 5 | 5 | 1.00 | 17 | 0.294 |
| 112 | A | 6 | 6 | 1.00 | 27 | 0.222 |
| 113 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 114 | A | 10 | 10 | 1.00 | 20 | 0.500 |
| 115 | A | 17 | 6 | 1.69 | 24 | 0.250 |
| 116 | A | 14 | 8 | 1.00 | 19 | 0.421 |

# Chapter 3

# Listing of integrals

## Local contents

## 3.1 $\int \frac{1}{\sqrt{1-ax}}\,dx$

**Optimal**. Leaf size=15

$$-\frac{2\sqrt{1-ax}}{a}$$

[Out] -2*(-a*x+1)^(1/2)/a

**Rubi** [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {32}

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - a*x],x]

[Out] (-2*Sqrt[1 - a*x])/a

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-ax}}\,dx = -\frac{2\sqrt{1-ax}}{a}$$

**Mathematica** [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2\sqrt{1-ax}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - a*x],x]

[Out] (-2*Sqrt[1 - a*x])/a

**Maple** [A]
time = 0.09, size = 14, normalized size = 0.93

| method | result | size |
|---|---|---|
| gosper | $-\frac{2\sqrt{-ax+1}}{a}$ | 14 |
| derivativedivides | $-\frac{2\sqrt{-ax+1}}{a}$ | 14 |
| default | $-\frac{2\sqrt{-ax+1}}{a}$ | 14 |
| trager | $-\frac{2\sqrt{-ax+1}}{a}$ | 14 |
| risch | $\frac{2ax-2}{a\sqrt{-ax+1}}$ | 19 |
| meijerg | $-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-ax+1}}{\sqrt{\pi}\,a}$ | 28 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-a*x+1)^(1/2)/a
```

**Maxima** [A]
time = 0.27, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-a*x + 1)/a
```

**Fricas** [A]
time = 0.39, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-a*x + 1)/a
```

**Sympy** [A]
time = 0.01, size = 12, normalized size = 0.80

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/2),x)
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

**Giac** [A]
time = 1.11, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{-ax+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-a*x + 1)/a
```

**Mupad** [B]
time = 0.03, size = 13, normalized size = 0.87

$$-\frac{2\sqrt{1-a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - a*x)^(1/2),x)
```

```
[Out] -(2*(1 - a*x)^(1/2))/a
```

## 3.2 $\int \frac{-2 \log \left(-\sqrt{-1+a x}\right)+\log (-1+a x)}{2 \pi \sqrt{-1+a x}} \, dx$

**Optimal**. Leaf size=15

$$-\frac{2 \sqrt{1-a x}}{a}$$

```
[Out] -2*(-a*x+1)^(1/2)/a
```

**Rubi** [C] Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.
time = 0.20, antiderivative size = 52, normalized size of antiderivative = 3.47, number of steps used = 5, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {12, 2332}

$$\frac{\sqrt{a x-1} \log (a x-1)}{\pi a}-\frac{2 \sqrt{a x-1} \log \left(-\sqrt{a x-1}\right)}{\pi a}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Log[-Sqrt[-1 + a*x]])/(a*Pi) + (Sqrt[-1 + a*x]*Log[-1 +
a*x])/(a*Pi)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-2 \log \left(-\sqrt{-1+a x}\right)+\log (-1+a x)}{2 \pi \sqrt{-1+a x}} \, dx &= \frac{\int \frac{-2 \log \left(-\sqrt{-1+a x}\right)+\log (-1+a x)}{\sqrt{-1+a x}} \, dx}{2 \pi} \\
&= \frac{\operatorname{Subst}\left(\int \left(-2 \log (-x)+\log \left(x^2\right)\right) \, dx, x, \sqrt{-1+a x}\right)}{a \pi} \\
&= \frac{\operatorname{Subst}\left(\int \log \left(x^2\right) \, dx, x, \sqrt{-1+a x}\right)}{a \pi}-\frac{2 \operatorname{Subst}\left(\int \log (-x) \, d\right.}{a \pi} \\
&= -\frac{2 \sqrt{-1+a x} \log \left(-\sqrt{-1+a x}\right)}{a \pi}+\frac{\sqrt{-1+a x} \log (-1+}{a \pi}
\end{aligned}
$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 3 vs. order 2 in optimal.
time = 0.02, size = 37, normalized size = 2.47

$$\frac{\sqrt{-1+ax}\left(-2\log\left(-\sqrt{-1+ax}\right)+\log(-1+ax)\right)}{a\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x])/(2*Pi*Sqrt[-1 + a*x]),x
]
```

```
[Out] (Sqrt[-1 + a*x]*(-2*Log[-Sqrt[-1 + a*x]] + Log[-1 + a*x]))/(a*Pi)
```

**Maple** [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.10, size = 42, normalized size = 2.80

| method | result | size |
|---|---|---|
| gosper | $\frac{\sqrt{ax-1}\left(\ln(ax-1)-2\ln\left(-\sqrt{ax-1}\right)\right)}{a\pi}$ | 34 |
| derivativedivides | $\frac{-2\ln\left(-\sqrt{ax-1}\right)\sqrt{ax-1}+\sqrt{ax-1}\ \ln(ax-1)}{\pi a}$ | 42 |
| default | $\frac{-2\ln\left(-\sqrt{ax-1}\right)\sqrt{ax-1}+\sqrt{ax-1}\ \ln(ax-1)}{\pi a}$ | 42 |
| meijerg | $\frac{i\sqrt{-\mathrm{signum}\left(ax-1\right)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\ \sqrt{-ax+1}\right)}{\sqrt{\pi}\ \sqrt{\mathrm{signum}\left(ax-1\right)}\ a}$ | 47 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(ln(a*x-1)-2*ln(-(a*x-1)^(1/2)))/Pi/(a*x-1)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/Pi/a*(-2*ln(-(a*x-1)^(1/2))*(a*x-1)^(1/2)+(a*x-1)^(1/2)*ln(a*x-1))
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.
time = 0.26, size = 41, normalized size = 2.73

$$\frac{\sqrt{ax-1}\ \log\left(ax-1\right)-2\sqrt{ax-1}\ \log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="maxima")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="fricas")
```

```
[Out] 0
```

**Sympy** [A]
time = 29.28, size = 42, normalized size = 2.80

$$\frac{\begin{cases} \frac{-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)+\sqrt{ax-1}\log\left(ax-1\right)}{a} & \text{for}\ a\neq 0 \\ \pi x & \text{otherwise} \end{cases}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(ln(a*x-1)-2*ln(-(a*x-1)**(1/2)))/pi/(a*x-1)**(1/2),x)
```

```
[Out] Piecewise(((-2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)) + sqrt(a*x - 1)*log(a*x -
1))/a, Ne(a, 0)), (pi*x, True))/pi
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.
time = 0.76, size = 41, normalized size = 2.73

$$\frac{\sqrt{ax-1}\log\left(ax-1\right)-2\sqrt{ax-1}\log\left(-\sqrt{ax-1}\right)}{\pi a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(log(a*x-1)-2*log(-(a*x-1)^(1/2)))/pi/(a*x-1)^(1/2),x, algori
thm="giac")
```

```
[Out] (sqrt(a*x - 1)*log(a*x - 1) - 2*sqrt(a*x - 1)*log(-sqrt(a*x - 1)))/(pi*a)
```

**Mupad** [B]
time = 0.51, size = 43, normalized size = 2.87

$$-\frac{2\ln\left(-\sqrt{a\,x-1}\right)\,\sqrt{a\,x-1}-\ln\left(a\,x-1\right)\,\sqrt{a\,x-1}}{\Pi\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x - 1)/2 - log(-(a*x - 1)^(1/2)))/(Pi*(a*x - 1)^(1/2)),x)
```

```
[Out] -(2*log(-(a*x - 1)^(1/2))*(a*x - 1)^(1/2) - log(a*x - 1)*(a*x - 1)^(1/2))/(
Pi*a)
```

## 3.3 $\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx$

**Optimal**. Leaf size=82

$$\frac{4x}{3(1-3x^2)}-\frac{2\sqrt{1+x^2}}{3(1-3x^2)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{1+x^2}\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x/(-3*x^2+1)-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(1/2*3^(1/2)*(x^
2+1)^(1/2))*3^(1/2)-2/3*(x^2+1)^(1/2)/(-3*x^2+1)
```

**Rubi** [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6874, 205, 213, 455, 43, 65}

$$\frac{4x}{3(1-3x^2)}-\frac{2\sqrt{x^2+1}}{3(1-3x^2)}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{x^2+1}\right)}{3\sqrt{3}}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (4*x)/(3*(1 - 3*x^2)) - (2*Sqrt[1 + x^2])/(3*(1 - 3*x^2)) - ArcTanh[Sqrt[3]
*x]/(3*Sqrt[3]) + ArcTanh[(Sqrt[3]*Sqrt[1 + x^2])/2]/(3*Sqrt[3])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(2x+\sqrt{1+x^2}\right)^2}\,dx &= \int \left(\frac{8}{3\left(-1+3x^2\right)^2}-\frac{4x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}+\frac{5}{3\left(-1+3x^2\right)}\right)dx\\
&= \frac{5}{3}\int \frac{1}{-1+3x^2}\,dx+\frac{8}{3}\int \frac{1}{\left(-1+3x^2\right)^2}\,dx-4\int \frac{x\sqrt{1+x^2}}{\left(-1+3x^2\right)^2}\,dx\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{5\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{4}{3}\int \frac{1}{-1+3x^2}\,dx-2\text{Subst}\left(\int \frac{\sqrt{1+x}}{(-1+3x)^2}\right.\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+x}\,(-1+3x)}\right.\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-4+3x^2}\,dx,x,\sqrt{1}\right.\\
&= \frac{4x}{3\left(1-3x^2\right)}-\frac{2\sqrt{1+x^2}}{3\left(1-3x^2\right)}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{1}{2}\sqrt{3}\,\sqrt{1+x^2}\right)}{3\sqrt{3}}
\end{aligned}$$

**Mathematica** [A]

time = 0.21, size = 60, normalized size = 0.73

$$\frac{6\left(-2x+\sqrt{1+x^2}\right)-2\sqrt{3}\,(-1+3x^2)\tanh^{-1}\left(\frac{x-\sqrt{1+x^2}}{\sqrt{3}}\right)}{-9+27x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sqrt[1 + x^2])^(-2),x]
```

```
[Out] (6*(-2*x + Sqrt[1 + x^2]) - 2*Sqrt[3]*(-1 + 3*x^2)*ArcTanh[(x - Sqrt[1 + x^
2])/Sqrt[3]])/(-9 + 27*x^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(60) = 120.
time = 0.16, size = 370, normalized size = 4.51

| method | result |
|---|---|
| trager | $-\frac{4x}{3(3x^2-1)}+\frac{2\sqrt{x^2+1}}{3(3x^2-1)}+\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)+3\sqrt{x^2+1}}{\mathrm{RootOf}\left(\_Z^2-3\right)x+1}\right)}{9}$ |

| | |
|---|---|
| default | $-\frac{x}{2(3x^2-1)}-\frac{\operatorname{arctanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{5x}{18\left(x^2-\frac{1}{3}\right)}-\sqrt{3}\left(-\frac{\left(\left(x-\frac{\sqrt{3}}{3}\right)^2+\frac{2\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{3}+\frac{4}{3}\right)^{\frac{3}{2}}}{12\left(x-\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\left(\ldots\right.}{\ldots}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+(x^2+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(3*x^2-1)-1/9*arctanh(x*3^(1/2))*3^(1/2)-5/18*x/(x^2-1/3)-3^(1/2)*(-
1/12/(x-1/3*3^(1/2))*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(3
/2)+1/36*3^(1/2)*(1/3*(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1
/2)+1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3+2/3*3^(1/2)*(x-1/3*
3^(1/2)))*3^(1/2)/(9*(x-1/3*3^(1/2))^2+6*3^(1/2)*(x-1/3*3^(1/2))+12)^(1/2))
)+1/12*x*((x-1/3*3^(1/2))^2+2/3*3^(1/2)*(x-1/3*3^(1/2))+4/3)^(1/2)+1/12*arc
sinh(x))+3^(1/2)*(-1/12/(x+1/3*3^(1/2))*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1
/3*3^(1/2))+4/3)^(3/2)-1/36*3^(1/2)*(1/3*(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+
1/3*3^(1/2))+12)^(1/2)-1/3*3^(1/2)*arcsinh(x)-2/3*3^(1/2)*arctanh(3/4*(8/3-
2/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(9*(x+1/3*3^(1/2))^2-6*3^(1/2)*(x+1/3*
3^(1/2))+12)^(1/2)))+1/12*x*((x+1/3*3^(1/2))^2-2/3*3^(1/2)*(x+1/3*3^(1/2))+
4/3)^(1/2)+1/12*arcsinh(x))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(x^2 + 1))^(-2), x)
```

**Fricas** [A]
time = 0.44, size = 100, normalized size = 1.22

$$\frac{\sqrt{3}\,(3\,x^2-1)\log\left(\frac{3\,x^2-2\,\sqrt{3}\,x+1}{3\,x^2-1}\right)+\sqrt{3}\,(3\,x^2-1)\log\left(\frac{3\,x^2+4\,\sqrt{3}\,\sqrt{x^2+1}+7}{3\,x^2-1}\right)-24\,x+12\,\sqrt{x^2+1}}{18\,(3\,x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(3)*(3*x^2 - 1)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + sqrt
(3)*(3*x^2 - 1)*log((3*x^2 + 4*sqrt(3)*sqrt(x^2 + 1) + 7)/(3*x^2 - 1)) - 24
*x + 12*sqrt(x^2 + 1))/(3*x^2 - 1)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2x+\sqrt{x^2+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x**2+1)**(1/2))**2,x)
```

```
[Out] Integral((2*x + sqrt(x**2 + 1))**(-2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(60) = 120.
time = 0.88, size = 177, normalized size = 2.16

$$\frac{1}{18}\,\sqrt{3}\,\log\left(\frac{\left|6\,x-2\,\sqrt{3}\right|}{\left|6\,x+2\,\sqrt{3}\right|}\right)-\frac{1}{18}\,\sqrt{3}\,\log\left(-\frac{\left|-6\,x-8\,\sqrt{3}+6\,\sqrt{x^2+1}-\frac{6}{x-\sqrt{x^2+1}}\right|}{2\left(3\,x-4\,\sqrt{3}-3\,\sqrt{x^2+1}+\frac{3}{x-\sqrt{x^2+1}}\right)}\right)-\frac{4\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)}{3\left(3\left(x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}\right)^2-16\right)}-\frac{4\,x}{3\,(3\,x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+(x^2+1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-6*x - 8*sqrt(3) + 6*sqrt(x^2 + 1) - 6/(x - sqrt(x^2 + 1)))/(3
*x - 4*sqrt(3) - 3*sqrt(x^2 + 1) + 3/(x - sqrt(x^2 + 1)))) - 4/3*(x - sqrt(
x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/(3*(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 +
 1)))^2 - 16) - 4/3*x/(3*x^2 - 1)
```

**Mupad** [B]
time = 0.57, size = 204, normalized size = 2.49

$$\frac{\sqrt{3}\left(\ln\left(x-\frac{\sqrt{3}}{3}\right)-\ln\left(x+\sqrt{3}+2\sqrt{x^2+1}\right)\right)}{18}-\frac{4x}{9\left(x^2-\frac{1}{3}\right)}+\frac{\sqrt{3}\left(\ln\left(x+\frac{\sqrt{3}}{3}\right)-\ln\left(x-\sqrt{3}-2\sqrt{x^2+1}\right)\right)}{18}-\frac{\sqrt{3}\left(6\ln\left(x-\frac{\sqrt{3}}{3}\right)-6\ln\left(x+\sqrt{3}+2\sqrt{x^2+1}\right)\right)}{54}-\frac{\sqrt{3}\left(6\ln\left(x+\frac{\sqrt{3}}{3}\right)-6\ln\left(x-\sqrt{3}-2\sqrt{x^2+1}\right)\right)}{54}+\frac{\sqrt{3}\sqrt{x^2+1}}{9\left(x-\frac{\sqrt{3}}{3}\right)}-\frac{\sqrt{3}\sqrt{x^2+1}}{9\left(x+\frac{\sqrt{3}}{3}\right)}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + (x^2 + 1)^(1/2))^2,x)
```

```
[Out] (3^(1/2)*(log(x - 3^(1/2)/3) - log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/18 +
(3^(1/2)*atan(3^(1/2)*x*1i)*1i)/9 - (4*x)/(9*(x^2 - 1/3)) + (3^(1/2)*(log(x
 + 3^(1/2)/3) - log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/18 - (3^(1/2)*(6*log
(x - 3^(1/2)/3) - 6*log(x + 3^(1/2) + 2*(x^2 + 1)^(1/2))))/54 - (3^(1/2)*(6
*log(x + 3^(1/2)/3) - 6*log(x - 3^(1/2) - 2*(x^2 + 1)^(1/2))))/54 + (3^(1/2
)*(x^2 + 1)^(1/2))/(9*(x - 3^(1/2)/3)) - (3^(1/2)*(x^2 + 1)^(1/2))/(9*(x +
3^(1/2)/3))
```

## 3.4 $\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx$

**Optimal**. Leaf size=43

$$\frac{3x\sqrt{-1+x^2}}{8(4-3x^2)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)$$

```
[Out] 5/16*arctanh(1/2*x/(x^2-1)^(1/2))+3/8*x*(x^2-1)^(1/2)/(-3*x^2+4)
```

**Rubi** [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {390, 385, 213}

$$\frac{3\sqrt{x^2-1}\,x}{8(4-3x^2)}+\frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (3*x*Sqrt[-1 + x^2])/(8*(4 - 3*x^2)) + (5*ArcTanh[x/(2*Sqrt[-1 + x^2])])/16
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)^2}\,dx = \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8}\int \frac{1}{\sqrt{-1+x^2}\,(-4+3x^2)}\,dx$$

$$= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)$$

$$= \frac{3x\sqrt{-1+x^2}}{8(4-3x^2)} + \frac{5}{16}\tanh^{-1}\left(\frac{x}{2\sqrt{-1+x^2}}\right)$$

**Mathematica** [**A**]
time = 0.09, size = 71, normalized size = 1.65

$$-\frac{3x\sqrt{-1+x^2}}{8(-4+3x^2)} + \frac{5}{32}\log\left(2-x^2+x\sqrt{-1+x^2}\right) - \frac{5}{32}\log\left(2-3x^2+3x\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(-4 + 3*x^2)^2),x]
```

```
[Out] (-3*x*Sqrt[-1 + x^2])/(8*(-4 + 3*x^2)) + (5*Log[2 - x^2 + x*Sqrt[-1 + x^2]]
)/32 - (5*Log[2 - 3*x^2 + 3*x*Sqrt[-1 + x^2]])/32
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(33) = 66.
time = 0.18, size = 172, normalized size = 4.00

| method | result |
|---|---|
| trager | $-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)} - \frac{5\ln\left(-\frac{4\sqrt{x^2-1}\,x-5x^2+4}{3x^2-4}\right)}{32}$ |
| risch | $-\frac{3x\sqrt{x^2-1}}{8(3x^2-4)} + \frac{5\,\text{arctanh}\left(\frac{3\left(\frac{2}{3}+\frac{4\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x-\frac{2\sqrt{3}}{3}\right)^2+12\sqrt{3}\left(x-\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32} - \frac{5\,\text{arctanh}\left(\frac{\cdots}{2\sqrt{9\left(x+\cdots\right.}}\right.}{\cdots}$ |

| | |
|---|---|
| default | $-\frac{5\,\mathrm{arctanh}\left(\frac{3\left(\frac{2}{3}-\frac{4\sqrt{3}\,\left(x+\frac{2\sqrt{3}}{3}\right)}{3}\right)\sqrt{3}}{2\sqrt{9\left(x+\frac{2\sqrt{3}}{3}\right)^{2}-12\sqrt{3}\,\left(x+\frac{2\sqrt{3}}{3}\right)+3}}\right)}{32}-\frac{\sqrt{\left(x-\frac{2\sqrt{3}}{3}\right)^{2}+\frac{4\sqrt{3}\,\left(x-\frac{2\sqrt{3}}{3}\right.}{3}}}{16\left(x-\frac{2\sqrt{3}}{3}\right)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-4)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/32*arctanh(3/2*(2/3-4/3*3^(1/2)*(x+2/3*3^(1/2)))*3^(1/2)/(9*(x+2/3*3^(1/
2))^2-12*3^(1/2)*(x+2/3*3^(1/2))+3)^(1/2))-1/16/(x-2/3*3^(1/2))*((x-2/3*3^(
1/2))^2+4/3*3^(1/2)*(x-2/3*3^(1/2))+1/3)^(1/2)+5/32*arctanh(3/2*(2/3+4/3*3^
(1/2)*(x-2/3*3^(1/2)))*3^(1/2)/(9*(x-2/3*3^(1/2))^2+12*3^(1/2)*(x-2/3*3^(1/
2))+3)^(1/2))-1/16/(x+2/3*3^(1/2))*((x+2/3*3^(1/2))^2-4/3*3^(1/2)*(x+2/3*3^
(1/2))+1/3)^(1/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 4)^2*sqrt(x^2 - 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(33) = 66$.
time = 0.54, size = 80, normalized size = 1.86

$$-\frac{12\,x^{2}+5\,(3\,x^{2}-4)\log\left(3\,x^{2}-3\,\sqrt{x^{2}-1}\,x-2\right)-5\,(3\,x^{2}-4)\log\left(x^{2}-\sqrt{x^{2}-1}\,x-2\right)+12\,\sqrt{x^{2}-1}\,x-16}{32\,(3\,x^{2}-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(12*x^2 + 5*(3*x^2 - 4)*log(3*x^2 - 3*sqrt(x^2 - 1)*x - 2) - 5*(3*x^2
 - 4)*log(x^2 - sqrt(x^2 - 1)*x - 2) + 12*sqrt(x^2 - 1)*x - 16)/(3*x^2 - 4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x-1)\,(x+1)}\,\left(3x^{2}-4\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-4)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(3*x**2 - 4)**2), x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(33) = 66.
time = 0.74, size = 94, normalized size = 2.19

$$\frac{5\left(x-\sqrt{x^2-1}\right)^2-3}{4\left(3\left(x-\sqrt{x^2-1}\right)^4-10\left(x-\sqrt{x^2-1}\right)^2+3\right)}-\frac{5}{32}\log\left(\left|3\left(x-\sqrt{x^2-1}\right)^2-1\right|\right)+\frac{5}{32}\log\left(\left|\left(x-\sqrt{x^2-1}\right)^2-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(5*(x - sqrt(x^2 - 1))^2 - 3)/(3*(x - sqrt(x^2 - 1))^4 - 10*(x - sqrt(x
^2 - 1))^2 + 3) - 5/32*log(abs(3*(x - sqrt(x^2 - 1))^2 - 1)) + 5/32*log(abs
((x - sqrt(x^2 - 1))^2 - 3))
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,\left(3\,x^2-4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 - 4)^2), x)
```

## 3.5 $\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx$

**Optimal**. Leaf size=74

$$\frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}\log(1-3x)$$

```
[Out] 8/9/(1-3*x)-8/9*arcsinh(x^(1/2))+10/9*arctanh(2*x^(1/2)/(1+x)^(1/2))+5/9*ln
(1-3*x)-4/3*x^(1/2)*(1+x)^(1/2)/(1-3*x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {6874, 99, 163, 56, 221, 95, 213}

$$-\frac{4\sqrt{x}\,\sqrt{x+1}}{3(1-3x)}+\frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] 8/(9*(1 - 3*x)) - (4*Sqrt[x]*Sqrt[1 + x])/(3*(1 - 3*x)) - (8*ArcSinh[Sqrt[x
]])/9 + (10*ArcTanh[(2*Sqrt[x])/Sqrt[1 + x]])/9 + (5*Log[1 - 3*x])/9
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(2\sqrt{x}+\sqrt{1+x}\right)^2}\,dx &= \int \left(\frac{8}{3(-1+3x)^2}-\frac{4\sqrt{x}\,\sqrt{1+x}}{(-1+3x)^2}+\frac{5}{3(-1+3x)}\right)dx\\
&= \frac{8}{9(1-3x)}+\frac{5}{9}\log(1-3x)-4\int \frac{\sqrt{x}\,\sqrt{1+x}}{(-1+3x)^2}\,dx\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{3}\int \frac{\frac{1}{2}+x}{\sqrt{x}\,\sqrt{1+x}\,(-1+3x)}\,d\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{4}{9}\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx-\frac{10}{9}\int\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}+\frac{5}{9}\log(1-3x)-\frac{8}{9}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{}\right.\\
&= \frac{8}{9(1-3x)}-\frac{4\sqrt{x}\,\sqrt{1+x}}{3(1-3x)}-\frac{8}{9}\sinh^{-1}\left(\sqrt{x}\right)+\frac{10}{9}\tanh^{-1}\left(\frac{2\sqrt{x}}{\sqrt{1+x}}\right)+\frac{5}{9}
\end{aligned}$$

**Mathematica** [A]
time = 0.08, size = 69, normalized size = 0.93

$$\frac{2\left(-4+6\sqrt{x}\,\sqrt{1+x}+(-1+3x)\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)+5(-1+3x)\log\left(1-x+\sqrt{x}\,\sqrt{1+x}\right)\right)}{-9+27x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (2*(-4 + 6*Sqrt[x]*Sqrt[1 + x] + (-1 + 3*x)*ArcTanh[Sqrt[x/(1 + x)]] + 5*(-
1 + 3*x)*Log[1 - x + Sqrt[x]*Sqrt[1 + x]]))/(-9 + 27*x)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(54) = 108$.
time = 0.02, size = 115, normalized size = 1.55

| method | result |
|---|---|
| default | $-\frac{8}{9(3x-1)}+\frac{5\ln(3x-1)}{9}-\frac{\sqrt{x}\,\sqrt{1+x}\,\left(12\ln\left(\frac{1}{2}+x+\sqrt{x\,(1+x)}\right)x-15\,\mathrm{arctanh}\left(\frac{5x+1}{4\sqrt{x\,(1+x)}}\right)x-4\ln\left(\frac{1}{2}+x-\right.\right.}{9\sqrt{x\,(1+x)}\,(3x-1)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -8/9/(3*x-1)+5/9*ln(3*x-1)-1/9*x^(1/2)*(1+x)^(1/2)*(12*ln(1/2+x+(x*(1+x))^(
1/2))*x-15*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))*x-4*ln(1/2+x+(x*(1+x))^(1/2
))+5*arctanh(1/4*(5*x+1)/(x*(1+x))^(1/2))-12*(x*(1+x))^(1/2))/(x*(1+x))^(1/
2)/(3*x-1)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + 2*sqrt(x))^(-2), x)
```

**Fricas** [A]
time = 0.68, size = 105, normalized size = 1.42

$$-\frac{5\,(3\,x-1)\log\left(3\,\sqrt{x+1}\,\sqrt{x}-3\,x-1\right)-4\,(3\,x-1)\log\left(2\,\sqrt{x+1}\,\sqrt{x}-2\,x-1\right)-5\,(3\,x-1)\log\left(\sqrt{x+1}\,\sqrt{x}-x+1\right)-5\,(3\,x-1)\log\left(3\,x-1\right)-12\,\sqrt{x+1}\,\sqrt{x}-12\,x+12}{9\,(3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(5*(3*x - 1)*log(3*sqrt(x + 1)*sqrt(x) - 3*x - 1) - 4*(3*x - 1)*log(2*
sqrt(x + 1)*sqrt(x) - 2*x - 1) - 5*(3*x - 1)*log(sqrt(x + 1)*sqrt(x) - x +
1) - 5*(3*x - 1)*log(3*x - 1) - 12*sqrt(x + 1)*sqrt(x) - 12*x + 12)/(3*x -
1)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2\sqrt{x}+\sqrt{x+1}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((2*sqrt(x) + sqrt(x + 1))**(-2), x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,[0]%%%},0,1] at parameters
values [5.38357630698]Warning, choosing root of [1,0,%%%{-4,[1]%%%}+%%%{-2,
[0]%%%},0,1] at
```

**Mupad [B]**
time = 1.71, size = 82, normalized size = 1.11

$$\frac{10\,\mathrm{atanh}\left(\frac{2662400\sqrt{x}}{81\left(\frac{665600\,x}{81\left(\sqrt{x+1}-1\right)^2}+\frac{665600}{81}\right)\left(\sqrt{x+1}-1\right)}\right)}{9}+\frac{5\ln\left(x-\frac{1}{3}\right)}{9}-\frac{16\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)}{9}-\frac{8}{27\left(x-\frac{1}{3}\right)}+\frac{4\sqrt{x}\,\sqrt{x+1}}{3\,(3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + 2*x^(1/2))^2,x)
```

```
[Out] (10*atanh((2662400*x^(1/2))/(81*((665600*x)/(81*((x + 1)^(1/2) - 1)^2) + 66
5600/81)*((x + 1)^(1/2) - 1))))/9 + (5*log(x - 1/3))/9 - (16*atanh(x^(1/2)/
((x + 1)^(1/2) - 1)))/9 - 8/(27*(x - 1/3)) + (4*x^(1/2)*(x + 1)^(1/2))/(3*(
3*x - 1))
```

## 3.6 $\int \frac{\sqrt{-1+x^2}}{(-i+x)^2} \, dx$

**Optimal**. Leaf size=64

$$\frac{\sqrt{-1+x^2}}{i-x} - \frac{i \tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

```
[Out] arctanh(x/(x^2-1)^(1/2))-1/2*I*arctan(1/2*(1-I*x)*2^(1/2)/(x^2-1)^(1/2))*2^
(1/2)+(x^2-1)^(1/2)/(I-x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {747, 858, 223, 212, 739, 210}

$$-\frac{i\text{ArcTan}\left(\frac{1-ix}{\sqrt{2}\sqrt{x^2-1}}\right)}{\sqrt{2}} + \frac{\sqrt{x^2-1}}{-x+i} + \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) - (I*ArcTan[(1 - I*x)/(Sqrt[2]*Sqrt[-1 + x^2])])/Sqr
t[2] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+x^2}}{(-i+x)^2}\,dx &= \frac{\sqrt{-1+x^2}}{i-x}+\int \frac{x}{(-i+x)\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}+i\int \frac{1}{(-i+x)\sqrt{-1+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx \\
&= \frac{\sqrt{-1+x^2}}{i-x}-i\text{Subst}\left(\int \frac{1}{-2-x^2}\,dx,x,\frac{-1+ix}{\sqrt{-1+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\ldots}{\sqrt{-1\ldots}}\right. \\
&= \frac{\sqrt{-1+x^2}}{i-x}-\frac{i\tan^{-1}\left(\frac{1-ix}{\sqrt{2}\sqrt{-1+x^2}}\right)}{\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}
$$

**Mathematica** [A]
time = 0.20, size = 66, normalized size = 1.03

$$
\frac{\sqrt{-1+x^2}}{i-x}+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1+ix-i\sqrt{-1+x^2}}{\sqrt{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/(-I + x)^2,x]
```

```
[Out] Sqrt[-1 + x^2]/(I - x) + ArcTanh[x/Sqrt[-1 + x^2]] - Sqrt[2]*ArcTanh[(1 + I
*x - I*Sqrt[-1 + x^2])/Sqrt[2]]
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(52) = 104$.
time = 0.12, size = 150, normalized size = 2.34

| method | result |
|---|---|
| risch | $-\frac{\sqrt{x^2-1}}{x-i} + \ln\left(x+\sqrt{x^2-1}\right) + \frac{i\sqrt{2}\,\arctan\left(\frac{(-4+2i(x-i))\sqrt{2}}{4\sqrt{(x-i)^2+2i(x-i)-2}}\right)}{2}$ |
| default | $\frac{\left((x-i)^2+2i(x-i)-2\right)^{\frac{3}{2}}}{2x-2i} - \frac{i\left(\sqrt{(x-i)^2+2i(x-i)-2}+i\ln\left(x+\sqrt{(x-i)^2+2i(x-i)-2}\right)-\sqrt{2}\,\mathrm{arctan}\right.}{2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x-I)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(x-I)*((x-I)^2+2*I*(x-I)-2)^(3/2)-1/2*I*(((x-I)^2+2*I*(x-I)-2)^(1/2)+I*
ln(x+((x-I)^2+2*I*(x-I)-2)^(1/2))-2^(1/2)*arctan(1/4*(-4+2*I*(x-I))*2^(1/2)
/((x-I)^2+2*I*(x-I)-2)^(1/2)))-1/2*x*((x-I)^2+2*I*(x-I)-2)^(1/2)+1/2*ln(x+(
(x-I)^2+2*I*(x-I)-2)^(1/2))
```

**Maxima** [A]
time = 0.50, size = 53, normalized size = 0.83

$$\frac{1}{2}i\sqrt{2}\,\arcsin\left(\frac{i\,x}{|x-i|}-\frac{1}{|x-i|}\right)-\frac{\sqrt{x^2-1}}{x-i}+\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*I*sqrt(2)*arcsin(I*x/abs(x - I) - 1/abs(x - I)) - sqrt(x^2 - 1)/(x - I)
 + log(2*x + 2*sqrt(x^2 - 1))
```

**Fricas** [A]
time = 0.86, size = 89, normalized size = 1.39

$$-\frac{\sqrt{2}\,(x-i)\log\left(-x+i\,\sqrt{2}+\sqrt{x^2-1}+i\right)-\sqrt{2}\,(x-i)\log\left(-x-i\,\sqrt{2}+\sqrt{x^2-1}+i\right)+2\,(x-i)\log\left(-x+\sqrt{x^2-1}\right)+2\,x+2\,\sqrt{x^2-1}-2i}{2\,(x-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(x - I)*log(-x + I*sqrt(2) + sqrt(x^2 - 1) + I) - sqrt(2)*(x
- I)*log(-x - I*sqrt(2) + sqrt(x^2 - 1) + I) + 2*(x - I)*log(-x + sqrt(x^2
- 1)) + 2*x + 2*sqrt(x^2 - 1) - 2*I)/(x - I)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}}{(x-i)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(-I+x)**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/(x - I)**2, x)
```

**Giac** [A]
time = 1.16, size = 84, normalized size = 1.31

$$i\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2-1}-i\right)\right)+\frac{2\left(ix-i\sqrt{x^2-1}-1\right)}{\left(x-\sqrt{x^2-1}\right)^2-2ix+2i\sqrt{x^2-1}+1}-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(-I+x)^2,x, algorithm="giac")
```

```
[Out] I*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 - 1) - I)) + 2*(I*x - I*sqrt(x^
2 - 1) - 1)/((x - sqrt(x^2 - 1))^2 - 2*I*x + 2*I*sqrt(x^2 - 1) + 1) - log(a
bs(-x + sqrt(x^2 - 1)))
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^2-1}}{(x-\mathrm{i})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/(x - 1i)^2,x)
```

```
[Out] int((x^2 - 1)^(1/2)/(x - 1i)^2, x)
```

## 3.7 $\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx$

**Optimal**. Leaf size=48

$$-\frac{x\sqrt{-1+x^2}}{4\,(1+x^2)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))*2^(1/2)-1/4*x*(x^2-1)^(1/2)/(x^2+1)
```

**Rubi** [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {390, 385, 212}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2-1}}{4\,(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
```

```
[Out] -1/4*(x*Sqrt[-1 + x^2])/(1 + x^2) + (3*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 + x^2]])
/(4*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)^2}\,dx &= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3}{4}\int \frac{1}{\sqrt{-1+x^2}\,(1+x^2)}\,dx \\
&= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right) \\
&= -\frac{x\sqrt{-1+x^2}}{4(1+x^2)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)}{4\sqrt{2}}
\end{aligned}
$$

**Mathematica** [A]
time = 0.11, size = 55, normalized size = 1.15

$$\frac{1}{8}\left(-\frac{2x\sqrt{-1+x^2}}{1+x^2}+3\sqrt{2}\,\tanh^{-1}\left(\frac{1+x^2-x\sqrt{-1+x^2}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(1 + x^2)^2),x]
[Out] ((-2*x*Sqrt[-1 + x^2])/(1 + x^2) + 3*Sqrt[2]*ArcTanh[(1 + x^2 - x*Sqrt[-1 +
 x^2])/Sqrt[2]])/8
```

**Maple** [A]
time = 0.14, size = 45, normalized size = 0.94

| method | result | size |
|---|---|---|
| risch | $\frac{3\,\text{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}-\frac{x\sqrt{x^2-1}}{4(x^2+1)}$ | 37 |
| default | $-\frac{x}{8\sqrt{x^2-1}\left(\frac{x^2}{x^2-1}-\frac{1}{2}\right)}+\frac{3\,\text{arctanh}\left(\frac{x\sqrt{2}}{\sqrt{x^2-1}}\right)\sqrt{2}}{8}$ | 45 |
| trager | $-\frac{x\sqrt{x^2-1}}{4(x^2+1)}+\frac{3\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\,\text{RootOf}\left(\_Z^2-2\right)x^2+4\sqrt{x^2-1}\,x-\text{RootOf}\left(\_Z^2-2\right)}{x^2+1}\right)}{16}$ | 66 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/8*x/(x^2-1)^(1/2)/(x^2/(x^2-1)-1/2)+3/8*arctanh(x*2^(1/2)/(x^2-1)^(1/2))
*2^(1/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 1)^2*sqrt(x^2 - 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.
time = 0.73, size = 83, normalized size = 1.73

$$\frac{3\sqrt{2}\left(x^2+1\right)\log\left(\frac{9x^2+2\sqrt{2}\left(3x^2-1\right)+2\sqrt{x^2-1}\left(3\sqrt{2}x+4x\right)-3}{x^2+1}\right)-4x^2-4\sqrt{x^2-1}\,x-4}{16\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 1)*log((9*x^2 + 2*sqrt(2)*(3*x^2 - 1) + 2*sqrt(x^2 -
 1)*(3*sqrt(2)*x + 4*x) - 3)/(x^2 + 1)) - 4*x^2 - 4*sqrt(x^2 - 1)*x - 4)/(x
^2 + 1)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x-1)(x+1)}\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(x**2 + 1)**2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(36) = 72.
time = 0.98, size = 101, normalized size = 2.10

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2-1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2-1}\right)^2+2\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2-1}\right)^2+1}{2\left(\left(x-\sqrt{x^2-1}\right)^4+6\left(x-\sqrt{x^2-1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 - 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 -
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 - 1))^2 + 1)/((x - sqrt(x^2
- 1))^4 + 6*(x - sqrt(x^2 - 1))^2 + 1)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2-1}\,\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2),x)
```

```
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 + 1)^2), x)
```

## 3.8 $\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2 \sqrt{-1+x}}\,dx$

**Optimal**. Leaf size=30

$$2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}-\frac{4x^{3/2}}{3}$$

```
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6821, 45}

$$-\frac{4x^{3/2}}{3}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 - (4*x^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{-1+x}+\sqrt{x}\right)^2\sqrt{-1+x}}\,dx = \int \left(-\frac{1}{\sqrt{-1+x}}-2\sqrt{x}+\frac{2x}{\sqrt{-1+x}}\right)dx$$
$$= -2\sqrt{-1+x}-\frac{4x^{3/2}}{3}+2\int \frac{x}{\sqrt{-1+x}}\,dx$$
$$= -2\sqrt{-1+x}-\frac{4x^{3/2}}{3}+2\int \left(\frac{1}{\sqrt{-1+x}}+\sqrt{-1+x}\right)dx$$
$$= 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}-\frac{4x^{3/2}}{3}$$

**Mathematica** [A]
time = 0.19, size = 26, normalized size = 0.87

$$-\frac{4x^{3/2}}{3}+\frac{2}{3}\sqrt{-1+x}\,(1+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((Sqrt[-1 + x] + Sqrt[x])^2*Sqrt[-1 + x]),x]
[Out] (-4*x^(3/2))/3 + (2*Sqrt[-1 + x]*(1 + 2*x))/3
```

**Maple** [A]
time = 0.08, size = 21, normalized size = 0.70

| method | result | size |
|---|---|---|
| default | $\frac{4(-1+x)^{\frac{3}{2}}}{3}-\frac{4x^{\frac{3}{2}}}{3}+2\sqrt{-1+x}$ | 21 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 4/3*(-1+x)^(3/2)-4/3*x^(3/2)+2*(-1+x)^(1/2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - 1)*(sqrt(x - 1) + sqrt(x))^2), x)
```

**Fricas** [A]
time = 0.81, size = 18, normalized size = 0.60

$$\frac{2}{3}\,(2\,x+1)\sqrt{x-1}-\frac{4}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(2*x + 1)*sqrt(x - 1) - 4/3*x^(3/2)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(26) = 52$.
time = 0.27, size = 53, normalized size = 1.77

$$-\frac{4\sqrt{x}}{6\sqrt{x}\,\sqrt{x-1}+6x-3}-\frac{2\sqrt{x-1}}{6\sqrt{x}\,\sqrt{x-1}+6x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/((-1+x)**(1/2)+x**(1/2))**2,x)
```

```
[Out] -4*sqrt(x)/(6*sqrt(x)*sqrt(x - 1) + 6*x - 3) - 2*sqrt(x - 1)/(6*sqrt(x)*sqr
t(x - 1) + 6*x - 3)
```

**Giac** [A]
time = 1.17, size = 20, normalized size = 0.67

$$\frac{4}{3}\,(x-1)^{\frac{3}{2}}-\frac{4}{3}\,x^{\frac{3}{2}}+2\,\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/((-1+x)^(1/2)+x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 4/3*(x - 1)^(3/2) - 4/3*x^(3/2) + 2*sqrt(x - 1)
```

**Mupad** [B]
time = 0.38, size = 21, normalized size = 0.70

$$\frac{4\,x\,\sqrt{x-1}}{3}+\frac{2\,\sqrt{x-1}}{3}-\frac{4\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x - 1)^(1/2) + x^(1/2))^2*(x - 1)^(1/2)),x)
```

```
[Out] (4*x*(x - 1)^(1/2))/3 + (2*(x - 1)^(1/2))/3 - (4*x^(3/2))/3
```

## 3.9 $\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx$

**Optimal**. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right)-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\Bigg($$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

**Rubi** [A]
time = 0.33, antiderivative size = 365, normalized size of antiderivative = 1.66, number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6874, 750, 840, 1180, 213, 209, 1032, 1048, 739, 212, 210, 999}

$$-\frac{2}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\,\text{ArcTan}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\,\text{ArcTan}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\frac{2\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)}+\frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)}-\frac{2}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)+\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - (2*(1 - 2*x)*Sqrt[-1 + x^2])/(5*(
1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])
]*Sqrt[x]])/5 + Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sq
rt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])] - (2*Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTan[
(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(
2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[x]])/5 + Sqrt[2/(
5*(1 + Sqrt[5]))]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt
[-1 + x^2])] - (2*Sqrt[(2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sq
rt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 750

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 999

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x^2}\left(\sqrt{x}+\sqrt{-1+x^2}\right)^2}\,dx = \int \left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2} + \frac{2x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2} + \frac{}{\sqrt{-1+x}}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right) + 2\int \frac{x}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} - \frac{2}{5}\int \frac{-\frac{1}{2}-x}{\sqrt{x}\left(-1-x+x^2\right)}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} - \frac{4}{5}\text{Subst}\left(\int \frac{-\frac{1}{2}-x^2}{-1-x^2+}\right.$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} + \sqrt{\frac{2}{5\left(-1+\sqrt{5}\right)}}\tan^{-}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)} - \frac{2(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)} + \frac{1}{5}\sqrt{\frac{2}{5}\left(-11+5\sqrt{5}\right)}\ \text{t}$$

**Mathematica** [**A**]
time = 6.48, size = 195, normalized size = 0.89

$$\frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2} + \sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right) - \sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right) - \sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right) + \sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x^2]*(Sqrt[x] + Sqrt[-1 + x^2])^2),x]
```

```
[Out] ((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5]]
*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqrt[5
]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*ArcTanh
[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(158) = 316$.
time = 0.12, size = 902, normalized size = 4.10

| method | result | size |
|---|---|---|
| default | Expression too large to display | 902 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)
+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)+2/5/(1/2-1/2*5^(1/2))/(-2+2*
5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*
5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+
2-2*5^(1/2))^(1/2))*5^(1/2)-6/5/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arct
an(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(
x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2))
+1/5*5^(1/2)/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(
-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)-1/5/(1/2+1/2*5^(1/2)
)/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2
)+1/2+1/2*5^(1/2))^(1/2)+6/5/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(
2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2
*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+2/5/(
1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/
2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*
(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))*5^(1/2)-1/5*5^(1/2)/(1/2+1/2*5^(1/2
))/(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/
2)+1/2+1/2*5^(1/2))^(1/2)-6/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(
1/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)
-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))-6/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x+1/2*5^(1/2)-1/2)+4/5/(-2+2*5^(1/2)
)^(1/2)*arctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))-8/25/(-2+2*5^(1/2))^(1/2)*ar
ctan(2*x^(1/2)/(-2+2*5^(1/2))^(1/2))*5^(1/2)+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2
)-4/5/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))-8/25/(2+2*
5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))*5^(1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*(sqrt(x^2 - 1) + sqrt(x))^2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(153) = 306$.
time = 1.13, size = 424, normalized size = 1.93

$$\frac{4\sqrt{5}\left(x^{2}-x-1\right)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{2}\sqrt{2x^{2}-\sqrt{x^{2}-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\frac{1}{4}\left(\sqrt{5}(2x+1)-2\sqrt{x^{2}-1}\left(\sqrt{5}+2\right)+4x+3\right)\sqrt{10\sqrt{5}-22}\right)-4\sqrt{5}\left(x^{2}-x-1\right)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{4}\left(\sqrt{2}\sqrt{2x+\sqrt{5}-1}\left(\sqrt{5}+2\right)-2\sqrt{x}\left(\sqrt{5}+2\right)\right)\sqrt{10\sqrt{5}-22}\right)-\sqrt{5}\left(x^{2}-x-1\right)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^{2}-1}+2\right)+\sqrt{5}\left(x^{2}-x-1\right)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)+\sqrt{5}\left(x^{2}-x-1\right)\sqrt{10\sqrt{5}+22}\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^{2}-1}+2\right)-\sqrt{5}\left(x^{2}-x-1\right)\sqrt{10\sqrt{5}+22}\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)-40x^{2}-20\sqrt{x^{2}-1}(2x-1)+20(2x-1)\sqrt{x}+40x+40}{50\left(x^{2}-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\left(\sqrt{x}+\sqrt{x^{2}-1}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2)/(x**(1/2)+(x**2-1)**(1/2))**2,x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*(sqrt(x) + sqrt(x**2 - 1))**2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(153) = 306$.
time = 2.04, size = 367, normalized size = 1.67

$$\frac{2}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^{2}-1}-1}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\log\left(\left|-153040x+22956\sqrt{5}\sqrt{50\sqrt{5}+110}+76520\sqrt{5}+153040\sqrt{x^{2}-1}-38260\sqrt{50\sqrt{5}+110}+76520\right|\right)-\frac{1}{5}\sqrt{\frac{1}{10}}\sqrt{5\sqrt{5}+11}\log\left(\left|-153040x-22956\sqrt{5}\sqrt{50\sqrt{5}+110}+76520\sqrt{5}+153040\sqrt{x^{2}-1}+38260\sqrt{50\sqrt{5}+110}+76520\right|\right)+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\sqrt{x}+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\left|\sqrt{x}-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{4\left(\left(x-\sqrt{x^{2}-1}\right)^{3}+2\left(x-\sqrt{x^{2}-1}\right)^{2}+3x-3\sqrt{x^{2}-1}-2\right)}{5\left(\left(x-\sqrt{x^{2}-1}\right)^{4}-2\left(x-\sqrt{x^{2}-1}\right)^{3}-2\left(x-\sqrt{x^{2}-1}\right)^{2}-2x+2\sqrt{x^{2}-1}+1\right)}+\frac{2\left(2x^{\frac{3}{2}}-\sqrt{x}\right)}{5\left(x^{2}-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2)/(x^(1/2)+(x^2-1)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/5*sqrt(1/10)*sqrt(5*sqrt(5) - 11)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1)
 - 1)/sqrt(2*sqrt(5) - 2)) + 1/5*sqrt(1/10)*sqrt(5*sqrt(5) + 11)*log(abs(-1
```

```
53040*x + 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 76520*sqrt(5) + 153040*sqr
t(x^2 - 1) - 38260*sqrt(50*sqrt(5) + 110) + 76520)) - 1/5*sqrt(1/10)*sqrt(5
*sqrt(5) + 11)*log(abs(-153040*x - 22956*sqrt(5)*sqrt(50*sqrt(5) + 110) + 7
6520*sqrt(5) + 153040*sqrt(x^2 - 1) + 38260*sqrt(50*sqrt(5) + 110) + 76520)
) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*sqrt(5) - 1/2)) - 1
/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5) + 1/2)) + 1/50*sq
rt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1/2*sqrt(5) + 1/2))) + 4/5*((x
- sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2 + 3*x - 3*sqrt(x^2 - 1) - 2)/(
(x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))^3 - 2*(x - sqrt(x^2 - 1))^2 -
 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2) - sqrt(x))/(x^2 - x - 1)
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x^2-1}\,\left(\sqrt{x^2-1}+\sqrt{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2),x)
[Out] int(1/((x^2 - 1)^(1/2)*((x^2 - 1)^(1/2) + x^(1/2))^2), x)
```

## 3.10 $\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{(1+x-x^2)^2\sqrt{-1+x^2}}\,dx$

**Optimal**. Leaf size=220

$$\frac{2-4x}{5\left(\sqrt{x}+\sqrt{-1+x^2}\right)}+\frac{1}{25}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\,\sqrt{x}\right)-\frac{1}{50}\sqrt{-110+50\sqrt{5}}\,\tan^{-1}\left(\frac{\sqrt{}}{}\right.$$

```
[Out] 1/5*(2-4*x)/(x^(1/2)+(x^2-1)^(1/2))-1/50*arctan((x^2-1)^(1/2)*(-2+2*5^(1/2)
)^(1/2)/(2-x*(-5^(1/2)+1)))*(-110+50*5^(1/2))^(1/2)+1/25*arctan(1/2*x^(1/2)
*(2+2*5^(1/2))^(1/2))*(-110+50*5^(1/2))^(1/2)-1/25*arctanh(1/2*x^(1/2)*(-2+
2*5^(1/2))^(1/2))*(110+50*5^(1/2))^(1/2)-1/50*arctanh((x^2-1)^(1/2)*(2+2*5^
(1/2))^(1/2)/(2-x-x*5^(1/2)))*(110+50*5^(1/2))^(1/2)
```

**Rubi** [B] Leaf count is larger than twice the leaf count of optimal. 541 vs. 2(220) = 440.
time = 0.40, antiderivative size = 541, normalized size of antiderivative = 2.46, number of steps used = 25, number of rules used = 13, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {6874, 750, 840, 1180, 213, 209, 989, 1048, 739, 212, 210, 1032, 1079}

$$\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\,\text{ArcTan}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{10}\left(5\sqrt{5}-11\right)}\,\text{ArcTan}\left(\frac{2-\left(1-\sqrt{5}\right)x}{\sqrt{2\left(\sqrt{5}-1\right)}\sqrt{x^2-1}}\right)+\frac{1}{5}\sqrt{\frac{2}{5}\left(5\sqrt{5}-11\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x}\right)-\frac{\sqrt{x^2-1}(1-2x)}{5(-x^2+x+1)}+\frac{2\sqrt{x}(1-2x)}{5(-x^2+x+1)}-\frac{(3-x)\sqrt{x^2-1}}{5(-x^2+x+1)}+\frac{(x+2)\sqrt{x^2-1}}{5(-x^2+x+1)}+\frac{1}{5}\sqrt{\frac{1}{10}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(2+5\sqrt{5}\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{1}{5}\left(5\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{2-\left(1+\sqrt{5}\right)x}{\sqrt{2\left(1+\sqrt{5}\right)}\sqrt{x^2-1}}\right)-\frac{1}{5}\sqrt{\frac{2}{5}\left(11+5\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (2*(1 - 2*x)*Sqrt[x])/(5*(1 + x - x^2)) - ((1 - 2*x)*Sqrt[-1 + x^2])/(5*(1
+ x - x^2)) - ((3 - x)*Sqrt[-1 + x^2])/(5*(1 + x - x^2)) + ((2 + x)*Sqrt[-1
 + x^2])/(5*(1 + x - x^2)) + (Sqrt[(2*(-11 + 5*Sqrt[5]))/5]*ArcTan[Sqrt[2/(
-1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-11 + 5*Sqrt[5])/10]*ArcTan[(2 - (1 - S
qrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(-2 + 5*Sqrt
[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(2 + 5*Sqrt[5])/5]*ArcTan[(2 - (1 - Sqrt[5])*x)/(Sqrt[2*(-1 +
 Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2*(11 + 5*Sqrt[5]))/5]*ArcTanh[Sqrt
[2/(1 + Sqrt[5])]*Sqrt[x]])/5 - (Sqrt[(-2 + 5*Sqrt[5])/5]*ArcTanh[(2 - (1 +
 Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2])])/5 - (Sqrt[(2 + 5*Sqrt
[5])/5]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1 + Sqrt[5])]*Sqrt[-1 + x^2]
)])/5 + (Sqrt[(11 + 5*Sqrt[5])/10]*ArcTanh[(2 - (1 + Sqrt[5])*x)/(Sqrt[2*(1
 + Sqrt[5])]*Sqrt[-1 + x^2])])/5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 750

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)
*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p + 1
), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 0] && (L
tQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c,
d, e, m, p, x]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
```

```
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1079

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2
)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(
(-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*((-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*
```

```
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(\sqrt{x}-\sqrt{-1+x^2}\right)^2}{\left(1+x-x^2\right)^2\sqrt{-1+x^2}}\,dx = \int \left(-\frac{2\sqrt{x}}{\left(-1-x+x^2\right)^2}-\frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}+\frac{x}{\sqrt{-1+x^2}\,(-1 \ldots}\right.$$

$$= -\left(2\int \frac{\sqrt{x}}{\left(-1-x+x^2\right)^2}\,dx\right)-\int \frac{1}{\sqrt{-1+x^2}\left(-1-x+x^2\right)^2}\,dx+\int \ldots$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{\ldots}}{5\left(1+x\ldots\right.}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{\ldots}}{5\left(1+x\ldots\right.}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{\ldots}}{5\left(1+x\ldots\right.}$$

$$= \frac{2(1-2x)\sqrt{x}}{5\left(1+x-x^2\right)}-\frac{(1-2x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}-\frac{(3-x)\sqrt{-1+x^2}}{5\left(1+x-x^2\right)}+\frac{(2+x)\sqrt{\ldots}}{5\left(1+x\ldots\right.}$$

**Mathematica** [A]

time = 6.59, size = 195, normalized size = 0.89

$$\frac{1}{25}\left(-\frac{10(-1+2x)\left(-\sqrt{x}+\sqrt{-1+x^2}\right)}{-1-x+x^2}+\sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{x}\right)-\sqrt{-110+50\sqrt{5}}\tan^{-1}\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)-\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\sqrt{x}\right)+\sqrt{110+50\sqrt{5}}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{-1+x^2}}{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - Sqrt[-1 + x^2])^2/((1 + x - x^2)^2*Sqrt[-1 + x^2]),x]
```

```
[Out] ((-10*(-1 + 2*x)*(-Sqrt[x] + Sqrt[-1 + x^2]))/(-1 - x + x^2) + Sqrt[-110 +
50*Sqrt[5]]*ArcTan[Sqrt[(1 + Sqrt[5])/2]*Sqrt[x]] - Sqrt[-110 + 50*Sqrt[5]]
*ArcTan[(Sqrt[-2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)] - Sqrt[110 + 50*Sqrt[5
]]*ArcTanh[Sqrt[(-1 + Sqrt[5])/2]*Sqrt[x]] + Sqrt[110 + 50*Sqrt[5]]*ArcTanh
[(Sqrt[2 + Sqrt[5]]*Sqrt[-1 + x^2])/(1 + x)])/25
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(158) = 316$.
time = 0.25, size = 1637, normalized size = 7.44

| method | result | size |
|---|---|---|
| default | Expression too large to display | 1637 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/25*5^(1/2)/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^
(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1
/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2))+(2/5-2/5*5^(1/2))*(-1/4/(1/2-1/2*5^(1/2
))/(x+1/2*5^(1/2)-1/2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1
/2)+1/2-1/2*5^(1/2))^(1/2)-1/4*(-5^(1/2)+1)/(1/2-1/2*5^(1/2))/(-2+2*5^(1/2)
)^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2)
)^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(
1/2))^(1/2)))+(2/5+2/5*5^(1/2))*(-1/4/(1/2+1/2*5^(1/2))/(x-1/2*5^(1/2)-1/2)
*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1
/2)+1/4*(5^(1/2)+1)/(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1
/2)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-
1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+2/25*5^(1/2)/
(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/
(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)
-1/2)+2-2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x-1/2*5^(1/2)-1/2)-8/25*(5/2+5^(1/2)
)/(2+2*5^(1/2))^(1/2)*arctanh(2*x^(1/2)/(2+2*5^(1/2))^(1/2))+2/5*x^(1/2)/(x
+1/2*5^(1/2)-1/2)-8/25*(-5/2+5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*x^(1/2)
/(-2+2*5^(1/2))^(1/2))-4/25*5^(1/2)*(1/4*(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2
)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/4*(5^(1/2)+1)*ln(x+((x-1/2*5^
(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2))-(1/2+1
/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2)+(5^(1/2)+1)*(x-1/2*5^(
```

```
1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/
2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))-1/5/(1/2-1/2*5^(1/2))/(x+1/2*5^(1/2)-1/
2)*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))
^(3/2)+1/10*(-5^(1/2)+1)/(1/2-1/2*5^(1/2))*(1/2*(4*(x+1/2*5^(1/2)-1/2)^2+4*
(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)+1/2*(-5^(1/2)+1)*ln(x+(
(x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/
2))+2*(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arctan(2*(1-5^(1/2)+(-5^(1/2)+
1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5
^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)))+2/5/(1/2-1/2*5^(1/2))*(1
/2*x*((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2
))^(1/2)+1/8*(2-2*5^(1/2)-(-5^(1/2)+1)^2)*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(
1/2)+1)*(x+1/2*5^(1/2)-1/2)+1/2-1/2*5^(1/2))^(1/2)))-1/5/(1/2+1/2*5^(1/2))/
(x-1/2*5^(1/2)-1/2)*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+
1/2+1/2*5^(1/2))^(3/2)+1/10*(5^(1/2)+1)/(1/2+1/2*5^(1/2))*(1/2*(4*(x-1/2*5^
(1/2)-1/2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)+1/2*(5^(1
/2)+1)*ln(x+((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*
5^(1/2))^(1/2))-2*(1/2+1/2*5^(1/2))/(2+2*5^(1/2))^(1/2)*arctanh(2*(1+5^(1/2
)+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2))/(2+2*5^(1/2))^(1/2)/(4*(x-1/2*5^(1/2)-1/
2)^2+4*(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+2+2*5^(1/2))^(1/2)))+2/5/(1/2+1/2*5^
(1/2))*(1/2*x*((x-1/2*5^(1/2)-1/2)^2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/
2*5^(1/2))^(1/2)+1/8*(2+2*5^(1/2)-(5^(1/2)+1)^2)*ln(x+((x-1/2*5^(1/2)-1/2)^
2+(5^(1/2)+1)*(x-1/2*5^(1/2)-1/2)+1/2+1/2*5^(1/2))^(1/2)))+4/25*5^(1/2)*(1/
4*(4*(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^
(1/2)+1/4*(-5^(1/2)+1)*ln(x+((x+1/2*5^(1/2)-1/2)^2+(-5^(1/2)+1)*(x+1/2*5^(1
/2)-1/2)+1/2-1/2*5^(1/2))^(1/2))+(1/2-1/2*5^(1/2))/(-2+2*5^(1/2))^(1/2)*arc
tan(2*(1-5^(1/2)+(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2))/(-2+2*5^(1/2))^(1/2)/(4*
(x+1/2*5^(1/2)-1/2)^2+4*(-5^(1/2)+1)*(x+1/2*5^(1/2)-1/2)+2-2*5^(1/2))^(1/2)
))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/5*(x^(5/2) - 3*x^(3/2))/(x^2 - x - 1) + integrate(1/5*(x^(3/2) + sqrt(x)
)/(x^2 - x - 1), x) + integrate((x^2 + x - 1)/((x^4 - 2*x^3 - x^2 + 2*x + 1
)*sqrt(x + 1)*sqrt(x - 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(153) = 306$.
time = 0.90, size = 424, normalized size = 1.93

$$\frac{4\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{2}\sqrt{2x^2-\sqrt{x^2-1}\left(2x+\sqrt{5}-1\right)+\sqrt{5}x-x}\sqrt{10\sqrt{5}-22}\left(\sqrt{5}+2\right)+\frac{1}{4}\left(\sqrt{5}(2x+1)-2\sqrt{x^2-1}\left(\sqrt{5}+2\right)+4x+3\right)\sqrt{10\sqrt{5}-22}\right)-4\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}-22}\arctan\left(\frac{1}{4}\left(\sqrt{2}\sqrt{2x+\sqrt{5}-1}\left(\sqrt{5}+2\right)-2\sqrt{x}\left(\sqrt{5}+2\right)\right)\sqrt{10\sqrt{5}-22}\right)-\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)+\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)+\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)-4x+2\sqrt{5}+4\sqrt{x^2-1}+2\right)-\sqrt{5}(x^2-x-1)\sqrt{10\sqrt{5}+22}\log\left(-\sqrt{10\sqrt{5}+22}\left(\sqrt{5}-3\right)+4\sqrt{x}\right)-40x^2-20\sqrt{x^2-1}(2x-1)+20(2x-1)\sqrt{x}+40x+40}{50(x^2-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/50*(4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22)*arctan(1/2*sqrt(2*x^2 -
 sqrt(x^2 - 1)*(2*x + sqrt(5) - 1) + sqrt(5)*x - x)*sqrt(10*sqrt(5) - 22)*(
sqrt(5) + 2) + 1/4*(sqrt(5)*(2*x + 1) - 2*sqrt(x^2 - 1)*(sqrt(5) + 2) + 4*x
 + 3)*sqrt(10*sqrt(5) - 22)) - 4*sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) - 22
)*arctan(1/4*(sqrt(2)*sqrt(2*x + sqrt(5) - 1)*(sqrt(5) + 2) - 2*sqrt(x)*(sq
rt(5) + 2))*sqrt(10*sqrt(5) - 22)) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
+ 22)*log(sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(x^
2 - 1) + 2) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(sqrt(10*sqrt(
5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) + sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5)
 + 22)*log(-sqrt(10*sqrt(5) + 22)*(sqrt(5) - 3) - 4*x + 2*sqrt(5) + 4*sqrt(
x^2 - 1) + 2) - sqrt(5)*(x^2 - x - 1)*sqrt(10*sqrt(5) + 22)*log(-sqrt(10*sq
rt(5) + 22)*(sqrt(5) - 3) + 4*sqrt(x)) - 40*x^2 - 20*sqrt(x^2 - 1)*(2*x - 1
) + 20*(2*x - 1)*sqrt(x) + 40*x + 40)/(x^2 - x - 1)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**2-1)**(1/2))**2/(-x**2+x+1)**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(153) = 306.
time = 1.88, size = 358, normalized size = 1.63

$$\frac{2}{5}\sqrt{\frac{1}{2}\sqrt{5}-\frac{11}{10}}\arctan\left(\frac{2x+\sqrt{5}-2\sqrt{x^2-1}-1}{\sqrt{2\sqrt{5}-2}}\right)+\frac{1}{25}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{\sqrt{x}}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\sqrt{x}+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(\left|-520x-78\sqrt{5}\sqrt{50\sqrt{5}+110}+260\sqrt{5}+520\sqrt{x^2-1}+130\sqrt{50\sqrt{5}+110}+260\right|\right)+\frac{1}{5}\sqrt{\frac{1}{2}\sqrt{5}+\frac{11}{10}}\log\left(\left|-1040x+156\sqrt{5}\sqrt{50\sqrt{5}+110}+520\sqrt{5}+1040\sqrt{x^2-1}-260\sqrt{50\sqrt{5}+110}+520\right|\right)+\frac{1}{50}\sqrt{50\sqrt{5}+110}\log\left(\left|\sqrt{x}-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{4\left(\left(x-\sqrt{x^2-1}\right)^3+2\left(x-\sqrt{x^2-1}\right)^2+3x-3\sqrt{x^2-1}-2\right)}{5\left(\left(x-\sqrt{x^2-1}\right)^4-2\left(x-\sqrt{x^2-1}\right)^3-2\left(x-\sqrt{x^2-1}\right)^2-2x+2\sqrt{x^2-1}+1\right)}+\frac{2\left(2x^{\frac{3}{2}}-\sqrt{x}\right)}{5\left(x^2-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^2-1)^(1/2))^2/(-x^2+x+1)^2/(x^2-1)^(1/2),x, algorithm
="giac")
```

```
[Out] 2/5*sqrt(1/2*sqrt(5) - 11/10)*arctan((2*x + sqrt(5) - 2*sqrt(x^2 - 1) - 1)/
sqrt(2*sqrt(5) - 2)) + 1/25*sqrt(50*sqrt(5) - 110)*arctan(sqrt(x)/sqrt(1/2*
sqrt(5) - 1/2)) - 1/50*sqrt(50*sqrt(5) + 110)*log(sqrt(x) + sqrt(1/2*sqrt(5
) + 1/2)) - 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-520*x - 78*sqrt(5)*sqrt(
50*sqrt(5) + 110) + 260*sqrt(5) + 520*sqrt(x^2 - 1) + 130*sqrt(50*sqrt(5) +
 110) + 260)) + 1/5*sqrt(1/2*sqrt(5) + 11/10)*log(abs(-1040*x + 156*sqrt(5)
*sqrt(50*sqrt(5) + 110) + 520*sqrt(5) + 1040*sqrt(x^2 - 1) - 260*sqrt(50*sq
```

```
rt(5) + 110) + 520)) + 1/50*sqrt(50*sqrt(5) + 110)*log(abs(sqrt(x) - sqrt(1
/2*sqrt(5) + 1/2))) + 4/5*((x - sqrt(x^2 - 1))^3 + 2*(x - sqrt(x^2 - 1))^2
+ 3*x - 3*sqrt(x^2 - 1) - 2)/((x - sqrt(x^2 - 1))^4 - 2*(x - sqrt(x^2 - 1))
^3 - 2*(x - sqrt(x^2 - 1))^2 - 2*x + 2*sqrt(x^2 - 1) + 1) + 2/5*(2*x^(3/2)
- sqrt(x))/(x^2 - x - 1)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sqrt{x^2-1}-\sqrt{x}\right)^2}{\sqrt{x^2-1}\,\left(-x^2+x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2),x)
[Out] int(((x^2 - 1)^(1/2) - x^(1/2))^2/((x^2 - 1)^(1/2)*(x - x^2 + 1)^2), x)
```

## 3.11 $\int \left(\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}\,(1+x)^2\sqrt{i+x^2}}\right)dx$

**Optimal**. Leaf size=138

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}\,(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}\,(1+x)}+\frac{\tanh^{-1}\left(\frac{i+x}{\sqrt{1-i}\,\sqrt{-i+x^2}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{i-x}{\sqrt{1+i}\,\sqrt{i+x^2}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

```
[Out] 1/2*arctanh((I+x)/(1-I)^(1/2)/(-I+x^2)^(1/2))/(1-I)^(3/2)*2^(1/2)-1/2*arcta
nh((I-x)/(1+I)^(1/2)/(I+x^2)^(1/2))/(1+I)^(3/2)*2^(1/2)-(1/4+1/4*I)*(-I+x^2
)^(1/2)/(1+x)*2^(1/2)+(-1/4+1/4*I)*(I+x^2)^(1/2)/(1+x)*2^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {745, 739, 212}

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^2-i}}{\sqrt{2}\,(x+1)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{x^2+i}}{\sqrt{2}\,(x+1)}+\frac{\tanh^{-1}\left(\frac{x+i}{\sqrt{1-i}\,\sqrt{x^2-i}}\right)}{(1-i)^{3/2}\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{-x+i}{\sqrt{1+i}\,\sqrt{x^2+i}}\right)}{(1+i)^{3/2}\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[I + x^
2]),x]
```

```
[Out] ((-1/2 - I/2)*Sqrt[-I + x^2])/(Sqrt[2]*(1 + x)) - ((1/2 - I/2)*Sqrt[I + x^2
])/(Sqrt[2]*(1 + x)) + ArcTanh[(I + x)/(Sqrt[1 - I]*Sqrt[-I + x^2])]/((1 -
I)^(3/2)*Sqrt[2]) - ArcTanh[(I - x)/(Sqrt[1 + I]*Sqrt[I + x^2])]/((1 + I)^(
3/2)*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
```

```
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rubi steps

$$\int \left(\frac{1}{\sqrt{2}(1+x)^2\sqrt{-i+x^2}}+\frac{1}{\sqrt{2}(1+x)^2\sqrt{i+x^2}}\right)dx = \frac{\int \frac{1}{(1+x)^2\sqrt{-i+x^2}}dx}{\sqrt{2}}+\frac{\int \frac{1}{(1+x)^2\sqrt{i+x^2}}dx}{\sqrt{2}}$$

$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+$$

$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+$$

$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-i+x^2}}{\sqrt{2}(1+x)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{i+x^2}}{\sqrt{2}(1+x)}+$$

**Mathematica** [A]
time = 3.74, size = 126, normalized size = 0.91

$$-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{-i+x^2}+\sqrt{i+x^2}+\frac{2(1+x)\tan^{-1}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\left(1+x-\sqrt{-i+x^2}\right)\right)}{\sqrt{1-i}}+(1+i)^{3/2}(1+x)\tan^{-1}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\left(1+x-\sqrt{i+x^2}\right)\right)\right)}{\sqrt{2}(1+x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2]*(1 + x)^2*Sqrt[-I + x^2]) + 1/(Sqrt[2]*(1 + x)^2*Sqrt[
I + x^2]),x]
```

```
[Out] ((-1/2 + I/2)*(I*Sqrt[-I + x^2] + Sqrt[I + x^2] + (2*(1 + x)*ArcTan[Sqrt[-1
/2 - I/2]*(1 + x - Sqrt[-I + x^2])])/Sqrt[1 - I] + (1 + I)^(3/2)*(1 + x)*Ar
cTan[Sqrt[-1/2 + I/2]*(1 + x - Sqrt[I + x^2])]))/(Sqrt[2]*(1 + x))
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(101) = 202.
time = 0.12, size = 278, normalized size = 2.01

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{\sqrt{2}\,\sqrt{(1+x)^{2}-2x-1-i}}{4(1+x)}-\frac{i\sqrt{2}\,\sqrt{(1+x)^{2}-2x-1-i}}{4(1+x)}-\frac{\sqrt{2}\,\ln\left(\frac{-2i-2x+2\sqrt{1-i}\,\sqrt{(1+}{1+x}\right.}{4\sqrt{1-i}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(1+x)^2*2^(1/2)/(x^2-I)^(1/2)+1/2/(1+x)^2*2^(1/2)/(x^2+I)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)/(1+x)*((1+x)^2-2*x-1-I)^(1/2)-1/4*I*2^(1/2)/(1+x)*((1+x)^2-2*x
-1-I)^(1/2)-1/4*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((1+x)^2-2*x
-1-I)^(1/2))/(1+x))-1/4*I*2^(1/2)/(1-I)^(1/2)*ln((-2*I-2*x+2*(1-I)^(1/2)*((
1+x)^2-2*x-1-I)^(1/2))/(1+x))-1/4*2^(1/2)/(1+x)*((1+x)^2-2*x-1+I)^(1/2)+1/4
*I*2^(1/2)/(1+x)*((1+x)^2-2*x-1+I)^(1/2)-1/4*2^(1/2)/(1+I)^(1/2)*ln((2*I-2*
x+2*(1+I)^(1/2)*((1+x)^2-2*x-1+I)^(1/2))/(1+x))+1/4*I*2^(1/2)/(1+I)^(1/2)*l
n((2*I-2*x+2*(1+I)^(1/2)*((1+x)^2-2*x-1+I)^(1/2))/(1+x))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  1which is not of the expected type LIST
```

**Fricas [A]**
time = 0.77, size = 161, normalized size = 1.17

$$\frac{\sqrt{-\frac{1}{2}i+\frac{1}{2}}\,(-(i-1)\,x-i+1)\log\left(\sqrt{2}\,\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^{2}-i}-1\right)+\sqrt{-\frac{1}{2}i+\frac{1}{2}}\,((i-1)\,x+i-1)\log\left(-\sqrt{2}\,\sqrt{-\frac{1}{2}i+\frac{1}{2}}-x+\sqrt{x^{2}-i}-1\right)+\sqrt{-\frac{1}{2}i-\frac{1}{2}}\,(-(i+1)\,x-i-1)\log\left(i\sqrt{2}\,\sqrt{-\frac{1}{2}i-\frac{1}{2}}-x+\sqrt{x^{2}+i}-1\right)+\sqrt{-\frac{1}{2}i-\frac{1}{2}}\,((i+1)\,x+i+1)\log\left(-i\sqrt{2}\,\sqrt{-\frac{1}{2}i-\frac{1}{2}}-x+\sqrt{x^{2}+i}-1\right)+\sqrt{2}\,(-(i+1)\,x-i-1)-\sqrt{2}\,\sqrt{x^{2}+i}-i\sqrt{2}\,\sqrt{x^{2}-i}}{(2i+2)\,x+2i+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-1/2*I + 1/2)*(-(I - 1)*x - I + 1)*log(sqrt(2)*sqrt(-1/2*I + 1/2) - x
 + sqrt(x^2 - I) - 1) + sqrt(-1/2*I + 1/2)*((I - 1)*x + I - 1)*log(-sqrt(2)
*sqrt(-1/2*I + 1/2) - x + sqrt(x^2 - I) - 1) + sqrt(-1/2*I - 1/2)*(-(I + 1)
*x - I - 1)*log(I*sqrt(2)*sqrt(-1/2*I - 1/2) - x + sqrt(x^2 + I) - 1) + sqr
t(-1/2*I - 1/2)*((I + 1)*x + I + 1)*log(-I*sqrt(2)*sqrt(-1/2*I - 1/2) - x +
```

```
 sqrt(x^2 + I) - 1) + sqrt(2)*(-(I + 1)*x - I - 1) - sqrt(2)*sqrt(x^2 + I)
- I*sqrt(2)*sqrt(x^2 - I))/((2*I + 2)*x + 2*I + 2)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)**2*2**(1/2)/(-I+x**2)**(1/2)+1/2/(1+x)**2*2**(1/2)/(I+x
**2)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real I
```

**Giac [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(82) = 164.
time = 0.88, size = 547, normalized size = 3.96

$$\sqrt{2}\left(\frac{-(i-1)\sqrt{2x^2+2\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+(2i-2)x+2i+2}{\left(\sqrt{2x^2+2\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2x\right)^2-4\sqrt{2x^2+2\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+8x-4i}-\frac{(i-1)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2x^2+2\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2x\right)-\sqrt{2x^2+2\sqrt{x^4+1}}\left(\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+2x-2\sqrt{2}+2}{\sqrt{2}\sqrt{2\sqrt{2}-2}-(i+1)\sqrt{2\sqrt{2}-2}}\right)}{\sqrt{2\sqrt{2}-2}\left(-\frac{i}{\sqrt{2}-1}+1\right)}\right)+\sqrt{2}\left(\frac{(i+1)\sqrt{2x^2+2\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-(2i+2)x-2i+2}{\left(\sqrt{2x^2+2\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2x\right)^2-4\sqrt{2x^2+2\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+8x+4i}+\frac{(i+1)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2x^2+2\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)-2x\right)-\sqrt{2x^2+2\sqrt{x^4+1}}\left(-\frac{i}{x^2+\sqrt{x^4+1}}+1\right)+2x-2\sqrt{2}+2}{\sqrt{2}\sqrt{2\sqrt{2}-2}+(i-1)\sqrt{2\sqrt{2}-2}}\right)}{\sqrt{2\sqrt{2}-2}\left(\frac{i}{\sqrt{2}-1}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)^2*2^(1/2)/(-I+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(I+x^2)^(1
/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*((-(I - 1)*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqrt(x^4 + 1)) +
 1) + (2*I - 2)*x + 2*I + 2)/((sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqrt
(x^4 + 1)) + 1) - 2*x)^2 - 4*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(I/(x^2 + sqrt(x
^4 + 1)) + 1) + 8*x - 4*I) - (I - 1)*arctan((sqrt(2)*(sqrt(2*x^2 + 2*sqrt(x
^4 + 1))*(I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x) - sqrt(2*x^2 + 2*sqrt(x^4 + 1
))*(I/(x^2 + sqrt(x^4 + 1)) + 1) + 2*x - 2*sqrt(2) + 2)/(sqrt(2)*sqrt(2*sqr
t(2) - 2) - (I + 1)*sqrt(2*sqrt(2) - 2)))/(sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2)
 - 1) + 1))) + sqrt(2)*(((I + 1)*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + s
qrt(x^4 + 1)) + 1) - (2*I + 2)*x - 2*I + 2)/((sqrt(2*x^2 + 2*sqrt(x^4 + 1))
*(-I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x)^2 - 4*sqrt(2*x^2 + 2*sqrt(x^4 + 1))*
(-I/(x^2 + sqrt(x^4 + 1)) + 1) + 8*x + 4*I) + (I + 1)*arctan((sqrt(2)*(sqrt
(2*x^2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) - 2*x) - sqrt(2*x^
2 + 2*sqrt(x^4 + 1))*(-I/(x^2 + sqrt(x^4 + 1)) + 1) + 2*x - 2*sqrt(2) + 2)/
(sqrt(2)*sqrt(2*sqrt(2) - 2) + (I - 1)*sqrt(2*sqrt(2) - 2)))/(sqrt(2*sqrt(2
) - 2)*(I/(sqrt(2) - 1) + 1)))
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2}}{2\sqrt{x^2-\mathrm{i}}\,(x+1)^2}+\frac{\sqrt{2}}{2\sqrt{x^2+1\mathrm{i}}\,(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2),x)
```

```
[Out] int(2^(1/2)/(2*(x^2 - 1i)^(1/2)*(x + 1)^2) + 2^(1/2)/(2*(x^2 + 1i)^(1/2)*(x
 + 1)^2), x)
```

## 3.12 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx$

**Optimal**. Leaf size=125

$$-\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

```
[Out] -1/4*(1-I)^(3/2)*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))-1/4*(1+I)^(3/
2)*arctanh((1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))-1/2*(1-I*x^2)^(1/2)/(1+x)-1
/2*(1+I*x^2)^(1/2)/(1+x)
```

**Rubi** [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2158, 745, 739, 212}

$$-\frac{\sqrt{1-ix^2}}{2(x+1)}-\frac{\sqrt{1+ix^2}}{2(x+1)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)^{3/2}\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*Sqrt[1 - I*x^2]/(1 + x) - Sqrt[1 + I*x^2]/(2*(1 + x)) - ((1 - I)^(3/2)
*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])])/4 - ((1 + I)^(3/2)*ArcTa
nh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)^2\sqrt{1+x^4}}\,dx &= \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1-ix^2}}\,dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)^2\sqrt{1+ix^2}}\,dx\\ &= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx+\frac{1}{2}i\int \frac{1}{(1+x)\sqrt{1+ix^2}}\\ &= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}+\frac{1}{2}i\text{Subst}\left(\int \frac{1}{(1-i)-x^2}\,dx,x,\frac{1+ix}{\sqrt{1-ix^2}}\right)-\frac{1}{2}i\text{Su}\\ &= -\frac{\sqrt{1-ix^2}}{2(1+x)}-\frac{\sqrt{1+ix^2}}{2(1+x)}-\frac{1}{4}(1-i)^{3/2}\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{4}(1+i)\end{aligned}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(125) = 250.
time = 2.31, size = 272, normalized size = 2.18

$$\frac{1}{2}\left(\frac{-1-2x^4-\sqrt{1+x^4}-x^2\left(1+2\sqrt{1+x^4}\right)}{(1+x)\left(x^2+\sqrt{1+x^4}\right)^{3/2}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{-1+\sqrt{2}}}-\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)-\frac{\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\,\sqrt{x^2+\sqrt{1+x^4}}\right)}{\sqrt{1+\sqrt{2}}}+\sqrt{-1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)^2*Sqrt[1 + x^4]),x]
```

```
[Out] ((-1 - 2*x^4 - Sqrt[1 + x^4] - x^2*(1 + 2*Sqrt[1 + x^4]))/((1 + x)*(x^2 + S
qrt[1 + x^4])^(3/2)) + ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/
Sqrt[-1 + Sqrt[2]] - Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqr
t[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])] - ArcTanh[Sqrt[-1 + Sqrt
[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[1 + Sqrt[2]] + Sqrt[-1 + Sqrt[2]]*ArcT
anh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 +
 x^4])])/2
```

**Maple** [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\left(1+x\right)^2\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(81) = 162$.
time = 2.28, size = 394, normalized size = 3.15

$$\frac{4\,(x+1)\sqrt{\sqrt{2}+1}\,\arctan\left(\frac{2\left(x^3+x^2-\sqrt{2}\,(x^3+1)+\sqrt{x^4+1}\left(\sqrt{2}\,x-x-1\right)-x+1\right)\sqrt{x^2+\sqrt{x^4+1}}\,\sqrt{\sqrt{2}+1}+\left(2\,x^2-\sqrt{2}\,(x^2+1)+2\,\sqrt{x^4+1}\left(\sqrt{2}-1\right)+2\right)\sqrt{2\,\sqrt{2}+2}\,\sqrt{\sqrt{2}+1}}{2\,(x^2-2\,x+1)}\right)+(x+1)\sqrt{\sqrt{2}-1}\,\log\left(-\frac{\left(2\,x^3-\sqrt{2}\,(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}\,(x-1)-2\,x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(\sqrt{2}\,(x^2+1)+2\,\sqrt{x^4+1}\right)\sqrt{\sqrt{2}-1}}{x^2+2\,x+1}\right)-(x+1)\sqrt{\sqrt{2}-1}\,\log\left(-\frac{\left(2\,x^3-\sqrt{2}\,(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}\,(x-1)-2\,x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(\sqrt{2}\,(x^2+1)+2\,\sqrt{x^4+1}\right)\sqrt{\sqrt{2}-1}}{x^2+2\,x+1}\right)+4\,\sqrt{x^2+\sqrt{x^4+1}}\left(x^2-\sqrt{x^4+1}-1\right)}{8\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*(4*(x + 1)*sqrt(sqrt(2) + 1)*arctan(1/2*(2*(x^3 + x^2 - sqrt(2)*(x^3 +
1) + sqrt(x^4 + 1)*(sqrt(2)*x - x - 1) - x + 1)*sqrt(x^2 + sqrt(x^4 + 1))*s
qrt(sqrt(2) + 1) + (2*x^2 - sqrt(2)*(x^2 + 1) + 2*sqrt(x^4 + 1)*(sqrt(2) -
1) + 2)*sqrt(2*sqrt(2) + 2)*sqrt(sqrt(2) + 1))/(x^2 - 2*x + 1)) + (x + 1)*s
qrt(sqrt(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)
*(sqrt(2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) + (sqrt(2)*(x^2 + 1
) + 2*sqrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) - (x + 1)*sqrt(sqr
t(2) - 1)*log(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(
2)*(x - 1) - 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (sqrt(2)*(x^2 + 1) + 2*s
qrt(x^4 + 1))*sqrt(sqrt(2) - 1))/(x^2 + 2*x + 1)) + 4*sqrt(x^2 + sqrt(x^4 +
 1))*(x^2 - sqrt(x^4 + 1) - 1))/(x + 1)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)^2\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)**2/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)**2*sqrt(x**4 + 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)^2/(x^4+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)^2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,{\left(x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)^2), x)
```

## 3.13 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx$

**Optimal**. Leaf size=81

$$-\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

```
[Out] -1/2*arctanh((1+I*x)/(1-I)^(1/2)/(1-I*x^2)^(1/2))*(1-I)^(1/2)-1/2*arctanh((
1-I*x)/(1+I)^(1/2)/(1+I*x^2)^(1/2))*(1+I)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2158, 739, 212}

$$-\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right)-\frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*(Sqrt[1 - I]*ArcTanh[(1 + I*x)/(Sqrt[1 - I]*Sqrt[1 - I*x^2])]) - (Sqrt
[1 + I]*ArcTanh[(1 - I*x)/(Sqrt[1 + I]*Sqrt[1 + I*x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{(1+x)\sqrt{1+x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1-ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(1+x)\sqrt{1+ix^2}}\,dx$$
$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{(1+i)-x^2}\,dx, x, \frac{1-ix}{\sqrt{1+ix^2}}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right)\text{Subst}\left(\int \frac{}{(1}\right.$$
$$= -\frac{1}{2}\sqrt{1-i}\,\tanh^{-1}\left(\frac{1+ix}{\sqrt{1-i}\,\sqrt{1-ix^2}}\right) - \frac{1}{2}\sqrt{1+i}\,\tanh^{-1}\left(\frac{1-ix}{\sqrt{1+i}\,\sqrt{1+ix^2}}\right.$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(81) = 162.
time = 0.91, size = 205, normalized size = 2.53

$$\frac{\sqrt{-1+\sqrt{2}}\left(\tan^{-1}\left(\sqrt{1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)-\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)\right)-\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\sqrt{-1+\sqrt{2}}\sqrt{x^2+\sqrt{1+x^4}}\right)+\sqrt{1+\sqrt{2}}\,\tanh^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}\,x\sqrt{x^2+\sqrt{1+x^4}}}{1+x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/((1 + x)*Sqrt[1 + x^4]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[2]]*(ArcTan[Sqrt[1 + Sqrt[2]]*Sqrt[x^2 + Sqrt[1 + x^4]]] -
ArcTan[(Sqrt[2*(-1 + Sqrt[2])]*x*Sqrt[x^2 + Sqrt[1 + x^4]])/(1 + x^2 + Sqrt
[1 + x^4])]) - Sqrt[1 + Sqrt[2]]*ArcTanh[Sqrt[-1 + Sqrt[2]]*Sqrt[x^2 + Sqrt
[1 + x^4]]] + Sqrt[1 + Sqrt[2]]*ArcTanh[(Sqrt[2*(1 + Sqrt[2])]*x*Sqrt[x^2 +
 Sqrt[1 + x^4]])/(1 + x^2 + Sqrt[1 + x^4])])/Sqrt[2]
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(1+x)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

```
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(49) = 98$.
time = 5.35, size = 369, normalized size = 4.56

$$\frac{1}{2}\sqrt{2\sqrt{2}-2}\arctan\left(\frac{\left(2x^2-\sqrt{2}(x^3-x^2+x+1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2\right)-2x\right)\sqrt{x^2+\sqrt{x^4+1}}\sqrt{2\sqrt{2}-2}+\left(x^2+\sqrt{2}\sqrt{x^4+1}+1\right)\sqrt{2\sqrt{2}+2}\sqrt{2\sqrt{2}-2}}{2(x^2-2x+1)}\right)-\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}+\left(x^2-\sqrt{2}(x^2+1)+\sqrt{x^4+1}\left(\sqrt{2}-2\right)+1\right)\sqrt{2\sqrt{2}+2}}{x^2+2x+1}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+2}\log\left(-\frac{\left(2x^3-\sqrt{2}(x^3-x^2-x-1)+\sqrt{x^4+1}\left(\sqrt{2}(x-1)-2x\right)-2\right)\sqrt{x^2+\sqrt{x^4+1}}-\left(x^2-\sqrt{2}(x^2+1)+\sqrt{x^4+1}\left(\sqrt{2}-2\right)+1\right)\sqrt{2\sqrt{2}+2}}{x^2+2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(2*sqrt(2) - 2)*arctan(1/2*((2*x^2 - sqrt(2)*(x^3 - x^2 + x + 1) +
sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2) - 2*x)*sqrt(x^2 + sqrt(x^4 + 1))*sqrt(2
*sqrt(2) - 2) + (x^2 + sqrt(2)*sqrt(x^4 + 1) + 1)*sqrt(2*sqrt(2) + 2)*sqrt(
2*sqrt(2) - 2))/(x^2 - 2*x + 1)) - 1/8*sqrt(2*sqrt(2) + 2)*log(-((2*x^3 - s
qrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1) - 2*x) - 2)*sqr
t(x^2 + sqrt(x^4 + 1)) + (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4 + 1)*(sqrt(2)
- 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1)) + 1/8*sqrt(2*sqrt(2) + 2)*l
og(-((2*x^3 - sqrt(2)*(x^3 - x^2 - x - 1) + sqrt(x^4 + 1)*(sqrt(2)*(x - 1)
- 2*x) - 2)*sqrt(x^2 + sqrt(x^4 + 1)) - (x^2 - sqrt(2)*(x^2 + 1) + sqrt(x^4
 + 1)*(sqrt(2) - 2) + 1)*sqrt(2*sqrt(2) + 2))/(x^2 + 2*x + 1))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{(x+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(1+x)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + sqrt(x**4 + 1))/((x + 1)*sqrt(x**4 + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(1+x)/(x^4+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/(sqrt(x^4 + 1)*(x + 1)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\sqrt{x^4+1}+x^2}}{\sqrt{x^4+1}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)),x)
```

```
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/((x^4 + 1)^(1/2)*(x + 1)), x)
```

## 3.14 $\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}} \, dx$

**Optimal**. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)/(x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2157, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}+x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

**Mathematica** [**A**]
time = 0.15, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

**Maple** [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+\sqrt{x^4+1}}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
[Out] int((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.
time = 0.96, size = 60, normalized size = 1.94

$$\frac{1}{4}\sqrt{2}\log\left(4x^4 + 4\sqrt{x^4+1}\,x^2 + 2\left(\sqrt{2}\,x^3 + \sqrt{2}\sqrt{x^4+1}\,x\right)\sqrt{x^2+\sqrt{x^4+1}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(4*x^4 + 4*sqrt(x^4 + 1)*x^2 + 2*(sqrt(2)*x^3 + sqrt(2)*sqrt
(x^4 + 1)*x)*sqrt(x^2 + sqrt(x^4 + 1)) + 1)
```

**Sympy** [A]
time = 0.91, size = 15, normalized size = 0.48

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix} 1, 1 & \frac{1}{2} \\ \frac{1}{4}, \frac{3}{4} & 0 \end{matrix} \middle| x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1, 1), (1/2,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^4+1} + x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) + x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

## 3.15 $\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx$

**Optimal**. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(-x^2+(x^4+1)^(1/2))^(1/2))*2^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = $\{2157, 209\}$

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{-x^2+\sqrt{1+x^4}}}{\sqrt{1+x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

**Mathematica** [A]
time = 0.15, size = 33, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x^2 + Sqrt[1 + x^4]]/Sqrt[1 + x^4],x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-x^2 + Sqrt[1 + x^4]]]/Sqrt[2]
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.10, size = 22, normalized size = 0.67

| method | result | size |
|---|---|---|
| meijerg | $-\frac{\sqrt{2}\ \text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4},\frac{5}{4}\right],\left[\frac{3}{2},\frac{3}{2}\right],-\frac{1}{x^4}\right)}{4x^2}$ | 22 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*2^(1/2)/x^2*hypergeom([1/2,3/4,5/4],[3/2,3/2],-1/x^4)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

**Fricas [A]**
time = 0.80, size = 29, normalized size = 0.88

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^4+1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^4 + 1))/x)
```

**Sympy [A]**
time = 0.40, size = 15, normalized size = 0.45

$$\frac{G_{3,3}^{2,2}\left(\begin{matrix}\frac{1}{2},1 & 1\\ \frac{1}{4},\frac{3}{4} & 0\end{matrix}\middle|\,x^4\right)}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+(x**4+1)**(1/2))**(1/2)/(x**4+1)**(1/2),x)
[Out] meijerg(((1/2, 1), (1,)), ((1/4, 3/4), (0,)), x**4)/(4*sqrt(pi))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+(x^4+1)^(1/2))^(1/2)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + sqrt(x^4 + 1))/sqrt(x^4 + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\sqrt{x^4+1}-x^2}}{\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2),x)
[Out] int(((x^4 + 1)^(1/2) - x^2)^(1/2)/(x^4 + 1)^(1/2), x)
```

# 3.16 $\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} \, dx$

**Optimal**. Leaf size=19

$$-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$$

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

**Rubi** [A]
time = 0.24, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {6820}

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+x)^{3/2}+(1+x)^{3/2}}{(-1+x)^{3/2}(1+x)^{3/2}} \, dx &= \int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{(1+x)^{3/2}}\right) dx \\ &= -\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}\end{aligned}$$

**Mathematica** [A]
time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^(3/2) + (1 + x)^(3/2))/((-1 + x)^(3/2)*(1 + x)^(3/2)),x
]
```

```
[Out] -2/Sqrt[-1 + x] - 2/Sqrt[1 + x]
```

**Maple** [A]
time = 0.16, size = 16, normalized size = 0.84

| method | result | size |
|---|---|---|
| default | $-\frac{2}{\sqrt{-1+x}}-\frac{2}{\sqrt{1+x}}$ | 16 |
| meijerg | $\frac{2\sqrt{\pi}-\frac{2\sqrt{\pi}}{\sqrt{1+x}}}{\sqrt{\pi}}-\frac{2(-\mathrm{signum}(-1+x))^{\frac{3}{2}}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1-x}}\right)}{\sqrt{\pi}\,\mathrm{signum}(-1+x)^{\frac{3}{2}}}$ | 56 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -2/(-1+x)^(1/2)-2/(1+x)^(1/2)
```

**Maxima** [A]
time = 0.29, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"maxima")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

**Fricas** [A]
time = 0.65, size = 28, normalized size = 1.47

$$-\frac{2\left((x+1)\sqrt{x-1}+\sqrt{x+1}\,(x-1)\right)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"fricas")
```

```
[Out] -2*((x + 1)*sqrt(x - 1) + sqrt(x + 1)*(x - 1))/(x^2 - 1)
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.
time = 1.06, size = 56, normalized size = 2.95

$$-\frac{2x\sqrt{x-1}}{x^2-1}-\frac{2x\sqrt{x+1}}{x^2-1}-\frac{2\sqrt{x-1}}{x^2-1}+\frac{2\sqrt{x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**(3/2)+(1+x)**(3/2))/(-1+x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] -2*x*sqrt(x - 1)/(x**2 - 1) - 2*x*sqrt(x + 1)/(x**2 - 1) - 2*sqrt(x - 1)/(x
**2 - 1) + 2*sqrt(x + 1)/(x**2 - 1)
```

**Giac** [A]
time = 0.70, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x+1}}-\frac{2}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^(3/2)+(1+x)^(3/2))/(-1+x)^(3/2)/(1+x)^(3/2),x, algorithm=
"giac")
```

```
[Out] -2/sqrt(x + 1) - 2/sqrt(x - 1)
```

**Mupad** [B]
time = 0.40, size = 15, normalized size = 0.79

$$-\frac{2}{\sqrt{x-1}}-\frac{2}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)^(3/2) + (x + 1)^(3/2))/((x - 1)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] - 2/(x - 1)^(1/2) - 2/(x + 1)^(1/2)
```

# 3.17 $\int \left(x+\sqrt{a+x^2}\right)^b\,dx$

**Optimal**. Leaf size=52

$$-\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x+(x^2+a)^(1/2))^(1+b)/(1+b)
```

**Rubi** [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2142, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{b+1}}{2(b+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] -1/2*(a*(x + Sqrt[a + x^2])^(-1 + b))/(1 - b) + (x + Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x+\sqrt{a+x^2}\right)^b dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+b}(a+x^2)\, dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int (ax^{-2+b}+x^b)\, dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)} + \frac{\left(x+\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

**Mathematica** [A]
time = 0.15, size = 43, normalized size = 0.83

$$\frac{\left(x+\sqrt{a+x^2}\right)^{-1+b}\left(ab+(-1+b)x\left(x+\sqrt{a+x^2}\right)\right)}{-1+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-1 + b)*(a*b + (-1 + b)*x*(x + Sqrt[a + x^2])))/(-1 +
 b^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(44) = 88.
time = 0.03, size = 120, normalized size = 2.31

| method | result | s |
|---|---|---|
| meijerg | $\frac{a^{\frac{b}{2}+\frac{1}{2}} b \left(\frac{8\sqrt{\pi}\, x^{1+b} a^{-\frac{b}{2}-\frac{1}{2}} \left(\frac{ab}{x^2}+b-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+b}}{(1+b)b(-2+2b)} + \frac{4\sqrt{\pi}\, x^{1+b} a^{-\frac{b}{2}-\frac{1}{2}} \sqrt{1+\frac{a}{x^2}}\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+b}}{(1+b)b}\right)}{4\sqrt{\pi}}$ | 1 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^(1/2*b+1/2)/Pi^(1/2)*b*(8*Pi^(1/2)/(1+b)/b*x^(1+b)*a^(-1/2*b-1/2)*(a*
b/x^2+b-1)/(-2+2*b)*((1+1/x^2*a)^(1/2)+1)^(-1+b)+4*Pi^(1/2)/(1+b)/b*x^(1+b)
*a^(-1/2*b-1/2)*(1+1/x^2*a)^(1/2)*((1+1/x^2*a)^(1/2)+1)^(-1+b))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

**Fricas [A]**
time = 0.66, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+a}\,b-x\right)\left(x+\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*b - x)*(x + sqrt(x^2 + a))^b/(b^2 - 1)
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(37) = 74$.
time = 1.56, size = 2147, normalized size = 41.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(b/2)*b**2*x*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a
)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2)
 + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) +
a**(9/2)*a**(b/2)*b*x*cosh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2
*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1
- b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - a**(7/2)*a**(b/2)*b**2*x**3*sqrt
(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1
- b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
2*a**(7/2)*x**2*gamma(1 - b/2)) + a**(7/2)*a**(b/2)*b*x**3*cosh(b*asinh(x/s
qrt(a)))*gamma(-b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 -
 b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2
)) + 2*a**5*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(
7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b/2)) - 2*a**5*a*
*(b/2)*b*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(
1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 -
b/2)) - 2*a**4*a**(b/2)*b*x**2*sqrt(a/x**2 + 1)*sinh(b*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2
)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*ga
mma(1 - b/2)) + 4*a**4*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sq
```

```
rt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9/2)*gamma(1
 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*gamma(1 - b
/2)) - 2*a**4*a**(b/2)*b*x**2*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) - 2*a**4*a**(b/2)*x**2*sqrt(a/x**2 + 1)*sinh(b*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1
 - b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) -
 2*a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**4*a**(b/2)*x**2*cosh(b*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2
*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)
*x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*b*x**4*sqrt(a/x**2 + 1)*sinh(b*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 -
b/2) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*
a**(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*x**4*cosh(b*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*
a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*
x**2*gamma(1 - b/2)) - 2*a**3*a**(b/2)*x**4*sqrt(a/x**2 + 1)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2
) - 2*a**(9/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**
(7/2)*x**2*gamma(1 - b/2)) + 2*a**3*a**(b/2)*x**4*cosh(b*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(9/2)*b**2*gamma(1 - b/2) - 2*a**(9
/2)*gamma(1 - b/2) + 2*a**(7/2)*b**2*x**2*gamma(1 - b/2) - 2*a**(7/2)*x**2*
gamma(1 - b/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(b/2)*b*x*sqrt(1 + x**2/
a)*sinh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b
**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)) + a**(5/2)*a**(b/2)*b*x*cos
h(b*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5
/2)*gamma(1 - b/2)) - 2*a**(5/2)*a**(b/2)*x*sqrt(1 + x**2/a)*sinh(b*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2
) - 2*a**(5/2)*gamma(1 - b/2)) - a**3*a**(b/2)*b**2*sqrt(1 + x**2/a)*sinh(b
*asinh(x/sqrt(a)))*gamma(-b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)
*gamma(1 - b/2)) + 2*a**3*a**(b/2)*b*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 -
 b/2)) + 2*a**2*a**(b/2)*b*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)
) + 2*a**2*a**(b/2)*x**2*cosh(b*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - b/2)/(2*a**(5/2)*b**2*gamma(1 - b/2) - 2*a**(5/2)*gamma(1 - b/2)), True
))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x+\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^b,x)
[Out] int((x + (a + x^2)^(1/2))^b, x)
```

# 3.18 $\int \left(x-\sqrt{a+x^2}\right)^b \, dx$

**Optimal**. Leaf size=56

$$-\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+b)/(1-b)+1/2*(x-(x^2+a)^(1/2))^(1+b)/(1+b)
```

**Rubi** [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{2142, 14\}$

$$\frac{\left(x-\sqrt{a+x^2}\right)^{b+1}}{2(b+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{b-1}}{2(1-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b,x]
```

```
[Out] -1/2*(a*(x - Sqrt[a + x^2])^(-1 + b))/(1 - b) + (x - Sqrt[a + x^2])^(1 + b)
/(2*(1 + b))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x-\sqrt{a+x^2}\right)^b dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+b}\left(a+x^2\right) dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+b}+x^b\right) dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+b}}{2(1-b)} + \frac{\left(x-\sqrt{a+x^2}\right)^{1+b}}{2(1+b)}$$

**Mathematica** [**A**]
time = 0.15, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{-1+b}\left(\frac{a}{-1+b}+\frac{\left(x-\sqrt{a+x^2}\right)^2}{1+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + b)*(a/(-1 + b) + (x - Sqrt[a + x^2])^2/(1 + b)))
/2
```

**Maple** [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{x^2+a}\right)^b dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b,x)
[Out] int((x-(x^2+a)^(1/2))^b,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

**Fricas** [A]
time = 0.73, size = 33, normalized size = 0.59

$$-\frac{\left(\sqrt{x^2+a}\,b+x\right)\left(x-\sqrt{x^2+a}\right)^b}{b^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="fricas")
```

```
[Out] -(sqrt(x^2 + a)*b + x)*(x - sqrt(x^2 + a))^b/(b^2 - 1)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(x-\sqrt{a+x^2}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**b, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b,x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(x^2 + a))^b, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\left(x-\sqrt{x^2+a}\right)^b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b,x)
```

```
[Out] int((x - (a + x^2)^(1/2))^b, x)
```

# 3.19 $\int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx$

**Optimal**. Leaf size=17

$$\frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] (x+(x^2+a)^(1/2))^b/b
```

**Rubi** [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\left(x+\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}} \, dx &= \text{Subst}\left(\int x^{-1+b} \, dx, x, x+\sqrt{a+x^2}\right) \\ &= \frac{\left(x+\sqrt{a+x^2}\right)^b}{b} \end{aligned}$$

**Mathematica [A]**
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\left(x+\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] (x + Sqrt[a + x^2])^b/b
```

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

**Fricas [A]**
time = 0.65, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + a))^b/b
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$.
time = 1.38, size = 311, normalized size = 18.29

$$\begin{cases} \frac{\sqrt{a}\, a^{\frac{b}{2}} \sinh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{bx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{b}{2}} \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}} x \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\, b} + \frac{a^{\frac{b}{2}} x \sinh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\, b\sqrt{\frac{a}{x^2}+1}} & \text{for } \left|\frac{x^2}{a}\right| > 1 \\ \frac{a^{\frac{b}{2}} \sinh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{b\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{b}{2}} \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{b}{2}\right)}{b^2\Gamma\left(-\frac{b}{2}\right)} + \frac{a^{\frac{b}{2}} x^2 \sinh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{ab\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{b}{2}} x \cosh\left(b \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\, b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a)*a**(b/2)*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(b*
x*sqrt(a/x**2 + 1)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a)))*gamma(1 - b/2)/(b
**2*gamma(-b/2)) + a**(b/2)*x*cosh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(
sqrt(a)*b) + a**(b/2)*x*sinh(b*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sqrt(a
)*b*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (a**(b/2)*sinh(b*asinh(x/sqrt(a))
- asinh(x/sqrt(a)))/(b*sqrt(1 + x**2/a)) - 2*a**(b/2)*cosh(b*asinh(x/sqrt(a
)))*gamma(1 - b/2)/(b**2*gamma(-b/2)) + a**(b/2)*x**2*sinh(b*asinh(x/sqrt(a
)) - asinh(x/sqrt(a)))/(a*b*sqrt(1 + x**2/a)) + a**(b/2)*x*cosh(b*asinh(x/s
qrt(a)) - asinh(x/sqrt(a)))/(sqrt(a)*b), True))
```

**Giac** [A]
time = 0.69, size = 15, normalized size = 0.88

$$\frac{\left(x + \sqrt{x^2 + a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (x + sqrt(x^2 + a))^b/b
```

**Mupad** [B]
time = 0.30, size = 15, normalized size = 0.88

$$\frac{\left(x + \sqrt{x^2 + a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
```

```
[Out] (x + (a + x^2)^(1/2))^b/b
```

## 3.20 $\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx$

**Optimal**. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

```
[Out] -(x-(x^2+a)^(1/2))^b/b
```

**Rubi** [A]

time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^b/b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^b}{\sqrt{a+x^2}}\,dx &= -\text{Subst}\left(\int x^{-1+b}\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}\end{aligned}$$

**Mathematica** [A]
time = 0.02, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^b}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^b/Sqrt[a + x^2],x]
[Out] -((x - Sqrt[a + x^2])^b/b)
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^b}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
[Out] int((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^b/sqrt(x^2 + a), x)
```

**Fricas** [A]
time = 0.84, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^b/b
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.90, size = 36, normalized size = 1.80

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^b}{b} & \text{for } b \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**b/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**b/b, Ne(b, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

**Giac** [A]
time = 0.86, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(x - sqrt(x^2 + a))^b/b
```

**Mupad** [B]
time = 0.30, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^b/(a + x^2)^(1/2),x)
```

```
[Out] -(x - (a + x^2)^(1/2))^b/b
```

## 3.21 $\int \frac{1}{(a+be^{px})^2} \, dx$

**Optimal**. Leaf size=42

$$\frac{1}{a\left(a+be^{px}\right)p}+\frac{x}{a^2}-\frac{\log\left(a+be^{px}\right)}{a^2p}$$

```
[Out] 1/a/(a+b*exp(p*x))/p+x/a^2-ln(a+b*exp(p*x))/a^2/p
```

**Rubi** [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2320, 46}

$$-\frac{\log\left(a+be^{px}\right)}{a^2p}+\frac{x}{a^2}+\frac{1}{ap\left(a+be^{px}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(p*x))^(-2),x]
```

```
[Out] 1/(a*(a + b*E^(p*x))*p) + x/a^2 - Log[a + b*E^(p*x)]/(a^2*p)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+be^{px}\right)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2} \, dx, x, e^{px}\right)}{p} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) \, dx, x, e^{px}\right)}{p} \\
&= \frac{1}{a\left(a+be^{px}\right)p}+\frac{x}{a^2}-\frac{\log\left(a+be^{px}\right)}{a^2p}
\end{aligned}$$

**Mathematica [A]**
time = 0.05, size = 39, normalized size = 0.93

$$\frac{\frac{a}{a+be^{px}}+\log\left(e^{px}\right)-\log\left(a+be^{px}\right)}{a^2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(p*x))^(-2),x]
[Out] (a/(a + b*E^(p*x)) + Log[E^(p*x)] - Log[a + b*E^(p*x)])/(a^2*p)
```

**Maple [A]**
time = 0.02, size = 43, normalized size = 1.02

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{\ln(a+b\,e^{px})}{a^2}+\frac{1}{a(a+b\,e^{px})}+\frac{\ln(e^{px})}{a^2}}{p}$ | 43 |
| default | $\frac{-\frac{\ln(a+b\,e^{px})}{a^2}+\frac{1}{a(a+b\,e^{px})}+\frac{\ln(e^{px})}{a^2}}{p}$ | 43 |
| risch | $\frac{x}{a^2}+\frac{1}{a(a+b\,e^{px})p}-\frac{\ln(e^{px}+\frac{a}{b})}{a^2p}$ | 43 |
| norman | $\frac{\frac{x}{a}+\frac{bx\,e^{px}}{a^2}-\frac{b\,e^{px}}{a^2p}}{a+b\,e^{px}}-\frac{\ln(a+b\,e^{px})}{a^2p}$ | 59 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(p*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/p*(-1/a^2*ln(a+b*exp(p*x))+1/a/(a+b*exp(p*x))+1/a^2*ln(exp(p*x)))
```

**Maxima [A]**
time = 0.35, size = 40, normalized size = 0.95

$$\frac{x}{a^2}+\frac{1}{\left(abe^{(px)}+a^2\right)p}-\frac{\log\left(be^{(px)}+a\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="maxima")
[Out] x/a^2 + 1/((a*b*e^(p*x) + a^2)*p) - log(b*e^(p*x) + a)/(a^2*p)
```

**Fricas [A]**
time = 0.72, size = 52, normalized size = 1.24

$$\frac{bpxe^{(px)}+apx-\left(be^{(px)}+a\right)\log\left(be^{(px)}+a\right)+a}{a^2bpe^{(px)}+a^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(p*x) + a*p*x - (b*e^(p*x) + a)*log(b*e^(p*x) + a) + a)/(a^2*b*p*e
^(p*x) + a^3*p)
```

**Sympy** [A]
time = 0.05, size = 36, normalized size = 0.86

$$\frac{1}{a^2p + abpe^{px}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{px}\right)}{a^2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))**2,x)
```

```
[Out] 1/(a**2*p + a*b*p*exp(p*x)) + x/a**2 - log(a/b + exp(p*x))/(a**2*p)
```

**Giac** [A]
time = 1.04, size = 47, normalized size = 1.12

$$\frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(px)}+a}+1\right|\right)}{a^2b} + \frac{1}{(be^{(px)}+a)ab}\right)}{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*e^(p*x) + a) + 1))/(a^2*b) + 1/((b*e^(p*x) + a)*a*b))/p
```

**Mupad** [B]
time = 0.41, size = 58, normalized size = 1.38

$$\frac{\frac{x}{a} + \frac{b\,x\,e^{p\,x}}{a^2} - \frac{b\,e^{p\,x}}{a^2\,p}}{a + b\,e^{p\,x}} - \frac{\ln\left(a + b\,e^{p\,x}\right)}{a^2\,p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*exp(p*x))^2,x)
```

```
[Out] (x/a + (b*x*exp(p*x))/a^2 - (b*exp(p*x))/(a^2*p))/(a + b*exp(p*x)) - log(a
+ b*exp(p*x))/(a^2*p)
```

## 3.22 $\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx$

**Optimal**. Leaf size=22

$$-\frac{1}{2a\,(b+ae^{2px})\,p}$$

[Out] -1/2/a/(b+a*exp(2*p*x))/p

**Rubi** [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2320, 267}

$$-\frac{1}{2ap\,(ae^{2px}+b)}$$

Antiderivative was successfully verified.

[In] Int[(b/E^(p*x) + a*E^(p*x))^(-2),x]

[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(be^{-px}+ae^{px})^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(b+ax^2)^2}\,dx,x,e^{px}\right)}{p}\\ &= -\frac{1}{2a\,(b+ae^{2px})\,p}\end{aligned}$$

**Mathematica** [A]
time = 0.03, size = 22, normalized size = 1.00

$$-\frac{1}{2a\left(b+ae^{2px}\right)p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(p*x) + a*E^(p*x))^(-2),x]
```

```
[Out] -1/2*1/(a*(b + a*E^(2*p*x))*p)
```

**Maple** [A]
time = 0.02, size = 21, normalized size = 0.95

| method | result | size |
|---|---|---|
| risch | $-\frac{1}{2a(b+a\,e^{2px})p}$ | 20 |
| derivativedivides | $-\frac{1}{2a(b+a\,e^{2px})p}$ | 21 |
| default | $-\frac{1}{2a(b+a\,e^{2px})p}$ | 21 |
| norman | $-\frac{1}{2a(b+a\,e^{2px})p}$ | 21 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/p/a/(a*exp(p*x)^2+b)
```

**Maxima** [A]
time = 0.30, size = 20, normalized size = 0.91

$$\frac{1}{2\left(b^2e^{(-2\,px)}+ab\right)p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2/((b^2*e^(-2*p*x) + a*b)*p)
```

**Fricas** [A]
time = 0.66, size = 19, normalized size = 0.86

$$-\frac{1}{2\left(a^2pe^{(2\,px)}+abp\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2/(a^2*p*e^(2*p*x) + a*b*p)
```

**Sympy** [A]
time = 0.04, size = 22, normalized size = 1.00

$$-\frac{1}{2a^2pe^{2px}+2abp}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] -1/(2*a**2*p*exp(2*p*x) + 2*a*b*p)
```

**Giac** [A]
time = 0.67, size = 19, normalized size = 0.86

$$-\frac{1}{2\left(ae^{(2\,px)}+b\right)ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] -1/2/((a*e^(2*p*x) + b)*a*p)
```

**Mupad** [B]
time = 0.39, size = 24, normalized size = 1.09

$$\frac{e^{2\,p\,x}}{2\,b\,p\,\left(b+a\,e^{2\,p\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

```
[Out] exp(2*p*x)/(2*b*p*(b + a*exp(2*p*x)))
```

# 3.23 $\int \frac{x}{(be^{-px}+ae^{px})^2} \, dx$

**Optimal**. Leaf size=62

$$\frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}$$

```
[Out] 1/2*x/a/b/p-1/2*x/a/(b+a*exp(2*p*x))/p-1/4*ln(b+a*exp(2*p*x))/a/b/p^2
```

**Rubi** [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2321, 2222, 2320, 36, 29, 31}

$$-\frac{\log\left(ae^{2px}+b\right)}{4abp^2}+\frac{x}{2abp}-\frac{x}{2ap\left(ae^{2px}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] x/(2*a*b*p) - x/(2*a*(b + a*E^(2*p*x))*p) - Log[b + a*E^(2*p*x)]/(4*a*b*p^2
)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2222

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Dist[d*(m/(b*f*g*n*(p + 1)*Log[F])), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2321

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(be^{-px}+ae^{px}\right)^2}\,dx &= \int \frac{e^{2px}x}{\left(b+ae^{2px}\right)^2}\,dx\\ &= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\int \frac{1}{b+ae^{2px}}\,dx}{2ap}\\ &= -\frac{x}{2a\left(b+ae^{2px}\right)p}+\frac{\text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{2px}\right)}{4ap^2}\\ &= -\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\text{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{2px}\right)}{4bp^2}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2px}\right)}{4abp^2}\\ &= \frac{x}{2abp}-\frac{x}{2a\left(b+ae^{2px}\right)p}-\frac{\log\left(b+ae^{2px}\right)}{4abp^2}\end{aligned}$$

**Mathematica [A]**
time = 0.07, size = 49, normalized size = 0.79

$$\frac{\frac{2e^{2px}px}{b+ae^{2px}}-\frac{\log\left(b+ae^{2px}\right)}{a}}{4bp^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b/E^(p*x) + a*E^(p*x))^2,x]
```

```
[Out] ((2*E^(2*p*x)*p*x)/(b + a*E^(2*p*x)) - Log[b + a*E^(2*p*x)]/a)/(4*b*p^2)
```

**Maple [A]**
time = 0.06, size = 50, normalized size = 0.81

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{\ln\left(b+a\,e^{2px}\right)}{4ba}+\frac{px\,e^{2px}}{2b\left(b+a\,e^{2px}\right)}}{p^{2}}$ | 50 |
| default | $\frac{-\frac{\ln\left(b+a\,e^{2px}\right)}{4ba}+\frac{px\,e^{2px}}{2b\left(b+a\,e^{2px}\right)}}{p^{2}}$ | 50 |
| norman | $\frac{x\,e^{2px}}{2bp(b+a\,e^{2px})}-\frac{\ln(b+a\,e^{2px})}{4ab\,p^{2}}$ | 51 |
| risch | $\frac{x}{2abp}-\frac{x}{2a(b+a\,e^{2px})p}-\frac{\ln\left(e^{2px}+\frac{b}{a}\right)}{4ab\,p^{2}}$ | 57 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b/exp(p*x)+a*exp(p*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/p^2*(-1/4/b/a*ln(a*exp(p*x)^2+b)+1/2*p*x*exp(p*x)^2/b/(a*exp(p*x)^2+b))
```

**Maxima [A]**
time = 0.36, size = 51, normalized size = 0.82

$$\frac{xe^{(2\,px)}}{2\,(abpe^{(2\,px)}+b^2p)}-\frac{\log\left(\frac{ae^{(2\,px)}+b}{a}\right)}{4\,abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(2*p*x)/(a*b*p*e^(2*p*x) + b^2*p) - 1/4*log((a*e^(2*p*x) + b)/a)/(a
*b*p^2)
```

**Fricas [A]**
time = 0.82, size = 58, normalized size = 0.94

$$\frac{2\,apxe^{(2\,px)}-\left(ae^{(2\,px)}+b\right)\log\left(ae^{(2\,px)}+b\right)}{4\,(a^2bp^2e^{(2\,px)}+ab^2p^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - (a*e^(2*p*x) + b)*log(a*e^(2*p*x) + b))/(a^2*b*p^2
*e^(2*p*x) + a*b^2*p^2)
```

**Sympy [A]**
time = 0.07, size = 49, normalized size = 0.79

$$-\frac{x}{2a^2pe^{2px}+2abp}+\frac{x}{2abp}-\frac{\log\left(e^{2px}+\frac{b}{a}\right)}{4abp^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))**2,x)
```

```
[Out] -x/(2*a**2*p*exp(2*p*x) + 2*a*b*p) + x/(2*a*b*p) - log(exp(2*p*x) + b/a)/(4
*a*b*p**2)
```

**Giac** [A]
time = 0.81, size = 74, normalized size = 1.19

$$\frac{2\,apxe^{(2\,px)} - ae^{(2\,px)}\log\left(-ae^{(2\,px)} - b\right) - b\log\left(-ae^{(2\,px)} - b\right)}{4\,(a^2bp^2e^{(2\,px)} + ab^2p^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b/exp(p*x)+a*exp(p*x))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*a*p*x*e^(2*p*x) - a*e^(2*p*x)*log(-a*e^(2*p*x) - b) - b*log(-a*e^(2*
p*x) - b))/(a^2*b*p^2*e^(2*p*x) + a*b^2*p^2)
```

**Mupad** [B]
time = 0.41, size = 47, normalized size = 0.76

$$\frac{x\,\mathrm{e}^{2\,p\,x}}{2\,b\,p\,\left(b + a\,\mathrm{e}^{2\,p\,x}\right)} - \frac{\ln\left(b + a\,\mathrm{e}^{2\,p\,x}\right)}{4\,a\,b\,p^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*exp(p*x) + b*exp(-p*x))^2,x)
```

```
[Out] (x*exp(2*p*x))/(2*b*p*(b + a*exp(2*p*x))) - log(b + a*exp(2*p*x))/(4*a*b*p^
2)
```

## 3.24 $\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx$

**Optimal**. Leaf size=86

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2}+\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{\sqrt{1-x+x^2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{1-x+x^2}}\right)}{\sqrt{6}}$$

```
[Out] arctan((1+x)*2^(1/2)/(x^2-x+1)^(1/2))*2^(1/2)-1/6*arctanh(1/3*(1-x)*6^(1/2)
/(x^2-x+1)^(1/2))*6^(1/2)+(1+x)*(x^2-x+1)^(1/2)/(x^2+x+1)
```

**Rubi** [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1074, 1049, 1043, 212, 210}

$$\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,(x+1)}{\sqrt{x^2-x+1}}\right)+\frac{\sqrt{x^2-x+1}\,(x+1)}{x^2+x+1}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,(1-x)}{\sqrt{x^2-x+1}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) + Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x)
)/Sqrt[1 - x + x^2]] - ArcTanh[(Sqrt[2/3]*(1 - x))/Sqrt[1 - x + x^2]]/Sqrt[
6]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1-x+3x^2}{\sqrt{1-x+x^2}\,(1+x+x^2)^2}\,dx = \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{12}\int \frac{18-6x}{\sqrt{1-x+x^2}\,(1+x+x^2)}\,dx$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \frac{1}{48}\int \frac{24+24x}{\sqrt{1-x+x^2}\,(1+x+x^2)}\,dx - \frac{1}{48}\int \cdots$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + 24\text{Subst}\left(\int \frac{1}{1728-2x^2}\,dx, x, \frac{-24+24x}{\sqrt{1-x+x^2}}\right) \cdots$$

$$= \frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} + \sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{\sqrt{1-x+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\cdots}{\sqrt{1-\cdots}}\right)}{\sqrt{6}} \cdots$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.32, size = 239, normalized size = 2.78

$$\frac{(1+x)\sqrt{1-x+x^2}}{1+x+x^2} - \text{RootSum}\left[3+6\#1+\#1^2-2\#1^3+\#1^4\&, \frac{19\log\left(-x+\sqrt{1-x+x^2}-\#1\right)+6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1}{3+\#1-3\#1^2+2\#1^3}\&\right] - \frac{1}{2}\text{RootSum}\left[3+6\#1+\#1^2-2\#1^3+\#1^4\&, \frac{-36\log\left(-x+\sqrt{1-x+x^2}-\#1\right)-6\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{1-x+x^2}-\#1\right)\#1^2}{3+\#1-3\#1^2+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(Sqrt[1 - x + x^2]*(1 + x + x^2)^2),x]
```

```
[Out] ((1 + x)*Sqrt[1 - x + x^2])/(1 + x + x^2) - RootSum[3 + 6*#1 + #1^2 - 2*#1^
3 + #1^4 & , (19*Log[-x + Sqrt[1 - x + x^2] - #1] + 6*Log[-x + Sqrt[1 - x +
 x^2] - #1]*#1)/(3 + #1 - 3*#1^2 + 2*#1^3) & ] - RootSum[3 + 6*#1 + #1^2 -
2*#1^3 + #1^4 & , (-36*Log[-x + Sqrt[1 - x + x^2] - #1] - 6*Log[-x + Sqrt[1
 - x + x^2] - #1]*#1 + Log[-x + Sqrt[1 - x + x^2] - #1]*#1^2)/(3 + #1 - 3*#
1^2 + 2*#1^3) & ]/2
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(71) = 142$.
time = 0.74, size = 455, normalized size = 5.29

| method | result |
|---|---|
| risch | $\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1} + \frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\left(6\sqrt{2}\,\arctan\left(\frac{2\sqrt{2}\,(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,(1-x)}\right)-\sqrt{6}\,\text{arctanh}\left(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\,\sqrt{6}\cdots}{4}\cdots\right.\right.}{6\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\left(\frac{1+x}{1-x}+1\right)}\cdots$ |

| | |
|---|---|
| default | $\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\left(3\sqrt{2}\arctan\left(\frac{2\sqrt{2}(1+x)}{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}(1-x)}\right)-\sqrt{6}\operatorname{arctanh}\left(\frac{\sqrt{\frac{(1+x)^2}{(1-x)^2}+3}\sqrt{6}}{4}\right)\right)}{2\sqrt{\frac{\frac{(1+x)^2}{(1-x)^2}+3}{\left(\frac{1+x}{1-x}+1\right)^2}}\left(\frac{1+x}{1-x}+1\right)}-\frac{9\sqrt{2}\sqrt{\frac{(1+x)}{(1-x)}\cdots}}{\cdots}$ |
| trager | $\frac{(1+x)\sqrt{x^2-x+1}}{x^2+x+1}-\frac{24\ln\left(\frac{1728x\operatorname{RootOf}\left(576\_Z^4+528\_Z^2+169\right)^5-744\operatorname{RootOf}\left(576\_Z^4+528\_Z^2+169\right)^3x+1344\operatorname{RootOf}\left(5\cdots\right.}{\cdots}\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((1+x)^2/(1-x)^2+3)^(1/2)*(3*2^(1/2)*arctan(2*2^(1/2)/((1+x)^2/(1-x)^2+
3)^(1/2)*(1+x)/(1-x))-6^(1/2)*arctanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2)
))/((((1+x)^2/(1-x)^2+3)/((1+x)/(1-x)+1)^2)^(1/2)/((1+x)/(1-x)+1)-1/6*(9*2^(
1/2)*((1+x)^2/(1-x)^2+3)^(1/2)*arctan(2*2^(1/2)/((1+x)^2/(1-x)^2+3)^(1/2)*(
1+x)/(1-x))*(1+x)^2/(1-x)^2-6*6^(1/2)*((1+x)^2/(1-x)^2+3)^(1/2)*arctanh(1/4
*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*(1+x)^2/(1-x)^2+3*2^(1/2)*arctan(2*2^(1
/2)/((1+x)^2/(1-x)^2+3)^(1/2)*(1+x)/(1-x))*((1+x)^2/(1-x)^2+3)^(1/2)-2*6^(1
/2)*arctanh(1/4*((1+x)^2/(1-x)^2+3)^(1/2)*6^(1/2))*((1+x)^2/(1-x)^2+3)^(1/2
)-12*(1+x)^3/(1-x)^3-36*(1+x)/(1-x))/((((1+x)^2/(1-x)^2+3)/((1+x)/(1-x)+1)^2
)^(1/2)/((1+x)/(1-x)+1)/(3*(1+x)^2/(1-x)^2+1)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 - x + 1)/((x^2 + x + 1)^2*sqrt(x^2 - x + 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(69) = 138$.
time = 1.06, size = 358, normalized size = 4.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(2/3*sqrt(6)*sqrt(3)*(x - 1) +
 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - sqrt(6)*(x + 1) +
 4)*(sqrt(6)*sqrt(3) + 3*sqrt(3)) - 2/3*sqrt(x^2 - x + 1)*(sqrt(6)*sqrt(3)
+ 3*sqrt(3)) + sqrt(3)*(2*x - 1)) + 8*sqrt(6)*sqrt(3)*(x^2 + x + 1)*arctan(
2/3*sqrt(6)*sqrt(3)*(x - 1) + 2/3*sqrt(2*x^2 - sqrt(x^2 - x + 1)*(2*x + sqr
t(6) + 1) + sqrt(6)*(x + 1) + 4)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - 2/3*sqrt(x
^2 - x + 1)*(sqrt(6)*sqrt(3) - 3*sqrt(3)) - sqrt(3)*(2*x - 1)) - sqrt(6)*(x
^2 + x + 1)*log(12168*x^2 - 6084*sqrt(x^2 - x + 1)*(2*x + sqrt(6) + 1) + 60
84*sqrt(6)*(x + 1) + 24336) + sqrt(6)*(x^2 + x + 1)*log(12168*x^2 - 6084*sq
rt(x^2 - x + 1)*(2*x - sqrt(6) + 1) - 6084*sqrt(6)*(x + 1) + 24336) - 12*x^
2 - 12*sqrt(x^2 - x + 1)*(x + 1) - 12*x - 12)/(x^2 + x + 1)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 - x + 1}{\sqrt{x^2 - x + 1}\,(x^2 + x + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**2+x+1)**2/(x**2-x+1)**(1/2),x)
```

```
[Out] Integral((3*x**2 - x + 1)/(sqrt(x**2 - x + 1)*(x**2 + x + 1)**2), x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(69) = 138.
time = 0.85, size = 304, normalized size = 3.53

$$-\frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x + \sqrt{6} - 2\sqrt{x^2 - x + 1} + 1}{\sqrt{3} + \sqrt{2}}\right) + \frac{1}{3}\sqrt{6}\sqrt{3}\arctan\left(-\frac{2x - \sqrt{6} - 2\sqrt{x^2 - x + 1} + 1}{\sqrt{3} - \sqrt{2}}\right) + \frac{1}{12}\sqrt{6}\log\left(4\left(\sqrt{6}\sqrt{3} + 3\sqrt{3}\right)^2 + 36\left(2x + \sqrt{6} - 2\sqrt{x^2 - x + 1} + 1\right)^2\right) - \frac{1}{12}\sqrt{6}\log\left(4\left(\sqrt{6}\sqrt{3} - 3\sqrt{3}\right)^2 + 36\left(2x - \sqrt{6} - 2\sqrt{x^2 - x + 1} + 1\right)^2\right) + \frac{\left(x - \sqrt{x^2 - x + 1}\right)^3 + 4\left(x - \sqrt{x^2 - x + 1}\right)^2 - 10x + 10\sqrt{x^2 - x + 1} + 5}{\left(x - \sqrt{x^2 - x + 1}\right)^4 + 2\left(x - \sqrt{x^2 - x + 1}\right)^3 + \left(x - \sqrt{x^2 - x + 1}\right)^2 - 6x + 6\sqrt{x^2 - x + 1} + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^2+x+1)^2/(x^2-x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(6)*sqrt(3)*arctan(-(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)/(sqr
t(3) + sqrt(2))) + 1/3*sqrt(6)*sqrt(3)*arctan(-(2*x - sqrt(6) - 2*sqrt(x^2
- x + 1) + 1)/(sqrt(3) - sqrt(2))) + 1/12*sqrt(6)*log(4*(sqrt(6)*sqrt(3) +
3*sqrt(3))^2 + 36*(2*x + sqrt(6) - 2*sqrt(x^2 - x + 1) + 1)^2) - 1/12*sqrt(
6)*log(4*(sqrt(6)*sqrt(3) - 3*sqrt(3))^2 + 36*(2*x - sqrt(6) - 2*sqrt(x^2 -
 x + 1) + 1)^2) + ((x - sqrt(x^2 - x + 1))^3 + 4*(x - sqrt(x^2 - x + 1))^2
- 10*x + 10*sqrt(x^2 - x + 1) + 5)/((x - sqrt(x^2 - x + 1))^4 + 2*(x - sqrt
(x^2 - x + 1))^3 + (x - sqrt(x^2 - x + 1))^2 - 6*x + 6*sqrt(x^2 - x + 1) +
3)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2 - x + 1}{\sqrt{x^2 - x + 1}\,{\left(x^2 + x + 1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2),x)
[Out] int((3*x^2 - x + 1)/((x^2 - x + 1)^(1/2)*(x + x^2 + 1)^2), x)
```

## 3.25 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx$

**Optimal**. Leaf size=19

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

```
[Out] 2*(x+(a^2+x^2)^(1/2))^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2147, 30}

$$2\sqrt{\sqrt{a^2+x^2}+x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}} \, dx, x, x+\sqrt{a^2+x^2}\right) \\ &= 2\sqrt{x+\sqrt{a^2+x^2}} \end{aligned}$$

**Mathematica** [A]
time = 0.03, size = 19, normalized size = 1.00

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a^2 + x^2],x]
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]]
```

**Maple** [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
[Out] int((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/sqrt(a^2 + x^2), x)
```

**Fricas** [A]
time = 0.97, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

**Sympy** [A]
time = 0.07, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/(a**2+x**2)**(1/2),x)
```

```
[Out] 2*sqrt(x + sqrt(a**2 + x**2))
```

**Giac** [A]
time = 0.86, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + sqrt(a^2 + x^2))
```

**Mupad** [B]
time = 0.42, size = 15, normalized size = 0.79

$$2\sqrt{x+\sqrt{a^2+x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/(a^2 + x^2)^(1/2),x)
```

```
[Out] 2*(x + (a^2 + x^2)^(1/2))^(1/2)
```

# 3.26 $\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx$

**Optimal**. Leaf size=26

$$\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

```
[Out] 2*(b*x+(b^2*x^2+a)^(1/2))^(1/2)/b
```

**Rubi** [A]
time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2147, 30}

$$\frac{2\sqrt{\sqrt{a+b^2x^2}+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx+\sqrt{a+b^2x^2}}}{\sqrt{a+b^2x^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, bx+\sqrt{a+b^2x^2}\right)}{b}\\ &= \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 26, normalized size = 1.00

$$\frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x + Sqrt[a + b^2*x^2]]/Sqrt[a + b^2*x^2],x]
```

```
[Out] (2*Sqrt[b*x + Sqrt[a + b^2*x^2]])/b
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+\sqrt{x^2b^2+a}}}{\sqrt{x^2b^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

```
[Out] int((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(b*x + sqrt(b^2*x^2 + a))/sqrt(b^2*x^2 + a), x)
```

**Fricas** [A]
time = 1.03, size = 22, normalized size = 0.85

$$\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

**Sympy** [A]
time = 0.47, size = 27, normalized size = 1.04

$$\begin{cases} \frac{2\sqrt{bx+\sqrt{a+b^2x^2}}}{b} & \text{for } b \neq 0 \\ \frac{x}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b**2*x**2+a)**(1/2))**(1/2)/(b**2*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(b*x + sqrt(a + b**2*x**2))/b, Ne(b, 0)), (x/a**(1/4), Tru
e))
```

**Giac** [A]
time = 1.19, size = 22, normalized size = 0.85

$$\frac{2\sqrt{bx+\sqrt{b^2x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+(b^2*x^2+a)^(1/2))^(1/2)/(b^2*x^2+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] 2*sqrt(b*x + sqrt(b^2*x^2 + a))/b
```

**Mupad** [B]
time = 0.51, size = 22, normalized size = 0.85

$$\frac{2\sqrt{\sqrt{b^2\,x^2+a}+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^2)^(1/2) + b*x)^(1/2)/(a + b^2*x^2)^(1/2),x)
```

```
[Out] (2*((a + b^2*x^2)^(1/2) + b*x)^(1/2))/b
```

## 3.27 $\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx$

**Optimal**. Leaf size=63

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))/a^(3/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))/a^(3/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2145, 335, 218, 212, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2) - (2*ArcTanh[Sqrt[x
+ Sqrt[a^2 + x^2]]/Sqrt[a]])/a^(3/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2145

```
Int[(x_)^(p_.)*((g_) + (i_.)*(x_)^2)^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_) +
(c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + p + 1)*e^(p + 1)*f^(2*
m)))*(i/c)^m, Subst[Int[x^(n - 2*m - p - 2)*((-a)*f^2 + x^2)^p*(a*f^2 + x^2
)^(2*m + 1), x], x, e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, e, f, g, i
, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegersQ[p, 2*m] &
& (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{a^2+x^2}\sqrt{x+\sqrt{a^2+x^2}}}\,dx = 2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)$$
$$= 4\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)}{a} - \frac{2\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x,$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

**Mathematica** [A]
time = 0.07, size = 56, normalized size = 0.89

$$-\frac{2\left(\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]*Sqrt[x + Sqrt[a^2 + x^2]]),x]
```

```
[Out] (-2*(ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + ArcTanh[Sqrt[x + Sqrt[a^2
+ x^2]]/Sqrt[a]]))/a^(3/2)
```

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a^2+x^2}\,\sqrt{x+\sqrt{a^2+x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

**Fricas [A]** Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.
time = 0.99, size = 198, normalized size = 3.14

$$\left[-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\,\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right)-\sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}\,a-\left(\sqrt{-a}\,(a+x)-\sqrt{a^2+x^2}\,\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="frica
s")
```

```
[Out] [-(2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) - sqrt(a)*log((a^2 +
 sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + s
qrt(a^2 + x^2)))/x))/a^2, (2*sqrt(-a)*arctan(sqrt(-a)*sqrt(x + sqrt(a^2 + x
^2))/a) - sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a - (sqrt(-a)*(a + x) - sqrt
(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x))/a^2]
```

**Sympy** [C] Result contains complex when optimal does not.
time = 1.00, size = 46, normalized size = 0.73

$$-\frac{\Gamma^{2}\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_{3}F_{2}\left(\begin{matrix}\frac{3}{4},\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4}\end{matrix}\middle|\frac{a^{2}e^{i\pi}}{x^{2}}\right)}{\pi x^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2)/(x+(a**2+x**2)**(1/2))**(1/2),x)
```

```
[Out] -gamma(3/4)**2*gamma(5/4)*hyper((3/4, 3/4, 5/4), (3/2, 7/4), a**2*exp_polar
(I*pi)/x**2)/(pi*x**(3/2)*gamma(7/4))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2)/(x+(a^2+x^2)^(1/2))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a^2 + x^2)*sqrt(x + sqrt(a^2 + x^2))*x), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{x+\sqrt{a^2+x^2}}\,\sqrt{a^2+x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(x + (a^2 + x^2)^(1/2))^(1/2)*(a^2 + x^2)^(1/2)), x)
```

## 3.28 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x} \, dx$

**Optimal**. Leaf size=82

$$2\sqrt{x+\sqrt{a^2+x^2}}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((x+(a^2+x^2)^(1/2))^(1/2)/a^(1/2))*a^(1/2)-2*arctanh((x+(a^2+x^2)
^(1/2))^(1/2)/a^(1/2))*a^(1/2)+2*(x+(a^2+x^2)^(1/2))^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2144, 470, 335, 218, 212, 209}

$$-2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2144

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*((-a)*f^2*h + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx &= \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} + (2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-a^2+x^2)}\,dx, x, x+\sqrt{a^2+x^2}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} + (4a^2)\,\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} - (2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right) - (2a)\text{Su}\\
&= 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x}}{}\right.
\end{aligned}$$

**Mathematica** [A]
time = 0.01, size = 82, normalized size = 1.00

$$2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.02, size = 25, normalized size = 0.30

| method | result | size |
|---|---|---|
| meijerg | $2\sqrt{2}\,\sqrt{x}\,\operatorname{hypergeom}\left(\left[-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\right],\left[\frac{1}{2},\frac{3}{4}\right],-\frac{a^{2}}{x^{2}}\right)$ | 25 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-a^2/x^2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

**Fricas** [A]
time = 1.12, size = 216, normalized size = 2.63

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}\,a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a^2+x^2}},\ 2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{x+\sqrt{a^2+x^2}}}{a}\right)+\sqrt{-a}\log\left(-\frac{a^2-\sqrt{a^2+x^2}\,a+\left(\sqrt{-a}\,(a+x)-\sqrt{a^2+x^2}\sqrt{-a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a^2+x^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

**Sympy** [C] Result contains complex when optimal does not.
time = 1.69, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\,\Gamma^{2}\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_{3}F_{2}\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\\frac{1}{2},\frac{3}{4}\end{matrix}\right|\frac{a^{2}e^{i\pi}}{x^{2}}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a^2 + x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] int((x + (a^2 + x^2)^(1/2))^(1/2)/x, x)
```

# 3.29 $\int x^3 \log^3(2+x) \log(3+x) \, dx$

**Optimal**. Leaf size=606

$$-\frac{302177x}{1152}+\frac{8029x^2}{2304}-\frac{763x^3}{3456}+\frac{3x^4}{256}+\frac{377}{64}(2+x)^2-\frac{71}{216}(2+x)^3+\frac{3}{256}(2+x)^4+\frac{2069}{144}\log(2+x)-\frac{187}{64}x^2\log(2+x)+$$

```
[Out] -302177/1152*x+3/256*x^4+8029/2304*x^2-763/3456*x^3+377/64*(2+x)^2-71/216*(
2+x)^3+3/256*(2+x)^4-5609/96*polylog(2,-2-x)-563/8*polylog(3,-2-x)-195/2*po
lylog(4,-2-x)-43/12*ln(2+x)^2+3891/128*ln(3+x)+2069/144*ln(2+x)+1/4*x^4*ln(
2+x)^3*ln(3+x)-25*x*ln(2+x)*ln(3+x)+13/4*x^2*ln(2+x)*ln(3+x)-7/12*x^3*ln(2+
x)*ln(3+x)+3/32*x^4*ln(2+x)*ln(3+x)+6*x*ln(2+x)^2*ln(3+x)-3/2*x^2*ln(2+x)^2
*ln(3+x)+1/2*x^3*ln(2+x)^2*ln(3+x)-3/16*x^4*ln(2+x)^2*ln(3+x)-187/64*x^2*ln
(2+x)+83/288*x^3*ln(2+x)-3/128*x^4*ln(2+x)+6733/32*(2+x)*ln(2+x)-377/32*(2+
x)^2*ln(2+x)+71/72*(2+x)^3*ln(2+x)-3/64*(2+x)^4*ln(2+x)-17/48*x^3*ln(2+x)^2
+3/64*x^4*ln(2+x)^2-1251/16*(2+x)*ln(2+x)^2+273/32*(2+x)^2*ln(2+x)^2-3/4*(2
+x)^3*ln(2+x)^2+3/64*(2+x)^4*ln(2+x)^2+65/4*(2+x)*ln(2+x)^3-33/8*(2+x)^2*ln
(2+x)^3+3/4*(2+x)^3*ln(2+x)^3-1/16*(2+x)^4*ln(2+x)^3-115/48*x^2*ln(3+x)+37/
144*x^3*ln(3+x)-3/128*x^4*ln(3+x)+415/12*(3+x)*ln(3+x)-4083/32*ln(2+x)*ln(3
+x)+963/16*ln(2+x)^2*ln(3+x)-81/4*ln(2+x)^3*ln(3+x)+563/8*ln(2+x)*polylog(2
,-2-x)-195/4*ln(2+x)^2*polylog(2,-2-x)+195/2*ln(2+x)*polylog(3,-2-x)
```

**Rubi** [A]
time = 2.78, antiderivative size = 606, normalized size of antiderivative = 1.00, number of steps used = 359, number of rules used = 30, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 2.143$, Rules used = {2489, 2463, 2436, 2333, 2332, 2448, 2437, 2342, 2341, 2443, 2481, 2421, 2430, 6724, 2458, 2388, 2339, 30, 2367, 2356, 45, 2372, 2338, 6874, 2479, 2440, 2438, 2441, 2442, 2445}

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-302177*x)/1152 + (8029*x^2)/2304 - (763*x^3)/3456 + (3*x^4)/256 + (377*(2
 + x)^2)/64 - (71*(2 + x)^3)/216 + (3*(2 + x)^4)/256 + (2069*Log[2 + x])/14
4 - (187*x^2*Log[2 + x])/64 + (83*x^3*Log[2 + x])/288 - (3*x^4*Log[2 + x])/
128 + (6733*(2 + x)*Log[2 + x])/32 - (377*(2 + x)^2*Log[2 + x])/32 + (71*(2
 + x)^3*Log[2 + x])/72 - (3*(2 + x)^4*Log[2 + x])/64 - (43*Log[2 + x]^2)/12
 - (17*x^3*Log[2 + x]^2)/48 + (3*x^4*Log[2 + x]^2)/64 - (1251*(2 + x)*Log[2
 + x]^2)/16 + (273*(2 + x)^2*Log[2 + x]^2)/32 - (3*(2 + x)^3*Log[2 + x]^2)/
4 + (3*(2 + x)^4*Log[2 + x]^2)/64 + (65*(2 + x)*Log[2 + x]^3)/4 - (33*(2 +
x)^2*Log[2 + x]^3)/8 + (3*(2 + x)^3*Log[2 + x]^3)/4 - ((2 + x)^4*Log[2 + x]
^3)/16 + (3891*Log[3 + x])/128 - (115*x^2*Log[3 + x])/48 + (37*x^3*Log[3 +
x])/144 - (3*x^4*Log[3 + x])/128 + (415*(3 + x)*Log[3 + x])/12 - (4083*Log[
```

```
2 + x]*Log[3 + x])/32 - 25*x*Log[2 + x]*Log[3 + x] + (13*x^2*Log[2 + x]*Log
[3 + x])/4 - (7*x^3*Log[2 + x]*Log[3 + x])/12 + (3*x^4*Log[2 + x]*Log[3 + x
])/32 + (963*Log[2 + x]^2*Log[3 + x])/16 + 6*x*Log[2 + x]^2*Log[3 + x] - (3
*x^2*Log[2 + x]^2*Log[3 + x])/2 + (x^3*Log[2 + x]^2*Log[3 + x])/2 - (3*x^4*
Log[2 + x]^2*Log[3 + x])/16 - (81*Log[2 + x]^3*Log[3 + x])/4 + (x^4*Log[2 +
 x]^3*Log[3 + x])/4 - (5609*PolyLog[2, -2 - x])/96 + (563*Log[2 + x]*PolyLo
g[2, -2 - x])/8 - (195*Log[2 + x]^2*PolyLog[2, -2 - x])/4 - (563*PolyLog[3,
 -2 - x])/8 + (195*Log[2 + x]*PolyLog[3, -2 - x])/2 - (195*PolyLog[4, -2 -
x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2367

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2388

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Dist[d, Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x),
 x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
```

```
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2479

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
```

```
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[x*((a +
 b*Log[c*(d + e*x)^n])^p/(i + j*x)), x], x] - Dist[b*e*n*p, Int[x*(a + b*Lo
g[c*(d + e*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2489

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[x^(
r + 1)*(a + b*Log[c*(d + e*x)^n])^p*((f + g*Log[h*(i + j*x)^m])/(r + 1)), x
] + (-Dist[g*j*(m/(r + 1)), Int[x^(r + 1)*((a + b*Log[c*(d + e*x)^n])^p/(i
+ j*x)), x], x] - Dist[b*e*n*(p/(r + 1)), Int[x^(r + 1)*(a + b*Log[c*(d + e
*x)^n])^(p - 1)*((f + g*Log[h*(i + j*x)^m])/(d + e*x)), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \log^3(2+x)\log(3+x)\,dx &= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int \frac{x^4\log^3(2+x)}{3+x}\,dx - \frac{3}{4}\int \frac{x^4\log^2(2+x)\log}{2+x} \\
&= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int \Big(-27\log^3(2+x) + 9x\log^3(2+x) - 3x^2 \\
&= \frac{1}{4}x^4\log^3(2+x)\log(3+x) - \frac{1}{4}\int x^3\log^3(2+x)\,dx + \frac{3}{4}\int x^2\log^3(2+x)\,dx \\
&= 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+x) + \frac{1}{2}x^3\log^2(2+x)\log \\
&= \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x) - \frac{3}{2}x^2\log^2(2+x)\log(3+ \\
&= -\frac{81}{4}(2+x)\log^2(2+x) + \frac{27}{4}(2+x)\log^3(2+x) + 6x\log^2(2+x)\log(3+x \\
&= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{81}{4} \\
&= -\frac{81x}{2} + \frac{81}{2}(2+x)\log(2+x) - \frac{17}{48}x^3\log^2(2+x) + \frac{3}{64}x^4\log^2(2+x) - \frac{765}{16} \\
&= -\frac{765x}{8} + \frac{27}{32}(2+x)^2 - \frac{1}{6}(2+x)^3 + \frac{3}{512}(2+x)^4 + \frac{765}{8}(2+x)\log(2+x) - \\
&= -\frac{857x}{8} + \frac{79}{32}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) + \frac{8}{28} \\
&= -\frac{16463x}{96} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(2+x)^4 - \frac{75}{64}x^2\log(2+x) - \\
&= -\frac{213473x}{1152} + \frac{6013x^2}{2304} - \frac{763x^3}{3456} + \frac{3x^4}{256} + \frac{185}{64}(2+x)^2 - \frac{71}{216}(2+x)^3 + \frac{3}{256}(
\end{aligned}
$$

**Mathematica [A]**
time = 0.23, size = 412, normalized size = 0.68

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[2 + x]^3*Log[3 + x],x]
```

```
[Out] (-195984 - 558290*x + 17705*x^2 - 1050*x^3 + 54*x^4 + 910528*Log[2 + x] + 4
00008*x*Log[2 + x] - 22836*x^2*Log[2 + x] + 2072*x^3*Log[2 + x] - 162*x^4*L
og[2 + x] - 302016*Log[2 + x]^2 - 118800*x*Log[2 + x]^2 + 11880*x^2*Log[2 +
 x]^2 - 1680*x^3*Log[2 + x]^2 + 216*x^4*Log[2 + x]^2 + 48384*Log[2 + x]^3 +
 15552*x*Log[2 + x]^3 - 2592*x^2*Log[2 + x]^3 + 576*x^3*Log[2 + x]^3 - 144*
```

```
x^4*Log[2 + x]^3 + 309078*Log[3 + x] + 79680*x*Log[3 + x] - 5520*x^2*Log[3
+ x] + 592*x^3*Log[3 + x] - 54*x^4*Log[3 + x] - 293976*Log[2 + x]*Log[3 + x
] - 57600*x*Log[2 + x]*Log[3 + x] + 7488*x^2*Log[2 + x]*Log[3 + x] - 1344*x
^3*Log[2 + x]*Log[3 + x] + 216*x^4*Log[2 + x]*Log[3 + x] + 138672*Log[2 + x
]^2*Log[3 + x] + 13824*x*Log[2 + x]^2*Log[3 + x] - 3456*x^2*Log[2 + x]^2*Lo
g[3 + x] + 1152*x^3*Log[2 + x]^2*Log[3 + x] - 432*x^4*Log[2 + x]^2*Log[3 +
x] - 46656*Log[2 + x]^3*Log[3 + x] + 576*x^4*Log[2 + x]^3*Log[3 + x] - 24*(
5609 - 6756*Log[2 + x] + 4680*Log[2 + x]^2)*PolyLog[2, -2 - x] + 288*(-563
+ 780*Log[2 + x])*PolyLog[3, -2 - x] - 224640*PolyLog[4, -2 - x])/2304
```

**Maple [F]**
time = 0.09, size = 0, normalized size = 0.00

$$\int x^3 \ln(2+x)^3 \ln(3+x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(2+x)^3*ln(3+x),x)
```

```
[Out] int(x^3*ln(2+x)^3*ln(3+x),x)
```

**Maxima [A]**
time = 0.50, size = 518, normalized size = 0.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="maxima")
```

```
[Out] 3/128*x^4 + 1/16*(4*x^4*log(x + 3) - x^4 + 4*x^3 - 18*x^2 + 108*x - 324*log
(x + 3))*log(x + 2)^3 - 65/4*log(x + 3)*log(x + 2)^3 + 195/4*log(x + 3)*log
(x + 2)^2*log(-x - 2) - 175/384*x^3 + 1/96*(9*x^4 - 70*x^3 + 495*x^2 - 6*(3
*x^4 - 8*x^3 + 24*x^2 - 96*x)*log(x + 3) + 4680*log(x + 3)*log(-x - 2) - 49
50*x + 4680*dilog(x + 3) + 5778*log(x + 3) + 6048*log(x + 2))*log(x + 2)^2
+ 195/4*dilog(x + 3)*log(x + 2)^2 - 195/4*dilog(-x - 2)*log(x + 2)^2 + 563/
16*log(x + 3)*log(x + 2)^2 + 21*log(x + 2)^3 + 17705/2304*x^2 + 1/8*(780*lo
g(x + 2)^2 - 563*log(x + 2))*dilog(-x - 2) - 1/1152*(27*x^4 - 296*x^3 - 187
20*log(x + 2)^3 + 2760*x^2 + 40536*log(x + 2)^2 - 39840*x - 67308*log(x + 2
))*log(x + 3) - 1/1152*(81*x^4 - 1036*x^3 + 56160*log(x + 3)*log(x + 2)^2 +
 112320*log(x + 3)*log(x + 2)*log(-x - 2) + 11418*x^2 - 12*(9*x^4 - 56*x^3
+ 312*x^2 + 4680*log(x + 2)^2 - 2400*x - 6756*log(x + 2))*log(x + 3) + 1123
20*dilog(x + 3)*log(x + 2) + 112320*dilog(-x - 2)*log(x + 2) - 81072*log(x
+ 3)*log(x + 2) + 72576*log(x + 2)^2 - 200004*x - 81072*dilog(-x - 2) + 146
988*log(x + 3) + 302016*log(x + 2) - 112320*polylog(3, -x - 2))*log(x + 2)
+ 563/8*dilog(-x - 2)*log(x + 2) - 5609/96*log(x + 3)*log(x + 2) + 1573/12*
log(x + 2)^2 - 279145/1152*x - 5609/96*dilog(-x - 2) + 17171/128*log(x + 3)
```

```
 + 14227/36*log(x + 2) - 195/2*polylog(4, -x - 2) - 563/8*polylog(3, -x - 2
)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(x + 3)*log(x + 2)^3, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\left(\frac{x^{4}\log{\left(x+2\right)}^{3}}{4}-\frac{3x^{4}\log{\left(x+2\right)}^{2}}{16}+\frac{3x^{4}\log{\left(x+2\right)}}{32}-\frac{3x^{4}}{128}+\frac{x^{3}\log{\left(x+2\right)}^{2}}{2}-\frac{7x^{3}\log{\left(x+2\right)}}{12}+\frac{37x^{3}}{144}-\frac{3x^{2}\log{\left(x+2\right)}^{2}}{2}+\frac{13x^{2}\log{\left(x+2\right)}}{4}-\frac{115x^{2}}{48}+6x\log{\left(x+2\right)}^{2}-25x\log{\left(x+2\right)}+\frac{415x}{12}-4\log{\left(x+2\right)}^{3}+25\log{\left(x+2\right)}^{2}-\frac{415\log{\left(x+2\right)}}{6}+\frac{10955281}{240000}\right)\log{\left(x+3\right)}-\frac{\int\frac{24900000x}{x+3}\,dx+\int\left(-\frac{1725000x^{2}}{x+3}\right)dx+\int\frac{185000x^{3}}{x+3}\,dx+\int\left(-\frac{16875x^{4}}{x+3}\right)dx+\int\left(-\frac{49800000\log{\left(x+2\right)}}{x+3}\right)dx+\int\frac{18000000\log{\left(x+2\right)}^{2}}{x+3}\,dx+\int\left(-\frac{2880000\log{\left(x+2\right)}^{3}}{x+3}\right)dx+\int\left(-\frac{18000000x\log{\left(x+2\right)}}{x+3}\right)dx+\int\frac{4320000x\log{\left(x+2\right)}^{2}}{x+3}\,dx+\int\frac{2340000x^{2}\log{\left(x+2\right)}}{x+3}\,dx+\int\left(-\frac{1080000x^{2}\log{\left(x+2\right)}^{2}}{x+3}\right)dx+\int\left(-\frac{420000x^{3}\log{\left(x+2\right)}}{x+3}\right)dx+\int\frac{360000x^{3}\log{\left(x+2\right)}^{2}}{x+3}\,dx+\int\frac{67500x^{4}\log{\left(x+2\right)}}{x+3}\,dx+\int\left(-\frac{135000x^{4}\log{\left(x+2\right)}^{2}}{x+3}\right)dx+\int\frac{180000x^{4}\log{\left(x+2\right)}^{3}}{x+3}\,dx+\int\frac{32865843}{x+3}\,dx}{720000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(2+x)**3*ln(3+x),x)
```

```
[Out] (x**4*log(x + 2)**3/4 - 3*x**4*log(x + 2)**2/16 + 3*x**4*log(x + 2)/32 - 3*
x**4/128 + x**3*log(x + 2)**2/2 - 7*x**3*log(x + 2)/12 + 37*x**3/144 - 3*x*
*2*log(x + 2)**2/2 + 13*x**2*log(x + 2)/4 - 115*x**2/48 + 6*x*log(x + 2)**2
 - 25*x*log(x + 2) + 415*x/12 - 4*log(x + 2)**3 + 25*log(x + 2)**2 - 415*lo
g(x + 2)/6 + 10955281/240000)*log(x + 3) - (Integral(24900000*x/(x + 3), x)
 + Integral(-1725000*x**2/(x + 3), x) + Integral(185000*x**3/(x + 3), x) +
Integral(-16875*x**4/(x + 3), x) + Integral(-49800000*log(x + 2)/(x + 3), x
) + Integral(18000000*log(x + 2)**2/(x + 3), x) + Integral(-2880000*log(x +
 2)**3/(x + 3), x) + Integral(-18000000*x*log(x + 2)/(x + 3), x) + Integral
(4320000*x*log(x + 2)**2/(x + 3), x) + Integral(2340000*x**2*log(x + 2)/(x
+ 3), x) + Integral(-1080000*x**2*log(x + 2)**2/(x + 3), x) + Integral(-420
000*x**3*log(x + 2)/(x + 3), x) + Integral(360000*x**3*log(x + 2)**2/(x + 3
), x) + Integral(67500*x**4*log(x + 2)/(x + 3), x) + Integral(-135000*x**4*
log(x + 2)**2/(x + 3), x) + Integral(180000*x**4*log(x + 2)**3/(x + 3), x)
+ Integral(32865843/(x + 3), x))/720000
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(2+x)^3*log(3+x),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(x + 3)*log(x + 2)^3, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \ln(x+2)^3 \ln(x+3)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(x + 2)^3*log(x + 3),x)
[Out] int(x^3*log(x + 2)^3*log(x + 3), x)
```

## 3.30 $\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx$

**Optimal**. Leaf size=17

$$\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

```
[Out] (x+(x^2+b)^(1/2))^a/a
```

**Rubi** [A]

time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 30}

$$\frac{\left(\sqrt{b+x^2}+x\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
```

```
[Out] (x + Sqrt[b + x^2])^a/a
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{b+x^2}\right)^a}{\sqrt{b+x^2}}\,dx &= \text{Subst}\left(\int x^{-1+a}\,dx, x, x+\sqrt{b+x^2}\right)\\ &= \frac{\left(x+\sqrt{b+x^2}\right)^a}{a}\end{aligned}$$

**Mathematica** [A]
time = 0.03, size = 17, normalized size = 1.00

$$\frac{\left(x+\sqrt{b+x^2}\right)^a}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a/Sqrt[b + x^2],x]
[Out] (x + Sqrt[b + x^2])^a/a
```

**Maple** [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+b}\right)^a}{\sqrt{x^2+b}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
[Out] int((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + b))^a/sqrt(x^2 + b), x)
```

**Fricas** [A]
time = 0.56, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + b))^a/a
```

**Sympy** [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$.
time = 1.20, size = 311, normalized size = 18.29

$$\begin{cases} \frac{\sqrt{b}\, b^{\frac{a}{2}} \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ax\sqrt{\frac{b}{x^2}+1}} + \frac{b^{\frac{a}{2}} x \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} + \frac{b^{\frac{a}{2}} x \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}\sqrt{\frac{b}{x^2}+1}} - \frac{2b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)\Gamma\left(1-\frac{a}{2}\right)}{a^2\Gamma\left(-\frac{a}{2}\right)} & \text{for } \left|\frac{x^2}{b}\right| > 1 \\ \frac{b^{\frac{a}{2}} \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{1+\frac{x^2}{b}}} + \frac{b^{\frac{a}{2}} x^2 \sinh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{ab\sqrt{1+\frac{x^2}{b}}} + \frac{b^{\frac{a}{2}} x \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)}{a\sqrt{b}} - \frac{2b^{\frac{a}{2}} \cosh\left(a \operatorname{asinh}\left(\frac{x}{\sqrt{b}}\right)\right)\Gamma\left(1-\frac{a}{2}\right)}{a^2\Gamma\left(-\frac{a}{2}\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a/(x**2+b)**(1/2),x)
```

```
[Out] Piecewise((sqrt(b)*b**(a/2)*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*
x*sqrt(b/x**2 + 1)) + b**(a/2)*x*cosh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b))
)/(a*sqrt(b)) + b**(a/2)*x*sinh(a*asinh(x/sqrt(b)) - asinh(x/sqrt(b)))/(a*s
qrt(b)*sqrt(b/x**2 + 1)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt(b)))*gamma(1 - a/
2)/(a**2*gamma(-a/2)), Abs(x**2/b) > 1), (b**(a/2)*sinh(a*asinh(x/sqrt(b))
- asinh(x/sqrt(b)))/(a*sqrt(1 + x**2/b)) + b**(a/2)*x**2*sinh(a*asinh(x/sqr
t(b)) - asinh(x/sqrt(b)))/(a*b*sqrt(1 + x**2/b)) + b**(a/2)*x*cosh(a*asinh(
x/sqrt(b)) - asinh(x/sqrt(b)))/(a*sqrt(b)) - 2*b**(a/2)*cosh(a*asinh(x/sqrt
(b)))*gamma(1 - a/2)/(a**2*gamma(-a/2)), True))
```

**Giac** [A]
time = 0.94, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")
```

```
[Out] (x + sqrt(x^2 + b))^a/a
```

**Mupad** [B]
time = 0.31, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+b}\right)^a}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a/(b + x^2)^(1/2),x)
```

```
[Out] (x + (b + x^2)^(1/2))^a/a
```

## 3.31 $\int \left(x+\sqrt{b+x^2}\right)^a\,dx$

**Optimal**. Leaf size=52

$$-\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)}+\frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}$$

```
[Out] -1/2*b*(x+(x^2+b)^(1/2))^(-1+a)/(1-a)+1/2*(x+(x^2+b)^(1/2))^(1+a)/(1+a)
```

**Rubi** [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2142, 14}

$$\frac{\left(\sqrt{b+x^2}+x\right)^{a+1}}{2(a+1)}-\frac{b\left(\sqrt{b+x^2}+x\right)^{a-1}}{2(1-a)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[b + x^2])^a,x]
```

```
[Out] -1/2*(b*(x + Sqrt[b + x^2])^(-1 + a))/(1 - a) + (x + Sqrt[b + x^2])^(1 + a)
/(2*(1 + a))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x+\sqrt{b+x^2}\right)^a dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+a}(b+x^2)\ dx, x, x+\sqrt{b+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int (bx^{-2+a}+x^a)\ dx, x, x+\sqrt{b+x^2}\right)$$
$$= -\frac{b\left(x+\sqrt{b+x^2}\right)^{-1+a}}{2(1-a)} + \frac{\left(x+\sqrt{b+x^2}\right)^{1+a}}{2(1+a)}$$

**Mathematica** [A]
time = 0.15, size = 46, normalized size = 0.88

$$\frac{1}{2}\left(x+\sqrt{b+x^2}\right)^{-1+a}\left(\frac{b}{-1+a}+\frac{\left(x+\sqrt{b+x^2}\right)^2}{1+a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[b + x^2])^a,x]
[Out] ((x + Sqrt[b + x^2])^(-1 + a)*(b/(-1 + a) + (x + Sqrt[b + x^2])^2/(1 + a)))
/2
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(44) = 88.
time = 0.03, size = 120, normalized size = 2.31

| method | result | size |
|---|---|---|
| meijerg | $\frac{b^{\frac{a}{2}+\frac{1}{2}} a\left(\frac{8\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}}\left(\frac{ab}{x^2}+a-1\right)\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}}{(1+a)a(2a-2)}+\frac{4\sqrt{\pi}\, x^{1+a} b^{-\frac{a}{2}-\frac{1}{2}}\sqrt{1+\frac{b}{x^2}}\left(\sqrt{1+\frac{b}{x^2}}+1\right)^{a-1}}{(1+a)a}\right)}{4\sqrt{\pi}}$ | 120 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+b)^(1/2))^a,x,method=_RETURNVERBOSE)
[Out] 1/4*b^(1/2*a+1/2)/Pi^(1/2)*a*(8*Pi^(1/2)/(1+a)/a*x^(1+a)*b^(-1/2*a-1/2)*(a*
b/x^2+a-1)/(2*a-2)*((1+1/x^2*b)^(1/2)+1)^(a-1)+4*Pi^(1/2)/(1+a)/a*x^(1+a)*b
^(-1/2*a-1/2)*(1+1/x^2*b)^(1/2)*((1+1/x^2*b)^(1/2)+1)^(a-1))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

**Fricas [A]**
time = 0.73, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+b}\,a-x\right)\left(x+\sqrt{x^2+b}\right)^a}{a^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + b)*a - x)*(x + sqrt(x^2 + b))^a/(a^2 - 1)
```

**Sympy [B]** Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(37) = 74$.
time = 1.19, size = 2147, normalized size = 41.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+b)**(1/2))**a,x)
```

```
[Out] Piecewise((-a**2*b**(9/2)*b**(a/2)*x*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b
)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamm
a(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) -
a**2*b**(7/2)*b**(a/2)*x**3*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)))*gamma
(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(9/2
)*b**(a/2)*x*cosh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + a*b**(7/2)*b**(a/2)*x**3*cosh(a*asinh(x/s
qrt(b)))*gamma(-a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2
*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2
)) + 2*a*b**5*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1
- a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2
) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**5*
b**(a/2)*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x
**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 -
a/2)) - 2*a*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*asinh(x/sqrt(b)) + a
sinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b*
*(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*ga
mma(1 - a/2)) + 4*a*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sq
```

```
rt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*b**(7/2)*x*
*2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*gamma(1 - a
/2)) - 2*a*b**4*b**(a/2)*x**2*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) - 2*a*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a
*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma
(1 - a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2)
 - 2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*a*b**3*b**(a/2)*x**4*cosh(a*asinh(x/
sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2)
 + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(
7/2)*x**2*gamma(1 - a/2)) - 2*b**4*b**(a/2)*x**2*sqrt(b/x**2 + 1)*sinh(a*as
inh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1
- a/2) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) -
2*b**(7/2)*x**2*gamma(1 - a/2)) + 2*b**4*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(
b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*
a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*
x**2*gamma(1 - a/2)) - 2*b**3*b**(a/2)*x**4*sqrt(b/x**2 + 1)*sinh(a*asinh(x
/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2
) + 2*a**2*b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**
(7/2)*x**2*gamma(1 - a/2)) + 2*b**3*b**(a/2)*x**4*cosh(a*asinh(x/sqrt(b)) +
 asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(9/2)*gamma(1 - a/2) + 2*a**2*
b**(7/2)*x**2*gamma(1 - a/2) - 2*b**(9/2)*gamma(1 - a/2) - 2*b**(7/2)*x**2*
gamma(1 - a/2)), Abs(x**2/b) > 1), (-a**2*b**3*b**(a/2)*sqrt(1 + x**2/b)*si
nh(a*asinh(x/sqrt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(
5/2)*gamma(1 - a/2)) - 2*a*b**(5/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asin
h(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 -
a/2) - 2*b**(5/2)*gamma(1 - a/2)) + a*b**(5/2)*b**(a/2)*x*cosh(a*asinh(x/sq
rt(b)))*gamma(-a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 -
a/2)) + 2*a*b**3*b**(a/2)*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma
(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) + 2*
a*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(1 -
a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)) - 2*b**(5
/2)*b**(a/2)*x*sqrt(1 + x**2/b)*sinh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))
*gamma(1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)
) + 2*b**2*b**(a/2)*x**2*cosh(a*asinh(x/sqrt(b)) + asinh(x/sqrt(b)))*gamma(
1 - a/2)/(2*a**2*b**(5/2)*gamma(1 - a/2) - 2*b**(5/2)*gamma(1 - a/2)), True
))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+b)^(1/2))^a,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + b))^a, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x+\sqrt{x^2+b}\right)^a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (b + x^2)^(1/2))^a,x)
[Out] int((x + (b + x^2)^(1/2))^a, x)
```

# 3.32 $\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx$

**Optimal**. Leaf size=34

$$\frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6(1+a)}$$

[Out] 1/6*x^(1+a)*(6+3*x^a+2*x^(2*a))^(1+1/a)/(1+a)

**Rubi** [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {1608, 1761}

$$\frac{x^{a+1}\left(2x^{2a}+3x^a+6\right)^{\frac{1}{a}+1}}{6(a+1)}$$

Antiderivative was successfully verified.

[In] Int[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]

[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6*(1 + a))

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1761

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.)*
((d_) + (e_.)*(x_)^(n_.) + (f_.)*(x_)^(n2_.)), x_Symbol] :> Simp[d*(g*x)^(m
 + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*g*(m + 1))), x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[a*e*(m + 1) - b*d*(m + n*
(p + 1) + 1), 0] && EqQ[a*f*(m + 1) - c*d*(m + 2*n*(p + 1) + 1), 0] && NeQ[
m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} \left(x^a+x^{2a}+x^{3a}\right) dx &= \int x^a\left(1+x^a+x^{2a}\right)\left(6+3x^a+2x^{2a}\right)^{\frac{1}{a}} dx \\
&= \frac{x^{1+a}\left(6+3x^a+2x^{2a}\right)^{1+\frac{1}{a}}}{6(1+a)}
\end{aligned}$$

**Mathematica** [A]
time = 0.10, size = 33, normalized size = 0.97

$$\frac{x^{1+a}(6+3x^a+2x^{2a})^{1+\frac{1}{a}}}{6+6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 + 3*x^a + 2*x^(2*a))^a^(-1)*(x^a + x^(2*a) + x^(3*a)),x]
```

```
[Out] (x^(1 + a)*(6 + 3*x^a + 2*x^(2*a))^(1 + a^(-1)))/(6 + 6*a)
```

**Maple** [A]
time = 0.04, size = 44, normalized size = 1.29

| method | result | size |
|---|---|---|
| risch | $\frac{x\,x^{a}\left(6+3x^{a}+2x^{2a}\right)\left(6+3x^{a}+2x^{2a}\right)^{\frac{1}{a}}}{6+6a}$ | 44 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/6*x*x^a*(6+3*x^a+2*(x^a)^2)/(1+a)*(6+3*x^a+2*(x^a)^2)^(1/a)
```

**Maxima** [A]
time = 0.61, size = 48, normalized size = 1.41

$$\frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a})(2\,x^{2\,a}+3\,x^{a}+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="max
ima")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

**Fricas** [A]
time = 0.72, size = 48, normalized size = 1.41

$$\frac{(2\,xx^{3\,a}+3\,xx^{2\,a}+6\,xx^{a})(2\,x^{2\,a}+3\,x^{a}+6)^{\left(\frac{1}{a}\right)}}{6\,(a+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="fri
cas")
```

```
[Out] 1/6*(2*x*x^(3*a) + 3*x*x^(2*a) + 6*x*x^a)*(2*x^(2*a) + 3*x^a + 6)^(1/a)/(a
+ 1)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x**a+2*x**(2*a))**(1/a)*(x**a+x**(2*a)+x**(3*a)),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6+3*x^a+2*x^(2*a))^(1/a)*(x^a+x^(2*a)+x^(3*a)),x, algorithm="gia
c")
```

```
[Out] integrate((2*x^(2*a) + 3*x^a + 6)^(1/a)*(x^(3*a) + x^(2*a) + x^a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(x^a+x^{2\,a}+x^{3\,a}\right)\,{\left(3\,x^a+2\,x^{2\,a}+6\right)}^{1/a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a),x)
```

```
[Out] int((x^a + x^(2*a) + x^(3*a))*(3*x^a + 2*x^(2*a) + 6)^(1/a), x)
```

# 3.33 $\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx$

**Optimal**. Leaf size=58

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))+1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 57, 632, 210, 31}

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]])/2 - Log[x]/2 + (3*Log[1 -
 (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^2\right)\\
&= -\frac{\log(x)}{2} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\
&= -\frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - \frac{\log(x)}{2} + \frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)
\end{aligned}
$$

**Mathematica** [A]
time = 0.05, size = 77, normalized size = 1.33

$$
\frac{1}{4}\left(2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^2}\right) - \log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)^(
1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.09, size = 65, normalized size = 1.12

| method | result |
|---|---|

| | |
|---|---|
| meijerg | $\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{2}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^{2}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{4\pi}$ |
| trager | $\frac{\ln\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2}x^{2}+15\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)\left(-x^{2}+1\right)^{\frac{2}{3}}+17\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)x^{2}+24\left(-x^{2}+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z\right.}{x^{2}}\right.}{2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*Pi*
3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^2*hypergeom([1,1,4/3],[2,2],
x^2))
```

**Maxima [A]**
time = 0.49, size = 62, normalized size = 1.07

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

**Fricas [A]**
time = 0.84, size = 64, normalized size = 1.10

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{2}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((-
x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

**Sympy [C]** Result contains complex when optimal does not.
time = 0.39, size = 36, normalized size = 0.62

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{1}{x^{2}}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-2))/(2*x**(2/3)*gam
ma(4/3))
```

**Giac [A]**
time = 0.71, size = 64, normalized size = 1.10

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

**Mupad [B]**
time = 0.54, size = 86, normalized size = 1.48

$$\frac{\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-9\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-9\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(1/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/4 - 9*((3^(1/2
)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 - 1/4) - log((9*(1 - x^2)^(1/3))/4 - 9*((
3^(1/2)*1i)/4 + 1/4)^2)*((3^(1/2)*1i)/4 + 1/4)
```

# 3.34 $\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx$

**Optimal**. Leaf size=58

$$-\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)$$

```
[Out] -1/2*ln(x)+3/4*ln(1-(-x^2+1)^(1/3))-1/2*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(
1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {272, 59, 632, 210, 31}

$$-\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(2/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]) - Log[x]/2 + (3*Log[
1 - (1 - x^2)^(1/3)])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-x^2\right)^{2/3}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^2\right)\\ &= -\frac{\log(x)}{2}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right)-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right)\\ &= -\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^2}\right)\\ &= -\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-\frac{\log(x)}{2}+\frac{3}{4}\log\left(1-\sqrt[3]{1-x^2}\right)\end{aligned}$$

**Mathematica [A]**
time = 0.04, size = 77, normalized size = 1.33

$$\frac{1}{4}\left(-2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+2\log\left(-1+\sqrt[3]{1-x^2}\right)-\log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(2/3)),x]
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^2)^
(1/3)] - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/4
```

**Maple [C]** Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.09, size = 48, normalized size = 0.83

| method | result |
|---|---|
| meijerg | $\frac{\left(\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+2\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{2\Gamma\left(\frac{2}{3}\right)x^2\,\text{hypergeom}\left(\left[1,1,\frac{5}{3}\right],[2,2],x^2\right)}{3}}{2\Gamma\left(\frac{2}{3}\right)}$ |

| | |
|---|---|
| trager | $\frac{\ln\left(\frac{-4\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2+15\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^2-9\left(-x^2+1\right)^{\frac{2}{3}}+9\,\mathrm{RootOf}\left(\_Z^2\right.}{x^2}\right.}{2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/GAMMA(2/3)*((1/6*Pi*3^(1/2)-3/2*ln(3)+2*ln(x)+I*Pi)*GAMMA(2/3)+2/3*GAMM
A(2/3)*x^2*hypergeom([1,1,5/3],[2,2],x^2))
```

**Maxima [A]**
time = 0.50, size = 62, normalized size = 1.07

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

**Fricas [A]**
time = 0.64, size = 64, normalized size = 1.10

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{4}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/4*log((
-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log((-x^2 + 1)^(1/3) - 1)
```

**Sympy [C]** Result contains complex when optimal does not.
time = 0.41, size = 37, normalized size = 0.64

$$-\frac{e^{-\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{1}{x^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(2/3),x)
```

```
[Out] -exp(-2*I*pi/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**(-2))/(2*x**(4/3)*g
amma(5/3))
```

**Giac** [A]
time = 0.68, size = 64, normalized size = 1.10

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{4}\log\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{2}\log\left(-\left(-x^{2}+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(2/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/4*log((-x^2 +
 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/2*log(-(-x^2 + 1)^(1/3) + 1)
```

**Mupad** [B]
time = 0.46, size = 76, normalized size = 1.31

$$\frac{\ln\left(\frac{9\left(1-x^{2}\right)^{1/3}}{4}-\frac{9}{4}\right)}{2}+\ln\left(\frac{9\left(1-x^{2}\right)^{1/3}}{2}+\frac{9}{4}-\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\ln\left(\frac{9\left(1-x^{2}\right)^{1/3}}{2}+\frac{9}{4}+\frac{\sqrt{3}\,9\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(2/3)),x)
```

```
[Out] log((9*(1 - x^2)^(1/3))/4 - 9/4)/2 + log((9*(1 - x^2)^(1/3))/2 - (3^(1/2)*9
i)/4 + 9/4)*((3^(1/2)*1i)/4 - 1/4) - log((3^(1/2)*9i)/4 + (9*(1 - x^2)^(1/3
))/2 + 9/4)*((3^(1/2)*1i)/4 + 1/4)
```

# 3.35 $\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$

**Optimal**. Leaf size=49

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*ln(x+(-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(
1/2)
```

**Rubi** [A]
time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {245}

$$\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(-1/3),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + Log[x + (1 - x^3)^
(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}}\,dx=-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)$$

**Mathematica** [A]
time = 0.03, size = 86, normalized size = 1.76

$$\frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(-1/3),x]
```

```
[Out] ArcTan[(-1 + (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[1 + x^2/(1 - x^3
)^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.20, size = 12, normalized size = 0.24

| method | result |
|---|---|
| meijerg | $x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)$ |
| trager | $\frac{\ln\left(-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^3+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x-5\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+3x\left(-x^3+1\right)^{\frac{2}{3}}+3x^2\left(-x\right.\right.}{3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/3,1/3],[4/3],x^3)
```

**Maxima** [A]
time = 0.51, size = 78, normalized size = 1.59

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(\frac{2\,\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)+\frac{1}{3}\,\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{6}\,\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) + 1/3*log((-x^3
 + 1)^(1/3)/x + 1) - 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1
)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.
time = 0.60, size = 82, normalized size = 1.67

$$-\frac{1}{3}\,\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,x-2\,\sqrt{3}\,\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{1}{3}\,\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{6}\,\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*
log((x + (-x^3 + 1)^(1/3))/x) - 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 +
 1)^(2/3))/x^2)
```

**Sympy** [C] Result contains complex when optimal does not.
time = 0.37, size = 29, normalized size = 0.59

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3
))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(-1/3), x)
```

**Mupad** [B]
time = 0.33, size = 10, normalized size = 0.20

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^3)^(1/3),x)
```

```
[Out] x*hypergeom([1/3, 1/3], 4/3, x^3)
```

## 3.36 $\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx$

**Optimal**. Leaf size=55

$$\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] -1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(
1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {272, 57, 632, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/2 + Log[1 - (1 - x
^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1-x^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)\\ &= -\frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)\\ &= -\frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\ &= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)\end{aligned}$$

**Mathematica** [A]
time = 0.04, size = 79, normalized size = 1.44

$$\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1 + (1 - x^3
)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.93, size = 65, normalized size = 1.18

| method | result |
|---|---|

| | |
|---|---|
| meijerg | $\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^{3}\,\mathrm{hypergeom}\left(\left[1,1,\frac{4}{3}\right],[2,2],x^{3}\right)}{9\Gamma\left(\frac{2}{3}\right)}\right)}{6\pi}$ |
| trager | $\frac{\ln\left(\frac{-211\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)^{2}x^{3}-3126\,\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)x^{3}+5502\left(-x^{3}+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^{2}+\_Z+1\right)-11543x^{3}-14247\left(-x^{3}+1\right)}{x^{3}}\right.}{3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*
3^(1/2)/GAMMA(2/3)+2/9*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1,1,4/3],[2,2],
x^3))
```

**Maxima [A]**
time = 0.49, size = 62, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

**Fricas [A]**
time = 0.70, size = 64, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\log\left(\left(-x^{3}+1\right)^{\frac{2}{3}}+\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^{3}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

**Sympy [C]** Result contains complex when optimal does not.
time = 0.39, size = 32, normalized size = 0.58

$$-\frac{e^{-\frac{i\pi}{3}}\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{1}{x^{3}}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**(-3))/(3*x*gamma(4/3)
)
```

**Giac [A]**
time = 2.29, size = 63, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 +
1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

**Mupad [B]**
time = 0.51, size = 80, normalized size = 1.45

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/3)),x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6)
```

## 3.37 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

**Optimal**. Leaf size=97

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln((1-x)*(1+x)^2)*2^(2/3)+3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-
1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

**Rubi** [A]

time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2174\}$

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(1
/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)])/(4*2^(1/3))
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-\right.}{4\sqrt[3]{2}}$$

**Mathematica [A]**
time = 0.78, size = 148, normalized size = 1.53

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)-\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-1+x)\sqrt[3]{2-2x^3}+4(1-x^3)^{2/3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^3
)^(1/3))] + 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] - Log[2^(2/3) -
 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x^3)^(2/
3)])/(4*2^(1/3))
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.24, size = 1163, normalized size = 11.99

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 1163 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(-(12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_
Z^3-4)^2*x+10*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z
^3-4)^3*x+8*RootOf(_Z^3-4)^2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)-8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-13*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x+8*Root
Of(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^
2)+42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+13*RootOf(_Z^
3-4)^2*(-x^3+1)^(1/3)+35*RootOf(_Z^3-4)*x^2+36*RootOf(RootOf(_Z^3-4)^2+2*_Z
*RootOf(_Z^3-4)+4*_Z^2)*x+30*RootOf(_Z^3-4)*x+52*(-x^3+1)^(2/3)+42*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+35*RootOf(_Z^3-4))/(1+x)^2)*Roo
tOf(_Z^3-4)-1/2*ln(-(12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
^2*RootOf(_Z^3-4)^2*x+10*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*RootOf(_Z^3-4)^3*x+8*RootOf(_Z^3-4)^2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4
)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-13*RootOf(_Z^3-4)^2*(-x^3+1)^(1/
3)*x+8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)+42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+13
*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+35*RootOf(_Z^3-4)*x^2+36*RootOf(RootOf(_Z^
3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+30*RootOf(_Z^3-4)*x+52*(-x^3+1)^(2/3)+
42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+35*RootOf(_Z^3-4))/(
1+x)^2)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/2*RootOf(Root
Of(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln(-(12*RootOf(RootOf(_Z^3-4)^2+2*
```

```
_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-4*RootOf(RootOf(_Z^3-4)^2+2*
_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x-8*RootOf(_Z^3-4)^2*(-x^3+1)^(2
/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+8*RootOf(_Z^3-4)*(-
x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-9*RootOf
(_Z^3-4)^2*(-x^3+1)^(1/3)*x-8*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*x^2+9*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+14*RootOf(_Z^3-4)*x^
2-12*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+4*RootOf(_Z^3-4)
*x+36*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
+14*RootOf(_Z^3-4))/(1+x)^2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(71) = 142$.
time = 3.05, size = 301, normalized size = 3.10

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}(13\,x^6+2\,x^5+19\,x^4-4\,x^3+19\,x^2+2\,x+13)-4\sqrt{2}\,(5\,x^5-5\,x^4+6\,x^3-6\,x^2+5\,x-5)(-x^3+1)^{\frac{1}{3}}+16\cdot 2^{\frac{1}{6}}(x^4+2\,x^3+2\,x^2+2\,x+1)(-x^3+1)^{\frac{2}{3}}\right)}{6\,(3\,x^6-18\,x^5-3\,x^4-28\,x^3-3\,x^2-18\,x+3)}\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(\frac{4\cdot 2^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}(x^2+1)+2^{\frac{1}{3}}(5\,x^4+6\,x^2+5)-2\,(3\,x^3-x^2+x-3)(-x^3+1)^{\frac{1}{3}}}{x^4+4\,x^3+6\,x^2+4\,x+1}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+2\,x+1)-2\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4\,(-x^3+1)^{\frac{2}{3}}}{x^2+2\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(13*x^6 + 2*x^5 +
19*x^4 - 4*x^3 + 19*x^2 + 2*x + 13) - 4*sqrt(2)*(5*x^5 - 5*x^4 + 6*x^3 - 6*
x^2 + 5*x - 5)*(-x^3 + 1)^(1/3) + 16*2^(1/6)*(x^4 + 2*x^3 + 2*x^2 + 2*x + 1
)*(-x^3 + 1)^(2/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) -
 1/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 +
6*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
+ 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(1-x^3\right)}^{1/3}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

## 3.38 $\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$

**Optimal**. Leaf size=145

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}}+\frac{1}{2}\log\left(x+\sqrt[3]{1-x^3}\right)-\frac{3\log\left(-\right.}{}$$

```
[Out] 1/8*ln((1-x)*(1+x)^2)*2^(2/3)+1/2*ln(x+(-x^3+1)^(1/3))-3/8*ln(-1+x+2^(2/3)*
(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1
/2)+1/4*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3
)
```

**Rubi** [A]
time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2177, 245, 2174}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 - x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3
)) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[(1 - x)*(1 +
 x)^2]/(4*2^(1/3)) + Log[x + (1 - x^3)^(1/3)]/2 - (3*Log[-1 + x + 2^(2/3)*(
1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
```

```
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{1-x^3}}\,dx - \int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \ldots$$

**Mathematica** [F]
time = 5.53, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/((1 + x)*(1 - x^3)^(1/3)),x]
[Out] Integrate[x/((1 + x)*(1 - x^3)^(1/3)), x]
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 12.29, size = 1790, normalized size = 12.34

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 1790 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(-(10*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^
3+4)^3*x+12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z
^3+4)^2*x-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3
```

+4)^2*(-x^3+1)^(2/3)-13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x+13*(-x
^3+1)^(1/3)*RootOf(_Z^3+4)^2+8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-35*x^2*RootOf(_Z^3+4)-42*RootOf(RootO
f(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-30*x*RootOf(_Z^3+4)-36*RootOf(R
ootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+52*(-x^3+1)^(2/3)-35*RootOf(_
Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(1+x)^2)*Roo
tOf(_Z^3+4)-1/2*ln(-(10*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
*RootOf(_Z^3+4)^3*x+12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^
2*RootOf(_Z^3+4)^2*x-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*
RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x-8*(-x^
3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+
4)*x+13*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3
+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)-35*x^2*RootOf(_Z^3+4)-42*R
ootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2-30*x*RootOf(_Z^3+4)-
36*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+52*(-x^3+1)^(2/3)-
35*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2))/(
1+x)^2)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+1/2*RootOf(Root
Of(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((4*RootOf(RootOf(_Z^3+4)^2+2*_Z
*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x-12*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x-8*RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)+9*(-x^3+1)^(1/3)*Roo
tOf(_Z^3+4)^2*x-8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4
)+4*_Z^2)*RootOf(_Z^3+4)*x-9*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+8*(-x^3+1)^(1/
3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+14*x^
2*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2
+4*x*RootOf(_Z^3+4)-12*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*
x-36*(-x^3+1)^(2/3)+14*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2))/(1+x)^2)-1/6*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)*ln(-RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4
*_Z^2)^2*RootOf(_Z^3+4)^4*x^3+2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*
_Z*RootOf(_Z^3+4)+4*_Z^2)+12*x*(-x^3+1)^(2/3)-12*x^2*(-x^3+1)^(1/3)+8*x^3-4
)+1/6*ln(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+
4)^4*x^3+6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+
4)^2*(-x^3+1)^(2/3)*x-6*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootO
f(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)*x^2+4*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-2*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+
4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+4*x^3-4)*RootOf(_Z^3+4)^2*RootOf(RootOf(_Z
^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)-1/3*ln(RootOf(RootOf(_Z^3+4)^2+2*_Z*Roo
tOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^4*x^3+6*RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)*x-6*RootOf(_Z^3+4)^2*
RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*(-x^3+1)^(1/3)*x^2+4*Ro
otOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^3-2*Ro
otOf(_Z^3+4)^2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+4*x^3-4)

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x**3+1)**(1/3),x)
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(x/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(1-x^3)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x + 1)),x)
[Out] int(x/((1 - x^3)^(1/3)*(x + 1)), x)
```

## 3.39 $\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx$

**Optimal**. Leaf size=110

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{\sqrt[3]{2}(2-x)}{\sqrt{3}\sqrt[3]{2-3x+x^2}}\right)}{2\sqrt[3]{2}}-\frac{\log(2-x)}{4\sqrt[3]{2}}-\frac{\log(x)}{2\sqrt[3]{2}}+\frac{3\log\left(2-x-2^{2/3}\sqrt[3]{2-3x+x^2}\right)}{4\sqrt[3]{2}}$$

```
[Out] -1/8*ln(2-x)*2^(2/3)-1/4*ln(x)*2^(2/3)+3/8*ln(2-x-2^(2/3)*(x^2-3*x+2)^(1/3)
)*2^(2/3)+1/4*arctan(-1/3*3^(1/2)-1/3*2^(1/3)*(2-x)/(x^2-3*x+2)^(1/3)*3^(1/
2))*3^(1/2)*2^(2/3)
```

**Rubi** [A]
time = 0.01, antiderivative size = 176, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {769, 124}

$$-\frac{\sqrt{3}\sqrt[3]{x-2}\sqrt[3]{x-1}\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}(x-2)^{2/3}}{\sqrt{3}\sqrt[3]{x-1}}\right)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}+\frac{3\sqrt[3]{x-2}\sqrt[3]{x-1}\log\left(-\frac{(x-2)^{2/3}}{\sqrt[3]{2}}-\sqrt[3]{2}\sqrt[3]{x-1}\right)}{4\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}-\frac{\sqrt[3]{x-2}\sqrt[3]{x-1}\log(x)}{2\sqrt[3]{2}\sqrt[3]{x^2-3x+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] -1/2*(Sqrt[3]*(-2 + x)^(1/3)*(-1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2^(1/3)*(-2
 + x)^(2/3))/(Sqrt[3]*(-1 + x)^(1/3))])/(2^(1/3)*(2 - 3*x + x^2)^(1/3)) + (
3*(-2 + x)^(1/3)*(-1 + x)^(1/3)*Log[-((-2 + x)^(2/3)/2^(1/3)) - 2^(1/3)*(-1
 + x)^(1/3)])/(4*2^(1/3)*(2 - 3*x + x^2)^(1/3)) - ((-2 + x)^(1/3)*(-1 + x)^
(1/3)*Log[x])/(2*2^(1/3)*(2 - 3*x + x^2)^(1/3))
```

Rule 124

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)*((e_.) + (f_.)*(x_))
^(1/3)), x_Symbol] :> With[{q = Rt[b*((b*e - a*f)/(b*c - a*d)^2), 3]}, Simp
[-Log[a + b*x]/(2*q*(b*c - a*d)), x] + (-Simp[Sqrt[3]*(ArcTan[1/Sqrt[3] + 2
*q*((c + d*x)^(2/3)/(Sqrt[3]*(e + f*x)^(1/3)))]/(2*q*(b*c - a*d))), x] + Si
mp[3*(Log[q*(c + d*x)^(2/3) - (e + f*x)^(1/3)]/(4*q*(b*c - a*d))), x])] /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - b*c*f - a*d*f, 0]
```

Rule 769

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(b + q + 2*c*x)^(1/3)*((b - q
+ 2*c*x)^(1/3)/(a + b*x + c*x^2)^(1/3)), Int[1/((d + e*x)*(b + q + 2*c*x)^(
1/3)*(b - q + 2*c*x)^(1/3)), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^
2 - 4*a*c, 0] && EqQ[c^2*d^2 - b*c*d*e - 2*b^2*e^2 + 9*a*c*e^2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{2-3x+x^2}}\,dx = \frac{\left(\sqrt[3]{-4+2x}\,\sqrt[3]{-2+2x}\right)\int \frac{1}{x\sqrt[3]{-4+2x}\,\sqrt[3]{-2+2x}}\,dx}{\sqrt[3]{2-3x+x^2}}$$
$$= -\frac{\sqrt{3}\,\sqrt[3]{-2+x}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[3]{2}\,(-2+x)^{2/3}}{\sqrt{3}\,\sqrt[3]{-1+x}}\right)}{2\sqrt[3]{2}\,\sqrt[3]{2-3x+x^2}}+\frac{3\sqrt[3]{-2+x}\,\sqrt[3]{-1+}}{4}$$

**Mathematica** [**A**]
time = 0.19, size = 162, normalized size = 1.47

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2-3x+x^2}}{2\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{2-3x+x^2}}\right)+2\log\left(-2\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{2-3x+x^2}\right)-\log\left(4\,2^{2/3}-4\,2^{2/3}x+2^{2/3}x^2-2\sqrt[3]{2}(-2+x)\sqrt[3]{2-3x+x^2}+4(2-3x+x^2)^{2/3}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x + x^2)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(2 - 3*x + x^2)^(1/3))/(2*2^(1/3) - 2^(1/3)*x +
(2 - 3*x + x^2)^(1/3))] + 2*Log[-2*2^(1/3) + 2^(1/3)*x + 2*(2 - 3*x + x^2)^
(1/3)] - Log[4*2^(2/3) - 4*2^(2/3)*x + 2^(2/3)*x^2 - 2*2^(1/3)*(-2 + x)*(2
- 3*x + x^2)^(1/3) + 4*(2 - 3*x + x^2)^(2/3)])/(4*2^(1/3))
```

**Maple** [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.38, size = 1069, normalized size = 9.72

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 1069 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-3*x+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*RootOf(_Z^3-4)*ln(-(112*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_
Z^2)^2*RootOf(_Z^3-4)^2*x^2+68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*RootOf(_Z^3-4)^3*x^2-504*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4
)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-216*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3
-4)+4*_Z^2)*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(2/3)-306*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+504*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2+258*RootOf(RootOf(_Z^3-4)^2+
2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)*x+306*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3-108*RootOf(_Z^
3-4)^2*(x^2-3*x+2)^(1/3)*x-516*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+
4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)+196*RootOf(RootOf(_Z^3-4)^2+2*_Z*R
```

```
ootOf(_Z^3-4)+4*_Z^2)*x^2+216*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(1/3)+119*RootOf
(_Z^3-4)*x^2-1680*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-948
*(x^2-3*x+2)^(2/3)-1020*RootOf(_Z^3-4)*x+1680*RootOf(RootOf(_Z^3-4)^2+2*_Z*
RootOf(_Z^3-4)+4*_Z^2)+1020*RootOf(_Z^3-4))/x^2)+1/2*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*ln((68*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x^2+28*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf
(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x^2-306*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root
Of(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-108*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(2/3)-126*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3*x+306*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2-237*RootOf(RootOf(_
Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)*x+126
*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^3-54*Ro
otOf(_Z^3-4)^2*(x^2-3*x+2)^(1/3)*x+474*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)*(x^2-3*x+2)^(1/3)+17*RootOf(RootOf(_Z^3-4)^2
+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2+108*RootOf(_Z^3-4)^2*(x^2-3*x+2)^(1/3)+7*R
ootOf(_Z^3-4)*x^2+408*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x
+258*(x^2-3*x+2)^(2/3)+168*RootOf(_Z^3-4)*x-408*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2)-168*RootOf(_Z^3-4))/x^2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(81) = 162$.
time = 2.30, size = 277, normalized size = 2.52

$$-\frac{1}{12}\,\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^{6}+36\,x^{5}-612\,x^{4}+2880\,x^{3}-5760\,x^{2}+5184\,x-1728\right)+12\,\sqrt{2}\left(x^{5}-38\,x^{4}+252\,x^{3}-648\,x^{2}+720\,x-288\right)\left(x^{2}-3\,x+2\right)^{\frac{1}{3}}+48\cdot 2^{\frac{1}{6}}\left(x^{4}-6\,x^{3}+6\,x^{2}\right)\left(x^{2}-3\,x+2\right)^{\frac{2}{3}}\right)}{6\left(x^{6}-108\,x^{5}+972\,x^{4}-3456\,x^{3}+6048\,x^{2}-5184\,x+1728\right)}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}x^{2}+6\cdot 2^{\frac{1}{3}}\left(x^{2}-3\,x+2\right)^{\frac{1}{3}}\left(x-2\right)+12\left(x^{2}-3\,x+2\right)^{\frac{2}{3}}}{x^{2}}\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}\left(x^{2}-3\,x+2\right)^{\frac{2}{3}}\left(x^{2}-6\,x+6\right)+2^{\frac{1}{3}}\left(x^{4}-36\,x^{3}+180\,x^{2}-288\,x+144\right)-6\left(x^{3}-14\,x^{2}+36\,x-24\right)\left(x^{2}-3\,x+2\right)^{\frac{1}{3}}}{x^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 36*x^5 - 6
12*x^4 + 2880*x^3 - 5760*x^2 + 5184*x - 1728) + 12*sqrt(2)*(x^5 - 38*x^4 +
252*x^3 - 648*x^2 + 720*x - 288)*(x^2 - 3*x + 2)^(1/3) + 48*2^(1/6)*(x^4 -
6*x^3 + 6*x^2)*(x^2 - 3*x + 2)^(2/3))/(x^6 - 108*x^5 + 972*x^4 - 3456*x^3 +
 6048*x^2 - 5184*x + 1728)) + 1/12*2^(2/3)*log((2^(2/3)*x^2 + 6*2^(1/3)*(x^
2 - 3*x + 2)^(1/3)*(x - 2) + 12*(x^2 - 3*x + 2)^(2/3))/x^2) - 1/24*2^(2/3)*
log((12*2^(2/3)*(x^2 - 3*x + 2)^(2/3)*(x^2 - 6*x + 6) + 2^(1/3)*(x^4 - 36*x
```

```
^3 + 180*x^2 - 288*x + 144) - 6*(x^3 - 14*x^2 + 36*x - 24)*(x^2 - 3*x + 2)^
(1/3))/x^4)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{(x-2)(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-3*x+2)**(1/3),x)
[Out] Integral(1/(x*((x - 2)*(x - 1))**(1/3)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-3*x+2)^(1/3),x, algorithm="giac")
[Out] integrate(1/((x^2 - 3*x + 2)^(1/3)*x), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(x^2-3\,x+2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2 - 3*x + 2)^(1/3)),x)
[Out] int(1/(x*(x^2 - 3*x + 2)^(1/3)), x)
```

# 3.40 $\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx$

**Optimal**. Leaf size=81

$$\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\,\sqrt[3]{-5+7x-3x^2+x^3}}\right)+\frac{1}{4}\log(1-x)-\frac{3}{4}\log\left(1-x+\sqrt[3]{-5+7x-3x^2+x^3}\right)$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+(x^3-3*x^2+7*x-5)^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*
(-1+x)/(x^3-3*x^2+7*x-5)^(1/3)*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2092, 2036, 335, 281, 245}

$$\frac{\sqrt{3}\,\sqrt[3]{(x-1)^2+4}\,\sqrt[3]{x-1}\,\text{ArcTan}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{(x-1)^2+4}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3+4(x-1)}}-\frac{3\sqrt[3]{(x-1)^2+4}\,\sqrt[3]{x-1}\,\log\left((x-1)^{2/3}-\sqrt[3]{(x-1)^2+4}\right)}{4\sqrt[3]{(x-1)^3+4(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (Sqrt[3]*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)^(2/3
))/(4 + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(4*(-1 + x) + (-1 + x)^3)^(1/3)) -
(3*(4 + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(4 + (-1 + x)^2)^(1/3) + (-1
+ x)^(2/3)])/(4*(4*(-1 + x) + (-1 + x)^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{-5+7x-3x^2+x^3}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[3]{4x+x^3}}\,dx, x, -1+x\right)\\
&= \frac{\left(\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{4+x^2}}\,dx, x, -1+x\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{4+x^6}}\,dx, x, \sqrt[3]{-1+x}\right)}{\sqrt[3]{4(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{4+x^3}}\,dx, x, (-1+x)^{2/3}\right)}{2\sqrt[3]{4(-1+x)+(-1+x)^3}}\\
&= \frac{\sqrt{3}\,\sqrt[3]{4+(-1+x)^2}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{4+(-1+x)^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-4(1-x)+(-1+x)^3}} - \frac{3\sqrt[3]{4+
\end{aligned}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.01, size = 85, normalized size = 1.05

$$\frac{3\sqrt[3]{(2-i)+ix}\,\sqrt[3]{i(-1+x)}\,((-1+2i)+x)F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{1}{4}i((-1+2i)+x),-\frac{1}{2}i((-1+2i)+x)\right)}{4\sqrt[3]{-5+7x-3x^2+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 7*x - 3*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*((2 - I) + I*x)^(1/3)*(I*(-1 + x))^(1/3)*((-1 + 2*I) + x)*AppellF1[2/3,
1/3, 1/3, 5/3, (-1/4*I)*((-1 + 2*I) + x), (-1/2*I)*((-1 + 2*I) + x)])/(4*(-
5 + 7*x - 3*x^2 + x^3)^(1/3))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.90, size = 433, normalized size = 5.35

| method | result |
|---|---|
| trager | $-\frac{\ln\left(92\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)^2 x^2+624\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-3x^2+7x-5\right)^{\frac{2}{3}}-675\,\mathrm{RootOf}\left(\_Z^2-\_Z+1\right)\left(x^3-3x^2+7x-\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-3*x^2+7*x-5)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(92*RootOf(_Z^2-_Z+1)^2*x^2+624*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^
(2/3)-675*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x-184*RootOf(_Z^2-_Z+1)
^2*x-41*RootOf(_Z^2-_Z+1)*x^2+51*(x^3-3*x^2+7*x-5)^(2/3)+675*RootOf(_Z^2-_Z
+1)*(x^3-3*x^2+7*x-5)^(1/3)+624*(x^3-3*x^2+7*x-5)^(1/3)*x+82*RootOf(_Z^2-_Z
+1)*x-583*x^2-624*(x^3-3*x^2+7*x-5)^(1/3)-713*RootOf(_Z^2-_Z+1)+1166*x-1643
)+1/2*RootOf(_Z^2-_Z+1)*ln(212*RootOf(_Z^2-_Z+1)^2*x^2-624*RootOf(_Z^2-_Z+1
)*(x^3-3*x^2+7*x-5)^(2/3)-51*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)*x-42
4*RootOf(_Z^2-_Z+1)^2*x+463*RootOf(_Z^2-_Z+1)*x^2+675*(x^3-3*x^2+7*x-5)^(2/
3)+51*RootOf(_Z^2-_Z+1)*(x^3-3*x^2+7*x-5)^(1/3)-624*(x^3-3*x^2+7*x-5)^(1/3)
*x-926*RootOf(_Z^2-_Z+1)*x+161*x^2+624*(x^3-3*x^2+7*x-5)^(1/3)+1643*RootOf(
_Z^2-_Z+1)-322*x+713)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

**Fricas** [A]
time = 1.10, size = 120, normalized size = 1.48

$$-\frac{1}{2}\,\sqrt{3}\,\arctan\left(\frac{22791076\,\sqrt{3}\,(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}(x-1)+\sqrt{3}\,(20389537\,x^2-40779074\,x+53222437)+17987998\,\sqrt{3}\,(x^3-3\,x^2+7\,x-5)^{\frac{2}{3}}}{7204617\,x^2-14409234\,x-20666867}\right)-\frac{1}{4}\,\log\left(3\,(x^3-3\,x^2+7\,x-5)^{\frac{1}{3}}(x-1)-3\,(x^3-3\,x^2+7\,x-5)^{\frac{2}{3}}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(3)*arctan((22791076*sqrt(3)*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1)
 + sqrt(3)*(20389537*x^2 - 40779074*x + 53222437) + 17987998*sqrt(3)*(x^3 -
 3*x^2 + 7*x - 5)^(2/3))/(7204617*x^2 - 14409234*x - 20666867)) - 1/4*log(3
*(x^3 - 3*x^2 + 7*x - 5)^(1/3)*(x - 1) - 3*(x^3 - 3*x^2 + 7*x - 5)^(2/3) +
4)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x^3 - 3x^2 + 7x - 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-3*x**2+7*x-5)**(1/3),x)
[Out] Integral((x**3 - 3*x**2 + 7*x - 5)**(-1/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-3*x^2+7*x-5)^(1/3),x, algorithm="giac")
[Out] integrate((x^3 - 3*x^2 + 7*x - 5)^(-1/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^3 - 3\,x^2 + 7\,x - 5\right)}^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3),x)
[Out] int(1/(7*x - 3*x^2 + x^3 - 5)^(1/3), x)
```

# 3.41 $\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$

**Optimal**. Leaf size=66

$$\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2x}{\sqrt{3}\,\sqrt[3]{x\left(-q+x^2\right)}}\right)+\frac{\log(x)}{4}-\frac{3}{4}\log\left(-x+\sqrt[3]{x\left(-q+x^2\right)}\right)$$

```
[Out] 1/4*ln(x)-3/4*ln(-x+(x*(x^2-q))^(1/3))+1/2*arctan(1/3*3^(1/2)+2/3*x/(x*(x^2
-q))^(1/3)*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2004, 2036, 335, 281, 245}

$$\frac{\sqrt{3}\,\sqrt[3]{x}\,\sqrt[3]{x^2-q}\,\text{ArcTan}\left(\frac{\frac{2x^{2/3}}{\sqrt[3]{x^2-q}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{x^3-qx}}-\frac{3\sqrt[3]{x}\,\sqrt[3]{x^2-q}\,\log\left(x^{2/3}-\sqrt[3]{x^2-q}\right)}{4\sqrt[3]{x^3-qx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*x^(1/3)*(-q + x^2)^(1/3)*ArcTan[(1 + (2*x^(2/3))/(-q + x^2)^(1/3))
/Sqrt[3]])/(2*(-(q*x) + x^3)^(1/3)) - (3*x^(1/3)*(-q + x^2)^(1/3)*Log[x^(2/
3) - (-q + x^2)^(1/3)])/(4*(-(q*x) + x^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx &= \int \frac{1}{\sqrt[3]{-qx+x^3}}\,dx\\
&= \frac{\left(\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-q+x^2}}\,dx}{\sqrt[3]{-qx+x^3}}\\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt[3]{-q+x^6}}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{-qx+x^3}}\\
&= \frac{\left(3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{-q+x^3}}\,dx,x,x^{2/3}\right)}{2\sqrt[3]{-qx+x^3}}\\
&= \frac{\sqrt{3}\,\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\,\tan^{-1}\left(\frac{1+\frac{2x^{2/3}}{\sqrt[3]{-q+x^2}}}{\sqrt{3}}\right)}{2\sqrt[3]{-qx+x^3}}-\frac{3\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\,\log\left(x^{2/3}-\sqrt[3]{-\right.}{4\sqrt[3]{-qx+x^3}}
\end{aligned}$$

**Mathematica** [A]
time = 0.62, size = 130, normalized size = 1.97

$$\frac{\sqrt[3]{x}\,\sqrt[3]{-q+x^2}\left(2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}\,x^{2/3}}{x^{2/3}+2\sqrt[3]{-q+x^2}}\right)-2\log\left(-x^{2/3}+\sqrt[3]{-q+x^2}\right)+\log\left(x^{4/3}+x^{2/3}\sqrt[3]{-q+x^2}+\left(-q+x^2\right)^{2/3}\right)\right)}{4\sqrt[3]{-qx+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-q + x^2))^(-1/3),x]
```

```
[Out] (x^(1/3)*(-q + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x^(2/3))/(x^(2/3) + 2*
(-q + x^2)^(1/3))] - 2*Log[-x^(2/3) + (-q + x^2)^(1/3)] + Log[x^(4/3) + x^(
2/3)*(-q + x^2)^(1/3) + (-q + x^2)^(2/3)]))/(4*(-(q*x) + x^3)^(1/3))
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x \left(x^{2}-q \right)\right)^{\frac{1}{3}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^2-q))^(1/3),x)
```

```
[Out] int(1/(x*(x^2-q))^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 - q)*x)^(-1/3), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(52) = 104$.
time = 1.78, size = 415, normalized size = 6.29

$$\frac{1}{2} \sqrt{3} \arctan\left(\frac{4 \sqrt{3} (q^{12} - 15 q^{10} + 90 q^{8} - 351 q^{6} + 810 q^{4} - 1215 q^{2} + 729)(x^{3} - qx)^{\frac{1}{3}} x - 2 \sqrt{3} (q^{12} + 6 q^{11} - 15 q^{10} - 54 q^{9} + 90 q^{8} + 270 q^{7} - 351 q^{6} - 810 q^{5} + 810 q^{4} + 1458 q^{3} - 1215 q^{2} - 1458 q + 729)(x^{3} - qx)^{\frac{2}{3}} - \sqrt{3} (q^{13} + 10 q^{12} - 15 q^{11} - 282 q^{10} + 90 q^{9} + 2178 q^{8} - 351 q^{7} - 6534 q^{6} + 810 q^{5} + 7614 q^{4} - 1215 q^{3} - (q^{12} - 6 q^{11} - 15 q^{10} + 54 q^{9} + 90 q^{8} - 270 q^{7} - 351 q^{6} + 810 q^{5} + 810 q^{4} - 1458 q^{3} - 1215 q^{2} + 1458 q + 729)x^{2} - 2430 q^{2} + 729 q)}{q^{13} + 18 q^{12} + 81 q^{11} - 162 q^{10} - 1350 q^{9} + 810 q^{8} + 6561 q^{7} - 2430 q^{6} - 12150 q^{5} + 4374 q^{4} + 6561 q^{3} - 9 (q^{12} + 2 q^{11} - 15 q^{10} - 18 q^{9} + 90 q^{8} + 90 q^{7} - 351 q^{6} - 270 q^{5} + 810 q^{4} + 486 q^{3} - 1215 q^{2} - 486 q + 729)x^{2} - 4374 q^{2} + 729 q}\right) - \frac{1}{4} \log\left(-3 (x^{3} - qx)^{\frac{1}{3}} x + q + 3 (x^{3} - qx)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((4*sqrt(3)*(q^12 - 15*q^10 + 90*q^8 - 351*q^6 + 810*q^4
- 1215*q^2 + 729)*(x^3 - q*x)^(1/3)*x - 2*sqrt(3)*(q^12 + 6*q^11 - 15*q^10
- 54*q^9 + 90*q^8 + 270*q^7 - 351*q^6 - 810*q^5 + 810*q^4 + 1458*q^3 - 1215
*q^2 - 1458*q + 729)*(x^3 - q*x)^(2/3) - sqrt(3)*(q^13 + 10*q^12 - 15*q^11
- 282*q^10 + 90*q^9 + 2178*q^8 - 351*q^7 - 6534*q^6 + 810*q^5 + 7614*q^4 -
1215*q^3 - (q^12 - 6*q^11 - 15*q^10 + 54*q^9 + 90*q^8 - 270*q^7 - 351*q^6 +
 810*q^5 + 810*q^4 - 1458*q^3 - 1215*q^2 + 1458*q + 729)*x^2 - 2430*q^2 + 7
29*q))/(q^13 + 18*q^12 + 81*q^11 - 162*q^10 - 1350*q^9 + 810*q^8 + 6561*q^7
 - 2430*q^6 - 12150*q^5 + 4374*q^4 + 6561*q^3 - 9*(q^12 + 2*q^11 - 15*q^10
- 18*q^9 + 90*q^8 + 90*q^7 - 351*q^6 - 270*q^5 + 810*q^4 + 486*q^3 - 1215*q
```

```
^2 - 486*q + 729)*x^2 - 4374*q^2 + 729*q)) - 1/4*log(-3*(x^3 - q*x)^(1/3)*x
 + q + 3*(x^3 - q*x)^(2/3))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x\left(-q+x^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x**2-q))**(1/3),x)
```

```
[Out] Integral((x*(-q + x**2))**(-1/3), x)
```

**Giac [A]**
time = 0.85, size = 67, normalized size = 1.02

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{4}\log\left(\left(-\frac{q}{x^2}+1\right)^{\frac{2}{3}}+\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{2}\log\left(\left|\left(-\frac{q}{x^2}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(x^2-q))^(1/3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-q/x^2 + 1)^(1/3) + 1)) + 1/4*log((-q/x
^2 + 1)^(2/3) + (-q/x^2 + 1)^(1/3) + 1) - 1/2*log(abs((-q/x^2 + 1)^(1/3) -
1))
```

**Mupad [B]**
time = 0.39, size = 37, normalized size = 0.56

$$\frac{3\,x\,{\left(1-\frac{x^2}{q}\right)}^{1/3}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \frac{x^2}{q}\right)}{2\,{\left(x^3-q\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(q - x^2))^(1/3),x)
```

```
[Out] (3*x*(1 - x^2/q)^(1/3)*hypergeom([1/3, 1/3], 4/3, x^2/q))/(2*(x^3 - q*x)^(1
/3))
```

# 3.42 $\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx$

**Optimal**. Leaf size=79

$$\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2(-1+x)}{\sqrt{3}\,\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\right)+\frac{1}{4}\log(1-x)-\frac{3}{4}\log\Big(1-x+\sqrt[3]{(-1+x)\,(q-2x+x$$

```
[Out] 1/4*ln(1-x)-3/4*ln(1-x+((-1+x)*(x^2+q-2*x))^(1/3))+1/2*arctan(1/3*3^(1/2)+2
/3*(-1+x)/((-1+x)*(x^2+q-2*x))^(1/3)*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.84, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2092, 2036, 335, 281, 245}

$$\frac{\sqrt{3}\,\sqrt[3]{x-1}\,\sqrt[3]{q+(x-1)^2-1}\,\text{ArcTan}\left(\frac{\frac{2(x-1)^{2/3}}{\sqrt[3]{q+(x-1)^2-1}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{(x-1)^3-(1-q)(x-1)}}-\frac{3\sqrt[3]{x-1}\,\sqrt[3]{q+(x-1)^2-1}\,\log\left((x-1)^{2/3}-\sqrt[3]{q+(x-1)^2-1}\right)}{4\sqrt[3]{(x-1)^3-(1-q)(x-1)}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] (Sqrt[3]*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*ArcTan[(1 + (2*(-1 + x)
^(2/3))/(-1 + q + (-1 + x)^2)^(1/3))/Sqrt[3]])/(2*(-((1 - q)*(-1 + x)) + (-
1 + x)^3)^(1/3)) - (3*(-1 + q + (-1 + x)^2)^(1/3)*(-1 + x)^(1/3)*Log[-(-1 +
 q + (-1 + x)^2)^(1/3) + (-1 + x)^(2/3)])/(4*(-((1 - q)*(-1 + x)) + (-1 + x
)^3)^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{(-1+x)\left(q-2x+x^2\right)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[3]{-(1-q)x+x^3}}\,dx, x, -1+x\right)\\
&= \frac{\left(\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,\sqrt[3]{-1+q+x^2}}\,dx, x\right.}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{x}{\sqrt[3]{-1+q+x^6}}\,dx, x, \right.}{\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}\\
&= \frac{\left(3\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-1+q+x^3}}\,dx, x, (\right.}{2\sqrt[3]{(-1+q)(-1+x)+(-1+x)^3}}\\
&= \frac{\sqrt{3}\,\sqrt[3]{-1+q+(-1+x)^2}\,\sqrt[3]{-1+x}\,\tan^{-1}\left(\frac{1+\frac{2(-1+x)^{2/3}}{\sqrt[3]{q-(2-x)x}}}{\sqrt{3}}\right)}{2\sqrt[3]{(1-q)(1-x)+(-1+x)^3}} -
\end{aligned}$$

**Mathematica [A]**
time = 0.73, size = 145, normalized size = 1.84

$$\frac{\sqrt[3]{-1+x}\,\sqrt[3]{q+(-2+x)x}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,(-1+x)^{2/3}}{(-1+x)^{2/3}+2\sqrt[3]{q+(-2+x)x}}\right)-2\log\left(-(-1+x)^{2/3}+\sqrt[3]{q+(-2+x)x}\right)+\log\left((-1+x)^{4/3}+(-1+x)^{2/3}\sqrt[3]{q+(-2+x)x}+(q+(-2+x)x)^{2/3}\right)\right)}{4\sqrt[3]{(-1+x)(q+(-2+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)*(q - 2*x + x^2))^(-1/3),x]
```

```
[Out] ((-1 + x)^(1/3)*(q + (-2 + x)*x)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(-1 + x)^
(2/3))/((-1 + x)^(2/3) + 2*(q + (-2 + x)*x)^(1/3))] - 2*Log[-(-1 + x)^(2/3)
 + (q + (-2 + x)*x)^(1/3)] + Log[(-1 + x)^(4/3) + (-1 + x)^(2/3)*(q + (-2 +
 x)*x)^(1/3) + (q + (-2 + x)*x)^(2/3)]))/(4*((-1 + x)*(q + (-2 + x)*x))^(1/
3))
```

**Maple [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(-1+x\right)\left(x^{2}+q-2x\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

```
[Out] int(1/((-1+x)*(x^2+q-2*x))^(1/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(65) = 130.
time = 1.42, size = 665, normalized size = 8.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan((2*sqrt(3)*(q^12 - 18*q^11 + 117*q^10 - 346*q^9 + 414*q^
8 - 18*q^7 + 69*q^6 - 774*q^5 - 234*q^4 + 1058*q^3 + 621*q^2 + 378*q - 539)
*(x^3 + (q + 2)*x - 3*x^2 - q)^(2/3) + 4*sqrt(3)*(q^12 - 12*q^11 + 51*q^10
- 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5 - 810*q^4 + 320*q^3 + 291*q^2
 - (q^12 - 12*q^11 + 51*q^10 - 70*q^9 - 90*q^8 + 288*q^7 - 57*q^6 + 54*q^5
- 810*q^4 + 320*q^3 + 291*q^2 + 714*q + 49)*x + 714*q + 49)*(x^3 + (q + 2)*
x - 3*x^2 - q)^(1/3) - sqrt(3)*(q^13 - 22*q^12 + 177*q^11 - 514*q^10 - 434*
```

```
q^9 + 5346*q^8 - 8247*q^7 - 4542*q^6 + 19638*q^5 - 8050*q^4 - 10343*q^3 + (
q^12 - 6*q^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 -
 1386*q^4 - 418*q^3 - 39*q^2 + 1050*q + 637)*x^2 + 6186*q^2 - 2*(q^12 - 6*q
^11 - 15*q^10 + 206*q^9 - 594*q^8 + 594*q^7 - 183*q^6 + 882*q^5 - 1386*q^4
- 418*q^3 - 39*q^2 + 1050*q + 637)*x + 1501*q + 32))/(q^13 - 22*q^12 + 249*
q^11 - 1546*q^10 + 4702*q^9 - 4230*q^8 - 10623*q^7 + 25338*q^6 - 3546*q^5 -
 31306*q^4 + 18817*q^3 + 9*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 1
86*q^7 - 15*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x^2
+ 9714*q^2 - 18*(q^12 - 14*q^11 + 73*q^10 - 162*q^9 + 78*q^8 + 186*q^7 - 15
*q^6 - 222*q^5 - 618*q^4 + 566*q^3 + 401*q^2 + 602*q - 147)*x - 995*q + 8))
 - 1/4*log(3*(x^3 + (q + 2)*x - 3*x^2 - q)^(1/3)*(x - 1) + q - 3*(x^3 + (q
+ 2)*x - 3*x^2 - q)^(2/3) - 1)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(x-1)\,(q+x^2-2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x**2+q-2*x))**(1/3),x)
[Out] Integral(((x - 1)*(q + x**2 - 2*x))**(-1/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)*(x^2+q-2*x))^(1/3),x, algorithm="giac")
[Out] integrate(((x^2 + q - 2*x)*(x - 1))^(-1/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\left(x-1\right)\,\left(x^2-2\,x+q\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)*(q - 2*x + x^2))^(1/3),x)
[Out] int(1/((x - 1)*(q - 2*x + x^2))^(1/3), x)
```

# 3.43 $\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$

**Optimal**. Leaf size=118

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{q}\,(-1+x)}{\sqrt{3}\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\right)}{2\sqrt[3]{q}}+\frac{\log(1-x)}{4\sqrt[3]{q}}+\frac{\log(x)}{2\sqrt[3]{q}}-\frac{3\log\left(-\sqrt[3]{q}\,(-1+x)+\sqrt[3]{(-1\right.}}{4\sqrt[3]{q}}$$

[Out] 1/4*ln(1-x)/q^(1/3)+1/2*ln(x)/q^(1/3)-3/4*ln(-q^(1/3)*(-1+x)+((-1+x)*(-2*q*x+x^2+q))^(1/3))/q^(1/3)+1/2*arctan(1/3*3^(1/2)+2/3*q^(1/3)*(-1+x)/((-1+x)*(-2*q*x+x^2+q))^(1/3)*3^(1/2))*3^(1/2)/q^(1/3)

**Rubi** [F]
time = 10.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]

[Out] ((-1 - 2*q - (1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 + q)^3*q)])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[-((-1 + q)^3*q)])^(1/3) + 3*x)^(1/3)*(-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (3*(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*((-1 - 2*q)/3 + x))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) + 9*((-1 - 2*q)/3 + x)^2)^(1/3)*Defer[Subst][Defer[Int][1/(((1 + 2*q)/3 + x)*(-1/3*(1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3) + x)^(1/3)*((-1 + 5*q - 4*q^2 + ((1 - 4*q)^2*(1 - q)^2)/(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3) + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))/9 + ((1 - 5*q + 4*q^2 + (1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(2/3))*x)/(3*(1 + 6*q - 15*q^2 + 8*q^3 + 3*Sqrt[3]*Sqrt[(1 - q)^3*q])^(1/3)) + x^2)^(1/3)), x], x, (-1 - 2*q)/3 + x])/(3*(-q + 3*q*x + (-1 - 2*q)*x^2 + x^3)^(1/3))

Rubi steps

$$\int \frac{1}{x\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{3}(1+2q)+x\right)\sqrt[3]{-\frac{2}{27}(1-q)^2(1+8q)-\frac{1}{3}(1-4q)(1-\cdots}}\right.$$

$$\left(\sqrt[3]{-1-2q-\frac{1-5q+4q^2+\left(1+6q-15q^2+8q^3+3\sqrt{3}\,\sqrt{-(-\cdots}\right.}{\sqrt[3]{1+6q-15q^2+8q^3+3\sqrt{3}\,\sqrt{-(-1+q)\cdots}}}}\right.$$

$$= \overline{\qquad\qquad\qquad}$$

**Mathematica** [A]
time = 2.23, size = 189, normalized size = 1.60

$$\frac{\sqrt[3]{-1+x}\,\sqrt[3]{q-2qx+x^2}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{q}(-1+x)^{2/3}}{\sqrt[3]{q}(-1+x)^{2/3}+2\sqrt[3]{q-2qx+x^2}}\right)-2\log\left(-\sqrt[3]{q}(-1+x)^{2/3}+\sqrt[3]{q-2qx+x^2}\right)+\log\left(q^{2/3}(-1+x)^{4/3}+\sqrt[3]{q}(-1+x)^{2/3}\sqrt[3]{q-2qx+x^2}+\left(q-2qx+x^2\right)^{2/3}\right)\right)}{4\sqrt[3]{q}\,\sqrt[3]{(-1+x)\left(q-2qx+x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((-1 + x)*(q - 2*q*x + x^2))^(1/3)),x]
```

```
[Out] ((-1 + x)^(1/3)*(q - 2*q*x + x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*q^(1/3)*
(-1 + x)^(2/3))/(q^(1/3)*(-1 + x)^(2/3) + 2*(q - 2*q*x + x^2)^(1/3))] - 2*L
og[-(q^(1/3)*(-1 + x)^(2/3)) + (q - 2*q*x + x^2)^(1/3)] + Log[q^(2/3)*(-1 +
 x)^(4/3) + q^(1/3)*(-1 + x)^(2/3)*(q - 2*q*x + x^2)^(1/3) + (q - 2*q*x + x
^2)^(2/3)]))/(4*q^(1/3)*((-1 + x)*(q - 2*q*x + x^2))^(1/3))
```

**Maple** [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(-1+x\right)\left(-2qx+x^2+q\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

```
[Out] int(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(100) = 200.
time = 10.68, size = 1496, normalized size = 12.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(3)*q*sqrt((-q)^(1/3)/q)*log(-((q^3 - 30*q^2 - 51*q - 1)*x^6 + 5
4*(q^3 + 6*q^2 + 2*q)*x^5 - 27*(17*q^3 + 26*q^2 + 2*q)*x^4 + 486*q^3*x + 54
0*(2*q^3 + q^2)*x^3 - 81*q^3 - 135*(8*q^3 + q^2)*x^2 + 9*((2*q^2 - q - 1)*x
^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^
(2/3)*(-q)^(1/3) + 9*((q^2 + 7*q + 1)*x^5 - (19*q^2 + 25*q + 1)*x^4 + 9*(7*
q^2 + 3*q)*x^3 + 45*q^2*x - 9*(9*q^2 + q)*x^2 - 9*q^2)*(-(2*q + 1)*x^2 + x^
3 + 3*q*x - q)^(1/3)*(-q)^(2/3) + sqrt(3)*(3*((4*q^2 + 13*q + 1)*x^4 - 6*(7
*q^2 + 5*q)*x^3 - 72*q^2*x + 3*(31*q^2 + 5*q)*x^2 + 18*q^2)*(-(2*q + 1)*x^2
 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^3 - 5*q^2 - 5*q)*x^5 + 5*(q^3
+ 7*q^2 + q)*x^4 - 45*q^3*x - 45*(q^3 + q^2)*x^3 + 9*q^3 + 15*(5*q^3 + q^2)
*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) + ((q^3 + 24*q^2 + 3*q - 1)*
x^6 - 54*(q^3 + 2*q^2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^
3 + q^2)*x^3 + 27*q^3 + 27*(14*q^3 + q^2)*x^2)*(-q)^(1/3))*sqrt((-q)^(1/3)/
q))/x^6) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*(-(2*q + 1)*x^2 + x
^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q + 1)*x^2 + x^3 + 3*q*
x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2 - 6*q*x + 3*q)*(-(2
*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^2 + 2*q)*x^3 + 9*q^
2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) -
 ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*(5*q^2 + q)*x^2 + 9
*q^2)*(-q)^(1/3))/x^4))/q, 1/12*(2*sqrt(3)*q*sqrt(-(-q)^(1/3)/q)*arctan(1/3
*sqrt(3)*(6*((2*q^2 - q - 1)*x^4 - 6*(q^2 - q)*x^3 + 3*(q^2 - q)*x^2)*(-(2*
q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) - 6*((q^3 + 7*q^2 + q)*x^5 -
 (19*q^3 + 25*q^2 + q)*x^4 + 45*q^3*x + 9*(7*q^3 + 3*q^2)*x^3 - 9*q^3 - 9*(
9*q^3 + q^2)*x^2)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3) - ((q^3 - 12*q^2
 - 15*q - 1)*x^6 + 18*(q^3 + 6*q^2 + 2*q)*x^5 - 9*(17*q^3 + 26*q^2 + 2*q)*x
^4 + 162*q^3*x + 180*(2*q^3 + q^2)*x^3 - 27*q^3 - 45*(8*q^3 + q^2)*x^2)*(-q
```

```
)^(1/3))*sqrt(-(-q)^(1/3)/q)/((q^3 + 24*q^2 + 3*q - 1)*x^6 - 54*(q^3 + 2*q^
2)*x^5 + 81*(3*q^3 + 2*q^2)*x^4 - 162*q^3*x - 108*(4*q^3 + q^2)*x^3 + 27*q^
3 + 27*(14*q^3 + q^2)*x^2)) - 2*(-q)^(2/3)*log(((-q)^(2/3)*(q - 1)*x^2 + 3*
(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(1/3)*(q*x - q)*(-q)^(1/3) + 3*(-(2*q +
1)*x^2 + x^3 + 3*q*x - q)^(2/3)*q)/x^2) + (-q)^(2/3)*log((3*((2*q + 1)*x^2
- 6*q*x + 3*q)*(-(2*q + 1)*x^2 + x^3 + 3*q*x - q)^(2/3)*(-q)^(2/3) + 3*((q^
2 + 2*q)*x^3 + 9*q^2*x - (7*q^2 + 2*q)*x^2 - 3*q^2)*(-(2*q + 1)*x^2 + x^3 +
 3*q*x - q)^(1/3) - ((q^2 + 7*q + 1)*x^4 - 18*(q^2 + q)*x^3 - 36*q^2*x + 9*
(5*q^2 + q)*x^2 + 9*q^2)*(-q)^(1/3))/x^4))/q]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{(x-1)\,(-2qx+q+x^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x**2+q))**(1/3),x)
[Out] Integral(1/(x*((x - 1)*(-2*q*x + q + x**2))**(1/3)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((-1+x)*(-2*q*x+x^2+q))^(1/3),x, algorithm="giac")
[Out] integrate(1/((-(2*q*x - x^2 - q)*(x - 1))^(1/3)*x), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(\left(x-1\right)\,\left(x^2-2\,q\,x+q\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)),x)
[Out] int(1/(x*((x - 1)*(q - 2*q*x + x^2))^(1/3)), x)
```

# 3.44 $\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$

**Optimal**. Leaf size=111

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{k}\,x}{\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{\sqrt[3]{k}}+\frac{\log(x)}{2\sqrt[3]{k}}+\frac{\log(1-(1+k)x)}{2\sqrt[3]{k}}-\frac{3\log\left(-\sqrt[3]{k}\,x+\sqrt[3]{(1-x)x(1-kx)}\right)}{2\sqrt[3]{k}}$$

```
[Out] 1/2*ln(x)/k^(1/3)+1/2*ln(1-(1+k)*x)/k^(1/3)-3/2*ln(-k^(1/3)*x+((1-x)*x*(-k*
x+1))^(1/3))/k^(1/3)+arctan(1/3*(1+2*k^(1/3)*x/((1-x)*x*(-k*x+1))^(1/3))*3^
(1/2))*3^(1/2)/k^(1/3)
```

**Rubi** [F]
time = 0.35, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
```

```
[Out] (3*(1 - x)^(1/3)*x*(1 - k*x)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, x, k*x])/(2
*((1 - x)*x*(1 - k*x))^(1/3)) + ((1 - x)^(1/3)*x^(1/3)*(1 - k*x)^(1/3)*Defe
r[Int][1/((1 - x)^(1/3)*x^(1/3)*(1 + (-1 - k)*x)*(1 - k*x)^(1/3)), x])/((1
- x)*x*(1 - k*x))^(1/3)
```

Rubi steps

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx=\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{2-(1+k)x}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\,(1-(1+k)x)}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}$$

$$=\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}\right)\int \frac{1}{\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-kx}}\,dx}{\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-\right.}{}$$

$$=\frac{3\sqrt[3]{1-x}\,x\sqrt[3]{1-kx}\,F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};x,kx\right)}{2\sqrt[3]{(1-x)x(1-kx)}}+\frac{\left(\sqrt[3]{1-x}\,\sqrt[3]{x}\,\sqrt[3]{1-\right.}{}$$

**Mathematica** [F]
time = 41.16, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)x}{\sqrt[3]{(1-x)x(1-kx)}\,(1-(1+k)x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)),x]
[Out] Integrate[(2 - (1 + k)*x)/(((1 - x)*x*(1 - k*x))^(1/3)*(1 - (1 + k)*x)), x]
```

**Maple** [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{2-(1+k)\,x}{\left((1-x)\,x\,(-kx+1)\right)^{\frac{1}{3}}\,(1-(1+k)\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
[Out] int((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="ma
xima")
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

**Fricas** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="fr
icas")
[Out] Timed out
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{kx+x-2}{\sqrt[3]{x\,(x-1)\,(kx-1)}\,(kx+x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))**(1/3)/(1-(1+k)*x),x)
```

```
[Out] Integral((k*x + x - 2)/((x*(x - 1)*(k*x - 1))**(1/3)*(k*x + x - 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(1+k)*x)/((1-x)*x*(-k*x+1))^(1/3)/(1-(1+k)*x),x, algorithm="gi
ac")
```

```
[Out] integrate(((k + 1)*x - 2)/(((k*x - 1)*(x - 1)*x)^(1/3)*((k + 1)*x - 1)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(k+1\right)-2}{\left(x\,\left(k+1\right)-1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)),x)
```

```
[Out] int((x*(k + 1) - 2)/((x*(k + 1) - 1)*(x*(k*x - 1)*(x - 1))^(1/3)), x)
```

## 3.45 $\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$

**Optimal**. Leaf size=176

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-kx)}{\sqrt[3]{1-k}\sqrt[3]{(1-x)x(1-kx)}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-(2-k)x)}{2^{2/3}\sqrt[3]{1-k}}+\frac{\log(1-kx)}{2\,2^{2/3}\sqrt[3]{1-k}}-\frac{3\log\left(-1+kx+2^{2}\right.}{2}$$

```
[Out] 1/2*ln(1-(2-k)*x)*2^(1/3)/(1-k)^(1/3)+1/4*ln(-k*x+1)*2^(1/3)/(1-k)^(1/3)-3/
4*ln(-1+k*x+2^(2/3)*(1-k)^(1/3)*((1-x)*x*(-k*x+1))^(1/3))*2^(1/3)/(1-k)^(1/
3)-1/2*arctan(1/3*(1+2^(1/3)*(-k*x+1)/(1-k)^(1/3)/((1-x)*x*(-k*x+1))^(1/3))
*3^(1/2))*3^(1/2)*2^(1/3)/(1-k)^(1/3)
```

**Rubi** [F]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

```
[Out] ((1 - x)^(2/3)*x^(2/3)*(1 - k*x)^(2/3)*Defer[Int][(1 - k*x)^(1/3)/((1 - x)^
(2/3)*x^(2/3)*(1 + (-2 + k)*x)), x])/((1 - x)*x*(1 - k*x))^(2/3)
```

Rubi steps

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx = \frac{\left((1-x)^{2/3}x^{2/3}(1-kx)^{2/3}\right)\int \frac{\sqrt[3]{1-kx}}{(1-x)^{2/3}x^{2/3}(1+(-2+k)x)}\,dx}{((1-x)x(1-kx))^{2/3}}$$

**Mathematica** [F]
time = 10.46, size = 0, normalized size = 0.00

$$\int \frac{1-kx}{(1+(-2+k)x)((1-x)x(1-kx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)),x]
```

[Out] Integrate[(1 - k*x)/((1 + (-2 + k)*x)*((1 - x)*x*(1 - k*x))^(2/3)), x]

**Maple [F]**
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{-kx+1}{(1+(-2+k)\,x)\,((1-x)\,x\,(-kx+1))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)

[Out] int((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x)

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="maxima")

[Out] -integrate((k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(132) = 264.
time = 36.70, size = 932, normalized size = 5.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(24*sqrt(3)*2^(1/3)*((k^5 - 3*k^4 - 4*k^3 + 22*k^2 - 24*k + 8)*x^4 - 2*(k^4 - 10*k^3 + 27*k^2 - 26*k + 8)*x^3 - 6*(k^3 - 4*k^2 + 4*k - 1)*x^2 - 2*(k^2 - 1)*x + k - 1)*(k*x^3 - (k + 1)*x^2 + x)^(2/3)/(k - 1)^(1/3) - 6*sqrt(3)*2^(2/3)*((k^6 + 27*k^5 - 40*k^4 - 20*k^3 + 48*k^2 - 16*k)*x^5 - (33*k^5 + 55*k^4 - 220*k^3 + 132*k^2 + 16*k - 16)*x^4 + 2*(55*k^4 - 55*k^3 - 66*k^2 + 82*k - 16)*x^3 - 2*(55*k^3 - 99*k^2 + 38*k + 6)*x^2 + (33*k^2 - 61*k + 28)*x - k + 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3)/(k - 1)^(2/3) + sqrt(3)*((k^6 - 48*k^5 - 192*k^4 + 416*k^3 - 48*k^2 - 192*k + 64)*x^6 + 6*(7*k^5 + 104*k^4 - 80*k^3 - 176*k^2 + 176*k - 32)*x^5 - 3*(139*k^4 + 256*k^3 - 768*k^2 + 352*k + 16)*x^4 + 4*(203*k^3 - 192*k^2 - 120*k + 104)*x^3 - 3*(139*k^2 - 208*k + 64)*x^2 + 6*(7*k - 8)*x + 1))/((k^6 + 96*k^5 - 48*k^4 - 160*k^3 + 240*k^2 - 192*k + 64)*x^6 - 6*(17*k^5 + 64*k^4 - 1

```
12*k^3 + 80*k^2 - 80*k + 32)*x^5 + 3*(149*k^4 + 32*k^3 - 96*k^2 - 160*k + 8
0)*x^4 - 4*(157*k^3 - 24*k^2 - 168*k + 40)*x^3 + 3*(149*k^2 - 128*k - 16)*x
^2 - 6*(17*k - 16)*x + 1))/(k - 1)^(1/3) - 1/12*2^(1/3)*log((12*2^(2/3)*(k*
x^3 - (k + 1)*x^2 + x)^(2/3)*((k^3 + k^2 - 4*k + 2)*x^2 - 2*(2*k^2 - 3*k +
1)*x + k - 1)/(k - 1)^(2/3) + 6*((k^3 + 8*k^2 - 8*k)*x^3 - (11*k^2 - 8)*x^2
 + (11*k - 8)*x - 1)*(k*x^3 - (k + 1)*x^2 + x)^(1/3) + 2^(1/3)*((k^4 + 28*k
^3 - 12*k^2 - 32*k + 16)*x^4 - 4*(8*k^3 + 15*k^2 - 30*k + 8)*x^3 + 6*(13*k^
2 - 10*k - 2)*x^2 - 4*(8*k - 7)*x + 1)/(k - 1)^(1/3))/((k^4 - 8*k^3 + 24*k^
2 - 32*k + 16)*x^4 + 4*(k^3 - 6*k^2 + 12*k - 8)*x^3 + 6*(k^2 - 4*k + 4)*x^2
 + 4*(k - 2)*x + 1))/(k - 1)^(1/3) + 1/6*2^(1/3)*log((6*2^(1/3)*(k*x^3 - (k
 + 1)*x^2 + x)^(1/3)*(k*x - 1)/(k - 1)^(1/3) - 2^(2/3)*((k^2 - 4*k + 4)*x^2
 + 2*(k - 2)*x + 1)/(k - 1)^(2/3) - 12*(k*x^3 - (k + 1)*x^2 + x)^(2/3))/((k
^2 - 4*k + 4)*x^2 + 2*(k - 2)*x + 1))/(k - 1)^(1/3)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{kx}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\,dx-\int\left(-\frac{1}{kx\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}-2x\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}+\left(kx^3-kx^2-x^2+x\right)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))**(2/3),x)
```

```
[Out] -Integral(k*x/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x*
*2 - x**2 + x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x) - Integral
(-1/(k*x*(k*x**3 - k*x**2 - x**2 + x)**(2/3) - 2*x*(k*x**3 - k*x**2 - x**2
+ x)**(2/3) + (k*x**3 - k*x**2 - x**2 + x)**(2/3)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-k*x+1)/(1+(-2+k)*x)/((1-x)*x*(-k*x+1))^(2/3),x, algorithm="giac
")
```

```
[Out] integrate(-(k*x - 1)/(((k*x - 1)*(x - 1)*x)^(2/3)*((k - 2)*x + 1)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{k\,x-1}{\left(x\,\left(k-2\right)+1\right)\,{\left(x\,\left(k\,x-1\right)\,\left(x-1\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)),x)
```

```
[Out] -int((k*x - 1)/((x*(k - 2) + 1)*(x*(k*x - 1)*(x - 1))^(2/3)), x)
```

## 3.46 $\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

**Optimal**. Leaf size=493

$$\frac{(a+b)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{(a-c)\tan^{-1}\left(\frac{1-\cdots}{\cdots}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/24*(a+b)*ln((1-x)*(1+x)^2)*2^(2/3)-1/12*(a-c)*ln(x^3+1)*2^(2/3)-1/12*(b+c
)*ln(x^3+1)*2^(2/3)+1/12*(a+b)*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*
(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*(a+b)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*
2^(2/3)+1/4*(b+c)*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/4*(a-c)*ln(-2^(1/3)*
x-(-x^3+1)^(1/3))*2^(2/3)+1/2*c*ln(x+(-x^3+1)^(1/3))-1/8*(a+b)*ln(-1+x+2^(2
/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*(a+b)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*(a+b)*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/3*c*arctan(1/3*(1-2*x/(-x^3+1)^(1/
3))*3^(1/2))*3^(1/2)-1/6*(a-c)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*2^(2/3)*3^(1/2)+1/6*(b+c)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1
/2))*2^(2/3)*3^(1/2)
```

**Rubi** [A]
time = 0.28, antiderivative size = 570, normalized size of antiderivative = 1.16, number of steps used = 19, number of rules used = 14, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57, 494, 245}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] ((a + b)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3
)*Sqrt[3]) + ((a + b)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3
]])/(2*2^(1/3)*Sqrt[3]) - (c*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/S
qrt[3] - (a*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*S
qrt[3]) + (c*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*
Sqrt[3]) + ((b + c)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]) + ((a + b)*Log[(1 - x)*(1 + x)^2])/(12*2^(1/3)) - (a*Log[1 + x^3]
)/(6*2^(1/3)) + (c*Log[1 + x^3])/(6*2^(1/3)) - ((b + c)*Log[1 + x^3])/(6*2^
(1/3)) + ((a + b)*Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1
 - x))/(1 - x^3)^(1/3)])/(6*2^(1/3)) - ((a + b)*Log[1 + (2^(1/3)*(1 - x))/(
1 - x^3)^(1/3)])/(3*2^(1/3)) + ((b + c)*Log[2^(1/3) - (1 - x^3)^(1/3)])/(2*
2^(1/3)) + (a*Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) - (c*Log[-(2
```

```
^(1/3)*x) - (1 - x^3)^(1/3)])/(2*2^(1/3)) + (c*Log[x + (1 - x^3)^(1/3)])/2
- ((a + b)*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)])/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
```

```
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{c}{\sqrt[3]{1-x^3}}+\frac{a-c+(b+c)x}{(1-x+x^2)\sqrt[3]{1-x^3}}\right)dx$$

$$= c\int \frac{1}{\sqrt[3]{1-x^3}}\,dx+\int \frac{a-c+(b+c)x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right)+\int \left(\frac{b-\frac{i(2a+b}{\sqrt{3}}}{\left(-1-i\sqrt{3}+\right.}\right.$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}c\log\left(x+\sqrt[3]{1-x^3}\right)+\frac{1}{3}\left(3b-i\sqrt{3}(2a+b\right.$$

$$= -\frac{c\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\left(2a+b-i\sqrt{3}b-c-i\sqrt{3}c\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}}{}}{}\right.}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)}$$

**Mathematica [F]**
time = 10.25, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] Integrate[(a + b*x + c*x^2)/((1 - x + x^2)*(1 - x^3)^(1/3)), x]
```

**Maple [F]**
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{c\,x^{2}+bx+a}{\left(x^{2}-x+1\right)\left(-x^{3}+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

```
[Out] int((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^{2}}{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1
)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate((c*x^2 + b*x + a)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^2+b\,x+a}{{\left(1-x^3\right)}^{1/3}\,\left(x^2-x+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
[Out] int((a + b*x + c*x^2)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

# 3.47 $\int \frac{1}{(3-2x)^{11/2}(1+x+2x^2)^5}\,dx$

**Optimal**. Leaf size=407

$$-\frac{19255}{395136(3-2x)^{9/2}}-\frac{462025}{30118144(3-2x)^{7/2}}-\frac{38491}{8605184(3-2x)^{5/2}}-\frac{141045}{120472576(3-2x)^{3/2}}-\frac{38225}{240945152\sqrt{3-2}}$$

```
[Out] -19255/395136/(3-2*x)^(9/2)-462025/30118144/(3-2*x)^(7/2)-38491/8605184/(3-
2*x)^(5/2)-141045/120472576/(3-2*x)^(3/2)+1/28*x/(3-2*x)^(9/2)/(2*x^2+x+1)^
4+1/1176*(23+73*x)/(3-2*x)^(9/2)/(2*x^2+x+1)^3+1/32928*(1387+3049*x)/(3-2*x
)^(9/2)/(2*x^2+x+1)^2+5/153664*(3049+4377*x)/(3-2*x)^(9/2)/(2*x^2+x+1)-3822
5/240945152/(3-2*x)^(1/2)+5/13492928512*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+
2*14^(1/2))^(1/2))*(-298093007954+81630132224*14^(1/2))^(1/2)-5/13492928512
*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(-298093007954+81630
132224*14^(1/2))^(1/2)+5/6746464256*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))*(298093007954+81630132224*14^(1/2))^(1/2)-5/
6746464256*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1
/2))*(298093007954+81630132224*14^(1/2))^(1/2)
```

**Rubi** [A]

time = 0.40, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {754, 836, 842, 840, 1183, 648, 632, 210, 642}

$$\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\,\text{ArcTan}\left(\frac{\sqrt{7+2\sqrt{14}}-2\sqrt{3-2x}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}-\frac{5\sqrt{\frac{1}{2}\left(149046503977+40815066112\sqrt{14}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{3-2x}+\sqrt{7+2\sqrt{14}}}{\sqrt{2\sqrt{14}-7}}\right)}{3373232128}+\frac{x}{28(3-2x)^{9/2}(2x^2+x+1)^4}+\frac{5(4377x+3049)}{153664(3-2x)^{9/2}(2x^2+x+1)}+\frac{3049x+1387}{32928(3-2x)^{9/2}(2x^2+x+1)^2}+\frac{73x+23}{1176(3-2x)^{9/2}(2x^2+x+1)^3}-\frac{38225}{240945152\sqrt{3-2x}}-\frac{141045}{120472576(3-2x)^{3/2}}-\frac{38491}{8605184(3-2x)^{5/2}}-\frac{462025}{30118144(3-2x)^{7/2}}-\frac{19255}{395136(3-2x)^{9/2}}+\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x-\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}-\frac{5\sqrt{\frac{1}{2}\left(40815066112\sqrt{14}-149046503977\right)}\log\left(-2x+\sqrt{7+2\sqrt{14}}\sqrt{3-2x}+\sqrt{14}+3\right)}{6746464256}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -19255/(395136*(3 - 2*x)^(9/2)) - 462025/(30118144*(3 - 2*x)^(7/2)) - 38491
/(8605184*(3 - 2*x)^(5/2)) - 141045/(120472576*(3 - 2*x)^(3/2)) - 38225/(24
0945152*Sqrt[3 - 2*x]) + x/(28*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^4) + (23 + 7
3*x)/(1176*(3 - 2*x)^(9/2)*(1 + x + 2*x^2)^3) + (1387 + 3049*x)/(32928*(3 -
 2*x)^(9/2)*(1 + x + 2*x^2)^2) + (5*(3049 + 4377*x))/(153664*(3 - 2*x)^(9/2
)*(1 + x + 2*x^2)) + (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTa
n[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/33732321
28 - (5*Sqrt[(149046503977 + 40815066112*Sqrt[14])/2]*ArcTan[(Sqrt[7 + 2*Sq
rt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/3373232128 + (5*Sqrt[(-1
49046503977 + 40815066112*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[7 + 2*Sqrt[1
4]]*Sqrt[3 - 2*x] - 2*x])/6746464256 - (5*Sqrt[(-149046503977 + 40815066112
*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])
/6746464256
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
```

```
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{11/2}\left(1+x+2x^2\right)^5}\,dx = \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{1}{784}\int \frac{700-644x}{(3-2x)^{11/2}\left(1+x+2x^2\right)^4}\,dx$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} +$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} +$$

$$= \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} + \frac{23+73x}{1176(3-2x)^{9/2}\left(1+x+2x^2\right)^3} +$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} + \frac{x}{28(3-2x)^{9/2}\left(1+x+2x^2\right)^4} +$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} +$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} +$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} -$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} -$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} -$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} -$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} -$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} -$$

$$= -\frac{19255}{395136(3-2x)^{9/2}} - \frac{462025}{30118144(3-2x)^{7/2}} - \frac{38491}{8605184(3-2x)^{5/2}} -$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 4.41, size = 178, normalized size = 0.44

$$\frac{-\frac{14(40289347-429812744x+135202154x^2-1073855156x^3+1627773523x^4-1470758860x^5+2888625656x^6-3106712560x^7+2343370048x^8-2443779648x^9+1873554048x^{10}-677249280x^{11}+88070400x^{12})}{(3-2x)^{9/2}(1+x+2x^2)^4}-45\sqrt{149046503977+12577271771i\sqrt{7}}\tan^{-1}\left(\frac{1}{2}\sqrt{-1-\frac{i}{\sqrt{7}}}\sqrt{3-2x}\right)-45\sqrt{149046503977-12577271771i\sqrt{7}}\tan^{-1}\left(\frac{1}{2}\sqrt{-1+\frac{i}{\sqrt{7}}}\sqrt{3-2x}\right)}{30359089152}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((-14*(40289347 - 429812744*x + 135202154*x^2 - 1073855156*x^3 + 1627773523
*x^4 - 1470758860*x^5 + 2888625656*x^6 - 3106712560*x^7 + 2343370048*x^8 -
2443779648*x^9 + 1873554048*x^10 - 677249280*x^11 + 88070400*x^12))/((3 - 2
*x)^(9/2)*(1 + x + 2*x^2)^4) - 45*Sqrt[149046503977 + (12577271771*I)*Sqrt[
7]]*ArcTan[(Sqrt[-1 - I/Sqrt[7]]*Sqrt[3 - 2*x])/2] - 45*Sqrt[149046503977 -
 (12577271771*I)*Sqrt[7]]*ArcTan[(Sqrt[-1 + I/Sqrt[7]]*Sqrt[3 - 2*x])/2])/3
0359089152
```

**Maple** [A]
time = 0.89, size = 415, normalized size = 1.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/151263/(3-2*x)^(9/2)+5/235298/(3-2*x)^(7/2)+19/470596/(3-2*x)^(5/2)+185/2
823576/(3-2*x)^(3/2)+505/3294172/(3-2*x)^(1/2)+1/6588344*(567651623/32*(3-2
*x)^(1/2)-6194606411/192*(3-2*x)^(3/2)+9801432515/384*(3-2*x)^(5/2)-8763772
549/768*(3-2*x)^(7/2)+149630663/48*(3-2*x)^(9/2)-200063633/384*(3-2*x)^(11/
2)+18969965/384*(3-2*x)^(13/2)-526135/256*(3-2*x)^(15/2))/((3-2*x)^2-7+14*x
)^4+5/13492928512*(-146319*(7+2*14^(1/2))^(1/2)*14^(1/2)+569986*(7+2*14^(1/
2))^(1/2))*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))+5/33732321
28*(-115739*14^(1/2)+1/2*(-146319*(7+2*14^(1/2))^(1/2)*14^(1/2)+569986*(7+2
*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-
2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))+5/13492928512*(1463
19*(7+2*14^(1/2))^(1/2)*14^(1/2)-569986*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(
1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))+5/3373232128*(-115739*14^(1/2)-1/2
*(146319*(7+2*14^(1/2))^(1/2)*14^(1/2)-569986*(7+2*14^(1/2))^(1/2))*(7+2*14
^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))
^(1/2))/(-7+2*14^(1/2))^(1/2))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(-2*x + 3)^(11/2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(298) = 596$.
time = 1.32, size = 957, normalized size = 2.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/852282865707923134247251378176*(2263908918780*22241759018113166^(1/4)*sqr
t(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10
+ 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3
 - 1242*x^2 - 162*x - 243)*sqrt(21292357711*sqrt(14) + 81630132224)*arctan(
1/10052187156951869469526908685753437228729401815040*22241759018113166^(3/4
)*sqrt(12577271771)*sqrt(79716926)*sqrt(-2089731384934400*22241759018113166
^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 8163013222
4)*(7645*sqrt(14) - 115739) - 4190418993502514995568679111884800*x + 209520
9496751257497784339555942400*sqrt(14) + 6285628490253772493353018667827200)
*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357711*sqrt(14) + 816
30132224) - 1/1958184534851295802906658902*22241759018113166^(3/4)*sqrt(797
16926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2*x + 3)*sqrt(21292
357711*sqrt(14) + 81630132224) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 22639089
18780*22241759018113166^(1/4)*sqrt(79716926)*sqrt(14)*sqrt(7)*(512*x^13 - 2
816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^
6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)*sqrt(2129235771
1*sqrt(14) + 81630132224)*arctan(1/24628619072593968384668700756050455442*2
2241759018113166^(3/4)*sqrt(12577271771)*sqrt(22241759018113166^(1/4)*sqrt(
79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)*(7645*sqr
t(14) - 115739) - 2005242886101391892*x + 1002621443050695946*sqrt(14) + 30
07864329152087838)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(21292357
711*sqrt(14) + 81630132224) - 1/1958184534851295802906658902*22241759018113
166^(3/4)*sqrt(79716926)*(115739*sqrt(14)*sqrt(7) - 107030*sqrt(7))*sqrt(-2
*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224) + 2/7*sqrt(14)*sqrt(7) + s
qrt(7)) + 315*22241759018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 -
 229870452342784*x^12 + 459740904685568*x^11 - 480638218534912*x^10 + 55900
3145469952*x^9 - 734018148958208*x^8 + 498923368153088*x^7 - 34611176062976
0*x^6 + 407660880326656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 10
1384624222208*x^2 - 21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11
- 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x
^4 + 936*x^3 - 1242*x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)
*sqrt(21292357711*sqrt(14) + 81630132224)*log(2089731384934400/12577271771*
22241759018113166^(1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt
(14) + 81630132224)*(7645*sqrt(14) - 115739) - 3331739243453861593088 00*x +
```

```
 166586962172693079654400*sqrt(14) + 499760886518079238963200) - 315*222417
59018113166^(1/4)*sqrt(79716926)*(41794627698688*x^13 - 229870452342784*x^1
2 + 459740904685568*x^11 - 480638218534912*x^10 + 559003145469952*x^9 - 734
018148958208*x^8 + 498923368153088*x^7 - 346111760629760*x^6 + 407660880326
656*x^5 - 139342635706368*x^4 + 76405803761664*x^3 - 101384624222208*x^2 -
21292357711*sqrt(14)*(512*x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x
^9 - 8992*x^8 + 6112*x^7 - 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*
x^2 - 162*x - 243) - 13224081420288*x - 19836122130432)*sqrt(21292357711*sq
rt(14) + 81630132224)*log(-2089731384934400/12577271771*22241759018113166^(
1/4)*sqrt(79716926)*sqrt(-2*x + 3)*sqrt(21292357711*sqrt(14) + 81630132224)
*(7645*sqrt(14) - 115739) - 333173924345386159308800*x + 166586962172693079
654400*sqrt(14) + 499760886518079238963200) + 393027605675872810832*(880704
00*x^12 - 677249280*x^11 + 1873554048*x^10 - 2443779648*x^9 + 2343370048*x^
8 - 3106712560*x^7 + 2888625656*x^6 - 1470758860*x^5 + 1627773523*x^4 - 107
3855156*x^3 + 135202154*x^2 - 429812744*x + 40289347)*sqrt(-2*x + 3))/(512*
x^13 - 2816*x^12 + 5632*x^11 - 5888*x^10 + 6848*x^9 - 8992*x^8 + 6112*x^7 -
 4240*x^6 + 4994*x^5 - 1707*x^4 + 936*x^3 - 1242*x^2 - 162*x - 243)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Timed out
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(298) = 596.
time = 1.28, size = 795, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] -5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqr
t(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 5
3515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/4)*sqrt(2*
sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8
) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1/4)*s
qrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2))
- 5/1511207993344*sqrt(7)*(22935*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sq
rt(14) + 8) + 7645*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) +
53515*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 160545*14^(3/4)*sqrt(2
```

```
*sqrt(14) + 8)*(sqrt(14) - 4) + 925912*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) +
8) + 6481384*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4)*(14^(1/4)
*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)
) - 5/3022415986688*sqrt(7)*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sq
rt(14) + 4) + 22935*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) -
160545*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) - 53515*14^(3/4)*(sqrt
(14) - 4)*sqrt(-2*sqrt(14) + 8) + 925912*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) +
 8) - 6481384*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/2)*sqrt(-
2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 5/3022415986688*sqrt(7)
*(7645*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 22935*14^(3/4
)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) - 160545*14^(3/4)*(sqrt(14) +
 4)*sqrt(-2*sqrt(14) + 8) - 53515*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14)
+ 8) + 925912*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) - 6481384*14^(1/4)*sqrt
(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4
) - 2*x + sqrt(14) + 3) + 1/5059848192*(1578405*(2*x - 3)^7*sqrt(-2*x + 3)
+ 37939930*(2*x - 3)^6*sqrt(-2*x + 3) + 400127266*(2*x - 3)^5*sqrt(-2*x + 3
) + 2394090608*(2*x - 3)^4*sqrt(-2*x + 3) + 8763772549*(2*x - 3)^3*sqrt(-2*
x + 3) + 19602865030*(2*x - 3)^2*sqrt(-2*x + 3) - 24778425644*(-2*x + 3)^(3
/2) + 13623638952*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^4 + 1/59295096*(
9090*(2*x - 3)^4 - 3885*(2*x - 3)^3 + 2394*(2*x - 3)^2 - 2520*x + 4172)/((2
*x - 3)^4*sqrt(-2*x + 3))
```

**Mupad [B]**
time = 0.47, size = 343, normalized size = 0.84

$$-\frac{\frac{272\,x}{441}-\frac{164\,{\left(2\,x-3\right)}^2}{441}+\frac{1966\,{\left(2\,x-3\right)}^3}{3087}-\frac{9091\,{\left(2\,x-3\right)}^4}{3087}-\frac{32070727\,{\left(2\,x-3\right)}^5}{5531904}-\frac{41014777\,{\left(2\,x-3\right)}^6}{11063808}-\frac{141921511\,{\left(2\,x-3\right)}^7}{154893312}+\frac{23262655\,{\left(2\,x-3\right)}^8}{309786624}+\frac{1571659\,{\left(2\,x-3\right)}^9}{15059072}+\frac{468427\,{\left(2\,x-3\right)}^{10}}{17210368}+\frac{394105\,{\left(2\,x-3\right)}^{11}}{120472576}+\frac{38225\,{\left(2\,x-3\right)}^{12}}{240945152}-\frac{520}{441}}{38416\,{\left(3-2\,x\right)}^{9/2}-76832\,{\left(3-2\,x\right)}^{11/2}+68600\,{\left(3-2\,x\right)}^{13/2}-35672\,{\left(3-2\,x\right)}^{15/2}+11809\,{\left(3-2\,x\right)}^{17/2}-2548\,{\left(3-2\,x\right)}^{19/2}+350\,{\left(3-2\,x\right)}^{21/2}-28\,{\left(3-2\,x\right)}^{23/2}+{\left(3-2\,x\right)}^{25/2}}+\frac{\mathrm{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977+\sqrt{7}\,12577271771{}\mathrm{i}}\,1572158971375{}\mathrm{i}}{391663056253676053933850624\,\left(-\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125{}\mathrm{i}}{195831528126838026966925312}\right)}-\frac{1572158971375\,\sqrt{7}\,\sqrt{3-2\,x}\,\sqrt{-149046503977+\sqrt{7}\,12577271771{}\mathrm{i}}}{391663056253676053933850624\,\left(-\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125{}\mathrm{i}}{195831528126838026966925312}\right)}\right)\,\sqrt{-149046503977+\sqrt{7}\,12577271771{}\mathrm{i}}\,5{}\mathrm{i}}{3373232128}-\frac{\mathrm{atan}\left(\frac{\sqrt{3-2\,x}\,\sqrt{-149046503977-\sqrt{7}\,12577271771{}\mathrm{i}}\,1572158971375{}\mathrm{i}}{391663056253676053933850624\,\left(\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125{}\mathrm{i}}{195831528126838026966925312}\right)}+\frac{1572158971375\,\sqrt{7}\,\sqrt{3-2\,x}\,\sqrt{-149046503977-\sqrt{7}\,12577271771{}\mathrm{i}}}{391663056253676053933850624\,\left(\frac{230036728532618625}{27975932589548289566703616}+\frac{\sqrt{7}\,181960107187971125{}\mathrm{i}}{195831528126838026966925312}\right)}\right)\,\sqrt{-149046503977-\sqrt{7}\,12577271771{}\mathrm{i}}\,5{}\mathrm{i}}{3373232128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(11/2)*(x + 2*x^2 + 1)^5),x)
```

```
[Out] (atan(((3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^(1/2)*15721589
71375i)/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/1958315
28126838026966925312 - 230036728532618625/27975932589548289566703616)) - (1
572158971375*7^(1/2)*(3 - 2*x)^(1/2)*(7^(1/2)*12577271771i - 149046503977)^
(1/2))/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/19583152
8126838026966925312 - 230036728532618625/27975932589548289566703616)))*(7^(
1/2)*12577271771i - 149046503977)^(1/2)*5i)/3373232128 - ((272*x)/441 - (16
4*(2*x - 3)^2)/441 + (1966*(2*x - 3)^3)/3087 - (9091*(2*x - 3)^4)/3087 - (3
2070727*(2*x - 3)^5)/5531904 - (41014777*(2*x - 3)^6)/11063808 - (141921511
*(2*x - 3)^7)/154893312 + (23262655*(2*x - 3)^8)/309786624 + (1571659*(2*x
- 3)^9)/15059072 + (468427*(2*x - 3)^10)/17210368 + (394105*(2*x - 3)^11)/1
20472576 + (38225*(2*x - 3)^12)/240945152 - 520/441)/(38416*(3 - 2*x)^(9/2)
 - 76832*(3 - 2*x)^(11/2) + 68600*(3 - 2*x)^(13/2) - 35672*(3 - 2*x)^(15/2)
 + 11809*(3 - 2*x)^(17/2) - 2548*(3 - 2*x)^(19/2) + 350*(3 - 2*x)^(21/2) -
28*(3 - 2*x)^(23/2) + (3 - 2*x)^(25/2)) - (atan(((3 - 2*x)^(1/2)*(- 7^(1/2)
```

```
*12577271771i - 149046503977)^(1/2)*1572158971375i)/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 230036728532618625/27975932589548289566703616)) + (1572158971375*7^(1/2)*(3 - 2*x)^(1/2)*(- 7^(1/2)*12577271771i - 149046503977)^(1/2))/(391663056253676053933850624*((7^(1/2)*181960107187971125i)/195831528126838026966925312 + 230036728532618625/27975932589548289566703616)))*(- 7^(1/2)*12577271771i - 149046503977)^(1/2)*5i)/3373232128
```

# 3.48 $\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx$

**Optimal**. Leaf size=648

$$\frac{4718120139975}{351733660450816(3-2x)^{19/2}}-\frac{815900548375}{629418129227776(3-2x)^{17/2}}-\frac{3029508823715}{1555033025150976(3-2x)^{15/2}}-\frac{13\ldots}{13476952\ldots}$$

```
[Out] 4718120139975/351733660450816/(3-2*x)^(19/2)-815900548375/629418129227776/(
3-2*x)^(17/2)-3029508823715/1555033025150976/(3-2*x)^(15/2)-13515743021825/
13476952884641792/(3-2*x)^(13/2)-5846828446875/14513641568075776/(3-2*x)^(1
1/2)-37283626871975/261245548225363968/(3-2*x)^(9/2)-132355162272575/284467
3747342852096/(3-2*x)^(7/2)-11557581705725/812763927812243456/(3-2*x)^(5/2)
-46601678385075/11378694989371408384/(3-2*x)^(3/2)+1/63*x/(3-2*x)^(19/2)/(2
*x^2+x+1)^9+1/7056*(53+173*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^8+1/691488*(8477+2
1409*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^7+5/6453888*(21409+47471*x)/(3-2*x)^(19/
2)/(2*x^2+x+1)^6+41/90354432*(47471+92875*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^5+4
1/5059848192*(3436375+5677637*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^4+451/101196963
84*(811091+998691*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^3+451/283351498752*(2896203
9+14627273*x)/(3-2*x)^(19/2)/(2*x^2+x+1)^2+11275/3966920982528*(14627273-35
058731*x)/(3-2*x)^(19/2)/(2*x^2+x+1)-24229218097975/227573899787428l6768/(3
-2*x)^(1/2)+11275/127441383880959773900 8*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7
+2*14^(1/2))^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)
-11275/1274413838809597739008*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2)
)^(1/2))*(9756589235-2148932869*14^(1/2))*(-14+4*14^(1/2))^(1/2)+11275/6372
06919404798869504*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(
1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)-11275/6
37206919404798869504*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14
^(1/2))^(1/2))*(9756589235+2148932869*14^(1/2))*(14+4*14^(1/2))^(1/2)
```

**Rubi** [A]
time = 0.71, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {754, 836, 842, 840, 1183, 648, 632, 210, 642}

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] 4718120139975/(351733660450816*(3 - 2*x)^(19/2)) - 815900548375/(6294181292
27776*(3 - 2*x)^(17/2)) - 3029508823715/(1555033025150976*(3 - 2*x)^(15/2))
 - 13515743021825/(13476952884641792*(3 - 2*x)^(13/2)) - 5846828446875/(145
```

```
13641568075776*(3 - 2*x)^(11/2)) - 37283626871975/(261245548225363968*(3 -
2*x)^(9/2)) - 132355162272575/(2844673747342852096*(3 - 2*x)^(7/2)) - 11557
581705725/(812763927812243456*(3 - 2*x)^(5/2)) - 46601678385075/(1137869498
9371408384*(3 - 2*x)^(3/2)) - 24229218097975/(227573899787428167 68*Sqrt[3 -
 2*x]) + x/(63*(3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9) + (53 + 173*x)/(7056*(3
- 2*x)^(19/2)*(1 + x + 2*x^2)^8) + (8477 + 21409*x)/(691488*(3 - 2*x)^(19/2
)*(1 + x + 2*x^2)^7) + (5*(21409 + 47471*x))/(6453888*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^6) + (41*(47471 + 92875*x))/(90354432*(3 - 2*x)^(19/2)*(1 + x +
 2*x^2)^5) + (41*(3436375 + 5677637*x))/(5059848192*(3 - 2*x)^(19/2)*(1 + x
 + 2*x^2)^4) + (451*(811091 + 998691*x))/(10119696384*(3 - 2*x)^(19/2)*(1 +
 x + 2*x^2)^3) + (451*(28962039 + 14627273*x))/(283351498752*(3 - 2*x)^(19/
2)*(1 + x + 2*x^2)^2) + (11275*(14627273 - 35058731*x))/(3966920982528*(3 -
 2*x)^(19/2)*(1 + x + 2*x^2)) + (11275*Sqrt[(7 + 2*Sqrt[14])/2]*(9756589235
 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqr
t[-7 + 2*Sqrt[14]]])/318603459702399434752 - (11275*Sqrt[(7 + 2*Sqrt[14])/2
]*(9756589235 + 2148932869*Sqrt[14])*ArcTan[(Sqrt[7 + 2*Sqrt[14]] + 2*Sqrt[
3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/318603459702399434752 + (11275*(975658923
5 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] - Sqrt[
7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504 - (11275*(97565
89235 - 2148932869*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + S
qrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/637206919404798869504
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{21/2}\left(1+x+2x^2\right)^{10}}\,dx = \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{\int \frac{1680-1484x}{(3-2x)^{21/2}(1+x+2x^2)^9}\,dx}{1764}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{\int}{}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{}{69}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{}{69}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{}{69}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{}{69}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{}{69}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{}{69}$$

$$= \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{53+173x}{7056(3-2x)^{19/2}\left(1+x+2x^2\right)^8} + \frac{}{69}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} + \frac{x}{63(3-2x)^{19/2}\left(1+x+2x^2\right)^9} + \frac{}{7056}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} + \frac{}{63(}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{}{155}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{}{155}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{}{155}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{}{155}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{}{155}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{}{155}$$

$$= \frac{4718120139975}{351733660450816(3-2x)^{19/2}} - \frac{815900548375}{629418129227776(3-2x)^{17/2}} - \frac{}{155}$$

$$\frac{4718120139975}{} \quad \frac{815900548375}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 7.15, size = 253, normalized size = 0.39

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] ((14*(-488441710017235774973 7 + 205702452014540322797289*x + 11192676869760
2999806116*x^2 + 136258708960392543166 4856*x^3 - 809990362095044210054958*x
^4 + 367330305827782222538692 6*x^5 - 868597398807984037770570 0*x^6 + 107181
3172591689315155506 8*x^7 - 272466042510766895520439 53*x^8 + 416138849372553
03086792337*x^9 - 597911026814941175721491 76*x^10 + 10203157363431783454797
6132*x^11 - 1333125413772463861158902 40*x^12 + 172649692294614969274168896*
x^13 - 229408132984166521977166336*x^14 + 258819256815163249845447936*x^15
- 282644664539994827031006720*x^16 + 304010591010966811155955200*x^17 - 287
279159180291305208156160*x^18 + 253788172995391086570485760*x^19 - 21663422
8326470609547509760*x^20 + 162290307223249502039654400*x^21 - 1067017258251
02321939251200*x^22 + 65360120291258796757811200*x^23 - 3396989006438128411
1155200*x^24 + 12365045055896811105484800*x^25 - 2621948941596237063782400*
x^26 + 240031204937714427494400*x^27))/((3 - 2*x)^(19/2)*(1 + x + 2*x^2)^9)
 - 426093525*Sqrt[2293002953699236822393 + (30540258843957888971*I)*Sqrt[7]
]*ArcTan[(Sqrt[-1 - I/Sqrt[7]]*Sqrt[3 - 2*x])/2] - 426093525*Sqrt[229300295
3699236822393 - (30540258843957888971*I)*Sqrt[7]]*ArcTan[(Sqrt[-1 + I/Sqrt[
7]]*Sqrt[3 - 2*x])/2])/12040343345613377038712832
```

**Maple** [A]
time = 0.91, size = 550, normalized size = 0.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5367029731/(3-2*x)^(19/2)+5/4802079233/(3-2*x)^(17/2)+73/23727920916/(3-2
*x)^(15/2)+165/25705247659/(3-2*x)^(13/2)+2365/221460595216/(3-2*x)^(11/2)+
30349/1993145356944/(3-2*x)^(9/2)+854095/43406276662336/(3-2*x)^(7/2)+75933
/3100448333024/(3-2*x)^(5/2)+8519225/260437659974016/(3-2*x)^(3/2)+891605/1
2401793332096/(3-2*x)^(1/2)+1/86812553324672*(-16557498921138789448 1/65536*
(3-2*x)^(23/2)+45406001689183688581/131072*(3-2*x)^(25/2)-43462358811134257
841/1179648*(3-2*x)^(27/2)+192384852501874197/65536*(3-2*x)^(29/2)-13528410
99712333/8192*(3-2*x)^(31/2)+4606702222670185/786432*(3-2*x)^(33/2)-2586532
0405815/262144*(3-2*x)^(35/2)+544765170330150812273/1024*(3-2*x)^(1/2)-3476
987783905860258979/1536*(3-2*x)^(3/2)+9364999706478908741137/2048*(3-2*x)^(
5/2)-23851905772903279054347/4096*(3-2*x)^(7/2)+192983613795383541041317/36
864*(3-2*x)^(9/2)-57758421475348449750643/16384*(3-2*x)^(11/2)+603330358695
84695411551/32768*(3-2*x)^(13/2)-14977088508349397804072 3/196608*(3-2*x)^(1
5/2)+6625689994458215569681 1/262144*(3-2*x)^(17/2)-17729978841543630405471/
```

262144*(3-2*x)^(19/2)+2869878271121283060373/196608*(3-2*x)^(21/2))/((3-2*x)^2-7+14*x)^9+11275/1274413838809597739008*(18352320711*(7+2*14^(1/2))^(1/2)*14^(1/2)-69111417106*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))+11275/318603459702399434752*(-9756589235*14^(1/2)-1/2*(18352320711*(7+2*14^(1/2))^(1/2)*14^(1/2)-69111417106*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))-11275/1274413838809597739008*(18352320711*(7+2*14^(1/2))^(1/2)*14^(1/2)-69111417106*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))-11275/318603459702399434752*(9756589235*14^(1/2)+1/2*(18352320711*(7+2*14^(1/2))^(1/2)*14^(1/2)-69111417106*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")

[Out] integrate(1/((2*x^2 + x + 1)^10*(-2*x + 3)^(21/2)), x)

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. 2(491) = 982.
time = 1.73, size = 1563, normalized size = 2.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")

[Out] 1/1094755373086200603246995644663447631605361478665641987670016*(4732002380085251586622550100*4787936175075825342943147314686^(1/4)*sqrt(1169607525756986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936)*arctan(1/36562170851931970248855340113387035354417457241870626866024945379489008832725311219252*4787936175075825342943147314686^(3/4)*sqrt(2776387167632535361)*sqrt(12865682783326846)*sqrt(1169607525756986)*sqrt(4787936175075825342943147314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14

) - 9756589235) - 71440233164918992209696826631202812*x + 28280279689505005
187146*sqrt(22335021272086100802556094) + 107160349747378488314545239946804
218)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3275718505284
62403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615767666910014
4258228441327447900096742*4787936175075825342943147314686^(3/4)*sqrt(116960
7525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(-2*x +
3)*sqrt(327571850528462403199*sqrt(14) + 1226422380928157351936) + 2/7*sqr
t(14)*sqrt(7) + sqrt(7)) + 4732002380085251586622550100*4787936175075825342
943147314686^(1/4)*sqrt(1169607525756986)*sqrt(14)*sqrt(7)*(524288*x^28 - 5
505024*x^27 + 24772608*x^26 - 64684032*x^25 + 119734272*x^24 - 194052096*x^
23 + 295206912*x^22 - 386777088*x^21 + 449261568*x^20 - 515594240*x^19 + 54
0503040*x^18 - 496581120*x^17 + 467712000*x^16 - 411828480*x^15 + 303534720
*x^14 - 248434368*x^13 + 186495624*x^12 - 105219828*x^11 + 83621482*x^10 -
49793667*x^9 + 19105065*x^8 - 20036484*x^7 + 5497632*x^6 - 2235114*x^5 + 32
76126*x^4 + 734832*x^3 + 826686*x^2 + 137781*x + 59049)*sqrt(32757185052846
2403199*sqrt(14) + 1226422380928157351936)*arctan(1/39296670234816303076555
330542603297083388480635973027797585697454399143598928370335464344780800*47
87936175075825342943147314686^(3/4)*sqrt(2776387167632535361)*sqrt(11696075
25756986)*sqrt(-14862107440409842545228890767360000*47879361750758253429431
47314686^(1/4)*sqrt(1169607525756986)*sqrt(-2*x + 3)*sqrt(32757185052846240
3199*sqrt(14) + 1226422380928157351936)*(2148932869*sqrt(14) - 9756589235)
- 1061752420864956548109093061495542399038192585561809435358469816320000*x
+ 420304555190263689316852795001664341102416628348354560000*sqrt(2233502127
2086100802556094) + 1592628631297434822163639592243313598557288878342714153
037704724480000)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))*sqrt(3
27571850528462403199*sqrt(14) + 1226422380928157351936) - 1/102357367080615
7676669100144258228441327447900096742*4787936175075825342943147314686^(3/4)
*sqrt(1169607525756986)*(9756589235*sqrt(14)*sqrt(7) - 30085060166*sqrt(7))
*sqrt(-2*x + 3)*sqrt(327571850528462403199*sqrt(14) + 122642238092815735193
6) - 2/7*sqrt(14)*sqrt(7) - sqrt(7)) + 271150425*47879361750758253429431473
14686^(1/4)*sqrt(1169607525756986)*(642998537252061761731821568*x^28 - 6751
484641146648498184126464*x^27 + 30381680885159918241828569088*x^26 - 793299
44533473119853663485952*x^25 + 146844790944939604835504750592*x^24 - 237989
833600419359560990457856*x^23 + 362048363881489025715123781632*x^22 - 47435
2077153419437787597242368*x^21 + 550984441886077267281495195648*x^20 - 6323
636315413643784471854448640*x^19 + 662885025215707070319757885440*x^18 - 609
018199514371017360613048320*x^17 + 573612464628670331388690432000*x^16 - 50
5075664975624031448627937280*x^15 + 372261773996761581935835217920*x^14 - 3
04685469106942025132773736448*x^13 + 228722407218762404519491928064*x^12 -
129043951976611196927641387008*x^11 + 102555257051181053298083889152*x^10 -
61068067637283818105902989312*x^9 + 23430879305087206538965155840*x^8 - 24
573192412708929931548033024*x^7 + 6742418926906827559038615552*x^6 - 274119
3833525857491515080704*x^5 + 4017914249140640432768679936*x^4 + 90121441102
2199723237834752*x^3 + 1013866212399974688642564096*x^2 - 32757185052846240
3199*sqrt(14)*(524288*x^28 - 5505024*x^27 + 24772608*x^26 - 64684032*x^25 +

```
 119734272*x^24 - 194052096*x^23 + 295206912*x^22 - 386777088*x^21 + 449261
568*x^20 - 515594240*x^19 + 540503040*x^18 - 496581120*x^17 + 467712000*x^1
6 - 411828480*x^15 + 303534720*x^14 - 248434368*x^13 + 186495624*x^12 - 105
219828*x^11 + 83621482*x^10 - 49793667*x^9 + 19105065*x^8 - 20036484*x^7 +
5497632*x^6 - 2235114*x^5 + 3276126*x^4 + 734832*x^3 + 826686*x^2 + 137781*
x + 59049) + 16897770206662448107094016*x + 72419015171426763474468864)*sq
rt(327571850528462403199*sqrt(14) + 1226422380928157351936)*log(14862107440
409842545228890767360000/2776387167632535361*47...
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(21/2)/(2*x**2+x+1)**10,x)
```

```
[Out] Timed out
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(491) = 982.
time = 1.75, size = 1000, normalized size = 1.54

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
```

```
[Out] -11275/1427343499466749467688896*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt(
14) + 4)*sqrt(-2*sqrt(14) + 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) - 4)
*sqrt(-2*sqrt(14) + 8) - 15042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14
) + 4) - 45127590249*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 7805271
3880*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sqrt(2*
sqrt(14) + 8))*arctan(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4)
+ 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 11275/14273434994667494676
8896*sqrt(7)*(6446798607*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) +
 8) + 2148932869*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) - 15
042530083*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 45127590249*14^(3/
4)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 78052713880*14^(1/4)*sqrt(7)*sqrt(
-2*sqrt(14) + 8) - 546368997160*14^(1/4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28
*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-
1/8*sqrt(14) + 1/2)) - 11275/285468699893349893537792*sqrt(7)*(2148932869*1
4^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 6446798607*14^(3/4)*s
qrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 45127590249*14^(3/4)*(sqrt(14)
 + 4)*sqrt(-2*sqrt(14) + 8) + 15042530083*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*s
qrt(14) + 8) + 78052713880*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 54636899
```

```
7160*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/2)*sqrt(-2*x + 3)*
sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 11275/2854686998933498935377792*s
qrt(7)*(2148932869*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 6
446798607*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 4512759024
9*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 15042530083*14^(3/4)*(sqr
t(14) - 4)*sqrt(-2*sqrt(14) + 8) + 78052713880*14^(1/4)*sqrt(7)*sqrt(2*sqrt
(14) + 8) + 546368997160*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt
(1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/204816509
808685350912*(232787883652335*(2*x - 3)^17*sqrt(-2*x + 3) + 13820106668010
55*(2*x - 3)^16*sqrt(-2*x + 3) + 389618236717151904*(2*x - 3)^15*sqrt(-2*x
 + 3) + 6925854690067471092*(2*x - 3)^14*sqrt(-2*x + 3) + 869247176222685156
82*(2*x - 3)^13*sqrt(-2*x + 3) + 817308030405306394458*(2*x - 3)^12*sqrt(-2
*x + 3) + 5960699611609964201316*(2*x - 3)^11*sqrt(-2*x + 3) + 344385392534
55396724476*(2*x - 3)^10*sqrt(-2*x + 3) + 159569809573892673649239*(2*x - 3
)^9*sqrt(-2*x + 3) + 596312099501239401271299*(2*x - 3)^8*sqrt(-2*x + 3) +
1797250621001927736488676*(2*x - 3)^7*sqrt(-2*x + 3) + 43439785826100980696
31672*(2*x - 3)^6*sqrt(-2*x + 3) + 8317212692450176764092592*(2*x - 3)^5*sq
rt(-2*x + 3) + 12350951282904546626644288*(2*x - 3)^4*sqrt(-2*x + 3) + 1373
8697725192288735303872*(2*x - 3)^3*sqrt(-2*x + 3) + 10788479661863702869789
824*(2*x - 3)^2*sqrt(-2*x + 3) - 5340653236079401357791744*(-2*x + 3)^(3/2)
 + 1255138952440667471476992*sqrt(-2*x + 3))/((2*x - 3)^2 + 14*x - 7)^9 + 1
/3280733202692679552*(235862511885*(2*x - 3)^9 - 107316677325*(2*x - 3)^8 +
 80348352084*(2*x - 3)^7 - 64554208290*(2*x - 3)^6 + 49954696792*(2*x - 3)^
5 - 35035280280*(2*x - 3)^4 + 21058773120*(2*x - 3)^3 - 10093321056*(2*x -
3)^2 + 6831901440*x - 10859127552)/((2*x - 3)^9*sqrt(-2*x + 3))
```

**Mupad [B]**
time = 0.56, size = 567, normalized size = 0.88

---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(21/2)*(x + 2*x^2 + 1)^10),x)
```

```
[Out] ((184192*(2*x - 3)^2)/47481 - (18944*x)/2261 - (15552*(2*x - 3)^3)/4199 + (
5666272*(2*x - 3)^4)/1440257 - (63490768*(2*x - 3)^5)/12962313 + (533495672
*(2*x - 3)^6)/70572593 - (1111521492*(2*x - 3)^7)/70572593 + (78007323158*(
2*x - 3)^8)/1482024453 - (250239440467*(2*x - 3)^9)/494008151 + (1118693654
785651073*(2*x - 3)^10)/453254454575104 + (1624300450152249301*(2*x - 3)^11
)/97125954551808 + (35048653520674948897*(2*x - 3)^12)/906508909150208 + (9
5527511967437577915*(2*x - 3)^13)/1813017818300416 + (564066299973141561054
7*(2*x - 3)^14)/114220122552926208 + (1737142288764447500149*(2*x - 3)^15)/
50764498912411648 + (12971210667229097601055*(2*x - 3)^16)/7107029847737630
72 + (32723441206946795665235*(2*x - 3)^17)/4264217908642578432 + (10264579
7034777710681325*(2*x - 3)^18)/39799367147330732032 + (14609317874302006653
```

15*(2*x - 3)^19)/2094703534070038528 + (687618468821894139745*(2*x - 3)^20)
/452825616923964211 2 + (39968995676603847725*(2*x - 3)^21)/1509418723079880
704 + (594013294361384987 5*(2*x - 3)^22)/1625527855624486912 + (57179785036
20010375*(2*x - 3)^23)/14629750700620382208 + (178056995818325525*(2*x - 3)
^24)/5689347494685704192 + (179665281323275*(2*x - 3)^25)/10159549097653043
2 + (1433237383402275*(2*x - 3)^26)/22757389978742816768 + (24229218097975*
(2*x - 3)^27)/22757389978742816768 + 37120/2261)/(20661046784*(3 - 2*x)^(19
/2) - 92974710528*(3 - 2*x)^(21/2) + 199231522560*(3 - 2*x)^(23/2) - 270069
397248*(3 - 2*x)^(25/2) + 259475340096*(3 - 2*x)^(27/2) - 187609683744*(3 -
2*x)^(29/2) + 105782451264*(3 - 2*x)^(31/2) - 47554666992*(3 - 2*x)^(33/2)
+ 17278167438*(3 - 2*x)^(35/2) - 5111496103*(3 - 2*x)^(37/2) + 1234154817*
(3 - 2*x)^(39/2) - 242625852*(3 - 2*x)^(41/2) + 38550456*(3 - 2*x)^(43/2) -
4883634*(3 - 2*x)^(45/2) + 482454*(3 - 2*x)^(47/2) - 35868*(3 - 2*x)^(49/2
) + 1890*(3 - 2*x)^(51/2) - 63*(3 - 2*x)^(53/2) + (3 - 2*x)^(55/2)) - (atan
(((- 7^(1/2)*30540258843957888971i - 229300295369923682239 3)^(1/2)*(3 - 2*x
)^(1/2)*437746180358291443303165206406 25i)/(330008698047761583560870082619 2
638064300936005891581238312 96*((7^(1/2)*427090967094607473872427449424977 17
8671875i)/165004349023880791780435041309631903215046800294579061915648 + 80
336582919506134555067610693840117548437 5/235720498605543988257764344728045 5
76021495428992255802736 64)) + (437746180358291443303165206406 25*7^(1/2)*(-
7^(1/2)*30540258843957888971i - 229300295369923682239 3)^(1/2)*(3 - 2*x)^(1/
2))/(3300086980477615835608700826192638064300936005891581238312 96*((7^(1/2)
*427090967094607473872427449424977178671875i)/165004349023880791780435041 30
963190321504680029457906191564 8 + 803365829195061345550676106938401175484 37
5/2357204986055439882577643447280455760214954289922558027366 4)))*(- 7^(1/2)
*30540258843957888971i - 229300295369923682239 3)^(1/2)*11275i)/318603459702
399434752 + (atan(((7^(1/2)*30540258843957888971i - 229300295369923682239 3)
^(1/2)*(3 - 2*x)^(1/2)*437746180358291443303165206406 25i)/(3300086980477615
83560870082619263806430093600589158123831296*((7^(1/2)*427090967094607473 87
2427449424977178671875i)/16500434902388079178043504130963190321504680029457
9061915648 - 803365829195061345550676106938401175484375/2357204986055439882
577643447280455760214954289922558027366 4)) - (43774618035829144330316520640
625*7^(1/2)*(7^(1/2)*30540258843957888971i - 229300295369923682239 3)^(1/2)*
(3 - 2*x)^(1/2))/(3300086980477615835608700826192638064300936005891581238 31
296*((7^(1/2)*427090967094607473872427449424977178671875i)/165004349023880 7
917804350413096319032150468002945790619156 48 - 803365829195061345550676106 9
38401175484375/235720498605543988257764344728045576021495428992255802736 64)
))*(7^(1/2)*30540258843957888971i - 229300295369923682239 3)^(1/2)*11275i)/3
18603459702399434752

## 3.49 $\int \frac{1}{(3-2x)^{41/2}(1+x+2x^2)^{20}}\,dx$

**Optimal**. Leaf size=1058

result too large to display

```
[Out] -143401467550777247627940437025/73985542663511997461099839851260280832/(3-2
*x)^(9/2)-4611053278117143010907562317585/7250583181024175751187784305423507521536/(3-2*x)^(7/2)-405965372440630510720926890227/2071595194578335928910795515835287863296/(3-2*x)^(5/2)-4986681479187781853417316522775/87006998172290109014253411665082090258432/(3-2*x)^(3/2)+173441368149804378661935869705/896508488907352010051592447177261056/(3-2*x)^(19/2)-22724090823469905152713519545/1604278348571050965355481221264572416/(3-2*x)^(17/2)-101190274412779618678573275245/3963511214116714149701777134888943616/(3-2*x)^(15/2)-460503190416958283087439337135/34350430522344855964082068502370844672/(3-2*x)^(13/2)-2211619588790911794826342607495/406920484649315986036049119181931544576/(3-2*x)^(11/2)-13056959628363355534285785425/106924014357253562723941220352/(3-2*x)^(39/2)-3948194343291401740321996415/202881463139404195937734623232/(3-2*x)^(37/2)-304688229262620222736480811/537361713180043545997243056128/(3-2*x)^(35/2)+2124315846756567455653862925/1688851098565851144562763890688/(3-2*x)^(33/2)+47657515074514118796095929535/66632852434325399703658138959872/(3-2*x)^(31/2)+34911619993974714062172751985/124667917457770102671360389021696/(3-2*x)^(29/2)+149066309808794760843017404825/1624981820656451683095663001731072/(3-2*x)^(27/2)+15848613964169066543734380171/601845118761648771516912222863360/(3-2*x)^(25/2)+11155168222970774232376891145/1685166332532616560247354224017408/(3-2*x)^(23/2)+14011818498091020272474956375/10110997995195699361484125344104448/(3-2*x)^(21/2)+115/3248261265098830736532127368829731369648128*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(30297118912219360725028693061-8061110911143276053983022787*14^(1/2))*(-14+4*14^(1/2))^(1/2)-115/3248261265098830736532127368829731369648128*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))*(30297118912219360725028693061-8061110911143276053983022787*14^(1/2))*(-14+4*14^(1/2))^(1/2)+115/1624130632549415368266063684414865684824064*arctan((-2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(30297118912219360725028693061+8061110911143276053983022787*14^(1/2))*(14+4*14^(1/2))^(1/2)-115/1624130632549415368266063684414865684824064*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))*(30297118912219360725028693061+8061110911143276053983022787*14^(1/2))*(14+4*14^(1/2))^(1/2)-927027754781476746208047620505/58004665448193406009502274443388060172288/(3-2*x)^(1/2)+5/595601664*(751303+1831285*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^16+1/25015269888*(184959785+429411497*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^15+1/4902992898048*(41652915209+92630823167*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^14+1/297448235814912*(2871555518177+6100156355517*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^13+1/7138757659557888*(77559130805859+156274047129113*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^12+5/1099368679571914752*(2656658801194921+5020880176134289*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^11+1/3420258114223734784*(45
```

```
187921585208601+78752911037377255*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^10+1/430952
522392190582784*(6063974149878048635+9477172618423641847*x)/(3-2*x)^(39/2)/
(2*x^2+x+1)^9+1/4826668250792534527180 8*(6918336011449258548 31+919498192874
055581221*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^8+23/157671162859222794554572 8*(919
498192874055581221+908287136092467468517*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^7+11
5/101879828309036267250647 04*(908287136092467468517+2982818849445222257 47*x
)/(3-2*x)^(39/2)/(2*x^2+x+1)^6+23/20375965661807253450129408*(2599313568802
265110081-1042614244862318737918 7*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^5-23/200184
925800211612843376 64*(10426142448623187379187+27513723463194262383705*x)/(3
-2*x)^(39/2)/(2*x^2+x+1)^4-115/7643424439644433994743808*(2651322442816901
6478843+30673415406553789342019*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^3-115/1258916
966529673030501662 72*(884116091130079810446 43-57122695362451521629 63*x)/(3-
2*x)^(39/2)/(2*x^2+x+1)^2+115/19583152812683802696692531 2*(2856134768122576
0814815+96593481283901949034610 7*x)/(3-2*x)^(39/2)/(2*x^2+x+1)+1/133*x/(3-2
*x)^(39/2)/(2*x^2+x+1)^19+1/33516*(113+373*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^18
+1/7976808*(40657+107329*x)/(3-2*x)^(39/2)/(2*x^2+x+1)^17
```

**Rubi** [A]

time = 1.63, antiderivative size = 1058, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {754, 836, 842, 840, 1183, 648, 632, 210, 642}

---

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] -13056959628363355534285785425/(10692401435725356272394122 0352*(3 - 2*x)^(3
9/2)) - 394819434329140174032199 6415/(2028814631394041959377346 23232*(3 - 2
*x)^(37/2)) - 304688229262620222736480811/(5373617131800435459972430 56128*(
3 - 2*x)^(35/2)) + 21243158467565674556538 62925/(16888510985658511445627638
90688*(3 - 2*x)^(33/2)) + 476575150745141187960959 29535/(666328524343253997
03658138959872*(3 - 2*x)^(31/2)) + 349116199939747140621727 51985/(124667917
457770102671360389021696*(3 - 2*x)^(29/2)) + 1490663098087947608430174 04825
/(16249818206564516830956630017310 72*(3 - 2*x)^(27/2)) + 158486139641690665
43734380171/(601845118761648771516912222863360*(3 - 2*x)^(25/2)) + 11155168
222970774232376891145/(16851663325326165602473542240174 08*(3 - 2*x)^(23/2))
 + 14011818498091020272474956375/(101109979951956993614841253441044 48*(3 -
2*x)^(21/2)) + 17344136814980437866193586 9705/(896508488907352010051592447 1
77261056*(3 - 2*x)^(19/2)) - 22724090823469905152713519545/(160427834857105
0965355481221264572416*(3 - 2*x)^(17/2)) - 10119027441277961867857327 5245/(
39635112141167141497017771348889436 16*(3 - 2*x)^(15/2)) - 46050319041695828
3087439337135/(3435043052234485596408206850237084467 2*(3 - 2*x)^(13/2)) - 2
2116195887909117948263426074 95/(40692048464931598603604911918193154457 6*(3
 - 2*x)^(11/2)) - 143401467550777247627940437025/(7398554266351199746109983 9
```

```
851260280832*(3 - 2*x)^(9/2)) - 46110532781171430109075623175585/(7250583181
02417575118778430542350752153 6*(3 - 2*x)^(7/2)) - 405965372440630510720926 8
90227/(207159519457833592891079551583528786329 6*(3 - 2*x)^(5/2)) - 49866814
791877818534173165227 75/(870069981722901090142534116650820902584 32*(3 - 2*x
)^(3/2)) - 927027754781476746208047620505/(580046654481934060095022744433 88
060172288*Sqrt[3 - 2*x]) + x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + (1
13 + 373*x)/(33516*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^18) + (40657 + 107329*x
)/(7976808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^17) + (5*(751303 + 1831285*x))/
(595601664*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + (184959785 + 429411497*x)
/(25015269888*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + (41652915209 + 9263082
3167*x)/(4902992898048*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + (287155551817
7 + 6100156355517*x)/(297448235814912*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^13)
+ (77559130805859 + 156274047129113*x)/(7138757659557888*(3 - 2*x)^(39/2)*(
1 + x + 2*x^2)^12) + (5*(2656658801194921 + 5020880176134289*x))/(109936867
9571914752*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^11) + (45187921585208601 + 7875
2911037377255*x)/(3420258114223734784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10)
+ (6063974149878048635 + 9477172618423641847*x)/(430952522392190582784*(3 -
 2*x)^(39/2)*(1 + x + 2*x^2)^9) + (691833601144925854831 + 9194981928740555
81221*x)/(48266682507925345271808*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^8) + (23
*(919498192874055581221 + 908287136092467468517*x))/(157671162859222794554 5
728*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^7) + (115*(908287136092467468517 + 298
281884944522225747*x))/(101879828309036267250647 04*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^6) + (23*(2599313568802265110081 - 1042614244862318737918 7*x))/(20
375965661807253450129408*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) - (23*(1042614
2448623187379187 + 2751372346319426238370 5*x))/(200184925800211612843376 64*
(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^4) - (115*(265132244281690164788 43 + 30673
415406553789342019*x))/(7643424439644443399474380 8*(3 - 2*x)^(39/2)*(1 + x
+ 2*x^2)^3) - (115*(88411609113007981044643 - 571226953624515216296 3*x))/(1
258916966529673030501662 72*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + (115*(2856
134768122576081481 5 + 9659348128390194903461 07*x))/(195831528126838026966 92
5312*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302
9711891221936072502869306 1 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[
(Sqrt[7 + 2*Sqrt[14]] - 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162
7470768413303184220743284241203 2 - (115*Sqrt[(7 + 2*Sqrt[14])/2]*(302971189
1221936072502869306 1 + 806111091114327605398302278 7*Sqrt[14])*ArcTan[(Sqrt[
7 + 2*Sqrt[14]] + 2*Sqrt[3 - 2*x])/Sqrt[-7 + 2*Sqrt[14]]])/8120653162747076
841330318422074328424120 32 + (115*(302971189122193607250286930 61 - 80611109
1114327605398302278 7*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] -
 Sqrt[7 + 2*Sqrt[14]]*Sqrt[3 - 2*x] - 2*x])/16241306325494153682660636844 14
865684824064 - (115*(302971189122193607250286930 61 - 806111091114327605398 3
022787*Sqrt[14])*Sqrt[(-7 + 2*Sqrt[14])/2]*Log[3 + Sqrt[14] + Sqrt[7 + 2*Sq
rt[14]]*Sqrt[3 - 2*x] - 2*x])/16241306325494153682660636844148656848240 64
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
```

```
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
```

```
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 842

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)
^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^2
)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && LtQ[m, -1]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(3-2x)^{41/2}\left(1+x+2x^2\right)^{20}}\,dx=\frac{x}{133(3-2x)^{39/2}\left(1+x+2x^2\right)^{19}}+\frac{\int \frac{3640-3164x}{(3-2x)^{41/2}(1+x+2x^2)^{19}}\,dx}{3724}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 16.12, size = 1100, normalized size = 1.04

Too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x)^(41/2)*(1 + x + 2*x^2)^20),x]
```

```
[Out] x/(133*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^19) + ((44296 + 146216*x)/(3528*(3
- 2*x)^(39/2)*(1 + x + 2*x^2)^18) + ((223125616 + 589021552*x)/(3332*(3 - 2
*x)^(39/2)*(1 + x + 2*x^2)^17) + ((865861681440 + 2110519336800*x)/(3136*(3
```

```
 - 2*x)^(39/2)*(1 + x + 2*x^2)^16) + ((2984274342235200 + 6928434268875840*
x)/(2940*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^15) + ((94088137371333390720 + 209
2401353236681 5360*x)/(2744*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^14) + ((2724306
5619141593598720 + 57873497074462503141120*x)/(2548*(3 - 2*x)^(39/2)*(1 + x
 + 2*x^2)^13) + ((7211037735478027891383 5520 + 1452953429486831061640166 40*
x)/(2352*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^12) + ((172901458108932896335179 8
01600 + 3267704166803014216810662144 00*x)/(2156*(3 - 2*x)^(39/2)*(1 + x + 2
*x^2)^11) + ((37055765251546181218632908712 9600 + 645802967231886306826540 4
24448000*x)/(1960*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^10) + ((6961755986759734
3875901057755494 4000 + 108802843783879062180944047308871 6800*x)/(1764*(3 -
2*x)^(39/2)*(1 + x + 2*x^2)^9) + ((1111965063471244015489248163496668569600
 + 1477884081820868038735185945420330393600*x)/(1568*(3 - 2*x)^(39/2)*(1 +
x + 2*x^2)^8) + ((14276360230389585254181896232760391602176 00 + 14102294542
80293592108580217248432347955200*x)/(1372*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
7) + ((12833088033950671688188079976960734366392 32000 + 4214391612869991217
7013558424620483623731 2000*x)/(1176*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^6) + (
(35990904373909724999169578894625893014666444 8000 - 1443636121324398194831 6
93460992758930913796096000*x)/(980*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^5) + ((
-11520216248168697594756913818722216268692092846 08000 - 3040089329780519199
03117016626095338157026025472 0000*x)/(784*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)^
4) + ((-22557462826971452456811282633656274091251331090022 40000 - 260969551
13255292554103826516650734708457329890099 20000*x)/(588*(3 - 2*x)^(39/2)*(1
+ x + 2*x^2)^3) + ((-1790251120769313069211522499042240401000172830460805 12
0000 + 115668033214143596894295804604678509924267822733393920000*x)/(392*(3
 - 2*x)^(39/2)*(1 + x + 2*x^2)^2) + ((7287086092491046604340635690094746125
228872832203816960 0000 + 24644670900872826929692130734587768100251906626103
4303488 0000*x)/(196*(3 - 2*x)^(39/2)*(1 + x + 2*x^2)) + (-5305505666658970 8
7493026465460148012491929957574880460800000/(3 - 2*x)^(39/2) + (-1708089006
2422412644804810732936117697712983887858137533644 80000/(37*(3 - 2*x)^(37/2)
) + (-6967409500899092000175397836924272167042711880384026979205 12000/(3 -
2*x)^(35/2) + (7573666677621473556024460064742611515974095257956818246615 04
000000/(3 - 2*x)^(33/2) + (6167726649054233403507372547934021941920835094 01
0816556282758758400000/(31*(3 - 2*x)^(31/2)) + (980445504127015992472138196
6457786103619439408616372746508906610688 00000/(29*(3 - 2*x)^(29/2)) + (4496
4233234365801798259356678072391756466292408034159102502223134 72000000/(3 -
2*x)^(27/2) + (487904184130260773926886832047572655461484781443782543411352
841560457216000/(3 - 2*x)^(25/2) + (429268867215238023064148871550918822599
0254208806769817062280254541824 0000000/(3 - 2*x)^(23/2) + (289369259398036 4
723231826294558630623656919099359688069727689450554368000000000/(3 - 2*x)^(
21/2) + (118767476492930264374166633243140666046068763101817907661320807641
1903590400000 00/(3 - 2*x)^(19/2) + (-23130641371662285970537372414163682847
2251691242315976748933281043780325376 0000000/(3 - 2*x)^(17/2) + (-992239519
6537908604226239489579648523559858468009362133384187617620979500236800000 00
/(3 - 2*x)^(15/2) + (-10941518315154632243157241587901809625083601209973176
690146784165460261475512320 0000000/(3 - 2*x)^(13/2) + (-8073268485314233063
```

```
8403379340954315600692165352258493007480189439306347456219136000000000/(3 -
2*x)^(11/2) + (-44337987226211231305207361494572283981715203938096393248399
6666511839997547213824000000000/(3 - 2*x)^(9/2) + (-183301908922166977441733
7067901437000873585615761361787541745447275781173253597919232000000000/(3 -
2*x)^(7/2) + (-5535412100027359570488442147160282454990867464017235233247800
66055766166841372505894912000000000/(3 - 2*x)^(5/2) + (-11332385663391839740 3
4397442837068388756677147138484115167264239399928318213926633984000000000/
(3 - 2*x)^(3/2) + (-1327220262908131487403839635355234271426655189754352930
6435677723641008864036246751334400000000000/Sqrt[3 - 2*x] + ((Sqrt[(7 - I*Sqr
t[7])/2]*(-1858108368071384082365375489497327979997317265656094102900994881
3097412409650745451868160000000000 - (38534140062781031467679876224014966993
3633555592186583754201688526589748283311569009254400000000000*I)*Sqrt[7])*Arc
Tanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 - I*Sqrt[7]]])/(-14 + (2*I)*Sqrt[7]) +
(Sqrt[(7 + I*Sqrt[7])/2]*(-1858108368071384082365375489497327979997317265656
0941029009948813097412409650745451868160000000000 + (385341400627810314676798
762240149669933633555592186583754201688526589748283311569009254400000000000
*I)*Sqrt[7])*ArcTanh[(Sqrt[2]*Sqrt[3 - 2*x])/Sqrt[7 + I*Sqrt[7]]])/(-14 - (
2*I)*Sqrt[7]))/7)/42)/70)/98)/126)/154)/182)/210)/238)/266)/294)/322)/350)/
378)/406)/434)/462)/490)/518)/546)/196)/392)/58...
```

<u>**Maple**</u> [**A**]
time = 1.18, size = 820, normalized size = 0.78

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 732 |
| risch | Expression too large to display | 761 |
| derivativedivides | Expression too large to display | 820 |
| default | Expression too large to display | 820 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30145677658996078082575630336*(8075977364926413789422689372179958353538 49
465/1048576*(3-2*x)^(1/2)+180866897114899220649017210287078795487454 1181/33
4114095890432*(3-2*x)^(57/2)-11968977253082880651292892111395530933265219/2
5701084299264*(3-2*x)^(59/2)+3395565446412935417599589886148144605 49873/982
6885173248*(3-2*x)^(61/2)-642433967191403749984730270090274852 63697/2948065
5519744*(3-2*x)^(63/2)+1298868527487271103574256186729223246 59/113387136614
4*(3-2*x)^(65/2)-503502693505289734438057515605193725/103079215104*(3-2*x)^
(67/2)+13388331332211939734879173298195 3297/824633720832*(3-2*x)^(69/2)-325
485074800348342966673885017837 9/824633720832*(3-2*x)^(71/2)+360433340020130
12394233506377914 5/5772436045824*(3-2*x)^(73/2)-928342237074576734557978321
305/1924145348608*(3-2*x)^(75/2)-447963293570690822971544737256709035462203
92558695/9070970929152*(3-2*x)^(43/2)+2860722331769322369839567258415059386
3016075796143/29480655519744*(3-2*x)^(45/2)-5059022664167725408892162874688
```

680417923742003781/29480655519744*(3-2*x)^(47/2)+73012476452577571533836489036461787385135079265/2680059592704*(3-2*x)^(49/2)-193924292090153482145402690313243308158022102373 7/501171143835648*(3-2*x)^(51/2)+4907385430648794239550771659874341524415632704 73/1002342287671296*(3-2*x)^(53/2)-550118352883612890020116931793783166990331026 75/1002342287671296*(3-2*x)^(55/2)-1006304725834560333245233940167063186576585913370455/10720238370816*(3-2*x)^(39/2)
+1380572274182261258625859209942856628019123019727140 5/39307540692992*(3-2*x)^(37/2)-17650942358963262675871173166229809316744939271143/51904512*(3-2*x)^(11/2)+118632384645382623721251719631219381945276176401882 2545/3915399561216*(3-2*x)^(21/2)+26722399847903378442920192943151823852165730773017 85/117922622078976*(3-2*x)^(41/2)-223975463212094869530620743747957372999570635 65/3145728*(3-2*x)^(3/2)+4045315666898833370484992335277819835991876340 17/12582912*(3-2*x)^(5/2)-11885980275522548300826832180646971886056129524 19/12582912*(3-2*x)^(7/2)+38315833791662940918235729539899936257724714453 45/18874368*(3-2*x)^(9/2)+9977850126168010187169130424774568330973123412551261/21592276992*(3-2*x)^(13/2)-12556967184995885809797263315720723203579692970777 45/2399141888*(3-2*x)^(15/2)-75593011640468285657019519019203244129463216094552 3631/3915399561216*(3-2*x)^(23/2)+85350850220721451198709386602112408790804163469724405 9/7830799122432*(3-2*x)^(25/2)-68861738098940055439945164424618714860070420051897 75/125627793408*(3-2*x)^(27/2)+13632998796724539514184825376514720827981414835295800 9/5527622909952*(3-2*x)^(29/2)-55066091420817590167865401986871791412011888132876913/5527622909952*(3-2*x)^(31/2)+27374875289284393578691387749101269233637917471416 75/755914244096*(3-2*x)^(33/2)-1166457217021587688420366823074349521448831011337110 5/9826885173248*(3-2*x)^(35/2)+1264662933338272271690443076373266517911961538955241 3/25098715136*(3-2*x)^(17/2)-2593673203685044441695042001860835122939346700333136537/6199382638592*(3-2*x)^(19/2))/((3-2*x)^2-7+14*x)^19-115/324826126509883073653212736882973136964812 8*(625415625567924649409607842 09*(7+2*14^(1/2))^(1/2)*14^(1/2)-23404402840488330765587709126 2*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(1/2)-(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))+115/32482612650988307365321273688297313 69648128*(62541562556792464940960784209*(7+2*14^(1/2))^(1/2)*14^(1/2)-23404402840488330765587709126 2*(7+2*14^(1/2))^(1/2))*ln(3-2*x+14^(1/2)+(3-2*x)^(1/2)*(7+2*14^(1/2))^(1/2))-115/812065316274707684133031842207432842412032*(30297118912219360725028693061*14^(1/2)+1/2*(62541562556792464940960784209*(7+2*14^(1/2))^(1/2)*14^(1/2)-234044028404883307655877091262*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2)*arctan((2*(3-2*x)^(1/2)-(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/2))^(1/2))+683151246370725/30145677658996078082575630336/(3-2*x)^(1/2)+355/52662895756423920 66/(3-2*x)^(33/2)+52865/277038748585308867472/(3-2*x)^(31/2)+14333/32395660116830472406/(3-2*x)^(29/2)+1478345/168904269298785083716 8/(3-2*x)^(27/2)+475387/312785683886639043920/(3-2*x)^(25/2)+16575515/700639931906071458380 8/(3-2*x)^(23/2)+246866015/735671928501375031299 84/(3-2*x)^(21/2)+1/3111898385606868039/(3-2*x)^(39/2)+10/2952313853011644037/(3-2*x)^(37/2)+143/7819642097165976098/(3-2*x)^(35/2)+122484655975/178523054649667007595427 84/(3-2*x)^(13/2)+10815878546425/148036809932570026298362470 4/(3-2*x)^(11/2)+8192823353/1863702218870150079 29

```
2928/(3-2*x)^(19/2)+8972680075/16675230379364500709463 04/(3-2*x)^(17/2)+102
495360575/16479051198430800701116416/(3-2*x)^(15/2)+769045155125/1009341885
90388654294338048/(3-2*x)^(9/2)+838467657280275/10550987180648627328901470 6
176/(3-2*x)^(7/2)+9270470094105/10766313449641456458062725 12/(3-2*x)^(5/2)+
320421783064625/301456776589960780825756303 36/(3-2*x)^(3/2)+115/81206531627
470768413303184220743284241 2032*(-302971189122193607250286930 61*14^(1/2)-1/
2*(625415625567924649409607842 09*(7+2*14^(1/2))^(1/2)*14^(1/2)-234044028404
883307655877091262*(7+2*14^(1/2))^(1/2))*(7+2*14^(1/2))^(1/2))/(-7+2*14^(1/
2))^(1/2)*arctan((2*(3-2*x)^(1/2)+(7+2*14^(1/2)...
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + x + 1)^20*(-2*x + 3)^(41/2)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 2763 vs. $2(821) = 1642$.
time = 39.48, size = 2763, normalized size = 2.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="fricas")
[Out] 1/392164866433134591452265700785383646021637848990793338705942972429050741 0
8177191061306767655456730731382076063194779070544791852259532242520616485 25
72160*(616525316537858546962128448983043227187951381815778781478549978900*5
795904991921858556653045419515717067178458593845454142080244780765852057823
32794174344701326^(1/4)*sqrt(128684608824630489703584217174321785034500513 9
4)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 107064944754
688*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*
x^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112
*x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 517079841212727
296*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 14912870282
33404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 27862
74020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 +
 3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*
x^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 28615445794952
97024*x^33 + 2477632938217930752*x^32 - 2088430257127768064*x^31 + 17127610
05459316736*x^30 - 1355447485390974976*x^29 + 1048940886155151360*x^28 - 79
0511024135089152*x^27 + 571750925528393856*x^26 - 408374103192240192*x^25 +
```

282845069599813728*x^24 - 186113897194906128*x^23 + 123982890381352520*x^2
2 - 78116367732251996*x^21 + 46488580159296898*x^20 - 29591055660829971*x^1
9 + 16200795673453545*x^18 - 8941894120163277*x^17 + 5578893209169441*x^16
- 2296849711499532*x^15 + 1448289882400788*x^14 - 756896247319212*x^13 + 18
2213447974992*x^12 - 240797810407770*x^11 + 25549234281774*x^10 - 265002817
27302*x^9 + 25520701332582*x^8 + 9965507230260*x^7 + 10389354811164*x^6 + 3
755740313808*x^5 + 1820618017974*x^4 + 463742325333*x^3 + 139858796529*x^2
+ 19758444939*x + 3486784401)*sqrt(378148402880167888800346812933915372766 2
345024772741260943*sqrt(14) + 141490223718487283855707890366841241012101616
40127797919744)*arctan(1/34885554762731597076008789349408244975617249636749
13242575009589894914045286581081812447079130476773106112671051669997871458 0
82291658322630168235582320931564879831926785152574881809490600509573163099 2
22783843446054688985482057622250395943920813921700*5795904991921858556653 04
541951571706717845859384545414208024478076585205782332794174344701326^(3/4)
*sqrt(16348573353231128508124926770926395033494513274184173 11)*sqrt(6434230
441231524485179210858716089251725025697 0)*sqrt(128684608824630489703584217 1
7432178503450051394)*sqrt(579590499192185855665304541951571706717845859384 5
454142080244780765852057823327941743447013 26^(1/4)*sqrt(128684608824630489 7
035842171743217850345005139 4)*sqrt(-2*x + 3)*sqrt(3781484028801678888003468
12933915372766234502477274126094 3*sqrt(14) + 14149022371848728385570789036 6
84124101210161640127797919744)*(806111091114327605398302278 7*sqrt(14) - 302
971189122193607250286930 61) - 21038097668013253556956344328723682390547871 9
25945120416845732487486521616208085674137074565089281534 0*x + 9637320505996
21794425456308219340060829468062999882820661390*sqrt(16678937196596395958 10
98742817586289130679764812156476721038706576007991289033281726) + 315571465
0201988033543451649308552358582180788891768062526859873122978242431212851 12
056118476339223010)*(302971189122193607250286930 61*sqrt(14)*sqrt(7) - 11285
555275600586475576231901 8*sqrt(7))*sqrt(378148402880167888800346812933915 37
2766234502477274126094 3*sqrt(14) + 14149022371848728385570789036684124101 21
016164012779791974 4) - 1/33164172268077541576042406944735803543071184128057
80544574064399284894720547513183329763987573259243427226688367795480452172 1
584006729715127306903510*579590499192185855665304541951571706717845859384 54
541420802447807658520578233279417434470132 6^(3/4)*sqrt(128684608824630489 70
358421717432178503450051394)*(3029711891221936072502869306 1*sqrt(14)*sqrt(7
) - 11285555275600586475576231901 8*sqrt(7))*sqrt(-2*x + 3)*sqrt(37814840288
016788880034681293391537276623450247727412609 43*sqrt(14) + 1414902237184872
838557078903668412410121016164012779791974 4) + 2/7*sqrt(14)*sqrt(7) + sqrt(
7)) + 6165253165378585469621284489830432271879513818157787814785499789 00*57
959049919218585566530454195157170671784585938454541420802447807658520578233
2794174344701326^(1/4)*sqrt(1286846088246304897035842171743217850345005139 4
)*sqrt(14)*sqrt(7)*(549755813888*x^58 - 11269994184704*x^57 + 1070649447546
88*x^56 - 630638638006272*x^55 + 2618521301286912*x^54 - 8342252417974272*x
^53 + 21849572376576000*x^52 - 49684091485814784*x^51 + 101394501297242112*
x^50 - 188583312363618304*x^49 + 323261995581177856*x^48 - 5170798412127272
96*x^47 + 778117896260812800*x^46 - 1105641165387988992*x^45 + 149128702823

```
3404416*x^44 - 1919929663119949824*x^43 + 2363050939901804544*x^42 - 278627
4020645928960*x^41 + 3161145685194047488*x^40 - 3453753931369283584*x^39 +
3634098467102523392*x^38 - 3697893960325791744*x^37 + 3640651752731836416*x
^36 - 3461798212247617536*x^35 + 3194540251789393920*x^34 - 286154457949529
7024*x^33 + 2477632938217930752*x^32 - 20884302...
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(41/2)/(2*x**2+x+1)**20,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7262 deep
```

**Giac** [A]
time = 2.53, size = 1410, normalized size = 1.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(41/2)/(2*x^2+x+1)^20,x, algorithm="giac")
```

```
[Out] -115/363805261691069042491598265308929913400590336*sqrt(7)*(241833327334298
28161949068361*14^(3/4)*sqrt(7)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 8061
1109111432760539830227877*14^(3/4)*sqrt(7)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) +
 8) - 56427776378002932377881159509*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(14)
 + 4) - 169283329134008797133643478527*14^(3/4)*sqrt(2*sqrt(14) + 8)*(sqrt(
14) - 4) + 242376951297754885800229544488*14^(1/4)*sqrt(7)*sqrt(-2*sqrt(14)
 + 8) - 1696638659084284200601606811416*14^(1/4)*sqrt(2*sqrt(14) + 8))*arct
an(1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqrt(14) + 4) + 2*sqrt(-2*x + 3))
/sqrt(-1/8*sqrt(14) + 1/2)) - 115/36380526169106904249159826530892991340059
0336*sqrt(7)*(24183332733429828161949068361*14^(3/4)*sqrt(7)*(sqrt(14) + 4)
*sqrt(-2*sqrt(14) + 8) + 8061110911143276053983022787*14^(3/4)*sqrt(7)*(sqr
t(14) - 4)*sqrt(-2*sqrt(14) + 8) - 56427776378002932377881159509*14^(3/4)*s
qrt(2*sqrt(14) + 8)*(sqrt(14) + 4) - 169283329134008797133643478527*14^(3/4
)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 242376951297754885800229544488*14^(
1/4)*sqrt(7)*sqrt(-2*sqrt(14) + 8) - 1696638659084284200601606811416*14^(1/
4)*sqrt(2*sqrt(14) + 8))*arctan(-1/28*14^(3/4)*(14^(1/4)*sqrt(1/2)*sqrt(sqr
t(14) + 4) - 2*sqrt(-2*x + 3))/sqrt(-1/8*sqrt(14) + 1/2)) - 115/72761052338
2138084983196530617859826801180672*sqrt(7)*(8061110911143276053983022787*14
^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 2418333273342982816194
9068361*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 169283329134
008797133643478527*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 56427776
378002932377881159509*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 24237
```

6951297754885800229544488*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 169663865
9084284200601606811416*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(14^(1/4)*sqrt(1/
2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 115/7276105233
82138084983196530617859826801180672*sqrt(7)*(80611091114327605398302278 7*1
4^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) + 4) + 241833327334298281619
49068361*14^(3/4)*sqrt(7)*sqrt(2*sqrt(14) + 8)*(sqrt(14) - 4) + 16928332913
4008797133643478527*14^(3/4)*(sqrt(14) + 4)*sqrt(-2*sqrt(14) + 8) + 5642777
6378002932377881159509*14^(3/4)*(sqrt(14) - 4)*sqrt(-2*sqrt(14) + 8) + 2423
7695129775488580022954448 8*14^(1/4)*sqrt(7)*sqrt(2*sqrt(14) + 8) + 16966386
5908428420060160681141 6*14^(1/4)*sqrt(-2*sqrt(14) + 8))*log(-14^(1/4)*sqrt(
1/2)*sqrt(-2*x + 3)*sqrt(sqrt(14) + 4) - 2*x + sqrt(14) + 3) + 1/2411259743
14794470715561049883908600016802938 88*(385912796294138623132486146144809805
*(2*x - 3)^37*sqrt(-2*x + 3) + 4994416662656937088431754278268478521 5*(2*x
- 3)^36*sqrt(-2*x + 3) + 31571043251901908187904170157686721002 51*(2*x - 3)
^35*sqrt(-2*x + 3) + 12986266353974282972701016844877225753779 3*(2*x - 3)^3
4*sqrt(-2*x + 3) + 390705603293305902738518568283243321795620 0*(2*x - 3)^33
*sqrt(-2*x + 3) + 916263423082400629136594690316769413288476 88*(2*x - 3)^32
*sqrt(-2*x + 3) + 1743051839783716654458570168808933730174627004*(2*x - 3)^
31*sqrt(-2*x + 3) + 27638544507622729125093621837291437830917462708*(2*x -
3)^30*sqrt(-2*x + 3) + 372498510070445411629537388290851713705080145718*(2*
x - 3)^29*sqrt(-2*x + 3) + 43299535169306873423374720142726663639696515873 1
4*(2*x - 3)^28*sqrt(-2*x + 3) + 43899444560112308623605331157143896725828 41
5934650*(2*x - 3)^27*sqrt(-2*x + 3) + 39160935736577378031615157845797245 36
48367489837454*(2*x - 3)^26*sqrt(-2*x + 3) + 30950317017588495750406269373 9
9363198202032753884252*(2*x - 3)^25*sqrt(-2*x + 3) + 217907196222246813794 1
6567825910093368668334676797780*(2*x - 3)^24*sqrt(-2*x + 3) + 1372614029241
9872579406216311605327709910696804658609 2*(2*x - 3)^23*sqrt(-2*x + 3) + 776
1711830556525453848713885531739126913521685009518 76*(2*x - 3)^22*sqrt(-2*x
+ 3) + 3950095526376994607880784338655934603802167995433166405*(2*x - 3)^21
*sqrt(-2*x + 3) + 18125803816832861597832766873339882118924015183338007655*
(2*x - 3)^20*sqrt(-2*x + 3) + 750834145086940501444266399776850855400388047
54309758915*(2*x - 3)^19*sqrt(-2*x + 3) + 2809326520733483435177609063127 1
8952356113432842758203 45*(2*x - 3)^18*sqrt(-2*x + 3) + 9494495163668915148 6
6641779309597536478490489987954462580*(2*x - 3)^17*sqrt(-2*x + 3) + 2896666
953760570249650513456393600983703549509654469117900*(2*x - 3)^16*sqrt(-2*x
+ 3) + 7968283692957988567650795129108295704483768260379820818752*(2*x - 3)
^15*sqrt(-2*x + 3) + 1972749457881227765860600971283186162692222652326643 57
34336*(2*x - 3)^14*sqrt(-2*x + 3) + 438441033794236958424800303207601166664
911722780351728704 00*(2*x - 3)^13*sqrt(-2*x + 3) + 871807724494537191124097
1585086169883527900473451529716249 6*(2*x - 3)^12*sqrt(-2*x + 3) + 154427451
62007985140301203501394992336719781489523913152972 8*(2*x - 3)^11*sqrt(-2*x
+ 3) + 24235172594435925434767071300022545098836579524787722007296 0*(2*x -
3)^10*sqrt(-2*x + 3) + 334646091432259174045261099248092390902126268663782 6
08549888*(2*x - 3)^9*sqrt(-2*x + 3) + 4030345196682619867089916868903813178
414701262373378021232 64*(2*x - 3)^8*sqrt(-2*x +...

**Mupad** [B]
time = 0.97, size = 2500, normalized size = 2.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(41/2)*(x + 2*x^2 + 1)^20),x)
```

```
[Out] ((64356352*(2*x - 3)^2)/38073 - (5767168*x)/1443 - (7517962240*(2*x - 3)^3)
/5444439 + (1357449428992*(2*x - 3)^4)/1181443263 - (34130408095744*(2*x -
3)^5)/34261854627 + (1965832636456960*(2*x - 3)^6)/2158496841501 - (9552588
571922432*(2*x - 3)^7)/10792484207505 + (69571472879183872*(2*x - 3)^8)/755
47389452535 - (5204838729946112*(2*x - 3)^9)/5036492630169 + (3250820527817
55904*(2*x - 3)^10)/257635969158645 - (461538785202937088*(2*x - 3)^11)/272
428464995505 + (17726678744562203264*(2*x - 3)^12)/6992330601551295 - (1432
471149647610304*(2*x - 3)^13)/332968123883395 + (2043463601243388704*(2*x -
 3)^14)/241114848329355 - (9697276847734397681 6*(2*x - 3)^15)/4840844262612
435 + (108338706701225459276 56*(2*x - 3)^16)/181389282075536535 - (44340157
049832305729324*(2*x - 3)^17)/181389282075536535 + (691509778132186261807 28
2*(2*x - 3)^18)/423241658176251915 - (13577358331537082239703407*(2*x - 3)^
19)/423241658176251915 + (50949594385895993964075303946506726149 81*(2*x - 3
)^20)/2035946169792430536236256 46080 + (4747534027372414822574988626088463 2
526403*(2*x - 3)^21)/20359461697924305362362564 6080 + (547362406727667345 86
81762307546007523414 99*(2*x - 3)^22)/518240843219891409223774371840 + (1363
217399168846741803250531443496167647559*(2*x - 3)^23)/438511482724523500112
424468480 + (40035704814224807138997531075224002020138 8159*(2*x - 3)^24)/59
856817391897457765345939947520 + (16780353218671061875177851217431652450855
3291*(2*x - 3)^25)/14964204347974364441336484986880 + (51108771060698315319
12486309354814419579941 5067*(2*x - 3)^26)/335198177394625763485937263706112
0 + (3939870831872067350820038893812216643460900 53*(2*x - 3)^27)/2280259710
16752220058460723609 60 + (19450991951225490080928815092282978539677719 5281*
(2*x - 3)^28)/11688962083504898418996786631802880 + (399049412174158598 4967
811280954752578787283887 1677*(2*x - 3)^29)/288717363462570990949220629805 53
11360 + (2982029082982520685654165296540313515739996589 54519*(2*x - 3)^30)/
29783475388770481171603812337833738240 + (172783707178371264987902065794355
725529860298244 11*(2*x - 3)^31)/270758867170640737923671021253033 9840 + (13
6589909140623157483229616961110867609087469195457*(2*x - 3)^32)/37906241403
889703309313942975424757760 + (121244485102821322131210667779257215167467 72
830847*(2*x - 3)^33)/668933671833347705458481346625142 7840 + (5268103225464
003924284598756770514565895682824129*(2*x - 3)^34)/645866993494266750097844
0588104826880 + (617176100928620262663130059020160395102873271141 3*(2*x -
3)^35)/187301428113337357528374777055039979520 + (236279120368028123256783 3
34911177879141056326577387*(2*x - 3)^36)/197226802936437285876032243873341 2
433920 + (10069182899664488193697417735778758301092236673480 01*(2*x - 3)^37
)/256394843817368471638841917035343616409 60 + (834315251412234134041251370 6
```

```
840068954518337251868859*(2*x - 3)^38)/717905562688631720588757367698962125
946880 + (6690164526112934310361705118130577674249448391954923*(2*x - 3)^39
)/215371668806589516176627210309688637784064 0 + (3055810652078339448493840 1
5433140408874881230574613*(2*x - 3)^40)/407459913958412598171997424910221 74
7159040 + (7318673393711958469818414571768081348141036133 09*(2*x - 3)^41)/4
477581472070468111780191482529909309440 + (98156536112115492322904146290693
53244130713267641*(2*x - 3)^42)/305594935468809448628998068682666310369280
+ (1119980151725948167868728714185939040414513261 7*(2*x - 3)^43)/1971580228
831028700832245604404298776576 + (136564747272427838170630719416707180545 54
74221*(2*x - 3)^44)/151422305976050793637586261153308213248 0 + (40305011659
049347862186541819167541945005655 01*(2*x - 3)^45)/314621902416905537891429 2
3150742928752640 + (14280096284455564909886672955220549158424336 31*(2*x - 3
)^46)/880941326767335506096001848220802005073 92 + (160089053926633694221849
846408842457682603621*(2*x - 3)^47)/880941326767335506096001848220802005073
92 + (210019981409672089241582716785447580068246038 9*(2*x - 3)^48)/11716519
646005562231076824581336666667483136 + (73152102949146076476299357236586179
9703833*(2*x - 3)^49)/474353022105488349436308687503508771962 88 + (14527825
011424680881745287944067060548347 7*(2*x - 3)^50)/126959191210586587643247 32
5184762641907712 + (30541762468911990334017682046220545959 17*(2*x - 3)^51)/
423197304035288625477491083949208806359 04 + (4322624121559696023583903787 64
52347793*(2*x - 3)^52)/113937735701808476090093753370940832481 28 + (1675721
416942126574649271075659765 75*(2*x - 3)^53)/103579759728916796445539775791 7
643931648 + (935756145095208333386444273642906999*(2*x - 3)^54)/17401399634
458021802850682333016418051686 4 + (325001551972552320039960952878829 9*(2*x
- 3)^55)/248591423349400311469295461900234543595 52 + (359910711199433658030
176367535945*(2*x - 3)^56)/17401399634458021802850682333016418051686 4 + (92
702775478147674620804762050 5*(2*x - 3)^57)/58004665448193406009502274443388
060172288 + 79953920/10101)/(5976303958948914397184*(3 - 2*x)^(39/2) - 5677
4887610014686773248*(3 - 2*x)^(41/2) + 26359769247506818859008 0*(3 - 2*x)^(
43/2) - 796876101097706139353088*(3 - 2*x)^(45/...
```

# 3.50 $\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx$

**Optimal**. Leaf size=378

$$-\frac{3450497-2004270x}{123480000\left(3-2x+x^2\right)^{9/2}}-\frac{4878869-2578034x}{411600000\left(3-2x+x^2\right)^{7/2}}-\frac{30316369-15043110x}{6860000000\left(3-2x+x^2\right)^{5/2}}-\frac{63043297-2\ldots}{41160000000\,(3\ldots}$$

```
[Out] 1/123480000*(-3450497+2004270*x)/(x^2-2*x+3)^(9/2)+1/411600000*(-4878869+25
78034*x)/(x^2-2*x+3)^(7/2)+1/6860000000*(-30316369+15043110*x)/(x^2-2*x+3)^
(5/2)+1/41160000000*(-63043297+29625922*x)/(x^2-2*x+3)^(3/2)+1/280*(-1+10*x
)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^4+1/1050*(28+67*x)/(x^2-2*x+3)^(9/2)/(2*x^2
+x+1)^3+1/117600*(5485+8878*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)^2+3/343000*(88
22+8233*x)/(x^2-2*x+3)^(9/2)/(2*x^2+x+1)-31/411600000000*(7434109-3088870*x
)/(x^2-2*x+3)^(1/2)-1/9604000000000*arctanh(1/7*(308108167+x*(932587773-620
347970*2^(1/2))-312239803*2^(1/2))*35^(1/2)/(-151363871237318045+1103204757
41093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-105954709866122631 50+772243330
1876572160*2^(1/2))^(1/2)+1/9604000000000*arctan(1/7*(308108167+312239803*2
^(1/2)+x*(932587773+620347970*2^(1/2)))*35^(1/2)/(151363871237318045+110320
475741093888*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(105954709866122631 50+772243
3301876572160*2^(1/2))^(1/2)
```

**Rubi** [A]

time = 0.49, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {988, 1074, 1049, 1043, 212, 210}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] -1/123480000*(3450497 - 2004270*x)/(3 - 2*x + x^2)^(9/2) - (4878869 - 25780
34*x)/(411600000*(3 - 2*x + x^2)^(7/2)) - (30316369 - 15043110*x)/(68600000
00*(3 - 2*x + x^2)^(5/2)) - (63043297 - 29625922*x)/(41160000000*(3 - 2*x +
 x^2)^(3/2)) - (31*(7434109 - 3088870*x))/(411600000000*Sqrt[3 - 2*x + x^2]
) - (1 - 10*x)/(280*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^4) + (28 + 67*x)/
(1050*(3 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^3) + (5485 + 8878*x)/(117600*(3
 - 2*x + x^2)^(9/2)*(1 + x + 2*x^2)^2) + (3*(8822 + 8233*x))/(343000*(3 - 2
*x + x^2)^(9/2)*(1 + x + 2*x^2)) + (Sqrt[(151363871237318045 + 110320475741
```

```
093888*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(151363871237318045 + 110320475741093
888*Sqrt[2]))]*(308108167 + 312239803*Sqrt[2] + (932587773 + 620347970*Sqrt
[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000 - (Sqrt[(-151363871237318045 +
110320475741093888*Sqrt[2])/70]*ArcTanh[(Sqrt[5/(7*(-151363871237318045 + 1
10320475741093888*Sqrt[2]))]*(308108167 - 312239803*Sqrt[2] + (932587773 -
620347970*Sqrt[2])*x))/Sqrt[3 - 2*x + x^2]])/137200000000
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(3-2x+x^2)^{11/2}(1+x+2x^2)^5}\,dx = -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} - \frac{\int \frac{-1235+1335x-800x^2}{(3-2x+x^2)^{11/2}(1+x+2x^2)^4}\,d}{1400}$$

$$= -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1}$$

$$= -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1}$$

$$= -\frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2)^4} + \frac{28+67x}{1050(3-2x+x^2)^{9/2}(1}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{1-10x}{280(3-2x+x^2)^{9/2}(1+x+2x^2}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{}{28}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{}{68}$$

$$= -\frac{3450497-2004270x}{123480000(3-2x+x^2)^{9/2}} - \frac{4878869-2578034x}{411600000(3-2x+x^2)^{7/2}} - \frac{}{68}$$

**Mathematica** [**C**] Result contains higher order function than in optimal. Order 9 vs. order

3 in optimal.
time = 2.67, size = 733, normalized size = 1.94

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x + x^2)^(11/2)*(1 + x + 2*x^2)^5),x]
```

```
[Out] ((-53205422447 + 261702502714*x - 266966654968*x^2 + 1002897791524*x^3 - 14
09335257371*x^4 + 2503427226914*x^5 - 3359813871472*x^6 + 4591320676952*x^7
 - 5134334619701*x^8 + 5380603084494*x^9 - 4915797913008*x^10 + 39996561325
32*x^11 - 2679143870481*x^12 + 1459208021718*x^13 - 606785954952*x^14 + 188
603773872*x^15 - 38639385552*x^16 + 4596238560*x^17)/((3 - 2*x + x^2)^(9/2)
*(1 + x + 2*x^2)^4) - 49392*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (-
6014*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 10727*Log[-x + Sqrt[3 - 2*x + x^2
] - #1]*#1 + 3229*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#
1^2 + 4*#1^3) & ] - 56448*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (737
81*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 60407*Log[-x + Sqrt[3 - 2*x + x^2]
- #1]*#1 + 13104*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#1
^2 + 4*#1^3) & ] - 504*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 & , (275935
046*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 208696097*Log[-x + Sqrt[3 - 2*x +
x^2] - #1]*#1 + 50007219*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^2)/(7 - 10*#
1 - 3*#1^2 + 4*#1^3) & ] + 1440*RootSum[14 + 7*#1 - 5*#1^2 - #1^3 + #1^4 &
, (3276009822*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 2447831621*Log[-x + Sqrt
[3 - 2*x + x^2] - #1]*#1 + 590084719*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1^
2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ] - 18*RootSum[14 + 7*#1 - 5*#1^2 - #1^3
 + #1^4 & , (254137663854*Log[-x + Sqrt[3 - 2*x + x^2] - #1] - 189631531133
*Log[-x + Sqrt[3 - 2*x + x^2] - #1]*#1 + 45801521671*Log[-x + Sqrt[3 - 2*x
+ x^2] - #1]*#1^2)/(7 - 10*#1 - 3*#1^2 + 4*#1^3) & ])/1234800000000
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 21027 vs. 2(298) = 596.
time = 1.52, size = 21028, normalized size = 55.63

| method | result |
|---|---|

| | |
|---|---|
| risch | $\frac{4596238560x^{17}-38639385552x^{16}+188603773872x^{15}-606785954952x^{14}+1459208021718x^{13}-2679143870481x^{12}+3999656132532x}{}$ |
| trager | Expression too large to display |
| default | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1873 vs. 2(298) = 596.
time = 1.13, size = 1873, normalized size = 4.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/7108652444723216758028075295024000000000*(26460206086876512301981559146074412800*x^18 - 211681648695012098415852473168595302400*x^17 + 1018717934344745723626290027123864892800*x^16 - 3214915039555496244690759436248041155200*x^15 + 7688343631118056605744516779381246569200*x^14 - 1398091139115337718
```

```
7559506313807067863200*x^13 + 2097798213825178490941475486049712039800 0*x^1
2 - 2571270526492225082945058010019781063840 0*x^11 + 2875728272779347952619
7333249442997761200*x^10 - 27283780001330543747380735174495978898400*x^9 +
25562212842803140665733059982554512415600*x^8 - 180458605512497813899514233
37622749529600*x^7 + 15206349685551845663545027271759639106000*x^6 - 726663
4096608462190931685680490685615200*x^5 - 3602042876982878244*33780221308347
3608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 + 616*x^16
- 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 17389*x^10
- 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*x^4 - 396*
x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 22064095148
2187776)*arctan(1/96439362234996391967746783551420544110289515227048435311 8
304*sqrt(205487899)*(12071210867722009415131100925112940*sqrt(4167294734812
9)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(205487899)*(5*337802213083473608
^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876
830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(201502465*sq
rt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) +
 24142421735444018830262201850225 88*sqrt(41672947348129)*sqrt(7)*(125*sqrt(
2) + 172))*sqrt(16448360508869491318497096 8*x^2 + sqrt(205487899)*(33780221
3083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*sqrt(2) - 4
2834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(89801606*x -
 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*sqrt(2) + 22
0640951482187776) - 411209012721737282962427 42*sqrt(x^2 - 2*x + 3)*(4*x + 1
) - 12336270381652118488872822 6*x + 20560450636086864148121371 0*sqrt(2) + 2
87846308905216098073699194) + 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10*x - 19) +
9*x - 29) + 1/11491792746071352963204808080707518 88*sqrt(205487899)*(5*3378
02213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349) - 573381349
*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(sqrt(2)*(20
1502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213083473608^(
3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*3378022130834736
08^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*x + 3))*sqr
t(151363871237318045*sqrt(2) + 220640951482187776) - 1/476*sqrt(x^2 - 2*x +
 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) + 172)) + 1/
476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) - 3602042876982878244*3378
02213083473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(2)*(16*x^18 - 128*x^17 +
 616*x^16 - 1944*x^15 + 4649*x^14 - 8454*x^13 + 12685*x^12 - 15548*x^11 + 1
7389*x^10 - 16498*x^9 + 15457*x^8 - 10912*x^7 + 9195*x^6 - 4394*x^5 + 4407*
x^4 - 396*x^3 + 1647*x^2 + 162*x + 243)*sqrt(151363871237318045*sqrt(2) + 2
20640951482187776)*arctan(-1/964393622349963919677467835514205441102895152 2
70484353118304*sqrt(205487899)*(12071210867722009415131100925112940*sqrt(41
672947348129)*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) - sqrt(205487899)*(5*3378022
13083473608^(3/4)*sqrt(41672947348129)*sqrt(35)*(534678000*sqrt(2) - 573381
349) + 2876830586*337802213083473608^(1/4)*sqrt(41672947348129)*sqrt(35)*(2
01502465*sqrt(2) + 108532744))*sqrt(151363871237318045*sqrt(2) + 2206409514
82187776) + 24142421735444018830262201850225 88*sqrt(41672947348129)*sqrt(7)
*(125*sqrt(2) + 172))*sqrt(16448360508869491318497096 8*x^2 - sqrt(205487899
```

```
)*(337802213083473608^(1/4)*sqrt(35)*sqrt(7)*sqrt(x^2 - 2*x + 3)*(89801606*
sqrt(2) - 42834985) - 337802213083473608^(1/4)*sqrt(35)*sqrt(7)*(sqrt(2)*(8
9801606*x - 132636591) - 42834985*x + 222438197))*sqrt(151363871237318045*s
qrt(2) + 220640951482187776) - 41120901272173728296242742*sqrt(x^2 - 2*x +
3)*(4*x + 1) - 123362703816521184888728226*x + 205604506360868641481213710*
sqrt(2) + 287846308905216098073699194) - 5/476*sqrt(7)*sqrt(2)*(sqrt(2)*(10
*x - 19) + 9*x - 29) + 1/11491792746071352963204808080707518888*sqrt(2054878
99)*(5*337802213083473608^(3/4)*sqrt(35)*(sqrt(2)*(534678000*x + 38703349)
- 573381349*x - 495974651) + 2876830586*337802213083473608^(1/4)*sqrt(35)*(
sqrt(2)*(201502465*x - 310035209) + 108532744*x - 511537674) - (5*337802213
083473608^(3/4)*sqrt(35)*(534678000*sqrt(2) - 573381349) + 2876830586*33780
2213083473608^(1/4)*sqrt(35)*(201502465*sqrt(2) + 108532744))*sqrt(x^2 - 2*
x + 3))*sqrt(151363871237318045*sqrt(2) + 220640951482187776) + 1/476*sqrt(
x^2 - 2*x + 3)*(5*sqrt(7)*sqrt(2)*(10*sqrt(2) + 9) + sqrt(7)*(125*sqrt(2) +
 172)) - 1/476*sqrt(7)*(25*sqrt(2)*(5*x - 1) + 172*x - 82)) + 9*33780221308
3473608^(1/4)*sqrt(205487899)*sqrt(35)*sqrt(7)*(3530255223715004416*x^18 -
28242041789720035328*x^17 + 135914826113027670016*x^16 - 428926009681373036
544*x^15 + 1025759783440690970624*x^14 - 186529...
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2-2x+3)^{\frac{11}{2}}\,(2x^2+x+1)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(11/2)/(2*x**2+x+1)**5,x)
```

```
[Out] Integral(1/((x**2 - 2*x + 3)**(11/2)*(2*x**2 + x + 1)**5), x)
```

**Giac** [C] Result contains complex when optimal does not.
time = 34.88, size = 2509, normalized size = 6.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(11/2)/(2*x^2+x+1)^5,x, algorithm="giac")
```

```
[Out] 1/19208000000000*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*l
og(3136*(2474301535988301451359142266380914651280177790712513272161012361812
934855593003307850244701148645840266042846227 00*sqrt(7)*sqrt(2)*sqrt(77224
33301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2)
- 151363871237318045)^2 + 144334256265984251329283298872220021324677037791 5
632742093923877724211999095918596245976075670043406821858326965750*sqrt(7)*
(110320475741093888*sqrt(2) - 151363871237318045)^3 + 2886685125319685026 58
566597744440042649354075583126548418784775544842399819183719249195215134008
```

6813643716653931500*sqrt(2)*(110320475741093888*sqrt(2) - 15136387123731804
5)^3 + 20619179466569178761326185553174288760668148255937610601341769681774
4571299416942320853725095720486688836903852250*sqrt(7722433301876572160*sqr
t(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318O
45)^3 - 1049138541122969625735220807296230414367334042656225923210840932892
59251415027575686933144355006438004151420024881229481000*sqrt(7)*sqrt(2)*(1
10320475741093888*sqrt(2) - 151363871237318045)^2 - 10491385411229696257352
208072962304143673340426562259232108409328925925141502757568693314435500643
800415142002488122948100*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 1059547
0986612263150)*(110320475741093888*sqrt(2) - 151363871237318045)^2 - 209827
708224593925147044161459246082873466808531245184642168186578518502830055151
37386628871001287600830284004976245896200*sqrt(2)*sqrt(7722433301876572160*
sqrt(2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 1513638712373
18045)^2 - 1223994964643464563357757608512268816761889716432263577079314421
70802459984198838301422001747507511004843323362361434394500*(11032047574109
3888*sqrt(2) - 151363871237318045)^3 + 724503996256958016683144110303659048
527226579099834730629600402015843465282345034154512602754539838193840458016
13291562600472200700*sqrt(7)*sqrt(2)*sqrt(7722433301876572160*sqrt(2) - 105
95470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045) + 6339
409967248382662472854538412359683674181009662984901543522122388718802293934
79427155097805557896986440201529880194683449362574125*sqrt(7)*(110320475741
093888*sqrt(2) - 151363871237318045)^2 + 1267881993449676532082187318351088
361508312490869111205095341459358991548431951565218821053012281909331172952
868319415989224917443750*sqrt(2)*(110320475741093888*sqrt(2) - 151363871237
318045)^2 + 126788199344967653538125603300215696332050217937699740680224518
030900924464663471430273419380295298646483255439984720301061537907975*sqrt(
7722433301876572160*sqrt(2) - 10595470986612263150)*(110320475741093888*sqr
t(2) - 151363871237318045)^2 - 21264528220686985082784156444749824400286141
322404339508073021441899306648522000622634130083780322911432744853704088500
185306368048860002880*sqrt(7)*sqrt(2)*(110320475741093888*sqrt(2) - 1513638
71237318045) - 354408803678116418845746257791902502469699160932438355128709
627960208195159816911628734559796548815662417323599879097891861163466576081
8080*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)*(1103
20475741093888*sqrt(2) - 151363871237318045) - 7088176073562328374916566029
889536476564991204751185360922127716049447858985212646112610216597117728735
334198568887900615291459333783931760*sqrt(2)*sqrt(7722433301876572160*sqrt(
2) - 10595470986612263150)*(110320475741093888*sqrt(2) - 151363871237318045
) - 62021540643670373420405091577929394267973114014403593823713156536262791
860899401705831007948594881042345033377486353135919026969012248710900*(1103
20475741093888*sqrt(2) - 151363871237318045)^2 + 32899118880973852710380417
819638428242533122398715783525424459521797423442868355883980260844124987688
79366524177397299680463206375263895127106986700*sqrt(7)*sqrt(2)*sqrt(772243
3301876572160*sqrt(2) - 10595470986612263150) + 575734580417042425329673296
871504559468760385065959616403976214712902997702584706455877479060569410162
15068838775424495834136105150442722719839966690*sqrt(7)*(110320475741093888

```
*sqrt(2) - 151363871237318045) + 115146916083408484993484259748605110296131
995582491774181190935500623259371276266538520099917051160383062730419039816
825148360727220332176154717624880*sqrt(2)*(110320475741093888*sqrt(2) - 151
363871237318045) + 19191152680568080928847909459028587443440938343434315974
646074253344559590752575102628626876766807523046716011108365196910710221166
349350116972946360*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
*(110320475741093888*sqrt(2) - 151363871237318045) - 4219083468134411357204
411829911575030935087599894152681923125521189411158761429808471398986722740
73729706037742726523339689588219472775435857599079592655320*sqrt(7)*sqrt(2)
 - 210954173406720568670297857045559135287403242478401319837374548657411401
676856640700014506909797905726420309651469993318236639490549988690120558780
166576108*sqrt(7)*sqrt(7722433301876572160*sqrt(2) - 10595470986612263150)
- 4219083468134411371380763977036231747195786343738197611526042097212408620
85099587557106516194625617695932035781267647104...
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2+x+1\right)^5\,\left(x^2-2\,x+3\right)^{11/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^5*(x^2 - 2*x + 3)^(11/2)), x)
```

# 3.51 $\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx$

**Optimal**. Leaf size=638

$$\frac{37358055634422583-14024622879097678x}{1840124479200000000\,(3-2x+x^2)^{19/2}}+\frac{476849951294984711-125181871472148210x}{10427372048800000000\,(3-2x+x^2)^{17/2}}+\frac{78517583754}{156410580}$$

```
[Out] 1/1840124479200000000*(37358055634422583-14024622879097678*x)/(x^2-2*x+3)^(
19/2)+1/10427372048800000000*(476849951294984711-125181871472148210*x)/(x^
2-2*x+3)^(17/2)+1/156410580732000000000*(7851758375483333511+194216499620
4584234*x)/(x^2-2*x+3)^(15/2)-11/4066675099032000000000*(7502325106308201
089-7813986379726516886*x)/(x^2-2*x+3)^(13/2)-3/11470109253680000000000*(
690532685152963590111-4484073619501828606*x)/(x^2-2*x+3)^(11/2)+1/938463484
392000000000000*(-83851943938029533565 7+46618939055585364387 0*x)/(x^2-2*x+
3)^(9/2)+1/3128211614640000000000000*(-1117646664729238460189+568839749685
437871554*x)/(x^2-2*x+3)^(7/2)+1/5213686024400000000000000*(-655140551156
5449301689+3127298559983309301910*x)/(x^2-2*x+3)^(5/2)+1/104273720488000000
0000000000*(-41790397823984598508 19+18869934455896524026 94*x)/(x^2-2*x+3)^(
3/2)+1/630*(-1+10*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^9+1/88200*(887+2218*x)/
(x^2-2*x+3)^(19/2)/(2*x^2+x+1)^8+1/1080450*(14453+29371*x)/(x^2-2*x+3)^(19/
2)/(2*x^2+x+1)^7+1/605052000*(8837931+17459234*x)/(x^2-2*x+3)^(19/2)/(2*x^2
+x+1)^6+1/26471025000*(447940041+813432205*x)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1
)^5+1/29647548000000*(592729157441+911061463974*x)/(x^2-2*x+3)^(19/2)/(2*x^
2+x+1)^4+1/12353145000000*(277010166219+310705340015*x)/(x^2-2*x+3)^(19/2)/
(2*x^2+x+1)^3+1/276710448000000*(5488221294349+1384103301166*x)/(x^2-2*x+3)
^(19/2)/(2*x^2+x+1)^2+1/2421216420000000*(-37857197792117-146548895467025*x
)/(x^2-2*x+3)^(19/2)/(2*x^2+x+1)+1/10427372048800000000000000*(-12105495
874518671061833+5117656435043679338190*x)/(x^2-2*x+3)^(1/2)-1/2259801992000
000000000000000*arctanh(1/7*(27294458952324838174 9+x*(65682664229653860143 1
-4648856159098934915 90*2^(1/2))-19194102638664510984 1*2^(1/2))*35^(1/2)/(-8
104222592127468960547894479780085484640 5+573059225230017071260263638786665 0
0308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(-567295581448922827238352613584 6
059839248350+401141457661011949882184547150665502162944 0*2^(1/2))^(1/2)+1/2
259801992000000000000000000*arctan(1/7*(27294458952324838174 9+1919410263866
45109841*2^(1/2)+x*(65682664229653860143 1+4648856159098934915 90*2^(1/2)))*3
5^(1/2)/(8104222592127468960547894479780085484640 5+573059225230017071260263
638786665 00308992*2^(1/2))^(1/2)/(x^2-2*x+3)^(1/2))*(567295581448922827238 3
526135846059839248350+401141457661011949882184547150665502162944 0*2^(1/2))^
(1/2)
```

**Rubi** [A]

time = 0.87, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {988, 1074, 1049, 1043, 212, 210}

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] (37358055634422583 - 14024622879097678*x)/(1840124479200000000*(3 - 2*x + x
^2)^(19/2)) + (476849951294984711 - 125181871472148210*x)/(1042737204880000
00000*(3 - 2*x + x^2)^(17/2)) + (7851758375483333511 + 1942164996204584234*
x)/(1564105807320000000000*(3 - 2*x + x^2)^(15/2)) - (11*(7502325106308201
089 - 781398637972651688б*x))/(40666750990320000000000*(3 - 2*x + x^2)^(13
/2)) - (3*(690532685152963590l1 - 44840736195018286006*x))/(114701092536800
0000000000*(3 - 2*x + x^2)^(11/2)) - (838519439380295335657 - 4661893905558
53643870*x)/(93846348439200000000000000*(3 - 2*x + x^2)^(9/2)) - (1117646664
729238460189 - 568839749685437871554*x)/(3128211614640000000000000*(3 - 2*
x + x^2)^(7/2)) - (65514055115654493016889 - 31272985599833093019l0*x)/(5213
68602440000000000000000*(3 - 2*x + x^2)^(5/2)) - (41790397823984598508l9 -
1886993445589652402694*x)/(104273720488000000000000000*(3 - 2*x + x^2)^(3/
2)) - (12105495874518671061833 - 5117656435043679338190*x)/(104273720488000
0000000000000*Sqrt[3 - 2*x + x^2]) - (1 - 10*x)/(630*(3 - 2*x + x^2)^(19/2
)*(1 + x + 2*x^2)^9) + (887 + 2218*x)/(88200*(3 - 2*x + x^2)^(19/2)*(1 + x
 + 2*x^2)^8) + (14453 + 29371*x)/(1080450*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*
x^2)^7) + (8837931 + 17459234*x)/(605052000*(3 - 2*x + x^2)^(19/2)*(1 + x +
 2*x^2)^6) + (447940041 + 813432205*x)/(26471025000*(3 - 2*x + x^2)^(19/2)*
(1 + x + 2*x^2)^5) + (592729157441 + 911061463974*x)/(29647548000000*(3 - 2
*x + x^2)^(19/2)*(1 + x + 2*x^2)^4) + (277010166219 + 310705340015*x)/(1235
3145000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^3) + (5488221294349 + 138
4103301166*x)/(276710448000000*(3 - 2*x + x^2)^(19/2)*(1 + x + 2*x^2)^2) -
(37857197792117 + 146548895467025*x)/(2421216420000000*(3 - 2*x + x^2)^(19/
2)*(1 + x + 2*x^2)) + (Sqrt[(81042225921274689605478944797800854846405 + 57
30592252300170712602636387866650030899 2*Sqrt[2])/70]*ArcTan[(Sqrt[5/(7*(810
42225921274689605478944797800854846405 + 573059225230017071260263638786665О
0308992*Sqrt[2]))]*(272944589523248381749 + 191941026386645109841*Sqrt[2] +
 (656826642296538601431 + 464885615909893491590*Sqrt[2])*x))/Sqrt[3 - 2*x +
 x^2]])/32282885600000000000000000 - (Sqrt[(-810422259212746896054789447978
00854846405 + 5730592252300170712602636387866650030899 2*Sqrt[2])/70]*ArcTan
h[(Sqrt[5/(7*(-81042225921274689605478944797800854846405 + 573059225230017О
71260263638786665003089 92*Sqrt[2]))]*(272944589523248381749 - 1919410263866
45109841*Sqrt[2] + (656826642296538601431 - 464885615909893491590*Sqrt[2])*
x))/Sqrt[3 - 2*x + x^2]])/32282885600000000000000000
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1043

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 1049

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
```

```
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{1}{(3-2x+x^2)^{21/2}(1+x+2x^2)^{10}}\,dx = -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} - \frac{\int \frac{-2960+3060x-1800x^2}{(3-2x+x^2)^{21/2}(1+x+2x^2)^9}\,d}{3150}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= -\frac{1-10x}{630(3-2x+x^2)^{19/2}(1+x+2x^2)^9} + \frac{887+2218x}{88200(3-2x+x^2)^{19/2}(}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} - \frac{1-}{630(3-2x+x^2)^{1}}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} + \frac{476849951294984}{104273720488000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} + \frac{476849951294984}{104273720488000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} + \frac{476849951294984}{104273720488000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} + \frac{476849951294984}{104273720488000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} + \frac{476849951294984}{104273720488000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} + \frac{476849951294984}{104273720488000}$$

$$= \frac{37358055634422583-14024622879097678x}{1840124479200000000(3-2x+x^2)^{19/2}} + \frac{476849951294984}{104273720488000}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 16.85, size = 1431, normalized size = 2.24

Too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 2*x + x^2)^(21/2)*(1 + x + 2*x^2)^10),x]
```

```
[Out] Sqrt[3 - 2*x + x^2]*((1 - x)/(11875000000*(3 - 2*x + x^2)^10) + (265 - 113*
x)/(403750000000*(3 - 2*x + x^2)^9) + (82361 - 4841*x)/(60562500000000*(3 -
 2*x + x^2)^8) + (1062937 + 1642511*x)/(1574625000000000*(3 - 2*x + x^2)^7)
 + (7*(-678331 + 833371*x))/(2220625000000000*(3 - 2*x + x^2)^6) + (7*(-731
61291 + 43964675*x))/(9084375000000000*(3 - 2*x + x^2)^5) + (-1340879383 +
 430593031*x)/(1816875000000000*(3 - 2*x + x^2)^4) - (11*(1626125723 + 11
2950205*x))/(3028125000000000*(3 - 2*x + x^2)^3) - (11*(3311570647 + 152
86717673*x))/(36337500000000000*(3 - 2*x + x^2)^2) - (11*(-411521923277
+ 484788625685*x))/(363375000000000000*(3 - 2*x + x^2)) + (251943 + 2217
70*x)/(630000000000*(1 + x + 2*x^2)^9) - (73*(-888423 + 1604678*x))/(88200
000000000*(1 + x + 2*x^2)^8) + (-2596903794 - 4965311863*x)/(1080450000000
0000*(1 + x + 2*x^2)^7) + (-539608494637 - 334647150510*x)/(121010400000000
0000*(1 + x + 2*x^2)^6) + (-40800462989458 + 56711874696335*x)/(26471025000
0000000000*(1 + x + 2*x^2)^5) + (42018358198215561 + 129196597088670934*x)/
(2964754800000000000000*(1 + x + 2*x^2)^4) + (62819559864314747 + 1696303
89653846945*x)/(3705943500000000000000*(1 + x + 2*x^2)^3) + (108242210919
6374795 + 4797048907791526114*x)/(8301313440000000000000*(1 + x + 2*x^2)
^2) + (65571203144429922747 + 367152793968978953465*x)/(363182463000000000
00000000*(1 + x + 2*x^2))) + ((232442807954946745795*I + 216341778311919248
41*Sqrt[7])*ArcTan[(-135063738860435016899586558948733259113515 + (18863089
4626466690216855285995045889396405*I)*Sqrt[7] - 150624136187268800855926877
6761430483700000*x - (105711500937472192718115651350352447938680*I)*Sqrt[7]
*x + 491153540508443587025809789813541985707360*x^2 - (46076406417713999339
9975100872663310399420*I)*Sqrt[7]*x^2 - 18008498514724668919944874526497767
8818020*x^3 + (197868296377913870863837680953446009396860*I)*Sqrt[7]*x^3 -
176004816500761880926774485599831047775825*x^4 - (2073428332284595771635570
43035558264835165*I)*Sqrt[7]*x^4 + (186244248199755548159585682605666126004
224*I)*Sqrt[10*(-5 + I*Sqrt[7])]*Sqrt[3 - 2*x + x^2] + (1146118450460034142
52052727757333000617984*I)*Sqrt[10*(-5 + I*Sqrt[7])]*x*Sqrt[3 - 2*x + x^2]
+ (300856093245758962411638410362999126622208*I)*Sqrt[10*(-5 + I*Sqrt[7])]*
x^2*Sqrt[3 - 2*x + x^2] - (14326480630750426781506590969666250772480*I)*Sq
rt[10*(-5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])/(2368773290838836979864678
493023884746594823*I + 423642940259238735473942663180025956729505*Sqrt[7] +
 (1890613486065620301760074218556745311646936*I)*x + 6150574559311228258394
328777942059796320*Sqrt[7]*x + (2511300259855822962340893027852239157667820
*I)*x^2 - 2027867550801106189867763431094227596320*Sqrt[7]*x^2 - (313421774
6230760357128318797499380812303788*I)*x^3 + 6343043160272004327919286696836
```

```
9397935660*Sqrt[7]*x^3 + (9447490648866264673283853691904607036697*I)*x^4
 + 1638131776510726478946291722103075063483 5*Sqrt[7]*x^4)])/(16141442800000
000000000000*Sqrt[70*(-5 + I*Sqrt[7])]) - ((I/16141442800000000000000000)*(
-2324428079549467457 95*I + 2163417783119192484 1*Sqrt[7])*ArcTan[(35*(436249
4290663946676585186218212607628595*I + 121040840074068210135412189480007416
20843*Sqrt[7] - (4091903159661733270719609450078323740 5000*I)*x + 175730701
694606521668409393655487422752*Sqrt[7]*x + (264872883292651275777339658533 6
4310310620*I)*x^2 - 579390728800316054247932408884065027 52*Sqrt[7]*x^2 - (1
52388941497528256839248140210078630706 20*I)*x^3 + 1812298045792001236548367
627667697083876*Sqrt[7]*x^3 - (7958372719599758089132442037656199635 95*I)*x
^4 + 4680376504316361368417976348865928752 81*Sqrt[7]*x^4))/(135063738860435
0168995865589487332591135 15 + (1886308946264666902168552859950458893964 05*I
)*Sqrt[7] + 150624136187268800855926877676143048 3700000*x - (10571150093747
2192718115651350352447938680*I)*Sqrt[7]*x - 4911535405084435870258097898135
41985707360*x^2 - (4607640641771399933999751008726633103994 20*I)*Sqrt[7]*x^
2 + 1800849851472466891994487452649776788180 20*x^3 + (19786829637791387086 3
8376809534460093968 60*I)*Sqrt[7]*x^3 + 1760048165007618809267744855998310 47
775825*x^4 - (2073428332284595771635570430355582648351 65*I)*Sqrt[7]*x^4 - (
1432648063075042678150659096966662507724 8*I)*Sqrt[70*(5 + I*Sqrt[7])]*Sqrt[
3 - 2*x + x^2] - (14326480630750426781506590969666625077248*I)*Sqrt[70*(5 +
 I*Sqrt[7])]*x^2*Sqrt[3 - 2*x + x^2] + (2865296126150085356301318193933325 0
154496*I)*Sqrt[70*(5 + I*Sqrt[7])]*x^3*Sqrt[3 - 2*x + x^2])])/Sqrt[70*(5 +
I*Sqrt[7])] - ((-232442807954946745795*I + 216341778311919248 41*Sqrt[7])*Lo
g[(-I + Sqrt[7] - (4*I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/(32282885600000 0
000000000*Sqrt[70*(5 + I*Sqrt[7])]) + ((I/32282885600000000000000000)*(2324
42807954946745795*I + 21634177831191924841*Sqrt[7])*Log[(-I + Sqrt[7] - (4*
I)*x)^2*(I + Sqrt[7] + (4*I)*x)^2])/Sqrt[70*(-5 + I*Sqrt[7])] - ((I/3228288
5600000000000000000)*(232442807954946745795*I + 21634177831191924841*Sqrt[7
])*Log[(1 + x + 2*x^2)*(-13*I + 15*Sqrt[7] + (22*I)*x - 10*Sqrt[7]*x + (9*I
)*x^2 + 5*Sqrt[7]*x^2 + I*Sqrt[70*(-5 + I*Sqrt[...
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 86792 vs. $2(518) = 1036$.

time = 4.55, size = 86793, normalized size = 136.04

| method | result |
|---|---|

| | |
|---|---|
| risch | $\frac{3372249001933422237824271360x^{37}-5350220539964003139479614 7712x^{36}+469149394082989701729494575872x^{35}-28474992209}{}$ |
| trager | Expression too large to display |
| default | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + x + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2775 vs. 2(518) = 1036.
time = 1.29, size = 2775, normalized size = 4.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="fricas")
```

```
[Out] 1/1434466423992337676564216536036272863645522031860857110126813332618112898
720320000000000000000000*(360459607762722366280837179749720551111906601728533
58396135728761934386631817942748278579200*x^38 - 5587123920322196677352976
286120668542234552326792270551401037958099829927931781125983179776OO*x^37 +
```

48121357636323435898491763496587693573439531330759233458841197897182406153
47695356895190323200*x^36 - 28710607758300836474268681367065241896063360827
677699962522107958880738952242991399003888332800*x^35 + 1315091821471322213
079849345669386715662902248081338714385016894218223678278587868742508613888
00*x^34 - 48615412586213217241775435956082154303807288316753404802358233229
5256693402493082467454965081600*x^33 + 150125118590038017914558770715112989
4284646412544039883821859865409233818038611634065993336166400*x^32 - 396012
076807241950819334539073291504431090617269457971847700920669696894988016137
7086262197766400*x^31 + 909142000002142860704234021157216421398787616081889
7418150894645435329015717575267031369642428000*x^30 - 184247649298721582706
989900442437618212098383039369354048250008452972140026730578162474910272628
00*x^29 + 33413073756673638925333625169011170445598811516221975115590200411
293389434416479555356860509015600*x^28 - 5481653256044929545971751700338269
9673242410936114304344629842103656622934490247108012261346586400*x^27 + 822
459830940635186677366276046635475885728402385815973257367014937498803836507
49401133206999014400*x^26 - 11372284806763969440259273586264909409387404544
3618754295078471234595964240139128161766283626302000*x^25 + 146086574413322
248286514192550522624098477614094095488624493581512454991258074867544318895
241990800*x^24 - 1750270940810010216829737529974120232517363052261271442728
11232619626419165679723993392477178363200*x^23 + 19688729160578415943345565
4443374481739030277196290989156609388218395099469530751149958413044135200*x
^22 - 208068683375682167383215047521697995267539026087882795784482813901791
360434798005710722616487282000*x^21 + 2081714449184784825196181653920157303
47012009814583465001141378703189206795143605224483243158516400*x^20 - 19622
755618454040835316742234157685550832000179582185155831117699557408106901596
9836642878534431200*x^19 + 176534941677723459681422280024952573032106299529
482816321219585323399086976471958310981405494523200*x^18 - 1491362557380113
805569548293989292587370076152040747303305658872207307833829238226195713407
37358000*x^17 + 12189081448358772438901196169673375625310538365442623433615
0913799569962877883235263704480534144400*x^16 - 919831860532221296355370692
78588580392985745730700928388526309371776740142438834607398588992195200*x^1
5 + 69317814132471559316390137037592557060398996838342232414889371690271398
098098738643314402130954400*x^14 - 4574307084113250024797073972709329687876
5897323708593659902862883667237249390700654758574610918000*x^13 + 329969655
216763949298031215090491433294517890491697894556446151291991903089176733485
18481311574800*x^12 - 17770083757788737971933739892049927033484890029804651
938270182161740937851280707834822272274354400*x^11 + 1354422526745145970196
036923825637435189936268397849855148372985225665526414709333739259622802880
0*x^10 - 481375973272848865172866855106995818624092546697867179982576756873
2599092879797201593187475517200*x^9 + 5091181133639025216832620106123280320
347641869015804163342220634415255665812683873707564839486000*x^8 - 46421311
850305640075834899457188406077339946202653776901797199608409580382714283736
3184426478400*x^7 + 1771233883264782126042267141811413849986971398265032235
916172889879027134542752439323372429279200*x^6 + 23911503454316320991841103
2521665649750496447867853609069487786445410804754998849116452338787600*x^5

```
+ 79817891129994413353362937273464455099835468*1264938752804265123815574105
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3)**(21/2)/(2*x**2+x+1)**10,x)
[Out] Timed out
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3)^(21/2)/(2*x^2+x+1)^10,x, algorithm="giac")
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(2\,x^2+x+1\right)}^{10}\,{\left(x^2-2\,x+3\right)}^{21/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)),x)
[Out] int(1/((x + 2*x^2 + 1)^10*(x^2 - 2*x + 3)^(21/2)), x)
```

## 3.52 $\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx$

**Optimal**. Leaf size=66

$$-\sqrt{2}\sqrt{a+\sqrt{1+a^2}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}\,(-a+x)}{\sqrt{(-a+x)\left(1+x^2\right)}}\right)$$

```
[Out] -arctan((-a+x)*2^(1/2)*(-a+(a^2+1)^(1/2))^(1/2)/((-a+x)*(x^2+1))^(1/2))*2^(
1/2)*(a+(a^2+1)^(1/2))^(1/2)
```

**Rubi** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.75, antiderivative size = 204, normalized size of antiderivative = 3.09, number of steps used = 9, number of rules used = 8, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6851, 6874, 733, 430, 946, 174, 552, 551}

$$\frac{2i\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\text{EllipticF}\left(\text{ArcSin}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right),\frac{2}{1-ia}\right)}{\sqrt{-\left(\left(x^2+1\right)(a-x)\right)}}+\frac{4\sqrt{a^2+1}\sqrt{x^2+1}\sqrt{\frac{a-x}{a+i}}\,\Pi\left(\frac{2}{1-i\left(a-\sqrt{a^2+1}\right)};\text{ArcSin}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\left(1-i\left(a-\sqrt{a^2+1}\right)\right)\sqrt{-\left(\left(x^2+1\right)(a-x)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*(1 + x
^2)]),x]
```

```
[Out] ((2*I)*Sqrt[(a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[1 - I*x]/S
qrt[2]], 2/(1 - I*a)])/Sqrt[-((a - x)*(1 + x^2))] + (4*Sqrt[1 + a^2]*Sqrt[(
a - x)/(I + a)]*Sqrt[1 + x^2]*EllipticPi[2/(1 - I*(a - Sqrt[1 + a^2])), Arc
Sin[Sqrt[1 - I*x]/Sqrt[2]], 2/(1 - I*a)])/((1 - I*(a - Sqrt[1 + a^2]))*Sqrt
[-((a - x)*(1 + x^2))])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-a-\sqrt{1+a^2}+x}{\left(-a+\sqrt{1+a^2}+x\right)\sqrt{(-a+x)\left(1+x^2\right)}}\,dx = \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{-a-\sqrt{1+a^2}+x}{\sqrt{-a+x}\,\left(-a+\sqrt{1+a^2}+x\right)\sqrt{}}}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int\left(\frac{1}{\sqrt{-a+x}\,\sqrt{1+x^2}} - \frac{}{\sqrt{-a}}\right.}{\sqrt{(-a+x)\left(1+x^2\right.}}$$

$$= \frac{\left(\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-a+x}\,\sqrt{1+x^2}}\,dx}{\sqrt{(-a+x)\left(1+x^2\right)}} - \frac{\left(2\sqrt{}\right.}{}$$

$$= -\frac{\left(2\sqrt{1+a^2}\,\sqrt{-a+x}\,\sqrt{1+x^2}\right)\int \frac{}{\sqrt{1-ix}\,\sqrt{1+ix}}}{\sqrt{(-a+x)\left(1+x^2\right)}}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(\right.}{}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{\left(\right.}{}$$

$$= \frac{2i\sqrt{\frac{a-x}{i+a}}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-ix}}{\sqrt{2}}\right)|\frac{2}{1-ia}\right)}{\sqrt{-(a-x)\left(1+x^2\right)}} + \frac{4}{}$$

**<u>Mathematica</u> [A]**
time = 1.28, size = 99, normalized size = 1.50

$$-\frac{\sqrt{2}\,\sqrt{-a+x}\,\sqrt{1+x^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{-a+\sqrt{1+a^2}}\,\sqrt{-a+x}}{\sqrt{1+x^2}}\right)}{\sqrt{-a+\sqrt{1+a^2}}\,\sqrt{(-a+x)\left(1+x^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a - Sqrt[1 + a^2] + x)/((-a + Sqrt[1 + a^2] + x)*Sqrt[(-a + x)*
(1 + x^2)]),x]
```

```
[Out] -((Sqrt[2]*Sqrt[-a + x]*Sqrt[1 + x^2]*ArcTan[(Sqrt[2]*Sqrt[-a + Sqrt[1 + a^
2]]*Sqrt[-a + x])/Sqrt[1 + x^2]])/(Sqrt[-a + Sqrt[1 + a^2]]*Sqrt[(-a + x)*(
1 + x^2)]))
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.17, size = 787, normalized size = 11.92

| method | result |
|---|---|
| default | $\frac{2i\sqrt{-i(x+i)}\,\sqrt{\frac{-a+x}{-i-a}}\,\sqrt{i(x-i)}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,\sqrt{-i(x+i)}}{2},\sqrt{2}\,\sqrt{-\frac{i}{-i-a}}\right)}{\sqrt{-a\,x^2+x^3-a+x}} - \frac{2\sqrt{a^2+1}\,\sqrt{-}}{}$ |
| elliptic | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),x,meth
od=_RETURNVERBOSE)
```

```
[Out] 2*I*(-I*(x+I))^(1/2)*((-a+x)/(-I-a))^(1/2)*(I*(x-I))^(1/2)/(-a*x^2+x^3-a+x)
^(1/2)*EllipticF(1/2*2^(1/2)*(-I*(x+I))^(1/2),2^(1/2)*(-I/(-I-a))^(1/2))-2*
(a^2+1)^(1/2)*(-(a-x)*(x^2+1)*(a^2+1))^(1/2)*(2*a*x-x^2+1)/(-a+x+(a^2+1)^(1
/2))/((-(a-x)*(x^2+1))^(1/2)*a^2+(-(a-x)*(x^2+1)*(a^2+1))^(1/2)*a-(-(a-x)*(
x^2+1)*(a^2+1))^(1/2)*x+(-(a-x)*(x^2+1))^(1/2))*(-I*(a^2+1)^(1/2)*(1-I*x)^(
1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^2*x^3-a^3+a^2
*x-a*x^2+x^3-a+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+
I))^(1/2),-2*I/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(a^2+1)^(1
/2)*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a^3*x^2+a^
2*x^3-a^3+a^2*x-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/2))*EllipticPi(1/2*2^
(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))
+I*(1-I*x)^(1/2)*(-1/(-I-a)*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a*x^2+x^3-a
+x)^(1/2)/(-I-a-(a^2+1)^(1/2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I
/(-I-a-(a^2+1)^(1/2)),2^(1/2)*(-I/(-I-a))^(1/2))+I*(1-I*x)^(1/2)*(-1/(-I-a)
*a+1/(-I-a)*x)^(1/2)*(1+I*x)^(1/2)/(-a*x^2+x^3-a+x)^(1/2)/(-I-a+(a^2+1)^(1/
2))*EllipticPi(1/2*2^(1/2)*(-I*(x+I))^(1/2),-2*I/(-I-a+(a^2+1)^(1/2)),2^(1/
2)*(-I/(-I-a))^(1/2)))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="maxima")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

**Fricas** [**A**]
time = 1.41, size = 546, normalized size = 8.27

$$\left[\frac{1}{4}\sqrt{-2a-2\sqrt{a^2+1}}\log\left(-\frac{8ax^7+x^8+4(2a^2+15)x^6-8(4a^3+15a)x^5+2(8a^4+80a^2+67)x^4+64a^4-8(20a^3+37a)x^3+4(16a^4+74a^2+15)x^2+48a^2-4\left(ax^6+2(2a^2+3)x^5-(4a^3-a)x^4-8a^3-(4a^3+29a)x^2+20x^3+2(10a^2+3)x-(4ax^5+x^6-(4a^2-15)x^4-16ax^3+(4a^2+15)x^2+8a^2-20ax+1)\sqrt{a^2+1}-5a\right)\sqrt{-ax^2+x^3-a+x}\sqrt{-2a-2\sqrt{a^2+1}}-8(24a^3+13a)x+16(ax^6-x^7+15ax^4-7x^5-(12a^2+7)x^3+4a^3+(4a^3+15a)x^2-(12a^2+1)x+a)\sqrt{a^2+1}+1}{8ax^7-x^8-4(6a^2-1)x^6+8(4a^3-3a)x^5-2(8a^4-24a^2+3)x^4-8(4a^3-3a)x^3-4(6a^2-1)x^2-8ax-1}\right),-\frac{1}{2}\sqrt{2a+2\sqrt{a^2+1}}\arctan\left(-\frac{\sqrt{-ax^2+x^3-a+x}\left(2a^2-2ax-x^2-2\sqrt{a^2+1}(a-x)-1\right)\sqrt{2a+2\sqrt{a^2+1}}}{4(ax^2-x^3+a-x)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(-2*a - 2*sqrt(a^2 + 1))*log(-(8*a*x^7 + x^8 + 4*(2*a^2 + 15)*x^6
- 8*(4*a^3 + 15*a)*x^5 + 2*(8*a^4 + 80*a^2 + 67)*x^4 + 64*a^4 - 8*(20*a^3 +
 37*a)*x^3 + 4*(16*a^4 + 74*a^2 + 15)*x^2 + 48*a^2 - 4*(a*x^6 + 2*(2*a^2 +
3)*x^5 - (4*a^3 - a)*x^4 - 8*a^3 - (4*a^3 + 29*a)*x^2 + 20*x^3 + 2*(10*a^2
+ 3)*x - (4*a*x^5 + x^6 - (4*a^2 - 15)*x^4 - 16*a*x^3 + (4*a^2 + 15)*x^2 +
8*a^2 - 20*a*x + 1)*sqrt(a^2 + 1) - 5*a)*sqrt(-a*x^2 + x^3 - a + x)*sqrt(-2
*a - 2*sqrt(a^2 + 1)) - 8*(24*a^3 + 13*a)*x + 16*(a*x^6 - x^7 + 15*a*x^4 -
7*x^5 - (12*a^2 + 7)*x^3 + 4*a^3 + (4*a^3 + 15*a)*x^2 - (12*a^2 + 1)*x + a)
*sqrt(a^2 + 1) + 1)/(8*a*x^7 - x^8 - 4*(6*a^2 - 1)*x^6 + 8*(4*a^3 - 3*a)*x^
5 - 2*(8*a^4 - 24*a^2 + 3)*x^4 - 8*(4*a^3 - 3*a)*x^3 - 4*(6*a^2 - 1)*x^2 -
8*a*x - 1)), -1/2*sqrt(2*a + 2*sqrt(a^2 + 1))*arctan(-1/4*sqrt(-a*x^2 + x^3
 - a + x)*(2*a^2 - 2*a*x - x^2 - 2*sqrt(a^2 + 1)*(a - x) - 1)*sqrt(2*a + 2*
sqrt(a^2 + 1))/(a*x^2 - x^3 + a - x))]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a**2+1)**(1/2))/(-a+x+(a**2+1)**(1/2))/((-a+x)*(x**2+1))**
(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x-(a^2+1)^(1/2))/(-a+x+(a^2+1)^(1/2))/((-a+x)*(x^2+1))^(1/2),
x, algorithm="giac")
```

```
[Out] integrate((a - x + sqrt(a^2 + 1))/(sqrt(-(x^2 + 1)*(a - x))*(a - x - sqrt(a
^2 + 1))), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{a-x+\sqrt{a^2+1}}{\sqrt{-\left(x^2+1\right)\,\left(a-x\right)}\,\left(x-a+\sqrt{a^2+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))),x)
```

```
[Out] int(-(a - x + (a^2 + 1)^(1/2))/((-(x^2 + 1)*(a - x))^(1/2)*(x - a + (a^2 +
1)^(1/2))), x)
```

## 3.53 $\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

**Optimal**. Leaf size=198

$$\frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}+\frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{a\tanh^{-1}}{}$$

```
[Out] -1/12*a*arctanh(x)*2^(1/3)+1/4*a*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1
/3)-1/8*b*ln(x^2+3)*2^(1/3)+3/8*b*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/12*a
*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)+1/12*a*arctan((1-2^(1/3)*(-x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)+1/4*b*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2)
)*3^(1/2)*2^(1/3)
```

**Rubi** [A]
time = 0.06, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 402, 455, 57, 631, 210, 31}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{a\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{a\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{b\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3b\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (a*ArcTan[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) + (Sqrt[3]*b*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (a*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2
)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[x])/(6*2^(2/3)) + (a*ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))])/(2*2^(2/3)) - (b*Log[3 + x^2])/(4*2^(2/3
)) + (3*b*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = a\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx + b\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}(x)}{6\,2^{2/3}} + \frac{a\tanh^{-1}}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} + \frac{a\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right.}{x}\right.}{2\,2^{2/3}\sqrt{3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.17, size = 145, normalized size = 0.73

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right) - \frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, x^2, -1/3*x^2])/6 - (9*a*x*AppellF1[1/2, 1/3,
 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)*(-9*AppellF1[1/2, 1/3,
1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] -
 AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(-x^2+1)^{\frac{1}{3}}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx}{\sqrt[3]{-\left(x - 1\right)\left(x + 1\right)}\left(x^{2} + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/3)/(x**2+3),x)
[Out] Integral((a + b*x)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,x}{{\left(1 - x^{2}\right)}^{1/3}\,\left(x^{2} + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int((a + b*x)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

## 3.54 $\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

**Optimal**. Leaf size=198

$$-\frac{a\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\ \sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2}\ \sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}\left(\right.}{}$$

```
[Out] -1/12*a*arctan(x)*2^(1/3)+1/4*a*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)
+1/8*b*ln(-x^2+3)*2^(1/3)-3/8*b*ln(2^(2/3)-(x^2+1)^(1/3))*2^(1/3)-1/12*a*ar
ctanh(1/x*3^(1/2))*2^(1/3)*3^(1/2)-1/12*a*arctanh((1-2^(1/3)*(x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)-1/4*b*arctan(1/3*(1+2^(1/3)*(x^2+1)^(1/3))*3^(1
/2))*3^(1/2)*2^(1/3)
```

**Rubi** [A]
time = 0.05, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 401, 455, 57, 631, 210, 31}

$$\frac{a\text{ArcTan}\left(\frac{x}{\sqrt[3]{2}\ \sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{a\text{ArcTan}(x)}{6\ 2^{2/3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\ \sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{\sqrt[3]{2}\ \sqrt[3]{x^2+1}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}+\frac{b\log\left(3-x^2\right)}{4\ 2^{2/3}}-\frac{3b\log\left(2^{2/3}-\sqrt[3]{x^2+1}\right)}{4\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*(a*ArcTan[x])/2^(2/3) + (a*ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))])/(2
*2^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2^(1/3)*(1 + x^2)^(1/3))/Sqrt[3]])/(2*2^
(2/3)) - (a*ArcTanh[Sqrt[3]/x])/(2*2^(2/3)*Sqrt[3]) - (a*ArcTanh[(Sqrt[3]*(
1 - 2^(1/3)*(1 + x^2)^(1/3)))/x])/(2*2^(2/3)*Sqrt[3]) + (b*Log[3 - x^2])/(4
*2^(2/3)) - (3*b*Log[2^(2/3) - (1 + x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = a\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx + b\int \frac{x}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\right.}{}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\right.}{}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{a\tanh^{-1}\left(\right.}{}$$

$$= -\frac{a\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{a\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} -$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.18, size = 153, normalized size = 0.77

$$\frac{1}{6}bx^2F_1\left(1;\frac{1}{3},1;2;-x^2,\frac{x^2}{3}\right) - \frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/3, 1, 2, -x^2, x^2/3])/6 - (9*a*x*AppellF1[1/2, 1/3, 1
, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3
/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF
1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(-x^2+3)\,(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx-\int \frac{bx}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(a/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x) - Integral(b
*x/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(x^2+1\right)}^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] int(-(a + b*x)/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

## 3.55 $\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx$

**Optimal**. Leaf size=97

$$-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+3x^2}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(x)*2^(1/3)+1/4*ln(6-3*x-3*2^(1/3)*(3*x^2-6*x+4)^(1/3))*2^(1/3)+1/6*
arctan(-1/3*3^(1/2)-1/3*2^(2/3)*(2-x)/(3*x^2-6*x+4)^(1/3)*3^(1/2))*2^(1/3)*
3^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {764}

$$-\frac{\text{ArcTan}\left(\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{3x^2-6x+4}}+\frac{1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-3\sqrt[3]{2}\sqrt[3]{3x^2-6x+4}-3x+6\right)}{2\ 2^{2/3}}-\frac{\log(x)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2^(2/3)*(2 - x))/(Sqrt[3]*(4 - 6*x + 3*x^2)^(1/3))]/(
2^(2/3)*Sqrt[3])) - Log[x]/(2*2^(2/3)) + Log[6 - 3*x - 3*2^(1/3)*(4 - 6*x +
 3*x^2)^(1/3)]/(2*2^(2/3))
```

Rule 764

```
Int[1/(((d_.) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)), x_Sy
mbol] :> With[{q = Rt[3*c*e^2*(2*c*d - b*e), 3]}, Simp[(-Sqrt[3])*c*e*(ArcT
an[1/Sqrt[3] + 2*((c*d - b*e - c*e*x)/(Sqrt[3]*q*(a + b*x + c*x^2)^(1/3)))]
/q^2), x] + (-Simp[3*c*e*(Log[d + e*x]/(2*q^2)), x] + Simp[3*c*e*(Log[c*d -
 b*e - c*e*x - q*(a + b*x + c*x^2)^(1/3)]/(2*q^2)), x])] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[2*c*d - b*e, 0] && EqQ[c^2*d^2 - b*c*d*e + b^2*e^2 - 3*a*
c*e^2, 0] && PosQ[c*e^2*(2*c*d - b*e)]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{4-6x+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2^{2/3}(2-x)}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2\ 2^{2/3}}+\frac{\log\left(6-3x-3\sqrt[3]{2}\sqrt[3]{4-6x+\right.}{2\ 2^{2/3}}$$

**Mathematica** [A]
time = 0.21, size = 168, normalized size = 1.73

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{2\ 2^{2/3}-2^{2/3}x+\sqrt[3]{4-6x+3x^2}}{\sqrt{3}\sqrt[3]{4-6x+3x^2}}\right)-2\log\left(-22^{2/3}+2^{2/3}x+2\sqrt[3]{4-6x+3x^2}\right)+\log\left(-4\sqrt[3]{2}+4\sqrt[3]{2}x-\sqrt[3]{2}x^2+2^{2/3}(-2+x)\sqrt[3]{4-6x+3x^2}-2(4-6x+3x^2)^{2/3}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 - 6*x + 3*x^2)^(1/3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(2*2^(2/3) - 2^(2/3)*x + (4 - 6*x + 3*x^2)^(1/3))/(S
qrt[3]*(4 - 6*x + 3*x^2)^(1/3))] - 2*Log[-2*2^(2/3) + 2^(2/3)*x + 2*(4 - 6*
x + 3*x^2)^(1/3)] + Log[-4*2^(1/3) + 4*2^(1/3)*x - 2^(1/3)*x^2 + 2^(2/3)*(-
2 + x)*(4 - 6*x + 3*x^2)^(1/3) - 2*(4 - 6*x + 3*x^2)^(2/3)])/2^(2/3)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 8.73, size = 1592, normalized size = 16.41

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 1592 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-6*x+4)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln((96*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)^2*RootOf(_Z^3-2)^2*x-12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z
^2)*RootOf(_Z^3-2)^3*x-96*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)-30*(3*x^2-6*
x+4)^(2/3)*x+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3-240
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2+480*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+96*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)-24*RootOf(_Z^3-2)^
2*(3*x^2-6*x+4)^(1/3)*x^2+96*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x-72*Root
Of(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4
)^(1/3)-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^
3-2)^2*x^2+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^
3-2)^3*x^2-3*RootOf(_Z^3-2)*x^3+30*RootOf(_Z^3-2)*x^2-60*RootOf(_Z^3-2)*x-4
8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x
^2-6*x+4)^(2/3)*x-18*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Ro
otOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x^2+72*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf
(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x-320*RootOf(RootOf(_Z^
3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3+40*RootOf(_Z^3-2)+16*RootOf(RootOf(_Z^3-2)^2
+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-2*RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^3+60*(3*x^2-6*x+4)^(2/3)-
64*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2)/
x^3)+1/3*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln((12*RootOf(
```

```
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x-24*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x+48*RootOf(
_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)+9*(3*x^2-6*x+4)^(2/3)*x-RootOf(RootOf(_Z^3-2)
^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^3+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_
Z^3-2)+4*_Z^2)*x^2-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x
-48*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3
*x^2-6*x+4)^(2/3)+12*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(1/3)*x^2-48*RootOf(_Z^
3-2)^2*(3*x^2-6*x+4)^(1/3)*x+60*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)
+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)-6*RootOf(RootOf(_Z^3-2)^2+2*_Z*
RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+12*RootOf(RootOf(_Z^3-2)^2+2*
_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2+2*RootOf(_Z^3-2)*x^3-48*Root
Of(_Z^3-2)*x^2+96*RootOf(_Z^3-2)*x+24*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_
Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*(3*x^2-6*x+4)^(2/3)*x+15*RootOf(RootOf(_Z^3
-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-6*x+4)^(1/3)*x^2-60
*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*(3*x^2-
6*x+4)^(1/3)*x+32*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+16*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3-64*RootO
f(_Z^3-2)+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z
^3-2)^2*x^3-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)^3*x^3-18*(3*x^2-6*x+4)^(2/3)-8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z
^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2)/x^3)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(74) = 148$.
time = 3.67, size = 171, normalized size = 1.76

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}x^3+2\cdot 4^{\frac{2}{3}}(3x^2-6x+4)^{\frac{2}{3}}(x-2)+4(3x^2-6x+4)^{\frac{1}{3}}(x^2-4x+4)\right)}{6(x^3-12x^2+24x-16)}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{1}{3}}(x-2)+2(3x^2-6x+4)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(3x^2-6x+4)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^2-4x+4)-2(3x^2-6x+4)^{\frac{1}{3}}(x-2)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3)*x^3 + 2*4^(2/3)*(3
*x^2 - 6*x + 4)^(2/3)*(x - 2) + 4*(3*x^2 - 6*x + 4)^(1/3)*(x^2 - 4*x + 4))/
(x^3 - 12*x^2 + 24*x - 16)) + 1/12*4^(2/3)*log((4^(1/3)*(x - 2) + 2*(3*x^2
- 6*x + 4)^(1/3))/x) - 1/24*4^(2/3)*log((4^(2/3)*(3*x^2 - 6*x + 4)^(2/3) +
4^(1/3)*(x^2 - 4*x + 4) - 2*(3*x^2 - 6*x + 4)^(1/3)*(x - 2))/x^2)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{3x^2-6x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-6*x+4)**(1/3),x)
[Out] Integral(1/(x*(3*x**2 - 6*x + 4)**(1/3)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-6*x+4)^(1/3),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 6*x + 4)^(1/3)*x), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(3\,x^2-6\,x+4\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)),x)
[Out] int(1/(x*(3*x^2 - 6*x + 4)^(1/3)), x)
```

# 3.56 $\int x\sqrt[3]{1-x^3}\,dx$

**Optimal**. Leaf size=73

$$\frac{1}{3}x^2\sqrt[3]{1-x^3}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-x-\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/3*x^2*(-x^3+1)^(1/3)-1/6*ln(-x-(-x^3+1)^(1/3))-1/9*arctan(1/3*(1-2*x/(-x^
3+1)^(1/3))*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {285, 337}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{3}\sqrt[3]{1-x^3}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^(1/3),x]
```

```
[Out] (x^2*(1 - x^3)^(1/3))/3 - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sq
rt[3]) - Log[-x - (1 - x^3)^(1/3)]/6
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x\sqrt[3]{1-x^3}\,dx = \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\int \frac{x}{(1-x^3)^{2/3}}\,dx$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[3]{1}\right.$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) +$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{18}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Su}$$
$$= \frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{18}\log\left(1+\frac{x^2}{(1-x^3)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{9}\text{lo}$$

**Mathematica** [A]
time = 0.15, size = 99, normalized size = 1.36

$$\frac{1}{18}\left(6x^2\sqrt[3]{1-x^3} - 2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2\sqrt[3]{1-x^3}}\right) - 2\log\left(x+\sqrt[3]{1-x^3}\right) + \log\left(x^2 - x\sqrt[3]{1-x^3} + \left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^(1/3),x]
```

```
[Out] (6*x^2*(1 - x^3)^(1/3) - 2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3
))] - 2*Log[x + (1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^
(2/3)])/18
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.00, size = 15, normalized size = 0.21

| method | result |
|---|---|
| meijerg | $\frac{x^2\,\text{hypergeom}\left(\left[-\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}$ |
| risch | $-\frac{x^2(x^3-1)}{3(-x^3+1)^{\frac{2}{3}}} + \frac{(x^3-1)^{\frac{2}{3}}(-\text{signum}(x^3-1))^{\frac{2}{3}}x^2\,\text{hypergeom}\left(\left[\frac{2}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{6\text{signum}(x^3-1)^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}}$ |
| trager | $\frac{x^2(-x^3+1)^{\frac{1}{3}}}{3} - \frac{\ln\left(-2\,\text{RootOf}\left(\_Z^2-\_Z+1\right)^2x^3+3\,\text{RootOf}\left(\_Z^2-\_Z+1\right)(-x^3+1)^{\frac{2}{3}}x-\text{RootOf}\left(\_Z^2-\_Z+1\right)x^3+3x^2(\right.}{9}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*hypergeom([-1/3,2/3],[5/3],x^3)
```

**Maxima** [A]
time = 0.49, size = 105, normalized size = 1.44

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x\left(\frac{x^3-1}{x^3}-1\right)}-\frac{1}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)+\frac{1}{18}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(1/3)/(x*((x^3 - 1)/x^3 - 1)) - 1/9*log((-x^3 + 1)^(1/3)/x + 1) + 1/18*lo
g(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

**Fricas** [A]
time = 2.04, size = 96, normalized size = 1.32

$$\frac{1}{3}\left(-x^3+1\right)^{\frac{1}{3}}x^2-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}}{3\,x}\right)-\frac{1}{9}\log\left(\frac{x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{18}\log\left(\frac{x^2-\left(-x^3+1\right)^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(1/3)*x^2 - 1/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
-x^3 + 1)^(1/3))/x) - 1/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/18*log((x^2 - (
-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

**Sympy** [C] Result contains complex when optimal does not.
time = 0.49, size = 32, normalized size = 0.44

$$\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(1/3),x)
```

```
[Out] x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)*x, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(1-x^3\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1 - x^3)^(1/3),x)
```

```
[Out] int(x*(1 - x^3)^(1/3), x)
```

## 3.57 $\int \frac{\sqrt[3]{1-x^3}}{x} \, dx$

**Optimal**. Leaf size=67

$$\sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] (-x^3+1)^(1/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))-1/3*arctan(1/3*(1+2*(-x^3
+1)^(1/3))*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 52, 59, 632, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \sqrt[3]{1-x^3} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x]/
2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1-x^3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{x}\,dx, x, x^3\right)\\
&= \sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right)\\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x\right.\\
&= \sqrt[3]{1-x^3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-x^3}\right)\\
&= \sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}$$

**Mathematica** [A]
time = 0.05, size = 88, normalized size = 1.31

$$\sqrt[3]{1-x^3} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(-1+\sqrt[3]{1-x^3}\right) - \frac{1}{6}\log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/x,x]
```

```
[Out] (1 - x^3)^(1/3) - ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] + Log[-1
+ (1 - x^3)^(1/3)]/3 - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)]/6
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.66, size = 49, normalized size = 0.73

| method | result |
|---|---|
| meijerg | $-\frac{-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\Gamma\left(\frac{2}{3}\right)x^3\,\mathrm{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,2],x^3\right)}{9\Gamma\left(\frac{2}{3}\right)}$ |
| trager | $(-x^3+1)^{\frac{1}{3}}+\frac{\ln\left(-\frac{211\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-2704\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502\left(-x^3+1\right)^{\frac{2}{3}}\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)+862\ldots}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/GAMMA(2/3)*(-3*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*GAMMA(2/3)+GA
MMA(2/3)*x^3*hypergeom([2/3,1,1],[2,2],x^3))
```

**Maxima** [A]
time = 0.67, size = 71, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(
1/3) - 1)
```

**Fricas** [A]
time = 0.98, size = 73, normalized size = 1.09

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + (-x^3 + 1
)^(1/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3
+ 1)^(1/3) - 1)
```

**Sympy** [C] Result contains complex when optimal does not.
time = 0.47, size = 37, normalized size = 0.55

$$-\frac{xe^{\frac{i\pi}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{1}{x^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/x,x)
```

```
[Out] -x*exp(I*pi/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), x**(-3))/(3*gamma(2/
3))
```

**Giac** [A]
time = 1.86, size = 72, normalized size = 1.07

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\left(-x^3+1\right)^{\frac{1}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + (-x^3 + 1)^(1/3
) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 +
1)^(1/3) - 1))
```

**Mupad** [B]
time = 0.37, size = 83, normalized size = 1.24

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}-\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(3\left(1-x^3\right)^{1/3}+\frac{3}{2}+\frac{\sqrt{3}\,3\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\left(1-x^3\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log(3*(1 - x^3)^(1/3) - (3^(1/2)*3i)/2 + 3/2)*
((3^(1/2)*1i)/6 - 1/6) - log((3^(1/2)*3i)/2 + 3*(1 - x^3)^(1/3) + 3/2)*((3^
(1/2)*1i)/6 + 1/6) + (1 - x^3)^(1/3)
```

## 3.58 $\int \frac{\sqrt[3]{1-x^3}}{1+x} \, dx$

**Optimal**. Leaf size=482

$$\sqrt[3]{1-x^3}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2}{\sqrt[3]{1}}}{\sqrt{}}\right)}{\sqrt{3}}$$

```
[Out] (-x^3+1)^(1/3)-1/3*2^(1/3)*ln(x^3+1)+1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*
2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3
))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)
+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(2^(1/3
)-(-x^3+1)^(1/3))*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/2*ln(-2^(1/3)*x-(-x^3
+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))
*2^(1/3)*3^(1/2)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3
*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*2^(
1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.20, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {2181, 420, 493, 298, 31, 648, 631, 210, 642, 495, 337, 503, 455, 52, 59}

$$\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[3]{1-x^3}-\frac{1}{3}\sqrt[3]{2}\log\left(x^3+1\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x),x]
```

```
[Out] (1 - x^3)^(1/3) + (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]
]/(2^(2/3)*Sqrt[3]) - ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
 (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2
^(1/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - (2^(1/3)*Lo
g[1 + x^3])/3 + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1
+ (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/
(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[
2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/
(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(2/3) - Log[-x - (1 - x^3)^(
1/3)]/2 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx = \int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

**Mathematica [F]**
time = 45.62, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1+x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x),x]
[Out] Integrate[(1 - x^3)^(1/3)/(1 + x), x]
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 12.52, size = 2990, normalized size = 6.20

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2990 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(1+x),x,method=_RETURNVERBOSE)
[Out] -(x^3-1)/(-x^3+1)^(2/3)+(1/2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2
)*ln(-(2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x
^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3
+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+4*R
ootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+2*Roo
tOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x+4*RootOf(Ro
otOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x+5*(x^6-2*x^3+1)
```

^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)+8*R
ootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^2+10*RootOf(RootOf(_Z^3-2)^2+_Z*RootO
f(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*x^2+7*RootOf(_Z^3-2)*x^4
+14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4+8*RootOf(_Z^3-2)^2*
(x^6-2*x^3+1)^(1/3)*x+10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*Ro
otOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*x+13*RootOf(_Z^3-2)*x^3+26*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3+8*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)
+10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x
^3+1)^(1/3)+20*RootOf(_Z^3-2)*x^2+40*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3
-2)+_Z^2)*x^2+13*RootOf(_Z^3-2)*x+26*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3
-2)+_Z^2)*x+8*(x^6-2*x^3+1)^(2/3)+7*RootOf(_Z^3-2)+14*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2))/(x^2+x+1)/(1+x)^2)-1/2*ln(-(2*RootOf(RootOf(_Z^
3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^3+4*RootOf(RootOf(_Z^3-2)
^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+2*RootOf(RootOf(_Z^3-2)^2
+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*
RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*Ro
otOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_
Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x-5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*Ro
otOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-2*RootOf(_Z^3-2)^2*(x^6-2*x^3
+1)^(1/3)*x^2-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^
3-2)*(x^6-2*x^3+1)^(1/3)*x^2-7*RootOf(_Z^3-2)*x^4-14*RootOf(RootOf(_Z^3-2)^
2+_Z*RootOf(_Z^3-2)+_Z^2)*x^4-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x-10*R
ootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)
^(1/3)*x-9*RootOf(_Z^3-2)*x^3-18*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+
_Z^2)*x^3-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-10*RootOf(RootOf(_Z^3-2)^2
+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)-16*RootOf(_Z^3-
2)*x^2-32*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-9*RootOf(_Z^3
-2)*x-18*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x-2*(x^6-2*x^3+1)^
(2/3)-7*RootOf(_Z^3-2)-14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2))/
(x^2+x+1)/(1+x)^2)*RootOf(_Z^3-2)-1/2*ln(-(2*RootOf(RootOf(_Z^3-2)^2+_Z*Roo
tOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^3+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^2*x^3+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z
^3-2)+_Z^2)*RootOf(_Z^3-2)^3*x^2+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2
)+_Z^2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+
_Z^2)*RootOf(_Z^3-2)^3*x+4*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^
2*RootOf(_Z^3-2)^2*x-5*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_
Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^2
-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x
^3+1)^(1/3)*x^2-7*RootOf(_Z^3-2)*x^4-14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_
Z^3-2)+_Z^2)*x^4-2*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x-10*RootOf(RootOf(
_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*x-9*Ro
otOf(_Z^3-2)*x^3-18*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^3-2*R
ootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-10*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z
^3-2)+_Z^2)*RootOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)-16*RootOf(_Z^3-2)*x^2-32*Roo
tOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-9*RootOf(_Z^3-2)*x-18*Root

```
Of(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x-2*(x^6-2*x^3+1)^(2/3)-7*RootO
f(_Z^3-2)-14*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2))/(x^2+x+1)/(1+
x)^2)*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)-1/3*ln((RootOf(RootOf
(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^6-RootOf(RootOf(_Z^
3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3+8*RootOf(RootOf(_Z^3-
2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^6-6*(x^6-2*x^3+1)^(2/3)*Roo
tOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*x^2-10*RootOf
(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^3+16*x^6-12*(x
^6-2*x^3+1)^(1/3)*x^4+2*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(
_Z^3-2)+_Z^2)-24*x^3+12*(x^6-2*x^3+1)^(1/3)*x+8)/(-1+x)/(x^2+x+1))+1/6*ln((
RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^6-Root
Of(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)^2*RootOf(_Z^3-2)^4*x^3+2*RootOf
(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*RootOf(_Z^3-2)^2*x^6-6*RootOf(_Z^
3-2)^2*RootOf(RootOf(_Z^3-2)^2+_Z*RootOf(_Z^3-2)+_Z^2)*(x^6-2*x^3+1)^(1/3)*
x^4+6*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*Root...
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(1+x),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(1/3)/(x + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-x^3\right)}^{1/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 1),x)
[Out] int((1 - x^3)^(1/3)/(x + 1), x)
```

## 3.59 $\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{2}(-1+x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-$$

```
[Out] -1/4*ln(-3*(-1+x)*(x^2-x+1))*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)
+3/4*ln(-2^(1/3)*(-1+x)+(-x^3+1)^(1/3))*2^(1/3)+1/2*ln(x+(-x^3+1)^(1/3))-1/
4*ln(2^(1/3)*x+(-x^3+1)^(1/3))*2^(1/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^
(1/2))*3^(1/2)-1/6*2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*3
^(1/2)+1/2*arctan(1/3*(1+2*2^(1/3)*(-1+x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*
2^(1/3)
```

**Rubi** [A]
time = 0.15, antiderivative size = 408, normalized size of antiderivative = 1.46, number of steps used = 19, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2183, 420, 493, 298, 31, 648, 631, 210, 642, 495, 337, 503}

$$\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left(x^3+1\right)}{3\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}+\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(1/3)*ArcTan[(
1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[1 + x^3]/(3*2^(2
/3)) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3
)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3
)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3)
 + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3
)) + Log[-x - (1 - x^3)^(1/3)]/2 - Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(2
/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx &= \int \left(\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(1+i\sqrt{3}-2x\right)}+\frac{2i\sqrt[3]{1-x^3}}{\sqrt{3}\left(-1+i\sqrt{3}+2x\right)}\right)dx\\ &= \frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{1+i\sqrt{3}-2x}\,dx}{\sqrt{3}}+\frac{(2i)\int \frac{\sqrt[3]{1-x^3}}{-1+i\sqrt{3}+2x}\,dx}{\sqrt{3}}\end{aligned}$$

**Mathematica [F]**
time = 9.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{1-x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(1 - x + x^2), x]
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 7.06, size = 925, normalized size = 3.30

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 925 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^2-x+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(-RootOf(_Z^6+108)^6*x^3+18*RootOf(_Z^6+108)^3*(-x^3+1)^(2/3)*x-18*Ro
otOf(_Z^6+108)^3*x^3+108*x*(-x^3+1)^(2/3)+216*x^2*(-x^3+1)^(1/3)+12*RootOf(
_Z^6+108)^3)-1/6*RootOf(_Z^6+108)*ln(-(RootOf(_Z^6+108)^5*x^4-2*RootOf(_Z^6
+108)^4*(-x^3+1)^(1/3)*x^3+2*RootOf(_Z^6+108)^5*x^3+6*RootOf(_Z^6+108)^4*(-
x^3+1)^(1/3)*x^2-x^2*RootOf(_Z^6+108)^5-2*RootOf(_Z^6+108)^4*(-x^3+1)^(1/3)
*x-2*RootOf(_Z^6+108)^5*x-6*RootOf(_Z^6+108)^2*x^4+36*RootOf(_Z^6+108)*(-x^
3+1)^(1/3)*x^3+RootOf(_Z^6+108)^5-12*RootOf(_Z^6+108)^2*x^3+144*(-x^3+1)^(2
/3)*x^2-108*RootOf(_Z^6+108)*(-x^3+1)^(1/3)*x^2+6*x^2*RootOf(_Z^6+108)^2-14
4*x*(-x^3+1)^(2/3)+36*RootOf(_Z^6+108)*(-x^3+1)^(1/3)*x+12*RootOf(_Z^6+108)
^2*x-6*RootOf(_Z^6+108)^2)/(x^2-x+1)^2)+1/72*ln(-(-3*RootOf(_Z^6+108)^4*(-x
^3+1)^(1/3)*x^3+RootOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x^2+RootOf(_Z^6+108)^4*(-
x^3+1)^(1/3)*x-15*RootOf(_Z^6+108)^2*x^4+6*RootOf(_Z^6+108)^2*x^3+72*(-x^3+
1)^(2/3)*x^2+3*x^2*RootOf(_Z^6+108)^2-36*x*(-x^3+1)^(2/3)+6*RootOf(_Z^6+108
)^2*x-3*RootOf(_Z^6+108)^2)/(x^2-x+1)^2)*RootOf(_Z^6+108)^4+1/12*ln(-(-3*Ro
otOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x^3+RootOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x^2+R
ootOf(_Z^6+108)^4*(-x^3+1)^(1/3)*x-15*RootOf(_Z^6+108)^2*x^4+6*RootOf(_Z^6+
108)^2*x^3+72*(-x^3+1)^(2/3)*x^2+3*x^2*RootOf(_Z^6+108)^2-36*x*(-x^3+1)^(2/
3)+6*RootOf(_Z^6+108)^2*x-3*RootOf(_Z^6+108)^2)/(x^2-x+1)^2)*RootOf(_Z^6+10
8)-1/36*ln(RootOf(_Z^6+108)^6*x^3-36*RootOf(_Z^6+108)^3*(-x^3+1)^(2/3)*x-36
*(-x^3+1)^(1/3)*RootOf(_Z^6+108)^3*x^2+216*x*(-x^3+1)^(2/3)-216*x^2*(-x^3+1
)^(1/3)+12*RootOf(_Z^6+108)^3-324*x^3+216)*RootOf(_Z^6+108)^3-1/6*ln(RootOf
(_Z^6+108)^6*x^3-36*RootOf(_Z^6+108)^3*(-x^3+1)^(2/3)*x-36*(-x^3+1)^(1/3)*R
ootOf(_Z^6+108)^3*x^2+216*x*(-x^3+1)^(2/3)-216*x^2*(-x^3+1)^(1/3)+12*RootOf
(_Z^6+108)^3-324*x^3+216)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 3085 vs. $2(218) = 436$.
time = 5.42, size = 3085, normalized size = 11.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(1/3)*arctan(1/3*(26795748*sqrt(3)*2^(2/3)*(586745*x^11 - 70
6109*x^10 - 191742*x^9 - 43779*x^8 + 396304*x^7 + 323715*x^6 - 462255*x^5 +
 73568*x^4 + 24102*x^3 + 2372*x^2 - 2008*x)*(-x^3 + 1)^(1/3) + 26795748*sqr
t(3)*2^(1/3)*(340975*x^10 + 46080*x^9 - 970873*x^8 + 685704*x^7 - 289743*x^
6 + 397966*x^5 - 203166*x^4 - 21912*x^3 + 29756*x^2 - 4016*x)*(-x^3 + 1)^(2
/3) + 7*sqrt(273426)*2^(1/6)*(6*sqrt(3)*2^(2/3)*(338078915*x^10 - 459916473
*x^9 - 111133574*x^8 + 235674676*x^7 + 297312537*x^6 - 494815414*x^5 + 2448
15194*x^4 - 34383000*x^3 - 8933924*x^2 + 2566224*x)*(-x^3 + 1)^(2/3) + sqrt
(3)*2^(1/3)*(2332175065*x^12 - 3283524318*x^11 + 1882024851*x^10 - 39193009
70*x^9 + 2796090405*x^8 + 610770276*x^7 + 98233512*x^6 + 140867400*x^5 - 11
45424564*x^4 + 430987096*x^3 + 108889824*x^2 - 54987072*x + 4032064) - 6*sq
rt(3)*(493920245*x^11 - 452201839*x^10 - 276972599*x^9 - 661557480*x^8 + 13
75964914*x^7 - 191435014*x^6 - 333786162*x^5 - 47180632*x^4 + 107411572*x^3
 - 13096840*x^2 - 2566224*x)*(-x^3 + 1)^(1/3)) - 3*sqrt(3)*(2247079524645*x
^12 - 5276442179264*x^11 + 3816306322874*x^10 - 3280399521884*x^9 + 6278089
258290*x^8 - 6181108351032*x^7 + 2698150339136*x^6 + 1210170331680*x^5 - 25
58541243960*x^4 + 1136906331664*x^3 - 42652634816*x^2 - 54080708992*x + 515
2977792))/(18230538112975*x^12 - 14115716188440*x^11 - 20854883745366*x^10
+ 1856205891292*x^9 + 11854156958820*x^8 + 23868971173080*x^7 - 27900743059
560*x^6 + 8785124358048*x^5 - 2880050871456*x^4 + 1047429829408*x^3 + 24296
4112512*x^2 - 141331907328*x + 8096384512)) + 1/18*sqrt(3)*2^(1/3)*arctan(-
1/3*(13397874*sqrt(3)*2^(2/3)*(18803*x^11 - 25367*x^10 - 203754*x^9 + 40802
1*x^8 - 139829*x^7 + 7128*x^6 - 233871*x^5 + 225275*x^4 - 47094*x^3 - 10225
*x^2 + 2921*x)*(-x^3 + 1)^(1/3) + 26795748*sqrt(3)*2^(1/3)*(10589*x^10 - 73
935*x^9 + 63883*x^8 + 142959*x^7 - 173613*x^6 - 31588*x^5 + 79410*x^4 - 437
7*x^3 - 13328*x^2 + 2921*x)*(-x^3 + 1)^(2/3) - 7*sqrt(273426)*(6*sqrt(3)*2^
(2/3)*(309683372*x^10 - 328552599*x^9 - 24698630*x^8 - 422031122*x^7 + 7021
64163*x^6 - 95703451*x^5 - 206316094*x^4 + 60985482*x^3 + 11167816*x^2 - 37
33038*x)*(-x^3 + 1)^(2/3) + sqrt(3)*2^(1/3)*(2345654785*x^12 - 2502234618*x
^11 - 252041853*x^10 - 4416416426*x^9 + 6899968311*x^8 - 1680852528*x^7 + 1
576960038*x^6 - 2990585436*x^5 + 642930363*x^4 + 528479914*x^3 - 117963261*
x^2 - 38399466*x + 8532241) - 6*sqrt(3)*(491687266*x^11 - 516958230*x^10 -
69305552*x^9 - 808934094*x^8 + 1418391515*x^7 - 385704187*x^6 - 112721241*x
^5 - 69510422*x^4 + 47121139*x^3 + 11465929*x^2 - 4799203*x)*(-x^3 + 1)^(1/
```

```
3))*sqrt((6*2^(2/3)*(4*x^10 - 27*x^9 + 32*x^8 + 6*x^7 + 12*x^6 - 65*x^5 + 4
8*x^4 - 6*x^3 - 4*x^2 + x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(35*x^12 - 66*x^11 -
201*x^10 + 338*x^9 + 90*x^8 - 90*x^7 - 249*x^6 - 18*x^5 + 306*x^4 - 166*x^3
 + 15*x^2 + 6*x - 1) - 6*(x^11 + 29*x^10 - 93*x^9 + 66*x^8 - 19*x^7 + 87*x^
6 - 99*x^5 + 10*x^4 + 27*x^3 - 11*x^2 + x)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11
 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 + 141*x^6 - 126*x^5 + 90*x^4 - 50*x^
3 + 21*x^2 - 6*x + 1)) - 3*sqrt(3)*(2995162579*x^12 + 315959718008*x^11 - 8
49682072424*x^10 + 177300060912*x^9 - 508006765899*x^8 + 3583876884636*x^7
- 3031033916540*x^6 - 1410763301208*x^5 + 2375077456341*x^4 - 546587071308*
x^3 - 175036021936*x^2 + 63861157012*x - 3114267965))/(367648430113*x^12 -
1408582980384*x^11 - 1269375810828*x^10 + 5714713216048*x^9 - 1087485936795
*x^8 - 126379999188*x^7 - 10319650860540*x^6 + 10854292018608*x^5 - 1383220
291365*x^4 - 1828745373668*x^3 + 426327416076*x^2 + 93479232396*x - 2492267
5961)) - 1/18*sqrt(3)*2^(1/3)*arctan(1/3*(13397874*sqrt(3)*2^(2/3)*(17344*x
^11 - 120304*x^10 + 110610*x^9 + 203214*x^8 - 213415*x^7 - 96387*x^6 + 3010
2*x^5 + 157561*x^4 - 101868*x^3 + 15151*x^2 + 913*x)*(-x^3 + 1)^(1/3) - 267
95748*sqrt(3)*2^(1/3)*(1277*x^10 + 57510*x^9 - 189677*x^8 + 108972*x^7 + 10
2426*x^6 - 47461*x^5 - 82155*x^4 + 56409*x^3 - 7301*x^2 - 913*x)*(-x^3 + 1)
^(2/3) + 7*sqrt(273426)*(6*sqrt(3)*2^(2/3)*(8733539*x^10 - 122586360*x^9 +
269810944*x^8 - 28009538*x^7 - 316185126*x^6 + 161786897*x^5 + 95479640*x^4
 - 80193978*x^3 + 11163982*x^2 + 1166814*x)*(-x^3 + 1)^(2/3) - sqrt(3)*2^(1
/3)*(1971824*x^12 - 78264612*x^11 + 705529692*x^10 - 1556393152*x^9 + 93384
9120*x^8 + 135726408*x^7 - 213906684*x^6 + 446158968*x^5 - 582881445*x^4 +
182390318*x^3 + 31120185*x^2 - 12999294*x - 833569) + 6*sqrt(3)*(12965988*x
^11 - 175265260*x^10 + 270273662*x^9 + 299814882*x^8 - 663644613*x^7 + 7755
3085*x^6 + 286893603*x^5 - 82332150*x^4 - 33723265*x^3 + 10863861*x^2 + 333
245*x)*(-x^3 + 1)^(1/3))*sqrt((6*2^(2/3)*(143*x^10 - 177*x^9 - 2*x^8 - 54*x
^7 + 141*x^6 - 31*x^5 - 18*x^4 - 6*x^3 + 7*x^2 - x)*(-x^3 + 1)^(2/3) + 2^(1
/3)*(1081*x^12 - 1338*x^11 - 15*x^10 - 1130*x^9 + 1962*x^8 - 234*x^7 + 33*x
^6 - 630*x^5 + 234*x^4 + 58*x^3 - 15*x^2 - 6*x + 1) - 6*(227*x^11 - 281*x^1
0 - 3*x^9 - 162*x^8 + 319*x^7 - 51*x^6 - 21*x^5 - 58*x^4 + 33*x^3 - x^2 - x
)*(-x^3 + 1)^(1/3))/(x^12 - 6*x^11 + 21*x^10 - 50*x^9 + 90*x^8 - 126*x^7 +
141*x^6 - 126*x^5 + 90*x^4 - 50*x^3 + 21*x^2 - ...
```

**Sympy** [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**2-x+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x**2 - x + 1), x)
```

**Giac** [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^2-x+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(1/3)/(x^2 - x + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1-x^3)^{1/3}}{x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^2 - x + 1),x)
[Out] int((1 - x^3)^(1/3)/(x^2 - x + 1), x)
```

## 3.60 $\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$

**Optimal**. Leaf size=232

$$\sqrt[3]{1-x^3}+\frac{1}{2}xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-\frac{x^3}{8}\right)-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[6]{3}\tan^{-1}\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-\sqrt[6]{3}\tan^{-}$$

```
[Out] (-x^3+1)^(1/3)+1/2*x*AppellF1(1/3,-1/3,1,4/3,x^3,-1/8*x^3)-3^(1/6)*arctan(2
/9*(-x^3+1)^(1/3)*3^(5/6)+1/3*3^(1/2))+3^(1/6)*arctan(1/3*(1-3^(2/3)*x/(-x^
3+1)^(1/3))*3^(1/2))-1/3*ln(x^3+8)*3^(2/3)+1/2*3^(2/3)*ln(3^(2/3)-(-x^3+1)^
(1/3))-ln(-x-(-x^3+1)^(1/3))+1/2*3^(2/3)*ln(-1/2*3^(2/3)*x-(-x^3+1)^(1/3))-
2/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2181, 440, 495, 337, 503, 455, 52, 59, 631, 210, 31}

$$\frac{1}{2}xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-\frac{x^3}{8}\right)-\frac{2\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[6]{3}\,\text{ArcTan}\left(\frac{1-\frac{3^{2/3}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)-\sqrt[6]{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}}{3\sqrt[6]{3}}+\frac{1}{\sqrt{3}}\right)+\sqrt[3]{1-x^3}-\frac{\log\left(x^3+8\right)}{\sqrt[3]{3}}+\frac{1}{2}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{1-x^3}\right)-\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{1}{2}3^{2/3}\log\left(-\sqrt[3]{1-x^3}-\frac{1}{2}3^{2/3}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] (1 - x^3)^(1/3) + (x*AppellF1[1/3, -1/3, 1, 4/3, x^3, -1/8*x^3])/2 - (2*Arc
Tan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + 3^(1/6)*ArcTan[(1 - (3^
(2/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - 3^(1/6)*ArcTan[1/Sqrt[3] + (2*(1 - x^3
)^(1/3))/(3*3^(1/6))] - Log[8 + x^3]/3^(1/3) + (3^(2/3)*Log[3^(2/3) - (1 -
x^3)^(1/3)])/2 - Log[-x - (1 - x^3)^(1/3)] + (3^(2/3)*Log[-1/2*(3^(2/3)*x)
- (1 - x^3)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx=\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

**Mathematica [F]**
time = 45.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{1-x^3}}{2+x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(1/3)/(2 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(1/3)/(2 + x), x]
```

**Maple [F]**
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-x^{3}+1\right)^{\frac{1}{3}}}{2+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(2+x),x)
```

```
[Out] int((-x^3+1)^(1/3)/(2+x),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-(x-1)(x^2+x+1)}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(2+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/(x + 2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(2+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x + 2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-x^3\right)^{1/3}}{x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x + 2),x)
[Out] int((1 - x^3)^(1/3)/(x + 2), x)
```

# 3.61 $\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$

**Optimal**. Leaf size=168

$$-\frac{x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}\,x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}+2\sqrt[3]{2+x^3}}{3^{5/6}}\right)}{3^{5/6}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\right.}{2}$$

```
[Out] -1/4*x^2*AppellF1(2/3,1,1/3,5/3,x^3,-1/2*x^3)*2^(2/3)+1/3*arctan(1/3*(3^(1/
3)+2*(x^3+2)^(1/3))*3^(1/6))*3^(1/6)+2/3*arctan(1/3*(1+2*3^(1/3)*x/(x^3+2)^
(1/3))*3^(1/2))*3^(1/6)+1/18*ln(-x^3+1)*3^(2/3)+1/6*ln(3^(1/3)-(x^3+2)^(1/3
))*3^(2/3)-1/3*ln(3^(1/3)*x-(x^3+2)^(1/3))*3^(2/3)
```

**Rubi** [A]
time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2183, 384, 524, 455, 57, 631, 210, 31}

$$-\frac{x^2F_1\left(\frac{2}{3};1,\frac{1}{3};\frac{5}{3};x^3,-\frac{x^3}{2}\right)}{2\sqrt[3]{2}}+\frac{2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{3}\,x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{x^3+2}+\sqrt[3]{3}}{3^{5/6}}\right)}{3^{5/6}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\log\left(\sqrt[3]{3}\,x-\sqrt[3]{x^3+2}\right)}{\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
```

```
[Out] -1/2*(x^2*AppellF1[2/3, 1, 1/3, 5/3, x^3, -1/2*x^3])/2^(1/3) + (2*ArcTan[(1
 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]])/3^(5/6) + ArcTan[(3^(1/3) + 2*(
2 + x^3)^(1/3))/3^(5/6)]/3^(5/6) + Log[1 - x^3]/(6*3^(1/3)) + Log[3^(1/3) -
 (2 + x^3)^(1/3)]/(2*3^(1/3)) - Log[3^(1/3)*x - (2 + x^3)^(1/3)]/3^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx = \int \left(\frac{1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}} + \frac{1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\right) dx$$
$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{2+x^3}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{}{\left(1+i\sqrt{3}\right.}$$

**Mathematica** [F]
time = 10.10, size = 0, normalized size = 0.00

$$\int \frac{2+x}{(1+x+x^2)\sqrt[3]{2+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)),x]
[Out] Integrate[(2 + x)/((1 + x + x^2)*(2 + x^3)^(1/3)), x]
```

**Maple** [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{2+x}{(x^2+x+1)(x^3+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
[Out] int((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] integral((x^3 + 2)^(2/3)*(x + 2)/(x^5 + x^4 + x^3 + 2*x^2 + 2*x + 2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2}{\sqrt[3]{x^3+2}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x+1)/(x**3+2)**(1/3),x)
[Out] Integral((x + 2)/((x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+1)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate((x + 2)/((x^3 + 2)^(1/3)*(x^2 + x + 1)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+2}{\left(x^3+2\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)),x)
[Out] int((x + 2)/((x^3 + 2)^(1/3)*(x + x^2 + 1)), x)
```

# 3.62 $\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx$

**Optimal**. Leaf size=25

$$\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

**Rubi** [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1601}

$$\frac{1}{8}\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

Rule 1601

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{3-3x+30x^2+160x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

**Mathematica** [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{8}\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x + 30*x^2 + 160*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4]/8
```

**Maple [A]**
time = 0.01, size = 24, normalized size = 0.96

| method | result | size |
|---|---|---|
| default | $\frac{\ln(320x^4+80x^3-12x^2+24x+9)}{8}$ | 24 |
| norman | $\frac{\ln(320x^4+80x^3-12x^2+24x+9)}{8}$ | 24 |
| risch | $\frac{\ln(320x^4+80x^3-12x^2+24x+9)}{8}$ | 24 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/8*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

**Maxima [A]**
time = 0.29, size = 23, normalized size = 0.92

$$\frac{1}{8}\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="maxima")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

**Fricas [A]**
time = 0.97, size = 23, normalized size = 0.92

$$\frac{1}{8}\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="fricas")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

**Sympy [A]**
time = 0.03, size = 22, normalized size = 0.88

$$\frac{\log\left(320x^4+80x^3-12x^2+24x+9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x**3+30*x**2-3*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9)/8
```

**Giac** [A]
time = 1.23, size = 23, normalized size = 0.92

$$\frac{1}{8} \log \left(320 x^4 + 80 x^3 - 12 x^2 + 24 x + 9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((160*x^3+30*x^2-3*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorith
m="giac")
```

```
[Out] 1/8*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

**Mupad** [B]
time = 0.07, size = 23, normalized size = 0.92

$$\frac{\ln \left(320 x^4 + 80 x^3 - 12 x^2 + 24 x + 9\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^2 - 3*x + 160*x^3 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9)/8
```

# 3.63 $\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} \, dx$

**Optimal**. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}+\frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

```
[Out] -1/22*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2)+1/22*arctan(1/66*(800*x^3-40*
x^2+30*x+57)*11^(1/2))*11^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2115}

$$\frac{\text{ArcTan}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\text{ArcTan}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] -1/2*ArcTan[(7 - 40*x)/(5*Sqrt[11])]/Sqrt[11] + ArcTan[(57 + 30*x - 40*x^2
+ 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])
```

Rule 2115

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rubi steps

$$\int \frac{3+12x+20x^2}{9+24x-12x^2+80x^3+320x^4} \, dx=-\frac{\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}+\frac{\tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)}{2\sqrt{11}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 86, normalized size = 1.46

$$\frac{1}{8}\text{RootSum}\left[9+24\#1-12\#1^2+80\#1^3+320\#1^4\&,\frac{3\log(x-\#1)+12\log(x-\#1)\#1+20\log(x-\#1)\#1^2}{3-3\#1+30\#1^2+160\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 12*x + 20*x^2)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (3*Log[x - #1] + 12*Lo
g[x - #1]*#1 + 20*Log[x - #1]*#1^2)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/8
```

**Maple** [C] Result contains complex when optimal does not.
time = 0.03, size = 62, normalized size = 1.05

| method | result | size |
|---|---|---|
| risch | $\frac{\sqrt{11}\,\arctan\left(\frac{(40x-7)\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\,\arctan\left(-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}\,x^3}{33}\right)}{22}$ | 52 |
| default | $\frac{i\sqrt{11}\,\ln\left(80x^2+\left(10i\sqrt{11}+10\right)x+3i\sqrt{11}-9\right)}{44}-\frac{i\sqrt{11}\,\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)}{44}$ | 62 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNVERBOSE)
```

```
[Out] 1/44*I*11^(1/2)*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)-1/44*I*11^(1
/2)*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="maxi
ma")
```

```
[Out] integrate((20*x^2 + 12*x + 3)/(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9), x)
```

**Fricas** [A]
time = 0.96, size = 43, normalized size = 0.73

$$\frac{1}{22}\sqrt{11}\,\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)+\frac{1}{22}\sqrt{11}\,\arctan\left(\frac{1}{55}\sqrt{11}\left(40\,x-7\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="fric
as")
```

```
[Out] 1/22*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) + 1/22*s
qrt(11)*arctan(1/55*sqrt(11)*(40*x - 7))
```

**Sympy** [A]
time = 0.06, size = 73, normalized size = 1.24

$$\frac{\sqrt{11}\cdot\left(2\operatorname{atan}\left(\frac{8\sqrt{11}\,x}{11}-\frac{7\sqrt{11}}{55}\right)+2\operatorname{atan}\left(\frac{400\sqrt{11}\,x^3}{33}-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right)\right)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x**2+12*x+3)/(320*x**4+80*x**3-12*x**2+24*x+9),x)
```

```
[Out] sqrt(11)*(2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) + 2*atan(400*sqrt(11)*x**
3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22))/44
```

**Giac** [A]
time = 1.29, size = 40, normalized size = 0.68

$$\frac{1}{22}\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}\left(800\,x^3-40\,x^2+30\,x+57\right)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}\left(40\,x-7\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((20*x^2+12*x+3)/(320*x^4+80*x^3-12*x^2+24*x+9),x, algorithm="giac
")
```

```
[Out] 1/22*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arcta
n(-1/55*sqrt(11)*(40*x - 7)))
```

**Mupad** [B]
time = 0.34, size = 53, normalized size = 0.90

$$\frac{\sqrt{11}\,\operatorname{atan}\left(\frac{8\,\sqrt{11}\,x}{11}-\frac{7\,\sqrt{11}}{55}\right)}{22}+\frac{\sqrt{11}\,\operatorname{atan}\left(\frac{400\,\sqrt{11}\,x^3}{33}-\frac{20\,\sqrt{11}\,x^2}{33}+\frac{5\,\sqrt{11}\,x}{11}+\frac{19\,\sqrt{11}}{22}\right)}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 20*x^2 + 3)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9),x)
```

```
[Out] (11^(1/2)*atan((8*11^(1/2)*x)/11 - (7*11^(1/2))/55))/22 + (11^(1/2)*atan((5
*11^(1/2)*x)/11 + (19*11^(1/2))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x
^3)/33))/22
```

# 3.64 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} \, dx$

**Optimal**. Leaf size=78

$$2\sqrt{11} \tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right)-2\sqrt{11} \tan^{-1}\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right)+2\log\left(9+24x-12x^2+80x^3+320x\right.$$

```
[Out] 2*ln(320*x^4+80*x^3-12*x^2+24*x+9)+2*arctan(1/55*(7-40*x)*11^(1/2))*11^(1/2
)-2*arctan(1/66*(800*x^3-40*x^2+30*x+57)*11^(1/2))*11^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2125, 2115}

$$-2\sqrt{11} \text{ArcTan}\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)+2\sqrt{11} \text{ArcTan}\left(\frac{7-40x}{5\sqrt{11}}\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^
4),x]
```

```
[Out] 2*Sqrt[11]*ArcTan[(7 - 40*x)/(5*Sqrt[11])] - 2*Sqrt[11]*ArcTan[(57 + 30*x -
 40*x^2 + 800*x^3)/(6*Sqrt[11])] + 2*Log[9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4]
```

Rule 2115

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C)))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

Rule 2125

```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Dist[1/(n*Coeff[Qn,
x, n]), Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x]
/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Rubi steps

$$\int -\frac{84+576x+400x^2-2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\log\left(9+24x-12x^2+80x^3+320x^4\right) - \frac{\int \frac{168960+675840x+1126400x^2}{9+24x-12x^2+80x^3+320x^4}}{1280}$$
$$= 2\sqrt{11}\tan^{-1}\left(\frac{7-40x}{5\sqrt{11}}\right) - 2\sqrt{11}\tan^{-1}\left(\frac{57+30x-40x^2+800}{6\sqrt{11}}\right.$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 99, normalized size = 1.27

$$\frac{1}{2}\text{RootSum}\left[9+24\#1-12\#1^2+80\#1^3+320\#1^4\&,\frac{-21\log(x-\#1)-144\log(x-\#1)\#1-100\log(x-\#1)\#1^2+640\log(x-\#1)\#1^3}{3-3\#1+30\#1^2+160\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x^4),x]
```

```
[Out] RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 144*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1 + 30*#1^2 + 160*#1^3) & ]/2
```

**Maple** [C] Result contains complex when optimal does not.
time = 0.03, size = 70, normalized size = 0.90

| method | result |
|---|---|
| default | $4\left(\frac{i\sqrt{11}}{4}+\frac{1}{2}\right)\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)+4\left(\frac{1}{2}-\frac{i\sqrt{11}}{4}\right)\ln\left(80x^2+(10i\sqrt{\right.$ |
| risch | $2\ln\left(6400x^4+1600x^3-240x^2+480x+180\right)-2\sqrt{11}\arctan\left(-\frac{20\sqrt{11}\,x^2}{33}+\frac{5\sqrt{11}\,x}{11}+\frac{19\sqrt{11}}{22}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(1/4*I*11^(1/2)+1/2)*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)+4*(1/2-1/4*I*11^(1/2))*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="maxima")
```

```
[Out] 4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9), x)
```

**Fricas** [A]
time = 0.76, size = 66, normalized size = 0.85

$$-2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)-2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}\left(40x-7\right)\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="fricas")
```

```
[Out] -2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(1
1)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*
x + 9)
```

**Sympy** [A]
time = 0.07, size = 100, normalized size = 1.28

$$\sqrt{11}\left(-2\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)-2\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right)+2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\frac{9}{320}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),x
)
```

```
[Out] sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x*
*3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(x*
*4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

**Giac** [A]
time = 1.66, size = 64, normalized size = 0.82

$$-2\sqrt{11}\left(\arctan\left(\frac{1}{66}\sqrt{11}\left(800x^3-40x^2+30x+57\right)\right)-\arctan\left(-\frac{1}{55}\sqrt{11}\left(40x-7\right)\right)\right)+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, alg
orithm="giac")
```

```
[Out] -2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arctan(
-1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

**Mupad** [B]
time = 0.09, size = 76, normalized size = 0.97

$$2\ln\left(320x^4+80x^3-12x^2+24x+9\right)-2\sqrt{11}\operatorname{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right)-2\sqrt{11}\operatorname{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
9),x)
```

```
[Out] 2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*x
)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2))
/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

## 3.65 $\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx$

**Optimal**. Leaf size=49

$$\frac{1}{2}\tan^{-1}\left(\frac{x(1+x^2)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

```
[Out] 1/2*arctan(x*(x^2+1)/(-x^4+1)^(1/2))+1/2*arctanh(x*(-x^2+1)/(-x^4+1)^(1/2))
```

**Rubi** [A]
time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{414\}$

$$\frac{1}{2}\text{ArcTan}\left(\frac{x(x^2+1)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/(1 + x^4),x]
```

```
[Out] ArcTan[(x*(1 + x^2))/Sqrt[1 - x^4]]/2 + ArcTanh[(x*(1 - x^2))/Sqrt[1 - x^4]
]/2
```

Rule 414

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[(-a)*b, 4]}, Simp[(a/(2*c*q))*ArcTan[q*x*((a + q^2*x^2)/(a*Sqrt[a + b*x
^4]))], x] + Simp[(a/(2*c*q))*ArcTanh[q*x*((a - q^2*x^2)/(a*Sqrt[a + b*x^4]
))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{1+x^4} \, dx = \frac{1}{2}\tan^{-1}\left(\frac{x(1+x^2)}{\sqrt{1-x^4}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{x(1-x^2)}{\sqrt{1-x^4}}\right)$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.13, size = 57, normalized size = 1.16

$$\left(\frac{1}{4}-\frac{i}{4}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{1-x^4}}\right)-\left(\frac{1}{4}+\frac{i}{4}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1-x^4}}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^4]/(1 + x^4),x]

[Out] (1/4 - I/4)*ArcTan[((1 + I)*x)/Sqrt[1 - x^4]] - (1/4 + I/4)*ArcTan[((1/2 + I/2)*Sqrt[1 - x^4])/x]

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.42, size = 113, normalized size = 2.31

| method | result |
|---|---|
| default | $\frac{\left(-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\sqrt{2}\,\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}\right)\sqrt{2}}{2}$ |
| elliptic | $\frac{\left(-\frac{\arctan\left(1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\arctan\left(-1+\frac{\sqrt{-x^4+1}}{x}\right)\sqrt{2}}{4}-\frac{\sqrt{2}\,\ln\left(\frac{1+\frac{-x^4+1}{2x^2}-\frac{\sqrt{-x^4+1}}{x}}{1+\frac{-x^4+1}{2x^2}+\frac{\sqrt{-x^4+1}}{x}}\right)}{8}\right)\sqrt{2}}{2}$ |
| trager | $\frac{\ln\left(-\frac{4\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)x+\sqrt{-x^4+1}}{4x^2\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)-x^2-1}\right)}{2}-\ln\left(-\frac{4\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)x+\sqrt{-x^4+1}}{4x^2\,\mathrm{RootOf}\left(8\_Z^2-4\_Z+1\right)-x^2-1}\right)\mathrm{RootOf}\left(8\_\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)

[Out] 1/2*(-1/4*arctan(1+(-x^4+1)^(1/2)/x)*2^(1/2)-1/4*arctan(-1+(-x^4+1)^(1/2)/x)*2^(1/2)-1/8*2^(1/2)*ln((1+1/2*(-x^4+1)/x^2-(-x^4+1)^(1/2)/x)/(1+1/2*(-x^4+1)/x^2+(-x^4+1)^(1/2)/x)))*2^(1/2)

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)

**Fricas [A]**
time = 1.05, size = 56, normalized size = 1.14

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}\,x}{x^2-1}\right)+\frac{1}{4}\log\left(-\frac{x^4-2\,x^2-2\,\sqrt{-x^4+1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)*x/(x^2 - 1)) + 1/4*log(-(x^4 - 2*x^2 - 2*sqrt(-x
^4 + 1)*x - 1)/(x^4 + 1))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**4+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**4 + 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^4+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/(x^4 + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{1-x^4}}{x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(x^4 + 1),x)
```

```
[Out] int((1 - x^4)^(1/2)/(x^4 + 1), x)
```

## 3.66 $\int \frac{\sqrt{1+x^4}}{1-x^4} \, dx$

**Optimal**. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*arctanh(x*2^(1/2)/(x^4+1)^(
1/2))*2^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {413, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^4]/(1 - x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ArcTanh[(Sqrt[2]*x)/Sqrt[1
+ x^4]]/(2*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 413

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^4}}{1-x^4}\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^4}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}\end{aligned}$$

**Mathematica** [A]
time = 0.17, size = 44, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^4]/(1 - x^4),x]
[Out] (ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]] + ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]])/(2
*Sqrt[2])
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.28, size = 365, normalized size = 6.89

| method | result |
|---|---|
| elliptic | $\frac{\left(-\frac{\ln\left(-1+\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)}{4}+\frac{\ln\left(1+\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)}{4}-\frac{\arctan\left(\frac{\sqrt{x^4+1}\,\sqrt{2}}{2x}\right)}{2}\right)\sqrt{2}}{2}$ |
| trager | $-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-2\right)x-\sqrt{x^4+1}}{(1+x)(-1+x)}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x-\sqrt{x^4+1}}{x^2+1}\right)}{4}$ |

| default | $-\frac{\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{i\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.\right.}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+1/2*I/(1/2*2^(1/2)+1/2*I*2^(1/
2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticF(x*(1/2*2^(1/2)+
1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*
EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4))-1/2*I/(1/2*2^(1/2)+1/2*I*
2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*(EllipticF(x*(1/2*2^
(1/2)+1/2*I*2^(1/2)),I)-EllipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I))-1/2*I/(
1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)*El
lipticE(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I*x^
2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + 1)/(x^4 - 1), x)
```

**Fricas [A]**
time = 1.46, size = 61, normalized size = 1.15

$$\frac{1}{4}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)+\frac{1}{8}\sqrt{2}\,\log\left(\frac{x^4+2\sqrt{2}\,\sqrt{x^4+1}\,x+2\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) + 1/8*sqrt(2)*log((x^4 + 2*sqrt
(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(x**4 + 1)/(x**4 - 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + 1)/(x^4 - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 + 1)^(1/2)/(x^4 - 1),x)
```

```
[Out] -int((x^4 + 1)^(1/2)/(x^4 - 1), x)
```

## 3.67 $\int \frac{\sqrt{1+px^2+x^4}}{1-x^4} dx$

**Optimal**. Leaf size=75

$$\frac{1}{4}\sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{1+px^2+x^4}}\right)+\frac{1}{4}\sqrt{2+p}\,\tanh^{-1}\left(\frac{\sqrt{2+p}\,x}{\sqrt{1+px^2+x^4}}\right)$$

```
[Out] 1/4*arctan(x*(2-p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2-p)^(1/2)+1/4*arctanh(x*(2+
p)^(1/2)/(x^4+p*x^2+1)^(1/2))*(2+p)^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2096, 1107, 211, 214}

$$\frac{1}{4}\sqrt{2-p}\,\text{ArcTan}\left(\frac{\sqrt{2-p}\,x}{\sqrt{px^2+x^4+1}}\right)+\frac{1}{4}\sqrt{p+2}\,\tanh^{-1}\left(\frac{\sqrt{p+2}\,x}{\sqrt{px^2+x^4+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (Sqrt[2 - p]*ArcTan[(Sqrt[2 - p]*x)/Sqrt[1 + p*x^2 + x^4]])/4 + (Sqrt[2 + p
]*ArcTanh[(Sqrt[2 + p]*x)/Sqrt[1 + p*x^2 + x^4]])/4
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2096

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
```

```
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rubi steps

$$\int \frac{\sqrt{1+px^2+x^4}}{1-x^4}\,dx = \text{Subst}\left(\int \frac{1}{1-2px^2+(-4+p^2)\,x^4}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right)$$
$$= \frac{1}{4}(-4+p^2)\,\text{Subst}\left(\int \frac{1}{-2-p+(-4+p^2)\,x^2}\,dx, x, \frac{x}{\sqrt{1+px^2+x^4}}\right) - \frac{1}{4}(-4+$$
$$= \frac{1}{4}\sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{2-p}\,x}{\sqrt{1+px^2+x^4}}\right) + \frac{1}{4}\sqrt{2+p}\,\tanh^{-1}\left(\frac{\sqrt{2+p}\,x}{\sqrt{1+px^2+x^4}}\right)$$

**Mathematica** [A]
time = 0.52, size = 81, normalized size = 1.08

$$\frac{1}{4}\left(-\sqrt{-2-p}\,\tan^{-1}\left(\frac{\sqrt{-2-p}\,x}{\sqrt{1+px^2+x^4}}\right) - \sqrt{2-p}\,\tan^{-1}\left(\frac{\sqrt{1+px^2+x^4}}{\sqrt{2-p}\,x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 + x^4]/(1 - x^4),x]
```

```
[Out] (-(Sqrt[-2 - p]*ArcTan[(Sqrt[-2 - p]*x)/Sqrt[1 + p*x^2 + x^4]]) - Sqrt[2 -
p]*ArcTan[Sqrt[1 + p*x^2 + x^4]/(Sqrt[2 - p]*x)])/4
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.11, size = 1512, normalized size = 20.16

| method | result | size |
|---|---|---|
| elliptic | $\frac{\left(\frac{4\left(\frac{1}{4}-\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{2p-4}}\right)}{\sqrt{2p-4}}+\frac{4\left(\frac{1}{4}+\frac{p}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{x^4+p\,x^2+1}\,\sqrt{2}}{x\sqrt{4+2p}}\right)}{\sqrt{4+2p}}\right)\sqrt{2}}{2}$ | 89 |
| default | Expression too large to display | 1512 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+p*x^2+1)^(1/2)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(1+p)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^
(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*Elliptic
```

```
F(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2
))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*
(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/
2))*(EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4)
^(1/2)))^(1/2))-EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-
1/2*(p^2-4)^(1/2)))^(1/2)))-1/4*(2+p)*(-1/2/(2+p)^(1/2)*arctanh(1/2*(p*x^2+
2*x^2+p+2)/(2+p)^(1/2)/(x^4+p*x^2+1)^(1/2))-1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1
/2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*
x^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*
x,1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2
*(p^2-4)^(1/2))^(1/2)))+1/2*(-1-p)/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1-(-1/2*p+
1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x^2)^(1/2)/(x^4
+p*x^2+1)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-
1/2*(p^2-4)^(1/2)))^(1/2))+1/4*(2+p)*(-1/2/(2+p)^(1/2)*arctanh(1/2*(p*x^2+2
*x^2+p+2)/(2+p)^(1/2)/(x^4+p*x^2+1)^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/
2)*(1-(-1/2*p+1/2*(p^2-4)^(1/2))*x^2)^(1/2)*(1-(-1/2*p-1/2*(p^2-4)^(1/2))*x
^2)^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x
,1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*
(p^2-4)^(1/2))^(1/2)))+1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*
(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1
)^(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2
-4)^(1/2)))^(1/2))*p-1/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p
^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^
(1/2)*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/2*p-1/2*(p^2-4
)^(1/2)))^(1/2))-2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4
)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2
)/(p+(p^2-4)^(1/2))*EllipticF(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1/2),(-1-p*(-1/
2*p-1/2*(p^2-4)^(1/2)))^(1/2))+2/(-2*p+2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2-
1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/(x^4
+p*x^2+1)^(1/2)/(p+(p^2-4)^(1/2))*EllipticE(1/2*x*(-2*p+2*(p^2-4)^(1/2))^(1
/2),(-1-p*(-1/2*p-1/2*(p^2-4)^(1/2)))^(1/2))+1/(-1/2*p+1/2*(p^2-4)^(1/2))^(
1/2)*(1+1/2*p*x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)
^(1/2))^(1/2)/(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/
2)*x,-1/(-1/2*p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p
+1/2*(p^2-4)^(1/2))^(1/2))-1/2*p/(-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*
x^2-1/2*x^2*(p^2-4)^(1/2))^(1/2)*(1+1/2*p*x^2+1/2*x^2*(p^2-4)^(1/2))^(1/2)/
(x^4+p*x^2+1)^(1/2)*EllipticPi((-1/2*p+1/2*(p^2-4)^(1/2))^(1/2)*x,-1/(-1/2*
p+1/2*(p^2-4)^(1/2)),(-1/2*p-1/2*(p^2-4)^(1/2))^(1/2)/(-1/2*p+1/2*(p^2-4)^(
1/2))^(1/2))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

**Fricas [A]**
time = 1.06, size = 359, normalized size = 4.79

$$\left[\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2x^2+1}\right)+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2x^2+1}\right), \frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right)+\frac{1}{8}\sqrt{p+2}\log\left(\frac{x^4+2(p+1)x^2+2\sqrt{x^4+px^2+1}\sqrt{p+2}x+1}{x^4-2x^2+1}\right), -\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)+\frac{1}{8}\sqrt{p-2}\log\left(\frac{x^4+2(p-1)x^2-2\sqrt{x^4+px^2+1}\sqrt{p-2}x+1}{x^4+2x^2+1}\right), \frac{1}{4}\sqrt{-p+2}\arctan\left(\frac{\sqrt{-p+2}x}{\sqrt{x^4+px^2+1}}\right)-\frac{1}{4}\sqrt{-p-2}\arctan\left(\frac{\sqrt{x^4+px^2+1}\sqrt{-p-2}}{(p+2)x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p
- 2)*x + 1)/(x^4 + 2*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x^4 + 2*(p + 1)*x^2 +
 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^2 + 1)), 1/4*sqrt(-p
 + 2)*arctan(sqrt(-p + 2)*x/sqrt(x^4 + p*x^2 + 1)) + 1/8*sqrt(p + 2)*log((x
^4 + 2*(p + 1)*x^2 + 2*sqrt(x^4 + p*x^2 + 1)*sqrt(p + 2)*x + 1)/(x^4 - 2*x^
2 + 1)), -1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*sqrt(-p - 2)/((p +
2)*x)) + 1/8*sqrt(p - 2)*log((x^4 + 2*(p - 1)*x^2 - 2*sqrt(x^4 + p*x^2 + 1)
*sqrt(p - 2)*x + 1)/(x^4 + 2*x^2 + 1)), 1/4*sqrt(-p + 2)*arctan(sqrt(-p + 2
)*x/sqrt(x^4 + p*x^2 + 1)) - 1/4*sqrt(-p - 2)*arctan(sqrt(x^4 + p*x^2 + 1)*
sqrt(-p - 2)/((p + 2)*x))]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{px^2+x^4+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+p*x**2+1)**(1/2)/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(p*x**2 + x**4 + 1)/(x**4 - 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+p*x^2+1)^(1/2)/(-x^4+1),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(x^4 + p*x^2 + 1)/(x^4 - 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\sqrt{x^4+p\,x^2+1}}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1),x)
[Out] -int((p*x^2 + x^4 + 1)^(1/2)/(x^4 - 1), x)
```

## 3.68 $\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx$

<u>**Optimal**</u>. Leaf size=171

$$-\frac{\sqrt{p+\sqrt{4+p^2}}\,\tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}}\,x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\,\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}}+\frac{\sqrt{-p+\sqrt{4+p^2}}\,\tanh^{-1}\left(\frac{\sqrt{-p+\cdots}}{2\cdots}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctanh(1/4*x*(p-2*x^2+(p^2+4)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/
(-x^4+p*x^2+1)^(1/2))*(-p+(p^2+4)^(1/2))^(1/2)*2^(1/2)-1/4*arctan(1/4*x*(p-
2*x^2-(p^2+4)^(1/2))*(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)/(-x^4+p*x^2+1)^(1/2))*
(p+(p^2+4)^(1/2))^(1/2)*2^(1/2)
```

<u>**Rubi**</u> [A]
time = 0.05, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{2097\}$

$$\frac{\sqrt{\sqrt{p^2+4}-p}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{p^2+4}-p}\,x\left(\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}-\frac{\sqrt{\sqrt{p^2+4}+p}\,\text{ArcTan}\left(\frac{\sqrt{\sqrt{p^2+4}+p}\,x\left(-\sqrt{p^2+4}+p-2x^2\right)}{2\sqrt{2}\,\sqrt{px^2-x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] -1/2*(Sqrt[p + Sqrt[4 + p^2]]*ArcTan[(Sqrt[p + Sqrt[4 + p^2]]*x*(p - Sqrt[4
 + p^2] - 2*x^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/Sqrt[2] + (Sqrt[-p +
Sqrt[4 + p^2]]*ArcTanh[(Sqrt[-p + Sqrt[4 + p^2]]*x*(p + Sqrt[4 + p^2] - 2*x
^2))/(2*Sqrt[2]*Sqrt[1 + p*x^2 - x^4])])/(2*Sqrt[2])
```

<u>Rule 2097</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, Simp[(-a)*(Sqrt[b + q]/(2*Sqrt[2]*Rt[(-
a)*c, 2]*d))*ArcTan[Sqrt[b + q]*x*((b - q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x] + Simp[a*(Sqrt[-b + q]/(2*Sqrt[2]*Rt[(-a)
*c, 2]*d))*ArcTanh[Sqrt[-b + q]*x*((b + q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d
+ a*e, 0] && NegQ[a*c]
```

<u>Rubi steps</u>

$$\int \frac{\sqrt{1+px^2-x^4}}{1+x^4}\,dx = -\frac{\sqrt{p+\sqrt{4+p^2}}\tan^{-1}\left(\frac{\sqrt{p+\sqrt{4+p^2}}\,x\left(p-\sqrt{4+p^2}-2x^2\right)}{2\sqrt{2}\sqrt{1+px^2-x^4}}\right)}{2\sqrt{2}} + \frac{\sqrt{-p+\sqrt{4+\cdots}}}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.26, size = 92, normalized size = 0.54

$$\frac{1}{4}i\left(\sqrt{-2i-p}\tan^{-1}\left(\frac{\sqrt{-2i-p}\,x}{\sqrt{1+px^2-x^4}}\right)-\sqrt{2i-p}\tan^{-1}\left(\frac{\sqrt{2i-p}\,x}{\sqrt{1+px^2-x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + p*x^2 - x^4]/(1 + x^4),x]
```

```
[Out] (I/4)*(Sqrt[-2*I - p]*ArcTan[(Sqrt[-2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]] - Sq
rt[2*I - p]*ArcTan[(Sqrt[2*I - p]*x)/Sqrt[1 + p*x^2 - x^4]])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(131) = 262.
time = 0.11, size = 625, normalized size = 3.65

| method | result |
|---|---|
| default | $\frac{\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\sqrt{p^2+4}\ln\left(\frac{-x^4+px^2+1}{x^2}-\frac{\sqrt{-x^4+px^2+1}\sqrt{2}\sqrt{p+\sqrt{p^2+4}}}{x}+\sqrt{p^2+4}\right)}{16}-\cdots\right.}{\cdots}$ |
| elliptic | $\frac{\left(-\frac{\sqrt{p+\sqrt{p^2+4}}\sqrt{p^2+4}\ln\left(\frac{-x^4+px^2+1}{x^2}-\frac{\sqrt{-x^4+px^2+1}\sqrt{2}\sqrt{p+\sqrt{p^2+4}}}{x}+\sqrt{p^2+4}\right)}{16}-\cdots\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+p*x^2+1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/16*(p+(p^2+4)^(1/2))^(1/2)*(p^2+4)^(1/2)*ln((-x^4+p*x^2+1)/x^2-(-x^
4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/8*(p^2+
4)^(1/2)*(p+(p^2+4)^(1/2))/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x
```

```
^2+1)^(1/2)*2^(1/2)/x-2*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))+
1/16*(p+(p^2+4)^(1/2))^(1/2)*p*ln((-x^4+p*x^2+1)/x^2-(-x^4+p*x^2+1)^(1/2)*2
^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^2+4)^(1/2))+1/8*p*(p+(p^2+4)^(1/2))/(-p
+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x-2*(p+(p^
2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))+1/16*(p+(p^2+4)^(1/2))^(1/2)*(
p^2+4)^(1/2)*ln((-x^4+p*x^2+1)/x^2+(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4
)^(1/2))^(1/2)+(p^2+4)^(1/2))-1/8*(p^2+4)^(1/2)*(p+(p^2+4)^(1/2))/(-p+(p^2+
4)^(1/2))^(1/2)*arctan(1/2*(2*(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+2*(p+(p^2+4)^(
1/2))^(1/2))/(-p+(p^2+4)^(1/2))^(1/2))-1/16*(p+(p^2+4)^(1/2))^(1/2)*p*ln((-
x^4+p*x^2+1)/x^2+(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x*(p+(p^2+4)^(1/2))^(1/2)+(p^
2+4)^(1/2))+1/8*p*(p+(p^2+4)^(1/2))/(-p+(p^2+4)^(1/2))^(1/2)*arctan(1/2*(2*
(-x^4+p*x^2+1)^(1/2)*2^(1/2)/x+2*(p+(p^2+4)^(1/2))^(1/2))/(-p+(p^2+4)^(1/2)
)^(1/2)))*2^(1/2)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2667 vs. 2(135) = 270.
time = 4.37, size = 2667, normalized size = 15.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*arctan(1/4
*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3 + 4*p)*x^8 + 2*(
3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)*x^2 - 2*((p^2 +
4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x^4 + (p*x^12 - (p
^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^2)*sqrt(p^2 + 4))
*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (
p^2 + 4)*x^4)*sqrt(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(2*(sqrt(2)*(
x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*(x^11 - 2
*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(
3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x)*sqrt(-x^4 + p*x
^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 + 4)*x^5 - 2*(p^3 +
 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)^(1/4)) - (2*((p^
3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^4 + p*x^2 + 1)*sq
```

rt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6 - (p^4 - 4*p^2 -
 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^2 + 1) - 2*((p*x^
10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt(-x^4 + p*x^2 + 1
)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p^2 + 4)*x^6 + (p^
3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt(p^2 + 4) - sqrt(
p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5 - x^3)*sqrt(p^2
+ 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p*x^7 - (p^2 - 6)*
x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 - 6)*x^9 - 10*p*x^7
 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^11
 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^2 + 4)^(1/4))*sq
rt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*s
qrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 -
4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 + 4*p)*x^10 + (2*p^
4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28)*x^4 - 3*(p^3 +
4*p)*x^2 - p^2 - 4)) + 8*sqrt(2)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^
(3/4)*arctan(-1/4*(2*(p^3 + 4*p)*x^12 - 2*(p^4 - 2*p^2 - 24)*x^10 - 20*(p^3
 + 4*p)*x^8 + 2*(3*p^4 + 4*p^2 - 32)*x^6 + 10*(p^3 + 4*p)*x^4 + 4*(p^2 + 4)
*x^2 - 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p^3 + 4*p)*x^6 - (p^2 + 4)*x
^4 + (p*x^12 - (p^2 - 6)*x^10 - 10*p*x^8 + (3*p^2 - 8)*x^6 + 5*p*x^4 + 2*x^
2)*sqrt(p^2 + 4))*sqrt(p^2 + 4) + 2*((p^2 + 4)*x^12 - (p^3 + 4*p)*x^10 - (p
^3 + 4*p)*x^6 - (p^2 + 4)*x^4)*sqrt(p^2 + 4) - sqrt(p^2 + sqrt(p^2 + 4)*p +
 4)*(2*(sqrt(2)*(x^9 - p*x^7 - x^5)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) +
sqrt(2)*(x^11 - 2*p*x^9 + (p^2 - 2)*x^7 + 2*p*x^5 + x^3)*sqrt(-x^4 + p*x^2
+ 1))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^9 + 8*x^7 - 6*p*x^5 + 2*p^2*x^3 + p*x
)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + sqrt(2)*((p^2 + 4)*x^9 + 4*(p^2 +
4)*x^5 - 2*(p^3 + 4*p)*x^3 - (p^2 + 4)*x)*sqrt(-x^4 + p*x^2 + 1))*(p^2 + 4)
^(1/4)) - (2*((p^3 + 4*p)*x^8 + 4*(p^2 + 4)*x^6 - (p^3 + 4*p)*x^4)*sqrt(-x^
4 + p*x^2 + 1)*sqrt(p^2 + 4) + 2*((p^4 + 6*p^2 + 8)*x^8 + 4*(p^3 + 4*p)*x^6
 - (p^4 - 4*p^2 - 32)*x^4 - 4*(p^3 + 4*p)*x^2 - 2*p^2 - 8)*sqrt(-x^4 + p*x^
2 + 1) - 2*((p*x^10 - (p^2 - 4)*x^8 - 6*p*x^6 + (p^2 - 4)*x^4 + p*x^2)*sqrt
(-x^4 + p*x^2 + 1)*sqrt(p^2 + 4) + ((p^2 + 4)*x^10 - (p^3 + 4*p)*x^8 - 2*(p
^2 + 4)*x^6 + (p^3 + 4*p)*x^4 + (p^2 + 4)*x^2)*sqrt(-x^4 + p*x^2 + 1))*sqrt
(p^2 + 4) + sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*((sqrt(2)*(x^11 - p*x^9 - p*x^5
 - x^3)*sqrt(p^2 + 4) + sqrt(2)*(2*x^13 - 5*p*x^11 + (3*p^2 - 8)*x^9 + 10*p
*x^7 - (p^2 - 6)*x^5 - p*x^3))*(p^2 + 4)^(3/4) - (sqrt(2)*(p*x^11 - (p^2 -
6)*x^9 - 10*p*x^7 + (3*p^2 - 8)*x^5 + 5*p*x^3 + 2*x)*sqrt(p^2 + 4) + sqrt(2
)*((p^2 + 4)*x^11 - (p^3 + 4*p)*x^9 - (p^3 + 4*p)*x^5 - (p^2 + 4)*x^3))*(p^
2 + 4)^(1/4))*sqrt(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x
^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 +
4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/((p^2 + 4)*x^12 - 3*(p^3 +
4*p)*x^10 + (2*p^4 + p^2 - 28)*x^8 + 10*(p^3 + 4*p)*x^6 - (2*p^4 + p^2 - 28
)*x^4 - 3*(p^3 + 4*p)*x^2 - p^2 - 4)) - (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*
(p^2 + 4))*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*
x^4 - (p^2 + 4)^(3/2)*x^2 + sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(
p^2 + 4)*p + 4)*(p^2 + 4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x

```
^4 + p^2 + 4)) + (sqrt(2)*sqrt(p^2 + 4)*p - sqrt(2)*(p^2 + 4))*sqrt(p^2 + s
qrt(p^2 + 4)*p + 4)*(p^2 + 4)^(1/4)*log(-((p^2 + 4)*x^4 - (p^2 + 4)^(3/2)*x
^2 - sqrt(2)*sqrt(-x^4 + p*x^2 + 1)*sqrt(p^2 + sqrt(p^2 + 4)*p + 4)*(p^2 +
4)^(3/4)*x - (p^3 + 4*p)*x^2 - p^2 - 4)/((p^2 + 4)*x^4 + p^2 + 4)))/(p^2 +
4)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{px^2 - x^4 + 1}}{x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+p*x**2+1)**(1/2)/(x**4+1),x)
[Out] Integral(sqrt(p*x**2 - x**4 + 1)/(x**4 + 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+p*x^2+1)^(1/2)/(x^4+1),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + p*x^2 + 1)/(x^4 + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4 + p\,x^2 + 1}}{x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1),x)
[Out] int((p*x^2 - x^4 + 1)^(1/2)/(x^4 + 1), x)
```

# 3.69 $\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

**Optimal**. Leaf size=80

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}-b\tan^{-1}\left(\sqrt[4]{-1+x^2}\right)+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}+b\tanh^{-1}\left(\sqrt[4]{-1+x^2}\right)$$

```
[Out] -b*arctan((x^2-1)^(1/4))+b*arctanh((x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(x^2-1
)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 407, 455, 65, 304, 209, 212}

$$\frac{a\text{ArcTan}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}-b\text{ArcTan}\left(\sqrt[4]{x^2-1}\right)+b\tanh^{-1}\left(\sqrt[4]{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTan[(-1 + x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTanh[
(-1 + x^2)^(1/4)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = a\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx + b\int \frac{x}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\text{Subst}\left(\int \frac{}{(2-}\right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + (2b)\text{Subst}\left(\int \frac{x}{1-}\right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + b\text{Subst}\left(\int \frac{1}{1-x^2}\right.$$
$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} - b\tan^{-1}\left(\sqrt[4]{-1+x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x}}\right.}{2\sqrt{2}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.19, size = 157, normalized size = 1.96

$$\frac{x\left(bx\sqrt[4]{1-x^2}\,(-2+x^2)\,F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right)-\frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)}\right)}{4\,(-2+x^2)\sqrt[4]{-1+x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (x*(b*x*(1 - x^2)^(1/4)*(-2 + x^2)*AppellF1[1, 1/4, 1, 2, x^2, x^2/2] - (24
*a*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/(6*AppellF1[1/2, 1/4, 1, 3/2, x^
2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5
/4, 1, 5/2, x^2, x^2/2]))))/(4*(-2 + x^2)*(-1 + x^2)^(1/4))
```

**Maple** [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(-x^2+2)\,(x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

```
[Out] int((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
[Out] -integrate((b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx-\int \frac{bx}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+2)/(x**2-1)**(1/4),x)
[Out] -Integral(a/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x) - Integral(b
*x/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-(b*x + a)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(x^2-1\right)}^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
[Out] int(-(a + b*x)/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

# 3.70 $\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx$

**Optimal**. Leaf size=88

$$\frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}}+b\tan^{-1}\left(\sqrt[4]{-1-x^2}\right)+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}}-b\tanh^{-1}\left(\sqrt[4]{-1-x^2}\right)$$

```
[Out] b*arctan((-x^2-1)^(1/4))-b*arctanh((-x^2-1)^(1/4))+1/4*a*arctan(1/2*x/(-x^2
-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*a*arctanh(1/2*x/(-x^2-1)^(1/4)*2^(1/2))*2^(1
/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1024, 407, 455, 65, 304, 209, 212}

$$\frac{a\text{ArcTan}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+\frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-x^2-1}}\right)}{2\sqrt{2}}+b\text{ArcTan}\left(\sqrt[4]{-x^2-1}\right)-b\tanh^{-1}\left(\sqrt[4]{-x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) + b*ArcTan[(-1 - x^2)^
(1/4)] + (a*ArcTanh[x/(Sqrt[2]*(-1 - x^2)^(1/4))])/(2*Sqrt[2]) - b*ArcTanh[
(-1 - x^2)^(1/4)]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx = a\int \frac{1}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx + b\int \frac{x}{\sqrt[4]{-1-x^2}\,(2+x^2)}\,dx$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{1}{2}b\text{Subst}\Bigg(\int \frac{}{\sqrt[4]{-}}$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - (2b)\text{Subst}\Bigg(\int \frac{x}{1-}$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} - b\text{Subst}\Bigg(\int \frac{1}{1-x^2}$$

$$= \frac{a\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x^2}}\right)}{2\sqrt{2}} + b\tan^{-1}\left(\sqrt[4]{-1-x^2}\right) + \frac{a\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1-x}}\right.}{2\sqrt{2}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.20, size = 162, normalized size = 1.84

$$\frac{x\left(bx\sqrt[4]{1+x^2}\,F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{24aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{(2+x^2)\left(-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)\right)}\right)}{4\sqrt[4]{-1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((-1 - x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (x*(b*x*(1 + x^2)^(1/4)*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2] - (24*a*Appe
llF1[1/2, 1/4, 1, 3/2, -x^2, -1/2*x^2])/((2 + x^2)*(-6*AppellF1[1/2, 1/4, 1
, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^2]
+ AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))))/(4*(-1 - x^2)^(1/4))
```

**Maple** [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(-x^2-1\right)^{\frac{1}{4}}\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="fricas")
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[4]{-x^2-1}\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2-1)**(1/4)/(x**2+2),x)
[Out] Integral((a + b*x)/((-x**2 - 1)**(1/4)*(x**2 + 2)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2-1)^(1/4)/(x^2+2),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 2)*(-x^2 - 1)^(1/4)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{{\left(-x^2-1\right)}^{1/4}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((- x^2 - 1)^(1/4)*(x^2 + 2)), x)
```

# 3.71 $\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx$

**Optimal**. Leaf size=149

$$\frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{b\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{1}{2}a\tanh^{-1}\left(\frac{1+\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)$$

```
[Out] 1/2*a*arctan((1-(-x^2+1)^(1/2))/x/(-x^2+1)^(1/4))+1/2*a*arctanh((1+(-x^2+1)
^(1/2))/x/(-x^2+1)^(1/4))+1/2*b*arctan(1/2*(1-(-x^2+1)^(1/2))/(-x^2+1)^(1/4
)*2^(1/2))*2^(1/2)+1/2*b*arctanh(1/2*(1+(-x^2+1)^(1/2))/(-x^2+1)^(1/4)*2^(1
/2))*2^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1024, 406, 450}

$$\frac{1}{2}a\text{ArcTan}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right)+\frac{1}{2}a\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{x\sqrt[4]{1-x^2}}\right)+\frac{b\text{ArcTan}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{1-x^2}+1}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*ArcTan[(1 - Sqrt[1 - x^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcT
an[(1 - Sqrt[1 - x^2])/(x*(1 - x^2)^(1/4))])/2 + (b*ArcTanh[(1 + Sqrt[1 - x
^2])/(Sqrt[2]*(1 - x^2)^(1/4))])/Sqrt[2] + (a*ArcTanh[(1 + Sqrt[1 - x^2])/(
x*(1 - x^2)^(1/4))])/2
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx = a\int \frac{1}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx + b\int \frac{x}{\sqrt[4]{1-x^2}\,(2-x^2)}\,dx$$
$$= \frac{b\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{\sqrt{2}\sqrt[4]{1-x^2}}\right)}{\sqrt{2}} + \frac{1}{2}a\tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x\sqrt[4]{1-x^2}}\right) + \frac{b\tanh^{-1}\left(\frac{1+\sqrt{1-x}}{\sqrt{2}\sqrt[4]{1-}}\right.}{\sqrt{2}}$$

**Mathematica [C]** Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.18, size = 144, normalized size = 0.97

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;x^2,\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{\sqrt[4]{1-x^2}\,(-2+x^2)\left(6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 - x^2)^(1/4)*(2 - x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, x^2, x^2/2])/4 - (6*a*x*AppellF1[1/2, 1/4, 1,
 3/2, x^2, x^2/2])/((1 - x^2)^(1/4)*(-2 + x^2)*(6*AppellF1[1/2, 1/4, 1, 3/2
, x^2, x^2/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/
2, 5/4, 1, 5/2, x^2, x^2/2])))
```

**Maple [F]**
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(-x^2+1)^{\frac{1}{4}}\,(-x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

```
[Out] int((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx-\int \frac{bx}{x^2\sqrt[4]{1-x^2}-2\sqrt[4]{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x**2+1)**(1/4)/(-x**2+2),x)
```

```
[Out] -Integral(a/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x) - Integral(b
*x/(x**2*(1 - x**2)**(1/4) - 2*(1 - x**2)**(1/4)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-x^2+1)^(1/4)/(-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x + a)/((x^2 - 2)*(-x^2 + 1)^(1/4)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{a+b\,x}{{\left(1-x^2\right)}^{1/4}\,\left(x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)),x)
```

```
[Out] int(-(a + b*x)/((1 - x^2)^(1/4)*(x^2 - 2)), x)
```

# 3.72 $\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx$

**Optimal**. Leaf size=135

$$-\frac{b\tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\,\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}-\frac{1}{2}a\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right)-\frac{b\tanh^{-1}\left(\frac{1+\sqrt{1+x^2}}{\sqrt{2}\,\sqrt[4]{1+x^2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*a*arctan((1+(x^2+1)^(1/2))/x/(x^2+1)^(1/4))-1/2*a*arctanh((1-(x^2+1)^(
1/2))/x/(x^2+1)^(1/4))-1/2*b*arctan(1/2*(1-(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(
1/2))*2^(1/2)-1/2*b*arctanh(1/2*(1+(x^2+1)^(1/2))/(x^2+1)^(1/4)*2^(1/2))*2^
(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1024, 406, 450}

$$-\frac{1}{2}a\text{ArcTan}\left(\frac{\sqrt{x^2+1}+1}{x\sqrt[4]{x^2+1}}\right)-\frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{x^2+1}}{x\sqrt[4]{x^2+1}}\right)-\frac{b\text{ArcTan}\left(\frac{1-\sqrt{x^2+1}}{\sqrt{2}\,\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{2}\,\sqrt[4]{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] -((b*ArcTan[(1 - Sqrt[1 + x^2])/(Sqrt[2]*(1 + x^2)^(1/4))])/Sqrt[2]) - (a*A
rcTan[(1 + Sqrt[1 + x^2])/(x*(1 + x^2)^(1/4))])/2 - (a*ArcTanh[(1 - Sqrt[1
+ x^2])/(x*(1 + x^2)^(1/4))])/2 - (b*ArcTanh[(1 + Sqrt[1 + x^2])/(Sqrt[2]*(
1 + x^2)^(1/4))])/Sqrt[2]
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rule 450

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx = a\int \frac{1}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx + b\int \frac{x}{\sqrt[4]{1+x^2}\,(2+x^2)}\,dx$$
$$= -\frac{b\tan^{-1}\left(\frac{1-\sqrt{1+x^2}}{\sqrt{2}\sqrt[4]{1+x^2}}\right)}{\sqrt{2}} - \frac{1}{2}a\tan^{-1}\left(\frac{1+\sqrt{1+x^2}}{x\sqrt[4]{1+x^2}}\right) - \frac{1}{2}a\tanh^{-1}\left(\frac{1-\sqrt{1+}}{x\sqrt[4]{1+}}\right.$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.14, size = 152, normalized size = 1.13

$$\frac{1}{4}bx^2F_1\left(1;\frac{1}{4},1;2;-x^2,-\frac{x^2}{2}\right) - \frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)}{\sqrt[4]{1+x^2}\,(2+x^2)\left(-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-x^2,-\frac{x^2}{2}\right)+x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-x^2,-\frac{x^2}{2}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)/((1 + x^2)^(1/4)*(2 + x^2)),x]
```

```
[Out] (b*x^2*AppellF1[1, 1/4, 1, 2, -x^2, -1/2*x^2])/4 - (6*a*x*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2])/((1 + x^2)^(1/4)*(2 + x^2)*(-6*AppellF1[1/2, 1/4
, 1, 3/2, -x^2, -1/2*x^2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -x^2, -1/2*x^
2] + AppellF1[3/2, 5/4, 1, 5/2, -x^2, -1/2*x^2])))
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{(x^2+1)^{\frac{1}{4}}\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

```
[Out] int((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="maxima")
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="fricas")
[Out] Timed out
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt[4]{x^2+1}\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x**2+1)**(1/4)/(x**2+2),x)
[Out] Integral((a + b*x)/((x**2 + 1)**(1/4)*(x**2 + 2)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(x^2+1)^(1/4)/(x^2+2),x, algorithm="giac")
[Out] integrate((b*x + a)/((x^2 + 2)*(x^2 + 1)^(1/4)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x}{{\left(x^2+1\right)}^{1/4}\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)),x)
[Out] int((a + b*x)/((x^2 + 1)^(1/4)*(x^2 + 2)), x)
```

## 3.73 $\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx$

**Optimal**. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}\,x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

**Rubi** [A]

time = 0.01, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {497}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}\,x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(2^(2/3)*Sqrt[3]) + Ar
cTan[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/S
qrt[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 497

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\,(4-x^3)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}\,x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\tanh$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.03, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.86, size = 164, normalized size = 1.29

| method | result |
|---|---|
| default | $\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\cdots\right.}{\cdots}\right)}{36\cdots}$ |
| elliptic | $\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^3-4\right)}\frac{\_\alpha^2\sqrt{2}\sqrt{i\left(-i\sqrt{3}+2x+1\right)}\sqrt{\frac{-1+x}{i\sqrt{3}-3}}\sqrt{-\frac{i\left(i\sqrt{3}+2x+1\right)}{2}}\left(-2\_\alpha^2+\_\alpha+1+i\cdots\right.}{\cdots}\right)}{36\cdots}$ |
| trager | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(-I*3^(1/2)+2*x+1))^(1/2)*((-1+x)/(I*3^(
1/2)-3))^(1/2)*(-1/2*I*(I*3^(1/2)+2*x+1))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2
```

```
+_alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1
/2)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^
3-4))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(92) = 184$.
time = 1.60, size = 1191, normalized size = 9.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
```

```
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

**Mupad** [B]
time = 0.45, size = 653, normalized size = 5.14

$$-\frac{2^{1/3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{2^{2/3}-1};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{1-x^3}\,\left(2^{2/3}-1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2^{1/3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)+1};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{1-x^3}\,\left(2^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)+1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2^{1/3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{1-x^3}\,\left(2^{2/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
```

```
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

# 3.74 $\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx$

**Optimal**. Leaf size=157

$$-\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}\,x}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}\,x\right)}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{-1+dx^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}$$

```
[Out] -1/6*arctan((1+2^(1/3)*d^(1/3)*x)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arc
tan((d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)-1/18*arctanh((1-2^(1/3)*d^(1/3)*x)*3^(
1/2)/(d*x^3-1)^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/18*arctanh(1/3*(d*x^3-1)^(1
/2)*3^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {498}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{2}\sqrt[3]{d}\,x+1}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}d^{2/3}}-\frac{\text{ArcTan}\left(\sqrt{dx^3-1}\right)}{9\ 2^{2/3}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}\,x\right)}{\sqrt{dx^3-1}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{dx^3-1}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] -1/3*ArcTan[(1 + 2^(1/3)*d^(1/3)*x)/Sqrt[-1 + d*x^3]]/(2^(2/3)*d^(2/3)) - A
rcTan[Sqrt[-1 + d*x^3]]/(9*2^(2/3)*d^(2/3)) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)
*d^(1/3)*x))/Sqrt[-1 + d*x^3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3)) - ArcTanh[Sqrt[-
1 + d*x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]*d^(2/3))
```

Rule 498

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(-q)*(ArcTan[Sqrt[c + d*x^3]/Rt[-c, 2]]/(9*2^(2/3
)*b*Rt[-c, 2])), x] + (-Simp[q*(ArcTan[Rt[-c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c
+ d*x^3])]/(3*2^(2/3)*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[Sqrt[c + d*x^3]/(
Sqrt[3]*Rt[-c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[-c, 2])), x] - Simp[q*(ArcTanh[
Sqrt[3]*Rt[-c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b
*Rt[-c, 2])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*
b*c - a*d, 0] && NegQ[c]
```

Rubi steps

$$\int \frac{x}{(4-dx^3)\sqrt{-1+dx^3}}\,dx = -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2}\sqrt[3]{d}\,x}{\sqrt{-1+dx^3}}\right)}{3\ 2^{2/3}d^{2/3}} - \frac{\tan^{-1}\left(\sqrt{-1+dx^3}\right)}{9\ 2^{2/3}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{d}\right.}{\sqrt{-1+dx^3}}\right.}{3\ 2^{2/3}\sqrt{3}\,d^{2/3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.04, size = 54, normalized size = 0.34

$$\frac{x^2\sqrt{1-dx^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};dx^3,\frac{dx^3}{4}\right)}{8\sqrt{-1+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((4 - d*x^3)*Sqrt[-1 + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - d*x^3]*AppellF1[2/3, 1/2, 1, 5/3, d*x^3, (d*x^3)/4])/(8*Sqrt[
-1 + d*x^3])
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 240, normalized size = 1.53

| method | result |
|---|---|
| default | $-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\left(-\right.}{}\right.}{}$ |

| | |
|---|---|
| elliptic | $-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^3-4\right)}\frac{\sqrt{-\frac{i\left(2x+\frac{1}{d^{\frac{1}{3}}}+\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}{2}}\sqrt{\frac{x-\frac{1}{d^{\frac{1}{3}}}}{-\frac{3}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}}}\sqrt{2}\sqrt{i\left(2x+\frac{1}{d^{\frac{1}{3}}}-\frac{i\sqrt{3}}{d^{\frac{1}{3}}}\right)d^{\frac{1}{3}}}\ (}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*I*2^(1/2)*sum(1/_alpha/d^(4/3)*(-1/2*I*(2*x+1/d^(1/3)+I*3^(1/2)/d^(1/3
))*d^(1/3))^(1/2)*((x-1/d^(1/3))/(-3/d^(1/3)-I*3^(1/2)/d^(1/3)))^(1/2)*(1/2
*I*(2*x+1/d^(1/3)-I*3^(1/2)/d^(1/3))*d^(1/3))^(1/2)/(d*x^3-1)^(1/2)*(-2*_al
pha^2*d+I*3^(1/2)*_alpha*d^(2/3)-I*3^(1/2)*d^(1/3)+_alpha*d^(2/3)+d^(1/3))*
EllipticPi(1/3*3^(1/2)*(-I*(x+1/2/d^(1/3)+1/2*I*3^(1/2)/d^(1/3))*3^(1/2)*d^
(1/3))^(1/2),1/3*I*3^(1/2)*d^(2/3)*_alpha^2-1/6*I*3^(1/2)*d^(1/3)*_alpha-1/
6*I*3^(1/2)+1/2*d^(1/3)*_alpha-1/2,(-I*3^(1/2)/d^(1/3)/(-3/2/d^(1/3)-1/2*I*
3^(1/2)/d^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-4))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1666 vs. $2(110) = 220$.
time = 1.21, size = 1666, normalized size = 10.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*
d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt
```

```
(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1) + (2*sq
rt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))^(1/3) + sqrt(3)*(d*x^4 - x) + 3*(s
qrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1
/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d
*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4
 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^
2)*(d^(-4))^(1/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3
)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*
d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^
6 + 48*d*x^3 - 64)))/(d*x^4 - x)) - 1/9*sqrt(3)*(1/432)^(1/6)*(d^(-4))^(1/6
)*arctan(1/3*(3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*sqrt(3)*(1/432)^(
1/6)*d*(d^(-4))^(1/6)*x^2 - 24*sqrt(3)*(1/432)^(5/6)*(d^4*x^3 - 4*d^3)*(d^(
-4))^(5/6))*sqrt(d*x^3 - 1) - (2*sqrt(3)*(1/2)^(1/3)*(d^2*x^3 - d)*(d^(-4))
^(1/3) + sqrt(3)*(d*x^4 - x) - 3*(sqrt(3)*sqrt(1/3)*d^2*sqrt(d^(-4))*x + 2*
sqrt(3)*(1/432)^(1/6)*d*(d^(-4))^(1/6)*x^2 + 24*sqrt(3)*(1/432)^(5/6)*(d^4*
x^3 + 2*d^3)*(d^(-4))^(5/6))*sqrt(d*x^3 - 1))*sqrt((d^3*x^9 - 60*d^2*x^6 -
24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1
/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6
)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^
(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*
x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)))/(d*x^4 - x)) + 1/18
*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/
2)^(2/3)*(d^5*x^7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4
*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) + 6*(1296*(1/432)^(5/6)*d^5*(d
^(-4))^(5/6)*x^5 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4))
 + 2*(1/432)^(1/6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^
3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/18*(1/432)^(1/6)*(
d^(-4))^(1/6)*log((d^3*x^9 + 66*d^2*x^6 - 72*d*x^3 + 48*(1/2)^(2/3)*(d^5*x^
7 + d^4*x^4 - 2*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5
 - 8*d^2*x^2)*(d^(-4))^(1/3) - 6*(1296*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5
 + sqrt(1/3)*(5*d^4*x^6 + 20*d^3*x^3 - 16*d^2)*sqrt(d^(-4)) + 2*(1/432)^(1/
6)*(d^3*x^7 + 16*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^
3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64)) - 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*lo
g((d^3*x^9 - 60*d^2*x^6 - 24*(1/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d
^(-4))^(2/3) + 12*(1/2)^(1/3)*(d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1
/3) + 12*(648*(1/432)^(5/6)*d^5*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 1
6*d^3*x^3 - 8*d^2)*sqrt(d^(-4)) - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*
x)*(d^(-4))^(1/6))*sqrt(d*x^3 - 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 -
 64)) + 1/36*(1/432)^(1/6)*(d^(-4))^(1/6)*log((d^3*x^9 - 60*d^2*x^6 - 24*(1
/2)^(2/3)*(d^5*x^7 - 5*d^4*x^4 + 4*d^3*x)*(d^(-4))^(2/3) + 12*(1/2)^(1/3)*(
d^4*x^8 + 7*d^3*x^5 - 8*d^2*x^2)*(d^(-4))^(1/3) - 12*(648*(1/432)^(5/6)*d^5
*(d^(-4))^(5/6)*x^5 - sqrt(1/3)*(d^4*x^6 + 16*d^3*x^3 - 8*d^2)*sqrt(d^(-4))
 - (1/432)^(1/6)*(d^3*x^7 - 2*d^2*x^4 - 8*d*x)*(d^(-4))^(1/6))*sqrt(d*x^3 -
 1) + 32)/(d^3*x^9 - 12*d^2*x^6 + 48*d*x^3 - 64))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3-1}-4\sqrt{dx^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+4)/(d*x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 - 1) - 4*sqrt(d*x**3 - 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+4)/(d*x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 - 1)*(d*x^3 - 4)), x)
```

**Mupad [B]**
time = 15.03, size = 331, normalized size = 2.11

$$\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(54\sqrt{dx^3-1}+54\sqrt{3}-54\,2^{1/3}\sqrt{3}\,d^{1/3}x\right)\left(\sqrt{dx^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}x\right)^3}{\left(2^{2/3}-d^{1/3}x\right)^6}\right)}{2916\,d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{dx^3-1}+2\sqrt{3}+2^{1/3}\sqrt{3}\,d^{1/3}x+2^{1/3}d^{1/3}x\,3i\right)^3\left(108\sqrt{3}-108\sqrt{dx^3-1}+54\,2^{1/3}\sqrt{3}\,d^{1/3}x+2^{1/3}d^{1/3}x\,162i\right)}{\left(2^{2/3}+2\,d^{1/3}x-2^{2/3}\sqrt{3}\,1i\right)^6}\right)\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}}{2916\,d^{2/3}}+\frac{\sqrt{3}\,314928^{1/3}\ln\left(\frac{\left(2\sqrt{dx^3-1}-2\sqrt{3}-2^{1/3}\sqrt{3}\,d^{1/3}x+2^{1/3}d^{1/3}x\,3i\right)^3\left(108\sqrt{dx^3-1}+108\sqrt{3}+54\,2^{1/3}\sqrt{3}\,d^{1/3}x-2^{1/3}d^{1/3}x\,162i\right)}{\left(2^{2/3}+2\,d^{1/3}x+2^{2/3}\sqrt{3}\,1i\right)^6}\right)\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}\,1i}{2916\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 - 1)^(1/2)*(d*x^3 - 4)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log(((54*(d*x^3 - 1)^(1/2) + 54*3^(1/2) - 54*2^(1/3)*
3^(1/2)*d^(1/3)*x)*((d*x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*d^(1/3)*x
)^3)/(2^(2/3) - d^(1/3)*x)^6))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*log((
(2*(d*x^3 - 1)^(1/2) + 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*d
^(1/3)*x)^3*(108*3^(1/2) - 108*(d*x^3 - 1)^(1/2) + 2^(1/3)*d^(1/3)*x*162i +
 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3) - 2^(2/3)*3^(1/2)*1i + 2*d^(1/3)*x
)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*d^(2/3)) + (3^(1/2)*314928^(1/3)*l
og(((2*(d*x^3 - 1)^(1/2) - 2*3^(1/2) + 2^(1/3)*d^(1/3)*x*3i - 2^(1/3)*3^(1/
2)*d^(1/3)*x)^3*(108*(d*x^3 - 1)^(1/2) + 108*3^(1/2) - 2^(1/3)*d^(1/3)*x*16
2i + 54*2^(1/3)*3^(1/2)*d^(1/3)*x))/(2^(2/3)*3^(1/2)*1i + 2^(2/3) + 2*d^(1/
3)*x)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916*d^(2/3))
```

## 3.75 $\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx$

**Optimal**. Leaf size=74

$$\frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)+\frac{1}{18}\tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}$$

```
[Out] 1/18*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))+1/18*arctan(1/3*(x^3-1)^(1/2))-1/18*
arctanh((1-x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {499, 455, 65, 210, 2163, 209, 2170, 212}

$$\frac{1}{18}\text{ArcTan}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)+\frac{1}{18}\text{ArcTan}\left(\frac{\sqrt{x^3-1}}{3}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(1-x)}{\sqrt{x^3-1}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
```

```
[Out] ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])]/18 + ArcTan[Sqrt[-1 + x^3]/3]/18 - Arc
Tanh[(Sqrt[3]*(1 - x))/Sqrt[-1 + x^3]]/(6*Sqrt[3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\,(8+x^3)}\,dx = -\left(\frac{1}{12}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx\right) - \frac{1}{12}\int \frac{-2-2x+x^2}{(4-2x+x^2)\sqrt{-1+x^3}}\,dx - \frac{1}{4}\int$$
$$= -\left(\frac{1}{12}\text{Subst}\left(\int \frac{1}{(-8-x)\sqrt{-1+x}}\,dx, x, x^3\right)\right) + \frac{1}{6}\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(}{\sqrt{}}\right.$$
$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{-9-x^2}\,dx,\right.$$
$$= \frac{1}{18}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) + \frac{1}{18}\tan^{-1}\left(\frac{1}{3}\sqrt{-1+x^3}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}(1-x)}{\sqrt{-1+x^3}}\right)}{6\sqrt{3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.03, size = 48, normalized size = 0.65

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{8}\right)}{16\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(8 + x^3)),x]
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -1/8*x^3])/(16*Sqrt[-1 +
 x^3])
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.23, size = 421, normalized size = 5.69

| method | result |
|---|---|
| default | $-\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{i\sqrt{3}}{6}+\frac{1}{2},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\sqrt{x^3-1}}$ |
| trager | Expression too large to display |
| elliptic | $\frac{\sqrt{-\frac{1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}+\frac{1}{3-i\sqrt{3}}-\frac{i\sqrt{3}}{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}}\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{i\sqrt{3}+3}+\frac{i\sqrt{3}}{i\sqrt{3}+3}}\,\text{E}}{6\sqrt{x^3-1}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+8)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/
6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+1/9*I*(1/2
-1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((
x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2
+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I
*3^(1/2)))^(1/2),1/6*I*(1+I*3^(1/2))*3^(1/2)+1/3*I*3^(1/2),((3/2+1/2*I*3^(1
/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/9*I*(1/2+1/2*I*3^(1/2))*(-3/2-1/2*I*3^(1
/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(
1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/6*I*(1-I*3^(1
/2))*3^(1/2)-2/3*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(51) = 102$.
time = 1.45, size = 547, normalized size = 7.39

$$\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3+6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)-\frac{1}{216}\sqrt{3}\log\left(\frac{4\left(x^6+48x^5+186x^4-56x^3-6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64\right)}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}\right)+\frac{1}{54}\arctan\left(\frac{\left(x^3-12x^2-6x-10\right)\sqrt{x^3-1}}{6\left(x^4-x^3-x+1\right)}\right)-\frac{1}{54}\arctan\left(-\frac{\left(\sqrt{x^3-1}\left(x^2-8x+10\right)+\left(3\sqrt{3}\left(x^3+x^2-2x\right)-\sqrt{x^3-1}\left(x^2+10x-8\right)\right)\sqrt{\frac{x^6+48x^5+186x^4-56x^3+6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}}\right)}{3\left(x^3-3x^2+2\right)}\right)-\frac{1}{54}\arctan\left(-\frac{\sqrt{x^3-1}\left(x^2-8x+10\right)-\left(3\sqrt{3}\left(x^3+x^2-2x\right)+\sqrt{x^3-1}\left(x^2+10x-8\right)\right)\sqrt{\frac{x^6+48x^5+186x^4-56x^3-6\sqrt{3}\left(x^4+12x^3+12x^2-16x\right)\sqrt{x^3-1}-120x^2-96x+64}{x^6-6x^5+24x^4-56x^3+96x^2-96x+64}}}{3\left(x^3-3x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 + 186*x^4 - 56*x^3 + 6*sqrt(3)*(x^4 + 12*
x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24
*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)) - 1/216*sqrt(3)*log(4*(x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
 + 1/54*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x +
 1)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) + (3*sqrt(3)*(x^3 +
 x^2 - 2*x) - sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 + 186*x^4
- 56*x^3 + 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) - 120*x^2
 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64)))/(x^3 -
 3*x^2 + 2)) - 1/54*arctan(-1/3*(sqrt(x^3 - 1)*(x^2 - 8*x + 10) - (3*sqrt(3
```

```
)*(x^3 + x^2 - 2*x) + sqrt(x^3 - 1)*(x^2 + 10*x - 8))*sqrt((x^6 + 48*x^5 +
186*x^4 - 56*x^3 - 6*sqrt(3)*(x^4 + 12*x^3 + 12*x^2 - 16*x)*sqrt(x^3 - 1) -
 120*x^2 - 96*x + 64)/(x^6 - 6*x^5 + 24*x^4 - 56*x^3 + 96*x^2 - 96*x + 64))
)/(x^3 - 3*x^2 + 2))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+2)\left(x^2-2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+8)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)*(x**2 - 2*x + 4)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+8)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 8)*sqrt(x^3 - 1)), x)
```

**Mupad [B]**
time = 0.21, size = 533, normalized size = 7.20

$$\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{9\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)2\mathrm{i}}{9\left(-1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\sqrt{3}\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{3};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)2\mathrm{i}}{9\left(1+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(x^3 + 8)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/(9*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (3^(1/2)*((3^(1/2)
*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*ellipticPi(-(3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*1i)/3, asi
n((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)
```

```
^(1/2)) - (3^(1/2)*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*((3^(1
/2)*1i)/2 + 3/2)*1i)/3, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*2i)/(9*(3^(1/2)*1i + 1)*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

# 3.76 $\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(1+d^(1/3)*x)^2/(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctanh(1/3*
(d*x^3+1)^(1/2))/d^(2/3)-1/18*arctan((1+d^(1/3)*x)*3^(1/2)/(d*x^3+1)^(1/2))
/d^(2/3)*3^(1/2)
```

**Rubi** [A]
time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {499, 455, 65, 212, 2163, 2170, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(\sqrt[3]{d}\,x+1\right)}{\sqrt{dx^3+1}}\right)}{6\sqrt{3}\,d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{d}\,x+1\right)^2}{3\sqrt{dx^3+1}}\right)}{18d^{2/3}}-\frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{dx^3+1}\right)}{18d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] -1/6*ArcTan[(Sqrt[3]*(1 + d^(1/3)*x))/Sqrt[1 + d*x^3]]/(Sqrt[3]*d^(2/3)) +
ArcTanh[(1 + d^(1/3)*x)^2/(3*Sqrt[1 + d*x^3])]/(18*d^(2/3)) - ArcTanh[Sqrt[
1 + d*x^3]/3]/(18*d^(2/3))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 499

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[d*(q/(4*b)), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{(8-dx^3)\sqrt{1+dx^3}}\,dx = -\frac{\int \frac{2d^{2/3}-2dx-d^{4/3}x^2}{\left(4+2\sqrt[3]{d}\,x+d^{2/3}x^2\right)\sqrt{1+dx^3}}\,dx}{12d} + \frac{\int \frac{1+\sqrt[3]{d}\,x}{\left(2-\sqrt[3]{d}\,x\right)\sqrt{1+dx^3}}\,dx}{12\sqrt[3]{d}} - \frac{1}{4}\sqrt[3]{d}\int \frac{}{(8}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{\left(1+\sqrt[3]{d}\,x\right)^2}{\sqrt{1+dx^3}}\right)}{6d^{2/3}} - \frac{1}{12}\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8-dx)\sqrt{1+dx}}\,dx, x\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{\right.}}{6d^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{d}\,x\right)}{\sqrt{1+dx^3}}\right)}{6\sqrt{3}\,d^{2/3}} + \frac{\tanh^{-1}\left(\frac{\left(1+\sqrt[3]{d}\,x\right)^2}{3\sqrt{1+dx^3}}\right)}{18d^{2/3}} - \frac{\tanh^{-1}\left(\frac{1}{3}\sqrt{1+dx^3}\right)}{18d^{2/3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.03, size = 32, normalized size = 0.31

$$\frac{1}{16}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-dx^3,\frac{dx^3}{8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((8 - d*x^3)*Sqrt[1 + d*x^3]),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -(d*x^3), (d*x^3)/8])/16
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.39, size = 383, normalized size = 3.72

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8\right)}\frac{\left(-d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^{2}\right)^{\frac{1}{3}}+\left(-d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$ |
| elliptic | $-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(d\_Z^{3}-8\right)}\frac{\left(-d^{2}\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{id\left(2x+\frac{-i\sqrt{3}\left(-d^{2}\right)^{\frac{1}{3}}+\left(-d^{2}\right)^{\frac{1}{3}}}{d}\right)}{\left(-d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{d\left(x-\frac{\left(-d^{2}\right)^{\frac{1}{3}}}{d}\right)}{-3\left(-d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/27*I/d^3*2^(1/2)*sum(1/_alpha*(-d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)*(d*(x-1/d*(-d^2)^(1/3))/(-
3*(-d^2)^(1/3)+I*3^(1/2)*(-d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3^(1/2)
*(-d^2)^(1/3)+(-d^2)^(1/3)))/(-d^2)^(1/3))^(1/2)/(d*x^3+1)^(1/2)*(I*(-d^2)^
(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-d^2)^(2/3)+2*_alpha^2*d^2-(-d^2)^(1/3)*_
alpha*d-(-d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-d^2)^(1/3))*3^(1/2)*d/(-d^2)^(1/3))^(1/2),-1/18/d*(2*I*(-d^2)^
(1/3)*_alpha^2*3^(1/2)*d-I*(-d^2)^(2/3)*_alpha*3^(1/2)-3*(-d^2)^(2/3)*_alph
a+I*3^(1/2)*d-3*d),(I*3^(1/2)/d*(-d^2)^(1/3)/(-3/2/d*(-d^2)^(1/3)+1/2*I*3^(
1/2)/d*(-d^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*d-8))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(73) = 146$.
time = 1.73, size = 497, normalized size = 4.83

$$\frac{2\sqrt{3}\left(d^2\right)^{\frac{1}{6}}d\arctan\left(-\frac{\left(9\sqrt{3}\,d^3x^5-\sqrt{3}\left(d^2x^6-40\,dx^3-32\right)\left(d^2\right)^{\frac{2}{3}}+3\sqrt{3}\left(5\,d^2x^4+8\,dx\right)\left(d^2\right)^{\frac{1}{3}}\right)\sqrt{dx^3+1}\left(d^2\right)^{\frac{1}{6}}}{9\left(d^4x^7-7\,d^3x^4-8\,d^2x\right)}\right)+2\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{d^4x^9+318\,d^3x^6+1200\,d^2x^3+18\left(5\,d^2x^7+64\,dx^4+32\,x\right)\left(d^2\right)^{\frac{2}{3}}+6\left(7\,d^3x^6+152\,d^2x^3+\left(d^2x^7+80\,dx^4+160\,x\right)\left(d^2\right)^{\frac{2}{3}}+6\left(5\,d^2x^5+32\,dx^2\right)\left(d^2\right)^{\frac{1}{3}}+64\,d\right)\sqrt{dx^3+1}+18\left(d^3x^8+38\,d^2x^5+64\,dx^2\right)\left(d^2\right)^{\frac{1}{3}}+640\,d}{d^3x^9-24\,d^2x^6+192\,dx^3-512}\right)-\left(d^2\right)^{\frac{2}{3}}\log\left(\frac{d^4x^9-276\,d^3x^6-1608\,d^2x^3-18\left(d^2x^7-52\,dx^4-80\,x\right)\left(d^2\right)^{\frac{2}{3}}-6\left(4\,d^3x^6+164\,d^2x^3+\left(d^2x^7-28\,dx^4-272\,x\right)\left(d^2\right)^{\frac{2}{3}}-24\left(d^2x^5+dx^2\right)\left(d^2\right)^{\frac{1}{3}}+160\,d\right)\sqrt{dx^3+1}+18\left(d^3x^8+20\,d^2x^5-8\,dx^2\right)\left(d^2\right)^{\frac{1}{3}}-1088\,d}{d^3x^9-24\,d^2x^6+192\,dx^3-512}\right)}{108\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/108*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(-1/9*(9*sqrt(3)*d^3*x^5 - sqrt(3)*(d^
2*x^6 - 40*d*x^3 - 32)*(d^2)^(2/3) + 3*sqrt(3)*(5*d^2*x^4 + 8*d*x)*(d^2)^(1
/3))*sqrt(d*x^3 + 1)*(d^2)^(1/6)/(d^4*x^7 - 7*d^3*x^4 - 8*d^2*x)) + 2*(d^2)
^(2/3)*log((d^4*x^9 + 318*d^3*x^6 + 1200*d^2*x^3 + 18*(5*d^2*x^7 + 64*d*x^4
 + 32*x)*(d^2)^(2/3) + 6*(7*d^3*x^6 + 152*d^2*x^3 + (d^2*x^7 + 80*d*x^4 + 1
60*x)*(d^2)^(2/3) + 6*(5*d^2*x^5 + 32*d*x^2)*(d^2)^(1/3) + 64*d)*sqrt(d*x^3
 + 1) + 18*(d^3*x^8 + 38*d^2*x^5 + 64*d*x^2)*(d^2)^(1/3) + 640*d)/(d^3*x^9
- 24*d^2*x^6 + 192*d*x^3 - 512)) - (d^2)^(2/3)*log((d^4*x^9 - 276*d^3*x^6 -
 1608*d^2*x^3 - 18*(d^2*x^7 - 52*d*x^4 - 80*x)*(d^2)^(2/3) - 6*(4*d^3*x^6 +
 164*d^2*x^3 + (d^2*x^7 - 28*d*x^4 - 272*x)*(d^2)^(2/3) - 24*(d^2*x^5 + d*x
^2)*(d^2)^(1/3) + 160*d)*sqrt(d*x^3 + 1) + 18*(d^3*x^8 + 20*d^2*x^5 - 8*d*x
^2)*(d^2)^(1/3) - 1088*d)/(d^3*x^9 - 24*d^2*x^6 + 192*d*x^3 - 512)))/d^2
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{dx^3\sqrt{dx^3+1}-8\sqrt{dx^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8)/(d*x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(d*x**3*sqrt(d*x**3 + 1) - 8*sqrt(d*x**3 + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8)/(d*x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + 1)*(d*x^3 - 8)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{d\,x^3+1}\,\left(d\,x^3-8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)),x)
```

```
[Out] -int(x/((d*x^3 + 1)^(1/2)*(d*x^3 - 8)), x)
```

## 3.77 $\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx$

**Optimal**. Leaf size=81

$$\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}$$

```
[Out] 1/4*arctan((1-(-3*x^2+1)^(1/3))/x)+1/12*arctanh(1/3*x*3^(1/2))*3^(1/2)-1/12
*arctanh(1/9*(1-(-3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{404\}$

$$\frac{1}{4}\text{ArcTan}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] ArcTan[(1 - (1 - 3*x^2)^(1/3))/x]/4 + ArcTanh[x/Sqrt[3]]/(4*Sqrt[3]) - ArcT
anh[(1 - (1 - 3*x^2)^(1/3))^2/(3*Sqrt[3]*x)]/(4*Sqrt[3])
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-3x^2}\,(3-x^2)}\,dx=\frac{1}{4}\tan^{-1}\left(\frac{1-\sqrt[3]{1-3x^2}}{x}\right)+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.95, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)}{\sqrt[3]{1-3x^2}\,(-3+x^2)\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};3x^2,\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};3x^2,\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};3x^2,\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - 3*x^2)^(1/3)*(3 - x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3])/((1 - 3*x^2)^(1/3)*(-3 + x^
2)*(9*AppellF1[1/2, 1/3, 1, 3/2, 3*x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, 3*x^2, x^2/3] + 3*AppellF1[3/2, 4/3, 1, 5/2, 3*x^2, x^2/3])))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.62, size = 651, normalized size = 8.04

| method | result |
|---|---|
| trager | $-48\ln\left(-\frac{18432\,\mathrm{RootOf}\left(2304\_Z^4+48\_Z^2+1\right)^5\left(-3x^2+1\right)^{\frac{1}{3}}x-36864\,\mathrm{RootOf}\left(2304\_Z^4+48\_Z^2+1\right)^5x+768\,\mathrm{RootOf}\left(230\right.}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+1)^(1/3)/(-x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -48*ln(-(18432*RootOf(2304*_Z^4+48*_Z^2+1)^5*(-3*x^2+1)^(1/3)*x-36864*RootO
f(2304*_Z^4+48*_Z^2+1)^5*x+768*RootOf(2304*_Z^4+48*_Z^2+1)^3*(-3*x^2+1)^(1/
3)*x-2304*RootOf(2304*_Z^4+48*_Z^2+1)^3*x-48*RootOf(2304*_Z^4+48*_Z^2+1)^2*
x^2-96*RootOf(2304*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)-48*RootOf(2304*_Z^4+4
8*_Z^2+1)^2-32*RootOf(2304*_Z^4+48*_Z^2+1)*x-2*(-3*x^2+1)^(2/3)-x^2-1)/(x^2
-3))*RootOf(2304*_Z^4+48*_Z^2+1)^3+RootOf(2304*_Z^4+48*_Z^2+1)*ln(-(9216*Ro
otOf(2304*_Z^4+48*_Z^2+1)^5*(-3*x^2+1)^(1/3)*x-18432*RootOf(2304*_Z^4+48*_Z
^2+1)^5*x+576*RootOf(2304*_Z^4+48*_Z^2+1)^3*(-3*x^2+1)^(1/3)*x-768*RootOf(2
304*_Z^4+48*_Z^2+1)^3*x+24*RootOf(2304*_Z^4+48*_Z^2+1)^2*x^2+48*RootOf(2304
*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)+8*RootOf(2304*_Z^4+48*_Z^2+1)*(-3*x^2+1
)^(1/3)*x+24*RootOf(2304*_Z^4+48*_Z^2+1)^2-(-3*x^2+1)^(2/3)+(-3*x^2+1)^(1/3
))/(x^2-3))-ln(-(18432*RootOf(2304*_Z^4+48*_Z^2+1)^5*(-3*x^2+1)^(1/3)*x-368
64*RootOf(2304*_Z^4+48*_Z^2+1)^5*x+768*RootOf(2304*_Z^4+48*_Z^2+1)^3*(-3*x^
2+1)^(1/3)*x-2304*RootOf(2304*_Z^4+48*_Z^2+1)^3*x-48*RootOf(2304*_Z^4+48*_Z
^2+1)^2*x^2-96*RootOf(2304*_Z^4+48*_Z^2+1)^2*(-3*x^2+1)^(1/3)-48*RootOf(230
4*_Z^4+48*_Z^2+1)^2-32*RootOf(2304*_Z^4+48*_Z^2+1)*x-2*(-3*x^2+1)^(2/3)-x^2
-1)/(x^2-3))*RootOf(2304*_Z^4+48*_Z^2+1)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1792 vs. $2(59) = 118$.
time = 2.21, size = 1792, normalized size = 22.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11
 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(
3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x^8 + 94152*x^6 - 13581*x^4 + 432*x^2
 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x^10 - 107*x^8 - 7262*x^6 + 2322*x^4
- 243*x^2 + 9) - 48*sqrt(3)*(5*x^9 - 245*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 +
 1)^(2/3) - 12*sqrt(3)*(29*x^11 + 293*x^9 - 2670*x^7 + 4986*x^5 - 1215*x^3
+ 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(49*x^10 - 5043*x^8 + 3658*x^6 + 378*x
^4 - 171*x^2 + 9) - 2*sqrt(3)*(x^11 + 917*x^9 - 40566*x^7 + 15786*x^5 - 204
3*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt((x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(
x^5 + 13*x^3) - 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-
3*x^2 + 1)^(2/3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2
 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)
*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2106*x^5 - 396*x^3 + 27*x) - 3*sqrt(3
)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4 - 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) -
3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8 + 4260*x^6 - 14337*x^4 + 2862*x^2 -
 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^11 - 1591*x^9 + 42426*x^7 - 15102*x^5
 + 1269*x^3 - 27*x) - 6*sqrt(3)*(27*x^10 + 2307*x^8 + 4574*x^6 - 2538*x^4 +
 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^12 - 4986*x^10 + 327519*x^8 - 159660*
x^6 + 25839*x^4 - 2106*x^2 + 81)) + 1/72*sqrt(6)*sqrt(3)*sqrt(2)*arctan(1/9
*(36*sqrt(6)*sqrt(3)*sqrt(2)*(3*x^11 - 1117*x^9 + 3918*x^7 - 1866*x^5 + 255
*x^3 - 9*x) + sqrt(3)*(sqrt(6)*sqrt(3)*sqrt(2)*(x^12 + 2184*x^10 - 211215*x
^8 + 94152*x^6 - 13581*x^4 + 432*x^2 + 27) + 12*(sqrt(6)*sqrt(3)*sqrt(2)*(x
^10 - 107*x^8 - 7262*x^6 + 2322*x^4 - 243*x^2 + 9) + 48*sqrt(3)*(5*x^9 - 24
5*x^7 + 183*x^5 - 15*x^3))*(-3*x^2 + 1)^(2/3) + 12*sqrt(3)*(29*x^11 + 293*x
^9 - 2670*x^7 + 4986*x^5 - 1215*x^3 + 81*x) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(4
9*x^10 - 5043*x^8 + 3658*x^6 + 378*x^4 - 171*x^2 + 9) + 2*sqrt(3)*(x^11 + 9
17*x^9 - 40566*x^7 + 15786*x^5 - 2043*x^3 + 81*x))*(-3*x^2 + 1)^(1/3))*sqrt
((x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)*
sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sqr
t(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 - 9
*x^4 + 27*x^2 - 27)) + 12*(2*sqrt(6)*sqrt(3)*sqrt(2)*(35*x^9 - 4860*x^7 + 2
```

```
106*x^5 - 396*x^3 + 27*x) + 3*sqrt(3)*(x^10 + 589*x^8 + 3946*x^6 - 774*x^4
- 27*x^2 + 9))*(-3*x^2 + 1)^(2/3) + 3*sqrt(3)*(x^12 + 3150*x^10 + 77991*x^8
 + 4260*x^6 - 14337*x^4 + 2862*x^2 - 135) - 6*(sqrt(6)*sqrt(3)*sqrt(2)*(x^1
1 - 1591*x^9 + 42426*x^7 - 15102*x^5 + 1269*x^3 - 27*x) + 6*sqrt(3)*(27*x^1
0 + 2307*x^8 + 4574*x^6 - 2538*x^4 + 279*x^2 - 9))*(-3*x^2 + 1)^(1/3))/(x^1
2 - 4986*x^10 + 327519*x^8 - 159660*x^6 + 25839*x^4 - 2106*x^2 + 81)) - 1/2
88*sqrt(6)*sqrt(2)*log(12*(x^6 - 93*x^4 + 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3)
- 117*x^2 - 2*(4*sqrt(6)*sqrt(2)*x^3 - 3*x^4 - 18*x^2 + 9)*(-3*x^2 + 1)^(2/
3) + (6*x^4 - sqrt(6)*sqrt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2
 + 1)^(1/3) + 9)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/288*sqrt(6)*sqrt(2)*log(1
2*(x^6 - 93*x^4 - 4*sqrt(6)*sqrt(2)*(x^5 + 13*x^3) - 117*x^2 + 2*(4*sqrt(6)
*sqrt(2)*x^3 + 3*x^4 + 18*x^2 - 9)*(-3*x^2 + 1)^(2/3) + (6*x^4 + sqrt(6)*sq
rt(2)*(x^5 - 10*x^3 - 27*x) - 108*x^2 - 18)*(-3*x^2 + 1)^(1/3) + 9)/(x^6 -
9*x^4 + 27*x^2 - 27)) + 1/72*sqrt(3)*log(-(x^12 + 2598*x^10 + 55143*x^8 + 1
14228*x^6 - 22113*x^4 - 7290*x^2 + 8*(3*x^10 + 576*x^8 + 5598*x^6 + 5832*x^
4 - 729*x^2 - sqrt(3)*(41*x^9 + 1368*x^7 + 4482*x^5 + 864*x^3 - 243*x))*(-3
*x^2 + 1)^(2/3) - 4*sqrt(3)*(25*x^11 + 2359*x^9 + 15426*x^7 + 6966*x^5 - 43
47*x^3 + 243*x) - 4*(84*x^10 + 4536*x^8 + 20880*x^6 + 5832*x^4 - 2916*x^2 -
 sqrt(3)*(x^11 + 521*x^9 + 7362*x^7 + 10746*x^5 - 1971*x^3 - 243*x))*(-3*x^
2 + 1)^(1/3) + 729)/(x^12 - 18*x^10 + 135*x^8 - 540*x^6 + 1215*x^4 - 1458*x
^2 + 729))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{1-3x^2}-3\sqrt[3]{1-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+1)**(1/3)/(-x**2+3),x)
[Out] -Integral(1/(x**2*(1 - 3*x**2)**(1/3) - 3*(1 - 3*x**2)**(1/3)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+1)^(1/3)/(-x^2+3),x, algorithm="giac")
[Out] integrate(-1/((x^2 - 3)*(-3*x^2 + 1)^(1/3)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^2-3\right)\,{\left(1-3\,x^2\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 - 3)*(1 - 3*x^2)^(1/3)),x)
[Out] -int(1/((x^2 - 3)*(1 - 3*x^2)^(1/3)), x)
```

# 3.78 $\int \frac{1}{(3+x^2)\sqrt[3]{1+3x^2}}\,dx$

**Optimal**. Leaf size=81

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

```
[Out] -1/4*arctanh((1-(3*x^2+1)^(1/3))/x)+1/12*arctan(1/3*x*3^(1/2))*3^(1/2)+1/12
*arctan(1/9*(1-(3*x^2+1)^(1/3))^2/x*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {403}

$$\frac{\text{ArcTan}\left(\frac{\left(1-\sqrt[3]{3x^2+1}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{3x^2+1}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(1 - (1 + 3*x^2)^(1/3))^2/(3*Sqrt[3]
*x)]/(4*Sqrt[3]) - ArcTanh[(1 - (1 + 3*x^2)^(1/3))/x]/4
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\left(3+x^2\right)\sqrt[3]{1+3x^2}}\,dx=\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+3x^2}\right)^2}{3\sqrt{3}\,x}\right)}{4\sqrt{3}}-\frac{1}{4}\tanh^{-1}\left(\frac{1-\sqrt[3]{1+3x^2}}{x}\right)$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.61, size = 126, normalized size = 1.56

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)}{(3+x^2)\sqrt[3]{1+3x^2}\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-3x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-3x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x^2)*(1 + 3*x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2])/((3 + x^2)*(1 + 3*x^2)^
(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, -3*x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/
2, 1/3, 2, 5/2, -3*x^2, -1/3*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -3*x^2, -1
/3*x^2])))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.18, size = 239, normalized size = 2.95

| method | result |
|---|---|
| trager | $-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2+3\right)\left(3x^2+1\right)^{\frac{1}{3}}x-\operatorname{RootOf}\left(\_Z^2+3\right)x^2+4\left(3x^2+1\right)^{\frac{2}{3}}+2\operatorname{RootOf}\left(\_Z^2+3\right)\left(3x^2+1\right)^{\frac{1}{3}}-2\left(3x^2+1\right.}{x^2+3}\right.}{12}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3)/(3*x^2+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*RootOf(_Z^2+3)*ln(-(2*RootOf(_Z^2+3)*(3*x^2+1)^(1/3)*x-RootOf(_Z^2+3)
*x^2+4*(3*x^2+1)^(2/3)+2*RootOf(_Z^2+3)*(3*x^2+1)^(1/3)-2*(3*x^2+1)^(1/3)*x
-4*RootOf(_Z^2+3)*x-x^2-2*(3*x^2+1)^(1/3)+RootOf(_Z^2+3)-4*x+1)/(x^2+3))+1/
8*ln(-(2*(3*x^2+1)^(2/3)+2*(3*x^2+1)^(1/3)*x+x^2+2*(3*x^2+1)^(1/3)+4*x-1)/(
x^2+3))-1/24*ln(-(2*(3*x^2+1)^(2/3)+2*(3*x^2+1)^(1/3)*x+x^2+2*(3*x^2+1)^(1/
3)+4*x-1)/(x^2+3))*RootOf(_Z^2+3)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(59) = 118.
time = 1.45, size = 345, normalized size = 4.26

$$\frac{1}{36}\sqrt{3}\arctan\left(\frac{4\sqrt{3}(3x^4-10x^3-36x^2+18x+9)(3x^2+1)^{\frac{2}{3}}-4\sqrt{3}(x^5+15x^4-26x^3-54x^2+9x-9)(3x^2+1)^{\frac{1}{3}}+\sqrt{3}(x^6-2x^5-105x^4-28x^3+63x^2+126x+9)}{x^6+126x^5-225x^4-828x^3-81x^2-162x+81}\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{2\left(2\sqrt{3}(23x^3+9x)(3x^2+1)^{\frac{2}{3}}+\sqrt{3}(x^5-80x^3-9x)(3x^2+1)^{\frac{1}{3}}+\sqrt{3}(11x^5+10x^3-9x)\right)}{x^6-657x^4-189x^2-27}\right)+\frac{1}{24}\log\left(\frac{x^6+108x^5+549x^4+360x^3+99x^2+6(3x^4+32x^3+42x^2+3)(3x^2+1)^{\frac{2}{3}}+6(x^5+27x^4+70x^3+18x^2+9x+3)(3x^2+1)^{\frac{1}{3}}+108x-9}{x^6+9x^4+27x^2+27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(3)*arctan((4*sqrt(3)*(3*x^4 - 10*x^3 - 36*x^2 + 18*x + 9)*(3*x^2
+ 1)^(2/3) - 4*sqrt(3)*(x^5 + 15*x^4 - 26*x^3 - 54*x^2 + 9*x - 9)*(3*x^2 +
1)^(1/3) + sqrt(3)*(x^6 - 2*x^5 - 105*x^4 - 28*x^3 + 63*x^2 + 126*x + 9))/(
x^6 + 126*x^5 - 225*x^4 - 828*x^3 - 81*x^2 - 162*x + 81)) - 1/36*sqrt(3)*ar
ctan(2*(2*sqrt(3)*(23*x^3 + 9*x)*(3*x^2 + 1)^(2/3) + sqrt(3)*(x^5 - 80*x^3
- 9*x)*(3*x^2 + 1)^(1/3) + sqrt(3)*(11*x^5 + 10*x^3 - 9*x))/(x^6 - 657*x^4
- 189*x^2 - 27)) + 1/24*log((x^6 + 108*x^5 + 549*x^4 + 360*x^3 + 99*x^2 + 6
*(3*x^4 + 32*x^3 + 42*x^2 + 3)*(3*x^2 + 1)^(2/3) + 6*(x^5 + 27*x^4 + 70*x^3
 + 18*x^2 + 9*x + 3)*(3*x^2 + 1)^(1/3) + 108*x - 9)/(x^6 + 9*x^4 + 27*x^2 +
 27))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2+3)\sqrt[3]{3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3)/(3*x**2+1)**(1/3),x)
```

```
[Out] Integral(1/((x**2 + 3)*(3*x**2 + 1)**(1/3)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3)/(3*x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 1)^(1/3)*(x^2 + 3)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^2+3)\,(3\,x^2+1)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)),x)
```

```
[Out] int(1/((x^2 + 3)*(3*x^2 + 1)^(1/3)), x)
```

# 3.79 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

**Optimal**. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(1/x*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1
/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

**Rubi** [A]

time = 0.01, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {402}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\,\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\,\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.57, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 11.55, size = 938, normalized size = 8.30

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 938 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*RootOf(_Z^6+108)*ln(-(-72*RootOf(_Z^6+108)^4*x^5+225*RootOf(_Z^6+108)^
4*x^4+72*RootOf(_Z^6+108)^4*x^3+1296*RootOf(_Z^6+108)*x^5-4050*RootOf(_Z^6+
108)*x^4-1296*RootOf(_Z^6+108)*x^3+3402*RootOf(_Z^6+108)*x^2-189*x^2*RootOf
(_Z^6+108)^4-1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2+1)^(2/3)*x^3-3888*(-x^2+1)^
(2/3)*x^2-3888*(-x^2+1)^(2/3)*x+6*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^5-108
*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^4+RootOf(_Z^6+108)^4*x^6-18*RootOf(_Z^
6+108)*x^6+144*RootOf(_Z^6+108)^5*(-x^2+1)^(1/3)*x^3+108*RootOf(_Z^6+108)^5
*(-x^2+1)^(1/3)*x^2-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5-54*RootOf(_Z^6
+108)^5*(-x^2+1)^(1/3)*x+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-864*Root
Of(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2
+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x-486*RootOf(_Z^6+108)+27*RootOf(_Z^
6+108)^4)/(x^2+3)^3)+1/432*ln((RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4
*x^5+225*RootOf(_Z^6+108)^4*x^4-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+72
*RootOf(_Z^6+108)^4*x^3+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-189*x^2*R
ootOf(_Z^6+108)^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3+648*(-x^2+1)^(2
/3)*x^4-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2-4536*(-x^2+1)^(2/3)*x^3+2
7*RootOf(_Z^6+108)^4+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+1944*(-x^2+1)^
(2/3)*x^2+1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+108)^4+1/72*ln((Roo
tOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5+225*RootOf(_Z^6+108)^4*x^4-36
*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+72*RootOf(_Z^6+108)^4*x^3+648*RootOf
(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-189*x^2*RootOf(_Z^6+108)^4-864*RootOf(_Z^6+
```

```
108)^2*(-x^2+1)^(1/3)*x^3+648*(-x^2+1)^(2/3)*x^4-648*RootOf(_Z^6+108)^2*(-x
^2+1)^(1/3)*x^2-4536*(-x^2+1)^(2/3)*x^3+27*RootOf(_Z^6+108)^4+324*RootOf(_Z
^6+108)^2*(-x^2+1)^(1/3)*x+1944*(-x^2+1)^(2/3)*x^2+1944*(-x^2+1)^(2/3)*x)/(
x^2+3)^3)*RootOf(_Z^6+108)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

**Fricas** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1943 vs. $2(81) = 162$.
time = 2.09, size = 1943, normalized size = 17.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
```

```
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

**Sympy [F]**

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

**Giac [F]**

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(1-x^2\right)}^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

# 3.80 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

**Optimal**. Leaf size=109

$$-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/1
2*arctanh(1/x*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{401\}$

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\text{ArcTan}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx=-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\right.}{}\right)}{2\ 2^{2/3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 4.64, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)+2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

**Maple** [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^2+3)(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. $2(77) = 154$.
time = 2.19, size = 1685, normalized size = 15.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2592*432^(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7
- 373248*x^5 - 69984*x^3 - sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 +
```

6030*x^7 + 7182*x^5 + 2511*x^3 + 243*x) + 3*432^(1/6)*sqrt(3)*(x^12 + 924*
x^10 - 33363*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) + (432^(5/6)*sqr
t(3)*(x^10 - 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) + 432*sqrt(3)*2^(1/3)*(13
*x^9 - 177*x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) + 36*(96*x^10 - 4032*x^
8 - 2592*x^6 + sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 2
43*x))*(x^2 + 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^
2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 -
18*x^2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 +
9*x))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8
+ 1206*x^6 + 756*x^4 + 81*x^2) + 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 207
0*x^5 - 756*x^3 - 81*x))*(x^2 + 1)^(2/3) + 18*sqrt(3)*(x^12 + 1422*x^10 + 2
1447*x^8 + 27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^
11 - 681*x^9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) + 3888*sqrt(3)*2^(1/
3)*(x^10 + 44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178
*x^10 + 46791*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/2592*432^
(5/6)*sqrt(3)*arctan(-1/54*(2592*x^11 - 393984*x^9 - 699840*x^7 - 373248*x^
5 - 69984*x^3 + sqrt(6)*(18*sqrt(3)*2^(2/3)*(19*x^11 + 111*x^9 + 6030*x^7 +
7182*x^5 + 2511*x^3 + 243*x) - 3*432^(1/6)*sqrt(3)*(x^12 + 924*x^10 - 3336
3*x^8 - 60912*x^6 - 36693*x^4 - 8748*x^2 - 729) - (432^(5/6)*sqrt(3)*(x^10
- 78*x^8 - 720*x^6 - 594*x^4 - 81*x^2) - 432*sqrt(3)*2^(1/3)*(13*x^9 - 177*
x^7 - 153*x^5 - 27*x^3))*(x^2 + 1)^(2/3) - 36*(96*x^10 - 4032*x^8 - 2592*x^
6 - sqrt(3)*(x^11 + 369*x^9 - 3654*x^7 - 5454*x^5 - 2187*x^3 - 243*x))*(x^2
+ 1)^(1/3))*sqrt((2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3
)*(432^(5/6)*(x^3 + x) - 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sq
rt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6
- 9*x^4 + 27*x^2 - 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 276*x^8 + 1206*x^6
+ 756*x^4 + 81*x^2) - 432^(1/6)*sqrt(3)*(31*x^9 - 1620*x^7 - 2070*x^5 - 756
*x^3 - 81*x))*(x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 + 1422*x^10 + 21447*x^8 +
27108*x^6 + 16767*x^4 + 6318*x^2 + 729) + (432^(5/6)*sqrt(3)*(x^11 - 681*x^
9 + 4338*x^7 + 6102*x^5 + 2349*x^3 + 243*x) - 3888*sqrt(3)*2^(1/3)*(x^10 +
44*x^8 + 94*x^6 + 60*x^4 + 9*x^2))*(x^2 + 1)^(1/3))/(x^12 - 2178*x^10 + 467
91*x^8 + 83268*x^6 + 47871*x^4 + 10206*x^2 + 729)) + 1/5184*432^(5/6)*log(-
(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2
) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 +
1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 -
9*x^4 + 27*x^2 - 27)) - 1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63
*x^2 + 27) - 864*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 43
2*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x
^3 + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10
368*432^(5/6)*log(31104*(2*2^(2/3)*(x^6 - 57*x^4 - 117*x^2 - 27) + (x^2 + 1
)^(2/3)*(432^(5/6)*(x^3 + x) + 24*2^(1/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^
2 + sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/3) - 8*432^(1/6)*(x^5 + 18*x^3 + 9*x)
)/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*log(31104*(2*2^(2/3)*(x^
6 - 57*x^4 - 117*x^2 - 27) - (x^2 + 1)^(2/3)*(432^(5/6)*(x^3 + x) - 24*2^(1
/3)*(x^4 + 9*x^2)) - 8*(6*x^4 - 18*x^2 - sqrt(3)*(x^5 - 9*x))*(x^2 + 1)^(1/

```
3) + 8*432^(1/6)*(x^5 + 18*x^3 + 9*x))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1} - 3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^2+1\right)^{1/3}\,\left(x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

# 3.81 $\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)x^2+x^3}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{2\sqrt{x}\,\sqrt{a^2-(1+a^2)\,x+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)\,x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

```
[Out] -2*arctan((1-a)*x^(1/2)/(a^2-(a^2+1)*x+x^2)^(1/2))*x^(1/2)*(a^2-(a^2+1)*x+x
^2)^(1/2)/(1-a)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2)
```

**Rubi** [A]
time = 0.54, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2081, 6865, 1712, 211}

$$-\frac{2\sqrt{x}\,\sqrt{-(a^2+1)\,x+a^2+x^2}\,\text{ArcTan}\left(\frac{(1-a)\sqrt{x}}{\sqrt{-(a^2+1)\,x+a^2+x^2}}\right)}{(1-a)\sqrt{-(a^2+1)\,x^2+a^2x+x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a^2 - (1 + a^2)*x + x^2]*ArcTan[((1 - a)*Sqrt[x])/Sqrt[a^2
 - (1 + a^2)*x + x^2]])/((1 - a)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{a+x}{(-a+x)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a^2-(1+a^2)\,x+x^2}\right)\int \frac{a+x}{\sqrt{x}\,(-a+x)\sqrt{a^2-(1+a^2)\,x+}}}{\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{a^2-(1+a^2)\,x+x^2}\right)\text{Subst}\left(\int \frac{a+x^2}{(-a+x^2)\sqrt{a^2+(-1-}}\right.}{\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

$$= \frac{\left(2a\sqrt{x}\,\sqrt{a^2-(1+a^2)\,x+x^2}\right)\text{Subst}\left(\int \frac{1}{-a-(-2a^2-a(-1-a^2))x^2}\right.}{\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

$$= -\frac{2\sqrt{x}\,\sqrt{a^2-(1+a^2)\,x+x^2}\,\tan^{-1}\left(\frac{(1-a)\sqrt{x}}{\sqrt{a^2-(1+a^2)\,x+x^2}}\right)}{(1-a)\sqrt{a^2x-(1+a^2)\,x^2+x^3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 18.69, size = 159, normalized size = 1.83

$$-\frac{2i(a^2-x)^{3/2}\sqrt{\frac{-1+x}{-a^2+x}}\sqrt{\frac{x}{-a^2+x}}\left((1+a)F\left(i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)-2\Pi\left(\frac{-1+a}{a};i\sinh^{-1}\left(\frac{\sqrt{-a^2}}{\sqrt{a^2-x}}\right)|1-\frac{1}{a^2}\right)\right)}{(-1+a)\sqrt{-a^2}\sqrt{(-1+x)x\,(-a^2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/((-a + x)*Sqrt[a^2*x - (1 + a^2)*x^2 + x^3]),x]
```

```
[Out] ((-2*I)*(a^2 - x)^(3/2)*Sqrt[(-1 + x)/(-a^2 + x)]*Sqrt[x/(-a^2 + x)]*((1 +
a)*EllipticF[I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)] - 2*EllipticP
i[(-1 + a)/a, I*ArcSinh[Sqrt[-a^2]/Sqrt[a^2 - x]], 1 - a^(-2)]))/((-1 + a)*
Sqrt[-a^2]*Sqrt[(-1 + x)*x*(-a^2 + x)])
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.10, size = 206, normalized size = 2.37

| method | result |
|---|---|

| | |
|---|---|
| default | $-\frac{2a^2\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{-1+x}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,\text{EllipticF}\left(\sqrt{-\frac{-a^2+x}{a^2}},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}-\frac{4a^3\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{-1+x}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,\text{Ellipti}}{\sqrt{-a^2x^2+a^2x+}}$ |
| elliptic | $-\frac{2a^2\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{-1+x}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,\text{EllipticF}\left(\sqrt{-\frac{-a^2+x}{a^2}},\sqrt{\frac{a^2}{a^2-1}}\right)}{\sqrt{-a^2x^2+a^2x+x^3-x^2}}-\frac{4a^3\sqrt{-\frac{-a^2+x}{a^2}}\,\sqrt{\frac{-1+x}{a^2-1}}\,\sqrt{\frac{x}{a^2}}\,\text{Ellipti}}{\sqrt{-a^2x^2+a^2x+}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2
+a^2*x+x^3-x^2)^(1/2)*EllipticF((-(-a^2+x)/a^2)^(1/2),(a^2/(a^2-1))^(1/2))-
4*a^3*(-(-a^2+x)/a^2)^(1/2)*((-1+x)/(a^2-1))^(1/2)*(x/a^2)^(1/2)/(-a^2*x^2+
a^2*x+x^3-x^2)^(1/2)/(a^2-a)*EllipticPi((-(-a^2+x)/a^2)^(1/2),a^2/(a^2-a),(
a^2/(a^2-1))^(1/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

**Fricas [A]**
time = 1.22, size = 85, normalized size = 0.98

$$\frac{\arctan\left(\frac{\sqrt{a^2x-(a^2+1)x^2+x^3}\,(a^2-2\,(a^2-a+1)x+x^2)}{2\,((a-1)x^3-(a^3-a^2+a-1)x^2+(a^3-a^2)x)}\right)}{a-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a^2 - 2*(a^2 - a + 1)*x + x^2
)/((a - 1)*x^3 - (a^3 - a^2 + a - 1)*x^2 + (a^3 - a^2)*x))/(a - 1)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+x}{\sqrt{x\left(-a^2+x\right)\left(x-1\right)}\,\left(-a+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a**2*x-(a**2+1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x)/(sqrt(x*(-a**2 + x)*(x - 1))*(-a + x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(-a+x)/(a^2*x-(a^2+1)*x^2+x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(a + x)/(sqrt(a^2*x - (a^2 + 1)*x^2 + x^3)*(a - x)), x)
```

**Mupad [B]**
time = 0.17, size = 217, normalized size = 2.49

$$\frac{4\,a\,\left(a^2-1\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}\,\Pi\left(-\frac{a^2-1}{a-a^2};\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)}{\left(a-a^2\right)\,\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}-\frac{2\,\left(a^2-1\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-a^2}{a^2-1}}\right)\middle|\frac{a^2-1}{a^2}\right)\,\sqrt{\frac{x}{a^2}}\,\sqrt{\frac{x-1}{a^2-1}}\,\sqrt{-\frac{x-a^2}{a^2-1}}}{\sqrt{a^2\,x-x^2\,\left(a^2+1\right)+x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x)/((a - x)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)),x)
```

```
[Out] (4*a*(a^2 - 1)*(x/a^2)^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1
))^(1/2)*ellipticPi(-(a^2 - 1)/(a - a^2), asin((-(x - a^2)/(a^2 - 1))^(1/2)
), (a^2 - 1)/a^2))/((a - a^2)*(a^2*x - x^2*(a^2 + 1) + x^3)^(1/2)) - (2*(a^
2 - 1)*ellipticF(asin((-(x - a^2)/(a^2 - 1))^(1/2)), (a^2 - 1)/a^2)*(x/a^2)
^(1/2)*((x - 1)/(a^2 - 1))^(1/2)*(-(x - a^2)/(a^2 - 1))^(1/2))/(a^2*x - x^2
*(a^2 + 1) + x^3)^(1/2)
```

## 3.82 $\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx$

**Optimal**. Leaf size=1

$$0$$

```
[Out] 0
```

**Rubi** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

time = 1.03, antiderivative size = 529, normalized size of antiderivative = 529.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2081, 6865, 1722, 1117, 1720}

$$\frac{2(1-a)\sqrt{x}\sqrt{-(-a^2+2a+1)\,x+(2-a)a+x^2}\,\text{ArcTan}\left(\frac{\sqrt{-a^2+2a-1}\sqrt{x}}{\sqrt{-(-a^2+2a+1)\,x+(2-a)a+x^2}}\right)}{a\sqrt{-a^2+2a-1}\sqrt{-(-a^2+2a+1)\,x^2+(2-a)ax+x^3}}+\frac{((2-a)a)^{3/4}\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)\,x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right)|\frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}+2\right)\right)}{a\sqrt{-(-a^2+2a+1)\,x^2+(2-a)ax+x^3}}+\frac{(2-a)\left(1-\sqrt{(2-a)a}\right)\sqrt{x}\left(\frac{x}{\sqrt{(2-a)a}}+1\right)\sqrt{\frac{-(-a^2+2a+1)\,x+(2-a)a+x^2}{(2-a)a\left(\frac{x}{\sqrt{(2-a)a}}+1\right)^2}}\,\Pi\left(\frac{\left(\sqrt{2-a}+\sqrt{a}\right)^2}{4\sqrt{(2-a)a}};2\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt[4]{(2-a)a}}\right)|\frac{1}{4}\left(\frac{-a^2+2a+1}{\sqrt{(2-a)a}}+2\right)\right)}{((2-a)a)^{3/4}\sqrt{-(-a^2+2a+1)\,x^2+(2-a)ax+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 + x^3]),
x]
```

```
[Out] (2*(1 - a)*Sqrt[x]*Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]*ArcTan[(Sqrt[-
1 + 2*a - a^2]*Sqrt[x])/Sqrt[(2 - a)*a - (1 + 2*a - a^2)*x + x^2]])/(a*Sqrt
[-1 + 2*a - a^2]*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + (((2 - a)
*a)^(3/4)*Sqrt[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)
*x + x^2)/((2 - a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticF[2*ArcTan[Sqrt[x]
/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)/Sqrt[(2 - a)*a])/4])/(a*Sqrt[(2 -
 a)*a*x - (1 + 2*a - a^2)*x^2 + x^3]) + ((2 - a)*(1 - Sqrt[(2 - a)*a])*Sqrt
[x]*(1 + x/Sqrt[(2 - a)*a])*Sqrt[((2 - a)*a - (1 + 2*a - a^2)*x + x^2)/((2
- a)*a*(1 + x/Sqrt[(2 - a)*a])^2)]*EllipticPi[(Sqrt[2 - a] + Sqrt[a])^2/(4*
Sqrt[(2 - a)*a]), 2*ArcTan[Sqrt[x]/((2 - a)*a)^(1/4)], (2 + (1 + 2*a - a^2)
/Sqrt[(2 - a)*a])/4])/(((2 - a)*a)^(3/4)*Sqrt[(2 - a)*a*x - (1 + 2*a - a^2)
*x^2 + x^3])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
```

```
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-2+a+x}{(-a+x)\sqrt{(2-a)ax+(-1-2a+a^2)\,x^2+x^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{(2-a)a+(-1-2a+a^2)\,x+x^2}\right)\int \frac{}{\sqrt{}}}{\sqrt{(2-a)ax+(-1}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{(2-a)a+(-1-2a+a^2)\,x+x^2}\right)\text{Sub}}{\sqrt{(2-a)ax}}$$

$$= \frac{\left(2\sqrt{(2-a)a}\,\sqrt{x}\,\sqrt{(2-a)a+(-1-2a+a^2)\,x}\right.}{a\sqrt{(2-a)}}$$

$$= \frac{2(1-a)\sqrt{x}\,\sqrt{(2-a)a-(1+2a-a^2)\,x+x^2}\ \text{t}}{a\sqrt{-1+2a-a^2}\,\sqrt{(2-a)ax}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 4 vs. order 1 in optimal.
time = 18.32, size = 127, normalized size = 127.00

$$\frac{2\sqrt{1+\frac{1}{-1+x}}\,\sqrt{1+\frac{(-1+a)^2}{-1+x}}\,(-1+x)^{3/2}\left(F\left(\sin^{-1}\left(\frac{\sqrt{-(-1+a)^2}}{\sqrt{-1+x}}\right)|\frac{1}{(-1+a)^2}\right)-2\Pi\left(\frac{1}{1-a};\sin^{-1}\left(\frac{\sqrt{-(-1+a)^2}}{\sqrt{-1+x}}\right)|\frac{1}{(-1+a)^2}\right)\right)}{\sqrt{-(-1+a)^2}\,\sqrt{(-1+x)x\,(-2a+a^2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + a + x)/((-a + x)*Sqrt[(2 - a)*a*x + (-1 - 2*a + a^2)*x^2 +
x^3]),x]
```

```
[Out] (2*Sqrt[1 + (-1 + x)^(-1)]*Sqrt[1 + (-1 + a)^2/(-1 + x)]*(-1 + x)^(3/2)*(El
lipticF[ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)] - 2*Elliptic
Pi[(1 - a)^(-1), ArcSin[Sqrt[-(-1 + a)^2]/Sqrt[-1 + x]], (-1 + a)^(-2)]])/(
Sqrt[-(-1 + a)^2]*Sqrt[(-1 + x)*x*(-2*a + a^2 + x)])
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 1.
time = 0.10, size = 317, normalized size = 317.00

| method | result |
|---|---|
| default | $\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\,\sqrt{\frac{-1+x}{-a^2+2a-1}}\,\sqrt{\frac{x}{-a^2+2a}}\ \text{EllipticF}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}} - \frac{2(-2a+2)(a^2-2a)\sqrt{\frac{a^2}{}}}{}$ |

| | |
|---|---|
| elliptic | $\frac{2(a^2-2a)\sqrt{\frac{a^2-2a+x}{a^2-2a}}\sqrt{\frac{-1+x}{-a^2+2a-1}}\sqrt{\frac{x}{-a^2+2a}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a^2-2a+x}{a^2-2a}},\sqrt{\frac{-a^2+2a}{-a^2+2a-1}}\right)}{\sqrt{a^2x^2-a^2x-2a\,x^2+x^3+2ax-x^2}}+\frac{2(2a-2)(a^2-2a)\sqrt{\frac{a^2-2a}{a^2-}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 2*(a^2-2*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-
a^2+2*a))^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)*EllipticF(((a^2
-2*a+x)/(a^2-2*a))^(1/2),((-a^2+2*a)/(-a^2+2*a-1))^(1/2))-2*(-2*a+2)*(a^2-2
*a)*((a^2-2*a+x)/(a^2-2*a))^(1/2)*((-1+x)/(-a^2+2*a-1))^(1/2)*(x/(-a^2+2*a)
)^(1/2)/(a^2*x^2-a^2*x-2*a*x^2+x^3+2*a*x-x^2)^(1/2)/(-a^2+a)*EllipticPi(((a
^2-2*a+x)/(a^2-2*a))^(1/2),(-a^2+2*a)/(-a^2+a),((-a^2+2*a)/(-a^2+2*a-1))^(1
/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="maxima")
```

```
[Out] -integrate((a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

**Fricas [C]** Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.69, size = 70, normalized size = 70.00

$$\frac{\log\left(-\frac{a^2-2\,(a^2-a)x-x^2+2\,\sqrt{(a^2-2\,a-1)x^2+x^3-(a^2-2\,a)x}\,a}{a^2-2\,ax+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="fricas")
```

```
[Out] log(-(a^2 - 2*(a^2 - a)*x - x^2 + 2*sqrt((a^2 - 2*a - 1)*x^2 + x^3 - (a^2 -
 2*a)*x)*a)/(a^2 - 2*a*x + x^2))/a
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+x-2}{\sqrt{x\,(x-1)\,(a^2-2a+x)}\,(-a+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a**2-2*a-1)*x**2+x**3)**(1/2),x)
```

```
[Out] Integral((a + x - 2)/(sqrt(x*(x - 1)*(a**2 - 2*a + x))*(-a + x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+a+x)/(-a+x)/((2-a)*a*x+(a^2-2*a-1)*x^2+x^3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(-(a + x - 2)/(sqrt(-(a - 2)*a*x + (a^2 - 2*a - 1)*x^2 + x^3)*(a -
 x)), x)
```

**Mupad [B]**
time = 0.48, size = 207, normalized size = 207.00

$$\frac{2\sqrt{\frac{x}{2\,a-a^2}}\,\sqrt{-\frac{x-1}{a^2-2\,a+1}}\,{\left(a-1\right)}^2\,\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\,\left(a\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)-2\,\Pi\left(-\frac{a^2-2\,a+1}{a-a^2};\mathrm{asin}\left(\sqrt{\frac{a^2-2\,a+x}{a^2-2\,a+1}}\right)\middle|-\frac{a^2-2\,a+1}{2\,a-a^2}\right)\right)}{a\,\sqrt{x^3+\left(a^2-2\,a-1\right)\,x^2+\left(2\,a-a^2\right)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + x - 2)/((a - x)*(x^3 - x^2*(2*a - a^2 + 1) - a*x*(a - 2))^(1/2)),
x)
```

```
[Out] (2*(x/(2*a - a^2))^(1/2)*(-(x - 1)/(a^2 - 2*a + 1))^(1/2)*(a - 1)^2*((x - 2
*a + a^2)/(a^2 - 2*a + 1))^(1/2)*(a*ellipticF(asin(((x - 2*a + a^2)/(a^2 -
2*a + 1))^(1/2)), -(a^2 - 2*a + 1)/(2*a - a^2)) - 2*ellipticPi(-(a^2 - 2*a
+ 1)/(a - a^2), asin(((x - 2*a + a^2)/(a^2 - 2*a + 1))^(1/2)), -(a^2 - 2*a
+ 1)/(2*a - a^2))))/(a*(x*(2*a - a^2) - x^2*(2*a - a^2 + 1) + x^3)^(1/2))
```

## 3.83 $\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx$

**Optimal**. Leaf size=46

$$\log\left(\frac{-a^2+2ax+x^2-2\left(x+\sqrt{(1-x)x\,(a^2+x-2ax)}\right)}{(a-x)^2}\right)$$

```
[Out] ln((-a^2+2*a*x+x^2-2*x-2*((1-x)*x*(a^2-2*a*x+x))^(1/2))/(a-x)^2)
```

**Rubi** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 0.96, antiderivative size = 180, normalized size of antiderivative = 3.91, number of steps used = 7, number of rules used = 7, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.137$, Rules used = {2081, 6865, 1724, 1118, 430, 1234, 551}

$$\frac{4(1-a)\sqrt{1-x}\,\sqrt{x}\,\sqrt{\frac{(1-2a)x}{a^2}+1}\;\Pi\left(\frac{1}{a};\text{ArcSin}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)\,x^2+a^2x-((1-2a)x^3)}}-\frac{2(1-2a)\sqrt{1-x}\,\sqrt{x}\,\sqrt{\frac{(1-2a)x}{a^2}+1}\;F\left(\text{ArcSin}\left(\sqrt{x}\right)|-\frac{1-2a}{a^2}\right)}{\sqrt{(-a^2-2a+1)\,x^2+a^2x-((1-2a)x^3)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 + (-1 +
 2*a)*x^3]),x]
```

```
[Out] (-2*(1 - 2*a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*EllipticF[Arc
Sin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^2)*x^2 - (1 - 2*
a)*x^3] + (4*(1 - a)*Sqrt[1 - x]*Sqrt[x]*Sqrt[1 + ((1 - 2*a)*x)/a^2]*Ellipt
icPi[a^(-1), ArcSin[Sqrt[x]], -((1 - 2*a)/a^2)])/Sqrt[a^2*x + (1 - 2*a - a^
2)*x^2 - (1 - 2*a)*x^3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1724

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rubi steps

$$\int \frac{-a+(-1+2a)x}{(-a+x)\sqrt{a^2x-(-1+2a+a^2)\,x^2+(-1+2a)x^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a^2-(-1+2a+a^2)\,x+(-1+2a)x^2}\right.}{\sqrt{a^2x-(-1+}}$$

$$= \frac{\left(2\sqrt{x}\,\sqrt{a^2-(-1+2a+a^2)\,x+(-1+2a)x^2}\right.}{\sqrt{a^2x-}}$$

$$= -\frac{\left(4(1-a)a\sqrt{x}\,\sqrt{a^2-(-1+2a+a^2)\,x+(-\right.}{\sqrt{}}$$

$$= -\frac{\left(4(1-a)a\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2a)x}{a^2}}\,\sqrt{}\right.}{}$$

$$= -\frac{2(1-2a)\sqrt{1-x}\,\sqrt{x}\,\sqrt{1+\frac{(1-2a)x}{a^2}}\,F(\mathrm{si}}{\sqrt{a^2x+(1-2a-a^2)\,x^2-(1-}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 20.77, size = 133, normalized size = 2.89

$$\frac{2i(-1+x)^{3/2}\sqrt{\frac{x}{-1+x}}\sqrt{-\frac{a^2+x-2ax}{(-1+2a)(-1+x)}}\left(-F\left(i\sinh^{-1}\left(\frac{1}{\sqrt{-1+x}}\right)|-\frac{(-1+a)^2}{-1+2a}\right)+2a\Pi\left(1-a;i\sinh^{-1}\left(\frac{1}{\sqrt{-1+x}}\right)|-\frac{(-1+a)^2}{-1+2a}\right)\right)}{\sqrt{-((-1+x)x\left(a^2+x-2ax\right))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + (-1 + 2*a)*x)/((-a + x)*Sqrt[a^2*x - (-1 + 2*a + a^2)*x^2 +
 (-1 + 2*a)*x^3]),x]
```

```
[Out] ((2*I)*(-1 + x)^(3/2)*Sqrt[x/(-1 + x)]*Sqrt[-((a^2 + x - 2*a*x)/((-1 + 2*a)
*(-1 + x)))]*(-EllipticF[I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*a)
)] + 2*a*EllipticPi[1 - a, I*ArcSinh[1/Sqrt[-1 + x]], -((-1 + a)^2/(-1 + 2*
a))]))/Sqrt[-((-1 + x)*x*(a^2 + x - 2*a*x))]
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.10, size = 536, normalized size = 11.65

| method | result |
|---|---|

| | |
|---|---|
| elliptic | $-\frac{2a^2\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}\right.}}\right.}{\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}$ |
| default | $-\frac{4a^3\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}}\sqrt{\frac{-1+x}{\frac{a^2}{-1+2a}-1}}\sqrt{\frac{x(-1+2a)}{a^2}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(x-\frac{a^2}{-1+2a}\right)(-1+2a)}{a^2}},\sqrt{\frac{a^2}{(-1+2a)\left(\frac{a^2}{-1+2a}\right.}}\right.}{(-1+2a)\sqrt{-a^2x^2+2a\,x^3+a^2x-2a\,x^2-x^3+x^2}}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -4*a^3/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a
)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)
^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),(a^2/(-1+2*a)/(a^2/
(-1+2*a)-1))^(1/2))+2*a^2/(-1+2*a)*(-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2)*(
(-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1/2)/(-a^2*x^2+2*a*x^3+a^2
*x-2*a*x^2-x^3+x^2)^(1/2)*EllipticF((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),
(a^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))-4*a^3*(a-1)/(-1+2*a)*(-(x-a^2/(-1+2*
a))/a^2*(-1+2*a))^(1/2)*((-1+x)/(a^2/(-1+2*a)-1))^(1/2)*(x/a^2*(-1+2*a))^(1
/2)/(-a^2*x^2+2*a*x^3+a^2*x-2*a*x^2-x^3+x^2)^(1/2)/(a^2/(-1+2*a)-a)*Ellipti
cPi((-(x-a^2/(-1+2*a))/a^2*(-1+2*a))^(1/2),a^2/(-1+2*a)/(a^2/(-1+2*a)-a),(a
^2/(-1+2*a)/(a^2/(-1+2*a)-1))^(1/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="maxima")
```

```
[Out] -integrate(((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

**Fricas [A]**
time = 0.70, size = 63, normalized size = 1.37

$$\log\left(-\frac{a^2-2\,(a-1)x-x^2+2\,\sqrt{(2\,a-1)x^3+a^2x-(a^2+2\,a-1)x^2}}{a^2-2\,ax+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="fricas")
```

```
[Out] log(-(a^2 - 2*(a - 1)*x - x^2 + 2*sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a -
 1)*x^2))/(a^2 - 2*a*x + x^2))
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2ax - a - x}{\sqrt{x(x-1)(-a^2 + 2ax - x)}\,(-a + x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a**2*x-(a**2+2*a-1)*x**2+(-1+2*a)*x**3)**
(1/2),x)
```

```
[Out] Integral((2*a*x - a - x)/(sqrt(x*(x - 1)*(-a**2 + 2*a*x - x))*(-a + x)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+(-1+2*a)*x)/(-a+x)/(a^2*x-(a^2+2*a-1)*x^2+(-1+2*a)*x^3)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate(-((2*a - 1)*x - a)/(sqrt((2*a - 1)*x^3 + a^2*x - (a^2 + 2*a - 1)*
x^2)*(a - x)), x)
```

**Mupad** [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - x*(2*a - 1))/((a - x)*(x^3*(2*a - 1) - x^2*(2*a + a^2 - 1) + a^2*x
)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

# 3.84 $\int \frac{1-\sqrt[3]{2}\, x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\, dx$

**Optimal**. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\, x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2162, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\, x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1-\sqrt[3]{2}\,x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = 2\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}\,x}{\sqrt{1+x^3}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

**Mathematica** [A]
time = 1.03, size = 34, normalized size = 1.06

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2^(1/3)*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
[Out] (-2*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.32, size = 258, normalized size = 8.06

| method | result |
|---|---|
| trager | $-\frac{2^{\frac{1}{3}}\,\text{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{x^3+1}\,x+3\,2^{\frac{2}{3}}\,\text{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^2-\text{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3+6\sqrt{x^3+1}\,2^{\frac{2}{3}}+6\,2^{\frac{1}{3}}\,\text{RootO}}{\left(2^{\frac{1}{3}}x+2\right)^3}\right.}{6}$ |
| default | $-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\left(\frac{3}{2}\right.}{}$ |
| elliptic | $-\frac{2\,2^{\frac{1}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\left(\frac{3}{2}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*2^(1/3)*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I
*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*(3/2-1/2*I*3^(1/2))*((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt[3]{2}\,x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx-\int\left(-\frac{1}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2**(1/3)*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(1/3)*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(-1/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2^(1/3)*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2^(1/3)*x - 1)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

**Mupad [B]**
time = 1.69, size = 67, normalized size = 2.09

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\,\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{{\left(x+2^{2/3}\right)}^6}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2^(1/3)*x - 1)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)*(3^(1/2
)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*1i)/3
```

## 3.85 $\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

**Rubi** [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2163, 212}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-2+x)\sqrt{1+x^3}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

**Mathematica [A]**
time = 0.66, size = 31, normalized size = 1.35

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-2 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.27, size = 240, normalized size = 10.43

| method | result |
|---|---|
| trager | $-\frac{\ln\left(\frac{x^3+6\sqrt{x^3+1}\,x+12x^2+6\sqrt{x^3+1}-6x+10}{(-2+x)^3}\right)}{3}$ |
| default | $\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$ |
| elliptic | $\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-2+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(
-3/2-1/2*I*3^(1/2)))^(1/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.
time = 0.73, size = 44, normalized size = 1.91

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2-6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 - 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{(x+1)\,(x^2-x+1)}\,\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - 2)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-2+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

**Mupad** [B]
time = 0.22, size = 204, normalized size = 8.87

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)

[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

# 3.86 $\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx$

**Optimal**. Leaf size=218

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\right.}{3\sqrt{2}\,\sqrt[4]{\phantom{3}}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2
))*3^(1/4)*2^(1/2)-1/18*arctan(1/6*(1-3^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2
))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2))*2
^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*
(1-2*x+3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)
```

**Rubi** [A]

time = 0.03, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {500}

$$-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(-2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/2*((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1
+ x^3])])/(Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[1 +
 x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) - ((2 - Sqrt[3])*ArcTanh[(3^
(1/4)*(1 + Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 + x^3])])/(3*Sqrt[2]*3^(1/4)) -
((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^
3])])/(6*Sqrt[2]*3^(1/4))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}} - \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right.}{}\right.}{3\sqrt{2}\,3^{3/4}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 47, normalized size = 0.22

$$\frac{x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,-\frac{x^3}{10+6\sqrt{3}}\right)}{20+12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 + 6*Sqrt[3] + x^3)),x]
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, -(x^3/(10 + 6*Sqrt[3]))])/(20 + 12*Sq
rt[3])
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.47, size = 353, normalized size = 1.62

| method | result |
|---|---|
| default | $-\frac{\sqrt{2}\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^2+\left(-1-\sqrt{3}\right)\_Z+2\sqrt{3}+4\right)}\frac{\left(-\sqrt{3}\,\_\alpha+\_\alpha-2\right)\left(3-i\sqrt{3}\right)\sqrt{\frac{1+x}{3-i\sqrt{3}}}\sqrt{\frac{-i\sqrt{3}+2x-1}{-3-i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}}{i\sqrt{3}}}}{}\right.}{18}$ |
| elliptic | $\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{\left(-1-\sqrt{3}\right)^2}{3}+\frac{2\left(-1-\sqrt{3}\right)^2\sqrt{3}}{9}-\frac{2}{3}-\frac{\sqrt{3}}{9}-\frac{2\sqrt{3}\left(\right.}{}\right.}{}$ |

| trager | Expression too large to display |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alpha-2)/(-1+2*_alpha-3^(1/2))*(3-I*3^(
1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-I*3^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)
*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/2)/(x^3+1)^(1/2)*(-1+2*_alpha-3^(1/2)
*_alpha)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/2*I*_alpha+1/3*I*_
alpha*3^(1/2)+1/2*3^(1/2)*_alpha-_alpha-1/6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^2+(-1-3^(1/2))*_Z+2*3^(1
/2)+4))+1/9*(-1-3^(1/2))/(2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+
1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi
(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 7739 vs. $2(148) = 296$.
time = 6.26, size = 7739, normalized size = 35.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/432*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3)
 + 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(-1/1296*(6
*sqrt(x^3 + 1)*((459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*
x^12 - 14760972*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 8437780
8*x^7 - 88427328*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 1260979
2*x^2 + sqrt(3)*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x
^12 - 8522268*x^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*
x^7 - 51053600*x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x
^2 - 2488832*x - 1889792) + (3691*x^16 - 6128*x^15 - 537864*x^14 + 1586477*
```

x^13 + 16210952*x^12 - 77181756*x^11 + 84218362*x^10 + 71018320*x^9 - 25445
5812*x^8 + 196076008*x^7 + 120105208*x^6 - 256326864*x^5 + 134645168*x^4 +
78464672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 - 3538*x^15 - 310536*x^14
+ 915953*x^13 + 9359398*x^12 - 44560908*x^11 + 48623494*x^10 + 41002448*x^9
 - 146910132*x^8 + 113204536*x^7 + 69342776*x^6 - 147990384*x^5 + 77737424*
x^4 + 45301600*x^3 - 45330624*x^2 + 12242560*x + 7598336) + 21204736*x + 13
160704)*sqrt(-672*sqrt(3) + 1164) - 4310784*x - 3273216)*(-672*sqrt(3) + 11
64)^(3/4) + 3*(984*x^15 - 30612*x^14 + 164676*x^13 - 205368*x^12 - 289200*x
^11 + 183720*x^10 + 886752*x^9 - 71568*x^8 - 1960992*x^7 + 1849440*x^6 + 15
58464*x^5 - 2478912*x^4 + 66432*x^3 + 750336*x^2 + 4*sqrt(3)*(142*x^15 - 44
19*x^14 + 23781*x^13 - 29608*x^12 - 41940*x^11 + 26454*x^10 + 128152*x^9 -
10692*x^8 - 283320*x^7 + 267064*x^6 + 224784*x^5 - 357936*x^4 + 9632*x^3 +
108288*x^2 - 96000*x - 33920) + (4945*x^15 - 88617*x^14 + 738528*x^13 - 186
0046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 154441
44*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704*x^2
 + sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 452980*x^
11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x^6 -
 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 545536)
+ 2674176*x + 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x - 235008)*(-672*
sqrt(3) + 1164)^(1/4))*sqrt(-2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) +
 24)*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 - 5976*x^16 + 5544*x^15 + 299664
*x^14 - 1062360*x^13 + 116712*x^12 + 3600000*x^11 - 4761216*x^10 - 1046592*
x^9 + 8676864*x^8 - 6592896*x^7 - 2641536*x^6 + 7016832*x^5 - 3699072*x^4 -
 1861632*x^3 + 1640448*x^2 + 12*sqrt(3)*(7*x^17 - 286*x^16 + 238*x^15 + 142
55*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11 - 226888*x^10 - 48920*x^9 +
415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x^5 - 177344*x^4 - 89152*x^3
+ 78784*x^2 - 39040*x - 18176) - (1164*x^17 - 6276*x^16 - 26052*x^15 + 3328
44*x^14 - 1632156*x^13 + 4149132*x^12 - 5805024*x^11 + 318696*x^10 + 126210
72*x^9 - 19878720*x^8 + 9619008*x^7 + 13361088*x^6 - 20168256*x^5 + 1093612
8*x^4 + 6434304*x^3 - 6426240*x^2 + 24*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^
15 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610
*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 154
784*x^3 - 154592*x^2 + 78464*x + 36544) + (2340*x^17 - 96354*x^16 + 84798*x
^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963744*x^11 - 7660368
0*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 42453216*x^6 + 1132
69536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 + sqrt(3)*(1351*x^17
 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13 + 1121030*x^12 + 3
3465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8 - 61330384*x^7 -
24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3 + 15297472*x^2 -
7571584*x - 3526400) - 13114368*x - 6107904)*sqrt(-672*sqrt(3) + 1164) + 32
61696*x + 1519104)*sqrt(-672*sqrt(3) + 1164) + 12*(97*x^17 - 523*x^16 - 217
1*x^15 + 27737*x^14 - 136013*x^13 + 345761*x^12 - 483752*x^11 + 26558*x^10
+ 1051756*x^9 - 1656560*x^8 + 801584*x^7 + 1113424*x^6 - 1680688*x^5 + 9113
44*x^4 + 536192*x^3 - 535520*x^2 + 2*sqrt(3)*(28*x^17 - 151*x^16 - 626*x^15
 + 8006*x^14 - 39266*x^13 + 99812*x^12 - 139652*x^11 + 7661*x^10 + 303610*x

```
^9 - 478214*x^8 + 231392*x^7 + 321412*x^6 - 485176*x^5 + 263080*x^4 + 15478
4*x^3 - 154592*x^2 + 78464*x + 36544) + 271808*x + 126592)*sqrt(-672*sqrt(3
) + 1164) - 811008*x - 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 + 1)*((45
9*x^16 - 1557*x^15 - 26415*x^14 - 1449954*x^13 + 4677912*x^12 + 12651948*x^
11 - 55684800*x^10 + 62834256*x^9 + 8526168*x^8 - 105313392*x^7 + 99605088*
x^6 - 18897984*x^5 - 42499296*x^4 + 37357632*x^3 - 8256960*x^2 + sqrt(3)*(2
65*x^16 - 899*x^15 - 15249*x^14 - 837130*x^13 + 2700776*x^12 + 7304604*x^11
 - 32149640*x^10 + 36277360*x^9 + 4922568*x^8 - 60802736*x^7 + 57507040*x^6
 - 10910784*x^5 - 24536992*x^4 + 21568448*x^3 - 4767168*x^2 + 1207168*x + 1
383424) + (3691*x^16 + 17731*x^15 - 951114*x^14 + 450359*x^13 + 4370159*x^1
2 + 30318522*x^11 - 78096668*x^10 + 9429316*x^9 + 146877876*x^8 - 197107784
*x^7 - 30834152*x^6 + 185125776*x^5 - 132260896*x^4 - 45545344*x^3 + 695175
36*x^2 + sqrt(3)*(2131*x^16 + 10237*x^15 - 5491...
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x^3+10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3+6*3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 + 10 + 6*sqrt(3))), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3+6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 + 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x^3+6\,\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)),x)
[Out] int(x/((x^3 + 1)^(1/2)*(6*3^(1/2) + x^3 + 10)), x)
```

# 3.87 $\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx$

**Optimal**. Leaf size=210

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1+x)}{\sqrt{2}\sqrt{1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\ \cdots}{\sqrt{\cdots}}\right)}{2\sqrt{2}\,3^{3/4}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*(1-2*x-3^(1/2))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))
*3^(3/4)*2^(1/2)-1/36*arctan(1/2*3^(1/4)*(1+x)*(1+3^(1/2))*2^(1/2)/(x^3+1)^
(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1+x)*(1-3^(1/2
))*2^(1/2)/(x^3+1)^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/6*(1+3
^(1/2))*(x^3+1)^(1/2)*3^(1/4)*2^(1/2))*(2+3^(1/2))*3^(1/4)*2^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{500\}$

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(-2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3+1}}\right)}{3\sqrt{2}\sqrt[4]{3}}-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(x+1)}{\sqrt{2}\sqrt{x^3+1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/3*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3] - 2*x))/(Sqrt[2]*Sqrt[1 +
x^3])])/(Sqrt[2]*3^(1/4)) - ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*(1
 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(6*Sqrt[2]*3^(1/4)) + ((2 + Sqrt[3])*ArcTa
nh[(3^(1/4)*(1 - Sqrt[3])*(1 + x))/(Sqrt[2]*Sqrt[1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) + ((2 + Sqrt[3])*ArcTanh[((1 + Sqrt[3])*Sqrt[1 + x^3])/(Sqrt[2]*3^(3/
4))])/(3*Sqrt[2]*3^(3/4))
```

Rule 500

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(-q)*(2 - r
)*(ArcTan[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[a, 2]*r^(3/2)))]/(3*Sqrt[2]*
Rt[a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTan[Rt[a, 2]*Sqrt[r]*(1 + r
)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2))), x
] - Simp[q*(2 - r)*(ArcTanh[Rt[a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*Sqrt
[a + b*x^3]))]/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(ArcTan
h[Rt[a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sqrt[2
]*Rt[a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^3}\left(10-6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}-2x\right)}{\sqrt{2}\sqrt{1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}} - \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1\right.}{\sqrt{2}}\right.}{6\sqrt{2}\sqrt[4]{3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.07, size = 50, normalized size = 0.24

$$-\frac{x^2 F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-x^3,\frac{1}{4}\left(5+3\sqrt{3}\right)x^3\right)}{4\left(-5+3\sqrt{3}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^3]*(10 - 6*Sqrt[3] + x^3)),x]
[Out] -1/4*(x^2*AppellF1[2/3, 1/2, 1, 5/3, -x^3, ((5 + 3*Sqrt[3])*x^3)/4])/(-5 +
3*Sqrt[3])
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.87, size = 350, normalized size = 1.67

| method | result |
|---|---|
| default | $\frac{\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right.}{3}\right.}{9\left(-2+\sqrt{3}\right)\sqrt{x^3+1}}$ |
| elliptic | $\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{2\left(\sqrt{3}-1\right)^2\sqrt{3}}{9}-\frac{\left(\sqrt{3}-1\right)^2}{3}+\frac{2\left(\sqrt{3}-1\right)\sqrt{3}}{9}+\right.}{}$ |

| trager | Expression too large to display |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^
(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/
(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alp
ha+2)/(1-2*_alpha-3^(1/2))*(3-I*3^(1/2))*((1+x)/(3-I*3^(1/2)))^(1/2)*((-I*3
^(1/2)+2*x-1)/(-3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-3))^(1/2)
/(x^3+1)^(1/2)*(2*_alpha-1+3^(1/2)*_alpha)*EllipticPi(((1+x)/(3/2-1/2*I*3^(
1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha-1/2*3^(1/2)*_alpha-_alpha-1/
6*I*3^(1/2)+1/2,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)),_alpha=R
ootOf(_Z^2+(3^(1/2)-1)*_Z-2*3^(1/2)+4))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 8237 vs. $2(146) = 292$.
time = 5.48, size = 8237, normalized size = 39.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/108*sqrt(3)*sqrt(sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*(67
2*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(1/324*(2
16*sqrt(3)*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 345
761*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^
7 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sq
rt(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12
- 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x
^6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
 271808*x + 126592)*(56*sqrt(3) + 97) - 36*sqrt(3)*(sqrt(3)*(2340*x^17 - 96
```

354*x^16 + 84798*x^15 + 4817124*x^14 - 17052930*x^13 + 1941678*x^12 + 57963
744*x^11 - 76603680*x^10 - 16678512*x^9 + 139922496*x^8 - 106227360*x^7 - 4
2453216*x^6 + 113269536*x^5 - 59694624*x^4 - 30025728*x^3 + 26496000*x^2 -
sqrt(3)*(1351*x^17 - 55630*x^16 + 48958*x^15 + 2781167*x^14 - 9845510*x^13
+ 1121030*x^12 + 33465376*x^11 - 44227144*x^10 - 9629336*x^9 + 80784280*x^8
 - 61330384*x^7 - 24510368*x^6 + 65396192*x^5 - 34464704*x^4 - 17335360*x^3
 + 15297472*x^2 - 7571584*x - 3526400) - 13114368*x - 6107904)*(56*sqrt(3)
+ 97) + 6*(97*x^17 - 523*x^16 - 2171*x^15 + 27737*x^14 - 136013*x^13 + 3457
61*x^12 - 483752*x^11 + 26558*x^10 + 1051756*x^9 - 1656560*x^8 + 801584*x^7
 + 1113424*x^6 - 1680688*x^5 + 911344*x^4 + 536192*x^3 - 535520*x^2 - 2*sqr
t(3)*(28*x^17 - 151*x^16 - 626*x^15 + 8006*x^14 - 39266*x^13 + 99812*x^12 -
 139652*x^11 + 7661*x^10 + 303610*x^9 - 478214*x^8 + 231392*x^7 + 321412*x^
6 - 485176*x^5 + 263080*x^4 + 154784*x^3 - 154592*x^2 + 78464*x + 36544) +
271808*x + 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) + 3*sqrt(sq
rt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 6)*((2*sqrt(3)*(3691*x^16 -
6128*x^15 - 537864*x^14 + 1586477*x^13 + 16210952*x^12 - 77181756*x^11 + 84
218362*x^10 + 71018320*x^9 - 254455812*x^8 + 196076008*x^7 + 120105208*x^6
 - 256326864*x^5 + 134645168*x^4 + 78464672*x^3 - 78514944*x^2 - sqrt(3)*(21
31*x^16 - 3538*x^15 - 310536*x^14 + 915953*x^13 + 9359398*x^12 - 44560908*x
^11 + 48623494*x^10 + 41002448*x^9 - 146910132*x^8 + 113204536*x^7 + 693427
76*x^6 - 147990384*x^5 + 77737424*x^4 + 45301600*x^3 - 45330624*x^2 + 12242
560*x + 7598336) + 21204736*x + 13160704)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) +
 (459*x^16 - 13425*x^15 - 33201*x^14 + 950652*x^13 - 997302*x^12 - 14760972
*x^11 + 47069892*x^10 - 49762248*x^9 - 8212536*x^8 + 84377808*x^7 - 8842732
8*x^6 + 25613856*x^5 + 27458496*x^4 - 36433344*x^3 + 12609792*x^2 - sqrt(3)
*(265*x^16 - 7751*x^15 - 19167*x^14 + 548864*x^13 - 575818*x^12 - 8522268*x
^11 + 27175852*x^10 - 28730312*x^9 - 4741560*x^8 + 48715600*x^7 - 51053600*
x^6 + 14788128*x^5 + 15853184*x^4 - 21034816*x^3 + 7280256*x^2 - 2488832*x
 - 1889792) - 4310784*x - 3273216)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672
*sqrt(3) + 1164)^(3/4) + 6*(sqrt(3)*(4945*x^15 - 88617*x^14 + 738528*x^13 -
 1860046*x^12 - 784596*x^11 + 7668708*x^10 - 6570680*x^9 - 6903864*x^8 + 15
444144*x^7 - 4312832*x^6 - 9559200*x^5 + 9359808*x^4 - 155968*x^3 - 3016704
*x^2 - sqrt(3)*(2855*x^15 - 51163*x^14 + 426388*x^13 - 1073898*x^12 - 45298
0*x^11 + 4427548*x^10 - 3793592*x^9 - 3985944*x^8 + 8916720*x^7 - 2490016*x
^6 - 5519008*x^5 + 5403904*x^4 - 90048*x^3 - 1741696*x^2 + 1543936*x + 5455
36) + 2674176*x + 944896)*sqrt(x^3 + 1)*(56*sqrt(3) + 97) + 2*(246*x^15 - 7
653*x^14 + 41169*x^13 - 51342*x^12 - 72300*x^11 + 45930*x^10 + 221688*x^9 -
 17892*x^8 - 490248*x^7 + 462360*x^6 + 389616*x^5 - 619728*x^4 + 16608*x^3
+ 187584*x^2 - sqrt(3)*(142*x^15 - 4419*x^14 + 23781*x^13 - 29608*x^12 - 41
940*x^11 + 26454*x^10 + 128152*x^9 - 10692*x^8 - 283320*x^7 + 267064*x^6 +
224784*x^5 - 357936*x^4 + 9632*x^3 + 108288*x^2 - 96000*x - 33920) - 166272
*x - 58752)*sqrt(x^3 + 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(1/4)
) + 108*(12*x^17 - 498*x^16 + 462*x^15 + 24972*x^14 - 88530*x^13 + 9726*x^1
2 + 300000*x^11 - 396768*x^10 - 87216*x^9 + 723072*x^8 - 549408*x^7 - 22012
8*x^6 + 584736*x^5 - 308256*x^4 - 155136*x^3 + 136704*x^2 - sqrt(3)*(7*x^17

```
 - 286*x^16 + 238*x^15 + 14255*x^14 - 50390*x^13 + 5942*x^12 + 171808*x^11
- 226888*x^10 - 48920*x^9 + 415384*x^8 - 315088*x^7 - 125600*x^6 + 336608*x
^5 - 177344*x^4 - 89152*x^3 + 78784*x^2 - 39040*x - 18176) - 67584*x - 3148
8)*sqrt(56*sqrt(3) + 97) + (144*sqrt(3)*(627*x^16 - 14286*x^15 + 39762*x^14
 + 50142*x^13 - 216816*x^12 + 112284*x^11 + 325707*x^10 - 586326*x^9 - 3294
*x^8 + 631752*x^7 - 539220*x^6 - 184392*x^5 + 483816*x^4 - 115296*x^3 - 108
576*x^2 - 2*sqrt(3)*(181*x^16 - 4124*x^15 + 11478*x^14 + 14474*x^13 - 62584
*x^12 + 32412*x^11 + 94021*x^10 - 169244*x^9 - 954*x^8 + 182368*x^7 - 15564
8*x^6 - 53232*x^5 + 139664*x^4 - 33280*x^3 - 31344*x^2 + 37024*x + 11584) +
 128256*x + 40128)*(56*sqrt(3) + 97) + 12*sqrt(3)*(sqrt(3)*(2340*x^17 - 358
50*x^16 - 106410*x^15 - 2064744*x^14 + 11945946*x^13 - 1710042*x^12 - 46293
732*x^11 + 59161524*x^10 + 18480192*x^9 - 122366520*x^8 + 81203856*x^7 + 45
222000*x^6 - 100598112*x^5 + 42207168*x^4 + 296...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x^3-6\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x**3-6*3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x**3 - 6*sqrt(3) + 10)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(10+x^3-6*3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 - 6*sqrt(3) + 10)*sqrt(x^3 + 1)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x^3-6\,\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)),x)
[Out] int(x/((x^3 + 1)^(1/2)*(x^3 - 6*3^(1/2) + 10)), x)
```

# 3.88 $\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx$

**Optimal**. Leaf size=222

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}+2x\right)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}}{\sqrt{2}}\right.}{2\sqrt{2}\,3^{3/4}}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2)
)*3^(3/4)*2^(1/2)+1/18*arctan(1/2*3^(1/4)*(1+2*x+3^(1/2))*2^(1/2)/(x^3-1)^(
1/2))*(2-3^(1/2))*3^(3/4)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*(1-x)*(1+3^(1/2)
)*2^(1/2)/(x^3-1)^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)-1/18*arctanh(1/6*(1-3^
(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2))*(2-3^(1/2))*3^(1/4)*2^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {501}

$$\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(2x+\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^
3])])/(6*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3] + 2
*x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(3*Sqrt[2]*3^(1/4)) + ((2 - Sqrt[3])*ArcTan
h[(3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(2*Sqrt[2]*3^(
3/4)) - ((2 - Sqrt[3])*ArcTanh[((1 - Sqrt[3])*Sqrt[-1 + x^3])/(Sqrt[2]*3^(3
/4))])/(3*Sqrt[2]*3^(3/4))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10-6\sqrt{3}+x^3\right)}\,dx = \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{6\sqrt{2}\sqrt[4]{3}} + \frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}}{\sqrt{2}}\right.}{3\sqrt{2}\sqrt[4]{3}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 65, normalized size = 0.29

$$-\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{10+6\sqrt{3}}\right)}{\left(20+12\sqrt{3}\right)\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 - 6*Sqrt[3] + x^3)),x]
[Out] -((x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/(10 + 6*Sqrt[3])])
/((20 + 12*Sqrt[3])*Sqrt[-1 + x^3]))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.97, size = 349, normalized size = 1.57

| method | result |
|---|---|
| default | $\frac{\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{}}{}\right.}{}\right.}{9\left(2+\sqrt{3}\right)\sqrt{x^3-1}}$ |
| elliptic | $\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(-\frac{2\left(1+\sqrt{3}\right)^2\sqrt{3}}{9}+\frac{\left(1+\sqrt{3}\right)^2}{3}-\frac{2\left(1+\sqrt{3}\right)\sqrt{3}}{9}+\right.}{}$ |

| trager | Expression too large to display |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(-1-3^(1/2))/(2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x
)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha+_alp
ha+2)/(-1-2*_alpha-3^(1/2))*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)*((
-I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))^(1
/2)/(x^3-1)^(1/2)*(1+2*_alpha-3^(1/2)*_alpha)*EllipticPi(((-1+x)/(-3/2-1/2*
I*3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)-1/2*I*_alpha-1/2*3^(1/2)*_alpha+_alp
ha+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alph
a=RootOf(_Z^2+(1+3^(1/2))*_Z+2*3^(1/2)+4))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 7910 vs. 2(146) = 292.
time = 4.94, size = 7910, normalized size = 35.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)*(56*sqrt(3) +
 97)*sqrt(-56*sqrt(3) + 97)*(-672*sqrt(3) + 1164)^(3/4)*arctan(1/1296*(6*sq
rt(x^3 - 1)*((459*x^16 + 13425*x^15 - 33201*x^14 - 950652*x^13 - 997302*x^1
2 + 14760972*x^11 + 47069892*x^10 + 49762248*x^9 - 8212536*x^8 - 84377808*x
^7 - 88427328*x^6 - 25613856*x^5 + 27458496*x^4 + 36433344*x^3 + 12609792*x
^2 + sqrt(3)*(265*x^16 + 7751*x^15 - 19167*x^14 - 548864*x^13 - 575818*x^12
 + 8522268*x^11 + 27175852*x^10 + 28730312*x^9 - 4741560*x^8 - 48715600*x^7
 - 51053600*x^6 - 14788128*x^5 + 15853184*x^4 + 21034816*x^3 + 7280256*x^2
+ 2488832*x - 1889792) - (3691*x^16 + 6128*x^15 - 537864*x^14 - 1586477*x^1
```

3 + 16210952*x^12 + 77181756*x^11 + 84218362*x^10 - 71018320*x^9 - 25445581
2*x^8 - 196076008*x^7 + 120105208*x^6 + 256326864*x^5 + 134645168*x^4 - 784
64672*x^3 - 78514944*x^2 + sqrt(3)*(2131*x^16 + 3538*x^15 - 310536*x^14 - 9
15953*x^13 + 9359398*x^12 + 44560908*x^11 + 48623494*x^10 - 41002448*x^9 -
146910132*x^8 - 113204536*x^7 + 69342776*x^6 + 147990384*x^5 + 77737424*x^4
- 45301600*x^3 - 45330624*x^2 - 12242560*x + 7598336) - 21204736*x + 13160
704)*sqrt(-672*sqrt(3) + 1164) + 4310784*x - 3273216)*(-672*sqrt(3) + 1164)
^(3/4) + 3*(984*x^15 + 30612*x^14 + 164676*x^13 + 205368*x^12 - 289200*x^11
- 183720*x^10 + 886752*x^9 + 71568*x^8 - 1960992*x^7 - 1849440*x^6 + 15584
64*x^5 + 2478912*x^4 + 66432*x^3 - 750336*x^2 + 4*sqrt(3)*(142*x^15 + 4419*
x^14 + 23781*x^13 + 29608*x^12 - 41940*x^11 - 26454*x^10 + 128152*x^9 + 106
92*x^8 - 283320*x^7 - 267064*x^6 + 224784*x^5 + 357936*x^4 + 9632*x^3 - 108
288*x^2 - 96000*x + 33920) - (4945*x^15 + 88617*x^14 + 738528*x^13 + 186004
6*x^12 - 784596*x^11 - 7668708*x^10 - 6570680*x^9 + 6903864*x^8 + 15444144*
x^7 + 4312832*x^6 - 9559200*x^5 - 9359808*x^4 - 155968*x^3 + 3016704*x^2 +
sqrt(3)*(2855*x^15 + 51163*x^14 + 426388*x^13 + 1073898*x^12 - 452980*x^11
- 4427548*x^10 - 3793592*x^9 + 3985944*x^8 + 8916720*x^7 + 2490016*x^6 - 55
19008*x^5 - 5403904*x^4 - 90048*x^3 + 1741696*x^2 + 1543936*x - 545536) + 2
674176*x - 944896)*sqrt(-672*sqrt(3) + 1164) - 665088*x + 235008)*(-672*sqr
t(3) + 1164)^(1/4))*sqrt(2*(7*sqrt(3) + 12)*sqrt(-672*sqrt(3) + 1164) + 24)
*sqrt(-56*sqrt(3) + 97) + 36*(144*x^17 + 5976*x^16 + 5544*x^15 - 299664*x^1
4 - 1062360*x^13 - 116712*x^12 + 3600000*x^11 + 4761216*x^10 - 1046592*x^9
- 8676864*x^8 - 6592896*x^7 + 2641536*x^6 + 7016832*x^5 + 3699072*x^4 - 186
1632*x^3 - 1640448*x^2 + 12*sqrt(3)*(7*x^17 + 286*x^16 + 238*x^15 - 14255*x
^14 - 50390*x^13 - 5942*x^12 + 171808*x^11 + 226888*x^10 - 48920*x^9 - 4153
84*x^8 - 315088*x^7 + 125600*x^6 + 336608*x^5 + 177344*x^4 - 89152*x^3 - 78
784*x^2 - 39040*x + 18176) + (1164*x^17 + 6276*x^16 - 26052*x^15 - 332844*x
^14 - 1632156*x^13 - 4149132*x^12 - 5805024*x^11 - 318696*x^10 + 12621072*x
^9 + 19878720*x^8 + 9619008*x^7 - 13361088*x^6 - 20168256*x^5 - 10936128*x^
4 + 6434304*x^3 + 6426240*x^2 + 24*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 -
8006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9
+ 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*
x^3 + 154592*x^2 + 78464*x - 36544) - (2340*x^17 + 96354*x^16 + 84798*x^15
- 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 57963744*x^11 + 76603680*x^
10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7 + 42453216*x^6 + 11326953
6*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^2 + sqrt(3)*(1351*x^17 + 5
5630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x^13 - 1121030*x^12 + 33465
376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280*x^8 - 61330384*x^7 + 2451
0368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360*x^3 - 15297472*x^2 - 7571
584*x + 3526400) - 13114368*x + 6107904)*sqrt(-672*sqrt(3) + 1164) + 326169
6*x - 1519104)*sqrt(-672*sqrt(3) + 1164) - 12*(97*x^17 + 523*x^16 - 2171*x^
15 - 27737*x^14 - 136013*x^13 - 345761*x^12 - 483752*x^11 - 26558*x^10 + 10
51756*x^9 + 1656560*x^8 + 801584*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x
^4 + 536192*x^3 + 535520*x^2 + 2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8
006*x^14 - 39266*x^13 - 99812*x^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 +

```
 478214*x^8 + 231392*x^7 - 321412*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^
3 + 154592*x^2 + 78464*x - 36544) + 271808*x - 126592)*sqrt(-672*sqrt(3) +
1164) - 811008*x + 377856)*sqrt(-56*sqrt(3) + 97) - (sqrt(x^3 - 1)*((459*x^
16 + 1557*x^15 - 26415*x^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 -
 55684800*x^10 - 62834256*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6
+ 18897984*x^5 - 42499296*x^4 - 37357632*x^3 - 8256960*x^2 + sqrt(3)*(265*x
^16 + 899*x^15 - 15249*x^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 3
2149640*x^10 - 36277360*x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 1
0910784*x^5 - 24536992*x^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 13834
24) - (3691*x^16 - 17731*x^15 - 951114*x^14 - 450359*x^13 + 4370159*x^12 -
30318522*x^11 - 78096668*x^10 - 9429316*x^9 + 146877876*x^8 + 197107784*x^7
 - 30834152*x^6 - 185125776*x^5 - 132260896*x^4 + 45545344*x^3 + 69517536*x
^2 + sqrt(3)*(2131*x^16 - 10237*x^15 - 549126*x...
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}\left(x^3-6\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3-6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 6*sqrt(3) - 10)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3-6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 - 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{x^3-1}\,\left(-x^3+6\,\sqrt{3}+10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)),x)
[Out] int(-x/((x^3 - 1)^(1/2)*(6*3^(1/2) - x^3 + 10)), x)
```

# 3.89 $\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx$

**Optimal**. Leaf size=214

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-1+x^3}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\right.}{6\sqrt{2}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*(1-x)*(1-3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2
))*3^(1/4)*2^(1/2)+1/18*arctan(1/6*(1+3^(1/2))*(x^3-1)^(1/2)*3^(1/4)*2^(1/2
))*(2+3^(1/2))*3^(1/4)*2^(1/2)+1/18*arctanh(1/2*3^(1/4)*(1+2*x-3^(1/2))*2^(
1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)+1/36*arctanh(1/2*3^(1/4)*(1
-x)*(1+3^(1/2))*2^(1/2)/(x^3-1)^(1/2))*(2+3^(1/2))*3^(3/4)*2^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {501}

$$-\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{2\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\text{ArcTan}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x^3-1}}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{x^3-1}}\right)}{6\sqrt{2}\sqrt[4]{3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(2x-\sqrt{3}+1\right)}{\sqrt{2}\sqrt{x^3-1}}\right)}{3\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
```

```
[Out] -1/2*((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1
 + x^3])])/(Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-1
 + x^3])/(Sqrt[2]*3^(3/4))])/(3*Sqrt[2]*3^(3/4)) + ((2 + Sqrt[3])*ArcTanh[(
3^(1/4)*(1 + Sqrt[3])*(1 - x))/(Sqrt[2]*Sqrt[-1 + x^3])])/(6*Sqrt[2]*3^(1/4
)) + ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[3] + 2*x))/(Sqrt[2]*Sqrt[-1
+ x^3])])/(3*Sqrt[2]*3^(1/4))
```

Rule 501

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[q*(2 - r)*(
ArcTanh[(1 - r)*(Sqrt[a + b*x^3]/(Sqrt[2]*Rt[-a, 2]*r^(3/2)))]/(3*Sqrt[2]*R
t[-a, 2]*d*r^(3/2))), x] + (-Simp[q*(2 - r)*(ArcTanh[Rt[-a, 2]*Sqrt[r]*(1 +
 r)*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2)))
, x] - Simp[q*(2 - r)*(ArcTan[Rt[-a, 2]*Sqrt[r]*((1 + r - 2*q*x)/(Sqrt[2]*S
qrt[a + b*x^3]))]/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r])), x] - Simp[q*(2 - r)*(Ar
cTan[Rt[-a, 2]*(1 - r)*Sqrt[r]*((1 + q*x)/(Sqrt[2]*Sqrt[a + b*x^3]))]/(6*Sq
rt[2]*Rt[-a, 2]*d*Sqrt[r])), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-1+x^3}\left(-10+6\sqrt{3}+x^3\right)}\,dx = -\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)(1-x)}{\sqrt{2}\sqrt{-1+x^3}}\right)}{2\sqrt{2}\,3^{3/4}} + \frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\left(\right.\right.}{3\sqrt{2}}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 10.05, size = 68, normalized size = 0.32

$$\frac{x^2\sqrt{1-x^3}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,-\frac{x^3}{-10+6\sqrt{3}}\right)}{4\left(-5+3\sqrt{3}\right)\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-1 + x^3]*(-10 + 6*Sqrt[3] + x^3)),x]
[Out] (x^2*Sqrt[1 - x^3]*AppellF1[2/3, 1/2, 1, 5/3, x^3, -(x^3/(-10 + 6*Sqrt[3]))
])/(4*(-5 + 3*Sqrt[3])*Sqrt[-1 + x^3])
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 37.46, size = 350, normalized size = 1.64

| method | result |
|---|---|
| default | $\frac{\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right.}{3}\right.}{9\left(-2+\sqrt{3}\right)\sqrt{x^3-1}}$ |
| elliptic | $\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\frac{2\left(1-\sqrt{3}\right)^2\sqrt{3}}{9}+\frac{\left(1-\sqrt{3}\right)^2}{3}+\frac{2\sqrt{3}\left(1-\sqrt{3}\right)}{9}\right.}{}$ |

| trager | Expression too large to display |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(3^(1/2)-1)/(-2+3^(1/2))*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*
3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x
)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-1/18*2^(1/2)*sum((-3^(1/2)*_alpha-_alph
a-2)/(-3^(1/2)+2*_alpha+1)*(-3-I*3^(1/2))*((-1+x)/(-3-I*3^(1/2)))^(1/2)*((-
I*3^(1/2)+2*x+1)/(3-I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x+1)/(I*3^(1/2)+3))^(1/
2)/(x^3-1)^(1/2)*(1+2*_alpha+3^(1/2)*_alpha)*EllipticPi(((-1+x)/(-3/2-1/2*I
*3^(1/2)))^(1/2),1/3*I*_alpha*3^(1/2)+1/2*I*_alpha+1/2*3^(1/2)*_alpha+_alph
a+1/6*I*3^(1/2)+1/2,((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)),_alpha
=RootOf(_Z^2+(1-3^(1/2))*_Z-2*3^(1/2)+4))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 8105 vs. 2(148) = 296.
time = 4.90, size = 8105, normalized size = 37.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/216*sqrt(3)*sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)*(7*sqrt(3) - 12) + 24)*
(672*sqrt(3) + 1164)^(1/4)*(56*sqrt(3) + 97)*(56*sqrt(3) - 97)*arctan(-1/64
8*(432*sqrt(3)*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
 345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 80158
4*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 -
2*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x
^12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 3214
12*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 3654
4) + 271808*x - 126592)*(56*sqrt(3) + 97) + 72*sqrt(3)*(sqrt(3)*(2340*x^17
```

+ 96354*x^16 + 84798*x^15 - 4817124*x^14 - 17052930*x^13 - 1941678*x^12 + 5
7963744*x^11 + 76603680*x^10 - 16678512*x^9 - 139922496*x^8 - 106227360*x^7
 + 42453216*x^6 + 113269536*x^5 + 59694624*x^4 - 30025728*x^3 - 26496000*x^
2 - sqrt(3)*(1351*x^17 + 55630*x^16 + 48958*x^15 - 2781167*x^14 - 9845510*x
^13 - 1121030*x^12 + 33465376*x^11 + 44227144*x^10 - 9629336*x^9 - 80784280
*x^8 - 61330384*x^7 + 24510368*x^6 + 65396192*x^5 + 34464704*x^4 - 17335360
*x^3 - 15297472*x^2 - 7571584*x + 3526400) - 13114368*x + 6107904)*(56*sqrt
(3) + 97) - 6*(97*x^17 + 523*x^16 - 2171*x^15 - 27737*x^14 - 136013*x^13 -
345761*x^12 - 483752*x^11 - 26558*x^10 + 1051756*x^9 + 1656560*x^8 + 801584
*x^7 - 1113424*x^6 - 1680688*x^5 - 911344*x^4 + 536192*x^3 + 535520*x^2 - 2
*sqrt(3)*(28*x^17 + 151*x^16 - 626*x^15 - 8006*x^14 - 39266*x^13 - 99812*x^
12 - 139652*x^11 - 7661*x^10 + 303610*x^9 + 478214*x^8 + 231392*x^7 - 32141
2*x^6 - 485176*x^5 - 263080*x^4 + 154784*x^3 + 154592*x^2 + 78464*x - 36544
) + 271808*x - 126592)*sqrt(56*sqrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(
1/2)*(288*sqrt(3)*(627*x^16 + 14286*x^15 + 39762*x^14 - 50142*x^13 - 216816
*x^12 - 112284*x^11 + 325707*x^10 + 586326*x^9 - 3294*x^8 - 631752*x^7 - 53
9220*x^6 + 184392*x^5 + 483816*x^4 + 115296*x^3 - 108576*x^2 - 2*sqrt(3)*(1
81*x^16 + 4124*x^15 + 11478*x^14 - 14474*x^13 - 62584*x^12 - 32412*x^11 + 9
4021*x^10 + 169244*x^9 - 954*x^8 - 182368*x^7 - 155648*x^6 + 53232*x^5 + 13
9664*x^4 + 33280*x^3 - 31344*x^2 - 37024*x + 11584) - 128256*x + 40128)*(56
*sqrt(3) + 97) + 24*sqrt(3)*(sqrt(3)*(2340*x^17 + 35850*x^16 - 106410*x^15
 + 2064744*x^14 + 11945946*x^13 + 1710042*x^12 - 46293732*x^11 - 59161524*x^
10 + 18480192*x^9 + 122366520*x^8 + 81203856*x^7 - 45222000*x^6 - 100598112
*x^5 - 42207168*x^4 + 29609472*x^3 + 22458240*x^2 - sqrt(3)*(1351*x^17 + 20
698*x^16 - 61436*x^15 + 1192081*x^14 + 6896998*x^13 + 987292*x^12 - 2672770
4*x^11 - 34156928*x^10 + 10669552*x^9 + 70648352*x^8 + 46883072*x^7 - 26108
944*x^6 - 58080352*x^5 - 24368320*x^4 + 17095040*x^3 + 12966272*x^2 + 47244
80*x - 2581504) + 8183040*x - 4471296)*(56*sqrt(3) + 97) - 6*(97*x^17 - 104
*x^16 - 20510*x^15 - 43181*x^14 + 217294*x^13 + 691762*x^12 + 584800*x^11 -
 521510*x^10 - 1780028*x^9 - 1416580*x^8 + 80528*x^7 + 1518056*x^6 + 132171
2*x^5 + 393392*x^4 - 501952*x^3 - 446848*x^2 - 4*sqrt(3)*(14*x^17 - 15*x^16
 - 2960*x^15 - 6232*x^14 + 31362*x^13 + 99844*x^12 + 84404*x^11 - 75267*x^1
0 - 256916*x^9 - 204458*x^8 + 11616*x^7 + 219104*x^6 + 190768*x^5 + 56784*x
^4 - 72448*x^3 - 64496*x^2 - 24480*x + 13376) - 169600*x + 92672)*sqrt(56*s
qrt(3) + 97))*sqrt(56*sqrt(3) + 97) - sqrt(-4*sqrt(3)*sqrt(56*sqrt(3) + 97)
*(7*sqrt(3) - 12) + 24)*((2*sqrt(3)*(3691*x^16 - 17731*x^15 - 951114*x^14 -
 450359*x^13 + 4370159*x^12 - 30318522*x^11 - 78096668*x^10 - 9429316*x^9 +
 146877876*x^8 + 197107784*x^7 - 30834152*x^6 - 185125776*x^5 - 132260896*x
^4 + 45545344*x^3 + 69517536*x^2 - sqrt(3)*(2131*x^16 - 10237*x^15 - 549126
*x^14 - 260015*x^13 + 2523113*x^12 - 17504406*x^11 - 45089132*x^10 - 544402
0*x^9 + 84799980*x^8 + 113800232*x^7 - 17802104*x^6 - 106882416*x^5 - 76360
864*x^4 + 26295616*x^3 + 40135968*x^2 + 7907648*x - 5562368) + 13696448*x -
 9634304)*sqrt(x^3 - 1)*(56*sqrt(3) + 97) - (459*x^16 + 1557*x^15 - 26415*x
^14 + 1449954*x^13 + 4677912*x^12 - 12651948*x^11 - 55684800*x^10 - 6283425
6*x^9 + 8526168*x^8 + 105313392*x^7 + 99605088*x^6 + 18897984*x^5 - 4249929

```
6*x^4 - 37357632*x^3 - 8256960*x^2 - sqrt(3)*(265*x^16 + 899*x^15 - 15249*x
^14 + 837130*x^13 + 2700776*x^12 - 7304604*x^11 - 32149640*x^10 - 36277360*
x^9 + 4922568*x^8 + 60802736*x^7 + 57507040*x^6 + 10910784*x^5 - 24536992*x
^4 - 21568448*x^3 - 4767168*x^2 - 1207168*x + 1383424) - 2090880*x + 239616
0)*sqrt(x^3 - 1)*sqrt(56*sqrt(3) + 97))*(672*sqrt(3) + 1164)^(3/4) + 6*(sqr
t(3)*(4945*x^15 + 37473*x^14 - 490698*x^13 - 2249468*x^12 + 474132*x^11 + 8
423784*x^10 + 5853520*x^9 - 8451720*x^8 - 15320016*x^7 - 768064*x^6 + 10405
056*x^5 + 6627744*x^4 - 700480*x^3 - 2799552*x^2 - sqrt(3)*(2855*x^15 + 216
35*x^14 - 283306*x^13 - 1298732*x^12 + 273748*x^11 + 4863472*x^10 + 3379536
*x^9 - 4879608*x^8 - 8845008*x^7 - 443456*x^6 + 6007360*x^5 + 3826528*x^4 -
 404416*x^3 - 1616320*x^2 - 1003648*x + 399360) - 1738368*x + 691712)*sqrt(
x^3 - 1)*(56*sqrt(3) + 97) - 2*(246*x^15 + 3678*x^14 - 13485*x^13 - 102933*
x^12 - 70062*x^11 + 81156*x^10 + 45204*x^9 + 12...
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x^3-10+6\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x**3+6*3**(1/2))/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x**3 - 10 + 6*sqrt(3))), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-10+x^3+6*3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/((x^3 + 6*sqrt(3) - 10)*sqrt(x^3 - 1)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{x^3-1}\,\left(x^3+6\,\sqrt{3}-10\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)),x)
[Out] int(x/((x^3 - 1)^(1/2)*(6*3^(1/2) + x^3 - 10)), x)
```

## 3.90 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx$

**Optimal**. Leaf size=65

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

**Rubi** [A]

time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1754, 213}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)^3\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}\,x^2}}\right.$$

**Mathematica [A]**
time = 8.31, size = 77, normalized size = 1.18

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{9+6\sqrt{3}}\,\sqrt{-4+4\sqrt{3}\,x^2+x^4}}{2+\left(-2-2\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-4 + 4*Sqrt[3]*x^2
+ x^4])/(2 + (-2 - 2*Sqrt[3])*x + (2 + Sqrt[3])*x^2)])/3
```

**Maple [C]** Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.42, size = 327, normalized size = 5.03

| method | result |
|---|---|
| default | $\frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\,\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4\sqrt{3}\,x^2}} -$ |
| elliptic | $\frac{\sqrt{1-\left(-1+\frac{\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\,\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4\sqrt{3}\,x^2}} -$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*
(1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*(-1-
3^(1/2))^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(4*(-1-3^(1/2))^2*3^(1/2)-8+4*3^(1/
2)*x^2+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*(-1-3^(1/2))^2*3^(1/2)-4)^(1
/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/2))-1/(-1+1/2*3^(1/2))^(1/2)/(-1-3^(1/2))*(1-
(-1+1/2*3^(1/2))*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*3^(1/2)
*x^2)^(1/2)*EllipticPi((-1+1/2*3^(1/2))^(1/2)*x,1/(-1+1/2*3^(1/2))/(-1-3^(1
/2))^2,(1+1/2*3^(1/2))^(1/2)/(-1+1/2*3^(1/2))^(1/2)))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(47) = 94$.
time = 1.12, size = 323, normalized size = 4.97

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37x^{12}-204x^{11}+804x^{10}-2408x^{9}+3708x^{8}-5472x^{7}+6432x^{6}+10944x^{5}+14832x^{4}+19264x^{3}+12864x^{2}+\left(54x^{10}-300x^{9}+1026x^{8}-2232x^{7}+3024x^{6}-3024x^{5}-1008x^{4}-2016x^{3}-2592x^{2}+\sqrt{3}\left(31x^{10}-176x^{9}+576x^{8}-1320x^{7}+1848x^{6}-1008x^{5}+1344x^{4}+1632x^{3}+1008x^{2}+832x+256\right)-1152x-480\right)\sqrt{x^{4}+4\sqrt{3}x^{2}-4}\sqrt{2\sqrt{3}-3}+3\sqrt{3}\left(7x^{12}-40x^{11}+160x^{10}-400x^{9}+924x^{8}-960x^{7}-1920x^{5}-3696x^{4}-3200x^{3}-2560x^{2}-1280x-448\right)+6528x+2368}{x^{12}+12x^{11}+48x^{10}+40x^{9}-180x^{8}-288x^{7}+384x^{6}+576x^{5}-720x^{4}-320x^{3}+768x^{2}-384x+64}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
```

```
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\left(x + 1 + \sqrt{3}\right) \sqrt{x^4 + 4\sqrt{3}\, x^2 - 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x - \sqrt{3} + 1}{\left(x + \sqrt{3} + 1\right) \sqrt{x^4 + 4\,\sqrt{3}\,x^2 - 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)
```

```
[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

## 3.91 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$

**Optimal**. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\,\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2
))*(3+2*3^(1/2))^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1754, 209}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\text{ArcTan}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\,\sqrt{x^4-4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\left(\right.}\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}\,x^2+}}\right.$$

**Mathematica** [A]
time = 8.20, size = 77, normalized size = 1.22

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-4-4\sqrt{3}\,x^2+x^4}}{-2+\left(2-2\sqrt{3}\right)x+\left(-2+\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-4 - 4*Sqrt[3]*
x^2 + x^4])/(-2 + (2 - 2*Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])
```

**Maple** [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.43, size = 311, normalized size = 4.94

| method | result |
|---|---|
| default | $\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4\sqrt{3}\,x^2}}+$ |
| elliptic | $\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(1-\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}\left(1-\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4\sqrt{3}\,x^2}}+$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x
^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I*
(1-4*3^(1/2)*(1-1/2*3^(1/2)))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*(3^(1
/2)-1)^2*3^(1/2)-4)^(1/2)*arctanh(1/2*(-4*(3^(1/2)-1)^2*3^(1/2)-8-4*3^(1/2)
*x^2+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*(3^(1/2)-1)^2*3^(1/2)-4)^(1/2)/(
-4+x^4-4*3^(1/2)*x^2)^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-1/
2*3^(1/2))*x^2)^(1/2)*(1-(1-1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^
(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)^2,
(1-1/2*3^(1/2))^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(45) = 90$.
time = 1.25, size = 112, normalized size = 1.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\left(2x^4-10x^3+3x^2-10x+2\right)+24\right)\sqrt{x^4-4\sqrt{3}x^2-4}\sqrt{2\sqrt{3}+3}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
```

```
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

# 3.92 $\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx$

**Optimal**. Leaf size=53

$$\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

```
[Out] ln(1+x)-3/2*ln(2+x-(x^3+2)^(1/3))+arctan(1/3*(1+2*(2+x)/(x^3+2)^(1/3))*3^(1
/2))*3^(1/2)
```

**Rubi** [A]

time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2176}

$$\sqrt{3}\,\mathrm{ArcTan}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)-\frac{3}{2}\log\left(-\sqrt[3]{x^3+2}+x+2\right)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]] + Log[1 + x] - (3
*Log[2 + x - (2 + x^3)^(1/3)])/2
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{-1+x}{(1+x)\sqrt[3]{2+x^3}}\,dx=\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)+\log(1+x)-\frac{3}{2}\log\left(2+x-\sqrt[3]{2+x^3}\right)$$

**Mathematica** [A]

time = 0.61, size = 92, normalized size = 1.74

$$-\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2+x^3}}{4+2x+\sqrt[3]{2+x^3}}\right)-\log\left(-2-x+\sqrt[3]{2+x^3}\right)+\frac{1}{2}\log\left(4+4x+x^2+(2+x)\sqrt[3]{2+x^3}+\left(2+x^3\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(Sqrt[3]*(2 + x^3)^(1/3))/(4 + 2*x + (2 + x^3)^(1/3))]) -
Log[-2 - x + (2 + x^3)^(1/3)] + Log[4 + 4*x + x^2 + (2 + x)*(2 + x^3)^(1/3)
 + (2 + x^3)^(2/3)]/2
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.30, size = 544, normalized size = 10.26

| method | result |
|---|---|
| trager | $-\ln\left(-\frac{787\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)^2 x^3+9008\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^3+2\right)^{\frac{2}{3}} x-9678\,\mathrm{RootOf}\left(\_Z^2-2\_Z+4\right)\left(x^3+2\right)^{\frac{1}{3}} x}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(-(787*RootOf(_Z^2-2*_Z+4)^2*x^3+9008*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)*
x-9678*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x^2-1574*RootOf(_Z^2-2*_Z+4)^2*x^2
-904*RootOf(_Z^2-2*_Z+4)*x^3+18016*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)+1340*x
*(x^3+2)^(2/3)-38712*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x+18016*(x^3+2)^(1/3
)*x^2-3148*RootOf(_Z^2-2*_Z+4)^2*x+23844*RootOf(_Z^2-2*_Z+4)*x^2-16208*x^3+
2680*(x^3+2)^(2/3)-38712*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)+72064*(x^3+2)^(1
/3)*x+47688*RootOf(_Z^2-2*_Z+4)*x-81040*x^2+72064*(x^3+2)^(1/3)+22036*RootO
f(_Z^2-2*_Z+4)-162080*x-113456)/(1+x)^2)+1/2*RootOf(_Z^2-2*_Z+4)*ln((1013*R
ootOf(_Z^2-2*_Z+4)^2*x^3+4504*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)*x+335*RootO
f(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x^2-2026*RootOf(_Z^2-2*_Z+4)^2*x^2-6865*RootOf
(_Z^2-2*_Z+4)*x^3+9008*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(2/3)-9678*x*(x^3+2)^(2/
3)+1340*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)*x+9008*(x^3+2)^(1/3)*x^2-4052*Roo
tOf(_Z^2-2*_Z+4)^2*x-14634*RootOf(_Z^2-2*_Z+4)*x^2+4722*x^3-19356*(x^3+2)^(
2/3)+1340*RootOf(_Z^2-2*_Z+4)*(x^3+2)^(1/3)+36032*(x^3+2)^(1/3)*x-29268*Roo
tOf(_Z^2-2*_Z+4)*x+12592*x^2+36032*(x^3+2)^(1/3)-28364*RootOf(_Z^2-2*_Z+4)+
25184*x+22036)/(1+x)^2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x**3+2)**(1/3),x)
[Out] Integral((x - 1)/((x + 1)*(x**3 + 2)**(1/3)), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
[Out] integrate((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-1}{(x^3+2)^{1/3}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)),x)
[Out] int((x - 1)/((x^3 + 2)^(1/3)*(x + 1)), x)
```

# 3.93 $\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$

**Optimal**. Leaf size=108

$$\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)-\frac{1}{2}\log(1+x)+\frac{3}{4}\log\left(2+x-\sqrt[3]{2+x^3}\right)-\frac{1}{4}\log\left(-x\right.$$

```
[Out] -1/2*ln(1+x)+3/4*ln(2+x-(x^3+2)^(1/3))-1/4*ln(-x+(x^3+2)^(1/3))+1/6*arctan(
1/3*(1+2*x/(x^3+2)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1/3*(1+2*(2+x)/(x^3+2
)^(1/3))*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2175, 245, 2176}

$$\frac{\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2(x+2)}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)+\frac{3}{4}\log\left(-\sqrt[3]{x^3+2}+x+2\right)-\frac{1}{4}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - (Sqrt[3]*ArcTan[(
1 + (2*(2 + x))/(2 + x^3)^(1/3))/Sqrt[3]])/2 - Log[1 + x]/2 + (3*Log[2 + x
- (2 + x^3)^(1/3)])/4 - Log[-x + (2 + x^3)^(1/3)]/4
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2175

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
```

```
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt[3]{2+x^3}}\,dx + \frac{1}{2}\int \frac{1-x}{(1+x)\sqrt[3]{2+x^3}}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2+x)}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right) - \frac{1}{2}\log(1+x) + \frac{3}{4}\log\Big($$

**Mathematica** [F]
time = 11.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\sqrt[3]{2+x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + x)*(2 + x^3)^(1/3)),x]
[Out] Integrate[1/((1 + x)*(2 + x^3)^(1/3)), x]
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.14, size = 1421, normalized size = 13.16

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 1421 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^3+2)^(1/3),x,method=_RETURNVERBOSE)
[Out] 1/6*ln((-4550781346817636-6728375859478224*x+4993190285176576*RootOf(_Z^2+_
Z+1)^2*x^3+9094739448000192*RootOf(_Z^2+_Z+1)^2*x^2+5884831407529536*RootOf
(_Z^2+_Z+1)^2*x-625895428788672*x^5-68457312523761*x^6-234710785795752*x^4-
469421571591504*x^2+2151515536461060*x^3+8816461926585488*RootOf(_Z^2+_Z+1)
*x^3+1055101552116528*RootOf(_Z^2+_Z+1)*x^2-21283128527537520*RootOf(_Z^2+_
Z+1)*x+16295099853018372*x*(x^3+2)^(2/3)-15559137585059152*RootOf(_Z^2+_Z+1
)+4346750471470680*RootOf(_Z^2+_Z+1)^2*x^5+153868976350327*RootOf(_Z^2+_Z+1
```

)*x^6+4547369724000096*RootOf(_Z^2+_Z+1)^2*x^4-868588603920114*RootOf(_Z^2+_Z+1)*x^5+527550776058264*RootOf(_Z^2+_Z+1)*x^4-9125490357912936*(x^3+2)^(1/3)-928201890361806*(x^3+2)^(2/3)*x^4-540325086981687*(x^3+2)^(1/3)*x^5-1959537324097146*(x^3+2)^(2/3)*x^3+36303984101745*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^4+288449229728205*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^5-1306943427662820*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^3-601904942144643*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4+1286927332633530*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^4-580773949503594*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^5-3775614346581480*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^2-3235955176589922*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3+1420057746354240*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^3-3304587786698814*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^4-2613886855325640*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x-1442113972435308*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^2-310637632014990*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^2-4286079775383252*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^3+7759251414704196*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x-798782482324260*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x+2571201069229632*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+7575270505902288*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x-480288966205944*(x^3+2)^(1/3)*x^4+5569211342170836*(x^3+2)^(2/3)*x^2+1200722415514860*(x^3+2)^(1/3)*x^3+7115580883942020*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)+3372637147591320*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)+10107087250606332*(x^3+2)^(2/3)+1326316169500028*RootOf(_Z^2+_Z+1)^2*x^6-4682817420507954*(x^3+2)^(1/3)*x^2-14648813469281292*(x^3+2)^(1/3)*x)/(1+x)^6)+1/6*RootOf(_Z^2+_Z+1)*ln((-15559137585059152-28889172364235904*x-1460422667173568*RootOf(_Z^2+_Z+1)^2*x^3-2660055572351856*RootOf(_Z^2+_Z+1)^2*x^2-1721212429168848*RootOf(_Z^2+_Z+1)^2*x-6486689165117784*x^5-1560371964117680*x^6-5349846734117760*x^4-10699693468235520*x^2+2362848974235344*x^3-4302097415889084*RootOf(_Z^2+_Z+1)*x^3-12224216591943552*RootOf(_Z^2+_Z+1)*x^2-14334419696176608*RootOf(_Z^2+_Z+1)*x+5921961582988536*x*(x^3+2)^(2/3)-4550781346817636*RootOf(_Z^2+_Z+1)-1271350089726990*RootOf(_Z^2+_Z+1)^2*x^5-1782698252991768*RootOf(_Z^2+_Z+1)*x^6-1330027786175928*RootOf(_Z^2+_Z+1)^2*x^4-6243995989986342*RootOf(_Z^2+_Z+1)*x^5-6112108295971776*RootOf(_Z^2+_Z+1)*x^4+9125490357912936*(x^3+2)^(1/3)-289992964115418*(x^3+2)^(2/3)*x^4-240144483102972*(x^3+2)^(1/3)*x^5-30525575170044*(x^3+2)^(2/3)*x^3+36303984101745*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^4-1068918799812864*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^5-1306943427662820*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^3+674512910348133*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^4-4769022337626624*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^4-1109367662334771*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^5-3775614346581480*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x^2+622068321264282*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^3-5262369476001792*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^3-7593321158119494*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^4-2613886855325640*(x^3+2)^(2/3)*RootOf(_Z^2+_Z+1)^2*x-6109114720727652*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x^2+1151143322875392*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x^2-10749171666899904*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^3-12987025125355476*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)*x+2960082830251008*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)^2*x+8405161812612978*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x^2+25184166805434588*(x^3+2)^(1/3)*RootOf(_Z^2+_Z+1)*x-3001806038787150*(x^3+2)^(1/3)*x^4+3

```
235710968024664*(x^3+2)^(2/3)*x^2-5043034145162412*(x^3+2)^(1/3)*x^3-711558
0883942020*RootOf(_Z^2+_Z+1)*(x^3+2)^(2/3)+12498127505504256*(x^3+2)^(1/3)*
RootOf(_Z^2+_Z+1)+2991506366664312*(x^3+2)^(2/3)-387924770967979*RootOf(_Z^
2+_Z+1)^2*x^6+5523323111368356*(x^3+2)^(1/3)*x^2+16810113817208040*(x^3+2)^
(1/3)*x)/(1+x)^6)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(84) = 168.
time = 1.48, size = 267, normalized size = 2.47

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{13910019318573948542\sqrt{3}\left(7114781247x^4+13663058416x^3-46178206896x^2-126842559344x-77084338088\right)\left(x^3+2\right)^{\frac{2}{3}}-27820038637147897084\sqrt{3}\left(1625757424x^5+16302821713x^4+26102613730x^3-26431113242x^2-80188343316x-42779182428\right)\left(x^3+2\right)^{\frac{1}{3}}+\sqrt{3}\left(93292570833559435663132301885x^6+382151535711085278859235047618x^5+673924074224408772959625384792x^4+889426563183087468015580290048x^3+888876515195959220955879945824x^2+351260598258508240019971964880x-47674000995597211057816884304\right)}{3\left(78905434814564721745708464883x^6+337746705836458222863347934450x^5+155989527760585878943360709 76x^4-895430525315100108684787964824x^3+361667862240477028869533375352x^2+254180230101163251064597209 0336x+1554815286823334092314485968880\right)}\right)+\frac{1}{12}\log\left(\frac{22x^6+6x^5-48x^4+44x^3+24x^2+3\left(7x^4-2x^3-32x^2-20x+4\right)\left(x^3+2\right)^{\frac{2}{3}}+3\left(7x^5-16x^3+34x^2+76x+32\right)\left(x^3+2\right)^{\frac{1}{3}}-192x-140}{x^6+6x^5+15x^4+20x^3+15x^2+6x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*(13910019318573948542*sqrt(3)*(7114781247*x^4 + 1366
3058416*x^3 - 46178206896*x^2 - 126842559344*x - 77084338088)*(x^3 + 2)^(2/
3) - 27820038637147897084*sqrt(3)*(1625757424*x^5 + 16302821713*x^4 + 26102
613730*x^3 - 26431113242*x^2 - 80188343316*x - 42779182428)*(x^3 + 2)^(1/3)
 + sqrt(3)*(93292570833559435663132301885*x^6 + 382151535711085278859235047
618*x^5 + 673924074224408772959625384792*x^4 + 889426563183087468015580290
048*x^3 + 888876515195959220955879945824*x^2 + 3512605982585082400199719648
80*x - 47674000995597211057816884304))/(78905434814564721745708464883*x^6 +
 337746705836458222863347934450*x^5 + 155989527760585878943360709 76*x^4 - 89
5430525315100108684787964824*x^3 + 361667862240477028869533375352*x^2 + 254
1802301011632510645972090336*x + 1554815286823334092314485968880)) + 1/12*l
og((22*x^6 + 6*x^5 - 48*x^4 + 44*x^3 + 24*x^2 + 3*(7*x^4 - 2*x^3 - 32*x^2 -
 20*x + 4)*(x^3 + 2)^(2/3) + 3*(7*x^5 - 16*x^3 + 34*x^2 + 76*x + 32)*(x^3 +
 2)^(1/3) - 192*x - 140)/(x^6 + 6*x^5 + 15*x^4 + 20*x^3 + 15*x^2 + 6*x + 1)
)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+1)\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x + 1)*(x**3 + 2)**(1/3)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^3+2\right)^{1/3}\,\left(x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 2)^(1/3)*(x + 1)),x)
```

```
[Out] int(1/((x^3 + 2)^(1/3)*(x + 1)), x)
```

# 3.94 $\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

**Optimal**. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}\,x-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3+a)^(1/3))/(a+b)^(1/3
)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/(a+b)^(1/3)*3
^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {384}

$$\frac{\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3)*x - (a + b*x^
3)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \text{Subst}\left(\int \frac{1}{1-(a+b)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{a+b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2+\sqrt[3]{a+b}\,}{1+\sqrt[3]{a+b}\,x+(a-}\right.$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{a+b}\,x+(a+b)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+}}\right.$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{a+b}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}} - \frac{\log\left(1-\frac{\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{a+b}} + \frac{\log\left(1+\frac{(a+b)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{a+}}{\sqrt[3]{a+}}\right.}{6\sqrt[3]{a+b}}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.90, size = 189, normalized size = 1.93

$$\frac{-2\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{3\sqrt[3]{a+b}\,x}{\sqrt{3}\sqrt[3]{a+b}\,x-\left(3i+\sqrt{3}\right)\sqrt[3]{a+bx^3}}\right)+\left(1+i\sqrt{3}\right)\left(2\log\left(2\sqrt[3]{a+b}\,x+\left(1+i\sqrt{3}\right)\sqrt[3]{a+bx^3}\right)-\log\left(\left(-\sqrt[3]{a+b}\,x+\sqrt[3]{a+bx^3}\right)\left(2i\sqrt[3]{a+b}\,x+\left(i+\sqrt{3}\right)\sqrt[3]{a+bx^3}\right)\right)\right)}{12\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(3*(a + b)^(1/3)*x)/(Sqrt[3]*(a + b)^(1
/3)*x - (3*I + Sqrt[3])*(a + b*x^3)^(1/3))] + (1 + I*Sqrt[3])*(2*Log[2*(a +
 b)^(1/3)*x + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)] - Log[(-((a + b)^(1/3)*x)
+ (a + b*x^3)^(1/3))*((2*I)*(a + b)^(1/3)*x + (I + Sqrt[3])*(a + b*x^3)^(1/
3))]))/(12*(a + b)^(1/3))
```

**Maple [F]**
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^3+1)\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(78) = 156.
time = 134.08, size = 1252, normalized size = 12.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log(-((a^3 - 27*a^2
*b - 108*a*b^2 - 81*b^3)*x^9 - 3*(10*a^3 + 54*a^2*b + 45*a*b^2)*x^6 - 3*(17
*a^3 + 18*a^2*b)*x^3 - a^3 + 9*((2*a^2 + 3*a*b)*x^7 - (a^2 + 3*a*b)*x^4 - a
^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(1/3) + 9*((a^2 + 9*a*b + 9*b^2)*x^8 + (7*
a^2 + 9*a*b)*x^5 + a^2*x^2)*(b*x^3 + a)^(1/3)*(-a - b)^(2/3) + 3*sqrt(1/3)*
(3*((4*a^2 + 21*a*b + 18*b^2)*x^7 + (13*a^2 + 15*a*b)*x^4 + a^2*x)*(b*x^3 +
 a)^(2/3)*(-a - b)^(2/3) + 3*((a^3 - 2*a^2*b - 12*a*b^2 - 9*b^3)*x^8 - 5*(a
^3 + 4*a^2*b + 3*a*b^2)*x^5 - 5*(a^3 + a^2*b)*x^2)*(b*x^3 + a)^(1/3) + ((a^
3 + 27*a^2*b + 54*a*b^2 + 27*b^3)*x^9 + 3*(8*a^3 + 18*a^2*b + 9*a*b^2)*x^6
+ 3*a^3*x^3 - a^3)*(-a - b)^(1/3))*sqrt((-a - b)^(1/3)/(a + b)))/(x^9 - 3*x
^6 + 3*x^3 - 1)) - 2*(-a - b)^(2/3)*log(-(3*(b*x^3 + a)^(1/3)*(a + b)*(-a -
 b)^(1/3)*x^2 + 3*(b*x^3 + a)^(2/3)*(a + b)*x + (a*x^3 - a)*(-a - b)^(2/3))
/(x^3 - 1)) + (-a - b)^(2/3)*log((3*((2*a + 3*b)*x^4 + a*x)*(b*x^3 + a)^(2/
3)*(-a - b)^(2/3) + 3*((a^2 + 4*a*b + 3*b^2)*x^5 + 2*(a^2 + a*b)*x^2)*(b*x^
3 + a)^(1/3) - ((a^2 + 9*a*b + 9*b^2)*x^6 + (7*a^2 + 9*a*b)*x^3 + a^2)*(-a
- b)^(1/3))/(x^6 - 2*x^3 + 1)))/(a + b), 1/18*(6*sqrt(1/3)*(a + b)*sqrt(-(-
a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(6*((2*a^2 + 3*a*b)*x^7 - (a^2 + 3*a
*b)*x^4 - a^2*x)*(b*x^3 + a)^(2/3)*(-a - b)^(2/3) - 6*((a^3 + 10*a^2*b + 18
*a*b^2 + 9*b^3)*x^8 + (7*a^3 + 16*a^2*b + 9*a*b^2)*x^5 + (a^3 + a^2*b)*x^2)
*(b*x^3 + a)^(1/3) - ((a^3 - 9*a^2*b - 36*a*b^2 - 27*b^3)*x^9 - 3*(4*a^3 +
18*a^2*b + 15*a*b^2)*x^6 - 3*(5*a^3 + 6*a^2*b)*x^3 - a^3)*(-a - b)^(1/3))*s
qrt(-(-a - b)^(1/3)/(a + b))/((a^3 + 27*a^2*b + 54*a*b^2 + 27*b^3)*x^9 + 3*
(8*a^3 + 18*a^2*b + 9*a*b^2)*x^6 + 3*a^3*x^3 - a^3)) - 2*(-a - b)^(2/3)*log
(-(3*(b*x^3 + a)^(1/3)*(a + b)*(-a - b)^(1/3)*x^2 + 3*(b*x^3 + a)^(2/3)*(a
+ b)*x + (a*x^3 - a)*(-a - b)^(2/3))/(x^3 - 1)) + (-a - b)^(2/3)*log((3*((2
*a + 3*b)*x^4 + a*x)*(b*x^3 + a)^(2/3)*(-a - b)^(2/3) + 3*((a^2 + 4*a*b + 3
```

```
*b^2)*x^5 + 2*(a^2 + a*b)*x^2)*(b*x^3 + a)^(1/3) - ((a^2 + 9*a*b + 9*b^2)*x
^6 + (7*a^2 + 9*a*b)*x^3 + a^2)*(-a - b)^(1/3))/(x^6 - 2*x^3 + 1)))/(a + b)
]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^{3}\sqrt[3]{a+bx^{3}}-\sqrt[3]{a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)/(b*x**3+a)**(1/3),x)
[Out] -Integral(1/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^3 + a)^(1/3)*(x^3 - 1)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(x^{3}-1\right)\,{\left(b\,x^{3}+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
[Out] -int(1/((x^3 - 1)*(a + b*x^3)^(1/3)), x)
```

# 3.95 $\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$

**Optimal**. Leaf size=154

$$\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}\,x-\sqrt[3]{a+b\right.}}{2\sqrt[3]{a+b}}$$

```
[Out] 1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)*x-(b*x^3
+a)^(1/3))/(a+b)^(1/3)+1/3*arctan(1/3*(1+2*(a+b)^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))/(a+b)^(1/3)*3^(1/2)+1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3)
)*3^(1/2))/(a+b)^(1/3)*3^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2183, 384, 455, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{a+b}}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}}+\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}-\frac{\log\left(x\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*(a + b)^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3)) + ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[
3]*(a + b)^(1/3)) + Log[(a + b)^(1/3) - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3)
) - Log[(a + b)^(1/3)*x - (a + b*x^3)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx = \int \left(\frac{1-\frac{i}{\sqrt{3}}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}+\frac{1+\frac{i}{\sqrt{3}}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\right)dx$$

$$= \frac{1}{3}\left(3-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{a+bx^3}}\,dx+\frac{1}{3}\left(3+i\sqrt{3}\right)\int \frac{}{\left(1+i\sqrt{\right.}\right.}$$

**Mathematica [F]**
time = 10.15, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(1+x+x^2)\sqrt[3]{a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)),x]
[Out] Integrate[(1 + x)/((1 + x + x^2)*(a + b*x^3)^(1/3)), x]
```

**Maple [F]**
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(x^2+x+1)\,(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
[Out] int((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{a+bx^3}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+x+1)/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral((x + 1)/((a + b*x**3)**(1/3)*(x**2 + x + 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+x+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((b*x^3 + a)^(1/3)*(x^2 + x + 1)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{\left(b\,x^3+a\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] int((x + 1)/((a + b*x^3)^(1/3)*(x + x^2 + 1)), x)
```

# 3.96 $\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx$

**Optimal**. Leaf size=96

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

```
[Out] 1/6*ln(-x^3+1)/(a+b)^(1/3)-1/2*ln((a+b)^(1/3)-(b*x^3+a)^(1/3))/(a+b)^(1/3)-
1/3*arctan(1/3*(1+2*(b*x^3+a)^(1/3)/(a+b)^(1/3))*3^(1/2))/(a+b)^(1/3)*3^(1/
2)
```

**Rubi** [A]

time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {455, 57, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a+b}}+\frac{\log\left(1-x^3\right)}{6\sqrt[3]{a+b}}-\frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*(a + b*x^3)^(1/3))/(a + b)^(1/3))/Sqrt[3]]/(Sqrt[3]*(a + b
)^(1/3))) + Log[1 - x^3]/(6*(a + b)^(1/3)) - Log[(a + b)^(1/3) - (a + b*x^3
)^(1/3)]/(2*(a + b)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-x^3)\sqrt[3]{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt[3]{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b)^{2/3}+\sqrt[3]{a+b}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right) + \frac{\text{Sub}}{\phantom{x}}$$

$$= \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+b}}{\sqrt[3]{a+}}\right.}{\sqrt[3]{a+b}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a+b}} + \frac{\log(1-x^3)}{6\sqrt[3]{a+b}} - \frac{\log\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a+b}}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.34, size = 182, normalized size = 1.90

$$\frac{2\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1+\frac{(-1-i\sqrt{3})\sqrt[3]{a+bx^3}}{\sqrt[3]{a+b}}}{\sqrt{3}}\right)-i\left(-i+\sqrt{3}\right)\left(\log\left(\left(\sqrt[3]{a+b}-\sqrt[3]{a+bx^3}\right)\left(2\sqrt[3]{a+b}+\sqrt[3]{a+bx^3}-i\sqrt{3}\sqrt[3]{a+bx^3}\right)\right)-2\log\left(2\sqrt[3]{a+b}+\left(1+i\sqrt{3}\right)\sqrt[3]{a+bx^3}\right)\right)}{12\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)*(a + b*x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[(1 + ((-1 - I*Sqrt[3])*(a + b*x^3)^(1/3)
)/(a + b)^(1/3))/Sqrt[3]] - I*(-I + Sqrt[3])*(Log[((a + b)^(1/3) - (a + b*x
^3)^(1/3))*(2*(a + b)^(1/3) + (a + b*x^3)^(1/3) - I*Sqrt[3]*(a + b*x^3)^(1/
3))] - 2*Log[2*(a + b)^(1/3) + (1 + I*Sqrt[3])*(a + b*x^3)^(1/3)]))/(12*(a
+ b)^(1/3))
```

**Maple [F]**
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(-x^{3}+1\right) \left(b \,x^{3}+a \right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

```
[Out] int(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x)
```

**Maxima [A]**
time = 0.62, size = 110, normalized size = 1.15

$$-\frac{\frac{2 \, \sqrt{3} \, b \arctan\left(\frac{\sqrt{3} \, \left(2 \, {\left(b x^{3} + a\right)}^{\frac{1}{3}} + {\left(a + b\right)}^{\frac{1}{3}}\right)}{3 \, {\left(a + b\right)}^{\frac{1}{3}}}\right)}{{\left(a + b\right)}^{\frac{1}{3}}} - \frac{b \log\left({\left(b x^{3} + a\right)}^{\frac{2}{3}} + {\left(b x^{3} + a\right)}^{\frac{1}{3}} {\left(a + b\right)}^{\frac{1}{3}} + {\left(a + b\right)}^{\frac{2}{3}}\right)}{{\left(a + b\right)}^{\frac{1}{3}}} + \frac{2 \, b \log\left({\left(b x^{3} + a\right)}^{\frac{1}{3}} - {\left(a + b\right)}^{\frac{1}{3}}\right)}{{\left(a + b\right)}^{\frac{1}{3}}}}{6 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/
(a + b)^(1/3))/(a + b)^(1/3) - b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) + 2*b*log((b*x^3 + a)^(1/3) -
(a + b)^(1/3))/(a + b)^(1/3))/b
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(75) = 150$.
time = 1.09, size = 387, normalized size = 4.03

$$\left[\frac{3 \, \sqrt{\frac{1}{3}} {\left(a + b\right)} \sqrt{\frac{{\left(-a - b\right)}^{\frac{1}{3}}}{a + b}} \log\left(\frac{2 \, b x^{3} + 3 \, \sqrt{\frac{1}{3}} {\left({\left(b x^{3} + a\right)}^{\frac{1}{3}} {\left(a + b\right)} - {\left(a + b\right)} {\left(-a - b\right)}^{\frac{1}{3}} - 2 \, {\left(b x^{3} + a\right)}^{\frac{2}{3}} {\left(-a - b\right)}^{\frac{2}{3}}\right)} \sqrt{\frac{{\left(-a - b\right)}^{\frac{1}{3}}}{a + b}} + 3 \, a - 3 \, {\left(b x^{3} + a\right)}^{\frac{1}{3}} {\left(-a - b\right)}^{\frac{2}{3}} + b}{x^{3} - 1}\right) + {\left(-a - b\right)}^{\frac{2}{3}} \log\left({\left(b x^{3} + a\right)}^{\frac{2}{3}} - {\left(b x^{3} + a\right)}^{\frac{1}{3}} {\left(-a - b\right)}^{\frac{1}{3}} + {\left(-a - b\right)}^{\frac{2}{3}}\right) - 2 \, {\left(-a - b\right)}^{\frac{2}{3}} \log\left({\left(b x^{3} + a\right)}^{\frac{1}{3}} + {\left(-a - b\right)}^{\frac{1}{3}}\right)}{6 \, {\left(a + b\right)}}, -\frac{6 \, \sqrt{\frac{1}{3}} {\left(a + b\right)} \sqrt{-\frac{{\left(-a - b\right)}^{\frac{1}{3}}}{a + b}} \arctan\left(\sqrt{\frac{1}{3}} {\left(2 \, {\left(b x^{3} + a\right)}^{\frac{1}{3}} - {\left(-a - b\right)}^{\frac{1}{3}}\right)} \sqrt{-\frac{{\left(-a - b\right)}^{\frac{1}{3}}}{a + b}}\right) - {\left(-a - b\right)}^{\frac{2}{3}} \log\left({\left(b x^{3} + a\right)}^{\frac{2}{3}} - {\left(b x^{3} + a\right)}^{\frac{1}{3}} {\left(-a - b\right)}^{\frac{1}{3}} + {\left(-a - b\right)}^{\frac{2}{3}}\right) + 2 \, {\left(-a - b\right)}^{\frac{2}{3}} \log\left({\left(b x^{3} + a\right)}^{\frac{1}{3}} + {\left(-a - b\right)}^{\frac{1}{3}}\right)}{6 \, {\left(a + b\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a + b)*sqrt((-a - b)^(1/3)/(a + b))*log((2*b*x^3 + 3*sqr
t(1/3)*((b*x^3 + a)^(1/3)*(a + b) - (a + b)*(-a - b)^(1/3) - 2*(b*x^3 + a)^
```

```
(2/3)*(-a - b)^(2/3))*sqrt((-a - b)^(1/3)/(a + b)) + 3*a - 3*(b*x^3 + a)^(1
/3)*(-a - b)^(2/3) + b)/(x^3 - 1)) + (-a - b)^(2/3)*log((b*x^3 + a)^(2/3) -
 (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) - 2*(-a - b)^(2/3)*log(
(b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b), -1/6*(6*sqrt(1/3)*(a + b)*sqr
t(-(-a - b)^(1/3)/(a + b))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (-a - b)
^(1/3))*sqrt(-(-a - b)^(1/3)/(a + b))) - (-a - b)^(2/3)*log((b*x^3 + a)^(2/
3) - (b*x^3 + a)^(1/3)*(-a - b)^(1/3) + (-a - b)^(2/3)) + 2*(-a - b)^(2/3)*
log((b*x^3 + a)^(1/3) + (-a - b)^(1/3)))/(a + b)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^3\sqrt[3]{a+bx^3}-\sqrt[3]{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)/(b*x**3+a)**(1/3),x)
[Out] -Integral(x**2/(x**3*(a + b*x**3)**(1/3) - (a + b*x**3)**(1/3)), x)
```

**Giac** [A]
time = 7.25, size = 113, normalized size = 1.18

$$-\frac{(a+b)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+(a+b)^{\frac{1}{3}}\right)}{3\,(a+b)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a+\sqrt{3}\,b}+\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}+(a+b)^{\frac{2}{3}}\right)}{6\,(a+b)^{\frac{1}{3}}}-\frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-(a+b)^{\frac{1}{3}}\right|\right)}{3\,(a+b)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] -(a + b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (a + b)^(1/3))/(a
+ b)^(1/3))/(sqrt(3)*a + sqrt(3)*b) + 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(a + b)^(1/3) + (a + b)^(2/3))/(a + b)^(1/3) - 1/3*log(abs((b*x^3
+ a)^(1/3) - (a + b)^(1/3)))/(a + b)^(1/3)
```

**Mupad** [B]
time = 0.59, size = 157, normalized size = 1.64

$$\frac{\ln\left((b\,x^3+a)^{1/3}-\frac{9\,a+9\,b}{9\,(-a-b)^{2/3}}\right)}{3\,(-a-b)^{1/3}}+\frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}}-\frac{\ln\left((b\,x^3+a)^{1/3}-\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2(9\,a+9\,b)}{36\,(-a-b)^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a-b)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^3 - 1)*(a + b*x^3)^(1/3)),x)
[Out] log((a + b*x^3)^(1/3) - (9*a + 9*b)/(9*(- a - b)^(2/3)))/(3*(- a - b)^(1/3)
) + (log((a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)^2*(9*a + 9*b))/(36*(- a - b)
^(2/3)))*(3^(1/2)*1i - 1))/(6*(- a - b)^(1/3)) - (log((a + b*x^3)^(1/3) - (
(3^(1/2)*1i + 1)^2*(9*a + 9*b))/(36*(- a - b)^(2/3)))*(3^(1/2)*1i + 1))/(6*
(- a - b)^(1/3))
```

# 3.97 $\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=88

$$-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}\,x-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

**Rubi** [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {384}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + x^3]/(6*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x,\right.$$

$$= \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subs}}{}$$

$$= -\frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{\sqrt[3]{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

**Mathematica** [A]
time = 0.24, size = 114, normalized size = 1.30

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right) - 2\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right) + \log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}\,(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[2
*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) - 2^
(1/3)*(1 - x^3)^(2/3)])/2^(1/3)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.59, size = 907, normalized size = 10.31

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 907 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-4)*ln(-(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2)*RootOf(_Z^3-4)^3*x^3+54*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
```

```
_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(_Z^3-4)^2*RootOf(Root
Of(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/
3)*x^2-6*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)*RootOf(_Z^3-4)*x^2-RootOf(_Z^3-4)*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+RootOf(_Z^3-4)+18*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)/(x^2-x+1))-1/6*ln((-3*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*
(-x^3+1)^(2/3)*RootOf(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-3*RootOf(_Z^
3-4)*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(
-x^3+1)^(2/3)+RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2))/(1+x)/(x^2-x+1))*RootOf(_Z^3-4)-ln((-3*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*Root
Of(_Z^3-4)^2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x+RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-3*RootOf(_Z^3-4)*x^3+36*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^(2/3)+RootO
f(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)/(x
^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

**<u>Maxima</u> [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**<u>Fricas</u> [B]** Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(67) = 134$.
time = 3.05, size = 253, normalized size = 2.88

$$-\frac{1}{18}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}\,(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6\,(109x^9-105x^6+3x^3+1)}\right)+\frac{1}{18}\cdot 2^{\frac{2}{3}}\log\left(\frac{6\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}x^2+2^{\frac{2}{3}}(x^3+1)+6\,(-x^3+1)^{\frac{2}{3}}x}{x^3+1}\right)-\frac{1}{36}\cdot 2^{\frac{2}{3}}\log\left(\frac{3\cdot 2^{\frac{2}{3}}(5x^4-x)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(19x^6-16x^3+1)-12\,(2x^5-x^2)(-x^3+1)^{\frac{1}{3}}}{x^6+2x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
```

```
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(1-x^3\right)}^{1/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

## 3.98 $\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=233

$$\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left((1-x)(1+x)^2\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

**Rubi** [A]

time = 0.06, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[
3]) + Log[(1 - x)*(1 + x)^2]/(12*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1
- x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^
(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - Log[-1 + x + 2^(2/3)*(1 - x^3
)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{2}x^2 F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

**Mathematica** [A]
time = 0.80, size = 283, normalized size = 1.21

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)+2\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+(1-x^3)^{2/3}\right)+\log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2-2(-1+x)\sqrt[3]{2-2x^3}+4(1-x^3)^{2/3}\right)}{12\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^
3)^(1/3))] - 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(
1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)]
- 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2^(2/
3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Log[2^
(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x
^3)^(2/3)])/(12*2^(1/3))
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(171) = 342$.
time = 2.46, size = 373, normalized size = 1.60

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)(-x^3+1)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(x^{16}-33x^{13}+110x^{10}-110x^{7}+33x^{4}-x\right)(-x^3+1)^{\frac{1}{3}}+\sqrt{6}\,2^{\frac{1}{3}}\left(x^{18}+42x^{15}-417x^{12}+812x^{9}-417x^{6}+42x^{3}+1\right)\right)}{6\left(x^{18}-102x^{15}+447x^{12}-628x^{9}+447x^{6}-102x^{3}+1\right)}\right)-\frac{1}{72}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{8}-4x^{5}+x^{2}\right)(-x^3+1)^{\frac{2}{3}}-2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{12}-32x^{9}+78x^{6}-32x^{3}+1\right)-6\left(x^{10}-11x^{7}+11x^{4}-x\right)(-x^3+1)^{\frac{1}{3}}}{x^{12}+4x^{9}+6x^{6}+4x^{3}+1}\right)+\frac{1}{36}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{12(-x^3+1)^{\frac{2}{3}}x^2-6\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^{4}-x\right)(-x^3+1)^{\frac{1}{3}}-2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^{6}+2x^{3}+1\right)}{x^{6}+2x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

# 3.99 $\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=82

$$\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(1+x^3)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 57, 631, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2} \cdots}\right.}{\cdots}$$

$$= -\frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3} \cdots\right.}{\sqrt[3]{2}}$$

$$= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}} - \frac{\log(1+x^3)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

**Mathematica [A]**
time = 0.10, size = 104, normalized size = 1.27

$$\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\log\left(-2+2^{2/3}\sqrt[3]{1-x^3}\right) - \log\left(2+2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,(1-x^3)^{2/3}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-2 + 2^(2/
3)*(1 - x^3)^(1/3)] - Log[2 + 2^(2/3)*(1 - x^3)^(1/3) + 2^(1/3)*(1 - x^3)^(
2/3)])/(6*2^(1/3))
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.31, size = 657, normalized size = 8.01

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 657 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(-(12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z
^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootO
f(_Z^3-4)^2*x^3+12*RootOf(_Z^3-4)*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-42*(-x^3+1)^(2/3)
-28*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))
/(1+x)/(x^2-x+1))*RootOf(_Z^3-4)-ln(-(12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootO
f(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*RootOf(_Z^3-4)*x^3-18*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^
3+1)^(1/3)-42*(-x^3+1)^(2/3)-28*RootOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z
^3-4)^2*x^3-5*RootOf(_Z^3-4)*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+35*Ro
otOf(_Z^3-4)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(1+x)
/(x^2-x+1))
```

**Maxima [A]**
time = 0.50, size = 86, normalized size = 1.05

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**Fricas [A]**
time = 0.74, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

**Giac** [A]
time = 1.12, size = 87, normalized size = 1.06

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

**Mupad** [B]
time = 0.55, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

## 3.100 $\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$

**Optimal**. Leaf size=135

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

**Rubi** [B] Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(135) = 270. time = 0.18, antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 16, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$\frac{2^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{\log((1-x)(x+1)^2)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[(1 -
 x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/3)*(1
 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) -
 (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3) - (1
- x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{\phantom{3}}\right.}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} + \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1\right.}{\sqrt[3]{1}}}{2}\right.}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}$$

**Mathematica** [A]
time = 0.92, size = 145, normalized size = 1.07

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)+\log\left(2^{2/3}-2\ 2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+\left(1-x^3\right)^{2/3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((1 - x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (1
 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(2/
3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/
3)])/(2*2^(1/3))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.21, size = 720, normalized size = 5.33

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 720 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*ln((RootOf(_Z^3+4)^2*Ro
otOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+RootOf(RootOf(_Z^3+4)^2
+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*(-x^3+1)^(2/3)-2*(-x^3+1)^(1/3)
*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x+2*(-x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)-x^2+x-1)/(x^2-x+1))-1/2*ln
(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+R
ootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1
)^(2/3)-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2
)*RootOf(_Z^3+4)*x-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+2*(-x^3+1)^(1/3)*RootO
f(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+(-x^3+1)^(1/3
)*RootOf(_Z^3+4)^2+x^2*RootOf(_Z^3+4)-3*x*RootOf(_Z^3+4)-2*(-x^3+1)^(2/3)+R
ootOf(_Z^3+4))/(x^2-x+1))*RootOf(_Z^3+4)-ln(-(RootOf(RootOf(_Z^3+4)^2+2*_Z*
RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^3*x+RootOf(RootOf(_Z^3+4)^2+2*_Z*Root
Of(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-2*(-x^3+1)^(1/3)*RootOf(
RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x-(-x^3+1)^(1/3
)*RootOf(_Z^3+4)^2*x+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_
Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2+x^2*RootOf(_Z
^3+4)-3*x*RootOf(_Z^3+4)-2*(-x^3+1)^(2/3)+RootOf(_Z^3+4))/(x^2-x+1))*RootOf
(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.
time = 6.90, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}\left(x^4-4x^3+5x^2-4x+1\right)(-x^3+1)^{\frac{2}{3}}-4\sqrt{2}\,(-1)^{\frac{1}{3}}\left(x^5-x^4-3x^3+3x^2+x-1\right)(-x^3+1)^{\frac{1}{3}}+2^{\frac{5}{6}}\left(x^6-7x^5+10x^4-7x^3+10x^2-7x+1\right)\right)}{6\left(3x^6-9x^5+6x^4-x^3+6x^2-9x+3\right)}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{2}{3}}(x^2-3x+1)+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(x^4-3x^2+1\right)+4(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2x^3+3x^2-2x+1}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^2-x+1\right)-2(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2-x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2-x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((-x^3 + 1)^(1/3)*(x^2 - x + 1)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x+1}{(1-x^3)^{1/3}\,(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)),x)
[Out] int((x + 1)/((1 - x^3)^(1/3)*(x^2 - x + 1)), x)
```

# 3.101 $\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

**Optimal**. Leaf size=135

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] 1/4*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/
3)-1/2*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*arctan(1/3*(1-2*2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

**Rubi** [**B**] Leaf count is larger than twice the leaf count of optimal. 383 vs. 2(135) = 270. time = 0.19, antiderivative size = 383, normalized size of antiderivative = 2.84, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1600, 2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$\frac{2^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(x^3+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[(1 -
 x)*(1 + x)^2]/(6*2^(1/3)) - Log[1 + x^3]/(3*2^(1/3)) + Log[1 + (2^(2/3)*(1
 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) -
 (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 + Log[2^(1/3) - (1
- x^3)^(1/3)]/(2*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(1/3))
 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{(1+x)^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \int \frac{1+x}{(1-x+x^2)\sqrt[3]{1-x^3}}\,dx$$

$$= \int \left(\frac{1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{1+i\sqrt{3}}{\left(-1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right) dx$$

$$= \left(1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx + \left(1+i\sqrt{3}\right)\int \frac{1}{\left(-1+i\sqrt{3}+\right.}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}+2x\right)}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} + \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\sqrt{3}\right.}{\sqrt[3]{1-x^3}}}{2\sqrt{3}}\right.}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}$$

**Mathematica [A]**
time = 0.91, size = 145, normalized size = 1.07

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right) - 2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right) + \log\left(2^{2/3}-2\,2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+\left(1-x^3\right)^{2/3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2*2^(1/3)*x + (1
 - x^3)^(1/3))] - 2*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/3)] + Log[2^(2/
3) - 2*2^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/
3)])/(2*2^(1/3))
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.14, size = 737, normalized size = 5.46

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 737 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^
3+4)^2*x+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)
^2*(-x^3+1)^(2/3)-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_
```

```
Z^3+4)^2-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+2*RootOf
(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3+4)^2+2
*_Z*RootOf(_Z^3+4)+4*_Z^2))/(x^2-x+1))*RootOf(_Z^3+4)-ln((2*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootOf(_Z^3+4)^2*x+RootOf(RootOf(_Z^
3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)^2*(-x^3+1)^(2/3)-(-x^3+1)
^(1/3)*RootOf(_Z^3+4)^2*x+(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2-2*RootOf(RootOf(_
Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*x^2+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*Roo
tOf(_Z^3+4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)
)/(x^2-x+1))*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)+1/2*RootOf
(_Z^3+4)*ln(-(2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)^2*RootO
f(_Z^3+4)^2*x+RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z
^3+4)^2*(-x^3+1)^(2/3)+2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf
(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)*x-2*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3+4)^2+
2*_Z*RootOf(_Z^3+4)+4*_Z^2)*RootOf(_Z^3+4)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*R
ootOf(_Z^3+4)+4*_Z^2)*x^2-6*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_
Z^2)*x-2*(-x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3+4)^2+2*_Z*RootOf(_Z^3+4)+4*_Z^
2))/(x^2-x+1))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(101) = 202$.
time = 6.85, size = 318, normalized size = 2.36

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(4\cdot 2^{\frac{1}{6}}(-1)^{\frac{2}{3}}(x^4-4x^3+5x^2-4x+1)(-x^3+1)^{\frac{2}{3}}-4\sqrt{2}(-1)^{\frac{1}{3}}(x^5-x^4-3x^3+3x^2+x-1)(-x^3+1)^{\frac{1}{3}}+2^{\frac{5}{6}}(x^6-7x^5+10x^4-7x^3+10x^2-7x+1)\right)}{6(3x^6-9x^5+6x^4-x^3+6x^2-9x+3)}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(-x^3+1)^{\frac{2}{3}}(x^2-3x+1)+2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(x^4-3x^2+1)+4(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2x^3+3x^2-2x+1}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-\frac{2\cdot 2^{\frac{1}{3}}(-1)^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)+2^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^2-x+1)-2(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*2^(2/3)*(-1)^(1/3)*arctan(1/6*sqrt(3)*2^(1/6)*(4*2^(1/6)*(-1)^(
2/3)*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) - 4*sqrt(2)*(-1)^(1/3
)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 2^(5/6)*(x^6 - 7*x
^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x
^2 - 9*x + 3)) - 1/12*2^(2/3)*(-1)^(1/3)*log(-(2^(2/3)*(-1)^(1/3)*(-x^3 + 1
)^(2/3)*(x^2 - 3*x + 1) + 2^(1/3)*(-1)^(2/3)*(x^4 - 3*x^2 + 1) + 4*(-x^3 +
1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/6*2^(2/3)*(-1)^(1/
3)*log(-(2*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) + 2^(2/3)*(-1)^(1/3)
*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((x + 1)/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x**2 - x + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^2/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^2}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int((x + 1)^2/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

# 3.102 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}} \, dx$

**Optimal**. Leaf size=119

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}-\frac{\log\left(1+\frac{2^{2/3}(1+x)^2}{(1+x^3)^{2/3}}-\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{2\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}(1+x)}{\sqrt[3]{1+x^3}}\right)}{\sqrt[3]{2}}$$

```
[Out] -1/4*ln(1+2^(2/3)*(1+x)^2/(x^3+1)^(2/3)-2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3
)+1/2*ln(1+2^(1/3)*(1+x)/(x^3+1)^(1/3))*2^(2/3)-1/2*arctan(1/3*(1-2*2^(1/3)
*(1+x)/(x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

**Rubi** [B] Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(119) = 238.
time = 0.18, antiderivative size = 357, normalized size of antiderivative = 3.00, number of steps used = 16, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {2183, 384, 502, 2174, 206, 31, 648, 631, 210, 642, 455, 57}

$$\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{2^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{x^3+1}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log(1-x^3)}{3\sqrt[3]{2}}-\frac{\log\left(\frac{2^{2/3}(x+1)^2}{(x^3+1)^{2/3}}-\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)}{3\sqrt[3]{2}}+\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(x+1)}{\sqrt[3]{x^3+1}}+1\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}x-\sqrt[3]{x^3+1}\right)}{2\sqrt[3]{2}}+\frac{\log\left(-2^{2/3}\sqrt[3]{x^3+1}+x+1\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)^2(x+1)\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*2^(1/3)*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - (2^
(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]])/Sqrt[3] -
ArcTan[(1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) -
 ArcTan[(1 + 2^(2/3)*(1 + x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[(1 -
 x)^2*(1 + x)]/(6*2^(1/3)) + Log[1 - x^3]/(3*2^(1/3)) - Log[1 + (2^(2/3)*(1
 + x)^2)/(1 + x^3)^(2/3) - (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)]/(3*2^(1/3)) +
 (2^(2/3)*Log[1 + (2^(1/3)*(1 + x))/(1 + x^3)^(1/3)])/3 - Log[2^(1/3) - (1
+ x^3)^(1/3)]/(2*2^(1/3)) - Log[2^(1/3)*x - (1 + x^3)^(1/3)]/(2*2^(1/3)) +
Log[1 + x - 2^(2/3)*(1 + x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1+x^3}}\,dx = \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}} + \frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\right) dx$$

$$= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1+x^3}}\,dx + \left(-1+i\sqrt{3}\right)\int \frac{\cdots}{\left(1+i\sqrt{\cdots}\right.}$$

$$= \frac{\left(3-i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1-i\sqrt{3}-2x\right)}{\sqrt[3]{1+x^3}}}{2\sqrt{3}}\right)}{2\sqrt[3]{2}\left(i+\sqrt{3}\right)} - \frac{\left(3+i\sqrt{3}\right)\tan^{-1}\left(\frac{2-\frac{\sqrt[3]{2}\left(1+i\cdots\right.}{\sqrt[3]{1-\cdots}}}{2\sqrt{3}\cdots}\right.}{2\sqrt[3]{2}\left(i-\sqrt{3}\right)}$$

**Mathematica** [A]
time = 0.90, size = 139, normalized size = 1.17

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1+x^3}}{-2\sqrt[3]{2}-2\sqrt[3]{2}x+\sqrt[3]{1+x^3}}\right)+2\log\left(\sqrt[3]{2}+\sqrt[3]{2}x+\sqrt[3]{1+x^3}\right)-\log\left(2^{2/3}+2\,2^{2/3}x+2^{2/3}x^2-\sqrt[3]{2}(1+x)\sqrt[3]{1+x^3}+(1+x^3)^{2/3}\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 + x^3)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 + x^3)^(1/3))/(-2*2^(1/3) - 2*2^(1/3)*x + (1
+ x^3)^(1/3))] + 2*Log[2^(1/3) + 2^(1/3)*x + (1 + x^3)^(1/3)] - Log[2^(2/3)
 + 2*2^(2/3)*x + 2^(2/3)*x^2 - 2^(1/3)*(1 + x)*(1 + x^3)^(1/3) + (1 + x^3)^
(2/3)])/(2*2^(1/3))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.45, size = 714, normalized size = 6.00

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 714 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(-(2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z
^3-4)^2*x+(x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
*RootOf(_Z^3-4)^2-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-2*RootOf(RootOf(_Z^3-4)^
2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2-2*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^2+2*_
Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+x+1))*RootOf(_Z^3-4)-ln(-(2*RootOf(RootOf(_Z
^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x+(x^3+1)^(2/3)*Root
Of(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2-(x^3+1)^(1
/3)*RootOf(_Z^3-4)^2*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2
)*x^2-(x^3+1)^(1/3)*RootOf(_Z^3-4)^2-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*x-2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x
^2+x+1))*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)+1/2*RootOf(_Z^
3-4)*ln((2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^
3-4)^2*x+(x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*
RootOf(_Z^3-4)^2+2*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-
4)+4*_Z^2)*(x^3+1)^(1/3)*x+2*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)*(x^3+1)^(1/3)+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z
^3-4)+4*_Z^2)*x^2+6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+2
*(x^3+1)^(2/3)+2*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x^2+
x+1))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(93) = 186.
time = 8.22, size = 268, normalized size = 2.25

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{1}{6}}\left(2^{\frac{5}{6}}\left(x^6+7x^5+10x^4+7x^3+10x^2+7x+1\right)-4\sqrt{2}\left(x^5+x^4-3x^3-3x^2+x+1\right)\left(x^3+1\right)^{\frac{1}{3}}+4\cdot 2^{\frac{1}{6}}\left(x^4+4x^3+5x^2+4x+1\right)\left(x^3+1\right)^{\frac{2}{3}}\right)}{6\left(3x^6+9x^5+6x^4+x^3+6x^2+9x+3\right)}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}\left(x^3+1\right)^{\frac{2}{3}}\left(x^2+3x+1\right)-2^{\frac{1}{3}}\left(x^4-3x^2+1\right)-4\left(x^3+1\right)^{\frac{1}{3}}\left(x^2+x\right)}{x^4+2x^3+3x^2+2x+1}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}\left(x^2+x+1\right)+2\cdot 2^{\frac{1}{3}}\left(x^3+1\right)^{\frac{1}{3}}\left(x+1\right)+2\left(x^3+1\right)^{\frac{2}{3}}}{x^2+x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(1/6)*(2^(5/6)*(x^6 + 7*x^5 + 10*x
^4 + 7*x^3 + 10*x^2 + 7*x + 1) - 4*sqrt(2)*(x^5 + x^4 - 3*x^3 - 3*x^2 + x +
 1)*(x^3 + 1)^(1/3) + 4*2^(1/6)*(x^4 + 4*x^3 + 5*x^2 + 4*x + 1)*(x^3 + 1)^(
2/3))/(3*x^6 + 9*x^5 + 6*x^4 + x^3 + 6*x^2 + 9*x + 3)) - 1/12*2^(2/3)*log((
2^(2/3)*(x^3 + 1)^(2/3)*(x^2 + 3*x + 1) - 2^(1/3)*(x^4 - 3*x^2 + 1) - 4*(x^
3 + 1)^(1/3)*(x^2 + x))/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1)) + 1/6*2^(2/3)*log(
(2^(2/3)*(x^2 + x + 1) + 2*2^(1/3)*(x^3 + 1)^(1/3)*(x + 1) + 2*(x^3 + 1)^(2
/3))/(x^2 + x + 1))
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\,dx-\int \left(-\frac{1}{x^2\sqrt[3]{x^3+1}+x\sqrt[3]{x^3+1}+\sqrt[3]{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3 + 1)**(1/
3)), x) - Integral(-1/(x**2*(x**3 + 1)**(1/3) + x*(x**3 + 1)**(1/3) + (x**3
 + 1)**(1/3)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(x^3+1)^(1/3),x, algorithm="giac")
[Out] integrate(-(x - 1)/((x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x-1}{(x^3+1)^{1/3}\,(x^2+x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)),x)
[Out] -int((x - 1)/((x^3 + 1)^(1/3)*(x + x^2 + 1)), x)
```

# 3.103 $\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2} \, dx$

**Optimal**. Leaf size=43

$$\frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2 \, _2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{5}{3}; x^3\right)$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

**Rubi** [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2183, 197, 371, 267}

$$x^2 \left(- \, _2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{5}{3}; x^3\right)\right) + \frac{x}{\sqrt[3]{1-x^3}} + \frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
```

```
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{(1+x+x^2)^2}\,dx = \int \left(-\frac{4(1-x^3)^{2/3}}{3\left(-1+i\sqrt{3}-2x\right)^2}+\frac{4i(1-x^3)^{2/3}}{3\sqrt{3}\left(-1+i\sqrt{3}-2x\right)}-\frac{4(1-x^3)^{2/3}}{3\left(1+i\sqrt{3}+2x\right)^2}+\frac{}{3\sqrt{}}\right.$$
$$= -\left(\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(-1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(1+i\sqrt{3}+2x\right)^2}\,dx+\frac{(4i)\int \frac{(1-x^3)^{2/3}}{-1+i\sqrt{3}-2x}}{3\sqrt{3}}$$

**Mathematica [A]**
time = 10.06, size = 43, normalized size = 1.00

$$\frac{(1+2x)\left(1-x^3\right)^{2/3}}{1+x+x^2}+x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x + x^2)^2,x]
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

**Maple [A]**
time = 0.21, size = 34, normalized size = 0.79

| method | result | size |
|---|---|---|
| risch | $-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}}+x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)$ | 34 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2+x+1)^2,x,method=_RETURNVERBOSE)
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x - 1\right)\left(x^{2} + x + 1\right)\right)^{\frac{2}{3}}}{\left(x^{2} + x + 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2+x+1)**2,x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 + x + 1)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2+x+1)^2,x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 + x + 1)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(1-x^3\right)}^{2/3}}{{\left(x^2+x+1\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2,x)
[Out] int((1 - x^3)^(2/3)/(x + x^2 + 1)^2, x)
```

# 3.104 $\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx$

**Optimal**. Leaf size=43

$$\frac{1}{\sqrt[3]{1-x^3}}+\frac{x}{\sqrt[3]{1-x^3}}-x^2\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)$$

```
[Out] 1/(-x^3+1)^(1/3)+x/(-x^3+1)^(1/3)-x^2*hypergeom([2/3, 4/3],[5/3],x^3)
```

**Rubi** [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2183, 197, 371, 267}

$$x^2\left(-\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)\right)+\frac{x}{\sqrt[3]{1-x^3}}+\frac{1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
```

```
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{1-x}{(1+x+x^2)\sqrt[3]{1-x^3}}\,dx = \int \left(\frac{-1-i\sqrt{3}}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}} + \frac{-1+i\sqrt{3}}{\left(1+i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\right)dx$$
$$= \left(-1-i\sqrt{3}\right)\int \frac{1}{\left(1-i\sqrt{3}+2x\right)\sqrt[3]{1-x^3}}\,dx + \left(-1+i\sqrt{3}\right)\int \frac{}{\left(1+i\sqrt{\right.}}$$

**Mathematica** [A]
time = 10.07, size = 43, normalized size = 1.00

$$\frac{(1+2x)\left(1-x^3\right)^{2/3}}{1+x+x^2} + x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((1 + x + x^2)*(1 - x^3)^(1/3)),x]
[Out] ((1 + 2*x)*(1 - x^3)^(2/3))/(1 + x + x^2) + x^2*Hypergeometric2F1[1/3, 2/3,
 5/3, x^3]
```

**Maple** [A]
time = 0.21, size = 34, normalized size = 0.79

| method | result | size |
|---|---|---|
| risch | $-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}} + x^2 \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)$ | 34 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\,dx-\int \left(-\frac{1}{x^2\sqrt[3]{1-x^3}+x\sqrt[3]{1-x^3}+\sqrt[3]{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x**2+x+1)/(-x**3+1)**(1/3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 - x**3)**(1/
3)), x) - Integral(-1/(x**2*(1 - x**3)**(1/3) + x*(1 - x**3)**(1/3) + (1 -
x**3)**(1/3)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(x^2+x+1)/(-x^3+1)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/((-x^3 + 1)^(1/3)*(x^2 + x + 1)), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x-1}{\left(1-x^3\right)^{1/3}\,\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)),x)
```

```
[Out] -int((x - 1)/((1 - x^3)^(1/3)*(x + x^2 + 1)), x)
```

# 3.105 $\int \frac{(1-x)^2}{(1-x^3)^{4/3}} \, dx$

**Optimal**. Leaf size=39

$$\frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2 \, {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)$$

```
[Out] (1+(1-2*x)*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3, 2/3],[5/3],x^3)
```

**Rubi** [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1868, 12, 371}

$$x^2 \, {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right) + \frac{(1-2x)x+1}{\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 + (1 - 2*x)*x)/(1 - x^3)^(1/3) + x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^
3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^2}{(1-x^3)^{4/3}}\,dx &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} - \int -\frac{2x}{\sqrt[3]{1-x^3}}\,dx\\ &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + 2\int \frac{x}{\sqrt[3]{1-x^3}}\,dx\\ &= \frac{1+(1-2x)x}{\sqrt[3]{1-x^3}} + x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)\end{aligned}$$

**Mathematica** [**A**]
time = 10.02, size = 43, normalized size = 1.10

$$\frac{1}{\sqrt[3]{1-x^3}} + \frac{x}{\sqrt[3]{1-x^3}} - x^2\,{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^2/(1 - x^3)^(4/3),x]
```

```
[Out] (1 - x^3)^(-1/3) + x/(1 - x^3)^(1/3) - x^2*Hypergeometric2F1[2/3, 4/3, 5/3,
 x^3]
```

**Maple** [**A**]
time = 0.13, size = 34, normalized size = 0.87

| method | result | size |
|---|---|---|
| risch | $-\frac{(-1+x)(1+2x)}{(-x^3+1)^{\frac{1}{3}}} + x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)$ | 34 |
| meijerg | $\frac{x}{(-x^3+1)^{\frac{1}{3}}} - x^2\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{4}{3}\right],\left[\frac{5}{3}\right],x^3\right) + \frac{x^3\,\mathrm{hypergeom}\left(\left[1,\frac{4}{3}\right],[2],x^3\right)}{3}$ | 41 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^2/(-x^3+1)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -(-1+x)*(1+2*x)/(-x^3+1)^(1/3)+x^2*hypergeom([1/3,2/3],[5/3],x^3)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="maxima")
```

```
[Out] x/(-x^3 + 1)^(1/3) - integrate((x^2 - 2*x)/((x^3 - 1)*(x^2 + x + 1)^(1/3)*(
-x + 1)^(1/3)), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^4 + 2*x^3 + 3*x^2 + 2*x + 1), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**2/(-x**3+1)**(4/3),x)
```

```
[Out] Integral((x - 1)**2/(-(x - 1)*(x**2 + x + 1))**(4/3), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^2/(-x^3+1)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)^2/(-x^3 + 1)^(4/3), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(x-1)^2}{\left(1-x^3\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^2/(1 - x^3)^(4/3),x)
```

```
[Out] int((x - 1)^2/(1 - x^3)^(4/3), x)
```

# 3.106 $\int (1-x^3)^{2/3} \, dx$

**Optimal**. Leaf size=67

$$\frac{1}{3}x\left(1-x^3\right)^{2/3}-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

[Out] 1/3*x*(-x^3+1)^(2/3)+1/3*ln(x+(-x^3+1)^(1/3))-2/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)

**Rubi** [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 245}

$$-\frac{2\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)^(2/3),x]

[Out] (x*(1 - x^3)^(2/3))/3 - (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + Log[x + (1 - x^3)^(1/3)]/3

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(1-x^3\right)^{2/3}\,dx = \frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2}{3}\int \frac{1}{\sqrt[3]{1-x^3}}\,dx$$
$$= \frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{3}\log\left(x+\sqrt[3]{1-x^3}\right)$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.
time = 0.09, size = 101, normalized size = 1.51

$$\frac{3(-1+x)\left(1-x^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},-\frac{2}{3};\frac{8}{3};-\frac{-1+x}{1-(-1)^{2/3}},-\frac{-1+x}{1+\sqrt[3]{-1}}\right)}{5\left(1+\frac{-1+x}{1+\sqrt[3]{-1}}\right)^{2/3}\left(1+\frac{-1+x}{1-(-1)^{2/3}}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3),x]
```

```
[Out] (3*(-1 + x)*(1 - x^3)^(2/3)*AppellF1[5/3, -2/3, -2/3, 8/3, -((-1 + x)/(1 -
(-1)^(2/3))), -((-1 + x)/(1 + (-1)^(1/3)))]])/(5*(1 + (-1 + x)/(1 + (-1)^(1/
3)))^(2/3)*(1 + (-1 + x)/(1 - (-1)^(2/3)))^(2/3))
```

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.03, size = 12, normalized size = 0.18

| method | result |
|---|---|
| meijerg | $x\,\mathrm{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)$ |
| risch | $-\frac{x(x^3-1)}{3(-x^3+1)^{\frac{1}{3}}}+\frac{2x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{3}\right],\left[\frac{4}{3}\right],x^3\right)}{3}$ |
| trager | $\frac{x(-x^3+1)^{\frac{2}{3}}}{3}+\frac{2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3+3\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)(-x^3+1)^{\frac{2}{3}}x-3\,\mathrm{RootOf}\left(\right.\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([-2/3,1/3],[4/3],x^3)
```

**Maxima** [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(52) = 104.
time = 0.48, size = 105, normalized size = 1.57

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(-x^3+1\right)^{\frac{1}{3}}}{x}-1\right)\right)-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3-1}{x^3}-1\right)}+\frac{2}{9}\log\left(\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+1\right)-\frac{1}{9}\log\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x}+\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*(-x^3 + 1
)^(2/3)/(x^2*((x^3 - 1)/x^3 - 1)) + 2/9*log((-x^3 + 1)^(1/3)/x + 1) - 1/9*l
og(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 + 1)
```

**Fricas [A]**
time = 1.04, size = 94, normalized size = 1.40

$$\frac{1}{3}\left(-x^{3}+1\right)^{\frac{2}{3}}x-\frac{2}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\,x-2\sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3\,x}\right)+\frac{2}{9}\log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{9}\log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}}x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(-x^3 + 1)^(2/3)*x - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x
^3 + 1)^(1/3))/x) + 2/9*log((x + (-x^3 + 1)^(1/3))/x) - 1/9*log((x^2 - (-x^
3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

**Sympy [C]** Result contains complex when optimal does not.
time = 0.49, size = 31, normalized size = 0.46

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3),x)
```

```
[Out] x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3), x)
```

**Mupad [B]**
time = 0.34, size = 10, normalized size = 0.15

$$x\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3),x)
```

```
[Out] x*hypergeom([-2/3, 1/3], 4/3, x^3)
```

# 3.107 $\int \frac{(1-x^3)^{2/3}}{x}\,dx$

**Optimal**. Leaf size=70

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{2}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)$$

```
[Out] 1/2*(-x^3+1)^(2/3)-1/2*ln(x)+1/2*ln(1-(-x^3+1)^(1/3))+1/3*arctan(1/3*(1+2*(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {272, 52, 57, 632, 210, 31}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/x,x]
```

```
[Out] (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - Log[x
]/2 + Log[1 - (1 - x^3)^(1/3)]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x^3)^{2/3}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)^{2/3}}{x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right)\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\right.\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1-\right.\\
&= \frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\log(x)}{2} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)
\end{aligned}$$

**Mathematica** [A]
time = 0.05, size = 90, normalized size = 1.29

$$\frac{1}{6}\left(3\left(1-x^3\right)^{2/3} + 2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1-x^3}\right) - \log\left(1+\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)^(2/3)/x,x]

[Out] (3*(1 - x^3)^(2/3) + 2*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 - x^3)^(1/3)] - Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/6

**Maple** [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 1.71, size = 66, normalized size = 0.94

| method | result |
|---|---|
| meijerg | $-\frac{\sqrt{3}\,\Gamma\left(\frac{2}{3}\right)\left(-\frac{\left(\frac{3}{2}-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+3\ln(x)+i\pi\right)\pi\sqrt{3}}{\Gamma\left(\frac{2}{3}\right)}+\frac{2\pi\sqrt{3}\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{3},1,1\right],[2,2],x^3\right)}{3\Gamma\left(\frac{2}{3}\right)}\right)}{9\pi}$ |
| trager | $\frac{(-x^3+1)^{\frac{2}{3}}}{2}+\frac{\ln\left(\frac{-211\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2x^3-3126\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)x^3+5502(-x^3+1)^{\frac{2}{3}}\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)-11543x^3}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)^(2/3)/x,x,method=_RETURNVERBOSE)

[Out] -1/9/Pi*3^(1/2)*GAMMA(2/3)*(-(3/2-1/6*Pi*3^(1/2)-3/2*ln(3)+3*ln(x)+I*Pi)*Pi*3^(1/2)/GAMMA(2/3)+2/3*Pi*3^(1/2)/GAMMA(2/3)*x^3*hypergeom([1/3,1,1],[2,2],x^3))

**Maxima** [A]
time = 0.59, size = 73, normalized size = 1.04

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)

**Fricas** [A]
time = 1.23, size = 75, normalized size = 1.07

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left(-x^3+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="fricas")

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/2*(-x^3
+ 1)^(2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x
^3 + 1)^(1/3) - 1)
```

**Sympy** [C] Result contains complex when optimal does not.
time = 0.49, size = 41, normalized size = 0.59

$$-\frac{x^2 e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{1}{x^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/x,x)
```

```
[Out] -x**2*exp(2*I*pi/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), x**(-3))/(3*gam
ma(1/3))
```

**Giac** [A]
time = 1.06, size = 74, normalized size = 1.06

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{2}\left(-x^3+1\right)^{\frac{2}{3}}-\frac{1}{6}\log\left(\left(-x^3+1\right)^{\frac{2}{3}}+\left(-x^3+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{3}\log\left(\left|\left(-x^3+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/x,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2*(-x^3 + 1)^(
2/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3
 + 1)^(1/3) - 1))
```

**Mupad** [B]
time = 0.40, size = 91, normalized size = 1.30

$$\frac{\ln\left(\left(1-x^3\right)^{1/3}-1\right)}{3}+\ln\left(\left(1-x^3\right)^{1/3}-9\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(1-x^3\right)^{1/3}-9\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{\left(1-x^3\right)^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/x,x)
```

```
[Out] log((1 - x^3)^(1/3) - 1)/3 + log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 - 1/6)
^2)*((3^(1/2)*1i)/6 - 1/6) - log((1 - x^3)^(1/3) - 9*((3^(1/2)*1i)/6 + 1/6)
^2)*((3^(1/2)*1i)/6 + 1/6) + (1 - x^3)^(2/3)/2
```

# 3.108 $\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$

**Optimal**. Leaf size=384

$$\frac{(1-x^3)^{2/3}}{2b}-\frac{(a^3+b^3)\,x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2}+\frac{a^2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}-\frac{(a^3+b^3)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a^3+}}{a\sqrt[3]{1-}}}{\sqrt{3}}\right.}{\sqrt{3}\,b^3}$$

[Out] 1/2*(-x^3+1)^(2/3)/b-1/2*(a^3+b^3)*x^2*AppellF1(2/3,1/3,1,5/3,x^3,-b^3*x^3/a^3)/a^2/b^2+1/2*a*x^2*hypergeom([1/3, 2/3],[5/3],x^3)/b^2-1/3*(a^3+b^3)^(2/3)*ln(b^3*x^3+a^3)/b^3+1/2*(a^3+b^3)^(2/3)*ln(-(a^3+b^3)^(1/3)*x/a-(-x^3+1)^(1/3))/b^3-1/2*a^2*ln(x+(-x^3+1)^(1/3))/b^3+1/2*(a^3+b^3)^(2/3)*ln((a^3+b^3)^(1/3)-b*(-x^3+1)^(1/3))/b^3+1/3*a^2*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))/b^3*3^(1/2)-1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1-2*(a^3+b^3)^(1/3)*x/a/(-x^3+1)^(1/3))*3^(1/2))/b^3*3^(1/2)+1/3*(a^3+b^3)^(2/3)*arctan(1/3*(1+2*b*(-x^3+1)^(1/3)/(a^3+b^3)^(1/3))*3^(1/2))/b^3*3^(1/2)

**Rubi** [A]
time = 0.28, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2178, 2177, 245, 2181, 384, 524, 455, 57, 631, 210, 31, 371}

$$-\frac{(a^3+b^3)^{2/3}\,\text{ArcTan}\left(\frac{1-\frac{2x\sqrt[3]{a^3+b^3}}{a\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}+\frac{(a^3+b^3)^{2/3}\,\text{ArcTan}\left(\frac{\frac{2b\sqrt[3]{1-x^3}}{\sqrt[3]{a^3+b^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}-\frac{(a^3+b^3)^{2/3}\log(a^3+b^3x^3)}{3b^3}+\frac{(a^3+b^3)^{2/3}\log\left(-\frac{x\sqrt[3]{a^3+b^3}}{a}-\sqrt[3]{1-x^3}\right)}{2b^3}+\frac{(a^3+b^3)^{2/3}\log\left(\sqrt[3]{a^3+b^3}-b\sqrt[3]{1-x^3}\right)}{2b^3}+\frac{a^2\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3}-\frac{a^2\log\left(\sqrt[3]{1-x^3}+x\right)}{2b^3}-\frac{x^2(a^3+b^3)\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-\frac{b^3x^3}{a^3}\right)}{2a^2b^2}+\frac{ax^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)}{2b^2}+\frac{(1-x^3)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] (1 - x^3)^(2/3)/(2*b) - ((a^3 + b^3)*x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -(
(b^3*x^3)/a^3)])/(2*a^2*b^2) + (a^2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^3) - ((a^3 + b^3)^(2/3)*ArcTan[(1 - (2*(a^3 + b^3)^(1/3)*x
)/(a*(1 - x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^3) + ((a^3 + b^3)^(2/3)*ArcTan[
(1 + (2*b*(1 - x^3)^(1/3))/(a^3 + b^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^3) + (a*
x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/(2*b^2) - ((a^3 + b^3)^(2/3)*Log
[a^3 + b^3*x^3])/(3*b^3) + ((a^3 + b^3)^(2/3)*Log[-(((a^3 + b^3)^(1/3)*x)/a
) - (1 - x^3)^(1/3)])/(2*b^3) - (a^2*Log[x + (1 - x^3)^(1/3)])/(2*b^3) + ((
a^3 + b^3)^(2/3)*Log[(a^3 + b^3)^(1/3) - b*(1 - x^3)^(1/3)])/(2*b^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx = \int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

**Mathematica** [F]
time = 26.63, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(a + b*x),x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(a + b*x), x]
```

**Maple** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(b*x+a),x)
```

```
[Out] int((-x^3+1)^(2/3)/(b*x+a),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(b*x+a),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(a + b*x), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(b*x+a),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)/(b*x + a), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1-x^3)^{2/3}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(a + b*x),x)
[Out] int((1 - x^3)^(2/3)/(a + b*x), x)
```

## 3.109 $\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$

**Optimal**. Leaf size=234

$$-\frac{(1-x^3)^{2/3}}{3(1+x^3)}+\frac{x(1-x^3)^{2/3}}{3(1+x^3)}+\frac{2x^2(1-x^3)^{2/3}}{3(1+x^3)}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}}+$$

```
[Out] -1/3*(-x^3+1)^(2/3)/(x^3+1)+1/3*x*(-x^3+1)^(2/3)/(x^3+1)+2/3*x^2*(-x^3+1)^(
2/3)/(x^3+1)+1/3*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/6*ln(2^(1/3)-(-x^3+1
)^(1/3))*2^(2/3)+1/6*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/9*arctan(1/3*(
1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/9*arctan(1/3*(1+2^
(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

**Rubi** [A]

time = 0.10, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2183, 386, 384, 480, 21, 371, 455, 43, 57, 631, 210, 31}

$$-\frac{2^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2^{2/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{(1-x^3)^{2/3}x}{3(x^3+1)}-\frac{(1-x^3)^{2/3}}{3(x^3+1)}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}+\frac{2(1-x^3)^{2/3}x^2}{3(x^3+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
```

```
[Out] -1/3*(1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(3*(1 + x^3)) + (2*x^2
*(1 - x^3)^(2/3))/(3*(1 + x^3)) - (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*x)/(1 - x
^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]])/(3*Sqrt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/3
 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(3*2^(1/3)) + Log[-(2^(1/3)*x) - (1 - x^3
)^(1/3)]/(3*2^(1/3))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx = \int \left(-\frac{4(1-x^3)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2}+\frac{4i(1-x^3)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)}-\frac{4(1-x^3)^{2/3}}{3\left(-1+i\sqrt{3}+2x\right)^2}+\frac{}{3\sqrt{}}\right.$$

$$= -\left(\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{(1-x^3)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2}\,dx+\frac{(4i)\int \frac{(1-x^3)^{2/3}}{1+i\sqrt{3}-2x}}{3\sqrt{3}}$$

**Mathematica [F]**

time = 30.23, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{(1-x+x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2,x]
[Out] Integrate[(1 - x^3)^(2/3)/(1 - x + x^2)^2, x]
```

**Maple** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
[Out] int((-x^3+1)^(2/3)/(x^2-x+1)^2,x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{(x^2-x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x**2 - x + 1)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^2 - x + 1)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1-x^3)^{2/3}}{(x^2-x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2,x)
```

```
[Out] int((1 - x^3)^(2/3)/(x^2 - x + 1)^2, x)
```

## 3.110 $\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx$

**Optimal**. Leaf size=199

$$\frac{\left(1-x^3\right)^{2/3}}{1-x+x^2}-\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\log\left(\sqrt[3]{2}\right.}{}$$

```
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+1/2*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(-2^(
1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+ln(x+(-x^3+1)^(1/3))-2/3*arctan(1/3*(1-2*x/(
-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3
))*3^(1/2))*2^(2/3)*3^(1/2)+1/3*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/
2))*2^(2/3)*3^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 250, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2183, 386, 384, 455, 43, 57, 631, 210, 31, 478, 544, 245}

$$-\frac{2\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2^{2/3}\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(1-x^3\right)^{2/3}x}{x^3+1}+\frac{\left(1-x^3\right)^{2/3}}{x^3+1}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{2}{3}2^{2/3}\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{3\sqrt[3]{2}}+\log\left(\sqrt[3]{1-x^3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] (1 - x^3)^(2/3)/(1 + x^3) + (x*(1 - x^3)^(2/3))/(1 + x^3) - (2*ArcTan[(1 -
(2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*
x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (2^(2/3)*ArcTan[(1 + 2^(2/3)*(1 - x
^3)^(1/3))/Sqrt[3]])/Sqrt[3] + Log[2^(1/3) - (1 - x^3)^(1/3)]/2^(1/3) + Log
[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(3*2^(1/3)) - (2*2^(2/3)*Log[-(2^(1/3)*x)
- (1 - x^3)^(1/3)])/3 + Log[x + (1 - x^3)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
```

```
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx &= \int \left(\frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}-\frac{2x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\right)dx\\
&= -\left(2\int \frac{x\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx\right)+\int \frac{\left(1-x^3\right)^{2/3}}{\left(1-x+x^2\right)^2}\,dx\\
&= -\left(2\int \left(-\frac{2\left(1+i\sqrt{3}\right)\left(1-x^3\right)^{2/3}}{3\left(1+i\sqrt{3}-2x\right)^2}+\frac{2i\left(1-x^3\right)^{2/3}}{3\sqrt{3}\left(1+i\sqrt{3}-2x\right)}-\frac{2\left(1-i\sqrt{3}\right)}{3\left(-1+i\sqrt{\right.}}\right.\right.\\
&= -\left(\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(1+i\sqrt{3}-2x\right)^2}\,dx\right)-\frac{4}{3}\int \frac{\left(1-x^3\right)^{2/3}}{\left(-1+i\sqrt{3}+2x\right)^2}\,dx+\frac{1}{3}\left(4\left(1-i\sqrt{\right.}\right.
\end{aligned}$$

**Mathematica** [F]
time = 25.50, size = 0, normalized size = 0.00

$$\int \frac{(1-2x)\left(1-x^3\right)^{2/3}}{(1-x+x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2,x]
```

```
[Out] Integrate[((1 - 2*x)*(1 - x^3)^(2/3))/(1 - x + x^2)^2, x]
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 24.62, size = 457, normalized size = 2.30

| method | result |
|---|---|
| trager | $\frac{(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}+\frac{\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^6+432\right)^4 x^2+\mathrm{RootOf}\left(\_Z^6+432\right)^4 x-\mathrm{RootOf}\left(\_Z^6+432\right)^4+12\,\mathrm{RootOf}\left(\_Z^6+432\right)^2\left(-x^3+1\right)^{\frac{1}{3}} x+72}{x^2-x+1}\right.}{72}$ |
| risch | Expression too large to display |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-x^3+1)^(2/3)/(x^2-x+1)+1/72*ln(-(RootOf(_Z^6+432)^4*x^2+RootOf(_Z^6+432)^
4*x-RootOf(_Z^6+432)^4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*(-x^3+1)^(
2/3))/(x^2-x+1))*RootOf(_Z^6+432)^4+1/6*ln(-(RootOf(_Z^6+432)^4*x^2+RootOf(
_Z^6+432)^4*x-RootOf(_Z^6+432)^4+12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+72*
(-x^3+1)^(2/3))/(x^2-x+1))*RootOf(_Z^6+432)+1/3*RootOf(_Z^6+432)*ln(-(RootO
f(_Z^6+432)^5*(-x^3+1)^(1/3)*x-RootOf(_Z^6+432)^4*x^2-RootOf(_Z^6+432)^4*x+
RootOf(_Z^6+432)^4-12*RootOf(_Z^6+432)^2*(-x^3+1)^(1/3)*x+36*RootOf(_Z^6+43
2)*x^2+36*RootOf(_Z^6+432)*x+144*(-x^3+1)^(2/3)-36*RootOf(_Z^6+432))/(x^2-x
+1))-1/12*ln((-x^3+1)^(2/3)-x*(-x^3+1)^(1/3)+x^2)*RootOf(_Z^6+432)^3-ln((-x
^3+1)^(2/3)-x*(-x^3+1)^(1/3)+x^2)+1/18*RootOf(_Z^6+432)^3*ln(-RootOf(_Z^6+4
32)^6*x^3+24*RootOf(_Z^6+432)^3+1728*x*(-x^3+1)^(2/3)-1728*x^2*(-x^3+1)^(1/
3)+1296*x^3-864)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -integrate((-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(163) = 326$.
time = 2.50, size = 1827, normalized size = 9.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(-1/6*(3822*4^(2/3)*sqrt(3)*(5
0*x^4 - 74*x^3 - 207*x^2 + 143*x + 19)*(-x^3 + 1)^(2/3) + 7644*4^(1/3)*sqrt
(3)*(19*x^5 - 150*x^4 + 43*x^3 + 112*x^2 + 57*x - 50)*(-x^3 + 1)^(1/3) - 7*
sqrt(39)*(6*4^(1/3)*sqrt(3)*(1150*x^4 - 3974*x^3 - 1911*x^2 + 1522*x + 3898
)*(-x^3 + 1)^(2/3) - 4^(2/3)*sqrt(3)*(1778*x^6 - 6366*x^5 - 8412*x^4 + 1725
4*x^3 + 15117*x^2 - 4227*x - 16105) + 12*sqrt(3)*(437*x^5 - 1539*x^4 - 333*
x^3 - 2074*x^2 + 372*x + 3261)*(-x^3 + 1)^(1/3))*sqrt((6*4^(1/3)*(5*x^4 + 4
*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^
4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-
x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6*sqrt(3
)*(29494*x^6 - 17582*x^5 + 153824*x^4 - 266248*x^3 - 129950*x^2 + 238106*x
- 29747))/(138718*x^6 - 463746*x^5 - 296508*x^4 - 115072*x^3 + 1093704*x^2
- 70446*x - 256859)) + 8*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(3822*4^(
2/3)*sqrt(3)*(19*x^4 - 181*x^3 + 36*x^2 + 169*x - 31)*(-x^3 + 1)^(2/3) - 76
44*4^(1/3)*sqrt(3)*(31*x^5 + 57*x^4 - 131*x^3 - 119*x^2 + 93*x + 19)*(-x^3
+ 1)^(1/3) + 7*sqrt(39)*(6*4^(1/3)*sqrt(3)*(3385*x^4 + 3574*x^3 - 1911*x^2
- 2948*x + 124)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(13027*x^6 + 16539*x^5 -
 8961*x^4 - 32644*x^3 - 2361*x^2 + 17139*x - 239) - 12*sqrt(3)*(2748*x^5 +
3450*x^4 - 4126*x^3 - 2385*x^2 + 1539*x - 76)*(-x^3 + 1)^(1/3))*sqrt((6*4^(
1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x
^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x
+ 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 6
*sqrt(3)*(53953*x^6 - 12994*x^5 - 396521*x^4 + 169424*x^3 + 300029*x^2 - 62
294*x - 41597))/(52723*x^6 + 682854*x^5 - 325173*x^4 - 1353400*x^3 + 193623
*x^2 + 640446*x - 16073)) + 16*4^(1/3)*sqrt(3)*(x^2 - x + 1)*arctan(1/6*(76
44*4^(2/3)*sqrt(3)*(5*x^4 - 107*x^3 - 243*x^2 + 26*x + 157)*(-x^3 + 1)^(2/3
) - 7644*4^(1/3)*sqrt(3)*(307*x^5 + 300*x^4 - 140*x^3 - 221*x^2 - 186*x - 9
8)*(-x^3 + 1)^(1/3) + 7*sqrt(39)*4^(1/3)*(6*4^(1/3)*sqrt(3)*(3109*x^4 + 400
*x^3 - 3822*x^2 + 1426*x + 3622)*(-x^3 + 1)^(2/3) + 4^(2/3)*sqrt(3)*(15505*
x^6 + 11493*x^5 - 22383*x^4 - 22720*x^3 - 5454*x^2 + 13032*x + 10888) - 12*
sqrt(3)*(2111*x^5 + 3450*x^4 - 941*x^3 - 1111*x^2 - 372*x - 2624)*(-x^3 + 1
)^(1/3)) + 6*sqrt(3)*(307479*x^6 + 239258*x^5 - 543668*x^4 - 607716*x^3 + 1
9112*x^2 + 232000*x + 343788))/(933353*x^6 + 1472754*x^5 + 285042*x^4 - 100
8596*x^3 - 1598208*x^2 - 560184*x + 468980)) + 48*sqrt(3)*(x^2 - x + 1)*arc
```

```
tan((4*sqrt(3)*(-x^3 + 1)^(1/3)*x^2 + 2*sqrt(3)*(-x^3 + 1)^(2/3)*x - sqrt(3
)*(x^3 - 1))/(9*x^3 - 1)) - 3*4^(1/3)*(x^2 - x + 1)*log(39626496*(6*4^(1/3)
*(5*x^4 + 4*x^3 - 3*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*
x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*
x^2 + 1)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1))
 - 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(5*x^4 + 4*x^3 - 3*x^2 -
4*x + 1)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12
*x^2 + 15*x + 1) - 12*(4*x^5 + 3*x^4 - 2*x^3 - 5*x^2 + 1)*(-x^3 + 1)^(1/3))
/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)
*log(39626496*(6*4^(1/3)*(x^4 - 4*x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) +
 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 - 25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 -
5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6
*x^2 - 3*x + 1)) + 3*4^(1/3)*(x^2 - x + 1)*log(9906624*(6*4^(1/3)*(x^4 - 4*
x^3 - 3*x^2 + 4*x + 5)*(-x^3 + 1)^(2/3) + 4^(2/3)*(x^6 + 15*x^5 - 12*x^4 -
25*x^3 - 12*x^2 + 15*x + 19) + 12*(x^5 - 5*x^3 - 2*x^2 + 3*x + 4)*(-x^3 + 1
)^(1/3))/(x^6 - 3*x^5 + 6*x^4 - 7*x^3 + 6*x^2 - 3*x + 1)) - 24*(x^2 - x + 1
)*log(3*(-x^3 + 1)^(1/3)*x^2 + 3*(-x^3 + 1)^(2/3)*x + 1) - 72*(-x^3 + 1)^(2
/3))/(x^2 - x + 1)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{4}-2x^{3}+3x^{2}-2x+1}\right)dx-\int\frac{2x\left(1-x^{3}\right)^{\frac{2}{3}}}{x^{4}-2x^{3}+3x^{2}-2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x**3+1)**(2/3)/(x**2-x+1)**2,x)
```

```
[Out] -Integral(-(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x) - Integ
ral(2*x*(1 - x**3)**(2/3)/(x**4 - 2*x**3 + 3*x**2 - 2*x + 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x)*(-x^3+1)^(2/3)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(-(-x^3 + 1)^(2/3)*(2*x - 1)/(x^2 - x + 1)^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$-\int\frac{(2\,x-1)\,{\left(1-x^3\right)}^{2/3}}{{\left(x^2-x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2,x)
[Out] -int(((2*x - 1)*(1 - x^3)^(2/3))/(x^2 - x + 1)^2, x)
```

# 3.111 $\int \frac{(1-x^3)^{2/3}}{1+x}\, dx$

**Optimal**. Leaf size=177

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log((1-x)(1}{2\sqrt[3]{2}}$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

**Rubi** [A]

time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2178, 2177, 245, 2174, 371}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx = \int \frac{(1-x^3)^{2/3}}{1+x}\,dx$$

**Mathematica [F]**
time = 46.20, size = 0, normalized size = 0.00

$$\int \frac{(1-x^3)^{2/3}}{1+x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x),x]
```

```
[Out] Integrate[(1 - x^3)^(2/3)/(1 + x), x]
```

**Maple [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{1+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(1+x),x)
```

```
[Out] int((-x^3+1)^(2/3)/(1+x),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x - 1\right)\left(x^{2} + x + 1\right)\right)^{\frac{2}{3}}}{x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(1+x),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(1+x),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x + 1), x)
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(1-x^3\right)^{2/3}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x + 1),x)
[Out] int((1 - x^3)^(2/3)/(x + 1), x)
```

## 3.112 $\int \frac{(1-x+x^2)(1-x^3)^{2/3}}{1+x^3}\,dx$

<u>**Optimal**</u>. Leaf size=177

$$\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log((1-x)(1}{2\sqrt[3]{2}}$$

```
[Out] 1/2*(-x^3+1)^(2/3)+1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/4*ln((1-x)*(1+
x)^2)*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+3/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*
2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*arctan(1
/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)
```

<u>**Rubi**</u> [A]

time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1600, 2178, 2177, 245, 2174, 371}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{2\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] (1 - x^3)^(2/3)/2 - (Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))
/Sqrt[3]])/2^(1/3) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] +
(x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 - Log[(1 - x)*(1 + x)^2]/(2*2
^(1/3)) - Log[x + (1 - x^3)^(1/3)]/2 + (3*Log[-1 + x + 2^(2/3)*(1 - x^3)^(1
/3)])/(2*2^(1/3))
```

<u>Rule 245</u>

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

<u>Rule 371</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int \frac{(1-x+x^2)\,(1-x^3)^{2/3}}{1+x^3}\,dx=\int \frac{(1-x^3)^{2/3}}{1+x}\,dx$$

**Mathematica [F]**
time = 39.09, size = 0, normalized size = 0.00

$$\int \frac{(1-x+x^2)\,(1-x^3)^{2/3}}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3),x]
```

```
[Out] Integrate[((1 - x + x^2)*(1 - x^3)^(2/3))/(1 + x^3), x]
```

**Maple [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 - x + 1)(-x^3 + 1)^{\frac{2}{3}}}{x^3 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x + 1), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(x - 1)(x^2 + x + 1))^{\frac{2}{3}}}{x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)*(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/(x + 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)*(x^2 - x + 1)/(x^3 + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x^3)^{2/3}\,(x^2-x+1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1),x)
```

```
[Out] int(((1 - x^3)^(2/3)*(x^2 - x + 1))/(x^3 + 1), x)
```

# 3.113 $\int \frac{(1-x^3)^{2/3}}{1+x^3} \, dx$

**Optimal**. Leaf size=132

$$\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(1+x^3)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(x+\right.$$

```
[Out] -1/6*ln(x^3+1)*2^(2/3)+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(
-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {399, 245, 384}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x^3+1)}{3\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - (2^(2/3)*ArcTan[(1 -
(2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] - Log[1 + x^3]/(3*2^(1/3))
 + Log[-(2^(1/3)*x) - (1 - x^3)^(1/3)]/2^(1/3) - Log[x + (1 - x^3)^(1/3)]/2
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(1-x^3)^{2/3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

**Mathematica [A]**
time = 0.34, size = 204, normalized size = 1.55

$$\frac{1}{6}\left(2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)-2\,2^{2/3}\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}x}{x-2^{2/3}\sqrt[3]{1-x^3}}\right)-2\log\left(x+\sqrt[3]{1-x^3}\right)+2\,2^{2/3}\log\left(2x+2^{2/3}\sqrt[3]{1-x^3}\right)+\log\left(x^2-x\sqrt[3]{1-x^3}+(1-x^3)^{2/3}\right)-2^{2/3}\log\left(-2x^2+2^{2/3}x\sqrt[3]{1-x^3}-\sqrt[3]{2}(1-x^3)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(2/3)/(1 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] - 2*2^(2/3)*Sqrt[3]*
ArcTan[(Sqrt[3]*x)/(x - 2^(2/3)*(1 - x^3)^(1/3))] - 2*Log[x + (1 - x^3)^(1/
3)] + 2*2^(2/3)*Log[2*x + 2^(2/3)*(1 - x^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^
(1/3) + (1 - x^3)^(2/3)] - 2^(2/3)*Log[-2*x^2 + 2^(2/3)*x*(1 - x^3)^(1/3) -
 2^(1/3)*(1 - x^3)^(2/3)])/6
```

**Maple [F]**
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int((-x^3+1)^(2/3)/(x^3+1),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

**Fricas [A]**
time = 1.15, size = 191, normalized size = 1.45

$$-\frac{1}{3} \cdot 4^{\frac{1}{3}} \sqrt{3} \arctan\left(-\frac{\sqrt{3} x - 4^{\frac{1}{3}} \sqrt{3} (-x^3+1)^{\frac{1}{3}}}{3 x}\right) + \frac{1}{3} \sqrt{3} \arctan\left(-\frac{\sqrt{3} x - 2 \sqrt{3} (-x^3+1)^{\frac{1}{3}}}{3 x}\right) + \frac{1}{3} \cdot 4^{\frac{1}{3}} \log\left(\frac{4^{\frac{2}{3}} x + 2 (-x^3+1)^{\frac{1}{3}}}{x}\right) - \frac{1}{6} \cdot 4^{\frac{1}{3}} \log\left(\frac{2 \cdot 4^{\frac{1}{3}} x^2 - 4^{\frac{2}{3}} (-x^3+1)^{\frac{1}{3}} x + 2 (-x^3+1)^{\frac{2}{3}}}{x^2}\right) - \frac{1}{3} \log\left(\frac{x + (-x^3+1)^{\frac{1}{3}}}{x}\right) + \frac{1}{6} \log\left(\frac{x^2 - (-x^3+1)^{\frac{1}{3}} x + (-x^3+1)^{\frac{2}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*4^(1/3)*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(-x^3 + 1)^(1
/3))/x) + 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/
x) + 1/3*4^(1/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/x) - 1/6*4^(1/3)*log(
(2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) - 1/
3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}}{(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(2/3)/(x^3 + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-x^3\right)}^{2/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(2/3)/(x^3 + 1),x)
```

```
[Out] int((1 - x^3)^(2/3)/(x^3 + 1), x)
```

# 3.114 $\int \frac{x(1-x^3)^{2/3}}{1+x^3} \, dx$

**Optimal**. Leaf size=250

$$\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left((1-x)(1+x)^2\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\right.}{}$$

[Out] -1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/12*ln((1-x)*(1+x)^2)*2^(2/3)+1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)

**Rubi** [A]
time = 0.09, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {495, 371, 502, 2174, 206, 31, 648, 631, 210, 642}

$$\frac{2^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{1}{3}2^{2/3}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{2\sqrt[3]{2}}+\frac{\log\left((1-x)(x+1)^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(1 - x^3)^(2/3))/(1 + x^3),x]

[Out] (2^(2/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/2 + Log[(1 - x)*(1 + x)^2]/(6*2^(1/3)) + Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(1/3)) - (2^(2/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[-1 + x + 2^(2/3)*(1 - x^3)^(1/3)]/(2*2^(1/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 502

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[b/a, 3]}, Dist[-q^2/(3*d), Int[1/((1 - q*x)*(a + b*x^3)^(1/3))
, x], x] + Dist[q/d, Subst[Int[1/(1 + 2*a*x^3), x], x, (1 + q*x)/(a + b*x^3
)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + a
*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{x(1-x^3)^{2/3}}{1+x^3}\,dx = \frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.01, size = 26, normalized size = 0.10

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] (x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3])/2
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x(-x^3+1)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(x*(-x^3+1)^(2/3)/(x^3+1),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(-(x-1)(x^2+x+1))^{\frac{2}{3}}}{(x+1)(x^2-x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x*(-(x - 1)*(x**2 + x + 1))**(2/3)/((x + 1)*(x**2 - x + 1)), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate((-x^3 + 1)^(2/3)*x/(x^3 + 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-x^3\right)^{2/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^3)^(2/3))/(x^3 + 1),x)
[Out] int((x*(1 - x^3)^(2/3))/(x^3 + 1), x)
```

# 3.115 $\int \frac{(1-x)\left(1-x^3\right)^{2/3}}{1+x^3}\,dx$

**Optimal**. Leaf size=383

$$-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-$$

[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/12*ln((1-x)*(1+x)^2)*2^(2/3)-1/6*ln(x^3+1)*2^(2/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/3*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/4*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/3*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)

**Rubi** [A]
time = 0.58, antiderivative size = 648, normalized size of antiderivative = 1.69, number of steps used = 17, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6857, 2178, 2177, 245, 2174, 371}

$$-\frac{2^{2/3}\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(1+(-1)^{2/3}\right)\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\left(1-\sqrt[3]{-1}\right)\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\left(1-\sqrt[3]{-1}\right)\text{ArcTan}\left(\frac{1-\frac{\sqrt[3]{2}\left(x+\sqrt[3]{-1}\right)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\left(1+(-1)^{2/3}\right)\text{ArcTan}\left(\frac{\frac{(-1)^{2/3}\sqrt[3]{2}\left(\sqrt[3]{-1}x+1\right)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{1}{6}\left(1+(-1)^{2/3}\right)x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{6}\left(1-\sqrt[3]{-1}\right)x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{1}{3}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{1}{6}\left(1+(-1)^{2/3}\right)\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{1}{6}\left(1-\sqrt[3]{-1}\right)\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\left(1-\sqrt[3]{-1}\right)\log\left(-(-2)^{2/3}\sqrt[3]{1-x^3}-(-1)^{2/3}x+1\right)}{2\sqrt[3]{2}}+\frac{\log\left(-2^{2/3}\sqrt[3]{1-x^3}-x+1\right)}{\sqrt[3]{2}}+\frac{\left(1+(-1)^{2/3}\right)\log\left(\sqrt[3]{-1}2^{2/3}\sqrt[3]{1-x^3}+\sqrt[3]{-1}x+1\right)}{2\sqrt[3]{2}}-\frac{\log\left(-\left((1-x)(x+1)^2\right)\right)}{3\sqrt[3]{2}}-\frac{\left(1+(-1)^{2/3}\right)\log\left(-(-1)^{2/3}\left(x+(-1)^{2/3}\right)^2\left(\sqrt[3]{-1}x+1\right)\right)}{6\sqrt[3]{2}}-\frac{\left(1-\sqrt[3]{-1}\right)\log\left((-1)^{2/3}\left(x+\sqrt[3]{-1}\right)\left((-1)^{2/3}x+1\right)^2\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

[In] Int[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]

[Out] -((2^(2/3)*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]) + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1 - (-1)^(1/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) + ((1 + (-1)^(2/3))*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]) - ((1 - (-1)^(1/3))*ArcTan[(1 - (2^(1/3)*((-1)^(1/3) + x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]) - ((1 + (-1)^(2/3))*ArcTan[(1 + ((-1)^(2/3)*2^(1/3)*(1 + (-1)^(1/3)*x))/(1 - x^3)^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]) + (x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/3 + ((1 - (-1)^(1/3))*x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/6 + ((1 + (-1)^(2/3))*x^2*Hypergeometric2F1[1/3, 2/3, 5/3, x^3])/6 - Log[-((1 - x)*(1 + x)^2)]/(3*2^(1/3)) - ((1 + (-1)^(2/3))*Log[-((-1)^(2/3)*((-1)^(2/3) + x)^2*(1 + (-1)^(1/3)*x))])/(6*2^(1/3)) - ((1 - (-1)^(1/3))*Log[(-1)^(2/3)*((-1)^(1/3) + x)*(1 + (-1)^(2/3)*x)^2])/(6*2^(1/3)) - Log[x + (1 - x^3)^(1/3)]/3 - ((1 - (-1)^(1/3))*Log[x + (1 - x^3)^(1/3)])/6 - ((1 + (-1)^(2/3))*Log[x + (1 - x^3)^(1/3)])/6 + ((1 - (-1)^(1/

```
3))*Log[1 - (-1)^(2/3)*x - (-2)^(2/3)*(1 - x^3)^(1/3)])/(2*2^(1/3)) + Log[1
 - x - 2^(2/3)*(1 - x^3)^(1/3)]/2^(1/3) + ((1 + (-1)^(2/3))*Log[1 + (-1)^(1
/3)*x + (-1)^(1/3)*2^(2/3)*(1 - x^3)^(1/3)])/(2*2^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rule 2178

```
Int[((a_) + (b_.)*(x_)^3)^(2/3)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(a +
 b*x^3)^(2/3)/(2*d), x] + (Dist[1/d^2, Int[(a*d^2 + b*c^2*x)/((c + d*x)*(a
+ b*x^3)^(1/3)), x], x] - Dist[b*(c/d^2), Int[x/(a + b*x^3)^(1/3), x], x])
/; FreeQ[{a, b, c, d}, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(1-x)(1-x^3)^{2/3}}{1+x^3}\,dx = \int \left(-\frac{2(1-x^3)^{2/3}}{3(-1-x)} + \frac{\left(-1-(-1)^{2/3}\right)(1-x^3)^{2/3}}{3\left(-1+\sqrt[3]{-1}\,x\right)} + \frac{\left(-1+\sqrt[3]{-1}\right)(1-x^3)^{2/3}}{3\left(-1-(-1)^{2/3}x\right)}\right)$$
$$= -\left(\frac{2}{3}\int \frac{(1-x^3)^{2/3}}{-1-x}\,dx\right) + \frac{1}{3}\left(-1+\sqrt[3]{-1}\right)\int \frac{(1-x^3)^{2/3}}{-1-(-1)^{2/3}x}\,dx + \frac{1}{3}(-1-(-1$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.17, size = 138, normalized size = 0.36

$$-\frac{1}{2}x^2F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right) - \frac{4x(1-x^3)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{(1+x^3)\left(-4F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)+x^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x)*(1 - x^3)^(2/3))/(1 + x^3),x]
[Out] -1/2*(x^2*AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]) - (4*x*(1 - x^3)^(2/3)*Ap
pellF1[1/3, -2/3, 1, 4/3, x^3, -x^3])/((1 + x^3)*(-4*AppellF1[1/3, -2/3, 1,
 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, x^3, -x^3] + 2*Appell
F1[4/3, 1/3, 1, 7/3, x^3, -x^3])))
```

**Maple** [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(1-x)\left(-x^3+1\right)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int((1-x)*(-x^3+1)^(2/3)/(x^3+1),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] -integrate((-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{(1-x^3)^{\frac{2}{3}}}{x^3+1}\right)dx-\int\frac{x(1-x^3)^{\frac{2}{3}}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x**3+1)**(2/3)/(x**3+1),x)
[Out] -Integral(-(1 - x**3)**(2/3)/(x**3 + 1), x) - Integral(x*(1 - x**3)**(2/3)/
(x**3 + 1), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(-(-x^3 + 1)^(2/3)*(x - 1)/(x^3 + 1), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$-\int\frac{(1-x^3)^{2/3}\,(x-1)}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1),x)
[Out] -int(((1 - x^3)^(2/3)*(x - 1))/(x^3 + 1), x)
```

# 3.116 $\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx$

**Optimal**. Leaf size=272

$$\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(1+\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

```
[Out] 1/6*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*ln(1+2^(2/3)*(1-x)^2/(-x^
3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*ln(1+2^(1/3)*(
1-x)/(-x^3+1)^(1/3))-1/12*ln(2*2^(1/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)
/(-x^3+1)^(1/3))*2^(1/3)+1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {420, 493, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log[1 + (2^(2/3)*(
1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]/(3*2^(2/3))
+ (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - Log[2*2^(1/3) +
(1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3)]/(6*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x^3}}{1+x^3}\,dx = xF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)$$

**Mathematica** [A]
time = 2.14, size = 283, normalized size = 1.04

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{-2\sqrt[3]{2}+2\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right)-4\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x-\sqrt[3]{1-x^3}\right)-2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right)+2\log\left(2^{2/3}-2\ 2^{2/3}x+2^{2/3}x^2+(-1+x)\sqrt[3]{2-2x^3}+(1-x^3)^{2/3}\right)+\log\left(2^{2/3}-2\ 2^{2/3}x+2^{2/3}x^2-2(-1+x)\sqrt[3]{2-2x^3}+4(1-x^3)^{2/3}\right)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)^(1/3)/(1 + x^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1
- x^3)^(1/3))] + 4*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(-2*2^(1/3) + 2
*2^(1/3)*x + (1 - x^3)^(1/3))] - 4*Log[-2^(1/3) + 2^(1/3)*x - (1 - x^3)^(1/
3)] - 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] + 2*Log[2^(2/3) - 2*2
^(2/3)*x + 2^(2/3)*x^2 + (-1 + x)*(2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + Lo
g[2^(2/3) - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1
 - x^3)^(2/3)])/2^(2/3)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.35, size = 681, normalized size = 2.50

| method | result | size |
|---|---|---|
| trager | Expression too large to display | 681 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)^(1/3)/(x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*RootOf(_Z^3-2)*ln(-(18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z
^2)^2*RootOf(_Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9
*_Z^2)*RootOf(_Z^3-2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9
*_Z^2)*x^6-2*RootOf(_Z^3-2)*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3
-2)+9*_Z^2)*RootOf(_Z^3-2)*(-x^3+1)^(2/3)*x^2+6*(-x^3+1)^(1/3)*x^4+18*RootO
f(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^3+12*RootOf(_Z^3-2)*x^3-6*
x*(-x^3+1)^(1/3)-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)-2*Ro
otOf(_Z^3-2))/(1+x)^2/(x^2-x+1)^2)+1/2*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(
_Z^3-2)+9*_Z^2)*ln((-18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)
^2*RootOf(_Z^3-2)^3*x^3-12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z
^2)*RootOf(_Z^3-2)^4*x^3-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z
^2)*RootOf(_Z^3-2)*x^6-2*RootOf(_Z^3-2)^2*x^6+18*RootOf(RootOf(_Z^3-2)^2+3*
_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*(-x^3+1)^(2/3)*x^2-18*RootOf(Roo
tOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*(-x^3+1)^(1/3)*x^4+6*RootOf(RootO
f(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)*x^3+4*RootOf(_Z^3-2)
^2*x^3+12*(-x^3+1)^(2/3)*x^2+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)
+9*_Z^2)*(-x^3+1)^(1/3)*x-3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)*RootOf(_Z^3-2)-2*RootOf(_Z^3-2)^2)/(1+x)^2/(x^2-x+1)^2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

**Fricas [A]**
time = 3.51, size = 341, normalized size = 1.25

$$\frac{1}{18}\,\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{6\,\sqrt{3}\,2^{\frac{2}{3}}(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}-24\,\sqrt{3}\,2^{\frac{1}{3}}(x^{14}-2\,x^{11}-6\,x^{8}-2\,x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}-\sqrt{3}\,(x^{18}+42\,x^{15}-417\,x^{12}+812\,x^{9}-417\,x^{6}+42\,x^{3}+1)}{3\,(x^{18}-102\,x^{15}+447\,x^{12}-628\,x^{9}+447\,x^{6}-102\,x^{3}+1)}\right)+\frac{1}{18}\cdot 2^{\frac{1}{3}}\log\left(-\frac{12\,(-x^{3}+1)^{\frac{2}{3}}x^{2}+2^{\frac{2}{3}}(x^{6}+2\,x^{3}+1)-6\cdot 2^{\frac{1}{3}}(x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}}{x^{6}+2\,x^{3}+1}\right)-\frac{1}{36}\cdot 2^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}(x^{8}-4\,x^{5}+x^{2})(-x^{3}+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(x^{12}-32\,x^{9}+78\,x^{6}-32\,x^{3}+1)+6\,(x^{10}-11\,x^{7}+11\,x^{4}-x)(-x^{3}+1)^{\frac{1}{3}}}{x^{12}+4\,x^{9}+6\,x^{6}+4\,x^{3}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(1/3)*arctan(-1/3*(6*sqrt(3)*2^(2/3)*(x^16 - 33*x^13 + 110*x
^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 24*sqrt(3)*2^(1/3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - sqrt(3)*(x^18 + 42*x^15 - 4
17*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 62
8*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/18*2^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^
2 + 2^(2/3)*(x^6 + 2*x^3 + 1) - 6*2^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3))/(x^6
+ 2*x^3 + 1)) - 1/36*2^(1/3)*log((12*2^(2/3)*(x^8 - 4*x^5 + x^2)*(-x^3 + 1)
^(2/3) + 2^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 6*(x^10 - 11*x^7 +
 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1))
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{-\left(x-1\right)\left(x^{2}+x+1\right)}}{\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral((-(x - 1)*(x**2 + x + 1))**(1/3)/((x + 1)*(x**2 - x + 1)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate((-x^3 + 1)^(1/3)/(x^3 + 1), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(1-x^3)^{1/3}}{x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^3)^(1/3)/(x^3 + 1),x)
```

```
[Out] int((1 - x^3)^(1/3)/(x^3 + 1), x)
```

# Chapter 4

# Appendix

## Local contents

# 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica_syntax.zip

**Maple and Mupad format** Maple_syntax.zip

**Sympy format** SYMPY_syntax.zip

**Sage math format** SAGE_syntax.zip

# 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

## 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

  #debug=False
  if debug:
     print ("Enter is_atom, expn=",expn)

  if not hasattr(expn, 'parent'):
     return False


  #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
  try:
     if expn.parent() is SR:
        return expn.operator() is None
     if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
     if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

     return False

  except AttributeError as error:
     print("Exception,AttributeError in  is_atom")
     print ("cought exception" , type(error).__name__ )
     return False


def expnType(expn):

  if debug:
     print (">>>>>Enter expnType, expn=", expn)
     print (">>>>>is_atom(expn)=", is_atom(expn))

  if is_atom(expn):
     return 1
  elif type(expn)==list:   #isinstance(expn,list):
     return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
     if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
        return 1
     else:
        return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
  elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
     if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
     elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```