

CAS integration tests regression report
Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 vs. Fricas 1.3.7
via sagemath 9.3

Nasser M. Abbasi

September 22, 2022

Compiled on September 22, 2022 at 3:52am

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1 Summary of regression test table

Table 1: Summary table of regression tests

#	test file #	integral #	Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6	Fricas 1.3.7 via sagemath 9.3
1	48	16	-1 (time out)	1 (pass)
2	48	17	-1 (time out)	1 (pass)
3	55	557	0 (not solved)	1 (pass)
4	55	558	0 (not solved)	1 (pass)
5	55	559	0 (not solved)	1 (pass)
6	55	560	0 (not solved)	1 (pass)
7	70	497	0 (not solved)	1 (pass)
8	103	687	-1 (time out)	1 (pass)
9	210	10134	0 (not solved)	1 (pass)

2 Test file number 48

Test folder name:

test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.3_General/48_1.2.3.4-f_x-
^m-d+e_x^n-q-a+b_x^n+c_x^-2_n-p

2.1 Problem number 16

$$\int \frac{x(d + ex^3)}{a + bx^3 + cx^6} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\ln\left(2^{\frac{1}{3}}c^{\frac{1}{3}}x + \left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}\right) \left(e + \frac{-be+2cd}{\sqrt{-4ac + b^2}}\right) 2^{\frac{1}{3}}}{6c^{\frac{2}{3}} \left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}} \\
+ & \frac{\ln\left(2^{\frac{2}{3}}c^{\frac{2}{3}}x^2 - 2^{\frac{1}{3}}c^{\frac{1}{3}}x\left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}} + \left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}\right) \left(e + \frac{-be+2cd}{\sqrt{-4ac + b^2}}\right) 2^{\frac{1}{3}}}{12c^{\frac{2}{3}} \left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}} \\
- & \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{\frac{1}{3}}c^{\frac{1}{3}}x}{\left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right) \left(e + \frac{-be+2cd}{\sqrt{-4ac + b^2}}\right) 2^{\frac{1}{3}}\sqrt{3}}{6c^{\frac{2}{3}} \left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}} \\
- & \frac{\ln\left(2^{\frac{1}{3}}c^{\frac{1}{3}}x + \left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}\right) \left(e + \frac{be-2cd}{\sqrt{-4ac + b^2}}\right) 2^{\frac{1}{3}}}{6c^{\frac{2}{3}} \left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}} \\
+ & \frac{\ln\left(2^{\frac{2}{3}}c^{\frac{2}{3}}x^2 - 2^{\frac{1}{3}}c^{\frac{1}{3}}x\left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}} + \left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}\right) \left(e + \frac{be-2cd}{\sqrt{-4ac + b^2}}\right) 2^{\frac{1}{3}}}{12c^{\frac{2}{3}} \left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}} \\
- & \frac{\arctan\left(\frac{\left(1 - \frac{2 \cdot 2^{\frac{1}{3}}c^{\frac{1}{3}}x}{\left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right) \left(e + \frac{be-2cd}{\sqrt{-4ac + b^2}}\right) 2^{\frac{1}{3}}\sqrt{3}}{6c^{\frac{2}{3}} \left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}}
\end{aligned}$$

command

```
integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

Timed out

Fricas 1.3.7 via sagemath 9.3 output

output too large to display

2.2 Problem number 17

$$\int \frac{d + ex^3}{a + bx^3 + cx^6} dx$$

Optimal antiderivative

$$\frac{\ln\left(2^{\frac{1}{3}}c^{\frac{1}{3}}x + \left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}\right)\left(e + \frac{-be+2cd}{\sqrt{-4ac + b^2}}\right)2^{\frac{2}{3}}}{6c^{\frac{1}{3}}\left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}}$$

$$- \frac{\ln\left(2^{\frac{2}{3}}c^{\frac{2}{3}}x^2 - 2^{\frac{1}{3}}c^{\frac{1}{3}}x\left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}} + \left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}\right)\left(e + \frac{-be+2cd}{\sqrt{-4ac + b^2}}\right)2^{\frac{2}{3}}}{12c^{\frac{1}{3}}\left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}}$$

$$- \frac{\arctan\left(\frac{\left(1 - \frac{2^{\frac{1}{3}}c^{\frac{1}{3}}x}{\left(b - \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\left(e + \frac{-be+2cd}{\sqrt{-4ac + b^2}}\right)2^{\frac{2}{3}}\sqrt{3}}{6c^{\frac{1}{3}}\left(b - \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}}$$

$$+ \frac{\ln\left(2^{\frac{1}{3}}c^{\frac{1}{3}}x + \left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}\right)\left(e + \frac{be-2cd}{\sqrt{-4ac + b^2}}\right)2^{\frac{2}{3}}}{6c^{\frac{1}{3}}\left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}}$$

$$- \frac{\ln\left(2^{\frac{2}{3}}c^{\frac{2}{3}}x^2 - 2^{\frac{1}{3}}c^{\frac{1}{3}}x\left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}} + \left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}\right)\left(e + \frac{be-2cd}{\sqrt{-4ac + b^2}}\right)2^{\frac{2}{3}}}{12c^{\frac{1}{3}}\left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}}$$

$$- \frac{\arctan\left(\frac{\left(1 - \frac{2^{\frac{1}{3}}c^{\frac{1}{3}}x}{\left(b + \sqrt{-4ac + b^2}\right)^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)\left(e + \frac{be-2cd}{\sqrt{-4ac + b^2}}\right)2^{\frac{2}{3}}\sqrt{3}}{6c^{\frac{1}{3}}\left(b + \sqrt{-4ac + b^2}\right)^{\frac{2}{3}}}$$

command

```
integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

Timed out

Fricas 1.3.7 via sagemath 9.3 output

output too large to display

3 Test file number 55

Test folder name:

test_cases/2_Exponentials/55_2.3_Exponential_functions

3.1 Problem number 557

$$\int \frac{\left(F \frac{\sqrt{1-ax}}{\sqrt{1+ax}} \right)^n}{1 - a^2 x^2} dx$$

Optimal antiderivative

$$\frac{\left(F \frac{\sqrt{-ax+1}}{\sqrt{ax+1}} \right)^n \operatorname{expIntegral} \left(\frac{n \ln(F) \sqrt{-ax+1}}{\sqrt{ax+1}} \right) F^{-\frac{n \sqrt{-ax+1}}{\sqrt{ax+1}}}}{a}$$

command

```
integrate((F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n/(-a^2*x^2+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

could not integrate

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\operatorname{Ei} \left(\frac{\sqrt{-ax+1} n \log(F)}{\sqrt{ax+1}} \right)}{a}$$

3.2 Problem number 558

$$\int \frac{F \frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}{1-a^2x^2} dx$$

Optimal antiderivative

$$\frac{\text{expIntegral}\left(\frac{3\ln(F)\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a}$$

command

```
integrate(F^(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

could not integrate

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\text{Ei}\left(\frac{3\sqrt{-ax+1}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

3.3 Problem number 559

$$\int \frac{F \frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}{1-a^2x^2} dx$$

Optimal antiderivative

$$\frac{\text{expIntegral}\left(\frac{2\ln(F)\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a}$$

command

```
integrate(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

could not integrate

Fricas 1.3.7 via sagemath 9.3 output

$$\frac{\text{Ei}\left(\frac{2\sqrt{-ax+1}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

3.4 Problem number 560

$$\int \frac{F \frac{\sqrt{1-ax}}{\sqrt{1+ax}}}{1-a^2x^2} dx$$

Optimal antiderivative

$$-\frac{\text{expIntegral}\left(\frac{\ln(F)\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a}$$

command

```
integrate(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

could not integrate

Fricas 1.3.7 via sagemath 9.3 output

$$-\frac{\text{Ei}\left(\frac{\sqrt{-ax+1} \log(F)}{\sqrt{ax+1}}\right)}{a}$$

4 Test file number 70

Test folder name:

test_cases/4_Trig_functions/4.1_Sine/70_4.1.1.2-g_cos-^p-a+b_sin-^m

4.1 Problem number 497

$$\int \sec(c+dx)(a+b\sin(c+dx))^{5/2} dx$$

Optimal antiderivative

$$-\frac{(a-b)^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a-b}}\right)}{d} + \frac{(a+b)^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{d} - \frac{2b(a+b\sin(dx+c))^{3/2}}{3d} - \frac{4ab\sqrt{a+b\sin(dx+c)}}{d}$$

command

```
integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```


Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

could not integrate

Fricas 1.3.7 via sagemath 9.3 output

output too large to display

5 Test file number 103

Test folder name:

test_cases/4_Trig_functions/4.3_Tangent/103_4.3.2.1-a+b_tan-^m-c+d_tan-ⁿ

5.1 Problem number 687

$$\int \cot^2(e + fx) \sqrt[3]{c + d \tan(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{d \ln(\tan(fx + e))}{6c^{\frac{2}{3}}f} + \frac{d \ln\left(c^{\frac{1}{3}} - (c + d \tan(fx + e))^{\frac{1}{3}}\right)}{2c^{\frac{2}{3}}f} \\
& - \frac{d \arctan\left(\frac{\left(c^{\frac{1}{3}} + 2(c + d \tan(fx + e))^{\frac{1}{3}}\right)\sqrt{3}}{3c^{\frac{1}{3}}}\right)\sqrt{3}}{3c^{\frac{2}{3}}f} \\
& + \frac{x\left(c - \sqrt{-d^2}\right)^{\frac{1}{3}}}{4} + \frac{d \ln(\cos(fx + e))\left(c - \sqrt{-d^2}\right)^{\frac{1}{3}}}{4f\sqrt{-d^2}} \\
& + \frac{3d \ln\left(\left(c - \sqrt{-d^2}\right)^{\frac{1}{3}} - (c + d \tan(fx + e))^{\frac{1}{3}}\right)\left(c - \sqrt{-d^2}\right)^{\frac{1}{3}}}{4f\sqrt{-d^2}} \\
& - \frac{d \arctan\left(\frac{\left(\left(1 + \frac{2(c + d \tan(fx + e))^{\frac{1}{3}}}{\left(c - \sqrt{-d^2}\right)^{\frac{1}{3}}}\right)\sqrt{3}\right)}{3}\right)\sqrt{3}\left(c - \sqrt{-d^2}\right)^{\frac{1}{3}}}{2f\sqrt{-d^2}} \\
& + \frac{x\left(c + \sqrt{-d^2}\right)^{\frac{1}{3}}}{4} - \frac{d \ln(\cos(fx + e))\left(c + \sqrt{-d^2}\right)^{\frac{1}{3}}}{4f\sqrt{-d^2}} \\
& - \frac{3d \ln\left(\left(c + \sqrt{-d^2}\right)^{\frac{1}{3}} - (c + d \tan(fx + e))^{\frac{1}{3}}\right)\left(c + \sqrt{-d^2}\right)^{\frac{1}{3}}}{4f\sqrt{-d^2}} \\
& + \frac{d \arctan\left(\frac{\left(\left(1 + \frac{2(c + d \tan(fx + e))^{\frac{1}{3}}}{\left(c + \sqrt{-d^2}\right)^{\frac{1}{3}}}\right)\sqrt{3}\right)}{3}\right)\sqrt{3}\left(c + \sqrt{-d^2}\right)^{\frac{1}{3}}}{2f\sqrt{-d^2}} \\
& - \frac{\cot(fx + e)(c + d \tan(fx + e))^{\frac{1}{3}}}{f}
\end{aligned}$$

command

```
integrate(cot(f*x+e)^2*(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

Timed out

Fricas 1.3.7 via sagemath 9.3 output

$$-\frac{(d \tan (fx + e) + c)^{\frac{1}{3}}}{f \tan (fx + e)}$$

6 Test file number 210

Test folder name:

test_cases/210_Hebisch

6.1 Problem number 10134

$$\int \frac{-5x - 4x^2 + x^3 + e^{7+x}(4 + 3x - x^2) + (-8 - 10x + 3x^2 + e^{7+x}(4x + 3x^2 - x^3)) \log(x) + (-4 - 3x + x^2) \log(x)}{-4x - 3x^2 + x^3}$$

Optimal antiderivative

$$\ln(x) (\ln(x) - e^{x+7} + \ln(1+x)) + x - \ln(4-x)$$

command

```
integrate(((x^2-3*x-4)*log(1+x)+((-x^3+3*x^2+4*x)*exp(x+7)+3*x^2-10*x-8)*log(x)+(-x^2+3*x+4)*exp(x+7)+x^3-4*x^2-5*x)/(x^3-3*x^2-4*x),x, algorithm="fricas")
```

Fricas 1.3.8 (sbcl 2.2.11.debian) via sagemath 9.6 output

could not integrate

Fricas 1.3.7 via sagemath 9.3 output

$$-e^{(x+7)} \log(x) + \log(x+1) \log(x) + \log(x)^2 + x - \log(x-4)$$